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**Probabilistic investigations on the watertightness of jet-grouted ground
considering geometric imperfections in diameter and position**

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ABSTRACT

The effect of geometric imperfections in both diameter and position of jet-grouted columns on the watertightness of underground cement-treated slab is investigated in this study. A three-dimensional discretized algorithm is proposed to facilitate the detection and measurement of untreated zones that penetrate the treated slab. The normalized flow rate of a cement-treated slab is then evaluated by calculating the harmonic average area of the penetrated defect. Statistical evaluation of the gross flow rate through the penetrated defects is carried out via Monte Carlo simulations. The results show that a more economic design is obtainable if intra-column variation of diameter is considered or multi-shaft jet-grouting is used. Based on the statistical results, a reliability-based design method is proposed for designers to strike a balance among various design parameters, including slab thickness, depth, column diameter and column spacing.

KEYWORDS: Jet-grouting; Watertightness; Probability; Monte Carlo Simulation; Random Field

INTRODUCTION

Jet-grouting is widely used in deep excavations and underground constructions, usually by a method of injecting high-momentum fluidal stabilizer into in-situ soils through rotating small-diameter nozzles. The final stabilizer and in-situ soil mixture is characterized by high stiffness, strength, and low permeability after curing. As a result, the ground treated with this technique is often used both as temporary support and as a temporary waterproof surface.

In a normal design, the treated ground can be altered into many desired shapes and dimensions by arranging the length, diameter, and spacing of columns. This facilitates the ground's application in various scenarios. For instance, when used for limiting soil and ground water inflow in deep excavation, a cement-treated layer of overlapping soil-cement columns is usually installed near the toe of a retaining wall prior to excavation (e.g. Madinaveitia 1996; Sondermann and Toth 2001; Modoni et al. 2016). Similarly, when tunnelling in urban areas, a pre-treatment on the soft clay surrounding the tunnelling face, or a layer of canopy or umbrella are usually required to prevent excessive disturbance of the environment (e.g. Pellegrino and Adams 1996; Lignola et al. 2008; Flora et al. 2011; Arroyo et al. 2012; Ochmansk et al. 2015a).

The columnar units are usually installed in an overlapped fashion to ensure that the treated ground can provide sufficient global strength, stiffness, and water-tightness. Although the jet-grouted columns are carefully installed to leave no zones untreated, untreated zones are frequently observed in various geotechnical applications, including in the bottom plug of a deep excavation (e.g. Madinaveitia 1996; Sondermann and Toth 2001; Flora et al. 2012; Modoni et al. 2016), and improved surrounding of a tunnel (e.g.

Shirlaw 1996; Pellegrino and Adams 1996; Morey and Campo 1999; Lignola et al. 2008; Flora et al. 2007, 2011; Arroyo et al. 2012). The existence of such untreated zones can potentially result in sudden inflows of ground water, and soils, which can draw down local water tables and cause significant surface settlement (Morey and Campo 1999). In extreme cases, piping running through untreated zones may lead to major collapse (Morey and Campo 1999).

The existence of untreated zones basically arises from two geometrical imperfections (e.g. Croce and Modoni 2007; Flora et al. 2011; Modoni et al. 2016): (1) imperfect column orientation, and (2) varying column diameter. The former flaw is a result of unavoidable inclination of the column's axis, even though the columns are installed with due discretion to maintain verticality. The latter flaw originates from two sources. The first one is the random variation of soil properties. As many prior studies have noted (e.g. Modoni et al. 2006; Ho et al. 2007; Shen et al. 2013; Flora et al. 2013), the diameter of jet-grout columns is affected significantly by various factors, such as soil type, strength, and permeability. More recently, data driven models can be found on the prediction of column diameter (e.g. Ochmański et al. 2015b; Tinoco et al. 2016). Therefore, for vertical columns, the diameter may vary along the column's axis corresponding to a layer of soil. On the other hand, the column's diameter may also vary spatially even in relatively homogeneous soil layer. This may be attributed to the uncertainty in craftsmanship and equipment function in different columns.

The jet-grouted waterproof ground can be essentially divided into two types according to the relative direction of potential seepage and column axis. For the first type, the seepage direction is approximately perpendicular to the column axis (Fig. 1(a)). Examples of this include canopies consisting of horizontal and sub-horizontal

columns, as well as cut-off consisting of vertical columns. Croce and Modoni (2007) examined the effect of geometric imperfections on the water-tightness of jet-grout cut-off. In this study, the cut-off consists of one or two layer(s) of overlapping jet-grout columns to prevent the ground water from seeping inside. For the second type, the seepage direction occurs approximately parallel to the column axis (Fig. 1(b)). An example of this is a cement-treated slab or a bottom plug installed near the toe level of an earth-retaining wall. Modoni et al. (2016) examined the effect of these two geometric imperfections on the waterproofing capability of a bottom plug treated with jet-grout columns using a probabilistic approach. The random variations of both types of geometric imperfections were considered using Monte Carlo simulations. In the above studies on both waterproof types, the axis orientation (azimuth and inclination angles) and column diameters were assumed to be independent random numbers. The size of geometric defects was estimated by measuring a representative cross-section in the middle layer. From a probabilistic point of view, this research provided valuable information on the size of geometric defects. However, when treating column diameters as random numbers, columns are still assumed to be perfectly cylindrical, which is frequently not the case. In practice, vertical jet-grout columns are usually "calabash-shaped" (Fig. 2(a)). Moreover, flow rate is estimated by the untreated area ratio at the treated layer's middle depth. This is based on the assumption that the size of geometric defects increases in an approximately linear relationship with depth, and that all the untreated zones penetrate the treated layer's thickness. However, due to the existence of intra-column diameter variation (calabash shape, Fig. 2(a)), the geometric defect size may not have a linear relationship to depth. Further, some untreated zones at a certain depth may be sealed in another depth (Fig. 2(b)).

To provide a more realistic simulation, and therefore a more accurate estimation of the effects of geometric imperfections on the water-tightness of jet-grouted waterproofs, a random process is employed to simulate intra-column diameter variation. This idea was originated from Modoni and Bzówka (2012) who noted that the random deviation from the mean diameter should be as the result of a random process. Additionally, a three-dimensional discretized algorithm is used to estimate the treated ground's flow rate. In the current study, the square layout of jet-grouting columns is considered (see Fig. 2(a)). The results based on this layout pattern will be on the conservative side, since the triangular layout is likely to be more efficient for waterproofing (see Modoni et al. 2016).

METHODOLOGY

Statistical characteristics

As many papers have addressed (e.g. Croce and Modoni, 2007; Flora et al., 2011; Modoni et al. 2016), geometric imperfections involve random orientations of column axis and diameter. Random orientations of column axis are usually described by two independent parameters (e.g. Eramo et al. 2011; Flora et al. 2011; Liu et al. 2015; Modoni et al. 2016); namely, azimuth (α) and inclination angle (β) (Fig. 3). Table 1 summarizes the statistical characteristics of the above parameters. Basically, the azimuth is assumed to be uniformly distributed within $[0, \pi]$, which is reasonable given that the mast's axis can incline to any direction. In prior studies (e.g. Croce et al. 2007; Arroyo et al. 2012), the inclination has been found to be normally distributed around zero, negative inclination angle indicates that the axis is oriented to the opposite direction. In some single-shaft jet-grouting productions, the jet grout machine's mast is

relocated each time that a jet-grout column is installed. Therefore, it is reasonable to assume that each column's pair of axis orientation parameters is independent from that of other columns. Similar assumptions are available in (e.g. Eramo et al. 2011; Flora et al. 2011; Modoni et al. 2016). However, in cases where multi-shaft jet grouting is employed, several masts are mounted on the same rigid beam or plate and are simultaneously calibrated. Multiple jet-grouted columns are then installed before the machine is relocated and recalibrated in the next operation step. Intuitively, the difference in orientation parameters among columns in the same operation step should be much smaller than the difference among different operation steps. For example, in a Kallang River ground improvement project, a group of jet-grout rigs was installed on sliding rails. In this case, one row of jet-grouted columns is installed and then the sliding rail is relocated to install another row. It can be expected that the relative orientation differences among the columns in the same row (operation) should be significantly less than the differences among different rows. For such circumstances, a random field (rather than independent random numbers) is suggested because it can reflect the correlation of orientation parameters among adjacent columns. The scale of fluctuation (SOF) of orientation parameters can mathematically reflect the scope of similarity. The SOF is loosely defined as the longest distance where a similarity between two points can be observed. In other words, if the distance between the centres of two columns is much smaller than the SOF, the orientation parameters of the columns are very close. For multi-shaft jet grouting, a longer SOF would be assigned in the along-row direction than in the cross-row direction. Liu et al. (2015) adopted similar assumptions in considering positioning error of multi-shaft cement-treated columns. In sum, for single-shaft jet-grouting, a set of independent random variables are used to simulate orientation parameters, while for multi-shaft jet-grouting, a random field with

spatial correlation is used.

Many studies discussed the prediction of jet-grouted columns' diameters (e.g. Modoni et al. 2006; Ho et al. 2007; Shen et al. 2013; Flora et al. 2013). Theoretical studies summarize that, apart from jet-grouting parameters, the average diameter of jet-grout columns is related to soil properties such as permeability and strength. Due to natural soil's stratified layout, the jet-grouted columns usually vary significantly along each column, creating a calabash shape. Table 1 summarizes the statistical characteristics of jet-grout columns' diameters. Assuming that the variation in column diameter is mainly attributed to the soil profile's stratified structure, the same random process can be generated for all columns. This should be reasonable, given that the SOF of soil properties in the horizontal direction is often much larger than the SOF in the vertical direction (Phoon and Kulhawy 1999). As a result, a random process with a vertical correlation (SOF) is used to simulate the variation of diameter.

To summarize, Monte Carlo simulations would be conducted using the above-mentioned random variables. In single-shaft jet grouting column, the azimuth is a uniformly distributed random number ranging from $[0, 2\pi]$. The inclination is a random number with normal distribution with zero mean. The azimuth-inclination pair for each column is regenerated, indicating that each column's orientation parameters are independent of each other. A jet-grouted column's diameter is established by a random process, and it remains the same for all columns. Once the random variables and random processes are generated, any untreated zones can be determined.

Discretized Algorithm of Geometric Imperfection Evaluation

The geometric imperfection of a cement-treated layer is essentially a three-

dimensional problem when the random variations of axis orientation and diameter are simultaneously considered. Although the treated and untreated zones can be readily drawn with modern graphic software, the following two issues still remain. The first issue involves the detection of any untreated zones that penetrate through the cement-treated layer. This penetration would result in seepage passages. Second, if such seepage passages exist, it is of practical interest to estimate their scale and associated flow rates.

Modoni et al. (2016) examined these issues by taking a representative slice from imperfectly treated ground. This simplifies the three-dimensional problem into a two-dimensional one. However, an untreated zone observed in a two-dimensional analysis may not necessarily penetrate the whole depth along the out-of-plane direction. This is because the untreated zone is likely to be sealed in other slices, which is particularly the case when the intra-column diameter variation is taken into account (Fig. 2(b)). Therefore, a digitalized three-dimensional model is established in the current study to address the two issues mentioned above.

Following the finite element method's discretization concept, the entire treated ground is discretized into small cuboids, each of which consists of eight nodes, as Figs. 4(a)-(c) show. Fig. 5 shows the algorithm's flow chart. Each node is examined to determine whether it is in the range of any adjacent column (Fig. 4(b)). If a node is not included in any column, then it is marked as untreated. In this way, it is easy to obtain all the nodes in untreated zones. An untreated zone's size of cross-sectional area in a slice should be proportional to the number of nodes located within the untreated zone (see Fig. 4(b)). Specifically, a node would represent an area that is equivalent to the area of the elementary cuboid's horizontal cross-section (see Fig. 4(b)). If any two arbitrarily

selected, successive slices have overlapped untreated zones, an untreated zone can penetrate through the treated layer's whole thickness. Fig. 4(d) shows three scenarios. In the first scenario, although the untreated areas reside in each slice, there is no overlap between the two highlighted adjacent slices. The second scenario cannot penetrate either, due to the fully treated (sealed) slice. This circumstance exists most likely when one layer is very soft, leading to a large column diameter and therefore a fully treated slice. It should be noted that the difference between adjacent slices is exaggerated in Fig 4(d) for the sake of illustration. In simulation, the vertical mesh should be sufficiently refined to ensure that the change of untreated area between adjacent slices are gradual. Otherwise, the overlapping of two adjacent untreated areas may be particularly restrictive and may lead to inaccurate estimation of untreated area and therefore flow rate.

Estimation of Flow Rate

Fig. 6 illustrates the geometric parameters. Although the estimation of the size of untreated zones is three-dimensional, the flow of ground water through the jet-grouted layer is assumed as a one-dimensional seepage as the seepage direction is mainly vertical as discussed by Modoni et al. (2016). According to Darcy's law, if a cell is penetrated by an untreated zone, the flow rate through the unit cell is given by:

$$Q = ki(z)A(z) \quad (1)$$

where:

k - coefficient of permeability of untreated soil, it is assumed to be constant in the untreated cohesionless soils;

$i(z)$ -hydraulic gradient at certain depth z ; $i(z) = \frac{dh}{dz}$, h is the water head.

$A(z)$ - area of cross-section at certain depth z ;

Given that $i(z) = \frac{dh}{dz}$, Eq. (1) becomes

$$\frac{Q}{A(z)} dz = k dh \quad (2)$$

According to the principle of continuity, the flow rate at each depth is a constant.

Therefore, integrating both sides of Eq. (2) gives

$$\int_0^t \frac{Q}{A(z)} dz = \int_0^H k dh \quad (3)$$

where:

t - the thickness of the cement-treated layer;

H - the water head difference between top and bottom of the treated layer.

As a result, the flow rate can be written as

$$Q = \frac{kH}{t} \tilde{A}(t) \quad (4)$$

where $\tilde{A}(t) = \frac{1}{\frac{1}{t} \int_0^t \frac{1}{A(z)} dz}$ is the harmonic average of the area along the seepage passage.

Compared to the arithmetic average, the harmonic average is dominated by low values.

If a seepage passage's minimum area is very close to zero, then the harmonic average of the area along the passage is almost zero, offering a near-zero flow rate. Although the harmonic average's theoretical mean and variance are difficult to derive, its numerical solution is easy to calculate, especially when the model is discretized. A discretized form of harmonic average $\tilde{A}(t)$ can be written as:

$$\tilde{A}(t) = \frac{1}{\frac{1}{n} \sum_{j=1}^n \frac{1}{A_j(z)}} \quad (5)$$

where $A_j(z)$ is the area of the seepage passage at the j^{th} slice, and the treated ground is equally discretized into n slices. Since the untreated zones are recorded and the continuous seepage passages are marked, the flow rate at each seepage passage can be estimated. To apply this method in broader scenarios, the flow rate can be normalized by the treated layer's geometric parameters (Modoni et al. 2016).

$$\Omega = \frac{Qt}{kHA} = \frac{\tilde{A}(t)}{A} \quad (6)$$

where A is the treated ground's total area. The normalized flow rate is actually equal to the ratio of the untreated area's harmonic average to the unit cell's area. In real-world situations, a treated layer consists of many unit cells, and the resulting flow rate through any given treated ground can be calculated by summing up each unit cell's flow rate.

The current method is designed particularly for water tightness of cement treated slab, this being different from finite element analysis (FEA) which is more general and versatile. It resembles the traditional finite element analysis in that it also discretizes the ground into small elements. However, it does not involve solving governing equations and therefore is significantly less time-consuming than traditional FEA. In one of the cases in subsequent parametric study, a 9 m X 9 m X 2 m treated ground is discretized into quadrilateral elements with dimensions 0.04 m X 0.04 m X 0.04 m, this creating 2.65 million elements. One typical realization of such finite element analysis using ABAQUS could take more than 10 hours. In contrast, in this study, the average calculation time of one realization is about 9 seconds. Results and discussions

Reference case

We used a reference case according to the statistical characteristics summarized in Table 1. Table 2 lists the cases' configuration parameters. The diameter of a jet-grouted column is simulated as a vertical random process with a mean value of 1.4 m and coefficient of variation (COV) of 0.1, which is used for all columns. The column radius' vertical SOF is selected as 0.5 m, within the range of vertical SOF of natural soils. The axis orientation's azimuth α is simulated as an independent random variable uniformly distributed over $[0, 2\pi]$ because the axis can incline in any direction without bias, this being validated by field data (Eramo et al. 2011). Similarly, the inclination angle β is assumed to follow a normal distribution with zero mean and a standard deviation of 0.3 degrees because the axis is calibrated to be vertical and the probability density function is bell-shaped (see Arroyo et al. 2012; Modoni et al. 2016). Compared to the field test results of standard deviation used in Modoni et al. (2016) and Eramo (2011), these assumptions are on the conservative side. The column spacing can be described by the ratio s/D , where s is the centre-to-centre distance (set constant as 1 m for simplicity), and D is the mean diameter. In the reference case, an s/D ratio of 0.7 is used. This ratio corresponds to the largest column space to ensure no untreated zones exist, as long as there are no variations in orientation or diameter. A squared treated ground consisting of 16 cells is simulated. Each cell's horizontal dimension is equal to the centre-to-centre distance. The mesh size of each cuboid is 1/25 of the centre-to-centre distance. Therefore, a seepage passage with a diameter larger than approximately 4 cm can be detected. The effects of the treated ground's scale and mesh size will be discussed later. In this study, 1000 realizations are considered in order to obtain a statistical representative result.

Convergence Study

Fig. 7 shows the effect of mesh size. A large portion of cumulative probability falls in a zero normalized flow rate when the mesh is coarser than 0.1 m. This is unsurprising, since untreated passages with small sizes cannot be detected. When the mesh size is under 0.04 m, the results converge. To ensure sufficient accuracy while maintaining computational efficiency, a mesh size of 0.04 m is selected.

Fig. 8 shows the effect of the model scale. As we can see, the normalized flow rate remains zero for over 80% of single-cell cases, which is a similar result to that obtained by Modoni et al. (2016). This is mainly because in most realizations the untreated zones cannot be captured by a limited-scope, single-cell model. Increasing the scale model mitigates this limitation. By doing so, cases where only a few cells are penetrated can be detected. Prior trials suggest that when the model scale is larger than 4×4 , the results converge. Therefore, a model with 4×4 cells is used.

Field Test Validation

Two indirect verifications are included as there is a lack of experimental or field measurement of such a problem. The first one comes from Madinaveitia (1996) in which a jet-grouted earth plug was installed as an inverted bench of a tunnel to resist both earth pressure and ground water inflow. However, water spouted at different points due to isolated faults in the treatment. The author did not measure the flow rate but indicated that the flow rate was relatively high. The excavation conditions are summarised in Table 3. Similarly, Eramo et al. (2011) reported a jet-grouted bottom plug in a subway station in Barcelona. In this case, no significant ground water inflow was reported. The excavation conditions are also summarised in Table 3. Table 3 shows that the Madinaveitia (1996) has shallower excavation (safer) and closer relative

spacing (safer) but thinner treated slab (more dangerous) than Eramo et al. (2012). The configurations are input into the model and Monte Carlo simulations are conducted for both cases. The Monte Carlo simulation results of the two cases are shown in Fig. 9(a) and (b). As can be observed, the normalised flow rate (Ω) in Madinaveitia (1996) is much higher than Eramo et al. (2011). When the flow rate per 1000 m² is considered, the difference is even greater. The median (50th percentile) of Madinaveitia (1996) case is about 0.38 L/s per 1000 m² which is very close to the limit drainage capacity (Modoni et al. 2016). In contrast, the Eramo et al.'s (2011) case show a very low flow rate. This is consistent with the fact that Madinaveitia (1996) encountered significant ground water inflow while Eramo et al.'s (2011) design did not encounter significant ground water inflow.

Parametric Study

As mentioned above, the column diameter variation comprises of inter-column variation and intra-column variation. In the present study, the jet-grout column's diameter was simulated as a random process that varies with depth, creating "calabash-shaped" columns. To replicate the case without intra-column variations of column diameter, a very long SOF of the random process is used to eliminate the intra-column diameter variations. Fig. 10 shows the comparison between cases with and without intra-column diameter variations. The case without intra-column variations in diameter predicts a larger normalized flow rate than the reference case. This is mainly because the existence of intra-column variation may lead to a discontinuity of untreated zones, as Fig. 2(b) shows. Therefore, taking intra-column variation into consideration potentially facilitates a more economic design. As mentioned above, it is assumed that the diametric variation was mainly contributed by the stratified structure of soils.

Therefore, intra-column variation instead of inter-column variation of diameter is considered of this study.

In real projects, jet grouting equipment features more than one shaft aligned in a row so as to produce multiple overlapping columns in one operation. In such a case, it is reasonable to expect only minor relative differences between the axis orientations of columns in the same operation. In contrast, the difference between columns installed in separate operations may be more pronounced. Thus, the along-row direction correlation is likely to be much larger than the along cross-row direction correlation. To reflect this correlation, two-dimensional random fields are generated for azimuth and inclination, respectively using spectral representation method (Shinozuka, et al. 1991) though other methods are also available (e.g. Karhunen–Loève expansion method by Phoon et al. 2005; Modified Linear-Estimation Method by Liu et al. 2014, 2016). The SOF of orientation mathematically represents this correlation value, which is conceptually the same as the SOF of the random process used to simulate the columns' diametric variation. Liu et al. (2015) adopted a similar concept when simulating the correlation of positioning errors of deep-mixed columns. Following Liu et al. (2015), the cross-row SOF of orientation parameters is held constant as 1 time radius, while the SOF of orientation parameters of the along-row is variable. Fig. 11(a) shows a typical graph of untreated zones in the bottom layer. As can be observed, untreated, adjacent zones in the same row have similar shape and size. As Fig. 11(b) shows, when the correlation of orientation parameters is considered, a slightly smaller, normalized flow rate of penetrated untreated zones can be obtained. Moreover, this normalized flow rate decreases when the SOF of orientation parameters in the along-row increases. This can be attributed to the fact that the existence of along-row correlation of orientation

parameters reduces any relative orientation difference within the same operation, therefore making the overlap more reliable. This demonstrates that a multi-shaft operation is favourable in terms of waterproofing.

Fig. 12 shows the effect of COV of diameter on normalized flow rate. When the COV of diameter increases, a larger probability concentrates on high flow rate than low flow rate. Intuitively, when the diameter variation is larger, it is more likely to generate large diameters which may seal the seepage passage. This explains why the empirical cumulative distribution function remains almost zero for a large portion. However, once the seepage passage forms, the flow rate would be very high because extremely low diameter may be generated due to the high COV of diameter. In reality, this happens when the soil profile is highly variable, creating significant intra-column variation in diameter.

Expectedly, as shown in Fig. 13, the normalized flow rate increases significantly as the standard deviation of inclination angle increases. The standard deviation of 0.3 degree is the upper bound value as those used in the parametric studies in Arroyo et al. (2012) and Modoni et al. (2016). Therefore, it is conservative to use standard deviation of 0.3 degree in the reference case.

We also examined the effect of dimensions (thickness and depth of treated layer and column spacing, as shown in Figs. 14-16). These dimension parameters are normalized by the mean diameter of column. Expectedly, normalized flow rate increases with a thinner treated layer, a larger embedment depth, and larger column spacing.

ENGINEERING IMPLICATIONS

We examined a variety of combinations of thickness and depth to form a design chart. Since normalized flow rate is a random variable, a rationally pessimistic value (e.g. quantile design value by Ching and Phoon 2013) is recommended as the representative value in the design. In many geotechnical engineering codes, the 5th percentile is selected as the representative value of the lowest allowable strength. Similarly, tending towards the conservative side in the present study, a representative normalized flow rate (Ω) higher than 95% cases is selected. In design, depth of excavation usually determines the treated layer depth. The water-head difference from bottom to top, and the untreated soil's permeability, are determined by the real situation. Although the water-head difference is related to the treated layer's thickness (which must yet be determined), a conservatively high water-head difference can be assumed first. The mean column diameter can be assumed using values from similar, adjacent projects or from theoretical or empirical relationships. Therefore, the designer needs to consider the treated layer's column spacing and thickness. In many codes, the drainage capability is limited by 0.5 L/s per 1000 m²; that is

$$\frac{Q}{A} \leq \delta = 5 \times 10^{-7} (\text{m}^3/\text{s})/\text{m}^2 \quad (7)$$

Substituting Eq. 6 into Eq. 7 gives,

$$\frac{\Omega D}{t} \leq \frac{\delta D}{kH} \quad (8)$$

Since the thickness (t) is unknown in the design, it is separated from $\frac{\delta D}{kH}$. The column diameter D is multiplied on both sides to normalize the treated layer's thickness. Assuming that single-shaft jet-grouting is used (more pessimistic than multi-shaft jet-

grouting) and the COV of diameter is 0.2 and standard deviation is 0.3° which is more conservative than most field data (e.g. Langhorst et al. 2007; Eramo et al. 2011; Arroyo et al. 2012), we carried out a series of Monte Carlo simulations with different combinations of normalised spacing (s/D), normalised depth (depth/ D) and normalised thickness (t/D). The $\frac{\Omega D}{t}$ value corresponding to 95th percentile of normalised flow rate Ω is plotted against normalized depth for different column's spacing and thickness of the treated layer (Fig. 17). The designers can determine the treated layer's thickness and the column spacing by the following process (flow chart shown in Fig. 18):

(1) Determine relevant excavation values, including excavation depth; water-head difference between the bottom and top of treated soil H ; permeability of untreated soil k . These values can be roughly evaluated by site investigation such as bore-holes and coring. Then, estimate the average diameter (D) of jet-grouted columns using either practical experience in adjacent sites or theoretical relationships (e.g. Modoni et al. 2006; Shen et al. 2013). As mentioned above, the drainage capacity (δ) is usually limited to 0.5 L/s per (1000 m²). Calculate the $\frac{\delta D}{kH}$.

(2) Assume a column spacing s to find the right chart in Fig. 17, according to the s/D ratio.

(3) Draw a horizontal line of $\frac{\Omega D}{t} = \frac{\delta D}{kH}$ and intersect a vertical line at the desired depth. Since the curves get closer when thickness increases, a linear interpolation of thickness is basically conservative.

Illustrative Example

A cement-treated layer is constructed in a deep excavation at 25 m below ground to

serve as an underground strut and waterproof surface. The ground water is 5 m below the ground level. Site investigation shows that the soil at 5 m below ground was sandy soil with a permeability coefficient up to 1×10^{-6} m/s. According to the results obtained in adjacent sites and the available jet-grouting equipment, the column diameter's mean is approximately 1 m. Column spacing and the treated layer's thickness must still be determined.

The excavation depth and water-head difference between the treated layer's bottom and top are 25 m and 20 m, respectively. Therefore, the value $\frac{\delta D}{kH}$ is equal to 0.025 and a horizontal line with $\frac{\Omega D}{t} = 0.025$ is drawn in Fig. 17(a). This is intersected with a vertical line depth/ $D=25$. The ratio of column spacing to diameter of 0.71 is first assumed. According to Fig. 17(a), the thickness is about 3.5 times the diameter (3.5 m in this case). Then, two additional ratios of column spacing to diameter are assumed (namely 0.67 and 0.625), providing a design thickness of 3.1 m and 2.4 m, respectively. It is clear that the required thickness can be smaller when columns are more closely installed. To make an optimal decision, the total treatment volume can be compared. The treatment volume for a square layout can be roughly evaluated via

$$V = \frac{A\pi D^2 t}{4s^2} \quad (9)$$

For a 100 m² ground, the treatment volumes of s/D ratio of 0.7, 0.67, and 0.63 are 556 m³, 406 m³, and 313 m³. As we can see, the volume with s/D 0.63 is the most economical design in terms of waterproofing. However, the spacing cannot be too close, since it is limited by multiple factors including the centre-to-centre distance of the multi-shaft jet-grout machine and the possible “masking effect” (Morey and Campo, 1999).

CONCLUDING REMARKS

This study establishes a three-dimensional numerical model of a cement-treated layer with multiple unit cells. The effects of random variations due to imperfect positioning and diameter are both considered. Compared to prior studies, the present work considers intra-column (rather than inter-column) diameter variation by simulating a column's diameter as a random process, leading to "calabash-shaped" columns. Although the horizontal SOF of natural soils is generally much greater than the size of a jet-grouting project, inter-column variations in diameter can still often be observed in reality, which may be caused by other factors, including the uncertainties in workmanship. In the current study, these kind of inter-column variations were not considered, and it will be the subject of future study. A discretized algorithm is implemented to facilitate the three-dimensional modelling and statistical evaluation of untreated zones. This enables the detection of untreated zones penetrating the entire layer in a three-dimensional scenario. The flow rate is related to the harmonic, rather than arithmetic, average of the areas along the seepage passage.

Throughout this study, a series of parametric studies are conducted and presented. It is found that a reliably stable cumulative distribution can be obtained only when the model's scale is large enough. By considering intra-column diameter variation, a more economic design can be achieved. Additionally, a multi-shaft treatment can help to reduce penetrated untreated zones. Considering the columns' orientation parameters as independent random variables is a conservative approach. The effects of an excavation's geometric parameters (embedment depth of treated layer, thickness, and column spacing) are examined. Based on these cumulative distributions, a rationally pessimistic estimation of normalized flow rate is extracted for various cases. A dimensionless

design chart considering different values of column spacing to diameter ratio, embedment depth, and thickness is also plotted. This study proposes a design guideline for a wide range of design parameters. The design chart enables a designer to obtain an optimized design via trade-offs among parameters such as column space and treated layer's thickness. The designers should bear in mind that the design chart is based on a static seepage analysis on single-shaft jet-grouting with COV of diameter of 0.2 and standard deviation of inclination angle of 0.3° which are meant to be on the conservative side. The calculated design value should be on the conservative side when lower COV of diameter, standard deviation of inclination angle, closer relative spacing and multi-shaft jet-grouting are used.

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Figure Captions

Fig. 1 Two types of waterproof (a) seepage direction perpendicular to the column axis; (b) seepage direction parallel the column axis

Fig. 2 A cement-treated layer with geometric imperfections (a) three-dimensional view of jet-grout columns with imperfect orientation and diameter variation; (b) Sectional view

Fig. 3 Definition of azimuth (α) and inclination angle (β) of a jet-grouting column

Fig. 4 Discretization of treated layer (a) elevation drawing (b) plan view at the j^{th} slice, and (c) judgement of penetration

Fig. 5 Flow chart for discretized algorithm

Fig. 6 Illustration of geometrical parameters

Fig. 7 Effect of mesh size (Ref. means Reference Case)

Fig. 8 Effect of model scale

Fig. 9 Empirical histogram of two field cases (a) normalised harmonic average of untreated area (b) Flow rate per 1000 m²

Fig. 10 Effect of intra-column variation and inclination

Fig. 11 Effect of SOF of orientation parameters of column axis (a) untreated zones at the bottom of the treated layer ($\delta_x = 4.5D$, $\delta_y = 0.5D$, δ_x is the SOF in along-row direction and δ_y is the SOF in cross-row direction) (b) empirical CDF of normalized flow rate.

Fig. 12 Effect of COV of diameter (COV(D) means diameter's coefficient of variation)

Fig. 13 Effect of inclination angle (std(β) means standard deviation of inclination angle, unit: degree)

Fig. 14 Effect of slab thickness

Fig. 15 Effect of depth

Fig. 16 Effect of column spacing

Fig. 17 Design chart for thickness of cement-treated layer, (a) $s/D=0.71$; (b) $s/D=0.67$; (c) $s/D=0.625$

Fig. 18 Design flow chart

Table 1 Variability of Geometric Imperfection of Jet-grout Column

No.	Type of Natural Soil	Average of Diameter (m)	COV(D)*	S.D. (β)**	Remarks	Source
1	Clay-Silt	-	0.02-0.05	-	-	Croce et al. (2004)
	Sands		0.02-0.10		-	
	Gravels		0.05-0.25		-	
2	Sandy Clay	1.1	0.06-0.19	-	Derived from field data of diameter at different depths, horizontal column	Langhorst et al. (2007)
3	Silty Sand	0.71-1.11	0.06	0.07***	Vertical columns in Vesuvius site	Croce et al. (2007)
	Sandy Gravel	1.06-1.20	0.19		Vertical columns in Polcevera site	
4	Silty Sand	2.5	-	0.16°	Vertical columns in Barcelona site	Eramo et al. (2011)
5	Sandy Clay	0.38	0.13	-	Vertical column with lower water content	Arroyo et al. (2012)
		0.48		-	Vertical column with lower water content	
		0.75	0.17	0.17°	(Sub) Horizontal column	

*COV(D) is the coefficient of variation of diameter;
**S.D. (β) means standard deviation of inclination;
***This standard deviation of inclination is obtained from different site (Isola Serafini).

Table 2 Reference case and parametric study

Remarks	Diameter D (m)	Spacing s (m)	Depth (m)	Water head difference H (m)	Thickness t (m)	COV (D)	S.D. (β)
Reference Case	1.4	1	20	20	2	0.1	0.3°
Effect of Diameter	1.4	1	20	20	2	0.1	0.3°
	1.5	1	20	20	2	0.1	0.3°
	1.6	1	20	20	2	0.1	0.3°
Effect of Depth	1.4	1	10	10	2	0.1	0.3°
	1.4	1	20	20	2	0.1	0.3°
	1.4	1	30	30	2	0.1	0.3°
	1.4	1	40	40	2	0.1	0.3°
Effect of thickness	1.4	1	20	20	1	0.1	0.3°
	1.4	1	20	20	2	0.1	0.3°
	1.4	1	20	20	3	0.1	0.3°
Effect of COV of Diameter	1.4	1	20	20	2	0.05	0.3°
	1.4	1	20	20	2	0.1	0.3°
	1.4	1	20	20	2	0.2	0.3°
Effect of Standard Deviation of Inclination	1.4	1	20	20	2	0.1	0.1°
	1.4	1	20	20	2	0.1	0.3°
	1.4	1	20	20	2	0.1	0.5°

*The reference case is bold and the varying parameters are highlighted

Table 3 Configuration of two case studies

	Diameter	Spacing	Depth	Water head difference	Thickness	COV (D)	S.D. (β)	Permeability Coefficient
	D (m)	s (m)	(m)	H (m)	t (m)		($^\circ$)	k (m/s)
Madinaveitia (1996)	1.4	1.2	10	8	3	0.1	0.3	1×10^{-5} *
Eramo et al. (2011)	2.5	2.0***	20	12	12	0.1	0.3	1×10^{-5} ***

* The untreated soil was described as very porous beach sand. The permeability coefficient of sand ranges from 1×10^{-4} to 1×10^{-6} m/s depending on the proportion of fine grains. In this case, the untreated sand was 10 m below the ground was assumed to be moderately dense. As a result, $1\text{e-}5$ m/s was used.

** The untreated soil was described as silty sand and deposit was buried 20 blow the ground. Therefore, it is assumed to be $1\text{e-}6$ m/s which is lower than that in Madinaveitia (1996).

*** The layout in Eramo et al. (2011) was triangular. However, by assuming the same spacing to be a square, one can obtain a more pessimistic estimation (higher untreated area)

Draft

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