Canadian Journal of Forest Research
Revue canadienne de recherche forestière

The challenge of estimating a residual spatial autocorrelation from forest inventory data

Journal:	<i>Canadian Journal of Forest Research</i>
Manuscript ID	cjfr-2017-0247.R1
Manuscript Type:	Article
Date Submitted by the Author:	17-Aug-2017
Complete List of Authors:	Magnussen, Steen; Natural Resources Canada, Canadian Forest Service Breidenbach, Dr. Johannes; Norsk Institutt for Skog og Landskap, Mauro Gutiérrez, Francisco; Oregon State University, College of forestry. Forest Engineering Resources and Management.
Keyword:	first-order autoregression, restricted maximum likelihood, simulation, stand effects, synthetic estimates
Is the invited manuscript for consideration in a Special Issue? :	N/A

SCHOLARONE™
Manuscripts

by

3. *Oregon State University. College of Forestry, Forest Engineering Resources and Management Department. 053 Peavy Hall, Corvallis, OR 97331, USA*

Abstract

Estimates of stand averages are needed by forest management for planning purposes. In forest enterprise inventories supported by remotely sensed auxiliary data, these estimates are typically derived exclusively from a model that does not consider stand effects in the study variable. Variance estimators for these means may seriously underestimate uncertainty, and confidence intervals may be too narrow when a model used for computing a stand mean omits a non-trivial stand-effect in one or more of the model-parameters, a non-trivial spatial distance dependent autocorrelation in the model residuals, or both. In simulated sampling from 36 populations with stands of different sizes, and differing with respect to: *i*) the correlation between a study variable (Y) and two auxiliary variables ($\mathbf{X}$); *ii*) the magnitude of stand effects in the intercept of a linear population model linking $\mathbf{X}$ to Y ; and *iii*) a first-order autoregression in Y and $\mathbf{X}$ – we learned that none of the tested designs provided reliable estimates of the within-stand autocorrelation among model residuals. More reliable estimates were possible from stand-wide predictions of Y . The anticipated bias in an estimated autoregression parameter had a modest influence on estimates of variance and coverage of nominal 95% confidence intervals for a synthetic stand mean.

Keywords: first-order autoregression, restricted maximum likelihood, simulation, stand effects, synthetic estimates.

Introduction

Forest management inventories play a key role in support of sustainable forestry practices (Hickey et al. 2007; Varma et al. 2000). Contemporary forest management inventories frequently employ airborne laser scanners to deliver a census of vegetation surface metrics ($\mathbf{X}$) correlated with attributes of interest (Y) at a spatial resolution commensurate with a forest inventory field plot (Latifi et al. 2015; Næsset 2004b; Wulder et al. 2013). From a probability sample of n field plots, the parameters in one or more models linking $\mathbf{X}$ to Y are estimated,

40 and the model(s) are then used to obtain either model-assisted (Opsomer et al. 2007) or
41 model-based (Gregoire 1998) population-level or strata estimates of the means and variances
42 of Y .

43 In the context of a forest management inventory, it has become routine to re-use the models
44 for a stand-level estimation of means, variances, and confidence intervals (Næsset 2004a).
45 Since field sampling is expensive, and models have, in some cases, lowered the required
46 sample size for meeting a precision target for population- or stratum-level mean, the n
47 samples rarely constitutes more than a tiny fraction of the sampling frame (Dymond et al.
48 2006; Hawbaker et al. 2009; Junttila et al. 2013; Parker and Evans 2004). As a consequence
49 one should expect a vast majority of stand-level estimates to be synthetic (Rao and Molina
50 2015, ch. 3.2). The properties (bias and accuracy) of stand-level synthetic estimators depend
51 strongly on the correctness –across all stands – of: the employed model; all assumptions
52 regarding model errors; covariance structures; and the distribution of model residuals
53 (Schreuder et al. 1993, ch. 8.1.1).

54 In forest management inventories one should expect, even in a stratum, some level of stand
55 effects in Y and $\mathbf{X}$ (Chen and Klinka 2003; Král et al. 2014), and possibly also in model
56 residuals (Breidenbach et al. 2008; Goerndt et al. 2011; Maltamo et al. 2012; Mauro et al.
57 2016). Also, a within-stand spatial distance-dependent autocorrelation (at the scale of a field
58 plot) in Y , $\mathbf{X}$, and possibly model residuals should not be ruled out (Breidenbach et al. 2015;
59 Finley et al. 2008; Lappi 2001; Magnussen et al. 2016b; Ver Hoef and Temesgen 2013). If
60 both stand effects and autocorrelation of residuals exist, but omitted in an analysis, the
61 variance of a (synthetic) stand mean would underestimate uncertainty (Fortin et al. 2016;
62 Mandallaz 2008, (6.7) p. 101). It is therefore important that a forest management inventory
63 design affords estimators for these effects by ensuring: a sufficient number of stands with

64 more than one sampled element (Grafström and Tillé 2013); and a spatial balance of sampled
65 stands (Grafström et al. 2014).

66 A recent simulation study suggested that a probability sampling with at least two samples
67 collected in a relatively small number of stands affords a reliable estimator of random stand
68 effects in the intercept of a linear model (Magnussen and Breidenbach 2017), which, in turn,
69 greatly improved estimates of variance and 95% confidence intervals for synthetic stand
70 means.

71 However, the study did not address the issue of a distance dependent autocorrelation in model
72 residual within a stand. Given the variance inflating potential of even a relatively weak
73 spatial distance dependent autocorrelation among within-stand model residuals (Fortin et al.
74 2012; Legendre 1993; Magnussen et al. 2016a; Smith 1938), its estimation justifies a directed
75 effort (Breidenbach et al. 2015; McRoberts et al. 2011; Nieschulze and Saborowski 2002).

76 For the estimation of the parameters in a spatial distance dependent autocorrelation process
77 one would ideally collect data by a design optimized for this purpose (de Gruijter 2002; Di
78 Zio et al. 2004; Falk et al. 2011; Stein and Ettema 2003). Typically these designs ensures a
79 balance in spatial distances among pairs of observations. Forest inventory designs are rarely
80 designed with consideration given to estimation of spatial processes.

81 In this study we demonstrate the challenge of bias in estimators of the parameter of a simple
82 first-order autoregressive process in model residuals with data from a forest enterprise
83 inventory design employing conventional circular fixed area primary selection units (PSUs
84 viz. plots viz. elements). The sampling designs considered here allow an estimation of stand
85 effects by random sampling $n_2 = \{2, 4, 6\}$ PSUs in each of n_1 stands selected with probability
86 proportional to size.

87 The demonstration is carried with simulated sampling from 36 fixed artificial tri-variate
88 populations with a linear model linking Y to $\mathbf{X}$ (Magnussen and Breidenbach 2017) and a
89 spatial first-order autoregressive process (at the scale of a field plot) among values of $\mathbf{X}$ and
90 Y . The levels of the distance dependent autocorrelation were intended to cover the range
91 anticipated for managed forests stands.

92 **Material and methods**

93 The simulation study, models, estimators, and performance indicators described in the
94 following sections contains a large number of symbols for parameters, variables, and
95 statistics. To assist readers, an alphabetically sorted list with definitions is provided in
96 Table 1.

97 *Sampling and inferential objectives*

98 A probability sample of size n elements is taken (without replacement) from a finite (fixed)
99 forest population composed of N elements (n is an order of magnitude less than N) and M
100 stands for the purpose of a design-based population-level inference and model-based
101 estimation of the average of Y in each stand. A variance of an expected stand mean, and a
102 95% confidence interval is also desired. The model-based paradigm (Gregoire 1998) is
103 adopted for stand-level inference because the number of elements to be sampled in most
104 stands is too low to afford more than a synthetic estimator (Rao and Molina 2015, ch. 3.2).

105 For our demonstration, we conveniently assume that an analyst adopts the actual model for a
106 population:

107
$$y_{ij} = \boldsymbol{\beta}'\mathbf{X}_{ij} + e_i + e_{ij}, i = 1, \dots, M, j = 1, \dots, m_i \tag{1}$$

108 where y_{ij} is the value of Y in the j th element in the i th stand; $\mathbf{X}_{ij} = \{1, x_{1ij}, x_{2ij}\}$ is a length 3
109 column vector of an intercept, and values of X_1 and X_2 in the j th element in the i th stand; $\boldsymbol{\beta}$ is
110 a length 3 column-vector of regression coefficients to be estimated from the sample; e_i is a

stand-specific intercept; and e_{ij} is an element specific error term. A superscript ' t ' in (1) is used for the transpose of a vector. Note, e_i is shared by all elements in a stand, which gives rise to a constant (distance independent) covariance among all pairs of elements in a stand. In contrast, e_{ij} is associated with a specific element within a stand. The stand (domain) terms e_i , $i = 1, \dots, M$ are, for the variable of interest, assumed independent and normally distributed with a mean of zero and a variance σ_{st}^2 equal to the covariance between any pair of elements ij and ij' ($j \neq j'$) within a stand (Casella and Berger 2002, ch. 11.2.6). The within-stand specific terms e_{ij} , $j = 1, \dots, m_i$ are assumed normally distributed with a mean of zero, a variance of σ_w^2 and correlated through an assumed stationary isotropic and distance dependent first-order autoregressive spatial process in $\mathbf{X}$ and Y with parameter α_e (see below).

By adopting the actual model for the relationship between $\mathbf{X}$ and Y and the covariance structure of model residuals within a stand, we have optimized the conditions for obtaining the actual values of the model parameters. Challenges encountered in this ideal setting will only increase in practice when the true model remains unknown, and the spatial process in model residuals have to be deduced from empirical residuals (Kim and Boos 2004). Hence, complex issues regarding the effects of variable and model selection are outside the scope of our work.

According to the model in (1), a model-based estimate of the stand mean becomes (Chambers and Clark 2012, ch. 7.1)

$$\hat{y}_i = m_i^{-1} \sum_{j=1}^{m_i} \hat{\beta}' \mathbf{X}_{ij} + \sum_{ij \in s_i} (y_{ij} - \hat{y}_{ij}) \quad (2)$$

where $\hat{\boldsymbol{\beta}}$ is an empirical best linear unbiased predictor (EBLUP, Fahrmeir et al. 2013, ch. 7.3) of $\boldsymbol{\beta}$ in the mixed linear model (1), $\hat{y}_{ij}$ is the prediction of y_{ij} given $\mathbf{X}_{ij}$ and $\hat{\boldsymbol{\beta}}$, and the summation is over sampled elements in the i th stand (s_i). For most stands, the second term is 0 as no stand element has been sampled. In our approach we have dropped the second term in order to mimic synthetic estimators which are applicable to the vast majority of stands in actual applications.

Formally the model-based estimator of the variance of $\hat{y}_i$ in (2) becomes (Chambers and Clark 2012, eq. 7.2)

$$\hat{V}(\hat{y}_i) = \bar{\mathbf{X}}_i \hat{\boldsymbol{\Omega}} \bar{\mathbf{X}}_i^t + \hat{\sigma}_{st}^2 + \left(m_i - n_{ij \in s_i}\right)^{-1} \hat{\sigma}_w^2 \sum_{j=1}^{m_i - n_{ij \in s_i}} \sum_{j'=1}^{m_i - n_{ij \in s_i}} \hat{\alpha}_e^{rnd(dist(ij, ij'))} \quad (3)$$

where $\hat{\boldsymbol{\Omega}}$ is the EBLUP estimate of the 3×3 covariance matrix of the parameters $\boldsymbol{\beta}$; $\hat{\sigma}_{st}^2$, $\hat{\sigma}_w^2$, and $\hat{\alpha}_e$ are restricted maximum likelihood (REML) estimators (Fahrmeir et al. 2013, ch. 7.3.2) of σ_{st}^2 , σ_w^2 , and α_e respectively, $n_{ij \in s_i}$ is the number of elements sampled in the i th stand, $\hat{\alpha}_e$ is the REML estimator of the first-order residual spatial autoregression parameter, and $rnd(dist(ij, ij'))$ is the Euclidean distance between elements ij and ij' in the i th stand rounded to the nearest integer. In (3) any $\alpha_e \geq 0$ raised to the power of 0 (i.e. a distance of zero) is, by definition, equal to 1. As done for the mean in (1) we have set the sample size in all stands to 0 in our calculations of variances. An alternative (improved) estimate of α_e was derived directly from all elements $\hat{y}_{ij}$ in the n_1 selected stands. Computing time issues dissuaded us from using all M stands for this estimation (see Appendix for details).

Note the variance estimator in (3) is valid only if the estimated population-level model is equal to both the true model for the population and for each stand in question. Otherwise it

underestimates the uncertainty by omitting model bias (the expected value of $\hat{\beta} - \beta$ over all possible samples in a finite population generated by a given probability design).

The desired nominal 95% confidence interval for a stand mean $\bar{y}_i$ was computed – based on normal-theory and application of the central-limit theorem (Fuller 2009, ch. 1.3.2) – as

$$CI95_{\bar{y}_i} = \left\{ \hat{y}_i - z_{0.025} \sqrt{\hat{V}(\hat{y}_i)}, \hat{y}_i + z_{0.975} \sqrt{\hat{V}(\hat{y}_i)} \right\} \quad (4)$$

where z_q is the q th percentile in the distribution function of a standard normal variate.

To compute the variance in (3) and the confidence interval in (4) an analyst requires a sampling design that affords a design-consistent (the error in a parameters goes to zero as the sample size increases, Mandallaz 2008, p. 6) and efficient estimation of $\hat{\Omega}$, $\hat{\sigma}_{st}^2$, $\hat{\sigma}_w^2$, and $\hat{\alpha}_e$.

Estimating two variance components and a first-order autoregression coefficient from a set of empirical residuals raise the sceptre of inconsistent results (Zhang 2004) due to the fact that for a fixed total within-stand covariance, say T_{cov} , the likelihood function can have multiple local maxima with very similar likelihoods since the equation

$$T_{cov} = m_i \sigma_{st}^2 + \sigma_w^2 \sum_{j=1}^{m_i} \sum_{j'=1}^{m_i} \alpha_e^{rnd(dist(ij, ij'))}$$

can have more than one solution in terms of

σ_{st}^2 , σ_w^2 , and α_e with practically equal likelihoods. The possible confounding may, however, be unimportant in the current context as long as the estimated variances are, on average, good approximations to their empirical counterparts in replicated sampling, and computed confidence intervals achieve a coverage (proportion of true means contained in a computed interval) close to the nominal target.

Sampling designs

For the purpose of estimating the parameters needed for an estimation of the M stand means in (2), their associated variances in (3), and associated confidence intervals in (4), a probability sampling with an overall sample size (n) of 120, 240, 360, and 480 and three within-stand sample sizes (n_2) of 2, 4, and 6, was simulated and replicated 200 times for each of the 36 populations. A within-stand sample size of 2 in a select number of stands is, of course, the minimum feasible for the stated estimation objectives.

In each of the 12 design combinations of n and n_2 , a total of $n_1 = n \times n_2^{-1}$ stands were selected with probability proportional to their size m_i . The within-stand sampling of n_2 elements was random without replacement. For the single purpose of estimating population-level parameters, a within-stand-sampling scheme that penalizes a selection of near-neighbour elements (Grafström et al. 2014; Grafström et al. 2012) would normally be preferred. However, such schemes would be detrimental to the estimation of the parameters in a distance dependent autocorrelation process in the model residuals e_{ij} (Nordman and Caragea 2008).

With our designs and sample sizes, 200 replications aimed at providing an average Monte Carlo error (the standard deviation of an estimator across replications of the simulation design, Koehler et al. 2009) of approximately 1% on the average (across stands) of $\sqrt{\hat{V}(\hat{y}_i)}$ and approximately 0.5% on the average coverage of a computed 95% confidence interval. In the forerunner to this study (Magnussen and Breidenbach 2017) we had 1200 replications and twice as many stands in a population. However, the simulations of a spatial autocorrelation process in Y and $\mathbf{X}$ meant that running this many replications across twice as many stands would have incurred impractical computing times when using an advanced workstation.

Performance assessment

196 After each completed sample, a mean, a variance, and a 95% confidence interval was
 197 computed for each of the 800 stands and 200 replications as per (2), (3), and (4), with
 198 REML estimates of β , σ_{st}^2 , σ_w^2 , and α_e . As mentioned above, the latter was also estimated
 199 directly from $\hat{y}_{ij}$ within the n_1 selected stands (Magnussen et al. 2016a).

200 We compare estimated averages (across stands and replicates) of
 201 $\hat{\sigma}_{st}^2$, $\hat{\sigma}_w^2$, $\hat{\alpha}_e$, $\hat{V}(\hat{y})$, and $CI95_{\bar{y}}$ to their known counterparts and explore the statistical
 202 significance of main effects and two-way interactions of sample designs and population
 203 parameters with weighted analyses of variance on cell means (over replications) (Searle et al.
 204 1992, ch. 4.4) and/or weighted linear regressions (Draper and Smith 2014, p. 108). The
 205 weight of a design entry was the inverse to the empirical variance of the estimates across the
 206 200 replications.

207 In practical forest enterprise inventories with dual inferential objectives (population and
 208 stands) an analyst must balance the advantages of a spatial balance in the sample (Grafström
 209 and Tillé 2013) against a need to ascertain spatial effects of consequence for stand-level
 210 estimators (Fortin et al. 2012). Typically, a balance has to be struck within the constraints of
 211 costs. We therefore focus our performance assessment on the apparent trade-offs between an
 212 increase in n_2 at the expense of a decrease in n_1 when the overall sample size is kept constant.

213 *Populations*
 214 We consider 36 tri-variate standard (mean zero, variance 1.0) Gaussian populations (forests
 215 or strata) each composed of N fixed area elements (the frame) subdivided into $M = 800$
 216 spatially compact (square) domains (forest stands) with $5^2, 6^2, \dots$, or 14^2 elements. An element
 217 is a square fixed area the size of a forest inventory PSU viz. sample plot (typically between
 218 200 and 800 m²). A study variable Y , and two auxiliary variables X_1 and X_2 correlated with Y ,
 219 are associated with each element. The values of X_1 and X_2 are known for each element in a

220 population while the value of Y is only observed for elements collected in a probability
 221 sample. The triplet $(y_{ij}, x_{1ij}, x_{2ij})$ denotes the values associated with the j th element ($j = 1, \dots,$
 222 m_i) in the i th stand ($i = 1, \dots, M$). Stand means in the populations are denoted $(\bar{y}_i, \bar{x}_{1i}, \bar{x}_{2i})$.
 223 The populations differentiate on: *i*) the correlation between Y and X_1 ($\rho(y, x_1)$), *ii*) the
 224 among-stand variance in Y ($\sigma_{\bar{y}_{st}}^2$), and *iii*) a distance-dependent first-order spatial
 225 autoregression in Y along rows and columns within a stand characterized by the coefficient
 226 α_y , controlling the correlation between two elements sharing a border across a row or column
 227 of elements. We generated populations with three levels of both $\rho^2(y, x_1)$ and $\sigma_{\bar{y}_{st}}^2$, and four
 228 levels of α_y . The population characteristics were crossed to all possible combinations;
 229 yielding $36(3 \times 3 \times 4 = 36)$ ‘fixed’ populations to be sampled repeatedly. Assigned values and
 230 ranges of the parameters in this study were designed to be representative of populations of
 231 even-aged mono-specific forest stands in the temperate zone. The levels of $\rho^2(y, x_1)$ were
 232 0.6, 0.7, and 0.8. Levels of $\sigma_{\bar{y}_{st}}^2$ were 0.05, 0.10, and 0.20 (i.e. the among-stand variance in Y
 233 accounts for 5%, 10%, and 20% of the total variance of 1.0 in Y). Finally, the levels of α_y
 234 were 0.00, 0.10, 0.20, and 0.30, which for an average stand with 90 elements, inflates a
 235 within-stand residual error variance by a factor of, 1.0, 1.8, 2.8, and 4.3 times, respectively
 236 (Magnussen 2001). The covariogram for elements in a stand separated by a distance $dist$ is
 237 $C_y(dist) = \alpha_y^{dist} (1 - \alpha_y^2)^{-1} (1 - \sigma_{\bar{y}_{st}}^2)$ (Cressie 1993, p. 200). The correlation between Y and X_2
 238 was fixed at 0.25 $\rho(y, x_1)$ while the correlation between X_1 and X_2 was fixed at 0.5. The
 239 among-stand (domain) variance in X_1 and X_2 was fixed at $0.7 \sigma_{\bar{y}_{st}}^2$. A first-order spatial
 240 autoregression coefficient of $0.7 \alpha_y$ was also imposed on X_1 and X_2 .

241 According to our control of the tri-variate correlations, a population census would generate
 242 an estimate of $\hat{\beta}_{census} = \{0.000, 0.904, -0.258\}$ for $\rho^2(y, x_1) = 0.6$,
 243 $\hat{\beta}_{census} = \{0.000, 0.976, -0.279\}$ for $\rho^2(y, x_1) = 0.7$, and $\hat{\beta}_{census} = \{0.000, 1.044, -0.298\}$ for
 244 $\rho^2(y, x_1) = 0.8$.
 245 Additional technical details of the simulations are in the Appendix.

246 Results

247 *Estimates of the within-stand residual variance σ_w^2*
 248 Estimated values of $\hat{\sigma}_w^2$ closely tracked, as apparent in Figure 1, their nominal values σ_w^2 (see
 249 Appendix). A linear regression of $\hat{\sigma}_w^2$ on σ_w^2 achieved a slope of 0.996 and an intercept of
 250 0.0119. With 200 replications of a design setting, the Monte Carlo error of an estimate of σ_w^2
 251 varied from 0.4% and 2.3% with a median of 0.9%. The error decreased with decreasing
 252 values of σ_w^2 , n , and n_2 , but increased with rising levels of α_y . As expected (Anderson 1976,
 253 p.14), α_y had a strong influence on the estimate of σ_w^2
 254 $(\alpha_y, \hat{F}_{3,362} = 425.0, P(\hat{F}_{3,362} | H_0) < 0.01)$. With $\alpha_y = 0.0$ (no autocorrelation) the apparent
 255 bias in $\hat{\sigma}_w^2$ was 2.8%, 7.6% when $\alpha_y = 0.1$, and 8.4 when $\alpha_y = 0.3$. The inflation is attributed
 256 to the fact that the estimated values are estimates of the process variance in e_{ij} , which, is
 257 greater than the nominal variance applicable to independent observations.
 258 From a practical perspective the most important result is that there was no important main
 259 effect of n or a significant two-way interactions between n and α_y
 260 $(\hat{F}_{9,362} = 0.43, P(\hat{F}_{9,362} | H_0) = 0.92)$. The within-stand sample size n_2 was statistically
 261 significant $(\hat{F}_{2,362} = 20.5, P(\hat{F}_{2,362} | H_0) < 0.01)$ but the influence on $\hat{\sigma}_w^2$ was a one percentage

increase in the apparent bias of $\hat{\sigma}_w^2$ as n_2 increased from 2 to 6. Interactions between n_2 , $\rho^2(y, x_1)$, and $\hat{\sigma}_{\bar{y}}^2$ were also statistically significant but an order of magnitude smaller than any two-way interaction, and two-orders of magnitude smaller than the main effects of n_2 and $\rho^2(y, x_1)$.

Estimates of the among-stand residual variance σ_{st}^2

Monte Carlo errors in estimates of σ_{st}^2 varied from 1.8 to 8.8% with a median of 4.4% in settings with a non-zero autocorrelation. In combination of low values of σ_{st}^2 , n , and n_2 , the estimated values of σ_{st}^2 were significantly inflated. Generally, as σ_{st}^2 increases, the estimates clustered tightly around the actual value. These trends are clearly visible in Figure 2. The apparent bias in $\hat{\sigma}_{st}^2$ was significantly influenced by n ($\hat{F}_{3,362} = 5.9, P(\hat{F}_{3,362} | H_0) < 0.01$), n_2 ($\hat{F}_{2,362} = 153.2, P(\hat{F}_{2,362} | H_0) < 0.01$), and α_y ($\hat{F}_{2,362} = 3.5, P(\hat{F}_{2,362} | H_0) = 0.03$), and interactions $n_2 \times \sigma_{\bar{y}_{st}}^2$ ($\hat{F}_{4,362} = 109.4, P(\hat{F}_{4,362} | H_0) < 0.01$) and $n_2 \times \rho^2(y, x_1)$ ($\hat{F}_{4,362} = 262.5, P(\hat{F}_{4,362} | H_0) < 0.01$). In general terms, the apparent bias dropped from 65% to 15% with an increase in n_2 from 2 to 4, from 73% to 10% with an increase in $\sigma_{\bar{y}_{st}}^2$ from 0.05 to 0.20, and from 102% to 4% as $\rho^2(y, x_1)$ drops from 0.8 to 0.6.

Estimates of the ARI coefficient α_e

The introduction of a first-order autoregressive spatial process in Y and $\mathbf{X}$ induced a distance dependent autocorrelation of the same magnitude in the true model residuals (e_{ij}) within a stand. In our simulations – and in agreement with theory (see Appendix) – the population level mean (across stands and replications) maximum likelihood estimate $\hat{\alpha}_e(e_{ij})$ of α_e in

the true residuals was, with the exception for $\alpha_e = 0$ very close to α_y . To wit: $\hat{\alpha}_e(e_{ij})$ was 0.036 ± 0.004 for $\alpha_y = 0.0$; 0.105 ± 0.007 for $\alpha_y = 0.1$; 0.196 ± 0.008 for $\alpha_y = 0.2$; and 0.293 ± 0.008 for $\alpha_y = 0.3$. Hence, only at $\alpha_y = 0.0$ was there an apparent discrepancy between $\hat{\alpha}_e(e_{ij})$ and α_y . Because $\alpha_y = 0$ is at the boundary of the parameter space for the first-order autoregression parameter, no test of the significance of the observed discrepancy was attempted.

In contrast, REML estimates $\hat{\alpha}_e(e_{ij} \in s)$ of α_e – when data were limited to the true residuals in the sample (s) – were significantly overestimated when $\alpha_y = 0$ and increasingly underestimated as α_y increases from 0.1 to 0.3 (Figure 3). All design effects and most two-way interactions were statistically significant. In general, the apparent negative bias in $\hat{\alpha}_e(e_{ij} \in s)$ increased with n , n_2 , and α_y . These results suggest that estimates obtained with empirical (observed) residuals $(y_{ij} - \hat{y}_{ij})$ would be worse since $\hat{\beta}_{sample} \neq \beta_{census}$. This was overwhelmingly confirmed, as can be taken from Figure 4 where estimates $\hat{\alpha}_e(\hat{e}_{ij})$ of α_e obtained from empirical residuals are plotted against estimates obtained with true residuals (e_{ij}) . Clearly, a sample-based estimate of α_e based on empirical residuals bears no relationship to an estimate obtained from actual (true) residuals or to the actual α_e . Monte-Carlo errors in $\hat{\alpha}_e(\hat{e}_{ij})$ varied from 1.0 to 15.4% with a median of 1.3%.

As expected from theory (see Appendix) and illustrated in Figure 5, vastly improved estimates of α_e could be obtained from the average maximum likelihood estimate of the first-order autoregression coefficient α_y in $\hat{y}_{ij}$ using all elements in the n_I sampled stands for estimation (a restriction to n_I was chosen in order to limit otherwise excessive computing

times). An overestimation of approximately 0.3 at $\alpha_y = 0$ is apparent, and as α_y increases the overestimation turns into a slight underestimation. Although the effects of n , n_2 , $\rho(y, x_1)$, and $\sigma_{\bar{y}_{st}}^2$ were statistically significant ($P < 0.01$), their marginal effects were minor ($< 1\%$) and without a linear trend. In the following we use these estimates ($\hat{\alpha}_{\hat{y}}$) of α_e to compute the variance and confidence interval for a synthetic stand mean.

Estimates of variance of a synthetic stand mean

The mean (across stands and replications) estimate of the variance of a synthetic stand mean $\hat{V}(\hat{y})$ tracked – when using $\hat{\alpha}_{\hat{y}}$ as the estimate of the first-order autocorrelation in within-stand residual errors – the actual $MSE(\hat{y})$ quite well ($R^2 = 0.96$) but with an increasing apparent (negative) bias in design settings with an $MSE(\hat{y}) > 0.035$ (Figure 6). Monte Carlo errors in estimates of the stand-average $\sqrt{\hat{V}(\hat{y})}$ varied from 0.05 to 0.63% with a median of 0.14%.

There was no interpretable trend in the difference between $MSE(\hat{y})$ and $\hat{V}(\hat{y})$ across levels of $\sigma_{\bar{y}_{st}}^2$, but the apparent bias changed from +5% to -7% as n increased from 120 to 480, and from +8% to -7% when n_2 increased from 2 to 6. As α_y increased from 0 to 0.3 the apparent bias changed from -8% to +8%. None of these trends in variance estimate can be viewed as important in light of the uncertainty in the estimates themselves and the associated Monte-Carlo errors.

Although REML sample-based estimates of α_e from empirical residuals were deemed of little value as estimates of the actual first-order autoregression coefficient, the ensuing estimates of variances of a synthetic stand mean was surprising close to those detailed above (Figure 7). In general, the two sets of estimates were strongly correlated ($R^2 = 0.97$) and the

average difference was approximately 4% across most settings. However, estimates obtained with $\hat{\alpha}_e$ were considerably less precise than estimates obtained with $\hat{\alpha}_y$. To wit, their among-replication standard deviation was approximately 4 times greater.

For an average stand with approximately 90 elements, the first-order autoregressive process in Y and $\mathbf{X}$ led to an average increase in $\hat{V}(\hat{y})$ – when using $\hat{\alpha}_y$ as the estimate of the first-order autocorrelation in within-stand residual errors – of 3% when α_y was 0.1, 9% with $\alpha_y = 0.2$, and 20% with $\alpha_y = 0.3$. These numbers compare reasonably with theoretical expectations of 1%, 10%, and 24%. An inflation greater than expected was, in general, caused by an overestimation of σ_{st}^2 or α_e . In contrast, with $\hat{\alpha}_e(\hat{e}_{ij})$ the average inflation was 1.2% but with essentially no trend in the inflation across levels of α_y .

Achieved coverage of a nominal CI95

Across all design settings the achieved coverage – when using $\hat{\alpha}_y$ as the estimate of α_e – was 0.95 but as indicated in Figure 8, the coverage was highest (mean = 0.98) at the lowest level of the among-stand residual variance (σ_{st}^2), and it then declined gradually (mean = 0.94) at the highest level of σ_{st}^2 . In addition to σ_{st}^2 , the effects of α_y and $\rho(y, x_1)$ were also statistically significant ($P < 0.001$). Coverage increased from 0.94 to 0.96 as α_y rose from 0 to 0.3 and from 0.94 to 0.97 as $\rho^2(y, x_1)$ increases from 0.6 to 0.8. No other trend attracted attention. Monte Carlo errors in stand-averages of coverage varied from 0.15 to 1.10% with a median of 0.46%.

Coverage computed with $\hat{\alpha}_e(\hat{e}_{ij})$ was slightly inferior to the coverage with $\hat{\alpha}_y$. Across all settings the achieved coverage was 0.94 with 90% of the estimates between 0.90 and 0.98.

The difference between the two sets of estimates increased with the strength of $\rho(y, x_1)$, from -1.0 at the weakest level to 0.04 at the strongest level.

Discussion

The focus of our discussion is on the apparent challenges in deriving restricted maximum likelihood estimates of a distance dependent autocorrelation process in model residuals when data comes from a forest inventory with fixed-area PSUs (plots) and supported by remotely sensed auxiliary variables correlated with a study variable of interest. Practical aspects and importance of estimating possible stand effects in forest enterprise inventories was discussed in a forerunner to this study (Magnussen and Breidenbach 2017).

The potential of a spatial distance dependent autocorrelation in model residuals to inflate a model-based variance of a stand mean is well established (Fortin et al. 2012; Griffith 2005; Legendre 1993; Magnussen 2001) and confirmed in this study. It is therefore natural that its estimation enters into the sampling and estimation strategy of a forest enterprise inventory (Gregoire and Valentine 2008, ch. 1.1).

The demonstrated failure to obtain reliable restricted maximum likelihood estimates of a single parameter in a simple first-order spatial autogressive process in model residuals should not come as a surprise. Sampling intensities in enterprise forest inventories are low and, everything else equal, set the stage for a poor precision of estimated parameters in a spatial process (Chilès and Delfiner 1999, ch. 2.9; Cressie 1993, p. 127-135; Marchant and Lark 2004; Nordman and Caragea 2008). A potential confounding of model parameters linked to variance components (Hodges and Reich 2010; Zhang 2004) is equally at play.

Our results therefore presage a poor confidence in (frequentist) estimates derived from actual forest enterprise inventory data when both the chosen model linking $\mathbf{X}$ to Y and the spatial process in model residuals may be miss-specified vis-à-vis the actual state (Breidenbach et al.

2008; Breidenbach et al. 2015; Gunnarsson et al. 1998; Mauro Gutiérrez et al. 2017; McRoberts et al. 2011; Rahlf et al. 2014; Ver Hoef and Temesgen 2013). Our failure would likely be exacerbated by more complex models for the spatial process (Di Zio et al. 2004; Hodges and Reich 2010; Kim and Boos 2004; Zhang et al. 2005; Zimmerman 2006) and by variance heteroscedasticity in the residuals (Rahlf et al. 2014). In sum, data suited for a model fitting in presence of spatial heterogeneity and spatial dependence (Beale et al. 2010; Falk et al. 2011; Fortin et al. 2012; Geniaux and Martinetti 2017; Jiang and Nguyen 2012; Salas et al. 2010; Salvati et al. 2012) are rarely available from forest enterprise inventories. A Bayesian approach may hold more promise (Babcock et al. 2015; Finley et al. 2008).

In forest enterprise inventories, practical and variance efficient sampling designs rarely provide empirical residuals suitable for estimation of a spatial correlation process (Mauro Gutiérrez et al. 2017). Fortunately, our results also suggest that even with an anticipated bias in an estimate of the parameter in a spatial autocorrelation process, a reasonable estimate of the variance of a synthetic stand mean and a 95% confidence interval with an anticipated coverage reasonably close to its nominal target is still possible.

From a practical perspective, our results may encourage an analyst to refrain from attempting an estimation of the autocorrelation process in model residuals. There are clearly many situations, which warrant this attitude. For example, ignoring a spatial autocorrelation that inflates the variance of a synthetic stand mean by at most 8% would seem permissible as it only lowers the coverage of a CI95 by approximately 0.01 (a result that can be obtained from the central limit theorem and the properties of a standard Gaussian distribution). As well, if the relative variance of model errors plus stand-effects accounts for 10% or less of the total variance in Y , a first-order spatial autoregression parameters of 0.06 or less can be ignored for a stand composed of 25 elements without adverse effect on variance and coverage. The corresponding threshold for a stand with 196 elements is 0.18. When the relative variance of

model errors and stand effects climb to 30%, the aforementioned thresholds increase to 0.27 and 0.39, respectively. Yet, a decision to ignore a possible spatial autocorrelation remains delicate because the potential variance inflation depends not only on the relative magnitude of the variances due to parameter errors, stand effects, and residuals, but also on stand size, model formulation (Kangas 1994), and spatial scale (Smith and Urban 1988).

Instead of pursuing a REML estimation of the parameter(s) in a spatial autocorrelation process in model residuals, our study suggests a computationally simpler approach by using element level model predictions in place of empirical residuals. These predictions are readily available for all elements of interest and not restricted by the sampling design (Kar et al. 2011; Zhang 2004; Zimmerman 2006). Our success obviously hinges on using the correct (actual) model for fitting (Engel 1984). If the model is misspecified, a realized autocorrelation in $\hat{y}_{ij}$ for a given α_y may be inflated. To guard against a model misspecification that generates an inflated estimate of α_y Magnussen et al. (2016a) recommended an ad-hoc reduction of $\hat{\alpha}_e(\hat{y}_{ij})$ by 60%.

Design recommendations by Magnussen and Breidenbach (2017) for a forest enterprise inventory with dual objectives of estimating a population mean (total) and synthetic stand means are valid even when the impact of a spatial autocorrelation process is considered. When stand effects in Y are likely, an efficient design is one that combines sampling of one element from a larger number of stands with sampling of two elements from a relatively small number of stands (say 40).

Conclusions

Forest enterprise inventories with population-level and stand-level inferential objectives resort to model-based estimators of a stand mean by choice or when the number of samples

representing a stand of interest is too low to afford an otherwise preferred design-based estimator. When an estimate of the variance of a stand mean is required, an analyst runs the risk of a serious underestimation unless in possession of reliable estimates of stand effects, and the strength of a possible spatial autocorrelation in the differences between model based expectations and the actual value (i.e. model residuals). While reliable estimates of stand effects can flow from sampling designs with as few as two samples taken in a relatively small subset of stands, this study made clear, that a restricted maximum likelihood approach to the estimation of a spatial autocorrelation in empirical residuals is likely to fail with practically affordable sampling designs. Fortunately, our results also suggest that a proxy for the spatial autocorrelation can come from model predictions without adverse effect on the coverage of nominal 95% confidence intervals.

Acknowledgements

Comments and critique from two journal reviewers and the Associate Editor prompted many improvements to our first submission. We gratefully appreciate their constructive reviews.

References

- Anderson, O.D. 1976. Time Series Analysis and Forecasting: The Box-Jenkins approach. Butterworths, London.
- Babcock, C., Finley, A.O., Bradford, J.B., Kolka, R., Birdsey, R., and Ryan, M.G. 2015. LiDAR based prediction of forest biomass using hierarchical models with spatially varying coefficients. *Remote Sens. Environ.* **169**: 113-127.
- Beale, C.M., Lennon, J.J., Yearsley, J.M., Brewer, M.J., and Elston, D.A. 2010. Regression analysis of spatial data. *Ecol. Let.* **13**(2): 246-264.
- Breidenbach, J., Kublin, E., McGaughey, R., Andersen, H.-E., and Reutebuch, S. 2008. Mixed-Effects Models for Estimating Stand Volume By Means of Small Footprint Airborne Laser Scanner Data. *Photogrammetric Journal of Finland* **21**(1): 4-15.
- Breidenbach, J., McRoberts, R.E., and Astrup, R. 2015. Empirical coverage of model-based variance estimators for remote sensing assisted estimation of stand-level timber volume. *Remote Sens. Environ.* **173**: 274-281. doi: 10.1016/j.rse.2015.07.026.
- Casella, G., and Berger, R.L. 2002. Statistical Inference. 2 ed. Duxbury Press, Pacific Grove. pp. 660.
- Chambers, R.L., and Clark, R.G. 2012. An introduction to model-based survey sampling with applications. Oxford University Press, New York. pp. 265.

- Chen, H.Y., and Klinka, K. 2003. Aboveground productivity of western hemlock and western redcedar mixed-species stands in southern coastal British Columbia. *For. Ecol. Manage.* **184**(1): 55-64.
- Chilès, J.P., and Delfiner, P. 1999. *Geostatistics: Modeling spatial uncertainty*. Wiley, New York. pp. 695.
- Christensen, R. 2001. *Linear models for multivariate, time series and spatial data*. Springer, New York. pp. 217.
- Cressie, N.A.C. 1993. *Statistics for spatial data*. Revised edition. 2nd ed. Wiley, New York. pp. 900.
- de Gruijter, J. 2002. Spatial sampling schemes for remote sensing. *In Spatial Statistics for Remote Sensing*. Springer. pp. 211-242.
- Di Zio, S., Fontanella, L., and Ippoliti, L. 2004. Optimal spatial sampling schemes for environmental surveys. *Environ. Ecol. Stat.* **11**(4): 397-414.
- Draper, N.R., and Smith, H. 2014. *Applied Regression Analysis*. 3 ed. Wiley, New York. pp. 736.
- Dymond, C.C., Wulder, M.A., Shore, T.L., Nelson, T., Boots, B., and Riel, B.G. 2006. Evaluation of risk assessment of mountain pine beetle infestations. *W. J. Appl. For.* **21**(1): 5-13.
- Engel, E. 1984. A unified approach to the study of sums, products, time-aggregation, and other functions of ARMA processes. *Journal of Time Series Analysis* **5**(3): 159-171.
- Fahrmeir, L., Kneib, T., Lang, S., and Marx, B. 2013. *Regression: models, methods and applications*. Springer, New York. pp. 698.
- Falk, M., Denham, R., and Mengersen, K. 2011. Spatially stratified sampling using auxiliary information for geostatistical mapping. *Environ. Ecol. Stat.* **18**(1): 93-108. doi: 10.1007/s10651-009-0122-3.
- Finley, A.O., Banerjee, S., Ek, A.R., and McRoberts, R.E. 2008. Bayesian multivariate process modeling for prediction of forest attributes. *J. Agric. Biol. Environ. Stat.* **13**(1): 60-83. doi: 10.1198/108571108x273160.
- Fortin, M.-J., James, P.M.A., MacKenzie, A., Melles, S.J., and Rayfield, B. 2012. Spatial statistics, spatial regression, and graph theory in ecology. *Spatial Statistics* **1**(0): 100-109. doi: <http://dx.doi.org/10.1016/j.spasta.2012.02.004>.
- Fortin, M., Manso, R., and Calama, R. 2016. Hybrid estimation based on mixed-effects models in forest inventories. *Can. J. For. Res.* **46**(11): 1310-1319. doi: 10.1139/cjfr-2016-0298.
- Fuller, W.A. 2009. *Sampling Statistics*. Wiley, New York. pp. 454.
- Geniaux, G., and Martinetti, D. 2017. A new method for dealing simultaneously with spatial autocorrelation and spatial heterogeneity in regression models. *Regional Science and Urban Economics*.
- Gneiting, T., Kleiber, W., and Schlather, M. 2010. Matérn Cross-Covariance Functions for Multivariate Random Fields. *J. Am. Stat. Assoc.* **105**(491): 1167-1177. doi: 10.1198/jasa.2010.tm09420.

- 491 Goerndt, M.E., Monleon, V.J., and Temesgen, H. 2011. A comparison of small-area
492 estimation techniques to estimate selected stand attributes using LiDAR-derived auxiliary
493 variables. *Can. J. For. Res.* **41**(6): 1189-1201. doi: 10.1139/x11-033.
- 494 Grafström, A., Lundström, N.L., and Schelin, L. 2012. Spatially balanced sampling through
495 the pivotal method. *Biometrics* **68**(2): 514-520.
- 496 Grafström, A., Saarela, S., and Ene, L.T. 2014. Efficient sampling strategies for forest
497 inventories by spreading the sample in auxiliary space. *Can. J. For. Res.* **44**(10): 1156-1164.
498 doi: 10.1139/cjfr-2014-0202.
- 499 Grafström, A., and Tillé, Y. 2013. Doubly balanced spatial sampling with spreading and
500 restitution of auxiliary totals. *Environmetrics* **24**(2): 120-131.
- 501 Gregoire, T.G. 1998. Design-based and model-based inference in survey sampling:
502 Appreciating the difference. *Can. J. For. Res.* **28**(10): 1429-1447.
- 503 Gregoire, T.G., and Valentine, H.T. 2008. Sampling strategies for natural resources and the
504 environment. Chapman & Hall/CRC, Boca Raton, FL. pp. 465.
- 505 Griffith, D.A. 2005. Effective Geographic Sample Size in the Presence of Spatial
506 Autocorrelation. *Annals of the Association of American Geographers* **95**(4): 740-760. doi:
507 10.1111/j.1467-8306.2005.00484.x.
- 508 Gunnarsson, F., Holm, S., Holmgren, P., and Thuresson, T. 1998. On the potential of kriging
509 for forest management planning. *Scand. J. For. Res.* **13**(1-4): 237-245.
- 510 Gupta, A.K., and Nagar, D.K. 1999. Matrix Variate Distributions. CRC. pp. 384.
- 511 Hawbaker, T.J., Keuler, N.S., Lesak, A.A., Gobakken, T., Contrucci, K., and Radeloff, V.C.
512 2009. Improved estimates of forest vegetation structure and biomass with a LiDAR-
513 optimized sampling design. *J. Geophys. Res.* **114**. doi: 10.1029/2008JG000870.
- 514 Hickey, G.M., Innes, J.L., and Kozak, R.A. 2007. Monitoring and information reporting for
515 sustainable forest management: A regional comparison of forestry stakeholder perceptions. *J.*
516 *Environ. Manage.* **84**(4): 572-585. doi: <http://dx.doi.org/10.1016/j.jenvman.2006.07.004>.
- 517 Hodges, J.S., and Reich, B.J. 2010. Adding Spatially-Correlated Errors Can Mess Up the
518 Fixed Effect You Love. *Am. Stat.* **64**(4): 325-334. doi: doi:10.1198/tast.2010.10052.
- 519 Jiang, J., and Nguyen, T. 2012. Small area estimation via heteroscedastic nested-error
520 regression. *Can J Statist* **40**(3): 588-603. doi: 10.1002/cjs.11148.
- 521 Johnson, C.R., Okubo, K., and Reams, R. 2001. Uniqueness of Matrix Square Roots and an
522 Application. *Linear and Multilinear Algebra* (323): 51-60.
- 523 Junttila, V., Finley, A.O., Bradford, J.B., and Kauranne, T. 2013. Strategies for minimizing
524 sample size for use in airborne LiDAR-based forest inventory. *For. Ecol. Manage.* **292**: 75-
525 85. doi: 10.1016/j.foreco.2012.12.019.
- 526 Kangas, A. 1994. Classical and model based estimators for forest inventory. *Silv Fenn*
527 *Monogr* **28**: 3-14.
- 528 Kar, S., Varshney, P.K., and Chen, H. 2011. Spatial whitening framework for distributed
529 estimation. *In Computational Advances in Multi-Sensor Adaptive Processing (CAMSAP),*
530 *2011 4th IEEE International Workshop on.* IEEE. pp. 293-296.
- 531 Kim, H.J., and Boos, D.D. 2004. Variance estimation in spatial regression using a non-
532 parametric semivariogram based on residuals. *Scand. J. Stat.* **31**(3): 387-401.

- 533 Koehler, E., Brown, E., and Haneuse, J.-P.A. 2009. On the assessment of Monte Carlo error
534 in simulation-based statistical analyses. *Am. Stat.* **63**(2): 155-162.
- 535 Král, K., Valtera, M., Janík, D., Šamonil, P., and Vrška, T. 2014. Spatial variability of
536 general stand characteristics in central European beech-dominated natural stands—effects of
537 scale. *For. Ecol. Manage.* **328**: 353-364.
- 538 Lappi, J. 2001. Forest inventory of small areas combining the calibration estimator and a
539 spatial model. *Can. J. For. Res.* **31**: 1551-1560.
- 540 Latifi, H., Fassnacht, F.E., Müller, J., Tharani, A., Dech, S., and Heurich, M. 2015. Forest
541 inventories by LiDAR data: A comparison of single tree segmentation and metric-based
542 methods for inventories of a heterogeneous temperate forest. *International Journal of Applied*
543 *Earth Observation and Geoinformation* **42**: 162-174.
- 544 Legendre, P. 1993. Spatial autocorrelation: Trouble or new paradigm? *Ecology* **74**: 1659-
545 1673.
- 546 Magnussen, S. 2001. Fast pre-survey computation of the mean spatial autocorrelation in large
547 plots composed of a regular array of secondary sampling units. *Mathematical Modelling and*
548 *Scientific Computing* **13**(3-4): 204-217.
- 549 Magnussen, S., and Breidenbach, J. 2017. Model-dependent forest stand-level inference with
550 and without estimates of stand-effects. *Forest. Oxf.* doi:
551 <https://doi.org/10.1093/forestry/cpx023>.
- 552 Magnussen, S., Frazer, G., and Penner, M. 2016a. Alternative mean-squared error estimators
553 for synthetic estimators of domain means. *J. Appl. Stat.* **43**(14): 1-24. doi:
554 [10.1080/02664763.2016.1142942](https://doi.org/10.1080/02664763.2016.1142942).
- 555 Magnussen, S., Mandallaz, D., Lanz, A., Ginzler, C., Næsset, E., and Gobakken, T. 2016b.
556 Scale effects in survey estimates of proportions and quantiles of per unit area attributes. *For.*
557 *Ecol. Manage.* **364**: 122-129. doi: <http://dx.doi.org/10.1016/j.foreco.2016.01.013>.
- 558 Maltamo, M., Mehtätalo, L., Vauhkonen, J., and Packalén, P. 2012. Predicting and
559 calibrating tree attributes by means of airborne laser scanning and field measurements. *Can.*
560 *J. For. Res.* **42**(11): 1896-1907.
- 561 Mandallaz, D. 2008. Sampling techniques for forest inventories. Chapman and Hall, Boca
562 Raton, Florida. pp. 251.
- 563 Marchant, B.P., and Lark, R.M. 2004. Estimating variogram uncertainty. *Mathematical*
564 *Geology* **36**(8): 867-898.
- 565 Mauro, F., Molina, I., García-Abril, A., Valbuena, R., and Ayuga-Téllez, E. 2016. Remote
566 sensing estimates and measures of uncertainty for forest variables at different aggregation
567 levels. *Environmetrics* **27**(4): 225-238. doi: [10.1002/env.2387](https://doi.org/10.1002/env.2387).
- 568 Mauro, F., Monleon, V.J., Temesgen, H., and Ruíz, L.Á. 2017. Analysis of spatial correlation
569 in predictive models of forest variables that use LiDAR auxiliary information. *Can. J. For.*
570 *Res.* **47**(6): 788-799.
- 571 McRoberts, R.E., Magnussen, S., Tomppo, E.O., and Chirici, G. 2011. Parametric, bootstrap,
572 and jackknife variance estimators for the *k*-Nearest Neighbors technique with illustrations
573 using forest inventory and satellite image data. *Remote Sens. Environ.* **115**(12): 3165-3174.
574 doi: [10.1016/j.rse.2011.07.002](https://doi.org/10.1016/j.rse.2011.07.002).

- 575 Næsset, E. 2004a. Accuracy of forest inventory using airborne laser scanning: evaluating the
576 first Nordic full-scale operational project. *Scand. J. For. Res.* **19**(6): 554-557.
- 577 Næsset, E. 2004b. Practical large-scale forest stand inventory using a small-footprint airborne
578 scanning laser. *Scand. J. For. Res.* **19**(2): 164-179.
- 579 Nieschulze, J., and Saborowski, J. 2002. Monitoring of Forests under Continuous Cover
580 System Management. *In* Continuous Cover Forestry: Assessment, Analysis, Scenarios.
581 *Edited by* K. von Gadow and J. Nagel and J. Saborowski. Springer Netherlands, Dordrecht.
582 pp. 53-66.
- 583 Nordman, D.J., and Caragea, P.C. 2008. Point and interval estimation of variogram models
584 using spatial empirical likelihood. *J. Am. Stat. Assoc.* **103**(481): 350-361.
- 585 Opsomer, J.D., Breidt, F.J., Moisen, G.G., and Kauermann, G. 2007. Model-assisted
586 estimation of forest resources with generalized additive models. *J. Am. Stat. Assoc.*
587 **102**(478): 400-409.
- 588 Parker, R.C., and Evans, D.L. 2004. An application of LiDAR in a double-sample forest
589 inventory. *W. J. Appl. For.* **19**(2): 95-101.
- 590 Rahlf, J., Breidenbach, J., Solberg, S., Næsset, E., and Astrup, R. 2014. Comparison of four
591 types of 3D data for timber volume estimation. *Remote Sens. Environ.* **155**: 325-333. doi:
592 <http://dx.doi.org/10.1016/j.rse.2014.08.036>.
- 593 Rao, J.N., and Molina, I. 2015. Small area estimation. 2 ed. John Wiley & Sons. pp. 480.
- 594 Salas, C., Ene, L., Gregoire, T.G., Næsset, E., and Gobakken, T. 2010. Modelling tree
595 diameter from airborne laser scanning derived variables: a comparison of spatial statistical
596 models. *Remote Sens. Environ.* **114**(6): 1277-1285.
- 597 Salvati, N., Tzavidis, N., Pratesi, M., and Chambers, R. 2012. Small area estimation via M-
598 quantile geographically weighted regression. *TEST* **21**(1): 1-28. doi: 10.1007/s11749-010-
599 0231-1.
- 600 Schreuder, H.T., Gregoire, T.G., and Wood, G.B. 1993. Sampling methods for multiresource
601 forest inventory. Wiley, New York. pp. 416.
- 602 Searle, S.R., Casella, G., and McCulloch, C.E. 1992. Variance components. Wiley, New
603 York. pp. 501.
- 604 Smith, H.F. 1938. An empirical law describing heterogeneity in the yields of agricultural
605 crops. *J. Agric. Sci.* **28**: 1-23.
- 606 Smith, T.M., and Urban, D.L. 1988. Scale and resolution of forest structural pattern. *Plant*
607 *Ecol.* **74**(2): 143-150.
- 608 Stein, A., and Ettema, C. 2003. An overview of spatial sampling procedures and experimental
609 design of spatial studies for ecosystem comparisons. *Agric. Ecosyst. Environ.* **94**(1): 31-47.
- 610 Varma, V.K., Ferguson, I., and Wild, I. 2000. Decision support system for the sustainable
611 forest management. *For. Ecol. Manage.* **128**: 49-55.
- 612 Ver Hoef, J.M., and Temesgen, H. 2013. A Comparison of the Spatial Linear Model to
613 Nearest Neighbor (k-NN) Methods for Forestry Applications. *PLoS ONE* **8**(3): e59129. doi:
614 10.1371/journal.pone.0059129.
- 615 Wolfram, S. 2016. The Mathematica Documentation Center (Version 11.1). Wolfram
616 Research, Champaign, IL. p. 1470.

- 617 Wulder, M., Coops, N., Hudak, A., Morsdorf, F., Nelson, R., Newnham, G., and Vastaranta,
618 M. 2013. Status and prospects for LiDAR remote sensing of forested ecosystems. *Can. J.*
619 *Rem. Sens.* **39**(sup1): S1-S5.
- 620 Zhang, H. 2004. Inconsistent Estimation and Asymptotically Equal Interpolations in Model-
621 Based Geostatistics. *J. Am. Stat. Assoc.* **99**(465): 250-261. doi:
622 10.1198/016214504000000241.
- 623 Zhang, L., Gove, J.H., and Heath, L.S. 2005. Spatial residual analysis of six modeling
624 techniques. *Ecol. Model.* **186**(2): 154-177. doi:
625 <http://dx.doi.org/10.1016/j.ecolmodel.2005.01.007>.
- 626 Zimmerman, D.L. 2006. Optimal network design for spatial prediction, covariance parameter
627 estimation, and empirical prediction. *Environmetrics* **17**(6): 635-652.
- 628
- 629

Draft

630 Table 1. Notation.

Symbol	Comment	Value(s) or Range
α_e	First-order autocorrelation in within-stand error terms (residuals). To be estimated from sample data.	
α_x	First-order autocorrelation in within-stand values of X_1 and X_2 .	$0.7\alpha_y$
α_y	First-order autocorrelation in within-stand values of study variable Y .	0.0-0.30
AR1	First-order autogression.	
β	Vector of regression coefficients ($\beta_0, \beta_1, \beta_2$) to an intercept, x_1 and x_2 . To be estimated from sample data.	$-0.01 \leq \beta_0 \leq 0.01$ $0.90 \leq \beta_1 \leq 1.04$ $-0.30 \leq \beta_2 \leq -0.26$
$\wedge$	A caret over a symbol indicates an estimate (or prediction).	
$CI95_\zeta$	A 95% confidence interval for the parameter ζ .	
$C_y(dist)$	Covariogram for elements of y separated by distance $dist$.	$\alpha_y^{dist} (1 - \alpha_y^2)^{-1} (1 - \sigma_{y_{st}}^2)$
e_i	A stand-specific intercept in the linear model linking	

	$\mathbf{X}$ to Y , $i=1,...,M$.	
e_{ij}	A stand and element specific error term in the linear model linking $\mathbf{X}$ to Y .	
$F_{k,l}$	F-ratio random variable with k and l degrees of freedom	
H_0	The null hypothesis	
M	Number of forest stands in a population.	
m_i	Number of elements in a stand.	25-196
n	Sample size in number of fixed area plots (elements).	120-480
n_1	Number of sampled forest stands.	20-240
n_2	Number of fixed area plots (elements) sampled within each of n_1 sampled forest stands.	2,4, and 6
N	Population size in number of elements (known).	≈ 97733
Ω	3x3 covariance matrix for coefficients in β .	
$P(\hat{\theta} H_0)$	The probability that the random variable θ is equal to $\hat{\theta}$ or greater under the null hypothesis.	
PSU	Primary sampling unit (a fixed area element viz. a sample plot).	
$\mathbf{R}$	A matrix of correlation coefficients.	

R^2	Coefficient of determination in regression of Y on $\mathbf{X}$.	$\approx 1.0833\rho^2(y, x_1)$
$\rho^2(y, x_1)$	Squared correlation coefficient between y and x_1 .	0.6, 0.7, and 0.8
$\rho(y, x_2)$	Correlation coefficient between y and x_2 .	$0.25\rho(y, x_1)$
$\rho(x_1, x_2)$	Correlation coefficient between x_1 and x_2 .	0.5
σ_{st}^2	The among-stand variance of e_i .	$\sigma_{\bar{y}_{st}}^2(1 - 1.0833\rho^2(y, x_1))$
σ_w^2	The within-stand variance of e_{ij} .	$(1 - R^2)(1 - \sigma_{\bar{y}_{st}}^2)$
$\sigma_{\bar{y}_{st}}^2$	The among-stand variance of stand means of Y .	0.05, 0.10, and 0.20
$\sigma_{\bar{x}_{st}}^2$	The among-stand variance of stand means of X_1 and X_2	$0.7\sigma_{\bar{y}_{st}}^2$
Σ	A variance-covariance matrix.	
T_{co}	Total within-stand covariance.	T_{co}
$V(\varsigma)$	The variance of a parameter ς .	
$\mathbf{X}$	<p>A matrix of known element-level auxiliary variables</p> <p>In this study $\mathbf{X}=(X_1, X_2)$. No specific attributes are associated with $\mathbf{X}$. Variables X_1 and X_2 are assumed normally distributed with a mean of zero and a (total) variance of 1.0.</p>	
$\mathbf{X}^t$	The transpose of $\mathbf{X}$.	
Y	A generic (unspecified) normally distributed	

element-level study variable assumed normally distributed with a mean of zero and a variance of 1.0.

y A realized value of *Y*. Values of *y* are simulated for all elements in a population but only *y* in elements in a sample, are known to an analyst.

z A standard Gaussian variable (mean zero, variance 1.0).

631

632

Draft

Appendix

Simulated populations

All populations used in this study are tri-variate (y , x_1 , and x_2) standard Gaussian populations (mean zero, variance 1). Each population is composed of $M = 800$ spatially compact forest stands with m_i ($i = 1, \dots, M$) square fixed area elements arranged in a regular array. The value of m_i was determined by a random draw from the set $\{5^2, 6^2, \dots, 14^2\}$. Consequently, the expected number of elements in a population (N) is 97733 with a standard deviation of 2072. The 36 populations vary with respect to the relative strength of stand effects in Y and $\mathbf{X}$, a tri-variate correlation structure, and a first-order autoregression in element values of Y and $\mathbf{X}$.

For the 9 populations with no first-order autoregression (i.e. $\alpha_y = 0$) in Y or $\mathbf{X}$, the values of X_1 , X_2 , and Y for a single stand were generated in a two-step process. In step one, m_i within-stand values were generated from a zero-centred tri-variate multivariate Gaussian distribution with a variances and correlation structure as specified by a population-level defined tri-variate correlation structure and within-stand variances (i.e. one minus the among-stand variances in Y and $\mathbf{X}$). The m_i simulated triplets were then mean-shifted to a mean of zero. In a second step, a stand-level effect was added by a single random draw from a zero-centred tri-variate multivariate Gaussian distribution with variances $\sigma_{y_{st}}^2$ for Y , and $0.7 \sigma_{y_{st}}^2$ for X_1 and X_2 , and otherwise the same correlation structure as used for the simulation of within-stand data. Note, the two error terms e_i and e_{ij} in (1) are not simulated explicitly; they remain unknown except in a full census of the population.

For the 27 populations with a positive first-order distance dependent autoregressive process in Y and $\mathbf{X}$, the within-stand values of X_1 , X_2 , and Y were generated through a different two-step process following the principles of linear co-regionalization (Chilès and Delfiner 1999,

657 ch. 5.6.5; Gneiting et al. 2010). In step one m_i within-stand values of X_1 , X_2 and Y , were
 658 generated from a zero-centred tri-variate multivariate Gaussian matrix distribution (Gupta
 659 and Nagar 1999) with row- and column covariance structures specified by population-level
 660 defined tri-variate correlation structures and the first-order autoregression along rows and
 661 columns. A Gaussian matrix distribution of a $\sqrt{m_i} \times \sqrt{m_i}$ random (univariate) matrix $\mathbf{z}$ is the
 662 distribution of $\boldsymbol{\mu}_z + \Sigma_{row}^{0.5} \mathbf{z} \Sigma_{col}^{0.5}$ where $\boldsymbol{\mu}_z$ is the expectation of $\mathbf{z}$ (here the null matrix $\mathbf{0}$) and
 663 $\Sigma_{dir}, dir = row, col$ is the covariance matrix of the $\sqrt{m_i}$ elements in a row (column) of the
 664 square matrix with m_i elements. Details on a matrix raised to a power of 0.5 are in Johnson et
 665 al. (2001). Afterwards, the m_i simulated triplets were mean-shifted to a mean of zero and
 666 combined to a $m_i \times 3$ matrix and post-multiplied by $\mathbf{R}^{0.5}$ where $\mathbf{R}$ is the 3×3 the correlation
 667 matrix of the three variables. In the second step, as before, stand-level effects were added by
 668 a single random draw from a zero-centred tri-variate multivariate Gaussian distribution with
 669 variances $\sigma_{y_{st}}^2$ for Y , and $0.7 \sigma_{y_{st}}^2$ for X_1 and X_2 , and otherwise the same correlation structure as
 670 used for the within-stand data. This procedure was implemented in the Mathematica® 11.0.1
 671 software (Wolfram 2016). A validation of the within-stand autoregression in Y and $\mathbf{X}$ was
 672 obtained by ordering the m_i elements to the shortest traverse through all elements in a stand
 673 followed by a maximum likelihood estimation of α_y and $\alpha_{x_k}, k = 1, 2$ (Christensen 2001, p.
 674 198). Two sets of estimates were computed and averaged; one by beginning the traverse from
 675 the upper left corner of a stand, and a second by starting it from the lower right corner. We
 676 learned that for a single stand the apparent realized first-order autoregression coefficient
 677 could be up to 50% from its nominal value, but the average over 800 stands was typically
 678 within 5% of a target, and the average over 800 stands and 200 replications was, typically,
 679 within 0.5% of a target. The largest deviations were consistently associated with the smallest
 680 stands with 25 elements.

681 The simulations generated 9 levels of the within-stand residual error σ_w^2 . With standard
 682 Gaussian data and the linear model in (1) we have $\sigma_w^2 = (1 - R^2)(1 - \sigma_{\bar{y}_{st}}^2)$ where R^2 is the
 683 coefficient of variation in the regression of Y on X_1 and X_2 . In our set up it holds that
 684 $R^2 \cong 1.0833\rho^2(y, x_1)$ where the error of approximation is ≤ 0.002 . Ignoring this error, we
 685 obtain – after rounding to the nearest 0.01 – the following levels of σ_w^2 : 0.11, 0.12, 0.13,
 686 0.19, 0.22, 0.28, 0.32, 0.33, each represented by 48 settings of n (4), n_2 (3), and α_y (4).

687 There is also 9 level of the among-stand residual error σ_{st}^2 . The nine levels can be obtained (to
 688 within 0.001) from $\sigma_{st}^2 \cong \sigma_{\bar{y}_{st}}^2 (1 - 1.0833\rho^2(y, x_1))$. Specifically, the levels are 0.007, 0.012,
 689 0.013, 0.018, 0.024, 0.027, 0.035, 0.048, and 0.070. It follows from the overall design that
 690 there are only 9 unique combinations of σ_{st}^2 and σ_w^2 , each represented by 48 settings of n , n_2 ,
 691 and α_y .

692 *Estimation of α from $\hat{y}_{ij}$*
 693 In our populations we introduced a first-order autoregressive process (AR1) in both Y and $\mathbf{X}$
 694 with parameters $\alpha_y = \{0.0, 0.1, 0.2, 0.3\}$ and $\alpha_{x_k} = 0.7\alpha_y, k = 1, 2$. Since as per (1) we have
 695 $y_{ij} = \boldsymbol{\beta}'\mathbf{X} + e_i + e_{ij}$ it follows (Engel 1984, (7.1) in proposition 4) that for a known $\boldsymbol{\beta}$ and e_i
 696 the prediction $\hat{y}_{ij} = \boldsymbol{\beta}'\mathbf{X} + e_i$ must also follows an AR1 process for all admissible lags (along
 697 rows or columns) of k elements (here 1 to $(1 + \sqrt{m_i})$). We can now state: for the elements
 698 within a stand e_{ij} is the difference between two AR1 processes which means that e_{ij} also
 699 follows an AR1 process (Engel 1984, (4.2) in proposition 1). The only solution to the AR1
 700 parameters satisfying the constraint on the autocorrelation functions of the involved random
 701 variables in (4.2) (Ibid.) requires $\alpha_y = \alpha_{\bar{y}} = \alpha_e$. Naturally, the equation only holds with a

702 census estimate of the regression coefficients, but it suggests an opportunity to estimate α_e
703 from α_j .

Draft

Figure 1. Estimates of the within-stand variance component in residual errors plotted against their nominal target values. A one-to-one line, is added for orientation.

708 Figure 2. Estimates of the among-stand variance component in residual errors plotted against
709 their nominal target value. A one-to-one line, is added for orientation.

710
711

Figure 3. Estimates of the first-order autoregressive coefficient α_e obtained from actual (true) model residuals (e_{ij}) plotted against the nominal first-order autoregressive coefficient α_y in y_{ij} . A one-to-one line, is added for orientation.

717 Figure 4. Scatter plot of the first-order autoregressive coefficient α obtained from empirical
718 residuals $(\hat{e}_{ij})$ plotted against estimates obtained from actual (true) residuals (e_{ij}) . A one-to-
719 one line, is added for orientation.

720
721

Figure 5. Estimates of the first-order autoregressive coefficient α_e obtained from model-based expectations of y_{ij} (i.e. $\hat{y}_{ij}$) plotted against the nominal first-order autoregressive coefficient α_y in y_{ij} . A one-to-one line, is added for orientation.

Figure 6. Estimates of the variance of a synthetic stand mean (see (3)) plotted against the actual mean squared error of the synthetic mean. Variances were obtained with α estimated from $\hat{y}_{ij}$. A one-to-one line, is added for orientation.

732 Figure 7. Estimates of the variance of a synthetic stand mean obtained with α_e estimated
 733 from empirical residuals ($\hat{e}_{ij}$) plotted against variances obtained with α_e estimated from
 734 model-based expectations ($\hat{y}_{ij}$). A one-to-one line, has been added for orientation.

735

736

737 Figure 8. Achieved coverage ($\hat{CO}$) of nominal CI95s plotted against nominal target values of
738 the among stand variance component of model residuals (σ_{st}^2). The target coverage of 0.95 is
739 highlighted (in bold).

740

741