Math Discourse in a Grade 2 Knowledge Building Classroom

by

Stacy Alexandra Costa
A thesis submitted in conformity with the requirements for the degree of Master of Arts
Graduate Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education
University of Toronto

© Copyright by Stacy A. Costa (2017)
Math Discourse in a Grade 2 Knowledge Building Classroom

Stacy A. Costa
Masters of Arts
Department of Curriculum, Teaching & Learning
University of Toronto
2017

Abstract

The goal of this study was to examine grade two Math Talk in geometry within a Knowledge Building community engaged in both face-to-face and computer-mediated discourse. Ontario Ministry of Education guidelines were used to identify grade two geometry concepts. Math vocabulary extracted from these guidelines was used, along with a content-based social network analysis tool, to explore the emergence of new domain-specific vocabulary in student discourse and to assess patterns of engagement surrounding use of those terms. A “Ways of Contributing” analytic framework was used to assess the nature of both teacher and student contributions to face to face and online discourse. Findings suggest that students as early as grade 2 can engage productively in Knowledge Building Math Talk in both face to face and online contexts.
Acknowledgments

To my Supervisor, Dr. Marlene Scardamalia, thank you immensely for believing in me, for introducing me to this project and Knowledge Building Pedagogy. Thank you for your continuous support in every endeavor I have taken an interest in. I am truly grateful for all the opportunities and experiences I have gained as a result of being one of your graduate students. I am forever indebted to you. You have taught me the importance of academic research in education and have always pushed me to excel. Without your guidance and persistent support, this thesis would not have been possible. You have also forever changed the course of my academic career. It is said that real leaders do not create followers; they create more leaders. This statement embodies that academic attitude you have instilled in me and the entire IKIT Team. You have always reminded me of the importance of innovation within education. Finally, thank you for your patience and encouragement during my many moments of academic perplexity, especially during the writing process. I consider myself fortunate and blessed to be able to contribute to and be a part of the Knowledge Building Research timeline.

Dr. Marcello Danesi – I am very indebted to you for all your academic and life advice. You have taught me more than I could ever give credit for here! Thank you for always challenging me. You have always believed in me since the first day I stepped into a classroom at the University of Toronto. Thank you for assisting me in pursuing my academic goals, and please always remain a part of my academic journey as a mentor. Thank you!

To the CSTD/CTL Department at OISE, thank you for your constant assistance in many academic matters, and for imparting to me your sense of community. Also, I want to thank the University of Toronto, for the countless opportunities I have and will experience for years to come.

To my Graduate Lab members, Leanne Ma, Derya Kici, Ahmad Khanlari, Gaoxia Zhu and Joel Wiebe, thank you for your COUNTLESS hours of input, suggestions, edits and constant challenges which have helped shaped me as a scholar and lab member. Your ongoing contributions to my ideas, works, and inclusion in your research have also enabled me to have the honor of calling you, friends! Especially to Leanne Ma, your ongoing support, advice and
scholarly discussions have provided me guidance and moral writing support! Also, for all our shared works, I am grateful for the research opportunities we have shared and will continue to share!

Susana La Rosa, thank you for always being available and always helping with everything needed! I cannot express enough how much I appreciate it and am grateful that you are only a few steps or a phone call away!

To the Grade Two Class and teacher, thank you for always making my visits so enjoyable and for the creation of the nickname “Stacy, the crazy computer lady." Truly my time in the classroom was a wealth of experience that will never be forgotten.

I also want to thank anyone who has made an educational impact on my life, from a small impression to those who have shaped me into what I am today. You have all allowed me to become an innovator and curator of ideas! Thank you! This journey has allowed me to replace fear of the unknown with curiosity.

To my three best friends: Alissa Seecharran, Alisha Babar, and Vanessa Compagnone, thank you for the endless coffee, lunches, support, advice and belief in me and our friendships. Each one of you has a special place in my heart and our friendships have lasted years and will last for decades to come. Thank you also for always being there! Love, you, Ladies!

To Anthony Santangelo, thank you for your support and needed encouragement at the end of this thesis writing process! Your motivation and constant reminder of hard work is inspiring. All your well wishes, support and advice will be cherished forever. Also, thank you for the laughs that have kept up my spirits, when the writing process became tedious. Thank you for everything thus far. You are wonderful!

Vavo Nobe Cabral – Thank you for always reminding me to take a break, and for always asking how’s my “book” going. For all the times, you’ve made sure I was okay, and for the daily check-ins. Obrigado!
Mom and Dad, (Alex & Maddie Costa). Honestly my thanks and dedication to you both should be longer than this entire thesis! Thank you both so much for putting up with my countless requests. Thank you for always being there when I needed you both most. Your constant check-ins, support, motivation, conversations, and all you both do, have helped my well-being and stamina throughout this process. Your support has been very much appreciated! You both have made countless sacrifices, and all your little actions (drives, dinners, jokes, prayers) will always be remembered and never forgotten! Times have not always been easy, but you both have proven that home is where the heart is! Your support will always be cherished and remembered. Thank you for instilling in me the importance of education, and for always advocating my best interests when I was younger. (and many times today too) Also, thank you for always believing in me! Thank you for everything! Love, you both so much xo! This thesis is dedicated to you both!

“Often when you think you're at the end of something, you're at the beginning of something else.” - Fred Rogers
TABLE OF CONTENTS

Abstract .............................................................................................................................................. ii
Acknowledgments ............................................................................................................................... iii
TABLE OF CONTENTS .......................................................................................................................... vi
List of Tables ........................................................................................................................................ viii
List of Figures ......................................................................................................................................... ix
List of Appendices ............................................................................................................................... x
Chapter 1 Introduction ......................................................................................................................... 1
  1.1 Thesis Rationale ........................................................................................................................... 1
  1.2 Thesis Overview ............................................................................................................................ 2
Chapter 2 Literature Review ................................................................................................................ 4
  2.1 Chapter Overview ........................................................................................................................... 4
  2.2 Math Instruction within Classrooms ............................................................................................... 4
  2.3 Math Talk .................................................................................................................................... 7
  2.4 Knowledge Building ....................................................................................................................... 11
  2.5 Knowledge Forum ........................................................................................................................ 15
  2.6 Knowledge Building in Mathematics ........................................................................................... 17
  2.7 Research Questions ....................................................................................................................... 20
Chapter 3 Methodology ....................................................................................................................... 21
  3.1 Conceptual Framework .................................................................................................................. 21
  3.11 Design-Based Research ............................................................................................................. 21
  3.12 Limitations and Challenges of Design-Based Research .............................................................. 23
  3.2 Participants .................................................................................................................................. 24
  3.3 Data Collection ............................................................................................................................ 24
  3.31 Data Analysis Procedures ......................................................................................................... 26
  3.32 Ways of Contributing .................................................................................................................. 28
  3.33 Knowledge Building Discourse Explorer (KBDeX) ................................................................... 30
  3.34 Wordlist ................................................................................................................................. 43
      43.4 Research Limitations and Ethical Considerations .................................................................. 30
  3.5 Chapter Summary ....................................................................................................................... 31
Chapter 4 Findings ............................................................................................................................... 32
  4.1 Chapter Overview ........................................................................................................................ 32
  4.2 Case 1: Knowledge Building Circle ............................................................................................ 32
  4.21 Knowledge Building Circle Turns ............................................................................................ 34
  4.3 How Math Talk Proceeds During the Knowledge Building Circle .............................................. 37
  4.4 Case 2: Knowledge Forum Notes ............................................................................................... 40
  4.41 Knowledge Forum Notes ......................................................................................................... 42
  4.5 How Math Talk Proceeded on Knowledge Forum ................................................................. 43
  4.6 Idea Development ..................................................................................................................... 46
  4.61 Introducing Math Concepts and using them to Support Math Talk ........................................ 46
  4.62 Heart as a Shape Inquiry ........................................................................................................... 49
  4.63 “Illegal Shapes”: Understanding Classification and Definition of a Shape ......................... 52
Chapter 5 Conclusion ........................................................................................................... 55
5.1 Summary ....................................................................................................................... 55
5.2 Future Studies & Directions ......................................................................................... 59
References .......................................................................................................................... 62
Appendix 1 - Grade Two Ontario Mathematics Geometry Word List ................................. 73
List of Tables

Table 1 - Ways of Contributing to Explanation-Seeking Discourse p.29

Table 2 - Knowledge Building Circle: Frequency of Speaking Turns for Students and Teacher p.33

Table 3 - Knowledge Building Circle: Frequency of Students’ Ways of Contributing to Classroom Discourse p.34

Table 4 - Knowledge Building Circle: frequency of teacher’s ways of contributing to classroom Discourse p.35

Table 5 - Knowledge Forum: Frequency of Student Notes p.41

Table 6 – Knowledge Forum Notes: Frequency of Students ways of contributing to classroom Discourse p.42

Table 7 - Knowledge Building Circle: Understanding a Parallelogram Transcript p.47

Table 8- Knowledge Building Circle: Defining Spatial Properties of a Three-Dimensional Shape Transcript p.48

Table 9 - Knowledge Forum Notes: Two-Dimension & Three Dimension Shape Comparison Transcript p.49

Table 10 - Knowledge Building Circle: Understanding a Heart as a Shape Transcript p.50

Table 11 - Knowledge Forum Notes: Dimensions of the Heart and Body Transcript p.52

Table 12 - Knowledge Building Circle: Illegal Shape Theory Transcript p.53

Table 13 - Knowledge Building Notes: Illegal Shape Build-Ons Transcript p.54
List of Figures

Figure 1 – Design Based Research Intervention Activities and Classroom Schedule Overview p.27
Figure 2 – Knowledge Building Circle: Knowledge Building Discourse Explorer (KBDeX) Visualization at End of the Study p.37
Figure 3 – Knowledge Building Circle: Knowledge Building Discourse Explorer (KBDeX) Visualization p.38
Figure 4 – Knowledge Building Circle: Knowledge Building Discourse Explorer (KBDeX) Visualization p.39
Figure 5 – Knowledge Forum: Knowledge Building Discourse Explorer (KBDeX) Visualization at End of the Study p.43
Figure 6 – Knowledge Forum Word Network at the End of the Study p.44
Figure 7 – Knowledge Forum Note Network at the End of the Study p.44

List of Appendices
Chapter 1
Introduction
The purpose of this chapter is to present the rationale and context of this study. First, the case is made for new pedagogies to meet demands for improved math understanding in a technology rich knowledge society. Two complementary pedagogies are presented with the research bases for each, followed by thesis objectives and chapter outline.

1.1 Thesis Rationale
As stated in the Ontario Ministry of Education’s document Growing Success (2009), “Education directly influences students’ life chances and life outcomes” (pg. 6). Today’s global, knowledge-based economy requires a citizenry with deep understanding of disciplinary knowledge as well as a broad range of what are popularly known as “21st century skills.”

Results of Ontario’s provincial standardized testing assessment known as EQAO show 63% of students meeting the provincial 2015 grade three mathematics standards (EQAO 2016). Bredekamp (2004) argues that educators should examine research and practices on how children actually learn mathematics and integrate relevant methods into school mathematics. Knowledge Building pedagogy is grounded in students’ natural curiosity and helps expand capacity through supporting community dynamics of the sort found in knowledge-creating organizations. It has been shown to boost achievement in disciplinary literacies in mathematics, engineering, science, and across the curriculum (Chen and Hong 2016). Knowledge Building supports students in designing, revising, discussing and applying ideas developed in community contexts and spaces so that they can take collective responsibility for knowledge advances.

The need for mathematics as a tool for creative work in a technology-rich knowledge society is widely recognized (Wagner & Dintersmith 2015; Ritchhart 2015). Mathematics is needed for problem solving, reasoning, questioning, computational strategies and creative application, well beyond the work required to solve well-defined problems characterized by “instructor input→student problem-solving→verification.” There is growing agreement that this instructional approach is not sufficient (Towers 1999; Schunk 2012; Baroody 2000), less agreement regarding effective alternatives. This thesis represents an exploratory effort to integrate community dynamics supported by Knowledge Building (Scardamalia & Bereiter,
2003) and “Math Talk” a form of classroom discussion that allows students to learn mathematics through discourse surrounding math ideas.

1.2 Thesis Overview
Knowledge Building researchers have identified pedagogical methods that support idea improvement (Zhang et al. 2009; Scardamalia et al. 2012) and “collective intelligence” (Broadbent & Gallotti 2015) which can only be described at the group level (Knowledge Building Gallery 2016). Studies have explored Knowledge Building in the area of mathematics (Moss & Beatty 2006, Moss & Beatty 2010, Nason, Brett & Woodruff 1996, Nason, Woodruff, & Lesh, 2002, Nason & Woodruff 2004 & Knowledge Building Gallery 2016) demonstrating impressive examples of “Math Talk.” This thesis seeks to contribute to the growing body of literature on Knowledge Building and Math Talk to consider possibilities for such work beginning as early as grade 2.

The research reported in this study is exploratory, using a blended Knowledge Building and Math Talk approaches across face-to-face and online discourse contexts to explore how mathematical ideas and artifacts can be productively shared in public, community contexts. The study aims to examine what ideas emerge in student’s Math Talk and how they help advance collective understanding. The thesis is organized as follows:

The Literature Review starts with a brief overview of the Ontario mathematics curriculum, followed by research regarding collaborative learning, “Math Talk,” and Knowledge Building pedagogy and technology, with a focus on mathematics. It ends with research questions that guide the research and analysis of math discourse.

The Methodology chapter starts with an overview of the research and design-based research, followed by a description of a “Ways of Contributing” Framework (Chuy, Resendes & Scardamalia, 2010) and Knowledge Building Discourse Explorer (KBDeX) software to analyze students’ face-to-face and computer-mediated discourse. The ways of contributing framework identifies six types of contribution and KBDeX produces content-based social network analyses based on the co-occurrence of terms among students of select vocabulary extracted from the Ontario Ministry Mathematics Guidelines. These analytic methods were used to explore the
emergence of domain-specific vocabulary and patterns of engagement to characterize Knowledge Building Math Talk.

Group and individual level discourse patterns are reported in the Findings chapter for both face-to-face/Knowledge Building Circle discourse and online discourse represented in Knowledge Forum notes. Ways of Contributing and KBDeX analyses are reported along with qualitative analyses of a segment of a teacher lesson and several student discourse threads.

Advances in geometry understanding in grade two face-to-face and online discussions are reported in the Discussion and Conclusions chapter, with implications and recommendations for future research. Results contribute to the growing literature on Knowledge Building Math Talk refinements in classroom pedagogy, and encourage the use of Knowledge Building within the elementary mathematics classroom.
Chapter 2
Literature Review

2.1 Chapter Overview

This chapter provides an overview of literature on Math Talk and its importance within the elementary classroom. Next is an overview of relevant literature on Knowledge Building pedagogy with a focus on mathematics. The chapter ends with a summary of ways in which this exploratory study contributes to the Knowledge Building mathematics literature and the research questions that guide the research and data analysis.

2.2 Math Instruction within Classrooms

Fostering mathematical thinking is a priority in many curriculum guidelines and professional development workshops for teachers. *Ontario Mathematics Guidelines* convey goals of “learning relevant facts, skills, and procedures” and developing “the ability to apply the processes of mathematics, and acquire a positive attitude towards mathematics” (Ontario Ministry of Education 2005; pg. 3). There are five mathematical strands: number sense and numeration, measurement, patterning and algebra, data management and probability, and geometry and spatial sense (Ontario Ministry of Education 2005). This subdivision applies to math education in all grades and provides a framework for subject matter to be covered during the school year. Leher & Chazan 1998, Whiteley 1999 in Sinclair 2005 indicate that geometry is often neglected for numeracy and algebra.

Osta (2014) defines curriculum as, “the pedagogical framework or philosophy underlying the teaching practices and materials, training programs for supporting teachers, and guidelines for assessing students’ learning” (pg. 417). According to Putnam in Martinovic (2004 p. 33) the educational curriculum is a prescriptive document which dictates an “ordered set of goals in which students are expected to learn, and recommended strategies for teaching material.” There is a growing tendency to support new pedagogies for deep learning, inquiry-based learning, project-based learning, and so forth. More generally, the literature on collaborative learning highlights advantages of students working together within the context of curriculum guidelines.
that encourage greater student engagement. Vygotsky (1978) suggested that learners can be assisted by working with others who are more knowledgeable. The more knowledgeable person need not be the teacher. Every student can contribute to the learning process by injecting his or her talents into the dialogical process.

Knowledge that may be out of reach for someone learning alone may be accessible if the learner has the support of peers or more knowledgeable others’ (Van de Walle et al., 2014, p. 6). In Ontario Mathematics Classrooms, and in many educational settings, students come from different cultural backgrounds. This allows them to bring distinctive ideas to the classroom and possibly fill gaps in knowledge by bringing different resources and perspectives to bear on interpreting ideas or problems. From this perspective diversity enhances learning. Boaler (2002) argues that the view that there is a common learning denominator in all students is erroneous and counterproductive as it leads to mathematics taught as if students have the same ability, preferred learning style and pace of working.

Boaler (1998) found that math students in an open “project-based” environment developed a stronger conceptual understanding to assist in mathematical learning both within and outside the classroom. Collaboration can attenuate risks associated with isolating and competitive tasks. Many confuse collaboration and working together with cheating. However, this conflation is often misleading and counterproductive. Demonstrations of different examples of learning and diverse interpretations of the same problem can allow for better understanding of complex ideas.

Scaffolding is widely used to bridge and expand a student’s capability by linking their prior knowledge to new knowledge (Wood et al. 1976; Wood and Wood 1996). Duncan et al. (2007) found that early number knowledge is a key predictor of later academic success and that by allowing number concepts to emerge in a collaborative manner, at an earlier age, students are better able to extend their knowledge to new topics. Students become more comfortable within the mathematics discipline and can interpret mathematics learning from a more engaging perspective.
Stigler and Hiebert (1999) reported results showing that students who worked independently risk having insufficient opportunities to translate their work into words, thus hindering the realization of mathematics understanding through discourse. Students can learn through exploring and emulating how mathematicians tackle unsolved problems. The Clay Mathematics Institute is dedicated to encouraging the use of historical artifacts and of getting mathematicians to engage in collaborative tasks. Many of the prizes handed out by the institute are to groups of mathematicians who have worked collaboratively on some project. The underlying idea is that scholars and experts often work collaboratively to advance mathematical knowledge in contrast to school mathematics that tends toward individual student work. Stahl (2005) argues that revising the mathematics curriculum would allow for deeper mathematics understanding in which groups tackle problems and learn in the process. Adapting the mathematics curriculum to the Clay Institute and Knowledge Building model, elaborated below, should enhance outcomes (Stahl 2005).

Pimm (1987) found that when students attempted to express their thoughts out loud in words, other students were helped in the classroom, allowing them to better self-organize their thoughts (pg. 23). By having students “turn up the volume on their own self-talk,” (Pimm 1987, p. 25), learning outcomes tend to be more positive and effective. Encouraging student self-talk allows for open-ended multi-step scaffolding of collaborative problem solving in mathematics classrooms. Through reflective self-talk and engagement with others students can gain more numeracy and procedural knowledge. Scardamalia and Bereiter (1983) note that "observation, practice and rule learning" (pg.63) are, the core aspects of acquiring procedural knowledge. Dialogical Learning has produced effective outcomes (Sinclair 2005) allowing students to share their relative strengths and address their weaknesses. Collaborative learning favors “enculturating” processes in a situation (Towers 1999), or the use of diverse terminologies based on different backgrounds that shed light on some structural problem. Finlayson (2014) found that mathematics structured to produce right or wrong answers from the students encourages formulaic approaches and inhibits the development of the higher order learning skills. Students need to connect previous and new knowledge; simply seeing new forms without understanding does not provide the cognitive foundation for understanding.
“Communication is an essential part of mathematics and mathematics education” (NCTM 2000, p. 60). By becoming familiar with technical mathematical language for expressing new concepts and ideas students can broaden and deepen their overall math competence (Baroody 2000; Ginsburg et al. 1999; NCTM 2000). Mathematical communication involves “adaptive reasoning” (Kilpatrick et al. 2001, p. 170) and argumentation (Andriessen 2006), which give students opportunities to think mathematically. Mathematical communication offers students opportunities and encouragement with important consequences, as elaborated below.

2.3 Math Talk

The discipline of mathematics consists of symbols, equations, and numbers, a type of language. required to interpret, teach and learn mathematical concepts. Mathematics depends fundamentally on language. Math Talk has been a tacit element of math education literature for over 25 years (NCTM 2000). “Math Talk” aims to tap into student curiosity and encourage students to talk about math in the classroom so that students can relate what they learn to what they know in a discourse-based manner. Teaching and learning mathematics at the elementary level always involves some form of discourse, both between students and between students and the teacher. The advantage of a “Math Talk” pedagogy is consistent use of discourse rather than a random and sporadic use., As Mountz (2011) characterizes it, “Math Talk is a style of discussion in the classroom that allows students to learn through discourse” (pg. 3). Students are encouraged to formulate their ideas concerning what math ideas are about and how to grasp them, providing valuable input to teachers. While allowing students to figure out solutions after some basic instructional time, students need to be able to formulate high-level ideas to work out problem-solving through cognitive ingenuity. Students will derive meaning from the discursive nature of the mathematics discourse taught to them, arguably connecting community ideas, the contents of discourse, thus extending the learning process considerably.

Morgan (2014) notes that some vocabulary is uniquely mathematical such as the term “parallelogram”; however mathematics also includes everyday language with subtly different
meanings in words such as prime, multiple and differentiate. This crisscrossing of semantic domains is very productive for Math Talk pedagogy. Students need stronger incentive to incorporate Math Talk so as to differentiate between meanings and to understanding and apply knowledge across disciplines. Students should not feel alienated nor intimidated by math but instead eager to participate. However, within the strand of geometry, not everything can be verbalized, since figures and geometric diagrams also play an essential role in communicating within mathematics. Some mathematical explanations will be explicated better by discussing the visual material. Martinovic (2004) refers to this type of learning as "vicarious learning," which refers to learning by overhearing an educational conversation (pg. 29). While students may not always be involved in discourse in this learning situation, they are reading and creating their own discourses and thus are dialoguing in a “vicarious” way. “Vicarious learning” provides students with foundational understandings as well as a basis of reference to refer to when clarifying their understanding or when sharing their answers in the classroom.

Judith Falle (2004) defines the overall goal of “Math Talk” pedagogy to have "students attempt to explore, investigate and solve problems together, resulting in more success their mathematics education" (p.19). One example of how students can solve problems together is through the usage of "Math Talk." "Math Talk" is (NCTM 2000) as “instructional conversation which references the crucial aspects of: questioning, explanation of math thinking, a source of math ideas and the responsibility of mathematical learning.” Cooke and Adam (1998) emphasize that Math Talk is effective in allowing students to analyze, justify, defend and reason, which are the first steps in developing stronger mathematic vocabulary. Falle (2004) examined how Math Talk constitutes a “small-scale strategy of learning that immerses students into speaking patterns which connect to larger-scale forms of learning unconsciously” (p.19). With students struggling not only to articulate their thoughts, they can also be confused in using the correct symbols. The importance of conversations must be in line with using a "mathematical register" (Pimm, 1987).

"The amount of ‘Math Talk’ produced by a child's input from the classroom is related to the growth of a child's acquisition of mathematically relevant language" (Boonen et al., 2011, p. 283). This observation implies that conversation should not be constrained to a child-teacher
system, but rather to a peer-to-peer one as well. The argument for a systematic type of classroom dialogue that includes everyone is that all, not just those inclined to talk, should be engaged. In a Math Talk model the interconnection of language and mathematics to describe shapes, charts, graphs and to describe how formulas and expressions are constructed produces a supportive context for understanding. Classrooms use language pedagogically, to conduct lessons as clearly as possible. As Resnick (1988) and Martinovic (2004) argue, mathematics is a poorly structured and taught as a discipline that removes the voice from the learners and should instead include debate, argumentation, and give space to multiple interpretations, allowing students to use mathematical ideas and argumentative form.

Falle (2004) found that "when teachers do listen in mathematics they do so to correct students rather than to reveal possible student cognitive development" (p.26). Math Talk is diagnostic; it reveals what students may or may not have grasped, how they value certain concepts on their terms and thus suggesting how they can modify their own responses more advantageously.

Mercer & Sams (2006) emphasize that talk-based group activities develop the individual’s mathematical reasoning skills, which serve to enhance the student’s problem-solving skills. Hufferd-Ackles et al. (2004) define a Math Talk community as "students who take responsibility for their own learning and the learning of others, which leads to a demonstration of their understanding of problems and of volunteering to assist struggling students" (p.106). Cooke & Adams (1998) discovered that Math Talk might be difficult to integrate into the classroom because most curricula envision the teacher as being at the center of the learning process. With teachers assuming a direct, authoritative role, there is a tendency for classroom learning to become a rigid competitive process that is hardly conducive to positive learning outcomes. Constraints are being placed upon students by using the textbook with no room for questions or additional interpretation (Finlayson, 2014).

As Pimm (1988) notes, “a distinctive feature of discourse about mathematics is the widespread use of technical vocabulary” (pg. 59). This provides a necessary space in a classroom to all participants, giving them a voice in the process. This is defined as “rich discourse.” Imm &
Stylianou (2012) characterized a "rich discourse" classroom as one which valued the student’s ideas as “rich, inclusive, and purposeful” (pg. 131). Students in the classroom are not incorporating pragmatic conversations because their perceived final goal is to answer the problem they have been presented in the math classroom. The reason students struggle is that math conversations for elementary students are rarely casual. Instead, they originate from within the classroom space and are guided completely by the teacher. Nathan & Knuth (2003) discuss "productive discourse" engendered in such situations to allow the teacher-student interactions to build upon ideas through branching out to include other class members or at least their individual contributions.

In Math Talk agreeing and thinking are encouraged to allow students to discover mathematical concepts and to work through misconceptions (Bruce 2007 in Hutton, Chen et Moss 2013). Hutton et al. (2013) note that while each student contributes math discourse, it becomes a significant part of the classroom and such individual contributions advance the classroom communities' learning. Pimm (1987) asserted that mathematical language and everyday language share words, so when students lacked mathematical terms they used everyday language to express themselves. Pimm explores the importance of using clear, consistent terminology, or knowledge of the discourse itself, but also understanding the context of the word and the context of how to correctly use the word within the mathematical discipline. Martinovic (2004) argues that within the domain of mathematics an important activity for learning is self-explanation, which is different from explanation to others; it is more focused than speaking aloud which may not be motivated by an effort to understand. Such learning strategies can be related to reflection and elaboration. Methods need to be introduced to include productive mathematic vocabulary within student's repertoire. Students need to express their ideas so that they can develop their understanding of mathematical concepts and language. Klibanoff et al. (2006) found that the more Math Talk the greater the acquisition of mathematically relevant language.
2.4 Knowledge Building

Schunk (2012) defines learning theories as attitudes, skills, strategies and the acquiring and modification of knowledge. Social Constructivism (Vygotsky 1978) is an intellectual by-product which is linked to the instructional approach of collaborative inquiry and Knowledge Building pedagogy. Vygotsky emphasized the need for social interaction as a critical vehicle for understanding. Vygotsky includes debating, dialoguing, and conversing, all of which enhance understanding because they involve a negotiation of meaning in which the student provides relevant insights. Students negotiate meaning and refine one another’s knowledge as core developmental tools to further the student’s learning. As Vygotsky stated, “Every function in the child’s cultural development appears twice: first, on the social level, and later, at the individual level; first between people, then inside the child” (Vygotsky 1978, p. 57).

Social constructivism became an area of interest in mathematics education in the 1980s and continues to evolve (Artzt and Newman 1997; Davidson 1990; Paul Earnest 1991, 1994, 1998). As elaborate above, collaborative learning in the field of mathematics is grounded in a social constructivist model of learning (Yackel et al. 2011). Collaborative learning envisions students as working together to solve a problem while the teacher facilitates the social construction of knowledge. Success within collaborative work depends on contributions from all students. Student accountability forces the community to trust and depend on each other while being respectful and contributing to advance their collective and individual understanding. This doctrine is a form of student-directed learning that engages students in interpretation, synthesis, and investigation through collaboration. The instructional method of collaborative inquiry allows two or more students to interpret together and achieve shared learning ideas and goals.

Knowledge Building is a principle-based approach in which students reframe and advance ideas in a manner that is conducive to idea improvement and in keeping with work and knowledge creating organizations. The term “Knowledge Building” was introduced by Bereiter and Scardamalia in the mid 1980’s, along with computer support for it (Scardamalia, Bereiter, Mclean, Swallow & Woodruff 1989; Scardamalia & Bereiter 1991, 1992;).
Scardamalia (2002) articulated twelve Knowledge Building principles to incorporate in practice:

1. Real Ideas, Authentic Problems
   Knowledge Problems arise from efforts to understand the World. These are problems that learners care about. Knowledge forum creates a culture for creative work with ideas. Notes, views, and Rise Aboves serve as direct reflections of the core work of the organization, and the ideas of its creators.

2. Improvable Ideas
   All ideas are treated as improvable. Participants work continuously to improve the quality, coherence, and utility of ideas. For such work to prosper, the culture must be one of psychological safety, so that people feel safe in taking risks. Knowledge Building supports all aspects of design, always an opportunity to revise. Background operations reflect change: continual improvement, revision, theory refinement.

3. Idea Diversity
   Idea diversity is essential to the development of knowledge advancement. To understand an idea is to understand the ideas that surround it, including those that stand in contrast to it. Idea diversity creates a rich environment for ideas to evolve. Knowledge Forum facilitates to link ideas, and to bring together different combinations of ideas together in different notes and views promoting the interaction that makes productive use of diversity.

4. Epistemic Agency
   Participants set forth their ideas and negotiate a fit between personal ideas and the ideas of others, using contrasts to spark and sustain knowledge advancement rather than depending on others to chart the course. They deal with a problem of goals, motivation, evaluation and long-range planning that are normally left to teachers or managers. Knowledge Forum provides support for theory construction and refinement and for viewing ideas. Scaffolds for high level knowledge processes are reflected in the use and variety of epistemological terms.
5. Community Knowledge, Collective Responsibility

Contributions to shared, top-level goals of the organization are prized and rewarded as much as individual achievements. Team members produce ideas of value to others and share responsibility for the overall advancement of knowledge in the community. Knowledge Forum’s open, collaborative workspace hold conceptual artifacts, that are contribute by community members. Community membership is defined in terms of reading, and building on the notes of others, ensuring views are informative and helpful for the community. Participants share responsibility for the highest levels of the organizations knowledge work.

6. Democratizing Knowledge

All participants are legitimate contributors to the shared goals of the community; all take pride in knowledge advances achieved by the group. The diversity and divisional differences represented in any organization do not lead to separations along knowledge have/have-not or innovator/non-innovator lines. All are empowered to engage in knowledge innovation. There is a way not the central knowledge space for all participants; analytic tools allow participants to assess evenness of contributions and other indicators of the extent to which all members to their part in a join enterprise.

7. Symmetric Knowledge Advancement

Expertise is distributed within and between communities. Symmetry in knowledge advancement results from knowledge exchange and from the fact to give knowledge is to get knowledge.

8. Pervasive Knowledge Building

Knowledge Building is not confined to particular occasions or subjects but pervades mental life – in and out of school.

9. Constructive Uses of Authoritative sources

To know a discipline is to be in touch with the present state and growing edge of knowledge in the field. This requires respect and understanding of authoritative sources, combined with a critical stance toward them. Knowledge building
encourages participants to use authoritative sources, along with other information sources as data for their own Knowledge Building and idea-improving processes.

10. Knowledge Building Discourse

The discourse of Knowledge Building communities’ results in more than the sharing of knowledge; the knowledge itself is refined and transformed through the discursive practices of the community – practices that have the advancement of knowledge as their explicit goal. Knowledge Forum supports rich intertextual and inter-team notes and views and emergent rather than predetermined goals and workspaces. Revision, reference, and annotation further encourage participants to identify shared problems and gaps in understanding and to advance understanding beyond the level of the most knowledgeable individual.

11. Embedded, Concurrent and Transformative Assessment

Assessment is part of the effort to advance knowledge – it is used to identify problems as the work proceeds and is embedded in the day-to-day workings of the organization. The community engages in its own internal assessment, which is both more fine-tuned and rigorous than external assessment, and serves to ensure that the community’s work will exceed the expectations of external assessors. Standards and benchmarks are objects of discourse in Knowledge Forum, to be annotated, built on, and risen above. Increases in literacy, twenty-first-century skills, and productivity are by-products of mainline knowledge work, and advance in parallel.

12. Rise above

Creative Knowledge Building entails working toward more inclusive principles and higher level formulations of problems. It means learning to work with diversity, complexity and messiness, to achieve new syntheses. Knowledge Builders transcend trivialities and oversimplifications and move beyond current best practices. Conditions to which people adapt are as a result of the success of other people in the environment. Notes and views support unlimited embedding of ideas, in advanced structures and support emergent rather than fixed goals.
Knowledge Building pedagogy supports students in posing questions, defining goals, acquiring and building knowledge and assuming collective responsibility for knowledge advancement. Collective responsibility is more complex than collaboration, requiring individual contributions to the community and individual reflective thought to enable individuals to achieve something greater than they could by working exclusively on their own, but never suppressing independent action. Knowledge Building should achieve a productive blend of individual and team work encouraging students to examine their own ideas while advancing the community’s knowledge.

Knowledge Building discourse is supported through “epistemological markers,” as elaborated below, to support a “process towards revelation and special problems” that can be easily identified and solved pedagogically (Guba & Lincoln, 2005, p.196). Co-creation assists students by allowing them to explore solutions in a space suited to their needs. In Knowledge Building Pedagogy, the classroom shifts focus from providing pre-validated answers, and instead focuses on thinking as a group interaction, facilitated through blended face-to-face and online discourse.

### 2.5 Knowledge Forum

Knowledge Forum is an asynchronous communication technology specially built to foster and facilitate Knowledge Building pedagogy (Scardamalia & Bereiter 1983). Knowledge Forum supports users in contributing ideas and artifacts to a shared discourse environment and to help advance ideas through writing, questioning, reflecting, problem solving, and theory improvement. Knowledge Forum supports multimedia contributions to an online community space. This technology, first piloted in 1983, was released as Computer Supported Intentional Learning Environments (CSILE). Scardamalia and Bereiter (1993) designed their system to address the fact that classrooms do not provide enough knowledge sustaining discourse. A networked system was used to provide students with a direct way to work together to advance their ideas, to identify learning gaps, to work through misconceptions, and more generally to engage in high level knowledge processes.

As Scardamalia (2002) explains, CSILE was designed to: “advance knowledge practices for all participants, foster continual improvement of community knowledge, as a framework for
Knowledge Building practices & pedagogies, and as well provide a space to conduct such work."
Knowledge Forum has undergone several revisions, incorporating various digital tools to
enhance participant learning and promote metacognition. The most recent edition, Knowledge
Forum 6 (KF6), can be used across devices and platforms (computers, tablets, and smartphones).
KF6 retains data by synchronizing online forums with a sign-in secured web access. These
modifications and enhancements create a rich educational experience that is in sync with 21st-
century student expectations.

By engaging in Knowledge Building activities, students can scaffold, build on, create new
knowledge, and address questions (Scardamalia 2002 & Bereiter and Scardamalia 2003). The
community space can be subdivided into “views.” Each community created view is a
collaborative page which allows users to add individual or co-authored notes. Notes can be
composed of text, images, drawings, and videos. Images can be embedded as the background of
a “view” and composed as a thought experiment. Scaffolds, a scaffolding feature within each
community can be designed as markers to allow students to organize and engage in high level
knowledge work, which is beneficial for the needs of the community.

Recently, keyword attachments to images and a tool that allows the author of a note to see words
being used by others in the online community supports new forms of connectedness. Students
can inspect adjacent words quickly to incorporate them into their own notes, to share ideas more
effectively with other students and to discuss and improve concepts through continual exchange.
Students are thus more inclined to read each other's notes, connect with shy students, and support
students demonstrating different levels of engagement. This further incorporates students who
wish to use speech to text which allows a more inclusive space, to incorporate well-being and
inclusion. The online community is meant to serve as a safe space for sharing ideas with no
biases or favouritisms, with all thoughts, misconceptions, and justifications part of the learning
process. The emphasis is on idea improvement with the technology designed to engage students
in the twelve Knowledge Building principles (Scardamalia 2002).

Students are encouraged to reflect on their notes and explanations and on how and why their
ideas add value to their class community. Through this reflection process they evaluate, support or negate certain ideas based on “explanatory coherence” (Thagard 1997). Thagard’s concept of “explanatory coherence” suggests that a student’s hypothesis is inherently thoughtful and not just randomly developed within the context of the community of learning. When a student creates a note, they contribute to the discussion in which all will discuss their ideas in the community.

Students read notes and can choose to read other’s notes without making any contributions. Most students write notes and build on to create threads to further thinking and address related issues. Students can also create a “rise above” (Scardamalia 2002), a meta-note that references the best, or most relevant ideas and thoughts from the network, combining them coherently and cohesively. Scardamalia and Bereiter (2006) characterize the “rise above” meta-note as an opportunity to “synthesize ideas, reduce redundancy and impose higher levels of organization” (pg. 27). The “rise above” should emerge spontaneously over time; new designs are being considered to address occasions where it does not. Scardamalia (2004) sees the “rise above” act not as a winning idea, but as one that cultivates the various ideas holistically. Moreover, “rise above” is a metacognitive synthesis to the continual progression of understanding and the lifecycle of an idea. Never should a “rise above” consist simply in a copying of ideas, but instead as an outcome of constructive dialoguing. This approach is in line with the view of Russian literary critic Mikhail Bakhtin (1986) “dialogic nature of learning”, in which the diverse thoughts become integrated into coherent structures or metacognitive syntheses of the ongoing dialogue. “Rise above” is a metacognitive synthesis to the continual progression of understanding and the lifecycle of an idea. This very notion of dialogue, interaction, and cognitive extrapolation makes the Knowledge Forum model intriguing and effective. The Knowledge Building learning model can be one incorporated into elementary mathematics classrooms.

2.6 Knowledge Building in Mathematics

Mathematics education must resonate with a new generation of students who experience the need for mathematics through many technological facets of their lives. The classroom should be able to incorporate technology as a complementary form of pedagogy. One area of direct relevance is in computer-assisted collaborative pedagogy. While learning is acquired ultimately
individualistically, as argued throughout this chapter the learning of the individual is enhanced through collaboration (Stahl 2005).

Confrey and Kazak (2006) and Steffe and Kieren (1994) argue that a curriculum focussed on only word problems is a complicated learning method for students, which may be riddled with misconceptions. Mathematical knowledge becomes subjective knowledge through construction in a collaborative framework; the individual student learning is understood as undergoing reformation and reconstruction by peer interactions.

When combined with the computer-assisted collaborative model, a process called “Distributed Cognition” emerges whereby knowledge can be spread across a group of people through the digital tools that they use to solve problems (Norman 1993 & Hutchins 1996 in Stahl 2006). This technological model thus not only supports student collaboration but promotes it, encouraging a process of mutual understanding, respect, and openness to the ideas shared and explored.

Before the current technological possibilities, Brown (1982) referred to the creation of an “immediate environment” needed to incorporate written symbols, construct diagrams on paper and enhance written accounts. In all these approaches, there seems to be an awareness that face-to-face communication is rendered more effective through written and technological support. Mathematics classrooms should consist of blended face-to-face communications with technological environments to enhance and complement one another. Student’s should not have the sole goal to publish more content but to grow their ideas. Teachers should ensure a balance between the use of paper-and-pencil learning with the digital tools. In this way, the student can come to see the abstract principles involved, independently of the medium used.


Nason & Woodruff (2002) noted the need for Knowledge Forum to provide symbolic math
representations to enhance the discourse space. The most recent version of Knowledge Forum (KF6) allows use of symbols and graphical representation of math ideas; students can now hand-write their equations using a blended method of typed and handwritten text, utilize voice-over command software to transform their spoken discourse into typed text, including equations, import pictures, and work more productively with other software environments.

By uploading an image from a smartphone or tablet, students can now share graphical representations of their ideas across platforms and across writing methods. In addition to the capacity to add pictures, students can build representations of mathematical structures and designs and other multimedia components can be attached and uploaded onto the group view, allowing students to build on ideas in various formats. The capacity to manipulate and present data in different forms supports students to constructing their own representations, comparing them, and extrapolating principles of mathematics. Some students benefit from showcasing their knowledge in a visual manner, others in a text-based manner; the technology is flexible as to cognitive learning style. Moreover, all uploads, and representations can be co-authored, allowing for collaborative production with potential for stronger motivation and engagement.

As Finlayson (2014) points out in her paper, Boaler (2002) has voiced the critique that “math is often taught as if students have the same ability, and pace of working” (p.101). Knowledge Building pedagogy and technology aim to accommodate diverse needs. Lazarus et al. (2013), explained that the Ontario mathematics curriculum (Ontario Ministry of Education, 2005) does not expand on what communication looks like in online environments, but identifies what it means to communicate in and do mathematics (p.34). Sinclair (2005) found students mathematical learning to be useful but found that students need additional sessions at which they can discuss their mathematical ideas, explore other mathematical definitions, and find further information to support the use of mathematical terminology. Students need to organize and consolidate their thinking in which “Math Talk” does not only occur within the mathematics classroom but can occur organically throughout any other classroom lesson. Moss & Beatty (2006) found that students on Knowledge Forum "interwove words, numbers, and formal
symbols in their interactions with one another” (pg. 460) but that more research was needed to analyze the significance of such discourse.

2.7 Research Questions
Leher and Chazan (1998) indicate that students’ knowledge of geometric concepts and terminology are sparse in comparison to concepts in numeracy. The current study is exploratory, focusing on math understanding in the area of geometry and, more specifically, on student mathematical discourse in both face-to-face and online contexts. The study aims to explore pedagogical and technological affordances to advance student understanding of geometry and knowledge construction as well as demonstrate possibilities for new analytic tools to advance the agenda.

The specific research questions that will be addressed are as follows:

1. How do students advance “Math Talk” in both face-to-face Knowledge Building Circles and online through Knowledge Forum? Are students working together as a community advancing their geometry knowledge?

2. What ideas were advanced through Math Talk in the grade 2 classroom under investigation and what does idea continuity and improvement look like in this context?

3. How are students’ "Ways of Contributing" (Chuy et al. 2011) and connectedness via math concepts manifested within both face-to-face Knowledge Building Circles and online Knowledge Forum contexts?
Chapter Three
Methodology

The methodology chapter begins with a conceptual overview of the research methodology and design-based research, followed by the selection process for participants, data collection and analysis procedures, and limitations and ethical considerations for this research.

3.1 Conceptual Framework
According to Bryman (2001) an interpretivist paradigm is one “in which human beings make sense of their subjective reality, attach meaning to human experience, in which world values and interest become a part of the research process” (p. 7). Creswell singles out research methods focused on local practices and action research: "You use action research when you have a particular educational problem to solve" (Creswell, 2012, p.577) and theoretical sampling involving “the simultaneous and sequential collection and analysis of data” (Creswell, 2012, p.433). Data-collection procedure involves ethnographic methods. This tripartite—interpretivist, action research, ethnographic approach made it possible to focus on the student’s voice, ideas, and mathematical understanding, as they are elicited directly.

3.11 Design-Based Research
This study utilizes design-based research methodology and its principles. Design-based research promotes “a process of testing new ideas” (Mills, 2011), while allowing for a rich description of the findings. Rather than testing an existing theory, the objective is to utilize, compile the collect observations and insights. Design-Based research is consistent with what Creswell (2012) describes as "an experimental study in which the experiment yields useful information about outcomes, with the additional collection of qualitative data developing a more in-depth understanding of how the experimental intervention worked" (p.535). These procedures demonstrate the relevance of Math Talk and Knowledge Building pedagogy in getting students to become what Penuel et al. (2007) call "potential adopters of innovation" (pg. 54).

Design-based research has a solid foundational as an emerging paradigm (e.g., Barab & Squire 2004, Collins, Joseph, & Bielaczyc, 2004; Dede, 2005; The Design-Based Research Collective,

Design-based research emerged in the context of the Learning Sciences. The goal of the International Society of Learning Sciences (2009) was to adopt design-based research to conduct inquiries into how people learn across tasks and settings, to establish evidence-based guidelines to predict new learning. The methodology links practice and formal education. Bell (2004) defined the method as “observation and documentation of the everyday practices of participants in the setting, before during and after the introduction of new designs. The analysis documents anticipated consequences or emergent designs or practices that were never anticipated” (pg. 249). By adopting this method, the approach of the thesis is to explore how a Knowledge Building learning environment influences mathematical learning.

Kali & Ronen (2005) adopted a peer evaluation activity, analyzing the phases of the study to build on existing knowledge. By utilizing design-based research this thesis seeks to provide the necessary tools, implementations, and suggestions for tackling practical problems in authentic mathematical settings. By analyzing procedures and outcomes the research will extract design-based principles and knowledge to guide new designs. The phases which were prioritized for this study were adopted from Kali & Ronen (2005): i) articulating design principles, ii) design-enactment-refinement iterations, and iii) revising pragmatic principles relevant to the Ontario Mathematics Curriculum, Math Talk, and Knowledge Building. Design-enactment-refinement iterations are distributed throughout the study, assessing impact on classroom mathematics learning as the work proceeded. Anderson and Shattuck (2012) state that design-based research's primary outcomes “do not produce measurable effect sizes that demonstrate what works” but instead provides rich descriptions of contexts, challenges of the “implementation, the development processes involved in creating and administrating the interventions and the design principles that emerged” (pg. 22). The results will be exploratory to better examine the research.
questions and have a better understanding of Knowledge Building pedagogy and mathematics to inform future work.

Design-based research was chosen to address the real learning problems that appear in the naturalistic classroom context (Barab & Squire 2004; Design-based research collective, 2003). By recognizing that variations in teaching are bound to occur, it is important to document such variations on how they relate to students learning and experience (Jen et al. 2015). By incorporating design-based learning into this thesis, the study aimed to collect and analyze students’ interpretation and conceptual understanding of shapes. This framework is flexible and adaptable to changing situations that were encountered in the geometry classroom. The research process was informed and guided by the current literature, with both qualitative and quantitative data used to assess and guide practice.

3.12 Limitations and Challenges of Design-Based Research

As with all methodologies, there are challenges that must be addressed. While the methodology of Design-Based Research is rigorous, Barab and Squire (2004) critique design-based methodology for being perhaps too rigid in “conceptualization, design, development, and implementation in which challenges the researcher to remain credible, and trustworthy becomes challenging” (p.10). These concerns can be overcome and were kept in mind throughout the study and data interpretation.

A primary disadvantage of design-based research is that it is time intensive and due to the sampling method elaborated below, results cannot be said to be representative of the broader grade 2 school population. Due to such factors, input and results can be contradictory (Penuel et al. 2007). Design-based research methodology is robust with respect to the complexities associated with classroom culture as it is meant for research in complex settings such as classrooms. A final concern, is that design-based research cannot be a logistically replicated intervention. Nevertheless, “emergent phenomena regularly lead to new lines of inquiry” (Design-based Research Collective 2003: pg. 7). Although this prior statement seems
problematic, this study envisions this as a challenge, in which the methods can be applied to understand better how to best organize the classroom space. This collaborative co-design aspect promotes and facilitates a better understanding of the classroom culture at hand, and allows researchers to devise appropriate intervention and discursive strategies.

3.2 Participants

The research site chosen has a long-standing relationship with the research team at the Institute for Knowledge Innovation and Technology (IKIT), University of Toronto. This team develops Knowledge Building theory, pedagogy, and technology and the school is a university lab school located on the University of Toronto, downtown campus. Teachers in this school are familiar with Knowledge Building practices and software, using it in many curricular areas, but not in mathematics. All students start in kindergarten using Knowledge Building pedagogy and in grade 1 using Knowledge Forum. The grade 2 teacher had several years of experience with Knowledge Building pedagogy and technology and her class with twenty-two students was selected. The researcher and teacher shared goals and worked collaboratively. Each student had been studying math within the framework of the Ontario curriculum and had access to the Knowledge Forum software used to record and evaluate their Math Talk. The sampling methodology afforded maximum opportunities for engagement and productive variations (Patton, 1980; Strauss & Corbin, 1998).

3.3 Data Collection

This eight-week investigation began March 2016. The teacher and researcher negotiated work to be conducted over two to three math periods a week. These time frames varied due to school-wide events or special activities that interrupted the regularly scheduled sessions. Visits consisted of structured one-to-two-hour intervals of classroom observation. During each observation, the class would begin with students forming a Knowledge Building circle to discuss a topic or theme. Once the discussion was concluded, the students continued work in Knowledge Forum. Thus Knowledge Building circles and Knowledge Forum were an integral part of the day-to-day activities. These students had a previous one-year experience with Knowledge Forum
technology so they worked with it easily. They were, however, introduced to a newer version (Knowledge Forum 6), but the shift was easy.

The class was divided into two groups of 11 students and remained this way for the duration of the study. The groups contained an equal number of boys and girls; after the math period ended for group one the same math session was enacted with group two. Both groups operated in similar environments and with similar input from the teacher. All participants completed the same units, on the same day, with the same teacher but their lines of inquiry, ideas and thoughts depended on their different participants. During each session, students had nine computers and twenty-two iPads available at their disposal. Student’s findings and additional ideas were posted in Knowledge Forum notes for further group discussion. Often ideas from previous sessions were re-introduced in a subsequent Knowledge Building circle. Some ideas from one group were introduced to the other for further discussion.

The complex intervention that occurred over the eight-weeks of this investigation involved hundreds of researcher-teacher decisions to promote innovative mathematics practice and understanding via mathematical discourse. These included conversations surrounding student abilities, how to engage all students to promote innovative practices and fine-tuning successful designs. With the feedback from students, the researcher and teacher decided next steps. Each week, student ideas, inquiries, and gaps in knowledge informed refinements in design. Penuel et al. (2007) state that "co-design process" involves teachers, and researchers working together to design educational innovations, to address educational needs and how to implement them in a systematic manner. The co-design process allowed for designs to elicit and promote students’ mathematical thinking during Knowledge Building. As students explored big ideas in geometry their patterns of understanding and suggestions became input for subsequent efforts. Concepts were incorporated to expand upon students previous understanding.

Before the beginning of each session an iPad was set up in the classroom to record student and teacher discourse within the Knowledge Building Circles. Students were reminded by the teacher to raise their hand. The most important rule: one student speaks at a time. This was meant to
allow all students to be heard equally and clearly. During group instruction and Knowledge Building circles the teacher would pose questions and incorporate mathematical statements. Students discussed ideas and identified strategies to advance them. Students shared ideas and were asked at times to clarify their thoughts and to build onto each other’s ideas, or alternatively pose questions to each other to clarify and further their understanding. Occasionally, some students worked individually or in small groups in separate activities. Their findings were then brought back to the group for further discussion. At times, certain students were prompted or called upon by the teacher to respond to help them feel comfortable responding on their own.

The primary source of data regarding students’ roles and experiences was discourse from speaking turns during the Knowledge Building circle and student notes created on Knowledge Forum. Video recordings of student discussions during the Knowledge Building circle made it possible to analyze face-to-face discourse turns: A discourse turns represents a student or teacher spoken statement or question occurring during a Knowledge Building circle.

3.31 Data Analysis Procedures

Occasionally, the researcher joined the students on the carpet in their circle formation, to ensure trust and openness. Table one describes the weekly interventions and actions that ensued during the study. These activities were designed to carry out exploratory lessons and were meant to address the specific needs of a mathematics Knowledge Building classroom while implementing mathematical instruction.
**Week 1**

The Study commenced by posing a question: "Why do we share knowledge?" in a Knowledge Building Circle. Students established that knowledge is related to the importance of why students learn & that knowledge should not be hoarded. Students discussed "good and bad" knowledge. There was consensus that knowledge needed to be shared or else civilization would not be functioning well. Students were introduced to Knowledge Forum Six & asked to be mindful when they post notes to think of their ideas as important knowledge. Students continued then guided this mindset into Knowledge Forum. The lesson ended by informing the students of the idea of using Knowledge Building pedagogy and technology to share knowledge within the discipline of mathematics.

**Week 2**

The week began with focus on geometry and mathematics. Students were posed the question "what is a shape"? Students began in a Knowledge Building circle which then shifted into a Knowledge Forum period. Students had various answers and could not as a community reach a consensus on what a shape is. Students were instructed to then draw what they believed was an example of a shape. Students used their understanding and provided a few mathematical terms to explain why they thought their artifact was a shape.

**Week 3**

Students were separated into two groups of three and one group of four. Students were given a handful of different shapes & were told to sort & classify these shapes in any manner they choose. The shapes were plastic & each shape was also a different colour. Once this activity was completed students presented their ideas and findings. Images of their results then were posted on Knowledge Forum & students began to write notes on their findings, ideas, and organizational patterns. They built on why they sorted in the ways they did, and built on alternative ways in which they could sort, but did not demonstrate these alternative ways during the activity.

**Week 4**

Students accessed a gif, which demonstrated the translations and transformation of shapes. Students were then asked to explain on Knowledge Forum what they saw, their thoughts on this, and what it meant. After this activity, students were given a sheet of paper and were asked to draw an example of something that was not a shape, and then explain again why they understood this not to be a shape by using mathematical words to describe their thinking.

**Week 5**

Students were introduced to two dimensional and three dimensional shapes and each given a three-dimensional block. They were asked to recreate it in a drawing and to explain their shape in as much detail as possible. Once they completed this activity they went on Knowledge Forum to explain their understanding of differences of two-dimensional to three-dimensional shapes. Students began to incorporate their shape, to contrast the difference in 2d as flat and 3D as a pop-up shape.

**Week 6**

On week six, students were given nine statements. These were the statements used by the students themselves in either describing whether an object was a shape or not. The purpose of the activity was to reach a consensus or to see if students would demonstrate their understanding of rules. This activity was conducted in Knowledge Forum but without students’ names shown within the view, so as not to bias responses.

**Week 7**

On week seven, students were presented with a view of their group’s previously sketched images. One of the two last images of the students was posted on the view. Each image was rotated and had the color removed. Some images represented shapes, while others did not. Students were asked to identify based on their understanding and to build knowledge if it was a shape or not.

**Week 8**

Students were shown the grade 2 mathematics curriculum words in a word cloud. They were told to write their thoughts on Knowledge Forum. Students began to recognize words they learned throughout the geometry unit and inquired or attempted to understand what other words meant.

*Figure 1. Design Based Research Intervention Activities and Classroom Schedule Overview*
This study incorporates qualitative and quantitative data. As Greene et al. (1989) noted, the mixed methods approach “seeks convergence, corroboration, correspondence of results and various methods” (p. 259). As noted by Greene & Caracelli (1997), mixed methods provide a “complex picture” of various interactive and cognitive phenomena. (p.7) and represents a type of convergent parallel design (Creswell, 2012) with quantitative and qualitative data used in a complementary way. By analyzing both datasets separately, the study can compare them in a convergent fashion. This study seeks to justify results from a convergent design utilizing the strongest designs of quantitative and qualitative methods. Quantitative and qualitative data are of equal weight, and are combined for an integrated model of the data.

Video recordings captured student face-to-face talk. The students were accustomed to video cameras being used extensively in this classroom, prior to this investigation. Students’ in-class discourse was downloaded onto a password-protected external hard drive and transcribed. Students wrote a total of 306 notes across nine views in Knowledge Forum: Why Do We Share our Knowledge, What is a Shape, Grouping Shapes, Shape Design, 2D & 3D Shapes, Shape, Identify Shapes, Math Scaffolds & Math Discourse.

### 3.32 Ways of Contributing

Resendes, Chuy, Resendes, Chen and Scardamalia (2011) devised a “Ways of Contribution” framework which focuses on students’ emergent contributions in a Knowledge Building classroom. As per Chuy, Resendes, Tarchi, Chen, Scardamalia, & Bereiter (2011), this framework focuses on “kinds of contributions students can make that move explanation-building dialogue forward” (p. 243). The framework is applied to students face-to-face and online discourse and is applied to the teacher’s oral discourse during the Knowledge Building circle session. Each statement is classified according to one of the six main modes of contribution, as indicated in Table 1.
<table>
<thead>
<tr>
<th>Main Category</th>
<th>Subcategory</th>
</tr>
</thead>
<tbody>
<tr>
<td>Formulating thought-provoking questions</td>
<td>1—explanatory questions</td>
</tr>
<tr>
<td></td>
<td>2—design questions</td>
</tr>
<tr>
<td></td>
<td>3—factual questions</td>
</tr>
<tr>
<td>2—Theorizing</td>
<td>4—proposing an explanation</td>
</tr>
<tr>
<td></td>
<td>5—supporting an explanation</td>
</tr>
<tr>
<td></td>
<td>6—improving an explanation</td>
</tr>
<tr>
<td></td>
<td>7—seeking an alternative explanation</td>
</tr>
<tr>
<td>3—Obtaining Information</td>
<td>8—asking or looking for evidence</td>
</tr>
<tr>
<td></td>
<td>9—testing hypotheses</td>
</tr>
<tr>
<td></td>
<td>10—reporting experimental results</td>
</tr>
<tr>
<td></td>
<td>11—introducing new information (source)</td>
</tr>
<tr>
<td></td>
<td>12—introducing new information (experience)</td>
</tr>
<tr>
<td></td>
<td>13—identifying a design problem</td>
</tr>
<tr>
<td></td>
<td>14—thinking of design improvements</td>
</tr>
<tr>
<td>4—Working with Information</td>
<td>15—providing an evidence or reference to support a particular explanation</td>
</tr>
<tr>
<td></td>
<td>16—providing an evidence or reference or to discard a particular explanation</td>
</tr>
<tr>
<td></td>
<td>17—weighing different explanations</td>
</tr>
<tr>
<td></td>
<td>18—accounting for conflicting explanations</td>
</tr>
<tr>
<td>5—Synthesizing and Comparing</td>
<td>19—synthesizing available knowledge</td>
</tr>
<tr>
<td></td>
<td>20—making a comparison or analogy</td>
</tr>
<tr>
<td></td>
<td>21—initiating a rise-above entry</td>
</tr>
<tr>
<td>6—Supporting Discussion</td>
<td>22—using diagrams to communicate or support ideas</td>
</tr>
<tr>
<td></td>
<td>23—giving an opinion</td>
</tr>
<tr>
<td></td>
<td>24—acting as a mediator</td>
</tr>
</tbody>
</table>

*Table 1. Ways of Contributing to Explanation-Seeking Discourse*
3.33 Knowledge Building Discourse Explorer (KBDeX)

KBDeX is an analytic tool designed to facilitate content-based Social Network Analysis for Knowledge Building discourse. As explained by Ma et al. (2015), "KBDeX produces visualization models of a word, note, and student networks based on the co-occurrence of words in a note. Edges in the student network show the strength of connections among students whose notes share the same word" (pg. 2) Student discourse entries on Knowledge Forum were exported into Knowledge Building Discourse Explorer (KBDeX) (Oshima, Oshima, & Matsuzawa, 2012). KBDeX was used to explore the emergence of select math terms in student discourse and to assess patterns of engagement surrounding those terms. For this investigation, the word list for KBDeX analysis was compiled from the Ontario Curriculum of Mathematics. Accordingly, results to be reported in the next chapter show connectedness between students based on their use of the math vocabulary extracted from these guidelines. Wordlists extraction is elaborated in the next section.

3.34 Wordlist

The word list consists of forty words and phrases extracted from the Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005), from the Grade Two mathematics strand of Geometry and Spatial Sense. These words served as “expert vocabulary” determine the extent to which students were using expert vocabulary. The terms are presented in Appendix 1. Expert vocabulary has been used in previous work reported by Resendes (2013) & Tam (2016). In this study vocabulary is used as a reference point to explore math-term usage and interconnectedness of students based on use of these terms.

3.4 Research Limitations and Ethical Considerations

A methodological challenge with design-based research is that the researcher is directly engaged in the process, creating possible bias. The sample size was small (22 students, one teacher) and not designed to address important considerations such as students with high needs or requiring Individual Education Plans.
As for ethical considerations, the proposed study was minimal risk, involving low group vulnerability. Participants were given the right to refuse to participate and they could withdraw at any point during the study. They were informed about their rights and data were collected with parental permission. Each participant’s family was informed of the study, via a letter explaining the role of the researcher. The teacher was a co-investigator, ensuring classroom practices were of value to students.

3.5 Chapter Summary

In this chapter, the conceptual framework for the research and iterative refinement process of design-based research was described, followed by research design, procedures, and research limitations and ethical considerations. Procedures included a co-design framework in which the teacher and researcher worked closely together with the goal of positive impact on classroom mathematics learning. Findings from this exploratory study are discussed in the next chapter.
Chapter Four
Findings

4.1 Chapter Overview

In this chapter results of face-to-face and online discourse are reported. First general findings are discussed face-to-face within Knowledge Building Circles followed by online discourse in the form of Knowledge Forum Notes. Second, two case studies are reported to provide more in-depth analysis of student engagement through each medium. KBDeX will be used to present quantitative findings; the Ways of Contribution scheme (Chuy et al. 2011) will be used to present qualitative findings. Key ideas will be elaborated through in-depth accounts of how Math Talk and Knowledge Building pedagogy proceeded. Data sources include student notes, and talking turns which can consist of any spoken statement or question.

4.2 Case 1: Knowledge Building Circle

In the Knowledge Building Circle, there were 477 discourse turns, 310 from twenty-two students, 167 from the teacher. As indicated in Table 2, several students only took three turns. The student, with the highest number had thirty-six turns, over twelve times that of the lowest input. The student mean talking turn was fourteen (310/22), with eight students at or above the mean.
<table>
<thead>
<tr>
<th>Students (groups 1 and 2) and teacher contributions to discourse</th>
<th>Knowledge Building Circle: number of discourse turns per participant (total 477 turns)</th>
<th>Percentage of contributions to the Knowledge Building Circle discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>4%</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>5%</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1%</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>1%</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>3%</td>
</tr>
<tr>
<td>6</td>
<td>23</td>
<td>5%</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>1%</td>
</tr>
<tr>
<td>8</td>
<td>36</td>
<td>8%</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>1%</td>
</tr>
<tr>
<td>10</td>
<td>12</td>
<td>3%</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
<td>3%</td>
</tr>
<tr>
<td>12</td>
<td>27</td>
<td>6%</td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td>2%</td>
</tr>
<tr>
<td>14</td>
<td>35</td>
<td>7%</td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td>2%</td>
</tr>
<tr>
<td>16</td>
<td>9</td>
<td>2%</td>
</tr>
<tr>
<td>17</td>
<td>12</td>
<td>4%</td>
</tr>
<tr>
<td>18</td>
<td>7</td>
<td>1%</td>
</tr>
<tr>
<td>19</td>
<td>19</td>
<td>4%</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>1%</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>1%</td>
</tr>
<tr>
<td>22</td>
<td>10</td>
<td>2%</td>
</tr>
<tr>
<td>0 - Teacher</td>
<td>167</td>
<td>35%</td>
</tr>
</tbody>
</table>

*Table 2.* Knowledge building circle: frequency of speaking turns for students and teacher
4.21 Knowledge Building Circle Turns

The “Ways of Contribution Framework” (Chuy et al 2011) includes six categories. Each student discourse turn was coded to reflect its contribution type. As noted in Table 3, students’ face-to-face discourse totalled 310 discourse turns: 28% coded as “Working with Information”, 39% as “Supporting Discussion.” Student 6 provides an example of “Supporting Discussion”:

Table 3. Knowledge building circle: frequency of students’ ways of contributing to classroom discourse

<table>
<thead>
<tr>
<th>Student ways of contributing to discourse</th>
<th>Knowledge building circle: number of discourse turns per student (total 310 turns)</th>
<th>Percentage of student ways of contributing to knowledge building circle discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – Formulating Thought-Provoking Questions</td>
<td>12</td>
<td>4%</td>
</tr>
<tr>
<td>2 – Theorizing</td>
<td>27</td>
<td>9%</td>
</tr>
<tr>
<td>3 – Obtaining Information</td>
<td>32</td>
<td>10%</td>
</tr>
<tr>
<td>4 – Working with Information</td>
<td>86</td>
<td>28%</td>
</tr>
<tr>
<td>5 – Synthesizing and Comparing</td>
<td>32</td>
<td>10%</td>
</tr>
<tr>
<td>6 – Supporting Discussion</td>
<td>121</td>
<td>39%</td>
</tr>
</tbody>
</table>

**Student 6:** “there is a line in the middle but, the edge of something is the perimeter.”

The student was referring to a shape drawn by students with a line “in the middle”, referring to the lines on the outside as “edges” and “perimeter.” The Math Talk includes discipline specific terms such as perimeter, edge, and line, with a declarative statement reflecting understanding of concepts used previously in class.

Knowledge Building Pedagogy and Math Talk in unison support expression of mathematical ideas and provide opportunities for expression of geometric thinking, as suggested by Students 11 and 8.

**Student 11:** “all shape are made up of lines but only some shape have corners.”

**Student 8:** “I’m building onto Student 11’s comment. All shapes if they have lines they must have corners, think of a triangle, there is threes lines and three corners, a hexagon has six lines and six corners, a square, four lines and four corners.”
The statement by Student 8 was coded as “Working with Information” within the “Ways of Contribution” Framework. In addition to math terminology the student used Knowledge Building terminology. A “build on” is the term used in Knowledge Forum to elaborate a previously written note. Here Student 8 listened to and built on a spoken statement reflecting understanding of basic shapes and providing a reasoned analysis of why some shapes have corners, supported with examples and student-generated evidence.

<table>
<thead>
<tr>
<th>Teacher ways of contributing to discourse</th>
<th>Knowledge building circle: number of teacher discourse turns (total 167 turns)</th>
<th>Percentage of teacher ways of contributing to knowledge building circle discourse</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – Formulating Thought-Provoking Questions</td>
<td>36</td>
<td>22%</td>
</tr>
<tr>
<td>2 – Theorizing</td>
<td>5</td>
<td>3%</td>
</tr>
<tr>
<td>3 – Obtaining Information</td>
<td>25</td>
<td>15%</td>
</tr>
<tr>
<td>4 – Working with Information</td>
<td>32</td>
<td>19%</td>
</tr>
<tr>
<td>5 – Synthesizing and Comparing</td>
<td>9</td>
<td>5%</td>
</tr>
<tr>
<td>6 – Supporting Discussion</td>
<td>60</td>
<td>36%</td>
</tr>
</tbody>
</table>

*Table 4. Knowledge building circle: frequency of teacher’s ways of contributing to classroom discourse*

As we see in Table 4, 36% of teacher discourse turns were coded as “Supporting Discussion”; the comparable number for students was 39%. Teacher discourse turns were coded at a higher rate of “Thought provoking questions” (22%) (the comparable number for students was 4 %). This reflects the high rate of questions asked by the teacher. Rather than asking additional questions students answered the questions; these were coded as “Supporting Discussion.” (39%), and “Working with information” (28%). The teacher’s role clearly influenced student discourse turns; as it represented half of total spoken discourse turns and was similar in kind to student discourse turns. By asking thought-provoking questions the teacher provided support for student discourse turns coded as “Theorizing.” Students actually theorized at a higher rate than the teacher (9% students; 3 % teacher). Analysis of teacher Knowledge Building Circle discourse turns suggests that she assumed monitoring and structuring roles reflected in supporting discussion, asking questions, and working with information. It would be fascinating to determine the effects of increased use by the teacher of “Synthesizing and Comparing” (currently
9% of her total discourse turns) and theorizing (currently 5% of her total discourse turns). Would that, for example, serve to further increase student theorizing and formulation of thought-provoking questions? Rather than provide answers to student questions the teacher encouraged students to clarify statements using open ended questions:

**Teacher:**

“Why do you think they are shapes?”

“So you’re saying that the shape that you are looking at on Student 9’s pants is a shape.”

“So what do you mean by that? So what is not a shape? you are on to something. let’s work with that idea.”

“Yes, we’ve had this conversation. you’ve held on to this idea. Student 10 can you demonstrate with your body what a shape is”

By connecting various student ideas, the teacher supports the generation of new ideas as well as encourage students to deepen explanations. The teacher conveys that she values students’ ideas, that they can advance their understanding, and she helps them connect their ideas to personal and group experiences. At times, the teacher would ask questions, at other times use a statement to prompt further conversation or inspire student’s thinking. This action is in line with Knowledge Building Pedagogical practises; teachers prize student ideas, give the floor to the students as frequently as possible, encourage discussion so that misconceptions that might arise become part of the conversation, create a safe space to build confidence and encourage participation and discussion of gaps in understanding.

KBDeX software (Oshima, Oshima, & Matsuzawa, 2012) was used to produce the image in Figure 2 below, a representation created at the end of the study. It is a Knowledge Building Circle Network Visualization, a bipartite graph based upon the co-occurrence of terms in the community’s Knowledge Building Circle discourse turns. The nodes (yellow circles) numbered 1-22 in Figure 2 represent the 22 students involved in the study; the teacher is represented as node 0. Each line demonstrates that there is a connection. This connection is represented with at least on shared vocabulary word set from Appendix 1, between the two individuals connected by the line. The thicker the line, the more shared words. The network visualization shows that all
students were connected, that the majority of connections between students were strong, and that the teacher has connections (thick line) to every student. The visualization conveys that the teacher had a strong presence within the Knowledge Building Circle; all nodes are connected conveying a strongly linked community.

Figure 2. Knowledge Building Circle: Knowledge Building Discourse Explorer (KBDeX) Visualization at End of the Study

4.3 How Math Talk Proceeds During the Knowledge Building Circle.

Students engaged in various Knowledge Building Circle talks during which they learned geometric terms in the course of engaging in mathematical conversations in which students justified their definitions of shapes.
Figure 3 is based on the word list of geometry specific terminology in Appendix 1. The words selected for inclusion are based on the Ontario Grade two mathematics geometry curricula. The Figure 3 visualization shows that the network contains thirty words. Three words: octagon, degree and angle, are not connected to any other words as they were used exclusively in notes that made no reference to other geometry specific terms in Appendix 1. In the centre of the graph, eight words/phrases are most strongly interconnected: square, side, prism, two-dimensional, three-dimensional, points, edge, and corner. Also, demonstrated in Figure 3 is student discourse including uncommon two-dimensional shape terms such as pentagon, octagon, decagon or hexagon. These terms are found in curriculum guidelines above the Ontario Grade Two Curriculum level, as are the three-dimensional shape terms such as prism, sphere and cube that students used. There are also many descriptive math terms used to describe shapes: “angle, symmetry, perimeter, edge, points, and side.”

*Figure 3. Knowledge Building Circle: Knowledge Building Discourse Explorer (KBDeX) Visualization*

Figure 4 demonstrates all the Knowledge Building Circle discourse notes throughout the study. The inter-connected word network to the right includes all the connected terms; on the periphery are remaining terms entered in one note with no connection to terms in other notes.
All Knowledge Building Discourse turns were coded with the “Ways of Contribution” framework (Chuy et al. 2011). The following four examples were coded, as “Working with Information.” Note that each example serves a different purpose in the community Knowledge Building Circle and each use math terms in ways that clarify student mathematical understanding. These examples also serve to demonstrate how Math Talk proceeded during the Knowledge Building Circle.

Example: Using real world artifacts to explain math concepts (Conducted in KB Circle)
Student 16 - “So this is the Perimeter of the carpet, and for example on this carpet this is the perimeter, these black lines which are the edge.”

The student looked at perimeter and edge and clarified the distinction. By incorporating the carpet into Math Talk students demonstrate understanding and show that they can use math terms to describe their surrounds and extend their repertoire.

Example: Defining student understanding of math concepts (Conducted in KB Circle)
Student 12 - “The perimeter is what has to hold area.”

Student 12 followed Student 16’s discovery of the perimeter of the carpet. Student 12 added to the information of Student 16, explaining the importance of perimeter in terms of area.

Example: Mathematical Reasoning. (Conducted in KB Circle)
Student 14 - “Sorted by taking the half shapes which go into the whole shapes and made them into a pile. So, these trapezoids go into a hexagon, and these equilateral triangles do into the diamond shape.”

This example demonstrates student understanding about reasoned regarding composition, construction, and naming several different shapes: trapezoids, hexagons and triangles. In addition, the student put together pieces using terminology such as whole and half. Thinking in terms of fractions is not part of the geometry unit, but the student incorporated additional mathematics knowledge. Math Talk not only advanced understanding of geometry, shapes and spatial awareness, it became part of explanations generated about shapes and objects in their everyday lives.

4.4 Case 2: Knowledge Forum Notes

In Knowledge Forum students, can write singly or co-authored notes. There were 295 singly authored, 306 when each author of a co-authored note is credited with writing that note. As can be seen in Table 5, Student 15 created 3 Knowledge Forum Notes, Student 22, created 42 notes or fourteen times more than that of the lowest input. The mean number was 14 (306/22 students).
<table>
<thead>
<tr>
<th>Students (Groups 1 and 2) contributions to discourse</th>
<th>Knowledge Forum: number of notes per participant (total 306 notes) *</th>
<th>Percentage of Contributions in Knowledge Forum</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31</td>
<td>10%</td>
</tr>
<tr>
<td>2</td>
<td>22</td>
<td>7%</td>
</tr>
<tr>
<td>3</td>
<td>7</td>
<td>2%</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
<td>11%</td>
</tr>
<tr>
<td>5</td>
<td>8</td>
<td>3%</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>2%</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>3%</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>4%</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>2%</td>
</tr>
<tr>
<td>10</td>
<td>7</td>
<td>2%</td>
</tr>
<tr>
<td>11</td>
<td>13</td>
<td>4%</td>
</tr>
<tr>
<td>12</td>
<td>8</td>
<td>3%</td>
</tr>
<tr>
<td>13</td>
<td>7</td>
<td>2%</td>
</tr>
<tr>
<td>14</td>
<td>8</td>
<td>3%</td>
</tr>
<tr>
<td>15</td>
<td>3</td>
<td>1%</td>
</tr>
<tr>
<td>16</td>
<td>11</td>
<td>4%</td>
</tr>
<tr>
<td>17</td>
<td>31</td>
<td>10%</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>3%</td>
</tr>
<tr>
<td>19</td>
<td>12</td>
<td>4%</td>
</tr>
<tr>
<td>20</td>
<td>5</td>
<td>2%</td>
</tr>
<tr>
<td>21</td>
<td>16</td>
<td>5%</td>
</tr>
<tr>
<td>22</td>
<td>42</td>
<td>14%</td>
</tr>
<tr>
<td>0 - Teacher</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

Table 5. Knowledge forum: frequency of student notes

Co-authored notes were produced by the following students:

Students (1, 11) = 6 co-authored notes
Students (2, 7) = 1 note
Students (3, 4) = 1 note
Students (3, 9) = 1 note
Students (5, 9) = 1 note
Students (8, 11) = 1 note
4.41 Knowledge Forum Notes

Knowledge Forum notes were coded based upon the Ways of Contribution” Framework. (Chuy et al. 2011). As demonstrated in Table 6, the majority of the notes were rated as “Supporting Discussion” (44%); the next most frequent category was “Working with Information” (22%)—the two most frequent categories and in the same order as found in the Knowledge Building Circle analysis. They were used at twice the rate as in the face-to-face Knowledge Building Circle analysis, despite slightly fewer Knowledge Forum notes than Knowledge Building Circle turns. Only in two categories were contributions lower in Knowledge Forum: Formulating Thought- Provoking Questions and Obtaining Information; higher levels of theorizing in Knowledge Forum seem to counter balance lower levels of question asking in Knowledge Forum and obtaining information, while not dominant in either medium, is higher in Knowledge Building Circles.

<table>
<thead>
<tr>
<th>Ways of Contribution Framework</th>
<th>Number of Notes (306 Notes total)</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 – Formulating Thought-Provoking Questions</td>
<td>11</td>
<td>4%</td>
</tr>
<tr>
<td>2 – Theorizing</td>
<td>52</td>
<td>17%</td>
</tr>
<tr>
<td>3 – Obtaining Information</td>
<td>15</td>
<td>5%</td>
</tr>
<tr>
<td>4 – Working with Information</td>
<td>68</td>
<td>22%</td>
</tr>
<tr>
<td>5 – Synthesizing and Comparing</td>
<td>25</td>
<td>8%</td>
</tr>
<tr>
<td>6 – Supporting Discussion</td>
<td>135</td>
<td>44%</td>
</tr>
</tbody>
</table>

*Table 6. Knowledge Forum Notes: Frequency of Students ways of contributing to classroom Discourse*

One member in the community was not connected through select math domain vocabulary, as indicated in Figure 5. Student 15 only contributed the following three notes:

“what a shape is Your heart is a shape because your muscle is a shape and your heart is a muscle shape is everything”

“me to shapes need to be complete”

“Look at this, I think that shapes turn into different shapes”
Figure 5. Knowledge Forum: Knowledge Building Discourse Explorer (KBDex) Visualization at End of the Study

Student 15 references ideas previously mentioned, presents no new mathematical terminology and no one built on Student 15’s notes. Student 15 contributed 1% of the total Knowledge Forum notes and 2% of the Knowledge Building Circle turns. The Knowledge Building Principle “democratizing knowledge” reflects the commitment to incorporate every member. Knowledge Forum’s analytic tools can provide the teacher with information regarding students needing more support. As indicated in Figure 5, students who produced the most Knowledge Forum notes (Student 22, Student 17 and Student 4) are at the centre of the figure.

4.5 How Math Talk Proceeded on Knowledge Forum

As we can see in the visualization of Figure 6, the word network contains thirty-two discourse terms. Six words are not connected in the network: equilateral, geometry, scalene, polygons, obtuse, and degree. The terms leading to the network’s strongest connections include: three-dimensional, two-dimensional, side, points and connect. As we can see in comparing Figure 6 and Figure 3 (the Knowledge Building Circle network) word used within Knowledge Forum but not the Knowledge Building Circle include “square, vertex, proportions, volume, multiplication, obtuse, polygon, scalene, geometry.” Other than the terms square, multiplication and geometry, all new math terminology references complex mathematical shape adjectives.
Figure 6. Knowledge Forum Word Network at the End of the Study

The Knowledge Forum Note network as visualized through KBDeX is shown in Figure 7. The image in the middle includes connected Knowledge Forum Notes, as does the image in the top right hand corner. There were no connections defined by words in the special math vocabulary list in Appendix 1 between the remaining Knowledge Forum Notes located along the periphery.

Figure 7. Knowledge Forum Note Network at the End of the Study

Below are Knowledge Forum Notes that demonstrate mathematical understanding and reasoning. As is evident in these Math Talk examples below, notes render student’s thinking visible to the whole community and serve to clarify their mathematical understanding.

Example: Using real world artifacts to explain math discourse
Student 1, Note 7 - “what is a heart -A shape has sides it can have points and corners your heart is almost a shape it mite even be but I've never seen a heart so I don't know what it looks like”
- Ways of Contribution Coding “4 – Working with Information”
In this example, Student 1 is writing in reference to an earlier inquiry, “if a heart was considered a shape”. This idea began within the Knowledge Building Circle and the student continued this line of inquiry online. The student explained why the heart should be considered a shape. By using mathematical terminology such as” sides, points and corners” the student defends the proposition that the heart is a shape. Student 4, built on Student 1’s note

Student 4 Note 12- “Anything in your body – anything in your body is a shape, a heart is a shape, anything in the world, even a piggys heart is a shape – anything!”
-  Ways of Contribution Coding “4 – Working with Information”

As the study progressed, Student 1 seemed to want more information on shapes, and contributed this note.

Student 1, Note 41 - “What is a shape – shapes are very fun to work with but very very interesting to can I hear what you think is a shape. Please build on if you can I’m looking forward to see you’re ideas of what a shape is.”
-  Ways of Contribution Coding “4 – Working with Information”

In note forty-one, Student 1 encouraged others to engage in Knowledge Building about shapes, helping to create an inclusive space in which all students feel their contributions have merit.

Student 14, Note 110 - “I think it is a black hole is a circle so it is a shape in the middle of the milky way.”
-  Ways of Contribution Coding “4 – Working with Information”

In this example Student 1 used mathematical terms to elaborate understanding of scientific phenomena and properties, incorporating interest in space to better understand shapes. This multi-disciplinary perspective reflects what Scardamalia and Bereiter (2016) refer to as “Criss-crossing landscapes.” The student incorporated their own interests, from the scientific discipline, and utilized other knowledge to enrichen the mathematical discussion.

Example: Defining Student understanding of Math Discourse.
Student 1, Note 75 - “2d shapes and 3d - 3D and 2d shapes are very different from each other 3D shapes pop out and they have always more of what 2d shapes have like a 2d shape is against some thing but a 3D shape pops out like if you put a 2d shape against the wall it will look like there’s only one side but then if you have a 3D shape it has 5 sides.”
-  Ways of Contribution Coding “2 - Theorizing”
Student 1, also contributed note 75 providing evidence of idea improvement. This note was written after the teacher’s lesson involving tactile experiences and demonstrates understanding of two and three-dimensional shapes.

**Example: Mathematical Reasoning**

**Student 15, Note 77** – “Look at this, I think that shapes can turn into different shapes.”

- Ways of Contribution Coding “4 – Working with Information”

Student 15 explains how a shape can be transformed into a different shape yet remain a shape. This analysis demonstrates that shapes do not need to remain static may well lead to a deeper understanding of rotations of shape and other geometric properties.

**Student 21, Note 91** – “Lines – lines make shapes some lines are curved to make a shape. All shapes are connected. Yes it has to be connected It doesn’t matter if its curve or straight.”

- Ways of Contribution Coding “2 - Theorizing”

Student 21 clarifies that a shape does not need to consist of straight lines.

### 4.6 Idea Development

In the following section, transcripts from the Knowledge Building Circle and Knowledge Forum Notes provide a discourse analysis of student Math Talk to convey idea development.

#### 4.61 - Introducing Math Concepts and using them to Support Math Talk

Transcript 1 provides an example of a teacher lesson within a Knowledge Building Circle. The teacher introduced shape names and passed around blocks in the Knowledge Building circle, encouraging students to make comparisons between two and three-dimensional shapes. Students touched, traced, and elaborated ideas regarding similar shapes as experienced in their own lives.
Table 7. Knowledge Building Circle: Understanding a Parallelogram Transcript

Most discourses turns in transcript 1 were coded as “Supporting Discussion” or “Obtaining Information.” Students classified shapes in reference to familiar objects (e.g., a diamond). One student built on using the term parallelogram. A third student came up with the specific description of a pike, referring to a rhombus. Thus, students’ examples and elaborations varied, even when all were introduced to the same object. The students were learning from one another and benefitting from unique and varied points of view. This community aspect supported both idea diversity and idea improvement.

Transcript 2 demonstrates student engagement in understanding math concepts as seen again in their efforts to understand and define three-dimensional shapes. They developed their understanding by classifying humans as three-dimensional and exploring spatial properties of Three-Dimensional objects in their environment.

<table>
<thead>
<tr>
<th>Ways of Contribution Code</th>
<th>Knowledge Building Circle – Transcript 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>Teacher: I want to show you something. This is a? (trapezoid)</td>
</tr>
<tr>
<td>6</td>
<td>Student 12: trapezoid.</td>
</tr>
<tr>
<td>3</td>
<td>Teacher: Yes, this is a trapezoid and it is a shape. This is a (parallelogram)</td>
</tr>
<tr>
<td>6</td>
<td>Student 14: Shape</td>
</tr>
<tr>
<td>6</td>
<td>Student 20: it is a diamond</td>
</tr>
<tr>
<td>3</td>
<td>Student 21: you mean parallelogram!</td>
</tr>
<tr>
<td>6</td>
<td>Student 14: shape</td>
</tr>
<tr>
<td>3</td>
<td>Student 19: no, a parallelogram specifically a pike.</td>
</tr>
<tr>
<td>3</td>
<td>Teacher: Now a parallelogram is about?</td>
</tr>
<tr>
<td>6</td>
<td>Student 17: they are parallel!</td>
</tr>
<tr>
<td>1</td>
<td>Teacher: yes, that means that what is running in the same direction?</td>
</tr>
<tr>
<td>4</td>
<td>Student 14: the lines are running in the same direction.</td>
</tr>
</tbody>
</table>
Table 8. Knowledge Building Circle: Defining Spatial Properties of a Three-Dimensional Shape Transcript

In Transcript 3, students understood two-dimensional shapes to be those on a flat surface and three-dimensional to “pop out” and occupy more space. Student 13 clarified that three-dimensional shapes have corners. Student 2, referring to incorporating several squares to build a cube, clarified that two-dimensional shapes could be put together to create three-dimensional shapes. Overall, through introduction and exploration of shapes and math concepts students engaged in Math Talk and Knowledge Building, as further elaborated below.
Table 9. Knowledge Forum Notes: Two-Dimension & Three Dimension Shape Comparison Transcript

4.62 Heart as a Shape Inquiry

In transcript 4, a student’s misunderstanding leads to a rich discussion. The teacher helps to keep the conversation moving forward, but the students are the ones using Math Talk to analyze whether the heart should be considered a shape.
<table>
<thead>
<tr>
<th>Ways of Contribution Code</th>
<th>Knowledge Building Circle Transcript 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Student 11: is a heart a shape?</td>
</tr>
<tr>
<td>6</td>
<td>Student 5: it can be</td>
</tr>
<tr>
<td>6</td>
<td>Student 10: it can be in some ways</td>
</tr>
<tr>
<td>1</td>
<td>Teacher: what do you mean it can be in some ways?</td>
</tr>
<tr>
<td>3</td>
<td>Student 10: so, like the heart we have here (points to inside of his chest)</td>
</tr>
<tr>
<td>6</td>
<td>Teacher: yes, so in our body</td>
</tr>
<tr>
<td>5</td>
<td>Student 5: yea so it is like an oval.</td>
</tr>
<tr>
<td>5</td>
<td>Student 3: yea it is like this (makes fist into a circle)</td>
</tr>
<tr>
<td>4</td>
<td>Student 1: it's a piece of meat!</td>
</tr>
<tr>
<td>4</td>
<td>Student 5: but there are some parts that are shapes.</td>
</tr>
<tr>
<td>6</td>
<td>Student 8: but there are some parts that are.</td>
</tr>
<tr>
<td>5</td>
<td>Student 10: like if you are talking about the heart on Student 9's pants then yes it's a shape. Like it is a triangle and then two circles</td>
</tr>
<tr>
<td>6</td>
<td>Student 5: yea that a shape.</td>
</tr>
<tr>
<td>4</td>
<td>Student 11 a: the bottom part is kind of a square, and the top part is two half circles.</td>
</tr>
<tr>
<td>5</td>
<td>Teacher: so, you're saying that the shape that you are looking at on Sofia's pants is a shape.</td>
</tr>
<tr>
<td>6</td>
<td>Student 10: semicircle</td>
</tr>
<tr>
<td>4</td>
<td>Teacher: but then you were saying it depends on the heart,</td>
</tr>
<tr>
<td>3</td>
<td>Student 5: it depends on what kind of heart you are talking about.</td>
</tr>
<tr>
<td>6</td>
<td>Teacher: so, if you are talking about...</td>
</tr>
<tr>
<td>2</td>
<td>Student 5: not all hearts are actually shapes</td>
</tr>
<tr>
<td>6</td>
<td>Teacher: So, tell me this heart</td>
</tr>
<tr>
<td>4</td>
<td>Student 5: this heart is an oval but it has different parts to it.</td>
</tr>
<tr>
<td>5</td>
<td>Student 1: well it isn't exactly an oval, it has blood and cells pumping out from it, so it can spread out</td>
</tr>
<tr>
<td>5</td>
<td>Student 2: so, like let's say you made one of two ovals and a triangle at the bottom, and that can still make a heart</td>
</tr>
<tr>
<td>2</td>
<td>Teacher: You have a lot of ideas of what a shape is and what a shape is not. Can I tell you that the others said that Student Nine's tights had hearts and that it is a shape, but not your heart in here?</td>
</tr>
<tr>
<td>5</td>
<td>Student 17: because the heart is not actually the shape of the heart. it is like a circle with a bunch of veins on it.</td>
</tr>
<tr>
<td>5</td>
<td>Student 15: it's not actually a circle, its starts out as a circle and then grows different parts, but it is a muscle.</td>
</tr>
<tr>
<td>1</td>
<td>Teacher: Can a muscle be a shape?</td>
</tr>
<tr>
<td>4</td>
<td>Student 15: Yes, your heart is a muscle.</td>
</tr>
<tr>
<td>4</td>
<td>Student 12: yes, it's like a heart is not actually.</td>
</tr>
<tr>
<td>4</td>
<td>Student 18: it is because it has bumps and is a shape</td>
</tr>
</tbody>
</table>
Table 10. Knowledge Building Circle: Understanding a Heart as a Shape Transcript

The discourse turns begins with a simple inquiry in the Knowledge Building circle by Student 11: “Is a heart a shape.” Students begin to justify and provide examples and describe what a heart looks like, suggesting that a heart can be considered a shape in some contexts. They consider the heart as an organ and as an image. Student 11 describes the heart using other shapes and they question if they are referring to the heart inside their bodies or an image they see, acknowledging that no student has seen this organ. This idea thread engages several students in collaborative understanding.

Within transcript 4, the teacher’s coded statements included three “Supporting Discussion” statements. One student suggested the heart was a circle and grew into different parts. The teacher respects the students input, with no effort to correct the student, and asks, “Can a muscle be a shape?” The student confirms that the heart is a muscle, and the conversation eventually shifted to the idea that the human body can also be a shape. Students noticed that shapes then are not static, and started to questions beliefs of what a shape could be and if a shape remains a shape, even through movement.

Students pursued this line of inquiry by subsequently posting notes in Knowledge Forum. As seen in Table 11, Student 15 explained that a heart was a muscle, and therefore a shape. Student 1, built on to this idea, and incorporated mathematical terms to describe a heart. By describing the heart’s points, corners and sides, the students come to the realization that they have never seen a human heart and Student 1 is referring to a symbolic image of a heart. Students 17, 4 & 21 reason that a heart can still be classified as a shape, even though it consists of curved sides. Finally, Student 4 decides that parts in the body can be shapes.
The student Math Talk spanned multiple science topics (biology/human body) and space and objects in their immediate surrounding environment. Scardamalia and Bereiter (2016) refer to power of “Criss-crossing landscapes” and having students find “their own paths rather than have directions forced upon them.” (Scardamalia & Bereiter, 2016, pg. 6). Students organically formulated ideas that transcended disciplines to make sense of the concept shape in the process engaging spatial awareness, movement, physical education and then biology.

### 4.63 “Illegal Shapes”: Understanding Classification and Definition of a Shape.

An idea that dominated the students’ discourse was that of an “illegal shape.” Students within the Knowledge Building Circle had a theory that everything could be a shape. They indicated that something might not “officially” be deemed a shape because it had not yet been classified as

<table>
<thead>
<tr>
<th>Ways of Contribution Code</th>
<th>Knowledge Forum Notes Transcript 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Note 6 – Student 15: what a shape- is Your heart is a shape because your muscle is a shape and your heart is a muscle shape is everything</td>
</tr>
<tr>
<td>4</td>
<td>Note 7 - Student 1: what is a heart - A shape has sides it can have points and corners your heart is almost a shape it might even be but I've never seen a heart so I don't know what it looks like</td>
</tr>
<tr>
<td>4</td>
<td>Note 12 – Student 4: anything in your body anything in your body is a shape a heart is a shape anything in the world even a piggy's heart is a shape anything!</td>
</tr>
<tr>
<td>2</td>
<td>Note 18 - Student 8: my theory - I think almost anything can be shape</td>
</tr>
<tr>
<td>2</td>
<td>Note 20 - Student 17: a shape is a thing that has sides and curve a shape is a thing that has sides and curve</td>
</tr>
<tr>
<td>4</td>
<td>Note 21 - Student 4: group a shape we learned how to group a shape and curves</td>
</tr>
<tr>
<td>2</td>
<td>Note 26 - Student 21: all shape are connected yes it has to be connected it doesn’t matter if its curve or straight</td>
</tr>
<tr>
<td>5</td>
<td>Note 138 - Student 4: shapes I learned shapes can be everywhere but some things are not shapes. well things in your body things in your body can be shapes</td>
</tr>
</tbody>
</table>

Table 11. Knowledge Forum Notes: Dimensions of the Heart and Body Transcript
such because “scientist/mathematicians could not name everything.” As demonstrated in Table 12, students recognized that classification was important. While this idea seemed to hinder their understanding of shapes early on (at one point, when presented with an octagon, students claimed it an illegal shape), it seems to have set the stage for their discussion of the heart and body organs as shapes. When the teacher’s discourse turn was coded as a “Thought Provoking Questions,” student responses were typically coded as “Supporting Discussion” with no Math Talk, as is demonstrated in Student 6’s response after the teacher’s question. The teacher’s question is also used to clarify the students thinking and to refine their idea.

<table>
<thead>
<tr>
<th>Ways of Contribution Code</th>
<th>Knowledge Building Circle Transcript 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Student 6: I think everything is a shape. They are illegal shapes.</td>
</tr>
<tr>
<td>1</td>
<td>Teacher: What is an illegal shape?</td>
</tr>
<tr>
<td>6</td>
<td>Student 6: something that is not named.</td>
</tr>
<tr>
<td>4</td>
<td>Student 8: Student 2 made it up, an illegal shape is a shape that does not have a name and that is made up and a legal shape is a square, triangle hexagon or something that is named</td>
</tr>
</tbody>
</table>

Table 12. Knowledge Building Circle: Illegal Shape Transcript

Students continued to reference “illegal shapes”, as indicated in Table 13. Student 10 compared an “illegal shape” and square. Student 8 discussed rules of shapes but felt naming a shape legitimized its existence. Student 8 understood that shapes have vertices, edges, and corners, as do “illegal shapes”, but due to lack of expert terminology felt the shape fell into the category of “illegal shape”.

Table 12. Knowledge Building Circle: Illegal Shape Transcript

Students continued to reference “illegal shapes”, as indicated in Table 13. Student 10 compared an “illegal shape” and square. Student 8 discussed rules of shapes but felt naming a shape legitimized its existence. Student 8 understood that shapes have vertices, edges, and corners, as do “illegal shapes”, but due to lack of expert terminology felt the shape fell into the category of “illegal shape”.
Table 13. Knowledge Building Notes: Illegal Shape Build-Ons Transcript

<table>
<thead>
<tr>
<th>Ways of Contribution Code</th>
<th>Knowledge Forum Notes Transcript 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>Note 22 - Student 6: illegal shapes a shape that is not named.</td>
</tr>
<tr>
<td>4</td>
<td>Note 31 – Student 11: unnamed - all things are shapes but some are unnamed</td>
</tr>
<tr>
<td>2</td>
<td>Note 32 – Student 4: some are illegal shapes everything is a shape but some are illegal shapes a illegal shape is a shape that are not named</td>
</tr>
<tr>
<td>3</td>
<td>Note 34 - Student 10: what legal shapes are illegal shapes are shapes that do not have names. a legal shape is different from a illegal because they have names such as rectangle and square</td>
</tr>
<tr>
<td>2</td>
<td>Note 82 – Student 2: Yes because what if scientists just found or made a shape and they named it but lot's of people did not know about it because they did not tell everybody yet because they were making sure that everything about it is a shape</td>
</tr>
<tr>
<td>5</td>
<td>Note 93 – Student 8: illegal shapes I think all of them are illegal shapes because it is closed shape but it doesn't have a name.</td>
</tr>
<tr>
<td>5</td>
<td>Note 100 – Student 8 &amp; Student 11: This shape we think that it is a illegal because it has vertices, edges and corners and is connected but it is not named</td>
</tr>
</tbody>
</table>

Students co-created the collaboratively defined concept of “illegal shapes” and adopted this terminology to assist in understanding definition and classification of shapes. This is consistent with Bereiter’s (2002b) notion of conceptual artifacts; “Illegal Shapes” became a conceptual artifact to advance knowledge in the community.
Chapter Five
Conclusion

This chapter begins with a summary of findings related to research questions and concludes with recommendations for future research regarding Mathematics Talk and Knowledge Building.

5.1 Summary

This thesis presents an exploratory study that incorporates Math Talk and Knowledge Building pedagogy. By adopting the “Ways of Contribution” (Chuy et al. 2011) conceptual framework, the study identified emerging patterns of contributions to Math Talk in student written discourse and face-to-face spoken discourse. Qualitative and the quantitative results derived from a Knowledge Building discourse analysis tool, KBDeX, showed the emergence of domain-specific vocabulary in student discourse and patterns of engagement surrounding use of those terms. The forty words and phrases used for the KBDeX analysis were extracted from the Ontario Mathematics Curriculum (Ontario Ministry of Education, 2005), from the Grade Two mathematics strand of Geometry and Spatial Sense. These words served as “expert vocabulary” to determine the extent to which students were including Geometry terms into their discourse, specifically the understanding of shapes. Together these analyses show engagement in Math Talk is reflected in a complex network of contribution types and concepts across online and face-to-face discourse.

Results indicated that the “Ways of Contribution” (Chuy et al. 2011) framework provided support for Math Talk, revealing that students were engaged in productive math talk and knowledge building discourse within the classroom community. Results show the full range of contribution types (formulating thought provoking questions, theorizing, obtaining information, working with information, synthesizing and comparing, and supporting discussion) in both face to face and online discourse. There were high levels of “Supporting discussion” in both (44% for Knowledge Forum notes and 39% of Knowledge Building Circle discourse) and greatest variation with theorizing (17% for Knowledge Forum notes and 3% for Knowledge Building Circle discourse). Students used 31 “expert vocabulary” math terms during spoken discourse.
turns within the Knowledge Building Circle and 32 in Knowledge Forum notes. This demonstrates a high level of student engagement with Geometry concepts, many above the Grade two Ontario Mathematics curriculum level; further the overview of use of these terms in transcript analysis suggests deep understanding of these concepts.

In line with the research question regarding working together to advance their community, KBDeX analytic results demonstrate that students were strongly connected as a community during both the Knowledge Building Circle and Knowledge Forum work. However, one student--Student 15--is not connected in the Knowledge Forum Community network. “Collaborative projects need not expect all members to share the same understanding of ideas to be successful. The possibility of generating inferences that are meaningful for the participants, whatever the content might be.” (Broadbent & Gallotti 2015: p.9). While the student was not connected, they still contributed to the community, and gained some understanding of shapes.

While these measures alone do not provide prescriptive benchmarks for math competency, these analytic tools allow for students and teachers to assess advances and needs to further pervasive Knowledge Building. Results are similar to Moss & Beatty (2010): Knowledge Building allows mathematical ideas and understanding to evolve through productive student interactions. While the results are exploratory they do open possibilities for future Math Talk and Knowledge Building research.

Jacobsen, Lock & Friesen (2013), argue that Knowledge Building environments promote intellectual engagement, which provides ongoing learning opportunities to ensure a richer learning experience for students. This was evident within this study where Math Talk provided an opportunity for the grade two students to engage in collaborative mathematical inquiry within Knowledge Building. Students were asked to work together in generating questions, ideas, and designing methods to foster their learning. By exploring geometrical topics, such as the nature of shapes students were incorporating and advancing their ideas, creating community knowledge by curating mathematical cognitive artifacts.

At times students met with difficulty, but difficult math issues were shared openly within the community. This resulted in collaborative investigation on how to tackle the problem. The
classroom community continually evaluated and revised the geometry theories of their own construction, especially with regard to the definition of a shape, the properties of shapes, and two-dimensional vs three-dimensional properties of shapes. As noted by Chuy et al. (2011), diverse ways of contributing give “students the opportunity to deepen their thinking about historical and scientific claims with an aspect of explanatory coherence.” (p.250). While Chuy et al. (2011) do not directly reference mathematics this also applied to mathematics discourse.

The Ontario Mathematics Curriculum defines the Big Ideas for geometry as “intuition about the characteristics of two and three-dimensional shapes and the effects and changes of shapes in relation to spatial sense. To recognize basic shapes and figures, to distinguish between the attributes of an object, while understanding and appreciating the geometric aspects of our world.” (Ontario Ministry of Education, 2005: pg. 9). The purpose of these Big Ideas is to guide the formulation of essential questions to help educators explore the topics more in depth with their students. Within the grade two math curricula geometry strand, these are the two Key Ideas to be understood:

**Key Idea 1:** Geometric objects have properties that allow them to be classified and described in a variety of different ways. (Ontario Ministry of Education, 2005)

**Key Ideas 2:** Understanding relationships between geometric objects allows us to create any geometric object by composing and decomposing other geometric objects. (Ontario Ministry of Education 2005)

These two Big Ideas emerged while students made contributions in their Knowledge Building Circle, and via the use of Knowledge Forum technology. In response to the research question regarding idea development within and between both pedagogical Knowledge Building mediums, results showed that Knowledge Forum was an entry point for ideas to become refined by students. Students made contributions in which they considered their bodies as geometric objects, explored shape properties and dimensions, and generated and explored both conceptual and plastic two-dimensional and three-dimensional shapes. By engaging analysis of body organs
and shapes in the local environment, students explored geometric algorithms and descriptive math terms, to extend their mathematical reasoning. Students also challenged the ideas of the community, their own ideas, and experts they referenced. Students understood the importance of experts work, by providing evidence within students own justifications. However, they were always thinking and questions their expert sources.

By incorporating pragmatic design thinking within lessons, students were able to grasp theoretical understanding surrounding geometry. They engaged in inquiries surrounding common problems in relation to shapes, incorporating spatial awareness. They sought to understand the world around them and to tackle increasingly deep problems that they encountered. Within students’ face-to-face and online discourse conversations they demonstrated “Collaborative Justification.” This theoretical phrase by Kopp et Mandl (2011), refers to learner’s justification for arguments supported during a collaborative task. Kopp et Mandl’s work is the first to investigate and measure collaborative argumentation justification. Their work only explores undergraduate students, and no further studies have considered work at the elementary level. “Collaborative Justification” utilizes communities’ contributions to allow for students to incorporate similar or opposing ideas to justify their understanding and definition of a term (see transcripts, chapter four, students’ justification of “illegal shapes” and the “is a heart a shape”).

This study suggests that the Knowledge Building Circle and Knowledge Forum software should be used in tandem. Due to the age of the students, it is important to note that the combination allowed ideas expressed first within the face-to-face community to be advanced in Knowledge Forum. At the group and individual level, the learning process allowed for sustained creative work with ideas. The combination of the two modalities allowed student to comprehend on their own terms with emerging discursive complexity. Students shifted their Math Talk between oral and written discourse, which allowed students to reiterate ideas; by contributing within both mediums they were becoming multi-literate citizens. “Students learning should be on display to inspire, invite and inform – not to serve as mere directions filling up a blank space. Documentation should inspire students to reach higher and achieve more as they look at learning to deepen it, modify it, or take it in new directions” (Ritchhart 2015 p.236)
Sinclair and Bruce (2015) argued that the applications of this approach to geometric learning in classrooms requires broader applications of learning through computer based tools and models, especially visual models. The use of Knowledge Forum--any technology-- is not meant to replace the creation and exchange of ideas, but instead assist the process. As Alavi et al. (2002) have argued, the amount of effort that students invest in learning may decrease as technology or task becomes more complex, hence simpler systems may outperform complex systems in certain contexts.

By building on their own ideas the students progressed well within the expected time frames of their class work and curriculum, and engaged in powerful dialogic forms of inquiry, in line with Bakhtin’s theory of Knowledge (Zack et Graves 2001). Students’ collective community thoughts and ideas are now integrated into one’s own. Through the construction and exchange of collective thoughts each single learner had the opportunity to develop these ideas in effective ways. An example of this from the study was a student who was having difficulty but finally understood what constituted a viable geometric form and an “illegal shape”. The teacher picked up the student’s theory and restated it for the entire class turning it into a dialogical object of thought.

Knowledge Building competence allowed students to enhance their Mathematical Talk and reasoning by providing a reflective community setting. Students learned about geometrical fluency. Students developed geometric ways to reason and become reflective in regards to anything that could be a shape. Having two different mediums allowed for recorded ideas to be validated and observed. This lead to further student mathematical discourse to initiate new ideas and work together to understand shapes. Knowledge Building and Math Talk allowed for “collective intelligence” (Broadbent & Gallotti’s 2015).

5.2 Future Studies & Directions

This study explores the complementarity of Math Talk and Knowledge Building. The research produced a productive interaction that might inform additional research. Future research should be designed to further corroborate this study’s findings and explore areas in need of refinement.
This study demonstrates advantages of design-based research; continuous iterations should enable increasingly impressive results.

Math Talk is just one way to show how spoken and written discourse can be integrated effectively. Future work should focus on advancing Knowledge Building Math Talk across the curriculum.

Through the evaluation and recording of student thinking it was possible to trace impressive advances. What happens to all the student thinking that occurs but is never recorded and available to be built upon by the community? How ever do students articulate, or express their ideas mathematically over time? There is a need to develop pedagogical mechanisms in mathematics classrooms and for assessment to allow students to facilitate their communication, and have a record of such accounts. Students need to have a transparent process in which their ideas become a powerful resource so that they come to feel that they can contribute and share their ideas mathematically. They need a public space to record ideas, to have them readily available for reference and to see their growth. Ideas not be isolated in the student’s mind where others cannot build on. Powerful learning occurs when ideas are shared, when they live in the world. Carpenter et al. (2003) maintain that the very nature of mathematics presupposes that students cannot learn mathematics with understanding without engaging in discussion and argumentation. Future research should include a community Knowledge Building setting that can help influence and inform mathematical assessment.

Future work should also explore how children’s drawings can be utilized in the mathematical learning process and how the physical act of drawing allows geometric understanding to emerge in the community setting. By incorporating more tactile designs into student learning, student understanding of Mathematical Talk and comprehension may be demonstrated?

Finally, one concern was the uneven contribution of student engagement within the community, in both media. Future work should explore issues and possibilities: are all students learning despite different rates of contribution? Will more even rates of contribution lead to more learning?
In a Knowledge Society students are not just to learn; they must also innovate (Wagner & Dintersmith 2015). Mathematics proficiency is a powerful tool for creative work in science, technology, art, gymnastics—all disciplines. Mathematics develops foundational thinking required in the 21st century. As mathematical language forms, basic conceptual foundation, students should be given the opportunity to invent, learn and adapt, developing a mathematical discourse culture to discover the joys and possibilities of mathematical thinking.
References


Ritchhart, R. (2015). *Creating Cultures of Thinking: The 8 forces We Must Master to Truly Transform our Schools.* Hoboken: Wiley.


Appendix 1 – Grade Two Ontario Mathematics Geometry Word List

Word List: Grade Two Mathematics & Geometry and Spatial Awareness - Based Upon the Ontario Mathematics Curriculum

2D
3D
addition
angle
area
circle
connect
corner
cube
curve
decagon
degree
division
edge
equilateral
fraction
heptagon
hexagon
measurements
multiplication
number
obtuse
octagon
pentagon
perimeter
points
polygons
prism
proportions
pyramid
quadrilateral
rectangle
round
scalene
side
sphere
square
subtraction
symmetry
triangle
vertex
volume