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A relook at radiation by a point charge- II

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Abstract: Efforts to suggest a classical model for hydrogen atom are discouraged by a conclusion, based on the principles of electrodynamics that an accelerating charged particle necessarily radiates. In this paper, we continue our re-examination of this conclusion. We examine the standard electric field formula and difficulties associated with it, for an accelerating source. Conventionally, the Larmor formula is obtained from the Poynting vector. We attempt this Larmor formula by finding rate of change of potential energy in the given volume. We feel that, in this task, the logic is weak and our assumptions are difficult to justify. We have to conclude that the statement, radiation makes classical circular orbits unstable is difficult to justify theoretically. Then circular orbits with all radii, with matching velocities, should be allowed classically. We try to give some justification for Bohr postulates, then only certain radii will be allowed resulting in discrete energy states. We conclude that, for radiation study, it is advisable to calculate change in total energy of the system instead of following a theorem based approach. We require complete and well defined systems to evolve a classical picture, which makes it desirable to study a simple two particle system of hydrogen atom.

1. Re-examination of the point charge analysis

We shall focus on the stability of hydrogen atom in this paper.

1.1. Difficulties with standard field expression and force by a moving source

In an *isolated* two particle system, both particles accelerate due to forces on each other. We call one of them as source and examine standard formulas for a static source, constant velocity source and accelerating source, step by step. We start with a *static* source (Paper I-Section 2.1). We do a single dimension analysis. Potential created by a static source q in primed frame B at observer q , can be written as a function of distance alone. $x' - x'_q = c(t' - t'_q)$.

$$\phi'(x', t') = \frac{q}{4\pi\epsilon_0(x' - x'_q)}, \quad \frac{dt'_q}{dt'} = \frac{c - v'}{c - v'_q}.$$

We assumed that the signal travels at speed of light. Observer speed at time t' is v' and source speed at t'_q is $v'_q=0$. Rate of reduction of field energy in the two particles system should be equal to rate of increase of kinetic energy of observer, as source is static.

$$q \frac{d\phi'}{dt'} = \frac{-q^2 \frac{d}{dt'}(x' - x'_q)}{4\pi\epsilon_0(x' - x'_q)^2} = \frac{-q^2 c(v' - v'_q)}{4\pi\epsilon_0(x' - x'_q)^2(c - v'_q)} = \frac{-q^2 v'}{4\pi\epsilon_0(x' - x'_q)^2}.$$

$$q \frac{d\phi'(x', t')}{dt'} = q \frac{dx'}{dt'} \frac{d\phi'}{dx'} = qv' \left(\frac{\partial \phi'}{\partial x'} + \frac{1}{v'} \frac{\partial \phi'}{\partial t'} \right) = qv' \frac{\partial \phi'}{\partial x'} = -v' F'_x = -\frac{dE'_{K.E.}}{dt'}. \quad (1)$$

We could write partial derivative instead of total derivative as for a static source in (1), we could write ϕ' as a function only of distance. Similar to Paper I- Section 1.3, we write force F'_x created by source on observer, when observer moves from (x'_1, t'_1) to (x'_2, t'_2) ,

$$F'_x = -q \frac{d\phi'}{dx'} = -q \frac{\phi'(x'_2, t'_2) - \phi'(x'_1, t'_1)}{x'_2 - x'_1} = -q \frac{\phi'(x'_2, t'_1) - \phi'(x'_1, t'_1)}{x'_2 - x'_1} = -q \frac{\partial \phi'}{\partial x'}.$$

We consider a *constant velocity* source now. Force created by a moving source of constant velocity v_q in un-primed lab frame A can be obtained, as discussed in Paper I- Section 2.1, by transforming the primed frame values to lab frame, assuming v_q as frame velocity.

$$F_x = F'_x = -q\gamma_{v_q} \left[\frac{\partial}{\partial x} + \frac{v_q}{c^2} \frac{\partial}{\partial t} \right] \phi' = -q \left[\frac{\partial}{\partial x} (\gamma_{v_q} \phi') + \frac{\partial}{\partial t} \left(\frac{\gamma_{v_q} v_q \phi'}{c^2} \right) \right] = -q \left(\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} \right),$$

$$F_x = -q \left[\frac{\partial}{\partial x} + \frac{v_q}{c^2} \frac{\partial}{\partial t} \right] \phi = -q \left[\left(\frac{\phi(x_2, t_1) - \phi(x_1, t_1)}{x_2 - x_1} \right) + \frac{v_q}{c^2} \left(\frac{\phi(x_1, t_2) - \phi(x_1, t_1)}{t_2 - t_1} \right) \right]. \quad (2)$$

Equation (2) gives force on observer by a constant velocity source. In a single dimension analysis, magnetic field $\vec{B}$ does not appear in force. In equation above ($F_x = F'_x = \dots$), the operator acting on ϕ' is a relativistic operator where v_q is frame velocity. In standard electrodynamics, this same formula is used even when the source is accelerating and v_q is varying. We examine this now. Refer to Fig. 1. The source with speed v_{q1} at (x_{q1}, t_{q1}) sent a signal which is received by observer at (x_1, t_1) . The source is of varying speed and moves to a new position (x_{q2}, t_{q2}) . This source at (x_{q2}, t_{q2}) with speed v_{q2} sent a signal received by the observer at (x_2, t_2) . Observer particle experiences force F_1 at (x_1, t_1) and F_2 at (x_2, t_2) which can be obtained using (2). The force on the observer particle as it travels a small distance, $(x_2 - x_1) = \Delta x \rightarrow 0$, should be equal to average of these forces.

$$F_{(Avg)x} = \lim_{t_2 \rightarrow t_1} \frac{F_1 + F_2}{2} = -q \lim_{t_2 \rightarrow t_1} \left(\frac{\partial \phi}{\partial x} + \frac{v_{q1} + v_{q2}}{2c^2} \frac{\partial \phi}{\partial t} \right) = -q \left(\frac{\partial}{\partial x} + \frac{v_q}{c^2} \frac{\partial}{\partial t} \right) \phi, \quad (3)$$

If we *desire* to use equation (2), even for a *varying velocity* source, force on observer will be,

$$F_x = -q \left[\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial t} \frac{v_q \phi}{c^2} \right] = -q \lim_{x_2 \rightarrow x_1, t_2 \rightarrow t_1} \left(\frac{\phi(x_2, t_1) - \phi(x_1, t_1)}{x_2 - x_1} + \frac{v_{q2} \phi(x_1, t_2) - v_{q1} \phi(x_1, t_1)}{c^2 (t_2 - t_1)} \right),$$

$$F_x = -q \left(\frac{\partial \phi}{\partial x} + \frac{v_q}{c^2} \frac{\partial \phi}{\partial t} + \frac{\phi}{c^2} \frac{\partial v_q}{\partial t} \right) = -q \left(\frac{\partial \phi}{\partial x} + \frac{\partial}{\partial t} \frac{v_q \phi}{c^2} \right) = -q \left(\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} \right) = qE_x. \quad (4)$$

Equation (4) gives the standard formula for electric field which is conventionally used for a constant or varying velocity source. With this modification, we can study the both particles accelerating situation. Equations (3) and (4) will give different values because of the varying velocity third term in (4). This is the *first difficulty* in using (4) for a varying velocity source situation. However, this is not the main difficulty when we wish to calculate rate of change of energy of a two particle system. We discuss this aspect now.

Observation: In (1), we could change total derivative to partial derivative, as source is static in primed frame, $\dot{v}_q = 0$. We introduced a condition $(\partial \phi' / \partial t') = 0$, based on property of a static source, that potential is a function of distance alone. In reality, this source should also experience force due to observer and has to be kept static by some external arrangement.

$$q \frac{d\phi'}{dt'} = q \left(\frac{dx'}{dt'} \right) \left(\frac{d\phi'}{dx'} \right) = qv' \left(\frac{d\phi'}{dx'} \right) = -qv' E'_{T.D.x} = -qv' E'_x = qv' \left(\frac{\partial \phi'}{\partial x'} \right).$$

The total and partial derivative will give same value only in primed frame. Transform the above equation to lab and the source will have constant velocity v_q equal to frame velocity. Here v is velocity of the observer particle and E_x is standard electric field.

$$q \frac{d\phi}{dt} = qv \left(\frac{d\phi}{dx} \right) = qv \left(\frac{\partial \phi}{\partial x} + \frac{1}{v} \frac{\partial \phi}{\partial t} \right) = -qv E_{T.D.x} \neq -qv E_x = qv \left(\frac{\partial \phi}{\partial x} + \frac{v_q}{c^2} \frac{\partial \phi}{\partial t} \right).$$

Note, $E_{T.D.x} \neq E_x$. For a two particle system, change in potential energy should be equal to total of change in kinetic energies of both particles (neglecting radiation). Some external arrangement is required to balance the force on source to keep its velocity constant. But, our aim is to study an isolated two particle system and if one charge is *assumed* to have constant velocity then the rate of change of field energy should be equal to change in kinetic energy of observer alone. From (1), for a static source, $q(d\phi' / dt') = -(dE'_{K.E.} / dt')$. Transform to lab frame with, $\phi' = \gamma_{v_q} (\phi - A_x v_q)$, $E'_{K.E.} = \gamma_{v_q} (E_{K.E.} - p_x v_q)$ and $t' = \gamma_{v_q} (t - x v_q / c^2)$.

$$q \frac{d\phi}{dt} \left(1 - \frac{dA_x}{d\phi} v_q \right) = - \frac{dE_{K.E.}}{dt} \left(1 - \frac{dp_x}{dE_{K.E.}} v_q \right).$$

$A_x = v_q \phi / c^2$ gives, $dA_x / d\phi = v_q / c^2$. Also $E_{K.E.}^2 = m_o^2 c^4 + p_x^2 c^2$ for the observer particle gives, $dp / dE_{K.E.} = 1 / v$. Then to write, $q(d\phi / dt) = -qv E_x = -dE_{K.E.} / dt$, requires an unrealistic observer velocity, $v = c^2 / v_q$. We discuss this again in Section 1.2.

We study the conventional approach for an *accelerating* source now. The operator acting on ϕ' in the equation below, is obtained by relativistic transformation with frame velocity v_q . For constant source velocity v_q , with $\phi = \gamma_{v_q} \phi'$ write,

$$-E'_x = \left[\gamma_{v_q} \left(\frac{\partial}{\partial x} + \frac{v_q}{c^2} \frac{\partial}{\partial t} \right) \right] (\phi') = \left[\frac{\partial}{\partial x} (\phi) + \frac{\partial}{\partial t} \left(\frac{v_q \phi}{c^2} \right) \right] = \left(\frac{\partial \phi}{\partial x} + \frac{\partial A_x}{\partial t} \right) = -E_x.$$

We modified the relativistic operator, to act on new functions ϕ and $\vec{A}$ which is mathematically possible when v_q is constant. Conventionally, same formula is used for a varying source velocity also. Modifying the operator has no relativistic justification for varying source velocity. In next Section 1.2, we understand why the electric field obtained using this modified operator can still satisfy requirements of *relativistic transformations* (Paper I- equation (11)). Relativistic force, given in the Paper I- Section 1.3, is rate of change of momentum linked to actual motion of the observer particle. But in (4), the motion of observer is of no consequence. In hydrogen, both particles are in motion and we need a method to incorporate motions of both the particles to calculate any change in energy of the combination.

We are studying a two particle system and there is no expression for potential energy which is symmetric with respect to interchange of velocities and positions of particles in the system as in potential function only source velocity of retarded time is reflected. The best way is to calculate the rate of change of total energy, by using the expression,

$$\frac{1}{2} \frac{d}{dt} (m_1 v_1^2 + m_2 v_2^2) + \frac{1}{2} \frac{d}{dt} (q_1 \phi_{21} + q_2 \phi_{12}) = \frac{d(E_{Total})}{dt}.$$

It is necessary, for a stable system, $d(E_{Total})/dt = 0$.

1.2. Writing standard electric field expression in source coordinates

We aim to rewrite *standard electric field* expression from (4) in source coordinates. We study a two dimensional system in which source is accelerating.

$$\vec{E} = - \left[\vec{\nabla} \phi + \frac{\partial \vec{A}}{\partial t} \right] = - \left[\left\{ \left(\frac{\partial \phi}{\partial x} + \frac{v_{qx}}{c^2} \frac{\partial \phi}{\partial t} \right) \hat{x} + \left(\frac{\partial \phi}{\partial y} + \frac{v_{qy}}{c^2} \frac{\partial \phi}{\partial t} \right) \hat{y} \right\} + \frac{\phi}{c^2} \frac{\partial \vec{v}_q}{\partial t} \right]. \quad (5)$$

ϕ is a function of independent variables $\vec{r}$ and t when the source-observer relationship is not defined. In a two particle system, observer coordinates $\vec{r}$ and t are related by velocity, $\vec{v}_o = v_{ox} \hat{x} + v_{oy} \hat{y}$. Source coordinates $\vec{r}_q$ and t_q are related by velocity $\vec{v}_q$. Source and observer coordinates have relation, $(r - r_q) = c(t - t_q)$. Write $(r - r_q) = r$ to shorten the expression. Then $dt_q/dt = (rc - \vec{r} \cdot \vec{v}_o) / (rc - \vec{r} \cdot \vec{v}_q)$. Each position, say $(x = x_1, y = y_1, t = t_1)$ on observer path is linked to one unique position $(x_q = x_{q1}, y_q = y_{q1}, t = t_{q1})$ on the source path. We

can therefore write (5) as a function of one independent variable t_q . Write for curly bracket in (5), initially assuming a constant velocity source (Section 1.1- Observation),

$$-q \frac{d\phi}{dt} = -q \left(\frac{\partial \phi}{\partial t} + \vec{v}_o \cdot \vec{\nabla} \phi \right) = \vec{v}_o \cdot \vec{F}_{T.D.} = q \vec{v}_o \cdot \vec{E}_{T.D.}$$

$$\vec{E}_{T.D.} = - \left(\vec{\nabla} \phi + \frac{\vec{v}_o}{v_o^2} \frac{\partial \phi}{\partial t} \right) = - \left\{ \left(\frac{\partial \phi}{\partial x} + \frac{v_{ox}}{v_o^2} \frac{\partial \phi}{\partial t} \right) \hat{x} + \left(\frac{\partial \phi}{\partial y} + \frac{v_{oy}}{v_o^2} \frac{\partial \phi}{\partial t} \right) \hat{y} \right\}.$$

Compare with (5). Requirement $v_{ox}^2 + v_{oy}^2 = v_o^2$, is satisfied with $v_{ox} = c^2 / v_q$, $v_{oy} = 0$ and $v_o = v_{ox} = c^2 / v_q$. Here, $v_{oy} = 0$ in (5) requires source velocity $\vec{v}_q = v_q \hat{x}$ only in X direction. We write (5) in a total derivative format by adding the varying velocity term similar to (4).

$$\vec{E} = - \left[\left\{ \left(\frac{d\phi}{dx} \right) \hat{x} + \left(\frac{d\phi}{dy} \right) \hat{y} \right\} + \frac{\phi}{c^2} \frac{dv_q}{dt} \hat{x} \right],$$

$$\vec{E} = - \left[\left\{ \left(\frac{\partial \phi}{\partial x} + \frac{1}{w_x} \frac{\partial \phi}{\partial t} \right) \hat{x} + \left(\frac{\partial \phi}{\partial y} + \frac{1}{w_y} \frac{\partial \phi}{\partial t} \right) \hat{y} \right\} + \frac{\phi}{c^2} \left(\frac{\partial}{\partial t} + w_{ax} \frac{\partial}{\partial x} \right) v_q \hat{x} \right].$$

Start with curly bracket of $\vec{E}$. Compare with terms along X and Y directions in (5) and $\vec{E}_{T.D.}$. For such components, $dx/dt \neq v_{ox}$ and $dy/dt \neq v_{oy}$ representing actual observer velocity.

$$\frac{dt}{dx} = \frac{v_{ox}}{v_o^2} = \frac{1}{w_x} = \frac{v_q}{c^2}, \quad \frac{dt}{dy} = \frac{v_{oy}}{v_o^2} = \frac{1}{w_y} = \frac{0}{c^2}.$$

We need not worry about values of these components at this stage. Term dt_q/dt will be different for X and Y direction components. But, term $d\phi/dt_q$ should be of same value in all the components, as it depends on actual motion of the source. We get,

$$\frac{d\phi}{dx} = \frac{d\phi}{dt_q} \frac{dt_q}{dt} \frac{dt}{dx} = \frac{d\phi}{dt_q} \frac{(rc - \vec{r} \cdot \vec{w}_x)}{(rc - \vec{r} \cdot \vec{v}_q)} \frac{1}{w_x} = \frac{(rv_q - (x - x_q)c)}{c(rc - (x - x_q)v_q)} \frac{d\phi}{dt_q}.$$

$$\frac{d\phi}{dy} = \frac{d\phi}{dt_q} \frac{dt_q}{dt} \frac{dt}{dy} = \frac{d\phi}{dt_q} \frac{(rc - \vec{r} \cdot \vec{w}_y)}{(rc - \vec{r} \cdot \vec{v}_q)} \frac{1}{w_y} = \frac{(rcv_{oy} - (y - y_q)c^2)}{c^2(rc - (x - x_q)v_q)} \frac{d\phi}{dt_q} = \frac{-(y - y_q)}{(rc - (x - x_q)v_q)} \frac{d\phi}{dt_q}.$$

Take the last term in (5). Write as a total derivative with, $w_{ax} = 0$. Finally add all terms.

$$\frac{\phi}{c^2} \frac{\partial v_q}{\partial t} = \frac{\phi}{c^2} \left(\frac{\partial}{\partial t} + w_{ax} \frac{\partial}{\partial x} \right) v_q = \frac{\phi}{c^2} \frac{dv_q}{dt} = \frac{\phi}{c^2} \frac{dv_q}{dt_q} \frac{dt_q}{dt} = \frac{a_q \phi}{c^2} \frac{(rc - \vec{r} \cdot \vec{w}_{ax})}{(rc - \vec{r} \cdot \vec{v}_q)} = \frac{a_q \phi r}{c(rc - (x - x_q)v_q)}.$$

$$\vec{E} = \frac{1}{c(rc - v_q(x - x_q))} \left((\vec{r}c - rv_q \hat{x}) \frac{d\phi}{dt_q} - r\phi a_q \hat{x} \right), \quad (6)$$

$$\vec{E} = \frac{r}{rc - v_q(x - x_q)} \left(\frac{d\phi}{dt_q} \hat{r} - c \frac{d}{dt_q} \left(\frac{v_q \phi}{c^2} \right) \hat{x} \right). \quad (7)$$

Equation (7) is same as the standard electric field (5) with operator in source coordinates, in a total derivative format. Equation (7) has total derivative terms $(d\phi/dt_q)$ and $d(v_q \phi / c^2) / dt_q$ similar to rate of change of kinetic energy $(dE_{K.E.} / dt)$ and momentum $(d\vec{p} / dt)$. These terms obey *relativistic transformation* rules similar to the rate of change of energy and momentum. This explains why the standard electric field with modified operators obtained using (5) obeys relativistic transformation requirements, even for varying source velocity. The total derivatives in source coordinates can be linked to actual motion of source. Equation (6) result should match with standard result (Paper I- Section 2.2). Study velocity and acceleration terms separately.

$$\left. \frac{d\phi}{dt_q} \right|_{\text{velocity}} = \frac{d}{dt_q} \left(\frac{q}{4\pi\epsilon_0 \left(r - \frac{(x - x_q)v_q}{c} \right)} \right) = \frac{-qc^2 (rc(cx - rv_q)(v_{ox} - v_q) + ryv_{oy}(c^2 - v_q^2))}{4\pi\epsilon_0 rc(rc - \vec{r} \cdot \vec{v}_q)^2 (rc - (xv_{ox} + yv_{oy}))}. \quad (8)$$

Now if we use specific values of observer velocity $v_{ox} = c^2 / v_q$ and $v_{oy} = 0$, we get,

$$\vec{E}_v = \frac{(\vec{r}c - rv_q \hat{x})}{c(rc - v_q(x - x_q))} \frac{d\phi}{dt_q} = \frac{q}{4\pi\epsilon_0} \frac{(1 - \frac{v_q^2}{c^2})(\vec{r} - \frac{rv_q}{c} \hat{x})}{\left(r - \frac{(x - x_q)v_q}{c} \right)^3}. \quad (9)$$

This matches with velocity terms of the standard result. Using (6), obtain acceleration portion $\vec{E}_A$. With $\vec{E} = \vec{E}_v + \vec{E}_A$, result matches with the standard result obtained in the Paper I- Section 2.2 [1,2]. We examine why the answer matches only with these specific values of observer velocity. Refer to Fig. 2. Draw a primed frame and lab frame picture suggesting classical motions of source and observer. The source is static in primed frame at P' and observer motion is from $Q'(x'_1, y'_1, t'_1)$ to $R'(x'_2, y'_1, t'_1)$. To calculate $\vec{E}_x = -\partial\phi / \partial x'$ in frame B, we need to take $\Delta x' = (x'_2 - x'_1) \neq 0$, $\Delta y' = 0$, $\Delta t' = (t'_1 - t'_1) = 0$ between two observer positions Q' and R' , to obtain partial derivative of ϕ' . This classically means observer speed is $v'_{ox} = (\Delta x' / \Delta t') \rightarrow \infty$. We need not worry about this infinite observer velocity at this stage. The static source sends two signals at (x'_{q1}, t'_{q1}) and (x'_{q1}, t'_{q2}) . The observer points $Q'(x'_1, y'_1, t'_1)$ and $R'(x'_2, y'_1, t'_1)$ receive these signals at same time t'_1 as shown in figure. This actually suggests two *simultaneous events* at $Q'(x'_1, y'_1, t'_1)$ and $R'(x'_2, y'_1, t'_1)$. Primed frame travels at speed $v_q \hat{x}$ with respect to lab frame. We now shift to lab frame and study observer motion first. Corresponding observer coordinates in lab are $Q(x_1, y_1, t_1)$ and

$R(x_1 + \gamma_{v_q} \Delta x', y_1, t_1 + \gamma_{v_q} v_q \Delta x' / c^2)$. If distance Δx between Q and R corresponds to observer particle motion in lab frame then X direction speed of observer can be obtained by Lorentz transformations as follows. Write for $\Delta x' \neq 0, \Delta t' = 0$, $\Delta x = \gamma_{v_q} (\Delta x' + v_q \Delta t') = \gamma_{v_q} \Delta x'$ and $\Delta t = \gamma_{v_q} (\Delta t' + v_q \Delta x' / c^2) = \gamma_{v_q} v_q \Delta x' / c^2$.

$$v_{ox} = \lim_{\Delta x' \rightarrow 0} \frac{(x_1 + \gamma_{v_q} \Delta x') - x_1}{(t_1 + \frac{\gamma_{v_q} v_q \Delta x'}{c^2}) - t_1} = \lim_{\Delta x' \rightarrow 0} \frac{c^2 \Delta x'}{v_q \Delta x'} = \frac{c^2}{v_q}. \quad (10)$$

This result v_{ox} can also be obtained by putting $v'_{ox} = (\Delta x' / \Delta t') \rightarrow \infty$ in the relativistic velocity transformation equation of Paper I- Section 1.1. As we assumed simultaneous events in primed frame, we get this unrealistic velocity in lab frame. It can be noted that only for a specific value of observer velocity, we get the standard formula for electric field $\vec{E}$. Fig. 2 tried to pictorially explain this requirement. The actual observer velocity value is unimportant.

We now study source in Fig. 2. Source is static at P' in primed frame and the signals were released by source at (x'_{q1}, t'_{q1}) and (x'_{q1}, t'_{q2}) . Then its motion in lab is described by $P_2(x_{q1} + \gamma_{v_q} v_q \Delta t'_q, t_{q1} + \gamma_{v_q} \Delta t'_q)$ to $P_1(x_{q1}, t_{q1})$. Here $\Delta x'_q = 0$, $\Delta t'_q = (t'_{q2} - t'_{q1})$ with $t'_{q2} < t'_{q1}$. The speed of source in lab frame is

$$\lim_{\Delta t'_q \rightarrow 0} \frac{(x_{q1} + \gamma_{v_q} v_q \Delta t'_q) - x_{q1}}{(t_{q1} + \gamma_{v_q} \Delta t'_q) - t_{q1}} = v_q \hat{x}. \quad (11)$$

The standard electrodynamics analysis ensures realistic value of source velocity in lab frame.

Observation: Converting total derivative to partial in (1) has removed any significance of actual observer speed. Introducing a condition $(\partial \phi' / \partial t') = 0$, based on source condition, is actually equivalent to taking observer velocity $v'_{ox} \rightarrow \infty$ in equation (1). But, this condition is true only for a static charge in primed frame and therefore equation (4) should give correct answer only for a constant velocity source in lab frame. This is the *second difficulty* in using (4) for a varying velocity source. Analysis explained why field $\vec{E}$ obtained by standard formula (4) or (5) satisfies requirements of relativistic transformations (Paper I- equation (11)). But, could not give any mathematical justification for using the modified operator when source has varying velocity. This is the *third difficulty* in using (4) for a varying velocity source.

Poynting's analysis starts with $q\vec{v} \cdot \vec{E} = \vec{J} \cdot \vec{E}$. This is not appropriate for a microscopic system such as hydrogen for two reasons. (a) We need change in total energy of the system. We saw in Paper I- Section 1.6 that $q\vec{v} \cdot \vec{E}$ gives rate of change of kinetic energy of the observer particle only. (b) Field expression $\vec{E}$ is not sufficient to give an accurate description of force due to the three difficulties mentioned above when source is *accelerating*.

1.3. Rate of change of field energy

As per electrodynamics, source at $(\vec{r}_q, t_q)$ creates field $\phi(\vec{r}, t)$ in space, described by a function of two independent variables $\vec{r}$ and t . Another charged point particle called the observer experiences this field and moves due to force created by source. Now, we say that in a two particle situation, both particles act as source and observer for each other and together create field ϕ in space. Both particles move due forces created by each other. The particle under study is called observer. Every position of observer $(\vec{r}, t)$ on its path is linked to one specific position of source $(\vec{r}_q, t_q)$ on its path in space. Condition linking observer position and source positions is $(r - r_q) = c(t - t_q)$. The distance and time coordinates get related for observer and source particles by their respective speeds given by $\vec{v}_o = d\vec{r} / dt$, $\vec{v}_q = d\vec{r}_q / dt_q$. Rate of change of field energy between the two particles at the observer is,

$$q \frac{d\phi}{dt} = q \left(\vec{v}_o \cdot \vec{\nabla} \phi + \frac{\partial \phi}{\partial t} \right) = \frac{q^2}{4\pi\epsilon_0} \left[\vec{v}_o \cdot \left(\frac{c\vec{v}_q}{(rc - \vec{r} \cdot \vec{v}_q)^2} - \frac{c\vec{r}(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^3} - \frac{c\vec{r}(\vec{a}_q \cdot \vec{r})}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right) - \frac{c^3}{(rc - \vec{r} \cdot \vec{v}_q)^2} + \frac{rc^2(c^2 - v_q^2)}{(rc - \vec{r} \cdot \vec{v}_q)^3} + \frac{rc^2(\vec{a}_q \cdot \vec{r})}{(rc - \vec{r} \cdot \vec{v}_q)^3} \right]. \quad (12)$$

Observation: $\vec{v}_o$ is measured at present time and $\vec{v}_q$ and $\vec{a}_q$ at retarded time. Rate of change of potential energy is not zero even when $\vec{v}_q = 0$ and $\vec{v}_o = 0$ but $\vec{a}_q \neq 0$. This change in energy cannot be kinetic energy of particles and has to be radiation energy in space. Problem is, we measure source acceleration with respect to a fixed point called as the origin in lab. There may not be any charge at this origin which can either experience the field or influence the motion. Then, a human observer sitting in an accelerating train can see a stationary charge on ground, radiating. Whereas, for another human observer on ground, there is no radiation. Radiation has to be a real physical phenomenon. Also, Lienard-Wiechert potentials only reflect source velocity. Hence, it is advisable to measure acceleration with respect to the observer charge in a two particle system where the observer charge is also causing the source to move. Radiation should depend upon change in the status between the two charges. Radiation will be possible only if this *acceleration* leads to *change/reduction* in total energy of the system due to change in relative position between the two particles ($E_{Total} = \text{Kinetic Energy} + \text{Potential Energy}$). Then transform the radiated photon energy to lab frame to get lab values.

Point charges create and experience $(1/r^2)$ type forces and they will be always accelerating irrespective of whether they are in a linear motion or circular motion. The electrodynamics field relations are based on relativistic force. When motion between frames is linear, then acceleration is dependent on force on the particle and can lead to change in energy.

1.4. Covariance of rate of change of potential

We do a two dimensional analysis with source velocity $\vec{v}_q$ and acceleration $\vec{a}_q$ in X direction and observer velocity $\vec{v}_o$ in X and Y direction. From (12), write for lab frame,

$$\frac{d\phi}{dt} = \frac{q}{4\pi\epsilon_0} \left[- \left(\frac{(x - r v_q / c)(v_{ox} - v_q)}{(r - \vec{r} \cdot \vec{v}_q / c)^3} + \frac{y(1 - v_q^2 / c^2)v_{oy}}{(r - \vec{r} \cdot \vec{v}_q / c)^3} \right) + \frac{a_q(x - x_q)(r - \vec{r} \cdot \vec{v}_o / c)}{c(r - \vec{r} \cdot \vec{v}_q / c)^3} \right]. \quad (13)$$

In primed frame, source velocity is zero at one instant but acceleration is not zero.

$$\frac{d\phi'}{dt'} = \frac{q}{4\pi\epsilon_0} \left[- \left(\frac{x'v'_{ox}}{r'^3} + \frac{y'v'_{oy}}{r'^3} \right) + \frac{a'_q(x' - x'_q)(r' - \vec{r}' \cdot \vec{v}'_o / c)}{cr'^3} \right]. \quad (14)$$

If frame velocity between primed frame and un-primed lab frame is $v_q \hat{x}$,

$$\frac{d\phi'}{dt'} = \frac{\gamma_{v_q} d(\phi - A v_q)}{\gamma_{v_q} d(t - x v_q / c^2)} = \frac{\frac{d\phi}{dt} - v_q \frac{dA}{d\phi} \frac{d\phi}{dt}}{(1 - v_{ox} v_q / c^2)}. \quad (15)$$

Write $\vec{A}$ using results from Paper I- Section 1.6 and take dot product of $\vec{A}$ with itself. Now $A\hat{x} = (v_q \phi / c^2)\hat{x} = (\gamma_{v_q} v_q \phi' / c^2)\hat{x}$ gives $\phi^2 = \phi'^2 + A^2 c^2$. Using $A\hat{x}$ write,

$$v_q \frac{dA}{d\phi} = \frac{v_q^2}{c^2} + \frac{v_q \phi}{c^2} \frac{dv_q}{dt} \frac{dt}{d\phi} = \frac{v_q^2}{c^2} + \frac{v_q \phi a_q}{c^2} \left(\frac{(r - \vec{r} \cdot \vec{v}_o / c)}{(r - \vec{r} \cdot \vec{v}_q / c)} \right) \frac{dt}{d\phi}.$$

Substituting in (15),

$$\frac{d\phi'}{dt'} = \frac{(1 - v_q^2 / c^2)}{(1 - v_{ox} v_q / c^2)} \frac{d\phi}{dt} - \frac{v_q \phi a_q}{c^2 (1 - v_{ox} v_q / c^2)} \left(\frac{(r - \vec{r} \cdot \vec{v}_o / c)}{(r - \vec{r} \cdot \vec{v}_q / c)} \right). \quad (16)$$

Start with primed frame equation (14) and get lab equation (13) using (16). Velocity part,

$$\left. \frac{d\phi}{dt} \right|_{\text{velocity}} = \frac{(1 - v_{ox} v_q / c^2)}{(1 - v_q^2 / c^2)} \frac{d\phi'}{dt'} = \frac{q(1 - v_{ox} v_q / c^2)}{4\pi\epsilon_0 (1 - v_q^2 / c^2)} \left(\frac{-x'v'_{ox} - y'v'_{oy}}{r'^3} \right). \quad (17)$$

Refer Paper I- Section 2.2, $x_p = (x - x_q) - v_q(t - t_q)$, $y_p = y$ gives $\gamma_{v_q}(r - \vec{r} \cdot \vec{v}_q / c) = r'$.

$$\begin{aligned} \frac{d\phi}{dt} &= \frac{q(1 - v_{ox} v_q / c^2)}{4\pi\epsilon_0 (1 - v_q^2 / c^2)} \left[\frac{-\gamma_{v_q}(x - v_q t)}{\gamma_{v_q}^3 (r - \vec{r} \cdot \vec{v}_q / c)^3} \left(\frac{v_{ox} - v_q}{1 - v_{ox} v_q / c^2} \right) - \frac{y}{\gamma_{v_q}^3 (r - \vec{r} \cdot \vec{v}_q / c)^3} \left(\frac{v_{oy}}{\gamma_{v_q} (1 - v_{ox} v_q / c^2)} \right) \right], \\ \left. \frac{d\phi}{dt} \right|_{\text{velocity}} &= \frac{-q}{4\pi\epsilon_0} \left(\frac{(x - r v_q / c)(v_{ox} - v_q)}{(r - \vec{r} \cdot \vec{v}_q / c)^3} + \frac{y(1 - v_q^2 / c^2)v_{oy}}{(r - \vec{r} \cdot \vec{v}_q / c)^3} \right). \end{aligned} \quad (18)$$

Put acceleration part of (14) in (16). Here $a'_q = \gamma_{v_q}^3 a_q$ and $\phi = q / 4\pi\epsilon_0 (r - \vec{r} \cdot \vec{v}_q / c)$. Get acceleration part in lab. Adding to (18) gives (13). Rate of change in potential is covariant.

Observation: We used real velocity of observer in obtaining equation (17). Directly finding rate of change of potential energy is superior when both source and observer are in motion.

2. Radiation analysis

We saw in Paper I- Section 2.6 that it is difficult to assign proper physical meaning to the Poynting vector method for a point particle. We use Poynting vector to obtain Larmor formula to explain radiation by a point charge. We now try to get this formula from the basic principle of rate of change of potential energy. In observation section, we separately obtain the Larmor formula by solving volume integral of Poynting's equation (33) of Paper I. This helps us to understand the *presumptions* and *logic* behind the Larmor formula.

2.1. Larmor formula

We now *attempt* to get the Larmor formula by calculating rate of change of potential energy in a given volume. We make certain assumptions which will tell us about the physical picture behind the formula. This method will help us to get *volume integral* by overcoming difficulty mentioned in the conclusion section of Paper I. We take advantage of the fact that we are mainly concerned about what happens at the *surface*.

Consider a fluid flow in a pipe, with temperature in the pipe given by a function $T = T(r, t)$. Temperature is a function of independent coordinates r and t . We study a particle in motion in this fluid which travels from (r_1, t_1) to (r_2, t_2) . Write the basic expression for partial derivatives [3]. For these two positions along the particle motion, write an exact equation.

$$T(r_2, t_2) - T(r_1, t_1) = \frac{T(r_2, t_2) - T(r_2, t_1)}{(t_2 - t_1)}(t_2 - t_1) + \frac{T(r_2, t_1) - T(r_1, t_1)}{(r_2 - r_1)}(r_2 - r_1).$$

When *higher* order differential terms are small (*or zero*) and $\Delta t \rightarrow 0, \Delta r \rightarrow 0$, we can write

$$T(r_2, t_1) - T(r_1, t_1) \cong T(r_2, t_2) - T(r_1, t_2).$$

$$\frac{dT}{dt} = \frac{T(r_2, t_2) - T(r_1, t_1)}{t_2 - t_1} = \lim_{\Delta t, \Delta r \rightarrow 0} \left(\frac{T(r_2, t_2) - T(r_2, t_1)}{(t_2 - t_1)} + \frac{(r_2 - r_1)}{(t_2 - t_1)} \cdot \frac{T(r_2, t_2) - T(r_1, t_2)}{(r_2 - r_1)} \right).$$

Rate at which temperature is changing with respect to a point moving along with the fluid is,

$$\frac{dT}{dt} = \frac{\partial T}{\partial t} \Big|_{(r_2, t_2)} + v_{or} \cdot \frac{\partial T}{\partial r} \Big|_{(r_2, t_2)} = \frac{\partial T}{\partial t} + \vec{v}_{or} \cdot \vec{\nabla} T. \quad (19)$$

Even if $\partial T / \partial t \neq 0$ and $\partial T / \partial r \neq 0$, we can get $dT / dt = 0$ for velocity of the particle,

$v_{or} = -(\partial T / \partial t) / (\partial T / \partial r)$. This requires $T(r_2, t_2) = T(r_1, t_1)$, which can happen in two possibilities: (1) Value of temperature function T is same throughout space at least in the

region of consideration. (2) Change in temperature T travels at the same speed as the particle so that the observer particle experiences no difference. Write a similar exact equation for potential and analyse it.

$$\frac{d\phi}{dt} = \frac{\phi(r_2, t_2) - \phi(r_1, t_1)}{(t_2 - t_1)} = \frac{\phi(r_2, t_2) - \phi(r_2, t_1)}{(t_2 - t_1)} + \frac{(r_2 - r_1)}{(t_2 - t_1)} \frac{\phi(r_2, t_1) - \phi(r_1, t_1)}{(r_2 - r_1)}. \quad (20)$$

Energy required for travelling from one position on a hilly terrain to another should depend upon difference between heights of these two positions measured from sea level, which is the common reference point. Similarly difference between the observer and source potentials can be measured with respect to a common reference point. A general expression for potential difference in primed frame B, between a static source q at (r'_q, t'_q) and observer location at (r', t') , can be written as a function of distance as

$$\phi'(r', t') - \phi'_q(r'_q, t'_q) = \frac{A}{(r' - r'_q)^n}.$$

Differentiating with A as constant, we get.

$$(r' - r'_q)^n d(\phi'(r', t') - \phi'_q(r'_q, t'_q)) + n(r' - r'_q)^{n-1} (\phi'(r', t') - \phi'_q(r'_q, t'_q)) d(r' - r'_q) = 0.$$

$$n \frac{(\phi'(r', t') - \phi'_q(r'_q, t'_q)) (t' - t'_q)}{(t' - t'_q) (r' - r'_q)} = \frac{-d(\phi'(r', t') - \phi'_q(r'_q, t'_q))}{d(t' - t'_q)} \left(\frac{d(t' - t'_q)}{d(r' - r'_q)} \right).$$

Suppose signal travels at speed $\vec{v}_s$ from source towards a point (r', t') in space.

$$r' - r'_q = v_s(t' - t'_q) \text{ gives } d(r' - r'_q) = v_s d(t' - t'_q) \text{ and } dr' - dr'_q = v_s(dt' - dt'_q).$$

For a static source $\vec{v}_q = 0$, we have $dr'_q = 0$. Suppose, we have another *special* observer O'_{bf} , travelling in the same direction as the signal, at speed $\vec{v}_O \hat{r}$ then $dr' = v_O dt'$. The source releases a signal at an instant $(r'_q, t'_q) = (0, 0)$. We suggest that this observer O'_{bf} travels along with this signal released by source at the instant $(r'_q, t'_q) = (0, 0)$, at same speed $v_O \hat{r} = v_s \hat{r}$. As $\phi'_q(r'_q, t'_q)$ will have one unique value corresponding to the instant $(r'_q, t'_q) = (0, 0)$, we get $dt'_q = 0$ and $d\phi'_q(r'_q, t'_q) = 0$. We have seen in equation (19) that total derivative of the function will be zero when both speeds match. Then the following equation can be written under peculiar conditions assumed for this example. Neglecting higher order differential terms similar to equation (19),

$$-n \frac{\phi'(r', t') - \phi'_q(r'_q, t'_q)}{(t' - t'_q)} = \frac{d\phi'(r', t')}{dt'} = \frac{\partial \phi'}{\partial t'} \bigg|_{(r'_2=r', t'_2=t')} + \vec{v}_O \cdot \frac{\partial \phi'}{\partial \vec{r}'} \bigg|_{(r'_2=r', t'_2=t')} = 0.$$

This equation helps to equate the derivative with the ratio of difference in values between (r'_q, t'_q) to (r', t') . As we get $\phi'(r', t') = \phi'_q(r'_q, t'_q)$ in above equation, any particular value of n

will not be important. We now try to obtain Larmor formula by calculating the rate of change of potential energy in space. We shall list all the assumptions.

We have a two particle system. One of them is called the source. The source is accelerating.

Assumption (1): The source creates potential and field in entire space irrespective of whether there is any observer available to experience it. Any observer particle, if present, can experience it. This assumption is as per conventional concepts of electrodynamics. Field is creation of source alone and not a joint effort by both source and observer charges.

This source in primed frame at $O'(r'_q = 0)$, with speed $\vec{v}'_q = 0$ and $\vec{a}'_q \neq 0$ at a time instant $t'_q = 0$ releases signals in all directions to create potential and field.

Assumption (2): We replace this source by an artificial static source at $r'_q = 0$ which can release similar signals for all times $t' \geq (t'_q = 0)$. It releases signals in all directions to create similar potential in space. These signals create an expanding sphere with outer radius $R' = c(t' - t'_q)$ at time t' as shown in Fig. 3. As potential function ϕ'_a within the sphere of radius R' is created by the artificial source, we presume terms $\vec{v}'_q = 0$ and $\vec{a}'_q$ are constant in function ϕ'_a within entire volume of this sphere. Here, ϕ'_a is that part which depends on *acceleration*. We have replaced a real source by an artificial source because signals created by the real source at $r'_q = 0$, at one time instant $t'_q = 0$ can create an expanding thin shell of zero width around this position. This source will move away due to acceleration at the next instant. Solving a *volume integral* around $r'_q = 0$, for a moving source is difficult.

Use relations in Paper I- Section 2.4, for $r' < R'$ and $(R' - r') = \Delta r' \rightarrow 0$, write $\vec{\nabla}'\phi'_a$ at outer radius R' of the shell at time t' with $\vec{v}'_q = 0$ and $\vec{a}'_q \neq 0$. Here $\hat{R}'_{r'_q}$ is a unit vector from r'_q to R' .

$$\vec{\nabla}'\phi'_a|_{(R',t')} = \left(\frac{\partial \phi'_a}{\partial r'} \right)_{(R',t')} \hat{R}'_{r'_q} = \frac{-q}{4\pi\epsilon_0} \left(\frac{\vec{a}'_q \cdot \hat{R}'_{r'_q}}{c^2 (R' - r'_q)} \right) \hat{R}'_{r'_q}.$$

As we have an artificial source, the term $\vec{a}'_q \cdot \hat{R}'_{r'_q}$ remains constant as $\vec{a}'_q$ remains same throughout the distance from origin at r'_q to R' along the signal path. Here $(R' - r'_q)$ is distance between source position r'_q at time t'_q and the position R' at time t' . Write following equation, using (20) and we take $n = 1$ in above obtained equation.

$$\begin{aligned} \frac{\partial \phi'_a}{\partial r'} &= - \left(\frac{\phi'_a(R', t'_q) - \phi'_a(r'_q, t'_q)}{(R' - r'_q)} \right) = \left(\frac{-q}{4\pi\epsilon_0} \frac{\vec{a}'_q \cdot \hat{R}'_{r'_q}}{c^2 (R' - r'_q)} \right), \\ \phi'_a(R', t'_q) - \phi'_a(r'_q, t'_q) &= \frac{q}{4\pi\epsilon_0} \left(\frac{\vec{a}'_q \cdot \hat{R}'_{r'_q}}{c^2} \right). \end{aligned} \quad (21)$$

With $(R' - r'_q) = c(t' - t'_q)$ and using values obtained in Paper I- Section 2.4.

$$\frac{\partial \phi'_a}{\partial t'} = - \left(\frac{\phi'_a(R', t') - \phi'_a(R', t'_q)}{(t' - t'_q)} \right) = \frac{q}{4\pi\epsilon_0} \left(\frac{\vec{a}'_q \cdot \hat{R}'_{r_q}}{c(R' - r'_q)} \right).$$

$$\phi'_a(R', t') - \phi'_a(R', t'_q) = \frac{-q}{4\pi\epsilon_0} \left(\frac{\vec{a}'_q \cdot \hat{R}'_{r_q}}{c^2(t' - t'_q)} \right) (t' - t'_q) = \frac{-q}{4\pi\epsilon_0} \left(\frac{\vec{a}'_q \cdot \hat{R}'_{r_q}}{c^2} \right). \quad (22)$$

The rate of change of potential between source and observer becomes zero, only for

$$\vec{v}'_O = c\hat{R}'_{r_q} = \frac{(R'(t') - r'_q(t'_q))}{(t' - t'_q)} \hat{R}'_{r_q} = \frac{(R' - r'_q)}{(t' - t'_q)} \hat{R}'_{r_q}.$$

$$\frac{d\phi'_a}{dt'} = \frac{\phi'_a(R', t') - \phi'_a(r'_q, t'_q)}{t' - t'_q} = \frac{\partial \phi'_a}{\partial t'} \bigg|_{(r'_2=R', t'_2=t')} + c\hat{R}'_{r_q} \cdot \frac{\partial \phi'_a}{\partial r'} \bigg|_{(r'_2=R', t'_2=t')} = 0.$$

Possibility (2) suggests that change in ϕ'_a travels at speed of light from the source to the shell. Therefore a fictitious observer O'_{bf} travelling at speed of light will experience the potential as a constant value. Therefore, $\phi'_a(R', t') = \phi'_a(r'_q, t'_q)$. Solving (21), (22) with $\phi'_a(R', t'_q) = 0$, as the source at r'_q emitted signals at time t'_q only. Dropping negative sign as it has no effect.

$$\phi'_a(R', t') = \frac{q}{4\pi\epsilon_0} \frac{\vec{a}'_q \cdot \hat{R}'_{r_q}}{c^2} = \frac{q}{4\pi\epsilon_0} \frac{a'_q \cos \theta'}{c^2}.$$

A signal coming out of source will travel along $\hat{R}'_{r_q}$ direction. However as per Assumption (1), field and potential ϕ'_a will be created in entire space within the sphere. This, ϕ'_a varies with θ' also. We have taken source at $r'_q = 0$. Field created in $\hat{\theta}'$ direction on shell of radius R' is

$$\vec{E}'_a(R', t') = \frac{-d\phi'_a(R', t')}{R'd\theta'} = -\vec{\nabla}'_{\theta'} \phi'_a(R', t') = -\vec{\nabla}'_{\theta'} \frac{q}{4\pi\epsilon_0} \frac{\vec{a}'_q \cdot \hat{R}'_{r_q}}{c^2} = \frac{q}{4\pi\epsilon_0} \frac{a'_q \sin \theta'}{R'c^2} \hat{\theta}'.$$

Note that in space around source, $(\vec{E}'_a) \cdot (c\hat{R}'_{r_q}) = 0$ and this field will not do any work in vacuum. This means that field will not do any work on itself. The standard electric field formula in Paper I- Section 2.4 gives same $\vec{E}'_a$ as above for $\vec{v}'_q = 0$. Use Maxwell equation, to get charge density.

$$\vec{\nabla}'_{\theta'} \cdot \vec{E}'_a(R', t') = \vec{\nabla}'_{\theta'} \cdot \frac{q}{4\pi\epsilon_0} \frac{a'_q \sin \theta'}{R'c^2} \hat{\theta}' = \frac{q}{4\pi\epsilon_0 c^2 R' \sin \theta'} \frac{\partial}{\partial \theta'} \left(\frac{a'_q \sin^2 \theta'}{R'} \right) = \frac{\rho'_a}{\epsilon_0}.$$

Then $\rho'_a = 2qa'_q \cos \theta' / 4\pi R'^2 c^2$. Volume integral of acceleration charge density ρ'_a in a given space will be zero, for $\theta' = 0$ to π . This charge density is same as Paper I- Section 2.5, obtained directly from Maxwell equations, if we consider *only* acceleration portion of electric field. But, we saw in Paper I that if we consider both velocity and acceleration parts of standard electric field then charge density $\rho = 0$ for $r - \vec{r} \cdot \vec{v}_q / c \neq 0$.

Assumption (3): By definition, potential energy between source and observer is $q_{ob} \phi_{s \rightarrow ob}$. Here q_{ob} is charge on observer and $\phi_{s \rightarrow ob}$ is potential created between source and observer. Now we suggest that the source creates an expanding sphere of self-energy which can be obtained by multiplication of self-charge and self-potential.

Consider a shell in the sphere (Fig. 3), of outer radius R' and inner radius R'_{in} . Then rate at which self-energy leaves a sphere of fixed radius R' can be calculated as follows.

$$P' = \frac{dE'}{dt'} = \frac{d}{dt'} \int_{V'_{in}}^{V'} (\rho'_a \phi'_a) dV' = \frac{d}{dt'} \int_{R'_{in}}^{R'} \frac{2qa'_q \cos \theta'}{4\pi r'^2 c^2} \frac{qa'_q \cos \theta'}{4\pi \epsilon_0 c^2} r'^2 \sin \theta' dr' d\theta' d\phi'.$$

We have obtained above that the signal is traveling at speed of light. Outer radius R' is fixed and the inner radius R'_{in} of shell will expand at speed of light.

$$P' = \frac{dE'}{dt'} = \frac{d}{dt'} \left(\frac{q^2 \mu_0 a_q'^2 (R' - R'_{in})}{6\pi c^2} \right) = \frac{-q^2 \mu_0 a_q'^2}{6\pi c^2} \frac{dR'_{in}}{dt'} = \frac{-q^2 \mu_0 a_q'^2}{6\pi c}. \quad (23)$$

Observation: Equation (23) gives same value as Larmor formula, which we obtained without using the Poynting's theorem. We now compare with standard Poynting's theorem method. Write energy equation in continuity equation format. In equation below, first curly bracket is said to give rate of change of energy stored in space. Use Paper I- equations (31) and (32).

$$\vec{E} \cdot \vec{J} = -\frac{1}{\mu_0} \left[\left\{ \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{E^2}{c^2} + B^2 \right) \right\} + \left\{ \vec{\nabla} \cdot (\vec{E} \times \vec{B}) \right\} \right],$$

$$\vec{E} \cdot \vec{J} = -\frac{1}{\mu_0} \left[\frac{1}{c^2} \frac{\partial}{\partial t} \left\{ E^2 - \frac{(\vec{E} \cdot \hat{r})^2}{2} \right\} + \frac{1}{c} \vec{\nabla} \cdot \left\{ E^2 \hat{r} - \vec{E} (\vec{E} \cdot \hat{r}) \right\} \right] = \frac{\partial \rho_{Eng}}{\partial t} + \vec{\nabla} \cdot \vec{J}_{Eng} = 0.$$

Current equation is $\vec{J}_{Eng} = \rho_{Eng} \vec{V}_{Eng}$. Use only acceleration portion of standard electric field from Paper I- Section 2.4 and dot product of this field and unit vector from source along signal direction $\hat{r}$ is $\vec{E}_A \cdot \hat{r} = 0$. Velocity of energy flow is $\vec{V}_{Eng}$.

$$\vec{V}_{Eng} = \frac{\vec{J}_{Eng}}{\rho_{Eng}} = \frac{\frac{1}{c} \left\{ E_A^2 \hat{r} - \vec{E}_A (\vec{E}_A \cdot \hat{r}) \right\}}{\frac{1}{c^2} \left\{ E_A^2 - \frac{(\vec{E}_A \cdot \hat{r})^2}{2} \right\}} = c \hat{r},$$

$$E_A^2 = \vec{E}_A \cdot \vec{E}_A = \left(\frac{q}{4\pi\epsilon_0 r} \frac{(\hat{r} \times (\hat{r} - \vec{v}_q/c) \times \vec{a}_q)}{c^2(1 - \hat{r} \cdot \vec{v}_q/c)^3} \right)^2 = \frac{K_A}{r^2}.$$

Velocity of energy flow is equal to speed of light. Get rate of change of energy stored in field from Poynting's equation discussed in Paper I- Section 2.6.

$$-\frac{1}{\mu_0} \left\{ \frac{1}{2} \frac{\partial}{\partial t} \left(\frac{E_A^2}{c^2} + B_A^2 \right) \right\} = -\frac{1}{\mu_0 c^2} \frac{\partial}{\partial t} \left\{ E_A^2 - \frac{(\vec{E}_A \cdot \hat{r})^2}{2} \right\} = -\frac{1}{\mu_0 c^2} \left(\frac{-2K_A}{r^3} \frac{\partial r}{\partial t} + \frac{1}{r^2} \frac{\partial K_A}{\partial t} \right).$$

We had written $\vec{r} - \vec{r}_q = \vec{r}$ in Paper I- Section 2.4, to shorten the expressions. Take $\hat{r}_{r_q}$ as a unit vector from r_q to r . Also r and t are measured at observer location and were taken as independent variables for obtaining standard fields. With $v_q \ll c$,

$$\frac{\partial(r - r_q)}{\partial t} = -\frac{\partial r_q}{\partial t} = \frac{-\hat{r}_{r_q} \cdot \vec{v}_q}{1 - \hat{r}_{r_q} \cdot \vec{v}_q/c} \cong -\hat{r}_{r_q} \cdot \vec{v}_q.$$

Integrate from 0 to R with $r_q(t_q) = 0$ for obtaining volume integral. Note, $\Delta r_q \ll \Delta r$ as $r_q(t_q)$ will change with velocity v_q and $r(t)$ will change with velocity c .

$$\begin{aligned} & -\int_0^R \frac{r^2}{\mu_0 c^2} \left(\frac{-2K_A}{(r - r_q)^3} \frac{\partial(r - r_q)}{\partial t} + \frac{1}{(r - r_q)^2} \frac{\partial K_A}{\partial t} \right) \sin \theta dr d\theta d\varphi \\ & \cong -\int_0^R \frac{r^2}{\mu_0 c^2} \left(\frac{-2K_A}{r^3} (-\hat{r}_{r_q} \cdot \vec{v}_q) + \frac{1}{r^2} \frac{\partial K_A}{\partial t} \right) \sin \theta dr d\theta d\varphi. \end{aligned}$$

K_A is a function of $\vec{v}_q$ and $\vec{a}_q$. As given in Paper I- equation (30), $\vec{v}_q$ and $\vec{a}_q$ change with x, y and z and hence with r, θ and φ in a complicated manner. Solving this integral will be complicated and method does not seem to converge. We now use a method based on fluid mechanics[3]. Write continuity equation integrating over volume V with surface S . For a fixed volume, write continuity equation by taking differentiation outside integration sign.

$$\frac{d}{dt} \int_V \rho_{Eng} dV + \int_S (\rho_{Eng} \vec{V}_{Eng}) \cdot d\vec{S} = 0.$$

Follow same method as in earlier work to get volume integral in primed frame B for $\vec{v}_q' = 0$ and acceleration constant. Alternatively, we can integrate in a small range $\Delta R' = R' - R'_{in} \rightarrow 0$ such that $\vec{a}_q'$ can be treated as constant in this small range.

$$-\frac{d}{dt'} \int_{R'_{in}}^{R'} \frac{r'^2}{\mu_0 c^2} (E_A'^2) \sin \theta' dr' d\theta' d\varphi' = -\frac{d}{dt'} \int_{R'_{in}}^{R'} \frac{r'^2}{\mu_0 c^2} \left(\frac{q^2 a_q'^2 \sin^2 \theta'}{(4\pi\epsilon_0)^2 r'^2 c^4} \right) \sin \theta' dr' d\theta' d\varphi' = \frac{q^2 \mu_0 a_q'^2}{6\pi c}.$$

This volume integral gives same result as Larmor formula[1,2]. In the work done equation, first term of volume integral and second term of Poynting vector surface integral are equal and opposite. Their *total becomes zero* as expected by equation (33) in Paper I-Section 2.6.

Conclusions of this Larmor formula analysis are:

(a) Paper I discussed difficulties in obtaining Larmor formula by Poynting's method. We attempted same formula by finding rate of change of potential energy. But, could not find any justification for restricting to only acceleration dependent potential ϕ_a and charge density ρ_a in this analysis. Note, Lienard-Wiechert potential by a moving point charge is $\phi = q / 4\pi\epsilon_0 (r - \vec{r} \cdot \vec{v}_q / c)$ and charge density is $\rho = 0$ for $r - \vec{r} \cdot \vec{v}_q / c \neq 0$ (Paper I- Section 2.5).

(b) This single particle analysis does not specify why the particle is accelerating and who is supplying energy to it. We are not studying a complete and well defined system.

Therefore, we cannot depend on Poynting's theorem or Larmor formula when we deal with microscopic systems such as a two particle bound system like hydrogen atom.

2.2. Lienard's generalization

We obtained (23) when source is instantaneously stationary, $\vec{v}'_q = 0$ but $\vec{a}'_q \neq 0$ in primed frame. In primed frame, we have assumed that signal travels at speed of light just like photons. Assuming that signal is photon-like with relativistic mass as m_0 in primed frame. If this photon has speed $\vec{v}'_{ph} = c\hat{r}'$, $p'_x = m_0 c \cos \theta'$ and $E' = m_0 c^2$. Primed frame travels at velocity $v_q \hat{x}$ with respect to lab frame. Then in lab frame source has velocity $\vec{v}_q = v_q \hat{x}$ and $\vec{a}_q = a_{qx} \hat{x} + a_{qy} \hat{y} + a_{qz} \hat{z}$ at the corresponding instant. The rate of energy loss in lab is,

$$E = \gamma_{v_q} (E' + v_q p'_x) = \gamma_{v_q} E' (1 + v_q m_0 c \cos \theta' / m_0 c^2), \quad t = \gamma_{v_q} t' (1 + v_q c \cos \theta' / c^2).$$

$$P = \frac{dE}{dt} = \frac{dE' (1 + v_q m_0 c \cos \theta' / m_0 c^2) \gamma_{v_q}}{dt' (1 + v_q c \cos \theta' / c^2) \gamma_{v_q}} = \frac{dE'}{dt'}.$$

Acceleration in lab frame can be obtained by relativistic transformations. Put this in (23).

$$a_q'^2 = a_{qx}'^2 + a_{qy}'^2 + a_{qz}'^2 = \gamma_{v_q}^6 a_{qx}^2 + \gamma_{v_q}^4 (a_{qy}^2 + a_{qz}^2) = \gamma_{v_q}^6 (a_q^2 - \frac{v_q^2}{c^2} (a_{qy}^2 + a_{qz}^2)).$$

$$P = \frac{dE}{dt} = \frac{-q^2 \mu_0 \gamma_{v_q}^6 (a_q^2 - \frac{v_q^2}{c^2} (a_{qy}^2 + a_{qz}^2))}{6\pi c} = \frac{-q^2 \mu_0 \gamma_{v_q}^6 (a_q^2 - \frac{|\vec{v}_q \times \vec{a}_q|^2}{c^2})}{6\pi c}. \quad (24)$$

Observation: This is Lienard's generalization formula[1,2].

Important conclusions of our *electrodynamics analysis from both papers* are:

- (a) We need the first two Maxwell equations as experimental conditions and the other results of electrodynamics can be obtained using rules of special relativity.
- (b) Paper I discussed the difficulties with a delta function current $\vec{J}$, in Poynting's theorem analysis[4]. The theorem based approach is not able to give adequate justification for radiation by an accelerating point charge.
- (c) It is therefore advisable to find change in total energy of the two particle system, instead of following a theorem based approach.

3. Hydrogen atom

The quantum equations are based on *total energy* relation whereas the Maxwell equations are based on components of *force* on the particle. Classically only energy equation may not be sufficient to describe path of a particle and we also need the force equation. Writing a force equation corresponding to the three dimensional quantum equation like Schrodinger equation will be a difficult task. We can study a simpler but somewhat approximate picture called Bohr's picture of hydrogen atom here for two reasons. (a) We can write both the force and energy equation for this classical picture. (b) The electrodynamics formulae have been obtained assuming particle as a point charge. A classical *point* particle under central force can revolve only in a *plane*. This classical picture is similar to the Bohr's hydrogen atom. It can explain some basic requirements of the hydrogen spectra[5].

3.1. Hydrogen atom as per standard electrodynamics formula

We now discuss the difficulty with standard force expression. Fig. 4 shows a typical classical picture of hydrogen atom in centre of mass frame. Both electron and proton rotate in circular orbits around the centre of mass, which is at the origin O of lab frame. Position of electron at retarded time t_r is $E_r(0, r_e, 0)$ and velocity $(v_e, 0, 0)$. Retarded proton position is $P_r(0, -r_p, 0)$ and velocity $(-v_p, 0, 0)$. Angular velocity of rotation of particles around centre of mass O is $\omega = \dot{\theta} = v_e / r_e = v_p / r_p = (v_e + v_p) / (r_e + r_p)$. In a small time $\Delta t = t - t_r$, for $\vec{r}_{12} = (v_e r_{12} / c, (r_e + r_p), 0)$ and $\hat{r}_{12} = (v_e / c, (r_e + r_p) / r_{12}, 0)$, the electron moves through a small angle θ . The $\vec{r}_{12}$ is a vector connecting the retarded position of source proton P_r to present position of electron E_t . Calculate field created at present position of electron due to retarded position of proton. Get direction of velocity part (Paper I- Section 2.3) of electric field $\vec{E}_v$ from standard field equation and find ratio between the X and Y components, as follows[1].

$$\vec{u} = c\hat{r}_{12} - \vec{v}_p = (v_e + v_p, \frac{c(r_e + r_p)}{r_{12}}, 0), \tan \theta = \frac{v_e \Delta t}{r_e} = \frac{v_e r_{12}}{r_e c} = \frac{E_{vX}}{E_{vY}} = \frac{u_x}{u_y} = \frac{(v_e + v_p)r_{12}}{(r_e + r_p)c}.$$

Get velocity part of magnetic field and obtain Lorentz force on electron due to proton. Direction of force vector is along the line $P_t O E_t$, passing through the present position of source proton, centre of mass and present position of electron (neglecting acceleration effect on distance). With source *acceleration*, the force on electron is *not* along a line from present

position of proton and passing through the centre of mass (Paper I- Section 2.3). Velocity of electron is tangential to orbit. Circular motion may not continue for long as velocity of electron and force on it, at present position, will not be perpendicular to each other, $\vec{E} \cdot \vec{J} \neq 0$. This makes the orbit unstable, irrespective of whether it is *Bohr's orbit* or any other orbit.

Now, study the proton frame where electron rotates around the proton (Section 1.3). The proton is at O in Fig. 5. B is present and A is retarded position of electron on the circular orbit. Electron velocity is $\vec{v}_q = \vec{v}_g$ at retarded time. Here, $\theta = \omega t_q$, $\vec{R} = R(-\sin(\omega t_q)\hat{x} + \cos(\omega t_q)\hat{y})$, $\vec{v}_q = d\vec{R}/dt_q$, $\vec{a}_q = d^2\vec{R}/dt_q^2 = -R\omega^2\hat{R}$, $\vec{R} \cdot \vec{v}_q = 0$. Also, $\vec{R} = -\vec{r}$ and $\vec{F} = q\vec{E}$. Get electric field at proton created by the rotating electron using Paper I- equation (31).

$$\vec{E} = \frac{-q(1 - v_q^2/c^2)}{4\pi\epsilon_0 r^2} \hat{r} + \frac{q\vec{v}_q}{4\pi\epsilon_0 r^2 c}.$$

Force on proton will not be along the line passing through the present position of electron and present position of centre of mass. We have already listed the *difficulties* with standard field expression for an *accelerated source* in Section 1.2. Then, as suggested in Sections 1.2 and 1.3, it is advisable to find rate of change of potential. With a fixed proton, potential ϕ_{pe} created by proton at electron is of (q/R) nature and $(d\phi_{pe}/dt) = 0$. The Lienard-Wiechert potential at proton due to electron in circular motion, will also be of (q/R) nature, as $\vec{R} \cdot \vec{v}_q = 0$. With, $\vec{v}_o = 0$, $\vec{r} \cdot \vec{v}_q = 0$, $dt_q/dt = 1$ and $-v_q^2 + \vec{a}_q \cdot \vec{r} = 0$ for a circular orbit, equation (12) is,

$$q \frac{d\phi_{ep}}{dt} = \frac{q^2}{4\pi\epsilon_0} \left(-\frac{c^3}{(rc)^2} + \frac{rc^2(c^2 - v_q^2)}{(rc)^3} + \frac{rc^2(\vec{a}_q \cdot \vec{r})}{(rc)^3} \right) = 0.$$

3.2. Total energy of the two particle system in a stable state

For a circular motion, there is no change in kinetic energy, $d(E_{K.E.})/dt = 0$. As both of them move together, relative position between them as seen by them, remains unchanged and there will be no change in potential between electron and proton. There is no change in total energy between the two positions on a particular circular orbit. As $\phi(\vec{r}_2, t_2) = \phi(\vec{r}_1, t_1)$, we have $d\phi/dt = 0$ and as discussed in Section 1.1, the $d(E_{Total})/dt = 0$.

Observation: Therefore, there should be no radiation, when electron rotates in a particular stable orbit with constant velocity. As per the classical electromagnetic analysis, where electron is a point charge, all circular orbits with various velocities of rotation are possible. Under such condition, photons of all energy values are possible if the electron goes from one orbit to another. Theory of electrodynamics is not sufficient to explain *discrete* energy levels.

3.3. Wave function to be single valued

Earlier, we suggested a mathematical model to explain some basic requirements of quantum mechanics[6]. Though, these papers are not for proposing any specific model, we

try to give justification for Bohr's postulate. We adopt some features of quantum mechanics along with electrodynamics. Study primed frame first. In Fig. 5, proton is fixed in lab at origin and the origins of lab and primed frame overlap. The plane-wave solution of Dirac equation can be written in primed frame using relativistic distance and time transformations as follows[5].

$$\psi' = \Delta y' = A e^{i(\omega' t')} = A e^{i\left(\frac{E'}{\hbar} t'\right)} = A e^{i\left(\frac{m_0 c^2}{\hbar} t'\right)}. \quad (25)$$

Here A' is not important in further analysis. Quantum wave function does not specify any specific position of a particle in given space. However, in electrodynamics, particle positions are important as forces depend upon distances. Though, physical nature of wave function is not important, the wave function is shown as some kind of oscillations $\Delta y'$ along Y axis. Hence, the electron is represented by a thick segment as shown in the Fig. 5. Let the state of a static electron in primed frame be described by $(\Delta x' = 0, \Delta t' \neq 0, \psi' = \Delta y')$. Primed frame has frame velocity v_g with respect to lab. Write transformations in Cartesian coordinates for small incremental motion of electron with respect to the proton at lab origin. Then $\Delta t = \gamma_{v_g} (\Delta t' + \Delta x' v_g / c^2) = \gamma_{v_g} \Delta t'$ and $\Delta x = \gamma_{v_g} (\Delta x' + v_g \Delta t') = \gamma_{v_g} (v_g \Delta t')$. Electron has velocity $\Delta x / \Delta t = v_g$ in lab frame. Transform above relation (25) to lab frame.

$$\psi = A e^{i\gamma_{v_g} \omega' (t - x v_g / c^2)} = A e^{i(\omega t - k x)} = A e^{\frac{i}{\hbar}(E t - p x)} = A e^{\frac{i}{\hbar}(m c^2 t - m v_g x)}. \quad (26)$$

Study the $(Et - px)$ portion. Relativistic distance transformations can be obtained from momentum and energy transformations (Paper I- Sections 1.1, 1.2). For a particle rotating in circle of fixed radius R , we have constant angular velocity $\dot{\phi} \neq 0$ and $\dot{R} = 0$, $\ddot{R} = 0$, $\ddot{\phi} = 0$. As acceleration is zero, same transformation relations may be applied in this case of a rotating particle also. Study $(\omega t - kx)$ and call wave velocity $v_p = E / p = \omega / k = c^2 / v_g$ as phase velocity. This velocity in lab is associated with *simultaneous events* occurring at some distance in primed frame (Section 1.2). Electron is stationary in primed frame at $(\phi'_1, t'_1) = (x'_1 = 0, t'_1)$. We equated Cartesian coordinates with the cylindrical ones so that we can replace them in relativistic transformations also. The same location of this stationary particle on the circle, at same time in primed frame, can also be described by coordinates $(\phi'_2 = \phi'_1 + 2\pi, t'_1) = (x'_2 = 2\pi R, t'_1)$. We can suggest that two simultaneous events are occurring at these two positions in primed frame. The corresponding two positions in lab are $(\phi_1, t_1) = (x_1 = 0, t_1)$ and $(\phi_2 = \phi_1 + 2\pi, t_2) = (x_2 = 2\pi R, t_2)$. In primed frame, wave function for both the simultaneous events is given by (25). Write ψ in lab using (26), for these two simultaneous events.

$$\psi = A e^{i(\omega t_1 - k x_1)} = A e^{i\left(\frac{m c^2}{\hbar} t_1 - \frac{m v_g}{\hbar} x_1\right)} = A e^{i\left(\frac{m c^2}{\hbar} t_1 - \frac{m v_g R}{\hbar} \phi_1\right)}, \quad (27)$$

$$\psi = Ae^{i(\omega t_2 - kx_2)} = Ae^{i(\frac{mc^2}{\hbar}t_2 - \frac{mv_g}{\hbar}x_2)} = Ae^{i(\frac{mc^2}{\hbar}t_2 - \frac{mv_g R}{\hbar}\phi_2)}.$$

In above equations, ψ has to be same in lab, as it corresponds to same particle position at same time in primed frame. Using time transformation relations,

$$\omega' t_1' = \gamma_{v_g} \omega' (t_1 - \frac{x_1 v_g}{c^2}) = (\omega t_1 - \frac{mv_g R}{\hbar} \phi_1) = (\omega t_1 - n \phi_1),$$

$$\omega' t_1' = \gamma_{v_g} \omega' (t_2 - \frac{x_2 v_g}{c^2}) = (\omega t_2 - \frac{mv_g R}{\hbar} \phi_2) = (\omega t_2 - n \phi_2).$$

Here, $\phi_2 = (\phi_1 + 2\pi)$. For ψ in equation (27) to be same, $mv_g R / \hbar = n$ has to be an integer. This is Bohr's postulate. A static particle in primed frame will have constant velocity and constant kinetic energy in lab frame. But, only a certain values of velocities are possible if electron has to rotate in circular orbits around proton with constant velocity. Then $(\omega t_1 - n \phi_1) = (\omega t_2 - n \phi_2) = (\omega t_2 - n(\phi_1 + 2\pi))$ gives $\omega t_2 = (\omega t_1 + 2\pi n)$. The wave function is,

$$\psi = Ae^{i(\omega t_1 - n \phi_1)} = Ae^{i(\omega t_2 - n \phi_2)} = Ae^{i(\omega t_1 + 2\pi n - n \phi_1 - 2\pi n)}. \quad (28)$$

Observation: Requirement of a single valued function is explained on the basis of relativistic treatment of *simultaneous events*. This with the balance of force relation gives discreet energy levels. Discrete energies are due to the fact that the wave function is single valued and goes to zero at infinity (boundary condition)[5]. Therefore photons of only certain energies are emitted when electron shifts from one stable state to another.

4. Conclusion

Analysis of various aspects of classical electrodynamics could not provide any theoretical justification for the widely accepted principle of “continuous radiation by an accelerating charge”. It can be concluded that: (a) The Poynting's theorem or Larmor formula has limitations in explaining radiation by a single point charge or point charges in a bound state. There is no theoretical justification to say that electron orbiting around proton in a circular orbit, in hydrogen atom, will go on radiating just because it is accelerating due to its motion in a circle. (b) Classical electrodynamics allows all orbits with various energy values. (c) The discrete energy states is a quantum mechanical effect. The fact that the wave function is single valued and goes to zero at infinity (boundary condition) allows only certain stable orbits. The radiated photon energy will depend upon the difference in total energies of such stable orbits.

For any radiation study, it is always advisable to find if there is any change in total energy of the system instead of a theorem based approach.

FIGURES

FIG. 1 Both source and observer in motion

FIG. 2 Simultaneous events

FIG. 3 An expanding radiation sphere

FIG. 4 Classical hydrogen atom in centre of mass frame

FIG. 5 Hydrogen atom in proton frame

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