Efficient Design of Wind Farm Layouts Utilizing Exact Gradient Information

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science
Mechanical and Industrial Engineering Department
University of Toronto

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Abstract

The Wind Farm Layout Optimization (WFLO) problem has attracted a lot of attention from researchers and industry practitioners, as it has been proven that better placement of wind turbines can increase the overall efficiency and the total revenue of a wind farm. Common approaches found in the WFLO literature focus on minimizing turbine wake interactions based on analytical models. However, the literature seems to have settled on using metaheuristics and stochastic optimization approaches. In this thesis, a gradient optimization approach is proposed to solve highly constrained WFLO problems, by using an interior point method with the exact gradients of the objective and constraints. The superiority of the proposed approach has been demonstrated, and the computational cost is reduced by 1 to 2 orders of magnitude, in terms of objective function evaluations. Additionally, it has been extended to solve the multi-criteria problem considering electrical infrastructure, land usage and environmental aspects.
Dedication

If you do not have voice or feel misrepresented, this page is yours.

Random or even systematic discrimination has nothing to do with your ambition. Your work could be your only supporter.

If you are excluded from access to higher education or were not awarded your academic degree, a thesis at the University of Toronto is written and dedicated to you.

David Guirguis
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Table of Contents

Dedication.......................................................................................................................... iii
Acknowledgments.................................................................................................................. iv
Table of Contents................................................................................................................... v
List of Tables ........................................................................................................................... vii
List of Figures ........................................................................................................................... viii
Chapter 1 ................................................................................................................................ 1
  1 Introduction ......................................................................................................................... 1
    1.1 Related Work ..................................................................................................................... 1
    1.2 Knowledge Gap and Contributions .................................................................................. 4
    1.3 Thesis Outline ................................................................................................................... 6
Chapter 2 ................................................................................................................................ 7
  2 Efficient Optimization of Wind Farms ................................................................................... 7
    2.1 Problem Formulation .......................................................................................................... 7
      2.1.1 Wake Modeling ............................................................................................................. 7
      2.1.2 Land-availability constraints ...................................................................................... 9
      2.1.3 Optimization Problem .................................................................................................. 11
    2.2 Design Optimization Approach ....................................................................................... 12
      2.2.1 Interior-point Method .................................................................................................. 12
      2.2.2 Gradients and Hessians .............................................................................................. 13
Chapter 3 ................................................................................................................................ 16
  3 Numerical Experiments and Discussions .......................................................................... 16
    3.1 Statistical comparison with GA ....................................................................................... 16
    3.2 Scalability and turbine density analyses ......................................................................... 23
    3.3 Land-availability constrained examples .......................................................................... 27
3.4 Further Notes .................................................................................................................. 30

Chapter 4 ............................................................................................................................... 32
4 Micrositing in Large-scale Wind Farms ........................................................................... 32
   4.1 Introduction ................................................................................................................... 32
   4.2 Problem Formulation ................................................................................................... 33
      4.2.1 Annual Energy Production ................................................................................. 33
      4.2.2 Wake Modelling ................................................................................................ 36
   4.3 Results and Discussions ............................................................................................... 37

Chapter 5 .................................................................................................................................. 44
5 Gradient Multi-Objective Optimization of Wind Farms ....................................................... 44
   5.1 Introduction ................................................................................................................... 44
   5.2 Problem Formulation ................................................................................................... 45
      5.2.1 Land Footprint ..................................................................................................... 45
      5.2.2 Electrical Infrastructure ..................................................................................... 46
      5.2.3 Environmental Considerations .......................................................................... 46
      5.2.4 Optimization Problem ....................................................................................... 46
   5.3 Results and Discussion ................................................................................................. 47

Chapter 6 .................................................................................................................................. 51
6 Conclusions and Future Work ............................................................................................ 51

References .............................................................................................................................. 53
List of Tables

Table 3-1 Specifications of wind farm and wind turbine ............................................................... 17
Table 3-2 Computational Budget allocated to GA .............................................................................. 18
Table 3-3 Statistical comparison between GA, HGA, 5S- IPM and 20S- IPM. Best solutions are in bold ........................................................................................................................................... 19
Table 3-4 Comparison between results of IPM starting by different initial configurations (USL-IPM and medians of 5S- IPM and 20S- IPM). Best solutions are in bold......................... 22
Table 3-5 Comparison between results of IPM starting by different initial configurations (USL-IPM and RSL- IPM) with 4, 5 and 6 turbine densities. Best solutions are in bold..... 24
Table 4-1 Results of land-use constrained instances ......................................................................... 40
Table 4-2 All optimized 400-turbines-layouts, with their associated AEP values and cabling costs. Note that each group of turbines connected to the same substation is color-coded. .............................................................................................................................................. 43
Table 5-1 Fractions of dominated solutions for each run..................................................................... 49
List of Figures

Figure 2-1 Schematic drawing of Jensen’s wake model with Gaussian velocity deficit profile... 8

Figure 2-2 Handling land-use constraints for internal and external forbidden zones inside the farm’s boundaries ............................................................................................................................................. 10

Figure 3-1 Wind roses for benchmark problems: (a) unidirectional wind regime; (b) constant frequency multidirectional wind regime; (c) variable frequency multidirectional wind regime ............................................................................................................................................. 16

Figure 3-2 Box plots for results of benchmark problems with different number of turbines ..... 20

Figure 3-3 Wind rose for used test examples in sections 3.1 and 3.2 ........................................ 25

Figure 3-4 Optimized layouts for 37 turbines with densities of 4, 5 and 6 Turbines/Km² respectively, starting by random layout (top) and uniform staggered layout (bottom). x: initial locations; +: optimized locations ........................................................................................................... 25

Figure 3-5 Optimized layouts for 50 turbines with densities of 4, 5 and 6 Turbines/Km² respectively, starting by random layout (top) and uniform staggered layout (bottom). x: initial locations; +: optimized locations ........................................................................................................... 26

Figure 3-6 Optimized layouts for 100 turbines with densities of 4, 5 and 6 Turbines/Km² respectively, starting by random layout (top) and uniform staggered layout (bottom). x: initial locations; +: optimized locations ........................................................................................................... 27

Figure 3-7 Initial layouts (left) and best optimized solutions (right) with convergence plots (far right), starting from a random layout (top) and from a uniform staggered layout (bottom) ............................................................................................................................................. 28

Figure 3-8 Initial layouts (left) and best optimized solutions (right) with convergence plots (far right), starting from a random layout (top) and from a uniform staggered layout (bottom) ............................................................................................................................................. 29
Figure 3-9 Example of land-availability constrained problem solved by IPM with finite-difference approximation of the gradients. .......................................................... 31

Figure 3-10 Schematic drawing of velocity distribution inside the wake region with (a) uniform distribution and (b) Gaussian distribution for velocity deficit................................. 31

Figure 4-1 Vestas V100-1.8MW turbine's manufacturer electric power curve, power coefficient and theoretical wind power curve as functions of wind speed ......................... 34

Figure 4-2 Fitted polynomials to the data points of the manufacturer’s power curve ............. 35

Figure 4-3 Adopted 4th order polynomial power curve in comparison with the manufacturer power curve................................................................................................................ 35

Figure 4-4 Land-use constraints of the numerical experiments ............................................. 37

Figure 4-5 The wind rose shows average wind speeds in each direction............................. 38

Figure 4-6 Optimizing 100 turbines with high severity of proximity constraints: (a) history of turbines movements, “x” indicates the initial positions of the turbines, “o” indicates their final position. (b) Convergence behavior during 2,893 function evaluations . 39

Figure 4-7 Sorted Annual Energy Production (AEP) [TWh] and computational cost of each optimization run, for different numbers of turbines................................................. 39

Figure 5-1 Problem setup for the multi-objective instance .................................................. 48

Figure 5-2 Normalized Pareto-front solutions obtained by five runs with different random starts .......................................................................................................................... 48

Figure 5-3 Normalized global Pareto-front solutions of all ε-constraint IPM runs in comparison with NSGA II Pareto-front................................................................. 49
Chapter 1

Introduction

Wind Farm Layout Optimization is concerned with optimal placement of turbines within the wind farm. Early work in this area discretized the wind farm into a finite set of square cells, using binary variables to represent whether a given cell had a turbine in it [1,2]. This discrete formulation of the WFLO problem, however, has some limitations in terms of its scalability and solvability for increasing problem sizes, which results on hard limits on the level of resolution that can be achieved. More recently, researchers have proposed a continuous-variable formulation [3,4] to avoid the limitations of discretized-domain formulations. In continuous-variable formulations, the location of each turbine is determined by two variables representing its Cartesian coordinates, and thus the turbines are allowed to be placed anywhere inside the farm. This results in much more flexibility in the placement of turbines, which leads to higher energy generation [5].

The main goal of the wind farm is generating electrical energy, which is proportional to the cubic power of the wind speed facing the turbine blades. Inside a wind farm, it is obviously advantageous to place the turbines at locations with higher wind speeds, i.e., at locations where wake effects and turbulence are at a minimum.

1.1 Related Work

In their pioneering work, Mosetti et al. [1] applied a Genetic Algorithm (GA) to a discrete formulation of the WFLO. Grady et al. [2] and Gao et al. [6,7] used multi-population genetic algorithms to enhance the diversity of solutions in an attempt to improve convergence, while Huang [8] proposed a hybrid of GAs with a hill-climbing approach targeting reductions in computational cost. Furthermore, Réthoré et al. [9] proposed a multi-fidelity approach that combines simple genetic algorithm and sequential linear programming, while Rahbari et al. [10] introduced a hybrid GA approach with a multilevel technique for improving the initial population. As another attempt to increase the quality of obtained solutions, Saavedra-Moreno et al. [11]
seeded the initial population of the genetic algorithm with a solution obtained by a greedy heuristic method. Perhaps unsurprisingly, in addition to GAs, other stochastic metaheuristics have been tested in the WFLO problem, such as Differential Evolution [12] and Particle Swarm Optimization [13–17]. In addition to population-based optimization methods, single-solution based methods such as Simulated Annealing [18] and Pattern Search [5] have also been implemented. Comprehensive reviews of wind farm optimization have been published recently, we will refer the reader to [19–21] for more details.

Although implementation of GAs has showed reasonable success in dealing with the WFLO problem, GAs have some limitations regarding computational cost, particularly because of their slow convergence behavior, which typically requires a large number of function evaluations. Additionally, although GAs are generally considered global-seeking optimization methods, there is no guarantee for optimality for any particular problem instance; they are also inadequate in solving certain problems [22–24]. Moreover, enhancing GAs with diversity-preserving operators to reduce premature convergence increases their computational cost [25]. In any case, it is curious that most of the WFLO research has relied on genetic algorithms and other global metaheuristics, despite the fact that early researchers in evolutionary computation did not recommend the use of GAs in analytical problems where classical mathematical methods can be used efficiently [26,27].

Local search approaches have also received attention from the WFLO community. For instance, Wagner et al. [28] developed a randomized problem-specific local search method as an attempt to speed up the farm layout’s evaluation and dealing with large farms efficiently. Another approach was proposed by Feng and Shen [29] who introduced a random search with a wake influence matrix for layout evaluation. In order to increase the likelihood of the local optimizer to reach the global optimum, gradient-based approaches have been used, either as sole solvers or hybridized with other heuristics. For example, Lackner and Elkinton [3] proposed an optimization framework for solving WFLO with a continuous-variable formulation, using gradient descent optimization method, albeit for a simple case of only two turbines. Pérez et al. [30], on the other hand, proposed a multi-start algorithm that comprises a heuristic search to generate random feasible initial solutions and a gradient-based local search to optimize generated solutions. The finite-difference approximation was used to approximate the gradients of the objective function, which can be computationally intensive and can lead to ill-conditioned Hessians [31]. From a practical perspective, this multi-start heuristic-gradient hybrid approach is not necessarily more efficient
than the traditional hybridization of genetic algorithms with gradient local search, since GAs as population-based algorithms work better in exploring the design domain, while the gradient-based local search is effective for local exploitation.

Most of these approaches highlighted the effectiveness of using local searches, although some of these articles [5,29,32] relied on comparisons between GA solutions of discrete-variable formulations of the WFLO and gradient-based solutions of continuous-variable formulations to issue their conclusions. Because of the higher flexibility that the continuous WFLO provides, it is bound to lead to layouts with higher energy generation, thus making these comparisons at least partially biased. In order to exploit the mathematical nature of the WFLO, optimization approaches with mixed-integer linear programs have also been proposed [33–36], albeit based on linearized models and a discrete set of allowable turbine locations.

In real-world layout design problems, the complexity of WFLO increases when layout constraints are considered, such as turbine proximity constraints where a minimum inter-turbine distance must be enforced, and land-use and setback constraints that arise from local environmental regulations [37].

In order to address this kind of constraints in the automated design optimization process, stochastic techniques and non-differentiable models have been proposed. For instance, Wagner et al. [28] started the optimization algorithm from an infeasible solution, and during its progress an optimization parameter called displacement vector is changed until a feasible solution is obtained. In a similar way, in González et al.’s implementation of GAs [38], infeasible solutions are mutated until a feasible solution is found. On the other hand, an explicit approach was proposed by Yamani et al. [39], in which a mathematical model has been introduced and added as a non-linear constraint. However, the applicability of this model is limited to convex, isolated infeasible areas located inside the wind farm domain. A similar approach, albeit with a wider applicability, using a different model which is a function of number of existing turbines in the infeasible areas, can be found in [40].
1.2 Knowledge Gap and Contributions

The focus in the WFLO literature has been given to stochastic metaheuristics and other iterative non-deterministic methods, leading to the perception that gradient-based local search methods are not the tool of choice for the design optimization of wind farms [19,21]. Some key reasons that have been mentioned in previous work are the nonlinearity, complexity and multimodal nature of the WFLO problem [19,21,41,42], in addition to the fact that the widely used Jensen [43] and Katic [44] wake models describe the wake-induced velocity deficits as discontinuous functions. Among these, the discontinuity of the velocity deficits has been the major roadblock for considering gradient-based optimization in this application. As a result, the wind farm layout optimization literature has not settled the question of whether global-seeking metaheuristics are indeed the best methods for this application, or whether they could be outperformed by systematic use of locally convergent gradient-based methods.

It is only very recently that researchers [45], working independently and concurrently with the work we present in this thesis, have optimized the layout of a wind farm with 80 turbines using sequential convex programming. In [45], the authors’ main focus was on demonstrating the usability of a novel continuous wake model that the authors proposed based on Jensen’s model [43], and which the authors calibrated with CFD simulation data. Although the authors solved a continuous–variable WFLO problem using a mathematical programming method, the numerical experiments were not sufficient to fully document the effectiveness of the proposed optimization approach. In particular, the authors considered only one wind regime, a fixed number of turbines, and a single starting point for the optimization, focusing instead on characterizing the influence of the wake decay constant on the attainable wind farm efficiency. In addition, the authors did not consider the presence of land-availability constraints. These constraints not only change the nature of the optimization problem, but also have been shown in previous studies to negatively affect the total energy production of wind farms [39].

In our group, we have worked on the modeling of wake interactions to solve the problem by using mathematical programming, albeit in a discrete formulation. For instance, we have proposed a mechanistic linear model [36,46], linear and quadratic approximations [47], in addition to mixed integer and constraint programs to solve the non-linear problem [48]. We also proposed an approach based on a mixed-integer programing model to optimize onshore wind farms in complex
terrains using computational fluid dynamics [49]. The trade-off between the energy production and noise propagation was explored in [50,51] using a multi-objective genetic algorithm. In order to avoid the limitation of the discrete formulation, we solved the bi-objective problem in the continuum domain [39,52].

In this thesis, we continue exploring the benefit of mathematical optimization in solving the WFLO problems efficiently, but with formulations that are more realistic. The main characteristics of the proposed model are: the unrestricted turbines’ positions; considering the turbine’s manufacturer power curve; using the nonlinear analytical wake model; accounting the irregular boundaries of the farm and land-use constraints. Other aspects that should be considered in turbine micrositing are added in a multi-objective formulation of the problem.

In detail, we rely on the well-known Jensen model for wake calculations. To express the wake-induced velocity deficits as continuous functions suitable for gradient-based optimization, we use a modified velocity profile that Jensen himself proposed and validated with experimental data [43]. In addition, a mathematical model for the land-availability constraints is introduced. We present explicit mathematical expressions for the gradient of the objective function and constraints, namely the velocity deficit functions, proximity constraints, and land-use constraints.

We demonstrate the effectiveness of nonlinear mathematical programing based on exact gradient information by performing a statistical comparison against a real-coded genetic algorithm using a comprehensive test suite that have been frequently used in the literature, as well as by solving test cases with significant land-use constraints and very large-scale farms. Furthermore, a scalability analysis with different turbine densities is performed, and the influence of the initial starting layout on the optimized solution is tested. Finally, land footprint, electrical infrastructure and setback constraints for environmental aspects are considered in a multi-objective formulation for the micrositing problem.

Portions of Chapters 1-3 were submitted for publication in [53], while Chapter 2 was accepted for inclusion in conference proceedings [54]. The work done in Chapter 5 is being extended for publication.
1.3 Thesis Outline

The remaining contents of the thesis are organized as follows:

- **Chapter 2**: Describing the used model for wake turbulence, and the formulation of the optimization problem, in addition to introducing the proposed model for land-use constraints and the optimization approach.

- **Chapter 3**: Presenting statistical comparisons with GA, and the carried numerical experiments and analyses.

- **Chapter 4**: Discussing the applicability of the proposed approach in solving very large-scale wind farms with validation by numerical experiments considering produced power and cabling cost.

- **Chapter 5**: Formulating the multi-objective problem, considering land footprint, electrical infrastructure, environmental aspects, and produced power.

- **Chapter 6**: Drawing conclusions and giving insights for future work.
Chapter 2

Efficient Optimization of Wind Farms

2.1 Problem Formulation

2.1.1 Wake Modeling

As a first step towards introducing nonlinear mathematical programing utilizing derivatives information in solving WFLO problem, Jensen model with a Gaussian modulation for the velocity deficit is considered [43]. A schematic drawing of Jensen’s wake model with the Gaussian velocity deficit profile is illustrated in Figure 2-1. Under the assumption of momentum conservation inside the wake region, we can write the following equation:

\[
\pi r_o^2 v_o + \pi (r^2 - r_o^2) u = \pi r^2 v
\]  

(2.1)

where \( r_o \) is the rotor radius, \( v_o \) is the velocity behind the turbine, \( u \) is the undisturbed wind speed, \( r \) is wake radius at a distance \( X \) behind the turbine and \( v \) is the velocity at distance \( X \). Then by assuming linear expansion of the wake:

\[
r = r_o + \alpha X
\]  

(2.2)

where the entrainment constant \( \alpha \) is a function of the hub height of the turbine \( z \) and the surface roughness of the ground \( z_o \):

\[
\alpha = \frac{1}{2 \left( \ln \frac{z}{z_o} \right)}
\]  

(2.3)

Moreover, by substituting of \( v_o \) by \( u/3 \) (Betz’ law), the velocity inside the wake can be solved to be:

\[
v = u \left[ 1 - \frac{2}{3} \left( \frac{r_o}{r_o + \alpha X} \right)^2 \right]
\]  

(2.4)
For more realistic model of the velocity profile inside the wake region, a modulation term is recommended by Jensen [43], where a Gaussian profile for the velocity deficit is used as an alternative to the uniform profile which is discontinuous at the wake boundary:

\[
f(\theta) = \begin{cases} 
\frac{1 + \cos(9\theta)}{2} & ; \theta \leq \frac{\pi}{9} \\
0 & ; \theta > \frac{\pi}{9}
\end{cases} \tag{2.5}
\]

where \( \theta \) is the angular (polar) location of where the velocity is calculated with respect to the central axis of the wake. With this modification, Eq. (2.4) becomes:

\[
v_k = u \left[ 1 - \frac{2}{3} f(\theta) \left( \frac{r_o}{r_o + \alpha X} \right)^2 \right] \tag{2.6}
\]

For turbine located inside the wake of \( n \) upstream turbines, the effective wind velocity is calculated after Mosetti et al. [1] as:

\[
v_{kd}(x, y) = u \left[ 1 - \sqrt{\sum_{k=1}^{n} \left( 1 - \frac{v_k}{u} \right)^2} \right] \tag{2.7}
\]

Figure 2-1 Schematic drawing of Jensen’s wake model with Gaussian velocity deficit profile
2.1.2 Land-availability constraints

In most of real-world design problems, the boundaries of the wind farm do not have regular shapes. Additionally, some environmental features (e.g. lakes, hills, and forests), roads, dwellings and other infrastructure can exist inside the farm. These aspects should be taken into consideration during the design process, so the turbines can be located only in the allowed areas inside the wind farm boundaries.

In the present study, differentiable mathematical models are developed to handle land availability constraints. There are two desired attributes that models to represent these constraints must have so that they are suitable for implementation with non-linear mathematical programing, namely the applicability in handling arbitrary shapes of the infeasible regions, and the continuity and differentiability of the mathematical functions.

The proposed approach involves two steps. At first, the boundaries of the infeasible areas are approximated to polygons or circles. Then, the polygons are partitioned into triangles and polygons in such a manner that each polygon shares only one edge with the feasible area. These edges represent the basins of attraction toward which turbines that have fallen inside infeasible areas will be moved during the optimization. As illustrated in Figure 2-2(a), the pentagon on the left represents a forbidden zone which is discretized into smaller triangles. Each one of these triangles has a single shared edge with the feasible area. These shared edges are basin of attraction where the constraint’s violation is diminishing as the turbine moves toward these edges. The constraint’s violation is defined as the summation of the distances between the turbine’s location and the end points of the shared edge, subtracted by its length. As shown, the turbines $T_1$ and $T_2$ are located inside the triangle ABC and at the ellipse whose focal points are the end points of the shared edge AC. These turbines are of equal constraint’s violation, while the turbine $T_3$ is feasible as the sum of distances to the end points equals the length of the shared edge. The model is simpler for circular shaped zones, where the constraint’s violation is defined as the circle’s radius, subtracted by the distance between the center of the circular forbidden zone and the turbine’s location. Thus, the infeasible turbine $T_4$ is being moved during the optimization process till reaching the bounding circle of the forbidden area at the position of $T_5$. 
Figure 2-2 Handling land-use constraints for internal and external forbidden zones inside the farm’s boundaries
Expressed mathematically, for each turbine $i$ located in a polygon shaped forbidden zone, the constraint function is defined as:

$$C_1(x_i, y_i) = \left[ \sqrt{(x_i - x_a)^2 + (y_i - y_a)^2} + \sqrt{(x_i - x_b)^2 + (y_i - y_b)^2} \right] - L_{ab}$$  \hspace{1cm} (2.8)

where $L_{ab}$ is the length of the shared edge with the eligible area; $(x_a, y_a)$ and $(x_b, y_b)$ are its coordinates.

In case of circular forbidden zones, for turbine $i$ located inside a circle of radius $R_c$ and center $(x_c, y_c)$, the constraint function is:

$$C_2(x_i, y_i) = R_c - \left[ \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right]$$  \hspace{1cm} (2.9)

The constraints of the external forbidden zones can be defined by the same way; an example is shown in Figure 2-2(b). In the forbidden polygon ABCDE, the constraint’s violation of $T_1$ is more than that of $T_2$, while $T_3$ is feasible. Finally, for circular farm’s boundary, the definition of internal forbidden zone is applicable but with an opposite sign, as the turbine moves in the opposite direction. Thus, the constraint function is expressed as:

$$C_3(x_i, y_i) = \left[ \sqrt{(x_i - x_c)^2 + (y_i - y_c)^2} \right] - R_c$$  \hspace{1cm} (2.10)

2.1.3 Optimization Problem

As discussed in the introduction, the continuous-variable formulation is used to avoid limitations of the discrete formulation. The array of decision variables involves two sub-arrays that represent the $x$ and $y$ Cartesian coordinates of the $N$ turbine inside the farm. For instances with multi-directional wind regimes, the turbines’ coordinates are rotated to align the layout with the wind directions (see [15]), so the downstream distance $X = |x_j - x_i|$. 

As the first step towards exploring the effectiveness of using nonlinear mathematical programeing with generalized derivatives, only a single objective optimization problem is considered in the current study. Naturally, the optimization objective of WFLO in this work is to maximize the generated power.
The optimization problem is defined as:

**Objective:** \( \text{max } f(x, y) \)

**Subject to:**

\[ g_m(x, y) \leq 0, \quad m = 1, \ldots, M \]  
\[ c_l(x, y) \leq 0, \quad l = 1, \ldots, L \]  
\[ lb \leq x_i \leq ub, \quad i = 1, \ldots, N \]  
\[ lb \leq y_i \leq ub, \quad i = 1, \ldots, N \]

with

\[ f(x, y) = \sum_{d=1}^{S} \left\{ \sum_{k=1}^{N} \left( \frac{1}{3} v_{kd}^3 \right) \right\} p_d \]  

Where \( M \) and \( L \) are the number of proximity and land-availability constraints, respectively, \( lb \) and \( ub \) are lower and upper bounds that represent the boundaries of the wind farm area, \( N \) is the number of turbines, \( S \) is the number of wind states (i.e., pairs of wind speed and direction), and \( p \) is the probability of occurrence for each wind state, satisfying \( \sum_{d=1}^{S} p_d = 1 \).

In industrial practice, a proximity constraint is enforced to maintain a minimum distance between turbines, usually set at five times the diameter of the turbine rotor. In the discrete formulation of the WFLO [1,2], the dimension of each cell is defined so that proximity constraints are implicitly, passively enforced during the optimization. For the continuous-variable formulation, on the other hand, the proximity constraint must be added explicitly.

For each pair of turbines with coordinates of \((x_i, y_i)\) and \((x_j, y_j)\), the non-linear proximity constraint can be defined as:

\[ g_m(x, y) = 5D - \sqrt{(x_j - x_i)^2 + (y_j - y_i)^2} \]  

### 2.2 Design Optimization Approach

#### 2.2.1 Interior-point Method

Nonlinear programming is a class of optimization methods that rely on the gradients of the objective and constraint functions to find a local optimal solution. In the current study, the interior-point
method [55] is used to solve the WFLO problem. In interior-point method, the original problem is approximated to a sequence of sub-problems by adding a barrier function and slack variables to be:

\[
\text{Objective: } \min_{x, y, S} f_{\mu}(x, y, S) = -f(x, y) - \mu \sum_{i=1}^{m} \ln S_i \tag{2.17}
\]

Subject to: \( C(x, y) + S = 0 \) \tag{2.18}

Where \( m \) is the number of the constraints, \( S_i > 0 \) are the slack variables and \( \mu > 0 \) is a barrier parameter. By introducing \( \nu \) as a penalty parameter, the merit function to be minimized becomes:

\[
f_{\mu}(x, y, S) + \nu \| C(x, y) + S \|
\]

At each optimization iteration, the algorithm uses a Newton step, to solve the Karush–Kuhn–Tucker (KKT) equations [56], or otherwise a conjugate gradient step, using a trust region if it does not succeed. An auxiliary Lagrangian function is used to satisfy the problem’s constraints. For a detailed description of the interior point method, the reader is referred to [55–57].

2.2.2 Gradients and Hessians

Differentiable mathematical functions for the objective and constraints are required to solve the problem efficiently, without the need to approximate the gradients by the finite-difference method, which not only would increase the number of function evaluations necessary to proceed, but would also introduce numerical errors that could affect the convergence of the algorithm, especially for highly constrained problems [31].

Following Jensen [43], a Gaussian modulation is added to the wake model as mentioned in Section 2.1.1. Besides increasing the accuracy of the wake model [43], the Gaussian modulation of the velocity profile inside the wake makes the wake boundary mathematically continuous, as illustrated in Figure 2-1, as opposed to the commonly used uniform (“top-hat”) velocity profile that is discontinuous at the wake boundary. By differentiating the objective function in (2.15) with respect to \( x_i \) and \( y_i \), the gradient of the objective function is given by:
\[
\frac{\partial f}{\partial x_i} = \sum_{d=1}^{S} \left\{ \sum_{k=1}^{N} \left( v_{kd}^2 \frac{\partial v_{kd}}{\partial x_i} \right) \right\} p_d \tag{2.20}
\]

\[
\frac{\partial f}{\partial y_i} = \sum_{d=1}^{S} \left\{ \sum_{k=1}^{N} \left( v_{kd}^2 \frac{\partial v_{kd}}{\partial y_i} \right) \right\} p_d \tag{2.21}
\]

where

\[
\frac{\partial v_{kd}}{\partial x_i} = - \left( \frac{\sum_{k=1}^{n} (v_k - u) \frac{\partial v_k}{\partial x_i}}{\sum_{k=1}^{n} (1 - \frac{v_k}{u})^2} \right) \tag{2.22}
\]

\[
\frac{\partial v_k}{\partial x_i} = \begin{cases} 
- \frac{2u}{3} \frac{(A-B)(x_j-x_i)}{|x_j-x_i|} & \text{if } \theta \leq \frac{\pi}{9} \\
0 & \text{otherwise} 
\end{cases} \tag{2.23}
\]

\[
A = \frac{\alpha r_o^{-2}}{(r_o + \alpha |x_j-x_i|)^3} \left( 1 + \cos \left( 9 \tan^{-1} \frac{|y_j-y_i|}{|x_j-x_i|} \right) \right) \tag{2.24}
\]

\[
B = \left( \frac{r_o}{r_o + \alpha |x_j-x_i|} \right)^2 \sin \left( 9 \tan^{-1} \frac{|y_j-y_i|}{|x_j-x_i|} \right) \left( \frac{4.5 |y_j-y_i|}{(x_j-x_i)^2+(y_j-y_i)^2} \right) \left( \frac{r_o}{r_o + \alpha |x_j-x_i|} \right)^2 \tag{2.25}
\]

and

\[
\frac{\partial v_{kd}}{\partial y_i} = - \left( \frac{\sum_{k=1}^{n} (v_k - u) \frac{\partial v_k}{\partial y_i}}{\sum_{k=1}^{n} (1 - \frac{v_k}{u})^2} \right) \tag{2.26}
\]

\[
\frac{\partial v_k}{\partial y_i} = \begin{cases} 
- \frac{3u E}{|y_j-y_i|} & \text{if } 0 < \theta \leq \frac{\pi}{9} \\
0 & \text{otherwise} 
\end{cases} \tag{2.27}
\]

\[
E = \sin \left( 9 \tan^{-1} \frac{|y_j-y_i|}{|x_j-x_i|} \right) \left( \frac{|x_j-x_i|}{(x_j-x_i)^2+(y_j-y_i)^2} \right) \left( \frac{r_o}{r_o + \alpha |x_j-x_i|} \right)^2 \tag{2.28}
\]

The derivatives of the wind velocity with respect to coordinates of turbine \( j \) are:

\[
\frac{\partial v_k}{\partial y_j} = - \frac{\partial v_k}{\partial y_i} \frac{\partial v_k}{\partial x_i} = - \frac{\partial v_k}{\partial x_i} \tag{2.29}
\]
In addition, gradients of the constrained functions are obtained by differentiating Eq. (2.8)-(2.10), (2.16) with respect to \( x_i \) and \( y_i \) as follows:

\[
\frac{\partial C_1}{\partial x_i} = \frac{x_i - x_a}{\sqrt{(x_i - x_a)^2 + (y_i - y_a)^2}} + \frac{x_i - x_b}{\sqrt{(x_i - x_b)^2 + (y_i - y_b)^2}} \tag{2.30}
\]

\[
\frac{\partial C_1}{\partial y_i} = \frac{y_i - y_a}{\sqrt{(x_i - x_a)^2 + (y_i - y_a)^2}} + \frac{y_i - y_b}{\sqrt{(x_i - x_b)^2 + (y_i - y_b)^2}} \tag{2.31}
\]

\[
\frac{\partial C_2}{\partial x_i} = \frac{x_c - x_i}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}} \tag{2.32}
\]

\[
\frac{\partial C_2}{\partial y_i} = \frac{y_c - y_i}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}} \tag{2.33}
\]

\[
\frac{\partial C_3}{\partial x_i} = \frac{x_i - x_c}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}} \tag{2.34}
\]

\[
\frac{\partial C_3}{\partial y_i} = \frac{y_i - y_c}{\sqrt{(x_i - x_c)^2 + (y_i - y_c)^2}} \tag{2.35}
\]

\[
\frac{\partial g}{\partial x_i} = \frac{x_i - x_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \tag{2.36}
\]

\[
\frac{\partial g}{\partial y_i} = \frac{y_i - y_j}{\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}} \tag{2.37}
\]

The derivatives of the proximity constraint with respect to coordinates of turbine \( j \) are:

\[
\frac{\partial g}{\partial x_j} = -\frac{\partial g}{\partial x_i} \frac{\partial g}{\partial y_i} = -\frac{\partial g}{\partial y_i} \tag{2.38}
\]

Because of limited memory, the Hessians are generated and updated at each iteration with the Broyden–Fletcher–Goldfarb–Shanno (BFGS) quasi-Newton approach [56].
Chapter 3

Numerical Experiments and Discussions

3.1 Statistical comparison with GA

In order to validate the effectiveness of using nonlinear programming in solving the WFLO problem, we compared it with (a) a global-seeking real-coded GA [58], applying Deb’s penalty function method [59] for handling the non-linear proximity constraints, and (b) a hybrid GA that uses Sequential Quadratic Programming (SQP) at the end of the run to improve for local convergence [56], denoted in this work as HGA. Three benchmark problems were used, taken from the WFLO literature [1,2,5,50]. The first problem includes a unidirectional wind regime with a constant speed of 12 m/s. The second problem involves a multidirectional regime wind with equal probability for each direction, at a constant speed of 12 m/s. In the third problem, a multidirectional wind regime with unequal probabilities and three wind speed levels (8, 12 and 17 m/s) is considered. For multidirectional cases, 36 directions are defined, with increments of 10° between them. Figure 3-1 shows the wind roses for these test cases.

Figure 3-1 Wind roses for benchmark problems: (a) unidirectional wind regime; (b) constant frequency multidirectional wind regime; (c) variable frequency multidirectional wind regime
The specifications of the wind farm and the turbines are listed in Table 3-1. We used the same parameters as previous work for consistency and ease of comparison. A wind farm domain of 2 km x 2 km with ground roughness of 0.3 m, is considered in the experiments. The turbine hub height is 60 m with a rotor radius of 20 m, which results in a proximity constraint limiting turbines to a minimum inter-turbine distance of 200 m. Number of turbines for each problem is kept constant. The test cases are solved, considering 10, 20 and 30 turbines.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Turbine hub height (Z)</td>
<td>60 m</td>
</tr>
<tr>
<td>Terrain length (Z₀)</td>
<td>0.3 m</td>
</tr>
<tr>
<td>Rotor diameter (D)</td>
<td>40 m</td>
</tr>
<tr>
<td>Farm area</td>
<td>2 km x 2 km</td>
</tr>
<tr>
<td>Proximity constraint</td>
<td>200 m</td>
</tr>
</tbody>
</table>

The convergence behavior of local optimization methods is highly dependent on the starting solution. Theoretically speaking, the initial solution should be located in the basin of attraction of a global optimum to get a global optimum solution. Because of that, and to study its influence on the optimized solutions, we have started the optimization algorithm by (a) a single uniform staggered layout (USL), and (b) multiple random solutions generated by Latin hypercube sampling [60] with 5 starts and 20 starts, other numbers of starting points were also tried but the differences observed in the results were negligible. We refer to these approaches as USL-IPM, 5S-IPM and 20S-IPM, respectively, with the IPM standing for “interior point method”.

The computational budget allocated to GA, in terms of total number of function evaluations, is listed in Table 3-2 for different problem sizes, i.e., for different numbers of turbines. A total of 10 runs were performed for each optimization method, with different random seeds, to control for the effects of randomness. All computational experiments were conducted in MATLAB. Box plots for the wind farm efficiencies (i.e., output power divided by the power that would be produced if
wakes were not present) are shown in Figure 3-2. We tabulated the results of the statistical comparisons in Table 3-3 as well.

<table>
<thead>
<tr>
<th>Number of turbines</th>
<th>Population size x generations</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>300 x 1000</td>
</tr>
<tr>
<td>20</td>
<td>400 x 1000</td>
</tr>
<tr>
<td>30</td>
<td>500 x 1000</td>
</tr>
</tbody>
</table>
Table 3-3 Statistical comparison between GA, HGA, 5S-IPM and 20S-IPM. Best solutions are in bold

<table>
<thead>
<tr>
<th></th>
<th>Case (1)</th>
<th>Case (2)</th>
<th>Case (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Best</td>
<td>Worst</td>
<td>Median</td>
</tr>
<tr>
<td>10 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GA</td>
<td>100</td>
<td>99.91</td>
<td>100</td>
</tr>
<tr>
<td>HGA</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>5S-IPM</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>20S-IPM</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>20 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GA</td>
<td>99</td>
<td>98.14</td>
<td>98.6</td>
</tr>
<tr>
<td>HGA</td>
<td>99.4</td>
<td>98.54</td>
<td>99.06</td>
</tr>
<tr>
<td>5S-IPM</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>20S-IPM</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>30 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GA</td>
<td>97.5</td>
<td>96.42</td>
<td>96.84</td>
</tr>
<tr>
<td>HGA</td>
<td>98</td>
<td>97.36</td>
<td>97.67</td>
</tr>
<tr>
<td>5S-IPM</td>
<td>99.1</td>
<td>98.67</td>
<td>98.87</td>
</tr>
<tr>
<td>20S-IPM</td>
<td>99.3</td>
<td>98.92</td>
<td>99.02</td>
</tr>
</tbody>
</table>
Figure 3-2 Box plots for results of benchmark problems with different number of turbines: (a) unidirectional wind regime; (b) constant frequency multidirectional wind regime; (c) variable frequency multidirectional wind regime
In the first case study, wind flows in only one direction from the north to the south at constant speed of 12 m/s. In this case, problem intuition indicates that the optimal configuration is to spread all turbines in a direction perpendicular to the wind direction, so as to avoid placing any turbines inside the wake of another. For 10 turbines, all tested methods succeeded to get the optimal solution. In the cases of 20 and 30 turbines, the results of GA have the lowest efficiencies, which increased after hybridization of GA with SQP (indicated by “HGA” hereafter). On the other hand, in case of 20 turbines, both of 5S-IPM and 20S-IPM succeeded to get a global optimal solution in all performed runs with zero standard deviation, while the standard deviation is found to be 0.24 and 0.22 for the GA and HGA runs, respectively.

The results obtained by 5S-IPM and 20S-IPM are the best among all results, with efficiencies up to 2.18 % higher than results produced by the standard GA, and 1.35% over HGA in the case of 30 turbines. The efficiencies obtained by 5S-IPM are slightly lower than those of 20S-IPM, with a difference of 0.15%. Although the cases of multidirectional wind are more complex, the qualitative behavior of the algorithms mirrors that of the first test case. For 10 turbines, the best obtained solutions from all optimization methods are truly optimal; however, the standard deviations for GA and HGA are higher than that of IPM. Finally, the results obtained from 5S-IPM and 20S-IPM are superior to the other approaches for all numbers of turbines, with standard deviations of less than 0.17 and 0.1, respectively.

Table 3-4 shows the comparison between USL-IPM, 5S-IPM and 20S-IPM in terms of total number of function evaluations and the quality of obtained solutions. For 10 turbines (i.e., a low turbine density), 20S-IPM and USL-IPM succeeded in obtaining optimal layouts; the same is observed in the unidirectional wind regime case with 20 turbines. Among all results, the differences in magnitude between the obtained efficiencies by all starting configurations do not exceed 0.53%, corresponding to the third case with 30 turbines. Regarding the computational cost, the total number of function evaluations does not exceed 15,748 for 20S-IPM; 4,058 for 5S-IPM; and 1,234 in case of USL-IPM.
<table>
<thead>
<tr>
<th></th>
<th>Case (1)</th>
<th>Case (2)</th>
<th>Case (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency %</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Obj. Eval.</td>
<td>4,672</td>
<td>1,430</td>
<td>439</td>
</tr>
<tr>
<td>20 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency %</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Obj. Eval.</td>
<td>15,463</td>
<td>4,058</td>
<td>1,274</td>
</tr>
<tr>
<td>30 Turbines</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Efficiency %</td>
<td>99.02</td>
<td>98.87</td>
<td>98.71</td>
</tr>
<tr>
<td>Obj. Eval.</td>
<td>15,748</td>
<td>3,766</td>
<td>1,234</td>
</tr>
</tbody>
</table>

**Table 3-4** Comparison between results of IPM starting by different initial configurations (USL- IPM and medians of 5S- IPM and 20S- IPM). Best solutions are in bold.
3.2 Scalability and turbine density analyses

As known and confirmed by the previous experiments, the WFLO problem becomes much more complex as the turbine density increases. In addition, increasing number of variables would make it harder for the optimization method to find an optimum solution. Because of that, additional experiments were done to validate the scalability of nonlinear programing in solving WFLO problems. A real wind regime from [61] is used in these experiments with an average wind velocity of 10 m/s. The wind rose is shown in Figure 3-3.

Regarding the wind turbine parameters, we use the dimensional specifications of the Vestas V80-2.0 MW turbine [62] which is used in constructing several wind farms, such as Horns Rev 1 [63]. The experiments are done for turbines densities of 4, 5 and 6 turbines/km² with different numbers of turbines such those in large farms, namely 37, 50 and 100. The wind farm sizes are scaled to keep the number of turbines fixed for the three levels of turbine densities. In order to test the sensitivity to the initial starting point, the interior-point method is implemented starting from a random start (denoted RSL-IPM) and also staring from a uniform staggered layout (USL-IPM). The results are tabulated in Table 3.5, and the optimized layouts are shown in Figure 3-4, Figure 3-5 and Figure 3-6. As observed from tabulated results, most of optimized layouts by USL-IPM are superior to those started from a random layout both in solution quality and computational cost. However, a multi-start IPM approach could be used for more confidence about the quality of the optimized layouts.
Table 3-5 Comparison between results of IPM starting by different initial configurations (USL- IPM and RSL- IPM) with 4, 5 and 6 turbine densities. Best solutions are in bold.

<table>
<thead>
<tr>
<th></th>
<th>4 Turbines/Km²</th>
<th>5 Turbines/Km²</th>
<th>6 Turbines/Km²</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RSL- IPM</td>
<td>USL- IPM</td>
<td>RSL- IPM</td>
</tr>
<tr>
<td>37 Turbines</td>
<td>Efficiency %</td>
<td>90.36</td>
<td>90.96</td>
</tr>
<tr>
<td></td>
<td>Obj. Eval's</td>
<td>243</td>
<td>583</td>
</tr>
<tr>
<td>50 Turbines</td>
<td>Efficiency %</td>
<td>89.12</td>
<td>90.05</td>
</tr>
<tr>
<td></td>
<td>Obj. Eval's</td>
<td>722</td>
<td>659</td>
</tr>
<tr>
<td>100 Turbines</td>
<td>Efficiency %</td>
<td>87.29</td>
<td>87.77</td>
</tr>
<tr>
<td></td>
<td>Obj. Eval's</td>
<td>821</td>
<td>219</td>
</tr>
</tbody>
</table>
Figure 3-3 Wind rose for used test examples in sections 3.1 and 3.2

Figure 3-4 Optimized layouts for 37 turbines with densities of 4, 5 and 6 Turbines/Km² respectively, starting by random layout (top) and uniform staggered layout (bottom). x: initial locations; +: optimized locations
**Figure 3-5** Optimized layouts for 50 turbines with densities of 4, 5 and 6 Turbines/Km² respectively, starting by random layout (top) and uniform staggered layout (bottom). x: initial locations; +: optimized locations
3.3 Land-availability constrained examples

As discussed in Chapter 1, in practical cases, the wind farm domains do not have regular shapes and may contain inner areas that are deemed infeasible due to land-use and setback constraints. In order to further test the proposed approach and constraint models, two additional test cases with different land-availability constraints are solved. The turbine specifications of the previous examples are used again for these cases. In the first case, the multi-directional, multi-speed wind regime from Section 3.2 is used, and the wind farm shape is adopted from the Copenhagen wind farm [64] with 37 turbines and 66.2% feasible area. In the second case, 20 turbines are distributed inside a ring of only 50m width, which results in a wind farm where only 6.5% of the area is feasible. A north unidirectional wind of 10 m/s speed is applied for easier comparison of the
obtained layouts with the first numerical example in Section 3.1.

Figure 3-7 and Figure 3-8 show the optimized layouts and convergence plots for the 5S-IPM and USL-IPM of the two instances, respectively. The optimized layouts by 5S-IPM are slightly better that those of USL-IPM, with marginal differences in efficiencies, namely 0.02% for the first case and 0.63% for the second case. As shown in the convergence plots, the constraint violations diminish rapidly, whereas the objective grows consistently after that. Sudden changes in objective growth are observed, a result of the non-smoothness of the objective function.

Figure 3-7 Initial layouts (left) and best optimized solutions (right) with convergence plots (far right), starting from a random layout (top) and from a uniform staggered layout (bottom)
Figure 3-8 Initial layouts (left) and best optimized solutions (right) with convergence plots (far right), starting from a random layout (top) and from a uniform staggered layout (bottom)
3.4 Further Notes

In order to validate the calculated derivatives and show the merit of utilizing the exact gradient information over gradients obtained via finite-differences, the first example in Section 3.3 is solved by IPM with finite-difference approximations. The solver was started with the same uniform staggered layout for comparison. As shown in Figure 3-9, the obtained layout is close to the optimal solution obtained with exact gradients, however, its efficiency is 0.36% lower. Nevertheless, the computational cost required to use IPM with approximate gradients approaches the computational cost required for GA to converge. The reported number of function evaluations to solve this problem with the interior point method with approximate gradients is 201,244, which is more than 20 times the number of function evaluations required when using exact gradients.

Another important issue for the solver to proceed is the velocity profile inside the wake region. Consider first the wake models that describe velocity as a uniform profile inside the wake, as illustrated in Figure 3-10 (a). In addition to the fact that this description of the wake is unrealistic [65–67], using this model prevents downstream turbines to move along the direction perpendicular to the wind direction. In other words, the gradient of the velocity deficit caused by the wake with respect to the cross-stream direction is zero. As an example, let us assume that there are two turbines and a single wind direction. By starting the solver with a layout where one of the turbines is located inside the wake region of the other one, the solver would lead the turbines to be moved in only the streamwise direction, because the velocity along the cross-stream direction is constant. To complicate matters further, the objective function would be discontinuous at the boundaries of the wake region, rendering continuous optimization methods unsuitable for this problem. On the other hand, the Gaussian velocity profile inside the wake approaches the observed far wake behavior [65–67] and makes the problem continuous.

Finally, it is important to note that, for uniform staggered layouts, it is expected that turbines could be aligned with upstream turbines (because of the geometry of uniform layouts). As a result, during the optimization iterations, turbines are unable to move in the cross-stream direction (i.e., perpendicular to the wind direction), because the objective function’s partial derivative with respect to this coordinate direction is zero. In order to avoid that, small random perturbations can be applied to the uniform layouts before using them as a starting point in the optimization.
**Figure 3-9** Example of land-availability constrained problem solved by IPM with finite-difference approximation of the gradients. Compare with Fig. 3-7

**Figure 3-10** Schematic drawing of velocity distribution inside the wake region with (a) uniform distribution and (b) Gaussian distribution for velocity deficit.
Chapter 4

4. Micrositing in Large-scale Wind Farms

4.1 Introduction

The crisis of climate change has received serious global attention, where the impacts and consequences of global warming on weather, sea levels, health, ecosystems, water and food security threaten our lives on the planet. In December 2015, leaders of 195 countries adopted a historic agreement for mitigation of climate change at the United Nations climate change conference in Paris [68]. The agreement aims to limit the rise in global temperature to below 2 °C, and promises were made to decrease greenhouse gas emissions [69].

The economic sector that emits the majority of carbon dioxide is the energy production sector; it contributes ~35% of the annual global gas emissions [70]. Therefore, constructing very large-scale wind farms, particularly onshore wind farms, as one of the lowest-cost renewable power generating solutions [71], should be a priority.

In case of very large-scale wind farms, hundreds of turbines are located inside a predefined domain while complying with setbacks and other land-use constraints, maintaining a minimum inter-turbine distance. For WFLO problems formulated in continuous-variable domains, the number of decision variables is at least double the number of wind turbines. Under limited computational resources, these nonlinear constraints, in addition to the high-dimensional decision vector, would limit the advantage of metaheuristics and population-based optimization methods in exploring the decision space. For that reason, randomized search approaches have been proposed [28,29]. Although using random-assisted searches may decrease the computational cost, they lack optimality guarantees and the quality of the solutions is primarily a function of the number of function evaluations required for the algorithm. The prohibitive computational cost limits the quality of optimized layouts.
In order to push the limits of micrositing, the optimization approach must require a lower computational cost without sacrificing the quality of the optimized solution, must be capable of tackling wake models that better approximate the real problem, and must provide optimality guarantees. Towards this goal, in this thesis, we demonstrate the capability of nonlinear mathematical programming in solving very large-scale WFLO problems. In contrast to some previous work, our formulations use the turbine’s manufacturer power curve, use the sum of squares approach to better model multiple wakes interactions [72], and include significant land-availability constraints, while enforcing lower computational budgets than previous work with randomized searches [28,29]. Additionally, the potential benefits of a multi-start local search approach is further demonstrated in the context of multi-disciplinary optimization. Namely, we solve for the optimal electrical infrastructure of large scale wind farms as a post-processing step for each of the local optima obtained during the multi-start gradient-based optimization of the annual energy production.

4.2 Problem Formulation

4.2.1 Annual Energy Production

The main objective of wind turbines micrositing is to maximize the real output electric power, however, most of proposed mathematical approaches consider maximizing the theoretical output power with the constant value of Betz limit (i.e., maximum theoretical limit that equals 16/27) for the power coefficient [33–36,45,53].

The power coefficient is a measure for the wind turbine’s performance, which is the ratio of power in the wind and turbine’s extracted power [73]. The power coefficient is not constant, but a function of wind speed. Thus, in order to capture the actual behavior of the wind turbine, it is required to consider the turbine’s manufacturer power curve. For the Vestas V100-1.8MW turbine, the manufacturer-supplied electric power curve, power coefficient and theoretical wind power curve, as functions of wind speed, are illustrated in Figure 4-1.

Theoretically, the turbine’s output power [73] can be calculated by the following equation:
where $\rho$ is the air density, $A$ is the swept area, $U$ wind speed at the hub height, $C_p$ is the power coefficient and $\eta$ is the efficiency of the turbine. The real electric power curve of pitch-controlled turbines can be divided into three sections with respect to the wind speed as follows:

$$P_{out} = \frac{1}{2} \rho A U^3 C_p \eta$$ (4.1)

$$P_{out} = \begin{cases} h(U) & \text{for } U_{\text{cut-in}} \leq U < U_{\text{rated}} \\ P_{\text{rated}} & \text{for } U_{\text{rated}} \leq U \leq U_{\text{cut-out}} \\ 0 & \text{otherwise} \end{cases}$$ (4.2)

**Figure 4-1** Vestas V100-1.8MW turbine's manufacturer electric power curve, power coefficient and theoretical wind power curve as functions of wind speed

The power in the section of the curve that lies between the cut-in speed $U_{\text{cut-in}}$ and the rated speed $U_{\text{rated}}$, is a function of wind speed $h(U)$. The power is nearly constant for speeds higher than the rated speed and less than the cut-out speed $U_{\text{cut-out}}$, while the turbine shuts down to prevent damage for speeds exceeding $U_{\text{cut-out}}$. 
In order to establish a differentiable mathematical function for the annual energy production, the turbine data from the manufacturer’s reference document [74], is fitted by a polynomial function. Figure 4-2 shows fitted polynomials with different orders for the Vestas V100-1.8MW [74]. In order to avoid oscillations in the fitted model (see the zoomed section in Figure 4-2), the few data points which are close to the rated power can be adjusted to the same power value of $P_{\text{rated}}$. Moreover, the fitted curve must be connected to the horizontal line of $P_{\text{rated}}$. The corrected 4th order polynomial model is shown in Figure 4-3.

**Figure 4-2** Fitted polynomials to the data points of the manufacturer’s power curve

**Figure 4-3** Adopted 4th order polynomial power curve in comparison with the manufacturer power curve
Hence, equation (4.2) can be rewritten as:

\[
\begin{align*}
P_{out} &= \begin{cases} 
  h(U), & \text{for } U_{\text{cut-in}} \leq U < U^* \\
  P_{\text{rated}}, & \text{for } U^* \leq U \leq U_{\text{cut-out}} \\
  0, & \text{otherwise} 
\end{cases} \\
(4.3)
\end{align*}
\]

\[
h(U) = \beta_1 U^4 + \beta_2 U^3 + \beta_3 U^2 + \beta_4 U + \beta_5 \\
(4.4)
\]

Where \( U^* \) is the speed coordinate of the point that connects the polynomial curve \( h(U) \) with the horizontal line of \( P_{\text{rated}} \), and \( \beta_1, \beta_2, \beta_3 \) and \( \beta_4 \) are the polynomial coefficients of the fitted model. Note that this representation of the turbine’s power curve is continuous and differentiable, but also more accurate than the ideal power curves used in some previous works.

Finally, for a wind farm with \( N \) turbines and a wind regime of \( S \) wind directions with a probability \( P \), the annual energy production is expressed as:

\[
AEP = 8766 \sum_{d=1}^{S} \left\{ \sum_{k=1}^{N} P_{out,kd}(U_{kd}) \right\} p_d \\
(4.5)
\]

### 4.2.2 Wake Modelling

We use the Jensen model [43], which is presented above in Section 2.1.1, for its simplicity and wide usage. However, for very large-scale wind farms, it is recommended to use models that take into account the two way interactions between the turbines and atmosphere, such as that by Frandsen et al. [75], or the deep-array wake model [76]. A future study may focus on the effects of using these alternative wake models on optimized solutions’ quality and computational cost.

It worth mentioning that, accounting the partial-wake in the model is more beneficial for “top-hat” models mainly because of the high overestimation of the wake deficient near the boundaries of the wake region. On the other hand, the inaccuracies in the velocity estimation near the wake boundaries are much less in case of models with Gaussian velocity profile as concluded by Bastankhah and Porté-Agel in [67].
4.3 Results and Discussions

The main objective of this study is to demonstrate the capability of nonlinear mathematical programming to push the computational limits of very large-scale wind turbines’ micrositing, without sacrificing the quality of optimized layouts, and without compromising their accuracy due to the use of oversimplified physical models. For that purpose, test cases of highly constrained wind farms with hundreds of turbines range from 100 to 400 are solved using the discussed approach in previous sections. The land-use constraints used in the test cases represent the shapes of the wind farm boundaries of the Danish Anholt offshore wind farm and the British West of Duddon Sands wind farm, with feasible areas of 51.2% and 59.6%, respectively (see Figure 4-4). The boundaries of the test cases are adopted from [77].

The power curve and dimensional specifications of the modern turbine Vestas V100-1.8MW [74] are used in all numerical experiments. A real wind data that exhibits variations in the average velocity for different wind directions is adopted from [4], albeit with a different preferential direction to ease the analysis of the resulting optimal solutions. We use 24 wind directions to facilitate the comparison with other approaches in the literature; however, more directions are recommended for more accurate power calculations [78]. The used wind rose in our test cases is shown in Figure 4-5.
A difficult instance is shown in Figure 4-6. In this example, 100 turbines are located inside a domain of 4.5x4.5 km. The severity of the proximity constraints is significant due to the large number of turbines. In this case, the WFLO problem looks similar to a packaging problem where a possible solution could be a grid of 10x10 turbines with a spacing of 0.5 km (the minimum inter-turbine distance allowed by the constraints). The solver is started with an infeasible random solution, however, during the optimization iterations some turbines are moved long distances, in cases exceeding 25% of the wind farm’s length, and end up distributed in a semi-uniform staggered layout. After 2,893 function evaluations, which were required for convergence, the annual energy production is improved by 9.1% and the maximum constraint violation of 452.2 m (at the starting infeasible solution) is diminished to zero.

Five runs for each land-use constrained instance were performed in parallel under the MATLAB environment on a standard desktop computer with a quad-core 3.6GHz processor, and 1600MHz memory speed. The obtained results are shown in Table 4-1.
Figure 4-6 Optimizing 100 turbines with high severity of proximity constraints: (a) history of turbines movements, “x” indicates the initial positions of the turbines, “o” indicates their final position. (b) Convergence behavior during 2,893 function evaluations.

Figure 4-7 Sorted Annual Energy Production (AEP) [TWh] and computational cost of each optimization run, for different numbers of turbines.
### Table 4-1 Results of land-use constrained instances

<table>
<thead>
<tr>
<th>Number of Turbines</th>
<th>Layout Shape</th>
<th>Annual Energy Production (TWh)</th>
<th>Average Improvement %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Standard Deviation</td>
<td>Best</td>
</tr>
<tr>
<td>100</td>
<td></td>
<td>0.0022</td>
<td>0.5546</td>
</tr>
<tr>
<td>150</td>
<td></td>
<td>0.0028</td>
<td>0.8128</td>
</tr>
<tr>
<td>200</td>
<td></td>
<td>0.0017</td>
<td>1.0562</td>
</tr>
<tr>
<td>300</td>
<td></td>
<td>0.0043</td>
<td>1.6111</td>
</tr>
<tr>
<td>350</td>
<td></td>
<td>0.0389</td>
<td>1.8515</td>
</tr>
<tr>
<td>400</td>
<td></td>
<td>0.0235</td>
<td>2.0954</td>
</tr>
</tbody>
</table>
Although all runs are started by random infeasible solutions, they are converged to feasible solutions with average improvements’ range of 3.14% to 5.09%. The computational costs range from 22 minutes for 100 turbines to 6.86 hours for 400 turbines. Due to the slow nature of the used programming language in comparison with other languages in addition to the variety of used computational facilities in the literature, the number of function evaluations is a more suitable comparison criterion. The mean numbers of function evaluations range from hundreds to less than a couple of thousands, which are 1 to 2 orders of magnitude less than those that were reported in previous work [28,29,79]. In Figure 4-7, the AEP values of the optimized layouts are sorted and plotted, along with the corresponding computational cost.

As observed from Figure 4-7, even though the AEP of the optimized turbine layouts does change significantly between runs (standard deviation does not exceed 0.039), the computational cost, measured in terms for the number of function evaluations, changes from a few hundreds to a few thousands, even for the same number of turbines. Thus, the sensitivity of the optimization algorithm to the initial solution affects the computational cost more than it does the quality of the optimized solutions. In contrast to population-based optimization methods such as GA’s where the population size should be increased as the number of variables increases for extensive exploration of the decision space, the number of function evaluations required for convergence seems to be independent of number of turbines. We hypothesize that this is due to the information that the gradient of the objective function provides, allowing for a more efficient trajectory towards the nearest local optima than random-assisted or metaheuristic search methods.

It is important to note that, if the optimization solver starts from a poor layout, the number of function evaluations required for convergence increases. Though it is generally not possible to determine a priori what a poor initial solution looks like in general optimization problems, domain-specific knowledge can be used to provide some guidance about what a good starting layout looks like. For example, in [53], we demonstrated the superiority of IPM utilizing exact gradients over GA and commonly used hybrid GA, under different wind regimes with moderate numbers of turbines. Therein, we showed that a uniform staggered layout provides a good starting point, frequently leading to a local optimum that is as good as or better than other local optimum, consuming less computational cost. The superiority over GA is expected to hold to the very large-scale wind farms because of the decline in performance at high dimensionality.
In the results shown in Figure 4-7 and Table 4-1, we have noted that the standard deviation of AEP ranges from 0.0022 to 0.039 for all instances. This indicates that the optimized solutions are close in terms of their objective function value; however, the actual turbine layouts that lead to these AEP values should differ because they depend on the initial solutions, as a consequence of the inherent multimodality of the problem. This property can be utilized by a multi-start, gradient-based, local search approach to provide the wind farm developer with a set of high quality layouts, with close AEP values. From this array of solutions to the wind farm layout optimization problem, the designer can undertake other quantitative or qualitative assessments, which cannot be easily considered during the optimization process, e.g., visual impact, shadow flicker, more accurate CFD-based assessments, or detailed infrastructure design that may be approached as a separate problem. In such cases, the multi-modality of the problem and the availability of an array of locally optimal solutions from a multi-start approach has the potential to inform the designer of designs tradeoffs.

To illustrate this, we calculated the optimal cabling costs of the obtained turbine layouts for the cases with 400 turbines. To this end, the turbines were clustered into four groups using k-means++ algorithm [80], where each group of turbines is supposed to be connected to an electrical substation. A cable network is established for each group by using Prim’s algorithm [81], where the objective is to find the minimum spanning tree. The cost parameters are taken from [9]. Although this implementation is unrealistic because the shortest path is not necessarily the most inexpensive way to connect the turbines in real wind farm design studies [9], it illustrates how the electrical costs vary for different wind farm layouts that have similar AEP values. All optimized layouts with their scores of AEP and cabling costs are listed in Table 4-2. As shown, a tradeoff is observed between the layouts 3 and 5. It can also be seen that layouts 1 and 5 are different but they have close objective values, which supports the above discussion.
Table 4-2 All optimized 400-turbines-layouts, with their associated AEP values and cabling costs. Note that each group of turbines connected to the same substation is color-coded.

<table>
<thead>
<tr>
<th>Layout</th>
<th>AEP</th>
<th>Cabling Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.0837 TWh</td>
<td>202.85 M €</td>
</tr>
<tr>
<td></td>
<td>2.0521 TWh</td>
<td>211.70 M €</td>
</tr>
<tr>
<td></td>
<td>2.0954 TWh</td>
<td>204.07 M €</td>
</tr>
<tr>
<td></td>
<td>2.0424 TWh</td>
<td>210.60 M €</td>
</tr>
<tr>
<td></td>
<td>2.0878 TWh</td>
<td>202.78 M €</td>
</tr>
</tbody>
</table>
Chapter 5

Gradient Multi-Objective Optimization of Wind Farms

5.1 Introduction

The design process of large-scale wind farms is a challenging task. There are many factors should be considered for optimal micrositing of wind turbines, such as wake turbulence of turning turbines that affect the wind streaming and so the produced energy; installation cost; cost of civil and electrical infrastructures; overall land-footprint; and the environmental considerations.

In addressing the multi-criteria Wind Farm Layout Optimization (WFLO) problem, the literature has been focused on the single-function formulation and simple weighted sum approach using single-objective stochastic and evolutionary methods, and Pareto formulation using multi-objective evolutionary algorithms. Because of the trade-offs between multiple objectives, the Pareto approach is useful to provide the developer with a non-dominated set of solutions. However, the evolutionary optimization algorithms are computationally prohibitive, in particular, for optimizing very large-scale wind farms. Additionally, most of WFLO problems are highly constrained, where many unfeasible zones can be existing inside the proposed land that would in turn complicate the optimization process.

To remedy these drawbacks, in this chapter, we are extending the proposed approach in Chapter 2 to solve the multi-criteria problem, using $\varepsilon$-constraint method in generating a Pareto of non-dominated set of solutions. The optimized layouts are compared with those by the Non-dominated Sorting Genetic Algorithm NSGA II.
5.2 Problem Formulation

The multi-criteria WFLO problem is formulated considering power generation, electrical infrastructure, land footprint and environmental considerations. Proposed approaches in the literature are divided into two categories: stochastic and metaheuristic approaches with continuous-variable formulation (e.g., [50,82,83]) or discretized domain formulation as in [9,38,41,84]; mathematical programing with restricted turbine-placement formulations, e.g., [34,85]. Whereas, in this chapter, we solve the multi-criteria problem in the continuous domain with the non-linear objective functions, computing a Pareto front to tackle the trade-off between objectives.

5.2.1 Land Footprint

To minimize the land usage, the objective is modeled as the area of the convex-hull that encloses all turbines’ positions. However, in order to determine the required land footprint for the project, the developer should consider the diameters of the turbines at the boundary of the hull and needed areas for construction and transmission. The objective is calculated by ordering the coordinates of the turbines at the bounding hull in the anticlockwise direction and forming a list that is ended by the first coordinate [86], as:

\[
\text{LFP} = \frac{1}{2} \left( \sum_{i=1}^{n} x_i y_{i+1} - \sum_{i=1}^{n} y_i x_{i+1} \right) \tag{5.1}
\]

Where \( n \) is the number of turbines at the hull boundary, \((x_i, y_i)\) is the coordinate of turbine “i”, and \((x_{n+1}, y_{n+1}) = (x_1, y_1)\).

By differentiating Eq. (5.1), the gradients of the LFP are calculated as:

\[
\frac{\partial \text{LFP}}{\partial x_i} = \frac{1}{2} (y_{i+1} - y_{i-1}) \tag{5.2}
\]

\[
\frac{\partial \text{LFP}}{\partial y_i} = \frac{1}{2} (x_{i-1} - x_{i+1}) \tag{5.3}
\]
5.2.2 Electrical Infrastructure

The costs of the electrical infrastructure and loses in transmitted electrical power can be minimized by siting the turbines close to the transformer or the collecting cables. Thus, the objective function is simply modeled as the summation of distances between collecting point and the turbines. However, other mathematical models can be formulated to fit a particular configuration for the used electrical infrastructure. The objective function is defined as:

$$ EI = \sum_{i=1}^{N} \sqrt{(x_i - x_T)^2 + (y_i - y_T)^2} $$

(5.4)

Where \((x_T, y_T)\) is the coordinate of the collecting point near the transformer or the exporting cables.

Hence, the gradients are calculated as:

$$ \frac{\partial EI}{\partial x_i} = \frac{x_i - x_T}{\sqrt{(x_i - x_T)^2 + (y_i - y_T)^2}} $$

(5.5)

$$ \frac{\partial EI}{\partial y_i} = \frac{y_i - y_T}{\sqrt{(x_i - x_T)^2 + (y_i - y_T)^2}} $$

(5.6)

5.2.3 Environmental Considerations

In order to mitigate the complexity of the optimization problem, the environmental aspects are dealt with by geometrical constraints in the domain of the wind farm, representing forbidden zones for habitants, wildlife, bird migration and other aspects, including required setback distances. The geometrical models of the land-availability constraints are introduced and presented above in Section2.1.2.

5.2.4 Optimization Problem

The multi-objective problem is solved by using the \(\epsilon\)-constraint method [87], where the Pareto front is obtained by decomposing the problem into single-objective sub-problems. In each sub-problem, the AEP is set as the objective function, while EI and LFP are approached as non-linear
constraints. Maximum and minimum values for EI and LFP are estimated and the limiting constraints $\varepsilon_1$ and $\varepsilon_2$ are set accordingly in each sub-problem to cover the intermediate space.

The sub-problem is defined as follows:

$$\text{Objective: max } AEP(x, y)$$

$$\text{Subject to: } g_m(x, y) \leq 0, \quad m = 1, \ldots, M$$

$$c_l(x, y) \leq 0, \quad l = 1, \ldots, L$$

$$EI(x, y) \leq \varepsilon_1$$

$$LFP(x, y) \leq \varepsilon_2$$

$$lb \leq x_i \leq ub, \quad i = 1, \ldots, N$$

$$lb \leq y_i \leq ub, \quad i = 1, \ldots, N$$

5.3 Results and Discussion

A test case with 50 turbines and 59.6% feasible area is solved using the problem formulation in the previous Section 5.2. The problem setup is shown in Figure 5-1. Because, there is no obvious trade-off between the objectives of the land footprint and the electrical infrastructure, the $\varepsilon$ values for both objectives are assigned the same value. The power curve and dimensional specifications of the modern turbine Vestas V100-1.8MW [74] are used in the numerical experiment. A real wind data that exhibits variations in the average velocity for different wind directions is adopted from [4], albeit with a different preferential direction to ease the analysis of the resulting optimal solutions. The used wind rose is shown in Figure 4-6.

In order to test the repeatability and the influence of the initial solution, five runs started by five random layouts, have been conducted. The Pareto-fronts of the normalized results are shown in Figure 5-2, and the fractions of dominated solutions by each run and globally are tabulated in Table 5-1. Figure 5-3 illustrates the comparison between NSGA II, starting by feasible solutions with 500 population-size and 1000 generations which are sufficient for convergence, and the non-dominated solutions by $\varepsilon$-constraint IPM.
Figure 5-1 Problem setup for the multi-objective instance

Figure 5-2 Normalized Pareto-front solutions obtained by five runs with different random starts. Note that the AEP normalized values are of reversed signs to make the Utopia points of all objectives at the origins.
Table 5-1 Fractions of dominated solutions for each run

<table>
<thead>
<tr>
<th>Dominated by</th>
<th>Fractions of dominated solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Run 1</td>
</tr>
<tr>
<td>Run 1</td>
<td>-</td>
</tr>
<tr>
<td>Run 2</td>
<td>0.1250</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.1875</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.1875</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.3125</td>
</tr>
<tr>
<td>Global</td>
<td>0.5625</td>
</tr>
</tbody>
</table>

Figure 5-3 Normalized global Pareto-front solutions of all $\epsilon$-constraint IPM runs in comparison with NSGA II Pareto-front. Note that the AEP normalized values are of reversed signs to make the Utopia points of all objectives at the origins.
As shown in Figure 5-2, the Pareto-fronts are relatively close, but start to diverge as the land footprint increases. For local optimization methods, the turbines move in their neighboring areas during the optimization run. Thus, the disadvantage of local methods appear clearly in low density wind farms. However, very low density farms are impractical as the power-density is one of the wind farms’ limitations. That means the area of interest in the objective space would be the left side at less land usage and where the superiority of the proposed local method.

In terms of quality dominance, only 2 to 4 solutions out of 16 to 18 Pareto solutions in each run are dominated by NSGA II, and the fraction of dominated solutions in the global Pareto of $\varepsilon$-constraint IPM by NSGA II is 0.0652. On the other hand, the solutions’ coverage of the NSGA II is limited to a narrow region in the objective space at very low density layouts. In order to improve the spread coverage and push the solutions to high density layouts, an additional constraint for the maximum land footprint can be added, but that would result in inferior solutions and sacrifice the advantage of multi-objective evolutionary algorithms.
In this thesis, we propose a nonlinear mathematical programing approach utilizing exact gradient information to solve the continuous-variable WFLO problem. A statistical comparison with standard genetic algorithms was conducted using an array of test cases from the literature with different dimensionalities, turbine densities and constraint severity.

The effectiveness of using the interior point method with exact derivatives of the objective function and constraints is demonstrated, where the proposed approach outperforms GA in terms of both quality and computational cost. Additionally, a new mathematical model for handling land-use constraints is introduced. Our results show that highly constrained WFLO problem instances can be solved efficiently using the proposed differentiable analytical model. On the other hand, black-box optimization methods may converge to infeasible solutions for highly constrained farms with high turbine densities, or may require a much higher number of function evaluations. It was also observed that using the interior point method to solve WFLO instances based on approximate gradient information obtained from finite differences leads to solutions that are close to the exact local optima, but at a much higher computational cost, even comparable to that of genetic algorithms.

In studying the effect of different strategies for generating initial layouts to feed into the optimization, no significant differences in the final solution were observed when comparing strategies that use 5 starting points and 20 starting points. Moreover, a single uniform staggered layout can result in good optimized solutions when used as a single starting point for the optimization. However, wind farm designers may want run the optimization solver with different initial layouts to increase their confidence in the final solution.

Additionally, the proposed approach has been extended to solve the multi-criteria WFLO problem, considering generated power, electrical infrastructure, land usage and environmental aspects. The
obtained solutions are compared with NSGA II, and confirmed its superiority in terms of Pareto coverage and computational cost.

As we continue to explore wind farm layout optimization based on exact gradient information, future work may focus on the implications of our findings for wind farms located in complex terrains, including different turbine’s features in the optimization model, and on other multi-objective gradient-based formulations for the problem.
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