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AdS/QCD with generalized warp factors and Stability

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Abstract

In this paper, we studied the thermodynamic behavior of generalized warp factor for the corresponding AdS metric background. In order to discussed the stability of theory we studied of thermodynamical properties of generalized metric background. Here also in order to obtain the quantity of sound for the measure of stability of system, we need to calculate the energy density and pressure. Such parameter of sound for the properties of stability constraint some conditions on the corresponding theory as \(\theta\). We used the corresponding conditions and rewrite the action of theory. The new action give us opportunity to study the AdS black hole and thermally charged AdS black hole.

Keywords: Stability; AdS black hole; Thermally charged AdS; Generalized warp factors.

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1 Introduction

As we know the gauge/gravity duality play important role in strongly-coupled systems near critical points. In that case, the corresponding system has a scaling symmetry [1]. Such system at a critical point can be described by a conformal field theory (CFT). From gauge/gravity point of view, the gravitational theory may be described by corresponding metric. In that case the pointed above metric is asymptotically Anti-de Sitter (AdS) space time. Here we note that, the critical points are governed by dynamical scalings in many physical systems, so that for the space and time we have different scale. The simple example for the such critical point is Lifshitz fixed point. The corresponding system is spatially isotropic and scale invariant for the time direction. There is anisotropic scaling which is characterized by a dynamical exponent, \(z\). So, we have following scale symmetry,

\[ t \rightarrow \lambda^z t, \quad x_i \rightarrow \lambda x_i, \]  

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where $t$ is time and $x_i$ are spatial coordinates of $d$ dimensional space. Ref.s. [2-3] found gravity description of Lifshitz fixed points from gauge/gravity duality point of view. In that case the corresponding metric will be as,

$$
\text{ds}^2 = -r^{2z} dt^2 + r^2 \sum_{i=1}^{d} dx_i^2 + \frac{dr^2}{r^{2z}},
$$

(2)

where such solution will be set up with Einstein-Maxwell-Dilaton theory. Also, nowadays there are numerous studies of Lifshitz-like black brane geometries as Ref.s. [4-16]. If we consider Abelian gauge fields to the Einstein-Maxwell-Dilaton (EMD) theory, it gives us anisotropic scaling and also an overall hyperscaling factor. So, the corresponding metric background for the Abelian gauge fields theory will be as [6],

$$
\text{ds}^2 = r^{-\frac{2\theta}{d}} \left( - r^{2z} dt^2 + r^2 \sum_{i=1}^{d} dx_i^2 + \frac{dr^2}{r^{2z}} \right)
$$

(3)

where the constant $z$ and $\theta$ are dynamical and hyperscaling violation exponents, respectively. So, the hyperscaling violating Lifshitz solutions with above metric background have been investigated in several papers [17-29]. The geometry (3) is general and covariant under following scale transformations,

$$
t \rightarrow \lambda^z t, \quad r \rightarrow \lambda^{-1} r, \quad x_i \rightarrow \lambda x_i, \quad ds_{d+2} \rightarrow \lambda^{\frac{\theta}{d}} ds_{d+2}.
$$

(4)

The distance in this theory is not invariant under the scaling due to non-zero $\theta$ in corresponding metric background. Also, here the context of AdS/CFT shows violations of hyperscaling in dual field theory. In this case with $(d+1)$-dimensional theories the entropy scales as $T^{\frac{d-1}{d}}$ [30-31]. This metric background play important role in condensed matter physics with finite charge density. Recently, the construction of finite temperature and finite charge density solutions with above metric background in probe D-branes is investigated by Ref. [34]. On the other hand, the study of gauge/gravity provides relation between the gravity theories in the AdS spacetime and conformal field theories on the boundary of AdS spacetime. In order to have dual scale invariant gauge theory, we need some generalization of the metric which is given by warp factors in the metric background. Therefore, the corresponding geometry will be as,

$$
\text{ds}^2 = r^{2\alpha} \left( - r^{2z} f(r) dt^2 + \frac{1}{r^2 f(r)} dr^2 + r^2 d\vec{x}^2 \right),
$$

(5)

where $\alpha = \frac{\theta}{d}$. Here the non invariant of distance in context of AdS/CFT lead to violation of hyperscaling in dual field theory. On the other hand the thermodynamical properties of black holes in Anti de Sitter (AdS) space are quite different from those of black holes in asymptotically flat or de Sitter space. The main reason is that the AdS space acts as a confined cavity so that black holes in AdS space can be thermodynamically stable. In particular, there exists a minimal Hawking temperature for a Schwarzschild black hole in
AdS space, below which there does not exist any black hole solution, instead a stable thermal gas solution exists. For a given temperature as a above mentioned the minimal one, there exist two black hole solutions. The black hole with smaller horizon is thermodynamically unstable with a negative heat capacity. The second black hole with larger horizon is thermodynamically stable with a positive heat capacity. As we know, Ref. [34] shown that in the lower temperature the grand potential increase, it means that the value of chemical potential for the constant $\theta$ increasing. In that case the corresponding paper [34] got the special solution of black hole (equation 2.10) and the first term of $f(r)$ is fixed by one. But, we take general solution and the first term of $f(r)$ the same as second term and third term will be depend to $r$. For the general solution we investigate the stability condition. So here we note that, the stability and instability form of black holes from thermodynamical point of view play important role in phase transition. Also stability properties will be important for the describing of $P - V$ critically of black holes with some critical points. So, all above information give us motivation to investigate the stability of system with some parameter as a $\theta$ in thermodynamics. So, the most important quantities for the investigation of stability here is energy density and corresponding pressure. So, we take advantage from two thermodynamics quantities and effect of warp factor in corresponding metric background. The interesting conditions for the stability of such corresponding system lead us to study the AdS black hole and thermally charged AdS black hole.

2 The solution of Einstein-Maxwell-Dilaton theory

Here, we are going to start with Einstein-Maxwell-Dilaton action with potential $V(\phi)$ which is given by,

$$S = -\frac{1}{16\pi G} \int d^{d+2}x \sqrt{g} \left[ R - \frac{1}{2} (\partial \phi)^2 + V(\phi) - \frac{1}{4} e^{\lambda \phi} F^2 \right], \quad (6)$$

where $V(\phi) = V_0 e^{\gamma \phi}$ and $\lambda$, $\gamma$ and $V_0$ are parameters of the model. Also, the corresponding potential $V$ will be appear in the following equation,

$$R_{\mu\nu} + \frac{V(\phi)}{d} g_{\mu\nu} = \frac{1}{2} \partial_{\mu} \phi \partial_{\nu} \phi + \frac{1}{2} e^{\lambda \phi} \left( F^\rho_{\mu} F_{\mu\nu} - \frac{F^2}{2d} g_{\mu\nu} \right). \quad (7)$$

So, the solution of equations (6) and (7) will be as,

$$ds^2 = r^{2\alpha} \left( r^{2\alpha} f(r) dt^2 + \frac{1}{r^{2\alpha} f(r)} dr^2 + r^2 d\vec{x}^2 \right), \quad \phi = \phi(r), \quad F_{\tau\tau} \neq 0. \quad (8)$$

which is hyperscaling violation metric [33]. Here $\alpha = -\theta/d$, $\theta$ is hyperscaling violation exponent. By using equations of motion, the metric function is given by [33],

$$f(r) = \frac{C}{r^{2\alpha - \gamma \beta}} - \frac{m}{r^{d(1+\alpha) + 1}} + \frac{Q^2}{r^{2d(1+\alpha) + \lambda \beta - 2\alpha}}. \quad (9)$$
We assume $\gamma = -\lambda$ and also one can rewrite equation (9) as,

$$f(r) = \frac{A}{r^{2\alpha + \lambda \beta}} - \frac{m}{r^{d(1 + \alpha) + 1}} + \frac{Q^2}{r^{2d(1 + \alpha)}}.$$  \hspace{1cm} (10)

In here we consider general case, because Ref. [33] assume the first term as $\frac{A}{r^{2\alpha + \lambda \beta}} = 1$. Also, note here the values of $C$, $A$ and $Q^2$ will be as,

$$C = \frac{V_0 e^{-\lambda \phi_0}}{d(1 + \alpha)(\alpha d + 2 \alpha + d + 1 - \lambda \beta)},$$  \hspace{1cm} (11)

$$A = C + Q^2,$$  \hspace{1cm} (12)

and

$$Q^2 = -\frac{\rho^2 e^{-\lambda \phi_0}}{2d(1 + \alpha)(\lambda \beta + d \alpha + d - 2 \alpha - 1)}.$$  \hspace{1cm} (13)

So, the components of metric are given by,

$$\sqrt{g} = r^{2\alpha + d(1 + \alpha)},$$  \hspace{1cm} (14)

$$g_{tt} = r^{2\alpha + 2} f(r),$$  \hspace{1cm} (15)

$$g_{rr} = \frac{f(r)}{r^{2\alpha - 2}},$$  \hspace{1cm} (16)

$$g_{xx} = r^{2\alpha + 2}.$$  \hspace{1cm} (17)

Next section we use the components of metric and investigate the thermodynamic properties of system.

### 3 Thermodynamics properties

Now we are going to compute the thermodynamics quantities as Hawking temperature, energy density and entropy density. In that case the corresponding Hawking temperature will be as,

$$T = \frac{\partial_r g_{tt}}{4 \pi \sqrt{g_{tt} g_{rr}}} \bigg|_{r = r_h},$$  \hspace{1cm} (18)

where $r_h$ is event horizon. In order to obtain $T$ one can calculate $\partial_r g_{tt}$ which is given by,

$$\partial_r g_{tt} = \partial_r (r^{2\alpha + 2} f(r)) \bigg|_{r = r_h} = \partial_r (r^{2\alpha + 2} f(r)) \bigg|_{r = r_h} + r^{2\alpha + 2} \partial_r f(r) \bigg|_{r = r_h}.$$  \hspace{1cm} (19)

So, the Hawking temperature will be as,

$$T = \frac{r^{2\alpha} \partial_r f(r)}{4 \pi} \bigg|_{r = r_h}. $$  \hspace{1cm} (20)
If we want to achieve definite form of $T$ we have to obtain mass $m$ in event horizon which is given by,

$$m = A r_h^{d\alpha - \lambda \beta + d + 2\alpha + 1} + Q^2 r_h^{-d\alpha - d + 1}. \quad (21)$$

By using equation (19) in $\partial_r f(r)|_{r=r_h}$ one can arrange following expression,

$$\partial_r f(r) = \frac{1}{r} \left[ Ar_h^{-\lambda \beta + 2\alpha}(-\lambda \beta + 2\alpha + d\alpha + d + 1) + Q^2 r_h^{-2d(1+\alpha)}(-d\alpha - d + 1) \right]|_{r=r_h}. \quad (22)$$

So, finally we substitute equation (22) in equation (20) the corresponding temperature will be as,

$$T = \frac{r_h}{4\pi} \left[ Ar_h^{-\lambda \beta + 2\alpha}(-\lambda \beta + 2\alpha + d\alpha + d + 1) - Q^2 r_h^{-2d(1+\alpha)}(d\alpha + d - 1) \right]. \quad (23)$$

Here, we back to calculate the entropy which can be obtained by the following Hawking-Bekenstein formula,

$$S = \frac{A}{4G_{d+2}}, \quad (24)$$

where $A$ is the area of the event horizon, which is given by,

$$A = \int d^d x \sqrt{g}|_{r=r_h, t=cte}, \quad (25)$$

so, we have,

$$A = r^d_h V_d, \quad (26)$$

where $V_d$ is the volume constant which is with respect to time, and $r$ hyper-surface with radius $r_h$. In that case the corresponding entropy will be as,

$$S = \frac{r^d_h}{4G_{d+2}} V_d. \quad (27)$$

We use the $s = \frac{S}{V_d}$ formula and obtain the entropy density which is given by,

$$s = \frac{r^d_h(1+\alpha)}{4G_{d+2}}. \quad (28)$$

We use the following formula for computing of the energy density,

$$\varepsilon = \int T ds. \quad (29)$$

So, the corresponding energy density will be as,

$$\varepsilon = \frac{d(1+\alpha)}{16\pi G} \left[ Ar_h^{-\lambda \beta + d\alpha + d + 2\alpha + 1} + Q^2 r_h^{-d\alpha - d + 1} \right]. \quad (30)$$
Next step we are going to calculate the pressure. In that case we use following relation,

\[ P = T s - \varepsilon, \]  

and obtain the corresponding pressure as,

\[ P = \frac{1}{16\pi G} \left[ A r_h^{-\lambda \beta + d \alpha + d + 2 \alpha + 1} (-\lambda \beta + 2 \alpha + 1) + Q^2 r_h^{-\alpha - d + 1} (-2d\alpha - 2d + 1) \right]. \]  

(32)

4 The stability of system

Here, we investigate the stability of system which is important for the discussion of phase transition and also QCD stuff. So, the stability of system lead us to obtain some important condition for the parameter of hyperscaling violation metric. In order to obtain the stability of condition one can arrange the speed of sound. Now we use the \( C_s^2 = \frac{dP}{d\varepsilon} = \frac{v}{c} \) for the speed of sound. So, we use the equation (30) and achieve the following equation,

\[ \frac{d\varepsilon}{d r_h} = \frac{d(1 + \alpha)}{16\pi G} \left[ A r_h^{-\lambda \beta + d \alpha + d + 2 \alpha + 1} (-\lambda \beta + d \alpha + d + 2 \alpha + 1) + Q^2 r_h^{-\alpha - d} (-d\alpha - d + 1) \right]. \]  

(33)

On the other hand, we take the equation (32) one can obtain the following,

\[ \frac{dP}{d r_h} = \frac{1}{16\pi G} \left[ A r_h^{-\lambda \beta + d \alpha + d + 2 \alpha + 1} (-\lambda \beta + d \alpha + d + 2 \alpha + 1)(-\lambda \beta + 2 \alpha + 1) + Q^2 r_h^{-\alpha - d} (-2d\alpha - 2d + 1)(-d\alpha - d + 1) \right]. \]  

(34)

The relation of \( C_s^2 = \frac{dP}{d\varepsilon} \) help us to obtain,

\[ \frac{dP}{d\varepsilon} = \frac{1}{d(1 + \alpha)} \left[ a_1 A r_h^{-\lambda \beta + d \alpha + d + 2 \alpha + 1} + a_2 Q^2 r_h^{-\alpha - d} \right], \]  

(35)

where \( a_1, a_2, b_1 \) and \( b_2 \) are given by,

\[ a_1 = (-\lambda \beta + d \alpha + d + 2 \alpha + 1)(-\lambda \beta + 2 \alpha + 1), \]
\[ a_2 = (-2d\alpha - 2d + 1)(-d\alpha - d + 1), \]
\[ b_1 = -\lambda \beta + d \alpha + d + 2 \alpha + 1, \]
\[ b_2 = -d\alpha - d + 1. \]  

(36)
Here, again we arrange equation (35) one can rewrite as,

\[
\frac{dP}{d\varepsilon} = \frac{a_2}{d(1 + \alpha)b_2} \left[ \frac{a_1}{a_2} A_r^{\lambda \beta + d\alpha + d + 2\alpha} + Q^2 r^{-\alpha - d} \right] \left[ \frac{b_1}{b_2} A_r^{\lambda \beta + d\alpha + d + 2\alpha} + Q^2 r^{-\alpha - d} \right].
\] (37)

In order to have definite form for the \( \frac{dP}{d\varepsilon} \), one can assume following condition,

\[
\frac{a_1}{a_2} = \frac{b_1}{b_2}. \tag{38}
\]

By using equation (38), we find very important following condition,

\[
\lambda \beta = 2d\alpha + 2d + 2\alpha. \tag{39}
\]

The condition (39) in equation (37) lead us to achieve the following speed of sound, which is given by,

\[
C_s^2 = \frac{2\theta - 2d + 1}{d - \theta}. \tag{40}
\]

In this relation, we used \( \alpha = -\frac{\theta}{d} \). Now we back to study the stability of black hole which is given by speed of sound. If we want to have black hole stability, one can need to apply the following condition,

\[
C_s^2 \geq 0. \tag{41}
\]

In that case, we have following condition for the \( \theta \),

\[
d - \frac{1}{2} \leq \theta < d. \tag{42}
\]

In case of \( \theta = d - \frac{1}{2} \) we have first order transition as \( C_s^2 = 0 \). All above condition give us motivation to investigate the AdS black hole and thermally charged black hole with corresponding conditions.

### 5 AdS Black hole with stability conditions

Now, we return to the equations (11) and (13) and put the above stability condition one can obtain,

\[
C = \frac{V_0 e^{-\lambda \phi_0}}{d(1 + \alpha)(-\alpha d - d + 1)}, \tag{43}
\]

\[
Q^2 = -\frac{\rho^2 e^{-\lambda \phi_0}}{2d(1 + \alpha)(3\alpha d + 3d - 1)}. \tag{44}
\]
Also, by using Einstein-Maxwell-Dilaton action [33,34] and solving the equation of motion for a gauge field and scalar field, we have following expressions

\[ e^\phi = e^{\phi_0} r^\beta. \]  
(44)
\[ F_{rt} = \rho e^{-\lambda \phi} r^{2\alpha - d - d}. \]  
(45)

Also, here we use the equations (39) and (45) the corresponding field strength is given by,

\[ F_{rt} = i\overline{Q} r^{-3d(1+\alpha)}, \]  
(46)

where \( \overline{Q} = Q e^{-\frac{\lambda \phi_0}{2}} \sqrt{2d(1+\alpha)(3d(3d + 3d - 1))} \) and the corresponding gauge field is,

\[ A_t(r) = \frac{i\overline{Q} r^{-3d(1+\alpha) + 1}}{1 - 3d(1 + \alpha)} + C_1, \]  
(47)

where \( C_1 \) is a constant and is related to the boundary value of \( A_t \) [34].

Here, by using the solution of \( A_t \) in (47), we apply the condition at the boundary \( (r \to \infty) \), the value of \( A_t \) is \( i\mu \) where \( i \) is due to the consideration of Euclidean signature. Therefore we find the solution of \( A_t \) which is given by,

\[ A_t(r) = i \left( \frac{\overline{Q} r^{-3d(1+\alpha) + 1}}{1 - 3d(1 + \alpha)} + \mu \right). \]  
(48)

At horizon, the \( A_t \) is zero. It leads us to obtain relation between \( \mu \) and \( Q \), which is given by,

\[ Q = \mu \left[ 3d(1 + \alpha) - 1 \right] \frac{e^\lambda \phi_0}{2d(1 + \alpha)} r_h^{3d(1+\alpha)}. \]  
(49)

By using the equations (6) and (7) the AdS black hole action will be as,

\[ S^{AdS BH} = \frac{1}{16\pi G} \int d^{d+2}x \sqrt{g} \left[ \frac{2V}{d} + \frac{e^\phi}{2d} F^2 \right] = \frac{1}{8\pi G d} V_d \beta \int dr \sqrt{g} \left[ V + \frac{1}{4} e^\phi F^2 \right], \]  
(50)

where \( V_d \) is volume in \( d \) dimensional and \( \beta \) is inverse of black hole temperature. So, finally the action of AdS black hole will be as,

\[ S^{AdS BH} = \frac{1}{8\pi G d} V_d \beta \int dr r^{d(1+\alpha) + 2\alpha} \left[ V_0 e^{-\lambda \phi_0 r^{-2d(1+\alpha) - 2\alpha}} - \frac{1}{4} \overline{Q}^2 e^{\lambda \phi_0} r^{-3d(1+\alpha) + 1} \right] r_h^{r_{max}}, \]  
(51)

this action is singular at \( r_{max} \to \infty \). To regularize this action we subtract thermal AdS from this action. In this case \( m = Q = 0 \), \( f(r) = \frac{C}{r^{2d(1+\alpha)}} \), and therefore we have,

\[ S_T^{AdS} = \frac{1}{8\pi G d} V_d \beta \left[ V_0 e^{-\lambda \phi_0} r^{-d(1+\alpha)} \right] r_{max}. \]  
(52)
So, the regularized action is given by,

\[ S_{AdSBH} = \frac{1}{8\pi G_d} V_d \beta \left[ \frac{V_d e^{-\lambda \phi_0}}{1 - d(1 + \alpha)} r_h^{1-d(1+\alpha)} - \frac{1}{4} \frac{Q^2 e^{\lambda \phi_0}}{-3d(1 + \alpha) + 1} r_h^{-3d(1+\alpha)+1} \right]. \]  

(53)

By using thermodynamical relation \( \Omega(\mu, T) = TS_{on-shell} \), we obtain grand potential for AdS black hole, which is given by,

\[ \Omega^{AdSBH} = -\frac{1}{8\pi G_d} V_d \left[ \frac{V_d e^{-\lambda \phi_0}}{1 - d(1 + \alpha)} r_h^{1-d(1+\alpha)} - \frac{1}{4} \frac{Q^2 e^{\lambda \phi_0}}{-3d(1 + \alpha) + 1} r_h^{-3d(1+\alpha)+1} \right]. \]  

(54)

6 Thermally charged AdS

Now, we consider special case of AdS charged black hole which is thermal charged AdS black hole. In that case the mass is zero, so \( f(r) = \frac{A}{r^{2d(1+\alpha)}} \) and therefore the corresponding metric is given by,

\[ ds^2 = r^{2\alpha} \left( Ar^{2-2d(1+\alpha)} dt^2 + \frac{1}{Ar^{2-2d(1+\alpha)}} dr^2 + r^2 d\Omega^2 \right), \]  

(55)

where \( A = C + Q^2 \).

Due to absence of horizon, we choose a lower cut-off as \( r_{IR} \) and integrate from \( r_{IR} \) to infinity. In this case, the \( Q' \) charge completely are different from the previous AdS black hole, because we have different boundary conditions.

The form of field strength and the gauge field same as before of AdS black hole just \( Q \) is replaced by \( Q' \).

\[ F_{rt} = iQ' r^{-3d(1+\alpha)} \]  

(56)

\[ A'_t(r) = iQ' r^{-3d(1+\alpha)+1} + C_2, \]  

(57)

where \( Q' = Q e^{-\lambda \phi_0} \sqrt{2d(1 + \alpha)(3\alpha d + 3d - 1)} \). At \( r \to \infty \), we have \( A_t = C_2 = i\mu \). At \( r = r_{IR} \), we apply Dirichlet boundary condition \( A'_t = i\zeta \mu \), where \( \zeta \) is a constant. Thus

\[ A'_t(r_{IR}) = i \left( \frac{Q' r_{IR}^{-3d(1+\alpha)+1}}{1 - 3d(1 + \alpha)} + \mu \right) = i\zeta \mu, \]

\[ \Rightarrow Q' = \mu(1 - \zeta)[3d(1 + \alpha) - 1] r_{IR}^{-3d(1+\alpha)}. \]  

(58)

We apply same method as a previous section and compute regularized action, which is given by,

\[ S_{AdS} = -\frac{1}{8\pi G_d} V_d \beta \left[ \frac{V_d e^{-\lambda \phi_0}}{1 - d(1 + \alpha)} r_{IR}^{1-d(1+\alpha)} - \frac{1}{4} \frac{Q'^2 e^{\lambda \phi_0}}{-3d(1 + \alpha) + 1} r_{IR}^{-3d(1+\alpha)+1} \right]. \]  

(59)
So, the grand potential will be as,

\[ \Omega_{AdS} = -\frac{1}{8\pi G d} V_d \left[ \frac{V_0 e^{-\lambda \phi_0}}{1 - d(1 + \alpha)} r_{IR}^{1-d(1+\alpha)} - \frac{1}{4} \frac{Q' e^{\lambda \phi_0}}{-3d(1 + \alpha) + 1} r_{IR}^{-3d(1+\alpha)+1} \right], \]  

(60)

where \( Q' \) is function of \( \mu \). In the other hand, we use \( N = -\frac{\partial \Omega}{\partial \mu} = -\frac{\partial \Omega}{\partial Q'} \frac{\partial Q'}{\partial \mu} \), and obtain

\[ N = -\frac{1}{16\pi G d} V_d \left[ \frac{Q'}{-3d(1 + \alpha) + 1} r_{IR}^{-3d(1+\alpha)+1} \right] \frac{\partial Q'}{\partial \mu}. \]  

(61)

By using (58) to (61), we have

\[ N = \frac{1}{16\pi G d} V_d e^{\lambda \phi_0} \left[ \mu (1 - \zeta) \right] \frac{\partial Q'}{\partial \mu}. \]  

(62)

As shown in [34], one can calculate the boundary of action. We obtain the boundary action for the thermally charged AdS, which is given by,

\[ S_{AdS}^{TCb} = \frac{1}{8\pi G d} \int_{\partial M} d^{d+1}x \sqrt{g^{(d+1)}} n^\sigma A^\mu F_{\mu\sigma} e^{\lambda \phi}, \]

(63)

where unit vector \( n^r = (0, -\sqrt{f'(r)/r^{\alpha-1}}, 0, 0, \ldots) \) and \( \sqrt{g^{d+1}} = r^{(1+\alpha)(d+1)} \sqrt{f'(r)}. \)

Again in order to obtain the \( \zeta \) from Dirichlet condition can obtain \( N \) from following approach. By using equation (63) we have,

\[ \Omega_{AdS}^{TCb} = \frac{1}{8\pi G d} V_d \mu e^{\lambda \phi_0} Q', \]  

(64)

Again one can calculate \( N = -\frac{\partial \Omega}{\partial \mu} \), and therefore \( N \) is given by,

\[ N = -\frac{1}{8\pi G d} V_d \mu e^{\lambda \phi_0} \frac{\partial Q'}{\partial \mu}. \]  

(65)

We compare this equation with equation (62) and we obtain \( \zeta = 3 \). Also, here note that the \( Q' \) will be following form,

\[ Q' = -2\mu \sqrt{\frac{3d(1 + \alpha) - 1}{2d(1 + \alpha)}} e^{\lambda \phi_0} r_{IR}^{1-3d(1+\alpha)}. \]  

(66)

So, in generally we say that the stability condition at first order transition helped us to made AdS and charged AdS black hole.
7 Conclusion

In this paper, we studied the thermodynamic generalized warp factor for the corresponding metric background. In that case we obtain two quantities as a pressure and energy density for investigation of stability. The stability of system lead us to obtain such conditions for the parameter of $\theta$. Here also we note that in case of $\theta = d - \frac{1}{2}$ we have first order transition as $C_s^2 = 0$. In that case there is phase transition between stability and instability cases. So, we have seen that the most important results for the action come from stability, instability and phase transition with warp factor. So, by using the corresponding conditions we made the corresponding action and investigated AdS and thermally charged AdS black hole with exact solutions.

References


