Canadian Journal of Physics
Revue canadienne de physique

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Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2017-0071.R2
Manuscript Type:	Article
Date Submitted by the Author:	04-Apr-2017
Complete List of Authors:	Gadre, Nitin; Private,
Keyword:	Electrodynamics, Special relativity, Classical theory, Theoretical physics, Radiation
Is the invited manuscript for consideration in a Special Issue? :	N/A

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A relook at radiation by a point charge- I

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PACS: 03.50.De, 03.30.+p

Keywords: Electrodynamics, Special relativity, Radiation.

Abstract: Efforts to suggest a classical model for hydrogen atom are discouraged by a conclusion, based on the principles of electrodynamics that an accelerating charged particle necessarily radiates. In this paper, we re-examine the steps leading to this conclusion. We start with the relativistic expressions for energy and momentum of a particle and establish the relationship between special relativity and electrodynamics. The standard field expression and its relativistic transformations are then studied for a point charge source, represented by a delta function. In conventional Poynting's theorem analysis, the rate of change of work done on a system of charges is written as addition of two terms, rate of change of stored energy and surface integral of Poynting vector. For a delta function source, the first two terms of this equation are either non-integrable or difficult to evaluate. Only the third surface integration term can be evaluated, which is said to give radiation by the point charge. Thus, the statement that an accelerated charge radiates is a conclusion based on this Poynting vector analysis. We examine it and realize that this statement, namely that a point charge radiates continuously just because it is accelerating, does not have adequate theoretical justification.

1. Relationship between relativity and electrodynamics

Efforts to describe a classical picture of hydrogen atom have been discouraged by a conclusion based on principles of electrodynamics that an accelerating charged particle necessarily radiates and therefore a classical picture of electron rotating in an orbit around proton is not acceptable. We are re-examining various concepts of electrodynamics to verify this conclusion.

In these two papers, we try to understand various conventional concepts of electrodynamics. We then analyse them critically, mainly to examine the issue "an accelerating electron radiates and therefore a classical picture of rotating electron in hydrogen atom cannot be acceptable". We proceed in three steps. (1) We first try to establish relationship between special relativity and electrodynamics. (2) We then understand limitations of conventional methods such as Poynting's theorem, Larmor formula in explaining radiation by a point charge. (3) We see if an alternate method of finding change of total energy can be superior in explaining radiation by a two particle system. In a bound system like hydrogen, the emitted photon energy will depend upon difference in total energies of the discrete energy states.

We are using SI units in this paper. We first establish relationship between special relativity and electrodynamics now.

1.1. Relativistic velocity transformations

A stationary particle of mass m_0 has energy $E_0 = m_0 c^2$ and momentum $\vec{p}_0 = 0$ in one frame. In another frame called laboratory or lab frame, we can obtain energy and momentum of this particle moving at constant velocity $\vec{v}$ as $E_{K.E.} = \gamma_v m_0 c^2$ and $\vec{p} = \gamma_v m_0 \vec{v}$. We can suggest a particle model which can satisfy this requirement [1,2]. Note, any particular particle model is not focus of these two papers. As a starting point of our analysis, we take these energy and momentum expressions as experimentally proven results and *property* of a *particle*. Relative velocity between two frames is $\vec{v}$ in X direction and $v < c$. Here, $\gamma_v = 1/\sqrt{1-v^2/c^2}$.

We conduct a two dimensional analysis now. We try to obtain the particle velocity transformations by conducting a collision experiment as in Fig. 1. The first particle of mass M_0 is initially stationary in laboratory frame A. Primed frame B travels at speed v in X direction, with respect to un-primed lab Frame A. We take frame velocity $v < c$. Second particle of stationary mass m_0 has velocity $\vec{u}'$ in frame B. The two particles collide and we study collision in the two frames.

Frame B: In this frame, the first particle has initial velocity $(-v)\hat{x}$ and second has initial velocity $\vec{u}'$. They return back with same velocity after collision as shown in Fig.1. The conservation of momentum equation for X component of momentum gives

$$\frac{m_0 u'_x}{\sqrt{1-u'^2/c^2}} = \frac{M_0 v}{\sqrt{1-v^2/c^2}} \quad (1)$$

The total energy of particles in frame B is $E_B = M_B c^2$ where

$$M_B = \frac{m_0}{\sqrt{1-u'^2/c^2}} + \frac{M_0}{\sqrt{1-v^2/c^2}} \quad (2)$$

Frame A: First particle M_0 is initially stationary and second has speed u in frame A. The total energy of particles in frame A is $E_A = M_A c^2$, where

$$M_A = \frac{m_0}{\sqrt{1-u^2/c^2}} + M_0 \quad (3)$$

Energy of particle in a frame should get enhanced by γ_v factor when measured in another frame.

$$M_A c^2 = \frac{M_B c^2}{\sqrt{1-v^2/c^2}} \quad (4)$$

$$\frac{m_0}{\sqrt{1-u^2/c^2}} + M_0 = \frac{m_0}{\sqrt{1-v^2/c^2} \sqrt{1-u'^2/c^2}} + \frac{M_0}{\sqrt{1-v^2/c^2}}$$

Substituting M_0 from (1), with $u'^2 = u_x'^2 + u_y'^2$,

$$\frac{1}{\sqrt{1-u'^2/c^2}} = \frac{1+u_x'v/c^2}{\sqrt{1-v^2/c^2}\sqrt{1-u'^2/c^2}}. \quad (5)$$

Y component of momentum need not change as frame velocity is only in X direction. We equate the Y components and write,

$$\frac{m_0 u_y}{\sqrt{1-u'^2/c^2}} = \frac{m_0 u_y'}{\sqrt{1-u'^2/c^2}},$$

$$u_y = \frac{u_y' \sqrt{1-v^2/c^2}}{1+u_x'v/c^2}. \quad (6)$$

Verify from (5) and (6),

$$u_x = \frac{u_x' + v}{1+u_x'v/c^2}. \quad (7)$$

Observation: The velocity transformations can be linked to particle energy and momentum transformations and can be treated as property of a particle[2]. The distance and time are modified suitably to get the desired value of momentum and energy in a particular frame. We study equation (7) for various values of particle velocity with frame velocity $v < c$.

If $u_x' < c$ then $u_x < c$. This gives a real and positive γ . Energy and momentum are real.

Particle velocity $u_x' = c$ gives $u_x = c$. Then γ is infinity and we take rest mass $m_0 = 0$ to describe a photon with non-zero real momentum and energy values.

If $u_x' > c$ in one frame, then in all frames $u_x > c$. Then γ will be imaginary and we will have to give rest mass m_0 an imaginary value to obtain real values for momentum and energy. Else, rest mass $m_0 = 0$ will give zero value to both energy and momentum.

Note that, velocity above speed of light for any material is prohibited not as a mathematical requirement but by the postulate of relativity[3]. This postulate ensures that we deal with real situations and not with fictitious particles having imaginary or zero mass.

1.2. Distance transformations

We use the velocity transformations (6) and (7) obtained earlier,

$$u_y = \frac{dy}{dt} = \frac{\frac{dy'}{dt'} \sqrt{1-v^2/c^2}}{1 + \frac{v}{c^2} \frac{dx'}{dt'}} = \frac{dy' \sqrt{1-v^2/c^2}}{dt' + dx' \frac{v}{c^2}} = \frac{dy'}{\gamma_v \left(dt' + dx' \frac{v}{c^2} \right)}.$$

The distance in Y direction need not change as frame velocity is in X direction. This requires time to change as follows,

$$dt = \gamma_v \left(dt' + dx' \frac{v}{c^2} \right).$$

$$u_x = \frac{dx}{dt} = \frac{\frac{dx'}{dt'} + v}{1 + \frac{v}{c^2} \frac{dx'}{dt'}} = \frac{\gamma_v (dx' + v dt')}{\gamma_v \left(dt' + \frac{v}{c^2} dx' \right)}.$$

Using equation for time transformation obtained above, get $dx = \gamma_v (dx' + v dt')$. If the interval measurements are with respect to four vector origins in both frames,

$$x = \gamma_v (x' + vt') , y = y' , t = \gamma_v \left(t' + x' \frac{v}{c^2} \right) . \quad (8)$$

Observation: The distance transformations can be obtained from particle energy and momentum transformations. We have taken frame velocity in X direction. A rigorous derivation can give Lorentz transformations with frame velocity in general direction[4].

1.3. Lorentz force

Momentum of a particle in lab frame is given by $\vec{p} = \gamma_v m_0 \vec{v}$ and energy is $E_{K.E.} = \gamma_v m_0 c^2$. There are no acceleration terms in both kinetic energy and momentum. These energy and momentum values measured at any particular instant are same even if the particle is accelerating or is in constant velocity motion. We define force as rate of change of momentum when a particle is shifted from one stable state of constant velocity $\vec{v}_1$ at $(\vec{r}_1, t_1)$ to another state of constant velocity $\vec{v}_2$ at $(\vec{r}_2, t_2)$. Values of momentum and energy for such stable initial or final states in lab frame can be obtained by using relativistic transformations from static particle values. The relativistic force on a particle is

$$\vec{F} = \frac{\vec{p}_{\vec{v}_2}(t_2) - \vec{p}_{\vec{v}_1}(t_1)}{t_2 - t_1} = \frac{\gamma_{v_2} m_0 \vec{v}_2 - \gamma_{v_1} m_0 \vec{v}_1}{t_2 - t_1} = \frac{d\vec{p}}{dt} = \frac{d}{dt} \frac{m_0 \vec{r}}{\sqrt{1 - \vec{r}^2 / c^2}},$$

$$\vec{F} = m_0 \left[\left(\frac{\ddot{\vec{r}}}{(1 - \vec{r}^2 / c^2)^{3/2}} \right) + \vec{r} \times \left(\frac{1}{c^2} \frac{\vec{r} \times \ddot{\vec{r}}}{(1 - \vec{r}^2 / c^2)^{3/2}} \right) \right]. \quad (9)$$

As per electrodynamics, the observer particle experiences above given force generated by another particle at some distance called as the source. Write this above equation (9) as

$$\vec{F} = q \left[\left(\vec{E} \right) + \vec{v} \times \left(\vec{B} \right) \right]. \quad (10)$$

In field theory, we say that the components of force on observer in (10), due to $\vec{E}$ and $\vec{B}$, are created by source on the observer. In this equation $\vec{v} = \dot{\vec{r}}$ is velocity of the observer.

Observation: Both the equations (10) and (9) give force on the observer particle. Hence, the expression (10) will have to follow same relativistic transformation rules as (9).

1.4. Relativistic transformations of $\vec{E}$ and $\vec{B}$

In primed frame B, the observer has speed $\vec{u}'$ and force on observer in this frame is $\vec{F}'$. Frame B has relative speed $\vec{v} = v\hat{x}$ with respect to un-primed frame A. Using standard force transformation, the force component in X direction in A frame is as follows [3].

$$F_x = F'_x + \frac{vu'_y}{c^2(1 + \frac{vu'_x}{c^2})} F'_y + \frac{vu'_z}{c^2(1 + \frac{vu'_x}{c^2})} F'_z.$$

Inserting primed frame values of X, Y and Z direction components of force from (10),

$$F_x = q \left[(E'_x + u'_y B'_z - u'_z B'_y) + \frac{vu'_y}{c^2(1 + \frac{vu'_x}{c^2})} (E'_y + u'_z B'_x - u'_x B'_z) + \frac{vu'_z}{c^2(1 + \frac{vu'_x}{c^2})} (E'_z + u'_x B'_y - u'_y B'_x) \right],$$

$$F_x = q \left[E'_x + \frac{u'_y}{1 + \frac{vu'_x}{c^2}} \left(B'_z + \frac{vE'_y}{c^2} \right) - \frac{u'_z}{1 + \frac{vu'_x}{c^2}} \left(B'_y - \frac{vE'_z}{c^2} \right) \right].$$

Compare with lab values of X component of force from (10), $F_x = q[E_x + u_y B_z - u_z B_y]$. By inserting γ_v , to adjust with velocity transformation relations obtained in Sec. 1.1, get

$$E_x = E'_x, B_z = \gamma_v \left(B'_z + \frac{vE'_y}{c^2} \right), B_y = \gamma_v \left(B'_y - \frac{vE'_z}{c^2} \right). \quad (11)$$

Write relativistic transformation for Y component of force and compare with Y component of force from equation (10). Similarly we can study F_z transformation also.

$$F_y = \frac{F'_y}{\gamma_v(1 + \frac{vu'_x}{c^2})} = \frac{q(E'_y + u'_z B'_x - u'_x B'_z)}{\gamma_v(1 + \frac{vu'_x}{c^2})},$$

$$F_y = q \left[\gamma_v (E'_y + vB'_z) + \frac{u'_z}{\gamma_v(1 + \frac{vu'_x}{c^2})} B'_x - \frac{(u'_x + v)}{1 + \frac{u'_x v}{c^2}} \gamma_v \left(B'_z + \frac{vE'_y}{c^2} \right) \right] = q(E_y + u_z B_x - u_x B_z).$$

$$E_y = \gamma_v (E'_y + vB'_z), B_x = B'_x, B_z = \gamma_v \left(B'_z + \frac{vE'_y}{c^2} \right), E_z = \gamma_v (E'_z - vB'_y).$$

1.5. Obtaining Maxwell equations

There is an observer particle which experiences force due to source. In primed frame B, the force is $\vec{F}'$ and in un-primed frame A, this force is measured as $\vec{F}$. The frame B travels at relative speed $v\hat{x}$ with respect to frame A. Suppose at time instant t' , the speed of observer is $\vec{u}'=0$. Then force on observer in frame B is $\vec{F}'=q[\vec{E}' + \vec{u}' \times \vec{B}']$. We are using SI units[5]. For the source in frame B, we add two conditions based on experiments in frame B,

$$\vec{\nabla}' \cdot \vec{E}'(\vec{r}', t') = \frac{\rho'(\vec{r}', t')}{\epsilon_0}, \quad \vec{\nabla}' \cdot \vec{B}'(\vec{r}', t') = \frac{\partial B'_x}{\partial x'} + \frac{\partial B'_y}{\partial y'} + \frac{\partial B'_z}{\partial z'} = 0. \quad (12)$$

Transform second equation to frame A, using (11). Assume distance and time as independent variables in the field functions. Frame velocity $v\hat{x}$ is constant and should not vary with present or *retarded* time to write following partial differentiation transformations using (8).

$$\gamma_v \left(\frac{\partial}{\partial x} + \frac{v}{c^2} \frac{\partial}{\partial t} \right) B_x + \frac{\partial}{\partial y} \left(\gamma_v (B_y + \frac{vE_z}{c^2}) \right) + \frac{\partial}{\partial z} \left(\gamma_v (B_z - \frac{vE_y}{c^2}) \right) = 0,$$

$$\gamma_v \left(\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \right) + \frac{\gamma_v v}{c^2} \left(\frac{\partial B_x}{\partial t} + \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right) = 0.$$

For equations (12) to be covariant in frame A, we need

$$\frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} = \vec{\nabla} \cdot \vec{B} = 0, \quad (13)$$

$$-\left(\frac{\partial \vec{B}}{\partial t} \right)_x = (\vec{\nabla} \times \vec{E})_x.$$

Writing a general three dimensional equation,

$$-\frac{\partial \vec{B}(\vec{r}, t)}{\partial t} = \vec{\nabla} \times \vec{E}(\vec{r}, t). \quad (14)$$

Similarly we start with following equation in frame A,

$$\vec{\nabla} \cdot \vec{E}(\vec{r}, t) = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} = \frac{\rho(\vec{r}, t)}{\epsilon_0}. \quad (15)$$

Transforming this to primed frame B and for the equations (12) to be covariant requires,

$$\vec{\nabla}' \cdot \vec{E}'(\vec{r}', t') = \frac{\rho'(\vec{r}', t')}{\epsilon_0}, \quad \rho' = \frac{\rho}{\gamma_v}, \quad (\vec{\nabla}' \times \vec{B}')_x = \frac{1}{c^2} \left(\frac{\partial \vec{E}'}{\partial t'} \right)_x.$$

Now transform this above equation to frame A. Use equation (15) to get following equation.

$$(\vec{\nabla} \times \vec{B})_x = \mu_0 \rho v \hat{x} + \frac{1}{c^2} \left(\frac{\partial \vec{E}}{\partial t} \right)_x = \mu_0 J \hat{x} + \frac{1}{c^2} \left(\frac{\partial \vec{E}}{\partial t} \right)_x .$$

Writing a general three dimensional equation,

$$\vec{\nabla} \times \vec{B}(\vec{r}, t) = \mu_0 \rho \vec{v} + \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = \mu_0 \vec{J}(\vec{r}, t) + \frac{1}{c^2} \frac{\partial \vec{E}(\vec{r}, t)}{\partial t} . \quad (16)$$

The relativistic transformations (Lorentz transformations) are obtained in special relativity by assuming speed of light in free space as invariant. This postulate also gets reflected in electrodynamics. The two constants μ_0 and ϵ_0 are treated as properties of free space and therefore invariant under transformation as properties of vacuum need not change. In electrodynamics, this becomes basis for velocity of light to be invariant as $c = 1/\sqrt{\mu_0 \epsilon_0}$.

Observation: Equations (13) to (16) are the Maxwell equations. It may be noted that we *cannot* obtain all the four equations directly from special relativity. We can start with two equations given in (12) and obtain remaining two equations assuming that these equations should be covariant under relativistic transformations. Note that, the force expression (9) is also a covariant relation. In current $\vec{J}$, the velocity $\vec{v}$ is frame velocity and taken as a constant in this analysis. If the source is static in frame B then it shall have constant velocity $\vec{v}$ in frame A. There is good correspondence between relativity and electrodynamics.

1.6. Scalar and vector potentials

If source is static in B frame then we presume that the entire space is filled by scalar potential ϕ , which is function of distance alone. As per electrodynamics, field is created by source and observer charge if present, experiences it. In frame A, this source has constant velocity and potential will be a function of time also. To satisfy (14), we define covariant relations,

$$\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \vec{\nabla} \times \vec{A}. \quad (17)$$

Using equation (11) and transformation relations, get transformations for potentials.

$$E'_x = \left(-\frac{\partial \phi'}{\partial x'} - \frac{\partial A'_x}{\partial t'} \right) = \left(-\frac{\partial \phi}{\partial x} - \frac{\partial A_x}{\partial t} \right) = E_x .$$

$$\phi = \gamma_v (\phi' + v A'_x), \quad A_x = \gamma_v (A'_x + \frac{v}{c^2} \phi') . \quad (18)$$

Similarly condition (11), $B'_x = B_x$ gives, $A_y = A'_y, \quad A_z = A'_z$.

Observation: The scalar field behaves like energy and vector field like momentum. Force on same observer particle measured in frames B and A can be described as follows,

$$\vec{F}' = \frac{d\vec{p}'}{dt} = \frac{d(\gamma_u m_0 \vec{u}')}{dt} = q \left[\vec{E}' + \vec{u}' \times \vec{B}' \right], \quad \vec{F} = \frac{d\vec{p}}{dt} = \frac{d(\gamma_u m_0 \vec{u})}{dt} = q \left[\vec{E} + \vec{u} \times \vec{B} \right]. \quad (19)$$

In relativistic force equation (9), the triple cross product term will lead to a square of observer velocity term. In electrodynamic force in (19), there is a cross product of observer velocity and magnetic field. As the magnetic field will have only source velocity term, the square of observer velocity term is not present in electrodynamic force. Electrodynamic force may have a product of source and observer velocity terms.

See equations (19). Strategy adopted is to equate the relativistic and electrodynamic forces in one frame, say primed frame. The observer velocity and acceleration will depend upon its mass and force it experiences. Then forces in other frames can be obtained, by relativistic transformations to the new frame. If we change source velocity in electrodynamic force on right side of this equation by shifting to another frame, the observer related terms such as observer velocity and acceleration present in relativistic force expression, given on left side of equation will also have to be transformed to this new frame.

We can calculate rate of change of kinetic energy of observer particle using equation (19). It depends only on electric field and not magnetic field as shown below. As our aim is to study radiation, which depends on rate of change of energy, the electric field is more important.

$$\frac{dE_{K.E.}}{dt} = \vec{u} \cdot \vec{F} = \vec{u} \cdot \frac{d\vec{p}}{dt} = q\vec{u} \cdot \left[\vec{E} + \vec{u} \times \vec{B} \right] = q\vec{u} \cdot \vec{E}.$$

2. Field by a charged point particle

2.1. Electric field and Lorentz gauge

We study a simple single dimension situation. An observer particle with charge q in primed frame B at position x' experiences force $\vec{F}'$ due to a static source at origin. Here static means a stationary source, which is not influenced by such external electric and magnetic fields, which can cause it to move. The observer has speed v' at time t' . Work done on the observer increases its kinetic energy which has to be equal to reduction in field energy.

$$F'_x = \frac{dE'_{K.E.}}{dx'} = -q \frac{d\phi'}{dx'} = -q \left(\frac{\partial \phi'}{\partial x'} + \frac{1}{v'} \frac{\partial \phi'}{\partial t'} \right).$$

For a static source, we suggest that scalar potential is a function of distance only. Therefore,

$$F'_x \hat{x} = -q \frac{d\phi'}{dx'} \hat{x} = -q \frac{\partial \phi'}{\partial x} \hat{x} = qE'_x \hat{x}. \quad (20)$$

$$\frac{\partial \phi'}{\partial t'} = 0. \quad (21)$$

Frame B has speed $v_q \hat{x}$ with respect to un-primed lab frame A. Then the source has constant speed $v_q \hat{x}$ in A frame. Transform the above equation (20) to frame A to get equation (17).

$$F_x \hat{x} = F'_x \hat{x} = -q \gamma_{v_q} \left(\frac{\partial}{\partial x} + \frac{v_q}{c^2} \frac{\partial}{\partial t} \right) \phi' \hat{x} = -q \left[\frac{\partial}{\partial x} (\gamma_{v_q} \phi') + \frac{\partial}{\partial t} \left(\frac{\gamma_{v_q} v_q \phi'}{c^2} \right) \right] \hat{x},$$

$$F_x \hat{x} = -q \left(\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} \right) \hat{x} = q E_x \hat{x}. \quad (22)$$

Here we introduced vector potential $\vec{A}$ in addition to scalar potential ϕ . Transform (21), to lab frame to get a condition called Lorentz gauge,

$$\frac{\partial \phi'}{\partial t'} = \left[\gamma_{v_q} \left(\frac{\partial}{\partial t} + v_q \frac{\partial}{\partial x} \right) \right] \phi' = \left[\frac{\partial}{\partial t} (\gamma_{v_q} \phi') + c^2 \frac{\partial}{\partial x} \left(\frac{\gamma_{v_q} v_q \phi'}{c^2} \right) \right] = \left(\frac{\partial \phi}{\partial t} + c^2 \frac{\partial A_x}{\partial x} \right) = 0,$$

$$\frac{\partial A_x}{\partial x} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0. \quad (23)$$

We shifted γ_{v_q} and v_q in the original relativistic operator acting on original function ϕ' , to the right side to get a modified operator comprising of two partial differential operators acting on two modified functions ϕ and $\vec{A}$. It is mathematically possible when $\vec{v}_q$ is constant.

Observation: Conditions (20) and (21) are true only for a static source. We obtained above two equations (22) and (23) by relativistic transformations which require $v_q \hat{x}$ to be constant. The Maxwell equations are experimental equations and therefore in practice, same equations are being used for an accelerating source also. Note that, we do not get mathematical justification for using the two equations (22) and (23) for varying source velocity. We discuss this issue again in Paper II. Till then, in this paper, we *assume* that these equations hold true for varying velocity also and continue our analysis to understand the conventional approach.

2.2. Field of a point charge moving with varying velocity in X direction

There is a static point charge source S' at O' in primed frame B (Fig. 2). This source S' at (x'_q, y'_q) in frame B releases signal at t'_q which reaches observer particle at position r' at time t' . Using (12) and (20), write field at observer particle position (x', y', t') . A charged point particle can be represented by a delta function.

$$\vec{\nabla}' \cdot \vec{E}'(\vec{r}', t') = -\nabla'^2 \phi'(\vec{r}', t') = \frac{q \delta(\vec{r}' - \vec{r}'_q)}{\epsilon_0}.$$

We do a simple two dimensional analysis. We can easily add the Z direction result similar to Y direction result to make it a three dimensional analysis. Solving above equation,

$$\phi'(x', y', t') = \frac{q}{4\pi\epsilon_0 \sqrt{(x' - x'_q)^2 + (y' - y'_q)^2}} = \frac{q}{4\pi\epsilon_0 (r' - r'_q)}.$$

Refer to Fig. 2. In lab frame A, we get ϕ and $\vec{A}$ using equation (18). Frame velocity is $v_q \hat{x}$. The static source in primed frame will have velocity $v_q \hat{x}$ in lab frame A. In lab frame, the source released signal at (x_q, y_q, t_q) corresponding to (x'_q, y'_q, t'_q) in primed frame. The O' and O overlapped at this instant. This signal reached the observer at (x, y, t) .

$$\phi(x, y, t) = \gamma_{v_q} \phi'(x', y', t') = \frac{\gamma_{v_q} q}{4\pi\epsilon_0 \sqrt{\gamma_{v_q}^2 ((x - x_q) - v_q(t - t_q))^2 + (y - y_q)^2}},$$

$$\phi = \frac{q}{4\pi\epsilon_0 \sqrt{((x - x_q) - v_q(t - t_q))^2 + (y - y_q)^2 (1 - v_q^2/c^2)}} = \frac{q}{4\pi\epsilon_0 \sqrt{x_p^2 + y_p^2 (1 - v_q^2/c^2)}}.$$

Here, we assume signal from source to observer travels at speed of light. We call,

$$x_p = ((x - x_q) - v_q(t - t_q)) \text{ and } y_p = (y - y_q). \text{ Also } (r - r_q) = c(t - t_q).$$

$$x_p^2 + y_p^2 (1 - v_q^2/c^2) = ((x - x_q) - \frac{v_q}{c}(r - r_q))^2 + (y - y_q)^2 (1 - v_q^2/c^2) = \frac{(r' - r'_q)^2}{\gamma_{v_q}^2}.$$

$$\phi = \frac{q}{4\pi\epsilon_0 \left((r - r_q) - \frac{v_q(x - x_q)}{c} \right)} = \frac{q}{4\pi\epsilon_0 \left(\sqrt{(x - x_q)^2 + (y - y_q)^2} - \frac{(x - x_q)v_q}{c} \right)}, \quad (24)$$

$$\vec{A} = \frac{\vec{v}_q \phi}{c^2} = \frac{v_q q}{4\pi\epsilon_0 c^2 \left(\sqrt{(x - x_q)^2 + (y - y_q)^2} - \frac{(x - x_q)v_q}{c} \right)} \hat{x}.$$

Equation (24) are Lienard-Wiechert potentials. Similar to discussion in Sec 1.3 about energy and momentum, there are no acceleration terms in ϕ and $\vec{A}$. Therefore it does not matter in ϕ and $\vec{A}$, if the source has constant velocity or accelerating. Under relativity ϕ behaves like kinetic energy and $\vec{A}$ like momentum. Suppose the source is accelerating. Both velocity $\vec{v}_q$ and acceleration $\vec{a}_q$ of source are in X direction. We say that $\vec{v}_q$ and $\vec{a}_q$ are at retarded time and velocity varies with retarded time t_q . The signal travels at speed of light. For simplicity, to shorten expression, let us write

$$(r - r_q) = \sqrt{(x - x_q)^2 + (y - y_q)^2} = c(t - t_q) = r, \quad \vec{r} - \vec{r}_q = (x - x_q)\hat{x} + (y - y_q)\hat{y} = \vec{r}.$$

We don't specify the observer speed in standard analysis as we only use partial derivatives with respect to observer coordinates. Actual displacement of observer Δx , in a small present time interval Δt has no role in deciding the force on observer, at least the electric field part of equation (19). There is only a vector multiplication of the magnetic field and the observer velocity. Whereas the fields depend upon source displacement Δx_q in time interval Δt_q .

$$\frac{\partial x_q}{\partial x} = \frac{-(x-x_q)v_q/c}{\left(r - \frac{(x-x_q)v_q}{c}\right)}, \quad \frac{\partial t_q}{\partial y} = \frac{-(y-y_q)/c}{\left(r - \frac{(x-x_q)v_q}{c}\right)}.$$

$$\vec{E} = -\left(\frac{\partial \phi}{\partial x} \hat{x} + \frac{\partial \phi}{\partial y} \hat{y} + \frac{\partial A}{\partial t} \hat{z}\right) = \frac{q}{4\pi\epsilon_0 c^2} \frac{\left[(c^2 - v_q^2)(\vec{r} - \frac{rv_q}{c} \hat{x}) + a_q(x-x_q)\vec{r} - r^2 a_q \hat{x}\right]}{\left(x_p^2 + y_p^2(1 - v_q^2/c^2)\right)^{3/2}},$$

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \left[\frac{(1 - v_q^2/c^2)((x-x_q)\hat{x} + (y-y_q)\hat{y} - \frac{(r-r_q)v_q}{c} \hat{x})}{\left((r-r_q) - \frac{(x-x_q)v_q}{c}\right)^3} + \frac{a_q(y-y_q)((x-x_q)\hat{y} - (y-y_q)\hat{x})}{c^2 \left((r-r_q) - \frac{(x-x_q)v_q}{c}\right)^3} \right], \quad \vec{B} = \vec{\nabla} \times \vec{A} = \frac{\hat{r}}{c} \times \vec{E}. \quad (25)$$

These are *standard* field formulas [6]. We obtain three dimensional results in Section 2.4.

Observation: In Green function method, we obtain the scalar and vector potentials same as (24) and the answer links one retarded instant and position of source with one present instant and position of observer. Velocity of source before and after this instant doesn't matter and acceleration has no role. Therefore our relativistic answers given by (24) based on constant source velocity, match with the Green function method results [7].

2.3. Relativistic force transformations

We test our results in (19). See the Fig. 3. Primed frame B has frame velocity $-v_o \hat{x}$ with respect to un-primed lab frame A. Source S' is in B frame and at one instant t'_q , the source velocity in B frame is $v'_q = 0$ and acceleration has X direction, $a'_q \hat{x} \neq 0$. Observer particle has $v_o \hat{x}$ velocity in frame B at instant t' . In frame B, the signal sent by source at time t'_q reaches observer at time t' . Using (25), write force on observer in frame B. $\vec{E}'$ and $\vec{B}'$ are written as addition of velocity and acceleration parts. Write as follows to shorten the expression.

$$\vec{r}' - \vec{r}'_q = (x' - x'_q)\hat{x} + (y' - y'_q)\hat{y} = x'\hat{x} + y'\hat{y} = \vec{r}'.$$

$$\vec{E}' = \frac{q}{4\pi\epsilon_0} \left[\frac{\vec{r}'}{r'^3} + \frac{\hat{r}' \times (\hat{r}' \times \vec{a}'_q)}{c^2 r'^3} \right] = \frac{q}{4\pi\epsilon_0} \left[\frac{x'\hat{x} + y'\hat{y}}{(x'^2 + y'^2)^{3/2}} + \frac{x'y'a'_q \hat{y} - a'_q y'^2 \hat{x}}{c^2 (x'^2 + y'^2)^{3/2}} \right] = \vec{E}'_v + \vec{E}'_a, \quad (26)$$

$$\vec{B}' = \frac{\hat{r}'}{c} \times \vec{E}' = \frac{-q}{4\pi\epsilon_0} \frac{(\vec{r}' \times \vec{a}'_q)}{c^3 r'^2} = \frac{q}{4\pi\epsilon_0} \frac{y'a'_q}{c^3 (x'^2 + y'^2)} \hat{z} = \vec{B}'_A, \quad (27)$$

$$\vec{F}' = q \left[\vec{E}' + \vec{v}_0 \times \vec{B}' \right] = \frac{q^2}{4\pi\epsilon_0} \left[\frac{\vec{r}'}{r'^3} + \frac{a'_q \left(y' \left(x' - \frac{v_0 r'}{c} \right) \hat{y} - y'^2 \hat{x} \right)}{c^2 r'^3} \right]. \quad (28)$$

We calculate force in un-primed lab frame A. Primed frame B has velocity $-v_0 \hat{x}$ with respect to frame A. In lab, at the instant t corresponding to t' in primed frame, observer velocity is zero. Source was instantaneously stationary in frame B when the signal was released by source at time t'_q . The O' of primed frame and O of lab frame overlapped at this instant when signal was released. At instant t_q in lab frame A, corresponding to t'_q , source will have velocity $v_q \hat{x} = -v_0 \hat{x}$ and acceleration $a_q \hat{x} = (dv_q / dt_q) \hat{x}$ in lab. This signal released by source at t_q will reach observer at time t in lab. Get electric field in lab frame by transforming (26).

$$(x - x_q) = \gamma_{v_0} ((x' - x'_q) - v_0 (t' - t'_q)), \quad (x' - x'_q) = \gamma_{v_0} ((x - x_q) + v_0 (t - t_q)) \text{ and } a_q = a'_q / \gamma_{v_0}^3.$$

To shorten the expression, write as follows. Obtain velocity part $\vec{E}_V$ first.

$$\vec{r}_p = x_p \hat{x} + y_p \hat{y} = ((x - x_q) + v_0 (t - t_q)) \hat{x} + (y - y_q) \hat{y} = (x + v_0 t) \hat{x} + y \hat{y}.$$

$$\vec{E}_V = E'_{xV} \hat{x} + \gamma_{v_0} E'_{yV} \hat{y} = \frac{q}{4\pi\epsilon_0} \left[\frac{\gamma_{v_0} (x + v_0 t) \hat{x} + \gamma_{v_0} y \hat{y}}{(\gamma_{v_0}^2 (x + v_0 t)^2 + y^2)^{\frac{3}{2}}} \right] = \frac{q(1 - v_0^2 / c^2)(x_p \hat{x} + y_p \hat{y})}{4\pi\epsilon_0 (x_p^2 + y_p^2 (1 - v_0^2 / c^2))^{\frac{3}{2}}}.$$

$\vec{r}_p$ has direction from present position of source to observer. We study acceleration part.

$$E_{xA} \hat{x} = E'_{xA} \hat{x} = \frac{-q}{4\pi\epsilon_0 c^2} \frac{a'_q y'^2 \hat{x}}{(x'^2 + y'^2)^{\frac{3}{2}}} = \frac{-q}{4\pi\epsilon_0 c^2} \frac{a_q y_p^2 \hat{x}}{(x_p^2 + y_p^2 (1 - v_0^2 / c^2))^{\frac{3}{2}}},$$

$$E_{yA} \hat{y} = \gamma_{v_0} (E'_{yA} - v_0 B'_{zA}) \hat{y} = \frac{q a'_q y' \left[\gamma_{v_0} \left(x' - \frac{v_0}{c} \sqrt{x'^2 + y'^2} \right) \right] \hat{y}}{4\pi\epsilon_0 c^2 (x'^2 + y'^2)^{\frac{3}{2}}} = \frac{q a_q x y_p \hat{y}}{4\pi\epsilon_0 c^2 (x_p^2 + y_p^2 (1 - v_0^2 / c^2))^{\frac{3}{2}}},$$

Finally, get $\vec{E}$ adding velocity and acceleration parts. The direction of velocity part is from present position of source to present position of observer, if acceleration effect on distance can be neglected. In frame A, observer velocity at instant t will be zero. Force using (19),

$$\vec{F} = q \vec{E} = \frac{q^2}{4\pi\epsilon_0} \frac{(1 - v_0^2 / c^2)(x_p \hat{x} + y_p \hat{y}) + (a_q / c^2)(x y_p \hat{y} - y_p^2 \hat{x})}{\left(r + \frac{(x - x_q) v_0}{c} \right)^3}. \quad (29)$$

We get same electric field as (25) by writing in the above equation $v_o \hat{x} = -v_q \hat{x}$.

Observation: In obtaining (29) from (28), we have used distance relation $x' = \gamma_{v_0} (x + v_0 t)$. If we wanted equation (29) to reflect real distances between accelerating source and observer then at least for large distances, we need some different type of relation such as

$$x' - x'_q = \gamma_{(v_0(t_q), a_q(t_q))} \left((x - x_q) + v_0(t_q)(t - t_q) + \frac{1}{2} a_q(t_q)(t - t_q)^2 + \dots \right).$$

With this change, the direction of velocity part of electric field $\vec{E}_v$ (equations (25) and (31)) will be along vector $\vec{u} = \vec{r} - r\vec{v}_q / c - r^2 \vec{a}_q / 2c^2$, which will be along the line connecting present position of source to present position of observer [6]. The denominator will also modify accordingly. However, we want relativistic field transformations required by equation (11) to be satisfied in both, constant velocity or accelerating source situation. Therefore, we are compelled to use standard relativistic distance transformations (8) to satisfy requirements (11). The distance gets adjusted to give proper transformed values of energy and momentum in the new frame, which requires appropriate transformed value of force in this particular frame. The main aim is to get correct value of force on a particle in a particular frame. We obtain force created by a moving source in laboratory by relativistic transformation from a frame in which source is either static or at least instantaneously stationary. All the terms in the force expression have to be transformed. For a constant velocity source, the frame and source velocity match giving transformed distance same as actual distance the source travels in the laboratory frame. For an accelerating source, these two values will be different. Therefore, direction of electric field will be along a line from present position of source to present position of observer for a constant velocity source. For an accelerating source, the direction of electric field will not be exactly from the present position of source to present position of observer.

2.4. Electric and magnetic field of a charge moving with velocity in general direction

Position, velocity and acceleration of source charge q at retarded time t_q are $\vec{r}_q$, $\vec{v}_q$ and $\vec{a}_q$. Position of observer charge q at present time t is $\vec{r}$. Relative distance between these two positions is $\vec{r} - \vec{r}_q$ and for simplicity to shorten the expression, we write

$$\vec{r} - \vec{r}_q = (x - x_q)\hat{x} + (y - y_q)\hat{y} + (z - z_q)\hat{z} = r_x\hat{x} + r_y\hat{y} + r_z\hat{z} = \vec{r}.$$

$$\sqrt{(x - x_q)^2 + (y - y_q)^2 + (z - z_q)^2} = c(t - t_q) = \sqrt{r_x^2 + r_y^2 + r_z^2} = r.$$

Differentiating we get various terms like some given below,

$$\frac{\partial \vec{r}_q}{\partial t_q} = \vec{v}_q, \quad \frac{\partial \vec{v}_q}{\partial t_q} = \vec{a}_q, \quad \frac{\partial t_q}{\partial x} = \frac{-r_x}{rc - \vec{r} \cdot \vec{v}_q}, \quad \frac{\partial r_x}{\partial x} = 1 + \frac{v_{qx} r_x}{rc - \vec{r} \cdot \vec{v}_q}, \quad \frac{\partial v_{qx}}{\partial x} = \frac{-a_{qx} r_x}{rc - \vec{r} \cdot \vec{v}_q},$$

$$\frac{\partial t_q}{\partial t} = \frac{rc}{rc - \vec{r} \cdot \vec{v}_q}, \quad \frac{\partial r}{\partial t} = \frac{-c(\vec{r} \cdot \vec{v}_q)}{rc - \vec{r} \cdot \vec{v}_q}, \quad \frac{\partial r}{\partial x} = \frac{cr_x}{rc - \vec{r} \cdot \vec{v}_q}, \quad \frac{\partial v_{qz}}{\partial t} = \frac{rca_{qz}}{rc - \vec{r} \cdot \vec{v}_q}. \quad (30)$$

$$\begin{aligned} \vec{\nabla} \phi &= \frac{q}{4\pi\epsilon_0} \left(\frac{c\vec{v}_q}{(rc - \vec{r} \cdot \vec{v}_q)^2} - \frac{c\vec{r}(c^2 - v_q^2 + \vec{a}_q \cdot \vec{r})}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right), \\ \frac{\partial \phi}{\partial t} &= \frac{q}{4\pi\epsilon_0} \left(\frac{-c^3}{(rc - \vec{r} \cdot \vec{v}_q)^2} + \frac{rc^2(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^3} + \frac{r(\vec{r} \cdot \vec{a}_q)c^2}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right), \\ \frac{\partial \vec{A}}{\partial t} &= \frac{q}{4\pi\epsilon_0} \left(\frac{r\vec{a}_q - c\vec{v}_q}{(rc - \vec{r} \cdot \vec{v}_q)^2} + \frac{r(c^2 - v_q^2 + \vec{a}_q \cdot \vec{r})\vec{v}_q}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right). \end{aligned}$$

Using equation (17), get the *standard* results for fields [6].

$$\vec{E} = -\vec{\nabla} \phi - \frac{\partial \vec{A}}{\partial t} = \frac{q}{4\pi\epsilon_0} \left(\frac{(c^2 - v_q^2)(c\vec{r} - r\vec{v}_q) + r(\vec{r} \times (c\hat{r} - \vec{v}_q) \times \vec{a}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right). \quad (31)$$

$$\vec{B} = \vec{\nabla} \times \vec{A} = \frac{\hat{r}}{c} \times \vec{E}. \quad (32)$$

2.5. To verify if the standard electric and magnetic field results satisfy Maxwell equations

We verify that equation (31) and (32) satisfy Maxwell equations. Write by separating velocity and acceleration portions as $\vec{E}_v$, $\vec{B}_v$ and $\vec{E}_a$, $\vec{B}_a$ as follows.

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \vec{\nabla} \cdot [\vec{E}_v + \vec{E}_a], \\ \vec{\nabla} \cdot \vec{E} &= \frac{q}{4\pi\epsilon_0} \vec{\nabla} \cdot \left[\left(\frac{(c^2 - v_q^2)(c\vec{r} - r\vec{v}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right) + \left(\frac{(\vec{a}_q \cdot \vec{r})(c\vec{r} - r\vec{v}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} - \frac{r\vec{a}_q}{(rc - \vec{r} \cdot \vec{v}_q)^2} \right) \right]. \end{aligned}$$

Equations (15) and (16) give various terms which cancel each other. Equation (16) gives various terms in $\vec{r}$, $\vec{v}_q$, $\vec{a}_q$ and $\vec{a}_q$ direction. Write a few important terms. Refer to Section 2.2. Use delta function property. We get ρ and $\vec{J}$ from Maxwell equations.

$$\rho = \epsilon_0 \vec{\nabla} \cdot \vec{E} = \frac{q}{4\pi} \left[(1 - v_q^2 / c^2) \left(\frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} - \frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} \right) + \left(\frac{2r(\vec{v}_q \cdot \vec{a}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} - \frac{2r(\vec{r} \cdot \vec{a}_q)(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^4} \right) \right]$$

$$+ \frac{q}{4\pi} \left[-\frac{2r(\vec{v}_q \cdot \vec{a}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} + \frac{2r(\vec{r} \cdot \vec{a}_q)(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^4} \right] = \frac{q(1 - v_q^2 / c^2)}{4\pi} \left(\frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} - \frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} \right),$$

$$\rho = \epsilon_0 \vec{\nabla} \cdot \vec{E} = [\rho_{v_q} + \rho_{a_q}] + [-\rho_{a_q}] = \frac{q\gamma_{v_q}}{4\pi} \left(\frac{3}{\gamma_{v_q}^3 (r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} - \frac{3}{\gamma_{v_q}^3 (r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} \right).$$

We can suggest that singularity lies on X axis.

$$\rho = \frac{q\gamma_{v_q}}{4\pi} \left(\frac{3}{r^3} - \frac{3}{r^3} \right) = q\gamma_{v_q} \delta(r) = q\gamma_{v_q} \delta(\gamma_{v_q} (r - \frac{\vec{r} \cdot \vec{v}_q}{c})) = q\delta\left(r - \frac{\vec{r} \cdot \vec{v}_q}{c}\right).$$

$$\vec{J} = \frac{1}{\mu_0} \left[\left(\vec{\nabla} \times \vec{B}_v - \frac{1}{c^2} \frac{\partial \vec{E}_v}{\partial t} \right) + \left(\vec{\nabla} \times \vec{B}_a - \frac{1}{c^2} \frac{\partial \vec{E}_a}{\partial t} \right) \right],$$

$$\vec{J} = \frac{q}{4\pi} \left[(1 - v_q^2 / c^2) \left(\frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} - \frac{3}{(r - \frac{\vec{r} \cdot \vec{v}_q}{c})^3} \right) \vec{v}_q + c \left(\frac{2r(\vec{v}_q \cdot \vec{a}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} - \frac{2r(\vec{r} \cdot \vec{a}_q)(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^4} \right) \hat{r} \right]$$

$$+ \frac{qc}{4\pi} \left[-\frac{2r(\vec{v}_q \cdot \vec{a}_q)}{(rc - \vec{r} \cdot \vec{v}_q)^3} + \frac{2r(\vec{r} \cdot \vec{a}_q)(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^4} \right] \hat{r} = [\vec{J}_{v_q} + \vec{J}_{a_q}] + [-\vec{J}_{a_q}] = q\delta\left(r - \frac{\vec{r} \cdot \vec{v}_q}{c}\right) \vec{v}_q.$$

For $r - \vec{r} \cdot \vec{v}_q / c \neq 0$, both $\rho = 0$ and $\vec{J} = 0$. At, $r - \vec{r} \cdot \vec{v}_q / c = 0$, the expression gives a delta function. We have assumed source as a point charge in Section 2.2.

Observation: For the acceleration dependent current $\vec{J}_{a_q}$ and charge density ρ_{a_q} , we get equal and opposite terms from velocity and acceleration portions of field. Also $\vec{J}_{a_q} = c\rho_{a_q} \hat{r}$. We find a similar expression when we examine radiation issue in Paper II.

2.6. Poynting's theorem

We now study the Poynting's theorem [7]. The Poynting vector becomes basis for the Larmor formula giving the standard formula for radiation by a point charge. We first study application of Poynting's theorem to a single charged particle. Using Maxwell equation, we can write work done by a particle on itself as

$$\vec{E} \cdot \vec{J} = \frac{1}{\mu_0} \vec{E} \cdot (\vec{\nabla} \times \vec{B}) - \epsilon_0 \vec{E} \cdot \frac{\partial \vec{E}}{\partial t} = -\frac{1}{\mu_0} \left[\left\{ \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{E^2}{c^2} + B^2 \right) \right\} + \left\{ \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \right\} \right].$$

We take volume integral of above equation. As per Poynting's theorem, the first bracket gives rate of reduction of stored field energy and second term has the Poynting vector. Replacing by scalar and vector potentials, write the above two brackets as follows,

$$(a) \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{E^2}{c^2} + B^2 \right) = -\vec{E} \cdot \left(\frac{1}{c^2} \vec{\nabla} \frac{\partial \phi}{\partial t} \right) - \vec{E} \cdot \left(\frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right) + \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = f(\vec{r}, t) - \vec{E} \cdot \left(\frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right) + g(\vec{r}, t),$$

$$(b) \vec{\nabla} \cdot (\vec{E} \times \vec{B}) = -\vec{E} \cdot \vec{\nabla} (\vec{\nabla} \cdot \vec{A}) + \vec{E} \cdot (\nabla^2 \vec{A}) - \vec{B} \cdot \frac{\partial \vec{B}}{\partial t} = -f(\vec{r}, t) + \vec{E} \cdot (\nabla^2 \vec{A}) - g(\vec{r}, t).$$

Adding (a) and (b). The first terms cancel due to Lorentz gauge. For $r - \vec{r} \cdot \vec{v}_q / c \neq 0$.

$$\vec{E} \cdot \vec{J} = -\frac{1}{\mu_0} \left[\left\{ \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{E^2}{c^2} + B^2 \right) \right\} + \left\{ \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \right\} \right] = -\frac{\vec{E}}{\mu_0} \cdot \left[\left\{ \nabla^2 \vec{A} \right\} - \left\{ \frac{1}{c^2} \frac{\partial^2 \vec{A}}{\partial t^2} \right\} \right] = 0. \quad (33)$$

We have already proved in Section 2.5 that $\vec{J} = 0$ for $r - \vec{r} \cdot \vec{v}_q / c \neq 0$ or we directly prove by substituting $\vec{A}$ in the current equation part of equation (33).

Observation: We take volume integral of (33). Theorem then suggests that first curly bracket in equation (33) gives the rate of reduction in stored energy in volume V and second part shows the radiated energy from surface of this volume. Only justification is that equation (33) looks like continuity equation in fluid mechanics. For this single particle analysis, we are essentially writing zero ($\vec{E} \cdot \vec{J} = 0$) equal to sum of two equal and opposite portions. We are then giving physical meaning to these portions without describing any physical picture similar to fluids. There is also another problem. Delta function current $\vec{J}$ is not a continuous distribution of charge and current [7]. $\vec{J}$ is infinity at $r - \vec{r} \cdot \vec{v}_q / c = 0$. Field $\vec{E}(\vec{r}, t)$ also has singularity at $r - \vec{r} \cdot \vec{v}_q / c = 0$. Term $\int_V (\vec{E}(\vec{r}, t) \cdot \vec{J}) dV$ will not be a meaningful answer.

Requirement that $\vec{E}(\vec{r}, t)$ should be well-behaved, when $\vec{J}$ is a delta function, is not satisfied [4]. Although, we can write for $\vec{r}' \neq 0$,

$$\nabla^2 \left(\frac{1}{r'} \right) = \frac{3}{r'^3} - \frac{3}{r'^3}.$$

The term $\vec{r}' = 0$ is always included in the integration calculation. Even if $f(0)$ is well behaved, breaking up of a delta function as a difference of two functions, in whatever form, is not acceptable. We cannot give any physical meaning to the two equal and opposite parts of the following equation.

$$f(0) = \int f(r') \delta(r') dr'^3 = \frac{-1}{4\pi} \left[\int f(r') \frac{3}{r'^3} dr'^3 - \int f(r') \frac{3}{r'^3} dr'^3 \right].$$

The analysis shows that the Poynting vector method is not valid for a single particle represented by a delta function as $\vec{E} \cdot \vec{J}$ is a non-integrable singularity at $r - \vec{r} \cdot \vec{v}_q / c = 0$ and zero everywhere else. Therefore theoretical justification for a single charge radiation is missing and it is incorrect to say that electrodynamics suggests such radiation.

3. Conclusion

The Poynting's theorem is conventionally being used for all types of field conditions. We have obtained an important relation (33) for a delta function source. First term of (33), $\vec{E} \cdot \vec{J}$ is non-integrable at $r - \vec{r} \cdot \vec{v}_q / c = 0$ and zero everywhere else. The second volume integral term giving rate of change in stored energy is very difficult to solve. We need knowledge of $\vec{E}$ and $\vec{B}$ at all points (positions) within the closed volume at same time instant t to take volume integral at the time t . As retarded source position is different corresponding to every such point at one instant t within the closed volume, the source will have different values of velocity and acceleration along its path. Therefore, knowledge of source acceleration and velocity only for one retarded instant at centre of the volume is not sufficient. Only the last, surface integral of cross product term called Poynting vector is supposed to give rate at which energy leaves the surface. This is then used to evolve the Larmor formula[6]. It is not appropriate to assign meaning only to one term in any mathematical equation, if it is not possible to evaluate the other terms.

Note, we did not study a complete and well defined system when we studied a single accelerating particle. We did not specify why it was accelerating and who was supplying energy to it. We have tried to understand limitations of Poynting's theorem in dealing with a single particle situation in this paper.

We started with energy and momentum transformation rules and then tried to evolve some important rules of electrodynamics from the relativistic force expression. We then analysed the Poynting's theorem and proved that it has limitations in dealing with a single point particle situation. Therefore, finding out change in *total energy* of the system may be a better option, especially when particles are in a bound system.

FIGURES