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Uncertainty assessment in aboveground biomass estimation at the regional scale using a new method considering both sampling error and model error

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22 **Abstract:** Uncertainty associated with multiple sources of error exists in biomass estimation
23 over large areas. This uncertainty affects the accuracy of the resultant biomass estimates. A
24 new method that introduces Taylor series principles into a Monte Carlo simulation procedure
25 was proposed and developed for estimating regional-scale aboveground biomass, along with
26 quantifying the corresponding uncertainty arising from both sampling and model predictions.
27 Additionally, the effect of sample size on estimates during model fitting was studied based on
28 the new method in order to determine whether the effect of the size of the calibration data set
29 can be neglected when the number of simulations is sufficiently large. The results revealed
30 that the proposed method not only produces more reliable estimates of both biomass and
31 uncertainty but also effectively and separately quantifies the uncertainties associated with
32 different sources of error. The new method also reduced the effect of model uncertainty on
33 final estimates. The uncertainty that was associated with model error increased significantly
34 with decreasing sample sizes during model fitting, and the error was not reduced by
35 increasing the number of Monte Carlo simulations.

36
37 **Keywords:** Aboveground biomass estimation; Uncertainty assessment; Monte Carlo
38 simulation; Model error; Sample size for model fitting

39

1. Introduction

To increase the ability to adapt to the adverse impacts of climate change and to mitigate greenhouse gases emissions, the Paris Agreement within the United Nations Framework Convention for Climate Change (UNFCCC) was adopted by consensus on 12 December 2015 (Sutter and Berlinger 2015). According to the UNFCCC, all the members that had signed the treaty were required to report estimates of their forest carbon stocks and carbon stock changes, because forests play an essential role in carbon emission reduction via carbon sink increments (Intergovernmental Panel on Climate Change (IPCC) 2006). In addition, quantified measures of all plausible sources of uncertainty, as well as the methods used to minimize uncertainty, were indispensable components of the previous IPCC Assessment Reports (IPCC 2006; Mastrandrea et al. 2010). Given that forest carbon stock estimates are largely derived from biomass values, the methodology and certainty underlying biomass estimation has attracted considerable attention (Peltoniemi et al. 2006; Lehtonen et al. 2007; Berger et al. 2014; Breidenbach et al. 2014).

Regional and national estimates of tree biomass are generally based on observations from national forest inventories, and they commonly involve biomass models (Lehtonen et al. 2007; Petersson et al. 2012; Breidenbach et al. 2014). Aboveground biomass (*agb*) at the regional level is generally estimated from forest inventory data by first summing the model predictions of individual trees within a plot. Then, forest biomass estimates at larger spatial scales can be produced by averaging over plots (Li and Zhao 2013; McRoberts and Westfall 2014). Uncertainties associated with multiple sources of error generated during this summation process are propagated and embedded in the final regional-based biomass estimates (Chave et al. 2004; Cohen et al. 2013).

Cunia (1965) suggested that three main sources of error are generally associated with forest inventory estimates: measurement error, sampling error and model error. Among these

65 error sources, measurement error has been the most extensively studied (Gertner 1987, 1990;
66 McRoberts and Lessard 2001; Berger et al. 2014) and is thus not addressed in the present
67 study.

68 Sampling-related uncertainty is introduced into regional estimates when inventory data
69 are based on forest samples over wide geographic areas, whereas model-related uncertainty is
70 propagated via the estimation of variables that cannot be directly measured in the field (eg.,
71 timber volume, biomass and carbon stocks) (Petersson et al. 2012; Breidenbach et al. 2014).
72 These uncertainties that are associated with forest inventory estimates are routinely ignored
73 or categorized as sampling variability (Dietze et al. 2008; McRoberts and Westfall 2014).
74 However, failure to account for these uncertainties will lead to considerable biases in regional
75 estimates that are derived from forest inventories (McRoberts and Westfall 2014; Ståhl et al.
76 2014). Therefore, uncertainty assessments and solutions for reducing error are necessarily
77 required in regional-scale estimation.

78 More generally, three main methodologies have been proposed to quantify uncertainties.
79 The first is the error propagation approach, which sums the variances of the component
80 sources of uncertainty produced using first-order Taylor series approximations (Gertner 1987,
81 1990; Chave et al. 2004). However, different error sources do not interact directly; therefore,
82 mechanically aggregating these sources may lead to inaccuracy (Ståhl et al. 2010). The
83 second approach is based on model analysis. This method was developed from mathematical
84 manipulations based on sampling theory combined with Taylor series principles, and it can be
85 used to separately estimate the uncertainties associated with sampling and model errors
86 (Cunia 1987; Ståhl et al. 2010; Gregoire et al. 2010; Ene et al. 2012; Gobakken et al. 2012;
87 Nelson et al. 2012; Ståhl et al. 2014). However, the estimates obtained by this approach
88 depend heavily on the characteristics of the data set for model fitting itself; thus, model
89 parameters and covariance matrixes used in estimations must vary with different calibration

90 data sets. The third approach uses simulation techniques such as the Monte Carlo methods,
91 which uses thousands of iterations to obtain the probability distributions and final estimates
92 of model parameters (McRoberts and Lessard 2001; Peltoniemi et al. 2006; McRoberts and
93 Westfall 2014). Monte Carlo simulation can effectively estimate and evaluate uncertainties
94 caused by multiple sources of error that propagate through complex models (Morgan and
95 Small 1992; McRoberts et al. 2013). Based on a review of the literature, it has not been
96 previously applied to determine uncertainties associated with both sampling-related and
97 model-related error until now, to the best of our knowledge.

98 The model analysis method can separately quantify different sources of uncertainty
99 associated with sampling and model errors. Therefore, introducing the Monte Carlo approach
100 to this method may offset its limitations associated with parameter variability. The first
101 objective of the present study is to propose and develop an alternative method that exploits
102 the advantages of these two methods, i.e., to determine uncertainties associated with different
103 error contributions, reduce the instability of model parameters and provide more reliable and
104 precise estimates of both regional biomass and uncertainty. McRoberts and Westfall (2014)
105 noted that using smaller data sets for model fitting resulted in greater covariance in the model
106 parameters and, therefore, larger standard errors in the final estimates. However, Monte Carlo
107 simulation may have the ability to reduce deviations in model parameters. Thus, our second
108 objective is to use different sample sizes for model fitting in the proposed method to
109 determine whether the effect of data set size can be neglected when the number of
110 simulations is sufficiently large.

111

2. Data and Methods

2.1 Data

The study area is located in Jiangxi Province (118°29'N, 113°34'E) of the People's Republic of China. This province is in the southeast region of China and has a warm and humid climate. In this area, Chinese fir (*Cunninghamia lanceolata*[Lamb.] Hook.) is the most common plantation tree species. For this study, 2610 permanent plots were systematically sampled throughout the province, and each covered an area of 0.067 ha. The data set in this study was divided into two different parts: a calibration data set for model fitting (DS_m) and a verification data set for large-area *agb* estimation (DS_e).

To build the DS_m, 150 sample trees were harvested from June to September 2009 near permanent plots in Jiangxi. To achieve extensive representativeness over a large area, the harvested trees selected for the analysis were distributed over a wide geographic area in Jiangxi province. Consequently, the effects of spatial autocorrelations among trees in the DS_m were ignored for they were relatively far away from each other. The samples were uniformly collected in 10 varying-sized diameter classes: above 2 cm, 4 cm, 6 cm, 8 cm, 12 cm, 16 cm, 20 cm, 26 cm, 32 cm and 38 cm, furthermore, the samples within each diameter class were selected according to the tree height from high to low as uniformly as possible.

Measurements of diameters at breast height (*dbh*), tree height (*h*) and ground diameter were taken from individual sample trees to formulate the DS_m. Fresh masses of bole, bark, branches and foliage were weighted in the field and then transported to the laboratory where they were oven-dried at 85°C until their weight stabilized. The biomass of each tree component was calculated according to the ratio of dry mass to fresh mass and was then summed to determine the *agb* of each tree. For the purposes of this analysis, these varieties and estimates were assumed to be obtained without error.

To minimize negative effects due to model misspecification and produce better model parameters for the following biomass estimation and uncertainty assessment, three forms of allometric equations were tried for *agb* model fitting: $g = \beta_0 \cdot dbh^{\beta_1} + \varepsilon$, $g = \beta_0 \cdot dbh^{\beta_1} \cdot h^{\beta_2} + \varepsilon$ and $g = \beta_0 \cdot (dbh^2 \cdot h)^{\beta_1} + \varepsilon$. These equations have been used in many studies (Ter-Mikaelian and Korzukhin 1997; Parresol 2001; Jalkanen 2005; Repola 2008, 2009; Petersson et al. 2012). The inclusion of *h* and *dbh* as predictive variables was found to improve the equations' accuracies effectively for the 150 sample trees in this study.

Among the 2610 permanent plots, 1128 plots with Chinese fir were considered the DS_e. All trees in the plots were continuously observed and their measurements were collected in 2009-2011. In each plot, trees with *dbh* larger than 5 cm were measured. Measuring the tree height (*h*) of individual trees is difficult, time consuming and often inaccurate across a survey plot because *h* varies with site quality, even for trees with the same *dbh* (Li and Fa 2011). Li and Fa (2011) classified tree height into 9 levels considering different sites across a large geographical area and including our verification data set. They selected the Chapman-Richard function $h = 1.3 + a \cdot (1 - e^{-b \cdot dbh})^c$ (Wykoff et al. 1982) to describe the height-diameter relationship for Chinese fir belonging to each level, where $a = (a_1, a_2, \dots, a_9)'$, *b*, *c* are model parameters. They then used the function to determine the height-diameter curve that each plot belonged to. This method can improve the accuracy of the height curve for Chinese fir; therefore, it was applied in the present study to estimate *h* for individual trees in the DS_e.

The DS_m and DS_e were independent from each other because of their different sources. The statistical characteristics of the sample trees in the DS_e and DS_m are listed in Tab.1.

Tab.1

2.2 Tree biomass estimates and residuals

The model chosen to estimate the *agb* of individual trees in this study was an allometric-based model that performed well in other studies (Brown et al. 1989; Chave et al. 2004; Zianis 2008; Djomo et al. 2010; Breidenbach et al. 2014) and achieved the highest coefficient of determination (R^2) among the three selected equation forms (equation (1)):

$$g = \beta_0 dbh^{\beta_1} h^{\beta_2} + \varepsilon \quad (1)$$

where *dbh* and *h* are predictive variables and *g* is a response variable. In this allometric model, *dbh* is the diameter at breast height (cm); *h* is tree height (m); *g* is the tree *agb* (kg); β_0 , β_1 and β_2 are model parameters. The heterogeneity in residual variance was overcome using a weighted nonlinear least squares technique to fit equation (1) to the data in the DS_m and estimate the model parameters. ε represents the residual deviations obtained as byproducts during model fitting using equation (1) and is calculated as

$$\varepsilon = \hat{g} - g \quad (2)$$

where $\hat{g} = \hat{\beta}_0 dbh^{\hat{\beta}_1} h^{\hat{\beta}_2}$ is the model prediction and $\hat{\beta}_0$, $\hat{\beta}_1$, $\hat{\beta}_2$ are unbiased estimates of β_0 , β_1 , β_2 . Specifically, ε is assumed to follow a Gaussian distribution with a mean of zero and heterogeneous variances; i.e., $\varepsilon \sim N(0, \sigma^2)$, where σ is the standard deviation of the distribution of residuals and the relationship between σ , *dbh* and *h* can be adequately described as follows: $E[\ln(\hat{\sigma})] = \gamma_1 + \gamma_2 \cdot \ln(dbh) + \gamma_3 \cdot \ln(h)$. In this equation, $E[\cdot]$ denotes the statistical expectation of $\ln(\hat{\sigma})$; $\hat{\sigma}$ is an unbiased estimate of σ ; and γ_1 , γ_2 and γ_3 are parameters (McRoberts and Lessard 2001). Thus, the residual term satisfies equation (3).

$$\varepsilon \sim N(0, \exp(\gamma_1 + \gamma_2 \cdot \ln(dbh) + \gamma_3 \cdot \ln(h))^2) \quad (3)$$

2.3 Simulating uncertainty

Generally, the uncertainty associated with model error has been ignored or combined

with sampling error during *agb* estimation based on data from sample-based surveys. To separate the model error from the total error, a first-order Taylor linearization was applied. This method is similar to that used by Ståhl et al. (2010, 2014).

Our new method combines the model analysis method and Monte Carlo simulation to benefit from the advantages of both methods while avoiding their respective weaknesses. The proposed method implements the model analysis approach in all rounds of the Monte Carlo simulations.

To focus specifically on assessing the model-related uncertainty and sampling-related uncertainty associated with biomass estimates, measurements of *dbh* and *h* in the DS_e were assumed to be free of error. Additionally, any effects caused by spatial autocorrelation among trees within plots or among different plots were ignored because they have little influence on the uncertainty of large area estimates (Breidenbach et al. 2014, Berger et al. 2014; Ståhl et al. 2014).

Five steps are used in our method to estimate the mean regional level of *agb* per unit area and assess the uncertainties in the estimates, including error sources associated with both modeling and sampling.

For convenience, we define main mathematical terms in Tab.2.

Tab.2

Step 1:

For each element in the DS_m, a simulated observation of *agb* (g_{pp}) was obtained as the sum of the prediction calculated by $\hat{g}_p = \hat{\beta}_0 \cdot dbh_p^{\hat{\beta}_1} \cdot h_p^{\hat{\beta}_2}$ and a residual term $\hat{\epsilon}_p$ randomly selected from a Gaussian distribution satisfying equation (3), i.e., $g_{pp} = \hat{g}_p + \hat{\epsilon}_p$ and

$$\hat{\varepsilon}_p \sim N(0, \exp(\gamma_1 + \gamma_2 \cdot \ln(dbh_p) + \gamma_3 \cdot \ln(h_p)))^2).$$

Step 2:

A new vector of model parameters was estimated by fitting the model to dbh_p , h_p and g_{pp} , where the dbh_p and h_p were used as the predictive variables, and the simulated g_{pp} was the response variable. The allometric form of this relationship is as follows:

$$g_{pp} = \alpha_0 \cdot dbh_p^{\alpha_1} \cdot h_p^{\alpha_2} + \varphi_p, \quad (4)$$

where α_0 , α_1 and α_2 are new model parameters estimated using a reweighted nonlinear least squares technique and φ_p is a residual term. The g_{pp} model will be used in the DS_e for large area *agb* estimation in the following steps.

Step 3:

Because the first order Taylor linearization can be expressed as

$$f(x, \alpha) \approx f(x, \hat{\alpha}) + (\alpha_1 - \hat{\alpha}_1) \cdot f'_1 + (\alpha_2 - \hat{\alpha}_2) \cdot f'_2 + \dots + (\alpha_q - \hat{\alpha}_q) \cdot f'_q,$$

where f'_q is the partial derivative of model $f(x, \alpha)$ in regard to parameter α , i.e.,

$$f'_q = \frac{\partial f_q(x, \alpha)}{\partial \alpha_q}; \quad q \text{ is the number of parameters. We applied the first order Taylor}$$

approximation to be the proxy of “true” value (unobservable) of *agb* for individual trees, based on the assumption that all model parameters $\hat{\alpha}$ s are sufficiently accurate.

The “true” *agb* values (g_{ij}) of all the individual trees in the DS_e based on the forest inventory were then expressed as equation (5):

$$g_{ij} \approx \hat{g}_{ij} + (\alpha - \hat{\alpha}) \cdot g'_{ij}. \quad (5)$$

$$\text{In this equation, } \hat{g}_{ij} = \hat{\alpha}_0 \cdot dbh_{ij}^{\hat{\alpha}_1} \cdot h_{ij}^{\hat{\alpha}_2} \text{ and } g'_{ij} = \frac{\partial g_{ij}(x_{ij}, \alpha)}{\partial \alpha},$$

where $x_{ij} = (dbh_{ij}, h_{ij})'$; $\alpha = (\alpha_0, \alpha_1, \alpha_2)'$ and $\hat{\alpha} = (\hat{\alpha}_0, \hat{\alpha}_1, \hat{\alpha}_2)'$ are the vectors of standard model parameters, with $\hat{\alpha}$ assumed to be an unbiased estimate of α ; and $(\hat{\alpha} - \alpha)$ is a column vector of bias between “true” parameters and the estimated parameters.

Then, the plot-level *agb* was calculated by aggregating all *agb* values of individual trees

226 within the same plot using equation (6):

$$G_j(x_{ij}, \alpha) \approx \hat{G}_j(x_{ij}, \hat{\alpha}) + G'_j \cdot (\alpha - \hat{\alpha}), \quad (6)$$

227 where $G'_j = \sum_i^{n_j} g'_{ij}$. The plot-level *agb* prediction was calculated using equation (7):

$$\hat{G}_j(x_{ij}, \hat{\alpha}) = \sum_i^{n_j} \hat{g}_{ij}(x_{ij}, \hat{\alpha}). \quad (7)$$

228 The plot-level *agb* per unit area was denoted by μ_j and calculated as follows:

$$\mu_j \approx \frac{\hat{G}_j(x_{ij}, \hat{\alpha})}{A_j} + \frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}. \quad (8)$$

229 The mean of “true” *agb* per unit area over all plots was assumed to be

$$\mu = \frac{\sum_j^n G_j(x_{ij}, \alpha)}{\sum_j^n A_j}, \quad (9)$$

230 and the predicted *agb* per unit area over all plots was calculated by

$$\bar{\mu} = \frac{\sum_j^n \hat{G}_j(x, \hat{\alpha})}{\sum_j^n A_j}. \quad (10)$$

231

232 Step 4:

233 The variance (or mean square error) of the mean *agb* over all plots was evaluated as

$$\begin{aligned} Var(\mu) &= E(\mu_j - \mu)^2 = E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \mu + \frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}\right)^2 \\ &= E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \mu\right)^2 + E\left(\frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}\right)^2 + 2 \cdot E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \mu\right) \cdot \left(\frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}\right). \end{aligned} \quad (11)$$

234 According to the assumption, $E(\mu) \approx \bar{\mu}$ when the sample size is sufficiently large and

235 the model parameters are sufficiently accurate; therefore, the first part of equation (10),

$$E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \mu\right)^2 \approx E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \bar{\mu}\right)^2 \approx E\left(\frac{\hat{G}_j(x, \hat{\alpha}) - \frac{\sum_j \hat{G}_j(x, \hat{\alpha})}{\sum_j A_j}}{A_j}\right)^2$$

reveals that the two items in this part share the same parameters ($\hat{\alpha}$) but different sources of predictive variables (x from different plots), which means the randomness of this variance depends only on sample error. Therefore, the sampling error contribution to the total variance is as follows:

$$Var_S = E\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \bar{\mu}\right)^2 = \frac{1}{n} \cdot \frac{\sigma_U^2}{A^2}, \quad (12)$$

where σ_U is the standard variance of the bias $(\hat{G}_j - \bar{\mu} \cdot A_j)$, which is the bias between *agb*

prediction of the j th plot and mean *agb* prediction over all plots, i.e., $\sigma_U^2 = \frac{\sum_j (\hat{G}_j - \bar{\mu} \cdot A_j)^2}{n-1}$.

Similarly, the second part $E\left(\frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}\right)^2$ indexes a variance item of which the randomness depends only on the differentiation in regard to parameters $\hat{\alpha}$ and α . Therefore, it can serve as the model uncertainty contribution to the total variance. The equation is as follows:

$$Var_M = E\left(\frac{G'_j \cdot (\alpha - \hat{\alpha})}{A_j}\right)^2 = P \Sigma P^T, \quad (13)$$

where Σ is the covariance matrix of the α -parameters, i.e., $\Sigma = \text{cov}(\hat{\alpha}, \alpha) = \text{cov}(\alpha_0, \alpha_1, \alpha_2)$;

and P is the mean of the partial derivative G'_j per unit area over all plots, i.e., $P = \frac{\sum_j G'_j}{\sum_j A_j}$.

Because the factor $\left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \mu\right) \approx \left(\frac{\hat{G}_j(x, \hat{\alpha})}{A_j} - \bar{\mu}\right)$ in the third term of equation (11) has a

zero expectation, Var_S and Var_M can be considered separate error components due to sampling error and model uncertainty, i.e.,

$$Var(\mu) = Var_S + Var_M, \quad (14)$$

and they are independent from each other (Ståhl et al. 2010, 2014; Chave et al. 2004). More details can be found in Ståhl et al. (2014).

Step 5:

Steps 1-4 were repeated and the total estimates of mean *agb* per unit area and their total variances incorporating both sampling error and model uncertainty contributions over replications were calculated as

$$\tilde{\mu} = \frac{1}{n_k} \sum_{k=1}^{n_k} \bar{\mu}^k, \quad (15)$$

$$Var(\tilde{\mu}) = \left(1 + \frac{1}{n_k}\right) \cdot U_1 + U_2, \quad (16)$$

where $\bar{\mu}^k$ is the estimate of mean *agb* per unit area during the *k*th replication, and $k=1, \dots,$

n_k ; n_k indexes the number of replications; $U_1 = \frac{1}{n_k - 1} \sum_{k=1}^{n_k} (\bar{\mu}^k - \tilde{\mu})^2$ is the variance among

simulations and $U_2 = \frac{1}{n_k} \sum_{k=1}^{n_k} Var(\bar{\mu}^k)$ is the mean of the variance within each simulation (Ståhl

et al. 2014). Results obtained from the simulations were compared with those obtained from the traditional model analysis method addressed by Ståhl et al. (2014).

One thousand replications were conducted using the Monte Carlo method because we verified that this number of simulations was sufficient for the $\tilde{\mu}$ and $Var(\tilde{\mu})$ estimates to be stabilized; i.e., both $\tilde{\mu}$ s and $Var(\tilde{\mu})$ s fluctuated among simulations within the triple standard deviation above or below their mean.

2.4 Impact of sample size

To analyze the impact of the sample size of the DS_m , four different sample sizes ($m=50,$ $m=80,$ $m=110$ and $m=140$) were used. The samples in each sub-data set were randomly

selected from the DS_m in each simulation such that the samples selected for model fitting were different in each simulation; the model parameters used in the large-area *agb* estimation varied accordingly. Thus, differences in the biomass and uncertainty estimates between simulations reflect how the calibration data set size affects the large area estimates and whether increasing the number of iterations can overcome an insufficient number of samples for model fitting. A five-step procedure was conducted as follows.

Step 1:

A new calibration data set of a certain size ($m=50$, $m=80$, $m=110$ and $m=140$) was produced by randomly resampling from the DS_m .

Step 2:

Sets of parameter estimates were obtained by fitting the model based on equation (1) to the measurements of *dbh*, *h* and *agb* in the calibration data set produced in Step 1.

Step 3:

The model parameters estimated in Step 2 were subsequently used for *agb* estimation for all individual trees in the DS_e .

Step 4:

The plot-level *agb* predictions were acquired by aggregating individual values using equation (7), and the estimate of mean *agb* per hectare for the *k*th simulation was calculated using equation (10). Then, the variances associated with sampling and model errors were obtained using equations (12) and (13), respectively, and the total variance associated with combined error was determined using equation (14).

Step 5:

Steps 1-4 were repeated 1000 rounds, and the final *agb* estimator $\tilde{\mu}$ and corresponding $Var(\tilde{\mu})$ were obtained using equations (15) and (16), respectively.

All parameter estimates in the biomass models and uncertainty estimates were

implemented using the R language (R. Core Team 2012).

3. Results

3.1 Tree biomass estimates and residual variables

The fitting results of equations (1) and (3) and the height-diameter relationship model are given in Tab.3. The coefficient of determination of the *agb* estimation equation is 0.982, which suggests that our biomass equation provides a good basis for the Monte Carlo simulation procedure. The parameters γ_1 , γ_2 and γ_3 are used to produce dummy residuals $\hat{\varepsilon}_p$ according to the relation of $\hat{\varepsilon}_p \sim N(0, \exp(\gamma_1 + \gamma_2 \cdot \ln(dbh_p) + \gamma_3 \cdot \ln(h_p))^2)$ in later simulations.

Tab.3

3.2 Biomass prediction and uncertainty estimates

As shown in Fig. 1, the *agb* estimator and relative *RMSE* (root mean square error) became stable after 1000 iterations. The black curve denotes the *agb* estimator, and the gray curve illustrates their relative *RMSE* values. The two curves initially fluctuate considerably and then gradually become smoother, especially after 400 time steps.

Fig. 1

The traditional model analysis method proposed by Ståhl et al. (2010, 2014) was implemented, and the results were compared with those of our method. As shown in Tab.4, the Chinese fir *agb* per hectare was 17.361 ± 1.088 t/ha using the traditional approach and

17.343±0.995 t/ha using the proposed method. The relative *RMSE* values due to sampling error were approximately 4% with each method, but those associated with model uncertainty decreased from 4.815% to 2.354% using our method, which caused the total relative *RMSE* to decrease from 6.267% to 5.736%.

Tab.4

3.3 Impacts of sample size on agb estimates and associated uncertainty

The *agb* and uncertainty estimates for various calibration data set sizes ($m=50$, $m=80$, $m=110$ and $m=140$) are shown in Tab.5. After 1000 time steps, the total relative *RMSE* increased from 6.631% to 12.917% as the sample size decreased from $m=140$ to $m=50$. The influence of sample size on the total *RMSE* was mainly reflected in the model uncertainty, of which the relative *RMSE* increased from 4.983% to 8.253% as the sample size decreased.

Tab.5

As shown in Fig. 2, the curves of both *agb* and relative *RMSE* initially fluctuated at all sample sizes and then subsequently stabilized. However, the duration of fluctuations varied among sample size levels; more time was required for stabilization as sample size decreased. For example, the values of relative *RMSE* stabilized after almost 100 time steps at $m=140$, but they required at least 600 times to stabilize at $m=50$.

Fig. 2

4. Discussion

The *agb* per unit area at the regional level and its uncertainty considering both sampling error and model uncertainty were estimated using a new method. As shown in Fig.1, the total estimates of mean *agb* per unit area and their relative *RMSE* values stabilized after a certain number of simulations. However, initially (e.g., at fewer than 200 replications), the *agb* estimators and their uncertainty fluctuated severely. The fluctuations can be attributed to introducing the varying dummy residuals into the *agb* observations when the allometric model for *agb* estimation was refitted during each simulation. When the model fitting is based on different groups of samples instead of a fixed calibration data set, as proposed in Ståhl et al. (2014), the parameter estimates of the model and their covariance matrixes must change, and the resulting estimates of mean *agb* and their variance, especially the model uncertainty contribution, are likely to be variable. As such, using the traditional model analysis method, the estimates of both *agb* and its uncertainty might be unreliable because they rely heavily on the data source. However, our method can generally reduce the variability in the estimates of *agb* and the corresponding uncertainty by averaging the model parameters and covariance matrixes after numerous iterations.

The influence of correlations among trees in the DS_m was neglected in our method because the sample trees that were used for model fitting were harvested over a large area throughout the province. In addition, the effects of correlations among trees in the DS_e were ignored because several studies have considered these effects to be negligible in uncertainty assessment for large-area estimation (Breidenbach et al. 2014; Berger et al. 2014; Ståhl et al. 2014). Thus, the conclusions in the present study were drawn based on the nil correlation assumption.

Compared with the estimates of uncertainty obtained from the traditional model analysis method, the total relative *RMSE* of our method was lower by approximately 0.531% (Tab.4).

Although this improvement appears slight, the *agb* estimator in this study is the estimates of *agb* per unit area, which means that the uncertainty estimate might be considerably larger when multiplied with area to obtain the total estimates at the regional level. Therefore, we consider the slight improvement to not be negligible.

Additionally, the method proposed in this article can significantly reduce the influence of model uncertainty on the variance of *agb* estimates, which plays an essential role in decreasing the total *RMSE*. This reduction might due to the implementation of 1000 Monte Carlo simulations, which effectively decreased the effects of uncertainty in the model parameters and their covariance.

As indicated in Tab.4, the *RMSEs* associated with model uncertainty were larger than those associated with sampling error regardless of the method used. This finding differs from those of Breidenbach et al. (2014), Berger et al. (2014) and Ståhl et al. (2014). This difference might reflect the much smaller size of the calibration data set in this study than those of previous studies. However, Zianis (2008) used small sample sizes for model fitting and made conclusions that were similar to those of the present study.

Applying our method, four levels of calibration data set size were implemented to study how the sample size for model fitting influenced the large area estimation and whether sample size could be ignored when the number of simulations performed by Monte Carlo approach was increased. Tab.5 shows that minor relative *RMSEs* values were contributed by sampling error for the larger calibration data set, which is expected in theory, although the sample size for modeling had little effect on the large-area estimates.

Nevertheless, the size of the calibration data set obviously affected the *RMSEs* due to model uncertainty. The model-related *RMSEs* increased as the size of the calibration data set decreased, and the effects could not be improved by increasing the simulation time (Tab.5). This finding is likely due to the fact that the larger calibration data sets resulted in smaller

covariances between model parameters, as supported by the results of McRoberts and Westfall (2014). Therefore, the sample size used for modeling is not be negligible in regional *agb* estimation, regardless of the number of simulations that are implemented using Monte Carlo approach.

Additionally, as illustrated in Fig.2, the estimates of both *agb* and uncertainty based on smaller calibration data set sizes fluctuated more severely throughout all simulations and required longer durations to stabilize than the estimates based on larger data set sizes. Moreover, the separation distance between the curves of the *agb* estimator and relative uncertainty indicates that smaller data sets for modeling have a wider range of *RMSE* values for large-area estimates than do larger data sets. Thus, increasing calibration data set size could shorten the calculation time required to obtain stable estimates and improve the reliability of total estimates at the regional scale.

The proposed method can be directly used to assess the effects of sampling error and model uncertainty on large-area estimates of *agb* in cases where the model for estimation must be refitted from the calibration data set; in such cases, it might not be possible to apply existing models because the covariance matrix of model parameters may not be available (Ståhl et al. 2014). In addition, differences between linear regression models and nonlinear regression models for biomass estimation, or differences between species-specific models and nonspecific models are negligible as addressed by McRoberts and Westfall (2014).

In future studies, our method could be applied to estimate belowground biomass, which is likely to generate larger model-related uncertainty than estimates of aboveground biomass because of the lower accuracy of model estimation (Ståhl et al. 2014).

5. Conclusion

This study proposed a new method of biomass and uncertainty estimation based on

forest inventory data and tree biomass equations at the regional level. Additionally, the traditional method of model analysis proposed by Ståhl et al. (2014) was compared with the proposed method. Three conclusions were drawn in this study. First, our method effectively and separately quantified sampling-related uncertainty and model-related uncertainty and decreased the effects caused by variability in model parameters. The results suggest that the reliability and stability of both biomass and uncertainty estimates can be increased by combining Monte Carlo simulation and the model analysis method based on forest inventory data. Second, the uncertainty in regional estimates of forest biomass due to model error can be effectively decreased using our method, which results in higher prediction accuracy. This conclusion suggests that more studies should focus on reducing the effects of model error and improving model performance. Third, larger calibration data sets significantly reduced the uncertainty associated with model error, which could not be offset by increasing the number of simulations. Therefore, increasing the sample size for model fitting can effectively decrease the uncertainty in biomass estimates and improve the accuracy of regional estimates of *agb*.

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1 **Table captions**

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3 **Tab.1 Statistical characteristics of trees in the DS_e and DS_m**

data set	Variable	Number of trees	Mean	Minimum	Maximum	SD
DS _e	<i>dbh</i> (cm)	58206	10. 23	5. 00	47. 20	3. 94
	<i>h</i> (m)		7. 83	4. 37	23. 10	2. 46
DS _m	<i>dbh</i> (cm)	150	16. 59	1. 60	42. 40	11. 89
	<i>h</i> (m)		11. 49	1. 90	27. 00	6. 87
	<i>agb</i> (kg)		114. 47	0. 35	596. 59	154. 91

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Tab.2 Mathematical terms used in simulating uncertainty

Num.	Sign	Meaning
1	dbh_p	$p=1, 2, \dots, m$. The diameter at breast height (cm) of the p th sample tree in the DS_m ; m is the number of sample trees in the DS_m and $m=150$ in this study.
2	h_p	$p=1, 2, \dots, m$. The tree height (m) of the p th sample tree in the DS_m .
3	g_{pp}	$pp=1, 2, \dots, m$. The simulated agb observation of the p th sample tree in the DS_m .
4	$\hat{g}_p$	$p=1, 2, \dots, m$. The agb prediction (kg) of the p th sample tree in the DS_m .
5	$\hat{\epsilon}_p$	$p=1, 2, \dots, m$. The residual term simulated for the p th sample tree in the DS_m .
6	dbh_{ij}	$i=1, \dots, n_j; j=1, \dots, n$. The diameter at breast height (cm) of the i th tree of a certain species group within the j th plot in the DS_e ; n_j is the number of trees of a certain species group in the j th plot in the DS_e ; n is the number of plots in the DS_e .
7	h_{ij}	$i=1, \dots, n_j; j=1, \dots, n$. The tree height (m) of the i th tree of a certain species group within the j th plot in the DS_e .
8	g_{ij}	$i=1, \dots, n_j; j=1, \dots, n$. The “true” value (unobservable) of agb of the i th tree of a certain species group within the j th plot in the DS_e .
9	$\hat{g}_{ij}$	$i=1, \dots, n_j; j=1, \dots, n$. The agb prediction of the i th tree of a certain species group within the j th plot in the DS_e .
10	G_j	$j=1, \dots, n$. The plot-level agb of the j th plot in the DS_e .

- 11 G'_j $j=1, \dots, n$. The partial derivative of model $G_j(x, \alpha)$ in regard to the parameter α s in the j th plot in the DS_e, where $\alpha = (\alpha_0, \alpha_1, \alpha_2)'$.
- 12 $\hat{G}_j$ $j=1, \dots, n$. The plot-level *agb* prediction of the j th plot in the DS_e.
- 13 A_j $j=1, \dots, n$. The area of the j th plot in the DS_e.
- 14 μ_j The *agb* per unit area in the j th plot in the DS_e.
- 15 $\bar{\mu}$ The predicted mean of *agb* estimates per unit area over all plots in the DS_e.
- 16 μ An dummy “true” value of *agb* estimator (estimates per unit area) in the DS_e.
- 17 $E[\cdot]$ The statistical expectation of the item in parentheses.
- 18 $RMSE$ Root mean square error; $RMSE_S$ is the uncertainty associated with sampling error; $RMSE_M$ is the uncertainty associated with model error; $RMSE_C$ is the combined uncertainty associated with both sampling error and model error.
- 19 $\bar{\mu}^k$ $k=1, \dots, n_k$. The $\bar{\mu}$ in the DS_e during the k th simulation using the Monte Carlo approach; n_k indexes the number of simulations.
- 20 $\tilde{\mu}$ The final *agb* estimator in the DS_e after plenty of simulation times.
- 21 $Var(\tilde{\mu})$ The total uncertainty of the final *agb* estimator.

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* mean R^2 of height-diameter equations of 9 levels

15 Tab.4 Estimates of mean *agb* per hectare and the corresponding uncertainty associated with different error sources based on the two methods

Calibration data set size	Method*	<i>Agb</i>	<i>RMSE</i> (t/ha)			<i>Relative RMSE</i> (%)**		
		Estimator (t/ha)	Sampling error	Model uncertainty	Total	Sampling error	Model uncertainty	Total
150	①	17.361	0.697	0.836	1.088	4.015	4.815	6.267
	②	17.343	0.696	0.408	0.995	4.014	2.354	5.736

16 *① is the traditional model analysis method and ② is the improved method

17 ** The ratio of the *RMSE* to the *agb* estimator

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Tab.5 Estimates of mean *agb* per hectare and the corresponding uncertainty associated with different error sources based on four levels of calibration data set size using the proposed method

Calibration data set size	<i>Agb</i> Estimator (t/ha)	<i>RMSE</i> (t/ha)			<i>Relative RMSE</i> (%)		
		Sampling error	Model uncertainty	Total	Sampling error	Model uncertainty	Total
140	17.354	0.697	0.865	1.151	4.014	4.983	6.631
110	17.328	0.696	0.972	1.373	4.015	5.609	7.921
80	17.311	0.695	1.136	1.698	4.015	6.561	9.809
50	17.178	0.690	1.418	2.219	4.019	8.253	12.917

Figure captions

Fig. 1 Simulated *agb* estimator and uncertainty using the improved method

Fig. 2 Simulated *agb* estimator and the corresponding uncertainty based on four sample sizes using the improved method