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Optimal Priority-Free Conditionally-Preemptive Real-Time Scheduling of Periodic Tasks Based on DES Supervisory Control

Xi Wang, ZhiWu Li, *Senior Member, IEEE*, and W. M. Wonham, *Life Fellow, IEEE*

Abstract—This study presents a general discrete event system (DES)-based hard periodic real-time task model. Based on supervisory control theory (SCT), an optimal priority-free real-time scheduling technique is proposed to process all the tasks running in uni-processor or multi-processor real-time systems (RTS). The preemption relation in this paper generalizes priority-based preemption. First, regular languages are utilized to describe the processor behavior related to each task's execution. Thereafter, the languages are represented by DES generators. Finally, the global processor behavior is generated as the synchronous product of these DES generators. A novel preemption policy, namely conditional-preemption, is developed. Two sets of conditional-preemption specifications are developed, on the processor level and task level, respectively. Moreover, in order to control the system to be nonblocking and also limit the worst-case response time (WCRT) of the tasks, two corresponding sets of specifications are presented. After generating the global specification as the synchronous product, by implementing SCT the calculated supervisor can provide all the safe real-time execution sequences. The supervisor calculation can be sped up by a three step algorithm. Finally, the real-time scheduling is implemented for real-world examples.

Index Terms—Real-time system, scheduling, discrete-event system, supervisory control, limited preemptive, conditional-preemptive.

I. INTRODUCTION

In past decades, most of the existing real-time scheduling algorithms are based on dynamic or fixed priorities [1]–[12]. The study in [7] shows that, with the tasks' periods equal to their deadlines, the preemptive earliest deadline first (EDF) scheduling algorithm is optimal. For uni-processor real-time system (RTS) scheduling, a timed discrete-event system (TDES)-based task model referred to as the Chen-Wonham (CW) model, is proposed in [13]. Based on supervisory control theory (SCT) [14]–[16], for non-preemptive scheduling, a supervisor [17], [18] that contains all the safe execution sequences is found. In these sequences, all the possible EDF and

fixed-priority (FP) sequences are included. If the supervisor is empty, the RTS is claimed to be non-schedulable. Otherwise, users can choose any sequence to schedule the RTS.

Compared with non-preemptive scheduling, preemptability can provide more flexibility to real-time scheduling. In fully preemptive systems, at any time, the execution of a running task can be interrupted by tasks with higher priorities, and it continues when all tasks with higher priorities have completed [19].

However, for the real-time scheduling in a processor, both preemptive and non-preemptive scheduling policies are too conservative. Generally, users may customize specific preemption plans that are neither preemptive nor non-preemptive. Recently, several studies are devoted to different preemption policies; these can be divided into two categories:

- Priority-based: Preemption thresholds scheduling (PTS) [20], under FP scheduling, allows a task to disable preemption up to a specified priority level. Each task is assigned a fixed priority and a preemption threshold, and only tasks with priorities higher than the threshold of the running task can preempt its execution.
- Task-based: Deferred preemptions scheduling (DPS) [21], under EDF scheduling, assigns each task a maximum interval that is free from preemption.

The two preemption policies are considered as the tradeoff between preemptive and non-preemptive real-time scheduling. They can significantly improve the EDF and FP real-time scheduling algorithms, respectively. However, both policies are still limited by priorities. In fact, for some real-time scheduling requirements, priorities cannot be assigned to real-time tasks. Moreover, to the best of our knowledge, no existing work can combine the two latter policies together into a real-time scheduling strategy and find all the safe execution sequences. For example, under FP or EDF scheduling, an RTS cannot be scheduled by satisfying PTS and DPS simultaneously.

In this paper, a discrete-event system (DES) real-time task model is presented to schedule the real-time tasks running in uni-processor or multi-processor systems. The preemption relation in this paper generalizes priority-based preemption. Regular languages are utilized to describe the processor behavior related to the execution and preemption of each task. Each language can be represented by a DES generator. The synchronous product [14] of these DES generators can integrate the models of the tasks running in a processor into a complex generator to represent the global processor behavior. For each processor, without considering the priori-

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ties, this present study extends the PTS and DPS scheduling policies to two general conditional-preemption specifications. They are utilized to customize scheduling and preemption requirements conditionally, such as FP, PTS, DPS, preemptive scheduling, and non-preemptive scheduling. Moreover, the worst-case response time (WCRT) of a task can also be restricted by a specification. Thereafter, similar to the work in [13], all the safe execution sequences generated by the synchronized specifications with respect to the tasks running in each processor can be calculated offline by SCT. Users can choose any sequence to schedule the processor. The real-time scheduling with conditional-preemption is applied to real-world uni-processor or multi-processor systems.

The rest of this paper is organized as follows. Section II presents the terminology used throughout the paper. The system model and motivation are described in Section III. The priority-free real-time scheduling with conditional-preemption principles is defined in Section IV. The DES model for the periodic tasks and the RTS are proposed in Section V. The specifications are formalized and established in Section VI. By applying to real-world systems, Section VII reports methodologies for the real-time scheduling with conditional-preemption. Further relevant issues are discussed in Section VIII. Finally, Section IX states our conclusions.

II. CONCEPTS AND TERMINOLOGY

A DES plant is a *generator*

$$\mathbf{G} = (Q, \Sigma, \delta, q_0, Q_m)$$

where

- Q is the finite *state set*,
- Σ is the finite *event set (alphabet)*, partitioned into the *controllable event* subset Σ_c and the *uncontrollable event* subset Σ_u ;
- $\delta : Y \times \Sigma \rightarrow Y$ is the *partial state transition function*;
- q_0 is the *initial state*; and
- $Q_m \subseteq Q$ is the subset of *marker states*.

In accordance with [14], Σ^+ denotes the set of all finite sequences of symbols in Σ . By adjoining the empty string ϵ , the set of strings over the alphabet Σ is written as Σ^* , i.e., $\Sigma^* = \Sigma^+ \cup \{\epsilon\}$. Function δ can be extended to $\delta : Y \times \Sigma^* \rightarrow Y$. We write $\delta(q, s)!$ to mean that $\delta(q, s)$ is defined, where state $q \in Q$ and string $s \in \Sigma^*$. The length $|s|$ of a string $s \in \Sigma^*$ is defined as

$$|\epsilon| = 0; |s| = k, \text{ if } s = \sigma_1 \sigma_2 \cdots \sigma_k \in \Sigma^+.$$

The *closed behavior* of $\mathbf{G}$ is represented by

$$L(\mathbf{G}) := \{s \in \Sigma^* | \delta(q_0, s)!\}$$

and the *marked behavior* is

$$L_m(\mathbf{G}) := \{s \in L(\mathbf{G}) | \delta(q_0, s) \in Q_m\} \subseteq L(\mathbf{G}).$$

Synchronous product [14] is a standard way to combine a finite set of DES into a single and more complex one. Suppose that we have n languages corresponding to n DES, $L_i \subseteq \Sigma_i^*$ with $\Sigma = \bigcup_{i \in \mathbf{n}} \Sigma_i$, $\mathbf{n} := \{1, 2, \dots, n\}$. The *natural projection* $P_i : \Sigma^* \rightarrow \Sigma_i^*$ is defined by

- $P_i(\epsilon) = \epsilon$,
- $P_i(\sigma) = \begin{cases} \epsilon, & \text{if } \sigma \notin \Sigma_i \\ \sigma, & \text{if } \sigma \in \Sigma_i \end{cases}$,
- $P_i(s\sigma) = P_i(s)P_i(\sigma)$, $s \in \Sigma^*$, $\sigma \in \Sigma$.

The *inverse image function* of P_i is

$$P_i^{-1} : Pwr(\Sigma_i^*) \rightarrow Pwr(\Sigma^*).$$

For $H \subseteq \Sigma_i^*$,

$$P_i^{-1}(H) := \{s \in \Sigma^* | P_i(s) \in H\}.$$

The *synchronous product* of $L_1, L_2, \dots, L_n$ is denoted by $L_1 || L_2 || \cdots || L_n$ with

$$L_1 || L_2 || \cdots || L_n := P_1^{-1}L_1 \cap P_2^{-1}L_2 \cap \cdots \cap P_n^{-1}L_n.$$

Suppose that the *specification language* is represented by $E \subseteq \Sigma^*$. Let $\mathcal{C}(E)$ be the family of sublanguages of E that are controllable with respect to $\mathbf{G}$. $\mathcal{C}(E)$ is nonempty and is closed under arbitrary unions. The (unique) *supremal element* within $\mathcal{C}(E)$, denoted by $\sup \mathcal{C}(E)$, always exists.

Suppose that a DES model is nonempty. Under supervisory control, all the uncontrollable events are automatically enabled. After adjoining a particular subset of the controllable events to be enabled, a set of all *control patterns* is defined as

$$\Gamma = \{\gamma \in Pwr(\Sigma) | \gamma \supseteq \Sigma_u\}$$

where $Pwr(\Sigma)$ is the *power set* of Σ . A *supervisory control* for $\mathbf{G}$ is any map $V : L(\mathbf{G}) \rightarrow \Gamma$. DES $\mathbf{G}$ under the supervision of V is written as $V/\mathbf{G}$. The *closed behavior* of $V/\mathbf{G}$ is defined to be $L(V/\mathbf{G}) \subseteq L(\mathbf{G})$ described as

- empty string $\epsilon \in L(V/\mathbf{G})$,
- if $s \in L(V/\mathbf{G})$, $\sigma \in V(s)$, and $s\sigma \in L(\mathbf{G})$ then $s\sigma \in L(V/\mathbf{G})$,
- no other strings belong to $L(V/\mathbf{G})$.

The *marked behavior* of $V/\mathbf{G}$ is

$$L_m(V/\mathbf{G}) = L(V/\mathbf{G}) \cap L_m(\mathbf{G}).$$

The *control map* V is *nonblocking* for $\mathbf{G}$ if

$$\overline{L_m(V/\mathbf{G})} = L(V/\mathbf{G}).$$

A language $K \in \Sigma^*$ is *controllable* (with respect to $\mathbf{G}$) if

$$\overline{K} \Sigma_u \cap L(\mathbf{G}) \subseteq \overline{K}$$

i.e.,

$$(\forall s, \sigma) s \in \overline{K} \ \& \ \sigma \in \Sigma_u \ \& \ s\sigma \in L(\mathbf{G}) \Rightarrow s\sigma \in \overline{K}.$$

Let $K \subseteq L(\mathbf{G})$ be nonempty and closed. There exists a supervisory control V for $\mathbf{G}$ such that $L(V/\mathbf{G}) = K$ iff K is *controllable* with respect to $\mathbf{G}$; this is referred to as a *nonblocking supervisory control* (NSC). Generally, if *marking* is also considered, then we select a sublanguage $M \subseteq L_m(\mathbf{G})$. A *marking nonblocking supervisory control* (MNSC) with respect to $\mathbf{G}$ exists, which is a map $V : L(\mathbf{G}) \rightarrow \Gamma$ with the behavior

$$L_m(V/\mathbf{G}) = L(V/\mathbf{G}) \cap M.$$

III. SYSTEM MODEL AND MOTIVATION

In the literature of real-time scheduling, the existing preemption algorithms are defined based on (dynamic or fixed) priorities, which means that when several tasks are simultaneously eligible to be processed, only a task with the highest priority is permitted to enter the processor. In order to relax this constraint and provide all the tasks an equal chance to be processed, an optimal policy, named priority-free scheduling policy, is presented. This policy allows users to define conditional-preemption requirements freely without assigning task priorities.

A. System Model

Suppose that an RTS $\mathbb{S}$ processes a set of independent synchronous/asynchronous periodic tasks, i.e., $\mathbb{S} = \{\tau_1, \tau_2, \dots, \tau_n\}$. For $i \in \mathbf{n}$, task $\tau_i \in \mathbb{S}$ is in the form of a four-tuple

$$\tau_i = (R_i, C_i, D_i, T_i)$$

with

- a release time R_i ,
- a worst-case execution time (WCET) C_i ,
- a hard deadline D_i , and
- a period T_i with $R_i \geq 0$ and $C_i \leq D_i \leq T_i$.

Here R_i , C_i , D_i , and T_i are non-negative integers representing the lengths of processor time units. A synchronous (resp., asynchronous) task τ_i has $R_i = 0$ (resp., $R_i > 0$). The processor utilization of τ_i is $U_i = \frac{C_i}{T_i}$.

Suppose that an RTS $\mathbb{S}$ is a uni-processor system; and the total processor utilization of $\mathbb{S}$ is $U_{\mathbb{S}} = \sum_{i=1}^n U_i$. Similarly, suppose that an RTS $\mathbb{S}$ is a multi-processor system that possesses a processor set Ξ that consist of s processors, i.e., $\Xi = \{\Xi_1, \Xi_2, \dots, \Xi_s\}$. The processor utilization of Ξ_j is the sum of the processor utilization of all the tasks running in it, i.e., $U_j^{\Xi} = \sum_i U_i$, $\tau_i \in \Xi_j$. In both uni-processor and multi-processor systems, a processor is not schedulable in case there is overload [1]. The present study allows a task to enter any processor randomly. However, once a task enters a processor, it cannot be shifted to another. In a processor, real-time scheduling policies can be divided into the following two categories:

- *preemptive*: a running task can be interrupted by the execution of other released tasks;
- *non-preemptive*: the execution of a running task cannot be interrupted.

B. Motivation

For some real-world preemption policies, the tasks running in a processor cannot be assigned with priorities. Next we present a motivating example.

Suppose that a uni-processor RTS $\mathbb{S}$ executes four tasks τ_1 , τ_2 , τ_3 , and τ_4 . Their parameters are shown in Table I. We assume that the execution of τ_1 can be preempted only by τ_2 , τ_2 only by τ_4 , and τ_4 only by τ_1 , respectively; moreover, τ_3 cannot be preempted. No priorities can be assigned to these tasks. In this case, the EDF, FP, and PTS algorithms cannot be

utilized to schedule this RTS. In order to solve such problems, we discard the priorities and refer to the real-time scheduling as priority-free.

TABLE I: Parameters of four tasks

task	R	C	D	T
τ_1	0	3	9	9
τ_2	3	3	6	6
τ_3	0	1	4	5
τ_4	0	2	14	18

Based on the priority-free real-time scheduling, conditional-preemption is presented in this study. By a preemption matrix that will be defined later, users can define any preemption relation among all the tasks running in the same processor. WCET-based conditional-preemption specification is presented in this study. Users can use it to designate how long after its execution is started, a task can be preempted by other tasks. Moreover, the WCRT of a task can also be restricted by a specification. Finally, all the safe execution sequences generated by the synchronized specifications with respect to the tasks running in each processor can be calculated offline by SCT. Users can choose any sequence to schedule the processor.

IV. PRIORITY-FREE HARD RTS WITH PERIODIC TASKS

Since both uni-processor and multi-processor real-time systems are considered, theoretical analysis in this section is based on processors instead of real-time systems.

A. Priority-Free Real-Time Scheduling

Definition 1: [priority-free] A scheduling policy is said to be *priority-free* if all the released tasks in a processor can be processed in any order.

From the perspective of an individual task τ_i , in each hyper-period [22], all the processor time units are partitioned into:

- *busy time*: the processor is occupied by other tasks, so τ_i cannot be executed;
- *running time*: τ_i is in process;
- *preemption time* (if any): after τ_i has started, its execution is interrupted by (a subset of) other tasks; and
- *free time*: the execution of τ_i is completed or τ_i has not arrived yet. These processor time units can be idle or utilized to execute other tasks.

Priority-free scheduling policy can be utilized to schedule all the periodic tasks randomly, i.e., the busy time and the preemption time of each task can be occupied by other tasks. Moreover, in accordance with traditional real-time scheduling, a task is not allowed to be interrupted if the system would thereby be left idle. In this case, the free time is also allowed to be left idle only when no task in process.

Example.

For τ_2 shown in Table I, a possible conditionally-preemptive real-time scheduling is illustrated in Fig. 1. By allowing preemption of other tasks, the first nine processor time units are partitioned into:

- busy time: time interval [3, 4];
- running time: time intervals [4, 6] and [7, 8];
- preemption time: time interval [6, 7]; and
- free time: time intervals [0, 3] and [8, 9].

The time intervals [0, 4], [6, 7], and [8, 9] can be occupied by other tasks running in the same processor. Moreover, only time intervals [0, 3] and [8, 9] could alternatively be idle.

Fig. 1: Real-time scheduling of task τ_2 .

B. Conditional-Preemption

From the perspective of processor and individual tasks, two sets of general policies are presented, respectively. They apply to any specific conditional-preemption plans.

1) Matrix-based conditional-preemption:

In this study, the priority-based preemption policy is extended to a general matrix-based policy. From the perspective of each processor, its preemption matrix is defined to describe the preemption relations among the tasks running in it.

Definition 2: [preemption matrix] An $n \times n$ matrix P is said to be a *preemption matrix* if $P_{i,j} = 1$ (resp., $P_{i,j} = 0$) represents that a task τ_i is allowed (resp., not allowed) to be preempted by τ_j .

Proposition 1: Preemption matrix P can provide up to 2^{n^2-n} conditionally-preemptive real-time scheduling plans.

Proof: Since τ_i cannot be preempted by itself, we have $P_{i,i} = 0$ for all $i \in \mathbf{n}$. If $i \neq j$, $P_{i,j}$ can be either 0 or 1. Since there are $n^2 - n$ *'s in an $n \times n$ matrix P , then P can provide up to 2^{n^2-n} scheduling plans. ■

Example.

Suppose that a processor executes all the tasks shown in Table I. Its preemption matrix P is in the form

$$P = \begin{pmatrix} 0 & * & * & * \\ * & 0 & * & * \\ * & * & 0 & * \\ * & * & * & 0 \end{pmatrix}$$

where *, either 0 or 1, can be predefined by users. According to preemptive real-time scheduling, each task can be preempted by other tasks. Thus, all the *'s in P are replaced by 1, as shown in P_1 . In contrast, according to non-preemptive real-time scheduling, no task can be preempted by other tasks. Thus, all the *'s in P are replaced by 0, as shown in P_2 .

Matrix P_3 describes that

- τ_1 can be preempted by τ_2 , τ_3 , and τ_4 ;
- τ_2 can be preempted by τ_3 and τ_4 ; and
- τ_3 can be preempted by τ_4 .

This is in accordance with the FP real-time scheduling by assigning priorities to tasks τ_1 , τ_2 , τ_3 , and τ_4 in an increasing order. Similarly, we can assign priorities to tasks τ_1 , τ_2 , τ_3 , and τ_4 in a decreasing order, as shown in P_4 .

By applying PTS to assign preemption threshold to each task, P_3 can be revised to be P_5 as an example. The preemption matrix for the motivating example is P_6 .

$$P_1 = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \quad P_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_4 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

$$P_5 = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad P_6 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Definition 3: [matrix-based conditional-preemption] A preemption policy is said to be *matrix-based conditionally-preemptive* if it can be represented by a preemption matrix.

2) WCET-based conditional-preemption:

In this study, the task-based preemption policy is extended to a general policy named WCET-based conditional-preemption. The execution of task τ_i takes C_i time units. From the perspective of an individual task, between any two adjacent processor time units the execution of τ_i can be preempted by a subset of other tasks. With respect to task τ_i , any two such subsets can be different from each other. Evidently, DPS studied in [21] can be considered as an application of the presented conditional-preemption policy.

In case that WCET-based conditional-preemption relations do not exist, the preemption relations defined in the matrix are applied to the real-time scheduling throughout the execution. Otherwise, the real-time scheduling should take both the WCET-based conditional-preemption and matrix-based conditional-preemption into account simultaneously.

Example.

According to P_1 in the last example, τ_2 can be preempted by τ_1 , τ_3 , and τ_4 , which represents that the execution of τ_2 can be interrupted by τ_1 , τ_3 , or τ_4 immediately upon their arrival. As shown in Table I, we have $C_2 = 3$, i.e., the execution of τ_2 takes three time units. Thus we can define two different WCET-based conditional-preemption plans for the execution of τ_2 , such as 1) between the first two time units, only τ_1 and τ_3 can interrupt the execution of τ_2 ; and 2) between the last two time units, only τ_1 and τ_4 can interrupt the execution of τ_2 .

V. DES MODEL FOR REAL-TIME SYSTEMS

In this research, regular languages are utilized to describe the processor behavior related to the periodic real-time tasks' execution. Thereafter, each language will be followed by a DES generator representation. Finally, the synchronized language, represented by a more complex DES generator, is utilized to describe the global processor real-time scheduling behavior. In a synchronized DES generator, all the enabled events can occur without considering their priorities. Thus,

if two or more events are eligible simultaneously, their synchronous product allows them to occur in any order. Since synchronous product can provide all the possible sequences that are not related to priorities, conditional-preemption is possible.

Example.

Suppose that we have three DES generators $\mathbf{G}_a$, $\mathbf{G}_b$, and $\mathbf{G}_c$. As depicted in Fig. 2, their alphabets are $\Sigma_i = \{\sigma_i, \sigma_d\}$, $i \in \{a, b, c\}$. Consequently, $L_m(\mathbf{G}_i) = (\sigma_i \sigma_d)^*$. The synchronous product, $\mathbf{G}_a || \mathbf{G}_b$ (resp., $\mathbf{G}_a || \mathbf{G}_b || \mathbf{G}_c$), is illustrated in Fig. 3(a) (resp., 3(b)), which has all the 4 (resp., 6) paths, in which σ_i can occur in any random sequence.

Fig. 2: DES model $\mathbf{G}_i$.

Fig. 3: Synchronous product.

A. DES Model for Periodic Tasks

For any task $\tau_i \in \mathbb{S}$ arriving periodically, its execution is represented by a DES generator $\mathbf{G}_i$ with marked language $L_m(\mathbf{G}_i)$ and (prefix) closed language $L(\mathbf{G}_i)$ satisfying

$$L(\mathbf{G}_i) = \overline{L_m(\mathbf{G}_i)} \quad (1)$$

that describes all possible executions and random preemptions of task τ_i .

1) Regular language representation:

The *alphabet (event labels)*, written as Σ_i to describe the behavior of $\mathbf{G}_i$, is the disjoint union of Σ_{oi} and Σ_e , i.e., $\Sigma_i = \Sigma_{oi} \dot{\cup} \Sigma_e$, with $\Sigma_{oi} \cap \Sigma_e = \emptyset$, $\Sigma_{oi} = \{\gamma_i, \alpha_i, \beta_i\}$, and $\Sigma_e = \{c_1, c_2, \dots, c_i, \dots, c_n, t\}$, where

- Σ_{oi} is the operation event set of task τ_i , with
 - γ_i : release event,
 - α_i : the execution of τ_i is started, and
 - β_i : the execution of τ_i is completed;
- Σ_e is the execution event set, with
 - $(i \in \mathbf{n})$ c_i : τ_i is running in the processor, and
 - t : no task is running in the processor, i.e., the corresponding processor time unit is idle.

For any $\sigma \in \Sigma_e$, the occurrence of σ takes a single processor time unit. The marked language $L_m(\mathbf{G}_i)$ describes all the

possible execution sequences of task τ_i 's execution within a period T_i . We have

$$L_m(\mathbf{G}_i) = L_i^R(\gamma_i L_i^T)^*. \quad (2)$$

This expression contains two parts:

- L_i^R : the processor behavior before task τ_i is released; and
- L_i^T : the processor behavior within a period T_i between the occurrence of two adjacent γ_i 's.

The events occurring within $s_r \in L_i^R$ take R_i time units in total, i.e.,

$$L_i^R = \{s_r \mid |s_r| = R_i\}. \quad (3)$$

If τ_i is a synchronous task, then L_i^R is empty. Otherwise, the set of strings $S_1 \subseteq (\Sigma_e - \{c_i\})^*$ represents the possible system behavior. Sublanguage L_i^R could be idle or other tasks being processed earlier than γ_i . For any $\sigma \in \Sigma_i$, let $\# \sigma(s_r)$ represent the number of occurrences of σ in a string s_r . We have

$$L_i^R = \{s_r \mid \#t(s_r) + \sum_{j=1, j \neq i}^n \#c_j(s_r) = R_i\}. \quad (4)$$

In a period T_i , events α_i and β_i must occur only once. Since their occurrences are instantaneous (take no time), and the events occurring within $s \in L_i^T$ take T_i time units in total, the length of every string in a period equals $T_i + 2$. Formally,

$$L_i^T = \{s \mid |s| = T_i + 2\}. \quad (5)$$

For any $s \in L_i^T$ representing the complete execution of task τ_i , the number of c_i 's is C_i , i.e.,

$$\#c_i(s) = C_i. \quad (6)$$

In a period T_i , i.e., in L_i^T , the processor runs T_i processor time units that are occupied by all the execution events. Formally, s satisfies

$$\#t(s) + \sum_{j=1}^n \#c_j(s) = T_i. \quad (7)$$

Any string $s \in \gamma_i L_i^T$ contains a substring $s' = \gamma_i s^e \beta_i$, in which s^e represents the system behavior since the arrival of τ_i until its execution is completed. Thus the response time of task τ_i is the processor time spent between the occurrences of γ_i and β_i . Since the occurrence of α_i is instantaneous, the response time of τ_i in s is

$$\mathbb{P}_i = |s^e| - 1. \quad (8)$$

The set of strings, $S_1 \subseteq (\Sigma_e - \{t, c_i\})^*$, occurring earlier than α_i is utilized to represent the busy time. Moreover, the preemption time, occurring between α_i and β_i randomly, is represented by a set of strings $S_2 \subseteq (\Sigma_e - \{t, c_i\})^*$. Furthermore, a set of strings $S_3 \subseteq (\Sigma_e - \{c_i\})^*$ that occur later than β_i , is utilized to represent the free time. Consequently, for any $s \in L_i^T$, s is structurally represented by

$$s = s_1 \alpha_i (s_2 c_i s_2) \dots (s_2 c_i) \beta_i s_3 \quad (9)$$

with $s_1 \in S_1$, $s_2 \in S_2$, and $s_3 \in S_3$, respectively. The strings in $(\Sigma_e - \{t, c_i\})^*$ represent the system behavior corresponding to the random preemption by other tasks. String s_2 occurs between α_i and c_i or any two adjacent c_i 's and represents that the execution of τ_i can be preempted at any time. After the occurrence of the last c_i , β_i occurs immediately to not delay the response time.

Strings in $(\Sigma_e - \{c_i\})^*$ represent the system behavior in the free time. These processor time units can be idle, or utilized to execute other tasks. In order to satisfy the hard deadline D_i , all the c_i 's must occur before β_i . Thus, before the occurrence of β_i , the preemption time cannot be longer than $D_i - C_i$. Formally, strings s_1 and s_2 within Eq. (9) form sublanguages S_1 and S_2 that also satisfy Eqs. (10) and (11) as follows:

$$S_1 = \{s_1 | 0 \leq \sum_{j=1, j \neq i}^n \#c_j(s_1) \leq D_i - C_i\} \quad (10)$$

$$S_2 = \{s_2 | 0 \leq \sum_{j=1, j \neq i}^n \#c_j(s_2) \leq D_i - C_i\} \quad (11)$$

By Eq. (7), the free time cannot be longer than $T_i - C_i$. Formally, sublanguage S_3 in Eq. (9) must also satisfy

$$S_3 = \{s_3 | 0 \leq \#t(s_3) + \sum_{j=1, j \neq i}^n \#c_j(s_3) \leq T_i - C_i\}. \quad (12)$$

Example.

The closed and marked languages to describe the processor behavior to execute task τ_1 are

$$L(\mathbf{G}_1) = \overline{L_m(\mathbf{G}_1)}$$

and

$$L_m(\mathbf{G}_1) = L_1^R(\gamma_1 L_1^T)^*.$$

Sublanguage L_1^T satisfies $L_1^R = \epsilon$, $\forall s \in L_1^T$, $|s| = 8$, $\#c_1(s) = 2$, and $\#t(s) + \sum_{j=1}^3 \#c_j(s) = 6$. String s is structurally represented by

$$s = s_1 \alpha_1 s_2 c_1 s_2 c_1 \beta_1 s_3$$

with $s_1 \in S_1$, $s_2 \in S_2$, and $s_3 \in S_3$ as follows.

- $S_1 = \{s_1 \in \{c_2, c_3\}^* | 0 \leq \sum_{j=2}^3 \#c_j(s_1) \leq 4\}$,
- $S_2 = \{s_2 \in \{c_2, c_3\}^* | 0 \leq \sum_{j=2}^3 \#c_j(s_2) \leq 4\}$, and
- $S_3 = \{s_3 \in \{c_2, c_3, t\}^* | 0 \leq \#t(s_3) + \sum_{j=2}^3 \#c_j(s_3) \leq 4\}$.

So far, the regular language description of processor behavior related to each task's execution is well defined. The next step is to utilize an appropriate DES model to represent the regular language satisfying Eqs. (1)-(12) simultaneously.

2) DES generator representation:

The DES generator for task τ_i is represented by

$$\mathbf{G}_i = (Q_i, \Sigma_i, \delta_i, q_{0i}, Q_{mi})$$

where

- Q_i is the finite state set,
- Σ_i is the alphabet with $\Sigma_i = \Sigma_{oi} \cup \Sigma_e$ it can also be partitioned into $\Sigma_i = \Sigma_{ci} \cup \Sigma_{ui}$, with
 - $\Sigma_{ci} = \{\alpha_i, c_1, c_2, \dots, c_i, \dots, c_n\}$: controllable event subset and
 - $\Sigma_{ui} = \{\beta_i, \gamma_i, t\}$: uncontrollable event subset;
- $\delta_i : Q_i \times \Sigma_i \rightarrow Q_i$ is the (partial) transition function,
- q_{0i} is the initial state, and
- Q_{mi} is the subset of marker states.

The controllability of α_i (resp. $i \in \mathbf{n}$, c_i) endows the uni-processor with the authority to choose and execute (resp. interrupt) any task among all the released ones. Disabling the controllable event α_i is utilized to delay the execution of task τ_i for the purpose of avoiding blocking. The general DES model for real-time periodic tasks is presented in Fig. 4. The subscript of each state's name consists of two parts. All the states and transitions are defined by:

- states $y_{0i,0}, y_{0i,1}, \dots, y_{0i,R_i-1}$ form the state set before the task releases for the first time;
- state y_{0i,R_i} : τ_i releases;
- for the other states except $y_{0i,-}$, the first subscript is the past processor time unit after the arrival of task τ_i in the current period, and the second is the corresponding operation related to τ_i or the amount of processor time units already utilized to process τ_i . These states are defined as
 - ($i \in \mathbf{n}, 0 \leq j \leq D_i - C_i$) y_{j,α_i} : the processor is ready to process τ_i ,
 - ($i \in \mathbf{n}, C_i \leq j \leq D_i$) y_{j,β_i} : the execution of τ_i is complete,
 - ($0 \leq j \leq T_i, 0 \leq k \leq C_i$) $y_{j,k}$: during the first j processor time unit after the release of τ_i , k time units have been utilized to process τ_i ,
 - $y_{T_i,\beta_i} = y_{0i,R_i}$,
 - $y_{T_i,\beta_i} = y_{0i,R_i}$, $D_i = T_i \Rightarrow y_{D_i,\beta_i} = y_{0i,R_i}$, and
 - the states with outgoing arrows are the marker states Q_{mi} ;
- the state with a double arrow is the initial and marker state $y_{0i,0}$;
- c_i (resp., c_j) represents the execution of τ_i (resp., $\tau_j \in \mathbb{S}$, $j \neq i$); and
- the function δ_i satisfies
 - ($0 \leq p \leq R_i - 1$) $\delta(y_{0i,p}, t) = y_{0i,p+1}$: the processor time units are idle;
 - ($0 \leq p \leq R_i - 1$) $\delta(y_{0i,p}, c_j) = y_{0i,p+1}$: $\tau_j \in \mathbb{S}$, $j \neq i$ is in process;
 - $\delta(y_{0i,R_i}, \gamma_i) = y_{0i,\alpha_i}$: τ_i is released;
 - ($0 \leq p \leq D_i - C_i$) $\delta(y_{p,\alpha_i}, \alpha_i) = y_{p,0}$: at state y_{p,α_i} , the execution of τ_i is started;
 - ($C_i \leq p \leq D_i$) $\delta(y_{p,C_i}, \beta_i) = y_{p,\beta_i}$: at state y_{p,C_i} , the processing of τ_i is completed;
 - ($0 \leq p \leq D_i - 1$, $0 \leq k \leq C_i - 1$) $k \leq p$, $\delta(y_{p,k}, c_i) = y_{p+1,k+1}$: τ_i is in process;
 - ($0 \leq p \leq T_i - 1$, $q \neq c_i$), $\delta(y_{p,q}, c_j) = y_{p+1,q}$: $\tau_j \in \mathbb{S}$, $j \neq i$ is in process;
 - ($C_i \leq p \leq T_i - 2$), $\delta(y_{p,\beta_i}, t) = y_{p+1,\beta_i}$: after the occurrence of β_i , the processor time units are idle;

and
 $-\delta(y_{T_i-1,\beta_i}, t) = y_{0i}$: the last processor time unit in T_i is idle.

Fig. 4: General DES model for task τ_i .

Example.

The DES model G_2 corresponding to tasks τ_2 is depicted in Fig. 5. Since $R_2 = 3$, event γ_2 occurs at the end of the third processor time unit. After each period of τ_2 is finished, γ_2 occurs immediately to repeat the process.

Fig. 5: DES model G_2 .

B. DES Task Model Creation in TCT

The software package TCT¹ is a tool utilized to create the DES generator related to each task. Thereafter, the DES model for RTS is established as the synchronous product [13] of the generated DES models. Further operations to edit the models and/or compute the supervisor can also be executed in TCT. The procedures utilized in this study are summarized in Appendix 1. All the operations in TCT and the names of the

generated files are recorded in an annotated file MAKEIT.TXT. In TCT, G_1 , G_2 , G_3 , and G_4 are named as TASK1, TASK2, TASK3, and TASK4, respectively. As an example, TASK2 is reported in Appendix 2. Furthermore, for $i \in \mathbf{n}$, events γ_i , α_i , and β_i are renamed $i0$, $i1$, and $i2$, respectively. Events c_i and t are represented by $i9$ and 0 , respectively. The system behavior corresponding to each processor is calculated by synchronous product. Suppose that we have an RTS $\mathbb{S}$ that possesses only one processor; three examples are discussed below.

1) Plan 1: τ_1 and τ_2 in process:

Suppose that tasks τ_1 and τ_2 are processed in $\mathbb{S}$, denoted by $\mathbb{S}^1$. It is generated by the procedure **sync** provided by TCT. Since tasks $\tau_3, \tau_4 \notin \mathbb{S}^1$, within τ_1 and τ_2 , we can eliminate c_3 and c_4 (events 39 and 49) by relabeling them to be c_2 (event 29) and c_1 (event 19), respectively. Thereafter, the DES model representing $\mathbb{S}^1$ can be generated by synchronous product. In TCT, $\mathbb{S}^1$ is represented by SYS1 that contains 71 states and 98 transitions. The corresponding TCT procedures are:

```
TEST1 = relabel (TASK1, [[39, 29], [49, 29]]) (42, 72)
TEST2 = relabel (TASK2, [[39, 19], [49, 19]]) (28, 45)
SYS1 = sync (TEST1, TEST2) (71, 98) 2
```

2) Plan 2: τ_1 , τ_2 , and τ_4 in process:

Suppose that tasks τ_1 , τ_2 , and τ_4 are processed in RTS $\mathbb{S}$, denoted by $\mathbb{S}^2$. Named to be SYS2 in TCT, it can be generated in a similar way, i.e.,

```
TEST1 = relabel (TASK1, [[39, 29]]) (42, 102)
TEST2 = relabel (TASK2, [[39, 19]]) (28, 63)
TEST4 = relabel (TASK4, [[39, 19]]) (69, 173)
SYS2 = sync (TEST1, TEST2, TEST4) (170, 279)
```

3) Plan 3: τ_1 , τ_2 , τ_3 , and τ_4 in process:

Suppose that all four tasks τ_1 , τ_2 , τ_3 , and τ_4 are processed in RTS $\mathbb{S}$, denoted by $\mathbb{S}^3$. Represented by SYS3 in TCT, $\mathbb{S}^3$ can be generated in a similar way, i.e.,

```
SYS3 = sync (TASK1, TASK2, TASK3, TASK4) (952, 2056)
Blocked_events = [0]
```

There is a blocked event 0 (t) in SYS3 ($\mathbb{S}^3$), which represents that in the real-time scheduling, there is no idle time unit. This means that the processor utilization of $\mathbb{S}^3$ is $U^3 \geq 1$. According to the study in [3], $\mathbb{S}^3$ is non-schedulable if $U^3 > 1$. By calculating the processor utilization of $\mathbb{S}^3$, we obtain $U^3 = 3/9 + 3/6 + 1/5 + 2/14 > 1$. Thus, $\mathbb{S}^3$ is non-schedulable. In the modeling phase, “bad decisions” made by synchronous product may block the priority-free conditionally-preemptive real-time scheduling. As a solution, in the rest of this paper, SCT is utilized to supervise the RTS to be nonblocking.

VI. SPECIFICATIONS FOR OPTIMAL PRIORITY-FREE CONDITIONALLY-PREEMPTIVE REAL-TIME SCHEDULING

In accordance with [14], all possible behaviors in a processor are generated by a DES, called the *plant*. A processor under control is a subset of the generated languages with respect

¹<http://www.control.utoronto.ca/DES>

²All the original message “Blocked events = None” in the original MAKEIT file are eliminated for readability.

to certain constraints that are contained in some specification languages. In order to schedule the processor to be non-blocking and conditionally-preemptive, we shall impose the synchronous product of proper specifications on the behavior of the processor. For each task running in a processor, four types of specifications are defined:

- Nonblocking specifications: nonblocking preemptive scheduling of real-time tasks;
- Matrix-based conditional-preemption specifications: the preemption relation among all the tasks;
- WCET-based conditional-preemption specifications: during the execution of each task, the exact preemption plan between two adjacent time units;
- WCRT-based conditional-preemption specifications: the WCRT in all the periods.

A. Nonblocking Specifications

In order to control the RTS to be nonblocking, the specification $\mathbf{S}_i^N$ for τ_i should allow the occurrence of any $s \in \Sigma_i^*$, i.e., $L(\mathbf{S}_i^N) = \Sigma_i^*$. The procedure **allevents** can be utilized to generate a DES representing Σ_i^* . As shown in Fig. 6, the nonblocking specification for task τ_i is a generator with only one state at which all $\sigma \in \Sigma_i$ are enabled.

Fig. 6: Nonblocking specification for task τ_i .

B. Matrix-Based Conditional-Preemption Specifications

In a processor, all the possible preemptions that may occur during the execution of τ_i are defined in the i -th row of the preemption matrix P . More precisely, task τ_i can be preempted by τ_j in case $P_{i,j} = 1$. The preemption occurs between the occurrence of α_i and β_i . Thus, a specification $\mathbf{S}_i^P$ is defined for each task τ_i with a generator

$$\mathbf{S}_i^P = (Q_i^P, \Sigma_i^P, \delta_i^P, q_{0i}^P, Q_{mi}^P).$$

Here

- Q_i^P : the state set contains two states:
 - y_0 : τ_i is not in process; and
 - y_1 : τ_i is in process;
- $\Sigma_i^P = \bigcup_{i \in \mathbf{n}} \Sigma_i$: the set of all the events appearing in the processor;
- δ_i^P : the (partial) transition function:
 - $\delta_i^P(y_0, \sigma) = y_0, \sigma \in \Sigma_e - \{c_i\}$: τ_i is not in process, the time unit can be taken by other tasks or idle;
 - $\delta_i^P(y_1, c_i) = y_1$: τ_i is in process;
 - $\delta_i^P(y_1, c_j) = y_1$: τ_i can be preempted by τ_j , i.e., $P_{i,j} = 1$;
 - $\delta_i^P(y_0, \alpha_i) = y_1$: the execution of τ_i is started; and
 - $\delta_i^P(y_1, \beta_i) = y_0$: the execution of τ_i is completed;
- $Q_{mi}^P = \{q_{0i}^P\} = \{y_0\}$ is the initial and marker state.

The DES model of specification $\mathbf{S}_i^P$ for τ_i is illustrated in Fig. 7, where c_j represents the execution of task τ_j that is allowed to preempt the execution of τ_i .

Fig. 7: Matrix-based conditional-preemption specification.

C. WCET-Based Conditional-Preemption Specifications

In this section, we define the exact preemption plans between two adjacent time units (c_i 's) of task τ_i . A specification $\mathbf{S}_i^C$ is defined for each task τ_i with a generator

$$\mathbf{S}_i^C = (Q_i^C, \Sigma_i^C, \delta_i^C, q_{0i}^C, Q_{mi}^C).$$

Here

- Q_i^C : the state set containing $C_i + 2$ states;
- $\Sigma_i^C = \bigcup_{i \in \mathbf{n}} \Sigma_i$: the set of all the events appearing in the processor;
- δ_i^C : the (partial) transition function:
 - $\delta_i^C(0_i, \sigma) = 0_i, \sigma \in \Sigma_e - \{c_i\}$: τ_i is not in process; and the time unit can be occupied by other tasks or idle;
 - $\delta_i^C(0_i, \alpha_i) = y_0$: τ_i is arrived;
 - $\delta_i^C(y_i, c_i) = y_{i+1}$: τ_i is in process;
 - $\delta_i^C(y_i, c_j) = y_i$: τ_i is preempted by the execution of τ_j ;
 - $\delta_i^C(y_{C_i+1}, \beta_i) = 0_i$: the execution of τ_i is completed;
- $Q_{mi}^C = \{q_{0i}^C\} = \{0_i\}$ is the initial and marker state.

The DES model of specification $\mathbf{S}_i^C$ for τ_i is shown in Fig. 8.

Fig. 8: WCET-based conditional-preemption specification.

D. WCRT-Based Conditional-Preemption Specifications

The preemption of real-time execution increases the response time of task τ_i . In order to constrain that the execution time of task τ_i to be no longer than a value WCRT $\mathbb{W}_i$, i.e., in a period T_i , the execution between γ_i and β_i is limited to be not greater than $\mathbb{W}_i$ time units. A specification $\mathbf{S}_i^W$ is defined for each task τ_i with a generator

$$\mathbf{S}_i^W = (Q_i^W, \Sigma_i^W, \delta_i^W, q_{0i}^W, Q_{mi}^W).$$

Here

- Q_i^W : the state set containing $\mathbb{W}_i + 2$ states;
- $\Sigma_i^W = \Sigma_i - \{t\}$: the set of all the events except t appearing in $\mathbf{G}_i$;
- δ_i^W : the (partial) transition function:
 - $\delta_i^W(0_i, \sigma) = 0_i, \sigma \in \Sigma_e - \{c_i\}$: τ_i is not in process; and the time unit can be taken by other tasks or idle;
 - $\delta_i^W(0_i, \gamma_i) = y_0$: τ_i is arrived;

- $(c_j \in \Sigma_e)\delta(y_i, c_j) = y_{i+1}$: τ_j is in process;
- $(1 \leq j \leq \mathbb{W} + 1)\delta_i^{\mathbb{W}}(y_j, \beta_i) = 0_i$: the execution of τ_i is completed;
- $Q_{mi}^{\mathbb{W}} = \{q_{0i}^{\mathbb{W}}\} = \{0_i\}$: the initial and marker state.

The DES model of specification $S_i^{\mathbb{W}}$ for τ_i is shown in Fig. 9. By Theorem 1 (Theorem 3.5.2 in [14]), K can be found by the procedure **supcon** (see Appendix 1).

Fig. 9: WCRT-based conditional-preemption specification.

Theorem 1: [14] Let $E \subseteq \Sigma^*$ and let $K = \text{supC}(E \cap L_m(\mathbf{G}))$. If $K \neq \emptyset$ there exists a *marking nonblocking supervisory control* (MNSC) for $\mathbf{G}$ such that $L_m(V/\mathbf{G}) = K$.

E. Specification Creation in TCT

The corresponding TCT operations to create specifications are listed in Appendix 3. In TCT, SN1, SN2, SN3, and SN4 are the nonblocking specifications S_1^N , S_2^N , S_3^N , and S_4^N , respectively. SP1 (resp. SP2) is the non-preemptive specification S_1^P (resp. S_2^P) for task τ_1 (resp. τ_2).

The matrix-based conditional-preemptions are created for the PTS example (P_5) and motivating example (P_6).

PTS example (P_5):

SP134: τ_1 can be preempted by τ_3 and τ_4 ; and

SP24: τ_2 can be preempted by τ_4 .

Motivating example (P_6):

SP12: τ_1 can be preempted by τ_2 ;

SP24: τ_2 can be preempted by τ_4 ; and

SP41: τ_4 can be preempted by τ_1 .

WCET-based conditional-preemption specifications:

SC1: only the last time unit of τ_1 can be preempted by τ_2 ;

SC2: only the first time unit of τ_2 can be preempted by τ_1 ;

SC3: only the last time unit of τ_1 can be preempted by τ_4 ;

SC4: only the first time unit of τ_2 can be preempted by τ_4 .

WCRT-based conditional-preemption specifications:

SR1: the WCRT of task τ_1 is $\mathbb{W}_1 = 4$; and

SR4: the WCRT of task τ_4 is $\mathbb{W}_4 = 2$.

VII. SUPERVISOR SYNTHESIS

So far, the priority-free scheduling policy with conditional-preemption can be described in regular language that can be represented by DES. It is well known that SCT can be used to find the supremal controllers that provide the minimally restricted controller of the systems. TCT is based on SCT, a well-recognized theory in DES modeling and control. By utilizing the procedure **sync** in TCT, all the specifications can be integrated into a unique one. The procedure **supcon** in TCT finds all the safe execution sequences within an RTS satisfying the synchronized specification. An offline technique is developed in Algorithm 1 to achieve this goal. Users need not be concerned with the mathematical calculations; and by

utilizing TCT, all the safe execution sequences can be provided in the supervisor. Each sequence can be utilized by users to schedule the RTS. All the EDF, FP, and other sequences can be found in the supervisor (Super).

Algorithm 1 Supervisory control of RTS

1. Build Plant by **sync**;
2. Build Spec by **sync**;
3. Super = **supcon** (Plant, Spec).

By comparison, under EDF real-time scheduling the tasks with the earliest deadlines are assigned with the highest priority. Moreover, EDF scheduling chooses only one task among them to execute without considering other possibilities. Other released tasks have no chance to be executed by the processor.

A. Example. 1.

In this study, as illustrated in Fig. 10, an example motor network similar to the one studied in [13] is considered as an RTS. Suppose that four electric motors are controlled by a uni-processor. Their deadlines and periods are represented by D and T, respectively. These parameters coincide with those of the tasks in the previous examples as

- Motor 1: τ_1 ,
- Motor 2: τ_2 ,
- Motor 3: τ_3 , and
- Motor 3: τ_4 .

The work plans of the motor network also coincide with the RTS models presented in Section V. Since $R_2 = 3$, Motor 2 will be ready three ms later than other tasks. We take the three work plans of $\mathbb{S}$, i.e., $\mathbb{S}^1$, $\mathbb{S}^2$, and $\mathbb{S}^3$, to find all the safe execution sequences under priority-free conditionally-preemptive scheduling.

Fig. 10: A motor network example.

B. Supervisory Control of $\mathbb{S}^1$

Since $\tau_1, \tau_2 \in \mathbb{S}^1$, we only consider the specifications corresponding to tasks τ_1 and τ_2 . The scheduling corresponding to P_1 is preemptive. Thus, we only need to control the RTS to be nonblocking. Denote this specification by PS1. According to Algorithm 1, in TCT, it is calculated by

$$PS1 = \text{sync} (SN1, SN2) (1, 11)$$

All the safe preemptive execution sequences are calculated by the procedure **supcon**, i.e.,

PSUP1 = **supcon** (SYS1, PS1) (71, 98)

The marker states in PSUP1 represent that the corresponding task is ready to be released, which is redundant information for the users. Thus they are unmarked as follows.

PSUP1 = **edit** (PSUP1, [mark -[all]]) (71, 98)

The safe execution sequence set in PSUP1 is represented by a DES with 84 states and 130 transitions. By projecting out all events but α_i , i.e.,

PMPSUP1 = **project** (PSUP1, Image [11, 21]) (6, 7)

we can obtain the preemptive release map of $\mathbb{S}^1$. All the safe release sequences are shown in Fig. 11. In PSUP1, by projecting out all events but c_i , i.e.,

PJPSUP1 = **project** (PSUP1, Image [19, 29]) (24, 33)

we can obtain the preemptive scheduling map of $\mathbb{S}^1$, as shown in Fig. 12(a), that contains all the safe execution sequences of c_1 and c_2 . The computed supervisor is shown in Table II, in which RM and SM represent the release map and the scheduling map, respectively. The numbers of the states and transitions are recorded in the form (number of states, number of transitions). By Algorithm 1, several other examples by considering different specifications are also listed in Table II. All the release maps are isomorphic with the release map depicted in Fig. 11. Moreover, all the scheduling maps are depicted in Fig. 12. This means that, based on a supremal release map, according to different preemption plans, the priority-free conditionally-preemptive scheduling policy can provide different results. To the best of our knowledge, no other scheduling algorithms can schedule an RTS by considering P_3 and SC1 simultaneously.

Fig. 11: Release map of $\mathbb{S}^1$.

TABLE II: Uni-processor scheduler behaviors of $\mathbb{S}^1$

Spec	Super	RM	SM
P_1	(71, 98)	(6, 7)	(24, 33)
P_2	(58, 70)	(6, 7)	(20, 21)
P_3	(69, 87)	(6, 7)	(24, 27)
P_4	(69, 87)	(6, 7)	(24, 27)
$P_1, SC1, SC2$	(68, 90)	(6, 7)	(23, 27)
$P_3, SC1$	(65, 82)	(6, 7)	(22, 24)

For comparison, the preemptive EDF scheduling result of $\mathbb{S}^1$ by Cheddar [23] is depicted in Fig. 13, which can also be found in Fig. 12(a). The execution sequence in Fig. 13 seems like a non-preemptive scheduling sequence. Evidently, none of the exact preemptive scheduling sequences in Fig. 12(a) can be generated by EDF. In case that at $t = 9$ in Fig. 13, τ_2 (with the earliest deadline) cannot arrive on time, and

then according to the multiple sequences, users can choose another available sequence shown in Fig. 12(a) to schedule task τ_1 first. Thus, recalculating the scheduling sequences is unnecessary. However, at $t = 9$, there is no EDF sequence to execute τ_1 first. If τ_2 cannot arrive on time, the EDF scheduling cannot schedule $\mathbb{S}^1$ successfully. The supervisory control technique provides a larger number of safe execution sequences as compared with EDF scheduling. The WCRT of τ_1 in the scheduling sequence shown in Fig. 13 is $\mathbb{W}_1 = 6$. By comparison, in the scheduling map shown in Fig. 12(a), when tasks τ_1 and τ_2 are released simultaneously, they can preempt another randomly. In all these sequences, we have $\mathbb{W}_1 \leq 6$.

C. Supervisory Control of $\mathbb{S}^2$

Since $\tau_1, \tau_2, \tau_4 \in \mathbb{S}^1$, we only consider the specifications corresponding to tasks τ_1, τ_2 , and τ_4 . According to Algorithm 1, several supervisors are calculated and listed in Table III. The release maps are isomorphic with each other, as shown in Fig. 14. The scheduling maps of $\mathbb{S}^2$ are depicted in Fig. 15. To the best of our knowledge, no sequence in Figs. 15(b) and 15(d) can be achieved by other scheduling algorithms.

TABLE III: Uni-processor scheduler behaviors of $\mathbb{S}^2$

Spec	Super	RM	SM
P_5	(128, 186)	(9, 12)	(34, 41)
$P_5, SC3, SC4$	(119, 173)	(9, 12)	(32, 37)
P_6	(145, 21)	(9, 12)	(44, 54)
$P_6, SC1$	(140, 204)	(9, 12)	(39, 47)

Fig. 14: Release map of $\mathbb{S}^2$.

For comparison, the PTS scheduling result of $\mathbb{S}^2$ is depicted in Fig. 16, and it can also be found in Fig. 15(a). Evidently, SCT provides a greater number of safe execution sequences as compared with PTS scheduling. The WCRT of τ_4 in the scheduling sequence shown in Fig. 16 is $\mathbb{W}_4 = 2$. In comparison, in all the sequences shown in Fig. 15(a), the WCRT of τ_4 varies from two to eight. This means that the WCRT in the conditional-preemption scheduling may increase. In this case, by also considering SR4 as a specification, we obtain a supervisor with 83 states and 118 transitions. The corresponding release map is isomorphic with the one shown in Fig. 14. Two scheduling sequences are depicted in Fig. 17; they form a subset of the safe executions shown in Fig. 15(a). The sequence in Fig. 16 can be found in Fig. 17.

Fig. 12: Scheduling map of $\mathbb{S}^1$. Fig. 13: Preemptive scheduling map of $\mathbb{S}^1$. Fig. 17: PTS and WCRT scheduling map of $\mathbb{S}^2$.

can be represented by specifications and the corresponding supervisors can be calculated by SCT. 842
843

TABLE IV: Parameters of five tasks

task	R	C	D	T
τ_1	0	2	9	9
τ_2	0	2	10	10
τ_3	0	2	12	12
τ_4	0	2	14	16
τ_5	3	2	16	18

1) Uni-processor system: 844 845

In a uni-processor system, the RTS processing these tasks is represented by a DES generator with 15572 states and 31333 events. Instead of calculating the supervisors of all the 2^{20} plans, we arbitrarily choose the following eight plans to calculate the corresponding supervisors. The corresponding supremal supervisors, release maps, and scheduling maps are recorded in Table V. 846
847
848
849
850
851
852

D. Example. 2. 833

Suppose that we have another motor network, denoted by $\mathbb{S}^4$ that executes the five tasks listed in Table IV. Suppose that in a uni-processor system, the preemption matrix takes the following form 834
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$$P = \begin{pmatrix} 0 & * & * & * & * \\ * & 0 & * & * & * \\ * & * & 0 & * & * \\ * & * & * & 0 & * \\ * & * & * & * & 0 \end{pmatrix}$$

which can provide 2^{20} matrix-based conditionally-preemptive real-time scheduling plans. It is unnecessary to calculate all the scheduling plans simultaneously. However, each of them 838
839
840
841

Fig. 15: Scheduling map of $\mathbb{S}^1$. Fig. 16: PTS scheduling map of $\mathbb{S}^2$.

$$\begin{aligned}
 P_1 &= \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} & P_2 &= \begin{pmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} & P_5 &= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} & P_6 &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} \\
 P_3 &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} & P_4 &= \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} & P_7 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{pmatrix} & P_8 &= \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}
 \end{aligned}$$

The numbers of the states in the release maps of each scheduling plan are close to each other. However, the schedul-

TABLE V: Uni-processor scheduler behaviors

P	Super	RM	SM
P_1	(15572, 31333)	(522, 787)	(2632, 5921)
P_2	(13051, 24466)	(521, 785)	(2354, 4364)
P_3	(6700, 10480)	(520, 774)	(1503, 1935)
P_4	(13508, 25868)	(522, 787)	(2385, 4602)
P_5	(13321, 25536)	(522, 787)	(2353, 4538)
P_6	(13011, 24428)	(522, 787)	(2339, 4326)
P_7	(13676, 26076)	(522, 787)	(2427, 4649)
P_8	(5140, 7396)	(515, 764)	(1279, 1528)

ing maps differ greatly. The preemption matrix P_1 (resp., P_8) represents that the RTS is under the preemptive (resp., non-preemptive) real-time scheduling; and the supervisor provides the most flexible (resp., restricted) behaviors. The preemption matrix P_2 indicates that the real-time scheduling plan is that the execution of τ_3 cannot be preempted by any other task. The preemption matrix P_3 represents that the execution of τ_1 can be preempted by τ_2 , τ_2 by τ_3 , τ_3 by τ_4 , τ_4 by τ_5 , and τ_5 by τ_1 , respectively. Importantly, the release maps for P_4 , P_5 , P_6 , and P_7 , are isomorphic with the release map for P_1 . However, the scheduling maps are differ substantially from one another. The reason is that P_4 , P_5 , P_6 , and P_7 show that the execution of τ_1 can only be preempted by τ_2 , τ_3 , τ_4 , and τ_5 , respectively.

2) Multi-processor system:

Suppose that the tasks are running in an RTS that possesses two processors and their corresponding preemption-matrices are identical. We need to calculate the processor behavior based on each processor individually. Suppose that tasks τ_1 and τ_2 are running in processor Ξ_1 , and τ_3 and τ_4 are running in processor Ξ_2 . Task τ_5 is a newly arrived task and is allowed to enter any processor. By applying preemption matrix P_4 to both Ξ_1 and Ξ_2 , the corresponding supremal supervisors, release maps, and scheduling maps are recorded in Table VI. If task τ_5 enters Ξ_1 (resp., Ξ_2), the corresponding new system behavior is represented by Ξ'_1 (resp., Ξ'_2).

TABLE VI: Multi-processor scheduler behaviors of of $\mathbb{S}^2$

Ξ	Super	RM	SM
Ξ_1	(183, 209)	(20, 21)	(45, 54)
Ξ_2	(87, 101)	(8, 9)	(18, 22)
Ξ'_1	(253, 313)	(25, 26)	(62, 76)
Ξ'_2	(344, 424)	(32, 35)	(76, 98)

E. Example. 3.

Consider a manufacturing cell as an example. As shown in Fig. 18, a robot R is utilized to transport two types of workpieces, W1 and W2, to a conveyor. Two pieces of W1 (resp., W2) are released to the input buffer B_1 (resp., B_2) simultaneously in every six (resp., three) seconds. The robot R has capacity one; and transporting each piece takes one second. Thus, we define two tasks $\tau_1 = \{0, 2, 6, 6\}$ (resp.,

$\tau_2 = \{0, 2, 3, 3\}$) to represent the transportation of the two pieces of W1 (resp., W2) by R , respectively. Consequently, we have a system $\mathbb{S} = \{\tau_1, \tau_2\}$.

Suppose that the preemption matrix with respect to $\mathbb{S}$ is

$$P_1 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

The scheduling can be considered as an FP scheduling, i.e., task τ_1 cannot be preempted by τ_2 ; and τ_2 is allowed to be preempted by τ_1 . In other words, the robot must transport the two W1's in two adjacent seconds, but it is not necessary for transporting W2. By utilizing SCT, the supervisor calculated by **supcon** is represented by a DES with 21 states and 27 transitions. As shown in Fig. 19, the release (resp., scheduling) map is represented by a generator with three (resp., six) states and three (resp., six) transitions. As shown in Fig. 19(b), no preemption scheduling sequences are found. This requires that the robot R must transport the workpieces in the following order periodically:

W2, W2, W1, W1, W2, W2.

Fig. 18: Manufacturing cell.

Fig. 19: FP scheduling.

Suppose that the preemption matrix is

$$P_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$

i.e., the real-time scheduling is preemptive. This means that the robot can transport W1 and W2 in any order. By using SCT to calculate the supervisor, it is represented by a DES with 37 states and 55 transitions. As shown in Fig. 20, the release map (resp., scheduling map) is represented by a generator with four (resp., ten) states and five (resp., 14) transitions. The scheduling map in Fig. 20(b) provides nine safe execution sequences.

If we also require that, based on P_2 , the WCRT of task τ_1 be $\mathbb{W}_1 = 5$, i.e., the two W1's must be transported in the first five seconds after their release, then we obtain a supervisor that is represented by a DES with 63 states and 91 transitions. The release map is isomorphic with the map depicted in 20(a); and the scheduling map is displayed in Fig. 21.

Fig. 20: Preemptive scheduling.

Fig. 21: Preemptive scheduling with $\mathbb{W}_1 = 5$.

VIII. DISCUSSION

A. Computational Complexity and Calculation Speed up

The real-time scheduling is based on the computation of the supremal controllable sublanguage with respect to a finite DES. According to [13] and [24], the computation of the supremal controllable sublanguage with respect to a finite DES can be completed in polynomial time. Similar to [13], the computational complexity of the presented method in this article is characterized by 1) the modeling of a processor time unit as a distinct event in the DES framework and 2) the exponential growth in the number of states when synchronous product is utilized to combine individual tasks into the plant. The computational complexity of the supremal sublanguage of a specification is $O(m^2n^2)$, where m and n are the sizes of the final state set of the plant $\mathbf{G}$ and the specification $\mathbf{S}$, respectively. Similar to [13], this remains a challenge in “scaling up” the proposed method for the real-time scheduling based on SCT. In the worst case, the synchronous product of two discrete-event systems, with x and y states, respectively, results in a more complex system with $x \cdot y$ states. A method to speed up the calculation is to reduce the number of states in the plant and specification. In order to achieve this goal, we divide the calculations into three steps. Each step considers different specifications as follows.

- Step 1: nonblocking and matrix-based conditional-preemption specifications,
- Step 2: nonblocking and WCET-based conditional-preemption specifications, and
- Step 3: nonblocking and WCRT-based conditional-preemption specifications.

As the commutative diagram shown in Fig. 22, Algorithm 1 is revised to be Algorithm 2, in which NS, PS, CS, and RS represent nonblocking specifications, matrix-based conditional-preemption specifications, WCET-based conditional-preemption specifications, and WCRT-based conditional-preemption specifications, respectively.

Fig. 22: Commutative diagram.

Algorithm 2 Supervisory control of RTS

1. Build Plant by **sync**;
2. Build S1, S2, and S3 by **sync**;
3. Super1 = **supcon** (Plant, S1);
4. Super2 = **supcon** (Super1, S2);
5. Super3 = **supcon** (Super2, S3).

For example, the scheduling of $\mathbb{S}^1$ based on P_3 and SC1 can be calculated in the first two steps as listed in Table VII. Moreover, if we require that the WCRT of task τ_1 is $\mathbb{W}_1 = 4$, only one such sequence exists, which is shown in Fig. 23. Accordingly, $\mathbb{S}^1$ can be scheduled in order $\tau_1\tau_2\tau_1\tau_2\tau_2$.

TABLE VII: Uni-processor scheduler behaviors of $\mathbb{S}^1$

Plant	Spec	Super	RM	SM
SYS1	NS	Super1 (69, 87)	(6, 7)	(24, 27)
Super1	P3	Super2 (65, 82)	(6, 7)	(22, 24)
Super2	SC1	Super3 (45, 52)	(5, 5)	(15, 15)

Fig. 23: Scheduling map of $\mathbb{S}^1$ with P_3 , SC1, and SR1.

B. Comparison with the CW Model

In [13], a TDES model is proposed to represent periodic real-time tasks. A task is represented by a TDES

$$\mathbf{G}_i = (Q_i, \Sigma_i, \delta_i, q_{0i}, Q_{mi})$$

where Σ_i consists of

- t : tick event,
- γ_i : the release event of τ_i ,
- α_i : the execution of τ_i is started, and
- β_i : the execution of τ_i is finished.

Suppose that after being enabled, events γ_i , α_i , and β_i should wait for t_{γ_i} , t_{α_i} , and t_{β_i} ticks, respectively, until they are eligible to occur. Formally, the timer bounds are

$$\begin{aligned}
& \bullet \gamma_i := \begin{cases} [0, 0], & \text{if } \tau_i \text{ releases at } r_{1,1} \\ [T_i - t_{\alpha_i} - t_{\beta_i}, T_i - t_{\alpha_i} - t_{\beta_i}], & \text{if } (\forall j > 1) \tau_i \text{ releases at } r_{i,j} \end{cases}, \\
& \bullet \alpha_i := [0, D_i - t_{\beta_i}], \text{ and} \\
& \bullet \beta_i := [t_{\beta_i}, t_{\beta_i}].
\end{aligned}$$

In the CW model [13], $t_{\beta_i} = C_i$. The *regular languages* utilized to describe the tasks represented by TDES/DES are different. In comparison, in the TDES (resp. DES) model, the processor execution of task τ_i is represented by t (resp. c_i). Consequently, in the TDES (resp. DES) model, the execution of different tasks is considered as the same (resp. different) events. Hence, in the *synchronous product*, they will occur simultaneously (resp. separately). For *preemptive* real-time scheduling, because of the nature of the real-time scheduling, the execution of two or more tasks within a uni-processor cannot happen simultaneously. Thus, the CW TDES model cannot be utilized to schedule real-time tasks *conditionally-preemptively*.

Remark:

Suppose that we have two substrings $s_1 = c_1 c_1$ and $s_2 = c_2$ under condition that exists $w_1, v_1 \in \Sigma_1^*$, $w_1 s_1 v_1 \in L(\mathbf{G}_1)$ and exists $w_2, v_2 \in \Sigma_2^*$, $w_2 s_2 v_2 \in L(\mathbf{G}_2)$. The execution of s_1 and s_2 within a uni-processor needs to take three time units. Let $L_1 = \{s_1\} \subset \Sigma_1'^*$ and $L_2 = \{s_2\} \subset \Sigma_2'^*$ with $\Sigma_1' = \{c_1\} \subset \Sigma_1$ and $\Sigma_2' = \{c_2\} \subset \Sigma_2$, respectively. Their *synchronous product* is $L_1 || L_2 = \{c_2 c_1 c_1, c_1 c_2 c_1, c_1 c_1 c_2\}$; it represents all the possible executions correctly. Similarly, according to the TDES model presented in [13], c_1 and c_2 are both replaced by the *tick* event t . Thus, we can obtain $s_1 = tt$, $s_2 = t$, $L_1 = \{s_1\} \subset \Sigma_1'^*$, $L_2 = \{s_2\} \subset \Sigma_2'^*$, $\Sigma_1' = \{t\} \subset \Sigma_1$, and $\Sigma_2' = \{t\} \subset \Sigma_2$. The *synchronous product* of L_1 and L_2 is $L_1 || L_2 = \emptyset$. Evidently, this model cannot actually represent real-time scheduling.

C. Comparison with (Time) Petri Nets Models

Recently, preemptive time Petri nets (pTPN) are developed in [25], [26], and [27] to dynamically schedule real-time systems. These studies have significant improvements in real-time scheduling based on time Petri nets. However, the real-time scheduling in [25], [26], and [27] can only be applied to the FP real-time scheduling. To the best of our knowledge, conditional-preemption is not addressed in these studies. For example, by using the models proposed in [25], [26], and [27], no fixed priority can be assigned to the tasks running in the motivating example of the present paper. Consequently, the real-time scheduling based on SCT is more general than the pTPN model.

Real-time scheduling can also be modeled by Petri nets (PN) with time constraints [28], [29]. However, the proposed approach in [28] faces the schedulability conditions of the offline scheduling problems; and finding the optimal offline periodic schedule is a challenge. The problems of [28] are addressed in [29]. However, it is arguable whether or not that a PN with time constraints is a sufficiently general model to schedule real-time systems. Furthermore, not all the

conditional specifications considered in the present paper are addressed in the PN model with time constraints.

In comparison, in this study, by utilizing SCT offline, all the safe execution sequences can be found if any exist. Otherwise, the supervisor of the system is empty. On the basis, we conclude that the developed conditionally-preemptive scheduling algorithm is more general than the scheduling based on (time) Petri nets models.

IX. CONCLUSION

This study reports a formal constructive method for real-time periodic tasks via a DES model. For both uni-processor and multi-processor systems, the behavior of a processor can be established by the synchronous product of the DES models of all tasks running in it. The tasks can be scheduled without considering their priorities. This paper presents two sets of conditional-preemption specifications, i.e., matrix-based conditional-preemptions, and WCET-based conditional-preemptions. Moreover, in order to control the system to be nonblocking and also limit the WCRT of the tasks, two corresponding sets of specifications are developed. The formal SCT of DES can be considered as a rigorous analysis and synthesis tool to schedule the RTS satisfying the hard deadlines. The procedure **sync** in TCT is utilized to generate the plant and global specifications. By utilizing the procedure **supcon**, all the conditionally-preemptive safe execution sequences can be calculated. These sequences can provide more choices than the EDF, PTS, DPS scheduling algorithms and the TDES SCT-based real-time scheduling proposed in [13]. In order to speed up the calculation, a commutative diagram is proposed to calculate the supervisors in three steps. The off-line scheduling algorithm presented in this study can be applied in a practical context to schedule the real-world uni-processor and multi-processor systems. In future work, we will focus on the real-time conditionally-preemptive scheduling of sporadic tasks.

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Appendix 1.

Synthesis procedures in TCT [14]:

DES2 = **allevents** (DES1) is a marked one-state DES selflooped with all the events of DES1.

DES = **create** (DES) is a new discrete-event system (DES). Option 1 allows fast user input via a sequence of prompts, resulting in direct creation of a .DES file. Option 2 allows the user to create a text (.ATS) file with any ASCII text editor; this file can be converted to a .DES file using the TCT procedure FD.

True/False = **isomorph** (DES1, DES2) tests whether DES1 and DES2 are identical up to renumbering of states (but with initial state held fixed at 0); if so, their state correspondence is displayed.

DES2 = **project** (DES1, [NULL/IMAGE EVENTS]) is a generator of the projected closed and marked languages of DES1, under the natural projection specified by the listed Null or Image events. In decentralized control, DES2 could be an observer's local model of DES1.

DES2 = **relabel** (DES1, [OLD-NEW EVENT LABEL PAIRS]) coincides with DES1, except for a mapping of specified event labels in DES1; unmapped labels are unchanged. Not for use with vocalized DES.

DES3 = **supcon** (DES1, DES2) is a trim generator for the supremal controllable sublanguage of the marked legal language generated by DES2 with respect to the plant DES1. DES3 provides a proper supervisor for DES1.

DES3 = **sync** (DES1, DES2) is the (reachable) synchronous product of DES1 and DES2.

Appendix 2.

Corresponding TCT MAKEIT file for TASK3 and TASK4:

TASK2 = **create** (TASK2, [mark 0, 1, 2, 3, 4, 9, 10, 15, 16, 21, 22], [tran [0, 0, 1], [0, 19, 1], [0, 39, 1], [0, 49, 1], [1, 0, 2], [1, 19, 2], [1, 39, 2], [1, 49, 2], [2, 0, 3], [2, 19, 3], [2, 39, 3], [2, 49, 3], [3, 20, 4], [4, 19, 10], [4, 21, 5], [4, 39, 10], [4, 49, 10], [5, 19, 11], [5, 29, 6], [5, 39, 11], [5, 49, 11], [6, 19, 12], [6, 29, 7], [6, 39, 12], [6, 49, 12], [7, 19, 13], [7, 29, 8], [7, 39, 13], [7, 49, 13], [8, 22, 9], [9, 0, 15], [9, 19, 15], [9, 39, 15], [9, 49, 15], [10, 19, 16], [10, 21, 11], [10, 39, 16], [10, 49, 16], [11, 19, 17], [11, 29, 12], [11, 39, 17], [11, 49, 17], [12, 19, 18], [12, 29, 13], [12, 39, 18], [12, 49, 18], [13, 19, 19], [13, 29, 14], [13, 39, 19], [13, 49, 19], [14, 22, 15], [15, 0, 21], [15, 19, 21], [15, 39, 21], [15, 49, 21], [16, 19, 22], [16, 21, 17], [16, 39, 22], [16, 49, 22], [17, 19, 23], [17, 29, 18], [17, 39, 23], [17, 49, 23], [18, 19, 24], [18, 29, 19], [18, 39, 24], [18, 49, 24], [19, 19, 25], [19, 29, 20], [19, 39, 25], [19, 49, 25], [20, 22, 21], [21, 0, 3], [21, 19, 3], [21, 39, 3], [21, 49, 3], [22, 21, 23], [23, 29, 24], [24, 29, 25], [25, 29, 26], [26, 22, 3]]) (28, 81)

Appendix 3.

The generated files for the specifications are recorded.

1. Nonblocking specifications:

SN1 = **allevents** (TASK1) (1, 8)
 SN2 = **allevents** (TASK2) (1, 8)
 SN3 = **allevents** (TASK3) (1, 8)
 SN4 = **allevents** (TASK4) (1, 8)

2. Matrix-based conditional-preemption specifications:

SP1 = **create** (SP1, [mark 0], [tran [0, 0, 0], [0, 11, 1], [0, 29, 0], [0, 39, 0], [0, 49, 0], [1, 12, 0], [1, 19, 1]]) (2, 7)
 SP2 = **create** (SP2, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 39, 0], [0, 49, 0], [0, 21, 1], [1, 22, 0], [1, 29, 1]]) (2, 7)
 SP4 = **create** (SP4, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 29, 0], [0, 39, 0], [0, 41, 1], [1, 42, 0], [1, 49, 1]]) (2, 7)
 SP12 = **create** (SP12, [mark 0], [tran [0, 0, 0], [0, 11, 1], [0, 29, 0], [0, 39, 0], [0, 49, 0], [1, 12, 0], [1, 19, 1], [1, 29, 1]]) (2, 8)
 SP24 = **create** (SP24, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 21, 1], [0, 39, 0], [0, 49, 0], [1, 22, 0], [1, 29, 1], [1, 49, 1]]) (2, 8)
 SP41 = **create** (SP41, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 29, 0], [0, 39, 0], [0, 41, 1], [1, 19, 1], [1, 42, 0], [1, 49, 1]]) (2, 8)
 SP134 = **create** (SP134, [mark 0], [tran [0, 0, 0], [0, 11, 1], [0, 29, 0], [0, 39, 0], [0, 49, 0], [1, 12, 0], [1, 19, 1], [1, 39, 1], [1, 49, 1]]) (2, 9)

3. WCET-based conditional-preemption specifications:

SC1 = **create** (SC1, [mark 0], [tran [0, 0, 0], [0, 11, 1], [0, 29, 0], [0, 39, 0], [0, 49, 0], [1, 19, 2], [1, 29, 1], [1, 39, 1], [1, 49, 1], [2, 19, 3], [2, 39, 2], [2, 49, 2], [3, 12, 0], [3, 19, 3], [3, 29, 3], [3, 39, 3], [3, 49, 3]]) (4, 17)
 SC2 = **create** (SC2, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 21, 1], [0, 39, 0], [0, 49, 0], [1, 19, 1], [1, 29, 2], [1, 39, 1], [1, 49, 1], [2, 19, 2], [2, 29, 3], [2, 39, 2], [2, 49, 2], [3, 29, 4], [3, 39, 3], [3, 49, 3], [4, 19, 4], [4, 22, 0], [4, 39, 4], [4, 49, 4]]) (5, 20)
 SC3 = **create** (SC3, [mark 0], [tran [0, 0, 0], [0, 11, 1], [0, 29, 0], [0, 39, 0], [0, 49, 0], [1, 19, 2], [1, 29, 1], [1, 39, 1], [1, 49, 1], [2,

1235 19, 3], [2, 29, 2], [2, 39, 2], [3, 12, 0], [3, 19, 3], [3, 29, 3], [3, 39,
 1236 3], [3, 49, 3])) (4, 17)
 1237 SC4 = **create** (SC2, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 21, 1],
 1238 [0, 39, 0], [0, 49, 0], [1, 19, 1], [1, 29, 2], [1, 39, 1], [1, 49, 1], [2,
 1239 19, 2], [2, 29, 3], [2, 39, 2], [2, 49, 2], [3, 29, 4], [3, 19, 3], [3, 39,
 1240 3], [4, 19, 4], [4, 22, 0], [4, 39, 4], [4, 49, 4])) (5, 20)
 1241 4. WCRT-based conditional-preemption specifications:
 1242 SR1 = **create** (SR1, [mark 0], [tran [0, 0, 0], [0, 10, 1], [0, 29,
 1243 0], [0, 39, 0], [0, 49, 0], [1, 19, 2], [1, 29, 2], [1, 39, 2], [1, 49, 2],
 1244 [2, 12, 0], [2, 19, 3], [2, 29, 3], [2, 39, 3], [2, 49, 3], [3, 12, 0], [3,
 1245 19, 4], [3, 29, 4], [3, 39, 4], [3, 49, 4], [4, 12, 0], [4, 19, 5], [4, 29,
 1246 5], [4, 39, 5], [4, 49, 5], [5, 12, 0]]) (6, 25)
 1247 SR4 = **create** (SR4, [mark 0], [tran [0, 0, 0], [0, 19, 0], [0, 29, 0],
 1248 [0, 39, 0], [0, 40, 1], [1, 19, 2], [1, 29, 2], [1, 39, 2], [1, 49, 2], [2,
 1249 19, 3], [2, 29, 3], [2, 39, 3], [2, 42, 0], [3, 42, 0]]) (4, 15)