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1 A NEURO-DYNAMIC PROGRAMMING APPROACH
2 TO THE OPTIMAL STAND MANAGEMENT PROBLEM

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10 ABSTRACT

11 Some ideas of neuro-dynamic programming are illustrated by considering the problem of optimally
12 managing a forest stand under uncertainty. Because reasonable growth models require state information
13 such as height (or age), basal area, and stand diameter, as well as an indicator variable for treatments
14 that have been performed on the stand, they can easily lead to very large state spaces that include
15 continuous variables. Realistic stand management policies include silvicultural options such as pre-
16 commercial and commercial thinning as well as post-harvest treatments. We are interested in problems
17 that are stochastic in their basic growth dynamics, in market prices and in disturbances, ranging from
18 insects to fire to hurricanes. Neuro-dynamic programming (NDP) algorithms are appropriate for
19 problems with large dimensions that may lack a simple model of dynamics and stochastic processes.
20 This paper looks at applying these ideas in the context of a multi species model. Results show that
21 policies obtained using NDP are optimal within a 95% confidence interval or better. The set of states
22 and controls incorporated into our NDP model allows us to develop optimal policies with a level of
23 detail not typically seen in the forestry literature.

24 Keywords: uncertainty, forestry, NDP, policy, approximation

1. INTRODUCTION

Forestry is full of problems that involve the control of dynamic, stochastic systems with the intent to either optimize some objective function or to attempt to steer the system to some desired state. Even-age stand harvesting has been represented as a control problem (Näslund, 1969) and the classic (Faustmann, 1849) formula can be seen as a control policy developed from a deterministic view of the system. Dynamic programming (DP) has been used extensively to solve deterministic control problems in forestry and Yoshimoto et al. (2016) give an excellent overview of that literature. DP models in the forestry literature generally use some type of approximations to state and control spaces. For example, Lembersky and Johnson (1975) use 48 discrete stand indicants which were combinations of diameter at breast height and number of trees per hectare. In Haight and Holmes (1991), silvicultural actions are reduced to two discrete options: cut or no cut. Models use a selection of discrete values of forest stand descriptors and silvicultural interventions which are continuous in nature because realistic representations of those control problems are too complex to solve analytically.

Early applications of DP in forestry used a single continuous state variable such as age, volume, residual basal area or number of trees (Amidon & Akin, 1968; Brodie & Kao, 1979; Chen et al., 1980; Haight et al., 1985; Schreuder, 1971). However, to properly model growth and response to various silvicultural interventions, a single state variable is inadequate. Most growth models require a representation of something like stand average diameter, basal area, height as well as an indicator variable of stand nature (natural, plantation, pre-commercially thinned, commercially thinned, etc.); thus discrete and continuous variables. In representing multiple species stands, the state dimensions grow to include variables such as species percentage and crown closure.

Historically, most forestry studies limit the number of control options either because of limitations to the size of the model being used (Haight et al., 1985) or for simplicity (Lien et al.,

2007). Amidon and Akin (1968), Brodie et al. (1978), Brodie and Kao (1979), Haight et al. (1985), Arthaud and Klemperer (1988), Peltola and Knapp (2001) all applied methods that optimize the management of a stand over a set of decisions that keep the stand type as a natural or plantation through the entire optimization horizon without changing from one to the other, or do not differentiate between treatment condition. Treatment conditions reflect the history of the past decisions that have been made about the management of the stand. A list of practical silvicultural interventions usually includes some types of partial thinning, land preparation and final felling which include discrete and continuous variables. Combining them all in the same model would allow us to consider multiple treatment conditions simultaneously.

In forest management, decision-makers face highly uncertain outcomes because of the long time horizons (Hildebrandt & Knoke, 2011) but risk is not the same as uncertainty. Risk has been defined as the expected loss due to a particular hazard thus uncertainty presents a risk if it results in an expected loss (von Gadow, 2001). Some risks such as loss of biodiversity are hard to evaluate financially but there is a very large literature that addresses financial risk and the sources of uncertainty that lead to expected losses. Readers should seek out review papers such as Hanewinkel et al. (2011), Hildebrandt and Knoke (2011) and Yousefpour et al. (2012). Textbooks such as Amacher et al. (2009) and Davis et al. (2005) are also excellent references.

Stochastic dynamic programming (SDP) has been used to investigate how control policies in a forestry context should be modified in the face of uncertain growth dynamics, market prices, interest rates, natural disasters and climate change (Ferreira et al., 2012; Haight & Holmes, 1991; Hool, 1966; Lembersky & Johnson, 1975; Rios et al., 2016; Steenberg et al., 2012; Yoshimoto, 2002)). In most studies, these uncertainties enter into forestry SDP models as transition probabilities between discrete states. For example, Ferreira et al. (2012) study the impact of wildfire occurrence by allowing a stand to transition from a non-burned to a burned / non-burned state according to

73 transition probabilities p and $(1 - p)$ respectively, which depend on past wildfire history for that
74 stand.

75 Models with complex state spaces, dynamics and stochastic processes are becoming more
76 common (Nicol & Chadès, 2011; Rios et al., 2016; Steenberg et al., 2012). The amount and quality
77 of data available to researchers and managers leads us to want to include more of that information
78 into models. However, Bertsekas (2001) explains the limitations of SDP in handling large high
79 dimensional continuous state spaces. When uncertainty leads to a large number of possible
80 outcomes for each state-action pair, the number of calculations can quickly get out of hand for a
81 problem with a large number of discrete values for a large number of state and control variables. In
82 an effort to reduce the number of discrete states in a forestry context, Nicol & Chadès (2011) use a
83 heuristic sampling method that yields near-optimal policies. The control literature contains many
84 examples of successfully using neuro-dynamic programming (NDP) to solve high dimensional
85 problems in contexts other than forestry (Castelletti et al., 2007; Ilkova et al., 2012). The basic idea
86 behind NDP is to evaluate the value function using a reduced number of discrete states and controls
87 and approximate the value function and policy on the continuous portion of state and control spaces.
88 A reduced number of discrete values for states and controls reduces the number of state-action pair
89 outcomes for stochastic problems. Gunn (2005) describes the basics of NDP in a forestry context
90 with the main ideas closely based on Bertsekas & Tsitsiklis (1996) and Sutton & Barto (1998). This
91 paper expands on Gunn (2005) and presents more details about NDP implementation in a forestry
92 context by using a case study of mix species even-aged stand management in Nova Scotia, Canada.

93 2. PROBLEM DESCRIPTION

94 The problem we want to solve is that of finding an optimal stationary policy μ of harvest and
95 silviculture for a two-species (red spruce and tolerant hardwood) even-aged stand growing in Nova
96 Scotia, Canada. We have chosen to solve this problem by using a model that maintains an eight

dimensional stand state i : age, stocking (st), initial planting density (ipd), diameter for each species (d^θ), percentage of each species in terms of the proportion of total stand basal area (pct^θ), crown closure (cc) which is directly proportional to spacing between trees and a discrete treatment condition (tt). θ refers to softwood (SW) or hardwood (HW). Any large open areas of a stand would reduce stocking below 100%. Crown closure is the percentage of ground, on the stocked portion of the stand, covered by the vertical projection of tree crowns. Definitions of these variables are those published in the national forestry database of silvicultural terms in Canada (1995). Each of the first seven variables is continuous and can take on a wide range of values. A finite set S^{Eval} of 1,127 discrete states i is divided into five (5) subsets by treatment condition S_{tt}^{Eval} , $tt = 1 \dots 5$ because growth and yield models differ depending on past treatment history. Table 1 shows the range of values for state variables used in the model and which of the state variables are used for each treatment condition. In some instances, it will be necessary to use the following notation for states: $i_{N_{tt}}^{tt}$ where N_{tt} is the number of discrete states for treatment tt . However, in order to simplify notation, where appropriate, we use i .

In this paper, uncertainty enters into the model as transition probabilities $p_{ij}(u)$ which represent uncertain market prices, uncertain impacts of natural disasters and, at harvest (clearcut), the stand is subject to an uncertain period of regeneration which can be eliminated through planting the stand. The financial risk associated with uncertain regeneration is the potential delay in growing stock being present on the stand thus delaying future harvests. Market prices are modeled as a stationary process and uncertain prices are assumed to follow a normal distribution with mean and variance chosen such that HW products are worth more than SW products and that there is enough difference between the highest and lowest prices for the policies to be different where price is a contributing factor. There is no good long term market price data available for Nova Scotia, therefore we make the assumption that the prices used in the model are reasonable. Haight (1990), Forboseh and

Pickens (1996), and Lu and Gong (2003) are examples of studies that use stationary price models. Because of their stationary nature, price is not included as a state variable in the model. The financial risk associated with the uncertainty of market prices is the expected profit that could be gained or lost by not harvesting in the current period and holding out for a better market price in a future period. Natural disasters include hurricanes, forest fires and insect outbreaks. When a stand is affected by a natural disaster, we model it as if it may or may not succumb to the disaster. If it does not succumb, no damage is done and the stand remains unchanged. If it does succumb, we model this as if it will be completely wiped out and the value of the wood products on the stand would only be enough to pay for the cost of removing them, therefore the stand has no salvage value. Therefore, the financial risk associated with the uncertainty of natural disaster occurrence is the loss of potential future profit because the stand has succumbed before a planned harvest. Modelling hurricanes and fires is a matter of determining the probabilities of a stand ending up in any one of the regeneration states in any given period. Stands have a higher probability of succumbing to hurricanes as they get older and stems become more rigid and break more easily. In the case of fires, the opposite occurs as older trees have thicker bark that protects them from the heat. If an insect outbreak occurs, the stand will be affected and older trees are modeled to have a higher probability of succumbing as they offer better food and shelter for invading insects. Uncertain market prices, all three types of natural disasters and uncertain regeneration are assumed independent of each other.

Unplanted stands, once regenerated, grow as natural stands. Within certain diameter limits, they can be pre-commercially thinned (PCT) where a preference for one species will remove more of the other. The forestry field handbook (1993) published by Nova Scotia Department of Natural Resources (NSDNR) recommends spacing between 2.1m and 3.0m depending on the type and condition of the stand. Plantations can have either early competition control or not. Those without early competition control have a certain probability of reverting to a well-stocked natural stand.

Plantations or PCT stands can be commercially thinned (CT) once they reach certain diameter limits. In the case of CT, there are a range of density reduction choices available with parameters including thinning intensity (basal area removal of 20%, 30% and 40%), species selection and removal method (thinning from above, from below and across the diameter distribution). Basal area removal was capped at 40% to protect the stand from windfall. Silvicultural options include those indicated above as well as the decision to do nothing and let grow for one period. All of these management options which include several continuous parameters are represented by 46 discrete options that form a finite set $U(i)$ where feasible actions depend on i .

Growth models $f(i, u)$ are based on the Nova Scotia Softwood and Hardwood Growth and Yield Models (GNY) (McGrath, 1993; O'Keefe & McGrath, 2006). These models are based on extensive field analysis using a series of research permanent sample plots established and maintained by the N.S. Department of Natural Resources. The main parameter is site capability expressed as Site Index height at age 50. Models maintain height, diameter and basal area from period to period and equations were developed to translate model variables to GNY variables and vice-versa. For yield calculations, merchantable and sawlog volumes are computed using Honer's equations (Nova Scotia Department of Natural Resources, 1996). GNY has essentially three growth models described below.

Natural unmanaged stands are grown using equations developed for the provincial revised normal yield tables (Nova Scotia department of lands and forests, 1990). Age implies height (and vice versa), height implies diameter and site density, which in turn implies basal area. Because the model mixes two species together and because each species had different age-height relationships, the model maintains age as a variable.

Natural stands that have been subject to pre-commercial and commercial thinning are grown using a variable density model. The age-height relationship characteristic of the site is maintained. The stand average diameters for each species are modeled as a function of site, spacing and free age.

Free age is the mechanism that we use to deal with density modifications. This function is invertible so that free age can be calculated as a function of site, spacing and diameter. Given a certain site, spacing and diameter, we use the inverse function to compute the free age. If we choose to leave the spacing as is, then growth amounts to increasing the free age by one period (five years) and calculating the resulting diameter. A thinning alters both spacing and diameter. The nature of the diameter change depends on the nature of the thinning (thinning from below, above or across the diameter distribution). Given the new diameter and spacing, we can recompute free age and continue to grow the stand by incrementing free age. Maximum density is a function of diameter and diameter and basal area define both density and spacing. Trees are grown by advancing age (height age) and free age by five years and recomputing diameter and height. Site density index (SDI) is computed for this new diameter and, if current density exceeds SDI, current density is reduced to SDI, spacing is recalculated and free age recomputed to correspond to the new diameter and spacing. Plantations may be thinned and they are also grown using a variable density growth model but maximum density for plantations is computed using a different equation.

As a stand grows from its initial state and silvicultural prescriptions are applied, the stand may change from one treatment type to another. Applying silvicultural treatment u to state i results in state j at the next time period, which are five year intervals because of the growth models used in Nova Scotia. For example, $tt = 1$ represents natural stands which have been growing without intervention since the last final felling. Applying a pre-commercial thinning to this stand results in the stand transferring from $tt = 1$ to $tt = 3$, the latter of which is a treatment type for pre-commercially thinned natural stands. For growth purposes, there is no necessity to distinguish between $tt = 2$ and $tt = 5$ or to distinguish between $tt = 3$ and $tt = 4$. However, the nature of revenue generated and silvicultural options are different in these circumstances.

Rewards $g(i, u)$ are calculated based on volume extracted from a stand at period k which is dependent upon volume equations developed by NSDNR and on equations developed for this study. Market prices for merchantable and board volume for each species as well as silvicultural costs are factored in. The reader should see $g(i, u)$ as a profit function that calculates the total profit in any given time period of applying a silvicultural action u to a forest stand i .

3. METHODOLOGY

There are many technicalities in properly detailing the nature of the problem to be solved which we will not present here but the notation used is closely based on Bertsekas and Tsitsiklis (1996) and interested readers should see Gunn (2005) for a succinct introduction to NDP. In light of our problem statement, it makes sense to think in terms of infinite horizon DP problems that yield stationary policies μ that map states i into management decisions u . For any stationary policy μ and all states i , we have a discounted value function:

$$(1) \quad J^\mu(i) = \sum_j p_{ij}(\mu(i)) [g(i, \mu(i)) + \alpha J^\mu(j)]$$

where $J^\mu(i)$ is the profit for state i and policy μ , $\mu(i)$ is the management decision for state i according to policy μ and α is the discount factor. If we state $J^*(i)$ as an approximation to the optimal profit for state i , we are looking for a stationary policy μ that satisfies $J^*(i) = \max_\mu J^\mu(i)$ for all states i . The problem of finding an optimal stationary policy or finding $J^*(i)$ is usually approached by way of recursive approximation algorithms. One of three classes of algorithms is usually applied: value iteration, policy iteration and linear programming (Bertsekas & Tsitsiklis, 1996). Policy iteration generates a sequence of stationary policies, each with improved profit over the preceding one. With a finite number of states and controls, policy iteration converges in a finite number of iterations but when the number of states and controls is large as it is the case in our problem, solving the linear system in the policy evaluation step of the policy iteration approach can

be time consuming. Using approximate linear programming to solve a stochastic dynamic programming forestry problem makes the linear programming problem difficult to handle given the large number of variables and constraints (Bertsekas & Tsitsiklis, 1996) therefore it will not be discussed. Value iteration has been a standard method for solving DP and SDP problems in forestry over five decades (Couture & Reynaud, 2011; Hool, 1966). This paper presents the details of using backward recursive value iteration in a NDP forestry problem for the first time.

Assuming that we know or can determine $p_{ij}(u)$ and α , and that we have a closed form version of $f(i, u)$ and $g(i, u)$, we want to solve eq. (1) by way of a value iteration algorithm such as eq. (2) where k indexes iterations.

$$(2) \quad J_k(i) = \max_{u \in U(i)} \left\{ \sum_j p_{ij}(u) [g(i, u) + \alpha J_{k-1}(j)] \right\}$$

The recursive nature of eq. (2) requires approximations to be made as we may have a case where j is not an element of S^{Eval} and we do not have an exact value for $J_{k-1}(j)$. Figure 1 illustrates a value function J plotted against a single state variable where the solid line represents the continuous nature of the value function, dots represent the values calculated at discrete states $i \in S^{Eval}$ and X's represent values approximated at discrete states j .

In the proposed approach, we replace $J_{k-1}(j)$ by $\tilde{J}_{k-1}(j, r)$ where r is the parameter vector of an approximation function. We rewrite eq. (2) as:

$$(3) \quad J_k(i) = \max_{u \in U(i)} \left\{ \sum_j p_{ij}(u) [g(i, u) + \alpha \tilde{J}_{k-1}(j, r)] \right\}$$

The fitting approach that leads to the name neuro-dynamic programming is to let r be the parameters of a neural network architecture. We generalize by letting r be the parameters of any appropriate continuous function approximating architecture. For example, after calculating $J_k(i)$ for

all $i \in S^{Eval}$ at iteration $k = 1$ of the algorithm, values of $J_k(i)$ are grouped by tt and five separate continuous functions $\tilde{J}_k^{tt}(i, r)$, one for each tt , are fit on values of $J_k(i)$. At iteration $k = 2$, these continuous functions are used to calculate an approximation $\tilde{J}_{k-1}^{tt}(j, r)$ for the value of being in state j which is of treatment condition tt . The sets of discrete states that describe each stand in each treatment condition vary immensely so it was decided to use five different approximants to increase the quality of the fit for each treatment condition.

The choice of an approximating architecture introduces a number of complications. The first is whether the approximating architecture $\tilde{J}_k(i, r)$ is capable of modeling the actual $J_k(i)$, the second is whether fitting of parameter vector r has been successful in minimizing the fitting error, and the third is whether the $i_1^{tt}, i_2^{tt}, \dots, i_{N_{tt}}^{tt}$ are numerous enough and chosen appropriately. Results in section 5.1 clearly demonstrate that Radial Basis Functions (RBF), presented below, answer affirmatively to each of these questions and can be an appropriate choice for NDP in a forestry context but care must be taken in choosing its parameters.

In this study, we use RBFs as function approximants which can be traced back to ideas put forward by Askey (1973) and further developed by Wendland (1995) and Buhmann (2000). RBFs have been successfully used for problems characterized by high dimensionality, in the hundreds in some cases, and can be incorporated into the NDP framework for value function approximations. RBFs are structured in such a way that the interpolant is forced to be equal to the function it is fitting at selected discrete points called centers and guarantees smooth transitions between points. Basis functions are radially symmetric about these centers which means their values depend only on the Euclidian distance from each center to each of the evaluation states, and on the shape of the basis function.

Four of the most mentioned forms of the RBF are the thin plate spline $\varphi = \delta^2 \log \delta$, the multiquadric $\varphi = (\delta^2 + \omega^2)^{1/2}$, the inverse multiquadric $\varphi = (\delta^2 + \omega^2)^{-1/2}$, and the Gaussian

257 $\varphi = e^{-\omega\delta^2}$ where ω is a shape parameter with its value set by the user, and δ is the scaled distance
 258 between centers (Buhmann, 2000; Franke & Schaback, 1998; Rippa, 1999). Each φ yields a
 259 different shape of the RBF function between centers. Distance between centers, calculated using a
 260 Euclidian norm, plays a central role in RBFs. A sparse set of centers means a greater distance and a
 261 higher probability that, between centers, the approximating function could behave in such a way that
 262 it returns an approximation that is very different than the real value. The first two are probably the
 263 best known and most often used versions of the RBF and this research reports results using the thin
 264 plate spline which eliminates the need for finding a good value for ω .

265 Before beginning the value iteration algorithm, we choose a subset of M_{tt} states i_n^{tt} , $n =$
 266 $1, \dots, M_{tt}$ from each subset tt of S^{Eval} which we call centers. After having calculated $J(i)$ for all
 267 $i \in S^{Eval}$ at iteration k and having determined that the value iteration algorithm has not converged,
 268 we force the interpolant $\tilde{J}^{tt} = J(i_n^{tt})$ at the centers where $J(i_n^{tt})$ are the values at those centers. M_{tt} is
 269 set as a function of the problem being solved and results will demonstrate the impact of choosing a
 270 different number of centers. The first step is to create, for every tt , a set of M_{tt} equations, one for
 271 each i_n^{tt} , using eq. (4).

$$(4) \quad J(i_n^{tt}) = \sum_{m=1}^{M_{tt}} r_m^{tt} \varphi(|i_n^{tt} - i_m^{tt}|_p) \quad \text{for } n = 1 \dots M_{tt}$$

272 We know φ and we use $p = 2$ as the dimension of the Euclidian norm thus, for every tt , we have
 273 M_{tt} unknowns, the r_m^{tt} , and M_{tt} equations giving a linear system $A \times r = f$, where the elements of
 274 the $M_{tt} \times M_{tt}$ square A matrix are given by $\varphi(|i_n^{tt} - i_m^{tt}|_2)$ for $n = 1 \dots M_{tt}$, and the M_{tt} elements of
 275 vector f are the values of $J(i_n^{tt})$. In the cases where φ is not defined for $i_n^{tt} = i_m^{tt}$, we set $\varphi = 0$. If A
 276 is invertible, we have $r = f \times A^{-1}$ which yields the weights r_m^{tt} . Micchelli (1986) gives general

conditions of φ that ensure nonsingularity of A and the RBF function chosen for implementation meets those conditions.

Given the set of weights above, we can approximate the value of $J(j, r)$ at the next iteration of the value iteration algorithm for any state j according to the treatment type of state j using eq. (5).

$$(5) \quad \tilde{J}^{tt}(j, r) = \sum_{m=1}^M r_m^{tt} \varphi(|j - i_m^{tt}|_2)$$

Parameters r_m^{tt} need to be recalculated at each iteration of the value iteration algorithm as they depend on the updated values of $J(i)$.

We need to choose a method for terminating the value iteration algorithm. Bertsekas (2001) proposes a method, with accompanying mathematical proof, for terminating value iteration algorithms that guarantees policy convergence as long as the stopping criteria ε is sufficiently small.

Without discussing the details of the proof, the upper (c_k^U) and lower (c_k^L) bounds on the change of the value function for all evaluation states between iterations of the value iteration algorithm are given by

$$c_k^L = \frac{\alpha}{1 - \alpha} \min_{i \in S^{Eval}} [J_k(i) - J_{k-1}(i)]$$

$$c_k^U = \frac{\alpha}{1 - \alpha} \max_{i \in S^{Eval}} [J_k(i) - J_{k-1}(i)]$$

The value iteration algorithm is terminated when $c_k^U - c_k^L \leq \varepsilon$ and policy μ is an approximation to the optimal policy. Results will show that policies converge before values with the stopping criteria ε used in this study.

What follows is an outline of the NDP algorithm implemented in this study. Treatment indicator tt is dropped to lighten notation but the reader is reminded that there is a function approximant for each tt and when approximating $J_{k-1}(j)$, the appropriate $\tilde{J}_{k-1}(j, r)$ must be used.

295 **Step 1:** List discrete states $i \in S^{Eval}$ and action set $U(i)$. Determine transition probabilities $p_{ij}(u)$,
 296 GNY function $f(i, u)$, profit function $g(i, u)$ and discount factor α . Set $k = 1$ and go to step 2.

297 **Step 2:** Calculate $J_k(i)$ for all $i \in S^{Eval}$ using eq. (2) and save optimal actions u and $J_k(i)$ for future
 298 retrieval. For $k = 1$, $\tilde{J}_{k-1}(j, r) = 0$ for all states j . For $k \geq 2$, use $\tilde{J}_{k-1}(j, r)$ to estimate $J_{k-1}(j)$ for
 299 states j . Go to step 3.

300 **Step 3:** Stop if $c_k^U - c_k^L \leq \varepsilon$. Current policy μ is an approximation to the continuous optimal policy
 301 and $J^*(i)$ are the values associated with $i \in S^{Eval}$ and μ . If $c_k^U - c_k^L > \varepsilon$, go to step 4.

302 **Step 4:** Fit an RBF function $\tilde{J}_k^{tt}(i, r)$ for each tt using values of $J_k(i)$ calculated at step 2. Set $k = k$
 303 $+ 1$ and go to step 2.

304 4. POLICY VALIDATION

305 There is no way of knowing the exact optimal policy as an analytical solution is not available
 306 thus it is important to remember that μ is a discrete approximation to the continuous optimal policy.
 307 While simulating μ , it is probable that states will be visited for which a discrete action is not
 308 available thus forcing us to approximate μ for the continuous portion of the state space between
 309 discrete states. During a simulation replication, we use a nearest-neighbor scheme where we choose
 310 the action for the closest discrete state where distance is measured using the Euclidian norm.

311 Given the same uncertainties that were used in the NDP model, the main idea is to simulate μ for
 312 a sufficient number of replications to build a confidence interval (CI) for the expected NPV of
 313 current and future profits for a given initial state i . If the CI contains $J^*(i)$ from the NDP
 314 optimization, we conclude that discrete states in S^{Eval} were chosen appropriately, that value function
 315 approximations were consistent in the value iteration algorithm and that there is no reason to believe
 316 that μ is not a good approximation to the actual continuous optimal policy. A CI that does not
 317 include $J^*(i)$ indicates that there may be inconsistencies in the value iteration algorithm and μ should
 318 not be taken to be a good approximation to the optimal policy.

What follows is a step by step description of how policy μ is simulated for state i where x_t is used to represent state i . The simulation advances in 5 year increments and subscript t represents the number of years since the beginning of the simulation replication.

Step 1: Choose state x_t and simulation parameter values. Set $t = 0$. Go to step 2.

Step 2: Take action u_t according to policy μ which results in state x_{t+5} at time $t+5$ according to (x_t, u_t) . In the case of stochastic prices, generate a random number and determine prices to be used for selecting u_t and for calculating $g_t(x_t, u_t)$. Calculate $NPV(g_t(x_t, u_t))$. Go to step 3.

Step 3: If a natural disaster occurs and the stand succumbs according to a generated random number, make state x_{t+5} a regeneration state. Go to step 4.

Step 4: If the stand is in a regeneration state, determine x_{t+5} based a generated random number and regeneration probabilities. Go to step 5.

Step 5: Store x_t , u_t , $NPV(g_t(x_t, u_t))$ and x_{t+5} for future retrieval. Go to step 6.

Step 6: If $NPV(g_t(x_t, u_t)) < \tau$ where τ is small enough to be financially inconsequential, stop the simulation. Otherwise, set $t = t+5$ and go to step 2.

Steps 1 to 6 make up one simulation replication where $X^m = \sum_t NPV(g_t(x_t, u_t))$ from that replication where m is the replication number. When enough replications have been done for the same i and μ , an average and a standard deviation are calculated and are used to build a $1 - \alpha$ confidence interval (CI) using the equations below:

$$S^2(n) = \frac{\sum_{m=1}^n [X^m - \bar{X}]^2}{n - 1} \quad \text{CI} = \bar{X} \pm t_{n-1, 1-\frac{\alpha}{2}} \sqrt{\frac{S^2(n)}{n}}$$

where $\bar{X}$ is the average of X^m for the n replications and $t_{n-1, 1-\frac{\alpha}{2}}$ is the value of the student- t distribution.

If the estimate $\bar{X}$ is such that $|\bar{X} - J^*(i)| = \vartheta$ then we say that $\bar{X}$ has an absolute error of ϑ . The confidence interval constructed with the formula above assumes that $S^2(n)$ will not change appreciably as the number of replications increases. The approximate number of replications, $n^*(\vartheta)$, required to obtain an error of ϑ is given by Law and Kelton (2000):

$$n^*(\vartheta) = \min \left\{ a \geq n \text{ such that } t_{a-1, 1-\frac{\alpha}{2}} \sqrt{\frac{S^2(n)}{a}} \leq \vartheta \right\}$$

We can determine $n^*(\vartheta)$ by iteratively increasing a by 1 until a value of a is obtained for which $t_{a-1, 1-\frac{\alpha}{2}} \sqrt{\frac{S^2(n)}{a}} \leq \vartheta$. In practice, the values of X^m may not be exactly normally distributed therefore Law and Kelton (2000) recommend the use of the t -distribution as it gives better coverage than a CI constructed using the normal distribution.

Since each discrete state has its corresponding $\mu(i)$ and $J^*(i)$, any state can be chosen as the starting point of the simulation and the corresponding $J^*(i)$ compared to the $(1-\alpha)$ CI above. Section 5.3 shows results of CIs constructed for a few discrete states and for different sets of parameters of the NDP model and discusses the absolute error ϑ for four simulations.

5. RESULTS AND DISCUSSION

All results in this section are for the base case scenario which uses the parameter values in table 2.

5.1 Consistency of function approximations

At each iteration of the value iteration algorithm, the RBF forces $\tilde{J} = J(i_n)$ at all centers and reducing the number of centers can reduce the accuracy of the approximations between the centers. Distances between centers are calculated using the Euclidian norm with all values of variables scaled to 0-1.

Inverting large ill-conditioned matrices can be difficult but scaling of the distances greatly increases the condition number of the matrix which is an indication of the precision with which it can be inverted. Reducing the number of centers from a full basis, where all evaluation states are used as centers, to a reduced basis improves the condition number of the A matrix by many levels of magnitude. For example, for $tt = 2$, we have 275 evaluation states. If we scale the distances between all these states and use all of them as centers in the RBF, the condition number of the matrix is 6.9496×10^7 . This is in contrast with an A matrix with 37 centers evenly distributed over the range of values of the state variables which has a condition number of 5.8042×10^4 , three orders of magnitude smaller. But the computational precision gained by having a much better conditioned A matrix comes at a great cost. Figures 2 and 3 show the difference between value function approximations using the RBF with 275 centers versus 37 centers.

In figure 2, the full basis RBF makes less than \$0.01 in approximation error over the entire set S_2^{Eval} . If the A matrix could be inverted without any loss of precision, those differences should all be null. By contrast, the approximation errors from the reduced basis RBF in figure 3 for those same 275 evaluation states range from \$4,300 to -\$2,700. Reducing the number of centers through which the RBF function is forced to pass, introduces errors in approximation at the evaluation states that were not included in the list of centers. Clearly, these approximation errors are not acceptable in a context of NDP where we are trying to iteratively build up values of being in any given state. For this reason, all results shown in this paper use the full basis thin plate spline RBF approximation architecture.

5.2 Value iteration algorithm convergence

All optimizations converged in a finite number of iterations and a few examples are presented below. Table 3 shows results for the base case scenario with the stopping criterion $\varepsilon = 0.2$. There is a clear relationship between the discount rate and the number of iterations to convergence. Table 3

shows the number of iterations to convergence versus the iteration number of the last policy change. The iteration number shown in column three is the last iteration during which a policy change was recorded for any evaluation state. The fourth column of the same table shows the largest value function change for the last iteration where there is a policy change. Clearly, regardless of the discount rate, values have not converged when the policy stops changing thus supporting the claim that as long as ε is sufficiently small, the value iteration algorithm will result in a stable policy that we show to be optimal through simulation in the next section.

5.3 Monte Carlo policy simulation

Table 4 gives the initial state for the four simulations discussed here. The first three simulations are done for softwood plantations because, given the parameters used in the NDP, all stands eventually become softwood plantations. It was important to use different discount rates as well as deterministic and stochastic prices. The fourth simulation starts with a young natural stand and allows us to simulate a larger portion of μ .

Simulations 1 and 2 have three price levels so they require the use of random market prices. Confidence interval (CI) information for the four simulations is given in table 4 as is the number of replications in each simulation ($n^*(\vartheta)$). CI's are built using the average $\bar{X}$ and standard deviation for all replications X^m for a given simulated μ and starting state i . Simulations 3 and 4 have one price level therefore the initial market state remains unchanged for the duration of each replication.

As recommended by Law and Kelton (2000), the confidence intervals for these simulations are constructed using a 95% confidence level and ϑ equivalent to 15% of the optimal value $J^*(i)$ from the NDP algorithm for the starting state i being simulated. In all cases, confidence intervals contain $J^*(i)$ for starting state i and policy μ but the intervals are quite wide because of the suggested large value of ϑ . The 15% value suggested by Law and Kelton is very conservative for our purposes and leads to very wide confidence intervals that are not very useful. If ϑ was reduced until the 95% CI

was too narrow to include $J^*(i)$, we would have $\vartheta < 5.2\%$ for all simulations. We can say with 95% confidence and a relatively small absolute error, that μ is a good approximation to the continuous optimal policy in those four cases.

5.4 Policy discussion

The policy discussion presented here is not meant to be a complete interpretation of the implications of using the policies calculated using NDP but rather some observations to reflect the level of detail obtained by using the methods described in this paper.

All states in the left side of table 5 are natural unmanaged stands ($tt = 1$) and they are discrete elements of S^{Eval} , and all states in the right side of the table are either pre-commercially ($tt = 2$) or commercially thinned ($tt = 5$) stands. Each of the colour-coded numbers makes reference to an action to be applied to the stand. A pre-commercial thinning action takes the stand from $tt = 1$ to $tt = 2$ and a commercial thinning action applied to a natural unmanaged stand takes the stand from $tt = 1$ to $tt = 5$. The reader is reminded that the natural stands on the left have 100% crown closure on a 100% stocked stand therefore the stand is supporting as many trees as it possibly can for its age. These natural stands have an average SW content of 75%. What follows is an example of the level of detail we can get from using NDP to develop individual stand management policies.

Action 1 is to do nothing and let the stand grow for one 5-year period. In all cases, taking action 1 simply means the stand will be 5 years older at the next decision time. For 5 year old natural stands, it is optimal to do nothing as the trees are too short to do a pre-commercial thinning and the diameters are too small to have any commercial value. Action 12 is a pre-commercial thinning that eliminates hardwood and spaces softwood to NS DNR recommended spacing and it is optimal to apply this action regardless of market price at ages 10 and 15. Taking action 12 with 10 and 15 year old natural stands, and letting them grow 5 years, results in transitions to the first two stands respectively (stands 1 and 2) in the right side of table 5. We notice that the new stands have had all

their hardwood removed ($pct^S = 100\%$) and that the new crown closure percentage is very low. In both cases, it is optimal to do nothing and let the stand grow (action 2) regardless of the observed state of the market. As random disturbances occur and the state of the forest stand and market evolve, the policies are used to continually make optimal decisions based on the state observed at decision time.

By age 20, an unmanaged natural stand has self-thinned to a point that investing in a PCT to thin out the stand is no longer the optimal action to take. Therefore, between the ages of 20 and 30 inclusively, it is optimal to do nothing and let the stand grow. At those ages, the average diameter of the trees is still too small to have any commercial value.

All optimal CT actions remove 40% of the total basal area on the stand and, aside from a few exceptions, CT actions are clustered into two groups. The first group, actions 19, 28 and 37, all remove the basal area by taking it as 25% SW and 75% HW. The only difference is the manner in which it is taken where 19 takes trees from the small diameters (from below), 28 takes the trees from across the diameter distribution and 37 takes the trees from the largest diameters (from above). In the second group, actions 38 and 39, CT is done from above with 50% basal area removal that is 75% SW with the balance in HW. Doing a commercial thinning from above yields slightly higher volumes for the same basal area but, more importantly, it creates a larger proportion of large logs which has a much higher market value, and higher market prices encourage the removal of larger trees because there is a high probability that prices will come down at the next period. The majority of the CT actions in the policy from the left hand side of table 5 lead to a state where the optimal action is to do nothing for at least 5 years. Stands 3 to 13 from the right side of table 6 are a sampling of the resulting forest stands after taking actions 19, 28, 37, 38 or 39 with the natural stands in the left side of table 5. Two characteristics are similar for all these stands: $pct^S = 100\%$ for all stands except stand 3 which still contains a small percentage of HW and cc varies within a narrow

range of 44% to 52%. At such low cc , it makes no sense to remove any trees as the amount of timber does not justify the removal. The policies in the right side of table 5 reflect this as it is optimal to do nothing for all stands up to 60 years of age (state 10). Starting at age 65, some regeneration harvests appear at very high prices with more appearing at age 75 (state 14). At this age, the stand diameter is high enough that it is optimal to do a regeneration harvest if the prices are simply above the mean. States 14 and 15 are shown to demonstrate that doing nothing when in state 14 yields state 15, and that with the rise in cc and diameter, there is a significant change in policy in just 5 years.

6. CONCLUSION

The main objective of this study as stated in the introduction is to demonstrate how to use a neuro-dynamic programming approach to the stand management problem when faced with high dimensional state and control spaces as well as multiple sources of uncertainty. For the same number of possible outcomes for each source of uncertainty, SDP typically requires a much higher number of discrete states because NDP can approximate the profit function between discrete states without sacrificing accuracy. In NDP, profit function approximations are built into the algorithm allowing for a much higher number of possible outcomes from all sources of uncertainty without needing to increase the number of discrete states. The set of state variables used in this study is not sufficient to fully model the management of real forest stands but it begins to approach the level of detail that forest managers might consider while developing policies. However, the detailed state/action spaces allow us to start considering the impact of uncertainty from sources such as the difference between planned harvest volumes and actual volumes, uncertain growth projections, climate change or uncertain quality of wood products at harvest.

Real world problems where many sources of uncertainty affect expected state/action outcomes and which would benefit from an additional level of detail in the optimal policies may be revisited using the ideas of NDP. Problems that could benefit from NDP typically are modeled using

479 stochastic dynamic programming and have a state space that can be represented by a sparse set of
480 discrete states without loosing the level of detail necessary to capture policy changes. To see NDP
481 gain wider use in forestry, we must take a closer look at problems that would typically be solved
482 using SDP algorithms but would be modeled with a simplified representation of uncertainty in its
483 SDP form. The main reason for incorporating uncertainty in forestry models is to quantify risk in
484 order to make informed decisions. Methodologies such as NDP can capture the dynamics of real
485 decision making while simultaneously considering as many sources of uncertainty as possible so that
486 policies are reasonable representations of real life decisions and can be used to create useful policy
487 discussions.

488 Forestry will continue to pose important problems that involve the solution of large scale
489 stochastic dynamic programming problems. Neuro-dynamic programming is an approach that shows
490 considerable promise.

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Table 1 – Range of values for state variables where *S* = softwood and *H* = hardwood

	<i>tt</i> = 1 Natural unmanaged		<i>tt</i> = 2 PCT natural		<i>tt</i> = 3 Softwood plantation		<i>tt</i> = 4 CT plantation		<i>tt</i> = 5 CT natural	
	min	max	min	max	min	max	min	max	min	max
Age (years)	0	95	5	95	5	95	10	100	25	105
<i>st</i> %	75	100	--	--	--	--	--	--	--	--
<i>ipd</i> (trees/hectare)	--	--	--	--	1000	4000	--	--	--	--
<i>d^S</i> (cm)	--	--	5.3	30.8	0	28.1	3.9	33.4	8.5	37.2
<i>d^H</i> (cm)	--	--	1.1	20.7	--	--	--	--	6.8	28.3
<i>pct^S</i> (%)	--	--	0	100	--	--	--	--	0	100
<i>cc</i> (%)	--	--	5.7	100	--	--	40	100	40	100
# discreet states	38		275		76		90		648	

Table 2 – Basic parameters of the NDP model

Annual discount rate	2%
Site index SW (in meters at age 50)	16.76m
Site index HW (in meters at age 50)	17m
Minimum dominant stand height (hardwood) for doing a pre-commercial thinning	6.1m
Minimum dominant stand height (softwood) for doing a pre-commercial thinning	2m
Maximum average stand height for doing a pre-commercial thinning	9m
Percentage of stand covered with softwood when a stand does naturally regenerate	75%
Natural stocking percentage of the forest	75%
Average selling price of softwood merchantable volume	\$13.07/m ³
Average selling price of softwood board volume	\$30.76/m ³
Average selling price of hardwood merchantable volume	\$22.40/m ³
Average selling price of hardwood board volume	\$46.23/m ³
Number of price level	1
Cost of planting less than 2500 trees on one hectare	\$1,350
Cost of planting 2500 or more trees on one hectare	\$1,500
Cost of surveying one hectare of newly harvested land	\$70
Cost of doing fill planting on one hectare	\$300
Cost of doing pre-commercial thinning on one hectare	\$750
Cost of one hour of labour for doing commercial thinning or final felling	\$40
Flat cost of doing a commercial thinning on one hectare	\$750

Approximate forested area in the west of the province of Nova Scotia (hectares)	1,691,300
Average number of fires per year in the area under study	3.5
Return interval of major hurricanes (years)	50
Average area of wind for a major hurricane (hectares)	400,000
Return interval of major insect outbreaks (years)	50

Table 3 – Example results for the base case scenario with thin plate spline RBF approximation and stopping criterion $\varepsilon = 0.2$

α (%)	# iterations for convergence	Iteration # of last policy change	Largest value function change at last policy change
1.0	138	105	\$110.55
2.0	101	42	\$287.11
4.0	39	23	\$14.11

Table 4 – Details of four Monte Carlo policy simulations including 95% confidence interval and number of replications

Sim	α	# of price breaks	Replication length	Starting state			Lower CI	Upper CI	DP value function J*	$n^*(\vartheta)$
				TRT	Age	Initial density				
1	0.04	3	295 years	3	25	2500	\$2,838.79	\$3,862.56	\$3,403.40	29
2	0.04	3	295 years	3	35	3250	\$3,263.89	\$10,376.53	\$6,945.36	2
3	0.03	1	295 years	3	30	2500	\$4,352.78	\$6,254.93	\$4,890.85	4
4	0.02	1	695 years	1	15	n/a	\$9,606.29	\$12,998.07	\$11,399.81	19

Table 5 – Partial policies for the base case scenario with 6 price levels

Natural unmanaged stands								Commercially or pre-commercially thinned natural stands											
Age (years)	Stocking (%)	Price						Stand number	Age (years)	d^S (cm)	d^H (cm)	pct^S (%)	cc (%)	Price					
		Price 1	Price 2	Price 3	Price 4	Price 5	Price 6							Price 1	Price 2	Price 3	Price 4	Price 5	Price 6
5	100	1	1	1	1	1	1	1	15	5.2	0	100	8.4	2	2	2	2	2	2
10	100	12	12	12	12	12	12	2	20	6.9	0	100	14.1	2	2	2	2	2	2
15	100	12	12	12	12	12	12	3	40	8.7	7.0	92	46.5	2	2	2	2	2	2
20	100	1	1	1	1	1	1	4	45	10.6	0	100	44.5	2	2	2	2	2	2
25	100	1	1	1	1	1	1	5	50	12.1	0	100	44.7	2	2	2	2	2	2
30	100	1	1	1	1	1	1	6	55	13.5	0	100	44.9	2	2	2	2	2	2
35	100	1	1	39	39	39	39	7	55	15.0	0	100	51.1	2	2	2	2	2	2
40	100	1	38	38	38	38	38	8	55	14.0	0	100	51.6	2	2	2	2	2	2
45	100	38	38	38	38	38	38	9	60	15.0	0	100	45.0	2	2	2	2	2	2
50	100	19	19	37	38	38	38	10	60	15.4	0	100	51.8	2	2	2	2	2	2
55	100	37	37	37	37	38	7	11	65	16.8	0	100	51.9	2	2	2	2	2	7
60	100	28	37	37	37	7	7	12	70	18.2	0	100	52.1	2	2	2	2	2	7
65	100	37	37	37	7	7	7	13	75	19.4	0	100	52.2	2	2	2	7	7	7
70	100	1	37	7	7	7	7	14	65	17.4	0	100	51.7	2	2	2	2	2	7
75	100	1	1	7	7	7	7	15	70	22.0	0	100	59	2	2	2	7	7	7
80	100	1	1	7	7	7	7	16	100	27.2	0	100	52.0	2	2	7	7	7	7
85	100	1	1	7	7	7	7												
90	100	1	1	7	7	7	7												
95	100	19	19	7	7	7	7												

1 - Let grow
2 - Let grow
7 - ReHar, plt 2500 tr/ha

12 - PCT, remove HW
19 - CT, rmv 40% BA, splt 25% (blw)
28 - CT, rmv 40% BA, splt 25% (cros)

37 - CT, rmv 40% BA, splt 25% (abv)
38 - CT, rmv 40% BA, splt 50% (abv)
39 - CT, rmv 40% BA, splt 75% (abv)

Figure 1 – Representation of a value function. The solid line represents the continuous nature of the value function, the dots represent the values calculated at a discrete states $i \in S^{Eval}$ and X's represent values approximated at discrete states j .

Figure 2 - Difference between approximate value and actual value, plotted against age, for the 275 discrete states for TRT=2 with those 275 states being used as centers in the RBF with distances scaled to 0-1

Figure 3 – Difference between approximate value and actual value, plotted against age, for the 275 discrete states for TRT=2 with 37 states being used as centers in the RBF with distances scaled to 0-1

Draft

Figure 1

Figure 2

Figure 3