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Experimental Measurements of the Paschen-Back Effect in Helium Triplet Transitions

Andrew S. Taylor and Oleg V. Batishchev

Abstract: Experimental confirmation of the theoretical calculations of the Zeeman and Paschen-Back Effects in the prominent $2^3P - 2^3P$ He I 1083 nm NIR transition by means of high-resolution spectroscopy is reported. A novel approach using a small-pixel-size fluorescence camera delivers required spectral resolution. In particular, it allows to quantify with good precision the spectral shift of the weak λ_0 1082.9 nm line from the zero magnetic field position. Measurements of the strong VIS He I 587 nm and 706 nm triplet line shapes are reported for $B = 0\text{--}1.0$ T interval, indicating non-linear wave function mixing as well.

Key words: Zeeman Effect, Paschen-Back Effect, High-Resolution Spectroscopy, Helium Triplet Transitions, VIS-NIR spectra.

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1. Introduction

The measurement of atomic energy level perturbations through emission spectroscopy is an important tool for studying the properties of physical matter. The splitting of lines due to the Zeeman Effect is the only method for measuring the magnetic field in interstellar objects, which cannot be reached with traditional *in situ* probes. This technique has been used on Mg I and Fe I emission in sunspots [1, 2] and Cr I and Fe II emission in rotating stellar media [3, 4]. The splitting of molecular band peaks has been used for the same purpose as well [5].

The NIR helium transition $2^3S_J - 2^3P_J$ at 1083 nm is particularly important for basic scientific research and practical applications. It has been used to test Quantum Electrodynamics and measure the fundamental fine-structure constant [6]. The triplet is sensitive to polarization effects, making it an ideal diagnostic tool for investigating plasma structures in the corona [7] or for measuring the direction of the magnetic field vector in solar prominences [8]. This transition has recently been shown to be beneficial in advanced physics laboratories to investigate the Zeeman Effect with high-resolution spectroscopy [9].

The same triplet in ^3He is used in Magnetic Resonance Imaging (MRI) measurements, requiring a systematic understanding of the energy levels. Saturated absorption experiment by optical pumping has measured the shifting of hyperfine ^3He levels, used in MRI, in a field of up to 1.5 T [10]. However, despite its importance to many fields of scientific research, the shifting and splitting of ^4He energy

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levels in a magnetic field have not been analyzed completely, while the unperturbed levels have been measured by microwave resonance absorption with unprecedented precision in ^4He [14, 15].

The spectrum of 1083 nm triplet was calculated by Ovsiannikov and Tchapyguine [11] for three characteristic values of the B -field: $B = 0.005$, 0.85 , and 10 T [12], corresponding physically to the Zeeman Effect, the Intermediate regime with both Paschen-Back and Zeeman characteristics, and the Paschen-Back limit. They solved the Schrödinger Equation considering the three 3P_J levels to be strongly interacting with each other and the field. The wave function for magnetic quantum number $m = 0$ is calculated as a superposition of states with $J = 0, 1$, and 2 . For $|m| = \pm 1$ the wave function is a superposition of the $J = 1$ and 2 states whereas the $|m| = 2$ function is composed of only $J = 2$ states. Energy levels and wavelengths are calculated to create theoretical spectra, and the relative intensity coefficients are reported. The $B = 0.85$ T value was selected to correct the two-level approximation data of Bethe and Salpeter [13] at that field strength.

In this work we first measure the 1083 nm triplet using a large spectrometer with medium-dispersion grating and small-pixel fluorescence CCD camera in $B = 0$ – 1 T range. This high-resolution spectroscopic method was successfully applied previously to Doppler [16] and isotopic shifts [17], and Zeeman Effect [9]. New results are compared to the theoretical predictions. Finally, the effects of magnetic field on VIS 706 nm and 587 nm He I multiplets are measured. The results are presented and discussed as well.

2. Experimental Apparatus and Calibration

To analyze emitted light, a McPherson M216 1 m focal-length asymmetric Czerny-Turner spectrometer was employed. The spectrometer is equipped with a 1200 g/mm grating and has a dispersion of $6.6 \text{ \AA/mm}$. To capture complex VIS and NIR spectral lines, a new high-contrast AmScope MU1203-FL fluorescence CCD camera was installed. The camera has a 4000×3000 pixel array with $1.85 \text{ }\mu\text{m}$ square pixel size. A fixed $10 \text{ }\mu\text{m}$ slit was installed at the entrance plane of the spectrometer. A Geissler spectral tube was used as a light emission source, placed in a custom-build permanent Magnet Holder, which produces a variable B -field up to $B = 1.07$ T. More details on the spectrometer setup, CCD positioning system, neodymium Magnet Holder design and operation, and data collection method can be found in ref. [9].

The spectrometer and camera must be accurately calibrated to obtain the per-pixel resolution Δp for every spectral interval, due to grating dispersion dependence on incident angle. In NIR it was done using the $2^3S_J - 2^3P_J$ transition at zero field. The 1083 nm triplet wavelengths are listed in Table 1. Data are taken from the NIST Basic Atomic Spectroscopic reference handbook [18]. They consist of two closely-spaced lines, $\lambda_2 = 1083.0340$ nm and $\lambda_1 = 1083.0250$ nm, and a separate $\lambda_0 = 1082.9091$ nm line. Since $\Delta\lambda_{21} \ll \Delta\lambda_{10}$, the pattern can be approximated as a two-level system, with the $J = 2, 1$ levels strongly coupled together and a separate $J = 0$ level. This fact is used for spectral resolution calibration.

Table 1. Fine structure triplet wavelengths of the prominent He I transitions at zero field [18]. The 3P_J can be considered a two-level approximation, with $J = 0$ and a strongly-coupled $J = 2, 1$ level.

Line	1083 nm	706 nm	587 nm
λ_0	1082.9091	706.5709	587.5966
λ_1	1083.0250	706.5215	587.5640
λ_2	1083.0340	706.5177	587.5615

Fig. 1. He I 1083 nm triplet spectrum at zero field. Each dot corresponds to an individual pixel on the CCD, approximately 0.89 pm apart. The flat top of the strong peak corresponds to combined λ_2 and λ_1 . The secondary peak corresponds to λ_0 .

The measured spectrum at $B = 0$ in Fig. 1 shows two peaks as expected from the two-level approximation, with the stronger line about three times more intense than the secondary line. The stronger line has a flat top of ~ 8 pixels in width, or ~ 7 pm. We know that the closely-spaced λ_2 and λ_1 are 9 pm apart, so the spectral resolution of the spectrometer can be estimated in the 7–9 pm range for a tight doublet. Such resolution is inadequate to capture the fine line structure. However our primary goal is to measure an individual line response to a magnetic field, i.e. the predicted λ_0 shift direction and magnitude. To reduce statistical error of an individual line position, multiple measurements can be taken, since the spectrometer has remarkable wavelength stability with negligible spectral drift. Therefore, for a single line wavelength, one-third of per-pixel resolution Δp is taken as systematic error, and the precision of a measurement is set to one-half of Δp . From the *a priori* known exact position of the three unperturbed peaks, and multiple measurements in the two-level approximation, the mean Δp value was found to be ~ 0.89 pm/pixel.

One important comment about a single line shape shall be made here. Though the asymmetrical spectrometer used here was designed to produce a coma-free image, the artifacts are still present. They are particularly pronounced near the upper wavelength limit of the spectrometer of ~ 1100 nm, see Fig. 1. Accordingly, the integral intensity of a line and relative amplitude are assumed to be measured with less accuracy than its spectral position. Furthermore, line intensity can be affected by radiation self-absorption, which was observed in a helium surface-wave discharge plasma [20]. In our case the gas density, discharge dimensions, and degree of ionization are approximately one order of magnitude less, but this effect cannot be completely excluded. Doppler broadening of emission line is on the order of several picometers and contributes to its shape as well. However, it does not affect the central wavelength value. Additional information on spectral calibration, resolving power, Doppler and optical aberration effects are given in ref. [21].

3. Results for NIR 1083 nm Triplet

Theoretical predictions were made for three separate domains of weak, intermediate, and strong magnetic field. The intermediate $B = 0.85$ T case is the most complicated, but also most interesting for applications. At the same time, our present equipment cannot resolve small spectral variations in weak fields under 0.1 T and deliver fields stronger than $B \sim 1$ T. Therefore, our measurements are focused on the transitional 0.1–1 T region with special emphasis on the $B = 0.85$ T point. The 19 possible transitions of He I 1083 nm in are listed in Table 2, following the nomenclature of ref. [12].

Table 2. Line number and state transitions for the $^3S_{Jm} - ^3P_{Jm}$ triplet. Theoretical wavelength values have been calculated from ref. [12] data. Measured wavelength values are given for $B = 0.85$ and 1.07 T. The last column shows the value change with sign in the wavelength.

Line number	Transition $^3S_{Jm} - ^3P_{Jm}$	Theoretical λ (nm) at $B = 0.85$ T	Experimental λ (nm) at		$\Delta\lambda$ (pm) for $B = 1.07$ vs 0.85 T
1	$^3S_{20} - ^3P_{11}$	—	—	—	
2	$^3S_{2-1} - ^3P_{10}$	—	—	—	
3	$^3S_{2-2} - ^3P_{1-1}$	1083.0812	1083.081	1083.097	16
4	$^3S_{21} - ^3P_{11}$	—	—	—	
5	$^3S_{20} - ^3P_{10}$	1083.0611	1083.061	1083.076	15
6	$^3S_{2-1} - ^3P_{1-1}$	1083.0297	1083.030	1083.030	stationary
7	$^3S_{22} - ^3P_{11}$	1082.9878	1082.987	1082.980	-7
8	$^3S_{21} - ^3P_{10}$	1082.9836	1082.981	1082.975	-6
9	$^3S_{20} - ^3P_{1-1}$	1082.9678	1082.970	1082.962	-8
10	$^3S_{10} - ^3P_{11}$	1083.1033	1083.107	1083.128	21
11	$^3S_{1-1} - ^3P_{10}$	1083.0756	1083.076	1083.093	17
12	$^3S_{11} - ^3P_{11}$	1083.0287	1083.030	1083.030	stationary
13	$^3S_{1-1} - ^3P_{1-1}$	—	—	—	
14	$^3S_{11} - ^3P_{10}$	—	—	—	
15	$^3S_{10} - ^3P_{1-1}$	—	—	—	
16	$^3S_{00} - ^3P_{11}$	1082.9913	1082.989	1083.014	25
17	$^3S_{00} - ^3P_{10}$	1082.8970	1082.901	1082.897	-4
18	$^3S_{00} - ^3P_{1-1}$	1082.8041	1082.805	1082.777	-28
19	$^3S_{10} - ^3P_{10}$	1083.0094	1083.010	1082.987	-23

Lines 1–9 are produced from the five states of the 3P_2 level, lines 10–15 from 3P_1 , and lines 16–18 from 3P_0 , respectively. Line 19 is forbidden by selection rules in the anomalous Zeeman Effect, but appears as the spin and orbit begin decoupling as the interaction with the B -field becomes stronger.

Figure 2 shows the spectra of the He I 1083 nm triplet for $B = 0$ – 1.0 T in 0.1 T intervals. Uncertainty of B is 0.02 T as given in ref. [9]. The spectra have features of the high-field Paschen-Back regime, the low-field anomalous regime, and an intermediate-field region between the two. Thirteen lines are partial-resolved, i.e. not all are visible for a given B value due to line overlap. A side-by-side comparison of spectra with increasing B helps to determine line dynamics more accurately. Cross-lines have been drawn to indicate peak positions, which allows us to measure wavelengths by means of interpolation for individual lines at various B -fields when peaks are overlapping. The tight 6/12 line does not shift in wavelength, which confirms the high reproducibility of wavelength measurements by this spectrometer.

The red dashed cross-lines correspond to the peaks of the Zeeman splitting for $\lambda_{2,1}$. The field interaction energy becomes comparable in magnitude to the λ_2 , λ_1 level splitting at $B > 0.1$ T. Above 0.4 T the three lines are resolvable and peak separation increases linearly with field. Below 0.4 T, the spectral resolution is insufficient to detect the splitting but is enough to show broadening.

The solid cross-lines show the anomalous Zeeman Effect from the λ_0 peak. At $B = 0.2$ T a clear Zeeman triplet is visible and the side peaks separation increases linearly with field. It is approximately twice as large as the Paschen-Back splitting, corresponding to the expected Landé factor $g_L = 2$ for this transition. The trend of the cross-line for line 16 suggests that it crosses the 6/12 line at $B > 1.2$ T.

Intermediate lines that do not correspond to either of the two regimes are shown by blue dotted cross-lines. At $B = 0.3$ T, a line is distinguishable between the 6/12 and 7 lines. This corresponds to the previously-forbidden line 19. Secondary shoulders on either side of lines 8 and 11, corresponding

Fig. 2. He I 1083 nm triplet for $B = 0\text{--}1.0$ T. Spectra are uniformly offset vertically so intensity is arbitrary. The double, dashed cross-lines correspond to the Zeeman split in the Paschen-Back limit. The solid cross-lines correspond to the anomalous Zeeman Effect triplet of the λ_0 peak. The dotted cross-lines correspond to other spectral lines. All are indicated by numbers following ref. [12] notation.

to lines 9 and 10, respectively, are resolved at $B > 0.7$ T.

Figure 3 shows the He I 1083 nm spectrum at 0.85 T, with arrows designating numbered lines. The spectrum matches the field strength used for calculations. Remarkably enough, we detected all 13 potentially visible lines and located them in the same order as theoretically predicted. The red arrows correspond to the Paschen-Back splitting, made up of lines 3 & 11, 6 & 12, and 7 & 8, respectively. The black arrows correspond to the anomalous Zeeman Effect triplet of lines 16, 17, and 18. The blue arrows show lines 5, 9, 10, and 19, corresponding to intermediate field transitions.

Unfortunately, ref. [12] does not provide calculated wavelengths. But by using their proportional scaling in their theoretical spectrum, we were able to calculate wavelength values. These are shown in column 3 of Table 2. Measured wavelength values with 1 pm accuracy are presented in column 4.

The movement of lines with field strength was studied. The line shape for the maximum $B = 1.07$ T is shown in Fig. 4. As B increases, lines 10, 3, 11, 5, and 16 increase in wavelength while 19, 7–9, 17, and 18 decrease. Between $0.85 < B < 1.0$ T, ref. [12] calculates that line 16 and 19 cross, and that 16 moves across lines 6 & 12 towards longer wavelengths. The spectra show that lines 16 and 19 cross at a field strength between Figs. 3 and 4. Line 16 has not yet crossed lines 6 & 12 at $B = 1.07$ T.

Very interestingly, though, ref. [12] calculates that line 17 is shifted away from λ_0 at $B = 0.85$ T. The spectra show the shift beginning at $\sim B = 0.5$ T and increasing with field strength. This is the exact position where ref. [12] states that the field interaction energy becomes comparable to the $J = 1, 0$ level splitting, resulting in wave function mixing of all three 3P levels. The wavelength shift also corresponds to a shift in the 3P_0 energy level at $\sim B = 0.5$ T shown in ref. [10] and ref. [19].

The results of the λ_0 change due to wave functions mixing is found in Fig. 5. The line 17 shift away from λ_0 is seen in Fig. 5a. At 1 T, the shift is ≈ 12.5 pm. In the $0.4 < B < 0.6$ T region a non-linear

Fig. 3. He I 1083 nm triplet at $B = 0.85$ T. Theoretical spectrum calculated in ref. [12]. The red arrows of lines 7/8, 6/12, and 3/11 show the Paschen-Back Effect triplet. The black arrows of lines 16, 17, and 18 show the anomalous Zeeman Effect triplet of λ_0 . Blue arrows of lines 9, 19, 5, and 10 show intermediate field transitions.

Fig. 4. He I 1083 nm triplet at $B = 1.07$ T. The red arrows of lines 7/8, 6/12, and 3/11 show the Zeeman split in the Paschen-Back limit. The black arrows of lines 16, 17, and 18 show the anomalous Zeeman Effect triplet. Blue arrows of lines 9, 19, 5, and 10 show intermediate field transitions. Lines 16 and 19 have crossed compared to the spectrum at $B = 0.85$ T, and line 17 has blue shifted farther from λ_0 .

dependence is observed, followed by a linear segment for $B > 0.6$ T.

The anomalous Zeeman triplet centered on line 17 can also be used to measure the Bohr magneton value $\mu_B \approx 5.79 \times 10^{-5}$ eV/T, as was done previously with this spectrometer for different lines [9]. Figure 5b shows the average wavelength spacing between triplet peaks as a function of the field. The data points for $0.5 < B < 0.7$ T deviate from the linear regression line, once again confirming the non-linear behavior and wave function mixing of the 3P_0 level in that magnetic field region. Using the expression for Paschen-Back spectral splitting amplitude

$$\Delta\lambda = \frac{\mu_B g_L}{hc} \lambda^2 B \quad (1)$$

and the expected $g_L = 2$ for this level, a value of $\mu_B = 6.03 \pm 0.26 \times 10^{-5}$ eV/T is calculated from the slope. This exercise confirms the accuracy of our spectral measurements to be about 5%. We can invert Eq. (1) and calculate the internal B from $\Delta\lambda$. For instance, if we insert $\Delta\lambda_{21}$ from Table 1 for each of the triplets, we find the same value of the internal atomic field $B \approx 0.16$ T in each case. This corroborates with the statement in ref. [19] that the Paschen-Back Effect is observed in helium starting with external field strength $B \sim 0.4$ T.

Fig. 5. Magnitude of wavelength shifts of the $J = 0$ level. (a) $\Delta\lambda_0$ of line 17 from λ_0 vs. B -field strength. Uncertainty is ~ 0.3 pm. (b) Plot of $\Delta\lambda$ vs. B -field for the Zeeman splitting in the Paschen-Back limit of the λ_0 line.

4. Results for VIS 706 nm and 587 nm Triplets

Since the 3P_J levels are involved in many other lines in the He I spectrum, and the spectrometer can resolve the Zeeman splitting throughout the VIS range, we can study wave function mixing in other helium transitions. The two strongest VIS transitions are the 2^3P-3^3S 706.5 nm and the 2^3P-3^3D 587.6 nm multiplets. The lines were spectrally calibrated via the same two-level approximation as for 1083 nm, with mean $\Delta p = 1.2$ and 1.3 pm/pixel for 706 nm and 587 nm, respectively.

The 706 nm triplet involves the same 3P and 3S levels as 1083 nm. This means the triplet includes the same 19 lines from Table 2, so we can use the same numbering as in Table 2. However, the lower level in the transition is 3P , so that the 706 nm $\lambda_{0,1,2}$ are in descending wavelengths. Figure 6a shows the staggered 706 nm spectra for $B = 0-1.0$ T. Again the $\lambda_{1,2}$ peak is split into a Paschen-Back triplet, the λ_0 peak is split into an anomalous Zeeman triplet, and line 17 increasingly shifts away from λ_0 after $B > 0.4$ T.

Fig. 6. Intensity vs. wavelength of the 2^3P VIS transitions, for $B = 0-1.0$ T. Spectra are uniformly offset vertically so intensity is arbitrary. (a) The 706 nm triplet. A red shift in wavelength of λ_0 begins at $B = 0.6$ T and continues to increase with the field strength; at 1.0 T it is red-shifted by ~ 7.2 pm. The anomalous Zeeman triplet of λ_0 is resolved at 0.3 T and increases linearly with applied B . (b) The 587 nm multiplet. The Paschen-Back Effect triplet of $\lambda_{2,1}$ is pronounced at high fields. The λ_0 line $^3P_{00} - ^3D_{10}$ red shifts from the zero field position starting at $B \sim 0.5$ T.

The intensity vs. wavelength for the 587 nm multiplet is shown in Fig. 6b, with $B = 0$ –1.0 T. The 587 nm $\lambda_{0,1,2}$ are in the same order as 706 nm, but the spectra consists of 27 lines instead of 19. Both the Paschen-Back and anomalous Zeeman triplets are once again visible, but intermediate lines are not resolved even at high field.

The spectral shifts of the secondary peak, $\Delta\lambda_0$, for both 706 nm and 587 nm are given in Fig. 7 as functions of B varied in 0–1 T range. Similarly to the 1083 nm line, the 706 nm line 17 increases non-linearly at $0.4 < B < 0.7$ T. At 1.0 T, the shift is ≈ 7.2 pm. The 587 nm line, on the other hand, also shifts from λ_0 but behaves differently, since it is created by the $^3P_{00} - ^3D_{10}$ transition. The shift magnitude $\Delta\lambda_0$ does not appear to increase at high field strengths. This is most noticeable as B is increased from 0.9 to 1 T: the 706 nm shifts by another 3 pm while the 587 nm line remains stationary. According to ref. [19], the $^3D_{10}$ state shifts linearly in energy with the field. Our data supports this conclusion for $B > 0.8$ T. Additional spectra at $B > 1.0$ T should be taken to confirm this behavior.

Fig. 7. Red shift in wavelength for both 706 nm and 587 nm of λ_0 line vs. B -field strength in the 0–1.0 T range. The 706 nm line deviates non-linearly from λ_0 with increasing field strength. For the 587 nm line, the spectral shift $\Delta\lambda_0$ appears to increase linearly in the B 0.3–1.0 T interval.

5. Conclusion

Spectra of the He I $2^3S_J - 2^3P_J$ 1083 nm triplet were collected in the $B = 0$ –1.07 T magnetic field range using a 1 m Czerny-Turner spectrometer with medium dispersion grating and small-pixel-size fluorescence CCD camera. The spectrum at $B = 0.85$ T was compared to the one calculated in ref. [12] for the same field strength. The position of all 13 spectral lines was verified, though not all lines could be resolved as individual peaks, due to mutual overlaps and optical aberrations. The crossing of lines 16 and 19 was observed at $B \sim 0.95$ T. The shift of line 17 from λ_0 was measured as a function of B , demonstrating complex behavior. The shift begins at $B = 0.5$ T, confirming the theoretical model that the wave functions of the three 3P levels begin mixing at that field strength.

The effect of magnetic field on 706 nm and 587 nm He I triplet transitions involving 3P was measured as well. We measured splitting associated with the Paschen-Back and Zeeman Effects. The wavelength shift from λ_0 was observed in the 706 nm line. It is in the opposite direction compared to the 1083 nm line. The 587 nm λ_0 peak also moves away from the zero field wavelength towards the red-shift direction as the 706 nm case, but at a lesser rate. A higher-resolution experimental setup and stronger magnetic field would be necessary to confirm the observed dynamics of this transition with better accuracy.

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