Evaluating Pension Funds Considering Unobservable Variables & Bridging Pension Funds & Mutual Funds through the Development of a New DEA Model

By

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy

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Abstract

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Private pension plans provide an important source of retirement income for employees and their families. The effective performance of private pension plans is an important issue to investigate because of the social and economic implications for investors, managers and governments. Typically, financial ratios which have inherent limitations, are used to evaluate pension funds’ performance. The objective of this research is to develop Data Envelopment Analysis (DEA) models which evaluate the private pension funds’ performance and suggest useful measures based on detailed data acquired from the federal regulator OSFI (Office of the Superintendent of Financial Institutions Canada). The research has three sections. The first section evaluates private pension funds’ performance by considering the effect of regulations which are not under the control of fund managers as well as comparing pension funds with mutual funds which have different characteristics. A new DEA model is developed that can evaluate different entities with different cultures from the same industry such as pension funds and mutual funds. The results show that the new DEA model provided a more realistic assessment of pension funds’
performance and comparison between pension funds and mutual funds. In section two, the reason for low minimum efficiency scores in DEA for pension funds is examined. It is found that the presence of very low efficiency scores is not uncommon in this industry. In section three, a new methodology is introduced which evaluates the pension funds’ performance by considering the importance of different variables based on an expert’s judgements as well as borrowing useful information from the mutual funds’ dataset. The results show that the discriminatory power of DEA increases after adding an expert’s opinions as well as mutual funds’ information to the pension funds’ DEA model and three different target levels are defined for inefficient plans. Since private pension funds have unique characteristics compared to other investment funds as well as the significant importance of retirement income to people, the results of this research will be of interest to government, financial and industrial managers.
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<td>AcSB</td>
<td>Accounting Standards Board</td>
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<tr>
<td>ALM</td>
<td>Asset Liability Management</td>
</tr>
<tr>
<td>AR</td>
<td>Assurance Region Model</td>
</tr>
<tr>
<td>BCC</td>
<td>Banker, Charnes, Cooper Model</td>
</tr>
<tr>
<td>CAPSA</td>
<td>Canadian Association of Pension Supervisory Authorities</td>
</tr>
<tr>
<td>CAT</td>
<td>Categorical DMUs</td>
</tr>
<tr>
<td>CCR</td>
<td>Charnes, Cooper, Rhodes Model</td>
</tr>
<tr>
<td>CICA</td>
<td>Canadian Institute of Chartered Accountants</td>
</tr>
<tr>
<td>Combo</td>
<td>Combination Pension Plan</td>
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<tr>
<td>CPI</td>
<td>Consumer Price Index</td>
</tr>
<tr>
<td>CPP</td>
<td>Canada Pension Plan</td>
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<tr>
<td>CRA</td>
<td>Canada Revenue Agency</td>
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<td>DB</td>
<td>Defined Benefit Plan</td>
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<tr>
<td>DC</td>
<td>Defined Contribution Plan</td>
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<tr>
<td>DEA</td>
<td>Data Envelopment Analysis</td>
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<tr>
<td>DFA</td>
<td>Distribution Free Approach</td>
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<tr>
<td>DMU</td>
<td>Decision Making Unit</td>
</tr>
<tr>
<td>EET</td>
<td>Exempt Contribution, Exempt Investment Income, Taxed Benefits</td>
</tr>
<tr>
<td>ERISA</td>
<td>Employee Retirement Income Security Act</td>
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<tr>
<td>Acronym</td>
<td>Full Form</td>
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<tr>
<td>FSCO</td>
<td>Financial Services Commission of Ontario</td>
</tr>
<tr>
<td>GAAP</td>
<td>Generally Accepted Accounting Principles</td>
</tr>
<tr>
<td>GIS</td>
<td>Guaranteed Income Supplement</td>
</tr>
<tr>
<td>IFRS</td>
<td>International Financial Reporting Standards</td>
</tr>
<tr>
<td>ITA</td>
<td>Income Tax Act</td>
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<tr>
<td>MF</td>
<td>Mutual Funds</td>
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<tr>
<td>MV-DEA</td>
<td>Mixed Variable DEA Model</td>
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<tr>
<td>Non-Dis-VRS</td>
<td>Non-discretionary Variable Return to Scale</td>
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<tr>
<td>OAS</td>
<td>Old Age Security</td>
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<tr>
<td>OECD</td>
<td>Organization for Economic Co-operation and Development</td>
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<td>OSFI</td>
<td>Office of Superintendent of Financial Institutions Canada</td>
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<tr>
<td>PBGC</td>
<td>Pension Benefit Guaranty Corporation</td>
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<td>PBSA</td>
<td>Pension Benefits Standards Act</td>
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<td>PF</td>
<td>Pension Funds</td>
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<td>QPP</td>
<td>Quebec Pension Plan</td>
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<tr>
<td>RPP</td>
<td>Registered Pension Plan</td>
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<tr>
<td>RRIF</td>
<td>Registered Retirement Income Fund</td>
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<tr>
<td>RRSP</td>
<td>Registered Retirement Saving Plan</td>
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<tr>
<td>SBM</td>
<td>Slacks Based Measure of Efficiency Model</td>
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<tr>
<td>SFA</td>
<td>Stochastic Frontier Analysis</td>
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<td>TFA</td>
<td>Thick Frontier Approach</td>
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<td>TO</td>
<td>Trade-Off Approach</td>
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<tr>
<td>VRS</td>
<td>Variable Return to Scale</td>
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Chapter 1: Introduction

1.1. Overview

Private pension plans manage assets that are used to provide workers with retirement income. In Canada, as of March 31, 2014, approximately 7% of pension plans are federally regulated which cover over 639,000 employees with a total value of $171 billion. Pension plans under provincial regulation represent 18 million employees and total assets of $1.08 trillion (OSFI, 2014). Pension fund managers operate in a different environment compared to other investment funds since they are subject to strict supervision as well as a different taxation system. In the typical North American pension plan, approximately 80% of ultimate benefit payments come from investment income versus only 20% from the original contributions (Ambachtsheer and Ezra, 1998). Therefore, the performance of pension fund managers is an important issue.

Also, low birth rates, together with higher average age of the population and increase in general life expectancy provide further motivation for investigating private pension funds’
performance. One of the key challenges facing Canada is an ageing population. In 2005, 13% of the population was older than 65. By 2031, this percentage is expected to exceed 25% (Department of Finance Canada, 2009). In light of the growing retiree population, ensuring that Canada’s retirement income system is productive and efficient in enabling Canadians to achieve sufficient means in retirement, is a vital goal.

Financial ratios, which have inherent limitations, have been used by governments, financial institutions and managers to evaluate pension funds’ performance for decades. However, financial ratio analysis does not consider the influence of different variables on each other simultaneously and it ignores any relationships, substitutions or trade-offs among various performance measures. As a result, financial ratio analysis does not indicate whether the resources used to provide services are being managed efficiently. The objective of this research is to adapt Operational Research tools, specifically Data Envelopment Analysis, to develop certain models which evaluate the efficiency in fund performance. From the findings we suggest useful measures based on real data to better manage the investments in a private pension fund. Moreover, we compare pension funds with mutual funds while retaining the main characteristics of each fund.

Data Envelopment Analysis (DEA) is a powerful analytical tool with specific characteristics that can calculate the efficiency score of a group of Decision Making Units (DMUs) by considering multiple inputs and multiple outputs simultaneously. DEA points to the efficient DMUs which can be considered as a target for the inefficient ones. Therefore, DEA is considered as one of the most useful techniques for managers to measure the efficiency of financial institutions. However, DEA has not been properly used for assessing pension funds’ performance due to the complexities of such funds. The few papers that can be found in literature as described in Chapter 4 do not consider the main characteristics of pension funds such as uncontrollable
variables for managers and regulations. Regulations affect these funds in many ways from investment strategy, tax status, reporting requirements and others. Therefore, one of the objectives of this research is to evaluate the pension funds’ performance through DEA considering the effects of regulations among other aspects. Another would be to consider the effect of these regulations in restricting the hand of pension fund managers by comparing their performance to other investment vehicles with different rules such as mutual funds.

For the purpose of this research, GAMS software and SPSS are used to carry out the programming for developing DEA models, statistical analyses and sensitivity analyses.

Because of the significant importance of adequate retirement income to people and the unique characteristics of private pension funds, the results of this research will not only add to the academic literature but may also be of interest to regulators, financial analysts and, of course, fund managers.

1.2. Research Objectives

The objective of this research is to develop new DEA models which evaluate pension funds’ performance considering the main characteristics of this investment vehicle as well as provide a meaningful environment in which different entities such as pension funds and mutual funds can be assessed relative to each other while maintaining their own characteristics. This thesis explores the differences between pension funds and mutual funds and develops two new DEA methodologies to bridge them directly and indirectly. These new models can also be applied to other industries.
To achieve this goal, this research investigates the performance of different pension plans by considering the effects of regulations on asset allocation and the managers’ authority. Also, this thesis explores the fundamental differences between pension funds and mutual funds and develops a new DEA model that takes into consideration their different cultural assumptions. This model ensures that all DMUs maintain their own characteristics while they are assessed relative to each other. In this first new DEA model, pension funds and mutual funds are compared relative to each other directly which means both pension funds and mutual funds’ DMUs are added together in one dataset. As there are different types of pension funds, the effects of fully funded and underfunded active pension plans on DEA efficiency scores are examined from different perspectives. Then a new methodology is introduced to provide a framework in which pension plans’ performance is evaluated by considering professional judgements and borrowing useful information from the mutual funds’ dataset. In the second new DEA model, pension funds and mutual funds are bridged together indirectly. Therefore, the dataset for this model has only pension funds’ DMUs and useful information from the mutual funds’ dataset is extracted and added to the pension funds’ model. The research evaluates the possibilities for pension fund managers to improve their performance under different rules.

1.3. Organization of Thesis

The structure of this thesis is as follows:

- **Chapter 2 - Private Pension Plans**: provides a literature review of private pension funds. In particular, the definitions, types of private pension funds, characteristics,
and the current mathematical methods used for evaluating their performance are presented.

- **Chapter 3 – Data Envelopment Analysis (DEA):** describes DEA and its different models with their mathematical formulations.

- **Chapter 4 – DEA Studies in Pension Funds:** introduces a detailed literature review of DEA pension funds’ performance analyses.

- **Chapter 5 – Mixed Datasets with Partially Deficient Variable Sets Embodied in Mixed Variable DEA (MV-DEA):** proposes the objective and theoretical development of the Mixed Variable DEA Model. The methodology is applied to the comparison of Canadian pension funds with Canadian mutual funds. Additionally, the data collection and preparation are reviewed. A discussion of the results is presented.

- **Chapter 6 – Low Minimum Efficiency Scores in Pension Funds Industry:** presents the investigation of fully funded and underfunded pension plans. A detailed explanation and interpretation of the results are also provided.

- **Chapter 7 – Improving Pension Funds’ Performance by Considering an Expert’s Opinions and Borrowing Mutual Funds’ Information Using DEA:** introduces the method in which expert’s judgements and useful information from mutual funds’ dataset are combined to bridge pension funds with mutual funds. Results are provided and discussed.

- **Chapter 8 – Conclusions, Contributions and Future Work:** presents conclusions of the dissertation and the main contributions of this research. Recommendations for future work are also provided.
• **Appendices** - includes glossary of terms and all references as well as the questionnaire and GAMS scripts for the developed DEA models.
Chapter 2: Private Pension Plans

2.1. Introduction

Private pension plans provide an important source of retirement income for employees and their families. Employers generally set up private pension plans voluntarily which must be funded and administered in compliance with applicable tax and strict pension laws (OSFI, 2009). Private pension plans are considered as one of the three pillars of retirement planning, along with government pensions (Canada/Quebec Pension Plans and Old Age Security) and personal savings (Registered Retirement Savings Plan (RRSP) and other tax friendly instruments offered by financial institutions).
2.2. History of Retirement Income System

The history of the Retirement Income System has been thoroughly described in Civilization.ca, 2002. A summary is presented below (Civilization, 2002):

The preliminary arrangement of the Canadian retirement income system dates back to the late 1800s. Before the industrialization period, most people were born, lived and worked on farms until they died. Families were responsible to care and support their elderly members. Those elderly without family support had to live on scarce public assistances which were difficult to obtain. Seniors had to work until they died unless they had medical evidence for work exemption in order to gain this minimum assistance. By industrialization, the traditional family unit and family needs changed and so did support for the elderly. Many people migrated from their farms and rural communities to the cities. Therefore, a new way of life began.

In 1887, the federal government created the Pension Fund Societies Act which enabled employees to set up a pension fund which an employer could contribute to. In 1908, the Government Annuities Act was established to encourage Canadians to save for their retirement through the purchase of a government annuity. However, the Government Annuities plan was not sufficient since few people could afford them.

In 1927, the government passed the Old Age Pensions Act that provided income to a person aged 70 or over with at least 20 years of residency. This program was subject to a means test which was used to determine the senior’s income from all possible sources of income even if no actual income was received from a particular source. However, the means test did not consider how much money a person needed to pay for food, shelter, clothing and etc. Therefore, if the assumed income
exceeded a certain amount, a senior would not receive any assistance, even if she/he in fact needed it. Moreover, the calculations were not consistent and varied from province to province.

In the 1930s, the Depression hit Canada and caused severe unemployment and poverty. The economy prospered in the Second World War. However, that period was a harsh time for seniors because of inflation. Therefore, more active government involvement was sought in the social security system to protect people. The government introduced Unemployment Insurance and Family Allowances Act in the 1940s.

In 1952, the Old Age Security (OAS) Act replaced the Old Age Pensions Act. OAS was a source of basic income for a person 70 years of age or over who had lived in Canada at least 20 years. This program was not based on a means test. Employer-sponsored pensions and private savings were supposed to supplement the OAS. In 1957, the government introduced the Registered Retirement Saving Plans (RRSPs) for self-employed people and employees without an employer-sponsored pension plan to save for their retirement. However, many retirees still had a low standard of living. In 1966, the Canada Pension Plan (CPP) and Quebec Pension Plan (QPP) were established. These plans protected employees and their families from loss of income due to retirement and based on employees’ contribution amounts, benefits were received. In 1967, the Guaranteed Income Supplement (GIS) was established as a part of OAS program in order to provide low-income OAS pensioners with additional money.

The period between the mid-1960s to the late 1980s was characterized by amendments in the government’s retirement income system because of social changes such as disability benefits, gender equality and women’s liberation.
In the 1990s, the public pension system had substantially improved the seniors’ situation. By 1997 only 19 percent of seniors had low income compared to 34 percent in 1980. However, uncertainty about the sustainability of Canada’s public pensions became an important issue because life expectancy was increasing and seniors were making up a large proportion of the population. Therefore, in 1998, the government increased the contribution rate to the CPP and changed the administration and calculation of benefits. Also, there was increased emphasis on the importance of private pensions and personal savings.

2.3. Canada’s Retirement Income System Today

In Canada, retirement income system has three key sources:

1. Government pension programs
2. Employer-Sponsored Pension Plans
3. Personal savings

2.3.1. Government Pension Programs

Retirement income from government programs can be referred to as public pensions. In general, government pension programs have at least four primary objectives (Pesando and Rea, 1977):

- Provide an income for seniors with minimum support
- Pass on some of the current increases in real income to elders
- Help individuals to save for their retirement
Ensure that the public or private plans do not distort the individuals’ motivations to work and save.

Retirement Income from Government sources fall under two programs (Hall, 1996):

1. Old Age Security (OAS) and Guaranteed Income Supplement (GIS) which provide the minimum retirement benefits to all people aged 65 or over who have lived in Canada for a sufficient number of years. Benefits are paid from the federal government’s revenue fund and recipients do not need to contribute. OAS benefits are reduced for those whose net income exceeds a certain amount. For those with small incomes, GIS may also be payable. OAS benefits are taxable but the benefits from GIS are tax exempt.

2. Canada and Quebec Pension Plans (CPP/QPP) which are earnings-based pension plans by employee and employer contributions. CPP/QPP are financed by contributions from employees, employers and the self-employed without any government subsidy. The benefits from CPP/QPP are payable in addition to OAS and other programs. Benefits from the CPP/QPP are taxable.

2.3.2. Employer-Sponsored Pension Plans

Retirement income from employer-sponsored pension plans can be referred to as private pension plans. There are two categories of private pension plans: registered and non-registered. Registered pension plans (RPPs) have a special tax treatment. RPPs are registered with the Canada Revenue Agency as well as federal or provincial pension authorities. Therefore, RPPs must comply with the Income Tax Act (ITA) and other pension regulations. Non-registered plans do not have the tax benefits and regulations of RPPs. The most common private pension plan is an RPP. RPPs
require employers to contribute. Employees make contribution as well if the plan is contributory. Contributions to an RPP are tax exempt, investment incomes in the RPP such as interest, dividends, capital gains, etc. are not taxed at the point of earning them but benefits are tax deferred until they are withdrawn from the plan. RPPs provide regular income to their members when they retire (Hall, 1996).

According to law, it is the employer who decides whether or not to create a pension plan and its conditions. Some employers establish pension plans as a result of collective bargaining or because of their competitors whom provide pension plans. A good pension plan improves the employer’s competitive position since it helps to recruit and retain skilled employees and facilitate their retirement. However, after the establishment of a plan, it must be funded and administered in compliance with applicable tax and pension rules.

Since private pension plans are the focus of this research, more explanations are addressed in the next sections.

### 2.3.3. Personal Savings

There are plans that people can contribute to on their own such as Registered Retirement Savings Plans (RRSPs). These plans were established for people who do not have pension plans. Today, even if a person has a pension plan, she/he may also have a RRSP. RRSPs are governed by the Income Tax Act which sets the maximum amount of RRSP contributions (18% of earned income in the previous calendar year up to $24,270 for 2014) that can be deducted from an individual’s taxable income. The tax treatment for RRSPs is the same as RPPs which are established by employers. The RRSP has to be converted into a stream of income when the contributor reaches age 71 (it changed from 69 to 71 in 2007). If funds are simply withdrawn from
a RRSP, the entire amount is fully taxable as ordinary income. In order to defer this taxation a contributor can transfer RRSP into a Registered Retirement Income Fund (RRIF\(^1\)) or a life annuity\(^2\). When the RRSP is converted into RRIF or a life annuity, a contributor has to pay tax on the amounts that are withdrawn. However, the applicable marginal tax rate may be lower (OSFI, 2009).

### 2.4. Types of Private Pension Plans

Private pension plans are mostly categorized in two fundamentally different groups:

1. Defined Benefit Plan
2. Defined Contribution Plan

#### 2.4.1. Defined Benefit Plans (DB Plans)

Defined Benefit Plan offers an employee the security of knowing what to expect at retirement based on their salary during their working years. In a DB plan, the investment risk of guaranteed retirement income is taken on by the employer. Defined benefit plans are generally offered by large companies, such as banks, railroads, and telecommunications companies. Different formulas can be used to calculate a member’s benefits. The formula operated in a specific plan is explained in the pension plan booklet provided to members.

---

1. Registered Retirement Income Fund (RRIF) is a personal retirement income fund offered by financial institutions. An RRIF is used to provide people with a constant income flow through retirement from the savings in their RRSPs. RRIFs are governed by the Income Tax Act which determines minimum withdrawal amounts.
2. Life annuity is an insurance product that indicates a predetermined periodic payout amount until the death of the annuitant. These products are frequently used to help retirees budget their money after retirement.
The most common types of benefit formulas applied in DB plans are:

1. Final or best average earnings: the benefit is calculated according to the member’s average earnings over the last (or highest paid) years of employment and total years of service. For example, 1.5% of average earnings by a member over the last 5 years of employment multiplied by the total years of his/her service.

2. Career average earnings: the benefit is based on the employee’s (member's) earnings over the entire period of plan membership. For example, 1.5% of the employee’s total earnings.

3. Flat benefit: the benefit is normally measured based on a fixed dollar (or flat) amount for each year of service, regardless of the member’s individual level of earnings. For example, $40 per month per year of service (OSFI, 2009).

2.4.2. Defined Contribution Plans (DC Plans)

Defined Contribution Plan specifies the amount of employer and employee contributions. The amount available to provide a pension income in this plan is affected by how successfully contributions are invested. Therefore, it is important for members to make informed investment decisions and the investment risk in a DC plan is with the employee. There are two types of defined contribution plans:

1. Money purchase pension plan: This is a predefined plan in which contributions are often a fixed percentage of an employee’s annual earnings or fixed amount per unit of time worked and are deposited monthly in an individual account in the member’s name.

2. Profit sharing pension plan: In this plan, only employer contributes based on the profitability of the company in proportion to the employee’s earnings (Hall, 1996).
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Defined contribution plan is usually sponsored by small companies because it is simple to start and to terminate and the members’ individual defined contribution accounts are easily transferable to individual or group locked-in RRSP\(^1\) accounts.

2.5. Comparison of DB and DC Plans

There are some factors which are important to consider in order to have a meaningful comparison of defined benefit plan and defined contribution plan such as the employee’s age, working years and career path. A defined contribution plan could be better for those younger employees who are frequently changing jobs because the benefits can be taken with them. A defined benefit plan might be better for those employees who work with the same employer until retirement because the expected pension amount is guaranteed and the benefits are often determined as a function of final average earnings.

Also, for employees who are risk averse, a DB plan is a better choice because all investment risks are taken by employers and benefit levels are guaranteed. However, in DC plans, the investment risk is shifted to employees. Moreover, longevity risk is another risk which is important to consider because of increasing life expectancy. In a DB plan, benefit levels are guaranteed for a retiree’s lifetime. Retirees generally have the option of including survivor benefits. In a DC plan, the length of payment depends on how investments are managed and employers do not promise

---

\(^1\) Locked-in Registered Retirement Savings Plan is a personal retirement saving account offered by financial institutions. A locked-in RRSP is used to hold money that is transferred out of a pension fund on termination of employment and this money cannot be used for any purpose other than to provide a retirement pension.
retirement benefits for lifetime. Therefore, DC plan retirees will have greater risk of outliving it if they do not have any other source of income.

Moreover, in a DB plan, the employer is responsible for contributing as much as necessary to provide the promised benefits. In a DC plan, both employee and employer contribute to the plan (Retirement Advisor, 2009). In addition to these key differences between DB and DC plans, there are some other differences which will be further addressed in the following section.

2.6. Shifting from DB Plans to DC Plans

Lately, defined contribution plans have increased compared to defined benefit plans. Improvements in standard of living and medical science have caused an increase in life expectancy in most developed countries which means people will survive to their retirement age and also will live longer during their retirement. Therefore, employers prefer DC plans because they do not need to provide pension benefits for retiree’s life time. Also, in order to be more successful and competitive in a global market, employers need to be more cost conscious and risk averse (Ostaszewski, 2001). As a result, DC plans are more desirable by the firm because the investment risk of the pension plan is taken by employees.

Moreover, volatile financial markets have caused the cost of funding retirement benefits to be less predictable which make it less favourable for employers to commit to a DB plan. Also, strict legal, funding and solvency laws were set by many governments for DB plans in order to protect contributions made to pension plans in the 1980s and 1990s (Bharmal, 1988). All these changes made DB plans more costly and complex to administer. However, the regulation of DC
plans is straightforward. Basically, the money is deposited in the individual accounts and this is the end of the liability for the company (Bertocchi and et al., 2010).

Another reason for the declining number of DB plans is that benefits earned at one employer continue to accumulate under his/her plan, but not under another employer’s plan. Employees are working at a greater number of companies during their careers than previous generations. Therefore, it is difficult for the majority of younger workers to manage their expected income under a defined benefit plan. For example, consider an employee who worked for 40 years at one company to another employee who worked 20 years at each of two employers. Each employer provides a benefit of 1.5% of the average of the final five years of salary multiplied by the number of years of service. The worker started with an income of $15,787 in 1972, and retired in 2012 with an income of $50,000 after receiving annual salary increases of 3% over 40 years. If the worker served his entire career with one employer, the annual benefit would be $28,302 (1.5% * 40 years * the final five-year salary average of $47,171). The benefits would be quite different had he worked for two employers. The retiree worked at the first employer from 1972 to 1992, with an average annual salary in the final five years of $26,117. The annual benefits of $7,835 (1.5% * 20 years * $26,117) are determined in 1992, but not paid until retirement in 2012. The second employer pays annual benefits in the amount of $14,151 (1.5%* 20 years * $47,171). Compared to the annual benefit of $28,302 after working the entire career for a single employer, the employee splitting careers between two companies earns an annual pension of only $21,986 ($7,835 + 14,151), which is $6,316 per year less than if he had worked for a single company (CICA, 2013). In contrast to DB plans, DC plans are portable which is better for employees who work multiple jobs in their career. For Canadian pension funds, under recently changed rules, investors can move between different DB plans at less or no cost. However, transition between
CHAPTER 2: PRIVATE PENSION PLANS

DB to DC plans has a large cost for an investor (Hussey, 2015). Furthermore, changes in culture, technology, and higher level of education have caused employees to be more independent thus making DC plans more desirable.

2.6.1. Observations in the U.S. and Canada

There has been a strong shift away from DB plans toward DC plans in the United States. However, in Canada, this transfer has not been as pronounced. The number of DB plans in the U.S. private sector decreased from 148,096 to 46,543 while the number of DC plans increased from 340,805 to 654,469 during 1980 to 2010. As shown in Table 2-1, the number of active members covered by a DB plan decreased from 30.1 million in 1980 to 17.2 million in 2010 while the number of active members covered by a DC plan increased from 18.9 million to 73.4 million respectively (U.S. Department of Labor, 2013).

Table 2-1: Number of Private Pension Plans and Active Members in the U.S.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Plans</th>
<th>Number of Members (millions)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>DB</td>
</tr>
<tr>
<td>1980</td>
<td>488,901</td>
<td>148,096</td>
</tr>
<tr>
<td>1990</td>
<td>712,308</td>
<td>113,062</td>
</tr>
<tr>
<td>2000</td>
<td>735,651</td>
<td>48,773</td>
</tr>
<tr>
<td>2010</td>
<td>701,012</td>
<td>46,543</td>
</tr>
</tbody>
</table>

1. A dataset for U.S. private pension plan’s information consists of plans with 100 or more members and a five percent sample of pension filings for plans with fewer than 100 members. From beginning of 2010, plans with fewer than 100 members are no longer considered. Since in this section the aim is to study the trend of shifting DB plans to DC plans, there is no need to be consistent for different years.
In Canada, this trend has not been as pronounced as in the U.S. According to Statistics Canada CANSIM database (Table 280-0016), in 1980, 2.3 million pension plan members were covered by 7,639 DB plans in the private sector while only 211,498 members were covered by 5,887 DC plans. In 2010, there were 11,330 DB plans covering approximately 1.6 million members and 5,978 DC plans covering almost 818,481 members. The plan members who were not covered by DB or DC plans, were covered by other types of plans such as a combination plans\(^1\) (Combo).

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Plans</th>
<th>Number of Members</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Total</td>
<td>DB</td>
</tr>
<tr>
<td>1980</td>
<td>13,896</td>
<td>7,639</td>
</tr>
<tr>
<td>1990</td>
<td>18,984</td>
<td>7,898</td>
</tr>
<tr>
<td>2000</td>
<td>14,281</td>
<td>6,654</td>
</tr>
<tr>
<td>2010</td>
<td>17,880</td>
<td>11,330</td>
</tr>
</tbody>
</table>

As indicated in Table 2-2, the number of DB plans decreased steadily from 1980 to 2010 in Canada compared to the U.S. where the shift was faster. This disparity can be attributed to differences in regulation, taxation, unionization and environmental conditions between the two countries (Brown and Liu, 2001). In Canada, pension regulations for DB plans make them more pleasant for employers because of less strict coverage rules than in U.S. Also, ancillary benefits such as death or disability before retirement are provided only under DB plans in Canada.

---

1. Combination pension plans incorporate both defined benefit and defined contribution plans’ concepts. This type of plan offers additional flexibility for plan members and employers by incorporating positive elements from both plans. In a combination plan, a pension must be promised (i.e. the defined benefit concept) and accordingly, from time to time, employers are able to use pension surplus (i.e. the defined contribution concept) to fund their defined benefit plan’s current service costs.
Moreover, the Canadian government has the same policies regarding taxation and funding for DB and DC plans while the U.S. government implements policies that favour DC plans in many ways. Another factor which may also explain the stable proportion of DB plans in Canada versus the rapid decline of DB plans in the U.S. is unionization. Unions have remained comparatively stronger in Canada than in the United States. Most of Canadian Governments are unionized. Unions have a significant proportion of their workers with pensions and have emphasized employment security and therefore they are in favor of DB plans (Brown and Liu, 2001). These differences cause the rapid increase of DC plans versus DB plans in the United States than in Canada.

2.7. Retirement Age

There are three kinds of retirement age in a pension plan:

1. Normal Retirement Age: is the age in which the employee has the right to retire and receives a full pension. The normal retirement age is 65 for both men and women in order to avoid any discrimination.

2. Early Retirement Age: is the time when the employee can retire before normal retirement age, ordinarily up to ten years before the normal retirement age. Then a reduced pension will be paid because the payments start earlier and will be longer compared to retirement benefits beginning at normal retirement age.

3. Postponed Retirement Age: is the time when an employee can choose to retire after the normal retirement age. The maximum age for postponing the retirement is stated in the
pension plan or in accordance with governing pension and tax regulations. A pension plan may allow the employee to continue to pay the required contributions after normal retirement age. Therefore, when delayed retirement occurs a larger monthly retirement benefit amount will commence. However, at the same time the number of months until death may be less.

2.7.1. Aging Population

One of the key challenges facing Canada is an ageing population. As evident in the Figure 2-1, the age structure of the Canadian population has changed considerably over time. The shape of the age structure is changing from a tree to a kite. In 2056, people aged 65-70 will constitute the largest age group in the population. This transformation is the result of the drop in fertility and the steady increase in life expectancy. It is concerning to find how few workers would be supporting the retirees. Therefore, it is a vital goal to ensure that Canada’s retirement income system is productive in enabling Canadians to achieve sufficient retirement benefits.
2.8. Regulations on Private Pension Plans

Pension funds have to be well governed and managed so that they can make money on a large scale and meet their obligations while keeping costs low (Ambachtsheer, 2007). Private pension plans have specific characteristics related to taxation and strict investment regulations. In Canada, taxation form is EET (Exempt contributions, Exempt investment income and capital gain, Taxed benefits when received) which means that the contributions by both employer and employee are tax deductible to each, the investment results of the pension fund are exempt and the benefits, when received, are taxed (Davis, 1998). Tax exemption is one of the reasons for the growth and popularity of pension plans. This system encourages plan members (investors) to invest in private pension plans and save for their old age, but there are limits to how much can be invested per year. In order to gain preferred tax treatment, a pension plan must be admitted for registration by the
Canada Revenue Agency (CRA). The registration rules are intended to limit contributions that can be deductible in order to ensure that not too much tax revenue is lost and also to limit benefits that can be paid (Morneau Sobeco, 2008).

Private pension plans must follow the standards set out in the applicable pension legislation, which typically involves minimum standards for funding, investment, membership eligibility, benefits, administration and information to members among others. For pension standards purposes, plans can be registered under federal or provincial jurisdiction. The Office of the Superintendent of Financial Institutions Canada (OSFI) regulates private pension plans federally subject to the Pension Benefits Standards Act, 1985 (PBSA) which sets minimum standards and rules to ensure that the rights of plan members, retirees and beneficiaries are protected. The plans which are not under federal jurisdiction are regulated at the provincial level. To decrease the conflicts between federal and provincial regulations, the Canadian Association of Pension Supervisory Authorities (CAPSA) was established in 1974 as a federal-provincial forum to discuss common issues faced by federal and provincial pension plan supervisory authorities.

In assessing the possible loss to members’ promised benefits, the tight regulations are designed to (OSFI, 2009):

- Provide several solvency restrictions requirements in order to maintain the financial health of the pension plans
- Identify those pension plans that may have problems meeting minimum funding requirements (For instance, plans are investigated to see whether they are fully funded or underfunded. Underfunded active plans in which estimated liabilities exceed assets need to be controlled)
• Ensure compliance with policies and procedures to control and manage risk
• Prompt communication with plan administrators advising them of material deficiencies
• Imply appropriate interventions to compel administrators to take corrective measures to address deficiencies identified
• Encourage people to save and invest in pension plans

Pension plans information must be provided for employees and their beneficiaries. Better disclosure of plan information gives plan members a sense of the plan’s health and improves their ability to raise concerns in an informed and timely manner. Moreover, whenever the plan is in a strong surplus or deficit situation, the sponsor may explain to members why certain actions must be taken.

There are differences between the allowable investment characteristics of pension funds and those of other investment funds. For instance, unlike mutual funds, according to the laws, an investor does not have access to his/her contributions at any time he/she wishes until the retirement age; and if access is allowed, the receipts are immediately taxed if received in the hands of the fund participant (some things are allowed such as roll-overs and portability). Also, under some regulations, the contribution amounts for employees and employers are fixed for each month; however, such regulations are not applicable to contributions in other investment funds. Moreover, due to the strict regulations of pension funds, their investment strategies are required to be publically disclosed which are not required for unregulated funds, for example, hedge funds (AIMA, 2004).

Generally, the pension assets should not be at risk when a business declares bankruptcy because according to the law the promised retirement benefits are required to be funded adequately
and be kept separate from an employer’s business assets. However, sometimes as a result of a downsizing or reorganization where a substantial number of plan members are laid off or when a business is shut down, a pension plan may be terminated fully or partially. In Canada, the federal agency OSFI also has the power to terminate a plan or revoke a plan’s registration. Moreover, the employer may simply decide to discontinue its pension plan as she/he set it up voluntarily, as long as the existing benefits of plan members are protected. If a defined benefit plan terminates in an underfunded or solvency deficient position, the retirement benefits of the plan members may have to be decreased accordingly. A defined benefit plan may have more assets than the expected cost of the promised pensions which should be refunded to the employer. However, this surplus amount can change considerably from time to time, based on interest rates, plan amendment and investment returns. Any refund of surplus from a DB plan to an employer must first gain the consent of OSFI. A defined contribution plan cannot have a surplus. All the assets are allocated to the members’ individual accounts (OSFI, 2009). For both DB and DC plans, if the plan is not locked in, any funds are available as a cash withdrawal. If not, members can transfer to other type of plans or locked in RRSP according to the law. In the United State, defined benefits plans are protected by the Pension Benefit Guaranty Corporation (PBGC) a federal agency created by the Employee Retirement Income Security Act of 1974 (ERISA). If a plan terminated because an employer has financial difficulty and the plan cannot support the promised benefits, the PBGC will pay the benefits up to the limits according to the law. Defined contribution plans are not secured by this organization (PBGC, 2013).
2.9. Private Pension Plan Performance Measurement

2.9.1. Ratios

In the real world, different ratios are applied to measure private pension funds’ performance (Antolin, 2008). Financial Ratio Analysis is a tool used to conduct a quantitative analysis of information in an organization’s financial statements (Fraser et al., 2009). Ratio analysis is simple to use and the accounting concepts are few and easy to understand, therefore, anyone who can understand basic math will have no trouble understanding financial ratios in attempting to assess the financial health of the underlying organization. Also, it provides percentage instead of absolute values which helps to investigate information in the financial statements rapidly (Costales and Szurovy, 1994).

One of the famous ratios is the Sharpe ratio. The Sharpe ratio is a measure of risk-adjusted performance developed by William F. Sharpe (Sharpe, 1966). The Sharpe ratio formula is presented in the Formulation (2.1):

\[ S_p = \frac{\bar{R}_p - R_f}{\sigma_p} \]  

(2.1)

Where

- \( S_p \) is the Sharpe ratio for the \( p \)th portfolio
- \( \bar{R}_p \) is the average return on portfolio \( p \)
- \( R_f \) is the risk free rate of return
- \( \sigma_p \) is the standard deviation for portfolio \( p \)

The risk free rate of return is the minimum return an investor expects for any investment over a specified period of time because he/she will not take on additional risk unless the potential
rate of return is greater than the risk free rate. However, the risk free rate does not exist because even the safest investments have a very small amount of risk. Therefore, the rate of return on riskless assets like Treasury Bills will be considered as the risk free rate of return (Muralidhar, 2001). A higher Sharpe ratio is more favorable because it shows that more return is achieved per unit of risk taken.

The Treynor ratio was proposed by Jack Treynor which is a risk-adjusted measure of return based on systematic risk\(^1\) (Treynor, 1965).

The Treynor ratio is calculated as:

\[
T_p = \frac{\bar{R}_p - R_f}{\beta_p}
\]  \hspace{1cm} (2.2)

Where

- \(T_p\) is the Treynor ratio for the \(p\)th portfolio
- \(\bar{R}_p\) is the average return on portfolio \(p\)
- \(R_f\) is the risk free rate of return
- \(\beta_p\) is the beta coefficient for portfolio \(p\)

1. Risk is the possibility of losing some or all of the original investment. The two general types of risk are “systematic risk” and “unsystematic risk”. Systematic risk is the probability of loss which is not predictable and impossible to avoid completely. It is inherent in a market and common to all businesses and investment opportunities. Inflation, recession and war represent sources of this type of risk since they affect the entire market. Unsystematic risk is variability of return caused by factors that are unique to an industry and can be reduced by diversification. By having investments in various companies and industries as well as different investment vehicles such as cash, mutual funds and real estate, investors can be less affected by unsystematic risk. Regulation amendments in an industry and labour strike are the examples of this type of risk.
This ratio is similar to the Sharpe ratio, with the difference being that the Treynor ratio uses beta\(^1\) instead of standard deviation\(^2\) in Sharpe ratio.

Generally, ratio analysis examines a part of the organization’s activities or combines the multiple dimensions into a single, unsatisfactory number. An unlimited number of ratios can be constructed but no causation factors derived from them and it is difficult to generalize about whether a ratio is good or not (Cooper et al., 2011). The financial ratio does not indicate whether the resources used to provide services are being managed efficiently. Also, financial ratio analysis investigates various ratios separately and does not consider the influence of different variables on each other simultaneously. The use of single measures ignores any relationships, substitutions or trade-offs among various performance measures (Zhu, 2003). Therefore, an organization may have some good and some bad ratios, making it hard to tell whether it’s a good organization or not.

2.9.2. OECD

Pension funds are measured by ratios and rates of return on a quarterly and yearly basis. Fund managers select different approaches to provide rates of return (Carmichael, 2005).

---

1. Beta represents the systematic risk and arises from the relationship between the return on a company and the return on the market. The beta for the \(i\)th company \((\beta_i)\) is the covariance of the company return and the market return divided by the variance of the market return: \(\beta_i = \frac{\text{Cov}_{im}}{\text{Var}_m}\). It can also be expressed as \(\beta_i = \rho_{im} \left(\frac{\sigma_i}{\sigma_m}\right)\) where \(\rho_{im}\) is the correlation between the company return and market return, and \(\sigma_i\) and \(\sigma_m\) are standard deviations of company and market returns. From this view, the beta can be explained as “correlated relative volatility” and can be referred to as a measure of the sensitivity of the company’s return to market’s return.

2. Standard deviation is a measure of volatility and reflects the dispersion of returns around their means and indicates the price swings. It is calculated as the square root of the variance. Standard deviation measures both systematic and unsystematic risks, which allows it to capture total risk.
to be consistent, the Organization for Economic Co-operation and Development\(^1\) (OECD) presented OECD-average rate of return (IRR) for the pension funds.

\[
Average \text{IRR}_N = \frac{Net \text{Investment Income}_N}{(Total \text{Investment}_{N-1} + Total \text{Investment}_N)/2} \times 100
\]  

(2.3)

The average IRR in each year \(N\) are given before management fees because different pension systems charge fees using different methods (OECD, 2013).

Employer-sponsored pension plans are asked to provide not only annual rates of returns, but also solvency ratios. The solvency ratio is the ratio of the assets of the plan to the expected cost of the promised pension benefits based on the assumption that the plan is terminating (OSFI, 2009).

### 2.9.3. Asset Liability Management

The Asset Liability Management (ALM) model is a risk management approach to find the optimal investment strategy by considering assets and liabilities. There are different models for assessing Asset Liability Management such as deterministic methods and stochastic programming. The deterministic models use known parameters and do not capture the random behaviour of the assets and liabilities. Stochastic programming is an approach to optimize problems under uncertainty. In the academic world, ALM models have been used for banks, mutual funds, hedge funds, university, insurance companies and pension funds. ALM models for pension funds appear

\[^1\) Organization for Economic Co-operation and Development (OECD) is an international economic organization of 34 countries (including Canada) established in 1961 to provide a platform in which governments can work together, compare policy experiences, seek answers to common problems and identify international policies of its members.\]

2.9.4. Frontier Efficiency Methods

Frontier efficiency analyses allow management to objectively identify best practices in complex operational environments. Five different approaches, namely, Data Envelopment Analysis (DEA), Free Disposal Hull (FDH), Stochastic Frontier Approach (SFA), Thick Frontier Approach (TFA) and Distribution Free Approaches (DFA) which are explained in Chapter 3, have been used in the literature for evaluating financial institutions’ efficiency. These approaches primarily differ in how much restriction is imposed on the specification of the best practice frontier and the assumption on random error and inefficiency. These methods are widely used in the literature for banks, mutual funds and financial institutions. However, there are only a few papers that use DEA and SFA methods to evaluate pension funds’ performance. The detailed explanations of these studies are described in Chapter 4.

For additional detailed information about evaluating the pension funds’ performance, the reader is encouraged to see Hinz et al. (2010) and Zenios and Ziemba (2007).
Chapter 3: Data Envelopment Analysis (DEA)

3.1. Introduction

Data Envelopment Analysis (DEA) is a non-parametric fractional linear programming technique that can be used to measure the relative performance of a group of entities called decision making units (DMUs) in a multi-dimensional framework. DEA is a powerful quantitative, analytical tool for measuring efficiency scores by accommodating multiple inputs and outputs at the same time (Cooper et al., 2007). DEA constructs an efficient frontier and generates scores between 0 (min) and 1 (max) for a group of DMUs in an input oriented model. The most efficient DMUs are given a maximum score of 1, while others are scored as a fraction of that (i.e. less than 1). The further from score 1 a DMU is, the greater the risk of inefficiency.
On a general level, DEA scores are calculated as a weighted sum of outputs over a weighted sum of inputs (Formulation 3.1):

\[
\text{Efficiency Score} = \frac{\sum_{r=1}^{s} u_r y_r}{\sum_{i=1}^{m} v_i x_i} = \frac{u_1 y_1 + u_2 y_2 + \cdots + u_s y_s}{v_1 x_1 + v_2 x_2 + \cdots + v_m x_m}
\] (3.1)

Where

- \( y_r \) is the amount of the output \( r \)
- \( u_r \) is the weight assigned to the output \( r \)
- \( x_i \) is the amount of the input \( i \)
- \( v_i \) is the weight assigned to input \( i \)

Note that the weights are not pre-assigned and the scores are calculated such that these weights are “optimized” for each DMU. Therefore, DEA attempts to maximize the outputs and minimize the inputs.

DEA provides important information that includes: 1) An efficiency score for each DMU, 2) An efficiency reference set with peer DMUs, 3) A target for an inefficient DMU, and 4) Information detailing by how much inputs can be decreased or outputs increased to improve performance (Gregoriou and Zhu, 2005). As a result, an efficient frontier consists of best performance units and a projection to the frontier that can be used as a “what to do” guide for fund managers. The efficiency reference set is composed of efficient DMUs that are used to build the goal for inefficient DMUs. The most commonly used DEA models are explained in the following sections.
3.2. Constant Return to Scale (CRS) Model

The constant return to scale model was proposed by Charnes, Cooper and Rhodes (also referred to as the CCR model) in 1978 as an extension of Farrell’s measure for efficiency \( \theta \) in the case of a single virtual input and a single virtual output to the case of multiple inputs (\( x_i \) for \( i = 1, \ldots, m \)) and multiple outputs (\( y_r \) for \( r = 1, \ldots, s \)) (Charnes et al, 1978).

\[
\theta = \frac{u_1y_1 + u_2y_2 + \cdots + u_sy_s}{v_1x_1 + v_2x_2 + \cdots + v_mx_m}
\]

The efficiency score and weights for each DMU \( o \) can be obtained by Formulation 3.2:

Maximize \[ \theta_o = \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{i=1}^{m} v_i x_{io}} = \frac{u_1y_{1o} + u_2y_{2o} + \cdots + u_sy_{so}}{v_1x_{1o} + v_2x_{2o} + \cdots + v_mx_{mo}} \] (3.2)

Subject to \[ \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{i=1}^{m} v_i x_{ij}} = \frac{u_1y_{1j} + \cdots + u_sy_{sj}}{v_1x_{1j} + \cdots + v_mx_{mj}} \leq 1 \quad (j = 1, \ldots, n) \]

\[ v_1, v_2, \ldots, v_m \geq 0 \]

\[ u_1, u_2, \ldots, u_s \geq 0 \]

Where \( y_{rj} \) is the amount of the output \( r \) from DMU \( j \)

\( u_r \) is the weight assigned to the output \( r \)

\( x_i \) is the amount of the input \( i \) from DMU \( j \)

\( v_i \) is the weight assigned to input \( i \)

This is the fractional program of the CCR model. This can be converted to its linear programming form of the primal (Multiplier form) and the dual (Envelopment form). Both the multiplier form and the envelopment form give the same optimal result.
CCR model is radial (change proportionally) and can be used as an “input oriented” model which aims to minimize inputs while satisfying at least the given output levels or “output oriented” model which attempts to maximize outputs without requiring an increase of input values (Cooper et al., 2007).

### 3.2.1. Input Orientation

In the input-oriented CCR model for a group of n DMUs with m inputs and s outputs, the objective is to obtain the best set of weights \((v^*, u^*)\) that maximize the efficiency of each DMU \(o\) \((o= 1, \ldots, n)\) by minimizing observed inputs while keeping the output levels constant. The optimal efficiency for each DMU, denoted by \(\theta^*\), is not greater than 1 \((\theta^* \leq 1)\) for an input oriented model. Therefore, a DMU \(o\) is CCR-efficient only if \(\theta^* = 1\) and there is at least one optimal \((v^*, u^*)\) with \(v^* > 0\) and \(u^* > 0\) (Cooper et al., 2007).

The primal form of the CCR model is presented as follows in the Formulation (3.3):

\[
\text{Maximize} \quad \sum_{r=1}^{s} u_r y_{ro} \\
\text{Subject to} \quad \sum_{i=1}^{m} v_i x_{io} = 1 \\
\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad (j = 1, \ldots, n) \\
\quad v_1, v_2, \ldots, v_m \geq 0 \\
\quad u_1, u_2, \ldots, u_s \geq 0
\]

The envelopment form of the input-oriented CCR model is expressed with a real variable \(\theta\) and a non-negative vector \(\lambda = (\lambda_1, \lambda_2, \ldots, \lambda_n)^T\) of variables as shown in the Formulation (3.4):
Minimize $\theta$  \hspace{1cm} (3.4)

Subject to $\theta x_{io} \geq \sum_{j=1}^{n} x_{ij} \lambda_j \quad (i = 1, \ldots, m)$

$y_{ro} \leq \sum_{j=1}^{n} y_{rj} \lambda_j \quad (r = 1, \ldots, s)$

$\lambda_j \geq 0$

For each DMU the optimization is performed with the objective function of decreasing all inputs proportionally while having fixed output levels. The optimal objective value is equal to $\theta^*$. However, there is a possibility to achieve more input decreases or output increases after proportional optimization. Therefore, to account for the input excesses $s^-$ and output shortfalls $s^+$ which are known as slack variables, extra linear programming is applied in a second phase by using the optimal solution of the dual form (phase I):

Maximize $w = \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+$ \hspace{1cm} (3.5)

Subject to $s_i^- = \theta^* x_{io} - \sum_{i=1}^{m} x_{ij} \lambda_j$

$s_r^+ = \sum_{r=1}^{s} y_{rj} \lambda_j - y_{ro}$

$\lambda_j \geq 0 \quad (j = 1, \ldots, n)$

$s_i^- \geq 0 \quad (i = 1, \ldots, m)$

$s_r^+ \geq 0 \quad (r = 1, \ldots, s)$
The DMU₀ is CCR-efficient if and only if:

i. \( \theta^* = 1 \) which is referred to as radial efficiency or technical efficiency; and

ii. \( s^- = s^+ = 0 \) which are called zero slacks.

If only the first condition \((\theta^* = 1)\) is satisfied, the DMU is weakly efficient. The inefficiencies associated with non-zero slacks are referred to as mix inefficiency. The conditions i and ii together describe the Pareto-Koopmans Efficiency which means a DMU is fully efficient if and only if it is not possible to improve any input or output without worsening some other input or output.

For an inefficient DMU₀, improvement can be achieved by referring its inefficiency to its reference set \( E₀ \) which contains the efficient DMUs. The formula for improvement which is called the CCR projection to the point \((\hat{x}_o, \hat{y}_o)\) on the frontier, is presented below in Formulation (3.6):

\[
\begin{align*}
\hat{x}_o &= \theta^* x_o - s^- = \sum_{j \in E_o} x_j \lambda^*_j \leq x_o \\
\hat{y}_o &= y_o + s^+ = \sum_{j \in E_o} y_j \lambda^*_j \leq y_o
\end{align*}
\]

3.2.2. Output Orientation

The objective of output-oriented CCR model is to maximize outputs without increasing the input values. The multiplier form of this model is presented in Formulation (3.7) (Cooper et al., 2007):
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Minimize \[ \sum_{i=1}^{m} p_i x_{i0} \] \hspace{1cm} (3.7)

Subject to \[ \sum_{r=1}^{s} q_r y_{ro} = 1 \]
\[ \sum_{r=1}^{s} q_r y_{rj} - \sum_{i=1}^{m} p_i x_{ij} \leq 0 \hspace{0.5cm} (j = 1, \ldots, n) \]
\[ p_1, p_2, \ldots, p_m \geq 0 \]
\[ q_1, q_2, \ldots, q_s \geq 0 \]

The optimal solution of the dual form can be derived from Formulation (3.8):

Maximize \( \eta \) \hspace{1cm} (3.8)

Subject to \( x_{i0} \geq \sum_{j=1}^{n} x_{ij} \mu_j \hspace{0.5cm} (i = 1, \ldots, m) \)
\[ \eta y_{ro} \leq \sum_{j=1}^{n} y_{rj} \mu_j \hspace{0.5cm} (r = 1, \ldots, s) \]
\[ \mu_j \geq 0 \]

An optimal solution of the output-oriented model related to that of the input-oriented model, \( p^* = \frac{v^*}{\theta^*} \), \( q^* = \frac{u^*}{\theta^*} \), \( \mu^* = \frac{\lambda^*}{\eta^*} \) and \( \eta^* = \frac{1}{\theta^*} \). Therefore, the higher the value of \( \eta^* \), the less efficient the DMU. And \( \theta^* \) describes the input reduction rate while \( \eta^* \) expresses the output expansion rate. The slacks of the output-oriented model are defined by Formulation (3.9):

\[ t_i^- = x_{i0} - \sum_{i=1}^{m} x_{ij} \mu_j \hspace{0.5cm} (i = 1, \ldots, m) \] \hspace{1cm} (3.9)
\[ t_r^+ = \sum_{r=1}^{s} y_{rj} \mu_j - \eta^* y_{ro} \hspace{0.5cm} (r = 1, \ldots, s) \]
These values are also related to the input-oriented CCR model, \( t^{--} = \frac{s^{-}}{\theta^*} \) and \( t^{++} = \frac{s^{++}}{\theta^*} \).

From all these relationships it can be concluded that an input-oriented CCR model will be efficient for any DMU if and only if it is also efficient when the output-oriented CCR model is applied to evaluate the performance. Therefore, a DMU is fully efficient if and only if \( \eta^* = 1 \) and all optimal slacks are zero.

The CCR projection for inefficient DMUs in the output-oriented form is:

\[
\hat{x}_o = x_o - t^{--} \\
\hat{y}_o = \eta^* y_o + t^{++}
\]  

(3.10)

3.3. **Variable Return to Scale (VRS) Model**

Banker, Charnes and Cooper modified the CCR model for variable return-to-scale (referred to as the BCC model). The frontiers of BCC model consist of piecewise linear and concave segments which present increasing returns-to-scale, followed by constant returns-to-scale, then decreasing returns-to-scale (Banker et al, 1984) as indicated in Figure 3-1.
BCC model (like CRR model) is radial (change proportionally) and can be used as “input oriented” and “output oriented” model. The BCC model contains more efficient scores than CCR, and a CCR efficient DMU is one of the BCC efficient DMUs (Cooper et al., 2007).

### 3.3.1. Input Orientation

The primal formulation for the input-oriented BCC model is described in below:

Maximize $z = \sum_{r=1}^{s} u_{r} y_{rj} - u_0$ \hspace{1cm} (3.11)

Subject to $\sum_{i=1}^{m} v_i x_{io} = 1$

$\sum_{r=1}^{s} u_{r} y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \leq 0 \hspace{1cm} (j = 1, ..., n)$

$v_1, v_2, ..., v_m \geq 0$

$u_1, u_2, ..., u_s \geq 0$

$u_0$ free in sign
The dual form of this linear program is expressed as:

\[\begin{align*}
\text{Minimize} & \quad \theta_B \\
\text{Subject to} & \quad \theta_B x_{io} \geq \sum_{j=1}^{n} x_{ij} \lambda_j \quad (i = 1, \ldots, m) \\
& \quad y_{ro} \leq \sum_{j=1}^{n} y_{rj} \lambda_j \quad (r = 1, \ldots, s) \\
& \quad \sum_{j=1}^{n} \lambda_j = 1 \\
& \quad \lambda_j \geq 0
\end{align*}\] (3.12)

From Formulations (3.11) and (3.12) it is clear that a difference between the CCR and BCC models are in the free variable \(u_0\) in the multiplier form and \(\sum_{j=1}^{n} \lambda_j = 1\) in the envelopment form.

In the phase II, slacks can be included as shown in Formulation (3.13):

\[\begin{align*}
s_i^- &= \theta_B^* x_{io} - \sum_{i=1}^{m} x_{ij} \lambda_j \quad (i = 1, \ldots, m) \\
s_r^+ &= \sum_{r=1}^{s} y_{rj} \lambda_j - y_{ro} \quad (r = 1, \ldots, s)
\end{align*}\] (3.13)

The DMU\(_o\) is BCC-efficient if and only if \(\theta_B^* = 1\) and \(s'^- = s'^+ = 0\).

The BCC projection for inefficient DMUs in the input-oriented form is (Cooper et al., 2007):

\[\begin{align*}
\hat{x}_o &= \theta_B^* x_o - s'^- \\
\hat{y}_o &= y_o + s'^+
\end{align*}\] (3.14)
3.3.2. Output Orientation

The primal form of the output-oriented BCC model is indicated as follows in Formulation (3.15):

\[
\text{Minimize } z = \sum_{i=1}^{m} v_i x_i o - v_0 \\
\text{Subject to } \sum_{r=1}^{s} u_r y_{r o} = 1 \\
\sum_{i=1}^{m} v_i x_{i j} - \sum_{r=1}^{s} u_r y_{r j} - v_0 \geq 0 \quad (j = 1, \ldots, n) \\
p_1, p_2, \ldots, p_m \geq 0 \\
q_1, q_2, \ldots, q_s \geq 0 \\
v_0 \text{ free in sign}
\]

The dual form of this model is presented in Formulation (3.16):

\[
\text{Maximize } \eta_B \\
\text{Subject to } x_{i o} \geq \sum_{j=1}^{n} x_{i j} \lambda_j \quad (i = 1, \ldots, m) \\
\eta_B y_{r o} \leq \sum_{j=1}^{n} y_{r j} \lambda_j \quad (r = 1, \ldots, s) \\
\sum_{j=1}^{n} \lambda_j = 1 \\
\lambda_j \geq 0
\]
From the optimal solution of $\eta_B$ slacks can be accounted for as mentioned in Formulation (3.17):

\[
t^-_i = x_{io} - \sum_{j=1}^{m} x_{ij} \lambda_j \quad (i = 1, ..., m) \tag{3.17}
\]

\[
t^+_r = \sum_{r=1}^{s} y_{rj} \lambda_j - \eta_B y_{ro} \quad (r = 1, ..., s)
\]

The DMU is BCC-efficient if and only if $\eta_B = 1$ and $t^- = t^+ = 0$. The BCC projection for inefficient DMUs in the output-oriented form is (Cooper et al., 2007):

\[
\hat{x}_o = x_o - t^-
\]

\[
\hat{y}_o = \eta^* B y_o + t^+
\]

### 3.4. Additive Model

Additive model, unlike CCR and BCC models which require distinction between input-oriented and output-oriented models, combines both orientations in a single model and captures all the scores of inefficiency (Charnes et al., 1985).

The envelopment form of this model is indicated in Formulation (3.19):
Maximize \[ z = \sum_{i=1}^{m} s_i^- + \sum_{r=1}^{s} s_r^+ \] \hspace{1cm} (3.19)

Subject to \[ s_i^- + \sum_{j=1}^{n} x_{ij}\lambda_j = x_{i0} \hspace{1cm} (i = 1, \ldots, m) \]
\[ \sum_{j=1}^{n} y_{rj}\lambda_j - s_r^+ = y_{ro} \hspace{1cm} (r = 1, \ldots, s) \]
\[ \sum_{j=1}^{n} \lambda_j = 1 \]
\[ \lambda_j, s_i^-, s_r^+ \geq 0 \]

A DMU is additive-efficient if and only if the DMU is \( s^- = s^+ = 0 \) and there is no constraint for \( z \). Additive projection for inefficient DMUs is shown in Formulation (3.20):
\[ \hat{x}_o = x_o - s^- \] \hspace{1cm} (3.20)
\[ \hat{y}_o = y_o + s^+ \]

Although the additive model captures all the scores of inefficiency compared to the radial models (CCR and BCC), this model is not unit invariant which means it is not independent of the units of measurement used (Cooper et al., 2007).

3.5. Slack-Based Measure of Efficiency Model

The slack-based measure of efficiency (SBM) is an extension of the Additive model which modifies the objective function of the additive model in order to create a unit invariant model. The SBM model is the non-radial model that considers all inefficiencies in the scores and can only handle semi-positive data. Also, although the SBM model is unit invariant, it is not translation
invariant which means that if the original input and/or output data values are translated, the efficient frontier and the position of the DMUs relative to the efficient frontier will be altered (Cooper et al., 2007).

The fractional program for the SBM model is presented in Formulation (3.21):

\[
\begin{align*}
\text{Minimize} \quad & \rho = \frac{1 - \frac{1}{m} \sum_{i=1}^{m} s_i^- / x_{i0}}{1 + \frac{1}{s} \sum_{r=1}^{s} s_r^+ / y_{ro}} \\
\text{Subject to} \quad & s_i^- + \sum_{j=1}^{n} x_{ij} \lambda_j = x_{i0} \quad (i = 1, \ldots, m) \\
& \sum_{j=1}^{n} y_{rj} \lambda_j - s_r^+ = y_{ro} \quad (r = 1, \ldots, s) \\
& \lambda_j, s_i^-, s_r^+ \geq 0
\end{align*}
\]

A DMU is SBM efficient if and only if \( \rho^* = 1 \). To solve the SBM model a positive scalar variable \( t \) can be introduced as shown in Formulation (3.22):

\[
\begin{align*}
\text{Minimize} \quad & \tau = t - \frac{1}{m} \sum_{i=1}^{m} ts_i^- / x_{i0} \\
\text{Subject to} \quad & 1 = t + \frac{1}{s} \sum_{r=1}^{s} ts_r^+ / y_{ro} \\
& s_i^- + \sum_{j=1}^{n} x_{ij} \lambda_j = x_{i0} \quad (i = 1, \ldots, m) \\
& \sum_{j=1}^{n} y_{rj} \lambda_j - s_r^+ = y_{ro} \quad (r = 1, \ldots, s) \\
& \lambda_j, s_i^-, s_r^+ \geq 0, t > 0
\end{align*}
\]

Then Formulation (3.22) can be linearized by defining \( S^- = ts^-, S^+ = ts^+ \), and \( \Lambda = t \lambda \) as shown in Formulation (3.23):
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Minimize \( \tau = t - \frac{1}{m} \sum_{i=1}^{m} S_i^- / x_{i0} \) \hspace{1cm} (3.23)

Subject to \( 1 = t + \frac{1}{s} \sum_{r=1}^{s} S_r^+ / y_{ro} \)

\[ s_i^- + \sum_{j=1}^{n} x_{ij} \Lambda_j = tx_{i0} \quad (i = 1, \ldots, m) \]

\[ \sum_{j=1}^{n} y_{rj} \Lambda_j - s_r^+ = ty_{ro} \quad (r = 1, \ldots, s) \]

\( \Lambda_j, S_i^-, S_r^+ \geq 0, t > 0 \)

The optimal solution of SBM is defined by \( p^* = \tau^*, \lambda^* = \Lambda^*/t^*, s^-^* = S^-^*/t^*, \) and \( s^+^* = S^+^*/t^* \). The optimal SBM \( p^* \) is not greater than the optimal CCR \( \theta^* \) because the SBM accounts for all inefficiencies whereas \( \theta^* \) accounts only for “purely technical” inefficiencies which can be eliminated without worsening any other inputs or outputs. SBM projection for inefficient DMUs is the same as the additive projection in Formulation (3.20) (Cooper et al., 2007).

In order to provide a more realistic description of the DMUs under consideration and to increase model accuracy, several extensions and modifications to the aforementioned DEA models are introduced in the literature. Some of these extensions, namely non-discretionary variables, categorical DMUs, assurance region model and trade-off approach which are used in this research, are explained in detail in the following sections.

3.6. Non-Discretionary Variables

In order to precisely represent a DMU’s production process, all variables should be considered in the model. However, sometimes there are one or more variables from the pertinent
variables that are not controllable by management and should be considered as non-discretionary variables. The non-discretionary variables should be removed from the objective function of the linear program but included in the constraints to ensure their presence and their values remain constant while the discretionary variables are optimized (Banker and Morey, 1986).

For additional information about the inclusion and treatment of non-discretionary variables, the reader is referred to Cooper et al. (2007).

3.7. Categorical DMUs

There are other managerial situations over which managers of particular organizations do not have complete control. For instance, in evaluating pension funds’ performance, it is necessary to consider the fund’s status (fully funded/underfunded). A hierarchical category is suitable for handling this situation. As for active pension funds examples, the fund’s status can be classified as two categories one for fully funded active plans and one for underfunded active plans. Then, a DEA model can evaluate the pension funds’ performance while taking into account their particular fund’s status and use this information to evaluate pension funds in the higher (fully funded) category.

There are two kinds of categorical variables, controllable and uncontrollable, which must be dealt with using different approaches. Controllable categorical variables refer to those which are selected by managers. A linear programming algorithm is implemented which allows the DEA model to produce a unique frontier for each hierarchical level providing the ability to assess each DMU against different frontiers. Also, the recommended reference set is restricted to DMUs which
exist within the same hierarchical level. A similar linear programming algorithm can be used for uncontrollable categorical variables to ensure that inefficient DMUs are only compared to efficient DMUs operating in the same or worse environments. Similar to the controllable categorical variable, separate frontiers would be produced and the reference set of each DMU would be restricted to those within its hierarchical level. It should be noted that these approaches should be used for non-comparable categories. For additional information and insight into categorical variables the reader is encouraged to consult Cooper et al. (2007).

3.8. Assurance Region Model

DEA does not require a priori knowledge about the inputs and outputs and the weights are chosen to maximise efficiency. However, sometimes the valuable information about how the factors of production used by the DMUs behave or the opinions of the relative worth of inputs or outputs should be taken into consideration (Dyson and Thanassoulis, 1988). In these cases, the additional information is imposed as a multiplier restriction which can result in more accurate efficiency estimation as well as more realistic depictions of relative efficiency and best performers. The Assurance Region (AR) model imposes constraints on the relative magnitude of the weights for inputs and outputs (Thanassoulis, 2001). These multiplier restrictions are based on managerial perspectives and expert’s opinion. These constraints are shown mathematically as:

\[ L_{1,2} \leq \frac{v_2}{v_1} \leq U_{1,2} \]  

(3.24)

Where \( L_{1,2} \) and \( U_{1,2} \) are lower and upper bounds respectively.
3.9. Trade-Off Approach

Extending to multiplier restrictions based on managerial judgements, Podinovsky (2004) suggests that “technology thinking” could be used in the construction of weight restrictions as well. The dual terms in the constraints of the envelopment model could be taken as production trade-offs that represent feasible simultaneous changes to the inputs and/or outputs of the technology. In this method, the radial target of any inefficient unit is technologically realistic and thus the efficiency measures retain their meaning of extreme radial improvement factors. The technological restrictions are extracted from data. There are two ways to incorporate production trade-offs into DEA models. One way is to use the envelopment models where the trade-offs can be incorporated as additional terms modifying the models. The second way is using weight restrictions in the multiplier models.

In the general form, for a group of n DMUs \((J = 1, 2, \ldots, n)\), input vector \(X_j \in \mathbb{R}^m\) and output vector \(Y_j \in \mathbb{R}^s\) represent each DMU \(j \in J\). The inputs and outputs are assumed nonnegative, and at least one input and one output of each DMU is strictly positive. Also, it is assumed that every input \(i\), is strictly positive for at least one DMU \(k \in J\), and every output \(r\) is strictly positive for at least one DMU \(j \in J\). Consider \(\tilde{X}\) to be the \(m \times n\) matrix with the columns \(X_j\) and \(\tilde{Y}\) the \(s \times n\) matrix with the columns \(Y_j\), \(j \in J\). If there are \(K\) technological values specifying production trade-offs, the trade-offs are represented in the following form (Podinovsky, 2004):

\[
(P_t, Q_t)
\]  

(3.25)

Where \(t = 1, 2, \ldots, K\). The vectors \(P_t \in \mathbb{R}^m\) and \(Q_t \in \mathbb{R}^s\) modify the inputs and outputs of production units respectively.
For the trade-off for the envelopment VRS model, the output oriented efficiency of $DMU_o$ is the optimal value of $\theta$ in the following program:

$$\text{Maximize } \theta \quad (3.26)$$

Subject to $\theta y_o \leq \sum_{t=1}^{k} \pi_t Q + \bar{y} \lambda$

$x_o \geq \sum_{t=1}^{k} \pi_t P_t + \bar{x} \lambda$

$\sum_{i=1}^{n} \lambda_i = 1$

$\lambda, \pi \geq 0$

The trade-off approach can be formulated to the construction of weight restrictions. To do so, consider the output-oriented envelopment model (Formulation 3.26) which incorporates trade-offs as introduced in Formulation (3.25). Its dual form is shown in the following multiplier program in Formulation (3.27):

$$\text{Minimize } v^T x_o - v_o \quad (3.27)$$

Subject to $u^T y_0 = 1$

$u^T \bar{y} - v^T \bar{x} \leq 0$

$u, v \geq 0$

$v_o \text{ sign free}$

With the additional weight restrictions:

$$u^T Q_t - v^T P_t \leq 0, \quad t=1,2,\ldots,K \quad (3.28)$$
This dual relationship signifies that the incorporation of trade-offs (3.25) into the envelopment model (3.26) is equivalent to the incorporation of weight restrictions (3.28) in the multiplier model (3.27) (Podinovski, 2004).


There are some advantages and disadvantages of incorporating value judgments’ methods (AR model and Trade-off) in DEA. The use of preference information local to some DMU rather than global may be easier for some managers. Also, the explicit modification of the production possibility set allows a more realistic set of targets for inefficient DMUs. However, the value judgments’ methods are demanding of manager’s geographical culture and current market time. For detailed information the reader is referred to Thanassoulis (2001), Førsund (2015) and Zhu (2015).

3.10. Comparing DEA with Other Frontier Analysis Techniques

In recent years, academic research on the performance of financial institutions has increasingly concentrated on frontier efficiency analysis. Frontier efficiency analysis techniques allow managers to identify best practices in complex operational environments and can be categorized in two classes: parametric and nonparametric methods. The main differences between these two methods are the functional form of the efficient frontier, the assumption of random error and inefficiency. Generally, nonparametric techniques cannot consider the potential errors and all deviations from the efficient frontier are presumed to be inefficiency (Berger and Humphrey,
There are three main parametric techniques, namely Stochastic Frontier Analysis (SFA), Distribution Free Approach (DFA) and Thick Frontier Approach (TFA). The main nonparametric method is Data Envelopment Analysis (DEA) which is explained in this chapter.

Stochastic Frontier Analysis (SFA) is a parametric frontier technique that specifies a priori restrictive assumptions of the best practice functional form and can handle either one input and multiple outputs or one output and multiple inputs. In SFA, the random errors are assumed to follow a symmetric distribution (usually the standard normal distribution) while inefficiencies follow an asymmetric distribution (usually the half-normal distribution) that can help to separate the evaluation of random errors and inefficiencies (Berger and Humphrey, 1997).

Distribution Free Approach (DFA) is similar to SFA but considers inefficiency and random error in a different way. This method assumes inefficiency is constant over time while the random error averages out to zero. However, DFA is criticized because it cannot capture the efficiency changes over time (Berger and Humphrey, 1997).

Thick Frontier Approach (TFA) assumes that deviations between the highest and lowest performance quartiles of firms represent only inefficiencies and the deviations within the highest and lowest represent random error. Generally, TFA indicates the overall efficiency and does not provide efficiencies for individual firms (Bauer et al., 1998).

Data Envelopment Analysis (DEA), as introduced in this chapter, is a nonparametric linear programming technique which establishes a best practice group of units and determines the inefficient units as well as targets for these units to improve their efficiencies. The characteristics of DEA are presented in Section 3.11.
3.11. Strengths and Limitations of DEA

DEA is a powerful technique which offers several advantages in comparison with other common performance measurements. DEA is able to handle multiple inputs and multiple outputs simultaneously. Also, it does not require any specific knowledge of relationships between inputs and outputs and does not require a priori specification of weights since DEA calculates the optimal weight for each DMU to maximize its efficiency score. Variables in DEA are unit invariant which means variables with different units can be used in the same model at the same time. Moreover, DEA provides a single performance score that can facilitate the comparison of DMUs in the model. Then the efficient DMUs can be considered as a target for inefficient DMUs. In summary, DEA offers an excellent guidance for fund managers.

Similar to all analysis techniques, DEA has some disadvantages. One of the limitations is that DEA does not account for random error which means all deviations from the frontier are assumed to be due to inefficiency not any error or statistical noise. Moreover, DEA is sample dependent which means the relative efficiency scores are based on the specific sample studied. In addition, DEA requires sufficient observations to allow good separation and discrimination amongst DMUs. A small sample size can reduce the accuracy of results. The rule of thumb is that the number of DMUs should be at least three times the total number of inputs plus outputs which are used in the model. Another similar rule is: \( n \geq \max\{m \times s, 3 \times (m + s)\} \), where \( n \) is the number of DMUs, \( m \) is the number of inputs and \( s \) is the number of outputs (Cooper et al., 2011). Another shortcoming in DEA is that it is sensitive to outlier data which can greatly impact the efficiency scores. Also, DEA is a retrospective performance measurement and cannot be used for future projections.
Chapter 4: DEA Studies in Pension Funds

This chapter presents a literature review about the use of DEA in the pension funds industry.

In 2005, Barrientos and Boussofiane studied the efficiency of pension fund managers in Chile by using DEA for the period of 1982-1999. They considered three inputs: marketing and sales costs, office personnel and executive fees, administration and computing costs. The outputs in their research were total revenues and the number of contributors. Both CCR and BCC models were used and the results indicated that the Chilean pension fund management companies exhibited significant inefficiency. There were changes over time but no continuous trends towards an efficiency improvement. They concluded that at the start of the reform of pension fund regulations in Chile, managers could have produced the same output using 40% of the resources. Then during a decade, sales and marketing costs increased which led to a decline in market efficiency.
Therefore, new regulations were introduced to restrict sales and marketing costs and improve efficiency scores. At the end of this period, according to DEA, managers could have produced the same output with 65% of the resources utilized (Barrientos and Boussofiane, 2005).

In 2006, Barros and Garcia evaluated Portuguese pension funds’ performance from 1994-2003 by using different DEA models such as CCR, BCC, Cross-Efficiency and Super-Efficiency and compared the results. The authors used three outputs: Number of funds managed, Value of the funds managed, Value of pensions paid to beneficiaries, and three inputs: Number of workers, Book value of fixed assets and Contributions received. Three hypotheses were tested. The first hypothesis was: Large pension funds management companies are more efficient than small pension funds management companies. The results indicated that the majority of Portuguese pension funds management companies displayed relatively high managerial skills, being VRS-efficient for the most part. However, there were some inefficient firms which could improve more. Hypothesis 2 assumed: private pension funds management companies are more efficient than public pension funds management companies. The results supported this hypothesis. However, they mentioned that the conclusion for this hypothesis should be interpreted with caution since the sample was very small. The third hypothesis the researchers tested was that institutions involved in mergers and acquisitions during the period were more efficient than those that were not involved in these processes. The results agreed with this hypothesis and small pension funds management companies which did not merge, had less efficient performance and their size acted as a restriction for them. In their research, only 12 pension funds management companies were studied (Barros and Garcia, 2006).

Barros and Garcia worked again on Portuguese pension funds’ performance for the same period of time using Stochastic Frontier Analysis (SFA) in 2007. The authors concluded that some
variables such as personnel fees, management fees and benefit payments as well as the number of participants had major roles on pension funds’ performance and managerial skills had effects on improving this performance (Barros and Garcia, 2007).

In 2010, Garcia analyzed changes in the productivity of Portuguese pension funds management institutions from 1994-2007 by using DEA and the Malmquist index. Input variables in this study were value of pensions paid, number of workers, net assets and contributions received and output variables were the number of funds, value of the funds, profits and number of participants in the funds. In all, 12 companies were investigated and the results indicated that increasing the governance and transparency of the pension funds management companies would increase their efficiency. Also, the investigator concluded that there was room for improvement for those companies that were performing badly (Garcia, 2010).

In 2011, Sathye estimated the production efficiency of pension funds in Australia for the years 2005 to 2009 by using CCR and BCC models. Two inputs were used, namely, operating expenses and contributions received and two outputs were considered, the value of the fund and benefits paid. The results indicated that the efficiency scores of Australia’s pension funds were too low. Also, the researcher carried out regression analysis on the variables and found that fund characteristics such as size and the proportion of funds invested in non-risk opportunities had a positive association while diversification and financial crises had negative association on the efficiency of pension funds (Sathye, 2011).

In 2015, Galagedera and Watson assessed pension funds in Australia by using DEA for year 2012. In the study the funds were classified under four categories: industry, public sector, corporate and retail. The results showed that retail funds were the best performers. However, each
of these categories have their own specific characteristics and it might not be proper to consider all of them in one model (Galagedera and Watson, 2015).

In 2015, Zamuee evaluated Namibian pension funds by using CCR model for years 2010 to 2014. The inputs were retirement funding contributions, administration costs, investment costs and total fund expenses. The outputs were fund credits at the end of 5 years, investment returns and average fund assets. The results showed that the majority of Namibian pension funds were performing with low efficiency score and urgent management intervention was required to improve levels of efficiency (Zamuee, 2015).

In summary, the previous studies on pension funds using DEA predominantly focused on comparing different models instead of having a clear methodology and framework. Moreover, as mentioned before, DEA requires sufficient sample size to allow good separation and discrimination amongst DMUs. Most of these studies had very few DMUs considering the number of inputs and outputs which decreases the accuracy of their results and they should be considered with caution. Also, a major flaw of these studies is that none of them consider any variables to indicate the effect of regulations on pension plans’ performance which restrict and impact managers’ control significantly and differentiate them from other investments that do not have such restrictions. Furthermore, none of them investigated pension plans in North America.
Chapter 5: Mixed Datasets with Partially Deficient Variable Sets Embodied in Mixed Variable DEA (MV-DEA)

The main objective of this chapter is to develop a model using Data Envelopment Analysis to examine the private pension funds’ performance by considering the specific characteristics of such funds in comparison with mutual funds. One of the DEA assumptions is that DMUs have to be from the same “culture”. However, in the real world environment, managers always want to compare their products with similar products with some differences in the same industry. It follows that there does not exist a model that can appropriately consider different environments for various products in the same industry. This chapter introduces a novel DEA model, namely Mixed Variable DEA that provides a meaningful environment where DMUs with different cultural assumptions are examined relative to each other while retaining their own specific characteristics. This chapter provides a model objective, a comprehensive explanation of the methodology, the
expansion of DEA models structure, along with data collection and preparation, experimental models and results. The GAMS\(^1\) scripts associated with this chapter are provided in Appendix A.

The road map for Chapters 5 to 7 is as follows: In Chapter 5, pension funds are compared with mutual funds directly which means both pension funds and mutual funds’ DMUs are added together in one dataset. Therefore, all pension funds and mutual funds are examined relative to each other by using the new MV-DEA model. In Chapter 6, the minimum efficiency scores ($\theta$) for different pension plans are investigated from various aspects. In Chapter 7, pension funds are evaluated by using external information from a pension funds’ expert and mutual funds’ dataset. Therefore, unlike Chapter 5 where both pension funds and mutual funds’ DMUs are combined in one dataset at the same time, the dataset in Chapter 7 has only pension funds’ DMUs and useful information from mutual funds’ dataset is extracted and added to the model which will be further explained in Chapter 7.

5.1. Objective

The objective of Mixed Variable DEA model (MV-DEA) is to provide an environment for different entities with different cultures and rules within the same industry to be evaluated relative to each other. This new model ensures that the specific characteristics of each entity is maintained while being compared to other entities. For instance, in the financial industry, different investment vehicles such as pension funds, mutual funds, hedge funds and etc. can be evaluated relative to

\(^1\) General Algebraic Modelling System (GAMS) is a high level modeling software for mathematical programming and optimization. GAMS is an extremely appropriate and flexible tool to build and solve DEA models compared to other optimizers (Olesen and Petersen, 1996). For more information, there is a tutorial by Richard E. Rosenthal in the GAMS website as well as “GAMS Users’ Guide” book by Toloo and Joshaghani (2010).
each other to see which one performs better while the main characteristics of each type of fund is protected. In summation, the proposed model is able to analyse the efficiency of DMUs with different cultures unlike the traditional DEA models.

5.2. Methodology

In this section, first the main characteristics of pension funds are investigated, calculated and considered in the model. Then, the new DEA model is developed to bridge pension funds and mutual funds.

One of the important issues in managing private pension funds is government regulations which are in place to protect the retirement income of the participants and affect the pension funds’ performance. Pension laws and regulations shape the unique legal investment environment in which pension funds operate. Therefore, from a managerial point of view, it is important to know how to effectively manage the funds by considering the impact of regulations. All regulations for pension funds are available on e-laws.gov.on.ca. In general, regulations can be categorized into two types.

One type of regulation deals with administration of the various types of pension plans. These rules are numerous and change from one situation to another based on age, mortality, conditions on fund transfer to spouse/common-law partner after death, etc. Therefore, the impacts of these rules are reflected in the data since the pension fund managers have to follow them when pursuing their goals.
The second type of regulation deals with the allocation of private pension fund assets. For instance, according to the Financial Services Commission of Ontario in compliance with Federal Investment Regulations, a maximum of 5% of the plan’s assets may be invested directly or indirectly in any Canadian resource property. Similarly, a maximum of 10% of the assets can be invested in or loaned to any one person/associated person (FSCO, 2004). Therefore, the only part of regulations that managers can somehow control is how to allocate their assets more effectively while observing these restrictions. As a result, if the effects of regulations on asset allocation can be quantified, then the managers’ performance for allocating their assets can be assessed. In order to achieve this goal, the standard deviation of returns is calculated for each of the pension plans based on their asset allocation. Also, some of the variables in pension funds are not completely under the control of managers which should be treated properly. Therefore, in order to provide a more realistic analysis of pension funds’ performance, in this study, standard deviation of returns is calculated and added to the model and some of the variables such as contribution amounts and benefit payments which are not under the control of pension fund managers, are considered as non-discretionary variables in the model.

Moreover, the goal of this chapter is to bridge private pension funds to mutual funds. Pension funds differ from mutual funds in some ways including regulations affecting investment strategy, the process of incoming and outgoing funds, tax considerations, reporting requirements and others. One of the DEA assumptions is that DMUs have to be from the same culture in the same industry (Dyson et al., 2001). However, from a managerial point of view, it is valuable to know how effectively pension funds perform in comparison with mutual funds which have

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1. For more information, Financial Services Commission of Ontario (FSCO) website provides detailed explanations of pension plans’ regulations. FSCO acts under the Financial Services Commission of Ontario Act, 1997 which regulates investment institutions as well as pension plans.
different characteristics. As mentioned in the objective, the goal of the MV-DEA model is to evaluate the performance of a group of DMUs with different cultural environments appropriately. Since some of the variables such as contribution amounts and benefit payments are not under the control of pension fund managers, but this is not the case for mutual funds, two different approaches should be used. In order to have pension funds and mutual funds together, a new DEA model should be established to consider non-discretionary variables for pension funds and discretionary variables for mutual funds at the same time. For the purpose of this study, in the new MV-DEA model, when a unit is identified as a pension fund, only the objective and constraints for non-discretionary situation are considered and the other objective and constraints in the model are skipped. When the model arrives at a mutual fund’s DMU, only the objective and constraints for the discretionary situation are considered and the other objective and constraints in the model are skipped. As a result, in this research, in one DEA model for some of the DMUs (pension funds) the non-discretionary VRS model (Non-Dis-VRS) is used while for the other DMUs (mutual funds) the VRS model is run.

Let’s consider DMUs (i=1,2,…,n), inputs (j=1,…,m) and outputs (r=1,…,s). The “D” and “ND” refer to the “Discretionary” and “Non-Discretionary” input and output. The mathematical model for the multiplier form is as follows:
Therefore, in the MV-DEA model, for pension funds’ DMUs the Non-Dis-VRS model (objective 1 and constraints 1 to 6 of the multiplier form) is used while for the mutual funds’ DMUs the VRS model (objective 2 and constraints 7 to 10 of the multiplier form) is used.

The envelopment form of the model is as follows:

Objective 1  \[ \text{Min } z = \sum_{j \in D} v_j x_{jo} + \sum_{j \in ND} v_j x_{jo} - \sum_{r \in ND} u_r y_{ro} - v_o \] (5.1)

Objective 2  \[ \text{Min } z = \sum_{j=1}^m v_j x_{jo} - v_o \]

Constraint 1  \[ \sum_{j \in D} v_j x_{ji} + \sum_{j \in ND} v_j x_{ji} - \sum_{r \in ND} u_r y_{ri} - \sum_{r \in D} u_r y_{ri} - v_o \geq 0 \]

Constraint 2  \[ \sum_{r \in D} u_r y_{ro} = 1 \]

Constraint 3  \[ v_j \geq \varepsilon, \quad j \in D \]

Constraint 4  \[ v_j \geq 0, \quad j \in ND \]

Constraint 5  \[ u_r \geq \varepsilon, \quad r \in D \]

Constraint 6  \[ u_r \geq 0, \quad r \in ND \]

Constraint 7  \[ \sum_{j=1}^m v_j x_{ji} - \sum_{r=1}^s u_r y_{ri} - v_o \geq 0 \]

Constraint 8  \[ \sum_{r=1}^s u_r y_{ro} = 1 \]

Constraint 9  \[ v_j \geq 0, \quad j = 1, \ldots, m \]

Constraint 10  \[ u_r \geq 0, \quad r = 1, \ldots, s \]

Therefore, in the MV-DEA model, for pension funds’ DMUs the Non-Dis-VRS model (objective 1 and constraints 1 to 6 of the multiplier form) is used while for the mutual funds’ DMUs the VRS model (objective 2 and constraints 7 to 10 of the multiplier form) is used.

The envelopment form of the model is as follows:

Objective 3  \[ \text{Max } \varphi + \varepsilon (\sum_{j \in D} s_j + \sum_{r \in D} t_r) \] (5.2)

Objective 4  \[ \text{Max } \varphi + \varepsilon (\sum_{j=1}^m s_j + \sum_{r=1}^s t_r) \]

Constraint 11  \[ \sum_{i=1}^n \lambda_i y_{ri} = t_r + \varphi y_{rio}, \quad r \in D \]

Constraint 12  \[ \sum_{i=1}^n \lambda_i y_{ri} = t_r + y_{rio}, \quad r \in ND \]

Constraint 13  \[ \sum_{i=1}^n \lambda_i x_{ji} = -s_j + x_{jio}, \quad j = 1, \ldots, m \]

Constraint 14  \[ \sum_{i=1}^n \lambda_i = 1 \]

Constraint 15  \[ \sum_{i=1}^n \lambda_i y_{ri} = t_r + \varphi y_{rio}, \quad r = 1, \ldots, s \]
For the envelopment form of the MV-DEA model, for pension funds’ DMUs the Non-Dis-VRS model (objective 3 and constraints 11 to 14) is used while for the mutual funds’ DMUs the VRS model (objective 4 and constraints 13 to 15) is run.

To further clarify, Formulation (5.3) is used for pension funds’ DMUs and Formulation (5.4) is used for mutual funds’ DMUs in the multiplier form of the MV-DEA model.

\[
\text{Min } z = \sum_{j \in D} v_j x_{jo} + \sum_{j \in ND} v_j x_{jo} - \sum_{r \in ND} u_r y_{ro} - v_o \\
\text{Subject to: } \sum_{j \in D} v_j x_{ji} + \sum_{j \in ND} v_j x_{ji} - \sum_{r \in ND} u_r y_{ri} - \sum_{r \in D} u_r y_{ri} - v_o \geq 0 \\
\sum_{r \in D} u_r y_{ro} = 1 \\
v_j \geq \varepsilon, \quad j \in D \\
v_j \geq 0, \quad j \in ND \\
u_r \geq \varepsilon, \quad r \in D \\
u_r \geq 0, \quad r \in ND
\]  

Non-Dis-VRS multiplier form for only pension funds’ DMUs in the data set

\[
\text{Min } z = \sum_{j=1}^m v_j x_{jo} - v_o \\
\text{Subject to: } \sum_{j=1}^m v_j x_{ji} - \sum_{r=1}^s u_r y_{ri} - v_o \geq 0 \\
\sum_{r=1}^s u_r y_{ro} = 1 \\
v_j \geq 0, \quad j = 1, ..., m \\
u_r \geq 0, \quad r = 1, ..., s
\]  

VRS multiplier form for only mutual funds’ DMUs in the data set

Also, in the envelopment form of the MV-DEA model, Formulation (5.5) is used for pension funds’ DMUs and Formulation (5.6) is used for mutual funds’ DMUs in the same dataset.
Moreover, in order to have a realistic insight into the pension funds industry, it is important to consider the funds’ status to see whether plans meet their obligations and are fully funded or they have deficits and are underfunded. However, mutual funds are not compelled to be fully funded. Therefore, the categorical DMUs are also considered for the MV-DEA model to indicate the funds’ status. As a result, fully funded plans are referenced only to their own category while the underfunded plans are referenced to their level as well as the higher level of hierarchy (fully funded plans).

It should be noted that all pension plans for this research are active plans. Therefore, if an active plan is flagged as underfunded it means that the plan will not meet its obligations based on OSFI tiers’ deadlines in the next 5 or 10 years. During these years, if the underfunded active plan’s manager compensates the financial deficits, the plan will move to the fully funded category. If not, after the deadline (5 or 10 years) the plan will be terminated. As a result, an underfunded active plan for the year of study for this research can be placed on the frontier as an efficient plan. However, only a few underfunded active plans become efficient. A detailed applicability of the MV-DEA model is explained in the following sections.
CHAPTER 5: MIXED DATASETS WITH PARTIALLY DEFICIENT VARIABLE SETS EMBODIED IN MIXED VARIABLE DEA (MV-DEA)

5.3. Data

5.3.1. Data Collection

Data collection is a crucial task. Extensive efforts were carried out in order to gather the data for private pension funds from OSFI. Our request was for private pension plans’ financial data which are supervised federally. Extracted data from annual reports submitted to OSFI by pension plan managers were used to select variables for year 2010. Data from all active DB plans, DC plans and the combination of both that were not terminated during the time period, and had 100 plan members or more are studied. Moreover, variables for mutual funds were chosen based on the available data for private pension funds and appropriate treatments like considering tax status were taken into account to provide a meaningful comparison between the two types of funds. The standard deviation of returns for a mutual fund was calculated based on asset allocation in order to guarantee consistency of the evaluation method for both pension funds and mutual funds.

After data preparation which is explained in Section 5.3.2., there are 173 pension plans which contain 90 DB plans (52%), 37 DC plans (21.4%), and 46 Combo plans (26.6%) and 61 Canadian open-ended\(^1\) mutual funds.

5.3.2. Data Preparation

Although OSFI is the best source for pension funds data, extensive work was performed to validate the data. Moreover, standard deviation of returns which is an important variable and is

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\(^1\) Open-ended mutual funds do not have restrictions on the amount of shares they issue. Therefore, the shares can be issued and redeemed at any time. Also, the shares are generally purchased directly from the fund rather than from the existing shareholders. Close-ended mutual funds have a fixed number of shares. New shares cannot be created by managers to meet investors’ demands and unlike open-ended funds, the shares can usually be traded between investors.
explained in Section 5.2., was not provided by OSFI. Therefore, standard deviation of returns for each pension plan and mutual fund was calculated from the available data for asset allocation.

Also, outliers which would have unwanted effects on the results of a DEA model were removed. Therefore, the variables were selected carefully and validated through various techniques such as statistical tests, sensitivity analysis and outlier removal which are explained below.

5.3.2.1. Number of DMUs

DEA requires a sufficient number of DMUs to allow good separation and discrimination amongst them. A small sample size can reduce the accuracy of results. The rule of thumb is that the number of DMUs should be at least three times the total number of inputs plus outputs which are used in the model. Another similar rule is: \( n \geq \max\{m \times s, 3 \times (m + s)\} \), where \( n \) is the number of DMUs, \( m \) is the number of inputs and \( s \) is the number of outputs (Cooper et al., 2011).

As presented in Section 5.4., there are 4 inputs and 2 outputs for DB plans, Combo plans and mutual funds and 3 inputs and 1 output for DC plans. There are 90 DB plans, 37 DC plans, 46 Combo plans and 61 mutual funds. Therefore, the number of pension plans and mutual funds are sufficient for this research.

5.3.2.2. Variable Selection

5.3.2.2.1. Efficiency Contribution Analysis

In this method, an input or output variable is included or excluded from a DEA model in order to determine the influence of each variable on the DEA scores (Smith, 1997). Therefore, first, each variable is removed and the model is re-run with the rest of the variables. Then, the difference in average scores between the original model and the re-run model are compared to determine whether the variable has a significant impact on the DEA scores.
5.3.2.2.2. Correlation Analysis

The correlation coefficient used here which represents the relationship between two variables is known as the Pearson correlation coefficient and can be calculated as shown in Formulation (5.7):

$$\rho_{x,y} = \frac{\text{cov}(x,y)}{\sigma_x \sigma_y} = \frac{E[(x-\mu_x)(y-\mu_y)]}{\sigma_x \sigma_y}$$  (5.7)

Where $x, y$ are the variables being compared

$\text{cov}(x, y)$ is the covariance of $x$ and $y$

$\sigma_x, \sigma_y$ is the standard deviation of $x$ and $y$

$E$ is the expected value

$\mu_x, \mu_y$ is the mean of variable $x$ and $y$

The coefficient value lies between -1 and +1. A coefficient of +1 indicates the two variables are perfectly positively correlated, so as one variable increases, the other increases proportionally. Conversely, a coefficient of -1 represents a perfect negative relationship which means if one variable increases, the other decreases proportionally. A coefficient of zero demonstrates no linear relationship at all and so if one of the variables changes, the other stays the same (Field, 2009).

Correlation analysis is a common method for selecting DEA variables. The correlation coefficient can be calculated for every combination of inputs and outputs and if two inputs or outputs are highly correlated with one another, it suggests that the relationship of each variable is very close when relating it to the other. Therefore, one of the two variables may be enough and one of them can be removed. However, both management’s perspective and the research’s objectives should be considered in this regard.
5.3.2.3. Outlier Detection

5.3.2.3.1. Manual Cleaning

A simple and necessary approach for removing outliers is checking the data manually. Although this technique is very time consuming, it is essential to examine if there are any outliers in the dataset (for instance, the investment expense for the pension fund is $2). Manual data cleaning together with other methods leads to a more precise and valuable dataset. For the purpose of this section, first data was examined manually using the histogram of each variable and the meaningless values were pinpointed. Then, the methods which are explained below were run. The results of these methods showed that indeed the manually pinpointed data were outliers.

5.3.2.3.2. Stripping the Efficient Frontier Approach

Stripping the frontier is an alternative method to determine whether the efficient DMUs on the frontier are, in fact, outliers (Cooper et al., 2011). In order to examine this method, all of the efficient DMUs on the frontier should be removed and the DEA model rerun. The frontier stripping is a good way to distinguish the obvious outliers on the frontier. However, it is better to consider the results of this method alongside the results of other methods.

5.3.2.3.3. Super Efficiency Test

Another approach to sensitivity analysis in DEA is removing DMUs that are referenced by many units as they might be super-efficient outliers that skew the frontier (Simar, 2003). Therefore, the supper-efficiency model was used to determine the impact of DMUs that skew the frontier and affect the DEA scores.
5.3.2.4. Sensitivity Analysis

5.3.2.4.1. Wilcoxon Rank-Sum Test

Wilcoxon Rank-Sum test is a nonparametric statistical test that examines whether the two groups belong to the same population or whether they differ significantly. It can be used when the population is not normally distributed. Since the theoretical distribution of the efficiency score in DEA is usually unknown, the Wilcoxon Rank-Sum test is a suitable method to test the DEA scores which are statistically independent (Cooper et al., 2007). For the purpose of this research, the Wilcoxon Rank-Sum test was run to check whether different pension plans can be examined in one group. The results show that only DB plans and Combo plans can be placed in one group.

According to Cooper et al., if the two independent datasets of efficiency scores are represented by $A = \{a_1, a_2, \ldots, a_m\}$ and $B = \{b_1, b_2, \ldots, b_n\}$, the combination of A and B would form C which contains $m+n$ observations in a new dataset that is arranged in descending order. Those identical values in C receive a mid-rank which means the sum of ranks divided by the number of identical values. If the sum of the ranks from A and B is $S$, then the normalized $S$ would be defined as:

$$T = \frac{S - m(m+n+1)/2}{\sqrt{mn(m+n+1)/12}}$$

(5.8)

$T$ has an approximately standard normal distribution and by using $T$, the null hypothesis that the two groups have the same population at a level of significance of $\alpha$ can be examined. The hypothesis is rejected if $T \leq -T_{\alpha/2}$ or $T \geq T_{\alpha/2}$, where $T_{\alpha/2}$ corresponds to the upper $\alpha/2$ percentile of the standard normal distribution (Cooper et al., 2007).
The null hypothesis of Wilcoxon Rank-Sum Test is that the two groups have the same population. The result of the hypothesis test should be 1 or 0. If h=1, this indicates rejection of the null hypothesis and if h=0, this represents a failure to reject the null hypothesis at a significance level of $\alpha=0.05$. Also, $\rho$-value of the test is a positive scalar from 0 to 1 with both extreme values expressing the complete separation of the distributions and 0.5 demonstrating full overlap.

In this research, the datasets were carefully examined by using all the methods mentioned above.

5.4. Application to Data

5.4.1. Pension Funds

According to the law, all pension plans have to report their annual financial statements to OSFI. Therefore, DB plans, DC plans and Combo plans have to fill in all the information requested in the form OSFI-60. However, for some of the variables such as benefit payments and management fees only DB plans and Combo plans have to enter the information, while this is optional for DC plans. The financial reporting requirements for DC plans are straightforward since there is no future obligation for them to report. As a result, financial reporting for DB and Combo plans is much more complicated than DC plans because the employer must estimate the value of future obligations to its employees. Consequently, the available data for DC plans is less than DB and Combo plans. Therefore, the study has to be carried out in two separate parts. Based on available data, one part focuses on DB plans and Combo plans since they report the same
information. The second part concentrates on DC plans which have a different reporting requirement.

5.4.1.1. DB and Combo Plans

One of the purposes of this research is to develop DEA models to measure the financial success of different pension plans. The variables in this research are selected based on the various methods introduced in Section 5.3.2., literature models in Chapter 4, and experts’ opinion. The inputs for this part are professional fees, investment expenses, standard deviation of returns and contribution amounts while the outputs are net investment income and benefit payments. The variables are measured in dollars.

**Table 5-1: Inputs and Outputs for DB Plans and Combo Plans**

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Outputs</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Investment Expenses</td>
<td>• Net Investment Income</td>
</tr>
<tr>
<td>• Management Fees</td>
<td>• Benefit Payments</td>
</tr>
<tr>
<td>• Contribution Amounts</td>
<td></td>
</tr>
<tr>
<td>• Standard Deviation of Returns</td>
<td></td>
</tr>
</tbody>
</table>

According to the definition of combination plans, managers must pay benefits to the retirees (i.e. the DB plan concept) and they may use pension surplus to fund their plan’s current service costs\(^1\) (i.e. the DC plan concept).

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1. Current service cost is defined as the present value of benefits earned by the employees during the current period which means all current employees get another year’s credit for their service (CFA, 2014).
5.4.1.2. DC Plans

DC plan managers do not need to file management fees and benefit payments with OSFI and these fields are left blank in their OSFI financial report. As a result, the inputs and outputs for DC plans are different from DB and Combo plans. The inputs for DC plans are investment expenses, standard deviation of returns and contribution amounts while the output is net investment income as shown in Table 5-2.

Table 5-2: Inputs and Outputs for DC Plans

<table>
<thead>
<tr>
<th>Inputs</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Investment Expenses</td>
<td>• Net Investment Income</td>
</tr>
<tr>
<td>• Contribution Amounts</td>
<td></td>
</tr>
<tr>
<td>• Standard Deviation of Returns</td>
<td></td>
</tr>
</tbody>
</table>

5.4.2. Mutual Funds

A mutual fund is an investment vehicle operated by an investment company that collects contributions from investors and invests the money in a variety of securities. A professional manages the fund and follows a particular investment style.

The advantages of mutual funds are (CSI, 2013):

- Low cost professional management: Usually small and middle class investors buy mutual funds because they do not have enough time and knowledge to monitor their portfolio of securities. This is an inexpensive way to access professional management to analyze the financial markets for the purpose of choosing those securities that best match a fund’s investment objectives.
• Diversification: A typical large fund might have a portfolio of 60 to 100 different securities in 15 to 20 investment vehicles. For individual investors, acquiring such a portfolio of securities is likely not feasible.

• Variety of funds and transferability: There are a wide range of mutual funds and mutual fund families which permit investors to transfer between two or more different funds under the same management with little or no additional fee.

• Liquidity: Open-ended mutual fund investors have a continuing right to redeem shares for cash at net asset value. In some cases, there are trailer fees. Close-ended mutual fund investors may find it difficult to buy or sell some close-ended funds if they are not listed on an exchange or they have a low volume of trading activity.

• Loan collateral: Fund shares are usually accepted as security for a bank loan.

• Ease of estate planning: Shares in a mutual fund continue to be professionally managed during the probate period until estate assets are distributed.

The disadvantages of mutual funds are (CSI, 2013):

• Cost: Historically, the cost of buying individual stocks or bonds from a broker was less than purchasing a mutual fund which charged a front-end load or sale commission and a management fee. However, competition in the market has reduced both load and management fees and investors are offered more variety of investment options.

• Unsuitable as a short term investment: Most mutual funds emphasize long-term investment and are unsuitable for investors looking for short-term performance since sales charges are often subtracted from an investor’s contributions.

1. Probate is the process of validating an individual’s will prior to distribution of estate assets. The term estate refers to all the assets owned by an individual at the time of death.
Professional investment management is not flawless: Mutual fund shares, like other securities, can suffer in down markets subject to market swings (systematic risk). Volatility in the market is extremely difficult to predict and is not controllable by the fund manager.

• Tax complication: Buying and selling by fund managers cause a series of taxable events that may not be suitable for individual investors.

DEA is considered as one the most useful techniques for managers to measure the efficiency of mutual funds. For instance, mutual funds’ performances have been assessed widely using DEA in Murthi et al. (1997), McMullan and Strong (1998), Morey and Morey (1999), Choi and Murthi (2001), Wilkens and Zhu (2001), Basso and Funari (2005), Malhotra et al. (2007), and Premachandra et al. (2012).

In this research, the inputs and outputs for mutual funds are chosen in the same manner as pension plans. The inputs are investment expenses, management fees, contribution amounts (amounts paid by mutual funds’ investors) and standard deviation of returns (which is calculated based on asset allocation in order to guarantee consistency of the evaluation method for both pension funds and mutual funds). The outputs are net investment income (after tax) and benefit payments (redemption plus dividends in mutual funds).

5.4.3. Comparing Pension Funds and Mutual Funds

To have a meaningful comparison between private pension funds and mutual funds the differences should be illustrated. The differences are represented bellow based on an interview with a pension fund and mutual fund manager as well as related literature:
• Purpose: As explained through this research, the objectives of mutual funds and pension funds are different, though both strive to maximize returns.

• Regulations: Pension funds operate under strict provincial and federal laws governing their investments. Interestingly, mutual funds generally have more regulations on their investment approaches than pension funds but have more flexibility (Hussey, 2015). In the US, mutual funds and their advisers operate under both the Investment Company Act of 1940 and the Investment Advisers Act of 1940. However, pension plans are subject only to the Investment Advisers Act of 1940 (ICI, 2006). In Canada, mutual funds advisers are subject to the Securities Act and the Commodities Futures Act (OSC, 2016). A detailed explanation for regulations on Canadian pension plans was provided in Section 2.8.

• Tax Status: One of the important characteristics of pension funds is that earnings are exempt from taxation as long as the money stays in the fund. Earnings in mutual funds, on the other hand are taxed annually. Tax-deferred investments are taxed when they are withdrawn (ICI, 2006) and (Hussey, 2015).

• Management Quality: To some extent, portfolio management is similar for mutual funds and pension funds since managers of both engage in research, buy and sell securities and allocate their assets based on their research. However, mutual fund managers have to comply with requirements on portfolio diversification, adhere to limits on portfolio holdings of illiquid assets, have restrictions on investments which are intended to reduce risks or limit conflicts of interest. To support purchase and redeem of shares in mutual funds on a daily basis, managers may carefully time
purchases and sales of securities. These requirements make the management more difficult and more costly for mutual funds (ICI, 2006).

- Investment Fees: The fees for Canadian mutual funds are higher than pension funds compared to US mutual funds and pension funds. This is due to the fact that the Canadian mutual fund market is less competitive, has a smaller selection of funds and more government regulations (Hussey, 2015).

- Inflow and Outflow of Funds: One of the fundamental differences between mutual funds and pension funds is the nature of inflow and outflow of funds. For instance, in an open-ended mutual fund, investors can buy more shares or sell them at will. However, there may be trailer fees to pay (CSI, 2013). In pension funds, members (employees and their employers) are obliged to pay into their pension fund and do not have access to their contributions until they reach retirement age. There are a few exceptions. For instance, in times of serious financial hardship, according to the Financial Services Commission of Ontario (FSCO) website, a member may qualify to access the contributions but as usual any money withdrawn may be immediately taxable and may affect the member’s eligibility for certain government benefits such as social assistance.

- Liquidity: Most mutual funds report daily liquidity while pension funds make some long term investments which are less liquid but have a more predictable cash flow (ICI, 2006) and (Hussey, 2015).

- Predictability: Mutual fund managers cannot predict the amount of incoming funds, but pension fund managers have an approximate estimate of incoming funds (Hussey, 2015).
• Variety and transferability: Mutual funds have more variety than pension funds. For mutual funds moving from one fund to another fund under the same management usually has no transaction fees. For pension funds, under recently changed rules, investors can move between different DB plans at less or no cost. However, transition between DB to DC plans has a large cost for an investor (Hussey, 2015).

• Transparency: Although there are lots of regulations on mutual funds, they are not required to be as transparent as pension funds (CSI, 2013) and (Hussey, 2015).

5.4.4. Model

Based on the goal of the model to maximize benefits, an output oriented model is chosen. Also, the VRS model is used as a base model for the new MV-DEA model since the VRS model is simple to execute and any modification or expansion can be added without introducing unnecessary complexity into the model and the results can be interpreted easily. Furthermore, unlike SBM which does not require specification of orientation, the output oriented model can be considered for VRS which is desirable for the objective of this study. It should be noted that this newly developed methodology can be used in conjunction with any DEA model while considering the objectives of the respective projects.

In order to investigate the MV-DEA model, different situations are considered:

• First, all data for pension funds and mutual funds are examined separately and then together. Different DEA models are used to compare the results of the traditional models with the new MV-DEA model. Therefore, in the first step, the VRS output oriented model is used for both funds considering all variables for all pension funds and mutual funds as discretionary variables. Then, the Non-Dis-VRS output oriented
model is used for all DMUs considering that there are some non-discretionary variables for all pension funds and mutual funds. Finally, the MV-DEA output oriented model is tested considering the Non-Dis-VRS model for pension funds and the VRS model for mutual funds.

- Second, the efficient DMUs from different models for pension funds and mutual funds are extracted. Then, these efficient DMUs are mixed together and the VRS, Non-Dis-VRS and MV-DEA models are tested and the results are analyzed.

5.5. Results and Discussion

5.5.1. Considering All DMUs

The total number of DMUs for each type of pension plan and mutual fund is as follows:

Defined Benefit Plans (DB): 90  Combination Plans (Combo): 46

Defined Contribution Plans (DC): 37  Mutual Funds (MF): 61

The results for the VRS, Non-Dis-VRS and MV-DEA models for all pension funds and mutual funds are represented in Tables 5-3, 5-4 and 5-5.

In Tables 5-3 and 5-4, the funds’ status (fully funded/underfunded) for DB and Combo plans are considered as categorical DMUs in the DEA models. In Table 5-5, since funds’ status is not an issue for DC plans the categorical DMUs are not included.
CHAPTER 5: MIXED DATASETS WITH PARTIALLY DEFICIENT VARIABLE SETS EMBODIED IN MIXED VARIABLE DEA (MV-DEA)

Table 5-3: Considering All DB and MFs for VRS, Non-Dis-VRS and MV-DEA Models

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O_CAT(^1)</th>
<th>Non-Dis-VRS_O_CAT(^2)</th>
<th>MV-DEA_O_CAT(^3)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#DMUs: 90</td>
<td>#DMUs: 90</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 33</td>
<td>#Efficient: 33</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average: 0.60552</td>
<td>Average: 0.57589</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max: 1</td>
<td>Max: 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min: 0.10629</td>
<td>Min: 0.10629</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ave of Lowest Quartile (22 DMUs): 0.17318</td>
<td>Ave of Lowest Quartile (22 DMUs): 0.17141</td>
<td></td>
</tr>
<tr>
<td>DB</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>#DMUs: 151</td>
<td>#DMUs: 151</td>
<td>#DMUs: 151</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 45</td>
<td>#Efficient: 45</td>
<td>#Efficient: 45</td>
</tr>
<tr>
<td></td>
<td>Average: 0.5811</td>
<td>Average: 0.49735</td>
<td>Average: 0.56416</td>
</tr>
<tr>
<td></td>
<td>Max: 1</td>
<td>Max: 1</td>
<td>Max: 1</td>
</tr>
<tr>
<td></td>
<td>Min: 0.07279</td>
<td>Min: 0.000436</td>
<td>Min: 0.07279</td>
</tr>
<tr>
<td></td>
<td>Ave of Lowest Quartile (38 DMUs): 0.16213</td>
<td>Ave of Lowest Quartile (38 DMUs): 0.11309</td>
<td>Ave of Lowest Quartile (38 DMUs): 0.16139</td>
</tr>
<tr>
<td>DB &amp; MF</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. VRS_O_CAT: VRS model for all DMUs (PFs and MFs), Output Oriented, Categorical DMUs (fully funded and underfunded).
2. Non-Dis-VRS_O_CAT: Non-Discretionary VRS model for all DMUs (PFs and MFs), Output Oriented, Categorical DMUs (fully funded and underfunded).
3. MV-DEA_O_CAT: Mixed Variable DEA model with Non-Discretionary VRS model for pension plans and at the same time VRS model for mutual funds (NEW DEA Model), Output Oriented, Categorical DMUs (fully funded and underfunded).
CHAPTER 5: MIXED DATASETS WITH PARTIALLY DEFICIENT VARIABLE SETS EMBODIED IN MIXED VARIABLE DEA (MV-DEA)

Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS & MV-DEA Models for DB & MF

**Figure 5-1**: Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS and MV-DEA Models for DB and MF

Descending Order of Efficiency Scores for VRS, Non-Dis-VRS and MV-DEA Models for DB & MF

**Figure 5-2**: Descending Order of Efficiency Scores for VRS, Non-Dis-VRS and MV-DEA Models for DB and MF
In Table 5-3, the results indicate that in the new MV-DEA model (having Non-Dis-VRS for PFs and VRS for MFs at the same time), the average efficiency scores for the new model is in-between the average efficiency scores for the VRS and the Non-Dis-VRS models. For instance, the average efficiency scores of the VRS model is 0.5811 and for the Non-Dis-VRS model it is 0.49735. In the MV-DEA model, the average efficiency scores is 0.56416.

As shown in Figure 5-1, the VRS model overestimates the efficiency scores for DB plans and the Non-Dis-VRS model underestimates the efficiency scores for MFs. As presented in Figure 5-2, in the descending order of efficiency scores for these three models, the MV-DEA model’s descending line of scores is in-between the VRS and the Non-Dis-VRS models’ descending lines.

Also, when the efficiency scores for pension funds’ DMUs from the Non-Dis-VRS model and mutual funds’ DMUs from the VRS model are compared to the efficiency scores of the DMUs in the MV-DEA model, the percentage change is zero for all DMUs in the DB and MFs’ datasets. Percentage change shows the relationship between the new value (the MV-DEA model’s result) and the old value (results of the VRS model for mutual funds’ DMUs and the Non-Dis-VRS model for pension funds’ DMUs) that can be calculated as \[\left(\frac{new\ X - old\ X}{old\ X}\right) \times 100\]. Percentage change is only one number and can be easily interpreted. Therefore, the result of the percentage change indicates that the MV-DEA model is working properly. In order to visualize this fact, in Figure 5-3, the results for mutual funds’ DMUs in the VRS model is the same as the results in the MV-DEA model. Also, the results for pension funds’ DMUs in the Non-Dis-VRS and the MV-DEA models are the same.
In summary, the results show that the average efficiency scores for the new MV-DEA model increased compared to the Non-Dis-VRS model for all DMUs (both pension funds and mutual funds) and decreased compared to the VRS model for all pension funds and mutual funds (without considering non-discretionary variables) based on Canadian DB pension funds and Canadian mutual funds’ datasets.

In Table 5-4, the results for the VRS, Non-Dis-VRS and MV-DEA models for DB, Combo and MFs are presented.
### Table 5-4: Considering All DB, Combo and MFs for VRS, Non-Dis-VRS and MV-DEA Models

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O_CAT</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>MV-DEA_O_CAT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#DMUs: 136</td>
<td>#DMUs: 136</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 39</td>
<td>#Efficient: 39</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(DB:34, Combo:5)</td>
<td>(DB:34, Combo:5)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Average: 0.53734</td>
<td>Average: 0.50828</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Max: 1</td>
<td>Max: 1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min: 0.06811</td>
<td>Min: 0.06441</td>
<td></td>
</tr>
<tr>
<td>Ave of Lowest Quartile (34 DMUs): 0.15551</td>
<td>Ave of Lowest Quartile (34 DMUs): 0.14821</td>
<td>Ave of Lowest Quartile (49 DMUs): 0.15146</td>
<td>Ave of Lowest Quartile (49 DMUs): 0.14647</td>
</tr>
</tbody>
</table>

|                              | #DMUs: 197                                         | #DMUs: 197                                               | #DMUs: 197                                            |
|                              | #Efficient: 50                                     | #Efficient: 50                                           | #Efficient: 50                                        |
|                              | Average: 0.53913                                   | Average: 0.46733                                         | Average: 0.51854                                      |
|                              | Max: 1                                              | Max: 1                                                    | Max: 1                                                |
|                              | Min: 0.06811                                       | Min: 0.000436                                            | Min: 0.064412                                        |
| Ave of Lowest Quartile (49 DMUs): 0.15146 | Ave of Lowest Quartile (49 DMUs): 0.11343 | Ave of Lowest Quartile (49 DMUs): 0.14647 | Ave of Lowest Quartile (49 DMUs): 0.14647 |
CHAPTER 5: MIXED DATASETS WITH PARTIALLY DEFICIENT VARIABLE SETS EMBODIED IN MIXED VARIABLE DEA (MV-DEA)

Figure 5-4: Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS and MV-DEA Models for DB, Combo and MF

Figure 5-5: Descending Order of Efficiency Scores for VRS, Non-Dis-VRS and MV-DEA Models for DB, Combo and MF
As shown in Table 5-4 and Figures 5-4 and 5-5, the average efficiency scores for the MV-DEA model for DB, Combo and MFs are in-between the average efficiency scores for the VRS and the Non-Dis-VRS models. Also, when the efficiency scores for DB and Combo plans’ DMUs from the Non-Dis-VRS model and mutual funds’ DMUs from the VRS model are compared to the efficiency scores of the DMUs in the MV-DEA model, the percentage change is zero for all DMUs in DB and Combo and MFs’ datasets as presented in Figure 5-3.

**Table 5-5: Considering All DC and MFs for VRS, Non-Dis-VRS and MV-DEA Models**

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O¹</th>
<th>Non-Dis-VRS_O</th>
<th>MV-DEA_O</th>
</tr>
</thead>
</table>
| DC       | #DMUs: 37  
#Efficient: 13  
Average: 0.69569  
Max: 1  
Min: 0.193545  
Ave of Lowest Quartile (9 DMUs): 0.32924 | #DMUs: 37  
#Efficient: 13  
Average: 0.69569  
Max: 1  
Min: 0.193545  
Ave of Lowest Quartile (9 DMUs): 0.32924 | N/A |
| DC & MF  | #DMUs: 98  
#Efficient: 15 (DC: 7, MF: 8)  
Average: 0.4617  
Max: 1  
Min: 0.0008  
Ave of Lowest Quartile (24 DMUs): 0.11519 | #DMUs: 98  
#Efficient: 15 (DC: 7, MF: 8)  
Average: 0.4617  
Max: 1  
Min: 0.0008  
Ave of Lowest Quartile (24 DMUs): 0.11519 | #DMUs: 98  
#Efficient: 15 (DC: 7, MF: 8)  
Average: 0.4617  
Max: 1  
Min: 0.0008  
Ave of Lowest Quartile (24 DMUs): 0.11519 |

¹. DC plans are very similar to mutual funds and the funds’ status (fully funded/underfunded) is not an issue for DC and MFs. Therefore, the categorical DMUs are not considered for DEA models for DC as well as DC and MFs.
In Table 5-5, the same results are obtained for all three DEA models for DC and MF since DC plan managers do not need to file management fees and benefit payments with OSFI and these fields are left blank in their OSFI financial report. Therefore, the inputs and outputs for DC plans are different from DB and Combo plans. As a result, since the model is output oriented and there is no output variable for benefit payment which is a non-discretionary variable, the Non-Dis-VRS model (without having the non-discretionary output) acts similarly to the VRS model. As such, the descending efficiency scores’ lines for these three models for DC and MFs are overlapped on each other.

The reason for having low minimum $\theta$, is that both fully funded and underfunded plans are considered in the models which will be examined and explained in Chapter 6.

5.5.2. Mixing Efficient DMUs

In this section, first, the efficient DMUs from the Non-Dis-VRS model for pension funds and the VRS model for mutual funds are extracted. Then, these efficient DMUs are added together and different DEA models are tested. The results are presented in Tables 5-6, 5-7 and 5-8:

The number of DMUs for each fund after extracting efficient DMUs based on the Non-Dis-VRS for pension funds and the VRS for mutual funds are:

- **DB**: 33 DMUs are efficient out of 90 DB plans
- **Combo**: 20 DMUs are efficient out of 46 Combo plans
- **DC**: 13 DMUs are efficient out of 37 DC plans
- **MF**: 22 DMUs are efficient out of 61 MFs
CHAPTER 5: MIXED DATASETS WITH PARTIALLY DEFICIENT VARIABLE SETS EMBODIED IN MIXED VARIABLE DEA (MV-DEA)

Table 5-6: Mixing Efficient DB and MF DMUs for VRS, Non-Dis-VRS and MV-DEA Models

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O_CAT</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>MV-DEA_O_CAT</th>
</tr>
</thead>
<tbody>
<tr>
<td>DB &amp; MF</td>
<td>#DMUs: 55</td>
<td>#DMUs: 55</td>
<td>#DMUs: 55</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 45</td>
<td>#Efficient: 45</td>
<td>#Efficient: 45</td>
</tr>
<tr>
<td></td>
<td>Average: 0.94084</td>
<td>Average: 0.89111</td>
<td>Average: 0.94084</td>
</tr>
<tr>
<td></td>
<td>Max: 1</td>
<td>Max: 1</td>
<td>Max: 1</td>
</tr>
<tr>
<td></td>
<td>Min: 0.15519</td>
<td>Min: 0.04523</td>
<td>Min: 0.15519</td>
</tr>
<tr>
<td></td>
<td>Ave of Lowest Quartile (14 DMUs): 0.7676</td>
<td>Ave of Lowest Quartile (14 DMUs): 0.5722</td>
<td>Ave of Lowest Quartile (14 DMUs): 0.7676</td>
</tr>
</tbody>
</table>

Figure 5-6: Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS & MV-DEA Models for Efficient DB & MF

Figure 5-6: Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS and MV-DEA Models for Efficient DB and MF
Figure 5-7: Descending Order of Efficiency Scores for VRS, Non-Dis-VRS and MV-DEA Models for Efficient DB and MF

In Table 5-6, Figures 5-6 and 5-7, the results indicate that when the efficient DMUs for pension funds and mutual funds from different models (Non-Dis-VRS and VRS) are extracted and combined together, the efficient pension funds perform better than the efficient mutual funds. Most of the efficient DMUs in the recent tests are from pension funds based on percentage of efficient DMUs from total DMUs for each pension funds and mutual funds.

Also, the results for the VRS and the MV-DEA models are the same since all DB plans become efficient in these two models (VRS is used for DB DMUs in the VRS model and Non-Dis-VRS is used for DB DMUs in the MV-DEA model) and VRS is used in both the VRS and the MV-DEA models for MFs’ DMUs. Therefore, the results are the same and their descending efficiency scores’ lines for the VRS and the MV-DEA models overlap on each other.
## Table 5-7: Mixing Efficient DB, Combo and MF DMUs for VRS, Non-Dis-VRS and MV-DEA Models

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O_CAT</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>MV-DEA_O_CAT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#DMUs: 75</td>
<td>#DMUs: 75</td>
<td>#DMUs: 75</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 50</td>
<td>#Efficient: 50</td>
<td>#Efficient: 50</td>
</tr>
<tr>
<td></td>
<td>(PF: 38, MF: 12)</td>
<td>(PF: 38, MF: 12)</td>
<td>(PF: 38, MF: 12)</td>
</tr>
<tr>
<td>Average: 0.85843</td>
<td>Average: 0.80799</td>
<td>Average: 0.84446</td>
<td></td>
</tr>
<tr>
<td>Max: 1</td>
<td>Max: 1</td>
<td>Max: 1</td>
<td></td>
</tr>
<tr>
<td>Min: 0.11074</td>
<td>Min: 0.04523</td>
<td>Min: 0.11074</td>
<td></td>
</tr>
<tr>
<td>Ave of Lowest Quartile</td>
<td>Ave of Lowest Quartile</td>
<td>Ave of Lowest Quartile</td>
<td></td>
</tr>
<tr>
<td>(19 DMUs): 0.4673</td>
<td>(19 DMUs): 0.3314</td>
<td>(19 DMUs): 0.4319</td>
<td></td>
</tr>
</tbody>
</table>

---

**Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS & MV-DEA Models for Efficient DB & Combo & MF**

**Figure 5-8**: Efficiency Scores’ Comparison amongst VRS, Non-Dis-VRS and MV-DEA Models for Efficient DB, Combo and MF
As presented in Table 5-7, Figures 5-8 and 5-9, the efficient DB and Combo plans perform better than the efficient mutual funds. For efficient DB, Combo and MF, since most of the pension funds’ DMUs are efficient (not all of them) and two different DEA models (VRS and Non-Dis-VRS) are used in the VRS and the MV-DEA models for pension funds’ DMUs, the results for these two models (VRS and MV-DEA) are slightly different.

**Figure 5-9:** Descending Order of Efficiency Scores for VRS, Non-Dis-VRS and MV-DEA Models for Efficient DB, Combo and MF
Table 5-8: Mixing Efficient DC and MF DMUs for VRS, Non-Dis-VRS and MV-DEA Models

<table>
<thead>
<tr>
<th>Plans</th>
<th>VRS_O</th>
<th>Non-Dis-VRS_O</th>
<th>MV-DEA_O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>#DMUs: 35</td>
<td>#DMUs: 35</td>
<td>#DMUs: 35</td>
</tr>
<tr>
<td></td>
<td>#Efficient: 15</td>
<td>#Efficient: 15</td>
<td>#Efficient: 15</td>
</tr>
<tr>
<td></td>
<td>Average: 0.72045</td>
<td>Average: 0.72045</td>
<td>Average: 0.72045</td>
</tr>
<tr>
<td></td>
<td>Max: 1</td>
<td>Max: 1</td>
<td>Max: 1</td>
</tr>
<tr>
<td></td>
<td>Min: 0.03062</td>
<td>Min: 0.03062</td>
<td>Min: 0.03062</td>
</tr>
<tr>
<td></td>
<td>Ave of Lowest Quartile</td>
<td>Ave of Lowest Quartile</td>
<td>Ave of Lowest Quartile</td>
</tr>
<tr>
<td></td>
<td>(9 DMUs): 0.27553</td>
<td>(9 DMUs): 0.27553</td>
<td>(9 DMUs): 0.27553</td>
</tr>
</tbody>
</table>

In Table 5-8, the reason for having the same results for all three DEA models for DC and MF is that for DC plans, it is optional for managers to report information about benefit payments and management fees to OSFI and these variables are left blank in the dataset. As a result, we do not have data for benefit payment which is a non-discretionary variable. Also, since the model is output oriented, the Non-Dis-VRS model (without having the non-discretionary output) acts similar to the VRS model. Moreover, the minimum score for DC and MF is low (0.0306). This DMU is a mutual fund which is an efficient DMU in the mutual funds’ dataset and it is referenced to itself. However, when efficient MFs are combined with efficient DCs, it becomes highly inefficient.

5.6. Conclusion

The work carried out in this chapter of the research is the first study to use DEA to evaluate pension funds’ performance in North America. The differences in regulations for different pension
plans are accounted for in the DEA models. This study has quantified pension funds’ regulations by using the standard deviation of returns and considered the discretionary and non-discretionary nature of the variables under the managers’ operation. Moreover, the funds’ status (fully funded and underfunded) has been considered in the DEA models as categorical DMUs. Also, a new DEA model, MV-DEA, is developed to provide a comparison between different funds and management practices under different cultures in the same industry. In order to validate the new model, both primal and dual formulations for the Non-Dis-VRS and the VRS models are applied to the pension funds and the mutual funds’ datasets and the results are compared with the MV-DEA model. The results show that the average efficiency scores for the new MV-DEA model (having Non-Dis-VRS for pension funds and VRS for mutual funds at the same time) increased compared to the Non-Dis-VRS model for all DMUs (both pension funds and mutual funds) and decreased compared to the VRS model for both fund types (without considering non-discretionary variables) based on Canadian private pension funds and Canadian mutual funds’ datasets.

Further investigation into private pension funds is carried out in the next chapter to examine the reasons for the very low minimum efficiency scores in the results. This would provide a better understanding of the pension funds industry.
Chapter 6: Low Minimum Efficiency Scores in Pension Funds

Industry

In this chapter the reason for very low minimum efficiency scores for pension funds is investigated. A hypothesis is proposed for this chapter and examined from various perspectives. The objective and the detailed explanation of the methodology are presented. The chapter concludes with a discussion of the obtained results.

6.1. Objective

As explained in Section 5.3., the datasets that are used for this research were selected carefully. However, the results provided in Section 5.5. show that some of the minimum efficiency scores are very low. The objective of this chapter is to provide an examination of the pension funds to find the reason for the very low minimum efficiency scores.
The techniques that are explained in the methodology and result sections, show that very low minimum efficiency scores for the pension funds industry are not as unexpected as they might be in studies in other domains.

6.2. Methodology

According to the law, all pension plans need to provide benefits for their retirees. However, only DB and Combo plans’ managers have to provide defined benefit payments for their retirees and the benefit payments for DC plans depend on how successfully DC plans’ managers invest their assets. Therefore, only DB and Combo plans’ managers have to file the information on benefit payments to OSFI in their annual reports. Also, to have a realistic insight into the pension funds industry, it is important to consider the funds’ status to see whether plans meet their obligations or they have deficits and are underfunded. The dataset that is used for this research is for year 2010. According to OSFI in 2011, approximately 10% of pension plans under federal regulations met their obligations while most of the pension plans were underfunded\(^1\). In this research, from 90 DB plans in the dataset, 29 plans are fully funded and the rest of the plans (61 plans) are underfunded. From 46 Combo plans, only 8 plans are fully funded and 38 plans are underfunded. In order to have a comprehensive study of this industry, both fully funded and underfunded plans are considered in the DEA model.

From the studies in pension funds using DEA that were explained in Chapter 4, two of the studies, Sathye (2011) and Zamuee (2015), mentioned the minimum efficiency scores for different

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DEA models for various years to be between 0.00 to 0.08. As evident in these two articles as well as this study, low minimums do occur in the pension funds industry.

Therefore, a hypothesis can be made as follows:

**Hypothesis A:** Since both fully funded and underfunded active pension plans should be considered in the DEA model and most active pension plans are underfunded, very low minimum efficiency scores are often found in pension funds industry.

In order to investigate this hypothesis, three different approaches are carried out:

In the first approach, the efficient DMUs are removed from DB and Combo plans which are obligated to pay defined benefits. The efficient DMUs are investigated to see how many of them are fully funded and how many of them are underfunded while considering that fully funded plans can be only referenced to fully funded plans and underfunded plans can be referenced to either fully funded or underfunded plans. The efficient DMUs are removed from the dataset in several steps until there are no more fully funded plans in the remaining dataset and the average and minimum efficiency scores are examined. As expected, this approach shows that fully funded plans perform better than underfunded plans and the majority of fully funded plans form the efficient frontier. Also, when the number of fully funded plans decrease due to stripping the frontiers, and most of the remaining plans are underfunded, the low minimum $\theta$ increases significantly.

In the second approach, the inefficient DMUs are examined to see how many of them are from underfunded plans. As anticipated, this approach again demonstrates from another perspective that most of underfunded plans are placed at the bottom of the efficiency scores’ ranking.
In the third approach, the DEA model is run for fully funded and underfunded pension plans separately to check the minimum efficiency score in each type of funds’ status. This approach displays that minimum $\theta$ in each category of fully funded and underfunded pension plans is in the typical range that can be found in literature in other areas of DEA studies. In various industries, the typical range for minimum $\theta$ is different. For instance:

- Airlines: 0.1 to 0.3 [(Barros and Peypoch, 2009) and (Wanke and Barros, 2016)]
- Agriculture: 0.2 to 0.4 [(Blancard and Martin, 2013) and (Vlontzos et al., 2014)]
- Banking: 0.4 to 0.6 [(Asmild et al., 2004) and (Chiu et al., 2008)]
- Mutual Funds: 0.1 to 0.3 [(Basso and Funari, 2005) and (Premachandra et al., 2012)]
- Pension Funds: 0.00 to 0.08 [(Sathye, 2011) and (Zamuee, 2015)]

Based on these three approaches, if most of the efficient DMUs are from fully funded plans as well as most of the inefficient DMUs with low efficiency scores are from underfunded plans and the minimum efficiency score for each type of funds’ status is in the typical range, hypothesis A is proven. Therefore, the low minimum efficiency score often found for the pension funds industry is acceptable since both fully funded and underfunded active plans should be included in the model.

### 6.3. Results and Discussion

In Figures 6-1 and 6-2 the efficiency scores’ distributions for pension plans are presented; sorted in descending order of their efficiency scores ($\theta$).
Figure 6-1: Efficiency Scores’ Distribution for DB Plans

As shown in Figure 6-1, approximately 30% of DMUs have $\theta$ between 0.1 to 0.3 and all these DMUs are underfunded plans.

Figure 6-2: Efficiency Scores’ Distribution for DB and Combo Plans
In Figure 6-2, 54 DMUs are placed at the bottom of ranking with $\theta$ between 0.06 to 0.3. Most of these DMUs are from underfunded plans.

The results for the three approaches that were mentioned in the methodology section are presented below.

6.3.1. Stripping Frontiers

6.3.1.1. DB Plans

There are 90 DB plans of which 29 are fully funded and 61 are underfunded:

a) In the first step, the efficient DMUs are extracted from the data. Out of 90 DB plans 33 DMUs are efficient (23 fully funded DMUs and 10 underfunded DMUs). As presented in Table 5-3 in Section 5.5.1., the average efficiency scores and minimum efficiency score for the output oriented Non-Dis-VRS model considering funds’ status for all 90 DB plans are 0.57589 and 0.10629 respectively. After removing the first layer of efficient DMUs, 57 DMUs remain that the average efficiency scores and minimum efficiency scores for the Non-Dis-VRS model with and without considering categorical situation are as follows:

---

1. It should be noted that all pension plans for this research are active plans. Therefore, if an active plan is flagged as underfunded it means that the plan will not meet its obligations based on OSFI tiers’ deadlines in the next 5 or 10 years. As a result, an underfunded active plan for the year of study for this research can be placed on the frontier as an efficient plan. However, only a few underfunded active plans become efficient.

2. Considering funds’ status (fully funded/underfunded) provides a more realistic analysis of the results. By having categorical DMUs, fully funded plans are referenced only to their own category while the underfunded plans are referenced to their level as well as the higher level of hierarchy (fully funded plans). Therefore, the main change between the Non-Dis-VRS model with and without categorical DMUs is in the DMUs’ reference sets. The results of the DEA model with categorical DMUs change slightly and one of the important aspects of pension funds is taken into consideration.
Table 6-1: Removing First Layer of Efficient DB Plans

<table>
<thead>
<tr>
<th>DB Plans</th>
<th>Non-Dis-VRS_O_CAT(^1)</th>
<th>Non-Dis-VRS_O(^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td># DMUs: 57</td>
<td>Average: 0.7747, Min: 0.3051</td>
<td>Average: 0.7671, Min: 0.3051</td>
</tr>
</tbody>
</table>

The results indicate that most of the efficient DMUs are from fully funded pension plans and perform better than other DMUs. As expected, when the efficient DMUs are extracted from the dataset, the efficiency scores of the highly inefficient DMUs (which are from underfunded plans) increase significantly.

b) In the second step, out of 57 DB plans, 25 DMUs are efficient (6 fully funded DMUs and 19 underfunded DMUs). After removing these 25 DMUs from the dataset (32 DMUs remaining), there are no more fully funded DB plans in the dataset.

Table 6-2: Removing Second Layer of Efficient DB Plans

<table>
<thead>
<tr>
<th>DB Plans</th>
<th>Non-Dis-VRS_O_CAT (^3)</th>
<th>Non-Dis-VRS_O</th>
</tr>
</thead>
<tbody>
<tr>
<td># DMUs: 32</td>
<td>N/A (^3)</td>
<td>Average: 0.8433, Min: 0.4137</td>
</tr>
</tbody>
</table>

The examination is stopped here since there are no more fully funded plans in the dataset and the results indicate that fully funded plans perform better than underfunded plans. Most of the fully funded plans are efficient in the first step and the rest become efficient in the second step.

---

1. Output Oriented Non-Dis-VRS model considering categorical DMUs (fully funded and underfunded pension plans)
2. Output Oriented Non-Dis-VRS model (without considering funds’ status)
3. There are no more fully funded plans in the dataset. Therefore, we do not have categorical DMUs in the model.
6.3.1.2. DB and Combo Plans

There are 136 DMUs for DB and Combo plans in which there are 37 fully funded plans and 99 underfunded plans:

a) In the first step for DB and Combo plans, out of 136 DMUs, 39 DMUs are efficient of which 27 DMUs are fully funded plans (23 DB plans and 4 Combo plans) and 12 DMUs are underfunded plans (10 DB plans and 2 Combo plans). As indicated in Table 5-4 in Section 5.5.1., the average efficiency scores and minimum efficiency score for the Non-Dis-VRS_O_CAT model for all 136 DB and Combo pension plans are 0.50828 and 0.06441 respectively. The results are shown in Table 6-3 after removing the first layer of efficient DMUs (39 DMUs) from the dataset (97 DMUs remaining).

<table>
<thead>
<tr>
<th>DB &amp; Combo Plans</th>
<th># DMUs: 97</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>Average: 0.7381</th>
<th>Min: 0.2279</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Non-Dis-VRS_O</td>
<td>Average: 0.7098</td>
<td>Min: 0.2107</td>
</tr>
</tbody>
</table>

b) In the second step, out of 97 DMUs from part a, 39 DMUs are efficient of which 11 DMUs are fully funded plans and 28 DMUs are underfunded plans. The results, after removing these 39 DMUs from the dataset (58 DMUs remaining), are presented in Table 6-4.

<table>
<thead>
<tr>
<th>DB &amp; Combo Plans</th>
<th># DMUs: 58</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>Average: 0.7631</th>
<th>Min: 0.2775</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Non-Dis-VRS_O</td>
<td>Average: 0.7134</td>
<td>Min: 0.2775</td>
</tr>
</tbody>
</table>
c) In the third step, out of 58 DMUs, 26 DMUs are efficient of which 5 DMUs are fully funded and 21 DMUs are underfunded. There are no more fully funded plans in the dataset after this step. The results, after removing these 26 DMUs from the dataset (32 DMUs remaining), are reported in Table 6-5.

<table>
<thead>
<tr>
<th>DB &amp; Combo Plans # DMUs: 32</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>N/A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Dis-VRS_O</td>
<td>Average: 0.9017</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Min: 0.4728</td>
<td></td>
</tr>
</tbody>
</table>

The experimental analysis is stopped here since there are no more fully funded plans in the dataset and the results indicate that all fully funded DB and Combo plans become efficient in three steps.

6.3.2. Stripping DMUs with Low θ

In this section, the aim is to demonstrate how many of the inefficient DMUs with low θ are from underfunded plans and if the test is started from the bottom of the efficiency scores’ ranking, after how many underfunded plans the next inefficient DMU is from fully funded plans.

6.3.2.1. DB Plans

When the DB plans are investigated from the bottom of the efficiency scores’ ranking, the first inefficient fully funded plan is reached after removing 29 inefficient underfunded DMUs. As expected, this is another indication that fully funded pension plans perform better than underfunded plans and the reason for having low θ is because both fully funded and underfunded categories of pension plans are considered in the DEA models.
The results, after removing these 29 DMUs from the dataset (61 DMUs remaining), are reported in Table 6-6.

**Table 6-6: Removing Inefficient Underfunded DB Plans from the Bottom of Efficiency Scores’ Ranking**

<table>
<thead>
<tr>
<th>DB Plans # DMUs: 61</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>Non-Dis-VRS_O</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Average: 0.8006</td>
<td>Average: 0.7893</td>
</tr>
<tr>
<td></td>
<td>Min: 0.3277</td>
<td>Min: 0.2819</td>
</tr>
</tbody>
</table>

In summary, the first inefficient fully funded plan is reached after removing 29 inefficient underfunded DMUs from the bottom of the efficiency scores’ ranking and the minimum efficiency score after removing these 29 DMUs increased significantly.

### 6.3.2.2. DB and Combo Plans

After removing the underfunded DB DMUs and reaching the first inefficient fully funded DB plan from the bottom of efficiency scores’ ranking as mentioned in Section 6.3.2.1., the Non-Dis-VRS models are run for mix of DB (61 plans) and Combo plans (46 plans) with 107 DMUs. There are 21 inefficient DMUs from the bottom of efficiency scores’ ranking which are all from underfunded Combo plans before reaching the first inefficient fully funded pension plan. In this part, the inefficient underfunded DMUs from the bottom of efficiency scores’ ranking are removed from the dataset. The results, after removing these 21 DMUs from the 107 DMUs in the dataset (86 DMUs remaining), are shown in Table 6-7.
Table 6-7: Removing Inefficient Underfunded DB and Combo Plans from the Bottom of Efficiency Scores’ Ranking

<table>
<thead>
<tr>
<th>DB &amp; Combo Plans</th>
<th>Non-Dis-VRS_O_CAT</th>
<th>Non-Dis-VRS_O</th>
</tr>
</thead>
<tbody>
<tr>
<td># DMUs: 86</td>
<td>Average: 0.7419</td>
<td>Average: 0.7318</td>
</tr>
<tr>
<td></td>
<td>Min: 0.3006</td>
<td>Min: 0.2819</td>
</tr>
</tbody>
</table>

In summary, working from the bottom of the efficiency scores’ ranking, the first inefficient fully funded plan is reached after removing 21 inefficient underfunded Combo DMUs. Note that 29 inefficient underfunded DB plans have already been removed.

6.3.3. Investigating Fully Funded and Underfunded Pension Plans Separately

6.3.3.1. DB Plans

In this section, each type of funds’ status is examined separately to investigate and visualize the average efficiency scores and minimum efficiency score for each category of fully funded and underfunded DB plans. The results are presented in Table 6-8.

Table 6-8: Examining Fully Funded and Underfunded DB Plans Separately

<table>
<thead>
<tr>
<th>DB Plans</th>
<th>Fully Funded DB Plans</th>
<th>Underfunded DB Plans</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Dis-VRS_O</td>
<td>#DMUs: 29</td>
<td>#DMUs: 61</td>
</tr>
<tr>
<td></td>
<td>Average: 0.8833</td>
<td>Average: 0.66</td>
</tr>
<tr>
<td></td>
<td>Min: 0.3036</td>
<td>Min: 0.1718</td>
</tr>
</tbody>
</table>

1. Since fully funded and underfunded pension plans are examined separately, the categorical DMUs do not need to be considered.
6.3.3.2. DB and Combo Plans

The results for each category of fully funded and underfunded DB and Combo plans are presented in Table 6-9.

| DB & Combo Non-Dis-VRS_O | Fully Funded DB & Combo Plans #DMUs: 37 | Average: 0.7592 |
| | | Min: 0.1828 |
| | Underfunded DB & Combo Plans #DMUs: 99 | Average: 0.6431 |
| | | Min: 0.1718 |

The results show that when fully funded and underfunded pension plans are tested separately, the average and minimum efficiency scores are higher than when both fully funded and underfunded categories of pension plans are added together.

In summation, all these three approaches indicate that when fully funded and underfunded pension plans are examined together the minimum efficiency score decreases significantly. Therefore, the hypothesis is accepted and since most pension plans are underfunded and both fully funded and underfunded pension plans should be considered together, very low minimum efficiency scores are often found.

6.4. Conclusion

The results from the two stripping approaches, stripping frontiers and stripping DMUs with low $\theta$, indicate that most fully funded plans are efficient and most underfunded plans are inefficient. By extracting the efficient DMUs (fully funded plans), the low $\theta$ of the highly inefficient DMUs (underfunded plans) increased significantly. Also, if the fully funded and
underfunded pension plans are tested separately, the minimum efficiency score increases significantly. In summation, in the pension funds industry, both fully funded and underfunded plans should be considered in the model and as a result, low $\theta$ in the DEA model is often found and acceptable for this industry.
Chapter 7: Improving Pension Funds’ Performance by Considering an Expert’s Opinions and Borrowing Mutual Funds’ Information

This chapter bridges pension funds and mutual funds. The ultimate objective of this chapter is to develop a model using DEA to evaluate the management performance of Canadian private pension funds by using an expert’s opinions and borrowing useful information from the Canadian mutual funds’ dataset. A brief background, explanation about the data as well as methodology for connecting pension funds with mutual funds and the results are provided.

7.1. Objective

A comprehensive review and comparison of pension funds and mutual funds was presented in Chapter 5. In this chapter, in addition to considering variables which were previously mentioned
and embodied the main characteristics of pension funds, managerial judgements are included in order to encompass the complexity of this industry. Furthermore, valuable information is extracted from the mutual funds’ dataset and incorporated into the envelopment form of the DEA model. The objective of this chapter is to evaluate pension funds’ performance using external information from an industry expert as well as mutual funds’ dataset thus providing different type of targets for inefficient pension plans.

### 7.2. Data

In this chapter, data from all DB plans, DC plans, the combination of both DB and DC, and open-ended mutual funds that were not terminated during year 2010 and had 100 plan members or more are studied. There are 173 pension plans which contain 90 DB plans, 37 DC plans, and 46 Combo plans which are regulated federally and 61 open-ended mutual funds. These are the same datasets which were used in Chapters 5 and 6. The procedure for data collection and preparation was presented in Section 5.3. In this chapter, the datasets are used in a different manner from previous chapters.

### 7.3. Methodology

This chapter has three steps. In the first step, the main characteristics of pension funds and the effect of regulations are investigated. Then, because of complexity of the pension funds
industry, the pension fund expert’s opinions are included in the model. In the third step, a few trade-offs are extracted from the mutual funds’ dataset and added to the pension funds’ model.

As explained in Section 5.2., one of the important issues in pension funds industry is government regulations that affect the pension funds’ performance. The only part of regulations that managers can somehow control is how to allocate their assets more effectively while observing the government restrictions. If the effects of regulations on asset allocation can be quantified, then the managers’ performance for allocating their assets can be assessed. Therefore, the standard deviation of returns is calculated for each of the pension plans based on their asset allocation. Also, for some of the variables such as contribution amounts and benefit payments there are some government restrictions. As a result, managers cannot control these variables completely.

Moreover, because of complexity of pension funds industry and improving the model’s discriminatory power managerial insights are incorporated into the model. According to the expert’s judgement with more than 20 years experience in pension funds industry, the weight for the standard deviation of returns to the investment expenses should be between 1 and 5 and the weight for the standard deviation of returns to the management fees is between 1 and 2. Also, the weight for the standard deviation of returns to the contribution amounts should be between 1 and 3. The weight for the net investment income to the benefit payments is between 1 and 3. The questionnaire given to the expert is presented in Appendix B.

1. The trade-off in this chapter is defined as the relationship between two variables from the mutual funds’ dataset. Similar variables between pension funds and mutual funds’ datasets such as standard deviation of returns and investment income are considered. The relationship between two such variables is extracted from the mutual funds’ dataset. Then, the reliability of this regression is tested statistically. If the regression is statistically significant, then it will be used as a trade-off from the mutual funds’ dataset in the pension funds’ model.
Furthermore, another objective of this chapter is to examine whether pension funds could perform better by using information from the mutual funds’ dataset. To this end, based on available data, similar variables are considered for pension funds and mutual funds. The characteristics of each type of fund are reflected in their datasets since fund managers have to follow the rules specified for the investment type to get the best performance. Afterwards, the relationships between the mutual funds’ variables are extracted from the dataset to see how different variables perform relative to each other and then added to pension funds’ model based on the trade-off approach in DEA which was introduced in Section 3.9. The aim of this method is to provide an environment for pension funds to use mutual funds’ information and gauge their performance against the best performers. By using this method in DEA, pension funds operate with their own characteristics while borrowing useful information from the mutual funds’ dataset. Therefore, new target levels for inefficient pension plans are defined. DEA has never been used in this manner in the financial investment industry.

Figure 7-1 and Table 7-1 provide an overview of the theoretical research objective with simple one input and two outputs.
First, the Non-Discretionary VRS (Non-Dis-VRS) model is used for pension funds as the base model and its frontier is shown in green. Second, the pension fund expert’s perspectives are incorporated into the multiplier form of the Non-Dis-VRS model as the Assurance Region (AR) model and its frontier is named Leadership and shown in blue. Third, some trade-offs are extracted from the mutual funds’ dataset and added to the envelopment form of the AR model and its frontier is named Outstanding and shown in red. As represented in Figure 7-1, pension plans (DMUs) A, B, C, D and E operate well and become efficient which can be considered as targets for inefficient units in the Non-Dis-VRS model. By adding expert’s opinions and using AR method to the Non-Dis-VRS model, only DMUs B, C and D operate very well and can be considered as targets even for efficient DMUs in the Non-Dis-VRS model as well as inefficient units. After adding the trade-offs from mutual funds, the discriminatory power of DEA increases further and only DMUs C and D become efficient which can be considered as targets for inefficient units and even efficient ones.
in the Non-Dis-VRS model and the AR model. Therefore, DMUs C and D are efficient in all three Non-Dis-VRS, AR and AR with trade-offs models. These two DMUs are considered to be Outstanding plans. DMU B is efficient in the Non-Dis-VRS and the AR models and inefficient for the AR with trade-offs model. Therefore, DMU B is considered to be a Leading plan. DMUs A and E are efficient only in the Non-Dis-VRS model and are thus Efficient plans. As a result, we have three different target levels and improvement schemes for inefficient units. The first improvement scheme is calculated based on the Basic frontier to remove the DMU’s pure inefficiency. The second improvement scheme is calculated based on the Leadership frontier and the third improvement scheme is calculated based on the Outstanding frontier. For instance, for DMU F there are three improvement schemes. However, for DMU E there are two improvement schemes. In Table 7-1, the classification of pension plans is presented.
Table 7-1: Theoretical Classification of Pension Plans

<table>
<thead>
<tr>
<th></th>
<th>Basic Frontier: Non-Dis-VRS</th>
<th>Leadership Frontier: Non-Dis-VRS+AR</th>
<th>Outstanding Frontier: Non-Dis-VRS+AR+MF’s TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outstanding Plans (DMUs C &amp; D)</td>
<td>Efficient</td>
<td>Efficient</td>
<td>Efficient</td>
</tr>
<tr>
<td>Leading Plans (DMU B)</td>
<td>Efficient</td>
<td>Efficient</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Efficient Plans (DMUs A &amp; E)</td>
<td>Efficient</td>
<td>Inefficient</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Inefficient Plans (one improvement Scheme is needed)</td>
<td>Efficient</td>
<td>Efficient (Its reference set is constructed based on Outstanding plans)</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Inefficient Plans (two improvement schemes are needed)</td>
<td>Efficient</td>
<td>Inefficient (Its first reference set is constructed based on Leading plans)</td>
<td>Inefficient</td>
</tr>
<tr>
<td>Inefficient Plans (all three improvement Schemes are needed)</td>
<td>Inefficient (Its first reference set is constructed based on Efficient plans to remove pure inefficiency)</td>
<td>Inefficient</td>
<td>Inefficient</td>
</tr>
</tbody>
</table>

In this chapter, the Non-Dis-VRS model and the AR model are used to evaluate the relative performance of different private pension plans. The mathematical DEA formulations for Non-Dis-VRS and the AR models are introduced in Chapter 3. Afterwards, the useful information from the mutual funds’ dataset is extracted and added as trade-off to the pension funds’ model. The
mathematics for the trade-off approach is explained in Section 3.9. The experimental results are presented in the next section.

7.4. Results and Discussion

There are 173 pension plans which contain 90 DB plans, 37 DC plans, and 46 Combo plans and 61 open-ended mutual funds. Moreover, as mentioned in Section 5.4.1., available data for DB and Combo plans are different from the available data for DC plans. For DB and Combo plans, investment expenses, management fees, contribution amounts and standard deviation of returns are the inputs and net investment income and benefit payments are the outputs. For DC pension plans, investment expenses, contribution amounts and standard deviation of returns are the inputs and net investment income is the output.

In this research, first the output oriented Non-Dis-VRS model is used to evaluate the pension funds’ performance by considering the effect of regulations on pension funds through calculating standard deviation of returns based on asset allocations and including controllable and uncontrollable variables. Also, the categorical DMUs are considered for the Non-Dis-VRS model to indicate the funds’ status. Therefore, fully funded plans are referenced only to their own category while the underfunded plans are referenced to their own level as well as fully funded plans’ level. Moreover, in order to have a better insight into the pension funds industry, a pension fund manager’s perspectives and opinions are considered to establish the actual decision making parameters for managers. Therefore, the AR model is used to evaluate the pension funds’ performance. According to the expert’s judgement with more than 20 years experience in pension funds industry, the weight for the standard deviation of returns ($v_1$) to the investment expenses...
\((v_2)\) should be between 1 and 5 which means \(1 \leq \frac{v_1}{v_2} \leq 5\) and the weight for the standard deviation of returns \((v_1)\) to the management fees \((v_3)\) is between 1 and 2 \((1 \leq \frac{v_1}{v_3} \leq 2)\). Also, the weight for the standard deviation of returns \((v_1)\) to the contribution amounts \((v_4)\) should be between 1 and 3 which means \(1 \leq \frac{v_1}{v_4} \leq 3\). The weight for the net investment income \((u_1)\) to the benefit payments \((u_2)\) is between 1 and 3 \((1 \leq \frac{u_1}{u_2} \leq 3)\).

For the purpose of this research, the same variables between mutual funds and pension funds such as investment expenses, standard deviation of returns and net investment income for the year 2010 are extracted from the mutual funds’ dataset. Since mutual funds managers should follow the rules and they work in the same financial markets as pension funds, the impacts of these rules are reflected in the data and the relationship between these variables from data can be considered as trade-off constraints in the model. The Durbin Watson test is used to test the statistical relationship between the mutual funds’ variables. If the Durbin Watson result by using SPSS statistical software is between 1.5 and 2.5, the regression between variables is meaningful (Field, 2009). In this study, the Durbin Watson for the regression between net investment income and standard deviation of returns is 1.98 and for net investment income and investment expenses is 1.59. The results show that the regressions between mutual funds’ variables are in the range of 1.5 and 2.5 and therefore can be considered as trade-offs for pension funds’ model. In order to interpret the regressions better and restrict the value between zero and one, the mutual funds’ data is scaled based on dividing the DMUs’ values for each variable by the maximum value of that variable. Therefore, the relationship between net investment income \((y_1)\) and standard deviation of returns \((x_1)\) for mutual funds is equal to \(y_1 = 0.30 \times x_1\) and the relationship between net investment income \((y_1)\) and investment expenses \((x_2)\) for mutual funds is equal to \(y_1 = 0.49 \times x_2\).
The trade-offs from the mutual funds’ dataset are incorporated in the envelopment form. In order to incorporate the trade-offs in the envelopment form into the AR model, the equivalent weight restrictions in the multiplier form should be constructed by modifying the dual form. Therefore, the equivalent weight restrictions for these two trade-offs are $v_1 u_1 \geq 0.3$ and $v_2 u_1 \geq 0.49$. 

The GAMS scripts for the DEA model using weight restrictions and trade-offs are presented in Appendix C.

The multiplier formulation for this approach is as follows:

$$\text{Minimize: } z = \sum_{j \in E} v_j x_{jo} + \sum_{j \in ND} v_j x_{jo} - \sum_{r \in ND} u_r y_{ro} - v_o$$

(7.1)

Subject to: $\sum_{j \in E} v_j x_{ji} + \sum_{j \in ND} v_j x_{ji} - \sum_{r \in ND} u_r y_{ri} - \sum_{r \in D} u_r y_{ri} - v_o \geq 0$

$\sum_{r \in D} u_r y_{ro} = 1$

$v_j \geq \epsilon, \ j \in D$

$v_j \geq 0, \ j \in ND$

$u_r \geq \epsilon, \ r \in D$

$u_r \geq 0, \ r \in ND$

$1 \leq \frac{v_1}{v_2} \leq 5$

$1 \leq \frac{v_1}{v_3} \leq 2$

$1 \leq \frac{v_1}{v_4} \leq 3$

$1 \leq \frac{u_1}{u_2} \leq 3$

$\frac{v_1}{u_1} \geq 0.3$

$\frac{v_2}{u_1} \geq 0.49$
For each target level based on Efficient, Leading and Outstanding plans, the values of the variables for the virtual references should be calculated to find the improvement schemes for inefficient units in each step. The virtual references for the Non-Dis-VRS model are determined by Formulation (7.2) as below:

\[ \text{Maximize: } \varphi \quad (7.2) \]
\[ \text{Subject to: } \sum_{i=1}^{n} \lambda_i y_{ri} \geq \varphi y_{rio} , \quad r \in D \]
\[ \sum_{i=1}^{n} \lambda_i y_{ri} \geq y_{rio} , \quad r \in ND \]
\[ \sum_{i=1}^{n} \lambda_i x_{ji} \leq x_{jio} , \quad j = 1, \ldots, m \]
\[ \lambda_i \geq 0 \]

The virtual references for the AR model in this approach are calculated by Formulation (7.3) as follows:

\[ \text{Maximize: } \varphi \quad (7.3) \]
\[ \text{Subject to: } \sum_{i=1}^{n} \lambda_i y_{ri} + \pi_9 \begin{pmatrix} 1 \\ -3 \\ -1 \\ 1 \end{pmatrix} + \pi_{10} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \geq \varphi y_{rio} , \quad r \in D \]
\[ \sum_{i=1}^{n} \lambda_i y_{ri} + \pi_9 \begin{pmatrix} 1 \\ -3 \\ -1 \\ 1 \end{pmatrix} \geq y_{rio} , \quad r \in ND \]
\[ \sum_{i=1}^{n} \lambda_i x_{ji} + \pi_3 \begin{pmatrix} -1 \\ 5 \\ 0 \\ 0 \end{pmatrix} + \pi_4 \begin{pmatrix} 1 \\ -1 \\ 0 \\ 0 \end{pmatrix} + \pi_5 \begin{pmatrix} -1 \\ 0 \\ 2 \\ 0 \end{pmatrix} + \pi_6 \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} + \pi_7 \begin{pmatrix} -1 \\ 0 \\ 0 \\ 3 \end{pmatrix} + \pi_8 \begin{pmatrix} 1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \leq x_{jio} \]
\[ \lambda_i, \pi_i \geq 0 \]
As represented in Formulation (7.4), the virtual references for the AR with mutual funds’ trade-offs model are evaluated as follows:

\[
\begin{align*}
\text{Maximize: } & \varphi \\
\text{Subject to: } & \sum_{i=1}^{n} \lambda_i y_{ri} + \pi_1 \begin{pmatrix} 0.49 \\ 0 \end{pmatrix} + \pi_2 \begin{pmatrix} 0.30 \\ 0 \end{pmatrix} + \pi_9 \begin{pmatrix} 1 \\ -3 \end{pmatrix} + \pi_{10} \begin{pmatrix} -1 \\ 1 \end{pmatrix} \geq \varphi y_{rio} \quad , \quad r \in D \\
& \sum_{i=1}^{n} \lambda_i y_{ri} + \pi_1 \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \pi_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \pi_3 \begin{pmatrix} -1 \\ 0 \end{pmatrix} + \pi_4 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \pi_5 \begin{pmatrix} 0 \\ 2 \end{pmatrix} + \pi_6 \begin{pmatrix} 1 \\ -1 \end{pmatrix} \geq y_{rio} \quad , \quad r \in ND \\
& \pi_7 \begin{pmatrix} -1 \\ 0 \\ 3 \end{pmatrix} + \pi_8 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \leq x_{jio} \\
& \lambda_i, \pi_i \geq 0
\end{align*}
\]

The results for DB and Combo and DC plans after adding the AR and trade-off constraints to the Non-Dis-VRS model are provided in Table 7-2.
Table 7-2: Results for DB, Combo and DC Plans

<table>
<thead>
<tr>
<th></th>
<th>Non-Dis-VRS¹</th>
<th>Non-Dis-VRS +AR</th>
<th>Non-Dis-VRS +AR+MF’s TO</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>DB Plans</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of DMUs</td>
<td>90</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>No. of Efficient DMUs</td>
<td>33</td>
<td>9</td>
<td>8</td>
</tr>
<tr>
<td>Average</td>
<td>0.5759</td>
<td>0.3028</td>
<td>0.3018</td>
</tr>
<tr>
<td>Maximum</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.1063</td>
<td>0.024</td>
<td>0.024</td>
</tr>
<tr>
<td><strong>DB and Combo Plans</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of DMUs</td>
<td>136</td>
<td>136</td>
<td>136</td>
</tr>
<tr>
<td>No. of Efficient DMUs</td>
<td>39</td>
<td>10</td>
<td>9</td>
</tr>
<tr>
<td>Average</td>
<td>0.5083</td>
<td>0.2570</td>
<td>0.2558</td>
</tr>
<tr>
<td>Maximum</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.0644</td>
<td>0.024</td>
<td>0.024</td>
</tr>
<tr>
<td><strong>DC Plans</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>No. of DMUs</td>
<td>37</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>No. of Efficient DMUs</td>
<td>13</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>Average</td>
<td>0.6957</td>
<td>0.5209</td>
<td>0.5173</td>
</tr>
<tr>
<td>Maximum</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.1935</td>
<td>0.1372</td>
<td>0.1372</td>
</tr>
</tbody>
</table>

¹. Non-Discretionary VRS Output Oriented Model. For DB and Combo plans, categorical DMUs are considered in the model. For DC plans, the funds’ status (fully funded/underfunded) is not an issue. Therefore, the categorical DMUs are not considered for DEA models for DC plans.
The results show that the discriminatory power of DEA for DB, Combo and DC plans increases by adding the AR constraints and then the trade-off constraints from mutual funds to the Non-Dis-VRS model. Adding the new constraints from the mutual funds’ dataset and the expert’s judgement for pension funds to the Non-Dis-VRS model prohibits extreme weighting divergences which results in a decline in efficiency scores. By having pension fund expert’s opinions and trade-offs from mutual funds, different target levels and improvement schemes are defined based on the Efficient, Leading and Outstanding plans for the inefficient pension plans. The improvement schemes for some of the DB inefficient plans are presented. For instance, DMU 22 in DB plans is efficient for the Non-Dis-VRS model and the AR model and its efficiency score decreases to 0.92 in the AR with trade-off constraints model. This DMU needs only one improvement scheme and its reference set is constructed based on Outstanding plans. The percentage change of the virtual output 1 based on Outstanding plans to the original output 1 is 8.2%. The percentage changes of the virtual output 2, inputs 1 to 4 based on Outstanding plans to the original variables are zero. It means that for improving the efficiency to the Outstanding plans’ level, output 1 should be increased by 8.2%. Also, DMU 53 in DB plans is efficient for the Non-Dis-VRS model and its efficiency score decreases to 0.992 in the AR model and the AR with trade-off constraints model. DMU 53 has two improvement schemes based on Leading plans and Outstanding plans. The percentage changes of the virtual output 1 based on Leading plans and Outstanding plans to the original output 1 are 0.73%. This indicates that for improving the efficiency to the Leading plans’ level and the Outstanding plans’ level, output 1 should be raised by 0.73%. For DMU 65, the efficiency score for the Non-Dis-VRS model is 0.736, for the AR model is 0.374 and for the AR with trade-off constraints model is 0.371. Therefore, for this DMU all three improvement schemes are needed and its first reference set is built based on the Basic frontier to remove the DMU’s pure
inefficiency. The percentage changes of the virtual output 1 to the original output 1 based on the Efficient, Leading and Outstanding target levels are 36%, 169% and 170% respectively. Also, the percentage change of the virtual output 2 to the original output 2 for the efficient level is 99% and for the Leading and Outstanding levels, the percentage changes are close to zero. For this DMU, the virtual input 3 based on the Efficient plans’ reference set should be decreased by 23%. The results can be interpreted in the same manner for the other DMUs in DB, Combo and DC plans.

7.5. Conclusion

In order to accomplish the theoretical objective of this chapter, the expert’s judgement is applied to the pension funds’ model to provide further insight into the available data. Afterwards, the same input and output variables between mutual funds and pension funds are selected. The relationship between the selected variables are extracted from the mutual funds’ dataset and added to the pension funds’ model. The results illustrate that after adding the expert’s opinions and information from the mutual funds’ dataset to the pension funds’ DEA model, the discriminatory power of the model is improved and the model can better detect the best pension funds’ performers and define different target levels for inefficient plans.
Chapter 8: Conclusions, Contributions and Future Work

This chapter provides a brief summary of the conclusions drawn from the examination and analysis of the results along with the main contributions and potential areas for future research.

8.1. Conclusions

8.1.1. Mixed Variable DEA Model

A novel Mixed Variable DEA model is developed in this study which detects DMUs with different characteristics. Therefore, different functional objectives and constraints are run for each category while all the DMUs with various cultures are evaluated relative to each other.
This approach was tested by using Canadian private pension funds and Canadian mutual funds’ datasets. Through this experimental analysis, the main characteristics of each type of fund were identified and included in the developed model. The MV-DEA model was able to compare and evaluate both pension funds and mutual funds successfully. The results indicated that the MV-DEA model can detect the best performers from both pension funds and mutual funds. The average efficiency scores of the MV-DEA was in the middle of the average efficiency scores of the DEA models based on each pension funds or mutual funds’ characteristics. Therefore, the new MV-DEA model provides an environment for evaluation and comparison of pension funds and mutual funds.

In summation, this work advanced both the theoretical and methodological components of DEA providing an environment for evaluation and comparison of different entities with different cultures in the same industry.

8.1.2. Analysis of Pension Funds’ Characteristics and Efficiency Scores

Government rules and restrictions are one of the main issues in the pension funds industry impacting asset allocation and managers’ control. This study quantified the impact of the regulations on asset allocation and incorporated the controllability of the variables from the managers’ perspective. Also, one of the issues in pension funds industry is that most pension plans are underfunded and in order to have a comprehensive study of this industry both fully funded and underfunded plans should be considered. An analysis was carried out to assess the fully funded and underfunded plans’ positions on the efficient frontier. The results indicated that most fully funded plans are efficient and after two or at most three times of removing the efficient DMUs from the dataset, only underfunded plans remain. As expected, the minimum efficiency score
increased significantly after the removal of each efficient frontier. Also, most underfunded plans were located at the bottom of the efficiency scores’ ranking. When fully funded and underfunded pension plans were evaluated separately, the minimum efficiency score increased compared to when they were considered together. Therefore, a low minimum efficiency score is often found for the pension funds industry since both fully funded and underfunded pension plans should be considered in the model. It follows that the examination of pension funds from different perspectives provides a theoretical contribution to the field of pension funds.

### 8.1.3. Borrowing Mutual Funds’ Information for Pension Funds

This study has provided a new framework in which pension plans’ performance was evaluated by considering professional judgements and borrowing useful information from the mutual funds’ dataset. Therefore, different target levels and improvement schemes for inefficient pension funds were defined. In the first step, pension funds’ DMUs were assessed by considering the main characteristics of this fund. In the second step, the expert’s opinions were included in the model. In the third step, useful information was extracted from the mutual funds’ dataset and added as trade-offs to the model. The discriminatory power of DEA increased in each step compared to the previous steps. In this approach, three target levels were obtained for inefficient pension plans and even efficient plans in the previous steps of the analysis.

### 8.2. Main Contributions

- Establishing a new DEA model, namely Mixed Variable DEA which considers non-discretionary variables for only some of the DMUs (pension funds) and at the same time
uses discretionary variables for the rest of the DMUs (mutual funds) while taking into account the funds’ status (categorical DMUs for fully funded and underfunded plans).

- For the first time, a meaningful comparison between pension funds and mutual funds is produced by considering the main characteristics and culture of each fund:
  - Regulation
  - Taxation
  - Uncontrollable nature of some pension funds’ variables
  - Fund’s status (fully funded or underfunded plans)

- Mixing and comparing the efficient DMUs from each fund (pension funds and mutual funds) to see which one performs better.
  - First efficient DMUs for pension funds and for mutual funds are identified separately based on different DEA models (VRS and Non-Dis-VRS). Then these efficient DMUs are combined in one dataset and various DEA models are tested such as VRS, Non-Dis-VRS and MV-DEA.
  - Efficient Canadian pension funds perform better than efficient Canadian mutual funds.

- Considering both fully funded and underfunded pension plans in the DEA model leads to a low minimum efficiency score.
  - Most fully funded plans are efficient and most inefficient DMUs are from underfunded plans. As predicted, by extracting the efficient DMUs (fully funded plans), the efficiency scores of the highly inefficient DMUs (underfunded plans) increased significantly.
When DEA models are run for both fully funded and underfunded pension plans, the minimum efficiency scores are very low. However, when DEA models are tested for fully funded pension plans and underfunded pension plans separately, the minimum efficiency scores increased significantly.

In order to have a comprehensive analysis of the pension funds industry, both fully funded and underfunded plans should be considered in the DEA model. Therefore, low efficiency scores for pension funds in the DEA model are often found in this industry.

• Providing an environment for pension funds to include mutual funds’ information by using trade-off approach while maintaining their own characteristics. In this part, there is no comparison carried out between pension funds and mutual funds. The relationships between similar variables for pension funds and mutual funds are extracted from the mutual funds’ dataset and added to the pension funds’ DEA model as trade-off constraints.

➢ First, experts’ judgments for Canadian pension funds industry are included in pension fund model (AR model).

➢ The relationships between variables for mutual funds are extracted and added as trade-offs to the pension funds’ model (AR + MF’s TO).

➢ The discriminatory power of the DEA model increased by adding pension fund expert’s judgements and also, the information from the mutual funds’ dataset.

➢ Three different target levels and improvement schemes (Efficient plans, Leading plans and Outstanding plans) for inefficient pension funds are provided.
8.3. Future Work

Several recommendations for further research in this domain are:

- The analysis can be carried out for multiple time periods in order to examine the changes in efficiency with time. The comparison between pension funds and mutual funds can change with time and its effects on best performers in these types of funds would be interesting to examine.

- The MV-DEA model can be used for any study and dataset. In this research, the MV-DEA model is used for two datasets (pension funds and mutual funds) considering different treatments. It would be interesting to use this methodology for one dataset with missing values in different fields.

- Although standard deviation of returns is the best proxy to evaluate asset allocation and quantify the effects of government regulations due to data availability, it is not the only factor that should be considered as the indicator for risk. In pursuing a more comprehensive look at the pension funds industry, it would be quite valuable to consider the proportion of liability and expected return for each pension plan if data could be obtained.

- Furthermore, similar performance analysis can be carried out comparing the public and private sectors for Canadian pension funds.
## Glossary

<table>
<thead>
<tr>
<th><strong>Additive Model</strong></th>
<th>A DEA model which measures efficiency using the slacks only and considers reduction in inputs with a simultaneous increase in outputs.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Assurance Region Model</strong></td>
<td>A DEA model which imposes constraints on the relative magnitude of weights for inputs and outputs.</td>
</tr>
<tr>
<td><strong>BCC or VRS Model</strong></td>
<td>A DEA model which considers a variable return to scale relationship between variables where a proportionate increase in inputs results in a proportionate increase or decrease in outputs.</td>
</tr>
<tr>
<td><strong>Categorical Variable</strong></td>
<td>A variable that assigns a DMU to a specific category with predefined discrete values.</td>
</tr>
<tr>
<td><strong>CCR or CRS Model</strong></td>
<td>A DEA model which assumes a constant return to scale relationship between inputs and outputs where a proportionate increase in inputs results in the same proportionate increase in outputs.</td>
</tr>
<tr>
<td><strong>Combination Plan</strong></td>
<td>A type of pension plan that incorporates both DB and DC plans’ concepts. Combination pension plan offers additional flexibility for plan members and employers by incorporating positive elements from both plans.</td>
</tr>
<tr>
<td><strong>DB Plan</strong></td>
<td>A type of pension plan that offers an employee the security of knowing what to expect at retirement based on their salary during their working years.</td>
</tr>
<tr>
<td><strong>DC Plan</strong></td>
<td>A type of pension plan that the amount available to provide a pension income is affected by how successfully contributions are invested.</td>
</tr>
<tr>
<td><strong>DEA</strong></td>
<td>Data Envelopment Analysis; A non-parametric, linear programming technique that could be used to measure the relative efficiency of a group of entities called decision making units.</td>
</tr>
<tr>
<td>-------------</td>
<td>--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>DMU</strong></td>
<td>Decision Making Unit; A term used to describe a unit being evaluated by DEA, in this research pension funds and mutual funds.</td>
</tr>
<tr>
<td><strong>Efficiency</strong></td>
<td>The ability to produce outputs using a minimum level of inputs.</td>
</tr>
<tr>
<td><strong>Efficient Frontier</strong></td>
<td>A frontier that represents the best performers or best practice units in the dataset.</td>
</tr>
<tr>
<td><strong>Input Oriented Model</strong></td>
<td>A DEA model whose objective is to minimize the inputs while keeping outputs constant.</td>
</tr>
<tr>
<td><strong>MV-DEA Model</strong></td>
<td>Mixed Variable DEA Model; A newly developed model introduced in this research that evaluates different entities with different cultures in the same industry.</td>
</tr>
<tr>
<td><strong>Non-discretionary</strong></td>
<td>A variable whose level of use or production is uncontrollable by managers.</td>
</tr>
<tr>
<td><strong>OECD</strong></td>
<td>Organization for Economic Co-operation and Development is an international economic organization of 34 countries (including Canada) established in 1961 to provide a platform in which governments can work together, compare policy experiences, seek answers to common problems and identify international policies of its members.</td>
</tr>
<tr>
<td><strong>OSFI</strong></td>
<td>The Office of the Superintendent of Financial Institutions Canada regulates private pension plans federally subject to the Pension Benefits Standards Act, 1985 (PBSA) which sets minimum standards and rules to ensure that the rights of plan members, retirees and beneficiaries are protected.</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Output Oriented Model</strong></td>
<td>A DEA model whose objective is to maximize the outputs while keeping inputs constant.</td>
</tr>
<tr>
<td><strong>Pure Inefficiency</strong></td>
<td>It can be removed from any input or output without changing the proportion of any other input and output.</td>
</tr>
<tr>
<td><strong>Reference Set</strong></td>
<td>A set of efficient units (peer group) to which the inefficient unit is compared in a DEA analysis.</td>
</tr>
<tr>
<td><strong>SBM</strong></td>
<td>Slack Based Model is a non-redial and unit invariant model which considers all inefficiencies in the scores.</td>
</tr>
<tr>
<td><strong>Trade-Off Approach</strong></td>
<td>A DEA model in which the dual terms in the constraints of the envelopment model could be taken as production trade-offs that represent feasible simultaneous changes to the inputs and/or outputs of the technology.</td>
</tr>
</tbody>
</table>
References


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Appendix A

GAMS Scripts for the Mixed Variable DEA (MV-DEA) Model for DB and Combo Plans

$Title Output oriented Non-Dis-VRS Model with Categorical DMUs for DB and Combo Plans and Output oriented VRS Model for Mutual Funds

$ontext

Mixed_Variable DEA Model (MV_DEA MODEL)

DB & Combo Plans- Output Oriented Non-Dis-VRS Model with Categorical DMUs and Discretionary and Non-Discretionary Variables

Mutual Funds- Output Oriented VRS Model with Categorical DMUs
(both Multiplier and Envelopment Forms)

$offtext

Sets

i /1*197/

nd_Cat1(i) "Corresponding to Underfunded Active DB and Combo Plans" /1*93/

nd_Cat2(i) "Corresponding to Fully Funded Active DB and Combo Plans" /94*136/

d_Cat2(i) "Corresponding to Mutual Funds" /137*197/

q(i)

Cat1(i) /1*93/
Cat2(i) /94*197/

PPS(i)

j    "Inputs" /Input_1, Input_2, Input_3, Input_4/
r    "Outputs" /Output_1, Output_2/

d(j) "Discretionary Inputs" /Input_1, Input_2, Input_3/
nd(j) "Non-discretionary Input" /Input_4/
c(r) "Discretionary Output" /Output_1/
nc(r) "Non-discretionary Output" /Output_2/;

parameter

RESULT_z_0(i)
RESULT_Mu(i,r)
RESULT_Nu(i,j)
RESULT_vo(i)
RESULT_z_1(i)
RESULT_w(i)
RESULT_Lambda(i,i)
RESULT_s(i,j)
RESULT_t(i,r);

Parameter

Input(i,j)
Output(i,r)
DMU_0_Output(r)
DMU_0_Input(j);

Scalar Epsilon /0.0000000000001/;
*=== Inputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Inputs.xls par = Input

*=== Inputs are imported from GDX
$GDXIN DMUs_Inputs.gdx
$LOAD Input
$GDXIN

*=== Outputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Outputs.xls par = Output

*=== Outputs are imported from GDX
$GDXIN DMUs_Outputs.gdx
$LOAD Output
$GDXIN

Variables
    z_0
    vo
    z_1
    Phi
    w ;

Positive Variables
    v(j)
    u(r)
    Lambda(i)
    s(j)
    t(r);
Equations

Objective1

Objective2

Objective3

Objective4

Objective5

Const1(i)

Const2

Const3(j)

Const4(r)

Const5(j)

Const6(r)

Const7(r)

Const8

Const9(i)

Const10

Const11(j)

Const12(r)

Const13

Const14(j)

Const15(r)

; 

Objective1 .. z_0 =e= sum(j$d(j), v(j)*DMU_0_Input(j))+sum(j$nd(j), v(j)*DMU_0_Input(j))- sum(r$nc(r), u(r)*DMU_0_Output(r))-vo;

Const1(pps) .. sum(j$d(j), Input(pps,j)*v(j))+sum(j$nd(j), Input(pps,j)*v(j))- sum(r$c(r), Output(pps,r)*u(r))-sum(r$nc(r), Output(pps,r)*u(r))-vo =g= 0;

Const2 .. sum(r$c(r), u(r)*DMU_0_Output(r)) =e= 1;
Const3(j)\$d(j) \quad \Rightarrow \quad v(j) = \text{Epsilon};

Const4(r)\$c(r) \quad \Rightarrow \quad u(r) = \text{Epsilon};

Objective2 \quad \Rightarrow \quad z_1 = \text{Phi} + \text{Epsilon} \times (\text{Sum}(j \text{d}(j), s(j)) + \text{Sum}(r \text{c}(r), t(r)));

Const5(j) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Input}(\text{pps},j) \times \text{Lambda}(\text{pps})) + s(j) = \text{DMU}_0 \text{Input}(j);

Const6(r)\$c(r) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Output}(\text{pps},r) \times \text{Lambda}(\text{pps})) - t(r) = \text{Phi} \times \text{DMU}_0 \text{Output}(r);

Const7(r)\$nc(r) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Output}(\text{pps},r) \times \text{Lambda}(\text{pps})) - t(r) = \text{DMU}_0 \text{Output}(r);

Const8 \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Lambda}(\text{pps})) = \text{1};

Objective3 \quad \Rightarrow \quad z_0 = \text{sum}(j, v(j) \times \text{DMU}_0 \text{Input}(j)) - \text{vo};

Const9(pps) \quad \Rightarrow \quad \text{sum}(j, \text{Input}(\text{pps},j) \times v(j)) - \text{sum}(r, \text{Output}(\text{pps},r) \times u(r)) - \text{vo} = \text{0};

Const10 \quad \Rightarrow \quad \text{sum}(r, u(r) \times \text{DMU}_0 \text{Output}(r)) = \text{1};

Objective4 \quad \Rightarrow \quad z_1 = \text{Phi};

Objective5 \quad \Rightarrow \quad w = \text{Sum}(j, s(j)) + \text{Sum}(r, t(r));

Const11(j) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Input}(\text{pps},j) \times \text{Lambda}(\text{pps})) = \text{DMU}_0 \text{Input}(j);

Const12(r) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Output}(\text{pps},r) \times \text{Lambda}(\text{pps})) = \text{Phi} \times \text{DMU}_0 \text{Output}(r);

Const13 \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Lambda}(\text{pps})) = \text{1};

Const14(j) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Input}(\text{pps},j) \times \text{Lambda}(\text{pps})) + s(j) = \text{DMU}_0 \text{Input}(j);

Const15(r) \quad \Rightarrow \quad \text{Sum}(\text{pps}, \text{Output}(\text{pps},r) \times \text{Lambda}(\text{pps})) - t(r) = z_1.1 \times \text{DMU}_0 \text{Output}(r);

Model Multiplier_ Non-Dis-VRSModel /Objective1, Const1, Const2, Const3, Const4/;
Model Envelopment_ Non-Dis-VRSModel /Objective2, Const5, Const6, Const7, Const8/;
Model Multiplier_VRSModel /Objective3, Const9, Const10/;
Model VRS_Phase1 /Objective4, Const11, Const12, Const13/;
Model VRS_Phase2 /Objective5, Const13, Const14, Const15/;
q(i) = No;
q(nd_cat1)=yes;

loop(q,

DMU_0_Output(r) = Output(q,r);
DMU_0_Input(j) = Input(q,j);

PPS(i)=no;
PPS(cat1)=yes;
PPS(cat2)=yes;

solve Multiplier_ Non-Dis-VRSModel using lp minimizing z_0;
RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

solve Envelopment_ Non-Dis-VRSModel using lp maximizing z_1;
RESULT_z_1(q) = z_1.l;
RESULT_t(q,r) = t.l(r);
RESULT_s(q,j) = s.l(j);
RESULT_Lambda(i,q) = Lambda.l(i);
);

q(i)=No;
q(nd_cat2)=yes;

loop(q,
DMU_0_Output(r) = Output(q,r);
DMU_0_Input(j) = Input(q,j);

PPS(i)=no;
PPS(cat2)=yes;

solve Multiplier_ Non-Dis-VRSModel using lp minimizing z_0;
RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

solve Envelopment_ Non-Dis-VRSModel using lp maximizing z_1;
RESULT_z_1(q) = z_1.l;
RESULT_t(q,r) = t.l(r);
RESULT_s(q,j) = s.l(j);
RESULT_Lambda(pps,q) = Lambda.l(pps);
);
q(i)=No;
q(d_cat2)=yes;
loop(q,

DMU_0_Output(r) = Output(q,r);
DMU_0_Input(j) = Input(q,j);

PPS(i)=no;
PPS(cat2)=yes;
solve Multiplier_VRSModel using lp minimizing z_0;
RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

solve VRS_Phase1 using lp maximizing z_1;
RESULT_z_1(q) = z_1.l;
RESULT_Lambda(pps,q) = Lambda.l(pps);

solve VRS_Phase2 using lp maximizing w;
RESULT_w(q) = w.l;
RESULT_t(q,r) = t.l(r);
RESULT_s(q,j) = s.l(j);

);  

***********************************************************************
RESULT_Mu(i,r)$(not RESULT_Mu(i,r)) = eps;
RESULT_Nu(i,j)$(not RESULT_Nu(i,j)) = eps;
RESULT_z_0(i)$(not RESULT_z_0(i)) = eps;
RESULT_vo(i)$(not RESULT_vo(i)) = eps;
RESULT_t(i,r)$(not RESULT_t(i,r)) = eps;
RESULT_s(i,j)$(not RESULT_s(i,j)) = eps;
RESULT_z_1(i)$(not RESULT_z_1(i)) = eps;

execute_unload  "TestVRSRes.gdx"  RESULT_z_0, RESULT_Mu, RESULT_Nu, RESULT_vo, RESULT_z_1, RESULT_Lambda, RESULT_t, RESULT_s;

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_z_0 rng = VRS_z! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_Mu rng = Mu! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_Nu rng = Nu! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_vo rng = VRS_vo! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_z_1 rng = VRS_z_1! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_t rng = t! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_s rng = s! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0  par = RESULT_Lambda rng = Lambda! Rdim = 1";
GAMS Scripts for the Mixed Variable DEA (MV-DEA) Model for DC Plans

$Title Output oriented VRS Model with Discretionary & Non-Discretionary Variables (Non-Dis-VRS for DC, VRS for MFs)

$ontext

Mixed Variable DEA Model (MV_DEA MODEL)

Output Oriented VRS Model with Discretionary & Non-Discretionary Variables (Non-Dis-VRS) for DC Plans

Output Oriented VRS Model (VRS) for MFs (both Multiplier and Envelopment Forms)

$offtext

$onsymxref

$onsymclist

$onuellist

$onuelxref

Sets

i /1*98/
p(i) "Corresponding to DC DMUs" /1*37/
M(i) "Corresponding to MF DMUs" /38*98/
q(i)

j "Inputs" /Input_1, Input_2, Input_3/
r "Outputs" /Output_1/
d(j) "Discretionary Inputs" /Input_1, Input_2/
nd(j) "Non-discretionary Input" /Input_3/;
parameter

RESULT_z_0(i)
RESULT_Mu(i,r)
RESULT_Nu(i,j)
RESULT_vo(i)
RESULT_z_1(i)
RESULT_w(i)
RESULT_Lambda(i,i)
RESULT_s(i,j)
RESULT_t(i,r);

Parameter

Input(i,j)
Output(i,r)
DMU_0_Output(r)
DMU_0_Input(j);

Scalar Epsilon /0.000000000001/;

*=== Inputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Inputs.xls par = Input
*=== Inputs are imported from GDX
$GDXIN DMUs_Inputs.gdx
$LOAD Input
$GDXIN
*=== Outputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Outputs.xls par = Output

*=== Outputs are imported from GDX
$GDXIN DMUs_Outputs.gdx

$LOAD Output

$GDXIN

Variables

z_0
vo
z_1
Phi
w;

Positive Variables

v(j)
u(r)
Lambda(i)
s(j)
t(r);

Equations

Objective1
Objective2
Objective3
Objective4
Objective5
const1(i)
const2
const3(j)
const4(r)
const5(j)
const6(r)
const7
const8(i)
const9
const10(j)
const11(r)
const12
const13(j)
const14(r)

objective1
z_0 =e= sum(j$d(j), v(j)*DMU_0_input(j))+sum(j$nd(j), v(j)*DMU_0_input(j))-vo;

const1(i)
sum(j$d(j), input(i,j)*v(j))+sum(j$nd(j), input(i,j)*v(j))-
sum(r, output(i,r)*u(r))-vo =g= 0;

const2
sum(r, u(r)*DMU_0_output(r)) =e= 1;

const3(j)$d(j)
v(j)=g=Epsilon;

const4(r)
u(r)=g=Epsilon;

objective2
z_1=e=Phi+Epsilon*(Sum(j$d(j),s(j))+Sum(r,t(r)));
Objective3  ..  \[ z_0 = \sum_j v(j) \cdot \text{DMU}_0\_\text{Input}(j) - v_0; \]
Const8(i)  ..  \[ \sum_j \text{Input}(i,j) \cdot v(j) - \sum_r \text{Output}(i,r) \cdot u(r) - v_0 = 0; \]
Const9  ..  \[ \sum_r u(r) \cdot \text{DMU}_0\_\text{Output}(r) = 1; \]

Objective4  ..  \[ z_1 = \Phi; \]
Objective5  ..  \[ w = \sum j s(j) + \sum r t(r); \]
Const10(j)  ..  \[ \sum_i \text{Input}(i,j) \cdot \Lambda(i) = \text{DMU}_0\_\text{Input}(j); \]
Const11(r)  ..  \[ \sum_i \text{Output}(i,r) \cdot \Lambda(i) = \Phi \cdot \text{DMU}_0\_\text{Output}(r); \]
Const12  ..  \[ \sum_i \Lambda(i) = 1; \]
Const13(j)  ..  \[ \sum_i \text{Input}(i,j) \cdot \Lambda(i) + s(j) = \text{DMU}_0\_\text{Input}(j); \]
Const14(r)  ..  \[ \sum_i \text{Output}(i,r) \cdot \Lambda(i) - t(r) = z_1.l \cdot \text{DMU}_0\_\text{Output}(r); \]

Model  Multiplier\_Non-Dis-VRSModel /Objective1, Const1, Const2, Const3, Const4/;
Model  Envelopment\_Non-Dis-VRSModel /Objective2, Const5, Const6, Const7/;
Model  Multiplier\_VRSModel /Objective3, Const8, Const9/;
Model  VRS\_Phase1 /Objective4, Const10, Const11, Const12/  
          VRS\_Phase2 /Objective5, Const12, Const13, Const14/;

q(i) = No;
q(p)=yes;
loop(q,

DMU_0\_Output(r) = Output(q,r);
DMU_0\_Input(j) = Input(q,j);

solve Multiplier\_Non-Dis-VRSModel using lp minimizing z_0;
RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

solve Envelopment\_Non-\_Dis-VRSModel using lp maximizing z\_1;
RESULT_z\_1(q) = z\_1.l;
RESULT_t(q,r) = t.l(r);
RESULT_s(q,j) = s.l(j);
RESULT_Lambda(i,q) = Lambda.l(i);
);

q(i)=No;
q(m)=yes;
loop(q,

DMU\_0\_Output(r) = Output(q,r);
DMU\_0\_Input(j) = Input(q,j);

solve Multiplier\_VRSModel using lp minimizing z\_0;
RESULT_z\_0(q) = z\_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

solve VRS\_Phase1 using lp maximizing z\_1;
RESULT_z\_1(q) = z\_1.l;
RESULT_Lambda(i,q) = Lambda.l(i);
solve VRS_Phase2 using lp maximizing w;
RESULT_w(q) = w.l;
RESULT_t(q,r) = t.l(r);
RESULT_s(q,j) = s.l(j);
);

******************************************************************************
RESULT_Mu(i,r)$(not RESULT_Mu(i,r)) = eps;
RESULT_Nu(i,j)$ (not RESULT_Nu(i,j)) = eps;
RESULT_z_0(i)$ (not RESULT_z_0(i)) = eps;
RESULT_vo(i)$ (not RESULT_vo(i)) = eps;
RESULT_t(i,r)$(not RESULT_t(i,r)) = eps;
RESULT_s(i,j)$ (not RESULT_s(i,j)) = eps;
RESULT_z_1(i)$ (not RESULT_z_1(i)) = eps;
execute_unload "TestVRSRes.gdx" RESULT_z_0, RESULT_Mu, RESULT_Nu, RESULT_vo, RESULT_z_1, RESULT_Lambda, RESULT_t, RESULT_s;
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_z_0 rng = VRS_z! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_Mu rng = Mu! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_Nu rng = Nu! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_vo rng = VRS_vo! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_z_1 rng = VRS_z_1! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_t rng = t! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_s rng = s! Rdim = 1"
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_Lambda rng = Lambda! Rdim = 1"
Appendix B

Variable Weighting Questionnaire for Pension Fund Expert

1. How would you weight “Net Investment Income ($u_1$)” to “Benefit Payments ($u_2$)” in the pension funds industry ($\frac{u_1}{u_2}$)? Please assign the appropriate and feasible weights (left bound has to be less than or equal to the right bound):

   \[ \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4, 5 \leq \frac{u_1}{u_2} \leq \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, 1, 2, 3, 4, 5 \]

   None of them: Please assign the appropriate weights: $\frac{u_1}{u_2}$

2. How would you weight “Standard Deviation of Returns ($v_1$)” to “Investment Expenses ($v_2$)” in the pension funds industry ($\frac{v_1}{v_2}$)? Please assign the appropriate and feasible weights (left bound has to be less than or equal to the right bound):

   \[ \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4, 5 \leq \frac{v_1}{v_2} \leq \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, 1, 2, 3, 4, 5 \]

   None of them: Please assign the appropriate weights: $\frac{v_1}{v_2}$

3. How would you weight “Standard Deviation of Returns ($v_1$)” to “Management Fees ($v_3$)” in the pension funds industry ($\frac{v_1}{v_3}$)? Please assign the appropriate and feasible weights (left bound has to be less than or equal to the right bound):

   \[ \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, 1, 2, 3, 4, 5 \leq \frac{v_1}{v_3} \leq \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, 1, 2, 3, 4, 5 \]

   None of them: Please assign the appropriate weights: $\frac{v_1}{v_3}$
4. How would you weight “Standard Deviation of Returns ($v_1$)” to “Contribution Amount ($v_4$)” in the pension funds industry ($---- \leq \frac{v_1}{v_4} \leq ----$)? Please assign the appropriate and feasible weights (left bound has to be less than or equal to the right bound):

\[
1/5 \quad 1/4 \quad 1/3 \quad 1/2 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad \frac{v_1}{v_4} \leq \frac{1}{5} \quad \frac{1}{4} \quad \frac{1}{3} \quad \frac{1}{2} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5
\]

None of them: Please assign the appropriate weights: $---- \leq \frac{v_1}{v_4} \leq ----$
Appendix C

GAMS Scripts for AR Model with Trade-Off Constraints from Mutual funds for DB and Combo Plans

$Title Assurance Region & Trade Off_Output oriented Non-Dis-VRS Model with Categorical DMUs for DB & Combo Plans
$ontext DB & Combo Plans- Assurance Region and Trade Off_Output Oriented Non-Dis-VRS Model with Categorical DMUs and Discretionary and Non-Discretionary Variables Assurance Region Constraints for PFs based on PF Expert's Opinion Trade-offs from All MFs Dataset $offtext $onsymxref $onsymlist $onuellist $onuelexref

Sets
i /1*136/
Cat1(i) "Corresponding to Underfunded Active DB and Combo Plans" /1*93/
Cat2(i) "Corresponding to Fully Funded Active DB and Combo Plans" /94*136/
q(i)
PPS(i)
j "Inputs" /Input_1, Input_2, Input_3, Input_4/
r "Outputs" /Output_1, Output_2/
d(j) "Discretionary Inputs" /Input_1, Input_2, Input_3/
nd(j) "Non-discretionary Input" /Input_4/
c(r) "Discretionary Output" /Output_1/
nc(r) "Non-discretionary Output" /Output_2/;

parameter
RESULT_z_0(i)
RESULT_Mu(i,r)
RESULT_Nu(i,j)
RESULT_vo(i)
lambda_AR_TO(i,i)
AR_TO_pi_1(i)
AR_TO_pi_2(i)
AR_TO_pi_3(i)
AR_TO_pi_4(i)
AR_TO_pi_5(i)
AR_TO_pi_6(i)
AR_TO_pi_7(i)
AR_TO_pi_8(i)
AR_TO_pi_9(i)
AR_TO_pi_10(i)
VirtInput(i,j)
VirtOutput(i,r)
Teta(i)

Output_Pi_1(r) /Output_1 0.49
  Output_2 0/

Input_Pi_1(j) /Input_1 0
  Input_2 1
  Input_3 0
  Input_4 0/

Output_Pi_2(r) /Output_1 0.30
  Output_2 0/

Input_Pi_2(j) /Input_1 1
  Input_2 1
  Input_3 0
  Input_4 0/

Input_Pi_3(j) /Input_1 -1
  Input_2 5
  Input_3 0
  Input_4 0/

Input_Pi_4(j) /Input_1 1
  Input_2 -1
  Input_3 0
  Input_4 0/

Input_Pi_5(j) /Input_1 -1
  Input_2 0
  Input_3 2
  Input_4 0/

Input_Pi_6(j) /Input_1 1
  Input_2 0
Parameter
   Input(i,j)
   Output(i,r)
   DMU_0_Output(r)
   DMU_0_Input(j);

Scalar Epsilon /0.000000000001/;

*** Inputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Inputs.xls par = Input
*** Inputs are imported from GDX
$GDXIN DMUs_Inputs.gdx
$LOAD Input
$GDXIN

*** Outputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Outputs.xls par = Output
*** Outputs are imported from GDX
$GDXIN DMUs_Outputs.gdx
$LOAD Output
$GDXIN

Variables
   z_0
   vo   ;
Positive Variables

\[ v(j) \]
\[ u(r) \]

Equations

Objective 1
Objective 2
Objective 3
Const 1(i)
Const 2
Const 3(j)
Const 4(r)
Const 5
Const 6
AR1_U_v2_1
AR1_L_v2_1
AR1_U_v3_1
AR1_L_v3_1
AR1_U_v4_1
AR1_L_v4_1
AR1_U_u2_1
AR1_L_u2_1

\[ \text{Objective 1} \quad .. \quad z_0 = e = \sum(j*d(j), v(j)*DMU_0\_Input(j)) + \sum(j*nd(j), v(j)*DMU_0\_Input(j)) - \sum(r*nc(r), u(r)*DMU_0\_Output(r)) - vo; \]

\[ \text{Const 1(pps)} \quad .. \quad \sum(j*d(j), Input(pps,j)*v(j)) + \sum(j*nd(j), Input(pps,j)*v(j)) - \sum(r*c(r), Output(pps,r)*u(r)) - \sum(r*nc(r), Output(pps,r)*u(r)) = g = 0; \]

\[ \text{Const 2} \quad .. \quad \sum(r*c(r), u(r)*DMU_0\_Output(r)) = e = 1; \]

\[ \text{Const 4(r)$c(r)$} \quad .. \quad u(r) = g = \text{Epsilon}; \]

\[ \text{Const 5} \quad .. \quad v('Input_2') = g = u('output_1')*0.49; \]

\[ \text{Const 6} \quad .. \quad v('Input_1') = g = u('output_1')*0.30; \]

\[ \text{AR1_U_v2_1} \quad .. \quad -1 * v('Input_1') = g = -5 * v('Input_2'); \]
\[ \text{AR1_L_v2_1} \quad .. \quad v('Input_1') = g = 1 * v('Input_2'); \]
\[ \text{AR1_U_v3_1} \quad .. \quad -1 * v('Input_1') = g = -2 * v('Input_3'); \]
\[ \text{AR1_L_v3_1} \quad .. \quad v('Input_1') = g = 1 * v('Input_3'); \]
\[ \text{AR1_U_v4_1} \quad .. \quad -1 * v('Input_1') = g = -3 * v('Input_4'); \]
\[ \text{AR1_L_v4_1} \quad .. \quad v('Input_1') = g = 1 * v('Input_4'); \]
\[ \text{AR1_U_u2_1} \quad .. \quad -1 * u('Output_1') = g = -3 * u('Output_2'); \]
\[ \text{AR1_L_u2_1} \quad .. \quad u('Output_1') = g = 1 * u('Output_2'); \]
Model Multiplier_AR_Non-Dis-VRSModel /Objective1, Const1, Const2, Const3, Const4, Const5, Const6, AR1_U_v2_1, AR1_L_v2_1, AR1_U_v3_1, AR1_L_v3_1, AR1_U_v4_1, AR1_L_v4_1, AR1_U_u2_1, AR1_L_u2_1/;

q(i) = No;
q(cat1)=yes;
loop(q,
 DMU_0_Output(r) = Output(q,r);
 DMU_0_Input(j) = Input(q,j);
)

PPS(i)=no;
PPS(cat1)=yes;
PPS(cat2)=yes;
solve Multiplier_AR_Non-Dis-VRSModel using lp minimizing z_0;

lambda_AR_TO(q,pps) = Const1.m(pps);
AR_TO_pi_1(q) = Const5.m;
AR_TO_pi_2(q) = Const6.m;
AR_TO_pi_3(q) = AR1_U_v2_1.m;
AR_TO_pi_4(q) = AR1_L_v2_1.m;
AR_TO_pi_5(q) = AR1_U_v3_1.m;
AR_TO_pi_6(q) = AR1_L_v3_1.m;
AR_TO_pi_7(q) = AR1_U_v4_1.m;
AR_TO_pi_8(q) = AR1_L_v4_1.m;
AR_TO_pi_9(q) = AR1_U_u2_1.m;
AR_TO_pi_10(q) = AR1_L_u2_1.m;
Teta(q) = Const2.m;

VirtInput(q,j) = sum(pps, Input(pps,j)*lambda_AR_TO(q,pps)) + AR_TO_pi_1(q)*
 Input_Pi_1(j)+ AR_TO_pi_2(q)* Input_Pi_2(j)+ AR_TO_pi_3(q)*
 Input_Pi_3(j)+AR_TO_pi_4(q)* Input_Pi_4(j)+AR_TO_pi_5(q)*
 Input_Pi_5(j)+AR_TO_pi_6(q)* Input_Pi_6(j)+AR_TO_pi_7(q)*
 Input_Pi_7(j)+AR_TO_pi_8(q)* Input_Pi_8(j);

VirtOutput(q,r)= sum(pps, Output(pps,r)*lambda_AR_TO(q,pps)) + AR_TO_pi_1(q)*
 Output_Pi_1(r)+ AR_TO_pi_2(q)* Output_Pi_2(r)-AR_TO_pi_9(q)* Output_Pi_9(r)-
 AR_TO_pi_10(q)* Output_Pi_10(r);

RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;
q(i)=No;
q(cat2)=yes;
loop(q,

DMU_0_Output(r) = Output(q,r);
DMU_0_Input(j) = Input(q,j);

PPS(i)=no;
PPS(cat2)=yes;
solve Multiplier_AR_ Non-Dis-VRSModel using lp minimizing z_0;

lambda_AR_TO(q,pps) = Const1.m(pps);
AR_TO_pi_1(q) = Const5.m;
AR_TO_pi_2(q) = Const6.m;
AR_TO_pi_3(q) = AR1_U_v2_1.m;
AR_TO_pi_4(q) = AR1_L_v2_1.m;
AR_TO_pi_5(q) = AR1_U_v3_1.m;
AR_TO_pi_6(q) = AR1_L_v3_1.m;
AR_TO_pi_7(q) = AR1_U_v4_1.m;
AR_TO_pi_8(q) = AR1_L_v4_1.m;
AR_TO_pi_9(q) = AR1_U_u2_1.m;
AR_TO_pi_10(q) = AR1_L_u2_1.m;
Teta(q) = Const2.m;

VirtInput(q,j) = sum(pps, Input(pps,j)*lambda_AR_TO(q,pps)) + AR_TO_pi_1(q)*
Input_Pi_1(j)+ AR_TO_pi_2(q)* Input_Pi_2(j)+ AR_TO_pi_3(q)*
Input_Pi_3(j)+AR_TO_pi_4(q)* Input_Pi_4(j)+AR_TO_pi_5(q)*
Input_Pi_5(j)+AR_TO_pi_6(q)* Input_Pi_6(j)+AR_TO_pi_7(q)*
Input_Pi_7(j)+AR_TO_pi_8(q)* Input_Pi_8(j);

VirtOutput(q,r)= sum(pps, Output(pps,r)*lambda_AR_TO(q,pps)) + AR_TO_pi_1(q)*
Output_Pi_1(r)+ AR_TO_pi_2(q)* Output_Pi_2(r)+AR_TO_pi_9(q)* Output_Pi_9(r)-
AR_TO_pi_10(q)* Output_Pi_10(r);

RESULT_z_0(q) = z_0.l;
RESULT_Mu(q,r) = u.l(r);
RESULT_Nu(q,j) = v.l(j);
RESULT_vo(q) = vo.l;

);
RESULT_Mu(i,r)$(not RESULT_Mu(i,r)) = eps;
RESULT_Nu(i,j)$(not RESULT_Nu(i,j)) = eps;
RESULT_z_0(i)$(not RESULT_z_0(i)) = eps;
RESULT_vo(i)$(not RESULT_vo(i)) = eps;
display VirtOutput;
execute_unload "TestVRSRes.gdx" RESULT_z_0, RESULT_Mu, RESULT_Nu, RESULT_vo, Teta, VirtInput, VirtOutput;
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_z_0 rng = VRS_z! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = Teta rng = Teta! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = VirtInput rng = VirtInput! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = VirtOutput rng = VirtOutput! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_Mu rng = Mu! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_Nu rng = Nu! Rdim = 1";
Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT_vo rng = VRS_vo! Rdim = 1";
GAMS Scripts for AR Model with Trade-Off Constraints from Mutual funds for DC Plans

$Title Assurance Region & Trade Off_Output oriented VRS Model with Discretionary & Non-Discretionary Variables for DC Plans

$ontext
DC Plans- Assurance Region and Trade Off_Output Oriented VRS Model with Discretionary & Non-Discretionary Variables
Assurance Region Constraints for PFs based on PF Expert's Opinion
Trade-offs from All MFs Dataset
$offtext
$onsymxref
$onsymlist
$onuellist
$onuelxref

Sets
i  /1*37/
q(i)
j  "Inputs" /Input_1, Input_2, Input_3/
r  "Output" /Output_1/
d(j) "Discretionary Inputs" /Input_1, Input_2/
nd(j) "Non-discretionary Input" /Input_3/

parameter
RESULT_z_0(i)
RESULT_Mu(i,r)
RESULT_Nu(i,j)
RESULT_vo(i)
lambda_AR_TO(i,i)
AR_TO_pi_1(i)
AR_TO_pi_2(i)
AR_TO_pi_3(i)
AR_TO_pi_4(i)
AR_TO_pi_7(i)
AR_TO_pi_8(i)
VirtInput(i,j)
VirtOutput(i,r)
Teta(i)

Output_Pi_1(r) /Output_1 0.49/
Input_Pi_1(j) /Input_1 0
  Input_2 1
  Input_3 0/
Output_Pi_2(r) /Output_1 0.30/
Input_Pi_2(j) /Input_1 1
  Input_2 0
  Input_3 0/
Input_Pi_3(j) /Input_1 -1
  Input_2 5
  Input_3 0/
Input_Pi_4(j) /Input_1 1
  Input_2 -1
  Input_3 0/
Input_Pi_7(j) /Input_1 -1
    Input_2 0
    Input_3 3/
Input_Pi_8(j) /Input_1 1
    Input_2 0
    Input_3 -1/ ;

Parameter
    Input(i,j)
    Output(i,r)
    DMU_0_Output(r)
    DMU_0_Input(j);
Scalar Epsilon /0.000000000001/;
*=== Inputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Inputs.xls par = Input
*=== Inputs are imported from GDX
$GDXIN DMUs_Inputs.gdx
$LOAD Input
$GDXIN
*=== Outputs are imported from Excel
$CALL GDXXRW.EXE DMUs_Outputs.xls par = Output
*=== Outputs are imported from GDX
$GDXIN DMUs_Outputs.gdx
$LOAD Output
$GDXIN
Variables

\[ z_0 \]
\[ v_0 \]

Positive Variables

\[ v(j) \]
\[ u(r) \]

Equations

Objective1

\[ z_0 = \sum_j \left( v(j) \cdot DMU_0 \_Input\_j \right) + \sum_j \left( v(j) \cdot DMU_0 \_Input\_j \right) - v_0; \]

Const1(i)

\[ \sum_j \left( Input(i,j) \cdot v(j) \right) + \sum_j \left( Input(i,j) \cdot v(j) \right) - \sum_r \left( Output(i,r) \cdot u(r) \right) - v_0 \geq 0; \]

Const2

\[ \sum_r \left( u(r) \cdot DMU_0 \_Output\_r \right) = 1; \]

Const3(j)$d(j)

\[ v(j) \geq \text{Epsilon}; \]

Const4(r)

\[ \text{AR1\_U\_v2\_1} \]
\[ \text{AR1\_L\_v2\_1} \]
\[ \text{AR1\_U\_v3\_1} \]
\[ \text{AR1\_L\_v3\_1}; \]
Const4(r) .. u(r) = g = Epsilon;
Const5 .. v('Input_2') = u('output_1') * 0.49;
Const6 .. v('Input_1') = u('output_1') * 0.30;
AR1_U_v2_1 .. -1 * v('Input_1') = -5 * v('Input_2');
AR1_L_v2_1 .. v('Input_1') = 1 * v('Input_2');
AR1_U_v3_1 .. -1 * v('Input_1') = -3 * v('Input_3');
AR1_L_v3_1 .. v('Input_1') = 1 * v('Input_3');

Model Multiplier_AR_TO_ Non-Dis-VRS Model /Objective1, Const1, Const2, Const3, Const4, Const5, Const6, AR1_U_v2_1, AR1_L_v2_1, AR1_U_v3_1, AR1_L_v3_1/;

q(i) = yes;
loop(q,

DMU_0_Output(r) = Output(q,r);
DMU_0_Input(j) = Input(q,j);
solve Multiplier_AR_TO_ Non-Dis-VRS Model using lp minimizing z_0;
lambda_AR_TO(q,i) = Const1.m(i);
AR_TO_pi_1(q) = Const5.m;
AR_TO_pi_2(q) = Const6.m;
AR_TO_pi_3(q) = AR1_U_v2_1.m;
AR_TO_pi_4(q) = AR1_L_v2_1.m;
AR_TO_pi_7(q) = AR1_U_v3_1.m;
AR_TO_pi_8(q) = AR1_L_v3_1.m;
Teta(q) = Const2.m;
VirtInput(q,j) = \sum(i, Input(i,j) \cdot \lambda_{AR\_TO}(q,i)) + AR\_TO\_pi\_1(q) \cdot Input\_Pi\_1(j) + AR\_TO\_pi\_2(q) \cdot Input\_Pi\_2(j) + AR\_TO\_pi\_3(q) \cdot Input\_Pi\_3(j) + AR\_TO\_pi\_4(q) \cdot Input\_Pi\_4(j) + AR\_TO\_pi\_7(q) \cdot Input\_Pi\_7(j) + AR\_TO\_pi\_8(q) \cdot Input\_Pi\_8(j);

VirtOutput(q,r) = \sum(i, Output(i,r) \cdot \lambda_{AR\_TO}(q,i)) + AR\_TO\_pi\_1(q) \cdot Output\_Pi\_1(r) + AR\_TO\_pi\_2(q) \cdot Output\_Pi\_2(r);

RESULT\_z\_0(q) = z\_0.l;
RESULT\_Mu(q,r) = u.l(r);
RESULT\_Nu(q,j) = v.l(j);
RESULT\_vo(q) = vo.l;

);  

*********************************************************

RESULT\_Mu(i,r)$(not RESULT\_Mu(i,r)) = eps;
RESULT\_Nu(i,j)$(not RESULT\_Nu(i,j)) = eps;
RESULT\_z\_0(i)$(not RESULT\_z\_0(i)) = eps;
RESULT\_vo(i)$(not RESULT\_vo(i)) = eps;

display VirtOutput;

execute_unload "TestVRSRes.gdx" RESULT\_z\_0, RESULT\_Mu, RESULT\_Nu, RESULT\_vo, Teta, VirtInput, VirtOutput;

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT\_z\_0 rng = VRS\_z! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = Teta rng = Teta! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = VirtInput rng = VirtInput! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = VirtOutput rng = VirtOutput! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT\_Mu rng = Mu! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT\_Nu rng = Nu! Rdim = 1";

Execute "GDXXRW.EXE TestVRSRes.gdx EpsOut =0 par = RESULT\_vo rng = VRS\_vo! Rdim = 1";