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VARIATIONAL FORMULATION INCORPORATING SPATIAL CAPACITY CHANGES TO RECONSTRUCT TRAJECTORY FOR HETEROGENEOUS TRAFFIC CONDITION

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This study deals with the reconstruction of vehicle trajectories incorporating a data fusion (DF) framework that combines video and probe sensors data in heterogeneous traffic condition. The framework is based on the application of Variational Formulation (VF) of kinematic waves for multiple lane condition. VF requires cumulative count and reference trajectory as boundary conditions. VF also requires generation of lopsided network using fundamental diagram (FD) parameters. In this regard, cumulative count and FD parameters are obtained from the video sensor, while reference vehicle trajectory is obtained from the probe sensor. The analysis shows that the framework can provide 83% accuracy in trajectory estimation from the nearest reference trajectory. However, the accuracy decreases as the reference trajectory gets apart from the estimated one. Additionally, an extension of the VF to accommodate roadway side friction is presented. FD as well as lopsided network reform when the roadway capacity varies due to side friction. Consequently, the vehicle trajectory bends to accommodate the capacity fluctuation.

**Keywords:** Data fusion, Variational Formulation, Heterogeneous Traffic, Vehicle Trajectories.
1. INTRODUCTION

Trajectories recorded from traffic sensors are direct representation of the traffic states, including traffic variations and different driving modes over the roadway. Trajectories have different applications in traffic engineering such as determining congestion level, monitoring vehicle activity, simultaneous localization, mapping, detecting vehicle, monitoring driver behavior etc. Vehicle trajectory measurement consists of discrete recording of its location, equally spaced in time, which represents the series of vehicle coordinates in 2D or 3D space. Trajectory of individual vehicle can be plotted in time space diagram, if position of that vehicle at certain time is identified. These time-space coordinates for a segment can easily be obtained using probe sensor data. However, to construct a meaningful time-space diagram, high frequency data from consecutive vehicles is a prerequisite. To illustrate the actual condition of the roadway, probe vehicles record information at frequent time steps. However, probe vehicle loses usability, when it exits the segment. Considering this fact, employing several probe vehicles at a small-time interval may not be feasible at all. On the contrary, video sensors can give high frequency data over a certain small spatial domain. Unfortunately, to provide wide spatial coverage and reliable data, numerous video sensors must be placed at small distances which also limit its practicality from cost considerations. Making an allowance to all-around limitations and accuracies of multifaceted data sources, data fusion (DF) technique is adopted now-a-days to generate more reliable and continuous traffic information.

Since data fusion (DF) technique has certain advantages, vehicle trajectory estimation incorporating a fusion framework has become an advanced research criterion for intelligent transportation systems (ITS). Accurate reconstruction of the future trajectories in signalized urban streets is challenging as traffic condition is based on the traffic signals, signal coordination, downstream traffic conditions and the incoming demand. Recent advancements in data collection
technologies and availability and reliability of multiple data sources, promote data fusion-based frameworks as an alternative approach for trajectory estimation and urban network traffic modeling. Among all available traffic models, kinematic wave model is the most widely recognized due to its ability to capture realistic traffic behavior such as spillover and propagation of shockwaves with adequate mathematical framework. The kinematic wave model was proposed by Lighthil and Whitham (1955) and Richards (1956), and therefore, the theory is known as the LWR (Lighthill–Whitham–Richards) model.

Daganzo (2005a, 2005b) proposed an alternative solution method to the LWR model which is known as variational method. Most of the preceding studies on the variational formation (VF) of kinematic waves are either theoretical analysis or applications on freeways, urban arterials and street networks (Chow et al. 2015, Mehran et al. 2012, Wada et al. 2015). For trajectory reconstruction, the application of DF, based on VF is a relatively new concept. Interestingly, there have not been many studies on urban street networks; especially for non-lane based heterogeneous traffic condition. More importantly, extending this application to investigate the suitability of a DF framework based on VF for non-lane based heterogeneous traffic condition, to the best of our knowledge has not been studied before. Therefore, there is an opportunity of VF based network modelling framework that considers various aspects of urban area including traffic signals, non-motorized vehicles, slow-moving buses, and side friction resulting from illegal on-street parking and maneuvering along the roadway. Thus, this study deals with the reconstruction of vehicle trajectories incorporating a data fusion (DF) framework, which combines video and probe sensor data in non-lane based heterogeneous traffic condition. It also explores potentials for multiple lanes and recommends certain extension considering side friction.

The paper is structured as follows: Section 2 reviews previous works on data fusion and trajectory estimation techniques; Section 3 presents the methodology behind the variational
formulation; Section 4 elaborates the data collection procedure using both probe and video sensor; Section 5 describes the generation of lopsided network; both analysis and results are presented in Section 6; Finally, concluding remarks and future research scopes are given in Section 7.

2. LITERATURE REVIEW

2.1 Data Fusion

Multiple source data fusion is the combination of multidisciplinary techniques, which is analogous to the cognitive process in humans. Different data fusion techniques are being developed with time to suit the application purview and data compliance of different research approaches. These techniques are being drawn from different scientific areas of artificial intelligence, pattern recognition, statistical estimation, etc. Without relating to the specific application, a variety of fusion techniques can be used ranging from a simple arithmetic mean to a more complex approach. A three-way split is suggested by El Faouzi et al. (2011), where the fusion techniques are divided into three categories of approaches, namely, Statistical, Probabilistic and Artificial Intelligence (AI). They are also investigated over the potential opportunities and challenges of intelligent transport system (ITS) data fusion per eight different application techniques.

To demonstrate the potential benefit of multi-sensor data fusion in traffic operation and management, Bachmann et al. (2012) investigated different types of techniques, such as, Distributed Techniques, Kalman Filter, Ordered Weighted Averaging (OWA), Fuzzy Integral, and Artificial Neural Network (ANN). Soriguera and Robusté (2011) proposed a methodology for reliable short-term travel time estimation using data fusion technique. They adopted a two-level processing fusion algorithm using both fuzzy logic and a probabilistic approach, which implements the Bayes rule. The study shows that the fused travel times are more reliable and accurate than the algorithms based on unique data sources, if the learning process is developed carefully.
Recent approaches also fuse various types of traffic information such as fixed data and signal timing. A model fusing loop detector data and signal phase, to extract high-resolution data, which in turn is essential to estimate travel time was proposed by Liu and ma (2009). Herrera and Bayen (2010) used Kalman filtering technique and Newtonian relaxation to integrate point detection and mobile sensor data to extract high resolution data for traffic state estimation of a freeway. Van Lint and Hoogendoorn (2010) proposed an extended generalized Treiber-Helbing filter (EGTF) for data fusion algorithm, which although heuristic in nature, uses basic notions from traffic flow theory.

Deng et al. (2013) investigated cumulative flow count based system modeling methods, which estimate macroscopic traffic states with heterogeneous data sources on a freeway segment, accommodating a novel use of the multinomial probit model and Clark’s approximation method. They proposed stochastic three-detector model to estimate the mean and variance–covariance estimates of cumulative vehicle counts on both ends of a traffic segment. Rehborn and Koller (2015) explored a data fusion approach, which combines stationary and mobile probe data and compares the reconstruction quality, which can be achieved with each of the two sources. The study shows that the reconstruction quality of a congested traffic situation using 2% communicating probe vehicles of the total flow rate is similar to the quality of a traffic situation, which was reconstructed based on data from stationary detectors 1–2 km apart. Bhaskar et al. (2015) proposed an innovative model to fuse loops with Bluetooth Media Scanners (BMSs) to estimate the trajectories of the Bluetooth fitted vehicles along the roadway. Mehran et al. (2012) suggested a novel approach towards fusion of fixed and probe sensor data to reconstruct trajectories of all vehicles considering signal timing parameters and traffic engineering concepts. Later on, Mehran and Kuwahara (2013) worked on a data fusion framework based on kinematic wave theory. The developed framework combines real-time and historical traffic data to predict
future traffic patterns at upstream and downstream boundaries considering the probe trajectory data more efficiently.

Data from different sensors are generally characterized through different temporal and spatial resolution. Datasets also differ in availability as a function of location, time, and conditions. Considering different conditions, research on the data fusion techniques addressing the non-lane based heterogeneous traffic condition is sparse. Addressing this research opportunity, this study develops a framework integrating the dataset from video sensor and probe vehicle or mobile sensor in an intricate urban setting where non-lane based heterogeneous traffic operates rather frantically than conventional approach.

2.2 Trajectory Estimation

Some researchers utilize sparse mobile sensor data to reconstruct trajectory and compute travel time distribution for traffic performance evaluation. Hofleitner et al. (2012) developed a Bayesian Network based arterial travel time prediction model which assumed a uniformly distributed intersection delay. Both models applied traffic flow theory to reconstruct the entire traffic flow of an arterial network from sparse data. However, the derivation of these models is usually accompanied by some fundamental assumptions, which may restrict their practicality. Sun and Ban (2013) developed a method to reconstruct short vehicle trajectories for the entire traffic flow at arterial signalized intersections using sample vehicle trajectories and sample travel times obtained from mobile traffic sensors. They applied the variation formulation (VF) method to solve the traffic flow problem. For accurate application of VF, the optimization-based method and the delay-based method are proposed to estimate the shockwave boundaries, which are then used to reconstruct short vehicle trajectories. Hao et al. (2014) proposed a probabilistic traffic state model to quantify the likelihood of each possible scenario. The travel time and distance between data point pairs were distributed to all driving modes, and computed the conditional probability, given a
calibrated priori distribution. A second-by-second trajectory is then reconstructed based on the optimal driving mode sequence.

Apart from the use of singular or multiple data sources for trajectory reconstruction and travel time estimation, researchers have visited alternative applications. Wada et al. (2015) proposed two alternative applications of vehicle trajectories on signalized arterials by using probe vehicles rather than fixed sensors. One of which relates to traffic signal timing estimation and the other involves traffic volume estimation. Both of these applications follow a simple methodology that combines vehicle trajectories and traffic engineering concept of shock wave. The method proposed by Mehran and Kuwahara (2013) is based on kinematic wave theory and is capable of using probe trajectory information completely. Such features enable reasonable estimation of vehicle trajectories using minimum input data. Moreover, techniques were introduced in the study to incorporate incoming or outgoing vehicles in the middle of the study section. The methodology was then applied to real world data and its performance was evaluated over a range of different scenarios. Feng et al. (2015) studied a vehicle trajectory reconstruction method for a large-scale network by using Automatic Vehicle Identification (AVI) and traditional detector data. The proposed method yields high estimation accuracy for the full trajectories of individual vehicles and the origin-destination (OD) matrix, which demonstrates significant potential for traffic-related applications. Montanino and Punzo (2015) devised a “traffic-informed” methodology to restore physical and platoon integrity of vehicle trajectories in a finite time–space domain and a general simulation-based validation framework for quantitative assessment of model performances.

There have been some efforts to develop a trajectory dataset in heterogeneous traffic condition at urban midblock using video sensors (Kanagaraj et al. 2015). In the study, trajectory data is extracted from the video sequences using specialized software and processed using the locally weighted regression method proposed by Toledo et al. (2007) to reduce measurement errors.
and to obtain continuous position, speed and acceleration functions. Finally, the authors listed three different characteristics of heterogeneous traffic condition which should be helpful for driving behavior models.

To the best of our knowledge, vehicle trajectory data in heterogeneous traffic condition has not received much attention compared to lane based conventional traffic condition. This may be due to the difficulty and high cost involved in data collection and extraction, and complexities associated with non-lane base movement and heterogeneous vehicles types with varying physical dimensions and dynamics. As of such condition, the application purview of our study deals with the non-lane based heterogeneous nature of the traffic and builds over the data fusion framework based upon VF of kinematic waves to estimate vehicle trajectory.

3. METHODOLOGY

The basic methodology of this study is divided into four stages, the seminal procedure starts with defining a discrete lopsided network. Initially, considering a triangular fundamental diagram (FD), the forward wave speed $u$, jam density $k_j$ and the maximum flow rate $q_{\text{max}}$ is obtained. Afterwards the backward wave speed $w$ is estimated from the extracted parameters using equation [1]. Using these parameters, the lopsided network is developed. Figure 1(a) shows the 3-D perspective of the lopsided network where orthogonal axes of the figure represent count, time and space respectively. Moreover, the time-space plane related to lopsided network is shown in Figure 1(b), which shows orientation of nodes in time-space plane. $t_{\text{step}}$ is a user defined variable which represents the horizontal distance between the nodes along time axis. $s_{\text{step}}$ is the vertical distance between network nodes along space axis, which is calculated from equation [2] for different locations. In both equations, $t$ represents locations 1, 2, 3, 4 etc.
Every node in the modified network also represents the height of the cumulative count surface on the time-space plane. Cumulative count surface accounts for the cumulative count values for different time-space. For setting the initial conditions on the network boundaries, the count along the first column in the network is assumed to be 1. Considering the passing times of the vehicles recorded by video sensor, cumulative traffic counts at the upstream and the downstream are assigned as the heights to the nodes along the lower and upper boundaries of the network. When probes are used as references to reconstruct other trajectories, some supplementary treatments are necessary. A probe trajectory can be deduced as a contour on a three-dimensional surface as in Figure 1(a) of cumulative curves. Consequently, a constant height should be assigned to the network nodes adjacent to the probe trajectory. Since the cumulative vehicle counts at the upstream are known, a constant height can be allocated once the intersection point of the probe trajectory with the lower boundary of the network is known.

Finally, optimization is done to find the height of each node in the network as shown in Figure 1(b). From the variational theory (Daganzo 2005a, Daganzo 2005b), calculation of the cumulative height for each of the network nodes is reduced to the shortest path with the link costs and considering the cumulative heights at the network boundaries. The height of any node is $N(i, j)$ and it is estimated from equation [3]. However, if any node $(i, j)$ is located on a link which represents a red interval in time-space, considering the shortcut effect of the red intervals, $N(i, j)$ is estimated from equation [4]. Once the cumulative heights are calculated, vehicle trajectories are obtained as a contour line of the cumulative heights.
Interestingly, the input variables required for VF represented by equation [3] and equation [4] are related to number of vehicles only. Moreover, the cumulative traffic counts at the upstream are only a numeric measure of vehicle passed through that point. On the other hand, the probe trajectory considers the distance travelled, not the displacement. Furthermore, the solution domain is generated on the scalar time-space plane in the form of lopsided network. Thus, it is noticeable that the proposed methodology is independent of lane discipline which represents the robustness.

4. DATA COLLECTION

In this section, the methodology behind real world measurement has been illustrated. It includes a study section which covers a 1 km multi-lane roadway stretch with two signalized intersection at Katabon, Dhaka, Bangladesh (Figure 2(a)). The site selection procedure is centered upon the merit of heterogeneous non-lane based characteristics of the roadway, which offers a significant amount of non-motorized vehicle (NMV) activity and side friction due to unauthorized parking. The experimental setup includes 8 mobile probe sensor data and 4 video sensor data. The video sensors are installed at 4 different locations along the study section to analyze traffic states at different locations. The study is conducted at an evening peak (5:00 PM to 6:00 PM) on 18 June 2015. Probe data are collected using GPS equipped mobile sensor along the study section. Probe data includes latitude-longitude of probe vehicles at 5-s interval. Required probe data are obtained using android software developed by the authors named as ‘Way Out’. Traffic signal data including phasing, cycle length and signal timings are obtained from field measurement. Notably, the traffic police maintain the operation of the traffic signal in the study section. Considering average signal phase time and phase sequence at peak and off-peak periods, 30 cycles are recorded to determine the
signal phase timing for the approach within the study section. For simplification, it is assumed that there are no vehicles entering or leaving the study area from midblock intersections.

For this study, video sensor data is used to obtain count data along with speed and density for FD. In contrast, using the probe sensors, reference points in the time-space plane are obtained. These probe data also act as real time vehicle trajectories to check the accuracy of the estimated trajectories using VF. Both of the dataset provides necessary boundary condition to formulate numerical solution and reconstruct vehicle trajectory. However, the accuracy of the trajectories is greatly dependent on the reference points.

4.1 Probe Software

A PHP framework server is developed to receive the data from an android application “WayOut”. It uses GPS of cell phone and transmits the location information to the server via internet. It stores the information into the server database. The database in the server has two tables for user and location. The users table stores identification number and name of users, while the location table stores time stamp, longitude and latitude along with identification number of users. A website is also developed to download the data stored in the database. Figure 2(b)-(d) show the user interface of the developed android application and Figure 2(e) shows the data stored in the server.

Temporal difference is used to indicate the minimum temporal difference and spatial difference to indicate minimum distance between two consecutive readings. The minimum time interval between updates is set to 5 seconds and the minimum spatial change for updates is fixed to 1 meter. For generating reliable and high resolution trajectory estimation these conditions are enough to create time-space data sets. A parametric study is conducted to check the effect of those factors. A subsequent analysis is also carried out for this study to find out the important features of probe travel behavior.
Latitude and longitude obtained from the location table are used to compute actual distance with respect to starting point of study area using equation [5] where, \( \phi_1 \) = latitude of location 1 (rad); \( \phi_2 \) = latitude of location 2 (rad); \( \Delta \lambda \) = difference in longitude between locations 1 and 2 (rad); and \( R \) = arc radius (6371 km). Location 1 and 2 are two arbitrary points on geodetic surface.

\[
d = \cos^{-1}(\sin \phi_1 \times \sin \phi_2 + \cos \phi_1 \times \cos \phi_2 \times \cos \Delta \lambda) \times R
\]

### 4.2 Video Image Processing

The video sensor data is extracted using video image processing (VIP) algorithm. Among all image processing techniques, the most common Background Subtraction (BGS) technique has been adopted. A software in MATLAB using Graphical User Interface (GUI) is developed, integrating BGS with some major extensions (see Figure 2(f)-(h)). These extensions enable the algorithm to overcome some major challenges. While developing the object detection algorithm, several environmental challenges, such as, non-lane based movement, NMV detection, camouflage, camera jitter, sudden illumination variation, dark car and shadow, low camera angle and elevation are addressed to ensure high quality data extraction. The software requires video data, vehicle geometry as input and provides speed, flow, and density at required interval. For flow, strip based counting method is used by combining successive incremental differentiation. For speed, the whole field of vision is segmented and the change in center of area of object in each segment is detected to find the corresponding pixel speed. The actual instantaneous speed is obtained through calibrating the pixel distance with field distance. Then, the instantaneous speed is converted into time mean speed and space mean speed. For measuring density, the binary pixel density of objects within the field of vision is determined. Afterwards, the pixel density is converted into actual density by using the actual field measurement. The algorithm is developed as such the traffic within the video is not considered as a scattered finite pixel, rather the pixel
information is converted into numeric one to detect a vehicle. The accuracy of estimation of speed, flow and density from synthetic video sequence is found to be 100%, where the root mean square error (RMSE) and mean absolute error (MAE) are zero. On the contrary, while dealing with the field video sequence, the algorithm resulted (%RMSE, %MAE) about (19.72, 14.01) in flow, (4.34, 3.51) in density, and (1.18, 0.88) in speed. For high quality data extraction, camera height is fixed at 25 feet to reduce the object details detected by the algorithm and the camera angle is maintained below 45 degree to avoid perception problem.

5. DEVELOPMENT OF LOPSIDED NETWORK

The data of the lopsided network are arranged in a way that simplifies the structure for solution formulation. However, the inclusion of location approach adds complexity in determining backward wave speed and \( s_{\text{step}} \). It induces change in \( s_{\text{step}} \) with different capacity, free flow speed and jam density. Thus, the following steps are used: (a) the capacity, forward wave speed, jam density are given inputs in different space location; (b) the time-space is generated using these data provided that data are obtained from analysis. Afterwards, \( \varepsilon \) is determined using the equation [6].

\[
\varepsilon = \frac{s_{\text{step}}}{w} 
\]

Following this, \( t_{\text{mid}} \) is determined using equation [7].

\[
t_{\text{mid}} = t_{\text{step}} - \varepsilon
\]

Above, \( t_{\text{step}} \) is the summation of \( t_{\text{mid}} \) and \( \varepsilon \). \( t_{\text{mid}} \) is the time required to cross \( s_{\text{step}} \) in free flow speed \( u \). It is very important parameter to create lopsided network.

In the next step, the time in lopsided network is determined using,

\[
t(i, j) = t(i, j-1) + t_{\text{mid}}
\]
One of the main challenges of this study is the placement of probe data into the lopsided network. The probes have regular time space properties, whereas the lopsided network has a time space inclined by an amount equivalent to forward wave speed. Since this difference restricts direct interaction, the probe data can be inserted into the lopsided network in two ways: (a) lag time approach; and (b) interpolation. In lag time approach, the regular position of probe in the time space is determined. Afterwards a specific time lag is deducted from the actual time. Let, a probe is at \( s \) position at time \( t \). The lag time can be calculated using equation [9] where, \( u \) is the forward wave speed. Later, the time location in lopsided network is determined using equation [10] where, \( t_{lp} \) is matched in zero distance and it will take the probe in \((s, t)\) co-ordinate in lopsided network.

\[
\text{[9]} \quad t_{lag} = \frac{s}{u}
\]

\[
\text{[10]} \quad t_{lp} = t - t_{lag}
\]

In interpolation approach, the lopsided time space is created using \( t_{mid} \) and \( \varepsilon \).

Subsequently, the time for each node in the lopsided network is calculated using,

\[
\text{[11]} \quad t_{lop}(i, j) = \begin{cases} \tau & i = 1 \\ \tau_{lop}(i-1, j) + \varepsilon_{mid} j \end{cases} \quad \forall j \geq 1
\]

In interpolation approach, using \( f(i, j) \), the position of the probe vehicle is calculated through interpolation for lopsided time. Then using the probe time space data and the position, it is placed in the lopsided network. After that the valid spaces are marked as ‘1’ if it matches with the corresponding space.
6. ANALYSIS AND RESULT

This section contains the comparative analysis between the actual probe trajectory collected from real world measurement (see Figure 3(a)) and the trajectory estimated using VF. The analysis has been portioned into two: (1) qualitative analysis (2) quantitative analysis. Qualitative analysis provides visual performance of the estimated probe in comparison to the actual probe collected from the field. In contrast, the quantitative analysis provides the numerical performance. As a measure of performance, the travel time of the actual and estimated probe has been considered.

6.1 Qualitative Analysis

Trajectories without reference probe

This section contains analysis of estimated trajectory, assuming fundamental diagram parameters as constant. The value of $q_{\text{max}}$, $u$ and $k$ are 1740 veh/hr, 10.5 km/hr and 350 veh/km respectively. Figure 3(b) shows estimated trajectories for the study area without using any of the probe trajectories as reference. The shaded lines are estimated trajectories of the study area. The figure shows that the trajectories after 5:25 PM have become flat due to error in estimation. It represents that if no reference probe is used the flow behavior cannot be estimated precisely. From the figure, it is also apparent that probe 1, 2, 3, 4 are in agreement with the estimated trajectory. While the 5th, 6th, 7th, 8th probe trajectories show considerable shockwaves due to the congested traffic condition and side friction, its corresponding estimated trajectory represents traveling at free-flow conditions between intersections. When probe trajectories are not used as reference, the impact of congested traffic conditions and the corresponding backward waves from the earlier time intervals are not revealed in the estimated trajectories.

Trajectories using different reference probe

When probe data are used as reference, the agreement between the estimated trajectory and the corresponding probe trajectory improves significantly. The trajectories estimated using different
probe vehicles as reference have been illustrated in Figure 3(c)-(f). When probe vehicle 1 is used as reference, it is observed that the estimated trajectories for probe 2 and 3 are well in agreement. However, for probe vehicles 4 to 8, it does not estimate accordingly. This can be explained from temporal changes in travel time for 1 km study section as shown in Figure 4(a). According to this figure, probe vehicles 1, 2, 3 are in congested region due to tidal flow of commuting time. Alternatively, probe vehicles 4, 5, 6, 7, 8 are facing totally different situation than the earlier probes. Hence, if vehicle trajectories are estimated using probe vehicle 1 as reference, existing situation becomes analogous to probe vehicle 2, 3. In case of reference probe vehicle 4, it is observed that estimated trajectories at the entry point of probe 5, 6, 7 and 8 are well in agreement (estimated and measured travel time almost same), although there is a significant difference in entry time about 30 minutes between the reference probe and probe 5, 6, 7 and 8. Nevertheless, it can reconstruct the trajectory well as the congested situation is almost same for all the estimated probes compared to the reference one. When reference probe vehicle 5 is used, it is observed that the estimated trajectories at the entry point of probe 6, 7 and 8 are in agreement with actual trajectories as well as travel time and it can detect every detail maneuver of other probes. Similar characteristics are observed for probe vehicle 6 when used as a reference.

6.2 Quantitative Analysis

Travel time of vehicles is the inherent element while estimating trajectory. It is the measure of quality of the estimated trajectory. In this study, travel time was recorded each time, when the probe sensor equipped vehicles rolled out for data transmission. Table 1 represents the travel time of 1 km distance of the study section for all the probes. Figure 4(a) shows the change in travel time with time estimated from the probe vehicles for 1 km segment. It can be inferred from this figure that the travel time decreases with time. It shows that a peak period was prevailing before 5:00 PM.
Figure 4(b) shows the travel time of 0.5 km stretch of the study section for each probe. This portion of the roadway segment does not contain any signal. As a result, vehicle can enjoy free flow speed in this stretch of the roadway. The straight line up to 0.5 km represents free flow condition of the vehicles. It can be seen from Figure 3(a) that the probe vehicle 5 and 6 suffer from sudden shockwave. On the contrary, Figure 4(b) shows that the travel time to pass this 0.5 km segment where the shockwave has occurred is increasing with time. It represents that, the diminishing of peak period results in decreasing the demand on the road. However, this decrement in demand promotes unauthorized on-street parking. This behavior induces side friction and diminishes capacity, thereby increasing travel time. This phenomenon cannot be explained using the analysis, where the fundamental diagram parameters are assumed to be constant.

6.3 Error Analysis

The estimated trajectory travel time is calculated relating to the entry time with the corresponding probe vehicles. Then the actual and estimated travel time from trajectory was compared, to determine root mean square error (RMSE) and mean absolute error (MAE). The error $e$ has been computed analyzing the difference between two travel times, where $t_e$ corresponds to estimated travel time from reference probe vehicle and $t_a$ relates to actual travel time.

$$\text{Error, } e = |t_e - t_a|; \text{ MAE} = \frac{\sum_{i=1}^{n} e_i}{n}; \text{ and RMSE} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}}$$

The error for estimating trajectory for different probes has been shown in Figure 4(c). It shows that probe 1 has the highest entry time difference with the last probe vehicle and produces the maximum error. In contrast, the error for probe 2 suffers less error than the probe 1 in estimating trajectory. So, the error has decreased to a minimum value when the probe 6 is used.
Interestingly, using probe 6, 83% accuracy in trajectory estimation has been achieved. However, the error grows as the entry time is more apart with respect to reference probe such as probe 1.

**Trajectories using different reference probe and changed traffic state**

This section explains and shows the effect of capacity fluctuation due to side friction. When variable flow parameters at different locations and time are used, the trajectory shifts according to the location of side friction. This change in trajectory shows the effect of shock wave due to side friction, which is absent at first. However, it transpires at certain period of time and affects the flow and speed parameters of the existing vehicles in the stream. It should be noted that the presence and effect of side friction are temporary. Travel time analysis is adopted to reveal the temporal and spatial position of the side friction. Figure 5(a)-(b) show the effects of capacity fluctuation over the trajectory. It is apparent that the trajectory bends to accommodate capacity fluctuation. Therefore, by applying spatio-temporal capacity fluctuation, the side friction along the study section can easily be expressed through estimated trajectory.

Figure 5(a)-(b) explain the change in FDs as well as the lopsided network. Let, the cumulative height through path $AB$ is $n$. On the contrary, the cumulative count of vehicles at $C$ is $n + m$. When the traffic state parameter changes after point $E$, the distance $s_1$ will be reduced to $s_2$ due to change in backward wave speed according to equation [2]. Now, using equation [3] the cumulative count at $F$ and $G$ can be computed. The cumulative count at $F$ is $N_F = n + k_j \times s_1$ provided that $m > k_j \times s_1$ and at $G$ is $N_G = n + k_j \times s_2$ as $s_1 > s_2$ ensures $n + k_j \times s_1 > n + k_j \times s_2$.

Therefore, a typical contour line having value $n + r$ where $r < m$ and $n + r > n + k_j \times s_2$, will pass through the connecting points $A$, $C$ and $E$, $F$. However, when the contour line moves forward it may face the condition $N_G < n + r < N_F$ and move on to intersect the path of $FG$. It should be noted that the cumulative count at point $F$ is greater than point $G$, which will not have occurred if the
capacity remains constant. If the capacity remains constant, the condition becomes \( s_1 = s_2 \) and the cumulative counts of \( N_G \) and \( N_F \) become equal. Consequently, the line \( n + r \) would go with its initial path if it does not face any signal.

6.4 Fundamental Diagram Analysis

In urban roadway, traffic parameters such as capacity, forward wave speed and jam density vary due to on-street parking, presence of NMV etc. As such neglecting the effect of this changing state and assuming it constant, will not represent the actual scenario of the road. Moreover, trajectories estimated using these parameters will not be reliable and representative output. Thus, we strive to find these traffic parameters essential to the fundamental diagram (FD) of traffic flow theory to ensure good estimation of trajectory. To find the \( q_{\text{max}}, u, k_j \) for different locations, the analysis of FD has been adopted in this study. The analysis is conducted with the combination of linear and quantile regression (Dervisoglu et al. 2009). Figure 6(a)-(b) show the fundamental diagrams considering 5 minutes’ resolution data of two typical locations. The analysis has been conducted on the dataset extracted over a period of 12 hours on June 18, 2015. The fundamental diagram parameters at different locations are also illustrated in Figure 6(c). The analytical values represent that the parameters are not spatially constant. It corroborates to the existence of side friction within the study segment especially in location 2. More importantly, the improvement in trajectory using variable fundamental curves for different locations are also presented in Figure 6(d)-(e).

7. CONCLUSION

This study investigates a data fusion framework based on the application of Variational Formulation (VF) to estimate vehicle trajectory for non-lane based heterogeneous traffic condition. For this purpose two data sources namely, video sensor and mobile probe sensor data are fused to derive a reliable and high quality dataset. Moreover, signal timings are also recorded
for reasonable estimation in signalized urban condition. In contrast to most of the previous studies,
this research focuses on trajectory reconstruction on multilane road. Absence of lane convention in
and abrupt maneuvering of vehicles generate a challenging situation in trajectory reconstruction. It
is a fact that more complete dataset over a wide temporal range would aid in the analysis.
However, one should acknowledge the constraints in data collection in non-lane based
heterogeneous traffic condition. As a result the scope of this study is limited to one hour data
collection period.

The analysis shows that there is a significant effect of capacity fluctuation over vehicle
trajectory. The trajectory bends to accommodate capacity fluctuation. Thus, lopsided network and
fundamental diagram (FD) are reformed when the capacity varies due to side friction. While
considering reference probe vehicle, the estimated trajectories of adjacent vehicles are in
agreement with the actual one unlike distant vehicles. This is because the surrounding condition
(congestion or free-flow) of probes along the roadway varies with time. The error estimated in
terms of MAE and RMSE is a measure of the quality of the estimated trajectory. It propagates
when the entry time of other probes is more apart with respect to reference probe. For this reason,
Probe 1 shows maximum error whereas, probe 8 shows minimum error. Traffic parameters
essential to FD of traffic flow theory are also estimated to ensure good evaluation of trajectory.

The proposed method is an extension of kinematic wave theory which can accommodate
different parameters related to fundamental diagram (FD). The requirement of obtaining trajectory
is small and the application of the method is simple. Besides, the proposed model can
accommodate side friction which is one of the major problems in non-lane based traffic stream.
Furthermore, since the method can predict vehicle trajectories in near future, it can provide drivers
useful information. A possible extension to this research approach would be to estimate traffic
state from trajectory data in non-lane based heterogeneous situation.
ACKNOWLEDGEMENT

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REFERENCES


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Hadiuzzaman, Haque, Musabbir, Islam, Hossain, and Muntakim
against macroscopic traffic patterns. Transportation Research Part B: Methodological. 80: 82-106.


Table 1. Travel time of probes.

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<th>Probe Name</th>
<th>Starting time (12 hour format)</th>
<th>Ending time (12 hour format)</th>
<th>Travel Time (minute)</th>
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Free Flow Condition (0.5 km of the study section)

| Probe 1    | 5:4.1358                      | 5:6.4650                    | 2.3292               |
| Probe 2    | 5:8.5802                      | 5:10.2457                   | 1.6655               |
| Probe 3    | 5:13.0247                     | 5:14.9338                   | 1.9091               |
| Probe 4    | 5:21.1728                     | 5:23.7807                   | 2.6079               |
| Probe 5    | 5:49.1975                     | 5:52.3575                   | 2.6361               |
| Probe 6    | 5:50.6790                     | 5:53.7240                   | 3.0450               |
| Probe 7    | 5:54.1358                     | 5:57.5803                   | 3.4445               |
| Probe 8    | 5:56.7284                     | 5:60.5293                   | 3.8009               |
List of Figure Captions

Figure 1: Lopsided network: (a) 3-D perspectives; and (b) Effect of surrounding node points on cumulative count calculation for a node.

Figure 2: (a) Study site; (b) Snapshot of probe software icon; (c) Snapshot of probe software interface; (d) Snapshot of user interface; (e) Snapshot of database transmitted by probe software; (f) Density interface of VIP; (g) Count interface of VIP; and (h) Speed interface of VIP.

Figure 3: (a) Actual trajectory of 8 probe vehicles; (b) Estimated trajectory of probe vehicles without reference probe; (c) Estimated trajectory using probe vehicle 1; (d) Estimated trajectory using probe vehicle 4; (e) Estimated trajectory using probe vehicle 5; and (f) Estimated trajectory using probe vehicle 6.

Figure 4: (a) Travel time required for probe vehicles to travel 1 km; (b) Travel time required for probe vehicles to travel 0.5 km; and (c) Errors using different probe vehicles as reference.

Figure 5: Effect of capacity fluctuation on estimated trajectory: (a) Change in fundamental diagram; and (b) change in lopsided network due to side friction.

Figure 6: (a) FD for location 1; (b) FD for location 3; (c) FD parameters at different locations of study area; (d) Estimated trajectory before using variable capacity; and (e) Estimated trajectory after using variable capacity.
(a) A map with two intersections labeled: Intersection 1 and Intersection 2.

(b) A smartphone screen showing a navigation application.

(c) A user interface from a navigation application.

(d) A blank user interface with fields for Name and OK button.

(e) A table with columns for No, User Name, User Time, Server Time, Longitude, Latitude, Created Time, and Updated Time.

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