ANALYSIS AND DESIGN OF SIMPLE, LOW LOSS AND LOW COST RECONFIGURABLE REFLECTARRAYS

by

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A thesis submitted in conformity with the requirements for the degree of Master of Applied Science Graduate Department of Department of Electrical and Computer Engineering University of Toronto

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Abstract

Analysis and design of simple, low loss and low cost reconfigurable reflectarrays

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With the growing of wireless technology, the need for lower-profile, electronically reconfigurable, highly-directive beam-steering antennas is increasing. This thesis proposes a simple low loss, low cost electronic beam-steering antenna architecture by combining a variety of concepts including the single bit phase shifter, sub-wavelength element spacing and delta-sigma modulation. The starting point in the design of reflectarray antennas is the derivation of the so-called S-curve, which maps changes in unit cell design parameters to the phase of the scattered field from the cell. Both numerical and analytical techniques for analyzing linear-polarized fixed and reconfigurable reflectarray unit cell with lumped element has been developed. A 16 × 16 element reflectarray prototype operating at 5.5 GHz is presented the low loss performance and a beam-steering range of -20 to +20 degrees with a single bit phase shifter.
Dedication

To my parents,
for all their love and support.
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Chapter 1

Introduction

In the last several decades the accelerating growth of the wireless industry and the need of internet access has exponentially increased the demand for faster data rates, more bandwidth, and more functionality. Though wireless systems have changed over the years, the general components within a wireless system remain the same. A wireless system typically consists of a transmitter, a channel, and a receiver. The antenna is the transitional structure between free-space and a guided-wave device (transmitter/receiver). It radiates radio waves from the transmitter and receives radio waves at the receiver. In addition to receiving or transmitting energy, an antenna in a more complex wireless system is usually required to optimize the radiation energy in some directions and suppress it in others. Thus the antenna must also serve as a directional device in addition to a probing device.

Antennas are becoming one of the most critical components in wireless communication systems. A good design of the antenna can relax system requirements and improve overall system performance. For most radar and long distance communications (satellite communication, deep-space communication and television broadcast systems), the need for high-gain antennas is unavoidable. The gain is a measure of the radiation intensity in a specific direction versus an isotropic antenna. Directional antennas focus the radiation in a specific direction, increasing the gain of the antenna, while minimizing the effect of interference from other directions. Traditionally, high-gain applications have relied upon parabolic reflectors or arrays. Another need in modern antenna design is the use of beam-steering antennas which are directive antennas capable of steering their beam to different directions. Without beam-steering, the performance of an antenna could only be achieved in fixed direction. However, in the most of the modern communications systems, antennas are tracking with constantly moving devices. Beam-steering antennas allow the system to achieve to this ability. Beam-steering antennas have been existed for many years, but still most of them achieve beam-steering by implemented mechanically rotating devices. This mechanical method of beam-steering is bulky and quite often impractical. Electronic beam-steering is the best replacement for mechanical steering. Unfortunately, traditionally electronic beam-steering antennas such as phased arrays are extremely expensive. Over the past few years, there has been interesting research on the electronic beam-steering antennas such as reflectarrays which are less complex and often lower profile. The following sections will discuss these antennas and then propose a simple, low loss and low cost electronic beam steering antenna.
1.1 Review of Directional Antennas

1.1.1 Parabolic Reflectors

In order to transmit and receive over very large distances, the reflector antenna needs to have very high gain, and therefore, needs to have a very large aperture. A very common reflector antenna is the parabolic reflector. A diagram of a reflector antenna is shown in Figure 1.1. It is composed of a feed and a reflector that collimates the incident waves into a desired direction in the far field. The reflector can be made very large and can also be made into different shapes in order to produce different antenna patterns. However, there are some distinct disadvantages to using large reflectors. Parabolic reflectors require a specific curved surface which can be quite bulky and difficult to manufacture. It also lacks the ability to achieve electronic beam scanning. This has inspired research to look for lower mass, lower volume and electronic beam scanning antenna alternatives.

![Figure 1.1: Schematic of parabolic reflector.](image)

1.1.2 Phased Arrays

It is often very difficult to achieve a desired radiation pattern using only a single element. By combining elements together in certain array geometries, and setting their electrical characteristics accordingly, one can achieve a high-gain antenna. The larger the number of elements is used, the greater the gain is. By changing the magnitude and phase of each element, a desired radiation pattern can be achieved.

Figure 1.2 shows a schematic of a transmitting phased array. Each element in the array is fed individually by means of the feed network. The magnitude and phase excitations of each element are then controlled by reconfigurable components embedded in the feed network which includes active phase shifters, filters, and power amplifiers. With electronic phase and amplitude control of each element, the array can produce highly directive beams and also produce scannable beams. For very large structures, the feed networks become quite large, complex and expensive. For very complex feed networks, there is also an increased amount of loss in the feed network, which further reduces the antenna’s achievable gain. As a result, another type of antenna, namely the “reflectarray”, has been investigated to mitigate the disadvantages associated with both the parabolic reflector and the phased array.
1.1.3 Reflectarrays

Reflectarrays is an antenna consisting of a flat reflecting surface and an illuminating feed antenna as shown in Figure 1.3. On the reflecting surface, there are many radiating elements (e.g. printed microstrip patches, dipoles, or rings) without any power division transmission lines. The feed antenna spatially illuminates these reflectarray elements that are predesigned to reradiate and scatter the incident field with electrical phases that are required to form a planar phase front in the far field distance. The predesigned phases of all elements are used to compensate for the different phases associated with the different path lengths from the illuminating feed. This operation is similar in concept to the use of a parabolic reflector that utilizes its curvature to reflect and form a planar phase front. Thus, the term “flat reflector” is sometimes used to describe reflectarrays.
feed networks. The reflectarray can be built using microstrip elements, which results in a very low-profile design. Each element of the reflectarrays acts as phase shifter which allows the reflectarray to collimate waves from the feed into a desired direction and even beam scanning. The required phase shift on each element can be achieved by different technologies, including electronically reconfigurable ones (discussed in Chapter 2). In this way, the elements do not require any complex active phase shifters. Therefore, reflectarrays have the advantage of low cost, low profile and electronic beam scanning over parabolic reflector and the phased array.

1.2 Motivation

There is a growing interest in designing simpler, cheaper beam-steering antennas which can be integrated in commercial systems. Phased arrays are in most cases much too costly and complex to incorporate in many systems due to their complicated and lossy beamforming networks. As previously discussed, reflectarrays antennas show potential for reducing cost in long distance communication system, which will enable development of larger aperture antennas. Low cost and simple electronically steerable antennas, such as reconfigurable reflectarray antennas, are attractive for beamforming and beam-steering. Several enabling technologies for the dynamic control of reconfigurable reflectarray antennas such as lumped element (PIN diodes, varactor diodes and microelectromechanical systems (MEMS)) and tunable materials (liquid crystal, graphene and photo-conductive) have been proposed. Most technologies exhibit loss (e.g. diodes and liquid crystal) or complexity (MEMS), or both (ferroelectric).

For example, in order to achieve electronic beam scanning, one of the well-known mechanisms which provides changes in the phase of the scattered field is by loading the microstrip patch with an electronically-controlled capacitance which can be achieved by implemented varactor tuning diodes. The model of the varactor diodes is a series combination of RLC circuits and the ohmic loss comes from the resistor in the varactor diodes. The series resistance of the varactor diodes has turned out to be the most significant source of loss in this design [3]. Additionally, in order to minimize the phase quantization loss, the reflectarrays are usually employed with at least 3-bit phase shifters which could potential increase the complexity of the biasing control network. There is thus a need for a low loss, low complexity reconfigurable reflectarrays antenna capable of beam steering.

What’s more, the starting point for any reflectarray design is the conception and subsequent analysis of the reflectarray unit cell. In virtually all designs, this process employs full-wave electromagnetic solvers employing periodic boundary conditions around a unit cell, invoking the infinite array (local periodic) analysis of a constitutive scatterer of the reflectarray. The output of this process is the generalized scattering matrix (GSM) of the reflectarray unit cell. The method of moments (MoM) is a popular method for conducting this analysis 4], and support analysis of periodic structures in order to derive the GSM. However, such methods do not usually handle lumped elements such as those found in diode-tuned reflectarray designs. Therefore, there is a need to extend the GSM to accommodate lumped elements.

While numerical approaches to analyzing reflectarray unit cells are fast and generally easy to carry out, a more fundamental design tool would be a fully analytical models of reflectarray unit cells. Such models can provide greater insight into the operation of reflectarray unit cells, and could be evaluated much faster than their full-wave counterparts. Furthermore, even if full-wave solvers are ultimately employed, analytical models can be used as a useful starting point, generating a “first cut” of the
reflectarray unit cell design before more detailed analysis is undertaken. However, the synthesis of detailed and accurate analytical models of reflectarray unit cells has only been addressed to a limited extent. Equivalent circuit modelling using simple LC models for reflectarray elements [5] have been used to analyze simple reflectarray elements, but do not consider reconfigurable or multi-layer elements. Meanwhile, circuit models for reconfigurable reflectarray elements are very limited or specific. Varactor-loaded dipoles have been considered as reflectarray unit cells, and an ECM for such a cell has been developed [6]. Another cell, using varactor-loaded patches, also can be represented using an equivalent circuit valid near the resonant frequency point [3], but this work is also specific to a certain cell type and requires full-wave simulations to component values in the underlying ECM. Hence, there is a need for a versatile model that can accommodate both fixed and reconfigurable reflectarray elements.

1.3 Thesis Goals

This thesis aims to make a contribution in two specific areas:

1. (a) Achieve simpler and less expensive designs using 1-bit phase shifting.
    (b) Achieving zero-power-consumption, through varactor-based designs.
    (c) Achieving lower cost designs by using Si diodes, and mitigation of losses through sub-wavelength cells.
    (d) Mitigation of quantization lobes and losses through sub-wavelength cells.

A dense reflectarray employing sub-wavelength unit cells will be developed to reduce the loss in reflectarrays elements, and reflectarrays elements with single-bit phase shifters will be implemented to further reduce the loss and simplify the beam forming network. Varactor diodes will be employed as single bit phase shifters instead of PIN diodes in this design. All 1-bit reflectarray elements employ PIN diodes, which consume power in their “ON” state. Conversely, continuously tunable designs are employ varactor diodes in a reversed-biased state, and hence there is essentially zero power consumption in such designs. This thesis will explore 1-bit designs based exclusively on varactor diodes to bring this quality to 1-bit reconfigurable reflectarrays. Low cost lossy (Si) varactor diodes will be employed instead of the expensive low loss (GaAs) diodes in order to mitigate the cost. We hypothesize that the loss introduced by the lossy varactor diodes could offset by employing sub-wavelength element spacing. Methods to reduce the quantization lobes and losses caused by the single-bit phase shifters have also not been explored for reflectarrays and will be investigated in this thesis. The development and fabrication of a single-bit dense reconfigurable reflectarray has not yet been presented in literature. The reconfigurability of this single bit dense reflectarray will be demonstrated. Reconfigurable reflectarray antennas will be an alternative choice of antenna offering many other flexible functions such as plane wave re-director. These goals will be realized through simulated characterization of the reflectarray. The prototype is used for experimental validation of theory and will show the possibility of using this reflectarray in wireless communication.

2. The development of both numerical and analytical techniques for analyzing linear-polarized reflectarray unit cell with tunable lumped elements.

(a) Extended GSM-MoM.
(b) Analytical model.

In order to simulate reflectarrays unit cell with lumped elements, existing numerical methods for computing the GSM will be extended to accommodate lumped elements loading along the edges of basis functions, and furthermore lumped elements will extend to lumped ports to reduce the simulation time. On the analytical side, the scattering behaviour of various reflectarray unit cells will be quickly and accurately predicted using an equivalent circuit model of the Floquet modes, and all the important phase curve of the reflectarray unit cell will be predicted using closed-form formulas. This Model works for linearly-polarized fixed and reconfigurable rectangular reflectarray elements. The ECM will be used to supplement, but not replace, full wave simulation tools.

1.4 Thesis Outline

This thesis is organized as follows. The second chapter of this thesis will first go through an overview of basic background theory used to design and analyze array antenna. Next, the literature on the design concepts applied in the proposed reflectarray design will be reviewed.

The third chapter will present two antenna design methods. It will begin by presenting the periodic MoM based full-wave electromagnetic solvers to derive the GSM with lumped port extension. It will next present a versatile and accurate equivalent circuit model for predicting the reflection coefficient for a linear-polarized reflectarray unit cells. Finally, ECM will apply to reveal the relationship between the parameters of the reflectarray element and the corresponding scattered field.

The fourth chapter will present the design of preliminary and final reflectarray unit cell, and a detailed analysis is performed on the reflection performance of the unit cell. Next a detailed discussion about the single bit phase optimization will be presented and followed by simulated beam-steering results for both one-dimensional and two-dimensional arrays. Finally measurement results of a prototype two-dimensional $16 \times 16$ element array will be presented in chapter fifth.

The final chapter will draw some conclusions about the proposed antenna design methods and the performance of the reconfigurable reflectarray, and discuss future directions.
Chapter 2

Background

The surface of the reconfigurable reflectarray is also define as reconfigurable electromagnetic surface. This chapter focuses on the fundamentals required to understand and design reconfigurable planar arrays and reconfigurable electromagnetic surfaces. The first section covers the basic theory behind array antennas and beam steering as well as fundamental parameters used to determine the performance of array antennas, including reflectarrays. The second section will discuss some design concepts used in this thesis to design and analyze reconfigurable electromagnetic surfaces.

2.1 Antenna Arrays

Most of the reconfigurable electromagnetic surfaces discussed in this thesis can be analyzed in the context of antenna arrays. It is necessary to design antennas with very directive characteristics (very high gains) to meet the demands of satellite communication, point-to-point terrestrial links, deep-space communication links, and radars [7]. One way to achieve this is by increasing their effective aperture size. Another way of providing large directivity, without necessarily increasing the size of the individual elements, is to combine an array of small radiating elements in an electrical and geometrical configuration. These antenna arrays can achieve similar effective aperture sizes and hence directivity as a fixed large antenna. In general, antenna arrays can be made very directive, which means power is focused in a small angular range. The total field of an array is determined by the vector summation of the radiated fields from the individual elements. In order to provide very directive patterns, the fields from the elements of the array should interfere constructively (add) in the desired directions and interfere destructively (cancel each other) in the remaining space. This section talks about the basic concept behind of the array antenna.

2.1.1 Array Theory

An antenna array is a regular or non-regular arrangement of individual radiating antenna elements. Each element is located in a particular spatial position and fed with particular phases and amplitudes to produce a collimated beam in a given direction. While arrays can be designed in any geometrical arrangement, a good starting point to their analysis is to consider a linear array of evenly spaced elements, which is shown in Figure 2.1. Here, we assume each element is isotropic, and if the actual elements are not isotropic sources, the total fields can be formed by multiplying the array factor of the
isotropic sources by the field of a single element as discussed later. represents the phase by which the current in each element leads the current of the preceding element. As shown in Figure 2.1, an $N$-element array of isotropic sources positioned along the $z$-axis with an equal spacing distance $d$. The elements are fed with equal amplitudes $I$ and a progressive phase shift $\beta$ represents the phase by which the current in each element leads the current of the preceding element. The total beam pattern can be analyzed by taking the superposition of the individual element beam pattern and the progressive phase. This pattern is given the name of array factor ($AF$) and is calculated using

$$AF(\theta) = 1 + e^{+j(k_0d \cos \theta + \beta)} + e^{+2j(k_0d \cos \theta + \beta)} + \ldots + e^{+j(N-1)(k_0d \cos \theta + \beta)},$$

(2.1a)

$$= \sum_{n=1}^{N} e^{j(n-1)(k_0d \cos \theta + \beta)},$$

(2.1b)

where $k_0$ is free space wavenumber. After some straightforward algebra manipulations, the normalized array factor for the linear array in Figure 2.1 can be written as

$$AF_n(\theta) = \frac{\sin\left(\frac{N}{2}(k_0d \cos \theta + \beta)\right)}{N \sin\left(\frac{1}{2}(k_0d \cos \theta + \beta)\right)}.$$  

(2.2)

To find the null of this array, Equation (2.2) is set to be zero giving,

$$\frac{N}{2}(k_0d \cos \theta + \beta) = \pm n\pi,$$

(2.3)

where $n = 1, 2, 3, \ldots$ but $n \neq N, 2N, 3N, \ldots.$

For $n = N, 2N, 3N, \ldots$, the normalized array factor attains its maximum values. According to Equa-
tion (2.2), the array factor produces its maximum when the argument
\[ \frac{1}{2}(k_0d \cos \theta + \beta) = \pm m\pi, \]  
(2.4)
where \( m = 0, 1, 2, 3, \ldots \). If \( d \) is less than half wavelength, the pattern only has one maximum within the angular range \(-\pi < \theta \leq \pi\) for any given \( \beta \). Therefore, to produce a beam at an angle \( \theta_b \), the phase difference between elements need simply be specified by the following formula:
\[ k_0d \cos \theta_b + \beta = 0, \]  
(2.5a)
\[ \beta = -k_0d \cos \theta_b. \]  
(2.5b)

We can easily scan the beam by only changing the progressive phase \( \beta \). This can be done in a fixed case or in electronically phase control case to steer the beam. This thesis will focus on the electronically beam scanning which is effectively implemented by modifying \( \beta \).

The antenna array is not always excited with linear phase gradient and uniform element spacing. In a general case, each element in the array could be excited with arbitrary phases. The array factor \((AF)\) can be calculated from
\[ AF(\theta) = \sum_{n=0}^{N-1} w_n e^{-jk_0z_n \cos \theta}, \]  
(2.6)
where \( w_n \) are the complex excitations of each element (magnitude \( I \) and phase \( e^{j\varphi_n} \)), and \( z_n \) is the position of the \( n^{th} \) element.

For the array in Figure 2.1 when \( d \) is greater than a half-wavelength, the pattern has multiple maxima which lead to the unwanted grating lobes. So, it is safer to use array element spacing smaller than the half wavelength to get rid of grating lobes. Figure 2.2 shows the normalized power pattern of linear array with length of 20 \( \lambda \) scanned to \( \theta = 70^\circ \) with an element spacing \( \lambda \) and \( \lambda/3 \), respectively. In the \( \lambda \) element spacing case \((d > \lambda/2)\), it is clearly shown there is a grating lobe. As expected, there is no grating lobe in the \( \lambda/3 \) element spacing case \((d < \lambda/2)\).

In the above discussion, only the array pattern assuming isotropic elements has been considered. When the antenna elements are identical, and assuming no significant mutual coupling between elements, the total pattern produced by the array is simply the array factor multiplied by the element pattern (element factor),
\[ PF(\theta) = EF(\theta)AF(\theta). \]  
(2.7)
If mutual coupling exists, exciting one element will also cause excitation of adjacent elements, disturbing the behaviour from the desired one. Therefore, if we calculate the far field radiating pattern based on array factor, the result will slightly different from the actual one.

For two-dimensional arrays, which are the focus of this thesis, the formulas and derivations for the array factor are very similar to the one dimensional arrays except that the summation in Equation (2.6) is now carried over elements positioned across two-dimensions and the array factor becomes a function of both \( \theta \) and \( \phi \). The array factor of an N-element 2-D array with a complex excitation of \( I_i = A_i e^{j\phi_i} \)
at each element $i$ can be expressed as,

$$AF(\theta, \phi) = \sum_{i=0}^{N-1} I_i e^{jk_0(x_i \sin \theta \cos \phi + y_i \sin \theta \cos \phi)}, \quad (2.8a)$$

$$= \sum_{i=0}^{N-1} A_i e^{j(\phi_i + k_0(x_i \sin \theta \cos \phi + y_i \sin \theta \cos \phi))}, \quad (2.8b)$$

where $x_i$ and $y_i$ are the coordinates of element $i$. In order to produce a beam in the $\theta = \theta_b$ and $\phi = \phi_b$ direction, the phase at each element should be

$$\phi_i = -k_0(x_i \sin \theta_b \cos \phi_b + y_i \sin \theta_b \cos \phi_b). \quad (2.9)$$

In the 2-D planar array discussed here, only the array pattern assuming isotropic elements has been considered. The element pattern needs to be considered as well and now, both the element factor and array factor are function of $\theta$ and $\phi$ and the pattern factor can be expressed as

$$PF(\theta, \phi) = EF(\theta, \phi) AF(\theta, \phi). \quad (2.10)$$

### 2.1.2 Electromagnetic Surface Background

In the previous section, array theory has been discussed. The array architectures discussed later rely on a simple, low loss and low profile electronically reconfigurable apertures. The operating principle of reflectarrays is discussed first and followed by an additional application of reflectarrays: a plane wave re-director.
Reflectarrays

As mentioned in Chapter 1, the required phase shift on each reflectarray element can be achieved by different technologies. The necessary phase shift at each element is obtained by varying one of the parameters in the reflectarray element.

The phase of each element in a non-reconfigurable reflectarray is typically varied using two methods. The first implementation of phase adjustment is to use identical patches with variable-length phase delay lines so that they can compensate for the phase delays over the different paths from the illuminating feed. Another is to use variable-size element, so that elements have different scattering phases to compensate for the phases of the different feed paths.

What’s more important, one significant advantage of reflectarrays over the traditional reflector is that they can easily be made electronically reconfigurable through tuning mechanisms. A brief summary of these approaches is discussed here. One of the methods to provide the phase shift is by tuning the resonant length of reflectarray element using devices such as PIN diodes, varactor diodes and RF microelectromechanical systems (MEMS).

The second way to accomplish the phase shift is through varying the dielectric constant of the substrate, which is the operating principle of reflectarray elements using dielectrics with tunable properties such as liquid crystals and ferro-electric films which have also been employed for in semi-distributed elements.

Reflectarrays achieve the absolute control over the radiated phase across the aperture. For pencil-beams, the phase distribution of the incident field is parabolic/paraboloidal across the surface and may wrap multiples times over the surface. The function of the reflectarray elements is then to change that phase distribution into a uniform phase distribution for a broadside beam or a linear phase gradient for a pencil beam in a certain direction.

Figure 2.3: Typical geometry of a printed two-dimensional reflectarray antenna

Figure 2.3 shows an image of a two-dimensional reflectarray antenna. The feed of the reflectarray is located in space above the array and several wavelengths away. A horn is usually used as feed to produce a spherical wave, and its radiation pattern can be approximately modelled as a $\cos^q(\theta)$ function in the feed coordinate system defined in Figure 2.3. In order to produce a beam in the direction $(\theta_b, \phi_b)$, the
progressive phase distribution on the reflectarray surface, as known from 2D-planar array theory, can be expressed as

\[ \phi(x_i, y_i) = -k_0 \sin \theta_b \cos \varphi_b x_i - k_0 \sin \theta_b \sin \varphi_b y_i, \] (2.11)

where \( k_0 \) is the propagation constant in vacuum. The elements are in the \( z = 0 \) plane and \( (x_i, y_i) \) is the coordinate of reflectarray element \( i \). Also, the phase of the reflected field at each reflectarray element is equal to the phase of the incident field plus the phase shift introduced by each element, and can be expressed as

\[ \phi(x_i, y_i) = -k_0 d_i + \phi_R(x_i, y_i), \] (2.12)

where \( \phi_R(x_i, y_i) \) is the phase of the scattered fields for element \( i \) and \( d_i \) is the distance from the center of the feed to the element \( i \). From Equations (2.11) and (2.12), the phase shift of each element needs satisfy

\[ \phi_R(x_i, y_i) = k_0 (d_i - (\cos \varphi_b x_i + \sin \varphi_b y_i) \sin \theta_b). \] (2.13)

Figure 2.4: Example of the phase distribution in a reflectarray.

Figure 2.4 shows the required phase shift on a centre fed reflectarray of 20 × 20 element with the focal point 5\( \lambda \) away from the aperture and produces a pencil beam in the normal direction to the surface. In the reflectarray design, the phase of the reflection coefficient must be adjusted in each element to match these required phases.

The formula for the array factor of two-dimensional reflectarray is very similar to Equation (2.5) and
includes the feed radiation pattern

\[ AF(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn}(\theta, \phi) \cos^q \theta_{mn} e^{j(\phi_{mn}(x_{mn}, y_{mn}))} e^{j(k_x x_{mn} + k_y y_{mn})}, \]  

(2.14)

where

\[ k_x = k_0 \sin \theta \cos \phi, \]  

(2.15a)

\[ k_y = k_0 \sin \theta \sin \phi. \]  

(2.15b)

\(x_{mn}\) and \(y_{mn}\) is the position of the reflectarray \(mn^{th}\) element, and \(k_x\) and \(k_y\) are the free space wavenumbers in \(x\) direction and \(y\) direction, respectively. \(A_{mn}\) is the magnitude of the reflection coefficient of the \(mn^{th}\) reflectarray element. Another interpretation of \(\cos^q \theta\) on reflectarray aperture is that \(\cos^q \theta\) presents the tapering of the amplitude excitation of the feed. The value of \(q\) is determined from the radiating characteristics of the actual feed used in the implementation of the reflectarray.

Reflectarrays combine the best features of parabolic reflectors and phased arrays. They offer the simplicity and high-gain associated with their reflector counterparts, while at the same time providing fast, adaptive beam-forming capabilities of phased arrays. The low profile nature, ease of manufacturing, low weight and good efficiency of reflectarrays promise them as high-gain antenna alternatives in satellite communication, point-to-point terrestrial links, deep-space communication links, and radars [7]. Additionally, reflectarray antennas can be also used as plane wave re-director because of its low profile nature.

**Plane Wave Re-directors**

Reflectarrays have been suggested to improve line-of-sight radio channel in telecommunication which is capable of scattering the incident wave from the base station to the blind spots or areas to improve the signal to noise ratio [12]. Their design is also expected to increase the path number of incident wave to optimize the channel capacity given prevailing traffic condition. A new frequency selective reflectarray (FSR) has also been proposed [13] with a crossed-dipole array and a frequency selective surface (FSS) of square loops printed on both sides of a dielectric substrate. The reflectarray functions as a reflector, and generates the desired reflected beam shape while steering the primary wave source in the desired direction. Here, the proposed reflectarray can be used to function as a reflector and reflect the incident field into the desired direction. The incident field of plane wave re-director is typically plane wave instead of spherical wave in the normal reflectarrays, because the feed antenna is very far away from the reflectarray aperture.

Figure 2.5 shows a conception image of a one-dimensional plane wave re-director (assuming the a normally incident wave). The feed for the plane-wave re-director is located sufficiently far away that the wavefronts are approximately planar. The calculation of the required phase shift from the plane wave re-director elements is similar to the calculation for reflectarrays. The only difference is that the incident field of plane-wave re-director is a plane wave instead of a spherical wave. In order to produce a beam in the direction \((\theta_b, \phi_b)\), the progressive phase distribution on the 2D plane wave re-director elements,
as known from 2D-planar array theory, has been found in (2.9), and can be expressed as

$$\phi(x_i, y_i) = -k_0 \sin \theta_b \cos \varphi_b x_i - k_0 \sin \theta_b \sin \varphi_b y_i.$$  \hspace{1cm} (2.16)

The phase of the reflected field at each plane wave re-director element is equal to the phase of the incident field plus the phase shift produced by each element, and can be expressed as

$$\phi(x_i, y_i) = k_0 \sin \theta_i \cos \varphi_i x_i k_0 \sin \theta_i \sin \varphi_i y_i + \phi_R(x_i, y_i),$$  \hspace{1cm} (2.17)

where $\phi_R(x_i, y_i)$ is the phase of the reflection coefficient for element $i$, $\theta_i$ and $\varphi_i$ is the angle of incident wave. From Equations (2.16) and (2.17), the phase shift on each element can be written as

$$\phi_R(x_i, y_i) = -k_0 \sin \theta_i (\cos \varphi_i x_i + \sin \varphi_i y_i) - k_0 \sin \theta_b (\cos \varphi_b x_i + \sin \varphi_b y_i).$$  \hspace{1cm} (2.18)

For this two-dimensional plane wave re-director, the array factor can be calculated from Equation (2.14) with $q = 0$, since the incident wave is a plane wave.

### 2.1.3 Array Performance

For every type of antenna array, the performance of antenna array can be determined by several parameters. Some of the most important performance parameters are directivity ($D$), gain ($G$), antenna efficiency ($\varepsilon_{AUT}$), bandwidth (BW) and side lobe level (SLL).

**Directivity**

Directivity ($D$) is a measure of how focused the pattern is in a specific direction. The directivity of an antenna is the ratio of its radiation intensity in a given direction over that of an isotropic source, and can be expressed as

$$D = \frac{U}{U_0} = \frac{4\pi U}{P_{\text{rad}}},$$  \hspace{1cm} (2.19)
where $U$ is the radiation intensity of an antenna which is defined as the power radiated per unit solid angle. $U_0$ is the radiation intensity of an isotropic source and $P_{\text{rad}}$ is the total radiated power. In antenna arrays, assuming the array element is isotropic source, and then directivity can be computed directly from the array factor based on Equation (2.8). $U$ and $P_{\text{rad}}$ can be calculated from

$$U = |AF(\theta)|^2,$$  \hspace{1cm} (2.20)

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi |AF(\theta)|^2 \sin(\theta) \, d\theta \, d\phi.$$ \hspace{1cm} (2.21)

If the direction is not specified, the direction of maximum radiation intensity is implied,

$$D_{\text{max}} = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}}.$$ \hspace{1cm} (2.22)

Most of the time, it is desirable to express the directivity in decibels (dB) instead of dimensionless quantities. The expressions for converting the dimensionless quantities of directivity to decibels (dB) is

$$D(\text{dB}) = 10 \log_{10} (D(\text{dimensionless})).$$ \hspace{1cm} (2.23)

As mentioned before, the practical antenna element in arrays is not an isotropic source, and then the element pattern has to be factored in. The directivity of antenna arrays (e.g. reflectarrays) can be easily computed from the array factor by multiplying it by the element factor of arrays element, and is expressed as

$$D_r = \frac{4\pi U_{\text{EF}} U_{\text{AF}}}{P_{\text{rad}}},$$ \hspace{1cm} (2.24a)

$$P_{\text{rad}} = \int_0^{2\pi} \int_0^\pi |AF(\theta, \varphi)|^2 |EF(\theta, \varphi)|^2 \sin \theta \, d\theta \, d\varphi,$$ \hspace{1cm} (2.24b)

$$= \int_0^{2\pi} \int_0^\pi U_{\text{EF}} U_{\text{AF}} \sin \theta \, d\theta \, d\varphi.$$ \hspace{1cm} (2.24c)

The radiation intensity $U_{\text{AF}}(\theta, \varphi)$ of the array factor can be calculated based on Equation (2.20) and $AF(\theta, \varphi)$ can be obtained from Equation (2.14). Assuming the reflectarray elements can be treated as uniform rectangular apertures mounted on an infinite ground plane, the radiation intensity $U_{\text{EF}}(\theta, \varphi)$ of the element factor using far-field electric and magnetic field components can be calculated from (2.14)

$$U_{\text{EF}}(\theta, \varphi) = \frac{1}{2\eta} (|E_\theta|^2 + |E_\phi|^2).$$ \hspace{1cm} (2.25)
$E_\theta$ and $E_\phi$ are the $\theta$ and $\phi$ components of the dominant E-field in the far field and can be written as

$$E_\theta = j \frac{abk_0E_0e^{-jk_0r}}{2\pi r} \left[ \sin \phi \left( \frac{\sin X}{X} \right) \left( \frac{\sin Y}{Y} \right) \right], \quad (2.26a)$$

$$E_\phi = j \frac{abk_0E_0e^{-jk_0r}}{2\pi r} \left[ \cos \theta \cos \phi \left( \frac{\sin X}{X} \right) \left( \frac{\sin Y}{Y} \right) \right], \quad (2.26b)$$

$$X = \frac{k_0a}{2} \sin \theta \cos \phi, \quad (2.26c)$$

$$Y = \frac{k_0b}{2} \sin \theta \sin \phi. \quad (2.26d)$$

$a$ and $b$ are the size of the rectangular aperture and $k_0$ is the free space wavenumber. $E_0$ is a constant which is the magnitude of the electric field on the aperture. The maximum radiation intensity ($U_{\text{max}}$) occurs at $\theta = 0$ according to Equation (2.26) and it is equal to

$$U_{\text{max}} = \left( \frac{ab}{X} \right)^2 \frac{|E_0|^2}{2\eta}. \quad (2.27)$$

When the antenna arrays collimates radiation towards $\theta = 0$, the maximum directivity is equal to

$$D_0 = \frac{4\pi U_{\text{max}}}{P_{\text{rad}}} = \frac{4\pi}{\lambda^2} ab. \quad (2.28)$$

### Gain

The gain of an antenna is closely related to its directivity. Directivity of an antenna represents how directed the power is compared to an antenna isotropically radiating the power, but gain measures the intensity in a given direction, over the radiation intensity considering the input power of the antenna. Then the gain of antenna can be expressed as

$$G = \frac{4\pi U}{P_{\text{in}}}. \quad (2.29)$$

where $P_{\text{in}}$ is the total input accepted power. The relationship between total radiated power ($P_{\text{rad}}$) and total input power ($P_{\text{in}}$) can be expressed as

$$P_{\text{rad}} = \varepsilon P_{\text{in}}, \quad (2.30)$$

where $\varepsilon$ is the radiation efficiency and usually includes reflection mismatch loss, conduction loss and dielectric loss \cite{14}. Then, the gain is related to the directivity and can be expressed as

$$G = \varepsilon D. \quad (2.31)$$

Since the radiation efficiency ($\varepsilon$) is smaller than one due to mismatches and loss in antennas, an antenna’s gain is always less than its directivity.

The gain of an aperture antenna can be calculated using

$$G_{\text{ap}} = \frac{4\pi A_{\text{ap}}}{\lambda^2} \varepsilon_{\text{ap}}, \quad (2.32)$$
where $A_{ap}$ is the physical area of the aperture antenna, and $\varepsilon_{ap}$ is the aperture efficiency which is composed of the product of various efficiency, and can be expressed as

$$\varepsilon_{ap} = \varepsilon_{cd}\varepsilon_{s}\varepsilon_{t}\varepsilon_{p}\varepsilon_{b}. \quad (2.33)$$

The radiation efficiency, $\varepsilon_{cd}$, includes the conductor loss and dielectric loss from the antenna arrays. In the proposed reflectarray, the element (conductor) loss will be significantly reduced and this element loss improvement will be discussed in the following chapters. Spillover ($\varepsilon_{s}$), taper ($\varepsilon_{t}$), phase ($\varepsilon_{p}$) and blockage ($\varepsilon_{b}$) efficiencies will be described in the following paragraphs.

Spillover efficiency, ($\varepsilon_{s}$), is the fraction of the total power that is radiated by the feed and captured by the reflecting surface as shown in Figure 2.6 and can be expressed as

$$\varepsilon_{s} = \frac{\text{Power captured by the surface}}{\text{Total power radiated by feed}}. \quad (2.34)$$

For example, for an equivalent parabolic reflector with an axisymmetric feed pattern $G_f(\theta)$, the spillover efficiency can be expressed as

$$\varepsilon_{s} = \frac{\int_{0}^{\theta_0} G_f(\theta) \sin(\theta) d\theta}{\int_{0}^{\pi} G_f(\theta) \sin(\theta) d\theta}, \quad (2.35)$$

where $\theta_0$ is the subtended angle to the edge of the reflector. The spillover efficiency depends on the characteristic of the feed. In order to minimize the spillover efficiency, the feed is generally chosen to be a horn antenna which can achieve a narrow beam pattern with low minor lobes.

Taper efficiency, ($\varepsilon_{t}$), is caused by the non-uniform amplitude distribution of the feed pattern over the reflecting surface. As the reflecting surface is illuminated with a narrow beam pattern, the elements in the centre will have higher amplitudes than those at the edges. This difference in amplitudes across the aperture surfaces reduces the efficiency compared to if the aperture were uniformly illuminated. This reduction in efficiency is referred to as taper efficiency. Taper efficiency also depends on the
characteristics of the feed. The taper efficiency can be found using

\[ \varepsilon_t = \frac{D_{\text{non-uniform}}}{D_{\text{uniform}}}, \] (2.36)

where \( D_{\text{non-uniform}} \) is the peak directivity of the reflectarray using the amplitudes incident on the reflectarray, and \( D_{\text{uniform}} \) is the peak directivity of the reflectarray using uniform amplitudes.

Spillover and taper losses are two main factors that contribute to the aperture efficiency. The design needs to be maximized the product of spillover and taper efficiency to achieve the best aperture efficiency, however, a compromise between spillover and taper efficiency must emerge. Very high spillover efficiency can be achieved by a narrow beam at the price at a very low taper efficiency. Therefore, as the spillover losses are reduced, the taper efficiency decreases. The maximum overall efficiency can be obtained by carefully choosing feed location and feed characteristics. In other words, there is an optimal feed that produces a compromise between taper and spillover losses, generally arising when the taper level is 10-12 dB on the reflector edges.

Phase efficiency (\( \varepsilon_p \)) is caused by the phase quantization errors of the field over the aperture plane which could be introduced by the differences between the desired phase and the actual phase. These phase quantization errors will be discussed in more details in the following sections.

Last, blockage efficiency (\( \varepsilon_b \)) is the percent power loss due to blockage provided by the feed and supporting structure. A diagram is shown in Figure 2.7. This blockage can be minimized by using an offset feed as shown in Figure 2.8.

![Figure 2.7: Feed blockage](image)

![Figure 2.8: Reducing feed blockage](image)
The blockage efficiency, $\varepsilon_b$, is the fraction of total power that is collimated by the reflectarray and not blocked by the feed or the supporting apparatus. It can be found using

$$\varepsilon_b = \frac{U_{\text{with feed}}}{U_{\text{no feed}}},$$

(2.37)

where $U_{\text{with feed}}$ is the radiation intensity of the reflectarray in the desired beam direction with the feed present, and $U_{\text{no feed}}$ is the peak radiation intensity of the reflectarray without the feed present.

### Bandwidth

The bandwidth can be considered to be the range of frequencies, on either side of a center frequency, where the antenna characteristics (such as input impedance, pattern, beamwidth, polarization, side lobe level, gain, beam direction, radiation efficiency) are within an acceptable value of those at the center frequency [14]. For beam steering antenna array, the bandwidth is defined as the range of frequency where the gain at the desired beam direction within a certain range of the center frequencies gain (usually 1-3 dB).

### Side Lobe Level

Minor lobes usually represent radiation in undesired directions, and side lobes are normally the largest of the minor lobes. The level of side lobes is usually expressed as a ratio of the power density in the lobe in question to that of the major beam. This ratio is often named as the side lobe ratio or side lobe level. Normally, side lobe level is presented as a negative number in dB. Side lobe levels of -20 dB or smaller are usually desirable in most applications [14].

Overall, many different properties of array antennas need to be considered during the array design processes. The goal of this thesis is to design a simple low profile, low loss electromagnetic surface with simple beamforming networks. Some design concepts to achieve these goals will be discussed in the next section.

### 2.2 Proposed Reflectarray Architecture

Electronically reconfigurable antenna arrays such as reflectarrays or transmitarrays are attractive due to their low profile, low loss and simple structure. One of the goals of this thesis is to design a simple, low loss and low cost electronically reconfigurable antenna. To do so, various design concepts used for potentially realizing the proposed reconfigurable reflectarrays is listed here and will be discussed in detail in the following sections.

1. The use of varactors in a switched mode (1 bit), rather than a continuous state, to simplify the biasing control and to mitigate the diodes (ohmic) loss.

2. The use of low-cost (Si) diodes to achieve the reconfiguration, and to mitigate the cost.

3. The use of the sub-wavelength element spacing to compensate the phase quantization loss from 1 bit phase shifting and reduce the diodes (ohmic) loss.

4. The potential use of alternative phasing techniques to mitigate problems related to 1-bit phase shifting.
2.2.1 Single Bit Phase Shifter

Normally, a phase shifter has finite number of quantized phase states. For an N-bit phase shifter, the number of quantized phase states is $2^N$. In theory, antenna array unit cells with a high phase resolution such as a large number of quantization phase states can reduce phase quantization errors and are desirable to achieve high beam scanning performance. However, such cells require complex control networks and increase the system cost, which will enlarge in the case of large arrays. For electronically reconfigurable antenna arrays, different phase states may require different biasing control, and then the more phase states are, the more complex biasing network is. Alternatively, antenna arrays employing phase shifters with a low number of bits use a simple biasing control (e.g. in the case of 1-bit control, simply ON or OFF) and low system cost, whereas the loss in antenna directivity increases because of an increase of phase quantization errors. Therefore, there is a tradeoff between two major factors: complexity of the antenna array design and antenna directivity reduction when choosing the number of bits for phase quantization. The studies of the tradeoff between the reduction in directivity and the number of bits of a digitally controlled reflectarray cell present that the directivity reduction is: 3.8 dB, 0.85 dB, 0.21 dB, and 0.05 dB for 1, 2, 3, and 4 bit, respectively [15]. This suggests that beyond 3 bit phase shifter design, the performance degradation is practically negligible, and a more detailed relationship between directivity reduction and number of bits will discuss later. But, a small number of bits design is still attractive due to its simple structure and researchers have implemented single bit phase shifter into both transmitarray and reflectarray designs to reduce the system complexity.

Transmitarrays, also known as array lenses, have also been proposed as low-cost alternatives to phased arrays. Single bit phase shifters were first applied in reconfigurable transmitarrays. An electronically reconfigurable unit cell with 1-bit phase quantization (PIN diodes) for X band linear polarization transmitarrays have been proposed [16]. The authors demonstrated $20 \times 20$ element fully reconfigurable transmitarray and 1 bit quantization leads to a reduction of the antenna directivity close to 4 dB.

Recently, some researchers have implemented single bit phase shifters into reflectarrays as well. Reconfigurable reflectarray antenna with digital elements are preferred in practical implementations due to the simplified biasing structure. The authors has proposed an electronically reconfigurable large reflectarray using single-bit phase shifters at 60 GHz and it is shown that the loss in directivity due to the phase-quantization errors is close to 3.9 dB [8]. A novel reconfigurable reflectarrays with single-bit phase resolution for Ku-band has been presented and phase errors caused by two phase states introduced about 5.4 dB reduction in directivity [17]. Optimization to minimize the directivity reduction caused by single bit phase quantization has been shown to achieve a 2.8 dB directivity loss by adding the phase offset.

Effect on Reflectarray: Directivity Loss

Here, the loss in directivity due to phase quantization errors of single bit phase shifters can be calculated from the array factor calculation in Section 2.1.3 where we are assuming the antenna array elements are uniform rectangular apertures mounted on an infinite ground plane and no coupling between the adjacent elements. A design of $20 \times 20$ reflectarray element has been analyzed. The reflectarray was illuminated with a spherical wave at broadside and produces a beam at $\theta_b = 20^\circ$ and $\phi_b = 0^\circ$. The unit cell size is half wavelength, leading to a total size of the reflectarray of $10\lambda \times 10\lambda$.

Figure 2.9 shows the directivity of antenna array employing both continuous phase shifters and single
bit phase shifters. The directivity of the reflectarray implemented with continuous phase shifters is 31.07 dB and the same reflectarray implemented with 1 bit phase shifters is 27.08 dB. Therefore, 1-bit phase resolution (180 phase steps) results in a quantization loss of 3.99 dB in directivity which agrees with [15].

Figure 2.10 shows the phase distribution on reflectarray elements of continuous phase shifters and 1 bit phase shifters, respectively. Figure 2.11 shows the phase plot of one dimensional reflectarray (a centre cut in x direction) in both continuous phase distribution and 1 bit phase distribution cases. Two phase states have been chosen as 75° and -105° in single bit phase shifters. Phase quantization errors due to the 1 bit phase shifter are clearly shown in Figure 2.10 and Figure 2.11. Later in this thesis, this 1 bit phase quantization loss can be reduced by decreasing the unit cell sizes and optimizing two phase states for each element.

Consequently, reflectarrays with single bit phase quantization leads to a reduction of antenna directivity around 4 dB, but it is still attractive because of low profile, less system complexity, and low
Effect on Plane Wave Re-director: Quantization Lobes and Directivity Loss

A phase shifter has a finite number of quantized phase states and introduces phase quantization errors because it cannot achieve the intermediate-phase values. In a beam scanning plan wave redirector and linear or planar arrays, phase quantization may introduce grating lobes in the visible region which is often called quantization lobes. The relative intensity of the quantization lobe is depended on the number of bits of the phase shifter and can be expressed as

\[
G(\theta_q) = \frac{1}{2^N - 1},
\]

where \( G(\theta_0) \) means the intensity of the main lobe occurs at \( \theta_0 \), \( G(\theta_q) \) is the intensity of the quantization lobe at \( \theta_q \), and N is the number of bits of the phase shifter. The larger number of bits of the phase shifter is, the less pronounced of the quantization lobe is. The gain loss due to the phase quantization is given by

\[
\text{Gain loss} = \frac{2^N}{\pi} \sin\left(\frac{\pi}{2^N}\right).
\]

The gain loss is also related to the number of bits of the phase shifter. Table 2.1 summarizes the reduction in gain caused by the phase quantization errors according to Equation (2.39).

<table>
<thead>
<tr>
<th>Gain loss [dB]</th>
<th>1-bit</th>
<th>2-bit</th>
<th>3-bit</th>
<th>4-bit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.92</td>
<td>0.91</td>
<td>0.22</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 2.1: Gain loss due to phase quantization errors

For the plane wave redirector, the gain loss and intensity of the quantization lobe could potentially
be reduced by employing sub-wavelength element spacing and delta sigma modulation which will be discussed later.

Overall, a small number of bits allows the reconfigurable reflectarray to have a simple biasing network structure and thereby a lower system cost. Low cost electronic beam scanning antennas represent good solutions for many applications such as remote sensing and imaging. In such application, a fast reconfigurability of the antenna pattern is needed to realize in real-time systems which require a simple biasing network. A reflectarray using single bit phase shifters is one of the best solutions because of the low system cost and simple biasing structure. Additionally, single bit phase quantization could reduce the insertion loss of the reflectarray elements which will be discussed in the next section.

### 2.2.2 Sub-wavelength Element Spacing

Microstrip reflectarrays are quickly becoming alternatives to parabolic reflectors due to numerous advantages such as their low profile, light weight and ease of transportation [19]. However, current reconfigurable reflectarray designs depend on expensive active components and exhibit loss, complexity or both. The goal of this thesis is to design a low-cost, low-loss, simplified system structure of reconfigurable reflectarray with zero-power consumption. Simplified biasing network and low cost have been achieved from using single bit phase shifters. It is also significantly important to obtain a relative high gain reconfigurable reflectarray and meanwhile retain the low cost characteristics of the architecture.

Radiation efficiency is mainly affected by the conductor loss and dielectric loss of the reflectarray. For the reflectarray loaded with tuning devices such as varactor diodes, the radiation efficiency highly depends on the ohmic loss from series resistor in the varactor diodes. This ohmic loss can be quite dramatic at frequencies where the element is resonant, absorbing 80% or more of the incident power [20]. Obviously, such high losses are unacceptable for a reflectarray when it is designed as an alternative to a reflector. Therefore, ohmic losses have to be minimized.

It is worthwhile to point out at this point that varactor diodes have been first employed as single bit phase shifters instead of PIN diodes in this design. PIN diodes consume power in their “ON” state and varactor diodes are zero power consuming in both states. No power consumption is the advantage of the varactor diodes over PIN diodes. From Table 2.2, the GaAs based varactor diodes could only introduce a little ohmic loss (small series resistor) and achieve high gain, but they are quite expensive! The alternative low-cost (typically large series resistor) Si based varactor diodes could introduce significant ohmic loss and lead to significant power losses that can dramatically reduce the gain of the reflectarray. Therefore, a compromise between the cost of the tuning device and the ohmic loss have to emerge. However, the proposed reflectarray will employ with low cost tuning device and achieve a low ohmic loss as well.

<table>
<thead>
<tr>
<th>Series Resistor (Ω)</th>
<th>Si</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>Cheap</td>
<td>Expensive</td>
</tr>
</tbody>
</table>

Table 2.2: Series resistor and price in different types of varactor diodes

In the literature, there are various approaches to improve the radiation efficiency on reconfigurable reflectarrays. We draw inspiration from these techniques and apply them to reconfigurable reflectarrays. One proposed method is to increase substrate thickness to reduce the dielectric loss. While increasing
substrate thickness does indeed reduce the loss, thicker substrates lead to low phase range which can cause phase error problems and higher material cost. Another approach is employing sub-wavelength element spacing which has been applied to reduce the conductor losses caused by transparent antenna \cite{21}. Sub-wavelength elements spacing have also been applied to reduce dielectric losses in reflectarrays and not incur significant gain drop due to the use of the lossy, low-cost substrate \cite{22}. Our proposed approach is also to introduce the sub-wavelength element spacing to reduce the diodes losses in reconfigurable reflectarrays. Consequently, sub-wavelength element spacing can reduce the ohmic loss from the high resistance and cheap Si based diodes (5 Ω) to achieve approximate the same loss level as the one introduced by low resistance and expensive GaAs based varactor diodes (0.7 Ω) with half wavelength element spacing. Employing the low cost tuning device in each reflectarray element could further reduce the cost of the system in addition to using the single bit phase shifter, and sub-wavelength element spacing will offset the extra loss introduced by the high resistance devices. The diodes loss has been first investigated in this thesis and others have focused on other sources of loss such as conductor loss and dielectric loss.

Researchers in the reflectarray research community have noted that reflectarray bandwidth is improved when using sub-wavelength patches as opposed to resonant patches (λ/2). A simple and effective method for broadband design has been introduced by using sub-wavelength elements instead of the conventional λ/2 elements \cite{23} \cite{24} \cite{25}. It has been demonstrated that the reflectarrays designed with sub-wavelength elements can achieve a significant improvement in gain bandwidth. Narrow bandwidth is particularly significant for single-layer microstrip reflectarrays as the phase curve changes rapidly around the resonant frequency. Sub-wavelength element spacing can obtain a more linear behavior of the phase response, therefore significantly enhancing the bandwidth.

In summary, sub-wavelength element spacing has the ability not only to reduce the diodes (ohmic) loss of reflectarrays and thereby achieve low gain reduction or low cost by employing cheap and lossy tuning device, but also to improve reflectarrays bandwidth. Therefore, the goal of designing a low loss or even low cost reflectarray can be achieve using sub-wavelength element spacing.

Reduction of Phase Quantization Errors

Sub-wavelength element spacing could potentially improve the directivity reduction of reflectarrays caused by the single bit phase distribution, because the sub-wavelength element spacing has a higher resolution in space over the half wavelength case, which may introduce less phase quantization errors. The study of the effect of sub-wavelength element spacing on directivity reduction of reflectarray is presented here. The directivity of reflectarrays is calculated from the array factor calculation presented in Section 2.1.2. A test case of 20 × 20 half wavelength spacing element has been analyzed by array theory with broadside incident (spherical wave) and designed to produce a beam at θ_b = 20° and φ_b = 0°. The total size of this reflectarray is 10λ × 10λ. An alternative reflectarray test case is with sub-wavelength elements spacing instead of half wavelength. 60 × 60 reflectarray element with an element spacing of approximate λ/6 has also been analyzed by array theory. The total size of this sub-wavelength reflectarray is also 10λ × 10λ which is the same as the 20 × 20 half wavelength reflectarray.

As shown in Figure 2.12 the loss in directivity of single bit phase quantization due to phase quantization errors with half wavelength element spacing is much larger than the one with sub-wavelength. Figure 2.13 shows the phase distribution on the reflectarray elements of 1 bit phase quantization with element spacing λ/2 and λ/6, respectively. It clearly shows that phase distribution with sub-wavelength (λ/6)
element spacing has higher resolution in space than the half wavelength one, and the high spacing resolution introduces less phase quantization errors over the reflectarray aperture. Therefore, sub-wavelength element spacing introduces less phase quantization errors compared to the half wavelength element spacing, and thereby improves the directivity of reflectarrays. The directivity of half wavelength reflectarray with only 1 bit phase shifters is 27.08 dB, and directivity reduction due to quantization losses is 3.99 dB. The directivity of sub-wavelength reflectarray with only 1 bit phase shifters is 28.49 dB and 1 bit phase resolution (180° phase steps) with sub-wavelength spacing results in a quantization loss of 2.58 dB which is less than the half wavelength case (3.99 dB). Table 2.3 summarizes the directivity of approximate 10λ × 10λ reflectarray with half wavelength (λ/2) and sub-wavelength (λ/6) element spacing in both 1 bit phase distribution and continuous phase distribution. As a result, the loss in directivity due to phase quantization errors has been dropped around 1.4 dB with approximate the same side lobe level.
Reducing Insertion Loss

Reconfigurable reflectarrays require their elements whose scattered field phase can be adjusted over a broad range (ideally $360^\circ$) and one way to achieve different phase states of the scattered field is employing varactor diodes. The insertion loss is caused by the series resistor in the varactor diodes and most of the insertion loss occurs around the resonant frequency. One potential solution to minimize this loss is to decrease the element spacing, and this approach to reduce the loss is to change the coupling coefficient with the resonator. For elements with a periodicity of $\lambda/2$, the fields are mostly concentrated in the substrate, directly below the element. However, for subwavelength elements, the fields are less concentrated below the element and are also present in between the elements in both the substrate and in the air. This reduces the reflection loss of the element near its resonance point [21]. The subwavelength elements exhibit a coupled resonance behavior, with fields being concentrated between the elements. It has also been observed that the currents on the conductors are also reduced in magnitude. Since the fields and currents are no longer largely concentrated in the lossy regions of the unit cell, the overall reflection loss is reduced [21]. Additionally, if the sub-wavelength element spacing is $\lambda/6$, an $(\lambda/2)^2$ area contains only one $\lambda/2$ short dipole and contains 9 $\lambda/6$ short dipoles as shown in Figure 2.14. When both the $\lambda/2$ and $\lambda/6$ short dipoles are loaded with the same resistors, the 9 $\lambda/6$ short dipoles could be interpreted to be paralleled with each other which means the smaller loaded resistors compared to the one $\lambda/2$ short dipole. Consequently, the resistance per unit area with $\lambda/6$ short dipoles is much smaller than the one with $\lambda/2$ short dipole and the smaller element spacing is, the less loaded resistor is. Therefore, sub-wavelength element spacing should be able to reduce the insertion loss caused by the embedded resistors. To prove this idea, two short dipoles of different sizes loaded with same resistors (5 $\Omega$) in an infinite environment have been analyzed by the Method of Moments (MoM). As shown in Figure 2.14, the size of one of dipoles is 19 mm $\times$ 2 mm and this dipole has been analyzed in an infinite environment.
environment with the periodicity of 30 mm × 30 mm (λ/2). The other dipole shrinks to 8.33 mm × 0.67 mm and has been simulated with unit cell size of 10 mm × 10 mm (λ/6). 9 (3 × 3) short dipoles with unit cell of λ/6 are shown in Figure 2.14 and then the total size of the short dipoles are the same as the λ/2 short dipole. An dipole loaded with 0.7 Ω resistor with half wavelength periodicity has also analyzed in MoM in order to compare against the reflection loss from the cheap lossy (5 Ω) varactor diodes with λ/6 periodicity. Table 2.4 summarizes the minimum magnitude of reflection coefficient, phase range of the reflectarray element and the average single bit reflection loss for all the cases shown in Figure 2.15 and Figure 2.16.

Figure 2.15: Magnitude and phase response of dipoles with λ/6 element spacing

Figure 2.16: Magnitude and phase response of dipoles with λ/2 element spacing

Varactor diodes are placed in the middle of each dipole and the different phase states can be achieved from varying the capacitance of varactor diodes. Figure 2.16 shows that the magnitude and phase responses of the scattered field as a function of capacitance with sub-wavelength (λ/6) element spacing and the minimum magnitude of reflection coefficient is -2.64 dB. Figure 2.16 shows the magnitude and
Table 2.4: Minimum reflection coefficient and phase range of the element

<table>
<thead>
<tr>
<th>Element spacing and type</th>
<th>Min Reflection Coefficient [dB]</th>
<th>Phase Range</th>
<th>Average 1 bit loss [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>λ/6 cheap lossy diodes</td>
<td>-2.64</td>
<td>256.14°</td>
<td>1.32</td>
</tr>
<tr>
<td>λ/2 cheap lossy diodes</td>
<td>-15.02</td>
<td>337.9°</td>
<td>5.44</td>
</tr>
<tr>
<td>λ/2 expensive low loss</td>
<td>-1.80</td>
<td>337.9°</td>
<td>0.87</td>
</tr>
<tr>
<td>diodes</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Phase responses of the scattered field as a function of capacitance with half wavelength (λ/2) element spacing and minimum magnitude of reflection coefficient is -15.02 dB. The loss comes from the series resistor (5 Ω) in the varactor diodes. Employing the single bit phase shifter, the average reflection loss of the two states in λ/6 and λ/2 element spacing are 1.32 dB and 5.44 dB, respectively. Therefore, the insertion loss of the reconfigurable reflectarray can be significantly reduced if the element spacing shrinks to sub-wavelength scales. The main drawback of sub-wavelength element spacing is that phase range is reduced and we may not obtain enough phase range for reconfigurable reflectarray. As shown in Figure 2.15 and 2.16, the phase range of the sub-wavelength dipoles is about 260° and the phase range of half wavelength dipoles is about 340°. Fortunately, the designed reconfigurable reflectarray is using 1 bit phase shifters which only requires 180° phase range. Also, sub-wavelength element spacing leads to more reflectarray cells which may potentially increase the system cost, but the cost of employing cheap tuning device will offset the need for more of them. It is worthwhile to point out that the insertion loss from cheap lossy (Si) varactor diodes (0.87 dB) is compatible with the one from expensive low loss (GaAs) varactor diodes (1.32 dB).

The insertion loss is further reduced in the case of employing single bit phase shifters. As mentioned earlier, single bit phase shifters only have two phase states which are 180° apart from each other. In Figure 2.15 and 2.16, two phase states can be carefully selected to omit the dip area and minimize the insertion loss. However, in the continuous phase shifters case, the required phase states are much more than the single bit phase shifters which means the insertion loss is significant in some phase states (in the dip zone). Therefore, the insertion loss could be further reduced by implemented single bit phase shifters. As mentioned before, single bit phase quantization introduces directivity reduction about 4 dB, however, this directivity reduction could be potentially improved by employing the sub-wavelength element spacing.

In summary, combining the benefits of the sub-wavelength element spacing such as low insertion loss and the reduced phase quantization loss and the benefits of the single bit phase shifter such as even lower insertion loss and simple biasing network structure, a simple, low loss and low cost reconfigurable reflectarray can be achieved.

### 2.2.3 Delta-Sigma Quantization

As mentioned before, the use of single bit phase shifter in beam scanning plane wave re-director causes pronounced phase quantization lobes as a function of space. An analogy with analog-to-digital converter (ADC) reveals the same thing happens with 1-bit ADCs as a function of time. Reducing the sampling time and using delta sigma can be used to reshape the quantization noise. We aim to apply the same idea here in the spatial domain through sub-wavelength element spacing and delta sigma to reduce the
phase quantization errors.

**Basic Concepts**

The delta sigma modulator has been widely applied in ADCs to provide the feedback from the output to the input, as a result the quantization errors are reduced. First, conventional ADCs will be introduced and a basic block diagram is shown in Figure 2.17 [26].

The input signal $x_i$ is mapped to one of a number of discretized values $y_i$ and this introduces an additive error $e_i$. The quantization error introduced at the output is $e_i = y_i - x_i$. One of potential solutions to reduce the quantization error $e_i$ is using delta sigma modulator. Figure 2.18 [26] describes a basic block diagram of ADC with delta sigma modulator.

The input signal $x_i$ passes first through an integrator, shown in the block diagram as the delay loop. The output of the integrator $w_i$ then feeds to the quantizer, and the quantized output $y_i$ is fed back and subtracted from the input signal. Whereas the output of the conventional ADC constantly tracks the input signal, this feedback forces the average value of the quantized output to track the average input. As a result, the error builds up in the integrator and in time will correct itself. The input and output relationship in delta sigma modulator is shown as follows. The integrator output is given by

$$w_i = x_{i-1} - y_{i-1} + w_{i-1}$$  \hspace{1cm} (2.40a)  

$$= x_{i-1} - y_{i-1} - e_{i-1} + w_{i-1}$$  \hspace{1cm} (2.40b)  

$$= x_{i-1} - e_{i-1}$$  \hspace{1cm} (2.40c)
The output signal is

\[
y_i = w_{i-1} + e_i \quad (2.41a)
\]

\[
y_{i} = x_{i-1} - e_{i-1} + e_i \quad (2.41b)
\]

Then the introduced quantization phase error with delta sigma modulator is \(e_i - e_{i-1}\) which is smaller than the conventional one \(e_i\). If we are assuming that the same white noise model \(e\) has been employed in both the conventional and delta sigma ADC, the signal-to-noise ratio of the output signal has been improved 3 dB in delta sigma modulator case compared to the conventional one \([26] [27]\).

**Linear Array**

Delta sigma modulation has been applied to linear array phase quantization \([26]\). Quantization errors are introduced due to mapping the continuous phase values to actual discretized phase values. Such errors could lead to a directivity reduction, increase side lobe level, and shift the main beam direction. Conventional phase quantization in linear arrays is same as phase quantization in ADCs. The N bit phase quantization adds quantization error \(q_n\) to the desired excitation phase \(w_n\) which results a phase quantized output, \(\hat{w}_n\),

\[
\hat{w}_n = w_n + q_n . \quad (2.42)
\]

After implementing delta sigma modulation, the input-output relation between the desired phase and delta sigma quantized phase is

\[
\hat{\tilde{w}}_n = w_n - q_n + q_{n-1}. \quad (2.43)
\]

\(\hat{\tilde{w}}_n\) is the output after delta sigma modulator, \(w_n\) is the desired phase without any quantization error, \(q_n\) is the phase quantization error generated in current stage and \(q_{n-1}\) is the phase quantization error generated in previous stage. Comparing Equations (2.42) and (2.43), the effect of the delta sigma cancellations would continue to eliminate the quantization noise due to the rest of the array to the point that the noise caused by this single element becomes the dominant source of quantization noise. Delta sigma quantized phased array could reduce the quantization errors compared to the conventional quantized phased arrays. Additionally, with the delta sigma modulator, the noise shaping could further reduce the noise when the array density increases, and much in the same way as increasing the temporal sample rate in 1-bit delta-sigma ADCs also reduces the quantization noise \([26]\).

Delta sigma modulator could reduce phase quantization errors, but how does it affect on directivity, side lobe levels, and the main beam direction? Now, let’s consider a linear array in Figure 2.1 of length \(L = 20\lambda\) with uniform element spacing in a 2 bit phase shifter case. Phase shifters with 2 bit of resolution can provide \(2^2\) values uniformly distributed over the range \([0, 2\pi]\). Therefore, a 2 bit phase shifter could provide four different phase states which could be 0°, 90°, 180° and 270°. The relation between the delta sigma phase quantization array and 2 bit phase quantization only array in three different element spacing cases \((\lambda/2, \lambda/4\) and \(\lambda/8\)) can be seen in Figure 2.19. In each plot, the normalized directivity pattern are shown for a scanning direction at an angle of \(\theta = 70^\circ\). In \(\lambda/2\) element spacing case, delta sigma quantization gives a slightly better normalized directivity pattern. In both \(\lambda/4\) and \(\lambda/8\) element spacing cases, delta sigma provides a significantly better directivity pattern with lower quantization.
lobes level compared to the one without it. Consequently, the more dense the linear array (the smaller element spacing) is, the better directivity pattern is. Therefore, in order to benefit from delta sigma quantization, the element spacing in the linear array should be sub-wavelength.

Now, we return our attention to the problem of phase quantization in reflectarrays. As mentioned before, enabling single bit phase shifter is able to dynamic control of electromagnetic waves in reconfigurable reflectarray antenna. But, single bit phase shifters only have two phase states (e.g. $0^\circ$ and $180^\circ$), and thereby introduce greater phase quantization errors. We propose to apply delta sigma modulation to reflectarrays for plane-wave redirection, which may reduce phase quantization errors and thereby the quantization lobes. Also, we try to apply delta sigma modulation to improve the collimated single bit reflectarray behaviour. The more details will be discussed in Chapter 4.
Chapter 2. Background

Figure 2.19: Normalized directivity patterns for linear arrays of length $L = 20 \lambda$

(a) $\lambda/2$ element spacing

(b) $\lambda/4$ element spacing

(c) $\lambda/8$ element spacing
Chapter 3

Reflectarray Antenna Design Methods

The need for low-cost, reconfigurable antenna beam-forming is widespread in many existing and next-generation wireless and sensing systems such as satellite communications, remote sensing and so on. Reflectarrays are capable of achieving beam scanning within a low-profile form factor. The starting point for any reflectarray design is the conception and subsequent analysis of the reflectarray unit cell. In virtually all designs, this process employs full-wave electromagnetic solvers with periodic boundary conditions around a unit cell, invoking the infinite-array (local periodic) analysis of a constitutive scatterer of the reflectarray. The output of this process is the generalized scattering matrix (GSM) of the reflectarray unit cell. Nowadays most commercial solvers based on the method of moments (MoM), the finite element methods (FEM), or other techniques, support analysis of periodic structures in order to derive the GSM.

While numerical approaches for analyzing reflectarray unit cells are fast and generally easy to carry out, a more fundamental design tool would be a fully analytical model of reflectarray unit cells. Such models can provide greater insight into the operation of reflectarray unit cells, and could be evaluated much faster than their full-wave counterparts. Furthermore, even if full-wave solvers are ultimately employed, analytical models can be used as a useful starting point, generating a “first cut” of the reflectarray unit cell design before more detailed analysis is undertaken.

The first section of this chapter will present full-wave simulation tool based on MoM to derive the GSM with lumped port extension using Rao-Wilton-Glisson (RWG) basis functions. The second section will then present an equivalent circuit model (ECM) tied to the expansion of the Floquet modes, which allows the scattering behaviour of various linearly-polarized (LP) reflectarray unit cells to be quickly and accurately predicted. This equivalent circuit allows the all-important phase curve of the reflectarray unit cell to be predicted using closed-form formulas. The final section will show how to utilize the ECM to analyze and characterize various reflectarray elements.
3.1 Method of Moments

3.1.1 Motivation

The first step to design a reflectarray is to analyze its unit cell. There are many numerical methods to conduct this process such as MoM, FEM, and so on. The finite element method is accurate, but requires volumetric meshing which can increase simulation time and resources. So, we can think of the memory required by such technique to be of order \(O(n^3)\), where \(n\) is the number of unknowns. The computational times can also be very high. Surface-based MoM methods are also accurate and require only surface meshing, as only conductors need be meshed. Layered dielectrics can be modelled through the Green’s function (discussed later). So, the complexity of MoM is generally order \(O(n^2)\). This makes the technique faster than volumetric based FEM. An efficient technique based on a spectral domain MoM directly computes the GSM of planar multi-layer structures realized from arbitrarily-shaped periodic metallizations has been developed [4].

A basic reflectarray collimates waves from a feeding antenna into a pencil beam by applying a phase correction to the scattered field at each element on the reflectarray surface. As discussed in Chapter 2, the phase of each fixed reflectarray element can be varied using phase delay lines or variable-size elements. Furthermore, a reconfigurable reflectarray antenna can be achieved through electronic tuning devices such as varactor diodes (lumped elements), however, lumped elements haven’t been implemented [4]. In this thesis, a method for extending the GSM to facilitate the inclusion of lumped ports in the model to reduce the simulation time is developed.

3.1.2 Generalized Scattering Matrix

Here we present an efficient technique for the computation of GSM of arbitrarily-shaped periodic metallization using RWG basis functions at the interface between two dielectric media. The general technique based on a spectral domain MoM is discussed first, and follow by the implementation to the reflectarray. This technique assumes the incident wave is a summation of space-harmonics. The geometry considered here is shown in Figure 3.1.

Here, we assume that a rectangular periodicity \(a \times b\) metallization resides on the interface between two dielectrics with permittivities \(\epsilon_0 \epsilon_{r1}\) and \(\epsilon_0 \epsilon_{r2}\) respectively, and permeability \(\mu_0\) for both. The metallizations can be resistive and the dielectrics lossy. These effects are represented by a surface impedance \(Z_s\) for the metallizations and by complex relative permittivities \(\epsilon_{r1}\) and \(\epsilon_{r2}\) for the dielectrics.

The incident field can be assumed to be a summation of infinite space harmonics,

\[
\vec{E}^i(x, y) = \sum_{l=1}^{L} \left[ d_l \vec{e}_{l,h}(k_{x1}, k_{y1}) + d_{L+l} \vec{e}_{l,e}(k_{x1}, k_{y1}) \right] e^{jk_{x1}x+jk_{y1}y},
\] (3.1)
where

\[ k_{xl} = \sqrt{\varepsilon_r} k_0 \sin \theta \cos \phi + 2m\pi/a, \]  \hspace{1cm} (3.2a)

\[ k_{yl} = \sqrt{\varepsilon_r} k_0 \sin \theta \sin \phi + 2n\pi/b, \]  \hspace{1cm} (3.2b)

\[ \vec{e}_{l,h}(k_{xl}, k_{yl}) = \frac{1}{\sqrt{k_{xl}^2 + k_{yl}^2}} (-k_{yl}a_x + k_{xl}a_y), \]  \hspace{1cm} (3.2c)

\[ \vec{e}_{l,e}(k_{xl}, k_{yl}) = \frac{1}{\sqrt{k_{xl}^2 + k_{yl}^2}} (k_{xl}a_x + k_{yl}a_y). \]  \hspace{1cm} (3.2d)

\[(\theta, \phi)\) describe the angle of incidence in spherical coordinates, and \(k_0\) is free space wavenumber. The index \(l\) represents \(l^{th}\) mode Floquet harmonic. The TE modes are indexed directly using \(l\) and the TM are indexed using index \(L + l\). \(k_{xl}\) and \(k_{yl}\) are defined as Floquet mode wavenumbers. \(\vec{e}_{l,h}\) and \(\vec{e}_{l,e}\) are the expressions of the normalized modal fields for the TE and TM Floquet harmonics, respectively. For normal incidence, \(\theta = \phi = 0\), the inconsistency of Equation (3.2) is solved by setting \(k_{x0} = 1\) and \(k_{y0} = 0\), which means that the TE wave is polarized with the electric field in the \(y\)-direction. We expand the excitation field, which is the sum of reflected and incident field, as the sum of the Floquet harmonics

\[ \vec{E}_x = \sum_{l=1}^{L} \left[ d_l \left( 1 + \Gamma_{L}^h \right) \vec{e}_{l,h}(k_{xl}, k_{yl}) + d_{L+l} \left( 1 + \Gamma_{L}^e \right) \vec{e}_{l,e}(k_{xl}, k_{yl}) \right] e^{jk_{xl}x + jk_{yl}y}, \]  \hspace{1cm} (3.3)

where \(\Gamma_{L}^e\) and \(\Gamma_{L}^h\) are the reflection coefficients from the two media interface. The excitation field induces currents on the surface of the metallization. Based on the boundary conditions on the conductive surface, the excitation electric field is related to the scattered electric field using

\[ \vec{E}_s(x, y) + \vec{E}_e(x, y) = Z_s \vec{J}(x, y), \]  \hspace{1cm} (3.4)

where \(\vec{J}(x, y)\) is the induced current and \(Z_s\) is the surface impedance of the metallization.
field from the metallization can be related to the induced current \( \vec{J} \) through the dyadic Green’s function \( \vec{G} \) according to,

\[
\vec{E}^s(x, y) = -\sum_{l=1}^{L} \vec{G}(k_{xl}, k_{yl}) \cdot \vec{J}(k_{xl}, k_{yl}) e^{jk_{xl}x + jk_{yl}y},
\]

where \( \vec{J}(k_{xl}, k_{yl}) \) and \( \vec{G}(k_{xl}, k_{yl}) \) are both in spectral domain. \( \vec{G}(k_{xl}, k_{yl}) \) is the associated semi-infinite dyadic Green’s function and the expression for the reflectarray is discussed later. The quantity \( \vec{J} \) represents the largest Floquet mode we consider (typically \( \vec{J} = 3600 \) is considered). The induced current distribution can be expanded as a summation of RWG basis functions as shown in Figure 3.2:

\[
\vec{J}(x, y) = \sum_{p=1}^{B} c_p \vec{\psi}_p(x, y),
\]

where \( c_p \) are the complex coefficients of the induced current. The \( p^{th} \) RWG basis function is defined as

\[
\vec{\psi}_p(r) = \begin{cases} 
\frac{l_n}{2A_n^+} \vec{\rho}_n^+ & \forall r \in T_n^+ \\
\frac{l_n}{2A_n^-} \vec{\rho}_n^- & \forall r \in T_n^- \\
0 & \text{otherwise}
\end{cases},
\]

where \( l_n \) is the length of the edge, and \( A_n^\pm \) is the area of triangle \( T_n^\pm \), and the position vector \( \vec{\rho}_n^\pm \) defined with respect to the free vertex of \( T_n^\pm \). After substituting Equations (3.1), (3.3), (3.5) and (3.6) into Equation (3.4), we obtain

\[
\begin{aligned}
&\sum_{l=1}^{L} \left[ d_l(1 + \Gamma_L^h) \vec{e}_{l,h}(k_{xl}, k_{yl}) + d_{L+l}(1 + \Gamma_L^e) \vec{e}_{l,e}(k_{xl}, k_{yl}) \right] e^{jk_{xl}x + jk_{yl}y} \\
&= \sum_{p=1}^{B} c_p \sum_{l=1}^{L} \vec{G}(k_{xl}, k_{yl}) \vec{\psi}_p(k_{xl}, k_{yl}) e^{jk_{xl}x + jk_{yl}y},
\end{aligned}
\]
on the equation above. The complex coefficients of the current, written as column matrix, $C$, can be derived as

$$ C = Z^{-1}F. $$

(3.9)

$C = \begin{bmatrix} c_1 & c_2 & \ldots & c_B \end{bmatrix}^T$ contains the coefficients of induced currents in Equation (3.6), and $F = \begin{bmatrix} F_1 & F_2 & \ldots & F_B \end{bmatrix}^T$ contains the incident fields with

$$ F_r = \sum_{l=1}^{L} \left[ d_l(1 + \Gamma^h_L)\vec{e}_{l,h}(k_{xl}, k_{yl}) + d_{L+1}(1 + \Gamma^e_L)\vec{e}_{l,e}(k_{xl}, k_{yl}) \right] \cdot \vec{\psi}_r^*(k_{xl}, k_{yl}). $$

(3.10)

Then, the column vector $F$ can be written as

$$ F = V(I + R)E^i, $$

(3.11)

$$ \begin{bmatrix} F_l \\ F_{L+1} \end{bmatrix} = \begin{bmatrix} V_h \\ V_e \end{bmatrix} (I + R)E^i, $$

(3.12)

where $R$ is a diagonal matrix with $\Gamma^h_L$ as the non-zero elements and its size is the number of the modes considered in the calculation, $I$ is the identity matrix with the same size as $R$, $E^i$ is a column matrix with the size of number of space harmonics representing the amplitudes of the incident space harmonics, and the element of $V_h$ and $V_e$ are given by

$$ v_h = \vec{e}_{l}^h \cdot \vec{\psi}_r^*(k_{xl}, k_{yl}), $$

(3.13a)

$$ v_e = \vec{e}_{l}^e \cdot \vec{\psi}_r^*(k_{xl}, k_{yl}). $$

(3.13b)

$Z$ is the impedance matrix with element $(r, p)$ and element given by

$$ Z_{r,p} = \sum_{q=1}^{B} \sum_{l=1}^{L} \vec{\psi}_r^*(k_{xl}, k_{yl}) \cdot \vec{G}_{l,q}(k_{xl}, k_{yl}) \cdot \vec{\psi}_p(k_{xl}, k_{yl}). $$

(3.14)

The scattered field can also expressed as a summation of space harmonics,

$$ \vec{E}^s(x, y) = \sum_{l=1}^{L} [a_l\vec{e}_{l}^h(k_{xl}, k_{yl}) + a_{L+1}\vec{e}_{l}^e(k_{xl}, k_{yl})] e^{jk_{xl}x+jk_{yl}y}. $$

(3.15)

Applying the orthogonality properties of the space harmonics of Equations (3.5) and (3.15), we obtain coefficients of the Floquet series describing the scattered field,

$$ a_l = \vec{e}_{l}^h \cdot \vec{G}(k_{xl}, k_{yl}) \cdot \vec{J}(k_{xl}, k_{yl}), $$

(3.16)

$$ a_{L+1} = \vec{e}_{l}^e \cdot \vec{G}(k_{xl}, k_{yl}) \cdot \vec{J}(k_{xl}, k_{yl}), $$

(3.17)
The above relations can be expressed as

\[
\mathbf{E}^s = \mathbf{W} \mathbf{C},
\]

\[
\begin{bmatrix}
\mathbf{E}_{s1}^s \\
\mathbf{E}_{sL+l}^s
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{W}_h \\
\mathbf{W}_e
\end{bmatrix} \mathbf{C},
\]

(3.18)

where \( \mathbf{E}^s \) is a column matrix representing the amplitudes of the scattered space harmonics. The elements of \( \mathbf{W}_h \) and \( \mathbf{W}_e \) are given by

\[
w_h = \bar{e}^h_l \cdot \bar{G}(k_{xl}, k_{yl}) \cdot \psi_p(k_{xl}, k_{yl}),
\]

(3.19a)

\[
w_e = \bar{e}^e_l \cdot \bar{G}(k_{xl}, k_{yl}) \cdot \psi_p(k_{xl}, k_{yl}).
\]

(3.19b)

Based on Equations (3.9) (3.12) and (3.18), the scattered field can be found as

\[
\mathbf{E}^s = \mathbf{WZ}^{-1} \mathbf{V}(\mathbf{I} + \mathbf{R}) \mathbf{E}^i,
\]

(3.20)

\[
= \mathbf{S}_0(\mathbf{I} + \mathbf{R}) \mathbf{E}^i.
\]

(3.21)

The GSM of the surface with metallizations at the interface of two dielectric media is expressed as

\[
\begin{bmatrix}
\mathbf{E}_1^s \\
\mathbf{E}_2^s
\end{bmatrix} = 
\begin{bmatrix}
\mathbf{S}_{11} & \mathbf{S}_{12} \\
\mathbf{S}_{21} & \mathbf{S}_{22}
\end{bmatrix} 
\begin{bmatrix}
\mathbf{E}_1^i \\
\mathbf{E}_2^i
\end{bmatrix},
\]

(3.22)

where \( \mathbf{E}_1^i \) and \( \mathbf{E}_2^i \) represent the incident fields from medium 1 and medium 2, respectively. \( \mathbf{E}_1^s \) and \( \mathbf{E}_2^s \) represent the scattered fields from medium 1 and medium 2, respectively. The total scattered fields should also include the field reflected by the dielectric interface, which is already considered in the excitation field. Therefore, the submatrix \( \mathbf{S}_{11} \) can be obtained from

\[
\mathbf{S}_{11} = \mathbf{S}_0(\mathbf{I} + \mathbf{R}) + \mathbf{R}.
\]

(3.23)

To calculate the submatrix \( \mathbf{S}_{22} \), we assume the incident fields are from the medium with dielectric constant \( \epsilon_{r2} \). Since \( \mathbf{R} \) is antisymmetric at the dielectric interface, \( \mathbf{S}_{22} \) can be written as

\[
\mathbf{S}_{22} = \mathbf{S}_0(\mathbf{I} - \mathbf{R}) - \mathbf{R}.
\]

(3.24)

Based on the boundary conditions for the transverse electric field in the periodic cell, \( \mathbf{S}_{21} \) and \( \mathbf{S}_{12} \) can be calculated from

\[
\mathbf{S}_{21} = \mathbf{S}_{11} + \mathbf{I},
\]

(3.25)

\[
\mathbf{S}_{12} = \mathbf{S}_{22} + \mathbf{I}.
\]

(3.26)

In the reflectarray case, the metallization resides on a grounded dielectric substrate instead of semi-infinite dielectric substrate. Also, in order to design a reflectarray, we need to analyze the behaviour of the amplitude and phase of the reflection coefficient. In other words, we are interested in the \( \mathbf{S}_{11} \) term in the GSM. The dyadic Green’s function implemented in the scattered field computation should
account for the finite dielectric substrate and ground plane. The dyadic Green function of microstrip patches has been found previously [28] and is presented in Appendix A. The reflection coefficient from the interface at two media can be calculated by considering the finite substrate and ground plane as a transmission line.

![Figure 3.3: Transmission line model of patch with grounded substrate](image)

As shown in Figure 3.3, the transmission line model of metallization with finite grounded substrate can be simplified to a transmission line with load impedance \(Z_{\text{Load}}\) and characteristic impedance equal to modal impedance. The input impedance at the air-dielectric interface for the \(l^{th}\) TE or TM mode can be calculated from

\[
Z_{\text{Load}}^{TE} = Z_{\text{Load}}^{TM} = \frac{Z_{TE} - |Z_{TM}| - \tanh(jk_z h)}{1 + Z_{TE} - |Z_{TM}| + Z_{Load}^{TE} - |Z_{Load}^{TM}|},
\]

(3.27)

where \(Z_{TE} - |Z_{TM}|\) is the modal impedance for TE and TM mode in medium \(z < 0\) respectively, \(k_z\) is the wavenumber in the \(z\)-direction, \(h\) is the thickness of the substrate. These parameters can be calculated from

\[
Z_{TE} = \frac{\omega \mu_0 k_z}{kJ},
\]

(3.28a)

\[
Z_{TM} = \frac{\omega \varepsilon_0 \varepsilon_r k_z}{kJ},
\]

(3.28b)

\[
k_z = \sqrt{\varepsilon_r k_z^2 - k_x^2 - k_y^2}.
\]

(3.28c)

The reflection coefficient for TE and TM at the dielectric interface is

\[
\Gamma^{TE|TM} = \frac{Z_{Load}^{TE} - Z_{Load}^{TM} + Z_{Load}^{TE} - |Z_{Load}^{TM}|}{Z_{Load}^{TE} + |Z_{Load}^{TM}| + Z_{Load}^{TM} + |Z_{Load}^{TE}|},
\]

(3.29)

where \(Z_{Load}^{TM} + |Z_{Load}^{TE}|\) is the modal impedance for TE and TM mode in medium \(z > 0\) respectively. Since the medium above the metallization is air, \(Z_{Load}^{TM} + |Z_{Load}^{TE}| = 120\pi\). \(S_{11}\) includes both the co-polarization and cross-polarization reflection coefficient to analyze the scattered behaviour of the reflectarray unit cell.

### 3.1.3 Extended GSM with Lumped Elements

The GSM described in Section 3.1.2 does not factor in lumped element loads such as capacitors, inductors and resistors that can be introduced in the scatterer to manipulate its characteristics (e.g. for recon-
There are two ways to implement lumped elements which are modifying the impedance matrix, and a post-processing approach, which is more efficient. In the first method, the proposed extension where the corresponding impedances of the lumped elements has been implemented into impedance matrix $Z$.

Lumped elements are introduced between two basis functions along their interior edges and the separation between the two edges is seen as infinitesimally small. For example, the physical model of load capacitor is shown in Figure 3.4a and Figure 3.4b indicates where the capacitor is loaded in RWG basis functions. Only the diagonal terms of impedance matrix need to be modified according to Equation (3.30)\cite{29}. Equation (3.30) shows how to modify the impedance matrix for capacitor, resistor, and inductor respectively,

\begin{align*}
Z_{nn} &= Z_{nn} + \frac{1}{j\omega C}, \quad (3.30a) \\
Z_{nn} &= Z_{nn} + (l_n^2) R, \quad (3.30b) \\
Z_{nn} &= Z_{nn} + (l_n^2) j\omega L. \quad (3.30c)
\end{align*}

The terms in Equation (3.30) are as follows: $n$ is the index of the edge, $l_n$ indicates the length of the $n^{th}$ edge, and $\omega$ is the angular frequency.

However, in this way, each lumped element requires impedance matrix recalculation and the new impedance matrix will result in a new current distribution to compute the GSM ($I = Z^{-1}V$). When the number of the lumped elements is large, re-inversion the impedance matrix is time consuming. Therefore, it is quite time consuming to analyze a large number of lumped elements such as capacitance sweep at various frequencies. We describe an alternative approach whereby lumped elements are replaced with lumped ports and manipulated to obtain the Floquet port GSM. In this approach, only one full-wave simulation is required for all the lumped elements.

### 3.1.4 Extended GSM with Lumped port

The GSM contains an arbitrary number of Floquet ports depending on the number of TE and TM modes considered. Only fundamental models are considered here, as usually the cell size is half a wavelength or less. More modes should be considered if the cell size exceeds half a wavelength to account for grating lobes. Therefore, let us assume that the extended GSM includes $P$ lumped ports and 2 Floquet
ports of fundamental modes \((TE_{00} \text{ and } TM_{00})\) as shown in Figure 3.5. The extended GSM of sizes \((P + 2) \times (P + 2)\) can be partitioned as

\[
S = \begin{bmatrix}
S_A & S_B \\
S_C & S_D
\end{bmatrix}.
\] (3.31)

Figure 3.5: two Floquet wave ports and P lumped port(s)

In the above partition, \(S_A\) is a \(2 \times 2\) matrix which captures the incident and scattered waves from Floquet ports. \(S_B\) is of size \(2 \times P\), and it relates the scattered Floquet modes when lumped ports are excited. \(S_C\) is of size \(P \times 2\), and it relates the outgoing wave from the lumped port when the Floquet ports are excited. Finally, \(S_D\) relates the incident and reflected waves from the lumped ports. After obtaining the scattering matrix of size \((P + 2) \times (P + 2)\), the next step is to obtain the two-port generalized scattering matrix linking the incident and scattered Floquet harmonics as shown in Figure 3.6.

Figure 3.6: P lumped port(s) and 2 Floquet wave ports to 2 Floquet wave ports

The first step is to compute the reflection coefficient depending on the load impedance at the lumped ports and after some straightforward algebraic manipulations, the Floquet wave ports GSM can be
calculated as

\[ S_{FL} = S_A + S_B \Gamma (1 - S_D \Gamma)^{-1} S_C, \]  

(3.32)

where

\[ \Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}. \]  

(3.33)

\( \Gamma \) is a diagonal matrix with reflection coefficient at the lumped ports as non-zero elements, \( Z_L \) is the load impedance at the corresponding lumped ports, and \( Z_0 \) is the corresponding characteristic impedance of the lumped ports (50 Ω is used here). Next, we consider an example with two lumped ports (\( P = 2 \)), then the size of the extended GSM with lumped port is \( 4 \times 4 \). In the following sections, how to calculate the elements in the extended GSM will be discussed.

**Calculation of GSM when Floquet Wave Ports are Excited**

In order to find the first column elements in the GSM, we should excite the first port (TE wave port) and terminate all the other ports with matched loads. Then the scattering parameters from all the ports are given by

\[ S_{1,1}^{\text{port1}} = \frac{b_1^{\text{port1}}}{a_1^{\text{port1}}} | a_2^{\text{port1}} = a_3^{\text{port1}} = a_4^{\text{port1}} = 0, \]  

(3.34a)

\[ S_{2,1}^{\text{port1}} = \frac{b_2^{\text{port1}}}{a_1^{\text{port1}}} | a_2^{\text{port1}} = a_3^{\text{port1}} = a_4^{\text{port1}} = 0, \]  

(3.34b)

\[ S_{3,1}^{\text{port1}} = \frac{b_3^{\text{port1}}}{a_1^{\text{port1}}} | a_2^{\text{port1}} = a_3^{\text{port1}} = a_4^{\text{port1}} = 0, \]  

(3.34c)

\[ S_{4,1}^{\text{port1}} = \frac{b_4^{\text{port1}}}{a_1^{\text{port1}}} | a_2^{\text{port1}} = a_3^{\text{port1}} = a_4^{\text{port1}} = 0, \]  

(3.34d)

where \( a_i \) indicates the incident wave into the ports and \( b_i \) indicates the reflected wave from the ports. Superscript port 1 indicates that the excited port is port 1. At port 1 (TE wave port), the incident wave \( a_1 \) and the reflected wave \( b_1 \) can be calculated using

\[ a_1^{\text{port1}} = V_1^i + Z_{\text{wave}} \times I_1^i \sqrt{Z_{\text{wave}}}, \]  

(3.35a)

\[ b_1^{\text{port1}} = V_1^r - Z_{\text{wave}} \times I_1^r \sqrt{Z_{\text{wave}}}, \]  

(3.35b)

where

\[ V_1^{ijr} = E_1^{ijr} \times a, \]  

(3.36a)

\[ I_1^{ijr} = \frac{V_1^{ijr}}{\sqrt{Z_{\text{wave}}}}. \]  

(3.36b)

\( a \) is the periodicity of the metallizations in the \( x \) direction as shown in Figure 3.1. \( E_1^i \) and \( E_1^r \) are the incident and reflected electrical fields from the TE wave port obtained from GSM. Since the waves are associated with the \( TE_{00} \) wave port, \( Z_{\text{wave}} \) is the \( TE_{00} \) wave impedance (120π). At port 2 (TM wave
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Here, $a_2 = 0$ because of a matched load $(120\pi)$ and the reflected wave $b_2$ can be calculated from

$$b_2^\text{port1} = \frac{V_2' - Z_{\text{wave}} \times I_2'}{\sqrt{Z_{\text{wave}}}},$$  \hspace{1cm} (3.37)

where

$$V_2' = E_2' \times b,$$ \hspace{1cm} (3.38a)

$$I_2' = \frac{V_2'}{Z_{\text{wave}}}. $$ \hspace{1cm} (3.38b)

$b$ is the periodicity of the metallizations in the $y$ direction as shown in Figure 3.1. $E_2'$ is the TM scattered fields from the metallization and can be also obtained from GSM. At port 3 and 4 (lumped ports), the reflected wave $a_3$ and $a_4$ are zero because these ports are terminated with matched loads (50 $\Omega$). The incident wave $b_3$ and $b_4$ can be calculated from

$$b_{3/4}^\text{port1} = \frac{V_{3/4}^\text{port1} - Z_{\text{wave}} \times I_{3/4}^\text{port1}}{\sqrt{Z_{\text{wave}}}},$$  \hspace{1cm} (3.39)

where

$$I_{3/4}^\text{port1} = C_{3/4}^\text{port1} \times l_{3/4}, $$ \hspace{1cm} (3.40a)

$$V_{3/4}^\text{port1} = -I_{3/4}^\text{port1} \times Z_0. $$ \hspace{1cm} (3.40b)

$C$ is a matrix of amplitudes of the current distribution which can be obtained from Equation (3.39). $l$ is the length of the lumped port edge in RWG basis function and $Z_0$ is the system characteristic impedance (50 $\Omega$). After obtaining $a_1, b_1, b_2, b_3$ and $b_4$, then $S_{1,1}, S_{2,1}, S_{3,1}$ and $S_{4,1}$ can be computed according to Equation (3.34). The elements $S_{i,2}$ in the extended GSM can be calculated in the same way.

Calculation of GSM when Lumped Ports are Excited

Each loaded lumped element can be considered as a lumped port. In order to calculate matrix elements for lumped port, we excite this lumped port (e.g. port 3) and terminate all the other ports with matched loads. The scattering parameters from all the ports are given by

$$S_{1,3}^{\text{port3}} = \frac{b_{3}^{\text{port3}}}{a_3} |_{a_1^{\text{port3}}=a_2^{\text{port3}}=a_4^{\text{port3}}=0},$$ \hspace{1cm} (3.41a)

$$S_{2,3}^{\text{port3}} = \frac{b_{3}^{\text{port3}}}{a_3} |_{a_1^{\text{port3}}=a_2^{\text{port3}}=a_4^{\text{port3}}=0},$$ \hspace{1cm} (3.41b)

$$S_{3,3}^{\text{port3}} = \frac{b_{3}^{\text{port3}}}{a_3} |_{a_1^{\text{port3}}=a_2^{\text{port3}}=a_4^{\text{port3}}=0},$$ \hspace{1cm} (3.41c)

$$S_{4,3}^{\text{port3}} = \frac{b_{3}^{\text{port3}}}{a_3} |_{a_1^{\text{port3}}=a_2^{\text{port3}}=a_4^{\text{port3}}=0}. $$ \hspace{1cm} (3.41d)

Superscript port 3 indicates that the excited port is port 3. The lumped port is excited by setting a voltage vector ($F^{\text{port3}}$) at the lumped port, and then the coefficient of the current at all the RWG edge
elements can be calculated from

\[ C_{\text{port}3} = Z^{-1} F_{\text{port}3}, \]
\[ A_{\text{port}3} = W \times C_{\text{port}3}. \]

\( F_{\text{port}3} \) denotes the \( F \) vector when it is only excited by port 3, \( C_{\text{port}3} \) indicates the coefficient of the current at all the edge elements when port 3 is excited, and \( A_{\text{port}3} \) contains the corresponding amplitude of scattered fields at all the edge elements. The incident wave \( a_3 \) and the reflected wave \( b_3 \) can be calculated using

\[ a_{\text{port}3}^3 = V_{\text{port}3}^3 + Z_0 \times I_{\text{port}3}^3 \sqrt{Z_0}, \]  
\[ b_{\text{port}3}^3 = V_{\text{port}3}^3 - Z_0 \times I_{\text{port}3}^3 \sqrt{Z_0}, \]

where

\[ V_{\text{port}3}^3 = \frac{F_{\text{port}3}}{l_3}, \]
\[ I_{\text{port}3}^3 = C_{\text{port}3} \times l_3. \]

At port 4 (lumped port), \( a_{\text{port}3}^4 = 0 \) because port 4 is terminated with a matched load (50 Ω) and the reflected wave \( b_{\text{port}3}^4 \) can be obtained from

\[ b_{\text{port}3}^4 = -2 \times \sqrt{Z_0} \times I_{\text{port}3}^4, \]
\[ I_{\text{port}3}^4 = C_{\text{port}3} \times l_4. \]

At port 1 and port 2 (Floquet ports), the reflected wave \( a_{\text{port}3}^1 \) and \( a_{\text{port}3}^2 \) are zero because Floquet ports are terminated with their modal impedances (matched loads), and the incident wave \( b_{\text{port}3}^1 \) and \( b_{\text{port}3}^2 \) can be calculated using

\[ b_{\text{port}3}^{1|2} = \frac{V_{\text{port}3}^{1|2} + Z_0 \times I_{\text{port}3}^{1|2}}{\sqrt{Z_0}}, \]

where

\[ V_{\text{port}3}^{1|2} = A_{\text{port}3}^{1|2} \times a|b, \]
\[ I_{\text{port}3}^{1|2} = -A_{\text{port}3}^{1|2} \times a|b \sqrt{Z_{\text{wave}}}. \]

After obtaining \( a_3, b_1, b_2, b_3, \) and \( b_4 \), then \( S_{1,3}, S_{2,3}, S_{3,3} \) and \( S_{4,3} \) can be computed according to Equation (3.41). The elements \( S_{i,4} \) in the extended GSM can be calculated in the same way.

### 3.1.5 Model Validation

For validating the GSM, the co-polarized reflection coefficients predicted by the GSM in fixed reflectarray case have been compared to a traditional reflectarray with variable size elements [2] in an infinite-array
environment. For validating the proposed extended GSM with lumped ports, the co-polarized reflection coefficients predicted by extended GSM have been compared against the results from the reconfigurable reflectarray element and the GSM with lumped elements where the impedance matrix is modified directly.

**GSM Validation**

Using the GSM presented in Section 3.1.2, a traditional non-reconfigurable reflectarray unit cell with variable size patches has been analyzed. The example considered here is a simple single-layer patch design shown in Figure 3.7.

A square patch \((l = w)\) resides on a grounded dielectric substrate with parameters \(h = 1\) mm and \(\varepsilon_r = 1.05\). The patch length is used to change the scattered field phase in order to achieve the desired phase shift. To generate the phase curve, the patch length was varied over a 6-14 mm range and analyzed at 11.5 GHz, 12 GHz and 12.5 GHz. The periodicity of the elements is 14 mm, which is slightly larger than half of a free space wavelength at those three frequencies. The phase of reflection coefficient as a function of frequency at normal incidence of these square patches in an infinite array environment is plotted in Figure 3.8. 50 TE and 50 TM Floquet harmonics were used to obtain convergent results for the GSM computation.

Figure 3.8 shows good correlations with predictions. There are some minor differences in the asymptotic phase values in Figure 3.8. One reason behind could be that the RWG basis function (triangular) was implemented in the proposed GSM, but predictions was implemented with rooftop basis function (rectangular) instead. Since the current on a square patch is flowing along the excited direction \((x\) or \(y)\), rectangular basis function is better to construct this linear current behaviour than the triangular basis function. Therefore, the results from different basis functions could introduce some mismatches.

**Extended GSM Validation**

To validate the extended GSM with lumped ports, we compare it to the GSM with loaded lumped element and FDTD results. A simple reconfigurable reflectarray example based on a previous varactor-tuned unit cell as shown in Figure 2.9 was analyzed.
Figure 3.8: Co-polarized reflection coefficient as a function of size of patch for the element shown in Figure 3.7. Legend: —MoM, — [2]

Figure 3.9: Reflectarray unit cell loaded with two varactor diodes

The cell has a periodicity of $a = b = 30 \text{ mm}$ and dimensions $l = 19 \text{ mm}$ and $w = 14 \text{ mm}$. The patch resides on a lossless dielectric substrate with a dielectric constant of $\epsilon_r = 3.02$ and a height of $h = 1.524 \text{ mm}$. A 1 mm gap between the patch-halves is introduced and varactor diodes connect the two patch halves as shown. This geometry introduces series ($C_s$) and parallel ($C_p$) gap capacitance as well as additional inductances $L_c$ due to to current crowding around the varactor connection points [3]. The parasitics associated with these discontinuities can be predicted, and introduced to change the effective load $Z_L$. The equivalent circuit model for the load impedance $Z_L$ is shown in Figure 3.10a.

$Z_v$ corresponds to the varactor diode impedance, which in this case is a series RLC circuit as shown in Figure 3.10b, and $Z_v$ is modelled as an Aeroflex MGV-100-20 varactor diode that develops a capacitance in the $1.80 - 0.12 \text{ pF}$ range over an applied bias voltage of $0 - 20 \text{ V}$. The parasitics of the diode were modelled using $R_v = 0.7 \Omega$ and $L_v = 0.4 \text{ nH}$. For a substrate height of $h = 1.524 \text{ mm}$, the parasitics in the patch model have been previously determined to be $C_s = 0.376 \text{ pF}$, $C_p = 0.0341 \text{ pF}$ and $\frac{1}{2} L_c = 0.317 \text{ nH}$ [3]. The co-polarized reflection coefficient as predicted by GSM with lumped element and
extended GSM with lumped port are plotted as a function of capacitance at 5.5 GHz in Figure 3.11, along with the FDTD results [3]. 60 TE and 60 TM Floquet harmonics are used to obtain a convergent result. The legend “Extended GSM” means the GSM with lumped port extension (a post-processing approach) to employ the lumped elements and the legend “GSM” represents that lumped elements are implemented by modifying the impedance matrix. Excellent correlation is seen between the GSM and the extended GSM results in terms of both amplitude and phase of the reflection coefficient. There is a slight variation between the GSM results and the FDTD results, and the reason could be that those are two different simulation methods which may result in a mismatch.

To conclude, the co-polarized reflection coefficients predicted by extended GSM have been compared to the GSM and FDTD results [3]. Very good agreements have been obtained in both cases. The extended GSM can be used to quickly and accurately determine the so-called S-curve of a reconfigurable reflectarray element for various load impedances and of a non-reconfigurable reflectarray for various patch sizes.
3.2 Equivalent Circuit Model

In Section 3.1 a full-wave method based on the periodic MoM in the spectral domain was presented for analyzing reflectarray unit cells. Alternatively, analytical approaches to analyzing reflectarray unit cells are faster and generally easier to carry out. Furthermore, even if full-wave solvers are ultimately employed, analytical models can be used as a useful starting point before more detailed analysis is taken.

However, the synthesis of detailed and accurate analytical models of reflectarray unit cells has only been previously addressed to a limited extent. Equivalent circuit modelling using simple LC models for reflectarray elements [5] have been used to analyze simple reflectarray elements, but do not consider reconfigurable or multi-layer elements. Meanwhile, circuit models for reconfigurable reflectarray elements are very limited or specific. Varactor-loaded dipoles have been considered as reflectarray unit cells, and an ECM for such a cell has been developed [6]. Another cell, using varactor-loaded patches, also can be represented using an equivalent circuit valid near the resonant frequency point [3], but this work is also specific to a certain cell type and requires full-wave simulations to component values in the underlying ECM. Hence, there is a need for a versatile model that can handle both fixed and reconfigurable elements.

Here we propose a ECM which is based on Floquet model expansion and it is a closed form formula to predict the scattering field of linearly-polarized reconfigurable and non-reconfigurable reflectarray unit cell. Such model can provide greater insight into the operation of reflectarray unit cells, and could be evaluated much faster than their full-wave counterparts. First, the ECM for an infinite driven dipole array is described in Section 3.2.1 and followed by the modified ECM for reconfigurable and non-reconfigurable reflectarray in Section 3.2.2. The ECM is validated by comparing the results to those derived from the MoM developed before in two test cases which are loaded reflectarray and single layer unloaded reflectarray in Section 3.2.3. Finally, the proposed ECM has been applied to analyze the effects of the available parameters in the reflectarray element design to the phase and amplitude response of the scattered fields of the reflectarray element in Section 3.3.

3.2.1 ECM for an Infinite Planar Dipole Array

![Figure 3.12: Infinite driven dipole array](image-url)
Consider a driven planar dipole in an infinite array environment as shown in Figure 3.12. The dipole has a length \( l \) and a width \( w \), and is placed in a periodic array employing rectangular cells of dimensions \( a \times b \). The dipole resides on a grounded dielectric substrate with thickness \( h \) and dielectric constant \( \varepsilon_r \). The dipole resides at \( z = 0 \) while the ground plane resides at \( z = -h \). The electric currents are \( x \)-directed, as shown in Figure 3.12, and the elements are linearly phased with uniform amplitudes. The current on the dipole can be expanded in a Floquet series as

\[
\vec{I}(x, y) = \hat{x} \sum_m \sum_n \hat{f}(k_{xmn}, k_{ymn})e^{-jk_{xmn}x-jk_{ymn}y}, \tag{3.48}
\]

where

\[
k_{xmn} = k_{x0} + \frac{2m\pi}{a} = k_0 \sin \theta \cos \phi + \frac{2m\pi}{a}, \tag{3.49a}
\]

\[
k_{ymn} = k_{y0} + \frac{2m\pi}{b} = k_0 \sin \theta \sin \phi + \frac{2m\pi}{b}. \tag{3.49b}
\]

\( \theta \) and \( \phi \) are the intended radiation direction in the spherical coordinate system. \( k_0 \) is the wavenumber in free space, and \( \hat{f} \) is the Fourier transform of the current distribution \( f(x, y) \) on the dipole. The Floquet impedance seen looking into a dipole in a periodic environment has been derived as \[18\]

\[
Z^{FL} = \frac{4l^2}{ab\pi^2} \sum_m \sum_n \left[ \frac{k_{y_{mn}}^2}{y_{TE_{mn}}} + \frac{k_{x_{mn}}^2}{y_{TM_{mn}}} \right] \frac{F_{k_{y_{mn}}}^2 G_{k_{x_{mn}}}^2}{k_0^2 - k_{z_{mn}}^2 - k_{z_{mn}}^2}, \tag{3.50}
\]

where

\[
k_{z_{mn}}^+ = \sqrt{k_0^2 - k_{x_{mn}}^2 - k_{y_{mn}}^2},
\]

\[
k_{z_{mn}}^- = \sqrt{\varepsilon_r k_0^2 - k_{x_{mn}}^2 - k_{y_{mn}}^2}. \tag{3.51}
\]

\( k_{z_{mn}}^+ \) and \( k_{z_{mn}}^- \) are the \( z \) components of the corresponding wavenumbers in the regions \( z > 0 \) and \( z < 0 \), respectively. \( G_{k_{x_{mn}}} \) and \( F_{k_{y_{mn}}} \) are the Fourier transforms of the current distribution in the \( x \) and \( y \) direction, respectively. \( k_{x_{mn}} \) and \( k_{y_{mn}} \) are the Floquet mode wavenumbers. The quantities \( y_{TE_{mn}} \) and \( y_{TM_{mn}} \) represent the equivalent admittance experienced by a \( TE_{zmn} \) and \( TM_{zmn} \) current source at the air-dielectric interface, respectively. This equivalent admittance can be viewed as the input admittance of a shunt source located at the junction of two transmission lines, as shown in Figure 3.13.

![Figure 3.13: Transmission line model of the driven planar dipole](image)

The transmission lines have characteristic admittances equal to the modal admittances and its length.
is equal to the media widths. Since the free-space side \((z > 0)\) of the transmission is infinite long, the associated infinite transmission line is equivalent to a matched termination and its characteristic admittance is the model admittance in free space. The input admittance at the current source can be calculated from the summation of the modal admittance in the \(z > 0\) region (free space) and the input admittance of the grounded dielectric slab,

\[
y_{mn}^{TE|TM} = Y_{mn}^{TE+|TM+} - jY_{mn}^{TE-|TM-} \cot (k_{zmn} h).
\]

The modal admittances are given by

\[
Y_{mn}^{TE+} = \frac{k_{zmn}^{+}}{\omega \mu_0},
\]

\[
Y_{mn}^{TM+} = \frac{\omega \epsilon_0}{k_{zmn}^{+}},
\]

\[
Y_{mn}^{TE-} = \frac{k_{zmn}^{-}}{\omega \mu_0},
\]

\[
Y_{mn}^{TM-} = \frac{\omega \epsilon_0 \epsilon_r}{k_{zmn}^{-}},
\]

where \(\omega\) is the angular frequency and \(\epsilon_r\) is the dielectric constant of the substrate. To explicitly show the contribution from TE and TM modes, Equation (3.50) can be organized as

\[
Z_{FL} = \frac{4L^2}{ab\pi^2} \sum_{m} \sum_{n} \frac{F_{k_{ymn}^2} G_{k_{xmn}^2}}{k_0^2 - k_{zmn}^2} \frac{1}{y_{mn}^{TE}} - \frac{4L^2}{ab\pi^2} \sum_{m} \sum_{n} \frac{F_{k_{ymn}^2} G_{k_{xmn}^2}}{k_0^2 - k_{zmn}^2} \frac{1}{y_{mn}^{TM}},
\]

or

\[
Z_{FL} = \sum_{m} \sum_{n} \frac{T_{mn}^{TE}}{y_{mn}^{TE}} + \sum_{m} \sum_{n} \frac{T_{mn}^{TM}}{y_{mn}^{TM}},
\]

where

\[
T_{mn}^{TE} = \frac{L^2}{ab \pi^2} \frac{k_{ymn}^2}{k_0^2 - k_{zmn}^2} F_{k_{ymn}^2} G_{k_{xmn}^2},
\]

\[
T_{mn}^{TM} = \frac{L^2}{ab \pi^2} \frac{k_{xmn}^2}{k_0^2 - k_{zmn}^2} F_{k_{ymn}^2} G_{k_{xmn}^2}.
\]

A close examination of Equation (3.50) reveals that the Floquet impedance \(Z_{FL}\) to the dipole is an infinite series summation of impedances \(1/y_{mn}^{TE|TM}\) scaled by transformers with turns ratios \(\sqrt{T_{mn}^{TE|TM}}\).

In fact, this interpretation of the ECM was inspired by earlier work in waveguides which also used a continuum of transformers to represent the coupling to rectangular waveguide modes \cite{30}. The remarkable aspect of this interpretation is that the current distribution on the dipoles manifests itself exclusively in the transformer turns ratios in the circuit model. Hence, knowing the current distribution on the dipoles, one can easily determine the turns ratios and subsequently the active Floquet impedance of the dipole. The current distribution on the metallization will be discussed later in this chapter.
3.2.2 Modification of ECM for Reflectarray

The equivalent circuit shown in Figure 3.14 is simply an expression of the well-known Floquet impedance given by Equation (3.50). However, we propose that this model can be modified for use in the analysis of dipole-like and even patch-based unit cells in a reflectarray as shown in Figure 3.15.

Analysis of reflectarray element requires us to first determine the scattering behaviour of the dipole, specifically, the reflection coefficient of the interested propagating Floquet mode. Assuming the dimensions $a$ and $b$ of the cell are such that only the two fundamental Floquet modes are propagating ($TE_{00}$ and $TM_{00}$ modes). To analyze linearly-polarized reconfigurable reflectarray, we turn this circuit around to predict the input impedance seen by the Floquet mode of interest, as shown in Figure 3.16. We look at the desired Floquet mode, while loading the $Z^{FL}$ port with impedance $Z_L$ which could potentially represent a tunable circuit component in the case of a reconfigurable reflectarray.
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Figure 3.16: Equivalent circuit for predicting the scattering of a loaded dipole into the $TE_{00}$ Floquet harmonic

The input impedance seen looking into the desired $TE_{00}$ mode at the air-dielectric interface is the sum of infinite summation of Floquet impedance except the desired mode ($TE_{00}$) and the load impedance ($Z_L$),

$$Z_{00}^{TE'} = Z_L + \sum_{m \neq 0} \sum_{n \neq 0} T_{mn}^{TE} \frac{1}{y_{mn}^{TE}} + \sum_m \sum_n T_{mn}^{TM} \frac{1}{y_{mn}^{TM}}. \quad (3.57)$$

Then, the input impedance $Z_{in,00}^{TE}$ of the desired $TE_{00}$ Floquet mode shown in the Figure 3.16 can be calculated based on transmission line theory.

Figure 3.17: Transmission line model of the loaded planar dipole

As shown in the Figure 3.17, the input impedance of the desired Floquet mode is the impedance just above the $z = 0$ interface and can be viewed as a parallel combination of Floquet impedance and a shorted transmission line with length $h$. Then, the input impedance of the desired $TE_{00}$ mode can be calculated from

$$Z_{in,00}^{TE} = \left[ \left( Z_{00}^{TE'} \right)^{-1} - j Y_{00}^{TE} \cot (k_{z00} h) \right]^{-1}. \quad (3.58)$$

After knowing the input impedance of the $TE_{00}$ mode, the associated reflection coefficient of the $TE_{00}$
Floquet port can be found,
\[ \Gamma_{00}^{TE} = \frac{Z_{TE,00}^{in} - Z_{00}^{TE+}}{Z_{TE,00}^{in} + Z_{00}^{TE+}}. \] (3.59)

The amplitude and phase of the reflection coefficient can be used to predict the co-polarized scattering behaviour of the unit cell. It is important to notice that the dipole current distribution was derived assuming an \( x \)-oriented current, so that only the \( TE_{00} \) input impedance can be predicted using the equivalent circuit as shown. To determine the \( TM_{00} \) input impedance would require that the current distribution in the \( y \)-direction be known. While in principle the equivalent circuit model is capable of determining the amplitude of the cross-polarized (TM) component when the cell is illuminated with a TE wave, the accuracy of this strongly depends on the current distribution assumed along the dipole, especially the \( y \)-direction.

### Current Distribution on the Element

The input impedance expression of Equation (3.57) is dependent on transformers with turns ratios which are a function of the current distribution on the metallization. Hence, the accuracy of ECM is dependent on the current distribution on the metallization. Therefore, we need to accurately determine the current distribution on the metallization, especially the current in non-resonant direction.

![3D current plot](image)

(a) 3D current plot

![Normalized current](image)

(b) The variation of \( x \)-oriented current in \( x \)-direction and \( y \)-direction

Figure 3.18: Current distribution on the metallization

Here, we assume the current is \( x \)-oriented as described in Equation (3.48). Figure 3.18a shows the 3D current plot on the metallization and the variation of \( x \)-oriented current with \( x \) and \( y \) are shown in Figure 3.18b. For the current in the \( x \) direction, half of the metallization can be treated as TEM transmission line by factoring in \( \epsilon_{reff} \) and \( C_f \) as shown in the Figure 3.19.

It is well known for reflectarray elements that an incident plane wave (especially one at broadside) will induce a current distribution on the patch similar to if it were fed from the centre in a driven mode. Therefore, this fact has been used to rationalize the use of a transmission line analogy whereby a half-patch is analyzed (because of symmetry). The fringing capacitance is important in that it allows for a nonzero current to flow at the open end of the patch. The length of the transmission line is half of the length in \( x \) direction. \( Z_0 \) is the characteristic impedance of the metallization and \( Z_{C_f} \) is the impedance
Figure 3.19: Transmission line model of the current in \( x \)-direction of fringing capacitance associated with the metallization. \( \epsilon_{ref} \) and \( C_f \) can be calculated using standard formulae assuming a quasi-static TEM wave \[14\]

\[
\epsilon_{ref} = \epsilon_r + \frac{1}{2} + \frac{\epsilon_r - 1}{2} \left[ 1 + \frac{12}{W} \frac{h}{W} \right]^{-1/2},
\]

\[
C_f = \frac{W}{240\pi\lambda_0} \left[ 1 - 0.636 \ln(k_0 h) \right],
\]

where \( \epsilon_r \) is the dielectric constant of the substrate, \( h \) is the height of substrate, and \( W \) is the width of the metallization. A capacitance \( C_f \) loads at each end of the transmission line. This allows the current distribution along the metallization to be found as

\[
I_x(x) = I_{x=0} \cosh(\gamma x) - \frac{V_{x=0}}{Z_0} \sinh(\gamma x),
\]

where

\[
Z_0 = \sqrt{\epsilon_{ref} \left[ \frac{W}{h} + 1.393 + 0.667 \ln \left( \frac{W}{h} + 1.44 \right) \right]}.
\]

Typically, the variation of \( J_x \) with \( y \) is not treated since it is not significant in thin dipoles \[13\]. However, assuming \( y \)-direction to be uniform is not accurate for patches according to Figure 3.18b. The assumed current distribution in the \( y \)-direction also significantly affects the accuracy of the phase curve because of the impact of that distribution on the Floquet modes. A function modelling the singular nature of the metallization currents on the edges is needed. Results from the analysis of metal strip gratings are useful for this purpose.

For a metal strip grating shown in Figure 3.20 subjected to an incident TE or TM wave, functions describing the tangential TE and TM electric fields in the gaps between the metal strips are \[31\]

\[
q^{TE}(y) = C^{TE}(\beta) \sqrt{(W/2)^2 - y^2} e^{-j\beta y},
\]

\[
q^{TM}(y) = \frac{C^{TM}(\beta) e^{-j\beta y}}{\sqrt{(W/2)^2 - y^2}},
\]

where \( \beta \) is the transverse wavenumber associated with the incident wave and \( C^{TE|TM} \) are constant. We recognize that when the local periodic assumption is invoke for reflectarrays, which is common in the
analysis of the unit cell, that in a transverse ($yz$) cross section the metallization layer of a reflectarray is just the complement of the metal strip grating shown in Figure 3.20. The metallization can be viewed as the gap between metal strips and the space between the adjunct metallizations can be treated as the metal strip. Therefore, for a TE-polarized incident wave, the magnetic field produced at the surface of the reflectarray metallizations is

\[ H(y) = \hat{y} \frac{C(\beta) e^{-j\beta y}}{\sqrt{(W/2)^2 - y^2}}. \]  

(3.64)

The presence of a ground plane and dielectric does not significantly modify the fields. Therefore, the surface current density on the metallization is equal to

\[ J_x(y) = \hat{n} \times H_y(y) = \hat{x} \frac{C(\beta) e^{-j\beta y}}{\sqrt{(W/2)^2 - y^2}}. \]  

(3.65)

As shown in Figure 3.21, there is a good correlation in $y$-direction of the current distribution between the proposed method and that predicted by MoM.

$G_{mn}$ and $F_{mn}$ are readily found by numerically taking the Fourier transform of Equation (3.61) and Equation (3.65), respectively. This model can be generalized to load-free reflectarray unit cells such as the standard patches and dipoles used in fixed-pattern reflectarrays [2]. This can be achieved simply by setting the load impedance to zero (a short circuit) in Figure 3.16. Then, Equation (3.57) can simplify to

\[ Z_{00}^{TE'} = \sum_{m \neq 0} \sum_{n \neq 0} T_{mE}^{TE} \frac{1}{y_{mE}^{TE}} + \sum_m \sum_n T_{mn}^{TM} \frac{1}{y_{mM}^{TM}}. \]  

(3.66)

This generalizes the model, making it highly useful for predicting the scattering properties of loaded and standard reflectarray unit cells alike.

### 3.2.3 Model Validation

To validate the ECM, the co-polarized reflection coefficient predicted by the ECM in an infinite-array environment has been compared to full-wave simulation results (MoM). There are two main test cases
which are loaded reflectarray unit cell and unloaded reflectarray unit cell.

**Loaded Reflectarray Unit Cells**

Using the ECM presented in Section 3.2.2, a simple reflectarray unit cell loaded with a lumped impedance can be analyzed. An initial and simplified version of the unit cell is shown in Figure 3.22.

A distributed load impedance $Z_L$ is placed across the load region which serially connects both patch-halves. The cell closely resembles reconfigurable reflectarray unit cells employing tunable lumped-element reactive loads (for example, varactor diodes) to tune the reflection phase of the unit cell [3], as discussed in Section 3.1.5. A simple example based on a previous varactor-tuned unit cell [3] was analyzed first. The load is a distributed capacitive load spanning the width of the patch exhibiting a net capacitance of approximately 0-600 pF/m. This value, when translated to lumped elements corresponds closely to those available from commercial varactor diodes. The co-polarized reflection coefficient as predicted by MoM and ECM is plotted as a function of frequency in Figure 3.23. Good correlation is seen between the ECM and MoM results in term of both the magnitude and phase of the reflection coefficient.

The design curve is formed by evaluating the co-polarized reflection coefficient at a fixed frequency.
Figure 3.23: Co-polarized reflection coefficient as a function of frequency for various varactor capacitances of element shown in Figure 3.22

and sweeping the load capacitance. The results of this process at 5.5 GHz are plotted in Figure 3.24. 50 TE (m) and 50 TM (n) space harmonics reused in the summation, Equation (3.57), to obtain convergent results. An additional curve has been included in this curve which shows the results of the ECM if a uniform current distribution is assumed in the y direction of x-oriented current instead of employing Equation (3.65). Clearly, a significant error is introduced in the phase curve if the nonuniform current distribution is not taken into account. Hence, the suggested adaptation to accommodate patches is accurate and models the change in Floquet harmonics resulting from the modified current distribution.

Figure 3.24: Co-polarized reflection coefficient as a function of load capacitance at 5.5 GHz for the element shown in Figure 3.22

A more realistic reflectarray unit cell loaded with varactors diodes at the edge of the patch is shown in Figure 3.9 which has been introduced in Section 3.1.5. The design curve is formed by evaluating the co-polarized reflection coefficient at four different load capacitances with sweeping the frequency. The inclusion and evaluation of the parasitics is done in the same way as Section 3.1.5. Excellent correlation between the ECM results and the MoM results is observed in Figure 2.25.

Another design curve is formed by evaluating the co-polarized reflection coefficient at a fixed frequency and sweeping the load capacitance. The results of this process at 5.5 GHz are plotted in Figure 3.26. An additional curve from ECM result has been included if a uniform current distribution is assumed in the
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(a) Reflection magnitude curves
(b) Reflection phase curves

Figure 3.25: Co-polarized reflection coefficient as a function of frequency for various capacitances, for the element shown in Figure 3.9. Legend: — MoM, – – ECM.

Figure 3.26: Co-polarized reflection coefficient as a function of capacitance at 5.5GHz, for the element shown in Figure 3.9.

Single-Layer Unloaded Reflectarray Unit Cells

As discussed in Section 3.2.2, the ECM can be generalized to deal with reflectarray elements without lumped-element loading, such as traditional non-reconfigurable reflectarray elements or reconfigurable elements realized with tunable materials. The first example considered is a simple single-layer patch design [2] shown in Figure 3.7 in Section 3.1.5. In this example, the patch length is used to change the scattered field phase. The relevant parameters of the design are \( h = 1 \text{ mm} \) and \( \varepsilon_r = 1.05 \). To generate the phase curve, a square patch \( (l = w) \) is considered where the patch length is varied over a 6 - 14 mm range and analyzed around 12 GHz. The results from ECM and MoM are plotted in Figure 3.27a. A separate analysis of various patch lengths versus frequency is produced in Figure 3.27b.

The non-uniform current distribution Equation (3.65) was assumed along the transverse patch dimension for the ECM results. Overall, the results correlate very well, especially in the vicinity of the design frequency. There are some minor differences in the asymptotic phase values in Figure 3.27 as both \( L \) and \( f \) become large, which is expected behaviour. As the patch length \( L \) becomes large, the fringing susceptance predicted by Equation (3.60b) becomes less accurate since it assumes the patch is in
isolation. In a periodic environment, the loading effect of adjacent patches becomes more apparent as $L$ increases. Similarly, as frequency increases, the accuracy of Equation (3.60b) decreases as the substrate becomes less electrically thin, and the fringing cannot be modelled as single fringing capacitance.

Fixed patches on tunable substrates can also be handled by the model. For example, liquid crystals exhibit an externally tunable anisotropic permittivity tensor when subjected to a static or quasi-static electric field [32]. This can be used to provide an electronically tunable phase shift. An example is considered here based on a previous cell design operating at 100 GHz [32]. The unit cell size is 0.9 mm × 0.9 mm. A patch with dimensions $L = 0.77$ mm and $W = 0.7$ mm resides on a 15 µm liquid crystal substrate. For simplicity, the quartz cover layer in the design is omitted, and an isotropic permittivity tensor is assumed since the ECM shown is scalar. The relative permittivity of the substrate sweeps from $\varepsilon_r = 3.2$ to $\varepsilon_r = 4.2$, which corresponds to a realistic range realizable from liquid crystal substrates. The results for the scattered phase are shown in Figure 3.28 which show excellent correlation with predictions by the ECM.

![Reflection magnitude curves](image1)
![Reflection phase curves](image2)

(a) Reflection magnitude curves  
(b) Reflection phase curves

Figure 3.28: Reflection phase curves for a liquid crystal-tuned element as a function of dielectric constant.

A separate analysis for various dielectric constant of the substrate versus frequency is produced in Figure 3.29. The results show excellent correlation with predictions by the ECM as well.

### 3.3 Reflectarray Element Design

For many linearly-polarized reflectarray cells, one of the well known mechanisms which provides changes in the phase of the scattered field is by loading the microstrip patch with an electronically-controlled
capacitance which can be achieved by implemented varactor tuning diodes. The model of the varactor diode is a series combination of RLC circuits and the ohmic loss comes from the resistor in the varactor diodes around the resonant frequency. The reflectarray element must be designed so that with a given frequency and range of capacitances, the minimum loss and reasonable phase range is produced over the range of capacitances offered by the varactor diode. To reduce the loss of the reflectarray element and meanwhile obtain the enough phase agility of the element, it is important to analyze the effects from the periodicity of the elements, the size of the element, the thickness of the dielectric substrate and the dielectric constant of the substrate. ECM has been applied to conduct this analysis because it is faster than MoM and its result is as accurate as MoM.

The size of the element and the element spacing could affect the phase and amplitude of scattered fields and a dipole antenna with various sizes has been analyzed using ECM. Figure 3.30a and Figure 3.30b illustrates the amplitude of scattered field as a function of capacitance from three different sizes dipole antenna with $\lambda/2$ and $\lambda/6$ periodicities, respectively and loaded varactor diodes with two resistances 0.7 $\Omega$ and 5 $\Omega$, respectively. Compared Figure 3.30a and Figure 3.30b, it can be seen that the choice of periodicity/operating frequency and varactor diode characteristics affect the ohmic loss of the element. As the periodicity of the dipoles becomes smaller, the ohmic loss of the dipole is much less when loaded with the same varactor diode. The reason behind it is that when we decrease the periodicity of the
dipoles, and this periodicity introduces capacitive loading between dipoles and modifies its resonance. Therefore, the dipoles operate far away from the corresponding resonant frequency, and the ohmic loss mostly comes from when the dipole is at resonance. Therefore, the ohmic loss is reduced. Additionally, the loss of the element is also determined by the size of the element. In both half wavelength element spacing and sub-wavelength element spacing case, when the length of the dipole increases, the ohmic loss will decrease.

Figure 3.31 shows that the phase of scattered field as a function of tuning capacitance from the same three dipole antennas with $\lambda/2$ and $\lambda/6$ periodicities. After the operating frequency is chosen, the size of the dipole needs to be carefully selected to achieve the enough phase agility of the element within the tuning range of $C_{\text{min}}$ to $C_{\text{max}}$ used in the varactor diodes. As the length of the dipole decreases with uniform step, the phase reflection coefficient curve versus capacitance shifts to the right which means that the phase range may constrains by the maximum capacitance $C_{\text{max}}$. On the other hand, if the phase reflection coefficient curve versus capacitance shifts to the left which means that the phase range may constrains by the minimum capacitance $C_{\text{min}}$. For example, the phase range for small dipole is constrained by the maximum capacitance $C_{\text{max}}$ of the diode. Conversely the phase range of a large dipole element will be limited by the diode’s minimum capacitance $C_{\text{min}}$ if the dipole is chosen to be too large. Therefore, the required phase range can be achieved by carefully selecting the length of the dipole. As shown in Figure 3.31, the phase range of the element reduces, while the periodicity of the elements becomes smaller. For $\lambda/2$ and $\lambda/6$ periodicities, the phase ranges are about 330 degrees and 270 degrees, respectively. Optimal pairing of the element geometry and the periodicity of the elements promotes the lowest ohmic loss and the required phase agility of the element.

The thickness of the dielectric substrate also has an impact on the phase and amplitude of scattered fields. The same dipole antenna with various thickness of the dielectric substrate has been analyzed using ECM. As shown in Figure 3.32, the slope of the phase curve is sharpest for $h = 1.905$ mm case with largest phase range and the phase curve slope is smoothest for $h = 3.175$ mm case with smallest phase range. Therefore, if the thickness of the dielectric substrate decreases, the slope of the phase curve becomes sharper and the corresponding phase range increases. From the loss perspective, the maximum ohmic loss of the element is lowest in $h = 3.175$ mm case and as the thickness of the substrate decreases, the maximum ohmic loss increases. As a summary, as the thickness of the dielectric substrate decreases, the phase range of the element increases with sharper slope and the ohmic loss of the element increases
The permittivity of the dielectric substrate also plays a role in both the phase range and the ohmic loss of the element. The same dipole antenna with various dielectric constant of the substrate has been analyzed using ECM. As shown in the Figure 3.33, when we increase the permittivity of the dielectric substrate, the ohmic loss of the element increases and the phase curve shifts to the right with the same phase range.

What’s more important, the capacitance range of the varactors diodes is limited, and therefore the periodicity of the elements, the size of the element, the thickness of the substrate and the dielectric constant of the substrate have to be carefully chosen in order to achieve the low loss and sufficient phase range within the reachable varactor diodes capacitances. The standard thickness and permittivity of the dielectric substrate are restricted, and then these two factors play less important roles compared to the size of the element and the periodicity of the elements. Optimization of the element geometry and the periodicity promotes the lowest ohmic loss and sufficient phase agility of the element.

Here we presented a adjustable and accurate equivalent circuit model for predicting the reflection coefficient for dipole-like reflectarray unit cells. The ECM is derived from Floquet model expansion for predicting the reflection coefficient for a dipole-like scatter. It can generate highly accurate results.
without the need for running full wave simulations for the components in the ECM. The versatility of the ECM allows it to accurately model both fixed and reconfigurable reflectarray unit cells.

Validation experiments against full-wave simulation tools show that the model is accurate for reflectarray elements based on changing the patch size, as well as the reconfigurable reflectarray based on lumped element loading and distributed tuning mechanisms. Predictions from ECM are close to those provided by the full-wave simulations. Evaluation of the ECM can be done very quickly because the reflection coefficient can be predicted in closed form. So, ECM can be a highly useful tool during the initial design stages of reflectarray prior to engaging a more detailed full-wave simulation of the cell. At the initial design stage, ECM is able to fast relate the ohmic loss and phase range of element to the element spacing, the size of the element, the thickness of the substrate, and permittivity of the dielectric substrate.

3.4 Conclusions

In this chapter, antenna design methods have been introduced which are full-wave simulation method (MoM) and analytical method (ECM). In order to design a low-loss reconfigurable reflectarray element, we need to analyze its element to achieve a lowest loss and good phase agility to minimize the absorption. The MoM was implemented to obtain the GSM at dielectric interface in an infinite array environment. This technique has been applied to the analysis of periodic surfaces involving rectangular and arbitrarily-shaped elements using RWG basis functions. We extended the GSM to analyze the scattering behaviour of reconfigurable reflectarray element employing lumped elements. Furthermore, the extended GSM with lumped ports has been developed to reduce the simulation time, especially when the number of lumped elements is large.

Analytical approaches to analyze reflectarray unit cells are fast and generally easy to carry out. Here, we proposed an equivalent circuit model which is able to predict the scattering behaviour for a linear polarized reflectarray unit cell. The ECM is based on Floquet modal expansion without running full wave simulations to obtain accurate results. The ECM is able to accurately analyze reconfigurable reflectarray and non-reconfigurable reflectarray unit cell. Therefore, the advantage is that the proposed ECM is more generic than other ECMs which are specific to certain reflectarray cells. The ECM tool can be used to supplement, but not replace the full wave simulation, which can be very useful during the initial design stages of a reflectarray.

The proposed antenna design procedures generally include two steps. First, the element will be analyzed using ECM and then followed by MoM. ECM can be used in the first step of designing reflectarray to obtain the relationship between parameters of the element and scattered fields characteristics of the element. After this initial stage, the element will be simulated using MoM to achieve more detailed information such as the performance of the cell at oblique incidence, cross polarization, and results with increased accuracy.
Chapter 4

Low-cost and Low-loss
Reconfigurable Electromagnetic Surface

This chapter presents reconfigurable reflectarray elements that are able to achieve adequate reconfigurable performance as well as low radiation loss and low cost. Two element designs will be presented and their performances will be analyzed using the MoM. Both 1D finite linear arrays and 2D finite planar arrays are designed and simulated in HFSS for use as reflectarray and plane wave re-director. A thorough investigation is carried out to improve the radiation performance of reconfigurable electromagnetic surface. The reflectarray designs are assessed experimentally in Chapter 5.

4.1 Reconfigurable Electromagnetic Surface Unit Cell

The goal of this reflectarray unit cell design is to achieve low loss and about 180° phase agility. For many linearly-polarized reflectarray cells, the fundamental mechanism which provides changes in the phase of the scattered field is the position of the operating frequency on the resonance curve of the microstrip patch element. In this design, the resonant frequency of a patch is modified electronically using varactor diodes. The basic idea is that by loading the microstrip patch with an electronically controlled capacitance, its resonant frequency can be changed to increase the usable frequency range of the patch. The resulting frequency agility, when applied to arrays of microstrip patches, can be used to change the scanning characteristics of the array.

4.1.1 Unit Cell Design Flow

The unit cell design is following as:

1. First, we take advantage of the fast and accurate ECM to determine the relationship between parameters of the element and scattered field characteristics of the element at this initial stage.

2. The MoM is utilized to achieve more detailed information such as the performance of the cell at oblique incidence, and conduct the analysis of the full reflectarray.
3. The final implemented reflectarray unit cell is simulated in HFSS to accommodate more sophisticated features such as the vias and the DC biasing network.

4.1.2 Unit Cell Modelling

In order to analyze different reflectarray element configurations, the MoM is used to analyze a single reflectarray element with periodic boundary conditions. The simulation setup is shown in Figure 4.1 and mimics an infinite-array scenario.

Figure 4.1: Simulation setup for a reflectarray element in MoM

The source is a Floquet wave port located at the top of the periodic boundary box. This setup is a quick and reliable way to determine the scattering parameters of a reflectarray element. However, it is assumed that the cell is embedded in an infinite array, surrounded by identical cells in the transverse plane, which is not typically the case in reflectarrays. So, this method makes two main assumptions. First, it assumes the structure is periodic and hence the mutual coupling between the element is assumed to be the same for all elements in the array. In reality, mutual coupling may vary between different elements, which is not addressed in this simulation. However, invoking this “local periodic assumption” has been shown not to be a major concern in the design of reflectarrays, since the deviations in mutual coupling do not markedly change the phase curve. Second, the edge effects of the reflectarray are not simulated. However, the edge effects of reflectarrays can usually be neglected, especially when the reflectarray is large. Furthermore, if the edge taper of the feed is included, this further reduces any edge effects. Therefore, overall infinite-array analysis is a fast and accurate method to design reflectarray unit cell element.

4.1.3 Unit Cell Design and Operation

A preliminary reflectarray element (dipole) and its scattered behaviour presents first, and followed by an analysis of an optimal reflectarray element (square loop) to improve the specular reflection performance.
Dipole

A simple design, which provides continuous phase tuning, is the varactor diode-loaded microstrip dipole. This element consists of a dipole split in half with varactor diode positioned across the gap. The diodes are oriented parallel to the direction of current flow on the dipole, and can be seen as tunable capacitive loading to the dipole. By tuning the varactors (through application of a reverse bias voltage), the effective resonant length of the dipole changes and hence resonant frequency of the dipole changes which leads to the change of reflection phase. This change of reflection phase is demonstrated through S-curves as shown in Figure 4.2. At a specific frequency, with a reasonable capacitance range where $C_{\text{max}}/C_{\text{min}}$ is around 10 for hyperabrupt varactor diodes, the reflection phase can be made to vary over close to a 360 degree range and this phase range is sufficient for basic reflectarray designs.

![Figure 4.2: Expected phase response provide by a dipole](image)

The proposed electromagnetic surface will take advantage of the benefit from sub-wavelength element spacing. As previously discussed in Chapter 2.2.2, sub-wavelength element spacing has the benefits of lower ohmic loss and reduced phase quantization loss compared to the half wavelength element spacing. One drawback of sub-wavelength element spacing is that it increases the number of the lumped components such as varactor diodes. Therefore, we have to balance the number of tuning devices and the ohmic loss to determine the size of element spacing. Here, the element spacing is chosen to be $\lambda/6$. $\lambda/6$ element spacing can significantly reduce the ohmic loss compared to $\lambda/2$ element spacing (discussed in Section 2.2.2) and also keep number of lumped elements (varactor diodes) in a reasonable range. The rationale behind choosing $\lambda/6$ element spacing is that while 9 times as many diodes are used per area, the cost of the diodes is more than 9 times less. This is why we cannot go much lower than $\lambda/6$, because then the additional cost of the devices is no longer worth it. Sub-wavelength element spacing also decreases the phase range and may not achieve required phase shift on each reflectarray element. But, as mentioned in Chapter 2, a single bit phase shifter will be applied in the proposed reflectarray element, and then only 180 degrees is sufficient to achieve a beam steering. As discussed in Section 2.2.2, the phase range of sub-wavelength element is about 250° which is sufficient for single bit phase shifter.

Figure 4.3 shows the initial unit cell design of reconfigurable electromagnetic surface. The element is a dipole located on a dielectric substrate with a ground plane. The geometry of unit cell is square with a width (periodicity) of 10 mm in both directions (around $\lambda_0/6$ at 5.5 GHz). The dipole itself is
8.33 mm × 0.67 mm. One varactor diode is placed in the middle of the dipole along the current flow direction. The varactor is chosen to be a Skyworks Silicon SMV2019-079LF hyperabrupt varactor diode, which is much cheaper than the GaAs ones, and this varactor possesses a capacitance range of roughly 0.3 to 2.2 pF when reverse biased from 0-20 V. By tuning the varactor (through application of a reverse bias voltage) the effective resonant length and hence resonant frequency of the dipole changes. At fixed frequency, the effect on the reflection phase versus capacitance from the length and width of the dipole, the thickness of the substrate and the dielectric constant of the substrate are summarized as following. First, increasing the length of the dipole will shift the phase curve to the right without changing the shape. Then, increasing the width of the dipole and the thickness of the substrate will reduce the ohmic loss and the phase range as well. And increasing the dielectric constant of the substrate will increasing the ohmic loss and shifts the phase curve to the right. In order to minimize the ohmic loss and obtain enough phase range in the tunable capacitances range, the dipole has been determined to reside on the grounded substrate with dielectric constant $\varepsilon_r = 12.85$ and thickness $h = 2.54$ mm. Such substrate can be obtained from Rogers TMM @13i Laminates. A summary of the parameters from the dipole unit cell is shown in Table 4.1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Dipole unit cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element spacing/ periodicity</td>
<td>10 mm $\lambda/6$</td>
</tr>
<tr>
<td>Substrate permittivity ($\varepsilon_r$)</td>
<td>12.85</td>
</tr>
<tr>
<td>Substrate height (h)</td>
<td>2.54 mm</td>
</tr>
<tr>
<td>Dipole length (L)</td>
<td>8.33 mm</td>
</tr>
<tr>
<td>Dipole width (W)</td>
<td>0.67 mm</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters of dipole unit cell
4.1.4 Unit Cell Simulation Results

The electromagnetic surface unit cell is analyzed using the periodic full-wave structure simulator described in Section 3.1. In the initial simulations, the incidence angle is chosen to be broadside to simplify the analysis and due to the small unit cell size ($\lambda/6$), only the fundamental TE and TM Floquet modes are excited, all others are evanescent. According to the data sheet, varactor diodes are modelled using a series resistance of 4.8 Ω, a series inductance of 0.45 nH and a variable capacitance corresponding to the reverse bias voltage across the junction. As mentioned in Section 3.1, the varactors diodes can be modelled in two ways in MoM. Extended GSM with lumped port is used for the design. In this method, only one full-wave simulation needs to be run and save lots of amount of time, the results now contain a $3 \times 3$ scattering matrix including the two Floquet modes and one lumped port for this design. After some straightforward manipulations Equation (3.32), the generalized scattering matrix can be obtained.

Figure 4.4: Phase and magnitude response provided by dipole unit cell

Figure 4.4 shows a plot of the amplitude and phase response of the scattered fields versus varactor diodes capacitances for an incidence wave polarized along the principal direction of the dipole at 5.5 GHz. As can be seen, the phase curve (S-curve) is truncated due to the small cell periodicity in the design. Nevertheless, the total phase range is about 190 degrees, which is enough for the required phase range of single bit phase shifter. Furthermore, the ohmic loss has been reduced from the $\lambda/2$ case. The peak ohmic loss at resonant frequency is about 1.4 dB. Generally, the reflection losses peak at a point roughly in the middle of the phase curve corresponding to resonance. This dip in reflection is due to power being absorbed by the varactors diodes, conductor loss, and dielectric losses in the element. The series resistance of the varactor diodes is the most significant source of loss in this design [3]. Therefore, the phase states of the reflectarray should be chosen carefully to minimize the impact of the diode loss on the performance. In order to achieve low ohmic loss and sufficient phase range, two phase states of reflection coefficient have been chosen as as 45° and −135° with the amplitude of reflection coefficient about -0.43 dB and -1.0 dB, respectively. It is worthwhile to point out that the reflection losses are not symmetric about resonance for the dipole design, and it would be better if the same loss was achieved in both states. The reflection loss is much less than the one with half-wavelength element spacing. In the
λ/2 element spacing electromagnetic surface unit cell case, a simple varactor diodes loaded reflectarray unit cell [3] was analyzed with the same loaded varactor diodes. Figure 4.5 shows a plot of the amplitude and phase response of the scattered fields versus varactor diodes capacitances. Table 4.2 summarizes the maximum reflection losses from the cell for λ/6 and λ/2 element spacing.

![Figure 4.5: Phase and magnitude response provide by rectangular patch unit cell](image)

<table>
<thead>
<tr>
<th>Element type</th>
<th>Dipole</th>
<th>Rectangular patch</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element spacing</td>
<td>λ/6</td>
<td>λ/2</td>
</tr>
<tr>
<td>Maximum reflection loss</td>
<td>-1.5 dB</td>
<td>-11.8 dB</td>
</tr>
</tbody>
</table>

Table 4.2: Maximum reflection loss in λ/2 and λ/6 element spacing

Therefore, sub-wavelength element spacing has been approved to significantly reduce the ohmic loss from the series resistor in the varactor diodes. Furthermore, the use of 1-bit phase shift allows the two phase states to be chosen far away the absorption resonance which further reduces the impact of the ohmic loss.

As can been seen from Figure 4.4, different phase states are related to the different varactor diode capacitances. In order to achieve two reflections phase states of 45° and −135°, the capacitance of the varactor diodes are set to be 0.37 pF and 2.22 pF, respectively. The revised bias voltage across the varactor junction controls the capacitance of varactor diodes, and thereby changing the voltage across the varactor diodes is able to achieve various capacitances. The relation between the varactor diodes capacitance and reverse voltage across the varactor junction is shown in Figure 4.6.

Figure 4.7 shows the relationship between the phase and amplitude response of reflection coefficient and the voltage across the varactor junction. As mentioned in Skyworks Silicon SMV2019-079LF hyperabrupt varactor diodes datasheet, the variance of its capacitances is about ±10%. These variances have to be taken into account to ensure that neither phase state was sensitive to variation in varactor diodes parameters (especially when cheap silicon diodes are used) in the practical design and the effects of capacitances variances (±10%) on the amplitude and phase response of the scattered fields also plot in Figure 4.7.

Table 4.3 summarizes the maximum and minimum phase and amplitude of the reflection coefficient.
Figure 4.6: Capacitance vs reverse voltage for SMV2019-079LF varactor diode

(a) Phase response

(b) Magnitude response

Figure 4.7: Phase and magnitude response of the reflection versus the voltage

<table>
<thead>
<tr>
<th>States</th>
<th>Desired</th>
<th>Min</th>
<th>Max</th>
<th>Desired</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.22 pF</td>
<td>-137.6</td>
<td>-140.8</td>
<td>-133.6</td>
<td>-0.97</td>
<td>-1.0</td>
<td>-0.92</td>
</tr>
<tr>
<td>0.37 pF</td>
<td>45.5</td>
<td>38.4</td>
<td>52.75</td>
<td>-0.46</td>
<td>-0.56</td>
<td>-0.36</td>
</tr>
</tbody>
</table>

Table 4.3: Phase and magnitude response to the variances

caused by ±10% capacitance variances at the two interested states. As can been seen, the amplitude of
the reflection coefficients vary a little bit and are still in a reasonable range. The maximum and minimum
phase ranges due to the ±10% capacitance variances are 193.5° and 172°, respectively. The effect of
these variances in phase range on the directivity pattern need to be found out and the corresponding
directivity can be computed using array factor calculation (discussed later). It turns out that this small
amount of phase between two states does not change the directivity pattern very much and therefore
the capacitances variances can be safely ignored.

Additionally, as previously discussed in Chapter 2 the feed of the electromagnetic surface need
to be offset to reduce the blockage, but an offset feed will introduce more oblique incidence to the
electromagnetic surface compared to a centre fed system. Therefore, the amplitude and phase response
of the scattered fields to various incidence angle (ϕ = 0° and θ = 0°, 10°, 20° and 30°) versus varactor
diodes capacitance need to be analyzed and the corresponding results are shown in Figure 4.8. A slight variation of the amplitude and phase response of reflection coefficient of the electromagnetic surface unit cell has been observed, while changing the angle of the incidence fields.

\[\Gamma = \frac{\sqrt{\epsilon_r \cos \theta_i} - \sqrt{\epsilon_0 \cos \theta_t}}{\sqrt{\epsilon_r \cos \theta_i} + \sqrt{\epsilon_0 \cos \theta_t}},\]  

where \(\epsilon_r\) is the dielectric constant of the substrate and \(\epsilon_0\) is the permittivity of free space. Therefore, when the dielectric constant \((\epsilon_r)\) of the substrate is 12.85, about 60% power potentially goes to the specular direction and this factor significantly reduces the radiation efficiency of the reflectarray antenna. Consequently, a reflectarray unit cell with a lower dielectric constant of its substrate needs to be designed to reduce the specular reflection with achieving the same amplitude response in both phase states.

**Square Loop**

After showing a simple working dipole reconfigurable electromagnetic surface unit cell, this sub-wavelength dipole unit cell has been revealed some drawbacks such as high dielectric constant \(\epsilon_r = 12.85\) which can lead to excessively large specular reflections. Additionally, it is desirable to produce the same reflection coefficient magnitude in both phase states. To address the first point, the unit cell should be design to use a substrate with a dielectric constant with a lower value. One potential solution is the loop which is electrically longer than the dipole, so to achieve the same electrical length of the dipole, a smaller dielectric constant can be used. Therefore, the optimized design is proposed using a square loop resided on a grounded dielectric substrate as shown in Figure 4.9.
Chapter 4. Low-cost and Low-loss Reconfigurable Electromagnetic Surface

Figure 4.9: Geometry of optimal reconfigurable electromagnetic surface unit cell

The operating frequency is still 5.5 GHz, and then the size of the unit cell stays the same. The optimized electromagnetic surface unit cell consists of a square loop split in half with varactor diodes and the RF short circuit positioned across the gap as shown in Figure 4.9. The varactor diodes are oriented parallel to the direction of current flow (y) on the square loop, and can be seen as tunable capacitive loading to the square loop. The RF short circuit is a capacitor which functions as a short circuit at the operating frequency (5.5 GHz) and high impedance circuit at all the others frequencies. In order to minimize the ohmic loss and obtain enough phase range over the capacitance range of the diode, the dielectric constant $\epsilon_r$ of the substrate has been chosen as 6.15 and thickness $h$ of the substrate is 1.524 mm. The inter and outer length of the square loop have been designed to be 7.5 mm and 6.5 mm, respectively. A summary of the parameters from the square loop unit cell are shown in Table 4.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Square loop unit cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element spacing/ periodicity</td>
<td>10 mm ($\lambda$/6)</td>
</tr>
<tr>
<td>Substrate permittivity ($\epsilon_r$)</td>
<td>6.15</td>
</tr>
<tr>
<td>Substrate height ($h$)</td>
<td>1.524 mm</td>
</tr>
<tr>
<td>Outer length ($L$)</td>
<td>7.5 mm</td>
</tr>
<tr>
<td>Inter length ($W$)</td>
<td>6.5 mm</td>
</tr>
</tbody>
</table>

Table 4.4: Parameters of square loop unit cell

Figure 4.10 shows the amplitude and phase response of reflection coefficient versus varactor diode capacitance in the square loop unit cell at 5.5 GHz. As can be seen, the total phase range is about 250 degrees, which is enough for the required phase range of single bit phase shifter and the amplitude response possesses small loss at resonance (dips in the amplitude curve). To minimize the ohmic loss and achieve sufficient phase range, two phase states have been chosen as $60^\circ$ and $-120^\circ$ in single bit phase shifter. Theses two states could be optimized later to compensate phase loss (discussed later). The corresponding capacitances values of varactors diodes, amplitude and phase of the scattered field...
are shown in Table 4.5 along with the comparison to the dipole reconfigurable electromagnetic surface unit cell. Compared scattered fields between the dipole unit cell and square loop unit cell, the loss from the series resistor in the varactor are almost the same and square loop unit cell achieves a slightly large phase range.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Square Loop</td>
<td>0.37</td>
<td>0.96</td>
<td>63</td>
<td>-118</td>
<td>-0.92</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-0.85</td>
<td>250</td>
<td>1.8</td>
</tr>
<tr>
<td>Dipole</td>
<td>0.38</td>
<td>2.22</td>
<td>45.5</td>
<td>-137.6</td>
<td>-0.43</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>-1.0</td>
<td>190</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 4.5: Unit cell performance comparison between square loop and dipole

As shown in Figure 4.11, when the incidence angle to the square loop surface increases, the ohmic loss from the varactor increases and also the phase range increases as well. Overall, the phase and magnitude response of the oblique incidence up to $\theta = 30^\circ$ still reach a reasonable loss and enough phase range.

The overall performance of the square loop reflectarray unit cell is acceptable with much lower dielectric constant of the substrate compared to the dipole case. The ohmic loss has been significantly reduced compared to $\lambda/2$ element spacing and meanwhile a sufficient phase range for a single bit phase shifter has been obtained.

### 4.1.5 Phase Error

As discussed in Section 4.1, the variances of the capacitances of the varactor diodes result in variances of phase difference between the two capacitance states. Here, we will analyze the effect of this phase variances on the directivity pattern using array factor calculation Equation (2.14). The two phase in the single bit phase shifters can be determined as follows. For example, if the phase difference between the two states is $180^\circ$, the two phase are selected as

$$
\Phi_{1\text{bit}} = \begin{cases} 
45^\circ & -45^\circ \leq \Phi_{\text{Continuous}} < 135^\circ \\
-135^\circ & \text{otherwise.}
\end{cases}
$$

(4.2)
Figure 4.11: Phase and magnitude response to oblique incidence angle.

To demonstrate this effect, a general 2D finite planar array with $30 \times 30$ element ($5\lambda \times 5\lambda$) will be analyzed. This finite planar array is fed with a spherical wave about $5\lambda$ away from the centre and the designed beam direction is $\theta_b = 20^\circ$ and $\phi_b = 0^\circ$. Figure 4.12a shows its directivity pattern with single bit phase shifter, where the phase differences between the two states are $160^\circ$, $180^\circ$ and $200^\circ$, respectively.

Figure 4.12: Directivity pattern with different phase range

(a) Directivity pattern with different phase range
(b) Directivity zoomed in

From Figure 4.12, the directivity for each case at the main beam direction are very close and the side lobe levels in both $160^\circ$ and $200^\circ$ cases are slightly increased. Overall, the variances of the phase difference between two states do not change the directivity pattern very much and therefore the impact of variances of capacitances is negligible.
4.2 Analysis of Finite Electromagnetic Surfaces

With the general design for the reconfigurable electromagnetic surface unit cell completed using infinite array analysis, the next step is to design and simulate a finite array. First, a linear array of the varactor loaded square loop elements will be designed and simulated in a parallel plate waveguide. The behaviour of single bit phase shifter is compared to the continuous phase shifter case and the directivity reduction due to both quantization and ohmic loss is also computed. Then, the reconfigurable ability of single bit phase shifter is demonstrated. Linear array in a parallel plate waveguide minimizes the edge effect compared to two dimension finite electromagnetic surface. Later, the array is made finite in two-dimensional. 2D finite electromagnetic surface will be designed and simulated. But, first of all, a thorough investigation, including the feed location and reference phase, is carried out to improve the radiation performance of reconfigurable electromagnetic surface such as directivity and side lobe level, assuming that the feed produces a good illumination efficiency in each case.

4.2.1 Focal Length

In the conventional reflectarray designs, the reflectarray aperture is usually placed in the far-field region of the feed. The incident field on each element can be considered as a local plane wave with the phase delay proportional to the spatial distance from the feed. Normally, the gain can be optimized by properly choosing the focal length to balance the spillover efficiency and tapper efficiency. If the feed is too far from the reflectarray, it suffers from significant spillover losses. If the feed is too close from the reflectarray, taper losses become pronounced. Although the focal length does not seem like a degree of freedom, it is still worthwhile to pointing out that the focal length also plays a role in the directivity reduction at the main beam in the single bit phase distribution, and to demonstrate this effect, the directivity with various focal lengths has been calculated based on pattern factor assuming that the feed produces a good illumination efficiency. Since both 1D finite linear array and 2D finite planar array will be designed and simulated later in this chapter, the effects of focal length on both 1D finite linear array and 2D finite planar array are discussed here.

1D Finite Linear Array

The length of the 1D finite linear array is chosen as 10\(\lambda\), and the element spacing is \(\lambda/6\), which is same as the proposed electromagnetic surface. Then, the number of element is 60. The offset feed is located at various distances \((f/D)\) above, where \(D\) is the size of the linear reflectarray and \(f\) is defined as the focal distance from the feed to the reflectarray aperture, above the aperture and generates a spherical wave as shown in Figure 4.13. This linear reflectarray is designed to produce a beam at \(\theta_b = 20^\circ\). The required phase shift on each element is calculated based on Equation (2.13). The two phase states in the single bit phase quantization are chosen to be \(60^\circ\) and \(-120^\circ\) for all the cases.

The directivity patterns with various \(f/D\) ratios are plotted in Figure 4.14. As can be seen, the directivity and side lobe level of the electromagnetic surface fed with various \(f/D\) ratios are different. In this particular test case with two phase states chosen as \(60^\circ\) and \(-120^\circ\), the smaller feed distance is, the better directivity performance is.

Table 4.6 summarizes the directivity and side lobe level for all four different \(f/D\) cases. In this 1D finite linear array, feed distance at \(f/D = 0.5\) achieved the best performance in terms of directivity and side lobe level using \(60^\circ\) and \(-120^\circ\) two phase states without considering the mutual coupling. The only
difference among all the case is the focal length which leads to the different phase distributions, and the reason behind is when the $f/D$ ratio increases, the incident phase variation on the electromagnetic surface becomes smaller. The smaller incident phase variation leads a closer required phase shift between each element and a small phase shift range over the aperture, which result in large phase quantization errors in single bit phase shifters (examples of phase distribution will be shown later in 2D finite planar array).


2D Finite Planar Array

The size of the 2D finite planar array is $3\lambda \times 3\lambda$, and the element spacing is $\lambda/6$. Then, the total number of the elements is $18 \times 18$. The feed is at the centre of the aperture with various distances above and also generate a spherical wave. This electromagnetic surface is designed to produce a beam at $\theta_b = 20^\circ$ and $\phi_b = 0^\circ$. The two phase states in the single bit phase quantization are chosen as $45^\circ$ and $-135^\circ$ for all cases.

![Figure 4.15: Directivity pattern of the 2D finite planar array with different $f/D$ ratios](image)

Figure 4.15 plots the directivity pattern with various focal lengths. As can be seen in Figure 4.15, this electromagnetic surface fed with different focal lengths produces different radiation performance in terms of directivity and side lobe level.

<table>
<thead>
<tr>
<th>$f/D$</th>
<th>Directivity [dB]</th>
<th>Side Lobe Level [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>19.49</td>
<td>-14.9</td>
</tr>
<tr>
<td>1.0</td>
<td>18.73</td>
<td>-10.3</td>
</tr>
<tr>
<td>1.5</td>
<td>18.44</td>
<td>-7.26</td>
</tr>
<tr>
<td>2.0</td>
<td>17.69</td>
<td>-2.77</td>
</tr>
</tbody>
</table>

Table 4.7: Directivity and side lobe level with various $f/D$ ratios

Table 4.7 summarizes the directivity and side lobe level for all the different $f/D$ ratios cases. As mentioned in 1D linear array case, the only difference is the phase distribution on the aperture. In order to find the reason, Figure 4.16 shows the required phase distribution on the electromagnetic surface elements with various $f/D$ for both continuous and 1 bit phase distribution ($45^\circ$ and $-135^\circ$).

As can be seen, the required phase shift on the reflectarray element in the vertical direction ($y$) with $f/D = 2$ has the smallest phase range among all the cases. If the phase range of the required phase shift in the vertical direction is too small ($f/D = 2$ case), the required phase shifts may quantize into the same phase state after the single bit phase quantization. Therefore, large single bit phase quantization errors are introduced in $f/D = 2$ case, and then its radiation performance should be the worse. It has been proved in Figure 4.15 when the feed is 2 D away from the aperture, the directivity and side lobe level
are the lowest among the four different focal lengths. On the other hand, the required phase shift on the reflectarray element in the vertical direction ($y$) with $f/D = 0.5$ has the largest phase range among all the cases, small single bit phase quantization errors are introduced, and then its radiation performance is the best as shown in Figure 4.15. Consequently, the radiation performance of an electromagnetic surface is related to the single bit phase distribution on the elements and the phase distribution is dependent on the focal length. In practice, the focal length is choosing based on the feed, and the selected feed needs to achieve a good illumination efficiency. But, if we have a choice on the focal length, and then it is worthwhile to point out that the focal length in 1 bit phase quantization could play a role on the directivity and the side lobe level in reflectarray designs.

### 4.2.2 Reference Phase

For a general reflectarray antenna, the required compensation phase $\phi_R(x_i, y_i)$ for the $i^{th}$ element must satisfy,

$$\phi_R(x_i, y_i) = -k_0(d_i - (\cos \varphi_b x_i + \sin \varphi_b y_i) \sin \theta_b) + \Delta \phi.$$  \hspace{1cm} (4.3)

where $k_0 d_i$ is the phase delay corresponding to the spatial distance from the feed to the $i^{th}$ element, $\varphi_b$ and $\theta_b$ are the designed beam direction, and $x_i$ and $y_i$ are the position of the $i^{th}$ element. It should be emphasized that a constant reference phase ($\Delta \phi$) provides additional design freedom because it is only the relative phasing between the elements that matters.

For a reflectarray using ideal elements with $360^\circ$ full-phase coverage, different reference phases do not change the phase difference between any two arbitrary elements on the aperture. However, there are only two phase states in a single bit phase shifter. After conducting 1 bit quantization of the required compensation phase on each element, different reference phases may result in different phase distributions. More importantly, the relative phase between some elements may change with different
reference phases after quantization. First, the effects of the reference phase to the radiation behaviour in the 1D finite linear array will discuss and followed by the effects to the 2D finite planar array.

1D Finite Linear Array

The offset feed of the same 1D finite linear array is location at \( f/D = 1.1 \) and illuminates with a spherical wave. The designed beam direction is still \( \theta_b = 20^\circ \). The required phase shift on each element is calculated from Equation (4.3) with different reference phases. After 1 bit phase quantization, the phase distribution will be different and results in different radiation performance (Details will be discussed later in 2D finite planar array). Figure 4.17 demonstrates the directivity pattern when reference phase is chosen as \(-45^\circ, 0^\circ, 45^\circ\) and \(90^\circ\), respectively.

![Figure 4.17: Directivity pattern of the 1D linear array with various reference phases](image)

Table 4.8: Directivity and side lobe level with various reference phases

<table>
<thead>
<tr>
<th>Reference phase</th>
<th>( \Delta \phi = -45^\circ )</th>
<th>( \Delta \phi = 0^\circ )</th>
<th>( \Delta \phi = 45^\circ )</th>
<th>( \Delta \phi = 90^\circ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side Lobe Level [dB]</td>
<td>-7.85</td>
<td>-7.46</td>
<td>-7.4</td>
<td>-6.38</td>
</tr>
</tbody>
</table>

It is observed that the maximum theoretical directivity with two phase distribution is 14.72 dB for \( \Delta \phi = 45^\circ \) and the minimum directivity is 12.78 dB for \( \Delta \phi = 90^\circ \). Optimization of the reference phase increases the directivity about 1.94 dB in this 1D finite linear array. Additionally, the side lobe level also has been improved from -6.38 dB to -7.4 dB.
2D Finite Planar Array

Now, let’s consider a 2D finite planar array and its size is $3\lambda \times 3\lambda$ with $\lambda/6$ element spacing as well. The feed is at the centre of the aperture with $f/D = 0.5$ and illuminates with a spherical wave. The designed beam direction is also $\theta_b = 20^\circ$ and $\phi_b = 0^\circ$.

Figure 4.18: Continuous and 1 bit phase distribution with different $\Delta \phi$

Figure 4.18a represents the required phase shift on each element of the array. Figure 4.18b, 4.18c and 4.18d show the 1 bit phase quantization with different $\Delta \phi = 0^\circ$, $45^\circ$, and $135^\circ$, respectively and as can been seen, the 1 bit phase distributions in these three cases are different. More importantly, the relative phase between some elements may change with different $\Delta \phi$ after quantization. For example, the phase difference between the element (1,1) and element (4,1) elements is $0^\circ$ when $\Delta \phi = 0^\circ$, but the phase difference between the same two elements is $180^\circ$ when $\Delta \phi = 45^\circ$ and $135^\circ$. As a result, the radiation performance of the reflectarray may changes accordingly. As shown in Figure 4.18 the 1 bit phase distribution with $\Delta \phi = 45^\circ$ is the most closed one to the continuous phase distribution, and therefore the corresponding directivity pattern should be the best one.

The directivity pattern is computed from pattern factor calculation and the directivity of the reflectarray with different reference phases using single bit phase distribution and continuous phase distribution at 5.5 GHz are plotted in the Figure 4.19.

Table 4.9 summarizes the directivity and the side lobe level for both continuous and 1 bit phase
quantization with different reference phases. It is observed that the maximum theoretical directivity out of the three considered phase distribution is 19.49 dB for $\Delta\phi = 45^\circ$, which proves the prediction, and the minimum directivity is 17.94 dB for $\Delta\phi = 135^\circ$. Optimization of the phase offset increases the directivity about 1.5 dB in this case. Additionally, the side lobe level also has been improved from -9.46 dB to -14.6 dB. Consequently, the radiation performance of an electromagnetic surface is related to the single bit phase distribution on the elements and this phase distribution is dependent on the reference phases. Therefore, it is also worthwhile to point out that the optimization of the reference phase in 1 bit phase quantization is particular useful in reconfigurable reflectarray designs to improve the directivity and the side lobe level.

In summary, as the focal length of the electromagnetic surface changes, the phase distribution on the aperture changes as well, and then the reference phase ($\Delta\phi$) needs to be optimized for this phase distribution on the aperture. In other words, as mentioned before, the focal length is constrained by the feed, and then the radiation performance can be optimized by the reference phase.

As shown in Figure 4.20a, the directivity in the 1D finite ($10\lambda$) linear array is a function of both focal length ($f/D$) and reference phase. The maximum and minimum directivity in all these cases are 15.16 dB and 12.5 dB, respectively, and thereby the directivity has been improved by 2.66 dB after theoretical optimization of the feed position and reference phase. The directivity of this 1D linear array using a continuous (un-quantized) phase distribution is 16.31 dB, and therefore the directivity reduction due to 1 bit phase quantization is only 1.15 dB. In the 2D finite ($3\lambda \times 3\lambda$) planar array case,
its directivity is also a function of both focal length \( (f/D) \) and reference phase. Figure 4.20b shows how the directivity changes with the reference phases after the focal length \( (f/D) \) is selected. The maximum and minimum directivity in all cases are 19.49 dB and 17.19 dB, respectively, and thereby the directivity has been improved by 2.66 dB after theoretical optimization of the focal length and reference phase. The directivity of this 2D planar array using a continuous phase distribution is 21.19 dB, and therefore the directivity reduction due to 1 bit phase quantization is only 1.6 dB. However, the analysis of the effect of the focal length and reference phase on the directivity is based on the array factor calculation. The effect of reference phase on the directivity may vary in the actual design, because this analytical analysis does not consider mutual coupling.

### 4.2.3 One-Dimensional Finite Electromagnetic Surface Design and Simulation

First, a 1D linear reflectarray with varactor loaded square loop elements will be designed and simulated in a parallel plate waveguide, and followed by a so called plane wave re-director. Plane wave re-direction is another application of the reflectarray, but as shown in its name, a plane wave re-director is excited with plane wave instead of spherical wave in the reflectarray. Both the reflectarray and plane wave re-director are modelled and simulated using Ansys HFSS software. The single bit phase quantization is implemented in both reflectarray and plane wave re-director. The radiation behaviour of single bit phase distribution is compared to the continuous phase distribution case and the directivity reduction due to the ohmic loss is also computed. Then, the reconfigurable ability of single bit phase shifters will be demonstrated.

**Reflectarray**

Figure 4.21 shows a linear square loop array model in HFSS. The two surfaces parallel to the \( yz \)-plane are set to be a perfect electric conductor (PEC), and then the array is mirrored (infinite) in the \( x \)-direction and finite in the \( y \)-direction. The size of this linear reflectarray is 600 mm (about 10\( \lambda \)) \( \times \) 10 mm. The element spacing is 10 mm, and then the number of cells is 60. Dipole is used as a feed. The offset dipole is excited with cylindrical wave source (line source) and located at 412 mm \( (f/D = 0.68) \) above the
reflectarray surface. The beam direction is first designed at θ_b = 0°. The height of the feed antenna was chosen based on the optimization among available reference phases and the reasonable incidence angle to the elements. According to the analysis of the square loop unit cell in Section 4.1, the phase range response of scattered fields in the available varactor diodes capacitances is about 250 degrees. Therefore, the available reference phase is limited. Besides this, if the feed of the reflectarray is too close to the aperture, the incidence angle to some elements may be too large, especially when the feed is offset. The large incidence angle may result a different phase and amplitude response of the scattered field. Two phase shift states of all elements on the 1D square loop reconfigurable reflectarray are chosen as 75° and −105° according to the analysis of reference phase. The height of the parallel plate waveguide is 500 mm, which is slightly higher than the feed antenna.

Because the incident wave is a cylindrical wave, the required phase shift on each element can be calculated based on Equation (2.13). A plot of the required phase shift on each element is shown in Figure 4.22a. The two desired phase states are 75° and −105°, and then any required phase shift which is between 165° and −15° in the continuous phase distribution quantizes to 75° in the single bit phase shifter, and others required phase shift quantizes to 105°. A plot of the 1 bit phase quantization on each element is shown in Figure 4.22b.
Figure 4.23: Phase and magnitude response to oblique incidence angle

<table>
<thead>
<tr>
<th>Directivity</th>
<th>Mean Beam [dB]</th>
<th>Side Lobe Level [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuous phase shifter</td>
<td>10.63</td>
<td>-13.31</td>
</tr>
<tr>
<td>1 bit phase shifter no loss</td>
<td>7.65</td>
<td>-7.65</td>
</tr>
<tr>
<td>1 bit phase shifter with loss</td>
<td>7.12</td>
<td>-5.42</td>
</tr>
<tr>
<td>1 bit phase shifter with loss and delta sigma</td>
<td>6.01</td>
<td>-8.02</td>
</tr>
</tbody>
</table>

Table 4.10: Characteristic of directivity pattern

Figure 4.23 shows the directivity pattern of this linear square loop array in three scenarios which are continuous phase distribution without loss, single bit phase distribution without loss and single bit phase distribution with loss. The directivity at the main beam and the side lobe level in all three cases are summarized in Table 4.10. As can be seen from Table 4.10, the directivity reduction due to the single bit phase quantization errors is 2.97 dB which is smaller than the $\lambda/2$ case in [8] [17] [16] [32].

The directivity of linear square loop array implemented with 1 bit phase shifters without loss (no series resistor in the varactor) is 7.65 dB and the one with loss case (5 Ω series resistor in the varactor) is 7.12 dB. Therefore, only a 0.53 dB directivity reduction has been introduced due to the 5 Ω series resistance of the loaded varactor diodes. The series resistance of the varactor diodes has been turned out to be the most significant source of loss in reflectarray design [8] and it states the radiation loss is 1.8 dB with only 0.7 Ω series resistance. The directivity reduction due to the radiation loss is already improved by 1.27 dB, although the series resistance of the varactor diodes is 5 Ω. If the series resistance of the varactor diodes in the proposed design is 0.7 Ω, the radiation loss due to the ohmic loss can be negotiable. Table 4.11 summarizes the radiation loss of the design [8] implemented GaAs varactor diodes (low resistance and expensive) and the proposed design employing Si varactor diodes (high resistance and cheap).

It is shown in Figure 4.23 that the minor lobe level around $\theta = 25^\circ$ is higher than the ones at the other angles. The reason behind could be the phase errors from the mutual coupling, and these phase errors maybe enlarged in the single bit phase quantization. Delta sigma modulation has been
applied to improve the directivity in the main beam and the poor side lobe, however as shown in Figure 4.23 the directivity reduces about 1 dB compared to the one without delta sigma. It turns out that delta sigma cannot improve the collimated single bit reflectarray behaviour and the possible reason could be the phase distribution on the reflectarray aperture which is a parabolic distribution. As discussed in Section 2.2.3 delta sigma modulation is able to reduce the quantization lobes from linear array and its phase distribution is linear periodic distribution which will introduce quantization lobes. However, in reflectarrays, the phase distribution is not periodical which won’t introduce quantization lobes. Therefore, delta sigma modulation is not able to improve the radiation behaviour.

Table 4.11: Radiation loss in GaAs and Si cases

<table>
<thead>
<tr>
<th></th>
<th>GaAs varactor diodes (λ/2)</th>
<th>Si varactor diodes (λ/6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radiation Loss [dB]</td>
<td>1.8</td>
<td>0.53</td>
</tr>
<tr>
<td>Cost</td>
<td>Expensive! × 10</td>
<td>Cheap</td>
</tr>
</tbody>
</table>

To demonstrate the reconfigurability of this linear reflectarray, the beam directions have been designed to be $\theta = -20^\circ$, $\theta = -10^\circ$, $\theta = 0^\circ$, $\theta = 10^\circ$, and $\theta = 20^\circ$. The directivity patterns of various beam directions are plotted in Figure 4.24. As can be seen, the 1D linear reflectarray with 1 bit phase shifters is able to produce a beam scanning with slightly high side lobe level. More importantly, the radiation loss (ohmic loss) is significantly reduced in this sub-wavelength element spacing and single bit phase quantization reconfigurable reflectarray.

**Plane Wave Re-director**

As shown in Figure 4.25 the HFSS simulation setup of the 1D finite plane wave re-director is same as the linear reflectarray one, except that plane wave re-director is excited with plane wave instead of spherical...
wave in reflectarrays. The plane wave re-director was illuminated with a plane wave at broadside, and the beam direction is designed at $\theta_b = -20^\circ$. The required phase shift on each element is calculated based on Equation (2.18), and the two phase quantization states are chosen as $60^\circ$ and $-120^\circ$ as a starting point. Figure 4.26a and Figure 4.26b show the required phase shift in each element and the corresponding single bit phase distribution, respectively.

![Demonstration of phase distribution](image)

Figure 4.26: The phase distribution on the 1D finite linear plan wave re-director

The directivity patterns of this 1D finite plane wave re-director using continuous phase distribution and single bit phase distribution have been computed, and are shown in Figure 4.27. As can be seen, an undesired beam was produced at broadside because HFSS cannot distinguish the incidence fields and scattered fields with plane wave excitation. An undesired quantization lobe was produced at the mirror angle of the design beam direction, and this quantization lobe has to be reduced. Delta sigma modulation was applied to reduce the quantization lobe in the 1D linear dipole array [26]. Therefore, delta sigma modulation has also been applied to the 1D finite plane wave re-director to reduce the quantization lobe.
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Figure 4.27: Directivity of 1D linear plane wave re-director

Figure 4.26c shows the single bit phase distribution on the aperture with delta sigma modulation. The phase distribution with delta sigma modulation breaks the periodical single bit phase distribution, which may reduce the quantization lobe. The directivity pattern of the 1D finite plane wave re-director with single bit phase distribution and delta sigma modulation is plotted in Figure 4.27 and compared to the one without delta sigma modulation. As can be seen, the quantization lobe has been successfully reduced after implementing delta sigma modulation.

In summary, a 1D linear finite electromagnetic surface has been designed and simulated in this section, the results show that the reconfigurable reflectarrays using single bit phase quantization with sub-wavelength element spacing are able to reduce the ohmic and quantization losses significantly. A plane wave re-director has been designed with delta sigma phase modulation and the quantization lobe has been reduced successfully.

4.2.4 Two-Dimensional Electromagnetic Surface Design and Simulation

A 2D finite electromagnetic surface is made finite in both the $x$-direction and $y$-direction shown in Figure 4.28a which is more practical than 1D linear array which is infinite in $x$-direction. Additionally, the 2D finite electromagnetic surface takes the edge effect into account. The 2D finite electromagnetic surface is also modelled and simulated using Ansys HFSS software. The directivity pattern obtained from HFSS is compared to the one calculated from an array factor calculation using both single bit phase distribution and continuous phase distribution.

Figure 4.28 shows the 2D finite square loop array model in HFSS. The array aperture has dimensions of $D$ along the $x$-axis and $y$-axis. The aperture size, $D$, is about $3\lambda$ and the element spacing is about $\lambda/6$. Then, the aperture size and the element spacing determined the number of reflecting elements, which is $18 \times 18$. A dipole is used as the feed antenna, placed at the centre of the surface with a focal length of $f$ above the reconfigurable electromagnetic surface, and excites a spherical wave directed towards the reflectarray. According to the analysis in the Section 4.2.1, the focal length $f$ has been chosen to be 0.68 $D$ as a starting point. The beam direction is designed at $\theta_b = 20^\circ$ and $\phi_b = 0^\circ$. 
Since the incident wave is a spherical wave, the required phase shift on each element is calculated based on Equation (2.13). A plot of the required phase shift on each element is shown in Figure 4.29a. Two phase shift states in the single bit phase shifter for all elements on the 2D electromagnetic surface has been chosen as $45^\circ$ and $-135^\circ$ in order to minimize the ohmic loss and optimize the reference phase. The corresponding 1 bit phase distribution is plotted in Figure 4.29b.

Table 4.12 summarizes the maximum directivity and side lobe level in all the three cases. By comparing the array factor results, single bit phase shifters result in a directivity reduction in the main beam about 3.08 dB compared to the continuous case. This directivity reduction is caused by the phase errors from the single bit phase quantization. As can be seen from Figure 4.30, the maximum directivity of the full wave simulation with single bit phase shifters is 1.55 dB lower than the array factor calculation. The
Chapter 4. Low-cost and Low-loss Reconfigurable Electromagnetic Surface

![Figure 4.30: Directivity of 2D reflectarray](image)

<table>
<thead>
<tr>
<th></th>
<th>Mean Beam [dB]</th>
<th>Side lobe level [dB]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Array factor with continuous phase shifter</td>
<td>21.09</td>
<td>-15.27</td>
</tr>
<tr>
<td>Array factor with 1 bit phase shifter</td>
<td>18.01</td>
<td>-14.45</td>
</tr>
<tr>
<td>Full wave simulation with 1 bit phase shifter</td>
<td>16.46</td>
<td>-13.02</td>
</tr>
</tbody>
</table>

Table 4.12: Characteristic of directivity pattern of 2D reflectarray

The side lobe level is also slightly higher and the main beam direction is slightly off the designed direction (20°). The reason behind could be the mutual coupling between the reflectarray elements.

In conclusion, an interesting discussion about the feed location and reference phase is carried out to improve the radiation performance of reconfigurable electromagnetic surface such as directivity and side lobe level, assuming that the feed is sized to produce a good aperture efficiency in each case. The 1D linear reflectarray with 1 bit phase shifters is able to produce a beam scanning with slightly high side lobe level. More importantly, the radiation loss (ohmic loss) is significantly reduced in this sub-wavelength element spacing and single bit phase quantization reconfigurable reflectarray. It was turned out that delta sigma modulation cannot improve the collimated single bit reflectarray behaviour. 1D plane wave re-director employing single bit phase shifters has been simulated with delta sigma modulation and the results show that the quantization lobe has been reduced. Finally, a 2D reflectarray has been demonstrated, along with the comparison to the array factor calculation.
Chapter 5

Reflectarray Fabrication and Measurements

The final chapter of this thesis discusses the fabrication and measurement of the 2-dimensional single bit reflectarray. The scattered behaviour of the reflectarray has been measured in two different methods, along with simulated and experimental comparisons. The reflectarray measurement setup is presented, including a unique phase shift calculation of 1D beamforming in a 2D reflectarray. A prototype reflectarray is measured with beam scanning and followed by a comprehensive analysis of efficiency and loss. The measurement results are compared with array factor predictions. Finally, the experimental performance is compared with the theoretical one.

5.1 Reflectarray Fabrication

The 2-dimensional reflectarray design is a 160 mm × 160 mm (16 × 16 element) reflectarray, as shown in Figure 5.1. The reflectarray element design is shown in Figure 5.1b. The element is a square loop operating at 5.5 GHz residing on a grounded dielectric substrate. The substrate was chosen to be Rogers RO4360G2 (\(\epsilon_r = 6.15\)) and the substrate height is 1.524 mm, as presented in Section 4.1. The unit cell is a square with a periodicity of 10 mm in both directions (about \(\lambda_0/6\) at 5.5 GHz). As shown in Figure 5.1b, one varactor diode is placed in the middle of one arm of the square loop across 0.5 mm gap cut along the principal direction, and one RF capacitor is placed in the middle of the other arm of the square loop across 1 mm gap cut along the principal direction. The varactor is chosen to be a Skyworks Silicon SMV2019-079LF hyperabrupt varactor diode, which is much cheaper than the GaAs ones, and this varactor diode possesses a capacitance range of 0.3 to 2.2 pF when reverse biased from 0-20 V. This RF capacitor works like a short circuit at the operating frequency (5.5 GHz), and functions as a high impedance circuit at the other frequencies. Two vias connect the top layer (elements) to the bottom layer (ground plane). One via connects to the ground plane (0 V) and the other via connects to the voltage controller. A DC bias network is located at the varactor diodes to isolate RF currents on the square loop from the biasing circuitry. A symmetrical biasing structure is used to reduce cross-polarization effects, as suggested by previous symmetrical element designs [33]. In our design, the DC bias network includes two high impedance resistors (10K \(\Omega\)) which connect two pads of the vias to the square loop element. A summary of the parameters of the reflectarray unit cell is shown in Table 5.1.
Chapter 5. Reflectarray Fabrication and Measurements

Figure 5.1: Reflectarray layout

(a) 16 × 16 reflectarray layout
(b) Reflectarray unit cell layout

Table 5.1: Parameters of reflectarray unit cell

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Reflectarray unit cell</th>
</tr>
</thead>
<tbody>
<tr>
<td>Element spacing/periodicity</td>
<td>10 mm (about λ₀/6)</td>
</tr>
<tr>
<td>Substrate permittivity (εᵣ)</td>
<td>6.15</td>
</tr>
<tr>
<td>Substrate height (h)</td>
<td>1.524 mm</td>
</tr>
<tr>
<td>Square loop outer length</td>
<td>8.33 mm</td>
</tr>
<tr>
<td>Square loop inner length</td>
<td>0.67 mm</td>
</tr>
<tr>
<td>Varactor diodes</td>
<td>0.3 - 2.22 pF (SMV2019-040LF)</td>
</tr>
<tr>
<td>RF capacitor</td>
<td>2 pF (0603)</td>
</tr>
<tr>
<td>High impedance resistors</td>
<td>10K Ω (0603)</td>
</tr>
</tbody>
</table>

A 40 pin cable connector was attached to a bus at the side of the reflectarray which supplies the reflectarray elements with analog control voltages. As mentioned before, the number of the reflectarray elements is 16 × 16, and to simplify the biasing control network, entire rows of elements are biased (phased) at a time. That is, each row of elements is connected to the same pin in the 40 pin cable connector as shown in Figure 5.2.

The reflectarray elements in the same row are connected using jumper wires and only 16 pins were using in the connector since this reflectarray only has 16 rows. This eliminated the need for a more complicated multilayer circuit board design. The fabricated reflectarray is shown in Figure 5.3. Figure 5.3a shows the front view of the fabricated reflectarray and Figure 5.3b shows the back view.

5.2 Experimental Element Scattered Behaviour

The next step was to experimentally measure the scattered behaviour of the reflectarray elements. A common way to characterize reflectarray unit cell is to use a metallic rectangular waveguide measurement setup where the reflectarray element is placed at the termination of the waveguide. A WR-187 waveguide
is the candidate with inner dimensions of 47.50 mm by 22.10 mm and its operating frequency range is from 3.95 GHz to 5.85 GHz. However, the size of the reflectarray unit cell is about $\lambda/6$, and it is necessary to include five unit cells in the long direction and two unit cells in the short direction, in order to provide a closer approximation to the actual periodicity of the design. This simulation has been analyzed in full wave simulator (HFSS), but the result exhibited significantly different behaviour as compared to an ideal unit cell simulation (periodic boundary condition). The reason behind this could be that the sub-wavelength reflectarray elements are sensitive to the unit cell size. It is not possible to pack a round number of unit cells in a standard waveguide with the desired periodicity, creating periodic errors in the element distribution. An alternative approach to measure the scattered behaviour of the reflectarray elements is to place an open-ended metallic rectangular waveguide close to the reflectarray aperture as shown in Figure 5.4a and measure the reflection coefficient. The other alternative approach is to measure the phase and magnitude of the main beam in the specular direction in the far field when the uniform voltage across all the varactors in the reflectarray is established, thereby creating a homogeneous reflecting surface. Both of the latter techniques were investigated.

In the open-ended waveguide method, the magnitude and phase of the reflection coefficient were measured using an Agilent N5244A network analyzer. The calibration of the network analyzer is only up to the waveguide connection point, and therefore, the measurement may exhibits high losses due to the loss in the waveguide and the small gap (6 mm) of the air between the waveguide and the reflectarray aperture. The proposed solution is that the reflection coefficient of a ground plane located at the same plane of the reflectarray aperture, was measured first shown in Figure 5.4b. The measured reflection coefficient was chosen as a reference which accounts for the loss and phase delay from the waveguide and the air gap. The capacitance of varactor diodes was controlled by the biasing voltage through a voltage controller. The voltage controller includes three Measurement Computing USB-3114 Analog Output Devices with 16 analog outputs each which were used to produce analog outputs in the range of 0-10 V and an op-amp array to amplify the 0-10 V signals to 0-20 V. The controller interfaces to the computer via USB and is easily controlled using MATLAB.
Chapter 5. Reflectarray Fabrication and Measurements

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(a) Front view of the fabricated reflectarray elements

(b) Back view of the fabricated reflectarray elements

Figure 5.3: The fabricated reflectarray

The magnitude and phase of reflection coefficient for four different reflectarray areas has been measured to make sure the measurements over different parts of the reflectarray are consistent and is shown in Figure 5.5. “Position 1”, “2”, “3” and “4” in Figure 5.5 represent the four areas as shown in Figure 5.3a. There is an excellent correlation among the phase response of the reflection coefficient for these four areas. The magnitude response of the reflection coefficient for these four areas is similar, and 1 dB discrepancy in magnitude could be caused by the accuracy of this measurement setup and the quality of the fabrication. Overall, there is good correlation among the four measurement results which means the reflectarray is working well in all the examination areas. Additionally, it also can be seen that the reflection losses increase as the phase curve becomes sharp as expected. After taking average of these four measurement results, the phase and magnitude of the reflection coefficient is shown in Figure 5.6 and compared to the simulated results.

The phase curve has been offset to match the starting point of the simulation result for the easy comparison. The measured phase range of this tunable square loop is 230°, and this square loop has a peak reflection loss close to 2.5 dB. The simulated phase range of this element is 213° with a peak reflection loss close to 1.6 dB. The discrepancy between the measured and the simulated results exists in both phase and magnitude response. There are several possible reasons for these differences. First, there may be discrepancies between the simulated and experimental dielectric properties of the substrate. The substrate was simulated using a dielectric constant of 6.15, but there may be a discrepancy in this constant during the varactor diodes assembling in the fabrication process. The varactor diodes were repeatedly soldered using a hot plate at 220°C and this hot environment may have changed the dielectric constant of the substrate. Also, discrepancies may come from the measurement method itself such as the air gap, because this method is not the best method to measure the reflection coefficient. Further analysis and more accurate method such as quasi-optical system is needed in order to find more accurate results. But, overall, there is good correlation between measured and simulated results.

In the second method, the far-field pattern of the reflectarray was measured using near field scanner with the reflectarray uniformly biased (all cells in the same state). Figure 5.7 shows the experimental
setup of the reflectarray with a near-field scanner that can measure the reflectarray’s far-field patterns. The details of this experimental setup will be described later in this chapter. As the uniform voltage cross the entire varactor diodes changed, the phase and magnitude of the main beam (in the specular direction) in the far-field varied accordingly. In order to appropriately estimate the magnitude of the main beam in the far field, the magnitude of main beam from an identical-size ground plane in the far-field was measured and chosen as a reference.

The magnitude and phase comparisons between the measured and the simulated results are shown in Figure 5.5. The phase curve has also been offset to match the starting point of the simulation result for the easy comparison. The measured phase range of this tunable square loop is 255°, and the element has a peak reflection loss close to 11 dB. Compared to the simulation results, this method introduces a large discrepancy in the magnitude of the reflection coefficient, but there is a good correlation in the phase response of the reflection coefficient between the measurement and simulation. The discrepancy in the magnitude mostly like comes from the measurement method itself, since it is strongly sensitive to stray reflections and diffraction effects which can destructively interfere with the desired reflectarray reflection. Therefore, the first method gives more accurate measurement results compared to the second method. For establishing the 1-bit phasing voltages, two voltage states were chosen as 2.5 V and 8.5 V based on the second method, but the phase curves between the first and second method are very close and as mentioned before, the phase range variation does not reduce the directivity of the reflectarray too much. Therefore, these two voltage states are acceptable.

Overall, the measured magnitude of the scattered field proves that the sub-wavelength element spacing significantly reduces the reflection loss (ohmic loss). It is worthwhile to point out that the series resistor in these varactors (Si diodes) is 4.8 Ω which only introduces the maximum reflection loss of about 2.3 dB. Meanwhile, 3.5 dB of reflection loss is observable in a half wavelength element spacing with 1.4 Ω series resistor (GaAs diodes) [3]. Consequently, the inexpensive Si diodes with sub-wavelength (λ/6) element spacing are able to achieve lower reflection loss than the expensive GaAs diodes with half wavelength (λ/2).
Figure 5.5: The magnitude and phase of the reflection coefficient of four areas on the reflectarray

(a) Comparison of phase curves.  
(b) Comparison of magnitude curves.

Figure 5.6: Simulated and experimental comparison of the reflectarray element

5.3 Phase Shift Calculation

In this section, the required phase shift calculation for 1D beamforming of this 2D reflectarray will be discussed.

The reflectarray was mounted on the support structure as shown in Figure 5.9. The reflectarray elements in the same column (in y direction) were connected together to simplify the biasing control network, and thereby we don’t have voltage (phase) control in y direction. Since the voltage (phase) can only vary in x direction, only 1D beamforming with this reflectarray can be achieved. Therefore, the formula to calculate the phase shift introduced by each element is slightly different from 2D reflectarray Equation (2.13), and can be expressed as

$$\phi_R(x_i) = -k_0 d_i + k_0 x_i \sin(\phi_0),$$

(5.1)

where $k_0 d_i$ is the phase of the incident field on the $i^{th}$ element of the reflectarray and $\phi_0$ is the designed beam direction. The phase shift introduced by each element is only a function of $x$ which means the reflectarray can only scan in the azimuth direction. The formula is calculated by first finding the phase
delay that is accrued at each element as the incident waves travel different distances from the feed to each element. Then the phase shift needed by each element can be found in order to produce a resultant beam at different $\phi$ angles. The elements were designed to achieve a desired phase-shift by varying the loaded capacitance of varactor diodes which can be achieve by varying the biasing voltage. However, single bit phase shifter has been implemented in this reflectarray design. As mentioned before, two phase states has been chosen to be 2.5 V and 8.5 V, which provide two actual phase shifts of $45^\circ$ and $-135^\circ$, respectively. After determined two phase states, the continuous phase distribution can be quantized into two phase states according to

$$
\phi_{\text{1bit}}(x_i) = \begin{cases} 
45^\circ & -45^\circ \leq \phi_R(x_i) < 135^\circ \\
-135^\circ & \text{otherwise}.
\end{cases}
$$

(5.2)

Because the voltages in the $y$ direction have to be same, and then we set the voltage values in the...
controller to be the voltages of the row with the maximum amplitude of the incident fields (10\textsuperscript{th} row from the bottom).

### 5.4 Reflectarray Design

The reflectarray is an offset design with a longitudinal distance (focal length) of 16 cm ($f/D = 1$) as shown in Figure 5.7. The feed used in the reflectarray design is an open-ended rectangular waveguide shown in Figure 5.7. Its aperture dimensions are 3.5 cm by 1.6 cm. The reflectarray is fed by this waveguide, offset by 25\degree with respect to the normal of the array. Figure 5.7 shows the experimental setup of the reflectarray on a near-field scanner that can measure its far-field patterns. The reflectarray design parameters are summarized in Table 5.2.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Reflectarray Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of element</td>
<td>256 ($16 \times 16$)</td>
</tr>
<tr>
<td>Offset angle</td>
<td>25\degree</td>
</tr>
<tr>
<td>Longitudinal distance (Feed to RA)</td>
<td>16 cm</td>
</tr>
<tr>
<td>Area of RA</td>
<td>256 cm\textsuperscript{2}</td>
</tr>
<tr>
<td>Theoretical aperture gain ($G_{ap}$)</td>
<td>20.34 dBi</td>
</tr>
</tbody>
</table>

Table 5.2: Parameters of the reflectarray design

From Equation (2.32), we know that the maximum gain of the reflectarray is proportional to area. The reflectarray design has an area of 256 cm\textsuperscript{2}, and then using Equation (2.32), the maximum theoretical aperture gain of the reflectarray is 20.34 dBi.

### 5.4.1 Incident Phase

Before determining the biasing voltage for each column, as shown in Figure 5.9 in the reflectarray, the required phase shift for each element needs to be calculated from Equation (5.1). To find the phase
delay that is accrued from the feed to each point along the reflectarray, the phase of incident fields from
the feed needs to be measured from the reversed setup shown in Figure 5.10.

Figure 5.10: The feed waveguide setup to measure the incident fields on the reflectarray

The waveguide feed rotated to face the scan plane instead of the reflectarray. The feed was also offset
by 25° with respect to the probe (scan plane) and the distance between the feed and scan plane was
also 16 cm. The fields from the feed on the scan plane can be measured by the NSI scanner, and then
the magnitude and phase of the fields at the scan plane can be computed. Therefore, the actual phase
and magnitude of incident fields on the reflectarray from the waveguide feed can be found. Figure 5.11a
shows the phase of the incident fields from the feed on the reflectarray and the normalized magnitude
of the incident fields are shown in Figure 5.11b.

(a) Phase of incident fields on the reflectarray  (b) Magnitude of incident fields on the reflectarray

Figure 5.11: Incident fields on the reflectarray
5.5 Reflectarray Measurement

5.5.1 Experimental Results

The resultant pattern from the reflectarray was measured using the NSI near field scanner. Measurements were made over a near-field scan window covering -50 to +50 degrees in both azimuth and elevation in the scanner coordinate system. Gain comparisons were made against a standard gain horn. Figure 5.12 shows measured co-polarized realized gains for the reflectarray with the cuts at 24.9° elevation and with 10 degree intervals between +20 and -20 degrees in the azimuth direction at 5.5 GHz. Figure 5.13 shows the same measured gains but at 10 degree intervals between +15 and -15 degrees in the azimuth direction at 5.5 GHz.

Figure 5.14 shows a comparison of the normalized measured co-polarized gains with pattern factor calculation produced by the combination of an array factor with an element factor at different scan angles. The array factor was taken as the same 16 × 16 elements reflectarray with the measured incident fields and the 1D beamforming with single bit phase quantization. The element factor was again chosen as a uniform rectangular aperture antenna with the dimension of 10 mm × 10 mm. All the plots shown in Figure 5.14 are the cuts at 24.9° elevation in both measurement results and pattern factor calculation.

Figure 5.12: Measured co-polarized gains in horizontal cuts at 24.9° elevation. Beams are steered from -20 to +20 degrees in 10 degree steps.

Overall, the beam can be seen to steer very well in the azimuth direction shown in Figure 5.12 and Figure 5.13. Since the voltage across the varactors in the elevation direction (vertical direction) is the same, the reflectarray does not have the ability to steer beam in the elevation direction. All measured patterns experienced small ripples. These ripples are not expected to be due to the antenna itself, but more likely an artifact of the measurement setup. One likely candidate is spurious reflections in the measurement chamber. Since the array is scanning relatively far off broadside (24.9° in the elevation
direction), the scan window was made as large as possible. This means that it is catching a lot of extra power that may be reflecting off or coming from other directions within the room. The impact of these ripples is minor in the overall patterns, however it slightly misrepresent the measurement of the gain pattern as can been seen from Figure 5.13 and Figure 5.12.

The maximum gain achieved across all beams is 8.54 dBi at broadside. As the beam was scanned to off-broadside, the maximum gain decreased about 1 dB which is acceptable. The reason behind is that as the beam is scanned to off-broadside, the gain could decrease a little bit because the effective reflectarray size becomes smaller. The direction of the peak gain at the broadside was slightly shifted to the left about 2°, and this happened to all the beams. The reason behind is the deviation of measurement setup and the measured results are sensitive to setup errors, in this case, possible misalignment of the reflectarray surface.

The side lobe levels can be found in Figure 5.14. On average, each scan angle produced similar side lobe levels between -4.5 dB and -7.2 dB. There are multiple possible reasons for the resultant high side lobe levels, the most likely and largest reason are the mutual coupling between the elements and the phase errors from the single bit phase distribution. Since the element spacing is only $\lambda/6$, the adjacent elements are very close to each other (2.5 mm apart), and mutual coupling could affect the radiation behaviour on each reflectarray element. Therefore, the overall radiation behaviour may be different from what we expected and exhibit high side lobes. As shown in Figure 5.14, the pattern factor calculation with 1D beam forming and 1 bit phase distribution also presents high side lobe levels, however, the 1 bit phase distribution with 2D beam forming doesn’t suffer from the poor side lobes as shown in Figure 5.15. As can been seen from Figure 5.15 in this 2D reflectarray, the 2D beam forming with 1 bit phase shifters achieves about 10 dB side lobe level, and the 1D beam forming with the same phase quantization only obtain about 5 dB side lobe level, which means that the 1D beam forming in a 2D reflectarray leads...
to high side lobes. Therefore, the poor side lobe level could be potentially reduced by implemented a 2D beam forming in this 2D reflectarray.

In the cuts at 24.9° in the elevation direction, the HPBW at the broadside is around 26.9° which is close to the pattern factor calculation with 1 bit phase distribution. As can been seen from Figure 5.14, the agreement between the measured HPBW and the HPBW from pattern factor calculation is good up to +/−15°. Beyond 15°, we experienced a narrow beamwidth. There may be multiple potential reasons for this. The most likely reason is the fact that the scanner is measuring near-fields over a truncated window and is therefore not capturing all the fields. As mentioned before, the array is scanning relatively far off broadside (24.9° in the elevation direction and up to 20° in the azimuth direction), and the maximum near fields scan plane is up to 50° in both elevation and azimuth directions due to the physical size of the chamber and the measurement setup. Therefore, there is a very large possibility that the scanner is measuring near fields over a truncated window for the larger scan angles in azimuth direction such as +/−15° and +/−20°. The fact that this is occurring may also contribute to less accurate results for extreme beam scanning angles.

![Figure 5.14: Normalized measured and pattern factor gains in horizontal cuts at 24.9° elevation at 5.5 GHz. Beams are steered from -20 to +20 degrees in 5 degree steps.](image-url)
5.5.2 Aperture Efficiency Calculation

As mentioned in Section 2.1.3, aperture efficiency ($\varepsilon_{ap}$) of an aperture antenna includes various efficiencies, and in the proposed reflectarray, the aperture efficiency can be represented as

$$\varepsilon_{ap} = \varepsilon_{cd}\varepsilon_{s}\varepsilon_{t}\varepsilon_{b}\varepsilon_{f}\varepsilon_{r}. \quad (5.3)$$

The radiation efficiency, $\varepsilon_{cd}$, mostly introduces by the conductor loss specific to ohmic loss from the series resistor in the varactor diodes. In the proposed reflectarray, the phase efficiency has been broken into two separate efficiency terms, which are fan beam efficiency ($\varepsilon_{f}$) and 1 bit phase quantization efficiency ($\varepsilon_{q}$). The proposed reflectarray lacks of the phase control in the $y$ direction, and then only 1D beamforming with this reflectarray can be performed. An extra efficiency term ($\varepsilon_{f}$) has been introduced as fan beam efficiency to account for the 1D beamforming in the 2D reflectarray. Single bit phase shifters have been employed in the proposed reflectarray and introduce phase quantization errors which gives rise to reduction in directivity. This directivity reduction is referred to as 1 bit phase quantization efficiency ($\varepsilon_{q}$). The subtended aperture efficiency ($\varepsilon_{r}$) introduces by the radiation of this reflectarray at off-broadside. $\varepsilon_{s}$, $\varepsilon_{t}$ and $\varepsilon_{b}$ are the spillover, taper and blockage efficiencies which have been introduced before. The efficiencies are examined in the following paragraphs.

The approximate spillover of the reflectarray was found using Equation (2.35). The spillover efficiency was found by computing the power captured by the reflectarray divided by the total power radiated by the feed. The power captured by the reflectarray is the summation of the power received by each element, and the total power radiated by the feed can be estimated as the total power received by a large area in the reflectarray plane. In Figure (5.16), the area within the small dotted rectangular represents the reflectarray aperture and the entire area was used to calculate the estimate total radiated fields. From this the approximate spillover efficiency was found to be -2.02 dB.

The taper efficiency is found as the fraction of power loss due to a non-uniform amplitude along the reflectarray. The taper efficiency was modelled by computing the radiation pattern of the reflectarray from array theory, as shown in Figure (5.17). Using equation (2.36) the array response is computed for
a reflectarray with a uniform feed illumination (no taper), which is compared to a reflectarray with non-uniform feed illumination (with taper). It can be seen from Figure 5.17 that the introduction of the taper reduces the main lobe by 1.05 dB.

In order to avoid feed blockage, the feed was offset by 25° as shown in Figure 5.7 and the resultant beam was pointed at θ = 25° (specular direction) off-broadside, away from the feed. Based on the reflectarray setup, the resultant beam can only scan in the x direction (azimuth direction) with a constant elevation angle (25°). Therefore, the feed didn’t affect the reflectarray radiation pattern, and then, the effect of blockage is assumed to be negligible.

The conductor loss (ohmic loss) can be determined from the measured scattered behaviour of the reflectarray elements. The two voltage-states were chosen to be 2.5 V and 8.5 V, which may lead to a measured reflection loss about 1.5 dB and 2 dB, respectively. Since the number of reflectarray element for each state varied for different scan angles, the impact of the ohmic loss on the reflectarray performance was estimably determined by taking average of the losses of the two states. Therefore, the estimate
radiation efficiency, $\varepsilon_{cd}$, was -1.75 dB.

To approximately determine the 1 bit phase quantization efficiency ($\varepsilon_q$), this quantization efficiency was also modelled by computing the radiation pattern of the reflectarray from array theory, as shown in Figure 5.18. The 1 bit phase quantization efficiency can be expressed as

$$\varepsilon_q = \frac{D_{1 \text{ bit}}}{D_{\text{continuous}}}.$$  \hfill (5.4)

Figure 5.18: Radiation pattern with continuous phase distribution and 1 bit phase distribution.

Using Equation (5.4), the array response is computed for a reflectarray with a single bit phase distribution, which is compared to a reflectarray with continuous phase distribution. As shown in Figure 5.18, the introduction of the single bit phase shifter reduces the main lobe by 3.02 dB.

The fan beam efficiency of this particular reflectarray can be calculated by computing the radiation pattern of the reflectarray from array theory, as shown in Figure 5.19. The fan beam efficiency can be expressed as

$$\varepsilon_f = \frac{D_{1 \text{D-beamforming}}}{D_{2 \text{D-beamforming}}}.$$  \hfill (5.5)

Using Equation (5.5), the array response is computed for this reflectarray with 1D beamforming, which is compared to this reflectarray with 2D beamforming. As shown in Figure 5.19, the introduction of the 1D beamforming in this 2D reflectarray reduces the main lobe by 2.37 dB.

Lastly, the subtended aperture efficiency is caused by the radiation of this reflectarray at off broadside direction. As mentioned before, the beam can only scan in the azimuth direction at 24.9$^\circ$ elevation. Therefore, the subtended aperture efficiency can be calculated by $\cos (24.9^\circ)$, which is -0.427 dB.

The losses for the reflectarray design are summarized in Table 5.3. Since the measured results were highly sensitive to setup error, we predict that the small discrepancies between the theoretical and experimental results are due to measurement and phase errors. Overall, there is a good agreement between the predicted gain measured gain, and the difference is only about 1 dB.
Figure 5.19: Radiation pattern 2D beamforming and 1D beamforming.

<table>
<thead>
<tr>
<th>Loss Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum theoretical aperture gain</td>
<td>20.34 dBi</td>
</tr>
<tr>
<td>Spillover loss</td>
<td>2.02 dB</td>
</tr>
<tr>
<td>Taper loss</td>
<td>1.05 dB</td>
</tr>
<tr>
<td>1 bit phase quantization loss</td>
<td>3.02 dB</td>
</tr>
<tr>
<td>1D beamforming loss</td>
<td>2.37 dB</td>
</tr>
<tr>
<td>Ohmic loss</td>
<td>1.75 dB</td>
</tr>
<tr>
<td>Blockage loss</td>
<td>0 dB</td>
</tr>
<tr>
<td>Subtended aperture loss</td>
<td>0.43 dB</td>
</tr>
<tr>
<td>Theoretical expected gain</td>
<td>9.70 dBi</td>
</tr>
<tr>
<td>Experimental measured gain</td>
<td>8.54 dBi</td>
</tr>
</tbody>
</table>

Table 5.3: Losses for reflectarray design
Chapter 6

Conclusions

6.1 Research Achievements

There were two primary goals presented at the beginning of this thesis. The first goal was the development of both numerical and analytical techniques for analyzing linear-polarized reflectarray unit cell with tunable lumped elements. The second goal was to develop and prototype a low loss, low complexity, and low cost reconfigurable reflectarray.

In the first goal, the development of numerical techniques for analyzing the reflectarray unit cell has been achieved by extended the existing numerical methods (MoM) for computing the GSM. The extended GSM accommodates lumped elements loading along the interior edges of RWG basis functions, and furthermore lumped elements have been extended to lumped ports to reduce the simulation time. For validating the GSM, the co-polarized reflection coefficients predicted by the GSM in fixed reflectarray case have been compared to a traditional reflectarray with variable size elements in an infinite-array environment. For validating the proposed extended GSM with lumped ports, the co-polarized reflection coefficients predicted by extended GSM have been compared against the results from the reconfigurable reflectarray element. On the analytical side, a versatile and accurate equivalent circuit model for predicting the reflection coefficient for linear-polarized reflectarray unit cells has been developed. The ECM was derived from Floquet modal expansion of a patch-like scatterer. As such, the ECM can generate highly accurate results without the need for running full-wave simulations to derive values for the components in the ECM. This is a major advantage over other ECMs. Additionally, the versatility of the ECM allows it to accurately model both fixed and reconfigurable reflectarray cells. Evaluation of the ECM can be done very quickly, since the reflection coefficient can be predicted in closed form using Floquet summations. Hence the ECM can be a highly useful tool during the initial design stages of reflectarrays prior to engaging in more detailed full-wave simulations of the cell. The ECM can be used to supplement, but not replace, full-wave simulation tools, since more detailed information such as performance of the cell at oblique incidence, or cross-polarization information, must still be obtained from full-wave simulations. Validation experiments with full-wave simulation tools show that the model is accurate for reflectarray elements based on changing the patch size, as well as reconfigurable reflectarray elements based on lumped-element loading as well as distributed tuning mechanisms. Predictions from the ECM are very close to those provided by full-wave simulations.

This thesis achieved the second goal by combining a variety of concepts including the single bit
phase shifter, sub-wavelength element spacing and delta-sigma modulation. A dense reflectarray employing sub-wavelength element spacing has been developed to reduce the conductor (ohmic) loss in reconfigurable reflectarrays elements, and reflectarrays elements with single-bit phase shifters have also be implemented to further mitigate the ohmic loss and simplify the beamforming control network. It is worth pointing out that the ohmic loss analysis in the reconfigurable reflectarray was first conducted in this thesis. Varactor diodes have been firstly employed as single bit phase shifters instead of PIN diodes in this design. PIN diodes consume power in their “ON” state and varactor diodes are zero power consuming in both states. Another advantage of sub-wavelength element spacing is to compensate the phase quantization loss from 1 bit phase shifting. Low cost lossy (Si) varactor diodes have been employed instead of the expensive low loss (GaAs) diodes in order to mitigate the cost with almost the same overall loss performance. Delta sigma has been applied to successfully reduce the quantization lobes in the plane wave re-director.

In the design of the reflectarray elements, many challenges were encountered. Throughout this work, a simple dipole unit cell was introduced first with high dielectric constant \( \epsilon_r \). This high dielectric constant made the reflectarray exhibit high specular reflection behaviour. Many different element geometries were tested in order to mitigate the specular reflection and ohmic losses. It turned out that loop elements are desirable, as they are electrically longer than dipoles, so to achieve the same electrical length of the dipole, a smaller dielectric constant can be used. The optimized element design was shown to introduce a maximum 1.8 dB of reflection loss with Si diodes and was observable at the design frequency 5.5 GHz.

Simulations of a linear reflectarray in the parallel plate waveguide at 5.5 GHz demonstrated a -20 to +20 degrees steering range with a constant gain. The series resistor (4.8 Ω) in the low-cost varactor diodes only introduces a 0.53 dB directivity reduction which is much lower than the 1.8 dB radiation loss with only 0.7 Ω series resistance. However, side lobe performance suffered and the degraded performance is likely due to the specular reflection and mutual coupling between the elements. Overall, the 1D linear reflectarray with 1 bit phase shifters was able to produce a beam scanning with slightly high side lobe level. More importantly, the radiation loss (ohmic loss) was significantly reduced in this sub-wavelength element spacing and single bit phase quantization reconfigurable reflectarray. Furthermore, a 1D linear plane wave re-director has been designed and simulated, the results show that the quantization lobe has been successfully reduced with delta sigma phase modulation.

Lastly, we prototyped a 16 × 16 element reflectarray and near-field measurements at 5.5 GHz showed a beam-steering range of -20 to +20 degrees in the azimuth direction with only a maximum 1 dB variation in measured gain. Side lobe levels were still fairly high in this design for the same reasons as in the linear reflectarray. Overall, there was a good correlation between the measured and simulated results. The series resistor introduced maximum measured reflection loss about 2.3 dB which proves that the sub-wavelength element spacing significantly reduces the reflection loss (ohmic loss) as well. The illumination efficiency, which is the product of the spillover and taper efficiency, was approximately 50%. The gain reduction caused by the single bit phase quantization errors was 3.02 dB with sub-wavelength element spacing which has been improve from 3.99 dB \[15\]. Overall, we were able to successfully demonstrate the low loss performance and beam steering of the single bit reflectarray design.

Finally, it is at this point worth pointing out the advantages of this design over a traditional reconfigurable reflectarray. The advantage of this design lies in the ohmic loss, the complexity of the beamforming network and the cost.
6.2 Thesis Contributions

This thesis is an exciting contribution to both numerical and analytical techniques for analyzing linear-polarized fixed and reconfigurable reflectarray unit cell. The single bit reflectarray design employing with sub-wavelength element spacing has not been discussed in literature.

The preliminary work of this analytical techniques for analyzing reflectarray element was presented at the 2016 IEEE International Symposium on Antennas and Propagation (APSURSI) [34]:


Another paper on the same topic has been written and was submitted to be published in the IEEE Transactions on Antennas and Propagation.


6.3 Future Work

Numerical techniques (MoM) for analyzing the reflectarray unit cell with tunable lumped elements could be further extended to analyze multilayer reflectarray elements and explore ability to simulate the high order modes in the future. On the analytical side, the flexibility of the model also enables the potential for analyzing multi-layer reflectarray elements. Since the transformer turns ratios in the ECM are solely determined by current distributions on the scatterers within each layer, ECMs for each layer can be derived easily and coupled through transmission lines modelling each substrate layer. The multi-modal nature of the ECM means that both propagating and evanescent modes will be accurately coupled in such formulations, yielding an even more versatile ECM.

For the single bit sub-wavelength element spacing reconfigurable reflectarray, the main challenge that was encountered in both simulation and measurement was the poor side lobe levels. The possible reason could be the strong specular reflection and mutual coupling between elements. In the future, different techniques should be examined to reduce the poor side lobes in the way of reducing the permittivity of the substrate to reduce the specular reflection and mitigating the mutual coupling. Further improvement in the measurement of the scattered behaviour of the reflectarray elements needs to be carried out in the quasi-optical setup to further improve the accuracy of the reflection coefficient of this reflectarray element. The gain reduction due to the single bit phase quantization errors could be further reduced by optimizing the reference phase in the two phase states as discussed in Section 4.2.2.
Appendix A

Green Function of the Grounded Dielectric Substrate

The Fourier transform of the dyadic Green's function $\tilde{G}(k_{xl}, k_{yl})$ of the grounded dielectric substrate has been derived as

$$\tilde{G}(k_{xl}, k_{yl}) = \begin{bmatrix} G_{xx}(k_{xl}, k_{yl}) & G_{xy}(k_{xl}, k_{yl}) \\ G_{yx}(k_{xl}, k_{yl}) & G_{yy}(k_{xl}, k_{yl}) \end{bmatrix}. \quad (A.1)$$

The matrix $G_{xx}(k_{xl}, k_{yl})$, $G_{xy}(k_{xl}, k_{yl})$, $G_{yx}(k_{xl}, k_{yl})$, and $G_{yy}(k_{xl}, k_{yl})$ can be calculated as

$$G_{xx} = \frac{C(K_2^2 K_2^2 K_1 T_e + K_0^2 K_2^2 T_m) \sin(K_1 d)}{(T_e T_m K_{xy})}, \quad (A.2a)$$

$$G_{yy} = \frac{C(K_2^2 K_2^2 K_1 T_e + K_0^2 K_2^2 T_m) \sin(K_1 d)}{(T_e T_m K_{xy})}, \quad (A.2b)$$

$$G_{xy} = \frac{C K_x K_y (K_1 K_2 T_e - K_0^2 T_m) \sin(K_1 d)}{(T_e T_m K_{xy})}, \quad (A.2c)$$

$$G_{yx} = G_{yy}, \quad (A.2d)$$

where

$$Z_0 = \sqrt{\frac{\mu_0}{\epsilon_0}}, \quad (A.3)$$

$$K_0 = \omega \sqrt{\frac{\mu_0 \epsilon_0}{\epsilon_0}}, \quad (A.4)$$

$$K_{xy} = (K_x^2) + (K_y^2), \quad (A.5)$$

$$K_1^2 = \epsilon_x K_0^2 - K_{xy}, \quad (A.6)$$
Appendix A. Green Function of the Grounded Dielectric Substrate

\[ K_2^2 = K_0^2 - K_{xy}, \]  \hspace{1cm} (A.7)

\[ T_e = K_1 \cos(K_1 d) + jK_2 \sin(K_1 d), \]  \hspace{1cm} (A.8)

\[ T_m = \epsilon_r K_2 \cos(K_1 d) + jK_1 \sin(K_1 d), \]  \hspace{1cm} (A.9)

\[ C = \frac{jZ_0}{K_0 AB}. \]  \hspace{1cm} (A.10)

\( \epsilon_r \) is the dielectric constant of the substrate, and \( K_1 \) and \( K_2 \) are the propagation constant in the \( z \) direction in the \( z < 0 \) and \( z > 0 \) zones with always positive real part and negative imaginary part. \( d \) is the thickness of the dielectric substrate. \( A \) and \( B \) are the periodicity in the \( x \) and \( y \) direction respectively.
Bibliography


