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An Investigation of Renyi Entropy in High energy Nucleus-nucleus collisions

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Abstract

A study of Renyi entropy of shower particles has been carried out using ^{16}O , ^{28}Si and ^{32}S projectiles in interaction with AgBr and CNO target present in nuclear emulsion at an incident momentum of 4.5 AGeV/c. The analysis reveals interesting experimental observations for both groups of targets. A comparison of the experimental results with the results obtained from analyzing the event sample generated by the UrQMD code is also presented. Renyi entropy values increase with the increase of projectile mass. This increase is weaker in case of CNO target. The calculated values of second order Renyi entropy H_2 are shown to vary linearly with the logarithm of average particle multiplicity $\ln\langle n \rangle$ for both targets. The dependence of Renyi entropy on logarithm of average particle multiplicity $\ln\langle n \rangle$ indicates the presence of long range correlation in high energy nucleus-nucleus interactions. The variation is found to be stronger in case of experimental events in comparison to the UrQMD simulated events indicating stronger long range correlation for the experimental data.

Introduction

Relativistic heavy ion collision has received an increased interest in the recent past and it is regarded as a promising source for providing information about the underlying particle production processes. With the advent of modern particle accelerators and the possibility to study nuclear collision at increasing beam energies, it became possible to compress nuclear matter to many times the density of normal nuclear matter. The heavy ion programmes at RHIC and more recently at LHC have furnished an unprecedented amount and quality of information about the ultra relativistic nucleus-nucleus collisions. The main motivation of these programmes is that they provide unique experimental access to the matter under conditions of extreme temperature and density **present** in hot early universe. Lattice quantum chromodynamics (QCD) calculations indicate that at zero net baryon density at a critical temperature of $T_c \sim 170$ MeV and energy density $\epsilon_c \sim 1 \text{ GeV fm}^{-3}$, a color deconfined and chirally restored QGP phase is formed. These energy densities may be attained in a relativistic heavy-ion collision where it is believed that a dense system of quarks and gluons is created. The system undergoes rapid collective expansion before the patrons hadronize and eventually decouple. Many scientists, both experimentalists and theoreticians, are searching for possible signals of QGP. So the primary goal of relativistic heavy-ion collisions is to create and study this deconfined state in the laboratory. The systematic measurements of local entropy in high-energy interactions may provide direct information about the internal degrees of freedom (DOF) of the QGP state and its evolution. In high-energy nucleus-nucleus collisions, entropy measurement of produced particles can provide important information about the multiparticle production mechanism. Different workers have investigated the evolution of entropy in multiparticle production in various high-energy interactions at different times [1-11]. The entropy of produced particles can easily be calculated from the experimental multiplicity distributions. If P_n is the probability of producing n particles in a high energy interaction the entropy S is defined as done by Simak et al [12] $S = -\sum_n P_n \ln P_n \dots\dots(1)$. Mathematically this entropy S is called the Shannon entropy [13].

A. Białas and W. Czyż [14-16] proposed to study the Renyi entropy [17-18] that are closely related with the thermodynamic entropy of the system, the Shannon entropy. Renyi entropy of order k can be defined as given in [14-16]

$$H_k = \frac{1}{1-k} \ln C_k \dots\dots\dots(2)$$

In terms of the probability of multiplicity distribution P_n , the values of C_k can be written as given in [19]

$$C_k = \sum_n (P_n)^k \dots\dots\dots(3)$$

Putting $k=2$ in relation (2) and relation (3) we can calculate the Renyi entropy of second order.

Renyi entropies provide an exact lower limit of Shannon entropy, which is very important for the understanding of the properties of QGP. Systematic measurements of the Renyi entropy in nucleus-nucleus collisions may provide direct information about the internal degrees of freedom of the QGP state and its evolution. In nucleus-nucleus collisions, Renyi entropy measurement can be used not only to search for QGP formation but it may also serve as an additional tool to investigate the correlations and event-by-event fluctuations. Renyi entropies themselves contain valuable information about the multiparticle system and can be utilized as a tool to study the dynamical system. The novel aspects of studying the Renyi entropies is that, these entropies are not related with the moments of the kinematic variables and /or the multiplicity distribution but rather they are related with the moments of the probability distributions themselves [14]. However it may be mentioned here that most of the published results on entropy analysis are on Shannon entropy. There are few published results on Renyi entropies [14-16, 20-21] available so far. In order to understand better the physical significance of Renyi entropy, it will be useful to study it in some details.

Goal of our present study is to carry out an investigation on the second order Renyi entropies of the shower particles produced in $^{16}O - AgBr$ interactions, $^{28}Si - AgBr$ interactions, $^{32}S - AgBr$ interactions, $^{16}O - CNO$ interactions, $^{28}Si - CNO$ and $^{32}S - CNO$ interactions at 4.5 AGeV/c. Our analysis is compared with the prediction of UrQMD model. It is true that at such a low incident momentum (4.5 AGeV/c) QGP is not expected to occur. Still Renyi entropy study at this energy will be interesting in the sense that previously no attempt has been made to study Renyi entropy using emulsion data at such a low energy.

The outcome of our analysis could be helpful in revealing the inner dynamics of multiparticle production mechanism.

Experimental Details

In this paper, data obtained from nuclear emulsion track detector has been used for the analysis. Nuclear emulsion technique is based on the recording of tracks of each ionizing particles in photographic emulsion. Nuclear emulsion detectors offer a good angular resolution ($\sim 0.1\text{mrad}$) and they are capable of registering all the produced particles in the 4π geometry. Possessing a complete 4π detector with a very good angular resolution, nuclear emulsion detectors are well suited both for investigation of event topology and for the measurements of angular distribution of different kinds of emitted or produced particles in high energy nucleus-nucleus interactions.

In order to get the required data for the present analysis, stacks of NIKFI-BR2 emulsion pellicles of dimension $20\text{cm} \times 10\text{ cm} \times 600\mu\text{m}$ were irradiated by the ^{16}O , ^{28}Si and ^{32}S beam at 4.5 AGeV/c obtained from the Synchrophasatron at Joint Institute of Nuclear Research (JINR), Dubna, Russia. Details of scanning of emulsion plates, data taking, and identification of target can be found from our earlier publication [22]. In nuclear emulsion detector, particles produced or emitted after an interaction can be classified into four categories, namely the shower particles, the grey particles, the black particles and the projectile fragments. Among these four types of particles our analysis has been performed on the shower particles which are mostly pions. Details of the characteristics of these particles can be found from our earlier publication [22].

Nuclear emulsion medium consists of variety of nuclei like H, C, N, O, Ag and Br. It has been pointed out in [22] that in emulsion experiment, it is very difficult to measure the charges of the fragments emitted from the target and hence exact identification of the target is not possible. However, we can divide the major constituent elements present in the emulsion into three broad target groups namely hydrogen (H), light nuclei (CNO) and heavy nuclei (AgBr) on the basis of the number of heavy (black+grey) tracks as described in [22]. Following the method of target identification total number of inelastic interactions for each projectile have been divided into three target groups H, CNO and AgBr. For the present analysis we have not considered the events which are found to occur due to collisions of the projectile beam with H target present in nuclear emulsion. Our analysis has been carried out for the interactions with the AgBr and CNO target only. According to the method of

identifying the AgBr and CNO target we have selected 1057 events of $^{16}\text{O} - \text{AgBr}$ and 1616 events of $^{16}\text{O} - \text{CNO}$ interactions, 514 events of $^{28}\text{Si} - \text{AgBr}$ and 769 events of $^{28}\text{Si} - \text{CNO}$ interactions, 434 events of $^{32}\text{S} - \text{AgBr}$ and 819 events of $^{32}\text{S} - \text{CNO}$ interactions for the present analysis. Average multiplicities of shower tracks ($\langle n \rangle$) of each interaction have been calculated and **are** presented in **table 1**.

Analysis and Results

We have calculated the second order Renyi entropy H_2 of the produced shower particles in $^{16}\text{O} - \text{AgBr}$, $^{28}\text{Si} - \text{AgBr}$, $^{32}\text{S} - \text{AgBr}$, $^{16}\text{O} - \text{CNO}$, $^{28}\text{Si} - \text{CNO}$ and $^{32}\text{S} - \text{CNO}$ interactions at 4.5 AGeV/c according to relation (2) and relation (3). The calculated values of second order Renyi entropy **are** tabulated in table 2 for all interactions. Errors shown in the table are statistical errors only. It is evident from table 2 that the Renyi entropy values are higher for the *AgBr* target in comparison to CNO target. From table 2 one can see that the Renyi entropy H_2 increases with the increase in the mass number of the projectile A_p . We have plotted the variation of Renyi entropy H_2 with the mass number of the projectile A_p in figure 1 and in figure 2 for *AgBr* and CNO target respectively. The dependence of Renyi entropy on the projectile mass is parameterized by the power-law relation $H_2 = aA_p^b$. Here a, b are the fitting parameters reflecting the nature of fitting for the experimental data. The values of a and b are calculated from the χ^2 minimization method. We have also calculated the values of the non-linear regression coefficient (R^2) in order to quantify the goodness of fitting of the plots. The values of the fitting parameters, χ^2 per degrees of freedom (χ^2 / DOF) value of the fit and the values of non-linear regression coefficient (R^2) of the fit are given in table 3 both *AgBr* and CNO target. From table 3 it is seen that, the value of b is smaller for the CNO target and the χ^2 / DOF value for the CNO target is also high. Therefore, it may be said that the dependence of Renyi entropy on the mass number of the projectile beam is stronger for the AgBr target. In case of CNO target the dependence is quite weak. The dependence of Renyi entropy on the mass number of the projectile beam is an interesting observation. As the studied projectile beams have the same incident momentum, the observed disorderness during the collision process increases with the increase of the size of the projectile nucleus.

It has been pointed out in [16] if Renyi entropy is proportional to $\log N$ where N is the total multiplicity or average multiplicity, long range correlation is indicated. Hence a study of dependence of Renyi entropy H_2 on $\ln\langle n \rangle$, where $\langle n \rangle$ is the average shower particle multiplicity, will be helpful to search the existence of long range correlation.

We have studied the variation of Renyi entropy H_2 with $\ln\langle n \rangle$ for all **reactions**. In figure 3 and in figure 4 we have depicted the variation of second order Renyi entropy H_2 with $\ln\langle n \rangle$ for AgBr and CNO target respectively. The variation of Renyi entropy H_2 with $\ln\langle n \rangle$ has been fitted with a linear relation of the form $H_2 = a + b \ln\langle n \rangle$ for both target group. Values of a and b obtained from the best linear behavior of the fit, are given in table 4. From table 4 it can easily be noticed that there is prominent dependence of Renyi entropy on $\ln\langle n \rangle$ for both AgBr and CNO target group. From table 4, it may be concluded that the presence of long range correlation is evident in our experimental data. As the slope value for the CNO events is less than that of the AgBr events, it may be concluded that long range correlation is stronger for the AgBr events.

The experimental results are compared with those obtained by analyzing events generated by the Ultra Relativistic Quantum Molecular Dynamics (UrQMD) model [23-24]. UrQMD model is a microscopic transport theory, based on the covariant propagation of all the hadrons on the classical trajectories in combination with stochastic binary scattering, color string formation and resonance decay. It represents a Monte Carlo solution of a large set of coupled partial integro-differential equations for the time evolution of various phase space densities. The main ingredients of the model are the cross sections of binary reactions, the two-body potentials and decay widths of resonances. The UrQMD collision term contains 55 different baryon species (including nucleon, delta and hyperon resonances with masses up to $2.25 \text{ GeV}/c^2$) and 32 different meson species (including strange meson resonances), which are supplemented by their corresponding anti-particle and all isospin-projected states. The states can either be produced in string decays, s-channel collisions or resonance decays. This model can be used in the entire available range of energies from the Bevalac region to RHIC. For more details about this model, **readers should consult** [23-24]. In order to generate the UrQMD data sample for ^{16}O , ^{28}Si and ^{32}S projectiles on interactions with H, CNO and AgBr target present in nuclear emulsion, events were generated separately

taking each of the H, Ag, Br, C, N and O nuclei as targets for each projectile. According to the proportional abundance of H, Ag, Br, C, N and O nuclei present in the nuclear emulsion, the generated events with Ag and Br nuclei as target were mixed with each other and events having C, N and O as target nuclei were mixed in order to get the desired UrQMD data sample for AgBr and CNO target respectively for each projectile [25-27]. As in the case of the experimental data in case of UrQMD data also we have not studied the events having H as target.

According to the procedure mentioned above, we have generated a large sample of events (ten times the experimental data sample of each interactions) using the UrQMD code (UrQMD-3.3p1) for $^{16}\text{O} - \text{AgBr}$ interactions, $^{28}\text{Si} - \text{AgBr}$ interactions, $^{32}\text{S} - \text{AgBr}$ interactions, $^{16}\text{O} - \text{CNO}$ interactions, $^{28}\text{Si} - \text{CNO}$ and $^{32}\text{S} - \text{CNO}$ interactions at 4.5 AGeV/c. We have also calculated the average multiplicities of the shower tracks for all interactions in case of the UrQMD data sample. Average multiplicities of the shower tracks in case of UrQMD data sample are presented in table 1 along with the experimental data.

As in case of the experimental data, we have calculated the values of second order Renyi entropy in case of UrQMD data also for both AgBr and CNO target. The calculated values of Renyi entropies for the UrQMD data sample are shown in table 2 for AgBr and the CNO target along with the experimental values. From table 2 it can be noticed that for the AgBr target, the UrQMD model predicted entropy values are smaller than their experimental counterparts. However, for the CNO target, the values of Renyi entropy H_2 calculated for the UrQMD model are equal to the experimental values within the experimental error. Table 2 indicates that the UrQMD simulated values of Renyi entropies increase with the increase of the mass number of the projectile. As before, we have tried to fit the variation of H_2 with A_p by the power law relation $H_2 = aA_p^b$. The variation is presented in figure 1 and in figure 2 for AgBr and CNO target respectively for the UrQMD data along with the experimental plot. The fitting parameters along with χ^2 per degrees of freedom (χ^2/DOF) value of the fit and the values of non-linear regression coefficient (R^2) of the fit are given in table 3 for both AgBr and CNO target. From table 3 it can be noted that the rise of Renyi entropy with the mass number of the projectile beam is weaker for UrQMD model analysis. We have studied the variation of Renyi entropy H_2 with $\ln \langle n \rangle$ in order to investigate the presence of long range correlation in UrQMD simulated data for both AgBr and CNO

targets. Figure 3 and figure 4 present the variation of second order Renyi entropy H_2 with $\ln \langle n \rangle$ for the UrQMD model calculations for AgBr and CNO target respectively. In case of UrQMD data sample also we have fitted the variation with the linear relation $H_2 = a + b \ln \langle n \rangle$. The calculated values of a and b are presented in table 4 for **both linear fits**. The slope values b for the UrQMD analyzed data sample are found to be smaller than the corresponding slope value of the experimental data for both targets. This observation suggests that the observed long range correlation is weaker in case of the simulated data sample. Although UrQMD model successfully reproduce the Renyi entropy values for the CNO target, the model cannot show the same kind of dependence of H_2 on $\ln \langle n \rangle$ for the CNO target as found in case of the experimental data. The discrepancy of the obtained results between the data and the model prediction in the analysis of long range correlation is significant and interesting. This discrepancy establishes that long range correlations observed in case of the experimental data are not due to mere statistics only. Rather the observed long range correlation may be attributed to the presence of dynamical fluctuations in the multiparticle production mechanism.

Conclusion

To summarize we recall that in the present paper a study of Renyi entropy of the shower particles produced in $^{16}\text{O} - \text{AgBr}$, $^{28}\text{Si} - \text{AgBr}$, $^{32}\text{S} - \text{AgBr}$, $^{16}\text{O} - \text{CNO}$, $^{28}\text{Si} - \text{CNO}$ and $^{32}\text{S} - \text{CNO}$ interactions at 4.5 AGeV/c has been carried out using nuclear emulsion track detector. We have compared the results of the experimental analysis with the results obtained from the analysis of events simulated by UrQMD model. Main conclusions of our analysis are given below.

1. Second order Renyi entropy values depend on the target of the collision. For heavier target entropy values are higher indicating higher disorderness. Experimental values of Renyi entropy for the AgBr target are higher than the corresponding UrQMD calculated values. UrQMD model can reproduce the Renyi entropy values for the CNO target.
2. For heavier projectile Renyi entropy values are higher. The dependence of Renyi entropy with the mass number of the projectile beam has been parameterized by the power-law relation $H_2 = aA_P^b$. For CNO target the dependence is quite weak.

3. Renyi entropy values depend on $\ln\langle n \rangle$ for the experimental data. The dependence of Renyi entropy H_2 on $\ln\langle n \rangle$ indicates the presence of long range correlation in high energy nucleus-nucleus interactions. The observed dependence is stronger for the AgBr target in comparison to the CNO target. In case of UrQMD events the dependence is weaker in comparison to experimental observation indicating that the long range correlation present in the experimental data sample is due to the non-statistical fluctuations in the multiparticle production process.

This is the first ever analysis of Renyi entropy using emulsion data. In that respect the study is interesting and demand attention. The experimental observations and outcome of such an analysis seem to be interesting. More data analysis at higher energy in this regard is necessary.

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Table 1

Interactions	Average Multiplicity of shower particles <n>	
	Experimental	UrQMD
¹⁶ O – AgBr (4.5AGeV/c)	18.05 ± 0.22	17.79 ± 0.21
²⁸ Si – AgBr (4.5AGeV/c)	23.62 ± 0.21	27.55 ± 0.22
³² S – AgBr (4.5AGeV/c)	28.04 ± .14	30.84 ± 0.17
¹⁶ O – CNO (4.5AGeV/c)	7.50 ± .21	9.28 ± 0.11
²⁸ Si -CNO (4.5AGeV/c)	8.84 ± .17	13.88 ± 0.16
³² S – CNO (4.5AGeV/c)	9.59 ± .17	15.31 ± 0.11

Table 1 represents the average multiplicities of the shower particles for all the interactions in case of the experimental and the UrQMD data.

Table 2

Interactions	Renyi Entropy Values	
	Second order Renyi entropy H_2 Experimental Values	Second order Renyi entropy H_2 UrQMD Model calculation
$^{16}O - AgBr$	$3.47 \pm .04$	$3.05 \pm .02$
$^{28}Si - AgBr$	$3.76 \pm .07$	$3.29 \pm .06$
$^{32}S - AgBr$	$3.91 \pm .07$	$3.36 \pm .08$
$^{16}O - CNO$	$2.72 \pm .03$	$2.74 \pm .02$
$^{28}Si - CNO$	$2.82 \pm .06$	$2.86 \pm .04$
$^{32}S - CNO$	$2.92 \pm .06$	$2.88 \pm .04$

Table 2 represents the values of Second order Renyi entropy H_2 for all the interactions in case of the experimental (AgBr and CNO events), as well as the UrQMD simulated data (AgBr and CNO events).

Table 3

Target	Type of data	a	b	χ^2 / DOF	R^2
$AgBr$	Experimental	$2.19 \pm .08$	$.16 \pm .03$	.97	.99
	UrQMD	$2.07 \pm .12$	$.14 \pm .01$	.96	.98
CNO	Experimental	$2.10 \pm .13$	$.09 \pm .01$	1.23	.78
	UrQMD	$2.23 \pm .08$	$.07 \pm .01$	.93	.95

Table 3 represents the values of the fitting parameters of the plot of Renyi entropy with the mass number of the projectile $H_2 = aA_p^b$ in case of the shower particle multiplicity, the χ^2 per degrees of freedom (χ^2 / DOF) value of the fit and the non-linear regression coefficient R^2 for the experimental and UrQMD data for both $AgBr$ and CNO target in our analysis.

Table 4

Second order Renyi entropy H_2	Target	Type of data	Values of Fitting parameters
	<i>AgBr</i>	Experimental	$a = .565 \pm .101$
			$b = 1.00 \pm .051$
		UrQMD	$a = 1.47 \pm .04$
			$b = .549 \pm .030$
	CNO	Experimental	$a = 1.16 \pm .14$
			$b = .768 \pm .103$
		UrQMD	$a = 2.10 \pm .25$
			$b = .284 \pm .041$

Table 4 represents the fitting parameters a and b calculated for the linear fit of $H_2 = a + b \ln \langle n \rangle$ for both the target for both experimental and UrQMD simulated data. Here H_2 is the second order Renyi entropy and $\langle n \rangle$ is the average multiplicity of the shower particles.

Fig 1 Shows the variation of Renyi entropy H_2 with the mass number of the projectile beam A_p for the AgBr target for both experimental and UrQMD simulation.

Fig 2 Shows the variation of Renyi entropy H_2 with the mass number of the projectile beam A_p for the CNO target for both experimental and UrQMD simulation.

Figure 3 the variation of second order Renyi entropy H_2 with $\ln\langle n \rangle$ for $AgBr$ target in case of experimental and UrQMD simulated data.

Figure 4 the variation of second order Renyi entropy H_2 with $\ln\langle n \rangle$ for CNO target in case of experimental and UrQMD simulated data.