Budgeting and Psychological Motivations in Sales Management

by

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A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Graduate Department of Management
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Sales force incentives are used to motivate, control, and reward salespeople, representing a substantial expenditure for many firms. In this dissertation, I examine the role that sales managers play in setting budgets for sales force incentives and I investigate how public recognition of performance in sales contests (a common form of incentive) affects salespeople’s motivations.

The standard analytical models used in the sales force incentives literature consist of a firm and a salesperson, with sales managers assumed (often implicitly) to be interchangeable with the firm. In my first study, I treat sales managers as distinct from their firms, in both their objectives and their information. I use this model to explore why many firms allow managers to participate in setting budgets for their salespeople’s incentives despite the apparent conflict of interest that results. I show that allowing such participation can be beneficial when it reveals useful information about the manager’s salesperson or their territory. I then propose a budget-setting approach that reveals such information and identify the conditions under which it is feasible and optimal.

In my next study, I consider alternative solutions to the challenge of revealing a sales manager’s private information about her salespeople or territories. In particular, I examine how the manager’s own compensation can be designed for this purpose. This can be achieved by offering the manager a menu of contracts, each consisting of a compensation plan for the manager and a budget constraint within which the manager must design the salesperson’s compensation plan.

In my final study, a joint work with Tanjim Hossain and Mengze Shi, I use a combination of analytical modeling and laboratory experiments to investigate how a salesperson’s psychological motivations are affected by the design of a sales contest. I am primarily interested in the public disclosure of contest results, such as the names and/or ranks of winners and losers. I find that recognizing winners generates positive psychological motivations, but that the relative performance of particular disclosure approaches can depend on the contest prize structure.
This thesis is dedicated to my wonderful wife, Jenna. I wouldn’t have started this journey without your absolute support and encouragement, and you haven’t wavered since. From the anxiety and uncertainty of PhD applications to...the anxiety and uncertainty of the job market, and everything in between, you have never failed to have my back. The degree may have my name on it, but it has absolutely been a team effort. And now...on to our next adventure!

Acknowledgements

I could fill another 100 pages thanking everyone who helped, guided, and supported me along the way, but I think that is frowned-upon, so I will name a select few and trust that the rest of you know who you are and how much I appreciate you.

To my parents: I can’t thank you enough for your never-ending love, support, and generosity. You have always been there to help in every possible way, and I truly, truly appreciate it. I’ve been ”absent-minded” since I was young, but I couldn’t have added the ”professor” part without you! I am both lucky and proud to be your son.

Sincerest thanks to my advisor, Mengze Shi, who has been a source of excellent guidance on my research and career development, but also a calming influence over the last five years. Whether it took 10 minutes or 2 hours, everything always seemed much clearer by the time I left your office. Thanks also to my committee of Tanjim Hossain, Sridhar Moorthy, and David Soberman. Your advice, patience, and trust throughout the Ph.D. program have undoubtedly made me a better scholar and I truly appreciate it. I am also grateful to my examination committee members, Ron Borkovsky and Anthony Dukes.

I am grateful to the rest of the Rotman Marketing faculty for your support and encouragement. Your teaching and feedback have been absolutely critical in helping me to develop my thinking in so many ways.

Thanks to my fellow Ph.D. students at Rotman, both past and present. I know I haven’t always been around much, but it has been a pleasure being on this ride with all of you. Special thanks to my former officemate, Avery Haviv, for providing friendship, advice, tech support, programming code, and someone to vent with.

Thanks to all of my friends and family who have bothered to read this far.

Last, but never, ever least, to Jacob and Faye: You have no idea how much you have contributed to daddy’s Ph.D. You got me out of bed every morning (figuratively and sometimes literally). You inspired me to take this journey so that I could be around more and then continued to inspire me to get my work done so that I could put it away and be with you. You help me to keep perspective on what is truly important and you never fail to put a smile on my face. Thank you!
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Chapter 1

Introduction

With the average company spending 10% of its total revenue on sales force costs and some industries spending as much as 40% (Zoltners et al., 2008), sales force management is a matter of no small concern to industry practitioners. However, the economic and practical importance of the sales function within firms is not fully reflected in academic research. In fact, while the amount spent annually on sales forces in the U.S. has been estimated at nearly three times the amount spent on advertising, sales-related papers made up less than 4% of articles published in marketing journals from 2001 to 2006, excluding the Journal of Personal Selling & Sales Management (Zoltners et al., 2008). Furthermore, there is agreement among researchers (e.g., Williams & Plouffe, 2007; Zoltners et al., 2008) that there exists a shortage of relevant research on particular topics of greatest concern to sales executives. These topics include sales force incentive compensation and budgeting / cost analysis, which are the areas of focus for this dissertation.

Sales force incentives can be broadly categorized into two types: absolute and relative. Under the former, a salesperson’s compensation is determined solely by his own performance, while under the latter, the performance of his peers plays a role. Examples of absolute incentives include commissions, by which a salesperson is paid a fixed amount per sale or a percentage of revenue generated, and quota-based plans, under which bonuses can be earned by meeting or exceeding predetermined targets. Relative incentive programs include forced-rank plans, in which each salesperson’s compensation is determined by his rank within the sales force on a particular metric, and sales contests, in which a limited number of top-performing salespeople earn prizes. Absolute and relative incentives each offer advantages and challenges, and many firms use a combination of the two to motivate, control and reward their salespeople. In this dissertation, I examine a common challenge faced by firms in designing each type of incentive plan.

In chapters 2 and 3, I investigate the role of sales managers in the design of absolute sales force incentives. In chapter 2, I focus on the budget-setting element of the design process. Most firms invest heavily in budgeting, yet many face the well-documented issue of managers manipulating the budget-setting process in order to obtain maximum resources for their own areas. I use an analytical model to examine why many firms allow managers to participate in budget setting for their salespeople’s incentives, despite the apparent conflict of interest that results. I show that this practice can be justified when a manager’s participation reveals private information that improves the firm’s decision making. Under given conditions, the firm can reveal such information by beginning with a restrictive budget constraint, then relaxing that constraint if and only if the manager chooses to participate in the budget-setting process.
process, at a cost specified by the firm. I describe the conditions under which this approach is not only feasible, but optimal for the firm to employ.

In chapter 3, I consider alternative solutions to the same problem of (absolute) incentive compensation design for salespeople when sales managers have relevant private information. In particular, I explore whether and how a firm can use the manager’s own compensation plan to reveal her information truthfully. I find that the firm can do so by offering the manager a menu of contracts, with each contract consisting of a compensation plan for the manager and a budget constraint within which the manager must design the salesperson’s compensation plan. I also consider an approach in which the firm offers the manager a single contract that includes an ‘unallocated budget bonus’, allowing her to keep a proportion of any budget that she does not allocate to the salesperson’s compensation. However, I show that this approach cannot outperform the optimal menu of contracts.

In chapter 4, co-authored with Tanjim Hossain and Mengze Shi, I study how a firm can manage a salesperson’s psychological motivations through the most common type of relative incentive program: a sales contest. In particular, I am interested in the joint design of a contest’s prize structure and its disclosure scheme - how contest outcomes are publicly announced. I run a set of laboratory experiments involving contests among salespeople and use the data to examine the levels of psychological motivations. I develop two new methods to separate the effect of psychological motivations from the effects of economic incentives and risk aversion. I find that both a) recognizing multiple top performers without revealing the ranking among them, and b) recognizing the single best performer, generate positive and significant psychological motivations. However, the relative performances of alternative disclosure schemes can depend on the contest prize structure. A smaller spread between rewards may induce higher levels of psychological motivations. Overall, I demonstrate the significant impact of an agent’s psychological motivations on her effort choice and highlight the importance to managers of jointly choosing incentive structures and disclosure schemes.

1.1 Literature Review

The following is a survey of the existing literature on sales force incentives, designed to provide a brief overview of the broader context into which this dissertation fits. A narrower (and deeper) review of related research is included in each of the subsequent chapters, including connections with other relevant areas of study, such as economics, accounting, and psychology.

Absolute incentives

In 1985, Basu et al. introduced the agency theoretic perspective on absolute sales force incentive design. Since then, agency theory has been the dominant research paradigm in the literature on this topic (Mantrala et al. 2010). Most of that literature is based on a standard moral hazard model (e.g., Holmstrom, 1979) of a risk-neutral firm and a risk-averse salesperson, with sales of a product determined by the salesperson’s effort (a one-dimensional action) along with some form of stochastic uncertainty. Agency theory enters through the assumption that the firm can observe the outcome (sales), but cannot directly observe the salesperson’s effort.

In the years since its publication, researchers have extended the core ideas and model from Basu et al. (1985) in a variety of directions. Lal and Staelin (1986) relax the assumptions of homogeneity among salespeople and information symmetry between the salesperson and the firm, to identify situations in
which it is optimal for the firm to offer a menu of compensation plans rather than a single plan. Rao (1990) assumes that salespeople are risk-neutral and heterogeneous in skill level, and also allows the optimal solution to include a menu of plans. Mantrala et al. (1994) and Raju and Srinivasan (1996) consider problems similar to Rao (1990), but with solutions restricted to a single plan using quotas set by the firm (rather than a menu of plans from which salespeople self-select). Basu and Kalyanaram (1990) restrict attention to linear compensation plans (consisting of a salary plus a constant commission rate) and compare their performance to the optimal plans found under the more general conditions of Basu et al. (1985). Lal and Srinivasan (1993) generalize the standard model to consider salespeople that sell multiple products and to reflect the reality that a salesperson makes continuous decisions about her effort level throughout the pay period, rather than deciding once at the beginning of the period. Zhang and Mahajan (1995) also consider salespeople that sell multiple products, but allow those products to have sales that are not necessarily independent of one another. Hauser, Simester and Wernerfelt (1994) incorporate additional incentives based on customer satisfaction measures, using a model that includes two types of salesperson effort (affecting the firm’s short-term sales and long-term reputation) and considers the response of customers over time. Joseph and Thevaranjan (1998) consider situations in which some sales effort can be observed and identify the conditions under which the optimal compensation plan combines incentives for non-observable effort with monitoring for observable effort. Godes (2004) considers situations in which sales uncertainty is endogenous (i.e. the salesperson’s effort impacts the degree of risk). More recently, Daljord, Misra and Nair (2016) impose the common desire of firms to have plans that are (at least partially) homogeneous across salespeople.

This is merely a sample of the wide variety of research that has followed Basu et al. (1985), but one thing that all of these models have in common (including those not mentioned) is that they represent two ‘levels’ of an organization: the firm itself and one or more salespeople. If mentioned at all, sales managers are treated interchangeably with the firm. In chapters 2 and 3 of this dissertation, I model a sales manager that is distinct from her firm, both in her objectives (to maximize personal earnings or utility rather than firm profit) and her information (which includes knowledge about her salespeople and/or their territories, acquired in the course of her managerial duties).

It is worth noting that not all of the recent work in sales force incentive design is based in agency theory. Anderson (1985), Anderson and Weitz (1996) and John and Weitz (1989) discuss sales force compensation to varying extents within the framework of transaction cost analysis (TCA), although Krafft et al. (2004) find that TCA has limited value in helping understand the design of compensation plans. Mantrala and Raman (1990) and Chen (2005) follow the lead of Gonik (1978) in using a model that combines forecasting and compensation by relying on a salesperson to forecast her own sales, then using that forecast as her quota while rewarding those who set themselves a higher target. Again, these models focus on the firm and the sales force, abstracting away from the role of the sales manager.

Relative incentives

Research on relative sales force incentives dates back as far as Haring and Myers (1953), who use a survey and interviews with firms to provide a summary of trends in “special incentives” (mostly sales contests and “honor awards”) offered to salespeople, and a summary of their effectiveness. Many years later, researchers continued to rely on surveys to study the characteristics of sales contests used by firms and how salespeople feel about and respond to those contests. These include surveys of managers (e.g., Wotruba & Schoel, 1983) as well as surveys of the salespeople themselves (e.g., Beltramini & Evans,
Chapter 1. Introduction

Other studies use surveys of salespeople to consider sales contests in light of particular cognitive theories, such as goal theory (Hart et al., 1989) and expectancy theory (Hastings et al., 1988).

An important body of analytical research on relative incentives (largely applicable, but not specific, to sales contests) began to emerge from the economics literature in the early 1980s. The seminal works of Lazear and Rosen (1981), Green and Stokey (1983), and Nalebuff and Stiglitz (1983) focus on the suitability or optimality of relative incentives, as compared to absolute incentives, under varying conditions and assumptions. Other studies consider elements of contest design. For example, Nalebuff and Stiglitz (1983) consider some nontraditional design features, such as penalizing the lowest performer or introducing “gaps” between participants that must be exceeded in order for one to place ahead of another. O’Keeffe, Viscusi and Zeckhauser (1984) analyze contest design when participants have unequal probabilities of winning due to either intentional bias or differences in ability. Krishna and Morgan (1998) examine the optimal contest prize structure, particularly when participants are risk averse.

A number of studies have since been conducted to test elements of the developing theory about contests, including both experimental tests (e.g., Bull et al., 1987; Weigelt et al., 1989) and empirical analyses of field data (e.g., Ehrenberg & Bognanno 1990; Knoeber & Thurman 1994). More recently, Güth et al. (2016) use a combination of analytical modeling and a laboratory experiment to develop and test theoretical predictions about a contest design with “outcome-dependent prizes”, which are endogenously determined based on the total productivity of all participants.

Despite this existing research and the widespread use of contests within sales organizations, the marketing literature contains relatively little study of sales contest design, with a few notable exceptions. Gaba and Kalra (1999) use analytical modeling and a laboratory experiment to examine how risk taking by salespeople is influenced by variations in either absolute (quota-based) or relative (contest) compensation schemes. Kalra and Shi (2001) analyze how the optimal prize structure of a sales contest depends on the number of participating salespeople, their risk attitudes, and the distribution of sales outcomes. Lim et al. (2009) use a series of laboratory and field experiments to test existing theories about the effects of varying the prize structure in a contest.

In recent years, the primary focus of contest research has been on incorporating behavioral considerations into theoretical models. Grund and Sliwka (2005) and Kräkel (2008) model the effects of emotions in contest participants (relating to inequity aversion in the former, social comparison in the latter). Gill and Stone (2010) model contest participants who care about receiving what they “deserve”. Yang et al. (2013) compare contests vs. quota-based sales incentives for salespeople in unbalanced territories (i.e. when one salesperson is advantaged over another), accounting for feelings of pride and disappointment. Lim (2010) uses a combination of analytical modeling and laboratory experiments to determine the optimal number of winners in a sales contest, accounting for “social loss aversion”. Chen et al. (2011) test theoretical predictions about the effects of varying the prize structure in a multiperson, asymmetric sales contest and explain their results using a model that accounts for social comparison.

Again, this is a sample of the literature on relative incentives, but none of the existing research focuses on the effects of public recognition of contest winners (and/or losers). In chapter 4 of this dissertation, such recognition is treated as an integral part of contest design, with various recognition schemes analyzed and tested in an experimental setting, along with variations in the contest prize structure.
Chapter 2

Should sales managers participate in budget setting?

This research was supported by the Social Sciences and Humanities Research Council of Canada.¹

2.1 Introduction

In a survey of senior finance managers at medium and large firms, the median US respondent indicated that managers in their firm spend an average of three to four weeks per year (approximately 6-8% of their time) on budget-related tasks, such as developing and revising budgets, conducting variance analyses and creating reports (Libby & Lindsay, 2010). Furthermore, the median respondent’s business unit takes 10 weeks to complete its annual formalized budgeting process.²

Despite this heavy investment of resources and attention, however, the budgeting process in many firms is susceptible to manipulation by the managers involved. Consider the following illustration:

Firm X sells its products through a team of salespeople whose selling effort is costly to observe or monitor. To motivate the sales force to exert effort, the firm offers each salesperson a bonus payment based on the sales they achieve relative to a quota set at the beginning of the year. As a district sales manager in the firm, Jane oversees nine salespeople. Among her other responsibilities, Jane is asked to estimate the sales potential for each of her territories each year, to be used as an input in setting her salespeople's quotas. This year, upon receiving the quotas for her territories, Jane approaches her sales director to explain why she believes some of them are unreasonably high. Although she makes a compelling case, the director is well aware that Jane’s own bonus payments are based on a district-level quota that is the sum

¹In addition to my advisor and thesis committee, valuable comments and suggestions for this chapter were provided by Paulo Albuquerque, Anne Coughlan, Duncan Simester, Birger Wernerfelt, and Juanjuan Zhang. Thanks also to seminar participants at HEC Montreal, INSEAD, London Business School, Northwestern University, Ohio University, Texas A&M University, University at Buffalo, University of Texas at Dallas, University of Guelph, and University of Toronto, as well as the 2014 Thought Leadership on the Sales Profession, 2014 & 2015 Marketing Science, and 2015 Enhancing Sales Force Productivity conferences for helpful feedback.

²These estimates of manager and firm time spent on budgeting are significantly lower than those previously reported by Hope and Fraser (2003) (20–30% of managers’ time and 4-5 months) and Umapathy (1987) (21–40% of management’s time), so one can think of them as lower bounds.
Although the details vary (e.g., the type of incentive plan, the nature of the manager’s involvement, her level in the organization, etc.), this example represents the conflict of interest that can arise when managers are involved in setting budgets. In the survey mentioned above, 86% of US respondents reported that ‘sandbagging’ (defined as “negotiating easier targets than one actually thinks can be accomplished, to make the budget easier to attain and to increase the odds of receiving a favorable evaluation and/or bonus”) occurs occasionally or frequently in their business units (Libby & Lindsay, 2010). Huang and Chen (2009) provide evidence from the accounting literature that “managers will use many strategies to obtain maximum funding in the budgeting process”. Steele and Albright (2004) outline a number of specific behaviors used by managers to subvert even well-designed firm budgeting processes (which include quota-setting). In the illustration above, Jane has two clear opportunities to attempt to manipulate her firm’s budgeting process: 1) when the firm seeks her input on sales potential, and 2) when she challenges the quotas assigned to her salespeople.

Taken together, firms invest heavily in a budgeting process, yet allow that process to be manipulated through managerial involvement, which raises questions. For example, it would seem relatively simple for a firm to eliminate (or severely limit) opportunities for managers to manipulate the process. So, given the level of investment, why do many firms choose to allow managers to participate in budget setting despite the apparent conflict of interest? One likely answer is that the firm values the manager’s private information; however, theory would suggest that any information she provides should be disregarded (or at least discounted), due to the misalignment of her personal incentives with those of the firm. Is there some way, then, that the firm can obtain reliable information from the manager?

Although these questions are highly relevant in the context of marketing and sales budgeting, the marketing literature has not made an attempt to address them. This appears to be due, at least in part, to the common use of models that omit or abstract away from both self-interested managers and budget constraints. For example, the standard model of sales force incentive compensation design, dating back to Basu et al. (1985), is a two-player agency-theoretic game involving a profit-maximizing firm and a utility-maximizing salesperson (e.g., Lal & Staelin, 1986; Rao, 1990; Hauser et al., 1994; Joseph & Thevaranjan, 1998). In such models, managers are (often implicitly) assumed to be interchangeable with their firms, making budget constraints superfluous and the questions above inapplicable.

In reality, however, neither a sales manager’s personal objectives nor her information typically align perfectly with those of her firm. Perhaps the most important driver of misaligned objectives between firms and managers is that while firms are primarily concerned with maximizing profits, sales managers are commonly evaluated (and thus compensated, promoted, etc.) based on sales volumes, as in the example of her territory-level quotas.\(^3\)

\(^3\)This illustration is not drawn from a single firm, but is based on the findings of a 2013 survey of US-based firms in the medical products and services industry, which shows that “sales potential [for setting territory-level quotas] is predominantly determined by manager judgment”, and that the vast majority of those managers’ own incentives are sales quota-based (ZS Associates, 2013).

4For convenience and clarity, I will use feminine pronouns (“she”, “her”) to refer to managers and masculine pronouns (“he”, “his”) to refer to salespeople throughout this chapter.

5There are a number of reasons for which firms may choose to compensate managers (and salespeople, for that matter) based on sales volumes rather than profits, including: i) compensation based on profits requires disclosure of sensitive financial details (e.g., gross margins) that are often kept confidential, even within firms; ii) using profits for compensation requires that they be calculated at a much more granular level than firms typically use (e.g., by district, territory, or even customer, instead of by country), which may be costly and difficult and require a number of subjective decisions (such as how to allocate firm- or country-level expenses); iii) even within acceptable accounting practices, profit calculations are subject to manipulation by the firm (Healy & Wahlen, 1999) or the perception thereof, posing a challenge to the credibility of profit-based contracts for non-executives; and iv) a risk-averse manager (or salesperson) would need to be compensated for the additional risk that they would assume under a profit-based incentive plan.
example above. Information asymmetry between a firm and a manager is of interest here when it favors the manager. As pointed out by McAfee and McMillan (1995), information that is valuable to a firm is acquired by individuals at all levels of the firm in the course of their day-to-day responsibilities. In the case of a sales manager, such information would likely include detailed knowledge of the salespeople and territories that she manages. For example, in my model, the manager’s private information relates to the difficulty of selling in a salesperson’s territory.

In this chapter, I use the context of sales force incentive plan design to offer an explanation of why many firms involve managers in setting their own budgets, despite the apparent conflict of interest that this involvement creates. To do this, I extend the standard agency-theoretic model of incentive compensation design to include a self-interested manager with sales-based incentives and private information about the difficulty of selling in her salesperson’s territory. I then analyze and compare degrees to which the firm can involve the manager in sales force compensation planning and design: 1) ‘constrained delegation’, in which the manager designs the compensation plan within a budget constraint imposed by the firm; 2) ‘full participation’, in which the manager designs the plan and is given an opportunity to participate in setting the budget constraint; and 3) ‘no involvement’, in which the manager does not participate in either plan design or budget setting.

Using the ‘constrained delegation’ model, I show that a self-interested manager compensated based on sales volumes will prefer to offer larger incentives to her salespeople than is profit-maximizing for the firm. Thus, if the firm wishes to delegate the design of the sales force compensation plan to the manager in order to take advantage of her private information, it must impose some form of budget restriction to constrain the manager’s tendency to overspend on incentives.

Of course, without access to the manager’s information, setting such a budget restriction optimally is non-trivial. In the ‘full participation’ model, I propose a budget-setting approach by which the firm induces the manager to reveal her private information by allowing her to participate in budget setting, but making it costly for her to do so. Specifically, the firm begins with a restrictive budget constraint, then relaxes it if and only if the manager chooses to incur the (firm-specified) cost of participating. This approach functions as a separating mechanism if the manager herself has more to gain from participating when the relaxed budget is profit-maximizing for the firm than when it is not.

Finally, for comparison, I consider the firm’s optimal approach without the manager’s involvement. I show that even when it is feasible for the firm to induce information revelation by allowing the manager to participate in budget setting, it is not necessarily optimal to do so. In some cases, the firm is better off either setting its budget without the manager’s private information or acquiring that information through other means.

In practical terms, this chapter provides a process that a firm can follow to elicit truthful information from a sales manager about the territories (or people) that she manages, offers guidance on when such a process is both feasible and optimal to employ, and shows how the firm can use that information to maximize the expected profitability of its sales force incentive plan. Returning to the illustration above, instead of the firm asking Jane (the sales manager) to estimate the sales potential of each of her territories for use in quota setting, the recommended process is akin to the firm presenting Jane with an aggressive quota for each territory, then giving her an opportunity to demonstrate that a lower quota is more appropriate. The firm then makes the requirements for such a demonstration sufficiently onerous that Jane will only choose to satisfy them when a lower quota is truly justified.

The chapter also provides guidance on what types of firms can and should allow their sales managers
to participate in budget setting in this way. One might think that the separating mechanism described would be most effective when there are large differences between territories. Surprisingly, however, I find that it is feasible as long as the difference between territory types does not exceed a particular threshold. This is an indirect consequence of the salesperson’s freedom to exit the firm if the compensation contract does not offer him sufficient expected utility. Furthermore, I show that allowing the manager to participate in budget setting (when the above mechanism is feasible) is optimal for the firm if: a) it is costly or impossible to obtain her information another way; b) the opportunity cost to the firm of her participation is low and/or unlikely to be incurred; and c) the profit that can be generated by a salesperson in the most difficult territory is substantially greater than the profit generated if he exits the firm. This last result arises because a salesperson in a difficult territory exits the firm if the budget is too restrictive. Thus, the manager’s information, which allows the firm to ensure that the budget is sufficient to retain that salesperson, increases in value with the profit he generates.

2.1.1 Literature Review

The objective of this research is to explore the impact of self-interested, privately-informed managers and/or firm-imposed budget constraints on the optimal design of sales force incentive compensation. Existing literature in this area addresses how the firm should design its compensation plan, and I take this literature a step further by examining firm policy on managerial participation in plan design and budget setting. Guidance on budget setting is important for practitioners. Despite budgets being among the most widely-used management tools in organizations worldwide (Hansen et al., 2003; Davila & Wouters, 2005), researchers have found that many firms base their budgeting decisions on simple heuristics and decision rules, including percentage of sales revenues, percentage change from the previous period, or competitive parity (Mantrala, 2002; Gupta & Steenburgh, 2008; Shankar, 2008), rather than profit maximization.

In addition to contributing to the sales management literature, this chapter relates to two different areas of economics: the first is the study of ‘influence activities’ and the second is the study of hierarchies in organizations. Most of the literature on influence activities, defined as ‘effort exerted by individuals to affect decisions within organizations’, focuses on the costs or inefficiencies associated with this effort and on ways to curtail it (e.g., Milgrom, 1988; Wulf, 2009). However, two recent exceptions point to the potential for influence activities to benefit firms by revealing private information held by the ‘influencing’ individual. Laux (2008) demonstrates that requiring a project manager to lobby for project approval can benefit a firm by inducing the manager to both acquire and reveal costly information. Simester and Zhang (2014) show that requiring salespeople to lobby for price discounts for their customers can induce them to reveal private information about customer demand. In my model, the manager’s involvement in budget setting is akin to participation in influence activities. I propose a similar mechanism to those of Laux (2008) and Simester and Zhang (2014), by which a firm can induce a sales manager to reveal private information about the territories she manages by requiring her to exert effort (i.e., to lobby) to obtain a larger incentive compensation budget. Both Laux (2008) and Simester and Zhang (2014) assume that the manager’s acquisition of private information is costly, with Simester and Zhang, in

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6This chapter’s model and findings can be adapted to apply to any context in which the following conditions exist: i) an agent, whose effort is unobserved, receives incentives based on an outcome other than firm profit; ii) the agent’s manager has relevant private information that she can choose to share with the firm; and iii) the manager is motivated (at least in part) by the same observable outcome as the agent. For example, these may apply to manufacturing settings in which a worker is paid a piece-rate for production.
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particular, relying on the assumption that evidence of low demand is more costly to acquire when true demand is high. I relax that assumption, allowing the sales manager to acquire private information about her territories through the regular performance of her job, making it essentially costless. Thus, in my model, the firm’s ability to use influence activities to reveal the manager’s private information stems not from a presumed difference in the cost, but from a difference in the benefit (to the manager) of exerting influence ‘truthfully’ vs. ‘untruthfully’. In this chapter, I derive the conditions under which such a difference exists.

The literature on hierarchies focuses mainly on the threat of collusion or ‘hidden gaming’ between individuals within a hierarchy (e.g., Tirole, 1986; Laffont, 1990; Imbeau, 2007) or on the inefficiencies introduced by delegating decision-making to lower-level individuals in a hierarchy, relative to centralizing those decisions at the top level (e.g., McAfee & McMillan, 1995; Faure-Grimaud & Martimort, 2001). One exception is Mookherjee & Reichelstein (1997), who show that delegation can be made as efficient as centralization in their setting, in which the agent’s production is deterministic and managers have no private knowledge about their subordinates, by allowing each individual to offer a menu of incentive schemes to their direct subordinates. I examine a setting with layers of private information (the salesperson’s effort is unobserved by both the manager and firm, while his territory ‘type’ is observed by the manager but not the firm) and offer a mechanism by which delegation can be made efficient with a single contract for each level.\footnote{In the specific context of sales management, previous research concerning delegation has mainly focused on the profitability, optimality and impact of delegating pricing decisions to the sales force (e.g., Lal, 1986; Bhardwaj, 2001; Hansen et al., 2008; Frenzen et al., 2010; Lim & Ham, 2014).}

This chapter also joins a small set of game theoretic research using models that combine moral hazard, adverse selection and cheap talk (e.g., Pei, 2013) and an even smaller subset that combines all of those elements in a multi-level principal-agent model (e.g., Tirole, 1986). The standard model of sales force incentive compensation design (e.g., Basu et al., 1985) is a classic example of moral hazard in a two-player principal-agent game (Holmstrom, 1979). Incorporating a sales manager in my model expands the game to three players (principal-manager-agent). The moral hazard problem remains, as the firm (and the manager) still do not observe the salesperson’s effort. A problem akin to adverse selection is introduced because, unlike in the standard model, the firm has incomplete knowledge about the salesperson’s utility function (lacking the manager’s private knowledge about the salesperson’s cost of effort). Furthermore, any communication from the manager to the firm about her private information is cheap talk, as it does not directly affect any player’s payoffs.

Finally, this chapter contributes to the accounting literature on budgetary slack and managerial participation in budget setting (e.g., Onsi, 1973; Young, 1985; Dunk, 1993; Huang & Chen, 2009), with budgetary slack defined as “the intentional overestimation of expenses and/or underestimation of revenues in the budgeting process” (Chartered Institute of Management Accountants, 2000). Most of the existing research in this area is based on either experiments or self-reported information from managers, and the results to-date have failed to establish a consensus on a number of questions, due to contradictory findings. My model provides theoretical support and an alternative explanation for certain empirical observations, without necessarily relying on unethical behavior or on other explanations mentioned in the recent literature (e.g., Davila & Wouters, 2005; Elmassri & Harris, 2011) to explain why managers exhibit a tendency to seek larger budgets.

The remainder of the chapter is structured as follows: In the next section, I introduce the model to be analyzed. In Section 3, I derive and discuss the results of the model. In Section 4, I show how the
model results generalize when two main assumptions are relaxed. In Section 5, I review the key findings and discuss some further research opportunities.

2.2 Model

I consider a firm that employs a single salesperson to help sell its product. For simplicity and tractability, I use a stylized setup in which the product’s sales are binary, taking one of two possible values: \( x \in \{x_L, x_H\} \), with \( \Delta x = x_H - x_L > 0 \). (Generalization of the model results to a continuous range of sales outcomes is discussed in §2.4.)

The salesperson is expected to exert effort to sell the product, but exerting effort is costly and cannot be observed by the firm. The salesperson’s effort is represented one-dimensionally and can take any non-negative real value, \( e \in \mathbb{R}_+ \). The salesperson’s effort affects the probabilities of the two sales outcomes, with an increase in effort increasing the probability of high sales and decreasing the probability of low sales. That relationship is represented by the sales response function \( p(e) \equiv \Pr(x = x_H|e) \), where \( p' > 0 \), \( p'' < 0 \) and \( p(e) \in (0, 1) \ \forall \ e \). In other words, \( p \) is increasing and concave\(^8\) and always includes a degree of uncertainty, regardless of the salesperson’s effort. Thus, the firm is never able to infer the salesperson’s effort from the sales outcome. Where necessary, I will assume the logistic form for the sales response function, \( p(e) = \frac{\exp(e)}{1 + \exp(e)} \), which has the desired properties.

Since the firm cannot incent the salesperson based on his effort, it induces effort instead by compensating him based on sales, which are mutually observed. Due to the binary nature of sales, a compensation plan is simply a pair of payout values \( (s_L, s_H) \), with each payout associated with a sales outcome. Thus, the salesperson receives compensation:

\[
s(x) = \begin{cases} 
  s_H & \text{if } x = x_H \\
  s_L & \text{if } x = x_L 
\end{cases}
\]

where \( s_H \geq s_L \).

When the salesperson exerts effort \( e \) and receives payout \( s(x) \), his utility is given by:

\[
U = u(s(x)) - v(e)
\]

where \( u(\cdot) \) represents the salesperson’s utility from income and is increasing and concave (\( u' > 0 \) and \( u'' < 0 \)) with \( u(0) \) normalized to 0. This reflects the standard assumption that the salesperson values income but is risk-averse.

The salesperson’s disutility from effort is represented by \( v(\cdot) = \beta v_0(\cdot) \), where \( v_0 \) is increasing and convex (\( v'_0 > 0 \) and \( v''_0 > 0 \)) with \( v_0(0) = 0 \), reflecting the standard beliefs that effort is costly and that the incremental cost of effort increases with the amount of effort exerted. \( \beta > 0 \) acts as a multiplier on the cost of effort, so a higher value implies higher effort costs for the salesperson. Where necessary, I will assume the following functional forms for \( u \) and \( v \), which have the desired properties:

\[
\begin{align*}
  u(s) &= \sqrt{s} \\
v(e) &= \beta [\exp(e) - 1]
\end{align*}
\]

\(^8\)The concavity of the probability function is reasonable under the assumptions that a) the salesperson is able to prioritize her tasks optimally, and b) the product being sold has a relatively short selling cycle and high sales volume.
The salesperson’s best alternative option provides utility $\bar{U}$. If the salesperson’s expected utility from the firm’s contract is less than $\bar{U}$, he rejects the contract and exits the firm, exerting no effort and receiving no compensation.\(^9\)

I begin by providing the optimal solution for a benchmark case that follows the standard sales force compensation models, with the firm setting the compensation contract, then the salesperson deciding whether to accept the contract and, if so, choosing his effort level. The firm is assumed to be fully informed about the salesperson’s utility and sales response functions, as well as his utility from the outside option.

In the remaining models, the firm is not fully informed about the salesperson’s utility. Specifically, the firm does not know the difficulty of the salesperson’s territory, which I refer to as its ‘type’. Instead, the firm has only a (correct) belief about the distribution of types. I allow the territory to take two possible types: low-difficulty and high-difficulty (or ‘easy’ and ‘hard’), with the types represented in the model by the parameter $\beta \in \{\beta_L, \beta_H\}$ with $\beta_L < \beta_H$. (Generalization to more than two territory types is discussed in §2.4.) In other words, territory difficulty is reflected in the salesperson’s cost of effort, with a less-difficult territory incurring less utility cost to the salesperson than would a more-difficult territory, for the same effect on sales outcomes.\(^10\) Despite a firm’s best efforts to design balanced territories, selling difficulty can vary across territories (even when sales potential is the same) for a number of reasons, including geographic constraints and differences in customer composition. Even within a particular geography, selling conditions and territory boundaries can change frequently (Zoltners & Sinha, 1983), making it difficult for a firm to maintain up-to-date information or to use historical performance to infer the current difficulty of selling within a given territory.

The territory type, while unknown to the firm, is known (costlessly) by the sales manager, having been revealed in the course of her managerial duties. The manager’s objective is driven by her own compensation plan, which has a similar structure to that of the salesperson:

$$s^M(x) = \begin{cases} 
   s^M_H & \text{if } x = x_H \\
   s^M_L & \text{if } x = x_L 
\end{cases}$$

where $s^M_H > s^M_L$. The manager’s compensation plan is set by the firm and is assumed to be exogenous to the salesperson’s compensation problem.\(^11\) Despite the similarities in their compensation structures, the manager’s motivations differ from those of the salesperson, because the manager does not incur the cost of exerting effort to drive sales. I assume that the manager is purely self-interested, seeking to maximize her expected compensation payout. The binary nature of the sales outcomes allows me to abstract away from whether she is risk-neutral or risk-averse, as her risk attitude does not affect the model solution.

This asymmetric information setup will be used to examine and compare three different ways in which the firm can involve (or not involve) the sales manager in the salesperson’s compensation plan design. In the ‘Constrained Delegation’ model, the firm delegates the design of the salesperson’s compensation plan to the manager, allowing her to set the payout values ($s_L, s_H$), but within a budget constraint

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\(^9\)For the purposes of this chapter, I assume that hiring and training a new salesperson requires a sufficient investment of money and/or time to prevent the firm from replacing an exiting salesperson within the time frame of the model.

\(^10\)Territory difficulty could also be modeled through the effectiveness of sales effort, by a multiplier on the probability of the high sales outcome. This would be effectively equivalent to the chosen specification.

\(^11\)Due to the binary nature of the sales outcomes in this model, the manager’s compensation can represent a commission on sales, a bonus for achieving a particular sales target, and/or a scaling of the salesperson’s payout, which are the most common sales-based compensation approaches for managers. It could also be interpreted as including non-monetary rewards associated with sales outcomes (such as career advancement).
imposed by the firm. (The nature of the budget constraint will be discussed in §2.3.2.) In the ‘Full Participation’ model, the manager again designs the salesperson’s compensation plan, but the firm also allows her to participate in setting the budget constraint. However, participation carries an effort cost for the manager, which will be discussed in §2.3.2. In the ‘No Involvement’ model, the firm designs the salesperson’s compensation plan, with no participation by the manager in the design or budget setting.

The firm is always assumed to maximize its expected profit. The marginal cost of production is normalized to 0.

The Constrained Delegation model represents the following sequence of steps:

1. The firm announces both the sales manager’s compensation terms, \((s^M_L, s^M_H)\), and the budget constraint for the salesperson’s compensation contract
2. The manager announces the salesperson’s compensation contract, \((s_L, s_H)\)
3. The salesperson decides whether to accept the contract and, if so, chooses his effort level \((e)\)
4. Sales are realized and the players receive their payouts

The benchmark model represents a similar sequence, except that step 1 is omitted and “manager” and “firm” can be used interchangeably in step 2. Similarly, in the No Involvement model, “firm” is substituted for “manager” in step 2 and step 1 is irrelevant. In step 1 of the Full Participation model, the firm also announces the cost to the manager of participating in the budget setting process. Additional steps are then included between steps 1 and 2, in which the manager chooses whether to participate in the budgeting process and, if so, the firm chooses whether to revise the budget accordingly.

I offer two comments on the manager’s private information about the salesperson’s territory type. First, I do not model any effort cost associated with acquiring it. Instead, I assume that it is a byproduct of her duties as manager. Relaxing this assumption and introducing an information acquisition cost here would complicate the model without revealing additional insights about the questions of interest. Second, I recognize that the firm may be able to acquire information about the territory type through some means other than the manager (e.g., by analyzing historical data). I allow for this possibility in §2.3.3, when considering the firm’s optimal budgeting approach.

Similarly, I recognize that managerial participation in budget setting may impose an opportunity cost to the firm, due to the time potentially spent away from the manager’s other responsibilities. Again, I abstract away from this cost for the initial analysis, but introduce it in §2.3.3.

Notation introduced in this section and throughout the chapter is summarized in a table at the end of the chapter.

2.3 Analysis and Results

For the sake of comparison, I begin by reviewing some established results from the benchmark case, in which the firm has full information and designs the salesperson’s compensation plan directly, with no manager and no budget constraint. I then analyze three asymmetric information models, in which a self-interested manager knows the salesperson’s territory type (i.e., his cost of effort), but the firm does not. Each of the models is solved using backward induction. For reference, these models are summarized in Table 2.1.

Using the asymmetric information models, I determine the expected outcomes when the firm delegates varying degrees of plan design and budget setting authority to the sales manager. In the case of ‘Full
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<table>
<thead>
<tr>
<th>Model</th>
<th>Players</th>
<th>Firm info</th>
<th>Manager involvement</th>
</tr>
</thead>
<tbody>
<tr>
<td>Benchmark</td>
<td>Firm, Salesperson</td>
<td>Complete</td>
<td>N/A</td>
</tr>
<tr>
<td>Constrained Delegation</td>
<td>Firm, Manager, Salesperson</td>
<td>Incomplete</td>
<td>Plan design</td>
</tr>
<tr>
<td>Full Participation</td>
<td>Firm, Manager, Salesperson</td>
<td>Incomplete</td>
<td>Budget &amp; plan design</td>
</tr>
<tr>
<td>No Involvement</td>
<td>Firm, Salesperson</td>
<td>Incomplete</td>
<td>None</td>
</tr>
</tbody>
</table>

Table 2.1: Model summary

Participation’, I identify a budget setting process by which the manager’s involvement ensures the best possible outcome, under given conditions. I then compare the outcomes of the different models to find the conditions (if any) under which each approach is optimal for the firm.

Salesperson’s optimal effort

The salesperson’s utility function and decision rule for determining his effort are the same across all of the models below, as his choices depend only on his compensation contract and not on who designs it or how.

Given a compensation contract \((s_L, s_H)\), the salesperson chooses effort \(e\) to maximize his expected utility:

\[
E(U|e) = p(e)u(s_H) + [1 - p(e)]u(s_L) - v(e)
\]

Thus, the salesperson will choose effort \(e^*\) such that

\[
\frac{v'(e^*)}{p'(e^*)} = u(s_H) - u(s_L) \tag{2.1}
\]

This has a unique, non-negative solution when \(u(s_H) - u(s_L)\) is sufficiently large \(\geq \frac{v'(0)}{p'(0)}\), because \(\frac{v'(e)}{p'(e)}\) is strictly increasing in \(e\) (proof in Appendix A). Intuitively, this result simply states that the salesperson will increase his effort until the marginal cost of doing so is equal to the marginal increase in his expected utility from income due to the additional effort.

2.3.1 Benchmark model

As described in §2.2, this model follows the standard models of sales force compensation, in which the firm is fully informed about the salesperson’s utility function and sets the compensation contract to maximize expected profit, with no self-interested manager or budget constraint. As such, the following results are well-established, but are included here for comparison.

Firm’s optimal contract

To induce a particular effort level \(e\), the firm must offer a plan that satisfies equation (2.1) for \(e^* = e\) (i.e., that meets the incentive compatibility (IC) constraint). Moreover, to ensure that the salesperson accepts the contract, the plan must also satisfy his individual rationality (IR) constraint at that effort level. In other words, his expected utility must meet or exceed the utility offered by his outside option.

Subject to these constraints, the firm chooses the plan that maximizes its expected profits. Thus,
for a given \( e \), the firm’s problem is:

\[
\max_{(s_L, s_H)} E[\pi(e)] = p(e)(x_H - s_H) + [1 - p(e)](x_L - s_L)
\]

s.t. \( E(U|e) \geq \bar{U} \) \hspace{2cm} (IR)

\[
\frac{v'(e)}{p'(e)} = u(s_H) - u(s_L)
\]

(2.1)

\[
\frac{v'(e)}{p'(e)} = \Delta u
\]

(2.2)

To simplify notation, I define: \( u_L = u(s_L) \), \( u_H = u(s_H) \) and \( \Delta u = u_H - u_L \). Furthermore, I define \( h(\cdot) = u^{-1}(\cdot) \), so \( h(u) \) represents the monetary payout required to provide utility \( u \).\(^{12}\) Thus, \( h(u_H) = s_H \) and \( h(u_L) = s_L \). This allows me to transform the problem into a more tractable form, in which the compensation plan is expressed in terms of the utility that the payouts provide to the salesperson, rather than their monetary value.\(^{13}\) The firm’s problem can then be re-written as follows:

\[
\max_{(u_L, u_H)} p(e)(x_H - h(u_H)) + [1 - p(e)](x_L - h(u_L))
\]

s.t. \( p(e)u_H + [1 - p(e)]u_L - v(e) \geq \bar{U} \) \hspace{2cm} (IR)

\[
\frac{v'(e)}{p'(e)} = \Delta u
\]

(2.3)

Since this problem has a concave objective function and linear constraints, it can be solved using Lagrangian optimization (details shown in Appendix B). The resulting optimal compensation plan (to induce a given effort \( e \)) is given by:

\[
u_L^*(e) = \bar{U} + v(e) - p(e)\frac{v'(e)}{p'(e)}
\]

(2.2)

\[
u_H^*(e) = \bar{U} + v(e) + [1 - p(e)]\frac{v'(e)}{p'(e)}
\]

(2.3)

This result is driven by the fact that the firm prefers to offer the salesperson just enough compensation to ensure that he accepts the contract (i.e., the IR constraint binds). Thus, with the IC constraint uniquely determining the necessary ‘reward spread’ \( (\Delta u) \) to induce effort \( e \), the firm chooses the unique compensation plan that provides the salesperson with expected utility equal to his outside option, \( \bar{U} \).

The firm’s expected profit is the average of its profits from the two sales outcomes, weighted by the probability of each outcome:

\[
E[\pi(e)] = p(e) \left( x_H - h \left[ \bar{U} + v(e) + [1 - p(e)]\frac{v'(e)}{p'(e)} \right] \right) + [1 - p(e)] \left( x_L - h \left[ \bar{U} + v(e) - p(e)\frac{v'(e)}{p'(e)} \right] \right)
\]

This can be maximized with respect to \( e \) to find the optimal solution for the firm. Unfortunately, the first-order condition does not allow a closed-form expression for the optimal \( e \). Thus, to proceed further, I use the functional forms for \( p(\cdot) \), \( v(\cdot) \) and \( u(\cdot) \) specified in §2.2. Substituting these into (2.2)

\(^{12}\) Note that \( h(\cdot) \) is increasing and convex, since \( u(\cdot) \) is increasing and concave.

\(^{13}\) For brevity, I will occasionally use the term “payouts” to refer to \( u_L \) and \( u_H \) (the utilities provided by the monetary payouts).
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and (2.3) gives:

$$u_L(e) = \bar{U} - \beta[\exp(2\epsilon) + 1]$$  \hspace{1cm} (2.4)

$$u_H(e) = \bar{U} + 2\beta\exp(e)$$  \hspace{1cm} (2.5)

I then have:

$$E[\pi(e)] = \frac{\exp(e)}{1 + \exp(e)}[x_H - h(\bar{U} + 2\beta\exp(e))] + \frac{1}{1 + \exp(e)}[x_L - h(\bar{U} - \beta[\exp(2\epsilon) + 1])]$$  \hspace{1cm} for \( \epsilon \geq 0 \)

Maximizing this with respect to \( \epsilon \) to find the optimal effort for the firm to induce, there is a unique non-negative solution when \( \Delta x \) is sufficiently large (\( \Delta x \geq 8\beta[4\beta + \bar{U}] \)) (details in Appendix C):

$$e^* = \ln \left( \frac{\sqrt[3]{\beta} \sqrt{\frac{(2\beta - \bar{U})^2 + 3\Delta x + 2\beta - \bar{U}}}{\beta}} - 1 \right)$$  \hspace{1cm} (2.6)

When \( \Delta x \) is not sufficiently large, the firm prefers to let the salesperson exit, since the difference in sales that can be achieved through his effort is not sufficient to justify his cost.

Substituting (2.6) back into (2.4) and (2.5) gives the firm’s optimal compensation plan for the salesperson:

$$u^*_L = 2\sqrt[3]{\beta} \sqrt{\frac{(2\beta - \bar{U})^2 + 3\Delta x + 2\beta - \bar{U}}{3}} - \frac{1}{3} \sqrt{(2\beta - \bar{U})^2 + 3\Delta x} - \frac{4}{3}(2\beta - \bar{U})$$  \hspace{1cm} (2.7)

$$u^*_H = 2\sqrt[3]{\beta} \sqrt{\frac{(2\beta - \bar{U})^2 + 3\Delta x + 2\beta - \bar{U} - (2\beta - \bar{U})}{\beta}}$$  \hspace{1cm} (2.8)

These expressions yield comparative statics that are largely intuitive. As the difference between the high and low sales outcomes (\( \Delta x \)) increases, with the other parameters remaining unchanged, the salesperson’s effort is worth more (in expectation), so the firm prefers to elicit more effort (i.e. \( e^* \) increases) by increasing the difference between the high and low payouts. Thus, \( u^*_H \) increases and \( u^*_L \) decreases with \( \Delta x \). As the salesperson’s outside option (\( \bar{U} \)) increases, the firm must offer more to satisfy the IR constraint, so \( u^*_H \) and \( u^*_L \) both increase, although \( e^* \) decreases. Finally, as \( \beta \) increases, effort is more costly to the salesperson, so the firm must offer more incentives to induce effort and to satisfy the IR constraint. Thus, \( e^* \) again decreases while \( u^*_H \) increases with \( \beta \). The behavior of \( u^*_L \) is less obvious, increasing with \( \beta \) if \( \Delta x \) is large, but decreasing if \( \Delta x \) is small.

2.3.2 Asymmetric information

In this section, I consider the case in which the manager has information about the salesperson’s territory that the firm does not have. This is likely to be more realistic in many selling situations than the symmetric information case. As stated by Chow et al. (1988), “In many organizational settings, a subordinate has more accurate information than his or her superior about factors influencing performance.” In the sales force compensation context represented in my model, it is likely that a sales manager has better information about the territories for which she is responsible than would her superiors.

I model this information asymmetry by assuming that there are two types of territories, defined by the difficulty of the selling environment and represented in the model by different levels of disutility incurred...
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by the salesperson due to selling effort, $\beta_L < \beta_H$. The manager knows which type the salesperson’s territory is, but the firm knows only the distribution of types:

$$\gamma_L \equiv \Pr(\beta = \beta_L) \quad \text{and} \quad \gamma_H \equiv 1 - \gamma_L = \Pr(\beta = \beta_H)$$

where $0 < \gamma_L, \gamma_H < 1$.

I restrict attention to the case in which employing a salesperson in either type of territory is preferred to not employing a salesperson at all. In other words, if the territory type were known, the firm would always choose to offer the salesperson a contract (designed as in the benchmark model) over letting him exit the firm.

Throughout the remainder of the chapter, I use the following notation:

$\sigma_L = (u_{\sigma L L}, u_{\sigma L H})$ and $\sigma_H = (u_{\sigma H L}, u_{\sigma H H})$ represent the firm’s optimal compensation plans (as determined using the benchmark model) when the salesperson’s territory type is $\beta_L$ and $\beta_H$, respectively.

$e_L$ ($e_H$) represents the effort level chosen by the salesperson with true territory type $\beta_L$ ($\beta_H$) under the optimally-designed compensation plan $\sigma_L$ ($\sigma_H$).

Constrained delegation

In this model, the firm delegates responsibility for designing the salesperson’s compensation plan (i.e., choosing $s_L, s_H$) to the manager, but within a budget constraint imposed by the firm.

To illustrate the necessity of the budget constraint, I first consider a version of the model without one. This is nearly the same as the benchmark model, with the manager needing to offer a plan that satisfies both the IC and IR constraints in order to induce effort $e$ from the salesperson. However, the manager’s objective function is different from the firm’s, in that the firm seeks to maximize profit while the manager seeks to maximize her own payout. This difference is critical, as it implies that the manager does not share the firm’s desire to minimize the expected payout to the salesperson, which is a key driver of the benchmark model solution.

The manager’s problem is:

$$\max_{(s_L, s_H)} E[s^M(x)] = p(e)s_H^M + [1 - p(e)]s_L^M$$

s. t. $E(U|e) \geq \bar{U}$ \hspace{1cm} (IR)

$$\frac{v'(e)}{p'(e)} = \Delta u$$ \hspace{1cm} (IC)

Since $s_H^M > s_L^M$, the manager’s objective here is equivalent to maximizing $p(e)$, or simply maximizing $e$ (since $p$ is strictly increasing). In other words, the manager seeks to induce the highest possible effort from the salesperson, because more selling effort leads to a greater probability of the high sales outcome, which results in a larger expected payout for the manager (as well as the salesperson).

I can use the notation introduced in §2.3.1 to transform the problem, re-writing it as:

$$\max_{(u_L, u_H)} e$$

s. t. $u_L + p(e)\Delta u - v(e) \geq \bar{U}$ \hspace{1cm} (IR)

$$\frac{v'(e)}{p'(e)} = \Delta u$$ \hspace{1cm} (IC)
Lemma 1. In the absence of a budget constraint, a self-interested manager always designs a compensation plan with the largest possible difference between the high and low payouts (i.e., the largest $\Delta u$).

Proof. This follows directly from the model formulation above. Since she is only concerned with maximizing $e$ and not with controlling the expected payout, the manager can choose $u_L$ sufficiently large to ensure that IR is satisfied for any $\Delta u$. Since $v'(e)$ is strictly increasing in $e$, the manager prefers to maximize $\Delta u$. □

This has obvious implications about what will happen if the firm does not impose a budget constraint.

Lemma 2. In the absence of a budget constraint, a self-interested manager always offers the salesperson a contract that is suboptimal for the firm. In particular, the high-sales payout offered will be too high and the low-sales payout too low.

In fact, Lemma 1 suggests that the manager would prefer to offer an infinitely high payout for the high sales outcome and an infinitely low payout for the low sales outcome, to achieve the ‘largest possible difference’ between the two. Thus, one can conclude that delegating the design of the salesperson’s compensation plan to a self-interested manager is not only suboptimal, but is effectively infeasible without a budget constraint.

The budget constraint that a firm imposes on a compensation plan can take many different forms, depending on the situation and on the degree and type of control that the firm seeks (and is able) to exert. The simplest type of constraint is a ‘hard cap’, or maximum value imposed on the total payout. In practice, caps are also imposed on metrics like the ratio of incentive pay to base salary, the ratio of incentive pay to a given target payout, and incentive pay per transaction or per account (Zoltners, Sinha & Lorimer, 2006). Firms also use a number of other, more-complex types of constraints. These include ‘soft cap’ constraints, such as a total payout pool that expands or contracts based on team performance at the regional or national level, and portfolio-level constraints, such as a restriction on the incentives paid for one product’s sales unless a minimum sales target is achieved for another product.

With the objective of identifying a profit-maximizing approach, I restrict my focus to budget constraints that allow the firm to achieve its optimal result (as determined in the benchmark model), at least when the salesperson’s territory type is known. In my stylized model, with its two-point distribution of sales outcomes, this can be achieved with a hard cap on the total payout (i.e., a limit on the value of $s_H$). However, that result does not generalize to a model with a greater number, or a continuum, of sales outcomes. Therefore, I model a slightly more complex form of budget constraint. Specifically, the manager is required to choose a compensation plan that is optimal for the firm for some existing territory type $\beta \leq \beta_{\text{max}}$, where $\beta_{\text{max}}$ is set by the firm. The requirement that the plan be optimal for some $\beta$ is intuitive, in the sense that choosing otherwise would be a transparent betrayal of the firm’s trust on the part of the manager. The imposition of $\beta_{\text{max}}$ is necessary and sufficient for the firm to induce its optimal result when the true territory type is known, as shown below.

In order to understand the implications of this type of budget constraint, I must first determine what happens when the compensation plan is optimized for the wrong territory type.

Lemma 3. If the salesperson’s compensation plan is optimally designed for a territory with type (\(\beta\)) lower than the true type, then the salesperson declines the contract and exits the firm.

If the plan is optimally designed for a territory with type higher than the true type, then the salesperson accepts the contract and receives a larger expected payout and more expected utility than if the plan were
optimized for the true type. Furthermore, he exerts more effort than if the plan were optimized for the true type and that effort increases in the type for which the plan is optimized.\textsuperscript{14}

A detailed proof can be found in Appendix D. Most of the claims in Lemma 3 follow from the fact that when the compensation plan is optimally designed for the correct territory type, the salesperson’s IR constraint binds (i.e., his expected utility just matches that provided by his outside option). For example, the intuition behind the first claim is as follows: If the plan is optimized for a low-cost territory and the territory is truly low-cost, then the salesperson is offered just enough expected utility (at his optimal effort) to accept the contract. If his cost of effort were any higher, then, he would prefer to decline the contract and exit. Conversely, if the plan is optimized for a high-cost territory, then the IR constraint must bind for such a territory. Lowering the salesperson’s cost of effort, then, relaxes that constraint, leads him to exert more effort, and increases his expected payout and utility.

**Proposition 1.** Under the given model specification, a self-interested manager always designs the salesperson’s compensation plan to be optimal for the highest possible territory type (i.e., the highest existing $\beta \leq \beta_{\text{max}}$). Furthermore, the manager always prefers that the firm set $\beta_{\text{max}}$ greater than or equal to the highest existing type.\textsuperscript{15}

*Proof.* This follows from Lemmas 1 and 3. Lemma 1 implies that the manager always prefers a plan that induces the highest effort from the salesperson. Lemma 3 indicates that the salesperson’s effort is maximized (within the budget constraint) by optimizing the plan for the highest possible territory type. Therefore, the manager prefers to design the plan optimally for the highest type $\leq \beta_{\text{max}}$ and benefits from an increase in that constraint up to the highest type. \hfill $\square$

It follows from Proposition 1 that if the firm and the manager have symmetric information, then the firm can induce the manager to choose the firm’s optimal compensation plan (and the salesperson to choose the firm’s optimal effort) by setting the budget constraint $\beta_{\text{max}}$ equal to the salesperson’s territory type. In that case, any attempt by the manager to lobby for a higher budget constraint would be non-credible and ineffective. This is consistent with the findings in the accounting literature on budgetary slack that when information asymmetry is low (and budget emphasis in evaluation is high), managers are not in a position to create slack.

**Full participation**

As shown in the previous section, a manager can always benefit from a higher budget constraint ($\beta_{\text{max}}$) for the salesperson’s compensation plan. It is safe to assume, then, that given an opportunity to participate in setting the budget, the self-interested manager will claim that the salesperson’s territory is the high-difficulty type, and that $\beta_{\text{max}}$ should therefore be equal to $\beta_H$, regardless of the true territory type. Of course, the firm is aware of this, so such claims (and, therefore, the manager’s participation) will rightfully be viewed with skepticism.

\textsuperscript{14}These results regarding effort and expected payout hold for all parameter values under the given model specification, but do not necessarily generalize to the complete parameter space under all model specifications. This is discussed further below.

\textsuperscript{15}The remainder of §2.3 refers to the model specified in §2.2, but the results hold for any specification in which $\bar{e} > e_L$ (i.e., in which a salesperson in a low-difficulty territory exerts more effort when his compensation plan is optimized for a high-difficulty territory than when it is optimized for his territory’s true type). If this does not hold (i.e., if $\bar{e} < e_L$), then the budget constraint $\beta_{\text{max}} = \beta_H$ is sufficient to induce the manager to choose the firm’s optimal compensation plan, regardless of the true territory type. While this outcome may be interesting in its own right, it is not the main focus of this chapter.
In this section, I suggest a participation mechanism by which the firm can induce the manager to reveal her true private information, by eliminating her motivation to make false claims. This mechanism requires that the firm impose a cost to the manager for participating in the budget setting process, but allow her to choose whether to participate (and incur the cost) or not. For example, if participation is time-consuming and/or difficult, then one expects the manager to incur disutility from the time/effort required to participate, similar to that incurred by the salesperson for his selling effort. In that case, the manager’s expected benefit from participating (and claiming that the salesperson’s territory is the high-difficulty type) must be weighed against her cost of participating. By making participation sufficiently onerous, then, the firm (under certain conditions) can use the manager’s participation as a separating mechanism, such that she prefers to participate only when her high-type claim is truthful.

Similar to Simester and Zhang (2014), the firm may impose a cost of participation on the manager by requiring her to provide support for her claims about the territory type. It can then control that cost by varying the degree and type of evidence required (e.g., administrative paperwork, analyses, formal reports, presentations to senior management, etc.). Unlike Simester and Zhang (2014), I do not assume that the manager’s cost of participation depends on whether her claims are truthful. Instead, similar to Laux (2008), the separation in my mechanism depends on the manager gaining more from participating when her claim is truthful than she gains when it is false. However, an assumption like that used by Simester and Zhang (that lobbying falsely is more costly than lobbying truthfully) would be expected to strengthen my result by expanding the parameter space under which the participation mechanism is effective.

The firm’s budgeting process using this mechanism is implemented as follows: The firm begins by setting the budget constraint $\beta_{max} = \beta_L$ (the optimal constraint for a low-difficulty territory). The firm then sets a threshold for the manager’s effort in the budget setting process and raises the constraint to $\beta_H$ if and only if the manager’s effort meets that threshold. The mechanism is successful when the manager prefers to exceed the threshold if and only if the higher constraint is optimal for the firm (i.e. the territory is truly the high-difficulty type).

In order for participation to be feasible as a separating mechanism, it must be the case that the manager gains more from an increase in the budget constraint from $\beta_L$ to $\beta_H$ when $\beta_H$ is the optimal constraint for the firm than when $\beta_L$ is optimal. In other words, the manager must gain more from participating when her claim (that the territory is the high-difficulty type) is truthful than when it is untruthful. That allows the firm to set the manager’s cost of participation greater than her expected benefit when her claim is false, but less than when her claim is true. The manager’s rational response, then, is to participate only when the salesperson’s territory is truly the high-difficulty type, thereby revealing her private information.

At this point, I assume that the manager’s participation carries no cost to the firm (including no opportunity cost of the manager’s time), but that assumption will be relaxed in §2.3.3.

**Proposition 2.** The participation mechanism described above is feasible as long as the territory types are sufficiently similar. Furthermore, the feasibility condition is more restrictive (i.e., the types must be more similar) when the salesperson’s optimal effort in a high-difficulty territory ($e_H$) is low.

Using the functional forms introduced above, the mechanism is feasible as long as the following sep-
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20

A detailed proof can be found in Appendix E.

The intuition behind Proposition 2, which states that participation can be used as a separating mechanism unless the territory types are too different, may not be immediately clear. In fact, it may be counterintuitive, as it seems like separation should be easier when the types being separated are less similar. The result is driven by the fact that under the lower budget constraint ($\beta_{\text{max}} = \beta_L$), a salesperson in a high-difficulty territory will exit the firm.

When the territory is the high-difficulty type, the benefit to the manager of obtaining the higher budget ($\beta_{\text{max}} = \beta_H$) is given by her own incentive pay ($s^M_H - s^M_L$) multiplied by the difference in the likelihood of the high sales outcome due to the effort of a salesperson with the optimal compensation plan ($\sigma_H$), relative to having no salesperson at all. When the territory is the low-difficulty type, on the other hand, the benefit of obtaining the higher budget is the manager’s incentive pay multiplied by the difference in the likelihood of the high sales outcome due to the effort of a salesperson with plan $\sigma_H$ vs. plan $\sigma_L$. In other words, the manager gains more from participating truthfully than untruthfully (i.e., the mechanism is feasible) when:

\[
(s^M_H - s^M_L)[p(e_H) - p(0)] > (s^M_L - s^M_L)[p(\hat{e}) - p(e_L)]
\]

where $\hat{e}$ represents the effort exerted by a salesperson with territory type $\beta_L$ under compensation plan $\sigma_H$.

This simplifies to:

\[
p(e_H) - p(0) > p(\hat{e}) - p(e_L)
\] (2.10)

As illustrated in Figure 2.1, since the sales response relationship, $p(e)$, is concave, the benefit to the manager of an increase in the budget constraint when the territory is high-difficulty (represented by the vertical distance between the red dots) occurs on the steeper part of the sales response curve than the corresponding effort increase when the territory is low-difficulty (represented by the vertical distance between the green dots). Thus, in general, one might expect an increase in the budget to generate a larger increase in the manager’s expected income when the territory is the high-difficulty type than when it is the low-difficulty type, thereby satisfying the separating condition. This can fail to be true, however, if the difference between the two types is large (resulting in a large difference between $e_L$ and $\hat{e}$), but $e_H$ is small, indicating that the budget increase has little impact on expected sales in a high-difficulty territory.

To see how the result is driven by the potential exit of the salesperson in a high-difficulty territory, consider what happens if I hold $\beta_L$ constant and vary $\beta_H$. From Lemma 3, I have that $\hat{e}$, and therefore $p(\hat{e})$, increases with $\beta_H$. Since $\sigma_L$ and $e_L$ depend only on $\beta_L$, $p(e_L)$ is unaffected. Therefore, the manager’s benefit from participating when the true territory type is $\beta_L$ (represented by the right-hand side of inequality 2.10) increases with $\beta_H$. The left-hand side of inequality 2.10 represents the manager’s benefit from participating when the true type is $\beta_H$, but only the first term, $p(e_H)$, is affected by changes in $\beta_H$, because the salesperson exits when the budget constraint is low (i.e., when the manager does not participate). From §2.3.1, $e_H$ decreases with $\beta_H$, so the left-hand side decreases with $\beta_H$. Therefore,
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the left-hand side decreases and the right-hand side increases with $\beta_H$. (In Figure 2.1, this would be represented by $\tilde{e}$ shifting to the right and $e_H$ to the left, increasing the vertical distance between the green dots and decreasing that between the red dots.) Thus, the inequality is satisfied for $\beta_H$ close to $\beta_L$\(^{16}\), but may be violated as $\beta_H$ gets large\(^{17}\).

When the separating condition does hold, the firm implements the participation mechanism by requiring the manager’s effort to exceed a particular threshold before increasing the budget from $\beta_L$ to $\beta_H$. This threshold is then set such that the manager’s disutility from exerting the threshold effort is larger than the increase in utility she enjoys from the budget increase when the territory is low-difficulty, but smaller than that increase in utility when the territory is high-difficulty.\(^{18}\) As a result, participation increases the manager’s net utility if and only if the territory is high-difficulty (i.e., if and only if increasing the budget is profitable for the firm). The resulting budget constraint is always equal to the salesperson’s true territory type ($\beta$) and the compensation plan chosen by the manager is the same as the one designed by the firm in the benchmark model.

No involvement

When the firm does not know the salesperson’s territory type, its expected profit from setting compensation plan $\sigma$ is the average of its expected profits from each type, weighted by the type probabilities:

$$E[\pi(\sigma)] = \gamma_L E[\pi(\sigma)|\beta_L] + \gamma_H E[\pi(\sigma)|\beta_H] \tag{2.11}$$

It is worth noting that if a salesperson’s IR constraint can be satisfied when his territory is the high-difficulty type, then it can also be satisfied when his territory is low-difficulty. To see this, suppose the IR constraint is satisfied under plan $\sigma = (u_L, u_H)$ at effort level $e$ when the territory is of type $\beta_H$:

$$p(e)u_H + [1 - p(e)]u_L \geq \bar{U} + \beta_H v_0(e)$$

$$\therefore p(e)u_H + [1 - p(e)]u_L > \bar{U} + \beta_L v_0(e) \quad \text{because } \beta_H > \beta_L$$

\(^{16}\)As $\beta_H \to \beta_L$, $\tilde{e} \to e_L$, so $p(\tilde{e}) - p(e_L) \to 0$, but $p(e_H) - p(0) \to p(e_L) - p(0) > 0$, so inequality 2.10 is satisfied.

\(^{17}\)There is an effective upper bound on $\beta_H$, because I have restricted attention to cases in which employing a salesperson in either type of territory is preferred to not employing a salesperson at all.

\(^{18}\)Although the participation mechanism is feasible over a range of thresholds, one would expect the firm to choose a threshold at the bottom of that range, to avoid imposing any unnecessary effort costs on the manager.
Thus, the IR constraint is satisfied (with slack) at effort level $e$, and therefore at the salesperson's optimal effort level as well, when the territory type is $\beta_L$.

Given that the salesperson will exit if his IR constraint cannot be satisfied under a given plan, there are two sets of potential solutions to consider: 1) the set of plans that the salesperson will accept only if his territory is the low-difficulty type, and 2) the set of plans that he will accept regardless of type. (I am not interested in plans that the salesperson will always reject, for obvious reasons.)

If a plan is in the first of these sets, then the salesperson will reject it when his territory is the high-difficulty type. Thus, for any $\sigma$ in that set, the firm's expected profit is given by:

$$E[\pi(\sigma)] = \gamma_L E[\pi(\sigma)|\beta_L] + \gamma_H [x_L + p(0)(x_H - x_L)]$$

with the last term (multiplied by $\gamma_H$) representing the firm's expected profit if the salesperson exits. Since that term does not depend on $\sigma$, optimizing $E[\pi(\sigma)]$ is equivalent to optimizing $E[\pi(\sigma)|\beta_L]$. In other words, the firm should design the compensation plan as though the salesperson's territory type is known to be $\beta_L$, resulting in plan $\sigma_L$.

If a plan is in the second set, then the firm seeks to maximize the expected profit given in (2.11), subject to the IR and IC constraints for both territory types. Under the optimal solution, the IR constraint for the high-difficulty territory type (IR$_H$) binds. To see this, suppose instead that it holds with slack under optimal solution $\sigma = (u_L, u_H)$. As shown above, then, the IR constraint for the low-difficulty type (IR$_L$) must also hold with slack under $\sigma$. However, the firm can then lower $u_L$ and $u_H$ incrementally and equally without violating either IR constraint, generating the same effort and expected sales with lower compensation payouts and thereby increasing its expected profit. This contradicts the optimality of $\sigma$, implying that (IR$_H$) must bind.

Thus, the optimal plan that the salesperson will accept regardless of his territory type, which I will call $\sigma^*$, is the solution to the following problem:

$$\max_{\sigma = (u_L, u_H)} \gamma_L E[\pi(\sigma)|\beta_L] + \gamma_H E[\pi(\sigma)|\beta_H]$$

s.t. $p(e_H)u_H + [1 - p(e_H)]u_L = U + \beta_H v_0(e_H)$ (IR$_H$)

$$\frac{\beta_L v'_0(e_L)}{p'(e_L)} = \Delta u$$

(\text{IC}_L)

$$\frac{\beta_H v'_0(e_H)}{p'(e_H)} = \Delta u$$

(\text{IC}_H)

where $e_L$ ($e_H$) represents the salesperson's effort if the territory is of type $\beta_L$ ($\beta_H$).

With a number of substitutions (see Appendix F), and the additional notation $z(e) = \frac{v'_0(e)}{p'(e)}$, this can be re-written as the following problem in terms of $e_H$:

$$\max_{e_H} x_L - h(\bar{U} + \beta_H v_0(e_H) - p(e_H)\beta_H z(e_H)) + \left[p(e_H) + \gamma_L \left[p \left(\frac{z^{-1} \left[\beta_H z(e_H)\right]}{\beta_L} - p(e_H)\right)\right]\right]$$

$$\left(\Delta x - [h(\bar{U} + \beta_H v_0(e_H) + [1 - p(e_H)]\beta_H z(e_H)) - h(\bar{U} + \beta_H v_0(e_H) - p(e_H)\beta_H z(e_H))]\right)$$
With the functional forms for \( h(\cdot) \), \( v_0(\cdot) \) and \( p(\cdot) \) introduced above, this becomes (see Appendix F):

\[
\max_{e_H^*} x_L - (\bar{U} - \beta_H[\exp(2e_H^*) + 1])^2 + \frac{\exp(e_H^*) + \gamma_L}{\exp(e_H^*) + 1} \left( 1 - \sqrt{\frac{\beta_L}{\beta_H}} \right) \exp(e_H^*) + \exp(e_H^*) + 1 \\
(\Delta x - [\bar{U} + 2\beta_H \exp(e_H^*)]^2 - (\bar{U} - \beta_H[\exp(2e_H^*) + 1])^2)
\]

The solution to this problem (which I will call \( e_H^{**} \)) is in terms of the effort that the salesperson will exert if his territory is of type \( \beta_H \). This is always less than \( e_H \), the optimal effort that the firm induces when the salesperson’s territory type is known to be \( \beta_H \) (see Appendix D). The intuition behind this result is as follows: A plan that induces \( e_H^{**} = e_H \) with (IR\(_H\)) binding is exactly \( \sigma_H \), the benchmark model solution when the salesperson’s territory type is known to be \( \beta_H \). From Lemma 3, if the territory type is \( \beta_L \), then \( \sigma_H \) induces effort \( \hat{e} > e_L \), the optimal effort when the type is known to be \( \beta_L \). Put another way, if \( e_H^{**} = e_H \), then \( \Delta u \) is optimal if the territory is high-difficulty, but is larger than optimal if the territory is low-difficulty. Since \( \gamma_L > 0 \) (i.e. there is a non-zero probability that the territory is low-difficulty), the firm’s expected profit can be increased by reducing \( e_H \) incrementally below \( e_H^{**} \), lowering the expected profit slightly if \( \beta = \beta_H \), but raising it if \( \beta = \beta_L \). However, it is clear that as \( \gamma_L \) approaches 0 (i.e., it is nearly certain that the territory is high-difficulty), the solution \( e_H^{**} \) approaches \( e_H \) and the firm’s optimal plan that ensures the salesperson’s acceptance (\( \sigma^{*} \)) approaches \( \sigma_H \). As \( \gamma_L \) approaches 1, on the other hand, the solution cannot approach \( \sigma_L \), because then the salesperson would exit if his territory is high-difficulty (shown in Lemma 3).

To determine the firm’s overall optimal compensation plan given \( \gamma_L \), I compare the optimal plans from each of the two potential solution sets, \( \sigma_L \) and \( \sigma^{*} \), as shown in Figure 2.2. For very small \( \gamma_L \) (approaching 0), the expected profits from these plans approach \( E[\pi(\sigma_L)|\beta_H] \) and \( E[\pi(\sigma_H)|\beta_H] \), respectively. Since \( \sigma_H \) is the firm’s optimal plan for territory type \( \beta_H \), \( \sigma^{*} \rightarrow \sigma_H \) is clearly optimal. Conversely, for very large \( \gamma_L \) (approaching 1), the expected profits approach \( E[\pi(\sigma_L)|\beta_L] \) and \( E[\pi(\sigma^{*})|\beta_L] \), respectively. Since \( \sigma^{*} \) cannot approach \( \sigma_L \), \( \sigma_L \) is clearly optimal in this case.

![Figure 2.2: Comparison of firm’s potential solutions without manager involvement](image)

With the difference \( E[\pi(\sigma_L)] - E[\pi(\sigma^{*})] \) increasing from negative when \( \gamma_L \) is very small to positive when \( \gamma_L \) is very large, I can identify the threshold value \( \gamma_L^0 \) at which \( E[\pi(\sigma_L)] = E[\pi(\sigma^{*})] \), indicating...
that the firm is indifferent between the two potential solutions.

\[
E[\pi(\sigma_L)] = E[\pi(\sigma^*)]
\]

\[
\Leftrightarrow \gamma^0_L E[\pi(\sigma_L)|\beta_L] + (1 - \gamma^0_L) E[\pi(\sigma_L)|\beta_H] = \gamma^0_L E[\pi(\sigma^*)|\beta_L] + (1 - \gamma^0_L) E[\pi(\sigma^*)|\beta_H]
\]

\[
\Leftrightarrow \gamma^0_L (E[\pi(\sigma^*)|\beta_L] - E[\pi(\sigma_L)|\beta_L]) = (1 - \gamma^0_L) (E[\pi(\sigma_L)|\beta_H] - E[\pi(\sigma^*)|\beta_H])
\]

Equation 2.12 gives the following expression for \(\gamma^0_L\):

\[
\gamma^0_L = \frac{E[\pi(\sigma^*)|\beta_H] - E[\pi(\sigma_L)|\beta_H]}{E[\pi(\sigma^*)|\beta_H] - E[\pi(\sigma_L)|\beta_H] - (E[\pi(\sigma^*)|\beta_L] - E[\pi(\sigma_L)|\beta_L])}
\]

When \(\gamma_L > \gamma^0_L\), the cost of choosing the ‘incorrect’ plan for a low-difficulty territory is weighted more heavily, so the firm’s optimal plan is \(\sigma_L\). Conversely, when \(\gamma_L < \gamma^0_L\), the cost of choosing incorrectly for a high-difficulty territory is weighted more heavily, so the firm’s optimal plan is \(\sigma^*\). Alternatively, one can think of this in the context of a sales force with many territories. When \(\gamma_L\) is large, there are many low-difficulty territories, so the firm chooses the plan \((\sigma_L)\) that is profit-maximizing for them, at the cost of losing the few salespeople facing high difficulty. However, as \(\gamma_L\) decreases, the proportion of high-difficulty territories grows until the firm chooses to compromise with the best possible plan \((\sigma^*)\) that is sufficient to retain all of its salespeople.

2.3.3 Optimal budget setting approach

Participation

By combining the analyses above, I can determine the value of allowing the sales manager to participate in designing the salesperson’s compensation plan and setting the compensation budget constraint.\(^{19}\) It is critical, however, to allow for the likelihood that such participation carries not only an effort cost for the manager, but an opportunity cost for the firm as well. After all, time that the manager spends on the budgeting process could potentially be spent on other productive activities. If participation in budget setting is considered to be part of the manager’s job, then either it displaces some of her other responsibilities or the firm must be prepared to compensate her for the expected effort it will require.

It seems natural to assume that this participation cost to the firm increases with the effort exerted by the manager. In considering the participation mechanism proposed in §2.3.2, however, the manager’s effort takes only two possible values: 0 or the threshold effort determined by the firm. Since there is no cost to the firm if the manager does not participate, I need only be concerned with the cost of the manager exerting the threshold effort. I will denote this cost to the firm by \(c_{\text{part}}\).

\(^{19}\)This section focuses on cases in which the participation mechanism described in §2.3.2 is feasible.
From §2.3.2, the firm’s optimal expected profit when it does *not* allow the sales manager to participate in budget setting is:

\[
E[\pi] = \begin{cases} 
\gamma_L E[\pi(\sigma_L)|\beta_L] + (1 - \gamma_L)[x_L + p(0)\Delta x] & \text{if } \gamma_L > \gamma_L^0 \\
\gamma_L E[\pi(\sigma^*)|\beta_L] + (1 - \gamma_L)E[\pi(\sigma^*)|\beta_H] & \text{if } \gamma_L < \gamma_L^0
\end{cases} \quad (2.14)
\]

If the firm *does* allow the manager to participate, using the proposed mechanism to reveal her private information, then the resulting plan is always optimal for the true territory type and the expected profit is given by:

\[
E[\pi] = \gamma_L E[\pi(\sigma_L)|\beta_L] + (1 - \gamma_L)(E[\pi(\sigma_H)|\beta_H] - c_{\text{part}}) \quad (2.16)
\]

Comparing 2.16 with 2.14 and 2.15, the firm should allow the manager to participate in the budgeting process if the cost of the manager’s participation is sufficiently low. Specifically, the condition for managerial participation is:

\[
c_{\text{part}} < \begin{cases} 
E[\pi(\sigma_H)|\beta_H] - [x_L + p(0)\Delta x] & \text{if } \gamma_L > \gamma_L^0 \\
\frac{\gamma_L}{(1 - \gamma_L)}E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L] + E[\pi(\sigma_H) - \pi(\sigma^*)|\beta_H] & \text{if } \gamma_L < \gamma_L^0
\end{cases} \quad (2.17)
\]

\[
\frac{\gamma_L^0}{1 - \gamma_L^0} = E[\pi(\sigma^*)|\beta_H] - [x_L + p(0)\Delta x] \\
E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L]
\]

Figure 2.3: Comparison of firm’s expected profits with and without manager participation (\(\gamma_L < \gamma_L^0\))

Figure 2.3 illustrates the firm’s expected profits with and without the manager’s participation, not including the participation cost. With participation, the expected profit is the average of the optimal expected profits for the two types, weighted by the probability of each type, as represented by the dashed line. The shaded area highlights the growing benefit of the manager’s participation as \(\gamma_L\) increases from 0 to \(\gamma_L^0\). If \(\gamma_L\) is very small, then this benefit is very small, as is the right-hand side of condition 2.18 (because \(\frac{\gamma_L}{(1 - \gamma_L)}\) is small and \(\sigma^*\) approaches \(\sigma_H\) as \(\gamma_L\) approaches 0, from §2.3.2). Therefore, the firm allows the manager to participate only if its cost when she does so is very low. This is intuitive, because when \(\gamma_L\) is small, the firm has less need for the manager’s private information and is more likely to incur the participation cost.

As \(\gamma_L\) increases, approaching \(\gamma_L^0\) from below, the benefit of the manager’s participation grows. From equation 2.13,
Substituting this into condition 2.18, it becomes clear that the right-hand side of condition 2.18 approaches that of condition 2.17 from below, so there is no discontinuity in the condition for managerial participation as $\gamma_L$ crosses the threshold $\gamma^0_L$.

If $\gamma_L > \gamma^0_L$ and the true territory type is $\beta_L$, then the plan that the firm sets with no involvement from the manager ($\sigma_L$) is the same as the plan that the manager sets (by choosing not to participate in budget setting). Therefore, allowing the manager to participate only makes a difference in the case that the territory type is $\beta_H$. As shown in inequality 2.17, then, the firm allows the manager to participate in budget setting if its cost when she participates is less than the incremental expected profit generated by an optimally-compensated salesperson in a high-difficulty territory, relative to having no salesperson at all. This difference is illustrated in Figure 2.4. Again, one can consider these results in the context of a sales force with many territories. When $\gamma_L$ is very small, most territories are high-difficulty and the manager’s involvement has minimal impact on the final outcome, relative to when she is not involved. Thus, managerial participation occurs frequently and offers little return, so the firm will tolerate only a small participation cost. As $\gamma_L$ increases, managerial participation occurs in fewer territories and has a larger impact when it does, so the firm is willing to tolerate an increasing participation cost until $\gamma_L$ reaches $\gamma^0_L$. Beyond that point, the outcome in low-difficulty territories is the same whether managers are invited to participate in budget setting or not. Therefore, the firm’s decision is determined by the value of the manager’s participation in high-difficulty territories vs. the participation cost, despite the fact that the number of high-difficulty territories is decreasing in $\gamma_L$.

Figure 2.4: Comparison of firm’s expected profits with and without manager participation ($\gamma_L > \gamma^0_L$)

Information acquisition

The primary reason for involving the manager in budget setting is to gain access to her private information. In some cases, however, it may be possible for the firm to acquire that information in other ways. In the context of my model, for example, information about the difficulty of the salesperson’s territory may be obtainable through analysis of historical sales and customer data or observation by someone other than the sales manager. Of course, any such approach carries a cost for the firm that could vary due to a number of factors, including data quality, territory location, organizational structure and firm culture.

In this section, I consider when the firm might prefer to acquire the territory type information without
the manager’s involvement\textsuperscript{20}. The firm’s expected profit by doing so is:

\[ E[\pi] = \gamma_L E[\pi(\sigma_L)|\beta_L] + (1 - \gamma_L)E[\pi(\sigma_H)|\beta_H] - c_{\text{info}} \]  

\text{(2.19)}

where \( c_{\text{info}} \) denotes the cost to the firm of acquiring the information.

Note that this is similar to the expression for the firm’s expected profit with managerial participation (equation 2.16), because the firm effectively ends up with the same information in both cases. The difference here is that the cost of information acquisition is always incurred, whereas the participation cost above is incurred only when the salesperson is in a high-difficulty territory. Thus, comparing these two outcomes, the firm prefers to acquire the information on its own if and only if:

\[ \frac{1}{1 - \gamma_L} c_{\text{info}} < c_{\text{part}} \]

Similarly, I can compare to the firm’s expected outcome without acquiring the territory type information by any means (equations 2.14 and 2.15). Here I find that the firm prefers to acquire the information (without the manager) if and only if:

\[ \frac{1}{1 - \gamma_L} c_{\text{info}} < \begin{cases} 
E[\pi(\sigma_H)|\beta_H] - [x_L + p(0)\Delta x] & \text{if } \gamma_L > \gamma_L^0 \\
\frac{\gamma_L}{(1 - \gamma_L)} E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L] + E[\pi(\sigma_H) - \pi(\sigma^*)|\beta_H] & \text{if } \gamma_L < \gamma_L^0 
\end{cases} \]  

\text{(2.20)} \quad \text{(2.21)}

These are very similar to the results above concerning the cost of participation. When \( \gamma_L \) is very small, the firm prefers to acquire information about the salesperson’s territory type without the manager only if the cost of doing so is very low. As \( \gamma_L \) increases, condition 2.21 approaches condition 2.20. In this case, however, the left-hand side of the conditions increases, indicating that the firm’s tolerance of the information acquisition cost decreases with \( \gamma_L \). The intuition here is that the territory type information has diminishing impact on the firm’s expected profit as \( \gamma_L \) increases, but the firm always incurs the cost of acquiring it (unlike the cost of the manager’s participation in budget setting).

Summary

I have considered three possible compensation plan design approaches for the firm:

1. Delegate plan design to the manager and allow her to participate in setting the budget constraint (using the mechanism described in \S 2.3.2)
2. Delegate plan design to the manager, set the budget constraint using territory type information acquired without the manager’s involvement
3. Design the plan without the manager’s involvement and without territory type information

Combining the expressions above, the optimal approach from among these is determined by:

\[ \begin{cases} 
\min \left( c_{\text{part}}, \frac{1}{1 - \gamma_L} c_{\text{info}}, E[\pi(\sigma_H)|\beta_H] - [x_L + p(0)\Delta x] \right) & \text{if } \gamma_L > \gamma_L^0 \\
\min \left( c_{\text{part}}, \frac{1}{1 - \gamma_L} c_{\text{info}}, \frac{\gamma_L}{(1 - \gamma_L)} E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L] + E[\pi(\sigma_H) - \pi(\sigma^*)|\beta_H] \right) & \text{if } \gamma_L < \gamma_L^0 
\end{cases} \]

with the alignment between the solution and the firm’s corresponding action shown in Table 2.2.

\textsuperscript{20}I consider only the case in which the firm fully acquires the information, resulting in certainty about the salesperson’s type.
Chapter 2. Should sales managers participate in budget setting?

<table>
<thead>
<tr>
<th>Minimum term</th>
<th>Firm’s optimal action</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{\text{part}}$</td>
<td>Delegation &amp; participation</td>
</tr>
<tr>
<td>$\frac{1-\gamma_L}{\gamma_L} c_{\text{info}}$</td>
<td>Delegation &amp; acquisition</td>
</tr>
<tr>
<td>$E[\pi(\sigma_H)</td>
<td>\beta_H] - [x_L + p(0)\Delta x]$</td>
</tr>
</tbody>
</table>

Table 2.2: Firm’s optimal approach

From these findings, it is now evident why some firms would continue to allow sales managers to participate in budget setting, even when doing so introduces an obvious conflict of interest. Furthermore, one can see that a significant investment is required to budget optimally, regardless of which of the above approaches is chosen. If a firm chooses to acquire information about its local selling environments directly (without managerial involvement), the resources required to do so would likely be substantial in all but the smallest firms. If a firm chooses to proceed without such information, even in my simplified setting with only two territory types, a number of parameter values must be determined (or estimated) in order to set the optimal budget, including the probability and cost of effort associated with each type. Lastly, if managerial participation in budget setting is permitted, the firm still needs to determine the optimal budget for each type of territory in order to use the proposed mechanism.

2.4 Model generalizations

In this section, I examine the robustness of the key findings presented above by relaxing two of the simplifying model assumptions. First, I model a continuum of sales outcomes. Then, I consider the impact of introducing a third territory type (medium-difficulty), represented by $\beta_M \in (\beta_L, \beta_H)$.

2.4.1 Continuous sales

In the main model used above, sales were either ‘high’ or ‘low’. In practice, however, the possible sales outcomes are generally expected to be nearly continuous over some interval. In this section, I allow sales to take any value $x$ on the interval $[x_L, x_H]$, with finite, non-negative endpoints. Given the salesperson’s effort $e$, sales follow the p.d.f. $f_e(x)$ and c.d.f. $F_e(x)$, where:

- $f_e(x) > 0 \ \forall \ x \in [x_L, x_H]$ for any $e \in \mathbb{R}_+$ (i.e., sales can take any value at any effort level, so effort cannot be inferred from any outcome);
- For any $e_1 < e_2$, $F_{e_1}(x) \geq F_{e_2}(x) \ \forall \ x \in [x_L, x_H]$ and $F_{e_1}(x) > F_{e_2}(x)$ for some $x \in [x_L, x_H]$ (i.e., higher effort first-order stochastically dominates lower effort).

For any $x \in [x_L, x_H]$, the salesperson’s compensation plan pays him $s(x)$ and the manager’s plan pays her $s^M(x)$, with both plans strictly increasing in $x$.

The remaining notation and assumptions are as in the main model. I will summarize the key results here, with the complete analysis in Appendix G.

The benchmark, in which the firm knows the salesperson’s territory type and designs his compensation plan, is now essentially the model introduced in Basu et al. (1985).

Beginning with the Constrained Delegation model, Lemmas 1 and 2 extend with minor adjustments to the continuous sales environment, and are combined in the following:
Lemma 4. In the absence of a budget constraint, a self-interested manager always designs a compensation plan that induces the highest possible effort from the salesperson. As a result, she always offers the salesperson a contract that is suboptimal for the firm.

In general, the manager will offer larger payouts for high sales outcomes and smaller payouts for low sales outcomes than are optimal for the firm. The extension of the reasoning here is quite straightforward from binary to continuous sales.

Next, I consider budget constraints of the same form imposed in the main model (i.e., the manager is required to choose a compensation plan that is optimal for the firm for some existing territory type \( \beta \leq \beta_{\text{max}} \)).

As in the main model, the remaining results hold under the condition that \( \tilde{e} > e_L \) (i.e., a salesperson in a low-difficulty territory will exert more effort under compensation plan \( \sigma_H \), optimally designed for a high-difficulty territory, than under \( \sigma_L \)). Under that condition, Lemma 3 continues to hold as stated in §2.3.2 when sales are continuous. Again, the proof is a straightforward extension from the main model.

Similarly, Proposition 1, stating that the budget constraint always binds when the manager designs the salesperson’s compensation plan, and that she always prefers the highest possible budget (i.e., \( \beta_{\text{max}} = \beta_H \)) continues to follow from Lemmas 3 and 4.

Moving on to the Full Participation model, perhaps most importantly, I find that the participation mechanism continues to be feasible under conditions similar to those in the main model. Again, the intuition is similar when sales are continuous.

Proposition 3. The participation mechanism described in Section 2.3.2 is feasible as long as the territory types are sufficiently similar. Furthermore, the feasibility condition is more restrictive (i.e., the types must be more similar) when the salesperson’s optimal effort in a high-difficulty territory \( e_H \) is low.

Lastly, the firm’s optimal compensation plan design without the manager’s involvement takes a similar form to that in the main model. There exists a similarly defined \( \gamma^0_L \) such that the firm’s optimal plan is \( \sigma_L \) when \( \gamma_L > \gamma^0_L \) and \( \sigma' \) when \( \gamma_L < \gamma^0_L \), with the salesperson accepting \( \sigma' \) (and the IR constraint binding) when the territory type is \( \beta_H \). As a result, the firm’s optimal budget setting approach as outlined in §2.3.3 extends naturally to the continuous sales model.

2.4.2 3 territory types

In this section, I show how the feasibility condition for the participation mechanism extends to an environment with three territory types: \( \beta_L < \beta_M < \beta_H \). While partial revelation might be possible (e.g., narrowing down to two of the three types), I will consider the mechanism to be feasible only if it fully reveals the manager’s private information, exposing the territory’s true type with certainty.

I begin by supposing that the two-type feasibility condition given in Proposition 2 holds for \( \{\beta_L, \beta_H\} \) (i.e., the participation mechanism could be used if only those two types existed), as this is a necessary condition for three-type feasibility. Thus, inequality 2.9 holds:

\[
\frac{\beta_L}{\beta_H} > 1 - \frac{[\exp(e_H) + 1][\exp(e_L) - 1]}{2[\exp(e_L) + 1]}
\]

where \( e_i \) continues to represent the effort exerted by a salesperson in a \( \beta_i \)-type territory under the firm’s optimally-designed compensation plan for that type.
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This implies that the two-type condition also holds for \( \{\beta_L, \beta_M\} \), by the following:

\[
\sqrt{\frac{\beta_L}{\beta_M}} > \sqrt{\frac{\beta_L}{\beta_H}} > 1 - \frac{[\exp(e_H) + 1][\exp(e_L) - 1]}{2[\exp(e_L) + 1]} > 1 - \frac{[\exp(e_M) + 1][\exp(e_L) - 1]}{2[\exp(e_L) + 1]}
\]

because \( \beta_M < \beta_H \) implies that \( \exp(e_M) > \exp(e_H) \), as shown in §2.3.1.

For the mechanism to be feasible for three types requires participation cost thresholds \( \delta_{LM} \) and \( \delta_{LH} \) that the firm can impose for raising the budget constraint from \( \beta_L \) to \( \beta_M \) and from \( \beta_L \) to \( \beta_H \), respectively, such that each of those thresholds is sufficient to separate its respective types. For example, threshold \( \delta_{LM} \) must fall between the manager’s expected benefit from a budget increase from \( \beta_L \) to \( \beta_M \) when the true type is \( \beta_L \) and her expected benefit when the true type is \( \beta_M \). Furthermore, the difference between those two thresholds (\( \delta_{MH} = \delta_{LH} - \delta_{LM} \)) must be sufficient to separate types \( \beta_M \) and \( \beta_H \). If such thresholds can be found, then the firm can begin with the budget constraint \( \beta_{\max} = \beta_L \) and allow the manager to participate. If the manager’s effort exceeds \( \delta_{LH} \), then the firm raises the budget constraint to \( \beta_{\max} = \beta_H \). If the manager’s effort exceeds \( \delta_{LM} \), but not \( \delta_{LH} \), then the firm raises the constraint to \( \beta_{\max} = \beta_M \). Otherwise, the constraint remains at \( \beta_{\max} = \beta_L \). This can be thought of as the manager exerting effort \( \delta_{LM} \) to raise the budget to \( \beta_M \), then exerting the difference, \( \delta_{MH} \), to further raise it to \( \beta_H \).

For example, the ‘intermediate’ effort required for the first budget increase might include tasks such as gathering evidence of territory difficulty and presenting it in a meeting with an immediate superior, while a full increase to the maximum budget requires a formal report and presentation to senior management.

**Proposition 4.** The three-type participation mechanism described above is feasible unless \( \frac{\beta_M}{\beta_H} \) and/or \( \frac{\beta_L}{\beta_M} \) is sufficiently small. Furthermore, the feasibility condition is more restrictive (i.e., these ratios must be higher) when the salesperson’s optimal effort in a high-difficulty territory \( (e_H) \) is low.

Using the functional forms introduced earlier, the mechanism is feasible as long as the following separating condition is satisfied:

\[
\frac{1}{2} > \frac{1}{\exp(e_H) + 1} \left( 1 - \sqrt{\frac{\beta_M}{\beta_H}} \right) + \frac{1}{\exp(e_L) + 1} \left( 1 - \sqrt{\frac{\beta_L}{\beta_M}} \right) + \frac{1}{\exp(e_L) + 1}
\] (2.22)

A detailed proof can be found in Appendix H.

There are a few observations worth noting here. First, the two-type feasibility condition (inequality 2.9) can be rewritten as:

\[
\frac{1}{2} > \frac{1}{\exp(e_H) + 1} \left( 1 - \sqrt{\frac{\beta_L}{\beta_H}} \right) + \frac{1}{\exp(e_L) + 1}
\]

In this form, it is easy to see how the two-type condition extends quite naturally to three types. Furthermore, if \( \beta_L \) and \( \beta_H \) are held constant and \( \beta_M \) is inserted between them, it is straightforward to show that the three-type condition is stronger (i.e., more restrictive) than the two-type condition, with the difference between the two decreasing as \( \beta_M \) approaches either \( \beta_L \) or \( \beta_H \). In other words, the three-type condition is harder to satisfy when the types are more evenly distributed.
2.5 Conclusions

Nearly all firms invest substantial resources in their internal budget-setting processes. The involvement of managers in these processes can result in conflicts of interest, particularly when, as is often the case, managerial performance measurement includes metrics based on sales (or production) volume rather than firm profits. The accounting and management literatures contain numerous references to the issue of managers maneuvering to obtain maximum resources for their own areas or projects. The marketing literature, on the other hand, is largely silent on this issue, partly due to the common use of stylized models that abstract away from the use of budgets and/or the involvement of self-interested managers.

It appears that firms continue to allow managers to participate in budget setting, despite the apparent risks, in order to make use of the managers’ private information. However, the resulting conflict of interest challenges the credibility of the information provided. This chapter explores whether and how a firm can extract reliable information from a manager for the purpose of setting her budget and identifies the conditions under which it should do so.

I model the specific context of a sales manager designing an incentive compensation contract for her salesperson, with private knowledge about the degree of difficulty of the salesperson’s territory (i.e., the cost to the salesperson of selling in that environment). When the territory difficulty is high (low), the firm’s optimal compensation budget is less (more) restrictive.

Among my key findings are that the firm can induce the manager to reveal her information by setting a restrictive budget and making it costly for her to seek a larger one. This approach is effective when the manager has more to gain from a less-restrictive budget if the salesperson’s territory is a high-difficulty one than if it is a low-difficulty one. I find that, counterintuitively, this holds as long as the difference between the two territory types is not too large. However, this approach, even when feasible, is not always optimal. In some cases, the firm is better off designing the compensation plan without the manager’s involvement (when it is not cost-effective to acquire her information through other means). Specifically, the firm should allow the manager to participate in budget setting when: a) it is costly or impossible to obtain her information another way; b) the opportunity cost to the firm of her participation is low and/or unlikely to be incurred; and c) the profit that can be generated by a salesperson in the most difficult territory is substantially greater than the profit generated if he exits the firm.

While this chapter uses participation in budget setting, there may be other mechanisms by which the firm can extract the manager’s private information. For example, throughout this chapter, I assume that the manager’s compensation contract is exogenous to the problem of designing the budget and the salesperson’s contract. In the next chapter, however, I consider how the manager’s own incentive contract could be designed to induce information revelation without her participation in budget setting.

2.6 Table of notation

Notation used in this chapter is summarized below, for reference. Definitions refer to notation used in the main model unless otherwise noted.
### Chapter 2. Should sales managers participate in budget setting?

#### Notation Definition

<table>
<thead>
<tr>
<th>Notation</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_L, x_H$</td>
<td>Possible sales outcomes (or bounds of sales range in continuous sales model, §2.4.1)</td>
</tr>
<tr>
<td>$e$</td>
<td>Effort exerted by salesperson</td>
</tr>
<tr>
<td>$p(\cdot)$</td>
<td>Probability of high sales outcome</td>
</tr>
<tr>
<td>$s(\cdot)$</td>
<td>Salesperson’s compensation payout</td>
</tr>
<tr>
<td>$s_L, s_H$</td>
<td>Salesperson’s possible compensation payouts</td>
</tr>
<tr>
<td>$s^M(x)$</td>
<td>Manager’s compensation payout</td>
</tr>
<tr>
<td>$s^L_M, s^H_M$</td>
<td>Manager’s possible compensation payouts</td>
</tr>
<tr>
<td>$U$</td>
<td>Salesperson’s total utility</td>
</tr>
<tr>
<td>$\bar{U}$</td>
<td>Salesperson’s utility from outside option</td>
</tr>
<tr>
<td>$u(\cdot)$</td>
<td>Salesperson’s utility from income</td>
</tr>
<tr>
<td>$u_L, u_H$</td>
<td>Utility provided to salesperson by payout $s_L, s_H$, respectively</td>
</tr>
<tr>
<td>$\Delta u$</td>
<td>Difference between salesperson’s utilities from high and low payouts ($= u_H - u_L$)</td>
</tr>
<tr>
<td>$h(u)$</td>
<td>Inverse of $u(\cdot)$; Payout required to provide salesperson with utility $u$</td>
</tr>
<tr>
<td>$v(\cdot)$</td>
<td>Salesperson’s disutility from effort (i.e., effort cost)</td>
</tr>
<tr>
<td>$v_0(\cdot)$</td>
<td>Salesperson’s ‘base’ effort cost (multiplied by $\beta$ to get effort cost)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Difficulty of selling in salesperson’s territory (i.e., territory type)</td>
</tr>
<tr>
<td>$\beta_L, \beta_H$</td>
<td>Possible territory types (low- and high-difficulty, respectively)</td>
</tr>
<tr>
<td>$\beta_M$</td>
<td>Additional territory type introduced in 3-type model (§2.4.2)</td>
</tr>
<tr>
<td>$\pi(\cdot)$</td>
<td>Firm’s profit</td>
</tr>
<tr>
<td>$\gamma_L, \gamma_H$</td>
<td>Probability that territory type is $\beta_L, \beta_H$, respectively</td>
</tr>
<tr>
<td>$\sigma_L, \sigma_H$</td>
<td>Optimal compensation plans when territory type is $\beta_L, \beta_H$, respectively</td>
</tr>
<tr>
<td>$\sigma^*$</td>
<td>Optimal compensation plan that salesperson will accept regardless of territory type</td>
</tr>
<tr>
<td>$e_L$</td>
<td>Salesperson’s effort under $\sigma_L$ when territory type is $\beta_L$</td>
</tr>
<tr>
<td>$e_H$</td>
<td>Salesperson’s effort under $\sigma_H$ when territory type is $\beta_H$</td>
</tr>
<tr>
<td>$\check{e}$</td>
<td>Salesperson’s effort under $\sigma_H$ when territory type is $\beta_L$</td>
</tr>
<tr>
<td>$\gamma^0_L$</td>
<td>Value of $\gamma_L$ at which firm is indifferent between $\sigma_L$ and $\sigma^*$</td>
</tr>
<tr>
<td>$\beta_{\text{max}}$</td>
<td>Budget constraint set by firm (see §2.3.2 for details)</td>
</tr>
<tr>
<td>$z(e)$</td>
<td>Simplified notation for $v''_0(e)$</td>
</tr>
<tr>
<td>$c_{\text{part}}$</td>
<td>Opportunity cost to firm of manager participating in budget setting</td>
</tr>
<tr>
<td>$c_{\text{info}}$</td>
<td>Cost to firm of acquiring territory type information without manager</td>
</tr>
<tr>
<td>$f_e(x), F_e(x)$</td>
<td>p.d.f. and c.d.f. of sales distribution, given $e$, in continuous sales case (§2.4.1)</td>
</tr>
<tr>
<td>$\delta_{AB}$</td>
<td>Lobbying cost threshold imposed by firm for raising budget constraint from $\beta_A$ to $\beta_B$</td>
</tr>
</tbody>
</table>

Table 2.3: Summary of notation
Chapter 3

Using managerial compensation to ensure optimal sales force compensation

3.1 Introduction

In the previous chapter, I introduced the challenge faced by a firm when its sales managers have private information about the territories or salespeople they manage, and that information is relevant to the optimal design of the sales force compensation plan. In particular, when a sales manager has a clear preference for higher sales over lower sales, she may benefit from misrepresenting her private information. I then proposed a budget-setting approach that uses a manager’s participation to reveal her private information truthfully.

As pointed out in that chapter, however, the proposed approach is not always feasible. In particular, it can fail if the range of the manager’s private information (e.g., the difference between territories or salespeople) is too large. Moreover, even when feasible, inviting sales managers to participate in budget setting can be quite costly in some cases, due to the time required for that participation. This can be an opportunity cost, if budget setting displaces effort spent on other important duties, or a more direct cost if the manager simply absorbs the additional workload and the firm must compensate her accordingly.

In this chapter, then, I examine an alternative approach to the same fundamental challenge. Again, I consider a self-interested manager with sales-based incentives and private information about her salesperson’s territory. Instead of using the manager’s participation in budget setting to reveal her private information, however, here I determine whether the firm can achieve that end through the design of the manager’s own compensation plan. If so, then such an approach might be suitable when it is costly or infeasible to allow the manager to participate in budget setting, or when allowing such participation is undesirable for other reasons, such as organizational culture or philosophy.

As shown in the previous chapter, a self-interested sales manager generally prefers to offer larger incentives to her salespeople than is profit-maximizing for the firm. One way to counter this tendency, then, might be to structure the manager’s compensation to effectively reward her for offering smaller incentives when appropriate. In this chapter, I analyze two possible models for the manager’s compen-
Chapter 3. Using managerial compensation to ensure optimal sales force compensation

The first model uses an idea similar to that of Lal and Staelin (1986) and Rao (1990), who examine the use of a menu of salesperson compensation contracts by firms with heterogeneous sales forces in which each salesperson’s selling ability is his private information. In my model, the firm faces similar information asymmetry, but instead offers a menu of contracts to the sales manager, with each contract consisting of compensation terms for the manager combined with a budget constraint within which the manager must design the salesperson’s compensation. For example, the firm could allow the manager to choose between a contract that offers a larger budget for the salesperson’s incentives, and one that offers a larger base salary for herself. From such a menu, the manager will no longer necessarily choose the option that maximizes the salesperson’s incentive pay.

In my second model, the firm offers the manager a single compensation plan along with a budget constraint for the salesperson’s compensation, but the manager’s payouts now include a percentage of the unallocated budget, in addition to the amounts tied to sales performance. Again, such a contract could mitigate the manager’s tendency to maximize the salesperson’s incentive pay.

If sales managers are risk averse, as is commonly assumed for salespeople, then in the context of this analysis, it is optimal for the (risk-neutral) firm to assume as much of the financial risk as possible. As a result, the optimal contracts offered to the manager under both models include minimal incentive pay. If the firm offers a menu of contracts to the manager, then that menu should include one option for each territory/salesperson type. Using high and low territory difficulty again to represent the manager’s private information, the optimal menu consists of two options. In the first option, the budget constraint is optimized for a hard territory and the manager receives only a base salary that is just sufficient to prevent her from exiting the firm. In the second option, the budget constraint is optimized for an easy territory and the manager receives a slightly lower base salary, but non-zero incentive pay.

Under this solution, the manager’s expected payout is higher when the territory is easy, so the firm is paying her information rent to ensure truth-telling. Similar to the previous chapter, it is not necessarily the case that this information revelation is more profitable for the firm than simply designing the salesperson’s compensation directly, without the manager’s information. In fact, the conditions under which the menu of contracts is preferred take a similar form to the analogous conditions in the previous chapter, with the following key distinction: When offering the manager a menu of contracts, the firm pays information rent only when the territory is easy; when allowing the manager to participate in budget setting, the firm incurs a “participation cost” only when the territory is hard. This suggests that either approach can be optimal, depending not only on the relative magnitude of those respective costs, but also on the nature of the sales environment (particularly the proportions of easy and hard territories).

Finally, I show that offering the manager a single contract that includes a proportion of any unallocated budget is effectively a special case of the menu-of-contracts approach, in which the premium that the manager receives for revealing that the territory is easy is constrained to be equal across sales outcomes. Thus, this approach cannot improve upon the optimal solution offered by a menu.

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1I acknowledge that this particular finding is due, in large part, to the representation of the manager’s responsibilities in my models as being limited to the selection of her salespeople’s compensation plans. In further development of this research, I plan to extend the analysis to capture sales managers’ (often unobserved) efforts spent on activities that drive sales more directly (e.g., coaching, hiring, working with customers, etc.). This will not only represent the true range of a manager’s responsibilities more accurately, but will also reflect why firms typically offer substantial incentive pay to sales managers, as opposed to the minimal-incentive solution discussed here. With that said, I believe that the key qualitative findings from the current, simplified model will prove to be robust to such an extension.
This chapter offers guidance to firms on how and when a sales manager’s compensation plan can be used to reveal her private information and ensure optimal compensation design for her salespeople. In addition to these practical insights, it provides a meaningful theoretical contribution as well. Krafft et al. (2012) mention the need for more research attention on “the interplay between superiors and subordinates across sales force management layers in the context of compensation / control”. In fact, Rouziès et al. (2009) point out that “research on compensation of sales managers is almost nonexistent”. While the latter provides an empirical investigation of factors affecting compensation plan design for salespeople and sales managers, I offer a game-theoretic model that explores the interplay mentioned by Krafft et al. (2012). While this research is not intended as a complete investigation of sales manager compensation design, it offers substantive insights into the connection between the compensation plans of managers and salespeople.

3.1.1 Literature Review

Due to the similarity of the subject matter, this chapter relates and contributes to some of the same areas of the literature as mentioned in the previous chapter. For example, this chapter extends my contribution to the literature on hierarchies in organizations, offering an alternative mechanism (managerial compensation) by which delegation of decision-making can be made as efficient as centralization. In this case, similarly to Mookherjee & Reichelstein (1997), I explore the use of menus of incentive schemes, but while that paper considers a setting with deterministic production and managers with no private knowledge about their subordinates, my environment includes stochastic production (sales) and private information between each layer of the hierarchy.

While not believed to be particularly common in practice even today, reports of companies offering a menu of contracts to their salespeople appear as early as Wikstrom (1968), later followed by Gonik (1978). The latter includes an example, used by IBM Brazil, in which the menu consists of a range of sales forecasts, with each forecast implying a particular compensation plan. Under this approach, later analyzed in detail by Mantrala and Raman (1990), a salesperson chooses a forecast, thereby also choosing the associated plan. Lal and Staelin (1986) and Rao (1990) use the agency-theoretic framework to consider the problem of sales force compensation design under asymmetric information more generally, and characterize the optimal solution using a menu of contracts. More recently, Chen et al. (2013) offer a menu-of-contracts solution to the joint problem of sales force compensation design under information asymmetry and delegation of pricing to the sales force. In all of these papers, the firm offers the menu of contracts directly to the salesperson, combining the dual objectives of revealing information about the salesperson and motivating him to exert sales effort. To my knowledge, this chapter includes the first analysis of a model in which a menu of contracts is offered to the sales manager, separating the information revelation mechanism from the motivational purpose of the salesperson’s contract.

3.2 Model

The models used in this chapter have a large degree of overlap with those used in the previous chapter, but all details are included here for the reader’s convenience.

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2A similar approach was applied in various contexts other than sales force compensation by the former Soviet Union, as discussed and analyzed by Weitzman (1977).
I consider a firm that employs a single salesperson to help sell its product. For simplicity and tractability, I use a stylized setup in which the product’s sales are binary, taking one of two possible values: \( x \in \{ x_L, x_H \} \), with \( \Delta x \equiv x_H - x_L > 0 \).

The salesperson is expected to exert effort to sell the product, but exerting effort is costly and cannot be observed by the firm. The salesperson’s effort is represented one-dimensionally and can take any non-negative real value, \( e \in \mathbb{R}_+ \). The salesperson’s effort affects the probabilities of the two sales outcomes, with an increase in effort increasing the probability of high sales and decreasing the probability of low sales. That relationship is represented by the sales response function \( p(e) \equiv \Pr(x = x_H | e) \), where \( p' > 0 \), \( p'' < 0 \) and \( p(e) \in (0, 1) \forall e \). In other words, \( p \) is increasing and concave and always includes a degree of uncertainty, regardless of the salesperson’s effort. Thus, the firm is never able to infer the salesperson’s effort from the sales outcome.

Since the firm cannot incent the salesperson based on his effort, it induces effort instead by compensating him based on sales, which are mutually observed. Due to the binary nature of sales, a compensation plan for the salesperson consists of a pair of payout values \( (s_L, s_H) \), with each payout associated with a sales outcome. Thus, the salesperson receives compensation:

\[
s(x) = \begin{cases} 
    s_H & \text{if } x = x_H \\
    s_L & \text{if } x = x_L 
\end{cases}
\]

where \( s_H \geq s_L \).

When the salesperson exerts effort \( e \) and receives payout \( s(x) \), his utility is given by:

\[
U = u(s(x)) - v(e)
\]

where \( u(\cdot) \) represents the salesperson’s utility from income and is increasing and concave \( (u' > 0 \text{ and } u'' < 0) \) with \( u(0) \) normalized to 0. This reflects the standard assumption that the salesperson values income but is risk-averse. The salesperson’s disutility from effort is represented by \( v(\cdot) = \beta v_0(\cdot) \), where \( v_0 \) is increasing and convex \( (v_0' > 0 \text{ and } v_0'' > 0) \) with \( v_0(0) = 0 \), reflecting the standard beliefs that effort is costly and that the incremental cost of effort increases with the amount of effort exerted. \( \beta > 0 \) acts as a multiplier on the cost of effort, so a higher value implies higher effort costs for the salesperson.

The firm is not fully informed about the salesperson’s utility. Specifically, the firm does not know the difficulty of the salesperson’s territory, which I refer to as its ‘type’. I allow the territory to take two possible types: low-difficulty and high-difficulty (or ‘easy’ and ‘hard’), with the types represented in the model by the parameter \( \beta \in \{ \beta_L, \beta_H \} \) with \( \beta_L < \beta_H \). In other words, territory difficulty is reflected in the salesperson’s cost of effort, with a less-difficult territory incurring less utility cost to the salesperson than would a more-difficult territory, for the same effect on sales outcomes.

While the firm does not know which type the salesperson’s territory is, it does have an accurate prior belief about the distribution of types:

\[
\gamma_L \equiv \Pr(\beta = \beta_L) \quad \text{and} \quad \gamma_H \equiv 1 - \gamma_L = \Pr(\beta = \beta_H)
\]

where \( 0 < \gamma_L, \gamma_H < 1 \).

I restrict attention to the case in which employing a salesperson in either type of territory is preferred to not employing a salesperson at all. In other words, if the territory type were known, the firm would
always choose to offer the salesperson a contract (assuming it is designed optimally) over letting him exit the firm.

The salesperson’s best alternative option provides utility $\bar{U}$. If the salesperson’s expected utility from a contract is less than $\bar{U}$, he rejects the contract and exits the firm, exerting no effort and receiving no compensation.\(^3\)

The territory type, while unknown to the firm, is known (costlessly) to the sales manager, having been revealed in the course of her managerial duties.

The sales manager’s role (within the context of this analysis) is to design the salesperson’s compensation plan (i.e., to choose the values of $s_L$ and $s_H$) within a budget constraint imposed by the firm. This budget constraint takes the form defined in the previous chapter: The manager is required to choose a compensation plan that is optimal for the firm for some existing territory type $\beta \leq \beta_{\text{max}}$, where the value of $\beta_{\text{max}}$ is chosen by the firm.

Similar to the salesperson, the manager is assumed to be risk-averse\(^4\), with increasing, concave utility from income, denoted $u^M(\cdot)$, with $u^M(0)$ normalized to 0. Since the manager does not exert direct effort in the context of this analysis, the manager’s utility is simply given by:

$$U^M = u^M(s^M)$$

where $s^M$ represents the manager’s compensation from the firm (discussed further below).

The sales manager’s best alternative option provides utility $\bar{U}^M$. This analysis focuses on just one of the many responsibilities of a sales manager (choosing compensation plans for her salespeople), making it difficult to model the impact of the manager exiting the firm. For simplicity, then, I assume that it is never optimal for the firm to offer the manager a contract (or menu of contracts) such that her expected utility falls below $\bar{U}^M$. In other words, the firm considers only those offers that satisfy the manager’s participation constraint.

The manager’s compensation is modeled in two different ways. In the ‘Menu of Contracts’ model, the manager’s compensation plan, $\sigma^M = (s^M_L, s^M_H)$, has a similar structure to that of the salesperson:

$$s^M(x) = \begin{cases} 
    s^M_H & \text{if } x = x_H \\
    s^M_L & \text{if } x = x_L 
\end{cases} \text{ where } s^M_H \geq s^M_L.$$ 

Unlike in the previous chapter, this analysis depends on explicit modeling of the cost of the manager’s compensation (to the firm), so these terms do not capture potential non-monetary rewards associated with the higher sales outcome, such as improved career outcomes. Instead, such rewards are assumed to be used by the manager as a “tiebreaker” in cases in which the monetary rewards are equal. In other words, if $s^M_H = s^M_L$, then the manager prefers the high sales outcome over the low sales outcome and behaves accordingly.

The firm allows the manager to choose from a menu of two or more contracts, with each contract $(\beta_{\text{max}}, \sigma^M)$ consisting of a budget constraint for the salesperson’s compensation and a compensation

\(^3\)Again, for the purposes of this chapter, I assume that hiring and training a new salesperson requires a sufficient investment of money and/or time to prevent the firm from replacing an exiting salesperson within the time frame of the model.

\(^4\)This differs from the previous chapter, in which the solution is unaffected by whether the manager is risk-averse or risk-neutral. Since this chapter deals more explicitly with the manager’s own compensation, it is necessary to make a distinction. In the absence of evidence to the contrary, it seems reasonable to assume similar risk attitudes between salespeople and sales managers (many of whom are former salespeople themselves).
plan for the manager.

In the 'Allocated Budget Bonus' model, the firm offers the manager a single compensation contract, again consisting of a budget constraint for the salesperson’s compensation and compensation terms for the manager. In this model, however, the manager’s compensation plan, written $\sigma^M = (s^M_L, s^M_H, q)$, takes the following form:

$$s^M(x) = \begin{cases} 
    s^M_H + q[s_H(\beta_{max}) - s_H] & \text{if } x = x_H \\
    s^M_L + q[s_H(\beta_{max}) - s_H] & \text{if } x = x_L 
\end{cases}$$

where $s^M_H \geq s^M_L$, $q \in [0, 1]$, and $s_H(\beta_{max})$ represents the firm’s optimal $s_H$ (high-sales payout to the salesperson) for territory type $\beta_{max}$. Therefore, $s_H(\beta_{max})$ is the highest payout that the manager can offer the salesperson under budget $\beta_{max}$, and $q[s_H(\beta_{max}) - s_H]$ represents a bonus to the sales manager, determined by taking a proportion ($q$) of the difference between the highest possible payout and the actual high-sales payout that she offers to the salesperson.

While the salesperson and the sales manager maximize their respective expected utilities, the firm is always assumed to maximize its expected profit. The marginal cost of production is normalized to 0.

The sequence of events in the Menu of Contracts model is as follows:

1. The firm announces the menu of contracts for the sales manager $\{(\beta_{max_1}, \sigma^M_1), \ldots \}$
2. The manager chooses a contract $(\beta_{max}, \sigma^M)$ from the menu
3. The manager announces the salesperson’s compensation contract, $\sigma = (s_L, s_H)$
4. The salesperson decides whether to accept the contract and, if so, chooses his effort level ($e$)
5. Sales are realized and the players receive their payouts

The Unallocated Budget Bonus model follows a similar sequence, except that the firm announces a single contract for the sales manager in step 1, making step 2 irrelevant.

Notation introduced in this section and throughout the chapter is summarized in a table at the end of the chapter.

### 3.3 Analysis and Results

I begin by analyzing the Menu of Contracts model (using backward induction) to identify the firm’s optimal solution and determine the conditions under which it can be used to ensure that the sales manager chooses the optimal compensation plan for the salesperson, regardless of territory type. I compare the optimal solution to that of the No Involvement model from the previous chapter, to discover when (if ever) the firm is better off designing the salesperson’s compensation plan directly, without the sales manager’s involvement.

I then analyze the Unallocated Budget Bonus model to identify the conditions (if any) under which it outperforms the Menu of Contracts model.

#### Salesperson’s effort

The salesperson’s utility function and decision rule for determining his effort are the same in both models here as they are in the previous chapter, because his preferences are consistent across all of these models.
Chapter 3. Using managerial compensation to ensure optimal sales force compensation

and depend only on his own compensation contract.

The previous chapter, then, gives the following result: Given a compensation contract \((s_L, s_H)\), the salesperson will choose effort \(e^*\) such that

\[
\frac{v'(e^*)}{p'(e^*)} = u(s_H) - u(s_L)
\]

This has a unique, non-negative solution when \(u(s_H) - u(s_L)\) is sufficiently large \((\geq \frac{v'(0)}{p'(0)})\).

Continuing with the notation used in the previous chapter:

\(\sigma_L = (s_L^L, s_L^H)\) and \(\sigma_H = (s_H^L, s_H^H)\) represent the firm’s optimal compensation plans when the salesperson’s territory type is known to be \(\beta_L\) and \(\beta_H\), respectively.

\(e_L(e_H)\) represents the effort level chosen by the salesperson with true territory type \(\beta_L\) (\(\beta_H\)) under the optimally-designed compensation plan \(\sigma_L (\sigma_H)\).

\(\hat{e}\) represents the effort exerted by a salesperson with territory type \(\beta_L\) under compensation plan \(\sigma_H\).

From Lemma 3 in the previous chapter, if the salesperson’s compensation plan is optimally designed for a territory with type lower than the true type, then the salesperson declines the contract and exits the firm. Thus, a salesperson in a \(\beta_H\)-type territory will exit if offered plan \(\sigma_L\). For consistency, I will continue to focus on model specifications in which \(\hat{e} > e_L\) (i.e., in which a salesperson in a low-difficulty territory exerts more effort when his compensation plan is optimized for a high-difficulty territory than when it is optimized for his territory’s true type).

3.3.1 Menu of Contracts

In this model, the firm offers the manager a menu of two or more contracts, each consisting of a compensation plan for the manager and a budget constraint within which the manager must design the salesperson’s compensation plan.

Manager’s design of salesperson’s compensation

Given a contract \((\beta_{\text{max}}, \sigma^M)\), the manager’s preferences in designing the salesperson’s compensation are consistent with those in the previous chapter. In particular, the first part of Proposition 1 from that chapter holds, stating that a self-interested manager always designs the salesperson’s compensation plan to be optimal for the highest possible territory type (i.e., the highest existing \(\beta \leq \beta_{\text{max}}\)).

That proposition also states that, all else being equal, the manager prefers \(\beta_{\text{max}}\) equal to the highest existing type. In this model, however, all else is not necessarily equal, as a menu option with a higher \(\beta_{\text{max}}\) might come with a less-desirable \(\sigma^M\).

Manager’s contract selection

The manager chooses the contract that provides her with the greatest expected utility, given the compensation plan she chooses to offer the salesperson under each contract and, in turn, the sales effort expected based on equation 3.1.

Suppose the firm offers the following menu: \(\{(\beta_1, (s_{LM1}, s_{MH1})), (\beta_2, (s_{LM2}, s_{MH2})), \ldots, (\beta_N, (s_{LMN}, s_{MHN}))\}\). Recall that the manager is informed about the salesperson’s territory type when choosing a contract.
from the menu. So, the manager’s expected utility from the $i^{th}$ contract in the menu is

\[
E[U^M | \beta_i, (s_{L_i}^M, s_{H_i}^M)] = \begin{cases} 
  p(e_{L_i})u^M(s_{H_i}^M) + [1 - p(e_{L_i})]u^M(s_{L_i}^M) & \text{if } \beta = \beta_L \\
  p(e_{H_i})u^M(s_{H_i}^M) + [1 - p(e_{H_i})]u^M(s_{L_i}^M) & \text{if } \beta = \beta_H 
\end{cases}
\]

where $e_{L_i}$ ($e_{H_i}$) represents the effort chosen by a salesperson in territory type $\beta_L$ ($\beta_H$) under the compensation plan chosen by the manager given $\beta_i$ (i.e., the plan optimized for the highest type $\leq \beta_i$).

Therefore, if the salesperson’s territory is easy, the manager chooses the contract that corresponds to

\[
\max \{p(e_{L_i})u^M(s_{H_i}^M) + [1 - p(e_{L_i})]u^M(s_{L_i}^M)\}_{i=1}^N
\]

and the expression is similar (replacing $e_{L_i}$ with $e_{H_i}$) if the territory is hard.

**Firm’s menu design**

Given a menu of contracts, the manager’s choice of contract is determined by the salesperson’s territory type. The optimal menu, then, does not need to include more than one option per type, as the manager will consistently choose the same option for each type (with any additional options never chosen). Thus, I need only consider menus consisting of two options.

Furthermore, any menu that results in the manager designing the same compensation plan for the salesperson in both territory types (i.e., a pooling solution) can be replaced by a non-menu-based solution, in which the firm offers that plan directly to the salesperson. In other words, a pooling solution can be replaced by a model in which the manager has no involvement, discussed below. Therefore, this section will focus on separating solutions, in which the manager designs a different compensation plan for the salesperson in each territory type.

**Lemma 5.** The optimal (separating) menu of contracts is composed of one option in which $\beta_{\text{max}} \in [\beta_L, \beta_H)$ and one in which $\beta_{\text{max}} \geq \beta_H$.

**Proof.** As noted above, if the salesperson’s compensation is optimized for a territory type lower than the actual type, then the salesperson exits the firm. Therefore, a budget constraint of $\beta_{\text{max}} < \beta_L$ prevents the manager from offering a plan that a salesperson in either type of territory would accept. With attention focused on cases in which the firm prefers to retain the salesperson in either territory type, such a contract is not of interest.

From above, the manager designs the salesperson’s compensation plan to be optimal for the highest possible territory type. So, if $\beta_{\text{max}} \in [\beta_L, \beta_H)$, then she chooses $\sigma = \sigma_L$, and if $\beta_{\text{max}} \geq \beta_H$, then she chooses $\sigma = \sigma_H$. Therefore, in order for the menu to result in a different compensation plan for the salesperson in each territory type, it must be composed of one option in which $\beta_{\text{max}} \in [\beta_L, \beta_H)$ and one in which $\beta_{\text{max}} \geq \beta_H$. \qed

Given any compensation plan ($\sigma^M$) for the manager, the firm is indifferent between budget constraints within each of these ranges (i.e. within $\beta_{\text{max}} \in [\beta_L, \beta_H)$ and within $\beta_{\text{max}} \geq \beta_H$), as are the manager.

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\( ^5 \)It is possible for a menu to include multiple options that provide the manager with equal expected utility given a particular territory type, resulting in the manager potentially choosing inconsistently between those options. In that case, however, the firm would prefer to offer only the option that maximizes its expected profit given that type. (Even if the options provide the same expected profits, the firm has no reason to offer more than one of them.)
and the salesperson. For brevity, then, I will assume that the firm chooses $\beta_{\text{max}} = \beta_L$ and $\beta_H$ from the respective ranges, to be the budget constraints in the two contracts that form the menu.

The next step is to identify the optimal compensation plan for the firm to offer the manager in each contract, maximizing expected profits while ensuring that she will always choose the contract with $\beta_{\text{max}}$ matching the true territory type. Given the 'correct' contract choice for each type, the firm maximizes its expected profits by minimizing its expected payout to the manager.

Thus, letting $\sigma_{M,\beta_L} \equiv (s^L_{\beta_L}, s^H_{\beta_L})$ and $\sigma_{M,\beta_H} \equiv (s^L_{\beta_H}, s^H_{\beta_H})$ denote the optimal compensation plans for the manager in the contracts with $\beta_{\text{max}}$ equal to $\beta_L$ and $\beta_H$, respectively, the firm’s problem is:

$$\min_{(s^L_{\beta_L}, s^H_{\beta_L}), (s^L_{\beta_H}, s^H_{\beta_H})} \left(1 - \gamma_L\right) \left(p(e_H)s^H_{\beta_L} + [1 - p(e_H)]s^H_{\beta_L}\right) + \gamma_L \left(p(e_L)s^H_{\beta_H} + [1 - p(e_L)]s^H_{\beta_H}\right)$$

s.t. $p(e_H)u^M(s^H_{\beta_L}) + [1 - p(e_H)]u^M(s^L_{\beta_L}) \geq \bar{U}^M$ \hspace{1cm} (IR$_{\beta_L}$)

$p(e_H)u^M(s^H_{\beta_H}) + [1 - p(e_H)]u^M(s^L_{\beta_H}) \geq \bar{U}^M$ \hspace{1cm} (IR$_{\beta_H}$)

$p(e_H)u^M(s^H_{\beta_L}) + [1 - p(e_H)]u^M(s^L_{\beta_L}) > p(\tilde{e})u^M(s^H_{\beta_L}) + [1 - p(\tilde{e})]u^M(s^L_{\beta_L})$ \hspace{1cm} (IC$_{\beta_L}$)

$p(e_H)u^M(s^H_{\beta_H}) + [1 - p(e_H)]u^M(s^L_{\beta_H}) > p(0)u^M(s^H_{\beta_H}) + [1 - p(0)]u^M(s^L_{\beta_H})$ \hspace{1cm} (IC$_{\beta_H}$)

$u^M(s^H_{\beta_L}) \geq u^M(s^H_{\beta_L})$ \hspace{1cm} (\Delta$_{\beta_L}$)

$u^M(s^H_{\beta_H}) \geq u^M(s^H_{\beta_H})$ \hspace{1cm} (\Delta$_{\beta_H}$)

(IR$_{\beta_L}$) and (IR$_{\beta_H}$) represent the manager’s participation constraints, given actual territory type $\beta_L$ and $\beta_H$, respectively, ensuring that the manager does not exit the firm. The incentive compatibility constraints, (IC$_{\beta_L}$) and (IC$_{\beta_H}$), require that the manager’s expected utility from the “correct” contract choice be greater than her expected utility from the “incorrect” choice, ensuring that she chooses the intended contract for each territory type. Lastly, (\Delta$_{\beta_L}$) and (\Delta$_{\beta_H}$) indicate that the manager’s high-sales payout must be at least as large as her low-sales payout under each contract.

To simplify notation, I define: $u^L_\beta \equiv u^M(s^L_{\beta})$ and $u^H_\beta \equiv u^M(s^H_{\beta})$. Furthermore, I define $\bar{h}(\cdot) \equiv u^{-1}(\cdot)$, as in the previous chapter, so $\bar{h}(u)$ represents the monetary payout required to provide utility $u$.

Thus, $\bar{h}(u^H_\beta) = s^H_\beta$ and $\bar{h}(u^L_\beta) = s^L_\beta$. This allows me to transform the problem into a more tractable form, in which the compensation plan is expressed in terms of the utility that the payouts provide to the sales manager, rather than their monetary value. The firm’s problem can be rewritten as follows:

$$\max_{u^L_{\beta L}, u^H_{\beta L}, u^L_{\beta H}, u^H_{\beta H}} - (1 - \gamma_L) \left(\bar{h}(u^H_{\beta L}) + p(e_H)[h(u^H_{\beta L} - h(u^L_{\beta L}))]\right) - \gamma_L \left(\bar{h}(u^L_{\beta H}) + p(e_L)[h(u^L_{\beta H} - h(u^L_{\beta L}))]\right)$$

s.t. $u^L_{\beta L} + p(e_L)(u^H_{\beta H} - u^L_{\beta L}) \geq \bar{U}^M$ \hspace{1cm} (IR$_{\beta_L}$)

$u^H_{\beta L} + p(e_H)(u^H_{\beta H} - u^H_{\beta L}) \geq \bar{U}^M$ \hspace{1cm} (IR$_{\beta_H}$)

$u^L_{\beta L} + p(e_H)(u^H_{\beta H} - u^L_{\beta L}) \geq u^H_{\beta L} + p(\tilde{e})(u^H_{\beta H} - u^H_{\beta L}) + \delta$ \hspace{1cm} (IC$_{\beta_L}$)

$u^H_{\beta L} + p(e_L)(u^H_{\beta H} - u^H_{\beta L}) \geq u^L_{\beta L} + p(0)(u^H_{\beta H} - u^L_{\beta L}) + \delta$ \hspace{1cm} (IC$_{\beta_H}$)

$u^L_{\beta L} - u^H_{\beta L} \geq 0$ \hspace{1cm} (\Delta$_{\beta_L}$)

$u^H_{\beta L} - u^H_{\beta H} \geq 0$ \hspace{1cm} (\Delta$_{\beta_H}$)

where $\delta$ is a small, positive value that allows the strict inequalities of the (IC) constraints to be expressed as weak inequalities.$^7$

$^6$Note that $h(\cdot)$ is increasing and convex, since $u(\cdot)$ is increasing and concave.

$^7$Intuitively, $\delta$ can be thought of as the smallest difference in expected utility that would drive a manager to consistently
Since this problem has a concave objective function and linear constraints, it can be solved using Lagrangian optimization (details shown in Appendix I). The resulting optimal compensation plans for the manager are:

\[
(u_L^β, u_H^β) = \left( \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta, \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta \right)
\]
\[
(u_L^β, u_H^β) = (\bar{U}^M, \bar{U}^M)
\]

The optimal menu, then, is characterized in Proposition 5.

**Proposition 5.** The firm’s optimal menu of contracts is given by:

\[
\left\{ (\beta_L, \left( h \left( \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta \right) \right), h \left( \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta \right)) \right\}, (\beta_H, (h(\bar{U}^M), h(\bar{U}^M)))
\]

When the salesperson’s territory is hard (\(\beta = \beta_H\)), the manager chooses the contract \((\beta_H, (h(\bar{U}^M), h(\bar{U}^M)))\), designing the optimal compensation plan \((\sigma_H)\) for the salesperson and receiving a fixed salary herself (providing utility \(\bar{U}^M\)), with no additional incentives based on the sales outcome.

When the territory is easy (\(\beta = \beta_L\)), the manager chooses the contract \((\beta_L, (h \left( \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta \right), h \left( \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta \right))\)) and designs the optimal compensation plan \((\sigma_L)\) for the salesperson. Since \(0 < p(e_L) - p(0) < p(e_L) + p(0) < 2\), the manager’s payout (in utility terms) is less than \(\bar{U}^M\) when sales are low and greater than \(\bar{U}^M\) when sales are high. In other words, her base salary is smaller than when the territory is hard, but she can earn positive incentive pay based on the sales outcome. Her expected payout under this contract, in terms of utility, is given by:

\[
E[U^M | \beta = \beta_L] = p(e_L) \left( \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta \right) + [1 - p(e_L)] \left( \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta \right)
\]
\[
= \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta + p(e_L) \left( \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta - \bar{U}^M + \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta \right)
\]
\[
= \bar{U}^M + \frac{\delta}{p(e_L) - p(0)} (2p(e_L) - [p(e_L) + p(0)])
\]

\[
: E[U^M | \beta = \beta_L] = \bar{U}^M + \delta
\]

So, the manager’s expected payout is higher when the territory is easy than when it is hard. In other words, the firm pays the manager information rent to ensure that she truthfully reveals when the salesperson’s territory is easy. This is similar to the finding in Rao (1990), except that here the rent is paid to the manager, rather than to the salesperson whose type is more desirable.

or reliably choose one contract option over another.
The firm’s expected profit from this menu is given by:

\[
(1 - \gamma_L) \left( p(e_H) \left[ x_H - h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} \right] + [1 - p(e_H)] \left[ x_L - h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} \right] \right) \\
+ \gamma_L \left( p(e_L) \left[ x_H - h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} \right] + [1 - p(e_L)] \left[ x_L - h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} \right] \right) \\
= (1 - \gamma_L) \left[ x_L - h(\bar{U}^M) - s_{L}^{\sigma_H} + p(e_H) \left( \Delta x - [h(\bar{U}^M) - h(\bar{U}^L)] - \Delta s_{L}^{\sigma_H} \right) \right] \\
+ \gamma_L \left[ x_L - h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} + p(e_L) \left( \Delta x - [h(u_{L}^{\beta_L}) - h(u_{L}^{\beta_L})] - \Delta s_{L}^{\sigma_L} \right) \right] \\
= x_L + (1 - \gamma_L) \left(-h(\bar{U}^L) - s_{L}^{\sigma_H} + p(e_H)[\Delta x - \Delta s_{L}^{\sigma_H}] \right) \\
+ \gamma_L \left(-h(u_{L}^{\beta_L}) - s_{L}^{\sigma_L} + p(e_L) \left[ \Delta x - [h(u_{L}^{\beta_L}) - h(u_{L}^{\beta_L})] - \Delta s_{L}^{\sigma_L} \right] \right)
\]

Optimality of manager’s involvement

While Proposition 5 outlines the optimal separating solution for the firm, I can also consider pooling solutions, in which the salesperson’s compensation plan is the same, regardless of territory type. If the firm chooses to implement such a solution, then there is no need to involve the manager in designing the salesperson’s plan. Thus, the optimal pooling solution is given by the ‘No Involvement’ model in section 2.3.2 (in the previous chapter).

For comparison with the ‘Menu of Contracts’ solution, it is necessary to be explicit here about the manager’s compensation, which was treated more abstractly in the previous chapter. When not involved in the design of the salesperson’s compensation, there is no reason (within the context of this model) to offer the manager incentives based on sales. Since she is risk-averse and the firm is risk-neutral, it is straightforward to conclude that the firm would prefer to pay her a fixed salary providing utility equal to that of her outside option ($\bar{U}^L$). Since this is a fixed cost to the firm, it has no effect on the optimal design of the salesperson’s compensation.

From the previous chapter, that optimal design is as follows:

\[
\sigma = \begin{cases} 
\sigma_L & \text{if } \gamma_L > \gamma_L^0 \\
\sigma^* & \text{if } \gamma_L < \gamma_L^0
\end{cases}
\]

where $\sigma^*$ represents the optimal plan that the salesperson will accept regardless of his territory type, and $\gamma_L^0$ is the threshold at which the firm is indifferent between $\sigma^*$ and $\sigma_L$, given by:

\[
\gamma_L^0 = \frac{E[\pi(\sigma^*)|\beta_H] - E[\pi(\sigma_L)|\beta_H]}{E[\pi(\sigma^*)|\beta_H] - E[\pi(\sigma_L)|\beta_H] - (E[\pi(\sigma^*)|\beta_L] - E[\pi(\sigma_L)|\beta_L])}
\]

Comparison when $\gamma_L > \gamma_L^0$

When $\gamma_L > \gamma_L^0$, this solution implies that the salesperson’s compensation will be optimal (and he will exert effort $e_L$) if the territory is easy, but that he will exit the firm if the territory is difficult. Thus,
the firm’s expected profit when \( \gamma_L > \gamma_L^0 \) is given by:

\[
E[\pi|\gamma_L > \gamma_L^0] = (1 - \gamma_L)(p(0)[x_H - h(U^M)] + [1 - p(0)][x_L - h(U^M)]) + \gamma_L(p(e_L)[x_H - h(U^M) - s^*_H] + [1 - p(e_L)][x_L - h(U^M) - s^*_L]) \]

\[
= (1 - \gamma_L)(x_L + p(0)\Delta x) + \gamma_L(x_L - s^*_L + p(e_L)[\Delta x - \Delta s^*_L]) - h(U^M) \]

\[
\therefore E[\pi|\gamma_L > \gamma_L^0] = x_L - h(U^M) + (1 - \gamma_L)p(0)\Delta x + \gamma_L(-s^*_L + p(e_L)[\Delta x - \Delta s^*_L]) \]

Comparing this to the firm’s expected profit under the optimal menu of contracts from Proposition 5, the menu is preferred if and only if:

\[
(1 - \gamma_L)((p(e_H) - p(0)[\Delta x - s^*_H + p(e_H)\Delta s^*_H]) > \gamma_L(h(u^*_L) + p(e_L)[h(u^*_L) - h(u^*_H)]) - h(U^M) \] (3.2)

If the territory is easy, then both solutions (a menu of contracts and the firm designing the salesperson’s compensation directly) result in the optimal compensation plan \((\sigma_L)\) for the salesperson. Therefore, the only difference in that case is the information rent that the firm must pay to the manager (i.e. the rent paid when the territory is easy multiplied by the probability of the easy type). If the territory is hard, on the other hand, the menu of contracts ensures that the salesperson is optimally compensated, while the firm’s own compensation design results in the salesperson exiting the firm. Thus, the left-hand side represents the expected benefit to the firm of compensating the salesperson optimally when the territory is hard, relative to losing the salesperson. Offering a menu of contracts is optimal when this expected benefit outweighs the expected information rent.

Above, I showed that the information rent is positive. Therefore, it is clear that the condition given by 3.2 fails as \( \gamma_L \) approaches 1. Intuitively, as the firm approaches certainty that the territory is easy, it is better off designing the salesperson’s plan itself (and assuming that the territory is easy), rather than offering the manager a menu of contracts.

**Comparison when \( \gamma_L < \gamma_L^0 \)**

The salesperson will accept the compensation plan \( \sigma^* \) regardless of the territory type, exerting effort \( e^*_L (e^*_H) \) if the true type is \( \beta_L (\beta_H) \). Thus, the firm’s expected profit from designing the salesperson’s plan itself when \( \gamma_L < \gamma_L^0 \) is given by:

\[
E[\pi|\gamma_L < \gamma_L^0] = (1 - \gamma_L)(p(e^*) [x_H - h(U^M) - s^*_H] + [1 - p(e^*)] [x_L - h(U^M) - s^*_L]) + \gamma_L \left(p(e^L) [x_H - h(U^M) - s^*_H] + [1 - p(e^L)] [x_L - h(U^M) - s^*_L]\right) \]

\[
\therefore E[\pi|\gamma_L < \gamma_L^0] = x_L - h(U^M) - s^*_L + [(1 - \gamma_L)p(e^*) + \gamma_L p(e^L)] (\Delta x - \Delta s^*) \]

Comparing this to the firm’s expected profit under the optimal menu of contracts, the menu is preferred if and only if:

\[
\gamma_L \left([p(e_L) - p(e^L)]\Delta x - [s^*_L + p(e_L)\Delta s^*_L - (s^*_L + p(e^L)\Delta s^*_L)]\right) + (1 - \gamma_L) \left([p(e_H) - p(e^H)]\Delta x - [s^*_H + p(e_H)\Delta s^*_H - (s^*_H + p(e^H)\Delta s^*_H)]\right) \]

\[
> \gamma_L \left(h(u^*_L) + p(e_L)[h(u^*_L) - h(u^*_H)] - h(U^M)\right) \] (3.3)

Once again, this inequality highlights the trade-off between the cost of using a menu of contracts
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(the expected information rent) on the right-hand side and its benefits on the left. The benefits are made up of the gains in expected profits when the territory is easy and when it is hard (from offering the respective optimal compensation plans to the salesperson rather than the ‘compromise’ of $\sigma^*$). As above, offering a menu of contracts to the manager is optimal when those benefits, weighted by the probabilities of the respective territory types, outweigh the expected information rent.

As $\gamma_L$ approaches 0, it is straightforward to see that condition 3.3 is satisfied. Thus, as the firm approaches certainty that the territory is hard, it is better off using a menu of contracts than designing the salesperson’s plan without the manager. Intuitively, this is true because the firm only pays information rent to the manager when the territory is easy. Therefore, when the probability of an easy territory is low, a menu of contracts allows the firm to optimize the salesperson’s compensation plan while rarely incurring any cost.

The results above are summarized in Proposition 6.

Proposition 6. The firm should offer the sales manager a menu of contracts, rather than design the salesperson’s compensation plan itself, if and only if the following is satisfied:

$$\text{information rent} < \begin{cases} \frac{1-\gamma_L}{\gamma_L} (E[\pi(\sigma_H)|\beta_H] - [x_L + p(0)\Delta]) & \text{if } \gamma_L > \gamma_L^0 \\ E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L] + \frac{1-\gamma_L}{\gamma_L} E[\pi(\sigma_H) - \pi(\sigma^*)|\beta_H] & \text{if } \gamma_L < \gamma_L^0 \end{cases}$$

where information rent $= h(u_{\beta L}^L) + p(e_L)[h(u_{\beta H}^L) - h(u_{\beta L}^L)] - h(U^M)$.

This condition holds as $\gamma_L$ approaches 0 and fails as $\gamma_L$ approaches 1.

For comparison, from the previous chapter, the conditions under which the firm should allow the manager to participate in the budgeting process (vs. excluding her and designing the salesperson’s compensation plan without her private information) are:

$$\text{participation cost} < \begin{cases} E[\pi(\sigma_H)|\beta_H] - [x_L + p(0)\Delta] & \text{if } \gamma_L > \gamma_L^0 \\ \frac{\gamma_L}{1-\gamma_L} E[\pi(\sigma_L) - \pi(\sigma^*)|\beta_L] + E[\pi(\sigma_H) - \pi(\sigma^*)|\beta_H] & \text{if } \gamma_L < \gamma_L^0 \end{cases}$$

These conditions are quite similar (with information rent in the menu-of-contracts solution akin to the participation cost in the previous analysis). However, the conditions differ because the information rent is incurred only when the territory is easy, while the participation cost is incurred only when the territory is hard.

For example, when $\gamma_L < 0.5$ (i.e., the territory is more likely to be hard), the firm will tolerate higher information rents (for a menu of contracts) than participation costs (for managerial participation in budget setting). Conversely, the opposite is true when the territory is more likely to be easy ($\gamma_L > 0.5$).

3.3.2 Unallocated Budget Bonus

As mentioned above, the idea of offering a menu of contracts to salespeople dates back to at least the 1960s, yet it is not believed to be very common in practice, which may indicate that firms have an aversion to menu-based compensation in general. Possible reasons for this could include organizational philosophy or culture, the complexity of administering different plans for individuals in similar roles, and concerns about fairness (or perceptions thereof). Therefore, some firms may prefer an alternative
approach to managerial compensation that reveals a manager’s private information without the use of a menu, particularly if such an approach does not sacrifice profitability.

In this section, I consider an ‘Unallocated Budget Bonus’ model (which I will refer to as “UBB”) in which the firm offers the manager a single compensation contract. As above, a contract consists of a budget constraint for the salesperson’s compensation along with compensation terms for the manager. However, the manager’s compensation plan now includes not only a payment based on the sales outcome, but also a bonus based on her unallocated budget, the difference between the highest possible payout she could offer and the actual high-sales payout that she offers to the salesperson.

The manager’s compensation plan, written \( \sigma^M = (s_L^M, s_H^M, q) \), takes the following form:

\[
s^M(x) = \begin{cases} 
  s_L^M + q[s_H(\beta_{\text{max}}) - s_H] & \text{if } x = x_L \\
  s_H^M + q[s_H(\beta_{\text{max}}) - s_H] & \text{if } x = x_H 
\end{cases}
\]

where \( s_H^M \geq s_L^M, q \in [0, 1] \), and \( s_H(\beta_{\text{max}}) \) represents the firm’s optimal \( s_H \) (high-sales payout to the salesperson) for territory type \( \beta_{\text{max}} \).

Once again, a budget constraint of \( \beta_{\text{max}} < \beta_L \) prevents the manager from offering a plan that a salesperson in either type of territory would accept. With attention focused on cases in which the firm prefers to retain the salesperson in either territory type, such a budget is not of interest.

If \( \beta_{\text{max}} \in [\beta_L, \beta_H] \), then according to the budget constraint, the manager must choose \( \sigma_L \), the compensation plan optimized for an easy territory (assuming that the manager’s own participation constraint is satisfied). Similar to a menu of contracts that results in a pooling equilibrium, this can be replaced by an approach in which the manager has no involvement in designing the salesperson’s compensation.

Thus, I can narrow my consideration to contracts in which \( \beta_{\text{max}} \geq \beta_L \), allowing the manager to choose either \( \sigma_L \) or \( \sigma_H \) (again assuming that the manager’s participation constraint is satisfied). If the manager chooses \( \sigma_L \), then the salesperson exerts effort \( e_L \) if the territory is easy and exits the firm if the territory is hard. Thus, the manager’s expected utility is:

\[
E[U^M|\sigma_L] = \begin{cases} 
  [1 - p(e_L)]u^M(s_L^M + q[s_H(\beta_{\text{max}}) - s_H^L]) + p(e_L)u^M(s_H^M + q[s_H(\beta_{\text{max}}) - s_H^L]) & \text{if } \beta = \beta_L \\
  [1 - p(0)]u^M(s_L^M + q[s_H(\beta_{\text{max}}) - s_H^L]) + p(0)u^M(s_H^M + q[s_H(\beta_{\text{max}}) - s_H^L]) & \text{if } \beta = \beta_H 
\end{cases}
\]

If she chooses \( \sigma_H \), then the salesperson exerts effort \( e_H \) if the territory is hard and \( \tilde{e} \) if it is easy. So, the manager’s expected utility is:

\[
E[U^M|\sigma_H] = \begin{cases} 
  [1 - p(\tilde{e})]u^M(s_L^M + q[s_H(\beta_{\text{max}}) - s_H^H]) + p(\tilde{e})u^M(s_H^M + q[s_H(\beta_{\text{max}}) - s_H^H]) & \text{if } \beta = \beta_L \\
  [1 - p(e_H)]u^M(s_L^M + q[s_H(\beta_{\text{max}}) - s_H^H]) + p(e_H)u^M(s_H^M + q[s_H(\beta_{\text{max}}) - s_H^H]) & \text{if } \beta = \beta_H 
\end{cases}
\]

The manager chooses \( \sigma_L \) or \( \sigma_H \) to maximize her expected utility, given the actual territory type.

**Comparison with menu of contracts**

**Proposition 7.** The Unallocated Budget Bonus model never outperforms the Menu of Contracts model (in terms of the firm’s expected profits).

**Proof.** Under the UBB model, the firm’s objective is to design the manager’s contract optimally, such
that she chooses the ‘correct’ plan for the salesperson given the territory type (i.e., $\sigma_L$ for an easy territory, $\sigma_H$ for a hard one).

Suppose the firm designs such a contract, $(\beta_{max}, (s_L^M, s_H^M, q))$. Then, the manager’s compensation plan can be written as:

$$s^M(x) = \begin{cases} 
\begin{align*}
\beta_L & \text{ if } \beta = \beta_L, x = x_L \\
\beta_H & \text{ if } \beta = \beta_H, x = x_H \\
\beta_L + Q & \text{ if } \beta = \beta_L, x = x_H \\
\beta_H + R & \text{ if } \beta = \beta_H, x = x_L \\
\beta_H + R & \text{ if } \beta = \beta_H, x = x_H
\end{align*}
\end{cases}$$

where $Q \equiv q(s_H(\beta_{max}) - s_H^M)$ and $R \equiv q[s_H(\beta_{max}) - s_H^M]$, with the constraint that $s_H^M \geq s_L^M$.

Equivalently, the manager’s plan can be re-written as:

$$s^M(x) = \begin{cases} 
\beta_L + Q' & \text{ if } \beta = \beta_L, x = x_L \\
\beta_H + Q' & \text{ if } \beta = \beta_L, x = x_H \\
\beta_H & \text{ if } \beta = \beta_H, x = x_L \\
\beta_H & \text{ if } \beta = \beta_H, x = x_H
\end{cases}$$

where $Q' \equiv Q - R$, $s_L^{M'} = s_L^M + R$, and $s_H^{M'} = s_H^M + R$, and the constraint is that $s_H^{M'} \geq s_L^{M'}$. In other words, the manager receives a fixed information rent of $Q'$ (independent of the sales outcome) when the territory is easy.

Similarly, the firm’s objective when using the Menu of Contracts model is to design the optimal menu such that the manager chooses the ‘correct’ compensation plan for the salesperson given the territory type. Under that model, the manager’s compensation plan can be written as:

$$s^M(x) = \begin{cases} 
\beta_L & \text{ if } \beta = \beta_L, x = x_L \\
\beta_L & \text{ if } \beta = \beta_L, x = x_H \\
\beta_H & \text{ if } \beta = \beta_H, x = x_L \\
\beta_H & \text{ if } \beta = \beta_H, x = x_H
\end{cases}$$

with the constraints that $s_H^{\beta_L} \geq s_L^{\beta_L}$ and $s_H^{\beta_H} \geq s_L^{\beta_H}$.

Equivalently, this can be re-written as:

$$s^M(x) = \begin{cases} 
\beta_L + A & \text{ if } \beta = \beta_L, x = x_L \\
\beta_H + B & \text{ if } \beta = \beta_L, x = x_H \\
\beta_L & \text{ if } \beta = \beta_H, x = x_L \\
\beta_H & \text{ if } \beta = \beta_H, x = x_H
\end{cases}$$

where $s_L^{M'} \equiv s_L^{\beta_L}$, $s_H^{M'} \equiv s_H^{\beta_L}$, $A \equiv s_H^{\beta_L} - s_L^{\beta_L}$, and $B \equiv s_H^{\beta_H} - s_L^{\beta_H}$, and the constraints are that $s_H^{M'} \geq s_L^{M'}$, and $B \geq A - (s_H^{M'} - s_L^{M'})$. This includes, but is not restricted to, the case in which $B = A$. In other words, the manager’s compensation again differs when the territory is easy, but in a way that may (and, in the optimal solution, does) depend on the sales outcome.

Comparing 3.4 and 3.5, it is clear that the set of possible compensation plans for the manager in the
Chapter 3. Using Managerial Compensation to Ensure Optimal Sales Force Compensation

UBB model is a subset of the possible plans in the Menu of Contracts model, specifically the subset in which \( A = B \). Furthermore, both models result in the manager choosing the same (optimal) compensation plan for the salesperson, regardless of territory type. Therefore, the UBB model is effectively a special case of the Menu of Contracts model, with an additional constraint on the design of the manager’s compensation plan. Thus, the UBB model can never outperform the Menu of Contracts model.

3.4 Conclusions

In the previous chapter, I discussed the conflict of interest that can arise when sales managers have private information about the people and/or territories they manage and are invited to participate in their firms’ budget-setting processes. I then proposed an approach to budget setting by which a manager’s participation itself could be used to reveal her information truthfully, thereby resolving the conflict of interest. However, that approach has the potential to be quite costly, as participating in budget setting pulls a manager away from the many other responsibilities that make her valuable to the firm.

In this chapter, I consider alternative ways in which a firm can reveal its sales managers’ private information, without their involvement in budgeting. In particular, I explore how a manager’s own compensation plan can be used to align her interests with those of the firm. Since the manager’s tendency under a typical sales-driven compensation plan is to over-incentivize her salespeople, I look for approaches that mitigate that tendency.

I find that a firm can reliably reveal a sales manager’s private information about a territory by offering her a menu of contracts, with each option corresponding a different territory type. Under such a menu, the manager’s expected compensation is higher when the optimal compensation for the salesperson includes smaller incentives. In other words, the firm offers the manager positive ‘information rent’ in exchange for revealing information that would otherwise work against her.

Comparing this approach to one in which the firm designs the salesperson’s compensation directly (i.e., without the manager’s information), I find that the conditions under which a menu of contracts is optimal are similar to those under which the managerial participation approach (from the previous chapter) is optimal, but with a key distinction. Under the participation approach, the firm incurs a cost only when the manager’s information leads to larger incentives for the salesperson (e.g., when the territory is hard, in my model context); under the menu-of-contracts approach, the firm pays information rent only when the information leads to smaller incentives for the salesperson (e.g., when the territory is easy). This suggests that either approach can be optimal for the firm, depending on the selling environment and the respective costs associated with the two approaches. For example, if the majority of territories are easy, then the participation approach becomes more attractive, while a menu of contracts is more appealing when the majority of territories are hard.

Lastly, I examine a compensation plan for the manager under which she receives not only a payment based on the sales outcome, but a proportion of any budget that she does not allocate to the salesperson’s compensation. Again, the intent of such a plan is to counter the manager’s tendency to over-incentivize the salesperson. However, I find that this approach is actually a special case of the menu-of-contracts approach, with an additional constraint placed on the manager’s compensation payments. As such, this type of contract can never out-perform the optimal menu of contracts, in terms of the firm’s expected profits.
The modeling in this chapter can be further developed by relaxing some of the simplifying constraints. In particular, I plan to extend the analysis to capture sales managers’ efforts spent on activities that drive sales more directly (e.g., coaching, hiring, working with customers, etc.). This will not only represent the true range of a manager’s responsibilities more accurately, but will also reflect why firms typically offer substantial incentive pay to sales managers. I also plan to examine how the results above extend to a greater number of territory types.

### 3.5 Table of notation

Notation used in this chapter is summarized below, for reference. Definitions refer to notation used in the main model unless otherwise noted.

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<th>Notation</th>
<th>Definition</th>
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<td>Possible sales outcomes</td>
</tr>
<tr>
<td>$e$</td>
<td>Effort exerted by salesperson</td>
</tr>
<tr>
<td>$p(e)$</td>
<td>Probability of high sales outcome</td>
</tr>
<tr>
<td>$s(x)$</td>
<td>Salesperson’s compensation payout</td>
</tr>
<tr>
<td>$s_L, s_H$</td>
<td>Salesperson’s possible compensation payouts</td>
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<tr>
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<td>Difficulty of selling in salesperson’s territory (i.e., territory type)</td>
</tr>
<tr>
<td>$\beta_L, \beta_H$</td>
<td>Possible territory types (low- and high-difficulty, respectively)</td>
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<tr>
<td>$\gamma_L, \gamma_H$</td>
<td>Probability that territory type is $\beta_L, \beta_H$, respectively</td>
</tr>
<tr>
<td>$U$</td>
<td>Salesperson’s utility from outside option</td>
</tr>
<tr>
<td>$u(\cdot)$</td>
<td>Salesperson’s utility from income</td>
</tr>
<tr>
<td>$v(\cdot)$</td>
<td>Salesperson’s disutility from effort (i.e., effort cost)</td>
</tr>
<tr>
<td>$v_0(\cdot)$</td>
<td>Salesperson’s ‘base’ effort cost (multiplied by $\beta$ to get effort cost)</td>
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<tr>
<td>$s^M(x)$</td>
<td>Manager’s compensation payout</td>
</tr>
<tr>
<td>$s^M_L, s^M_H$</td>
<td>Manager’s possible payouts (or sales-dependent payout components, in UBB)</td>
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<tr>
<td>$U^M$</td>
<td>Manager’s total utility</td>
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<tr>
<td>$\sigma^M$</td>
<td>Manager’s utility from outside option</td>
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<tr>
<td>$\sigma^M(\cdot)$</td>
<td>Manager’s ‘base’ effort cost (multiplied by $\beta$ to get effort cost)</td>
</tr>
<tr>
<td>$h(\cdot)$</td>
<td>Inverse of $u^M(\cdot)$; Payout required to provide manager with utility $u$</td>
</tr>
<tr>
<td>$\sigma_L, \sigma_H$</td>
<td>Optimal salesperson compensation plans when $\beta=$ $\beta_L, \beta_H$, respectively</td>
</tr>
<tr>
<td>$\sigma^*$</td>
<td>Optimal compensation plan that salesperson will accept regardless of type</td>
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<tr>
<td>$e_L$</td>
<td>Salesperson’s effort under $\sigma_L$ when territory type is $\beta_L$</td>
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<td>$e_H$</td>
<td>Salesperson’s effort under $\sigma_H$ when territory type is $\beta_H$</td>
</tr>
<tr>
<td>$\gamma_L$</td>
<td>Value of $\gamma_L$ at which firm is indifferent between $\sigma_L$ and $\sigma^*$</td>
</tr>
<tr>
<td>$\beta_{max}$</td>
<td>Budget constraint set by firm</td>
</tr>
<tr>
<td>$\sigma^M, \beta = (s^M_L, s^M_H)$</td>
<td>Optimal compensation plan for manager when $\beta_{max} = \beta$ (in Menu of Contracts)</td>
</tr>
<tr>
<td>$u^M_L, u^M_H$</td>
<td>Manager’s utility from payouts $s^M_L, s^M_H$, respectively</td>
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<tr>
<td>$q$</td>
<td>Proportion of ‘unallocated budget’ paid to manager (in UBB)</td>
</tr>
</tbody>
</table>

Table 3.1: Summary of notation
Chapter 4

Managing psychological motivations in contests

4.1 Introduction

Employee motivation has long been considered a major research area in both economics and marketing, particularly as it relates to sales management. The existing literature has overwhelmingly focused on economic incentives, investigating how well alternative compensation plans can align the incentives of principals and agents (see Mantrala et al., 2010 for a review). However, the literature has paid far less attention to the effects of psychological motivations, although their importance has long been recognized in motivation theory (e.g., Maslow, 1943; Bandura, 1986; Malone & Lepper, 1987). Psychological motivations can result from, for example, fun nature of work (e.g., playing video games), satisfaction from reaching goals (e.g., making a quota), or social recognition from achievement (e.g., winning “Employee of the Month” award).

In this chapter, I study the impact of psychological motivations on agents’ effort choices and how those motivations may depend on the design of incentive structures and disclosure schemes. I investigate this issue in the context of sales contests in which agents are rewarded and recognized based on their relative performances. Ranking employees by performance is a common practice among many reputable companies, including GE, Microsoft, and Cisco (Grote, 2005). Among the contests commonly observed in practice, incentive structures often differ by the spreads between rewards assigned to different ranks. Contests also differ by the format of disclosure, or how the outcome of a contest is announced to the participants. See Zoltners et al. (2011) for a discussion of disclosure schemes in sales management.

I consider four types of disclosure schemes: a no disclosure scheme, in which the outcome of the contest is never announced publicly; a winner disclosure scheme, in which only the top finisher is announced (e.g., an “Employee of the Month” or a “Salesperson of the Year” program); a partial disclosure scheme, in which all the winners are recognized, but the relative ranking between them is not disclosed (e.g., a “President’s Club” program); and a full disclosure scheme, in which the rankings of the winners and the identities of the losers are announced (e.g., a “Wall of Fame and Shame”). No disclosure provides a benchmark, consistent with the standard contest game typically represented in the literature, while the others represent the contest designs most commonly observed in practice. From a contestant’s point of view, these disclosure schemes vary in terms of how much other contestants know.
about her performance, but all four provide the same information about her own performance. Thus, any variation in the resulting level of psychological motivation comes from the effect on a contestant’s sense of achievement based on how peers regard her, not on how she regards herself.

My empirical study examines data collected from laboratory experiments, in which 192 subjects chose their levels of effort in simulated four-person sales contests, with rewards for the top two finishers in each contest. Each subject participated in 40 contest iterations under one of the four disclosure schemes described above (full, winner, partial, or no disclosure), with 20 iterations each under two different reward spreads (high and low). In a single contest iteration, each of the four subjects was endowed with an equal sum of points, of which they could choose to spend any number as “selling effort”, keeping the rest as income. Each participant’s sales revenue was then determined stochastically as a function of the effort she invested, and the two salespeople with the highest revenues had rewards added to their income. Finally, the results of the contest were revealed (or not), according to the disclosure scheme being employed. I also elicited each subject’s certainty equivalents for a series of lotteries, to calculate a risk-aversion parameter.

Results from the lab experiments show that contestants’ effort levels depend on both the reward spread and the disclosure scheme of a contest. While the mean effort levels under partial and winner disclosures are higher than that under no disclosure for each reward structure, they are higher than that under full disclosure only for the low reward spread. In general, some sort of public disclosure of the contest outcomes increases effort choice over having no public disclosure. Clearly, the efforts of the participating agents were influenced by their psychological motivations, which in turn depended on the disclosure scheme and reward spread.

In order to infer levels of psychological motivations from the selling efforts observed in the experiments, I develop a contest model that includes psychological motivations and derive each contestant’s equilibrium selling efforts. For the empirical analysis, I consider two alternative approaches. First, I take a direct approach to modeling an agent’s psychological values from winning rewards and/or earning recognition. I distinguish the sources of psychological values as “own” (induced by knowledge of one’s own achievement) versus “peer-generated” (induced by public recognition). Under this direct approach, I find that agents obtain positive and significant peer-generated psychological value from being recognized as one of the top performers under partial disclosure or as the single top performer under winner disclosure.

Second, I develop an indirect approach to estimate a unique psychological motivation parameter for each combination of incentive and disclosure schemes. The direct approach described above suffers from a plethora of psychological value parameters, not all of which can be individually identified. This drawback is further exacerbated as the number of contestants and ranks increases. The indirect approach takes advantage of the property that the equilibrium effort is determined by equating agents’ marginal gains and the marginal cost of their incremental efforts. When inferring psychological motivations from observed efforts, modeling the marginal increase (decrease) in direct utilities is equivalent to modeling the marginal decrease (increase) in effort costs. In essence, the effects of multiple psychological values on selling efforts can be captured by a single parameter reflecting the change in the marginal cost of effort. In this indirect approach, I develop a parsimonious and robust way of capturing the aggregate effect of multiple psychological values at work. I estimate the psychological motivation parameter among the contestants in my experiment, beginning with an assumption of risk neutrality and then relaxing that assumption using the risk-aversion parameter estimated from the lottery part of the experiment. Under
the assumption that the subjects are risk neutral, the medians of the psychological motivation parameter by treatment are mostly positive, suggesting that the sales contests generate positive psychological motivations. Furthermore, those estimates remain significant after accounting for risk aversion. In particular, the level of psychological motivation to work becomes higher when a partial disclosure or a winner disclosure scheme is employed, and is higher under the low reward spread than under the high reward spread.

The rest of this chapter is structured as follows: The following subsection discusses the contribution of this research in light of the existing literature. Section 2 explains the design of the laboratory experiments. In Section 3, I describe the experimental results regarding subjects’ effort choices. Section 4 provides an extended model of sales contests, directly incorporating the psychological values of winning rewards and receiving recognition into an agent’s utility function. My theoretical analysis leads to an equilibrium prediction for the level of effort under each treatment. I also use the data to make empirical inferences about the psychological values. In Section 5, I present a new method to indirectly quantify the value of psychological motivations and to estimate the resulting parameters using lab data. Finally, I conclude by discussing the implications of my results and some directions for future research.

4.1.1 Literature Review

This chapter contributes to several streams of research. First, it contributes to the sales management literature, by demonstrating not only the significance of psychological motivations, but also how they are affected by commonly-adopted sales management practices. The existing sales literature mainly focuses on the design of economic incentives. For example, Basu et al. (1985) derive the optimal compensation plan and examine how the shape of such a compensation plan should depend on salespeople’s characteristics (e.g., risk attitude) and product-market characteristics (e.g., sales uncertainty). Extensions to the compensation plan suggested by Basu et al. (1985) have been investigated in consideration of sales quotas (e.g., Mantrala et al., 1994), customer satisfaction (e.g., Hauser et al., 1994), over-selling (e.g., Kalra et al., 2003), territory allocation (e.g., Caldiero & Coughlan, 2009), and haggling (Dessai & Purohit, 2004). The existing studies of optimal compensation plans, by abstracting away from psychological motivations, implicitly assume an independent relationship between economic and psychological motivations. Under such an implicit assumption, when comparing alternative types of incentive schemes, researchers can assume that psychological motivations remain the same and, hence, focus solely on economic incentives. This chapter, along with related research on psychological motivations in sales management (e.g., Lim, 2010; Chen et al., 2011; Yang et al., 2013), suggests that future research may consider relaxing such independence assumptions. To the best of my knowledge, this is also the first study of the effects of recognition programs on the performance of salespeople.

Second, this chapter conducts an empirical study of sales contests and thus contributes new insights to the contest literature. Existing research has investigated the optimal design of sales contests (e.g., Lazear & Rosen, 1981; Nalebuff & Stiglitz, 1983) and the impact of sales contests on customer value (Garrett & Gopalakrishna, 2010). My model, with psychological motivation directly incorporated, is a more general version of the privilege contest model presented in Schroyen and Treich (2013). A well-known theoretical result on the design of reward structures is that a rank-order contest should provide a smaller spread between rewards when the agents are more risk-averse (Krishna & Morgan, 1998; Kalra & Shi, 2001; Lim et al., 2009). My analysis, as well as a number of others, suggests that the increase in effort choice from offering rewards that are closer to each other may not be attributable to risk aversion.
alone. In my model, an increase in the psychological motivation to exert effort may also make a contest with a smaller reward spread more effective in inducing effort from contestants. This finding can also be connected to inequity aversion, as suggested by Fehr and Schmidt (1999). There have been a number of papers that analyze the impact of information provided to players in a contest setting. For instance, Hyndman et al. (2012) explore the impact of disclosing the winning bid on regret by bidders and on their bidding behavior in an all-pay auction. In a field experiment by Barankay (2012), salespeople are informed of their own ranking in a bonus program based on the absolute performance of the salespeople. In contrast, this research focuses on the change in the level of a contestant’s psychological motivation coming from changes in how much information her peers have about her achievement.

Third, this chapter contributes to the research on interdependence between psychological and economic motivations, an area that has attracted growing interest in the economics and marketing literatures. Research has shown that changes in economic incentives can alter psychological motivation levels. With the presence of monetary incentives, the perceived nature of a task can change. For example, the task can cease to be fun or to reflect self-image, or it can lose its association with social norms (Kreps 1997). In some cases, adding monetary incentives can even crowd out psychological motivations. A number of papers, including Frey and Oberholzer-Gee (1997) and Gneezy and Rustichini (2000), find evidence that economic incentives can crowd out psychological motivations. For a survey of the literature on the impact of incentives in modifying agent behavior, see Gneezy et al. (2011). Also, see Kamenica (2012) for a detailed review of the literature on the psychology of incentives. To the best of my knowledge, the empirical work in this area remains qualitative, typically demonstrating the interdependence of the two types of motivation, but not identifying and quantifying the levels of psychological motivations. This chapter offers an analytical model and experimental procedure to empirically investigate the magnitude of psychological motivations arising from public recognition of agent performance. This methodology can be readily adapted to future studies in other contexts.

Finally, this chapter contributes to the behavioral and experimental economics literature by proposing a modification to the agent’s decision model. For example, in the marketing literature, Amaldoss and Jain (2005) study the effect of social comparisons in luxury goods markets, Cui et al. (2007) investigate the impact of fairness concerns in channel management, and Lim (2010) examines the effect of loss aversion in sales contests. In this chapter, the proposed modification allows me to quantify the extent of a behavioral bias due to psychological motivations. Following my method, one can augment theoretical models to estimate the extent of other types of deviations from observed or experimental data.

### 4.2 Experimental Design

In this section, I describe the design of a set of laboratory experiments, in which contestants make effort choices in sales contests. I choose the context of sales contests because of its well-established analytical framework and its practical relevance to the disclosure schemes of interest. I designed and conducted the experiments to observe and analyze how incentive structures and disclosure of contest outcomes may jointly affect contestants’ effort choices.

My experiments involved four-player contests in which the top two contestants earned prizes and the remaining two did not. The total prize payout was the same in all contests, but the difference between the prizes of the top two contestants varied. In high reward spread (HRS) contests, the first prize was nine times larger than the second prize, while in low reward spread (LRS) contests, the first and second
prizes were virtually equal. These were chosen not only to emphasize the difference between the high and low spreads, but also to represent situations that are managerially relevant. The high spread reflects cases in which there is one “true” winner, with a runner-up receiving little more than recognition. The low spread reflects cases in which multiple winners receive essentially the same reward, as is commonly the case with non-cash prizes such as President’s Club trips. For each reward spread, I applied four disclosure schemes, which are described below. Thus, I implemented a 2×4 experimental design with eight treatments in total.

I ran 16 sessions with 12 subjects in each session. Each subject participated in only one session, so there were 192 subjects in total. The sessions were run between March 2012 and February 2013 at McMaster University in Canada, with all subjects being students of the university. The experiments were programmed and conducted using the software z-Tree, developed by Fischbacher (2007). Each session consisted of 40 contest periods. In each period, the 12 subjects were randomly assigned into three 4-player groups.

To present the game in a relatable context, the subjects were asked to act as salespeople participating in a contest to generate revenue. The contest required each salesperson to choose their level of effort to sell an industrial product named “Product Beta”. The subjects were ranked within their 4-player groups based on the revenue they generated, and earned rewards based on their ranks. In each period, a subject was endowed with 100 points. She could use some or all of these points as effort to generate revenue, keeping the remainder as income. Suppose she used \( e \in \{1, 2, \ldots, 100\} \) points as effort to generate sales. She would then keep 100 – \( e \) points as income from that period and generate \( s(e) = 350 + \ln(e) + \varepsilon \) units of revenue, where \( \varepsilon \) was drawn from a logistic distribution with mean zero and variance \( \pi^2/3 \). For each subject, a new random term \( \varepsilon \) was independently drawn in each period.

Subjects chose their efforts simultaneously without knowing the identities of the other players in their group. From their chosen effort and their draw of the random term, each subject’s revenue for the period was calculated. The player who generated the highest revenue received a reward of \( R_1 \) points and the player who generated the second-highest revenue received a reward of \( R_2 \) points. The remaining two players did not receive any reward.\(^1\) A subject’s income from a period in which she used \( e \) points as effort was 100 – \( e + r \) points, where \( r \in \{R_1, R_2, 0\} \) represents her reward. As the effort cost directly enters the payoff function through a reduction in points, the effort cost can be considered a monetary cost.\(^2\)

The reward scheme was varied within each session. In half of the periods, subjects participated in high reward spread contests, in which \( R_1 \) equaled 360 and \( R_2 \) equaled 40. The other half were low reward spread contests, in which \( R_1 \) and \( R_2 \) equaled 204 and 196, respectively. To control for any order effects, the HRS contests came first in half of the sessions under each disclosure scheme and last in the other half. In a given period within a session, all players faced the same reward structure.

At the beginning of every session, each subject was assigned a unique username, which remained unchanged throughout the session. This username was of the form “Salesperson X” where X represents a letter from the English alphabet. A subject was identified and known to other players by this username. After each period, the results of the contest were announced to the contestants according to one of four disclosure schemes, each of which was employed in four experimental sessions. Under all of the regimes,\(^1\) Kalra and Shi (2001) show that the optimal number of winners in a sales contest should not exceed half the number of participants (unless necessary to induce participation, which is not a consideration in this experiment).\(^2\) As is common in the sales contest literature, I abstract away from other factors, such as heterogeneity in productivity among salespeople and non-contest incentive compensation (e.g., absolute incentives). This allows me to isolate and focus on the effects of contest design that are of primary interest.
each contestant learned their own reward (indicating whether they finished first, second, or in the bottom two). Under no disclosure, no further information was revealed. This provides a benchmark, as no disclosure best represents a standard one-shot contest game. Under winner disclosure, contestants also learned the identity of the winner of the first prize. Under partial disclosure, they learned the identities of the first and second prize winners, but not specifically who won which prize. Under full disclosure, they learned the specific identities of the first prize winner and the second prize winner and also learned the identities (but not the ranks) of the remaining two contestants who did not win a reward. A subject did not know the identities of the three other players she was participating with in a given period when choosing her effort. Moreover, a given subject experienced both reward-spread treatments, but only under a single disclosure scheme. As the usernames were somewhat mechanical-sounding and were chosen by me, subjects may not have identified with their usernames very strongly. Furthermore, recognition itself did not provide any possible monetary benefit, even in the long run. As a result, my recognition manipulation is potentially rather weak. Thus, any evidence of disclosure schemes affecting subject behavior would suggest quite a strong impact of public recognition in a real work place, in which people are closely attached to their identities and recognition may bring future benefits and opportunities.

After the 40 sales contest periods, each subject participated individually in a risk-attitude elicitation round with three periods. In each period, I elicited the subject’s certainty equivalent for a lottery. The subject was asked to report her willingness-to-pay (WTP) for a lottery that took a value of 20 points with probability $p$ and a value of 0 with probability $1 - p$. The values of $p$ in the three periods were 30%, 50%, and 80%, with the order varying across subjects to control for order effects. The subject reported an integer between 1 and 19 as her WTP and her earnings for the period were determined by the Becker-DeGroot-Marschak (1964) mechanism. An integer between 1 and 19 (inclusive) was chosen randomly with equal probability, independent of the subject’s reported WTP. If this integer was above the reported WTP, the subject received that many points as her earnings from the period. If it was below her reported WTP, the subject’s earnings were determined by the lottery. Thus, the subject’s unique weakly-dominant strategy was to truthfully report her certainty equivalent of the given lottery as her willingness to pay. I chose to use three different lotteries so that the measurements of subjects’ risk preferences are not specific to one particular lottery.

After the subjects participated in the contests and the lotteries, they completed a survey in which they reported some information about themselves, including their major, year of study, and experience with laboratory experiments. They also answered some questions about their playing strategy during the session. At the end of the survey, one contest from each reward spread and one lottery period were randomly chosen to determine the earnings of each subject in the session. Contest periods 1 to 5 and 21 to 25 were omitted from this selection, so that subjects could use those as practice periods for the two reward spreads. Total point earnings from the three selected periods — two contests and one lottery — were translated into Canadian dollars using an exchange rate of $1 per 15 points. Moreover, each subject earned $5 as a show-up fee. The subjects spent less than an hour and a half in the laboratory, including the reading of instructions, payment and debriefing. The average payment made to a subject was $25.71, paid in cash. Each subject was presented with a detailed instruction sheet, which included diagrams illustrating the logistic distribution and the logarithmic functions. The instructions were also verbally communicated using a recorded reading of the instruction sheet, with subjects listening on headphones. A sample of the instructions is presented in Appendix J.
4.3 Theoretical Model

In this section, I provide a theoretical model of contests in which agents have psychological motivations to exert effort, in addition to the economic motivation provided by the contest rewards. I do not focus on intricacies of optimal contest design, such as the relative efficiency of contests over other schemes (Lazear & Rosen, 1981; Nalebuff & Stiglitz, 1983) or the optimal prize structure (Kalra & Shi, 2001). Rather, the purpose of the model is to incorporate and identify the impact (positive or negative) of psychological motivations on effort choice in a simple competitive setting. I apply this model to analyze salespeople’s behavior in a sales contest based on my experimental design, incorporating the psychological effects of the disclosure scheme and the reward structure of the contest. My equilibrium analysis provides a closed-form solution that links chosen effort to this psychological effect.

4.3.1 General Model and Analysis

Consider a contest with four agents (denoted by \(i = 1, \ldots, 4\)), in which the ranking of the agents in the contest depends on the output they produce. The analysis can be extended easily to arbitrary \(N\) contestants. Agent \(i\) exerts effort \(e_i > 0\), which results in an output of \(s(e_i) + \epsilon_i\). The production function \(s\) is commonly known, identical for all four agents, and is increasing and non-convex in \(e_i\). Following the experiments, I assume that \(s(e) = K + \ln(e)\) for some positive constant \(K\) and the idiosyncratic random variable \(\epsilon_i\) is drawn from a logistic distribution function with mean 0 and variance parameter 1.\(^3\) The agents are compensated using a rank-order contest characterized by the reward structure \(R = R_1, \ldots, R_4\) where \(R_1\) is the prize awarded to the agent producing the \(j^{th}\)-highest level of output. Suppose the principal adopts a disclosure scheme, denoted by \(D\), which reveals the contest outcome to the contestants in a specific fashion. For an agent who has an initial wealth level of \(w_i\), expends effort \(e_i\), and earns a reward of \(r\), the net utility from the contest is denoted by \(U(w_i, R, e_i|R, D)\) where \(U\) is increasing in \(w_i\) and \(r\) and is decreasing in \(e_i\). The utility function \(U\) may include both economic and psychological payoffs from the contest.

The agent will choose \(e_i\) to maximize her expected utility, given by:

\[
\sum_{j=1}^{4} \Pr_j(e_i, e_{-i})U(w_i, R_j, e_i|R, D) \quad (4.1)
\]

where \(\Pr_j(e_i, e_{-i})\) denotes the probability that agent \(i\) attains rank \(j\) when she expends effort \(e_i\), and the efforts of the other agents are represented by the vector \(e_{-i}\). Thus, if agent \(i\) chooses effort \(e_i^*\), then

\[
e_i^* = \arg\max_{e_i} \sum_{j=1}^{4} \Pr_j(e_i, e_{-i})U(w_i, R_j, e_i|R, D) \quad (4.2)
\]

I restrict attention to symmetric equilibria, in which each agent expends the same amount of effort

\(^3\)The logistic distribution is frequently used in the literature as a good representative of bell-shaped distributions. Like the commonly-assumed normal distribution, the logistic distribution is symmetric and displays a central tendency in density. The density function with mean 0 and variance parameter 1 is \(f(\epsilon) = \frac{\exp(-\epsilon)}{(1+\exp(-\epsilon))^2}\) with variance \(\pi^2/3\).
The first-order condition is given by:

\[
\sum_{j=1}^{4} \left( \Pr_j(e_i, e^*_i) \frac{\partial U(w_i, R_j, e_i|R, D)}{\partial e_i} + \frac{\partial \Pr_j(e_i, e^*_i)}{\partial e_i} U(w_i, R_j, e_i|R, D) \right) \bigg|_{e_i=e^*_i} = 0
\]  

(4.3)

The second-order condition to ensure that \( e^* \) maximizes the expected utility is also standard.

Recall that, in the experiment, a subject was endowed with 100 points in every period, from which she expended effort \( e \) in the contest. Thus, her earnings from the contest equaled \( 100 - e + R_j \) if she won a reward of \( R_j \). This provided economic utility of \( u(100 - e + R_j) \) for some utility function \( u \) that is increasing and weakly concave. Moreover, \( u \) is at least twice differentiable and the values of the derivatives depend on the total monetary holding, \( w + r - e \).

Next, I model an agent’s psychological value from the contest, which potentially depends on both the reward structure and the disclosure scheme. I assume that the monetary and psychological payoffs are additively separable and propose the following utility specification:

\[
U(w, R_j, e|R, D) = u(w - e + R_j) + o^R_j + p^{R,D}_j
\]  

(4.4)

Equation 4.4 indicates that the source of the psychological values can be twofold — arising from a sense of one’s own achievement of a rank (denoted by \( o^R_j \), “o” indicating “own”) and from public disclosure of the rank (denoted by \( p^{R,D}_j \), “p” indicating “peer-generated”). These psychological values can be positive or negative. The \( o^R_j \) and \( p^{R,D}_j \) parameters can be thought of as the net effects of a (potentially complex) combination of psychological factors. While I can speculate on what those factors might be, I will restrict my focus to the net effects, which are sufficient for providing managerial direction on contest design.

Under all eight treatments in my experiment, a subject learns whether she was ranked first, second, or among the bottom two contestants. Hence, I assume that the value from knowing one’s own rank (\( o^R_j \)) does not depend on the disclosure scheme, but it may depend on the reward structure. Furthermore, since the ranking of the bottom two contestants is not known to any of the contestants and \( R_3 = R_4 \), I assume that \( o^R_3 = o^R_4 \) for any \( R \).

On the other hand, the peer-generated psychological value (\( p^{R,D}_j \)) depends on both the reward structure and the disclosure scheme. I assume that if the identity of an agent is not publicly disclosed, then she does not receive any peer-generated psychological value. Thus, under the no disclosure scheme, \( p^{R,ND}_j = 0 \) for all \( j \). Under the winner disclosure scheme, only the identity of the top contestant is publicly recognized. Thus, only the top contestant may derive peer-generated psychological value from disclosure, so I assume that \( p^{R,WD}_2 = p^{R,WD}_3 = p^{R,WD}_4 = 0 \). Under the partial disclosure scheme, the identities of the top two contestants, but not their rankings, are publicly announced, creating psychological value from public recognition. Hence, I assume that \( p^{R,PD}_1 = p^{R,PD}_2 \) and \( p^{R,PD}_3 = p^{R,PD}_4 = 0 \). Finally, under the full disclosure scheme, the rankings (winner, runner-up, and bottom two) of all contestants are publicly announced. As a result, a significant peer-generated psychological value can exist for all ranks.

The above assumptions on the psychological value parameters are summarized below:

**Assumption 1.** Peer-generated psychological values (\( p^{R,D}_j \)) have the following properties:

---

4This model can be thought of as a more general version of the privilege contest model in Schroyen and Treich (2013).
1. Under no disclosure, \( p_{1,ND}^R = p_{2,ND}^R = p_{3,ND}^R = p_{4,ND}^R = 0 \).

2. Under winner disclosure, \( p_{2,WD}^R = p_{3,WD}^R = p_{4,WD}^R = 0 \).

3. Under partial disclosure, \( p_{1,PD}^R = p_{2,PD}^R = p_{3,PD}^R = p_{4,PD}^R = 0 \).

4. Under full disclosure, \( p_{3,FD}^R = p_{4,FD}^R \neq 0 \).

Furthermore, I can develop some intuitive hypotheses about the non-zero peer-generated psychological values under different disclosure schemes. I expect that being recognized as a contest winner will provide a participant with positive psychological value. In other words, \( p_{1,WD}^R \) and \( p_{1,PD}^R \) should be positive. Recognition under WD informs the other participants that the winner finished in first place, while recognition under PD informs them only that she finished in the top two. Thus, the winner’s recognition under the WD scheme is more exclusive and valuable, so I expect \( p_{1,WD}^R \) to be greater than \( p_{1,PD}^R \). This discussion is summarized in the following hypothesis:

**Hypothesis 1.** For a given reward structure \( R \), \( p_{1,WD}^R > p_{1,PD}^R > 0 \).

My model extends Kräkel (2008), which considers psychological values from winning (losing) in a two-person contest. First, there is only one winner in Kräkel (2008), but I allow for multiple winners and different psychological values from achieving the first and second ranks. Second, Kräkel (2008) models only two contestants, so each player always knows the other’s status. The information structure in my 4-person contest is much richer. I introduce four types of disclosure schemes and permit disclosure-specific psychological values.

Next I examine the optimal choice of effort in these contests. The analytical method follows Kalra and Shi (2001). Given the assumptions on \( s(e) \), the distribution of \( \varepsilon \), incentive plan \( R \), and disclosure scheme \( D \), equilibrium effort level \( e^* \) can be determined by the following equation:

\[
\sum_{j=1}^{4} \left( (u(100 - e^* + R_j) + \sigma_j^R + p_{j,PD}^R) \frac{5 - 2j}{20e^*} + \frac{1}{4} u'(100 - e^* + R_j) \right) = 0
\]

This can be rewritten as:

\[
3 \left( u(100 - e^* + R_1) + \sigma_1^R + p_{1,PD}^R \right) + u(100 - e^* + R_2) + \sigma_2^R + p_{2,PD}^R - 4 \left( u(100 - e^* + \sigma_3^R + p_{3,PD}^R) \right) = 5e^* \sum_{j=1}^{4} u'(100 - e^* + R_j) \quad (4.5)
\]

### 4.3.2 Risk-Neutral Agents

Here, I consider the special case in which the agents are risk neutral; that is, \( u(x) = x \). Following equation 4.5, the contestants’ optimal effort level is given by the following equation:

\[
e^* = \frac{3R_1 + R_2 + 3\sigma_1^R + \sigma_2^R - 4\sigma_3^R + 3p_{1,PD}^R + p_{2,PD}^R - 4p_{3,PD}^R}{20}
\]

Now, given Assumption 1, I can characterize the optimal effort level under each disclosure scheme. The optimal effort choice under the no disclosure, winner disclosure, partial disclosure, and full disclosure
implies that the equilibrium efforts \( e^R \) values are unknown and hence I cannot make inferences about the values of \( e^R \). Lastly, by equations 4.7 and 4.8, if \( e^R \) is found to be greater than \( e^R \), then that hypothesis is confirmed. Similarly, according to equations 4.6 and 4.8, if \( p^R \) is found to be greater than \( p^R \), then that hypothesis is confirmed. Conversely, if \( p^R \) is found to be greater than \( p^R \), then the converse again holds. Lastly, by equations 4.7 and 4.8, if \( e^R \) and \( e^R \) are equal, then \( p^R = \frac{1}{3} p^R \), which would confirm the hypothesis that \( p^R > p^R \). I summarize the above in the following proposition:

**Proposition 8.** With risk-neutral agents, for a given reward structure \( R \),

\[
\begin{align*}
e^R_{ND} &= \frac{3R_1 + R_2 + 3o^R_1 + o^R_2 - 4o^R_3}{20} \quad (4.6) \\
e^R_{WD} &= \frac{3R_1 + R_2 + 3o^R_1 + o^R_2 - 4o^R_3 + 3p^R_{WD}}{20} \quad (4.7) \\
e^R_{PD} &= \frac{3R_1 + R_2 + 3o^R_1 + o^R_2 - 4o^R_3 + 4p^R_{PD}}{20} \quad (4.8) \\
e^R_{FD} &= \frac{3R_1 + R_2 + 3o^R_1 + o^R_2 - 4o^R_3 + 3p^R_{FD} + p^R_{FD} - 4p^R_{FD}}{20} \quad (4.9)
\end{align*}
\]

From these equations, it is clear that Hypothesis 1 has direct implications for the equilibrium efforts and vice versa. First, according to equations 4.6 and 4.7, the hypothesis \( p^R_{WD} > 0 \) implies that equilibrium efforts \( e^R_{WD} \) should be greater than \( e^R_{ND} \). Conversely, if \( p^R_{WD} > 0 \) is found to be greater than \( e^R_{ND} \), then that hypothesis is confirmed. Similarly, according to equations 4.6 and 4.8, \( p^R_{PD} > 0 \) implies that the equilibrium efforts \( e^R_{PD} \) should be greater than \( e^R_{ND} \) and the converse again holds. Lastly, by equations 4.7 and 4.8, if \( e^R_{PD} \) and \( e^R_{WD} \) are equal, then \( p^R_{WD} = \frac{1}{3} p^R_{PD} \), which would confirm the hypothesis that \( p^R_{WD} > p^R_{PD} \). I summarize the above in the following proposition:

**Proposition 8.** With risk-neutral agents, for a given reward structure \( R \),

\[
\begin{align*}
a) \ e^R_{WD} &> e^R_{ND} \text{ if and only if } p^R_{WD} > 0 \\
b) \ e^R_{PD} &> e^R_{ND} \text{ if and only if } p^R_{PD} > 0 \\
c) \ e^R_{PD} &> e^R_{WD} \text{ if and only if } 4p^R_{PD} > 3p^R_{WD}
\end{align*}
\]

Parts a) and b) indicate that efforts will be higher under winner and partial disclosures, respectively, than under no disclosure if and only if there is a positive psychological impact from being publicly recognized as a winner. This is rather intuitive, as the effect on utility of that positive impact is equivalent to that of an increase in the value of the first prize (under WD) or the first and second prizes (under PD). Part c) indicates that effort under partial disclosure will be higher than under winner disclosure unless the psychological effect of being recognized alone as the first-place finisher (under WD) is greater than that of being recognized as one of the top two finishers (under PD) by a factor of at least \( 4/3 \). The intuition behind that result is that PD is more likely to result in some form of recognition (and resulting psychological benefit), so it induces more effort unless the effect of shared recognition is substantially ‘diluted’ relative to that of exclusive recognition.

At this point, effort levels under full disclosure cannot be compared with those under any other disclosure scheme. I can hypothesize the signs of parameters \( p^R_{PD} \), \( p^R_{PD} \), and \( p^R_{PD} \), but their relative values are unknown and hence I cannot make inferences about the values of \( e^R_{PD} \) based on equation 4.9 without multiple additional assumptions. In essence, the multitude of psychological value parameters limits the scope of inferences.

As both own and peer-generated psychological values can be different when comparing two incentive structures for a given disclosure scheme, comparing across reward spread treatments is more difficult. Hence, I do not present any formal hypotheses in that context. In a symmetric equilibrium, all agents exert the same level of effort. Their rankings, then, will effectively depend on luck and one may expect that a more equitable contest will provide a greater level of psychological motivation. If this effect is strong enough, then agents may exert more effort under the low reward spread treatments.
Chapter 4. Managing psychological motivations in contests

4.4 Experimental Results

In this section, I analyze the data gathered in the experimental sessions using the model described above. First, I examine the effort levels chosen by the subjects, to identify differences among the eight treatments (two reward structures × four disclosure schemes). Using no disclosure as a baseline, I am interested in observing whether subjects choose higher effort levels under the partial disclosure and winner disclosure schemes. Second, I use the data to estimate the psychological value parameters and to test the predictions from Hypothesis 1 about the relationships between the peer-generated psychological values.

All of the data analysis omits contest periods 1-5 and 21-25 from each session, which are treated as practice periods because the subjects’ earnings from the sessions were not dependent on their performance in those rounds. As I do not find any effect of the order of the reward structures, I pool the data from all four sessions under each disclosure scheme.

4.4.1 Effort Decisions and Psychological Motivations

I begin by examining the subjects’ mean effort levels under each treatment, presented in Table 4.1. It is evident that subjects chose significantly higher effort levels under the winner disclosure and partial disclosure schemes than under the no disclosure scheme for both reward structures (significant at 5% or less). However, there was no significant difference in effort between winner disclosure and partial disclosure under either reward structure. By Proposition 8, this confirms that the experimental results are consistent with the predictions made in Hypothesis 1. Even my weak recognition manipulation results in significant differences in effort choice, indicating that psychological motivations have a significant impact on sales agents’ effort decisions.

<table>
<thead>
<tr>
<th>Mean Effort</th>
<th>No Disclosure</th>
<th>Winner Disclosure</th>
<th>Partial Disclosure</th>
<th>Full Disclosure</th>
<th>All Disclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High Reward Spread</td>
<td>55.0</td>
<td>60.3</td>
<td>59.2</td>
<td>58.5</td>
<td>58.3</td>
</tr>
<tr>
<td></td>
<td>(34.3)</td>
<td>(29.8)</td>
<td>(32.4)</td>
<td>(32.9)</td>
<td>(32.4)</td>
</tr>
<tr>
<td>Low Reward Spread</td>
<td>58.8</td>
<td>65.2</td>
<td>64.3</td>
<td>58.4</td>
<td>61.7</td>
</tr>
<tr>
<td></td>
<td>(33.5)</td>
<td>(26.3)</td>
<td>(28.6)</td>
<td>(31.0)</td>
<td>(30.1)</td>
</tr>
<tr>
<td>Observations</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>2,880</td>
</tr>
<tr>
<td>p-value for $H_0 : HRS - LRS = 0$</td>
<td>3.5%</td>
<td>0.1%</td>
<td>0.2%</td>
<td>99.1%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Notes: Standard deviations in parentheses
Sample sizes apply to each reward spread

Table 4.1: Summary of individual effort choice

As discussed following Proposition 8, the theoretical model does not offer any predictions about the relative effort levels under full disclosure and the other disclosure schemes without further assumptions. However, the experimental data offers observations about these comparisons and about the relative effort levels chosen under the high and low reward spreads. Continuing to use no disclosure as a baseline, effort tends to be higher under the full disclosure scheme when the reward spread is high (p-value = 5.4%), but not when the rewards are spread more evenly.

Lastly, higher effort is observed under the low reward spread than under the high reward spread under
all disclosure schemes, with the exception of full disclosure.\footnote{As a robustness check, I also analyze the impact of the reward spread on effort choice using a panel regression that controls for individual fixed effects. Observations regarding the impact of the reward spread remain unchanged.}

In summary, my experimental results show that effort levels are higher when the contest winner or winners (but not the losers) are publicly recognized than when they are not. This offers supporting evidence for Hypothesis 1, that these public recognition programs generate positive psychological motivations. Furthermore, the data shows that effort is generally higher when contest rewards are more evenly distributed. However, this effect is dependent on the extent to which the contest results are publicly disclosed.

### 4.4.2 Estimating Psychological Value Parameters

I can further use the experimental results to estimate some of the psychological value parameters ($o$’s and $p$’s) introduced above. Specifically, I can substitute the subjects’ effort choices under different experimental treatments into the appropriate equations 4.6 ~ 4.9 to estimate these parameters. First, for each contest period played under the no disclosure condition, I insert each subject’s effort choice and the appropriate rewards $R_1$ and $R_2$ (according to the reward scheme for the period) into equation 4.6 to calculate the value of $3o^R_1 + o^R_2 - 4o^R_3$ (which I will call the o-index) for that subject and period. I then take the mean of these values as the estimate of the o-index under each reward scheme. The o-index represents the net effect of an agent’s psychological motivations on her effort. As noted earlier, under the no disclosure condition, there is no public disclosure of contest outcomes, and therefore the psychological motivation represents a baseline level resulting only from the anticipation of learning her own rank. This baseline psychological motivation is assumed to be the same across all disclosure schemes.

Second, for each period played under the winner disclosure scheme, I substitute each subject’s effort choice, the rewards $R_1$ and $R_2$, and the estimate of the o-index into the equation for $e_{R,WD}$. This allows me to calculate the value of $p_{1,WD}^R$ for that subject and period. I then take the means of these values to estimate this psychological value parameter $p_{1,WD}^R$ under each reward structure. Following an almost-identical procedure, I can use the data under partial disclosure to estimate the psychological value parameter $p_{1,PD}^R$ under each reward structure.

Lastly, I take a similar approach for the periods played under full disclosure to estimate the value of $3p_{1,FD}^R + p_{2,FD}^R - 4p_{3,FD}^R$ (the FD-index) under each reward scheme. The FD-index represents the net effect of peer-generated psychological motivations on a participant’s effort under FD.

The psychological value index and parameter estimates (means) are summarized in Table 4.2. Since the index / parameter values do not appear to follow a normal distribution (i.e., they are not symmetric or unimodal), the table also includes medians and significance results based on the non-parametric sign test.

Table 4.2 shows that the estimates of $p_{1,WD}^R$ and $p_{1,PD}^R$ are positive and significant for both reward structures. This indicates that both winner disclosure and partial disclosure induced positive peer-generated psychological values and motivations. The participants desired the opportunity to be publicly recognized for their achievements. Furthermore, there is significant evidence (not shown in Table 4.2) that $p_{1,WD}^R$ is greater than $p_{1,PD}^R$. The Mann-Whitney test rejects the hypothesis (with $p < 1\%$ under LRS and $p < 5\%$ under HRS) that these estimates are from the same distribution. This suggests that being recognized as the sole winner provides greater peer-generated psychological value than being recognized as one of the top two finishers. This could be because disclosing the top two without dis-
tistinguishing between them dilutes the value of finishing in first place. These results support Hypothesis 1.

Table 4.2 also shows that the $o$-index, the net effect on effort of learning one’s own rank, is not significant under the high reward spread, but is quite large and highly significant under the low reward spread. This indicates that, apart from the peer-generated psychological motivations induced by public disclosure, there may be other sources of psychological motivations. However, such motivations seem to be crowded out when the reward spread is high. In other words, the psychological motivation provided by learning one’s own rank when the reward spread is low appears to be undermined by the economic incentives of the high reward spread.

Lastly, the net effect of full disclosure on effort (the $FD$-index) is positive under the high reward spread, but is not significant under the low reward spread. Thus, when the reward spread is low, fully disclosing the contest outcome does not lead to any significant peer-generated psychological motivation. However, when the reward spread is high, full disclosure induces positive and significant peer-generated psychological motivations. The $FD$-index ($3p_{R,FD}^1 + p_{R,FD}^2 - 4p_{R,FD}^3$) measures the net effect of positive emotions from being recognized as a winner and negative emotions from being revealed as a loser. The net effect mainly depends on peer-generated psychological values from being the top winner ($p_{R,FD}^1$) and a non-winner ($p_{R,FD}^3$). As full disclosure includes a distinction between the winner and runner-up, it is likely that the peer-generated psychological value for top winner ($p_{R,FD}^1$) is much greater under the high reward spread (when the winner gains far more than the runner-up) than under the low reward spread.

4.5 Quantifying Psychological Motivations: An Indirect Approach

So far, I have taken a direct approach to modeling and estimating psychological motivations and their impacts on selling effort. Using the no disclosure scheme as a baseline, I am able to identify and estimate the peer-generated psychological values induced by the winner disclosure and partial disclosure schemes, to test hypotheses regarding these peer-generated psychological values, and to demonstrate the effects of psychological motivations on selling efforts. However, my analysis also uncovers a critical drawback of the direct approach: the plethora of psychological value parameters leads to limited identification. Specifically, for the psychological values of knowing one’s own result ($o_R$), I can estimate an index (the $o$-index) but cannot separately identify each of the individual parameters. Similarly, for the peer-generated
psychological values under the full disclosure scheme, I can estimate an index (the FD-index), but not the psychological value of recognition for reaching ranks 1, 2, and 3 or 4. These identification issues will become worse as the number of contestants increases or as the contest becomes asymmetric (e.g., Chen et al., 2011). Even for the partial disclosure and winner disclosure schemes, identifying the peer-generated psychological values relies on the set of restrictions imposed in Assumption 1.

In this section, I present a novel, indirect approach to studying the impact of psychological motivations. The main objective of this approach is to estimate a single, unique psychological motivation parameter for each regime characterized by a reward structure and a disclosure scheme. Instead of directly modeling psychological values through an agent’s utility function, here I capture the psychological motivations through a change to the agent’s marginal cost of effort. More specifically, I assume that all psychological payoffs from exerting effort \( e \) in a contest characterized by \( R \) and \( D \) can be summarized by a parameter, \( \gamma_{R,D} \), representing the marginal benefit (or cost) from exerting each unit of effort. Thus, the psychological payoff from exerting effort \( e \) is \( \gamma_{R,D} \times e \). If \( \gamma_{R,D} \) is positive, then there is psychological benefit from exerting effort in addition to the economic benefit; otherwise, there is a psychological cost.

The psychological component is not outcome-specific in this formulation, as it represents the agent’s net expected psychological payoff when she chooses her effort level. Nevertheless, an agent’s effort directly affects the probability distributions of different contest outcomes, thereby indirectly affecting her net expected psychological payoff. When a contest’s reward or disclosure scheme changes, the psychological payoffs from different outcomes change. As effort directly affects the probability distribution of different outcomes, such a change in the contest design (either in reward or disclosure scheme) can affect the net marginal return from exerting effort, effectively making effort exertion more or less costly. Hence, modeling psychological motivation through a change in effort cost indeed captures changes in the outcome-dependent psychological payoffs, albeit indirectly. Importantly, modeling psychological motivation with a single parameter makes the model tractable and parsimonious. This benefit becomes more salient when the number of agents, \( N \), and hence the number of psychological value parameters resulting from the direct approach, described in Sections 4.3 and 4.4, increases.

The value of the psychological motivation parameter \( \gamma_{R,D} \) is specific to reward structure \( R \) and disclosure scheme \( D \). A change in the reward distribution or the adoption of a different disclosure scheme could change the level of psychological motivation, as suggested by the experimental results shown in Section 4.4. Thus, the total payoff from earning a reward of \( r \) by exerting effort \( e \) in a contest characterized by \( R \) and \( D \) is:

\[
u(w + r - e) + \gamma_{R,D} \times e\]

Hence, the agent will choose \( e_i \) to maximize her expected utility, given by:

\[
\sum_{j=1}^{N} Pr_j(e_i, e_{-i}) u(w + r - e) + \gamma_{R,D} \times e_i
\]

My analysis approach is similar to that in Section 4.4. I restrict attention to symmetric equilibria, in which each agent expends the same amount of effort \( (e_k = e^*, \forall k) \). The first-order condition is given by:

\[
\sum_{j=1}^{N} \left( Pr_j(e_i, e_{-i}) \frac{\partial u(w + R_j - e_i)}{\partial e_i} + \frac{\partial Pr_j(e_i, e^*)}{\partial e_i} u(w + R_j - e_i) \right) \bigg|_{e_i = e^*} + \gamma_{R,D} = 0
\]
Therefore,

\[ \sum_{j=1}^{N} \left( \frac{N - 2j + 1}{e^* N (N + 1)} u(w + R_j - e^*) - \frac{u'(w + R_j - e^*)}{N} \right) + \gamma_{R,D} = 0 \]

The above equation can be rewritten as follows:

\[ e^* = \frac{\sum_{j=1}^{N} \left( \frac{N - 2j + 1}{N (N + 1)} u(w + R_j - e^*) \right)}{\sum_{j=1}^{N} \frac{u'(w + R_j - e^*)}{N} - \gamma_{R,D}} \]

Equation 4.10 provides the equilibrium effort, given exogenously-determined reward structure \( R \) and disclosure scheme \( D \). Several features of this approach are worth noting. First, the indirect approach adopted here follows the property that at the equilibrium effort level, the marginal value of incremental effort equals the marginal cost. An increase in equilibrium effort due to psychological motivations can be modelled either through an increase in marginal value or a decrease in marginal cost. The direct approach in Sections 4.3 and 4.4 models changes in the marginal values of outcomes, and the indirect approach in this section models changes in the marginal cost of effort. Second, a benefit of the indirect approach is that if an agent’s utility function and initial wealth level from a given contest \( (R, D) \) are known, then I can back out her psychological motivation parameter \( \gamma_{R,D} \) from her optimal effort choice. I summarize this in Lemma 6.

**Lemma 6.** Suppose agents play symmetric strategies. If an agent’s optimal effort choice is observed and her utility function known, then the psychological motivation parameter is identified.

**Proof.** Equation 4.10 characterizes the unique symmetric equilibrium. It implies that

\[ \gamma_{R,D} = \sum_{j=1}^{N} \left( \frac{u'(w + R_j - e^*)}{N} - \frac{N - 2j + 1}{N (N + 1)} e^* u(w + R_j - e^*) \right) \]

In my experiment, \( w \) equals 100. If the utility function \( u \) and the agent’s effort choice from contest \( (R, D) \) are known, then all parameters on the right-hand side are known. Thus, the psychological motivation parameter \( \gamma_{R,D} \) can be calculated easily.

Next, I use Lemma 6 to infer the psychological motivation parameter values from my empirically-observed effort decisions. My goals are to characterize the effect of the contest design on the net impact of psychological motivations and to discuss the implications of such results on the optimal design of contests, including both the reward structure and the disclosure scheme.

### 4.5.1 Estimation of Psychological Motivation Parameters

Using Lemma 6 and the observed effort choices in my experiments, I now estimate \( \gamma_{R,D} \). To do so, however, I need to make an assumption about each agent’s economic utility function, \( u \). As is common in the literature, I first assume that all subjects are risk neutral, so \( u(x) = x \). I then relax that assumption, allowing subjects to have heterogeneous levels of risk aversion.
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Risk Neutrality

Under risk neutrality, an agent’s effort choice in the symmetric equilibrium of my experiment will be 
\[ e^* = \frac{3R_1 + R_2}{20(1 - \gamma_{R,D})}. \]
Therefore, the psychological motivation parameter in my experiment is given by the following equation: 
\[ \gamma_{R,D} = 1 - \frac{3R_1 + R_2}{20e^*}. \]
I use this to calculate the value of \( \gamma_{R,D} \) for each subject in each period. Assuming that each observed \( \gamma_{R,D} \) equals the true value of the parameter plus a zero-mean error term, I can estimate the psychological motivation parameter for each contest characterized by \( R \) and \( D \). One issue, however, is that the range of possible estimates skews strongly to the left. For example, under the \( HRS \), the maximum, median, and minimum efforts (100, 50.5 and 1) give estimates of 0.44, -0.11 and -55, respectively. As a result, a small set of observations with large negative values of \( \gamma_{R,D} \) pulls the mean of the estimated parameter far below the median under each treatment. Therefore, rather than considering the mean estimates to examine how psychological motivation differs across treatments, I present the medians and use the Mann-Whitney two-sample statistic to test the hypothesis that the estimates under different treatments are from the same distribution. More specifically, the test indicates whether one set of estimates tends to have higher values than the other.

Table 4.3 presents the median estimated value of \( \gamma_{R,D} \) under each treatment. The psychological motivation parameters mirror the mean individual effort choices directionally across disclosure schemes. For example, I find significantly higher values of the psychological motivation parameter under the winner disclosure and partial disclosure schemes than under the no disclosure scheme for both reward structures. Using no disclosure as a baseline, psychological motivation tends to be higher under full disclosure when the reward structure is \( HRS \), but not when it is \( LRS \). Also, across all disclosure schemes, the \( LRS \) invokes significantly higher values of the psychological motivation parameter than the \( HRS \). This suggests that psychological motivation, in part, drives the higher efforts that are observed under the \( LRS \) (as seen in Section 4.4), under the assumption that subjects are risk neutral. As an alternative to using the Mann-Whitney test, I can compare the means of the skewed distributions of parameter estimates by dropping extremely low values of \( \gamma_{R,D} \) from each treatment. For example, if the lowest 10% of values of \( \gamma_{R,D} \) are dropped from each treatment, the mean values of the parameter and the comparisons of means across treatments are qualitatively the same as those in Table 4.3.\(^6\)

<table>
<thead>
<tr>
<th>Median ( \gamma_{R,D} )</th>
<th>No Disclosure</th>
<th>Winner Disclosure</th>
<th>Partial Disclosure</th>
<th>Full Disclosure</th>
<th>All Disclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Reward Spread</td>
<td>-0.02</td>
<td>0.07**</td>
<td>0.07***</td>
<td>0.07***</td>
<td>0.07***</td>
</tr>
<tr>
<td>Low Reward Spread</td>
<td>0.34**</td>
<td>0.40***</td>
<td>0.42***</td>
<td>0.33***</td>
<td>0.38***</td>
</tr>
<tr>
<td>Observations</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>720</td>
<td>2,880</td>
</tr>
</tbody>
</table>

\( p \)-value for \( H_0: HRS - LRS = 0 \)  
\( 0.0\%          \)  \( 0.0\%          \)  \( 0.0\%          \)  \( 0.0\%          \)  \( 0.0\%          \)

Notes: ***, **, and * represent statistical significance at 1%, 5%, and 10%, respectively, based on sign test. \( p \)-values are based on Mann-Whitney test.

Sample sizes apply to each reward spread.

Table 4.3: Median of estimated psychological motivation parameter (\( \gamma_{R,D} \)) under risk neutrality

\(^6\)As an example, for contests with a high reward spread and full disclosure, the lowest 10% of \( \gamma_{HRS,FD} \) take values of -4.6 and lower.
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Risk Aversion

Above, I have presented the psychological motivation parameter estimates assuming risk neutrality. This assumption, which substantially simplifies the analysis, can be restrictive. Now I extend the analysis by relaxing the risk-neutrality assumption. In the experiment, I included a risk-attitude elicitation round, allowing me to measure the individual-level risk preferences of the subjects.

In the risk-attitude elicitation round, I elicited each subject’s certainty equivalent for each of three lotteries, using the Becker-DeGroot-Marschak mechanism. A subject’s reported willingness-to-pay, or certainty equivalent, for a lottery that took a value of 20 points with probability $p$ and a value of 0 with probability $1 - p$ is used to estimate a CRRA risk coefficient, as follows: Suppose a subject’s preference can be characterized by the utility function $u(x) = x^{1-\rho}$ and the subject reports a certainty equivalent of $c$ for that lottery. Then, $p 20^{1-\rho} = c^{1-\rho}$, implying that $\rho = \frac{\ln(20) + \ln(p) - \ln(c)}{\ln(20) - \ln(c)}$.

I estimate $\rho$ this way for each subject, for each of the three lotteries. Since I am interested in identifying the effects of risk aversion, I use the maximum of each subject’s three estimates (i.e., the most risk-averse estimate) to represent their risk attitude for my analysis.

Table 4.4 summarizes these risk coefficients for all of the experimental subjects.

<table>
<thead>
<tr>
<th>Risk coefficient ($\rho_i$)</th>
<th>No Disclosure</th>
<th>Winner Disclosure</th>
<th>Partial Disclosure</th>
<th>Full Disclosure</th>
<th>All Disclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.15</td>
<td>0.25**</td>
<td>0.23**</td>
<td>0.28***</td>
<td>0.23***</td>
</tr>
<tr>
<td></td>
<td>(0.89)</td>
<td>(0.73)</td>
<td>(0.65)</td>
<td>(0.43)</td>
<td>(0.69)</td>
</tr>
<tr>
<td>Median</td>
<td>0.31***</td>
<td>0.45***</td>
<td>0.37***</td>
<td>0.37***</td>
<td>0.37***</td>
</tr>
<tr>
<td>Observations</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>48</td>
<td>192</td>
</tr>
</tbody>
</table>

Notes: Standard deviations in parentheses
***, **, and * represent statistical significance at 1%, 5%, and 10%, respectively, using t-test for the means and sign test for the medians

Table 4.4: Summary of individual CRRA risk coefficients

Both the mean and the median values of the risk coefficients for all subjects together are significantly positive at the 1% level, indicating that subjects were largely risk-averse. Although the mean value of the coefficient is not significant for the no disclosure scheme, there are no statistically-significant differences in either the median or mean values across disclosure schemes. This is not surprising, given that subjects were assigned to them randomly. Furthermore, under my experimental parameters, the range of possible values of $\rho_i$ has its maximum at 0.93 and its minimum at -22.5, so it is not surprising that the mean value is below the median under each disclosure scheme. Since the theory developed in Section 4.3 applies only to risk-averse and risk-neutral individuals, I drop subjects with negative risk coefficients (i.e., those who are risk-seeking) from the empirical analysis presented below.

Before examining the psychological motivation parameters under risk aversion, I first re-visit the mean effort levels chosen by the subjects under each treatment. This is to ensure that restricting the sample to risk-neutral and risk-averse subjects does not change any of the key observations from Table 4.1. Table 4.5 confirms that the directional results discussed in Section 4.4 continue to hold.

Next, I estimate the psychological motivation parameters under the assumption that subjects are heterogeneously risk averse. Suppose again that each agent’s utility can be described by a CRRA utility function. That is, $u_i(x) = x^{1-\rho_i}$, where $\rho_i$ denotes agent $i$’s CRRA risk coefficient, as discussed above.

---

7The results do not change qualitatively if I use another function of the three risk-aversion parameters instead.
Table 4.5: Summary of individual effort choice excluding risk-seeking subjects

<table>
<thead>
<tr>
<th>Mean Effort</th>
<th>No Disclosure</th>
<th>Winner Disclosure</th>
<th>Partial Disclosure</th>
<th>Full Disclosure</th>
<th>All Disclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Reward Spread</td>
<td>53.5</td>
<td>60.0</td>
<td>60.1</td>
<td>58.6</td>
<td>58.1</td>
</tr>
<tr>
<td></td>
<td>(34.5)</td>
<td>(30.2)</td>
<td>(31.5)</td>
<td>(32.1)</td>
<td>(32.2)</td>
</tr>
<tr>
<td>Low Reward Spread</td>
<td>58.2</td>
<td>63.7</td>
<td>63.0</td>
<td>58.1</td>
<td>60.8</td>
</tr>
<tr>
<td></td>
<td>(34.1)</td>
<td>(26.9)</td>
<td>(28.5)</td>
<td>(30.4)</td>
<td>(30.2)</td>
</tr>
<tr>
<td>Observations</td>
<td>600</td>
<td>615</td>
<td>630</td>
<td>645</td>
<td>2,490</td>
</tr>
<tr>
<td>p-value for</td>
<td>1.7%</td>
<td>2.2%</td>
<td>8.8%</td>
<td>77.5%</td>
<td>0.3%</td>
</tr>
<tr>
<td>$H_0 : HRS - LRS = 0$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Standard deviations in parentheses
Sample sizes apply to each reward spread

Table 4.6 presents the median estimated values of $\gamma_{R,D}$ under CRRA for each treatment. First, although the values look relatively small, all are significant at 1% using the sign test, with at least 66% of the estimated parameter values greater than zero under each treatment, except for the no disclosure-HRS combination (57%). Second, comparisons of $\gamma_{R,D}$ across disclosure schemes yield results similar to...
those under risk neutrality, although somewhat weaker once risk aversion is accounted for. Specifically, there is no longer a significant difference in the psychological motivation parameters between the no disclosure and winner disclosure schemes under the HRS. One possible explanation for this result is that a high reward spread naturally creates a winner-take-all feeling and, as a result, adding a winner disclosure can become redundant.

<table>
<thead>
<tr>
<th>Median $\gamma_{R,D}$</th>
<th>No Disclosure</th>
<th>Winner Disclosure</th>
<th>Partial Disclosure</th>
<th>Full Disclosure</th>
<th>All Disclosures</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Reward Spread</td>
<td>0.01***</td>
<td>0.01***</td>
<td>0.03***</td>
<td>0.02***</td>
<td>0.01***</td>
</tr>
<tr>
<td>Low Reward Spread</td>
<td>0.03***</td>
<td>0.03***</td>
<td>0.04***</td>
<td>0.03***</td>
<td>0.03***</td>
</tr>
<tr>
<td>Observations</td>
<td>600</td>
<td>615</td>
<td>630</td>
<td>645</td>
<td>2,490</td>
</tr>
<tr>
<td>$p$-value for $H_0: HRS - LRS = 0$</td>
<td>0.0%</td>
<td>0.0%</td>
<td>0.0%</td>
<td>1.1%</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Notes: ***, **, and * represent statistical significance at 1%, 5%, and 10%, respectively, based on sign test
$p$-values are based on Mann-Whitney test
Sample sizes apply to each reward spread

Table 4.6: Median of estimated psychological motivation parameter ($\gamma_{R,D}$) under CRRA

Third, as in the analyses under the risk-neutral case, psychological motivations under the LRS are lower for no disclosure and full disclosure than for winner disclosure and partial disclosure, but some of those differences are now only marginally significant ($p$-values below 10%). More specifically, partial disclosure leads to a (weakly) higher psychological motivation parameter than any other scheme under both reward structures. Moreover, winner disclosure and full disclosure lead to (weakly) higher psychological motivation parameters than no disclosure under both reward structures.

Overall, recognizing the winners in some form clearly increases psychological motivation relative to not publicly announcing any information about the contest outcome. It also appears that recognizing both winners without divulging their ranking (partial disclosure) succeeds in providing the highest level of psychological motivation.

Furthermore, the psychological motivation parameter continues to be significantly higher under the LRS than under the HRS for each disclosure scheme. This suggests that, even after accounting for risk aversion, psychological motivation plays a part in driving the higher efforts that are observed under the LRS. Theory models show that when players are risk neutral, a winner-take-all reward (high reward spread) structure maximizes effort. The experimental literature, however, finds robust evidence that providing multiple large rewards leads to higher levels of effort. Typically, such results are attributed to risk aversion (Lim et al., 2009). My finding that the psychological motivation parameters are higher for the lower reward spread treatment, even after incorporating risk aversion, suggests that such results should not be solely attributed to risk attitudes.\(^\text{10}\)

The higher level of psychological motivation observed under the low reward spread reveals an interesting form of motivation crowding-out through incentives. In the existing literature on the crowding out of motivation, psychological motivation is often found to be undermined when the amount of mone-

\(^{10}\)Note that the magnitudes of the estimated parameter $\gamma$ are smaller in Table 4.6 than in Table 4.3. This does not necessarily imply that risk aversion leads to a smaller impact of psychological motivation on effort choice. How the effort level enters the equation for psychological motivation is different under risk neutrality and risk aversion. As a result, the numbers in the two tables may not be comparable. Nevertheless, this may also imply that psychological motivation goes hand-in-hand with loss aversion for such small-stakes situations, supporting my evidence of psychological motivations significantly affecting effort choice. Thanks to an anonymous referee for pointing this out.
tary incentive is increased. In my study, however, the total amount of rewards remains the same under the two schemes. Rather, the level of psychological motivation is reduced when the spread of prizes is increased, from $LRS$ to $HRS$. Thus, crowding out of motivation occurs in a contest with highly-unequal prizes, relative to one in which the prizes are more evenly distributed.

4.6 Conclusions and Discussion

My theoretical analysis and laboratory experiments have demonstrated how the reward structures and disclosure schemes in sales contests can affect the participating agents’ psychological motivations, and hence their effort decisions. My results show that, first, psychological motivations do contribute to effort decisions. The psychological motivation parameters that I estimate are significant under virtually all experimental conditions, including the no disclosure condition that is closest to the theoretical benchmark.

Second, I find that the incentive structure can affect the levels of psychological motivations. More specifically, when the reward spread is low, psychological motivations tend to be higher. It is worth noting that past research tends to attribute high efforts observed under low reward spreads to agents’ risk aversion. My result shows that, even after controlling for risk aversion, a low reward spread could lead to higher effort through enhanced psychological motivation.

Third, I show that the choice of disclosure scheme can affect the level of psychological motivation. Disclosure schemes, which do not change financial incentives, can affect an agent’s effort only through peer-generated psychological motivations. Among many results regarding disclosure schemes, I find that, overall, having a recognition program can enhance psychological motivation and increase effort (as compared to having no public disclosure). This result provides strong support for the wide acceptance of recognition programs in industry. I also find that, among the disclosure schemes examined in my study, across many conditions, partial disclosure consistently performs at least as well as any other disclosure scheme. This result is consistent with the advice of industry experts (Zoltners et al., 2011) and may help explain why “President’s Club”-style recognition programs are the most popular in practice.

Finally, I develop a flexible framework to estimate a single parameter that quantifies the scale and economic impact of psychological motivations. This framework can easily be extended in future research to investigate such motivations in other contexts. Within the sales management domain, future research may examine the level of psychological motivations associated with straight commission schemes, quota-based compensation schemes, and team-based incentive schemes. The framework can also be extended to study agent behavior in public economics (e.g., the psychological motivation to contribute to crowdfunding).

The contributions of this chapter have a number of implications for sales management. Most generally, the significant effect of psychological motivations on effort decisions indicates that managers must account for them in order to design truly optimal sales contests and other motivation programs. Specifically, consideration should be given to the psychological impacts of both the distribution of prizes and the public announcement of outcomes. In determining the optimal prize distribution for a contest, sales managers should watch for the possibility of crowding out psychological motivation not only through larger prizes, but also through more uneven distributions of a fixed prize pool. In considering whether, and to what extent, contest results should be announced publicly, managers should be aware that public recognition of contest winners appears to have a positive effect on participants’ effort levels,
but that announcing the identities of the non-winners may be counterproductive. Managers should also be mindful that the psychological effects of public announcement and prize distribution can be intertwined. Thus, sales managers should treat the decisions about contest prize distribution and mode of public announcement as joint rather than separate decisions. For example, a manager designing a sales contest can maximize psychological motivation by distributing rewards among multiple winners and publicly recognizing all of them (without “shaming” the lower finishers). Taking full advantage of the salespeople’s non-economic motivations allows her to induce greater effort with the same financial resources (or the same effort with less). Lastly, I believe that my indirect model can be a useful tool for managers interested in understanding the psychological motivations of salespeople in their own particular contexts. For example, beyond contest design, a manager could use this approach to study potential differences in psychological motivations for salespeople selling different products (e.g. new vs. established brands) or to different types of customers (e.g. large vs. small, new vs. repeat purchasers). The parsimonious nature of the model allows it to be applied broadly and it is designed to focus less on the psychological motivations themselves than on the practical outcome of those motivations — their net effect on effort.
Appendices for Chapter 2

A Proof that \( \frac{v'(e)}{p'(e)} \) is strictly increasing in \( e \)

\[
\left( \frac{v'(e)}{p'(e)} \right)' = \frac{v''(e)p'(e) - v'(e)p''(e)}{[p'(e)]^2}
\]

By assumption, \( v \) is strictly increasing and convex, so \( v'(e) > 0 \) and \( v''(e) > 0 \). Similarly, \( p \) is strictly increasing and concave, so \( p'(e) > 0 \) and \( p''(e) < 0 \).

Therefore, \( \left[ p'(e) \right]^2 > 0 \), \( v''(e)p'(e) > 0 \), and \( v'(e)p''(e) < 0 \).

Therefore, \( \left( \frac{v'(e)}{p'(e)} \right)' = \frac{v''(e)p'(e) - v'(e)p''(e)}{[p'(e)]^2} > 0 \), so \( \frac{v'(e)}{p'(e)} \) is strictly increasing in \( e \). \( \Box \)

B Derivation of optimal compensation plan for benchmark model

From §2.3.1, the firm’s problem (for a given \( e \)) is expressed as:

\[
\max_{(u_L, u_H)} \text{p}(e)(x_H - h(u_H)) + [1 - \text{p}(e)](x_L - h(u_L))
\]

s. t. \( \text{p}(e)u_H + [1 - \text{p}(e)]u_L - v(e) \geq \bar{U} \) (IR)

\[
\frac{v'(e)}{p'(e)} = \Delta u
\]

(IC)

Letting \( \lambda \) and \( \mu \) be the Lagrange multipliers for the (IR) and (IC) constraints, respectively, gives the following 1st-order conditions:

For \( u_L \):

\[
-[1 - \text{p}(e)]h'(u_L) + \lambda[1 - \text{p}(e)] - \mu = 0
\]

(12)

For \( u_H \):

\[
-p(e)h'(u_H) + \lambda p(e) + \mu = 0
\]

(13)

Note that \( h'(u) = \frac{1}{u'} \), so equations (12) and (13), respectively, give:

\[
\frac{-[1 - \text{p}(e)]}{u'(s_L)} + \lambda[1 - \text{p}(e)] - \mu = 0
\]

and

\[
\frac{-p(e)}{u'(s_H)} + \lambda p(e) + \mu = 0
\]

Adding these two equations gives:

\[
\lambda = \frac{1 - \text{p}(e)}{u'(s_L)} + \frac{p(e)}{u'(s_H)}
\]

\( \therefore \lambda > 0 \), since \( p(e) \in (0, 1) \) and \( u' > 0 \)

\( \therefore \) (IR) binds

\( \therefore p(e)u_H + [1 - p(e)]u_L - v(e) = \bar{U} \)

\( \therefore u_L + p(e)\Delta u = \bar{U} + v(e) \)

Substituting (IC) into this gives:

\[
u_L^f(e) = \bar{U} + v(e) - p(e) \cdot \frac{v'(e)}{p'(e)}
\]
Substituting this back into (IC) gives:

$$u_f^t(e) = u_f^s(e) + \frac{v'(e)}{p'(e)} = \bar{U} + v(e) + [1 - p(e)]\frac{v'(e)}{p'(e)}$$

C Derivation of firm’s optimal $e$ in benchmark model

Noting that $h(u) = u^2$ (because $u(s) = \sqrt{s}$), the following expression represents the firm’s expected profit when inducing effort $e$ from the salesperson:

$$E[\pi(e)] = \frac{\exp(e)}{1 + \exp(e)} [x_H - (\bar{U} + 2\beta \exp(e))^2] + \frac{1}{1 + \exp(e)} [x_L - (\bar{U} - \beta \exp(2e) + 1)^2]$$

Differentiating with respect to $e$ and setting to 0 gives:

$$0 = \frac{\exp(e)}{1 + \exp(e)} [-2(\bar{U} + 2\beta \exp(e))\beta \exp(e)] + \frac{\exp(e)}{(1 + \exp(e))^2} [x_H - (\bar{U} + 2\beta \exp(e))^2] + \frac{1}{1 + \exp(e)} [x_L - (\bar{U} - \beta \exp(2e) + 1)^2]$$

$$= \frac{-1}{1 + \exp(e)} \left[ -4\beta \exp(2e)(\bar{U} + 2\beta \exp(e)) + 4\beta \exp(2e)(\bar{U} - \beta \exp(2e) + 1) \right]$$

$$+ \frac{\exp(e)}{(1 + \exp(e))^2} \left[ \frac{\Delta x}{\beta} + [\exp(4e) - 2 \exp(2e) + 1] \right]$$

$$= \frac{-1}{1 + \exp(e)} \left[ \frac{2\beta \exp(e)[\exp(e) + 1]^2}{\beta} \frac{\Delta x}{\beta} + [\exp(4e) - 2 \exp(2e) + 1] \right]$$

$$= \frac{-1}{1 + \exp(e)} \left[ \frac{2\beta \exp(e)[\exp(e) + 1]^2}{\beta} \frac{\Delta x}{\beta} + [\exp(4e) - 2 \exp(2e) + 1]^2 - 2\bar{U} \exp(2e) + 2 \exp(e) + 1] \right]$$

$$\therefore 0 = \frac{\exp(e)}{(1 + \exp(e))^2} \left[ \Delta x + \beta^2 [1 - \exp(e)]^2 [1 + \exp(e)]^2 - 2\bar{U} [1 + \exp(e)]^2 - 4\beta^2 \exp(e)[1 + \exp(e)]^3 \right]$$

Since $\frac{\exp(e)}{(1 + \exp(e))^2} > 0$, this implies that:

$$0 = \Delta x + \beta^2 [1 - \exp(e)]^2 [1 + \exp(e)]^2 - 2\bar{U} [1 + \exp(e)]^2 - 4\beta^2 \exp(e)[1 + \exp(e)]^3$$

$$= \beta [1 + \exp(e)]^2 \beta [(1 - \exp(e)]^2 - 4 \exp(e)[1 + \exp(e))] - 2\bar{U} + \Delta x$$

$$\therefore 0 = \beta [1 + \exp(e)]^2 \beta (-3 \exp(2e) - 6 \exp(e) + 1) - 2\bar{U} + \Delta x$$

This is a quadratic polynomial in $\exp(e)$, with two real roots:

$$\exp(e) = \frac{\pm 1}{\sqrt{3\beta}} \sqrt{(2\beta - \bar{U})^2 + 3\Delta x + 2\beta - \bar{U}} - 1$$
However, \( e \) must be non-negative, so \( \exp(e) \geq 1 \), which results in the unique real solution:

\[
e^{f^*} = \ln \left( \frac{1}{\sqrt{3\beta}} \sqrt{\sqrt{(2\beta - \bar{U})^2 + 3\Delta x + 2\beta - \bar{U}} + 1} \right)
\]

with the further condition that \( \Delta x \geq 8\beta(4\beta + \bar{U}) \).

(It is straightforward to show that the second-order condition is satisfied everywhere, so this is indeed the maximum.)

\section{Proof of Lemma 3}

\textbf{Claim:} \textit{Salesperson in }\beta_H\textit{-type territory always exits under plan }\sigma_L\textit{.}

\textbf{Proof.} From §2.3.1, when determining }\sigma_L\text{, the IR constraint binds. Therefore,

\[
p(e_L)u^\sigma_H + [1 - p(e_L)]u^\sigma_L = \bar{U} + \beta_L v_0(e_L)
\]

Suppose that a salesperson in a }\beta_H\text{-type territory does not exit, and exerts effort }\tilde{e}\text{ under }\sigma_L\text{. This implies that his IR constraint is satisfied, so:

\[
p(\tilde{e})u^\sigma_H + [1 - p(\tilde{e})]u^\sigma_L \geq \bar{U} + \beta_H v_0(\tilde{e}) > \bar{U} + \beta_L v_0(\tilde{e}) \quad \text{because } \beta_H > \beta_L
\]

This implies that a salesperson in a }\beta_L\text{-territory would prefer to exert effort }\tilde{e}\text{ over }e_L\text{, contradicting the definition of }e_L\text{. Therefore, a salesperson in a }\beta_H\text{ territory must exit under }\sigma_L\text{.} \hfill \qed

\textbf{Claim:} \textit{Salesperson in }\beta_L\text{-type territory always accepts plan }\sigma_H\text{.}

\textbf{Proof.} From §2.3.1, when determining }\sigma_H\text{, the IR constraint binds. Therefore,

\[
p(e_H)u^\sigma_H + [1 - p(e_H)]u^\sigma_L = \bar{U} + \beta_H v_0(e_H) > \bar{U} + \beta_L v_0(e_H) \quad \text{because } \beta_H > \beta_L
\]

Therefore, a salesperson in a }\beta_L\text{-territory can satisfy his IR constraint by choosing }e_H\text{, so he would accept plan }\sigma_H\text{.} \hfill \qed

\textbf{Claim:} \textit{Salesperson in }\beta_L\text{-type territory receives more expected utility under }\sigma_H\text{ than under }\sigma_L\text{.}

\textbf{Proof.} As above, when determining }\sigma_L\text{, the IR constraint binds, so

\[
E(U|\sigma_L, e_L, \beta_L) = p(e_L)u^\sigma_H + [1 - p(e_L)]u^\sigma_L - \beta_L v_0(e_L) = \bar{U}
\]

is the highest possible expected utility for a salesperson in a }\beta_L\text{-type territory.

Under }\sigma_H\text{, if that salesperson chooses effort }e_H\text{, his expected utility is:

\[
E(U|\sigma_H, e_H, \beta_L) = p(e_H)u^\sigma_H + [1 - p(e_H)]u^\sigma_L - \beta_L v_0(e_H)
\]
Since the IR constraint binds when determining \( \sigma_H \) for a salesperson in a \( \beta_H \) territory:

\[
\hat{U} = p(e_H)u_H^{\sigma_H} + [1 - p(e_H)]u_L^{\sigma_H} - \beta_H v_0(e_H) < E(U|\sigma_H, e_H, \beta_L) \quad \text{because} \quad \beta_H > \beta_L
\]

Therefore, \( E(U|\sigma_L, e_L, \beta_L) < E(U|\sigma_H, e_H, \beta_L) \).

Furthermore, the salesperson’s optimal effort under \( \sigma_H \), which I will call \( \hat{e} \), must provide at least as much expected utility as effort \( e_H \) does.

Therefore, \( E(U|\sigma_L, e_L, \beta_L) < E(U|\sigma_H, e_H, \beta_L) \leq E(U|\sigma_H, \hat{e}, \beta_L) \), so a salesperson in a \( \beta_L \) territory receives more expected utility under \( \sigma_H \) than under \( \sigma_L \). \( \square \)

**Claim:** Salesperson in \( \beta_L \)-type territory exerts more effort under \( \sigma_H \) than under \( \sigma_L \) (i.e. \( \hat{e} > e_L \)) and that effort increases in \( \beta_H \).

**Proof.** From equation 2.1, \( v'(\hat{e}) = u_H^{\sigma_H} - u_L^{\sigma_H} \).

From equations 2.7 and 2.8, \( u_H^{\sigma_H} - u_L^{\sigma_H} = \frac{1}{3} \left[ \sqrt{(2\beta_H - U)^2 + 3\Delta x} + (2\beta_H - \hat{U}) \right] \)

\[
v'(\hat{e}) = \beta_L \exp(\hat{e}) \quad \text{and} \quad p'(\hat{e}) = \frac{\exp(\hat{e})}{1 + \exp(\hat{e})} \quad \text{so} \quad v'(\hat{e}) = \beta_L[1 + \exp(\hat{e})]^2
\]

Therefore, \( \beta_L [1 + \exp(\hat{e})]^2 = \frac{1}{3} \left[ \sqrt{(2\beta_H - U)^2 + 3\Delta x} + (2\beta_H - \hat{U}) \right] \)

\[\therefore \hat{e} = \ln \left( \frac{\beta_L}{3} \sqrt{\frac{(2\beta_H - U)^2 + 3\Delta x + 2\beta_H - \hat{U} - \beta_L}{\beta_L}} \right) \]

Combining this with equation 2.6 gives:

\[
\hat{e} - e_L = \ln \left( \frac{\beta_L}{3} \sqrt{\frac{(2\beta_H - U)^2 + 3\Delta x + 2\beta_H - \hat{U} - \beta_L}{\beta_L}} \right) - \ln \left( \frac{\beta_L}{3} \sqrt{\frac{(2\beta_L - U)^2 + 3\Delta x + 2\beta_L - \hat{U} - \beta_L}{\beta_L}} \right)
\]

\[
= \ln \left( \frac{\sqrt{(2\beta_H - U)^2 + 3\Delta x + 2\beta_H - \hat{U} - \beta_L}}{\sqrt{(2\beta_L - U)^2 + 3\Delta x + 2\beta_L - \hat{U} - \beta_L}} \right)
\]

It is straightforward to show that \( \sqrt{(2\beta - U)^2 + 3\Delta x + 2\beta - \hat{U}} \) increases in \( \beta \). Thus, \( \beta_H > \beta_L \Rightarrow \sqrt{(2\beta_H - U)^2 + 3\Delta x + 2\beta_H - \hat{U}} > \sqrt{(2\beta_L - U)^2 + 3\Delta x + 2\beta_L - \hat{U}} \) and \( \hat{e} - e_L \) increases in \( \beta_H \).

Therefore, \( \hat{e} - e_L > 0 \) and \( \hat{e} \) increases in \( \beta_H \), so a salesperson in a \( \beta_L \) territory exerts more effort under \( \sigma_H \) than under \( \sigma_L \) and that effort increases in \( \beta_H \). \( \square \)

**Claim:** Salesperson in \( \beta_L \)-type territory receives larger expected payout under \( \sigma_H \) than under \( \sigma_L \).

**Proof.** Since the IR constraint binds, the expected payout for a salesperson in a \( \beta_L \) territory under \( \sigma_L \) is:

\[
p(e_L)u_H^{\sigma_L} + [1 - p(e_L)]u_L^{\sigma_L} = \hat{U} + \beta_L v_0(e_L)
\]

As shown above, that salesperson would accept plan \( \sigma_H \) (and exert optimal effort \( \hat{e} \)), implying that his IR constraint is satisfied. In that case, his expected payout is:

\[
p(\hat{e})u_H^{\sigma_H} + [1 - p(\hat{e})]u_L^{\sigma_H} > \hat{U} + \beta_L v_0(\hat{e}) > \hat{U} + \beta_L v_0(e_L) \quad \text{because} \quad \hat{e} > e_L \quad \text{(from above)}
\]
Therefore, his expected payout is higher under \( \sigma_H \) than under \( \sigma_L \).

E  Proof of Proposition 2

The participation mechanism is feasible if and only if the firm can set a threshold for the manager’s effort such that the manager prefers to exert the threshold effort when the salesperson’s territory is the high-difficulty type, but not the low-difficulty type. This is possible if and only if the manager’s expected gains from participating are greater when the salesperson is the high-difficulty type than the low-difficulty type (in which case the firm can set the participation threshold such that the manager’s cost of exerting the threshold effort falls between her expected gains for the two types).

From §2.3.2, when the budget constraint is \( \beta_{\text{max}} = \beta_L \) (i.e. when the manager does not participate), the manager must choose \( \sigma_L \), the firm’s optimal compensation plan for territory type \( \beta_L \). Similarly, when the budget constraint is \( \beta_{\text{max}} = \beta_H \) (i.e. when the manager does participate), the manager chooses \( \sigma_H \), the firm’s optimal compensation plan for territory type \( \beta_H \).

The feasibility condition, then, is as follows:

\[
E[s^M(x)|\beta_H, \sigma_H] - E[s^M(x)|\beta_H, \sigma_L] > E[s^M(x)|\beta_L, \sigma_H] - E[s^M(x)|\beta_L, \sigma_L]
\]

where \( E[s^M(x)|\beta, \sigma] \) represents the manager’s expected payout when she sets plan \( \sigma \) for a salesperson in a territory of type \( \beta \).

Since the salesperson exits when he has a high-difficulty territory and is offered plan \( \sigma_L \) (from Lemma 3), this is equivalent to:

\[
p(e_H)s^M_H + [1-p(e_H)]s^M_L - p(0)s^M_H - [1-p(0)]s^M_L > p(\tilde{e})s^M_H + [1-p(\tilde{e})]s^M_L - p(e_L)s^M_H - [1-p(e_L)]s^M_L
\]

where \( \tilde{e} \) represents the effort exerted by a salesperson with territory type \( \beta_L \) under compensation plan \( \sigma_H \).

This can be simplified to:

\[
[p(e_H) - p(0)][s^M_H - s^M_L] > [p(\tilde{e}) - p(e_L)][s^M_H - s^M_L]
\]

\[
\Leftrightarrow p(e_H) - p(0) > p(\tilde{e}) - p(e_L)
\]  \hspace{1cm} (14)

By equation 2.1:

\[
\beta_L \frac{v'_0(\tilde{e})}{p'(\tilde{e})} = u_H^\sigma - u_L^\sigma
\]

and

\[
\beta_H \frac{v'_0(e_H)}{p'(e_H)} = u_H^\sigma - u_L^\sigma
\]

Combining these gives:

\[
\frac{v'_0(\tilde{e})}{p'(\tilde{e})} = \frac{\beta_H}{\beta_L} \frac{v'_0(e_H)}{p'(e_H)}
\]

Define \( z(e) = \frac{v'_0(e)}{p'(e)} \).

\[
\therefore z(e) = \frac{\beta_H}{\beta_L} z(e_H) \text{ and } \tilde{e} = z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e_H) \right]
\]
Making use of the functional forms for \( p(\cdot) \) and \( v_0(\cdot) \) specified in Section 2.2:

\[
z(e) = [\exp(e) + 1]^2
\]

and \( z^{-1}(x) = \ln(\sqrt{x} - 1) \)

\[
\therefore \hat{e} = \ln\left( \sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1]^2 - 1 \right)
\]

\[
= \ln\left( \sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1] - 1 \right)
\]

Therefore:

\[
p(\hat{e}) = \frac{\exp\left[ \ln\left( \sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1] - 1 \right) \right]}{\exp\left[ \ln\left( \sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1] - 1 \right) \right] + 1}
\]

\[
= \sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1] - 1
\]

\[
\frac{\sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1]}{\sqrt{\frac{\beta_H}{\beta_L}} [\exp(e_H) + 1]}
\]

\[
\therefore p(\hat{e}) = 1 - \frac{\sqrt{\frac{\beta_L}{\beta_H}}}{[\exp(e_H) + 1]}
\]

So, inequality 14 becomes:

\[
\frac{\exp(e_H)}{\exp(e_H) + 1} - \frac{1}{2} > 1 - \frac{\sqrt{\frac{\beta_L}{\beta_H}}}{[\exp(e_H) + 1]} - \frac{\exp(e_L)}{\exp(e_L) + 1}
\]

\[
\Leftrightarrow \frac{\exp(e_H) - 1}{2[\exp(e_H) + 1]} > \frac{1}{\exp(e_L) + 1} + \frac{\sqrt{\frac{\beta_L}{\beta_H}}}{\exp(e_H) + 1}
\]

\[
\Leftrightarrow \frac{\exp(e_H) - 1}{2} > \frac{\exp(e_H) + 1}{\exp(e_L) + 1} - \frac{\sqrt{\beta_L}}{\beta_H}
\]

\[
\Leftrightarrow \sqrt{\frac{\beta_L}{\beta_H}} > \frac{2[\exp(e_H) + 1] - [\exp(e_H) - 1][\exp(e_L) + 1]}{2[\exp(e_L) + 1]}
\]

\[
\Leftrightarrow \sqrt{\frac{\beta_L}{\beta_H}} > \frac{\exp(e_H) + 3 - \exp(e_H)\exp(e_L) + \exp(e_L)}{2[\exp(e_L) + 1]}
\]

\[
\Leftrightarrow \sqrt{\frac{\beta_L}{\beta_H}} > 1 - \frac{[\exp(e_H) + 1][\exp(e_L) - 1]}{2[\exp(e_L) + 1]}
\]

It is evident that the right-hand side of this inequality decreases with \( e_H \), indicating that the feasibility condition is more restrictive (i.e. \( \frac{\beta_L}{\beta_H} \) must be larger) when the salesperson’s optimal effort in a high-difficulty territory is low.

Note that the right-hand side decreases with \( e_L \) also, so I could similarly point out that the feasibility condition is more restrictive when \( e_L \) is small. However, it is known from \$2.3.1\ that \( e_H < e_L \), so \( e_L \) being small implies that \( e_H \) is small, which makes the claim about \( e_H \) sufficient.
F Derivation of optimization problem in No Involvement case

From §2.3.2, the optimal plan that the salesperson will accept regardless of his territory type is the solution to the following problem:

$$\max_{\sigma=(u_L, u_H)} \gamma_L E[\pi(\sigma)|\beta_L] + \gamma_H E[\pi(\sigma)|\beta_H]$$

s.t. $p(e^H)u_H + [1 - p(e^H)]u_L = \bar{U} + \beta_H v_0(e^H)$ \hspace{1cm} (IR_H)

$$\beta_L \frac{v_0'(e^L)}{p'(e^L)} = \Delta u$$ \hspace{1cm} (IC_L)

$$\beta_H \frac{v_0'(e^H)}{p'(e^H)} = \Delta u$$ \hspace{1cm} (IC_H)

(IR_H) can be rewritten as $u_L + p(e^H)\Delta u = \bar{U} + \beta_H v_0(e^H)$.

Substituting for $\Delta u$ from (IC_H) gives:

$$u_L = \bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H \frac{v_0'(e^H)}{p'(e^H)}$$ \hspace{1cm} (15)

Substituting this back into (IC_H) gives:

$$u^H = u^L + \beta_H \frac{v_0'(e^H)}{p'(e^H)}$$

$$\therefore u^H = \bar{U} + \beta_H v_0(e^H) + [1 - p(e^H)]\beta_H \frac{v_0'(e^H)}{p'(e^H)}$$ \hspace{1cm} (16)

Now, combining (IC_L) and (IC_H) gives:

$$\frac{v_0'(e^L)}{p'(e^L)} = \frac{\beta_H}{\beta_L} \frac{v_0'(e^H)}{p'(e^H)}$$

Using the notation $z(e) = \frac{v_0'(e)}{p'(e)}$:

$$z(e^L) = \frac{\beta_H}{\beta_L} z(e^H)$$

and $e^L = z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right]$ \hspace{1cm} (17)

Next, the objective function can be rewritten as follows:

$$\max_{u_L, u_H} \gamma_L \left( p(e^L)[x_H - h(u_H)] + [1 - p(e^L)][x_L - h(u_L)] \right)$$

$$+ \gamma_H \left( p(e^H)[x_H - h(u_H)] + [1 - p(e^H)][x_L - h(u_L)] \right)$$

$$\Leftrightarrow \max_{u_L, u_H} \left( x_L - h(u_L) + \gamma_L p(e^L) + (1 - \gamma_L)p(e^H) \right) (\Delta x - [h(u_H) - h(u_L)])$$

$$\Leftrightarrow \max_{u_L, u_H} \left( x_L - h(u_L) + (p(e^H) + \gamma_L[p(e^L) - p(e^H)]) (\Delta x - [h(u_H) - h(u_L)]) \right)$$
Substituting equations 15, 16 and 17 into this gives:

\[
\begin{align*}
\max_{e^H} x_L - h \left[ \bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H z(e^H) \right] &+ \left( p(e^H) + \gamma_L \left[ p \left( z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right] \right) - p(e^H) \right] \left( \Delta x - \left[ h\left( \bar{U} + \beta_H v_0(e^H) \right) + \left[ 1 - p(e^H)\beta_H z(e^H) \right] - h \left( \bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H z(e^H) \right) \right) \right) \\
&= \left( \Delta x - \left[ h\left( \bar{U} + \beta_H v_0(e^H) \right) + \left[ 1 - p(e^H)\beta_H z(e^H) \right] - h \left( \bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H z(e^H) \right) \right) \right)
\end{align*}
\]

(18)

Next, introducing the functional forms for \( h(\cdot), v_0(\cdot) \) and \( p(\cdot) \) gives the following:

From the proof of Proposition 2 (Appendix E):

\[
\begin{align*}
z(e) &= [\exp(e) + 1]^2 \\
\text{and } z^{-1}(x) &= \ln (\sqrt{x} - 1)
\end{align*}
\]

Therefore:

\[
\begin{align*}
z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right] &= \ln \left( \sqrt{\frac{\beta_H}{\beta_L} [\exp(e^H) + 1] - 1} \right) \\
p \left( z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right] \right) &= \sqrt{\frac{\beta_H}{\beta_L} [\exp(e^H) + 1] - 1} \\
\therefore p \left( z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right] \right) &= \frac{\exp(e^H) + 1 - \sqrt{\frac{\beta_L}{\beta_H}}}{\exp(e^H) + 1} \\
\therefore p(e^H) + \gamma_L \left[p \left( z^{-1} \left[ \frac{\beta_H}{\beta_L} z(e^H) \right] \right) - p(e^H) \right] &= \frac{\exp(e^H) + \gamma_L \left( 1 - \sqrt{\frac{\beta_L}{\beta_H}} \right)}{\exp(e^H) + 1}
\end{align*}
\]

Also:

\[
\begin{align*}
\bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H z(e^H) &= \bar{U} + \beta_H [\exp(e^H) - 1] - \beta_H \exp(e^H) [\exp(e^H) + 1] \\
\therefore \bar{U} + \beta_H v_0(e^H) - p(e^H)\beta_H z(e^H) &= \bar{U} - \beta_H [\exp(2e^H) + 1] \\
\text{and } \bar{U} + \beta_H v_0(e^H) + [1 - p(e^H)]\beta_H z(e^H) &= \bar{U} + 2\beta_H \exp(e^H)
\end{align*}
\]

Finally, substituting these expressions into the optimization problem 18 gives:

\[
\begin{align*}
\max_{e^H} x_L &- \left( \bar{U} - \beta_H [\exp(2e^H) + 1] \right)^2 + \frac{\exp(e^H) + \gamma_L \left( 1 - \sqrt{\frac{\beta_L}{\beta_H}} \right)}{\exp(e^H) + 1} \\
&\left( \Delta x - \left[ \left( \bar{U} + 2\beta_H \exp(e^H) \right)^2 - \left( \bar{U} - \beta_H [\exp(2e^H) + 1] \right)^2 \right] \right)
\end{align*}
\]

G  Analysis of continuous-sales model

Proof of Lemma 4:
In the absence of a budget constraint, the manager’s problem is:

$$\max_{s(\cdot)} E[s^M(x)] = \int_{x_L}^{x_H} s^M(x)f_e(x)dx$$

s.t. \[ \int_{x_L}^{x_H} u(s(x))f_e(x)dx - v(e) \geq \bar{U} \] (IR)

\[ \int_{x_L}^{x_H} u(s(x))\frac{df_e(x)}{de}dx - v'(e) = 0 \] (IC)

Since \( s^M(\cdot) \) is increasing in \( x \) and higher effort first-order stochastically dominates lower effort, the manager’s objective is equivalent to maximizing \( e \). Furthermore, since she is not concerned with controlling the expected payout, the manager can choose payouts sufficiently large to ensure that the IR constraint is satisfied at any effort level she chooses to induce. Thus, in the absence of a budget constraint, a self-interested manager always designs a compensation plan that induces the highest possible effort from the salesperson. Since the same is not true for the firm, the unconstrained manager always offers the salesperson a contract that is suboptimal for the firm.

\[ \square \]

**Proof of Lemma 3 with continuous sales:**

**Claim: Salesperson in \( \beta_H \)-type territory always exits under plan \( \sigma_L \).**

By the standard argument (as shown, for example, in Basu et al. (1985)), the IR constraint binds when determining \( \sigma_L \). Therefore,

\[ \int_{x_L}^{x_H} u[\sigma_L(x)]f_{e_L}(x)dx = \bar{U} + \beta_L v_0(e_L) \]

Suppose that a salesperson in a \( \beta_H \) territory does not exit under \( \sigma_L \), exerting effort \( \bar{e} \) instead. This implies that his IR constraint is satisfied, so:

\[ \int_{x_L}^{x_H} u[\sigma_L(x)]f_{\bar{e}}(x)dx \geq \bar{U} + \beta_H v_0(\bar{e}) > \bar{U} + \beta_L v_0(\bar{e}) \quad \text{because } \beta_H > \beta_L \]

This implies that a salesperson in a \( \beta_L \) territory would prefer to exert effort \( \bar{e} \) over \( e_L \), contradicting the definition of \( e_L \). Therefore, a salesperson in a \( \beta_H \) territory must exit under \( \sigma_L \). \( \square \)

**Claim: Salesperson in \( \beta_L \)-type territory always accepts plan \( \sigma_H \).**

As above, the IR constraint binds when determining \( \sigma_H \). Therefore,

\[ \int_{x_L}^{x_H} u[\sigma_H(x)]f_{e_H}(x)dx = \bar{U} + \beta_H v_0(e_H) > \bar{U} + \beta_L v_0(e_H) \quad \text{because } \beta_H > \beta_L \]

Therefore, a salesperson in a \( \beta_L \) territory can satisfy his IR constraint by choosing \( e_H \), so he would accept plan \( \sigma_H \). \( \square \)

**Claim: Salesperson in \( \beta_L \)-type territory receives more expected utility under \( \sigma_H \) than under \( \sigma_L \).**

As above, when determining \( \sigma_L \), the IR constraint binds, so \( E(U|\sigma_L,e_L,\beta_L) = \bar{U} \) is the highest possible expected utility for a salesperson in a \( \beta_L \) territory under \( \sigma_L \).

Under \( \sigma_H \), if that salesperson chooses effort \( e_H \), his expected utility is:

\[ E(U|\sigma_H,e_H,\beta_L) = \int_{x_L}^{x_H} u[\sigma_H(x)]f_{e_H}(x)dx - \beta_L v_0(e_H) \]
Since the IR constraint binds when determining $\sigma_H$ for a salesperson in a $\beta_H$ territory:

$$\bar{U} = \int_{x_L}^{x_H} u[\sigma_H(x)]f_{eH}(x)dx - \beta_Hv_0(\epsilon_H) < E(U|\sigma_H, \epsilon_H, \beta_L)$$

because $\beta_H > \beta_L$.

Therefore, $E(U|\sigma_L, \epsilon_L, \beta_L) < E(U|\sigma_H, \epsilon_H, \beta_L)$.

The salesperson’s optimal effort under $\sigma_H$, $\hat{e}$, must provide at least as much expected utility as effort $\epsilon_H$ does.

Therefore, $E(U|\sigma_L, \epsilon_L, \beta_L) < E(U|\sigma_H, \epsilon_H, \beta_L) \leq E(U|\sigma_H, \hat{e}, \beta_L)$, so a salesperson in a $\beta_L$ territory receives more expected utility under $\sigma_H$ than under $\sigma_L$.

Claim: Salesperson in $\beta_L$-type territory exerts more effort under $\sigma_H$ than under $\sigma_L$ (i.e. $\hat{e} > \epsilon_L$) and that effort increases in $\beta_H$.

In the continuous sales model, I restrict attention to cases in which $\hat{e} > \epsilon_L$, so the first part of the claim holds by assumption.

The second part of the claim follows intuitively because, roughly speaking, higher effort is induced by higher payouts for high sales outcomes and/or lower payouts for low sales outcomes. In other words, similar to the binary sales model, higher effort is exerted when the difference between payouts for high sales and low sales is larger. The assumption that $\hat{e} > \epsilon_L$, then, implies that the difference between the optimal high and low payouts increases with the territory type for which the plan is optimized. This, in turn, implies that $\hat{e}$ increases in $\beta_H$.

Claim: Salesperson in $\beta_L$-type territory receives larger expected payout under $\sigma_H$ than under $\sigma_L$.

Since the IR constraint binds, the expected payout for a salesperson in a $\beta_L$ territory under $\sigma_L$ is:

$$\int_{x_L}^{x_H} u[\sigma_L(x)]f_{eL}(x)dx = \bar{U} + \beta_Lv_0(\epsilon_L)$$

As shown above, that salesperson would accept plan $\sigma_H$ (and exert optimal effort $\hat{e}$), implying that his IR constraint is satisfied. In that case, his expected payout is:

$$\int_{x_L}^{x_H} u[\sigma_H(x)]f_{\hat{e}}(x)dx > \bar{U} + \beta_Lv_0(\hat{e}) > \bar{U} + \beta_Lv_0(\epsilon_L)$$

because $\hat{e} > \epsilon_L$.

Therefore, his expected payout is higher under $\sigma_H$ than under $\sigma_L$.

\[H \text{ Proof of Proposition 4}\]

From the proof of Proposition 2 (Appendix E), $\delta_{LH}$ must fall in the following interval:

$$\left( E[s^M(x)|\beta_L, \sigma_H] - E[s^M(x)|\beta_L, \sigma_L], E[s^M(x)|\beta_H, \sigma_H] - E[s^M(x)|\beta_H, \sigma_L] \right)$$

$$\therefore \delta_{LH} \in \left( \Delta s^M[p(\hat{e}_{LH}) - p(\epsilon_L)], \Delta s^M[p(\epsilon_H) - p(0)] \right)$$

(19)

where $\Delta s^M = s^M_H - s^M_L$ and $\hat{e}_{AB}$ is the effort chosen by a salesperson in territory type $\beta_A$ under compensation plan $\sigma_H$. 
Similarly,
\[
\delta_{LM} \in \left( \Delta s^M[p(\hat{e}_{LM}) - p(e_L)], \Delta s^M[p(e_M) - p(0)] \right) \tag{20}
\]
and
\[
\delta_{MH} \in \left( \Delta s^M[p(\hat{e}_{MH}) - p(e_M)], \Delta s^M[p(e_H) - p(0)] \right) \tag{21}
\]

Combining expressions 19 and 20 gives:
\[
\delta_{LM} - \delta_{LM} \in \left( \Delta s^M \left[p(\hat{e}_{LM}) - p(e_L) - (p(e_M) - p(0)) \right], \Delta s^M [p(e_H) - p(0) - (p(\hat{e}_{LM}) - p(e_L))] \right) \tag{22}
\]

If the intervals 21 and 22 overlap, then it is possible to find \(\delta_{LM}\) and \(\delta_{LM}\) that are sufficient to separate their respective types, and are such that the difference between them \((\delta_{LM} - \delta_{LM})\) is sufficient to separate types \(\beta_M\) and \(\beta_H\). When that is the case, the three-type participation mechanism is feasible.

By Lemma 3, \(\hat{e}_{LM} > e_L\), so \(p(\hat{e}_{LM}) - p(e_L) > 0\).

\[
\therefore \Delta s^M \left[p(e_H) - p(0) - (p(\hat{e}_{LM}) - p(e_L)) \right] < \Delta s^M [p(e_H) - p(0)]
\]

So, the upper bound of interval 22 < the upper bound of interval 21.

The intervals overlap, then, if and only if the upper bound of interval 22 > the lower bound of interval 21.

Equivalently, the mechanism is feasible if and only if
\[
p(e_H) - p(0) - [p(\hat{e}_{LM}) - p(e_L)] > p(\hat{e}_{MH}) - p(e_M) \tag{23}
\]

From the proof of Proposition 2 (Appendix E):
\[
p(\hat{e}_{LM}) = 1 - \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_M) + 1}
\]
and
\[
p(\hat{e}_{MH}) = 1 - \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_H) + 1}
\]

So, inequality 23 can be rewritten as:
\[
\frac{\exp(e_H)}{\exp(e_H) + 1} - \frac{1}{2} \left[ 1 - \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_M) + 1} - \frac{\exp(e_L)}{\exp(e_L) + 1} \right] > 1 - \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_H) + 1} - \frac{\exp(e_M)}{\exp(e_M) + 1}
\]
\[\Leftrightarrow \frac{1}{2} - \frac{1}{\exp(e_H) + 1} - \frac{1}{\exp(e_L) + 1} + \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_M) + 1} > 1 - \frac{\sqrt{\beta_L}}{\beta_M} \frac{1}{\exp(e_H) + 1} - \frac{\exp(e_M)}{\exp(e_M) + 1}
\]
\[\Leftrightarrow \frac{1}{2} > \frac{1}{\exp(e_H) + 1} \left( 1 - \sqrt{\frac{\beta_L}{\beta_M}} \right) + \frac{1}{\exp(e_M) + 1} \left( 1 - \sqrt{\frac{\beta_L}{\beta_M}} \right) + \frac{1}{\exp(e_L) + 1}
\]

Therefore, the three-type participation mechanism is feasible if this inequality is satisfied.

If the inequality is not satisfied, then at least one of the terms on the right-hand side must be large. The first term is large when \(\sqrt{\frac{\beta_L}{\beta_M}}\) is small, and larger when \(e_H\) is also small. The second term is large when \(\sqrt{\frac{\beta_L}{\beta_M}}\) is small, and larger when \(e_M\) is small. Note, however, that \(e_H < e_M\), so when \(e_M\) is small,
From §3.3.1, the firm’s problem is expressed as:

\[
\max_{u_L, u_H^L, u_H^H} \left( 1 - \gamma_L \right) \left( h(u_L^H) + p(e_L)\left(h(u_L^H) - h(u_H^L)\right) \right) - \gamma_L \left( h(u_H^L) + p(e_L)\left(h(u_L^H) - h(u_H^L)\right) \right)
\]

s.t.

\[
\begin{align*}
&u_L^H + p(e_L)(u_H^L - u_L^H) \geq \bar{U}^M \\
u_L^H + p(e_H)(u_H^H - u_L^H) \geq \bar{U}^M \\
u_L^H + p(e_H)(u_H^L - u_L^H) \geq \bar{U}^H \\
u_H^L + p(e_H)(u_H^H - u_L^H) \geq \bar{U}^L
\end{align*}
\]

Claim: The \((IR_{\beta_L})\) constraint is unnecessary.

Proof. Suppose \((IC_{\beta_L})\) and \((IR_{\beta_H})\) hold.

From \((IC_{\beta_L})\),

\[
u_L^H + p(e_L)(u_H^L - u_L^H) > u_L^H + p(e_H)(u_H^L - u_L^H)
\]

Since \(\tilde{e} > e_H\) (from Lemma 3 in Chapter 2) and \(p(\cdot)\) is increasing, this implies that

\[
u_L^H + p(e_L)(u_H^L - u_L^H) > u_L^H + p(0)(u_H^L - u_L^H)
\]

By \((IR_{\beta_H})\), then,

\[
u_L^H + p(e_L)(u_H^L - u_L^H) > \bar{U}^M
\]

So, if \((IC_{\beta_L})\) and \((IR_{\beta_H})\) hold, then \((IR_{\beta_L})\) must hold.

Letting \(\lambda, \mu_1, \mu_2, \rho_1\) and \(\rho_2\) be the Lagrange multipliers for \((IR_{\beta_H}), (IC_{\beta_L}), (IC_{\beta_H}), (\Delta_{\beta_L}),\) and \((\Delta_{\beta_H}),\) respectively, gives the following 1st-order conditions:

For \(u_L^H:\)

\[
-\gamma_L[1 - p(e_L)]h(u_L^H) + \mu_1[1 - p(e_L)] - \mu_2[1 - p(0)] - \rho_1 = 0
\]

For \(u_H^L:\)

\[
-\gamma_Lp(e_L)h(u_H^L) + \mu_1p(e_L) - \mu_2p(0) + \rho_1 = 0
\]

For \(u_H^H:\)

\[
-(1 - \gamma_L)[1 - p(e_H)]h'(u_H^H) + \lambda[1 - p(e_H)] - \mu_1[1 - p(\tilde{e})] + \mu_2[1 - p(e_H)] - \rho_2 = 0
\]
For $u^\beta_H$:

$$-(1 - \gamma_L)p(e_H)h'(u^\beta_H) + \lambda p(e_H) - \mu_1 p(\bar{e}) + \mu_2 p(e_H) + \rho_2 = 0$$  \hspace{1cm} (27)$$

Note that $h'(u) = \frac{1}{w''}$, so equations (24)-(27), respectively, give:

$$\frac{-\gamma_L[1 - p(e_L)]}{u^{M'}(s^\beta_L)} + \mu_1[1 - p(e_L)] - \mu_2[1 - p(0)] - \rho_1 = 0$$  \hspace{1cm} (28)$$

$$\frac{-\gamma_L p(e_L)}{u^{M'}(s^\beta_H)} + \mu_1 p(e_L) - \mu_2 p(0) + \rho_1 = 0$$  \hspace{1cm} (29)$$

$$\frac{-(1 - \gamma_L)[1 - p(e_H)]}{u^{M'}(s^\beta_L)} + (\lambda + \mu_2)[1 - p(e_H)] - \mu_1[1 - p(\bar{e})] - \rho_2 = 0$$  \hspace{1cm} (30)$$

$$\frac{-(1 - \gamma_L)p(e_H)}{u^{M'}(s^\beta_H)} + (\lambda + \mu_2)p(e_H) - \mu_1 p(\bar{e}) + \rho_2 = 0$$  \hspace{1cm} (31)$$

Adding equations 28 and 29 gives:

$$\mu_1 - \mu_2 = \gamma_L \left(\frac{1 - p(e_L)}{u^{M'}(s^\beta_L)} + \frac{p(e_L)}{u^{M'}(s^\beta_H)}\right)$$  \hspace{1cm} (32)$$

\therefore \mu_1 - \mu_2 > 0, since \gamma_L \in (0, 1), p(e_L) \in (0, 1), and u^M \text{ is increasing } (u^{M'} > 0)\n
\therefore \mu_1 > 0, since \mu_2 \geq 0\n
\therefore (IC_{\beta_L}) \text{ binds}\n
\therefore u^\beta_L + p(e_L)(u^\beta_H - u^\beta_L) = u^\beta_H + p(\bar{e})(u^\beta_H - u^\beta_L) + \delta$$

Similarly, adding equations 30 and 31 gives:

$$\lambda + \mu_2 - \mu_1 = (1 - \gamma_L) \left(\frac{1 - p(e_H)}{u^{M'}(s^\beta_L)} + \frac{p(e_H)}{u^{M'}(s^\beta_H)}\right)$$  \hspace{1cm} (33)$$

\therefore \lambda + \mu_2 - \mu_1 > 0, since \gamma_L \in (0, 1), p(e_H) \in (0, 1), and u^M \text{ is increasing}\n
\therefore \lambda > \mu_1 - \mu_2 > 0, from above\n
\therefore (IR_{\beta_H}) \text{ binds}\n
\therefore u^\beta_H + p(e_H)(u^\beta_H - u^\beta_L) = U^M$$  \hspace{1cm} (34)$$

Now, subtracting equation 30 from equation 31 gives:

$$\frac{(1 - \gamma_L)[1 - p(e_H)]}{u^{M'}(s^\beta_L)} - \frac{(1 - \gamma_L)p(e_H)}{u^{M'}(s^\beta_H)} + (\lambda + \mu_2)[2p(e_H) - 1] - \mu_1[2p(\bar{e}) - 1] + 2\rho_2 = 0$$

$$\therefore \rho_2 = \frac{1}{2} \left[1 - \gamma_L \left(\frac{p(e_H)}{u^{M'}(s^\beta_H)} - \frac{1 - p(e_H)}{u^{M'}(s^\beta_H)}\right) + 2\mu_1[p(\bar{e}) - p(e_H)] - [\lambda + \mu_2 - \mu_1][2p(e_H) - 1]\right]$$
Substituting for $\lambda + \mu_2 - \mu_1$ from equation 33 gives:

$$\rho_2 = \frac{1}{2} \left[ 1 - \gamma_L \right] \left[ \frac{p(e_H)}{u^{M'}(s_L^{\beta_H})} - \frac{1 - p(e_H)}{u^{M'}(s_L^{\beta_H})} \right] + 2 \mu_1 \left[ p(\tilde{e}) - p(e_H) \right]$$

$$= \frac{1}{2} \left[ 1 - \gamma_L \right] \left[ \frac{p(e_H)}{u^{M'}(s_L^{\beta_H})} + \frac{1 - p(e_H)}{u^{M'}(s_L^{\beta_H})} \right] [2p(e_H) - 1]$$

$$= \frac{1}{2} \left[ 1 - \gamma_L \right] \left[ \frac{[2 - 2p(e_H)]p(e_H)}{u^{M'}(s_L^{\beta_H})} - \frac{2p(e_H)[1 - p(e_H)]}{u^{M'}(s_L^{\beta_H})} + 2 \mu_1 (p(\tilde{e}) - p(e_H)) \right]$$

$$\therefore \rho_2 = (1 - \gamma_L)p(e_H)[1 - p(e_H)] \left( \frac{1}{u^{M'}(s_L^{\beta_H})} - \frac{1}{u^{M'}(s_L^{\beta_H})} \right) + \mu_1 [p(\tilde{e}) - p(e_H)]$$

Since $u^{M'}$ is concave and $s_L^{\beta_H} < s_L^{\beta_H}$, therefore $u^{M'}(s_L^{\beta_H}) > u^{M'}(s_L^{\beta_H})$.

$$\therefore \rho_2 > 0$$, because $\gamma_L \in (0, 1)$, $p(e_H) \in (0, 1)$, $\frac{1}{u^{M'}(s_L^{\beta_H})} > \frac{1}{u^{M'}(s_L^{\beta_H})}$, $p(\tilde{e}) > p(e_H)$, and $\mu_1 > 0$ (from above)

$$\therefore (\Delta_{\beta_H}) \text{ binds}$$

$$\therefore u_H^{\beta_H} - u_L^{\beta_H} = 0$$

Substituting this into (IR$_{\beta_H}$) gives:

$$u_H^{\beta_H} = u_L^{\beta_H} = \bar{H}$$

Substituting into (IC$_{\beta_L}$) gives:

$$u_L^{\beta_L} + p(e_L)(u_H^{\beta_L} - u_L^{\beta_L}) = \bar{M} + \delta$$

Similarly, subtracting equation 28 from equation 29 gives:

$$\frac{\gamma_L[1 - p(e_L)]}{u^{M'}(s_L^{\beta_H})} - \frac{\gamma_Lp(e_L)}{u^{M'}(s_L^{\beta_H})} + \mu_1 [2p(e_L) - 1] - \mu_2 [2p(0) - 1] + 2 \rho_1 = 0$$

$$\therefore \rho_1 = \frac{1}{2} \left[ \gamma_L \left[ \frac{p(e_L)}{u^{M'}(s_L^{\beta_H})} - \frac{1 - p(e_L)}{u^{M'}(s_L^{\beta_H})} \right] - \mu_1 [2p(e_L) - 1] + \mu_2 [2p(0) - 1] \right]$$

$$= \frac{1}{2} \left[ \gamma_L \left[ \frac{p(e_L)}{u^{M'}(s_L^{\beta_H})} - \frac{1 - p(e_L)}{u^{M'}(s_L^{\beta_H})} \right] - \mu_1 [2p(e_L) - 2p(0)] - [\mu_1 - \mu_2][2p(0) - 1] \right]$$

Substituting for $\mu_1 - \mu_2$ from equation 32 gives:

$$\rho_1 = \frac{1}{2} \left[ \gamma_L \left[ \frac{p(e_L)}{u^{M'}(s_L^{\beta_H})} - \frac{1 - p(e_L)}{u^{M'}(s_L^{\beta_H})} \right] - 2 \mu_1 [p(e_L) - p(0)] - \gamma_L \left[ \frac{p(e_L)}{u^{M'}(s_L^{\beta_H})} + \frac{1 - p(e_L)}{u^{M'}(s_L^{\beta_H})} \right] [2p(0) - 1] \right]$$

$$= \frac{1}{2} \left[ \gamma_L \left[ \frac{[2 - 2p(0)]p(e_L)}{u^{M'}(s_L^{\beta_H})} - \frac{2p(0)[1 - p(e_L)]}{u^{M'}(s_L^{\beta_H})} \right] - 2 \mu_1 [p(e_L) - p(0)] \right]$$

$$= \gamma_L \left[ \frac{[1 - p(0)]p(e_L)}{u^{M'}(s_L^{\beta_H})} - \frac{[1 - p(e_L)][p(0)]}{u^{M'}(s_L^{\beta_H})} \right] - \mu_1 [p(e_L) - p(0)]$$
Since \( u^M \) is concave and \( s_H^{\beta_L} < s_H^{\beta_L} \), therefore \( u^M(s_H^{\beta_L}) < u^M(s_H^{\beta_L}) \).

\[
\therefore \rho_1 > \gamma_L \left[ \frac{1 - p(0) [p(e_L) - [1 - p(e_L)] p(0)]}{u^M(s_H^{\beta_L})} - \mu_1 [p(e_L) - p(0)] \right]
\]

Substituting for \( \mu_1 \) from equation 32:

\[
\rho_1 > [p(e_L) - p(0)] \left[ \frac{\gamma_L}{u^M(s_H^{\beta_L})} - \left( \mu_2 + \gamma_L \left[ \frac{p(e_L)}{u^M(s_H^{\beta_L})} + \frac{1 - p(e_L)}{u^M(s_H^{\beta_L})} \right] \right) \right]
\]

\[
\therefore > [p(e_L) - p(0)] \left[ \frac{\gamma_L}{u^M(s_H^{\beta_L})} - \left( \mu_2 + \frac{\gamma_L}{u^M(s_H^{\beta_L})} \right) \right]
\]

as above, because \( u^M(s_H^{\beta_L}) < u^M(s_L^{\beta_L}) \)

\[
\therefore \rho_1 > - \mu_2 [p(e_L) - p(0)]
\]

\[
\therefore \text{At least one of } \mu_2 \text{ and } \rho_1 \text{ must be greater than } 0
\]

\[
\therefore \text{At least one of } (IC_{\beta_H}) \text{ and } (\Delta_{\beta_L}) \text{ must bind. If } \mu_2 = 0, \text{ then } \rho_1 > 0 \text{ and } (\Delta_{\beta_L}) \text{ binds; if } \rho_1 = 0, \text{ then } \mu_2 > 0 \text{ and } (IC_{\beta_H}) \text{ binds. If both } \mu_2 \text{ and } \rho_1 \text{ are greater than } 0, \text{ then both constraints bind.}
\]

Suppose \((\Delta_{\beta_L})\) binds:

Then \( u_{HL}^{\beta_L} - u_{HL}^{\beta_L} = 0 \) and, substituting this into equation 35 gives \( u_{HL}^{\beta_L} = u_{HL}^{\beta_L} = \bar{U}^M \)

Substituting this, along with equation 34 into \((IC_{\beta_H})\) gives \( \bar{U}^M \geq \bar{U}^M + 2\delta \)

This is a contradiction, because \( \delta > 0 \)

\[
\therefore (\Delta_{\beta_L}) \text{ cannot bind}
\]

\[
\therefore (IC_{\beta_H}) \text{ must bind}
\]

\[
\therefore u_{HL}^{\beta_L} + p(e_L)(u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) = u_{HL}^{\beta_L} + p(0)(u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) + \delta
\]

Substituting in from equation 34, this gives:

\[
\bar{U}^M = u_{HL}^{\beta_L} + p(0)(u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) + \delta \quad (36)
\]

\[
= u_{HL}^{\beta_L} + p(e_L)(u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) - [p(e_L) - p(0)](u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) + \delta
\]

\[
\therefore \bar{U}^M = \bar{U}^M + \delta - [p(e_L) - p(0)](u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) + \delta \quad \text{by substituting from equation 35}
\]

\[
\therefore u_{HL}^{\beta_L} - u_{HL}^{\beta_L} = \frac{2\delta}{p(e_L) - p(0)} \quad (37)
\]

Substituting this back into equation 36 gives \( \bar{U}^M = u_{HL}^{\beta_L} + p(0) \frac{2\delta}{p(e_L) - p(0)} + \delta \)

\[
\therefore u_{HL}^{\beta_L} = \bar{U}^M - \left( \frac{2p(0)}{p(e_L) - p(0)} + 1 \right) \delta
\]

\[
\therefore u_{HL}^{\beta_L} = \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta
\]

Now, \( u_{HL}^{\beta_L} = u_{HL}^{\beta_L} + (u_{HL}^{\beta_L} - u_{HL}^{\beta_L}) \)

Substituting from equation 37 again:

\[
u_{HL}^{\beta_L} = \bar{U}^M - \frac{p(e_L) + p(0)}{p(e_L) - p(0)} \delta + \frac{2}{p(e_L) - p(0)} \delta
\]
\[ u_H^{\beta} = \bar{U}^M + \frac{2 - [p(e_L) + p(0)]}{p(e_L) - p(0)} \delta \]

Appendix for Chapter 4

J Experimental instructions (as provided to subjects)

General Rules

This session is part of an experiment about sales force decision making. If you follow the instructions carefully and make good decisions, you can earn points during the session. Based on your points earning, you will be paid in cash at the end of the session.

There are twelve people (including yourself) in this laboratory who are participating in this session as subjects. They have all been recruited in the same way as you and are reading the same instructions as you are for the first time. It is important that you do not communicate to any of the other participants in any manner until the session is over.

The session will consist of 40 contest periods in each of which you can earn points. There will also be a 3-period long risk-attitude elicitation round where, in each period, you have to report your willingness-to-pay for a lottery. At the end of the experiment, two contest periods and one out lottery period will be randomly chosen to determine the earnings of all players. One of the two periods will be chosen from periods 6 to 20 and the other will be chosen from periods 26 to 40. You will be paid a show-up fee of $5 plus an amount based on your point earnings from the three chosen periods. For payments, 15 points are worth $1. Thus, if you earn \( y \) points in total from these randomly chosen periods, then your total income will be \( 5 + y/15 \). The more points you earn, the more cash you will receive.

Identification

At the beginning of the session, you will be assigned an identifying username as a sales person. This username will be of the form “Sales Person X” where X is a letter from the English alphabet. This username will be your identity for the entire session and you will be known to other players by this username.

Description of a Period

For this experiment, assume that you are employed as a sales person. Your job is to sell Product Beta which is an industrial product. In this task, you will have to make decisions on how much effort you expend in selling the product. At the beginning of each period, you will be randomly matched with exactly three other subjects. You and these three other subjects will participate in a 4-player sales contest. The winners of the contests will be determined by the amount of revenue each player brings. In each period, you will receive 100 points, parts (or all) of which you can use as effort to generate revenue. The remainder will be counted as part of your income (in points) from that period. Here using 1 point for effort represents expending very little effort in selling Product Beta and using 100 points represents expending the maximum possible level of effort. You can save the amount of points that you do not use as effort as your income. Suppose you use \( e \), out of 100, points as effort to generate sales. Then, you will keep \( 100 - e \) points as your income from that period and you will generate \( s(e) = 350 + \ln(e) + \varepsilon \) units of revenue. Here \( \varepsilon \) is distributed according to a logistic distribution with mean of zero and variance of \( \pi^2/3 \). Specifically, the probability distribution function (pdf) is \( f(x) = \frac{\exp(-x)}{(1+\exp(-x))^2} \). The attached figures graphically present the function 350 + \( \ln(e) \) and the pdf \( f(x) \).

Your revenue will be used in determining the reward you receive from the sales contest in a given
period. All four players (including yourself) will choose their efforts \((e)\) simultaneously. On the computer screen, you will choose how many points you want to use as effort. Your effort has to be an integer between 1 and 100 (inclusive). You have one minute to make this decision. If you do not make your decision within one minute, you will be forced to make an immediate choice. Once all 4 players choose their effort levels, the computer will independently generate a random \(\varepsilon\) for each player and the revenue amount of each player will be calculated. Then, the player who generated the highest revenue will receive a reward of \(A\) points and the player who generated the second highest revenue will receive a reward of \(B\) points. The remaining two players will not win any reward. Thus, your income from a period in which you use \(e\) points as effort will equal \(100 - e + R\) points where \(R\) is the reward you win. At the end of a period, you will learn how much revenue you generated and the amount of reward (if any) you received in that period.

Additionally, the following sentences were appended at the end of the above paragraph in the partial, winner, and full disclosure treatments:

**Partial Disclosure:** You will also learn the identities of the two players who received the rewards but not their ranking.

**Winner Disclosure:** You will also learn the identity of the winner of the contest.

**Full Disclosure:** You will also learn the identities of the winner and the runner-up of the contest and the two players who did not win any reward.

**Differences between Periods**

Recall that there will be 40 periods in this experiment and you will be randomly assigned to three other players in each period. You will participate in the above-mentioned 4-player sales contest in every period. However, the reward scheme will not be the same in every period. In periods 1 to 20, the rewards \(A\) and \(B\) will equal 360 and 40 points, respectively. In periods 21 to 40, they will equal 204 and 196 points, respectively. You will be reminded of the reward scheme before period 1 and before period 21 and it will also be listed on the effort choice screen. All twelve players in the session will face the same reward scheme in a given period.

**Risk-attitude Elicitation Round**

After the end of 40 periods of sales contest, you will individually participate in a risk-attitude elicitation round with three periods where, in each period, you will report your willingness-to-pay for a lottery. These are individual lotteries which take a value of 0 with probability 1 \(- p\) and a value of 20 points with probability \(p\). The probability of winning 20 points, \(p\), will be different for the 3 different lotteries – the possible values are 30%, 50%, and 80%. For each given lottery, you will report your willingness-to-pay, which has to be an integer between 1 and 19. Independent of your reported willingness-to-pay, the computer will choose a number between 1 and 19. If this number is above your reported willingness-to-pay, you will be paid this number in points. If it is below your reported willingness-to-pay, you will be paid according to the lottery. Thus, it is optimal to truthfully report your willingness-to-pay. You will be paid according to one randomly chosen period of this round at the end of the session.

**Ending the Session**

At the end of the risk-attitude elicitation round, you will see a screen displaying your earnings from each period. You will receive $5 for participating in this experiment. On top of that, you will earn an amount based on your point earnings from two randomly chosen periods from the sales contest periods and one randomly chosen period from the 3 lotteries in the risk-attitude elicitation round. Recall that,
if you earn $y$ points from these three periods, your total income from the experiment will be $\$5 + y/15$. You will be paid this amount in cash.
Bibliography


BIBLIOGRAPHY


