Inquiry-Based Learning: A Case Study of an Experienced Elementary Mathematics Teacher in Action

by

Genie Kim

A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Department of Curriculum, Teaching and Learning
Ontario Institute for Studies in Education of the
University of Toronto

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Department of Curriculum, Teaching and Learning
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Abstract

Currently, Inquiry-Based Learning (IBL) has become the central focus and method of delivery to support mathematics learning, especially within elementary schools in Ontario. Through undertaking a single case study of my colleague who teaches Grade 5, I investigated the Inquiry-Based Learning strategies she uses in her classroom to increase student understanding and proficiency in mathematics. Additionally, I explored the challenges teachers of Grades 4 to 6 may confront when seeking to implement IBL methods in their mathematics classroom. My participant is a former math coach and instructional leader of mathematics. My data was collected from interviews with my colleague, interviews with her students, classroom observations, and through student questionnaires, journals and classwork.

My findings indicate that various IBL related strategies and techniques should be implemented to promote students’ mathematical understanding and proficiency. Eight major findings have been summarized, which point to how my case study practices/exhibits the following: (1) she constructs/implements rich, open-ended inquiry-based tasks that permit multiple-entry points; (2) she asks deliberate and intentional questions to elicit students’ mathematical thinking and develop their conceptual understanding; (3) she makes students’ invisible thinking become visible through the process of questioning and recording students’ thinking; (4) she provides learning experiences where students must struggle with their
mathematical understanding; (5) she provides students will ample opportunities to experience repeated practice of procedural knowledge; (6) she promotes ongoing and active student mathematical discourse; (7) she possesses deep mathematical content knowledge to effectively implement IBL practices; and (8) she incorporates all five dimensions of mathematical proficiency into her IBL mathematics lessons (Kilpatrick, Swafford & Findell, 2001).

Implications from this study suggest that teachers as well as educational policy-makers need to consider a variety of measures to help deepen teachers’ mathematical content knowledge. Whether it occurs at the individual, pre-service or in-service level, literature as well as the findings from this study indicate the dire need for teachers to possess sufficient mathematical background knowledge and ability in order to promote mathematical proficiency among their students. Acquiring this deep mathematical content knowledge by teachers must then be married with the implementation of various key IBL strategies/techniques. Suggestions for areas of further research are included at the end of the study.
Acknowledgements

Reaching the metaphorical summit in one’s doctoral journey inescapably requires supportive individuals to assist in the inherently daunting and oftentimes seemingly insurmountable ascent. I was privileged and blessed to have had many within my midst.

I have a penchant for expressive words; yet, I am often fearful they will be received as glib or inconsequential. Notwithstanding this, I will boldly attempt to convey my deep appreciation to some very special people, who have each, in their unique and distinctive ways, helped me attain my mountaintop experience.

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Next, my participant, Marian, without whom this dissertation would not have been possible, I owe you a great debt of gratitude. In applying to graduate school, I hoped that
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This thesis is dedicated to my parents: First, to my mother, who inspired within me scholarly pursuits; thank you, mom, for pushing the boundaries of my understanding and beautifully shaping my thoughts and worldview. And to my father, who bred within me the practice of applying nothing short of pure diligence and integrity in all of my undertakings; Dad, it was your admirable example that paved the way for me. Mom and Dad…I did it!
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Chapter One: Introduction

1.1 Introduction

The paramount role mathematics plays in the life of a child cannot be overstated nor trivialized. Bryk and Treisman (2010) state, “Remedial math has become an insurmountable barrier for many students, ending their aspirations for higher education” (p. 1). Furthermore, Ashcraft (2002) maintains, “Highly math-anxious individuals are characterized by a strong tendency to avoid math, which ultimately undercuts their math competence and forecloses important career paths” (p. 181). Clearly, one’s recurring experiences with mathematics can create long-term repercussions that can function to either facilitate or hinder a student’s academic and life outcomes. In understanding that mathematics is a gateway to many career opportunities, it is disconcerting that a significant number of students within our schools feel frustrated, overwhelmed and disengaged with mathematics.

A myriad of studies are undertaken to support students’ learning of mathematics, yet the figurative pendulum shows no signs of mitigating its activity vis-a-vis the most effective approach to mathematics instruction. ‘Student-centered’ pedagogy is offered as a response. Herein, inquiry-based learning (IBL) becomes relevant (Maaß & Artigue, 2013).

Currently, IBL has become the central focus and method of delivery to support mathematics learning, especially within many elementary schools in Ontario. There exists no shortage of educational research conducted in mathematics, both past and present, which largely espouses this constructivist pedagogical method of instruction. IBL is a shift from the traditional, mainly transmission method of teaching, towards a more active form of teaching and learning that is student-centered (Engeln, Euler & Maass, 2013). In seeking how to optimally increase student achievement in mathematics, my research questions investigate a largely espoused
pedagogy among mathematics education scholars today: inquiry-based learning. Therefore, identifying key methods and strategies, specifically as they relate to IBL, that serve to promote student achievement in mathematics is the aim of my research.

In this thesis, I will describe the role of teachers in utilizing inquiry-based learning (IBL) methods to increase student understanding and proficiency in mathematics. My study specifically focuses on investigating how elementary teachers can implement IBL strategies/methods in their mathematics classrooms to promote mathematical understanding and proficiency among their students. In the context of my study, the term inquiry-based learning (IBL) is a form of student-centered learning where teachers take on the role of facilitator in promoting active engagement of learning through discovery, interaction and experiential problem-solving.

In my research, I conducted a case study of my colleague, focusing on how she implements IBL strategies/methods in her mathematics classroom to promote mathematical understanding and proficiency among her Grade 5 students.

1.2 Purpose of the Study

The purpose of this study is to determine how teachers of Grades 4 to 6 can effectively implement IBL practices to increase students’ understanding and proficiency in mathematics. Additionally, I investigated challenges/barriers teachers of Grades 4 to 6 may confront when utilizing IBL methods in their classrooms.

1.3 Statement of the Problem

In my research, I investigated two questions as they pertain to teaching students in Grades 4 to 6 mathematics using inquiry-based learning methods:

1) How does an experienced Grade 5 teacher in IBL, implement inquiry-based learning strategies/practices in her classroom to promote mathematical understanding and proficiency among her students?
2) What are some challenges teachers of Grades 4 to 6 may face when implementing IBL methods in their mathematics classes?

1.4 Significance of the Study

The importance and implications of this research are tremendous and expansive in its breadth and scope. The significance of this research can be seen in its potential to provide practical insights, as well as relevant and applicable strategies that may extend beyond the scope of my participant’s and my own classrooms, finding its way into all spaces where learning occurs.

The implications of this research are far-reaching. The findings reveal new insights and practical IBL strategies and techniques that teachers, at all levels of instruction, can implement to promote mathematical proficiency and engagement among students. Ashcraft’s (2002) compelling metaphor regarding the notion that mathematics serves as a gatekeeper to many career opportunities, heightens the demand and call for teachers to build and develop their mathematical content knowledge - for their students’ sake.

1.5 Personal Background of the Researcher

As a Korean-Canadian who arrived in Canada at the age of two, I received all of my formal education in various inner-city schools within Toronto. During this time, my parents lived the typical immigrant life, arduously shouldering more than one job, whilst often alternating the 'graveyard' shift. As a result, academic support at home was infeasible. Sadly, mathematics quickly became both a nightmare and a seeming curse to me. Many of my friends chided me about being the only Asian who didn't excel in math. Outwardly, I laughed along with them, but inside, there was a fair level of embarrassment and self-doubt -- I equated poor mathematical skills with being "stupid". I also experienced great trepidation that I might not be admitted to a reputable university, which my parents assumed would become an eventuality for me.
Fortunately, I was in high school during the time Calculus no longer became mandatory and Finite math was deemed acceptable for university admission. Unabashedly, I phoned my Finite math teacher near the end of the semester entreaty him to help me pass his course so I could fulfill the University of Toronto’s conditional acceptance of my application.

This seemingly arbitrary background was provided to demonstrate why and how mathematics has become a personal and passionate subject for me. I do not want any child to experience the same chronic angst, fear and dread that enveloped me when I was in school. Hence, the 'gateway' metaphor could not be more fitting and apt in its applicability to my life: Had I not passed Finite math, I would not be here on this amazing journey, pursuing doctoral studies in mathematics at OISE.

As for my professional career as an educator, I have been teaching now for 15 years. I have taught Grades 4-11 in both the public and private educational systems. The bulk of my career has been teaching Grades 4 and 5. I have come full-circle and am currently teaching Grade 5 at an inner-city school. It is at this juncture I will relate my personal experiences that have led me to my current investigation of IBL.

When inquiry methods were first introduced into the schools where I was teaching, a great amount of indignation and resistance ensued. Unabashedly, I confess I was one of the many vocal objectors. However, notwithstanding our opposition, professional development proceeded and was almost exclusively devoted to the understanding and implementation of IBL. Opportunities were afforded for teachers to develop comfort and confidence with this new pedagogical method of instruction. Over the years, up to the present, despite ongoing professional development of IBL, with the unremitting accompaniment of its resources, teachers continue to gather together to express their discontent and aversion for “discovery-learning”
methods. Water-cooler talk persists about IBL, centering on the impracticality of its methods in enabling students to effectively learn critical mathematical skills and concepts. Again, I was not exempt from ‘stirring the pot’.

As a result, I quietly persisted in using my own tried and true instructional methods of teaching, presumptuously relegating inquiry-based strategies to a secondary position. Admittedly, with a fair degree of smugness as well as a sense of superiority, I believed my subversive acts were defensible: all executed selflessly from my conviction that I had the best interest for all of my students. These past two years, to my incredulity, my thinking was dramatically and fundamentally shaken - by one individual.

Recently, a colleague at my school has confounded and challenged my prejudiced thinking about IBL. Marian (pseudonym) worked as a math coach for our Family of Schools for four years and as a math instructional leader for one year; she recently returned to our school as a classroom teacher. For several years now, including this past school year, Marian has provided IBL professional learning for teachers. Listening to her reinforce and endorse the practices of IBL served to disturb and provoke my thinking; more compellingly, witnessing Marian’s current and former students express their great enthusiasm for mathematics spurred me to re-evaluate the much touted merits of IBL, which I had previously disparaged.

The displays on Marian’s walls became concrete testimony of the rich and meaningful learning occurring inside of her classroom. The apparent success demonstrated by the vast majority of Marian’s students in mathematics contrasted with what I was experiencing with my own students. Whenever I had implemented IBL in my classroom, I consistently witnessed the following: 1) my mathematically proficient students appeared to be the only ones engaged and adept with the IBL tasks, and 2) the majority of my students seemed alarmingly reliant on their
mathematically inclined peers to accomplish IBL activities on their behalf. Bearing witness to the stark contrast between my colleague’s and my own experiences, I humbly submitted to re-assessing IBL with a new lens.

Out of genuine curiosity and a desire to discover best practices, in 2014/15, I asked Marian if I could observe her teach some lessons in mathematics. She consented. I was quite eager to see what Marian was doing differently within her mathematics lessons that I might have been remiss in doing whenever I had implemented discovery methods of teaching. I was excited and intrigued to discover where my philosophy and praxes may have fallen short. Unfortunately, with the many unforeseen events that can and do arise within any given school day, coordinating our schedules became a mammoth challenge. The end of the school year arrived, and to my dismay, these much anticipated observations failed to materialize.

In view of the foregoing, I shall return to articulating the purpose of my research. Originally, the intent for my thesis was to explore the role of two dichotomous approaches to learning mathematics: traditional vs reform. Serendipitously, a discussion with my supervisor became centered on my colleague’s inspiring work and role. Despite the fact that my research has taken on an unanticipated trajectory, I am delighted that I can still explore these two pedagogies, but now in a context that provides more objectivity and an auspicious chance to learn about best practices related to IBL. The context could not be more ideal: A highly competent, expert former math coach/instructional leader, well versed and proficient vis-a-vis IBL pedagogy, was teaching adjacent to me, and she had agreed to having me investigate her practices throughout the 2015-2016 school year.

To make the situation more favourable, we both were teaching grade 5. This afforded me the opportunity to observe her IBL approach while critically reflecting on my own more
traditional style of instruction. Despite my case study being ‘unique’ and atypical, researching my participant is relevant for scholarly research as understanding her case may illuminate the possible gaps that exist in teachers’ understanding and appreciation of how IBL in mathematics should be implemented. I am a case in point! Ultimately, I sought to heed Stake’s (1995) assertion that statistical generalizations should not be the primary aim of case studies; rather, the opportunity to learn from the case should be at the forefront.

When it comes to the realm of education, notwithstanding the possibility of being subjected to ridicule, I want to live out the Pollyanna Principle and believe the best about all educators. Therefore, I choose to presume that it is at the heart and intention of all teachers (doubtless, those who are intrinsically called into this vocation) to seek out, for the ultimate benefit of their students, the most effective and preeminent methods of teaching. Hence, the crux of my study is to discover what methods/strategies my colleague uses with IBL in her classroom, so I can, if possible, share this new knowledge with others, as well as incorporate her “best practices” into my own pedagogy. I am hoping this study will help me grow both as a scholar and as a professional practitioner.

1.6 Key Term: Inquiry-Based Learning

In this thesis, the term *inquiry-based learning* (IBL) will be understood to mean a pedagogical approach that places students’ questions, ideas, and observations at the forefront of their learning experiences. Teachers are viewed to be active agents and facilitators of the learning process, where they engage in practices that move their students from a position of wondering, to a position of established understanding, and then back to a place of further questioning (Scardamalia, 2002).
1.7 Plan of the Thesis

This thesis is comprised of five chapters. Chapter one provides an overview of my study, including the research context, research questions, and the significance of the study.

Chapter Two serves as a review of the existing literature; as well, it examines previous research that has been conducted in this area. ‘Mathematical proficiency’ is the undergirding framework; this concept is examined with the sole purpose of understanding how teachers can implement IBL to encourage mathematical proficiency among Grades 4 to 6 students. The Ten Dimensions of Mathematics Education (McDougall, 2004) serves as my overarching conceptual framework; this framework allows me to gauge and illuminate the effective IBL strategies utilized by my colleague in her mathematics class. As my study aims to investigate how IBL methods and strategies are effectively implemented by a Grade 5 teacher, Marian’s students are also described.

The methodology for my study will be described in Chapter Three. Throughout this section, I further elaborate on the research context and provide a comprehensive profile of my participants. I outline the qualitative data sources for my study as well as the methods that will be used for my analysis. I expound upon a well-suited methodological approach, case study research, that supports and aligns with my investigation. This chapter is concluded with a discussion of the ethical considerations of my study.

Chapter Four presents the findings from my case study. Chapter Five synthesizes the findings from my case study to answer each of my two research questions presented in Chapter One and links these findings to existing literature about IBL in mathematics. I also suggest areas for future research and discuss some limitations of my study.
Chapter Two: Literature Review

2.1 Introduction

In this chapter, I will review the existing literature related to my study. The purpose of this examination is to explore and connect findings in the literature that are related to the effectiveness of implementing IBL in Grades 4 to 6 mathematics classrooms. I first provide a brief overview of educational philosophy and the constructivist influence on the reform movement in mathematics. Engeln, Euler and Maass (2013) state that the tradition of IBL within the United States goes back to Dewey; hence, I would be remiss in neglecting to include Dewey in my discussion of constructivism and its intersection with IBL. Vygotsky’s influence is also accorded comparable space in my discussion as his theories and constructs are foundational to understanding how children conventionally learn.

I then define IBL and discuss the importance of this pedagogy in helping to improve the learning of mathematics. I also examine the role of the teacher, as well as the role student-tasks play in relation to their capacity to support students’ learning of mathematics. Then I review the importance of teachers’ mathematical content knowledge and its relationship to students’ mathematical understanding. In addition, I explore two key elements critical in IBL pedagogy: questioning techniques and student communication. I then openly discuss some challenges that exist with the IBL approach.

Next, I examine ‘mathematical proficiency’, a term used by the National Research Council (2001), which was formulated to represent what it means for a person to learn mathematics successfully. The five strands associated with Mathematical Proficiency will be explored in relation to its impact on students’ understanding and competence in mathematics. ‘Mathematical proficiency’ will serve as a broad-ranging framework for my study, since
improving students’ understanding and proficiency are both at the heart of all mathematical research.

I also review the Ten Dimensions Framework as it pertains to the effective teaching of mathematics. I devote a considerable section specifically on six of the ten dimensions, as these focused dimensions help shed light on various elements necessary for student understanding in mathematics.

This review will be seen to amalgamate literature from various fields; it develops a framework for understanding the myriad factors that influence the effective implementation of IBL in Grades 4 to 6 mathematics classrooms.

2.2 Educational Philosophy

Due to its strong influence, philosophical thought is fundamental to understanding education. Aspects of educational philosophy are derived from the roots of pragmatism. Pragmatism is contrasted with traditional philosophies. “Pragmatism, also referred to as experimentalism, is based on change, process, and relativity” (Ornstein & Hunkins, 2004, p. 34). Dewey was considered to be one of the foremost educational pragmatists. He understood education as being a process for advancing the human condition, and the school was seen as correlating with society. Thus, Dewey believed that instruction should be based on children’s experiences and interests in order to prepare them for the future (Dewey, 1938).

Progressivism has its underpinnings in pragmatic philosophy where focus is placed on the child rather than on the subject matter. The progressive education movement of America is most associated with Dewey (Mooney, 2000). As a progressivist, Dewey shared with Vygotsky, Montessori, and Piaget the belief that education should be child-centered, active and interactive (as cited in Mooney, 2000).
Dewey’s understanding of instruction is consistent with the central tenets and principles underpinning inquiry-based learning in mathematics. Hence, I will approach my study through the ‘Deweyan’ lens.

“If one examines curriculum documents across Canada, critical thinking heads the lists of essential skills. Yet ironically, these same documents have become increasingly prescriptive regarding what teachers must teach and what students must learn” (Watt & Colyer, 2014). This ‘prescriptive’ state of affairs is diametrically opposed to the staunch beliefs of two distinguished educational theorists, Dewey and Vygotsky, both of whom have greatly influenced the teaching and learning of mathematics.

2.3 Influence of Dewey and Vygotsky

Dewey and Vygotsky were both resolute champions against the transmission method of instruction. For instance, Dewey took exception to the kind of teaching that stems from a rigid form of instruction, one that produces static knowledge; he believed that the complex series of problems that should encompass a curriculum is one that views the child as being willful, purposive, curious and active as opposed to being a passive receptor of external data (as cited in Archambault, 1974). Analogously, Vygotsky firmly believed, “The role of instruction is not merely to transmit content-specific knowledge but to bring about a state of reflective understanding that enables the learner to control, monitor and master the learning process” (as cited in Miller, 2011, p. 119).

It is in this very context of instruction in which reflective knowing occurs that Vygotsky introduced one of his most important contributions to pedagogy: his concept of ZPD or the zone of proximal development (Miller, 2011). This theory postulates that the ZPD is created through negotiation between the expert learner and the novice (Newman, Griffin & Cole, 1989). Thus,
according to Vygotsky’s framework for how children learn, he believed that the higher mental processes are a function of socially meaningful activity; hence, all learning for Vygotsky was social (as cited in Smidt, 2009).

Consistent with Vygotsky’s view, Dewey also believed that learning was social, and that the teacher should serve as a guide who supports the student to achieve his or her goals. Dewey asserted, “For teaching, properly conceived, could not be said to consist of ‘instruction’ at all. The teacher should be a catalytic agent who, by providing materials, clues, information, suggestions, clarifications - could create a setting that would be conducive to learning” (as cited in Archambault, 1974, p. xxiv). Thus, Dewey saw the relationship between the teacher and student as one that is reciprocal where the teacher is seen as a friendly co-partner and guide in a common purpose (as cited in Archambault, 1974).

Clearly, both Dewey and Vygotsky believed that the centerpiece of teaching was to facilitate and encourage the child to become a principal and active agent in his or her educational experience. The study of IBL in mathematics dovetails with these eminent constructivist theorists’ viewpoint of foregrounding the child’s experiences in all matters of learning.

2.4 Constructivism and Math Reform

The theory of constructivism undergirds and pervades much of mathematics research today. Cobb (1988) suggests that constructivism primarily aims to support students’ learning through the construction of conceptual networks and seeks to encourage students to become autonomous and metacognitive mathematical thinkers.

Constructivism, as a realm, is concerned with how people learn, and it characterizes the individual as being active in the process of thinking and learning; this contrasts significantly with the behaviorist viewpoint. Behaviorists principally underscore the role of the teacher in the
learning process (Ornstein & Hunkins, 2004). Ornstein and Hunkins (2004) assert, “In constructivism, the learner is the key player; the learner must participate in generating meaning or understanding...the learner must engage himself or herself in internalizing and reshaping or transforming information via active consideration” (p. 117).

The philosophy of inquiry-based learning finds its roots in constructivist learning theories, such as the works of Dewey and Vygotsky, as was elaborated upon earlier. In the field of mathematics education specifically, IBL can be traced to constructivism and is reflected in the ‘reform’ movement spearheaded in North America by the US-based National Council of Teachers of Mathematics (NCTM 2000). Towers (2010) notes that inquiry-based learning has its basis in Dewey’s philosophy of learning.

Dewey is credited with contributing to the development of the concept of inquiry, particularly that of reflective inquiry as he believed inquiry helps formulate the groundwork for such a pedagogical practice (Artigue & Blomhoj, 2013). Dewey’s learning pedagogy is founded on the basis of learners actively participating in personal or authentic experiences in order to make meaning from such experiences (Archambault, 1964).

The increased emphasis of inquiry-based learning in mathematics is a direct result of dramatic changes that have occurred in math education over the years, which is pervasively characterized as ‘math reform’. The central debate in the ‘math wars’ that pits basic skills (traditional learning) against conceptual understanding (math reform) has a long-standing history within education (Hiebert & Stigler, 2004; Kilpatrick, Swafford, & Findell, 2001). Currently, math reform has been finding its way into all recesses of mathematics teaching and learning. In order to transform teaching to align with the reform initiatives, Hiebert and Stigler (2004)
suggest that teachers shift their priorities, with the rescheduling of time commitments and embedding critical reflection time on teaching strategies.

Egodawatte, McDougall and Stoilescu (2011) argue that the emphasis of mathematics education reform must be divorced from the transmission model of knowledge to one of transaction, where mathematical investigation and exploration are advanced. This perspective stems from the constructivist view of learning, which counters the transmission model of learning due to its ineffectiveness in promoting deep mathematical understanding. IBL is inherently student-centered in its approach, and it operates around a constructivist framework in order to promote mathematical proficiency among students.

2.5 Teachers’ Self-Efficacy and Reform Mathematics

‘Self-efficacy’ is generally defined as a person’s belief in his or her ability to succeed in various situations (Bandura, 1997). One’s sense of self-efficacy is seen to largely influence how one approaches goals, tasks, and challenges (Bandura, 1997). Teachers’ self-efficacy is used to describe teachers’ beliefs pertaining to their capability of instructing various subject matters, even to students who challenge them (Tschannen-Moran, Woolfolk Hoy, & Hoy, 1998). Teachers who possess a high sense of self-efficacy are believed to be more industrious, persistent and involved in informal learning activities (Bandura, 1997).

Numerous studies have been conducted to examine the relationship between teachers’ self-efficacy and student outcomes (Holzberger, Philipp, & Kunter, 2013). In a reform classroom, the teacher’s role is to facilitate their students’ learning through the meaningful selection of mathematical tasks; additionally, teachers encourage high-quality mathematics discourse which serve to build connections between students’ formal and informal mathematics knowledge (Fosnot & Dolk, 2002). Hence, teachers’ self-efficacy can be seen to play an
important role in contributing to their ability to navigate the vast demands exacted, when implementing an IBL mathematics program.

Bandura (1997) discusses four sources of self-efficacy beliefs. ‘Mastery experience’ is described to be the degree to which individuals have experienced prior success with a relevant task. Bandura’s mastery experience warrants mention as this term becomes useful in understanding teachers’ experiences with implementing IBL methods in mathematics. Tschannen-Moran et al. (1998) indicate that one’s self-efficacy may change in relation to one’s specific experiences. Thus, teacher self-efficacy (mastery experience) is relevant when considering how teachers’ experiences with IBL can directly or indirectly influence their ability to effectively teach mathematics utilizing inquiry-based methods. Self-efficacy beliefs can function to motivate teachers to expend effort and energy into the learning of new practices. The willingness to adopt unfamiliar teaching methods occurs through the teacher’s willingness to take risks and persist through difficulties that typically accompany the implementation of new learning processes (Gabriele & Joram, 2007).

Stein and Wang (1988) maintain a relationship exists between teachers’ self-efficacy and their inclination towards changing their classroom practices. They purport that teachers with high self-efficacy are more favourably disposed to adopting new teaching methods. When seeking to understand how teachers’ self-efficacy impacts on their implementation of new pedagogical practices, the ‘source(s)’ of teacher self-efficacy needs to be considered (Gabriele & Joram, 2007). Identifying the source(s) that contribute to self-efficacy becomes significant when seeking to understand a teacher’s willingness to adopt or persist in using inquiry-based learning methods in his or her classroom.
Bandura (1997) maintains that an individual’s past performance(s) largely determines the net effect of his or her self-efficacy beliefs. According to Bandura, what matters is not the performance itself, but rather the ‘interpretation’ of one’s performance that primarily influences the development of self-efficacy. Smith (1996) claims that, when teachers enact new practices, such as those involving reform-based mathematics teaching, discouragement can readily ensue when a teacher’s sense of self-efficacy is thwarted due to a seeming lack of appreciable progress in student outcomes. Smith (1996) explains that this occurs because reform-based pedagogy provides fewer explicit criteria for judging success compared to traditional methods. This becomes problematic when seen from Bandura’s (1997) lens: Teachers’ ‘interpretation’ of past performances impact and shape their self-efficacy beliefs. Smith (1996) further asserts that a teacher’s ability to accurately interpret his or her performance in a reform-based mathematics class may be impaired as ‘success’ is characterized differently from that of a traditional mathematics class.

Gabriele and Joram (2007) conducted a study to determine whether professional development could enhance self-efficacy among teachers, when they transitioned from a traditional to a reform-based pedagogy. These researchers found that, when professional development activities specifically focused on helping teachers recognize and interpret various examples of student thinking during a lesson, teacher self-efficacy was positively influenced. These professional development sessions, which aimed to increase self-efficacy, helped teachers successfully transition from using traditional ways of teaching to reform-oriented methods of pedagogy. Understanding that self-efficacy mediates between teachers’ beliefs and behaviours alerts us to the fact that supports must be put in place whenever IBL methods are newly introduced.
2.6 Inquiry-Based Learning

Improving student learning and understanding in mathematics cannot be separated from teacher pedagogy. The role inquiry-based learning (IBL) has played in mathematics learning in the recent past is tremendous. Maaß and Doorman (2013) argue that IBL supports the development of such competencies as acquiring new knowledge, creative problem solving and critical thinking skills, which align with both the five strands of mathematical proficiency and the Ten Dimensions Framework of Mathematics Education. These authors further argue, “IBL aims to develop and foster inquiring minds and attitudes that are vital for enabling students to face and manage uncertain futures” (p. 887).

Scardamalia (2002) defines inquiry-based learning as being a method of instruction that places the students’ questions, ideas and observations at the centre of the learning experience. Scardamalia contends that underlying this approach is the notion that students and teachers both share responsibility for learning. Despite there being a multitude of characterizations of IBL, undergirding each are common core understandings: learning must be student-centered, problem-based, and experiential.

In my investigation of IBL, it becomes relevant to discuss PRIMAS. PRIMAS is an acronym that stands for ‘Promoting Inquiry-Based Learning in Mathematics and Science across Europe’; it is a large-scale international project linked with the European Union. The PRIMAS project, which involved 12 European countries from 14 universities over a four-year period, sought to promote the implementation of IBL in both mathematics and science (PRIMAS, 2010). I am partial to PRIMAS’s broader definition of IBL, which strongly aligns with Scardamalia’s (2002) definition. It appeals to me for a variety of reasons. One is that they see IBL as a multi-faceted teaching and learning culture where the process of inquiry is seen as central for learning.
They also underscore the following: a) the role students play in constructing meaning, b) how meaningful learning takes place in a social context, c) how learning is supported by meaningful contexts (situated cognition), and d) how learning is a dialogic process.

I particularly appreciate PRIMAS’s focus on enriching a teacher’s repertoire of IBL knowledge and practices. PRIMAS recognizes the key role teachers play in implementing IBL pedagogies inside their classrooms. This definition further accords with the five strands of mathematical proficiency: “Thus, by engaging students actively in the construction, evaluation and reflection of knowledge, inquiry-based education promotes competencies that are relevant for lifelong learning and for a successful orientation in a complex world” (Barrow, 2006, p. 825).

Below is a broad yet detailed overview that helps succinctly delineate the breadth, scope and essence of IBL:

Inquiry-based learning is not meant to be prescriptive for the teacher or the student; it is an interactive, fluid, and recursive process responsive to the discipline, the teaching goals, and student learning needs. While the fundamental characteristics of inquiry do not change, there are different types of inquiry that may be best suited to different situations, depending on the question or problem posed and the needs of students. At one end of the continuum of student choice is an open inquiry. This is an inquiry where students choose the question and design and conduct the investigation independently. At the opposite end of the continuum is guided inquiry where you assist throughout the process, by selecting the question, providing specific frameworks and resources in the investigation, and modelling the critical analysis required for the accrued research. A blended inquiry represents all the possibilities in the middle of the continuum between ‘open’ and ‘guided’. This is where both open and guided inquiry co-exist. Blended inquiry is the form of inquiry most often attempted in classrooms since it allows for balance and flexibility in teacher and student direction. (Watt & Colyer, 2014, p. 11)

2.6.1 Why Focus on IBL Pedagogy?

In contrast to how students generally view mathematics, mathematicians, “see mathematics as a creative process in which real joy comes from grappling with difficult problems and (hopefully) solving them” (Capaldi, 2015, p. 283). Capaldi bemoans how today’s
students, unfortunately, do not possess the same viewpoint. Nevertheless, Capaldi (2015) optimistically offers IBL as a solution to this regrettable problem.

Strong endorsements by many prominent researchers and theorists have been expressed regarding the pedagogical value of IBL. Hattie (2009) states that inquiry-based instruction has been shown to generate transferable critical thinking skills, improved achievement, as well as improved attitudes among students. Moreover, Jang, Reeve, and Deci (2010) have found students to be more engaged and self-directed learners in inquiry-based classrooms. Similarly, Boaler (2016) argues that, when students see their role as that of being passive recipients of knowledge as opposed to active agents, students disengage from mathematics.

Kuhn et al. (2000) have discovered that inquiry skill development promotes critical thinking, personal responsibility for one’s own learning, and intellectual growth. Recent support for inquiry methods derive from cognitive science research, which provides evidence of the importance of social activity and authentic learning contexts (Greeno, Collins & Resnick, 1996). Palmer (2002) noted statistically, that significantly higher grades were earned by students who were instructed through the inquiry-based learning method compared to those who were taught via traditional methods.

Engeln, Euler and Maass (2013) claim inquiry-based learning is the method of choice to help overcome deficits in mathematical literacy and they describe both the potentials and challenges of implementing IBL from the perspective of teachers. However, in the same breath, these authors also question to what extent IBL is used in day-to-day instruction within the classroom. Engeln, Euler and Maass (2013) note:

There is a generally accepted consensus that a lack of basic competencies and interest in mathematics and science subjects will hinder young people in becoming active citizens and contributing adequately to the development of society. (p. 823)
Accordingly, they assert: “The quality of teaching and learning in mathematics and science is considered crucial and requires considerable improvements in order to comply with growing societal needs” (p. 823). Herein, they offer IBL as a means of improving education. Engeln, Euler and Maass (2013) define IBL as, “a shift from traditional, mainly deductive, teaching styles towards more appealing and activating forms of teaching and learning” (p. 823).

Hattie (2009) declares that it is not only the use of IBL that improves students’ performance, but the role of the teacher also plays a critical function, particularly in their capacity to be directive and provide ongoing and necessary feedback. Clearly, the role of the teacher must be given preeminence and value when considering the effective implementation of IBL.

### 2.6.2 Role of the Teacher in Implementing IBL

Although it is strongly believed that science and mathematics education can be improved through the implementation of IBL, it must be openly conceded that its effectiveness is not yet resolutely established (Bruder & Prescott, 2013). Barrow (2006) recognizes that the critical piece to enabling IBL to function well is the teacher. He maintains that teachers, through executing their subtle skills, must orchestrate and facilitate the learning process through modelling and coaching.

Dewey believed that teachers need to have confidence in their skills and abilities. He assumed that teachers need to trust in their knowledge and experience and appropriately nurture for their students inquiry for learning (as cited by Mooney, 2000). Engeln, Euler and Maass (2013) contend, “One of the main stumbling blocks for implementing IBL in the classroom is teachers’ beliefs about teaching and learning” (p. 826).
Artigue and Blomhoj (2013) argue, “In IBL, moreover, particular attention must be paid to the delicate role of the teacher in supporting and guiding the development of productive inquiry and on how forms of teacher-student(s) interaction contribute to the negotiation of meaning” (p. 802). These authors further assert that ‘realistic mathematics education’ (RME)-Freudenthal, and ‘theory of didactical situations’ (TDS) are both relevant to IBL. They contend that both theories, which are problem-solving theories, help conceptualize the role of the teacher, where the teacher’s guidance is essential.

Maaß and Doorman (2013) declare, “If teachers are to encourage students to participate in IBL, they must first have experienced inquiry themselves” (p. 890). Likewise, Artigue and Blomhoj (2013) also note: “In order to be able to plan for and support IBL for students, teachers need to experience and exercise inquiry in mathematics themselves. Moreover, they need to develop an inquiry stance towards their own teaching” (p. 807).

Schoenfeld (2002) states that, when teachers are provided with long-term opportunities to develop their skills and understanding, their students’ mathematical performance can improve significantly. With reference to the role of the teacher, Vygotsky (1978) contends that a teacher’s role is to close the gap between actual and potential development by supporting students through task-specific skills and engaging them in collaborative communication with their peers.

What is conspicuously evident in all of the theorists’ articulation of IBL is the underlining theme of the role of the teacher. Understanding how IBL is effectively implemented to bring about mathematical proficiency among students is precisely what I hope to distill through observing my colleague in action.
2.6.3 Student Tasks

The pivotal role of mathematical tasks for students’ learning cannot be overstated. If inquiry-based learning tasks are properly implemented by teachers, students can be supported in their understanding and learning of mathematics (Silver et al., 2009). Moreover, Hiebert et al. (2005) claim that teachers’ effective use of tasks can increase students’ achievement in mathematics. Raphael, Pressley and Mohan (2008) note that high-level tasks sustain students’ engagement, and they also provide the most academic instruction. It was found that teachers using these types of tasks were able to cover more concepts in greater depth. The National Council of Teachers of Mathematics (NCTM, 2000) has set standards of which teachers are required to utilize mathematical tasks that appeal to both the students’ interest and their intellect.

Students who engage with rich, inquiry-based tasks, those involving problem-solving, justifying, reasoning and evaluating their mathematical thinking, are more likely to experience a heightened appreciation and understanding of mathematics (Arbaugh & Brown, 2005; Stein & Lane, 1996). Additionally, to promote achievement in mathematics, teachers must provide their students with opportunities to construct meaning from the tasks with which they are engaged (Hiebert & Stigler, 2004). When constructing their mathematical understanding, Hiebert and Stigler (2004) also note that students need to be provided with ample time to construct their own algorithms and solutions.

Hiebert and Stigler (2004) compared the teaching approaches exercised by middle school teachers in Japan, Germany and the United States. Their findings revealed that Japanese lessons focused heavily on conceptual understanding, compared to the lessons employed by teachers in the United States, where minimal application was given to conceptual understanding. The results from the two divergent teaching approaches are noteworthy. The Japanese middle school
students’ deductive reasoning skills were clearly witnessed as they spent a significant amount of time proving, justifying and analyzing the various elements embedded within mathematical tasks. Contrariwise, students in the United States rarely explored the mathematical relationships that were present within the tasks. In their lack of conceptual focus, the teachers in the United States contributed to the deficits in students’ understanding of mathematics (Hiebert and Stigler, 2004).

Hiebert and Stigler (2004) argue that efforts must be made to ensure students are afforded opportunities to solve challenging problems that require them to construct mathematical relationships. They assert that conceptual understanding is a critical component of students’ mathematical learning. The results from Hiebert and Stigler’s (2004) study suggest that a balance is needed where time must concurrently be spent on practicing basic skills along with developing conceptual understanding. Additionally, Hiebert and Stigler (2004) conclude that teachers must learn how to avoid intervening and giving the answers to students, which was commonly found among American teachers. Instead, these researchers encourage teachers to provide students with opportunities to think deeply about mathematical ideas, which should then culminate with a discussion among their peers about the newly learned concepts. Similarly, Baumert et al. (2010) maintain that teachers should not declare the accuracy or inaccuracy of student responses, but rather, in their discussions, teachers must challenge their students to evaluate the validity of their solutions on their own.

Doyle (1988) proposes, “The work students do, which is defined in large measure by the tasks teachers assign, determines how they think about a curriculum domain and come to understand its meaning” (p. 167). Doyle (1988) uses the concept of ‘cognitive level’ to categorize tasks. Those tasks relying on memory, formulas and matching strategies are
considered to have a lower cognitive demand. Conversely, tasks that focus on comprehension, interpretation, and a flexible application of knowledge and skills have higher cognitive demands. Gainsburg (2008) laments that most teachers tend to, “Worry more about over-challenging their students than under-challenging them” (p. 216) and therefore implement lower cognitive demand tasks for their students.

Arbaugh and Brown (2005) acknowledge the great challenges many teachers face regarding the complexities surrounding mathematical tasks. They propose that teachers should be given opportunities to become well-versed vis-a-vis the intricacies of task selection and their appropriate implementation. Stein et al., (1996) suggest that task implementation can play a decisive role: either to support or inhibit students’ ability to learn mathematics.

Rich tasks are characterized as those allowing students to ‘enter into’ the task through different avenues; thus, they are often referred to as ‘multiple entry’ tasks. The benefit of such multiple-entry tasks is their ability to invite students of most if not all skill levels to participate.

Rich tasks are also characterized as “open”. This implies the responses allow for multiple solution strategies and a range of responses. Despite a mathematical task being deemed as rich, a teacher’s implementation of the supposedly rich task may not result in “rich” learning. Rich tasks can be transformed into low-level tasks during the implementation phase by the teacher (Doyle, 1988). Thus, it is evident that task implementation is just as important as task selection.

Evidently, difficulties abound vis-a-vis task selection and implementation. Diverse studies can be found that underscore the importance and merits of teachers using IBL methods and tasks in their mathematics classrooms; yet, numerous challenges with IBL still exist, particularly as it concerns teachers’ mathematical content knowledge.
2.6.4 Teachers’ Mathematical Content Knowledge

Various authors distinguish among some core dimensions of teacher knowledge, such as mathematical content knowledge (MCK) and pedagogical content knowledge (PCK) (Grossman, 1995; Sherin, 1996; Shulman, 1987). Mathematical content knowledge (MCK) refers to teachers’ knowledge of the subject matter at hand. Pedagogical content knowledge (PCK) involves a diversity of understandings that amalgamate knowledge of mathematics, of students, and of pedagogy. PCK, essentially relates to knowledge required for instructional processes (Baumert et al. 2010). For the purposes of this study, I will refer primarily to teachers’ mathematical content knowledge (MCK) as opposed to their pedagogical content knowledge (PCK). This is by virtue of PCK being implausible without the existence of sufficient MCK. These two components of professional knowledge are interdependent and both mutually indispensable, when seen from the lens of effective mathematical instruction (Baumert et al., 2010).

Baumert et al.’s (2010) investigation found that deficits in teachers’ MCK operated at the expense of the teachers’ PCK. When discussing pre-service teacher training, Baumert et al. (2010) conclude:

Programs that compromise on subject matter training, with the result that teacher candidates develop only a limited mathematical understanding of the content covered at specific levels, have detrimental effects on PCK and consequently negative effects on instructional quality and student progress. (p. 167)

Existing literature in the realm of teaching and teacher-education consistently point to the fact that domain-specific and general pedagogical knowledge and skills determine teachers’ instructional quality thereby influencing students’ mathematical achievement (Bransford, Darling-Hammond & LePage, 2005; Grossman & McDonald, 2008; Hiebert, Morris, Berk & Jansen, 2007). Studies conducted on motivational theories reveal that providing students with challenging tasks is counterproductive for motivating them unless they are provided with teacher
support and scaffolding. This scaffolding is only possible when teachers possess adequate MCK (Baumert et al. 2010).

The National Council of Teachers of Mathematics (2000) maintains that deep subject knowledge is a core component of teacher competence. Moreover, the NCTM (2000) argues that conceptual understanding of the material being taught must be present in teachers. Corroborating this claim, the National Mathematics Advisory Panel (2008) asserts that, in order to have positive effects on students’ learning, teachers must have detailed and advanced knowledge of the mathematical content in which they are responsible for teaching. Hill et al. (2005) explored the relationship between elementary school teachers’ mathematical knowledge and their students’ achievement, and found a significant positive correlation existed between the two. Their research findings lend support for stimulating policy initiatives that seek to improve teachers’ mathematical knowledge.

In a large-scale study, Hill et al. (2008) found teachers’ mathematical knowledge impacts the quality of mathematical instruction. Additionally, these authors identify factors that mediate the relationship between teachers’ mathematical content knowledge and students’ mathematical achievement. Hill et al. (2008) admit to being uncertain regarding ‘how’ teacher knowledge influences student learning. Hill et al. (2008) justify this lack of understanding by stating, “Large-scale educational production function studies never peer inside classrooms to compare the practice of higher-knowledge and lower-knowledge teachers” (p. 431).

The arsenal of teaching strategies as well as knowledge of possible alternative mathematical representations that become available to teachers are largely contingent on the breadth and depth of teachers’ prior conceptual understanding of the subject matter at hand (Baumert et al. 2010). Baumert et al. (2010) investigated the significance of teachers’ content
knowledge, as well as their pedagogical content knowledge in mathematics in relation to the quality of instruction. Their investigation revealed considerable positive effects of teachers’ pedagogical content knowledge on their students’ achievement in mathematics.

However, Baumert et al. (2010) cite further studies in which teachers’ lack of mathematical content knowledge limited their ability to explain and represent to their students the necessary conceptual understanding. It was found that teachers’ insufficient MCK could not be compensated by their pedagogical skills. In an earlier study, Hill et al. (2005) found that teachers’ mathematical content knowledge positively predicted students’ mathematical achievement in Grades One and Three. This demonstrated for the authors that teachers’ content knowledge plays an important role, even at the very early elementary years.

In a compelling study comparing teachers in China and the United States, Ma (1999) demonstrated that the Chinese teachers’ deep and broad understanding of fundamental mathematics allowed them to have flexibility and a more varied repertoire of pedagogical strategies for teaching their students mathematics, compared to their counterparts in the United States.

It must be noted that MCK is not sufficient by itself for teachers to successfully teach mathematics. Teachers’ MCK must be accompanied and complemented by pedagogical skills. Literature points to the need for teachers’ to have both MCK and PCK (Baumert et al., 2010).

Swafford, Jones and Thornton (1997) offer teachers, who lack deep mathematical knowledge, hope. Their study demonstrates that intervention and enhancement of mathematical content knowledge can improve and strengthen teachers’ instructional quality. Hill et al. (2008) offer a useful suggestion for teachers possessing little mathematical knowledge. They recommend that it is preferable that these teachers follow a textbook than generate their own
math lessons, as they state that implementing such materials are ‘chancy’ at best. The significance and import of teachers’ MCK becomes evident when understanding a key IBL strategy: questioning.

### 2.6.5 Questioning

‘The art and science of asking questions is the source of all knowledge’, this apt quote by American novelist, Thomas Berger, speaks to the importance of questioning. To educate students for ‘life’, Sanchez (2013) argues that we must move away from routine mathematical exercises, and instead offer ‘open-ended questions’ as a means of developing students’ ability to solve such complex problems. Sanchez (2013) defines open-ended questions as being, “Questions that can be solved or explained in a variety of ways, that focus on conceptual aspects of mathematics, and that have the potential to expose students’ understanding and misconceptions” (p. 206). Sanchez remarks that asking students for alternative explanations compels them to consider a different representation. The benefit of multiple representations is that it allows students to identify their errors. Open-ended questions can promote teachers to focus their pedagogy on NCTM’s Process Standards, as well as on reasoning and making sense of the math (Sanchez, 2013). Additionally, Sanchez (2013) suggests that answers to open-ended questions provide teachers with rich information about their students’ understandings and misconceptions.

Henningsen and Stein (1997) suggest that scaffolding occurs when students are provided with meaningful questions. These authors contend that, when these meaningful questions are asked, teachers must not provide a solution method for their students. Henningsen and Stein (1997) argue that the goal of scaffolding is to help probe students in their development of conceptual understanding. To deepen students’ mathematical conceptual understanding, teachers must encourage student communication.
2.6.6 Student Communication

The importance of student communication in mathematics is effectively conveyed by Marks-Krpan (2013): “Communication is a critical component of learning new concepts because through articulating their understanding of a concept, students often crystallize that understanding” (p. viii). Moreover, Marks-Krpan offers, “Communication, combined with meaningful, guided inquiry, can help students reach their full mathematical potential” (p. viii). The NCTM (2000) recommends that students, “Communicate their mathematical thinking coherently and clearly to peers, teachers, and others” (p. 60). Additionally, McDougall (2004) maintains that instruction in reform classes should focus on the construction of mathematical ideas through students’ communication in the following domains: oral, written, and graphic.

2.6.7 Challenges to IBL

It is important to address the challenges that exist within the inquiry-based learning approach. Firstly, the challenge commences with identifying a commonly agreed upon description of IBL as broached earlier. Due to the spectrum of IBL approaches being so vast and the research findings so highly contextual, it is difficult to make conclusive statements about IBL (Anderson, 2002). To provide an illustration of how complex and intricate the delineation of IBL can be, Walker and Leary (2009) claim that IBL can be differentiated according to the following: 1) the type and complexity of the problems presented, 2) the degree to which learning is student-centered, and 3) the order in which the problems and information are presented.

Similarly, Engeln, Euler and Maass (2013) opine that there are varying ways of describing inquiry pedagogies: inquiry-based teaching, inquiry-based method, inquiry-based education and inquiry-based pedagogy. The authors lament, “The failure to give a concrete definition has led to misunderstanding and is one reason for discussions about the effectiveness
of IBL” (p. 824). Notwithstanding the myriad of studies touting the benefits and advantages of implementing inquiry-based approaches to teaching mathematics, there are also many active detractors whose sole aim is to undermine the adoption and spread of IBL.

Kirschner, Sweller and Clark (2006) strongly oppose IBL, and they defend their position by pointing to some fundamental characteristics of the human cognitive architecture (i.e., working memory, long-term memory, and the intricate relations between them) that become disadvantaged and compromised when discovery methods are used. To illustrate, Young, Wu and Menon (2012) argue that those who suffer from math anxiety have their working memory disrupted because their attention is devoted to intrusive thoughts and worries, rather than the mathematical tasks at hand. According to Kirschner, Sweller and Clark (2006), “Learning, in turn, is defined as a change in long-term memory” (p. 75). The implication of their definition is that given our human cognitive architecture, direct instructional guidance becomes compatible and supportive of students’ learning if they acquire information that explicitly and directly explains the necessary concepts and procedures.

Kirschner, Sweller and Clark (2006) assert, “Our long-term memory incorporates a massive knowledge base that is central to all of our cognitively based activities” (p. 77). These researchers compare long-term memory to working memory and define working memory as the cognitive structure where conscious processing takes place. Kirschner, Sweller and Clark (2006) subsequently contend, “The onus should surely be on those who support inquiry-based instruction to explain how such a procedure circumvents the well-known limits of working memory when dealing with novel information” (p. 77). They further argue, “Inquiry-based instruction requires the learner to search a problem space for problem-relevant information. All problem-based searching makes heavy demands on working memory” (p. 77).
Sweller (1988) further cautions against IBL. He notes that, despite the alleged advantages of unguided environments to help students derive meaning from learning materials, cognitive load theory suggests that the free exploration of a highly complex environment may generate a heavy working memory load that is detrimental to learning; this is particularly important in the case of novice learners, who lack proper schemas to integrate the new information with their prior knowledge.

To repudiate Kirschner, Sweller and Clark’s (2006) arguments discrediting IBL, Hmelo-Silver, Duncan and Chinn (2006) wrote an impassioned article in direct response against their disparagement of ‘minimally guided approaches’. They counter that problem-based learning are not minimally guided approaches, but rather they offer extensive scaffolding and guidance.

Clearly, this debate will be ongoing with untold justifications and defenses volleyed back and forth by each camp. It is here where we must take heed of Dewey’s words: “The fundamental issue is not of new versus old education nor of progressive against traditional education but a question of what, if anything whatever, must be worthy of the name of education” (as cited in Mooney, 2000, p. 1).

In addition to philosophical challenges associated with IBL pedagogy, there exist practical challenges that can hinder IBL practices from gaining entrance into the classroom. It first occurs at the individual level – teachers’ knowledge and beliefs have a major impact on their mathematical teaching practices (Keys & Bryan, 2001). Dorier and Garcia (2013) assert that teachers play a critical role in that they essentially are the arbiters who decide whether or not IBL practices are implemented in their mathematics lessons. Thus, teacher buy-in is crucial lest resistance ensues.
In cases where teachers favourably view IBL practices, lack of school hours is often mentioned as serving as a barrier to its implementation (Deters, 2005). Dorier and Garcia (2013), through their analysis of conditions and constraints that either favour or thwart the implementation of IBL in mathematics, echo this notion that ‘time’ operates as a significant barrier, hindering the chances that IBL methods will be employed inside the mathematics classroom. Dorier and Garcia (2013) also suggest assessments can act as a potent barrier that restrict teachers from adopting IBL methods. They make this claim due to the fact that teachers often experience pressure to cover materials, and IBL is seen to be time-consuming.

Teachers’ lack of appropriate training is cited as being a significant constraint, impeding the consistent use of IBL methods. This is attributed to teachers’ general lack of adequate understanding of mathematics (Dorier & Garcia, 2013). These researchers suggest that the remedy for this problem can arise through in-service teachers’ training and professional development. However, through various studies implemented, particularly through the PRIMAS project partners, it was reported that the in-service training was very carelessly controlled and the training consisted of short sessions that were commonly organized by volunteer teachers who lacked specific qualifications to validate them as trainers (Dorier and Garcia, 2013).

In a similar manner, Anderson (2002) cites the limited and inadequate teacher training to function as a constraining barrier to the implementation of IBL. Dorier and Garcia (2013) state that this lack of training and knowledge among teachers about IBL practices and their resistance to change, hinder opportunities to effectively introduce inquiry methods inside the mathematics classroom. Dorier and Garcia (2013) have concluded from their research that the following are necessary to overcome the challenges and barriers to implementing IBL methods: policy changes
regarding IBL, long-term professional development, and ongoing and continuous teacher support must become a reality in schools that desire change.

2.7 Mathematical Proficiency

When it comes to exploring how teachers use IBL to increase achievement in mathematics, the theoretical framework of mathematical proficiency becomes instructive. The National Research Council (2000) published a report summarizing the work of various committee members from diverse backgrounds who reviewed and synthesized relevant research on mathematics learning from pre-kindergarten through to grade 8. The committee chose the term mathematical proficiency to identify what it means for anyone to learn mathematics successfully. This framework will help me with my investigation in identifying whether or not successful student-learning of mathematics has occurred through IBL pedagogy.

Kilpatrick, Swafford and Findell (2001) define mathematical proficiency as encompassing five strands: 1) conceptual understanding - the understanding of concepts, operations and relations; 2) procedural fluency - skill in carrying out procedures flexibly, accurately, efficiently, and appropriately; 3) strategic competence- the ability to formulate, represent and solve mathematical problems; 4) adaptive reasoning- the capacity for logical thought, reflection, explanation, and justification; and 5) productive disposition - positive attitudes about mathematics, and seeing it as sensible, useful, and worthwhile, coupled with a belief in one’s own efficacy.

In discussing the underpinnings of mathematical proficiency, Kilpatrick, Swafford, and Findell (2001) assert:

These strands are not independent; they represent different aspects of a complex whole. The most important observation we make here, one stressed throughout this report, is that the five strands are interwoven and interdependent in their development of proficiency in
mathematics. The five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency. (p. 116)

Hence, the authors purport that learning is not an all-or-none phenomenon; it requires the development and synchrony of each and every strand. It is only through the provision of opportunities to develop all strands that students are more likely to essentially become competent with each (Kilpatrick, Swafford & Findell, 2001). They further argue that in order to effectively help children acquire mathematical proficiency, it necessitates instructional programs that address all its strands. IBL pedagogy can be seen to cultivate and foster opportunities for students to develop each of the 5 critical mathematics strands. These authors note that, in the first half of the century, successful mathematics learning assumed the need for facility with computational skills. However, with the new math movement, successful mathematics learning is defined primarily in terms of understanding the structures of mathematics in concert with understanding its unifying ideas, which is in direct opposition to the earlier focus on computational skills.

Kilpatrick, Swafford and Findell (2001) cogently encapsulate the import of mathematical proficiency for the life of a student, which implicitly mirrors the ‘gatekeeper’ metaphor earlier referenced:

The currency of value in the job market today is more than computational competence. It is the ability to apply knowledge to solve problems. For students to be able to compete in today’s and tomorrow’s economy, they need to be able to adapt the knowledge they are acquiring. They need to be able to learn new concepts and skills. They need to be able to apply mathematical reasoning to problems. They need to view mathematics as a useful tool that must constantly be sharpened. In short, they need to be mathematically proficient. (p. 144)

Egodawatte, McDougall and Stoilescu (2011) argue:

The current emphasis of mathematics education reform documents is on the need to change the environment of mathematics classrooms from the transmission of knowledge
by the teacher to the transaction of knowledge between the teacher and the students which promotes mathematical investigation and exploration. (p. 189)

The above viewpoints stem from the constructivist perspective of learning which counters the transmission model of learning due to its ineffectiveness in promoting deep understanding. Herein, it becomes appropriate to delve into the Ten Dimensions of Mathematics Education.

2.8 **Ten Dimensions of Mathematics Education**

The Ten Dimensions of Mathematics Education (McDougall, 2004) serves as a practical and useful framework for effective teaching, especially as it incorporates reform initiatives. It derives from a multi-year mathematics education research program, where tools were developed to support the implementation of strategies for the improvement of mathematics education in elementary schools (McDougall, 2011). The Ten Dimensions of Mathematics Education distinguishes between traditional and standards-based mathematics instructional approaches. It was discovered that both teachers and administrators were better able to discern areas for improvement when they used these tools (McDougall, 2010).

The Ten Dimensions include the following: (i) Program Scope and Planning (encouraging teachers to observe all strands, expectations, and big ideas of the mathematics curriculum; (ii) Meeting Individual Needs (teachers vary instruction to accommodate all learning needs); (iii) Learning Environment (student groupings are varied and student input considered); (iv) Student Tasks (teachers use meaningful tasks that are varied in type); (v) Constructing Knowledge (various instructional strategies and reflective questioning techniques used for the construction of student knowledge); (vi) Communicating with Parents (parental communication is ongoing); (vii) Manipulatives and Technology (teaching tools are used to enhance student learning); (viii) Students’ Mathematical Communication (opportunities for oral, written, and physical forms of communication are provided to students); (ix) Assessment (a variety of
assessment strategies, including diagnostic, formative and summative data are used); and (x) Teacher’s Attitude and Comfort with Mathematics (teachers project positive attitudes towards mathematics for their students) [McDougall, 2004].

Six of the ten dimensions, in particular, which accord with reform initiatives, are found to support learning in the inquiry-based tradition. Hence, my study will focus specifically on six of McDougall’s (2004) dimensions, which are described below.

Dimension One, Scope and Planning, is where teachers identify the expected curricular outcomes that build on key ideas and integrative learning. It is here where teachers effectively plan their mathematics program, considering the processes that best support the construction of student understanding.

In Dimension Two, Meeting Individual Needs, teachers consider the whole child and use multiple approaches that lend themselves to accommodating various learning styles and needs. Scaffolding, varying pedagogical techniques, and using open-ended tasks are examples of how teachers differentiate their instruction.

Student Tasks is Dimension Four. Teachers are expected to instruct their students by implementing rich mathematical tasks. Procedural tasks should be introduced in conjunction with tasks that stimulate deep conceptual understanding. These rich tasks should engender the following: the construction of meaning through multiple forms of representations, necessitate higher-order thinking skills, and inspire student communication.

Dimension Five is the Construction of Knowledge. Student knowledge is to be created through the building of students’ prior knowledge through the appropriate use of questioning techniques that serve to deepen and broaden students’ mathematical thinking. Active student engagement is required for the construction of knowledge.
Manipulatives and Technology is Dimension Seven. Conceptual understanding is fostered and cultivated as students actively engage with the medium of concrete and tangible tools. Through such palpable means, the gap between the concrete and the abstract becomes bridged.

Student’s Mathematical Communication is Dimension Eight. This dimension emphasizes the need for teachers to encourage and stimulate student communication in its various forms – visual, oral, and written. Students are prompted to justify and prove their mathematical understandings using appropriate mathematical language.

Dimension Ten is Teacher’s Attitude and Comfort with Mathematics. Teachers’ positive views and zeal for mathematics are seen to foster similar sentiments within their own students. Teachers who are comfortable with mathematics are able to demonstrably convey for their students the important role mathematics plays in their lives; teachers are also able to elucidate for their students important insights and connections among mathematical concepts, thus deepening their students’ understanding.

By virtue of the Ten Dimensions embodying reform-based principles and being universal in its applicability for improving student achievement in mathematics, I shall use this framework to help me understand and evaluate the effectiveness of my colleague’s implementation of IBL practices during her instruction of mathematics with her Grade 5 students.

2.9 Summary of Literature Review

Kilpatrick, Sweller and Maass (2001) contend that, in order for people to participate fully in society, it is imperative that they possess basic knowledge of mathematics. They argue that citizens who are incapable of reasoning mathematically have opportunities closed off and are
thus disadvantaged as numeracy can impact competence in everyday tasks. Hence, it behooves all educational researchers to unearth optimal methods of teaching mathematics.

Ostensibly, IBL has been found by many to be a powerful pedagogical approach to helping students acquire proficiency in mathematics. Inquiry-based learning is recognized to be a powerful teaching and learning method. Yet, it should not exist in isolation from other methods of teaching, but rather other strategies and methods should be used in tandem to support and sustain a diversity of learners (Watt & Colyer, 2014). It is well recognized that the role of the teacher is paramount. Hence, it is to this very end that I am studying my colleague inside her classroom, in the hopes of gleaning the skillful subtleties of her teaching practices with IBL methods. My investigation is well-positioned to observe my colleague incorporate the IBL approach. I hope to understand how her strategies help promote and cultivate conjecturing, discovery, solving, exploring, collaborating and communicating among her Grade 5 students.
Chapter Three: Methodology

3.1 Introduction

For my study, I have selected a qualitative approach, wherein my investigation has foregrounded the voices of my participants as I inquired into the meaning(s) individuals ascribe to a social/human situation occurring within a natural setting (Creswell, 2007). Qualitative research, as a philosophy of knowing, focuses on understanding from the viewpoint of 'who' or 'what' is being studied; it starts from the premise that reality is subjective and is contextually dependent (McMillan & Wergin, 2002). In strong support of qualitative inquiry, Merriam (1998) maintains, "I believe that research focused on discovery, insight, and understanding from the perspectives of those being studied offers the greatest promise of making significant contributions to the knowledge base and practice of education" (p. 1). The key philosophical assumption underpinning qualitative research is that reality is constructed by individuals who interact with their social world (McMillan & Wergin, 2002; Merriam, 1998).

In selecting my research strategy, I have used Yin's (2003) advisement, "The first and most important condition for differentiating among the various research strategies is to identify the type of research question being asked" (p. 9). According to Yin (2003), the case study becomes the most advantageous research strategy when, "a 'how' or 'why' question is being asked about a contemporary set of events, over which the investigator has little or no control" (p. 9). Given that my research question exclusively focuses on a "how" question for which I have 'no' jurisdiction or sway, the case study has become the most appropriate method of research. Hence, I have conducted a case study research.

My objective was to determine ‘how’ a Grade 5 teacher implements inquiry-based learning strategies/practices in her classroom to promote mathematical understanding and
proficiency among her students. This chapter describes the research approach, research design, the research setting, participants, collection of data and the methods of analyses.

3.2 Research Design

A myriad of choices exists pertaining to research strategies that are at one's disposal. Each strategy possesses its own advantages, disadvantages, and strategies. The case study research strategy is an all-encompassing and comprehensive method of research, which includes the design, data collection techniques, and specific approaches to data analysis (Stoecker, 1991).

Discrepancies exist among some leading researchers regarding the nature and definition of a ‘case study’. For example, Yin (1994) defines case study in relation to the research process. In contrast, Stake (1995) underscores the unit of study, the case, as opposed to its process. Merriam (1998) defines the case study as "a thing, a single entity, a unit around which there are boundaries" (p. 27). Merriam (1998) further asserts, "Case studies are differentiated from other types of qualitative research in that they are intensive descriptions and analyses of a single unit or bounded system" (p. 19). What matters according to Yin (2003) is to find a strategy that best fits the research under question.

Guba and Lincoln (1981) bring attention to some limitations of case studies, "Case studies can oversimplify or exaggerate a situation, leading the reader to erroneous conclusions about the actual state of affairs" (p. 377). As a result, case studies have been denigrated from among the social science methods due to the seeming insufficiency in these key areas: quantification, objectivity, and rigour. Yin (2003) acknowledges the existence of this unfortunate stereotype. Therefore, in designing and implementing case studies, Yin (2003) cautions and implores investigators to exercise meticulous care and rigour so as to circumvent the criticisms traditionally applied to this method of inquiry.
In spite of there being many detractors, case studies have become commonly used as a research tool (Perry & Kraemer, 1986). Case studies are increasingly utilized among many scholars, particularly in the following disciplines: psychology, sociology, political science, and social work (Gilgun, 1994). Yin (2003) offers his endorsement, "The distinctive need for case studies arises out of the desire to understanding complex social phenomena. In brief, the case study method allows investigators to retain the holistic and meaningful characteristics of real-life events - such as individual life cycles" (p. 2).

For my investigation, I have used the case study as a methodology as articulated by Creswell (2007); “I choose to view it as a methodology, a type of design in qualitative research, or an object of study, as well as a product of inquiry” (p. 73). As outlined by Creswell (2007), I explored a bounded system throughout the course of the school year (2015-2016) using in-depth data collection that involved multiple sources of information.

As recommended by Yin (2003), I used six types of information for my data collection: participant observations, interviews, audiovisual material, documents, direct-observation, and physical artifacts. My research was situated in my school, inside of my colleague’s grade 5 classroom, who served as my case study. Minimal disturbance to the natural flow of the day for my colleague and her students was my top priority. A consent form was given to my colleague for her to sign (Appendix A). An additional consent form was distributed to my colleagues’s class for her students’ parents to sign (Appendix B).

3.3 Setting and Participants

I studied my colleague, Marian (pseudonym), a grade 5 teacher and former math coach as well as instructional leader. My participant is an atypical/unique case, due to her distinctive roles. Thus, my study design becomes an intrinsic case study as I have ultimately focused on a
‘within-site’ case: studying the implementation of IBL by a professionally trained mathematics coach/instructional leader within her own grade 5 classroom (Stake, 1995). Stake (1995) argues that learning from the case should be at the forefront of one’s study. Stake further contends that in order to achieve such prioritization, a researcher should endeavour to maximize the variance of characteristics studied by revealing the unique complexity of the case. Through my year-long engagement with my participant, I was able to unearth these complexities.

Marian’s students also served as participants in my study. The sole purpose of having my colleague’s students serve as participants was to better understand my ‘case’ under investigation. Their participation allowed a generation of data that helped to yield deeper insights into my colleague’s pedagogical practices. As recommended by Creswell (2007), the individual(s) selected to participate in one’s research should be based on the potential that exists for gleaning a rich understanding of the central phenomenon under study. Notwithstanding the fact that my study was not intended to look at the students’ perspectives, my colleague’s students’ responses provided insights into how teachers can increase student understanding and efficacy in mathematics.

There were 31 students in my colleague’s class. The entire class was recruited as participants for my study. I collected and included relevant material and data for my research from those students for whom consent was provided (Appendix B). All students who provided consent were asked to complete a questionnaire (Appendix E). Moreover, they completed journal entries subsequent to various math lessons conducted by their teacher - my colleague (Appendix F). The students’ work samples along with their oral and written responses have helped to shed light on the efficacy of the IBL practices of my colleague.
Apprehension vis-a-vis my participants feeling obligated to engage in my study did not become a consideration. It was clearly explicated that this study was solely for the purpose of exploring instructional strategies/tasks that positively support students’ learning of mathematics. Additionally, for those participants who voluntarily chose to participate in my study, concern regarding these participants potentially feeling compelled to provide responses that conformed to my personal beliefs or assumptions was unwarranted as it was plainly articulated that I held an impartial and neutral position with respect to the results and outcome of my study. I used the case study analytic procedures and provided detailed descriptions of the case that was set within a specific context (Creswell, 2007).

Notwithstanding my collecting data from my colleague’s entire class, I focused my investigation on 15 students with whom I conducted one-to-one semi-structured interviews; this sample of participants represented a cross-section of student mathematical abilities: low, medium, and high. This becomes what Creswell (2007) calls ‘stratified purposive sampling. The selection criteria for the 15 students whom I interviewed, were based on two factors: 1) my colleague’s recommendations based on an equitable cross-section of her students’ mathematical abilities and 2) the responses derived from the questionnaire as well as the content of the individual student journal entries. The express purpose of having chosen this sample group was to provide an in-depth understanding of students’ attitudes and perceptions based on my colleague’s IBL lessons/tasks. The rationale for my sampling strategy was to provide both a broadly representative as well as an in-depth view of students’ experiences with IBL, which have become useful and meaningful data for my study.
Stringer (2004) admonishes the interviewer to establish a relationship of trust before proceeding with an interview. Accordingly, the interview phase did not commence until a ‘relationship of trust’ was established between each respective student and myself.

3.4 Data Collection

The intent of my study was to identify and describe the specific IBL strategies and practices my colleague uses to teach mathematics. Creswell (2007) recommends that, if one’s own workplace does become the site of one’s research, then strategies to validate one’s study be used to ensure that the accounts are accurate and insightful. "A major strength of case study data collection is the opportunity to use many different sources of evidence" (Yin, 2003, p. 97). Taking to heart Yin’s (2003) words, I triangulated different data sources to validate the accuracy of my findings by cross-checking three different sources: 1) Interviews from my participant (colleague), 2) Questionnaire/journal entries/interviews from my colleague’s students, and 3) Researcher journal entries/field notes. Through triangulating my data, this has permitted corroboration, elaboration and illumination of the research problem, thus supporting my study’s rigour and reliability (Stringer, 2004).

I also incorporated Lather’s (1991) re-conceptualized notion of validation: wherein she asserts that paradigmatic uncertainty calls for new techniques and concepts for acquiring and defining trustworthy data. I appreciate how Lather (1991) champions open narratives which contain holes, questions, and an admission of situatedness and partiality. This is a refreshing paradigm shift that naturally promotes forthrightness and genuine transparency, which is what I wholeheartedly sought to offer in my study. I worked conscientiously to bring meaning, order, and trustworthy insights into my mass of collected data as I looked for recurring themes, categories and patterns. The foregoing was accomplished with the goal of fundamentally
discovering significant relationships among the discrete elements of data. Taking these interconnections, I aimed to build a coherent interpretation as I distilled the possible implications for education (Efron & Ravid, 2013).

3.4.1 Participant Interview/Journal Entry

I commenced my study by conducting an initial semi-structured interview with my participant, Marian (Appendix C). Throughout the course of the year, I conducted semi-structured interviews with my participant prior to and subsequent to my observing her IBL lessons (Appendix D). The interviews took place before and after school for approximately one hour in length inside of her classroom. The interviews were audio-recorded and then transcribed.

3.4.2 Student Questionnaire/Journal Entries:

At the outset of the school year, I obtained student thoughts and reflections about mathematics through a questionnaire (Appendix E). This questionnaire helped to generate and provide thick descriptions of the context under study. Following the instruction of various mathematics lessons, along with the implementation of IBL tasks by my colleague, I asked Marian to have her students complete journal entries regarding their experiences vis-a-vis her IBL lessons (Appendix F).

3.4.3 Student Interviews

I conducted one-to-one semi-structured qualitative interviews with 15 students throughout the course of my study. The purpose of my interviews was to obtain a body of relevant information regarding students’ thoughts, attitudes, and perspectives pertaining to their experiences with the various IBL mathematical strategies and tasks implemented. The interviews took place during both morning and afternoon recesses and were usually 10-15 minutes in length (Appendix G).
The short length of time was initially a concern given that 10-15 minutes may have been viewed to preclude the opportunity to obtain sufficiently rich and detailed information; nevertheless, due to the age of the students, I believe the time period considered was age-appropriate. A semi-structured approach was chosen due to the fact that this method was guided by pre-established interview questions, which permitted flexibility when further clarification was required. The interviews were audio-taped and transcribed by me. While conducting the interviews, I used Creswell’s (2007) interview protocol.

3.4.4 Classroom Observations

In my study, I also included my own personal reflections, through field notes and journal writings, which were based on the observations of my participants (my colleague and her students). I observed 17 different IBL lessons throughout the course of the school year. My observations specifically centered on my participant while she implemented IBL strategies/tasks during her mathematics lessons; I simultaneously observed her students as they engaged with various mathematical tasks. Observation was chosen as a means of data collection as this method permitted me to see what the participants did without having to depend on their words alone (Johnson & Turner, 2003). Classroom observations occurred during my preparation times, which were 40 minutes in length. I coordinated with my participant appropriate times to observe her teach her mathematics lessons. I used Creswell’s (2007) observational protocol to record the information I collected.

As my participant, Marian, was my “case” under study, I video-recorded her teaching. I did not video-record her students. Instead, I used student work samples as my physical artifacts/documents for analysis. Actual texts from the questionnaire, journal entries, and
interviews were used with an attempt to reflect the expressions of the participants; specific references to the participants’ identities were replaced through the use of pseudonyms.

3.5 Data Analysis

I used the ‘embedded analysis’ approach to analyze and interpret specific aspects of my case using detailed descriptions of my data, namely, my participant’s implementation of IBL in her classroom (Yin, 2003). I focused on a few key issues, not with the intent of generalizing the data beyond my case, but instead with the purpose of understanding the complexity of the case, and identifying common themes (Creswell, 2007). I primarily used Stake’s (1995) four forms of data analysis and interpretation in case study research. ‘Categorical aggregation’ was employed as a way to seek a collection of instances from my data, which allowed issue-relevant meanings to emerge.

Additionally, I sought out patterns and looked for a correspondence between two or more categories. As described by Creswell (2007), I aggregated my data into “meaning” categories and then collapsed them into a more manageable number of themes. When analyzing the data, I used emerging categories after I had coded and identified the elemental pieces (Efron, & Ravid, 2013). As advised by Bogdan and Biklen (2006), I revisited and refined my emerging categories by ensuring there was congruency between the data and the categories; this helped to inform the synthesis and interpretation of my data.

In conjunction with the foregoing, I also utilized Denzin’s (1989) epiphanic moments, illuminative and significant experiences, as my units of analysis. I analyzed and interpreted the ‘epiphanies’ by following Stringer’s (2004) steps toward interpretive data analysis. The key question that was continually asked was how this epiphany/event helped to illuminate or extend my understanding of the issue under investigation, i.e., types of IBL instructional strategies/tasks.
that influence students’ ability to learn mathematics (Stringer, 2004). Based on the emerging units of meaning, I unitized the data so that I was able to meaningfully categorize and code the information (Stringer, 2004).

All interview recordings were transcribed and the transcripts were reviewed against the recordings to ensure accuracy. The findings from this study have unfolded through identifying the ‘lessons learned’ from my case (Lincoln & Guba, 1985). I utilized Stake’s (1995) general reporting structure by providing entry and closing vignettes and thick and detailed descriptions and themes for my case. In the final analysis, I looked at the overall picture without getting mired in the details so that I was able to discover the significance of the story related to my research questions (Bogdan & Biklen, 2006).

3.6 Validity of Data Analysis

To ensure the trustworthiness of my findings, I searched my data for discrepancies and counterevidence that may have invalidated my initial assertions or provided alternative interpretations (Gibson & Brown, 2009). As a means of further increasing trustworthiness to my interpretations, I practiced reflexivity, and I openly discussed how my personal experiences, biases, and subjective judgements may have coloured and shaped my interpretations (Glesne, 2010).

Debriefing had become an integral part of my study. This was an important process as the credibility of my research was enhanced when my participant was afforded the opportunity to debrief (Stringer, 2004). Member checking was also embedded into my study; my participant was provided opportunities to question and review the information I have gathered as well as the outcomes of my study, to further validate my results (Stringer, 2004).
3.7 Ethical Considerations

My proposed study has been reviewed by the Office of Research Ethics of the University of Toronto as well as the Toronto District School Board ethics committee. Pseudonyms for the teacher and all student participants have been used in conformity to the confidentiality protocol. Specific details about the location of the school have been omitted. Participants first provided their verbal consent to be a part of this study, and they also signed a formal consent letter (Appendices A & B) to confirm their participation. The participants were recruited to join the study only when they met the selection criteria as described earlier.
Chapter Four: Findings

4.1 Introduction

This chapter describes the teaching beliefs, practices, and perspectives of Marian, a Grade 5 teacher, who was a participant serving as my case study. I first describe the school context of my participant to provide an appreciation of her teaching environment. Subsequent to this, Marian’s educational background and her experiences with being a math coach and an instructional leader are detailed. Additionally, Marian’s beliefs and practices vis-a-vis inquiry-based learning in mathematics are closely examined.

In order to provide a rich and comprehensive understanding of how Marian successfully implements Inquiry-Based Learning (IBL) in her mathematics classroom, the findings will be narrated and conveyed by means of three component parts: 1) section one will provide a detailed overview of Marian’s experiences, including her beliefs and practices pertaining to inquiry-based teaching in mathematics; 2) section two will depict six lesson vignettes; interwoven within these vignettes will be interviews with Marian and her Grade 5 students, and 3) section three will reveal my personal reflections that stem from my interviews, observations, and my efforts to implement IBL within my own classroom.

A clear understanding of Marian’s practices will surface through my recounting of six chronicled lesson vignettes. The distilled themes emerging from each lesson vignette will be framed around the Ten Dimensions of Mathematics Education as well as various elements of the Five Strands of Mathematical Proficiency. Each lesson vignette will give voice to Marian’s beliefs and practices surrounding the teaching and learning of mathematics that occur through her implementation of IBL.
4.2 School Context

Marian teaches at Mapleview Public School, a Junior kindergarten to Grade 5 school located within the Greater Toronto Area. There are approximately 700 students enrolled, representing 42 countries and over 25 languages. The population is culturally diverse, with the majority of the students coming from a South Asian background. More than 80% of the students speak a language other than English at home, and many of the families are unfamiliar with the Canadian school system. Many of Mapleview’s students enter kindergarten with very limited English, despite the fact that 80% of the students were born in Canada.

4.3 Case of Marian

4.3.1 Marian’s Background

Marian attended York University in Toronto, Ontario, where she specialized in Psychology and minored in English. She received her Bachelor of Education degree at the University of Toronto. Marian is a veteran teacher who has spent the majority of her career teaching in the primary grades. Currently she is teaching Grade 5 for her third year. Her teaching responsibilities include all of the Grade 5 subjects, with the exception of music and French.

Marian is a very intelligent and well-spoken teacher who is regarded as Mapleview’s ‘math expert’. Marian expressed that early in her career, she felt more confident and self-assured as an English teacher, and lacked confidence when teaching mathematics. Marian candidly shares that she feels comfortable teaching mathematics today, but admits that math did not come naturally to her. She relates in an interview about the early stages of her paradigm shift regarding her understanding of math pedagogy:

I could not understand why my students in math were having such difficulty so I began to do research. I had an inkling that it was the way I was teaching it. Rote was not flying anymore, and the students were not able or willing to memorize things. There were so many things that they did not have to memorize as Google was doing everything for
them. I was already suspecting that something was missing and that there was more to
math than just the rote. I knew there had to be some understanding and reasoning.
(Marian Interview, November 13, 2015)

This burgeoning curiosity, early in her career, would eventually lead Marian to become
proficient in mathematics today.

4.3.2 The Beginnings of Marian’s Journey to Mathematical Proficiency

Despite admitting to her lack of confidence with math prior to becoming a mathematics
coach, it bemuses Marian when she must concede to the fact that, today, she is regularly called
upon to provide mathematical support and guidance to many of her colleagues. When asked how
she felt about math prior to becoming a coach, Marian’s response was, “My answer is - not very
confident. I had done statistics courses in university, but the math we were doing was plugging
numbers into computers. The math I knew was traditional, memorized and rote” (Marian
Interview, November 13, 2015). When asked how a reversal of circumstances occurred with her
now becoming a math ‘expert’, Marian shares with me her personal journey from mathematics
novice to expert:

Seven years ago, Michelle (colleague) and I were “voluntold” by one of our vice-
principal to attend a series of Collaborative Inquiry and Learning (CIL) workshops. We both felt uncomfortable attending these workshops. On the first day we arrived,
we both believed that we were not the right people to be there. (Marian Interview, November 13, 2015)

Marian explains how both she and Michelle nervously disclosed to each other a deep fear that
enveloped them:

We both hoped we would not do anything harder than Grade 3 math, because we
knew we could not do it. We are only primary teachers and we are not comfortable
doing anything beyond Grade 3 math. This was the feeling among many of the
people in the group. I had very little confidence. (Marian Interview, November 13,
2015)
Marian admits to feeling like a student herself when she took part in ongoing workshops/courses where she was to re-learn math:

At the beginning, I was like my own students. I was telling the instructor not to ask me questions, and to just tell me what to do. A lot of us (teachers) were saying the same thing: just tell us what to do. The instructor said to us that it is obvious that we were the product of traditional education. (Marian Interview, November 5, 2015)

Marian continues, “We were like, what? What? We were very uncomfortable having to figure out our own way out. Then I had to think about how our students want us to tell them what to do because that is what they are used to” (Marian Interview, November 13, 2015). While sitting in the workshops, Marian shared how this is where she began to apprehend the need to rethink the way she was teaching mathematics.

4.3.3 Marian’s Own Misconceptions about Mathematics Prior to becoming a Coach

When asked to share her thoughts about math prior to becoming a coach, Marian gives a telling description:

Literacy came easily for me. However, I thought that math was this rigid, structured thing that could not be moved and that it was inflexible. That was my understanding, that math was a square box and that was it. (Marian Interview, November 13, 2015)

Marian further elaborated on her earlier misunderstandings and experiences with mathematics. She explained how she always utilized traditional algorithms, as she was incognizant of the fact that different strategies even existed. Marian then recounted her early experiences from when she was a student herself, which contributed to the cementing of her long-standing misconceptions of math:

Thinking back, when I did do it a different way in school, the teachers frowned upon it and said, ‘Oh, you are not following the rules, steps and procedures’. I thought that if I did something more quickly and more efficiently, then I was cheating. That is how I came away from elementary school math - thinking that I was cheating if I did not follow the procedures. (Marian Interview, November 13, 2015)
In Grade 12, Marian had a math teacher whose approach was perceived as being uncharacteristic - he expected his students to reason, justify and prove their mathematical work, which was a style and practice in which Marian was not accustomed. In this course, Marian confesses that she failed miserably as she could not understand nor see the point in having to learn math that way. Marian shares her inward frustration towards this math teacher:

I was so angry, inside I screamed, “Are you kidding! Just tell me what to do! I do not want to have to think about it. This is math, I do not have to think. I just want the numbers and formulas to use!” That was my thing about math as an adult. Just tell me what to do and I can do it. (Marian Interview, November 13, 2015)

Marian describes how she discovered, through her years of working as a coach, that many students possessed the same misconceptions she once held. To clarify this point, Marian provides an illustration of a boy named Peter, in whose class she worked while coaching. Marian had observed Peter for two weeks and she found it unusual and bewildering that Peter would respond to a simple math question by making it intentionally long and complicated. Befuddled, Marian decided to ask Peter if he knew another way to solve the question. Compliantly, Peter turned around and solved the question very quickly. Experiencing even greater confusion by this, Marian inquired of Peter why he had initially chosen to answer the question the long way. Marian states that she was not surprised by Peter’s response, “Oh, because this is math class and everything has to be long and complicated” (Marian Interview, November 13, 2015). Marian then sums up for me how she formerly held the same viewpoint as this student: “In math class, one is not supposed to understand, but rather, one is just to do” (Marian Interview, November 13, 2015).

4.3.4 Marian’s Role as Mathematics Coach & Instructional Leader

Marian’s role as a mathematics coach was to provide support to teachers who were neither comfortable teaching the new curriculum nor aware of its essential makeup. According to
Marian, “This is because the curriculum looks very different from when we went to school; it is a whole new way of thinking with the reasoning, justifying, proving and the strategies” (Marian Interview, November 13, 2015).

Marian shares that her role as a math coach was to provide a substantial amount of in-class support. As a coach, she always ensured that she embedded ample opportunities for co-teaching. Marian clarifies that she never did any ‘modelling’ for the teachers as she believes that, if you show teachers how to do something, the chances are high that they will not take anything away from the demonstration. Hence, Marian always guaranteed that time was spent co-planning together with the teacher, which included researching strategies and techniques for the upcoming ‘co-teaching’ lessons.

Marian provides a description that helps distinguish between the role of a math coach and that of an instructional leader:

As a coach, I was in the class working with kids. As an IL (Instructional Leader), I was more hands off because I had 125 schools to work with. The philosophy behind IL and coaching was very different. IL is the old, everyone gathers, I am still the expert and I am going to show you what to do. So, we did a lot of workshops and in-services and the teachers went back to their schools and there was no follow up; this model does not work. (Marian Interview, November 13, 2015)

Marian explains the most significant difference in outcomes between being a mathematics coach and an instructional leader:

As an instructional leader, because there is no follow-up, you cannot be sure what the message was and if they (teachers) are implementing it. As a coach, there is follow-up. You can have the workshops and in-services and then I can go into the classrooms and say, “Ok, let us try what was suggested at the workshop”. I can then support the teacher, which is the same way I learned. (Marian Interview, November 13, 2015)

Marian underscores the importance for teachers to work collaboratively with the coaches. She talks about a teacher whom she was coaching, who today has become well-equipped to
effectively coach others. Marian explains how this teacher took the initiative to work with Marian on a weekly basis for a three-year period. Quickly changing her tone, Marian then explains how intensive coaching is required for a teacher to become well-versed in implementing inquiry-based methods in mathematics.

4.4 Marian’s Beliefs about Teaching Mathematics

Marian believes that, when teaching mathematics, there are important factors that are critical for student-learning: 1) lesson planning, 2) questioning techniques, 3) constructing knowledge and meaning, and 4) student communication. Marian is insistent that both teachers and students have decisive roles to play in the acquisition of mathematical learning.

4.4.1 Lesson Planning

As planning always precedes teaching, I asked Marian what her practice was in preparing her inquiry-based mathematics lessons. Marian informs me that she commences with the curriculum guides. Marian explains that the curriculum is all inquiry-based, hence, she uses the curriculum expectations as her end goal, and navigates her trajectory by looking backwards. Marian explains:

How I am going to get to my goal is based on where the students are starting from. I kind of use backwards design. I create scenarios in order to elicit the understanding and based on that I go on to the next thing; it is not an easy, straightforward path. (June 13, 2016)

Marian elaborates further:

When I plan for the year, it is very skeletal at first. I spend the majority of my time on where their needs are. It is great to have a beautiful long range plan, but realistically, I am not going to follow it because it may not be right for the kids in front of me. (June 13, 2016)

4.4.2 Questioning Techniques

A key and central component of Marian’s IBL teaching pedagogy is asking meaningful questions to her students. Marian’s questions are posed with the purpose of drawing out and
eliciting from her students their ‘invisible’ understanding and learning. Marian maintains, when it comes to questioning, her philosophy is to ask herself where she wants to end up with her students. She relies on her students to decide how she will reach her end goal. Marian insists that students must construct and build their learning from one another as it will not occur through her explicitly telling them. Marian’s practice is to pose questions and see what her students already know. This allows her to know which path to take next.

When I asked Marian to explain how she uses her questioning technique, she responds:

My questions are so open that anyone in the class can answer them. It is not a question that anyone can do in a few seconds. It is a question that requires them to think and reason. If you could do multiplication the traditional way, you can do the question. If you can do repeated addition, you can do the question. Everyone can do it. (Marian Interview, November 13, 2015)

Marian continues to expound on her use of questions:

I pull questions from everywhere. It has to be open enough that everyone feels comfortable and confident that they can solve it. I may have students who skip count or divide to get at an answer. They can all get to the answer, but some ways are more efficient and sophisticated than others. Everyone can enter the question. (Marian Interview, November 13, 2015)

Marian then adds that when responding to questions, she hopes her students will use ‘efficient’ strategies. She clarifies, “When I say efficient, it is not about getting the answer in the fastest way, but answering the question so it leads to more accuracy” (Marian Interview, November 13, 2015).

When I asked Marian to explain her questioning method when a student does not provide the correct answer, she responds:

You do not just say, “No”. Instead, you have to say, “What do you think will happen if we do this?” Or, you can ask, “What were you thinking?” You almost have to anticipate what the stumbling blocks will be so you know what questions to ask. (Marian Interview, November 13, 2015)

Marian then shares something that shows me she reflects on the questions she poses:
Sometimes, if I am not getting the answers that I am expecting, I have to stop myself and ask if I need to change my question. Maybe the question is not right, and I have to stop and ask intentional questions and deliberate questions to elicit what I need. (Marian Interview, November 13, 2015)

Marian concludes by reminding me, “It is always about asking questions. If there is one way, can there be a better way? The more questions you ask, the deeper you delve, and the more you understand” (November 13, 2015).

### 4.4.3 Constructing Knowledge and Meaning

When implementing tasks for her students to solve, Marian underscores that she always provides choice for her students regarding their use of strategies. Marian’s rationale for furnishing her students with choice relates to a key element of Marian’s pedagogy: the importance she accords to the construction of meaning and understanding when solving problems. Marian laments the fact that many students use the traditional algorithm because that is what they are familiar with, however, when they use it, Marian contends, “Students learn to do it in their heads, but you know they do not know the meaning behind it because they do not carry” (Marian Interview, November 13, 2015). Marian then proceeds to explain how, in her practice, she always provides opportunities for her students to “understand” the meaning behind the steps they are using.

Marian believes that ‘practice’ and ‘repetition’ are vehicles that assist in the development of understanding. While discussing the construction of knowledge, Marian informs me:

It is supposed to be gradual and repetitious. The more I keep doing something, the more they are practicing, the more it is transferring over to new situations. Transferring is the application. I cannot say they have understanding unless they can transfer it; they need to be able to apply it on their own independently.” (November 26, 2015)

Marian chuckled as she reflected on how all of the teachers she had worked with would experience shock in witnessing their own students produce sophisticated ways of responding to
math problems. Marian smiled as she shared, “That is all the talk would be about during the
debrief - how the thinking really did come from the kids themselves. It is funny because before
the lesson, they (teachers) were adamant that their kids would not get it” (November 26, 2015).

4.4.4 Student Communication

Encouraging student communication is a significant component of Marian’s beliefs and
practices. Marian addresses the importance of peer communication in learning mathematics,
“They have to discuss with one another and talk before they can say, "Oh, we can do this”
(Marian Interview, November 13, 2015). Marian then adds, “If there is a different perspective, I
get them to convince the others. It is all about convincing their community” (Marian Interview,
November 13, 2015).

4.5 Marian’s Definition and Beliefs about Inquiry-Based Learning

When Marian is asked about her definition of IBL, she prefices her depiction of IBL by
asserting, “It is not a free for all!” She stresses, “It is not where children learn by themselves and
choose what happens. Ideally, that would be fantastic, but we have a curriculum to follow”
(Marian Interview, November 13, 2015).

Marian encapsulates her definition of IBL by way of illustrating her own practice:

It is inquiry-based learning, but the teacher sets the parameters. I will pose a lot of
questions and have the students figure out the answers. It is not like I know the answer,
but what I am more interested in are the processes by which they get there. What
strategies do they use? I know that by looking at the curriculum there is a lot that I need
to cover. (Marian Interview, November 13, 2015)

With emphasis stressed in her tone, Marian then adds:

True inquiry-based learning comes from the students. It has to be about something
they want to learn, but it is difficult to do with the curriculum. You as a teacher have
to intentionally and strategically set up circumstances and activities that align with the
curriculum objectives. (Marian Interview, November 13, 2015)
Marian underscores, “The learning has to come through without you telling them” (Marian Interview, November 13, 2015).

Marian then relates to me how things operate in the ‘practical’ world of teaching. She explains how she views and approaches IBL with her students:

Many will say that it is student driven. But it is a balance of both: it is inquiry-based, with the teacher guiding. I can pose a question, and the questions do not always have to be from the students. I can pose a question and students can ask questions and we can choose which path we are going to go on. Like, for science, I will set up an activity and see where it will lead us. But I know that I have a curriculum and I must meet the overall expectations. (Marian Interview, November 13, 2015)

Throughout my interview with her, Marian repeatedly mentions the fact that, “We have to go with the curriculum” (Marian Interview, November 13, 2015).

When I probed Marian about how she plans and prepares her IBL lessons, she expounds:

Looking at the curriculum, there is a lot I need to cover. I start by posing a question and by seeing what the students already know and then ask where do I have to go from there. I am going to teach through a series of questions. I know it sounds crazy, but it is based on where the children lead me to. So if the majority of the class can multiply, then where I start will be different from if the majority of the class could not multiply. It is all where they are starting from. (Marian Interview, November 13, 2015)

Marian explains how she teaches her lessons through ‘big ideas’. She elaborates, “It is not about bits and pieces, but it is all about the big ideas” (Marian Interview, November 13, 2015). Marian confesses, “Unfortunately, I do not understand what all the big ideas are; it takes time. So, I rely on Marian Small (an expert and prolific writer about Mathematics)” (Marian Interview, November 13, 2015).

Marian then addresses how IBL takes shape, and how mathematical proficiency occurs:

I talk about efficiency with students. When I say efficient, it is not about getting the answer the fastest way, but answering it so that it will lead to more accuracy. Everything is about building up, building up their knowledge and skills, with a lot of practice. At the end, we always have a congress (consolidation) so we can look at the work and discuss it. This gives an opportunity for students who are counting by ones to say, "Oh, maybe next time I will try counting by 10s". For those counting by 10s, maybe they will
multiply, and you move them up. It is like a continuum. (Marian Interview, November 13, 2015)

Marian proceeds to talk about integration, and how providing students with experiences in different skill areas helps to deepen their understanding:

When we did the traditional algorithm, a student noticed that it related to partial products and asked, “Is that a short-cut?” They never knew what they were doing when they did the traditional algorithm. Now they have a deeper understanding. So when I get to measurement, I have already done area. So from here, I leap right into volume. I am always integrating. (Marian Interview, November 13, 2015)

4.5.1 Challenges to Teaching Mathematics Through IBL Methods

Marian stresses and insists on the acute need for teachers to have sufficient background mathematical knowledge. She underscored this point by asking rhetorically, “How am I going to elicit ideas from the kids if I do not know what I am looking for?” (Marian Interview, November 13, 2015). I asked Marian how long it would take a teacher to advance their mathematical understanding. Marian apologetically tells me that it will take a very long time, and that she discovered how the more she learned, the more she discovered how little she really knew.

Marian shares a story about a colleague (Rosa) at Mapleview who approached her about wanting to learn how to teach mathematics using IBL methods. Rosa found time to observe Marian teach in her own classroom using IBL. After observing Marian teach a few lessons, Rosa informed Marian how she will now go back to her own classroom and teach all of the strategies she had learned. Marian expressed how horrified she felt. Marian tried to explain to Rosa that it is not about “teaching strategies” to students, but instead it is about, “Posing a problem and anticipating the various strategies and letting them go” (Marian Interview, November 13, 2015).

Marian conveys to me her frustration with the following interchange she had with Rosa:

**Marian:** You must let the kids come up with the strategies on their own.

**Rosa:** But the kids are not going to get it.
Marian: Yes, they are.

Rosa: What if none of them get it?

Marian: There is going to be at least one child who will, and you need to focus on the work of that one child (during consolidation time). (Marian Interview, November 13, 2015)

Marian explains how she tries to convince Rosa to recognize that her students will come up with the strategies on their own and that she needs to trust the students. Marian says to me, “Teachers need to understand they must set up the scenario where students do not have a choice but to think in a certain way. You are intentionally setting it up - picking the numbers that will force them to see it” (Marian Interview, November 13, 2015).

Marian shares how some principals try to force teachers to teach using this approach, to which Marian protests, “You cannot force it. Teachers will do it if they want to” (Marian Interview, November 13, 2015). Marian then discusses how those teachers who have worked for almost half a year with her, to a large degree, were able to teach mathematics using inquiry-based methods effectively. I asked Marian how she taught these teachers. Marian explains:

I started with what they wanted to do. You cannot force anything on anyone. I ask about the big ideas and we plan and co-teach the lesson(s). I am there as a guide. I give questions that the teacher could pose and get them to see what they may not have seen. After doing this for awhile, the teachers no longer needed me. (November 15, 2015)

When I asked Marian what she believes are the greatest obstacles and hindrances for teachers to become successful and effective with implementing IBL in mathematics, Marian begins by stating that she can only speak for herself, and confesses, “The biggest obstacle for me was I did not really understand the math thoroughly and deeply. I understood it from the rote perspective” (Marian Interview, November 13, 2015). Marian discloses how, with the new curriculum, she was having a difficult time explaining and extracting the depth of knowledge from her students as she did not know for herself what she was looking for. Marian reveals:
It is like you pose a question and say to the kids to answer it any way you want, but as a teacher, I did not know what I was looking for. All I was looking for was the traditional algorithm because that made sense to me. Meanwhile, you have students who are on the verge of a different way of thinking, and I could not pull them out or help them because I was unfamiliar with them. (Marian Interview, November 13, 2015)

Without my prodding or probing, Marian rephrases the same idea she just conveyed, perhaps to underscore her points:

I think for me and a lot of teachers, we have to get a really good sense of the math. We have to teach through problem-based learning, where you allow the children to struggle and figure out their own way, but the only way for you to help students, is you need to be aware of all the different ways so that you can ask the right questions. (Marian Interview, November 13, 2015)

I asked Marian whether she feels confident teaching mathematics through IBL methods today. Marian replies, “Everyday is a new learning experience for me. I let my students know that I do not know everything, and that I do not have all the answers. I think that content knowledge is the hardest” (Marian Interview, November 13, 2015). Marian then laments the fact that time is always an issue when implementing IBL in mathematics as it should occur. She groused, “We always run out of time” (November 15, 2016).

When I directed Marian to address challenges that she experiences herself in teaching mathematics through inquiry-based methods, she responds, “Finding questions”. Marian expresses that the greatest challenge is creating the “right” problem/question. She itemizes an exhaustive set of criteria required for the creation of meaningful, inquiry-based questions: 1) questions/problems need to be deep enough, (often they are shallow), 2) questions must be big enough to ensure they can be differentiated for all learners, 3) questions need to allow all students to have an access point into them, 4) questions need to be in line with the curriculum, and 5) questions/problems need to be what students will be interested/engaged in solving. Marian
concludes by sharing that she usually feels like she is “just flying through it all” as there is so much to cover.

4.6 How Marian Allows her Students to become Mathematically Proficient

Marian responds to the question of how she develops mathematical proficiency in her students with a simple phrase: all through practice. Marian emphasizes this point by stretching her words, “It really is through practice”. She adds, “Even my students know that once they have achieved understanding, now comes the practice. They are going to practice until it becomes automatic. What I am striving for is automaticity” (June 13, 2015). Marian defends her practice by arguing, “It almost looks like rote memorization, but it is not. It is a proficiency that comes through understanding: understanding at the conceptual level, and at the foundational level” (June 13, 2015).

Marian tells me that people walking into her classroom would assume that her students are proficient using decimals, but she would attribute their proficiency to the abundance of practice she affords them across many different contexts. Marian complains about the curriculum; she expresses that she is grief-stricken about how its expansiveness does not provide adequate time for her students to practice various, essential skills. Marian mentions that she always ensures she allotts time for her students to obtain the practice they need. Moreover, she throws in the fact that she does not provide practice until her students have achieved understanding. Intrigued, I ask Marian what this process of understanding looks like. She replies, “We learn it through open-ended activities, through problem-solving” (June 13, 2016).

When I ask Marian to explain her key IBL practices that have enabled her students to become successful in mathematics, she responds, “Math must make sense for the students”. Marian expounds on this idea:
You have to keep going over it until it makes sense. You cannot pretend that it is ok if it does not make sense. That is what the kids have learned. If something does not make sense, then just follow the procedure and it doesn’t matter. That is why we should work with partners. There are lots of ways to solve problems. (June 23, 2016)

4.7 How Marian Feels about Being a Grade 5 Math Teacher Today

Marian feels much more confident today as a math teacher than she had felt during her early years of teaching. She expresses how she still does not feel 100% about teaching mathematics. Marian divulges that there is still so much she does not know about the curriculum, especially the Grade 5 curriculum. Marian admits to frequently perusing the curriculum guides to obtain the “big ideas”, which she hopes her students will walk away with at the end of her lessons.

Marian confesses, “Some things come easily and there are some things where I know I still have weaknesses, like in the spatial geometry unit”. She then offers, “I am going straight to the books and going step by step because I am not as confident in it. The more confident I am, the more I can let go of the textbook because I have my own developmental continuum in my head” (Marian Interview, November 13, 2015). Being reminded of a truth, Marian declares:

Now that I think about it, when I worked with teachers, that is how they all started. Teachers really hold on tightly to the textbook in the beginning because they say they really need something. Once they develop confidence, they let go of the books, but not completely. (Marian Interview, November 13, 2015)

Marian ponders a little, she then talks about her own current experiences:

I still have my textbooks. There is this letting go, this release. I remind myself that I am going to take my cues from the students. I know the big ideas. I know where I want to go, but every year I am going to take another path as it is dependent on the students that are in front of me. (Marian Interview, November 13, 2015)

4.8 Vignettes

As addressed earlier, lesson vignettes will be used to help elucidate and bring to life Marian’s IBL teaching beliefs and practices in mathematics. The Ten Dimensions of
Mathematics Education will be used as a framework for identifying and illuminating the essential components of my participant's teaching strategies and techniques. In addition, the Five Strands of Mathematical Proficiency will be used as a secondary framework for understanding how Marian increases her students’ mathematical proficiency through the use of inquiry-based methods.

4.9 Vignette One

*November 5, 2015*

Marian follows the Three Part Lesson model/structure for all of her math lessons, which will become evident in many of the forthcoming vignettes. A Three Part Lesson is comprised of three constituent parts, as revealed by its name (Ontario College of Teachers, 2010). Part One (Minds On) is the preliminary part of the lesson where students are cognitively primed to solve a math problem by thinking about the strategies they have used in the past. Part Two (Action) is the stage where students are actively engaged in solving the problem at hand, whether it be in small groups, pairs, or independently. Part Three (Consolidation) is when students gather together as a whole group and share their strategies vis-a-vis the process(es) by which they solved the problem.

For the very first day of observation, I quickly and quietly slipped into Marian's class during my second period preparation time at 9:20 am to catch the first part of her math lesson for the morning. Marian gave me a subtle smile to acknowledge my presence and then, without skipping a beat, she continued talking to the students who were all sitting at the carpet seemingly engaged with her lesson. Earlier, Marian and I had a discussion about my upcoming year-long, weekly visits to her classroom. I underscored for her my desire that she ignore my physical presence during all observation periods, and that she expressly communicate the same message
to her Grade 5 students. This was strongly conveyed as I wanted to minimize any potential
disruption or disturbance to the flow of her math lessons. I had the opportunity to reinforce this
message to her students when I distributed the consent forms for their parents to sign.

Upon my arrival into Marian's class for my initial data collection session, some of the
students with whom I developed a genial relationship through various clubs, gave me discreet
waves and inconspicuous smiles to welcome me. Other than that, the majority of the students
were focused on Marian, who began writing numbers on the blackboard. Admittedly, I was
confounded by the seeming nonchalance and indifference with which the students acknowledged
me, given that I was accustomed to being greeted with vocal enthusiasm in the past. Strangely,
this sense of being respectfully ignored heightened my sense of professional duty and
responsibility as an observer; their disregard of my presence gave me an odd thrill as I somehow
felt I was being officially christened a bonafide ‘researcher’.

Unfortunately, I missed Marian’s introduction to her math lesson. Looking at the board, it
was apparent that her students were learning about multiplying digits using arrays. Subsequent to
my arrival, the students were sent off to work in pairs and instructed to answer the questions on
the board. Marian’s students hastened to the computer cart to retrieve an ipad. The students all
knew who their math partners were and each sat down, ostensibly eager to tackle the list of
questions on the board. The pairs each knowingly took turns using the ipads without incident.
Interestingly, the students seemed to want to have their own time solving the math questions
without having their peers provide feedback or support.

Marian permitted her students choice when selecting numbers for their open arrays.
Marian circulated around the classroom, visiting each pair of students while taking vigorous
notes on her clipboard. When I later inquired about her use of the clipboard, she shared that it
was her daily practice and method of collecting formative assessments. While circulating, Marian was heard asking such questions as, “How did you choose those numbers?” “What are you doing here?” (Marian Observation, November 5, 2015).

While employing the ipads, the students appeared deeply engaged as they actively practiced using a tool in mathematics called an open array to solve multiplication questions. An open array is a tool students use to connect partial products together to arrive at an answer for a multiplication question. After some practice with their pairs in answering questions from the board, Marian drew the students’ attention to herself by gently calling out, ‘eyes on me’. Having garnered their attention (although it took some time as the students were preoccupied with their ipads), Marian then proceeded to inform her students that she would be writing new questions on the board, which the students were expected to answer using the open array. Each pair of students had to take turns responding.

Marian emphasized that all students would have to place the ipad over their heads to demonstrate completion. Without calling it a competition, the students appeared motivated, with each pair striving to become the fastest in displaying their answers for their teacher to see. A fair level of clamour and commotion ensued. Marian’s students appeared to be vying to showcase their answers to Marian. In spite of the seeming pandemonium, Marian did not flinch; she continued with her measured pace and methodically looked over each response, giving a nod to signal affirmation. Whether they were first or last, Marian acknowledged each pair’s response. It was apparent that all of her students seemed eager for Marian to pose the next question on the board. In an interview that followed the lesson, Marian explained her purpose and rationale for using this activity: for the sake of practice.
Thirty minutes of math class quickly came to an end upon the sounding of the recess bell, which was met with audible groans. Consolidation time took place after recess. As I needed to return to my own classroom, I was unable to continue with my observation.

4.9.1 Program Scope and Planning

In planning for today’s lesson on open arrays, Marian said she had to refresh her knowledge by referring to Cathy Fosnot’s book about big ideas. This enabled Marian to remember that she needs to center her lessons around the theme of grids. Marian wanted to reinforce the concept of area through the use of open arrays. Marian explains that the importance of using the open array is for students to be given an opportunity to use multiples of 2, 5, and 10. These friendly numbers enable students to become more skilled and efficient in solving multiplication questions. Marian asserts, “It allows for efficient mental math to occur” (November 5, 2015). When asked how she knows all of this, Marian explains that she is always referring to teacher resources. She admits that even her students are aware that she will need to refer to a book when they ask her questions for which she has little or no familiarity.

Marian discussed with me how she believes in having students practice mathematical strategies over and over again. She apprised me that the open array is a tool she has been using with the students for the past 3 weeks. Marian is willing to belabour this mathematical skill as she wants her students to become proficient using this tool when multiplying numbers. Hence, Marian was very intentional with providing the friendly competition as she was confident that this activity would lead to student-engagement. Through this innocuous competition, Marian hoped to provide her students with ongoing practice using the open array. Marian further elaborates:

We are going to repeat it and practice it so that it becomes automatic. It is going to look like repetition. I am teaching for depth instead of breadth. I am thinking that if they
understand something, they are not going to forget it, and they are going to make it their own. I am going to provide the opportunities, exercises, and activities. (Marian Interview, November 5, 2015)

For this first lesson I had observed, Marian admitted that the practice phase or Part Two extended longer than her typical method of running her math class. Marian customarily spends the bulk of her math time on ‘consolidation’.

4.9.2 Meeting Individual Needs

The entire class appeared enthused while using the ipads, but they expressed greater excitement when they participated in the whole group activity. Throughout this math class, Marian always provided her students with a parallel task, which is an identical task in concept and form from the original task, but one that is either simpler or more difficult in terms of its complexity. Parallel tasks lend themselves naturally to differentiation for both her remedial and advanced students. This type of task is strongly advocated by a leading Canadian math educator and researcher, Marian Small, who my participant, Marian, frequently refers to when planning her lessons. Marian proudly states, “All my kids are using the same strategy, but it is differentiated so my HSP kids can use the same language, but with different numbers” (Marian Interview, November 5, 2015). HSP is an acronym for ‘Home School Program’. Students who function two or more grade levels below in the core subject areas (mathematics, reading and writing) are placed in this program. HSP students receive intensive academic support from a Special Education teacher in a small group context for half of the school day.

For this particular lesson, Marian provided smaller digits for her HSP students to practice with while using the array. For example, the HSP students were given the numbers 2 x 8, while their peers were given 24 x 14 or 235 x 126. Marian was explicit in stating that the smaller numbers were for Ralph, Kerry, and Sammie. She called out the names of those students
requiring the simpler numbers and explicitly directed them to tackle the second question. I was surprised by this action and would ask Marian about it at the end of the class.

During recess, when I broached this subject with Marian, she responded, “The parallel tasks make the HSP students feel like they are a part of the group; the language is the same, but it is just the numbers that are different, which lets them participate in the activity” (Marian Interview, November 5, 2015). It was eye-opening to watch Marian’s HSP students during math; they appeared to be complacent, comfortable, and satisfied members within the class. More significantly, I could see the excitement on their faces when Marian affirmed for them their success in obtaining the correct answers. To further corroborate the HSP students’ positive experiences with mathematics, Marian shares,

An HSP student came to school and said, “I made questions for myself at home”. They are so inspired that they go home and practice. For me, that is about success. So these HSP students feel that they are doing what the rest of the class is doing and it is a good feeling. (Marian Interview, November 5, 2015)

4.9.3 Manipulatives and Technology

The use of technology is embedded as part of the students’ learning experience in mathematics. Marian shares, “I am always looking up new educational apps and activities on the Internet to engage my kids” (Marian Interview, November 5, 2015). During one of her searches, Marian came across an app called ‘Show Me’. Marian explains, “It’s an app students can use to record their thinking and responses on the ipad. Instead of writing answers using the traditional paper and pencil method, students use the Show Me app. Show Me is an interactive whiteboard app, which also allows students to create presentations by adding text, drawings, photos and images. Marian introduced this app to her students, and for today’s activity, the students simply used it as a whiteboard by digitally recording their thinking processes on the ipad for their peers
and teacher to see. Marian enthused, “They are really into it” (Marian Interview, November 5, 2015).

4.9.4 Students’ Mathematics Communication

In addition to having students practice various mathematical skills repeatedly, a significant part of Marian’s practice is to have her students express their thinking using mathematical language. Marian explains, “Kids are like sponges, they pick things up quickly, so I use the language over and over until it becomes their own. I am very specific with them, for example, I will say, ‘Are you asking for the first factor, second factor or the product?’ I just build on an expectation and I find that they rise to the occasion” (Marian Interview, November 5, 2015).

Marian’s students often utilized sophisticated mathematical vocabulary. They were well-versed in expressing their thinking using various mathematical terms. I heard the following words and phrases uttered naturally from the students themselves: factoring, strings, open array, splitting, compensation and dimension. These terms were consistently used in the appropriate contexts and often expressed without Marian’s promptings.

4.9.5 Assessment

When I had inquired about the notes written on her clipboard, Marian proceeded to show me the collection of formative data she daily collects and accumulates. I asked Marian what she does with her set of notes. Marian states that these notes serve as her formative assessments, which enable her to see where all of her students are functioning. She further shares that these formative assessments enable her to glean what the teachable moment(s) may need to be for her upcoming lessons. She clarifies that, during consolidation or math congress, which is the time all of the students gather together to present and share their learning with one another, that she uses
these very notes to decide whose learning style and strategy she wants to showcase and highlight. In Vignette 3, a clear illustration of how Marian executes this through her use of inquiry-based techniques will become evident.

4.9.6 Mathematical Proficiency

With regards to the five strands of mathematical proficiency, in today’s lesson, Marian reinforced ‘procedural fluency’. Through the Show Me ipad activity, Marian’s students have been afforded the opportunity to develop their skills in carrying out the open array flexibly, accurately and efficiently. Again, Marian always ensures, whenever possible, her students are provided with ample opportunities to practice using various strategies. Marian is insistent that her students achieve confidence and skill mastery.

Adaptive reasoning is a skill we will come to see Marian focusing her attention on in all of her lessons. In today’s lesson, Marian is seen to continually ask her students to ‘justify’ and ‘explain’ their choice of numbers. Marian does not influence her students’ responses and thinking; rather, she guides their thinking processes through the types of questions she poses to her students. Marian demonstrates throughout all of her lessons that ‘proof’ is the cornerstone as well as the centerpiece of mathematical learning.

Productive disposition was highly evident throughout the day’s math class. While solving the open array questions, Marian’s students demonstrated positive attitudes about mathematics, which was evidenced through their clear excitement and confidence; it was apparent that each of Marian’s students believed in his/her own sense of self-efficacy. Confidence and excitement among the students were clearly revealed through their fist pumps, joyous laughter, as well as their demand for more questions. The students’ successful ability to accomplish each challenge presented to them appeared to generate deep enthusiasm and motivation for learning.
4.10  Vignette Two

November 12, 2015

The second day of data collection arrived. I entered Marian’s classroom, once again to find myself dutifully ignored by most of her students. I put on my researcher’s hat and carefully positioned myself, tightly clasping my clipboard and pencil, in preparation to maximize my new learning for the day.

Upon entering, I noticed the whole class sitting on the floor waiting for Marian’s question. Each student had on his/her lap, a white board the size of an iPod, and a dry erase marker with which to record his/her work.

Marian writes 10 x 10 on the board and asks, “How can we get the answer?” A student responds by saying, “Two 5 by tens”. Marian repeats the student’s words while writing the string on the board, “Two five by tens”. She then solicits the class’s thinking by asking, “Does that make sense?” to which all the students respond in unison, “Yes!” Marian then questions, “How else can it be done?” clearly she is seeking after an alternate solution. A student offers, “(5x5) + (5x5) + (5x5) + (5x5). Marian asserts, “Let us think about what Shivam said; let us look at his strategy”, and she writes it on the board. The class solves the equation together and discovers the accuracy of Shivam’s strategy.

Marian writes another question on the board and exhorts, “Ok, here is another question. Solve and justify it”. Throughout the lesson, Marian was neither found offering her students answers, nor heard directing them with prompts. She simply asked for new strategies and waited patiently. This form of questioning continued for another 30 minutes before I discreetly exited Marian’s room. I needed to sequester myself from potential distractions. Below are my reflections:
Prior to observing Marian, I mistakenly assumed that the data collection component of my study would be the least appealing part of the dissertation process; I simply viewed it as obligatory drudgery that needed to be followed through and completed. I was gravely misguided. When one wears the ‘researcher/observer’s hat, one’s lens seems to automatically change. I found myself being perceptive of things that normally would have unmindfully escaped my notice and awareness. For example, had I walked into Marian’s classroom to simply observe as a colleague, I would never have attended to such things as Marian’s ‘wait time’ nor strategic questioning techniques. Instead, I would have been more apt to focus on the content and activities involved in her lesson.

Marian’s students were solving mathematical questions using strategies that I would never have conceived possible by Grade 5 students. My take away from this particular math lesson is how lacking I am in mathematical knowledge. Through observing Marian’s second lesson, an epiphany occurred: It appears that mathematical knowledge and understanding must precede understanding of IBL strategies and techniques. I had no idea that simple addition, subtraction and multiplication of numbers could be expressed in so many different and exciting ways. (Genie Kim Journal, November 12, 2015)

Later, when I was able to debrief with Marian about her math lesson, I expressed to her my amazement pertaining to what I had seen her execute with her students. Her response was simply, “It is the kids; it comes from them”. The very notion that credit was applied to her students, in relation to the deep, flexible and critical thinking that emerged throughout Marian’s lesson, caused some cognitive dissonance within me.
4.10.1 Program Planning

Today’s math lesson solidified for me what I had apprehended in rudimentary form during my first lesson observation: the importance of the teacher’s role in eliciting knowledge and thinking from students. Marian is not disposed to giving her students the answers. She always solicits her students’ thinking through the skillful questions she asks. In doing so, Marian enables her students to formulate the answers. I needed an in-depth understanding of this technique, which I found to be incomprehensible. How does Marian draw out such varied and creative mathematical thinking from her students? Below is an excerpt surrounding our discussion about her process:

**Interviewer:** Do you always elicit answers from your students using this questioning technique?

**Marian:** I start September saying to my students, “I am not your typical teacher. I am not going to tell you what to do”. They do not get it until they see me waiting for them to give me a cue. I will pose a question and I will watch and I will wait because I know there are six years of school and there is understanding in there. They just do not know it; I have to pick it out and elicit it.

**Interviewer:** How do you do this?

**Marian:** I program my lessons so I can elicit that information that is already in their heads. I pose a question. I know where I want to end up and I know how I will get there based on the students in front of me. They are going to build from one another and it is not going to be my telling them. It is going to be my posing questions and seeing what they already know. I provide opportunities and they are going to do it on their own. (Marian Interview, November 13, 2015)

To the above response, I was naturally led to ask Marian ‘why’ she felt compelled to elicit the thinking from her students. I needed to understand why Marian spent time eliciting rather than efficiently ‘telling’. She responds:

As a teacher, I know where I want to end up. I know I want the kids to end up walking out of the room with one big idea that they can easily transfer to another situation. It is not about teaching specifics. It is about teaching transferable concepts and it is not about me showing them how to do it. (Marian Interview, November 13, 2015)
Marian likens this to how parents are often inclined to take over and show their children how to do things, “Because we want them to be perfect and we want it to be efficient and fast” (Marian Interview, November 13, 2015). However, Marian cautions, “People need to do things and figure their own way out. The more you show them, the less it becomes theirs and they do not own it, so they will not remember it” (Marian Interview, November 13, 2015). Marian elaborates on her line of thinking by way of a personal example:

I realized this when I taught Grade 3. I had spent the whole time teaching and showing them in order to prepare them for the EQAO. At the end of the year, I am walking around behind them and I am thinking, “What do you mean you do not know how to do this, I showed you.” Then I stepped back and I thought, “I did not really teach them anything. I went home and had a conversation with my husband. He was miles ahead of me in terms of philosophy and he said, “What do you think you have to do?” I hate when he asks me. I want to be told, and I just kept telling him, “Just tell me what to do”. Then he asked me, “What do you think you have to do for next year?” I said, “I think I actually have to teach them how to learn independently on their own”. (Marian Interview, November 13, 2015)

Marian then opines that with the introduction of the new curriculum, and the overwhelming and mammoth amount of content that requires coverage, “We forget about the child. So it just becomes about how the child has to be ‘shown’ the curriculum” (Marian Interview, November 13, 2015)

I slowly began to grasp the rationale behind Marian’s practice of ‘eliciting’ as opposed to ‘showing’ her students the process of arriving at the answers efficiently. With this emerging understanding, a new question surfaced. Exactly “how” does Marian effectively elicit responses and solutions from her Grade 5 students without earlier having shown it to them (I always show my students). During my second observation, appropriate and accurate responses from Marian’s students seemed to mysteriously emanate from nowhere; yet, this came across as natural and commonplace for Marian. To my query of ‘how’, Marian explains:
I know it is a hard thing. Teachers, I know, will say, “But they are so little. I have not taught it to them yet. I tell them to remember, “Your students had teachers before you; they know things, but they just do not know that they know. They do not know how to articulate it”. (Marian Interview, November 13, 2015)

Marian then describes something I had never heard of before - the notion of making the ‘invisible become visible’:

So, what I try to do is to make the invisible visible, like their thinking strategies - it is almost metacognitive. When they tell me how they solved 10 x12, I record it and I give them a template so the next time I give them a question, now they have a tool to tell me what they are thinking. If we tell them to tell us what they are thinking, but no one shows them how to record their thinking mathematically, then they will not know how to do it. Then as teachers we will say, “They do not know how to do it, and I am going to have to show them”. Well, not necessarily. You can record their thinking. It is all about the students and recording their thinking and providing them with tools, mathematical tools so they can show you next time what they are thinking. (Marian Interview, November 13, 2015)

As illustrated in both Marian’s interview responses as well as demonstrated in her method of questioning her students, Marian draws out from her students the ‘invisible’ mathematics that is inside of her students. In my personal reflections, which will follow later, I will describe my thoughts pertaining to Marian’s difficult to fathom words below:

I program my lessons so I can elicit the information that is already in their heads. I pose a question. I know where I want to end up and I know how I will get there based on the students in front of me. They are going to build from one another and it is not going to be my telling them. It is going to be my posing questions and seeing what they already know. I provide opportunities and they are going to do it on their own. (Marian Interview, November 13, 2015)

Marian’s students appeared to have unwittingly helped her to prove for me that they do ‘get it’ on their own, without their teacher having to explicitly tell them.

4.10.2 Learning Environment

To help with her students’ skill development and achievement, Marian often uses the white-board, in a very strategic manner. During an interview subsequent to my observation,
Marian explained her use of the white board, which provided insights into the kind of learning environment she seeks to create:

My students often use a white board because I want them to know that I am not judging their work. They know that if they made a mistake they can quickly go ‘voop’ and erase it. (Marian Interview, November 13, 2015)

For today’s lesson, Marian had her students record their thinking on the white boards during consolidation time. She explains how she likes to have her students flash their answers as it enables her to quickly, and in a non-threatening way, see who is ‘getting it’. Marian shares, “I find that when some students are confused, they do not know that they are confused” (Marian Interview, November 13, 2015). This process helps Marian to both discover for whom and where support will be needed. Then Marian takes this opportunity to discuss with me how she often uses this medium to identify and draw out from among the class those unassuming students who typically find themselves on the periphery - those students who generally evade being called upon:

There are some students who are so lost that they do not even know that they are lost as they think that is the way math is supposed to be. I then tell them, I don’t care if you are lost. You are going to have to show me you can understand this. They are shocked that my expectation is that they understand; and they are shocked that I even have this expectation as they are so used to just being quiet, good kids so teachers do not bother them as their teachers did not call on them because their hands are not up. They have learned a ‘learned helplessness’. They have learned that their teacher will never call on them. (Marian Interview, November 13, 2015)

Marian then illustrates how she uproots and quells this ‘learned helplessness’ from her students, which she views to be an anathema:

My kids now know that, if I call on a student who may be weak, everyone must put their hands down as it is time for that student to think. You even hear students start to say, “Let her think. It is hers”. I am not good on the spot. So I do not give pressure for them to answer the question. I have time to wait. If time passes, then I will move on, but I will let them know that I will come back to them. (Marian Interview, November 13, 2015)
4.10.3 Constructing Knowledge

In the midst of observing Marian elicit her students’ thinking and making it ‘visible’, Marian’s strategy is discerned. Once Marian draws out the responses and thinking from her students through her pointed questions, she is frequently seen to reiterate the newly constructed knowledge, which she then clarifies for the class.

During the lesson, Marian continued to probe and ask, “Did anyone do it a different way?” Then she allowed the students to explain their strategy through the mini-white board. When appropriate, Marian expressed for the students, the technical term for the given strategy. For instance, Marian explicated to the class after a student shared the process of her thinking, “She just used related facts”. Marian then continued to use this newly defined term to reinforce it for her students.

After writing another question on the board, Marian instructed her students to find the solution using their white boards. Anna offered her strategy, which she used for the question 12 x 9, by holding up her white board. Anna had written: (10 x 9) + 9 + 9. Marian then asked the class if there is a more efficient way to express Anna’s solution. Marian was guiding the students towards creating multiplication equations.

Marian then directed her students’ attention, leading them to notice that the open array and the strings they had just constructed are the same things expressed differently. Marian leads her students so the conclusion emerges from them. She clarifies in concrete terms what the learning is, only after the students have discovered it on their own. When I inquired if Marian would give the answer to the students had they not constructed the understanding on their own, Marian responds, “I will continue with the lesson the next day to draw it out from them. But if I feel it is beyond them, because of our large curriculum, I tell them” (November 13, 2015).
4.10.4 Teacher Attitude and Comfort with Mathematics

Marian’s comfort and facility with mathematics is evident as she engages with her students in the mathematics lesson. A conversation regarding her mathematical knowledge, as well as teachers’ mathematical knowledge in general, has been excerpted:

**Interviewer:** You use a lot of terms that I never use in my class, like partial products. Did you know and use these terms before you were involved with CIL (Collaborative Inquiry for Learning)?

**Marian:** No. CIL was a starting point, but where I really learned the most math were from the math AQ (Additional Qualification) courses. For example, in the AQ course, I saw the instructor organizing the teachers’ work and I did not understand how she did that. I realized that I was organizing by tools instead of by strategies. I have had the privilege of being a part of all this, but it was not until I took the math AQ that I really learned the math, and I now get it.

Marian then offered an explanation of her new understanding, which illuminated some suspicions that were percolating inside of me:

The thinking is what matters. How am I going to elicit ideas from the kids if I do not know what I am looking for? It is hard to get to that because you need a deep mathematical understanding before teaching. You can teach teachers Three-Part lessons, but it does not matter. It is the knowledge - the mathematics behind it. We are teaching teachers the structure, and the teachers go back to their rooms and say, “Yeah, it is not working”. And there is a reason why it is not working, because they lack the background knowledge. It is incumbent that teachers have the background knowledge. (Marian Interview, November 13, 2015)

I asked Marian how long it would take the average teacher like myself to get to where she is at mathematically. Marian relates to me her own personal journey to help answer my question:

Once you understand, you then learn how little you really know. And I am thinking, “My poor students from my previous years!” How I must have stunted their growth and put a ceiling on their learning because I did not see what they were capable of. I remember I said to one of my students, “James, I do not know what strategy you are using, but I am going to call it James’ strategy”. I told him that I was not familiar with the way he thinks. I have now learned that you have to make that thinking visible, like the way James did. He always understood the flexibility of numbers. He changed every thing to a friendly number so it was easier to add. But, I thought at that time, “I do not understand what you are doing, but it works!” The more I learned about math, I realized,
“Oh my gosh, there must be more that I should be learning!” (Marian Interview, November 13, 2015)

Marian then discusses the potential pitfalls when teachers lack mathematical knowledge:

Our lack of knowledge puts a glass ceiling on our kids. Because what I see with a lot of teachers, especially in grades 6-8, when they are not comfortable teaching math, they become poor teachers. You see that when a child’s answer deviates from what is in the textbook, they cannot help that child. It could be a simple misunderstanding, but they just say, “Wrong, you are wrong. Your answer does not match my formula, therefore you are wrong”. Meanwhile, the child really gets it, but he just does not know how to do it using the formula. (Marian Interview, November 13, 2015)

4.10.5 Mathematical Proficiency

In today’s lesson, Marian was seeking to develop her students’ mathematical proficiency. Marian recounts the process by which her students’ conceptual understanding and strategic competence are both advanced and strengthened:

As soon as I put someone’s thinking on the board, you hear a student say, “Hey, that’s what I was thinking, but I didn’t know how to say that”. You hear that a lot. You hear, “Oh wow, I didn’t know I was allowed to do that. Can you do that with numbers?” A student asked, “Can we do that with multiplication?” And I said, “I do not know. Let us try it”. I tell them I do not know, as it is for them to prove and justify it. I want the student to be the one to prove it to the other students in the class. Then you hear students say, “It works, it works!” They were shocked that it was working; that the strategy that we were using for addition also works for multiplication, which comes back to why I do not teach them how to do such task-specific things because when I teach them a strategy, they can use it anywhere. They can move it from concept to concept. They realize that these strategies are transferable. (Marian Interview, November 13, 2015)

Marian elaborates on this point to underscore the need for student understanding, which advances and solidifies their strategic competence:

Now Ayub owns it. Now he gets it. He understands it and the people around him understand it. We are now going to practice it. What is funny is that a week from now, another student will say the same thing as Ayub, because they just got it. Because for some students, it will dawn on them later. That is why we keep practicing and repeating, because even though you think they know it, they do not know it. They will repeat what you say, but do they really understand what it means? So I get right in there and ask them to prove and justify it, because they are good at just parroting the words. (Marian Interview, November 13, 2015)
Marian’s students’ positive perceptions about mathematics, or their productive disposition, are revealed in their active engagement throughout Marian’s lesson today. Marian’s students respectfully questioned each other and even questioned their teacher. Moreover, the students’ seemingly positive demeanor when being probed and pushed by Marian, points to their keen interest in learning mathematics.

4.11 Vignette Three

*November 20, 2015*

Employing word problems to reinforce multiplication skills is the mathematics lesson for this morning. Using magnets, Marian placed a colourful poster of very edible looking cupcakes on the blackboard. She had written on the board the following word problem: How many cupcake holders will you need if a box contains cupcakes in 6 groups of 24 boxes? Below the question, Marian created a parallel task for her remedial students, similar to what was earlier described in vignette 1. Marian inserted the numbers 2 and 4 for her HSP students so they would be able to use the same question framework while comfortably solving the problem through the use of smaller numbers. Once the problems were written on the board, Marian simply read over the questions, and without clarifying the expectations for the exercise, students were sent off to solve the problem with their math partners. The only instruction Marian provided for her students was that they select their own strategy with which to solve the multiplication word problem. Marian’s students all made their way to their desks without expressing any angst or confusion about their given tasks.

In keeping with her established practice, Marian walked around the class taking avid notes on her clipboard. She engaged with individual students, asking them questions. For example, Marian recognized that an HSP student, Ralph, was going down the wrong path, and
she advised, "Think about what I am asking you to do" (Marian Observation, November 20, 2015). With a low voice, Marian made this statement twice. Initially, Ralph looked at Marian quizzically, then, excitedly, he gave her a look demonstrating he discovered his error. Ralph erased his work and attacked the problem using a different approach. Marian did not hover over Ralph. Instead, once Marian recognized that Ralph understood his mistake, she walked away and continued to engage with other students.

I followed Marian, allowing enough distance between us to afford her some personal space; yet, I gave myself sufficient proximity to overhear the conversations she was having with her students. I continued to detect Marian actively recording on her clipboard notes about her students’ work. The only kinds of comments heard coming from Marian were, "Is there another way to do this? What is the question asking you?” And “What are you thinking here? Show me your thinking" (Marian Observation, November 20, 2015). Not once did I overhear Marian give tips or prompts to guide her students. Again, I was perplexed by Marian’s apparent reluctance to provide direct support for her students.

After the passage of time, Marian made a vocal signal to capture her students’ attention, summoning them to the carpet for consolidation time. The students brought their pieces of paper to the floor and handed them in to Marian. The students sat and chatted with their peers while waiting for Marian's next direction. The students appeared unfazed by Marian’s private activity; they seemed to be accustomed to having their teacher shuffle through their work prior to the whole group conference.

With seeming confidence and obvious ease, Marian looked at her clipboard and speedily organized the students’ work samples. In what appeared to be less than two minutes, Marian categorized and clustered the students’ work according to the type of strategy used. Marian
clutched a magnet and decisively pulled out one student’s paper, positioning it on the board for the whole class to see.

During consolidation time, a variety of student-generated strategies were presented and discussed. Throughout, Marian was observed to continually elicit from the students their mathematical thinking; she insisted that they provide proof and justification for their solutions. For example, Marian asserted, “I saw the debate you had with your partner and I saw your struggle. Explain to the class what happened” (Marian Observation, November 20, 2015). Marian’s students used divergent strategies when solving the problem. Marian makes her students’ divergent perspectives visible. She continually pushes her students, in front of their peers, to prove and justify their positions.

4.11.1 Program Planning

In planning for all of her math lessons, at the forefront of Marian’s mind are outcomes, processes and key concepts. When asked about her goal for today’s lesson, Marian states, “What I hoped they would all understand today is that in multiplication, you can break it up anywhere and anyway you want, and the answer will be right”. Marian then proceeds to stress, “I am always pushing for the thinking, the thinking, the thinking” (Marian Interview, November 20, 2015).

With respect to addressing the planning involved for today’s class, Marian shares that she already taught this lesson during the previous year. Thus, Marian was able to anticipate the kinds of student-generated strategies that were likely to emerge. Otherwise, Marian admits, she would have had to create a list on a piece paper of all the potential strategies before the start of the day’s lesson. She mentions, “Since I already knew what the possible categories would be today, I just
had to look at their work and figure out what they were doing. I saw that ok, they are doing this and they are doing that” (Marian Interview, November 20, 2015).

4.11.2 Meeting Individual Needs

Marian and I had a conversation surrounding the topic of meeting her students’ individual needs. Below are the highlights of that discussion:

**Interviewer:** You have students with varying abilities in your class; how do you deal with diverse needs?

**Marian:** You are talking about differentiated instruction. It is with overall expectations. If the overall expectation is the distributive properties of multiplication, I can do distributive properties with grade 5 numbers, with grade 4 numbers, and grade 3 numbers. It is just that I am taking the overall expectations and then individually figuring out where all the kids are, in terms of the overall expectations.

**Interviewer:** How do you do this?

**Marian:** To do DI (differentiated instruction), you look at the overall expectations as they give you the big picture. So, if I am looking at distributive properties of multiplication, you will see that it is in pretty much all grades, and so I am still teaching distributive properties, but the numbers are different. So, my grade 5s are doing it with 3 digits, my students who are at the grade 4 level do it with 2 digits and so on. Everyone is still doing distributive property, but at their level, within their zone of proximal development.

**Interviewer:** Ah Vygotsky!

**Marian:** Yes, and it works. They are not too frustrated; it is not too complicated and for some, it is not too easy. It is always with the overall expectations. When the curriculum first came out, I thought if I taught to each specific expectation, I would need to be teaching for two years. You cannot do it. Plus, I was learning that the kids were not retaining it because it does not make sense. So my whole way of designing is, it has to make sense. (Marian Interview, November 20, 2015)

Marian satisfied my curiosity. Yet I still pondered how differentiation worked during consolidation time.

**Interviewer:** Ok, now during your math congress (consolidation), while students are presenting their strategies, how do you know if your lower performing students are understanding?
Marian: You have to train them. That is something I try to build up. But with the math congress, I am going to start from where everyone can buy in. And then we build and we build so we get to the end and you have most of the students with you - not everybody, but the majority. What I do on day one, I will do on day three, so hopefully, you get more students on board with you. But I can also assure you that by the time I get to the end of the unit, I still will not have everyone getting it as everyone is at different stages in their learning. But I am hoping that all students will have moved. (Marian Interview, November 20, 2015)

4.11.3 Learning Environment

Marian’s students worked in pairs today as they tackled the word problem. Students working with a partner is a common observation I make whenever I visit Marian’s class. When I inquired about her method of pairing/grouping her students, Marian replies:

When students work on tasks, I have them in the same pair for the whole unit. So, when they work together the next day, they know where they left off and they are comfortable. Usually, the talk is focused and on-task. I like homogeneous groups. (Marian Interview, November 20, 2015)

I confessed to Marian that I am unclear about her rationale for using homogeneous as opposed to heterogeneous groups. Despite this being a polarizing topic, Marian explains why she generally advocates pairing her students according to their similar capabilities:

If the pair is mixed, the weaker student tends to feel insecure and relies completely on the student who gets it. It is very difficult to have a high and a low performing child together. The low performing child is so intimidated, even though they have good ideas, they become insecure and will cave into what the other student says. It is not entirely homogeneous, but maybe one is a little higher than the other, but they kind of pull each other up and help each other. (Marian Interview, November 20, 2015)

Marian shares another important factor when pairing her students together, “I have to see also what personalities work well together” (Marian Interview, November 20, 2015).

I then redirect our discussion to the topic of student errors. Marian speaks at length about the process by which she helps to rectify misunderstandings among her students. At times,
Marian explains, she will use one of her own student’s errors and turn it into a teachable moment. Yet, she cautions:

But you have to be careful of the child. Will that child be embarrassed if you point out the error or is it a child that is confident enough to say, “Yeah, we all make mistakes”. So you have to put feeling into it.

Marian then reveals her sentiments regarding safely using student errors during teachable moments:

It is a good moment when there are errors, as it is a teachable moment. For instance, when students had to subtract numbers, half the class got the answer wrong and it was mainly those who used the traditional algorithm, and the ones that used strategies that made sense to them got the answer right. I asked them, “What is the difference?” They said that they did not know. I asked them, “Why would you use a strategy that you do not know how to use? They all went, “What do you mean?” and I said, “Look at the answers. Are you comfortable with this strategy? Are you comfortable knowing that this is going to help you get to the right answer?” Because it was a big group of them, it was really safe to do this. I usually do not have them put their names on congress stuff. Nobody knows. It is nobody’s business. (Marian Interview, November 20, 2015)

4.11.4 Student Tasks

Today’s open-ended problem-solving task was one Marian utilized to elicit and invite a multitude of student-generated solutions. Through today’s problem-solving task, Marian wanted her students to be able to understand and discern the process behind every problem-solving strategy offered. The open-ended task enabled Marian to not only showcase multiple solutions to the same problem, but it also permitted her to reinforce for her students newly learned strategies. For example, every time a student presented his/her solution, Marian always asked, “Who’s strategy is this student’s work like?” (Marian Observation, November 20, 2015).

Today’s open-ended task allowed for many different strategies to be exhibited. Marian underscores for her students, “There are many strategies, but different ways to get there” (Marian Observation, November 20, 2015). While illuminating this fact, Marian points to the various strategies the class utilized, such as the splitting strategy.
4.11.5 Constructing Knowledge

Once again, Marian’s math lesson extends beyond the 40 minute block wherein my data collection period occurs. Feeling like déjà vu, the recess bell cut short my observation of Marian’s lesson. Consolidation time continued following recess - which I would not be able to observe, as I needed to return to my class and be with my own students.

I was intrigued by what I had earlier witnessed. I wanted to know what Marian hoped to achieve during consolidation time. Additionally, I was curious to know whether Marian generally permitted all of her students an opportunity to present their work during consolidation time. Marian informs me:

No. I get a sample of the kids whose work best typify most of the answers, and I go from there. And I question and ask, “What were you thinking?” It is through the consolidation that the generalizations come out. I ask, “What have we learned?” And I write down on chart paper what we have learned. They are so comfortable with all the strategies that they can use them in different formats and different applications. (Marian Interview, November 20, 2015)

For today’s lesson, a variety of questions rapidly floated around in my mind, which I took the liberty of asking Marian:

Interviewer: When you circulate around the class, what do you do when a child asks you, “Is this right?”

Marian: I would put it back to them and ask, “What do you think? What were the instructions?” I will intervene if there is an error. I will not say, “This is wrong”. I will say, “Maybe you need to check something. I don’t know that I quite follow you. I follow you all the way up to here, but I don’t follow you from here.”

Interviewer: So, you do not tell them it is wrong but instead let them remain with their error?

Marian: Sometimes I allow the students to come to the carpet with the errors. As I said, sometimes it is a good teaching opportunity,

Interviewer: During congress, do you talk to the strategies the students have come up with or do you expect your students to do the talking?
**Marian:** Sometimes I do and sometimes the kids do. Sometimes I have the kids present and then I repeat what they said. It is training. What I say is, “You can stay sitting there and I will record what you are thinking”. I do this so the invisible becomes visible. I will use whatever tool makes sense. The students will just start to ask, “Can we do what he did? Can we use an open number line?”

**Interviewer:** Is that why you said consolidation time was the most important?

**Marian:** Yes. It is where misconceptions get solved. (Marian Interview, November 20, 2015)

Marian then takes this moment to relate to me an event that helps exemplify how

‘powerful learning’ typically occurs during consolidation time:

It was funny because some students were familiar with the open number line, but they did not know how to use it properly. So, they were doing really strange things with them. I said to them that an open number line is supposed to help you not count on your fingers. You are supposed to choose the numbers that you can do in your heads. They said, “Oh.” They did not know this. The students were using the tools, but they did not understand them. I tell them that the whole point was to break down the numbers to what they knew, because numbers are flexible. They did not know that numbers are flexible. (Marian Interview, November 20, 2015)

Marian then openly confessed that she too did not know about the flexibility of numbers. She admits, “I thought you could not break up and manipulate numbers. Once I realized as an adult that you can manipulate numbers I wondered why nobody told us” (Marian Interview, November 20, 2015). “This is the very reason”, Marian explains, “why the whole thing is to push them to mental math. It will be a while for them to get to mental math. That is why we practice and we practice and we practice” (Marian Interview, November 20, 2015).

### 4.11.6 Students’ Mathematics Communication

The substantial part of Marian’s lesson(s) involves persuading, proving, and justifying oneself to the class community. Student communication, according to Marian, must be actively guided and practiced in a purposeful way:

You have to train them in how to speak to one another - how to ask for clarification, as some will sit there and not understand but act as if they do. So, I will say, “Can
you repeat what Ayub just said in your own words? What does he mean?” Ideally, the conversation should be about just the students. (Marian Interview, November 20, 2015)

It was noteworthy how a student comfortably corrected Marian’s misuse of terminology during consolidation time, “Mrs. Marian, it is a tool Sammie used, not a strategy”. Marian laughed and applauded Matt for catching her error. Matt’s ability to discern and articulate the distinction between a mathematical tool and strategy was impressive; this error escaped Marian, and it utterly eluded me too. Marian shares that she continually encourages her students to challenge their teachers and their peers. Marian then describes:

Whenever we take up work, they learn to be critical of one another’s strategies. I will ask whose strategy this is similar to. They will say, it is similar to so and so’s. Another boy objected and said, “No, they multiplied but he is adding”. They have to be critical of one another’s thinking. (Marian Interview, November 20, 2015)

4.11.7 Teacher Comfort with Mathematics

Marian admits that categorizing students’ work according to the type of strategy used, still presents her with some difficulty. I asked Marian how intuitive categorizing has become for her today compared to when she was first learning. This was Marian’s response,

It is still hard. Sometimes I have to stop the child and ask, “How did you do it?” because they may look the same, but the thinking is different. And what blows me away, is when I get things like this. [Marian pulls out one student’s answer sheet and speaks to it]. “His response was not intuitive. He added 2 numbers together first and when I saw his numbers, I had to ask him where he got the 48 from. He had to tell me. Then I knew what he was thinking. (Marian Observation, November 20, 2015)

Despite her words, Marian’s students’ apparent confidence in her knowledge and expertise as a math teacher was palpable throughout the lesson.

Through her candour in sharing with me about her mathematical inadequacies, Marian demonstrates for me her true comfort and confidence with mathematics. Marian discusses the work of one of her students, and she remarks, “I do not know what he was doing here. I have to
question his thinking later” (Marian Interview, November 20, 2015). Shortly thereafter, she confesses, “The kids know the strategies and they keep me on my toes, like what happened today (When Matt corrected Marian). They sometimes catch my mistakes and if they did not, I would have said the answer was right!” (Marian Interview, November 20, 2015). Marian’s comments surprise me: it usually takes a confident and self-assured individual to freely admit to uncertainty.

4.11.8 Mathematical Proficiency

Marian is consistently observed to strengthen her students’ conceptual understanding and adaptive reasoning skills. Marian states, “Once they get the concepts, they can do it. It is the conceptual understanding they need” (Marian Interview, November 20, 2015).

During today’s lesson, Marian declared, “Let us take a look”. She then asked the class what they believed this student had done (Names are not written on the papers). A student raised her hand and replied, "Repeated addition". Marian then asked the student to explain how the process works. Subsequent to the given explanation, Marian stated, “Ok, let us build on what Venita said” (Marian Observation, November 20, 2015).

In the foregoing example, where Marian emphasized her students’ need to prove and justify their thinking, it was evident that Marian’s students possess a strong ability to formulate, represent, and solve mathematical problems. These skills also point to Marian’s students’ procedural fluency and strategic competence.
4.12  Vignette Four

December 2, 2015

Background to Joint Lesson between Marian’s class and My Class

For this classroom observation, it was decided that we would have a joint lesson between my class and Marian’s class. After school one day, an impromptu interview took place in front of my class. Marian enthusiastically recounted a story about how two of her remedial students (one being an HSP student) were the only ones who accurately solved a challenging math problem. Marian emphasized to me that the fantastical nature of this story really stemmed from the fact that nobody else in the class was able to solve this very problem. Marian gloated about how when the two girls presented their solution to the whole class, it had sparked great excitement among all of their peers, with many students vocalizing congratulatory words.

After hearing about this account, I began to question Marian. I wanted Marian to give me her recipe of tried-and-true strategies that would enable me to develop my students to become as mathematically capable and confident as her students. Marian’s response was unwavering: “Your students are capable of doing the same. The mathematical thinking is inside of them” (Marian Interview, November 10, 2015). These ‘too-good-to-be-true’ words sounded like they had come out of a movie, which I did not buy into. Yet, I was profoundly curious to discover if there could possibly be any truth to Marian’s hopeful statement. Through this discussion, we both agreed that we would combine our classes together and put Marian’s words to the test. Smiling, Marian promised me, “You will see that your students will be able to do the same thing as mine...without prior teaching” (November 10, 2015). Unsuppressed, my retort was a doubtful, “We will see”.

Admittedly, I experienced trepidation and hesitation about this decision to combine both of our classes together. This was due to two major insecurities: 1) Would Marian come to
discover how lacking I am as a math teacher? and 2) Would my students see a clear distinction in mathematical abilities between Marian and myself and become disappointed in me? I had to consciously subdue and obliterate those fears. With some inner turmoil, I compelled myself to swallow my pride by reminding myself that important research needed to be conducted.

This joint lesson turned out to be ideal in that it would permit me to spend the entire morning watching Marian implement each component part of her typical Three-Part math lesson. I have been limited and would continue to experience limitations vis-a-vis the length and scope of my observations of Marian. Prior to starting my weekly observations, Marian and I took some time to coordinate a 40-minute block to observe her teach mathematics. Yet, given that I teach full-time at Mapleview, synchronizing my preparation schedule with Marian’s math block became an immense challenge. Marian’s math block always occurs in the mornings (when her students’ minds are fresh), and I only have one 40 minute preparation period in the mornings on any given week. Therefore, whenever Marian’s math lesson extends beyond 40 minutes, which it often does, my observations would become prematurely curtailed. Thus, joining our classes together almost became a necessity: the joint lesson afforded me an opportunity to observe the entirety of Marian’s lesson, without fear of the 40 minutes running out.

4.12.1 Joint Math Lesson

Marian and I mutually agreed that the joint lesson would be an introduction to division. This decision was made based on two criteria: 1) the math topic would have to meet the Grade 5 curricular expectation(s) and 2) it needed to be a topic in which both Marian and I did not already cover with our students. In order to substantiate Marian’s argument that my students possessed the same capabilities as her students, she and I needed both classes to be ‘tabula rasa’ prior to her lesson.
On the day of the joint lesson, Marian and I needed to take some time to methodically pair up our students together. Marian asked that the students be paired up homogeneously, according to their mathematical abilities. I completely understood why Marian requested this homologous grouping given my prior interview with her - Marian’s rationale was described in Vignette 3.

Marian started the joint lesson by stating she has a problem that requires solving. She then informed the students that she needs everyone’s help. Students from both classes ostensibly buy into Marian’s purported dilemma and, surprisingly, they seem to not be distracted by one another. To my amazement, everyone expressed genuine interest and committed their focus on Marian. My students appeared unfazed by their being in a different learning environment, among Grade 5 peers who were largely unfamiliar to them.

Before sending the students off to solve the problem, Marian asked the two classes what they can do if they are stuck. She informs the students that they can use manipulatives and draw pictures to solve the problem. Additionally, she reminded the students, “We always need to start solving a problem with what we know. What do we know? Remember, we never guess. Also, remember, too much information will confuse us. Make it simple” (Marian Observation, December 2, 2015).

Appearing to take their task/challenge seriously, the math partners first sought out and then secured a spot for themselves in the overcrowded classroom and immediately proceeded to attack the problem. In spite of the fact that some of the pairs of students were unfamiliar with one another, they acclimated well, and to my relief, all of the students worked collaboratively together.
During consolidation time, five strategies were elicited: 1) skip counting by six, 2) repeated addition, 3) reasoning (through using doubling, halving), 4) multiplication, and 5) traditional algorithm.

Below are samples of student-generated strategies:

At the conclusion of the lesson, Marian informed the two classes that the problem was a division question. However, she underscored the point that today’s problem, like many other kinds of problems, permit people to use a multitude of strategies to arrive at the answer. The expressions of most of the students demonstrated understanding of this concept.
4.12.2 Program Planning

During recess, when both classes were ushered out of the classroom, Marian immediately studied her clipboard and appeared deep in thought. I quietly let Marian peruse through her notes. Finally, she uttered, “I see six strategies that will come out today. See, look here” (Marian Interview, December 2, 2015). Noticing my confused expression, Marian clarifies, “I always listen to the conversations students have, but with the two groups it was a little too loud. Still, I was able to record the names of the students using different strategies”. Marian then pointed to different sections on her paper and stated that she will have specific students present their strategies during consolidation time. I asked Marian, “So, you already knew in advance the six strategies that would come from the students?” Marian replied, “I knew of four, but that is only because I know the math.” I ask Marian how students generate these strategies without explicitly being taught. Marian’s intonation and slow speech indicated to me that she wanted to emphasize her points carefully:

It is not for you to teach them. It is for us to elicit and pull. They are going to learn. You see the students sitting there that did not do anything? Well, tomorrow’s question, they are going to do this (Marian points to the easiest strategy, counting by ones) because they know it is ok to do this. (Marian Interview, December 2, 2015)

Seeing that today’s task came from a resource book, I asked Marian where she typically finds her tasks/problems. She replies:

I look at their work, and there is where my inspiration for the next lesson comes from. While I was looking at the work they did with your class, I noticed that there were a lot of similar errors, especially my struggling students. That is what I needed to correct. I need to correct that thinking before we move on. That is where you always look - at the work they have done and see what you are doing tomorrow. (Marian Interview, December 7, 2015)

Marian then says, “It is not about turning the page in the textbook. You cannot. So, I look at their work and move on” (Marian Interview, December 7, 2015).
4.12.3 Student Tasks

With the combined classes sitting quietly on the floor, focused on Marian, she personalized a story about “Muffles”, which comes from Cathy Fosnot’s book. Marian commenced the lesson by mentioning how she went into Muffles’ store. She engaged the students into her narrative. Marian explained how she saw that a truck was delivering drinks for Muffles’ vending machine. Marian recounted this fictitious tale, using a first-person, active voice. She continued by stating that she broached the delivery man and asked, “Excuse me sir, how many bottles does the vending machine take to fill up the machine?” Marian then comfortably assumed the assertive voice of the delivery man: “One hundred fifty-six bottles”. The students listened with clear fascination. Marian resumed, using her own voice, “Water bottles come in packs of six. Each pack needs to be put into the vending machine”. Marian proceeded to ask the students, rhetorically, how many 6 packs it would take to fill the vending machine. She then provided a more challenging parallel task. “Juice machine”, says Marian, “A juice machine is beside the water bottle vending machine and this juice machine holds 156 bottles. Marian then continued with her narrative, “I noticed that there were 6 flavours. How many bottles of each flavour will fit inside the machine?” Then Marian throws out a question to all the students, “What am I asking you to do? I’m not asking for the math. What is the problem that needs to be solved?” Again, this was a rhetorical question. Without eliciting a response, Marian sent the pairs off to solve the parallel tasks. She did not indicate to the students, who was to solve which parallel task.

I was thoroughly impressed with how the math partners worked diligently together towards seeking a solution to the Muffles problem. I slid over to Marian and discreetly enthused about this to her. Marian chuckled and reminded me, “The learning has to come through without
you telling them” (Marian Interview, December 2, 2015). In the midst of the loud chatter, Marian tells me:

It is all about building their confidence. Starting from where they are. Like we were talking about before, in their zone of proximal development, and making sure they can solve this problem so they feel confident enough so that when they get a similar problem tomorrow they go, “Yeah, I did it yesterday. I can do it again”. (Marian Interview, December 7, 2015)

4.12.4 Constructing Knowledge

During our discussion at recess time, Marian mentioned to me that she intentionally leaves students to their own devices, allowing them to solve the problem according to their own cognitive abilities. When I asked Marian if she believes students will solve the problem she has posed today, Marian responds:

Yes. When we congregate for our congress, is when I pull it all out. For me, that is the most important part. The congress. It is the understanding. That is where you pick up all the misconceptions and you go, “Ok, if you are still doing this, then I have to step back and go here. It lets me determine and make sure I cover this and make sure they understand. (Marian Interview, December 7, 2015)

Marian then discusses something that I, as an experienced teacher, have a difficult time conceiving and fathoming:

You have to trust that someone in the class will see the pattern or rule. You do not tell them. It is through the intentional use of numbers that I would create a situation, so that someone is bound to get the strategy I want to teach them. It means that I would sometimes have to wait the next day or the day after. (Marian Interview, December 2, 2015)

Here, Marian addresses the inner workings of her waiting strategy:

Students know that I will never tell them. They know that I will wait. I tell them that I can wait, so let us try again tomorrow. And my lesson for tomorrow is based on what happens in the classroom today. I can say whatever in my day plans, but it is driven by the students. (Marian Interview, December 2, 2015)
4.12.5 Students’ Mathematics Communication

As the students worked together in pairs, the two classes generated a fair level of controlled noise; the entire room was abuzz with active debate and discussion about the day’s problem. During consolidation time, students from both classes were seen to contribute to the discussion, nevertheless, it became apparent, looking at the raised hands, that Marian’s students would have easily dominated the conversation had Marian not ensured equal voice was accorded to both classes. Marian and I were both cognizant of the fact that it was her own students who had asked most of the questions; this fact could not be disguised. Consequently, this led to the following discussion:

**Interviewer:** Do you feel that because you promote conjecturing, your students naturally ask questions?

**Marian:** I think kids will ask questions no matter what class they are in. I think the top 5 percent of your class are the questioners. They will do well in spite of us. It is getting the other kids to start questioning. Like Sam, now he is constantly questioning (Marian Interview, December 7, 2015).

**Interviewer:** Did Sam question before?

**Marian:** No. He was a follower. He wears his emotions on his sleeve and when he is confused, you know it, so I look at him. He is my touchstone sometimes. I think they start questioning each other. And they say things like, “How can that be? It cannot be”. and they have debates. It is not led by me anymore. (Marian Interview, December 7, 2015)

**Interviewer:** So you led the questioning in the beginning?

**Marian:** Yes. I did a lot of modelling, and they felt it was ok to question one another as long as you do it professionally and respectfully and not to embarrass another. It is because they are naturally curious. (Marian Interview, December 2, 2015)

4.12.6 Teacher Attitude and Comfort with Mathematics

Again, Marian’s competence and aptitude for mathematics comes through as she recounts her observations pertaining to the joint lesson:
See these students who used the second least efficient strategy? They are most likely going to use this strategy next (Marian points to a complex strategy) because students always say, “I did not know I was allowed to do this”. They are thinking that there is only the traditional algorithm and that is the only way. You will see that they are going to see and talk about the reasoning that Harjot and Prabjot used, and they will go, “Oh yeah, I never thought about that”. (Marian Interview, December 2, 2015)

Marian was referring to two of our students: Harjot, who is my only HSP student, and Prabjot, Marian’s student, who is on an Individual Education Plan for all subject areas. These two struggling students were the only ones to arrive at the answer using the most complex solution (reasoning) for the water bottle problem. Please see below:

![Image of a math problem solving activity]

*Reasoning Strategy (Prabjot & Harjot)*

I expressed my amazement at Prabjot and Harjot’s unexpected use of the most complex method of finding a solution. Marian then explains to me that the HSP students are typically the ones who execute reasoning skills when solving problems. Surprised and incredulous, I asked Marian how and why this was the case, to which she responded:

It is because HSP students do not understand the traditional algorithm so they are forced to figure it out on their own. The teachers cannot teach them because these kids cannot follow the algorithm so the teachers get frustrated. So these kids are forced to figure things out on their own, and they have to. Your Harjot and my Prabjot both worked together to solve the problem through reasoning. (Marian Interview, December 2, 2015)

Marian then pulls out the work of two of her HSP girls, Sammie and Kerry, who were discussed in Vignette 2, and points to their work. Marian states, “Sammie and Kerry used a
higher order strategy because they have had practice and they know it. They have seen it before, so they know that if they cannot do it one way, they can do it another way” (Marian Interview, December 2, 2015).

Marian expanded on her line of thought regarding reasoning skills, “Very few students use ‘reasoning‘ (to solve problems) but once they get it, they do it often, like Manveer, who does reasoning all the time” [Manveer is a capable student who unfortunately earned a reputation for himself as being troublesome and disinterested in learning] (Marian Interview, December 2, 2015). Marian continues, “Manveer is engaged because now he knows that the strategies he uses are acceptable and they are” (Marian Interview, December 2, 2015).

Marian explains that, when she teaches, she always teaches ‘developmentally’. Marian offers an example, “I start with counting by ones, then skip counting. So today’s lesson was like a diagnostic to figure out where the kids are at now. I start from where they are at and then I push them. You have to push them; you cannot leave them where they are” (Marian Interview, December 2, 2015). I asked Marian to explain how she pushes her students. Marian replies:

Just by practicing. By practicing, they will see that it is a visual developmental continuum, and they realize that they do not always have to do it one way - next time they can solve it another way. I will be doing the same type of question until Christmas, except I will change the context of the question so it is about the teachers’ lounge vending machine. They become so comfortable with the content that they do not have to learn new content/context everyday. Now they just have to focus on the numbers and the flexibility of numbers. (Marian Interview, December 2, 2015)

Another burning question, which by now must come across as redundant to Marian, emerged from my lips, “What do you think that the average teacher like me needs, in order to do what you just did?” I expected Marian to laugh. Instead, she solemnly confesses:

I am not going to lie. You need a lot of background knowledge. The background knowledge came from teacher resources for me and by doing the questions at home. I have to see how many ways I can do them. And some of the strategies I am not comfortable doing myself, so with some resources, I would practice the questions at
home, and practice them with my own children and then with my students. So, I will throw out a problem like today’s and tell the students to show me what they can do. It is about practice, practice and practice. (Marian Interview, December 2, 2015)

4.12.7 Genie Kim’s Journal Reflections Post Joint Lesson

For Vignette Four, I will not be discussing the Mathematical Proficiencies observed from today’s joint lesson. Instead, I will briefly conclude this vignette by describing my personal reflections. Below is an excerpt of my musings, impressions, and conclusions from the joint lesson:

I could not believe what I saw during our first joint lesson. It was evident that Marian’s students were so much more comfortable and competent in mathematics than my own. Marian introduced a division word problem to the class. Neither Marian nor I have taught or introduced division to our students prior to today’s lesson. Nonetheless, there were a number of students who already knew how to use the traditional algorithm for division.

Marian’s students’ ability to problem-solve and show flexibility with numbers was striking; their critical mathematical skills sharply contrasted with my own students. My usually excitable students like Balraj were curiously quiet. Three other mathematically adept students of mine, who are normally confident and vocal in my mathematics class, sat silently during consolidation time. Clearly, from my impression, my students seemed intimidated; it appears that during the joint lesson, my students were compelled to silence.

Despite feeling somewhat discomfited by the clear distinction of abilities between my students and Marian’s, I felt a great sense of satisfaction in what had transpired with my HSP student. At this juncture, I need to share Harjot’s story. Harjot is my HSP student who visibly thrived in today’s joint lesson. He was able to solve the math problem using his reasoning abilities. Marian asked Harjot to present his strategy to the large audience. Harjot did a magnificent job: Harjot effectively conveyed to the two classes the process behind his strategy. Following his presentation, Marian complimented Harjot and validated him grandly. Harjot looked thrilled. After lunch, Harjot returned to my class, despite the fact that he was scheduled to go to his HSP class following lunch. He bashfully gave me a lined piece of paper. Harjot created a similar question to Marian’s, and he solved his question by applying the reasoning strategy he had earlier used during the joint lesson. I asked Harjot, “When did you do this?” He replied, “I did it at home”. I was in disbelief. In what universe does a student go home at lunchtime to create and solve a math problem when no such expectation is given? Needless to say, I was flabbergasted! (Genie Kim’s Journal, December 2, 2015)
At the conclusion of today’s joint lesson, I had to concede - Marian was right. The knowledge and thinking was indeed inside of the students. Now, it is high time that I reflect on Marian’s methods and practices of drawing out students’ “invisible” thinking.

4.13 Vignette Five

*February 23, 2016*

This morning, as I entered into Marian’s class, her students were energetically assembled together in groups of 3-4. The students were scattered all about, manipulating long, colourful elastic bands. Marian’s students were actively engaged in creating two-dimensional geometric figures through pulling different parts of the elastic band. From a fair distance away, Marian was seen talking to a group of students.

Marian then raised her head and instructed the class to create an isosceles triangle. All members within the groups maneuvered their bodies to fashion the said triangle. Despite appearing to possess knowledge of the defining characteristics of an isosceles triangle, the students struggled with this task. Arguments among different groups ensued, with such commands being overheard, “Move that way; it looks like a scalene triangle. You have to pull yours back more” (Lesson Observation, February 23, 2016). Evidently, the activity is not as simple as would be expected.

As Marian visited each group, she provided the students with feedback, whether to acknowledge their success, or to ask pointed questions regarding their chosen alignment. While the students waited for Marian to visit their group, they discussed the validity and soundness of their construction. Comments such as, “Yeah, this is right; two sides have the same angles” could be overheard (Lesson Observation, February 23, 2016).
4.13.1 Program Scope

The class was working on a geometry unit, and their focus for today’s lesson was two-dimensional figures - specifically the angle properties of various triangles. Marian explained to me that she wanted her students to understand the overall expectations within the geometry strand; students should be able to identify, classify and construct two-dimensional shapes according to their side and angle properties. It is the understanding of angles that Marian wanted solidified. She used this hands-on activity so her students could have a concrete grasp and awareness of angles. Marian explains, “I always go from concrete to semi-concrete to semi-abstract to abstract” (Marian Interview, February 23, 2016). She continues, “After they get a lot of practice, then they go to paper and pencil” (Marian Interview, February 23, 2016). When I asked Marian if she generally introduces her units with concrete, hands-on learning before introducing abstract concepts, Marian replies:

You cannot just jump there. It has to be gradual because kids do not get it. Sometimes it is pressure of time and report cards that make me jump, and then I regret it because they were not ready and you have lost them so it was just a waste of time. (Marian Interview, February 23, 2016)

4.13.2 Learning Environment

As I circulated around the class, I recognized Marian’s 3 HSP students, Ralph, Kerry and Sammie working together. This led me to presume that Marian had grouped her students homogeneously again. Intrigued and curious to see how the three students worked together on this kinesthetic activity, I listened in on their discussion. Each member safely contributed his/her thoughts, although, for this activity, Kerry was the dominant voice in the discussion. Kerry was convincing her group members of how to construct a 90-degree angle. At that moment, Marian joined us. Marian bent her knees and listened in to understand what Kerry was saying. Marian became visibly animated and requested Kerry to reiterate what she was doing to create the 90
degrees. Kerry, seeming to sense Marian’s approval, sat up, poising herself. Kerry then explained how she was using the tiles on the floor to help her create the 90 degree angle. Marian stopped the class, and drew everyone’s attention to Kerry’s group. Marian stated, “Many of you have been struggling to create a 90 degree angle. Kerry is going to tell you what she did to help her” (Marian Observation, February 23, 2016). Marian asked Kerry to restate to the whole class her strategy. Kerry beamed and announced in a clear voice what she had just earlier explained to Marian.

4.13.3 Constructing Knowledge

Today’s activity was evidently enjoyable for all of Marian’s students. Yet, it was also apparent that many of the tasks were considered challenging for them too. While observing her students struggle, Marian employed different groups to demonstrate how they have overcome their struggles with one another. She sent one group to explain to others the process they used to create various angles. Marian never tells the students where they have gone wrong. Instead, she probes and pushes her students to prove and justify how they arrived at their visual representation(s).

Marian questioned a student about how he knew his group’s construction was a right angle triangle. This student responded, “I know because it is straight” (Lesson Observation, February 23, 2016). This clearly did not satisfy Marian, as she continued to probe, “But how do you know that it is a right angle triangle?” (Marian Observation, February 23, 2016). Marian was observed to draw out from her students the understanding of ‘baseline’. She wanted her students to understand the importance of identifying the baseline so that they can appropriately articulate their justifications. She challenged all of her students to use benchmarks and use the appropriate
vocabulary. Throughout this activity, Marian repeatedly encouraged her students, “Prove it. Explain your thinking” (Marian Observation, February 23, 2016).

### 4.13.4 Manipulatives

Today’s math activity required Marian’s students to have some background knowledge and skill in creating a product that demonstrates conceptual understanding. This lesson was facilitated by a hands-on activity. Groups of students were given the use of large, stretchy, multi-coloured elastic bands. These colourful bands resemble bungee/shock cords, in thickness and texture, without hooks attached on each end. The use of this manipulative appeared to engage all of Marian’s students. The HSP students appeared deeply engrossed solving the problems. One would be hard pressed to find a group or individual using the elastic bands in a mischievous or irresponsible way. Each student seemed to take the math activity seriously. The colourful bands served only as a mathematical tool for the students.

This hands-on activity enabled all of Marian’s students to experience understanding and enjoyment from the task. Marian explains how, in spite of the broad spectrum of student abilities, hands-on manipulatives always have a way of captivating and capturing the students’ interest and creativity. Through such tools, Marian asserts that students’ understanding is increased. When I inquired about how often she uses manipulatives during her math class, Marian responds:

> The way my math lesson or unit progresses is I always start at the concrete level, so everyone has a manipulative, model or a tool. You can see the progression; it just happens naturally. It moves from the concrete to the semi-concrete, where now you are making models and then to the abstract. That is the flow. That is what children need to undergo before they learn and it becomes theirs. (Marian Interview, February 23, 2016)

Marian then addresses a misapprehension that is common among teachers:
I think as teachers we often go to the abstract, with junior teachers thinking the kids already have this and they understand, but no they do not. They still need manipulatives. (Marian Interview, February 23, 2016)

Here, Marian underscores for me the critical importance of manipulatives:

So they need the concrete - all of them. We think only the students who are struggling in math need it, but they all need it to express their thinking through the use of a model. A model can take any shape and form. It can be a base ten block and then a tool can be an open number line, which is semi-concrete, and then the numbers. Digits is your abstract. (Marian Interview, February 23, 2016)

4.13.5 Students’ Mathematics Communication

Marian’s students’ usage of advanced mathematical vocabulary continued to be impressive. Such terminology was heard originating from the students themselves: baseline, perpendicular, benchmark. Marian repeated and reinforced the words that emerged from her students. When Marian’s students were remiss about using specific mathematical terms, she drew it out of them through her questioning and probing.

As I stood adjacent to Marian, who was engaging a group into a debate about how they knew they had created a 45-degree angle, I observed her questioning technique. The group had successfully created the 45-degree angle, yet, Marian did not merely accept their visual portrayal to equate understanding. Marian probes, “I see your angle, but how did you know it was 45 degrees?” A student responded, “Well, I know 45 degrees is half of 90 degrees. So we made it half” (Lesson Observation, February 23, 2016). Marian asked this student to go over to a struggling group to support their thinking. I followed the student to see how he would convey this learning to others. The student walked over to the group that was experiencing difficulty figuring out how to create 45-degrees, and he casually inquired, “Hey, you know how to make a 90-degree angle, right?” This was a rhetorical question as the class just completed that challenge. Then he prompted the group to create the 90 degree angle. The student cleverly reminded them
that half of 90 degrees is 45 degrees. A few of the group members expressed understanding and agreement. Like a master teacher himself, the student directed them to the tiles on the floor to help guide their construction. At that point, the struggling group identified what they needed to do next. Understanding was achieved through peer support.

4.13.6 Mathematical Proficiency

Today’s hands-on, interactive geometry activity facilitated the development of many of the five mathematical proficiencies among Marian’s students. Firstly, conceptual understanding was fostered and deepened through the hands-on, visual construction of angles and triangles, using the elastic bands.

Strategic competence was enhanced through the nature of today’s activity. Marian’s students needed to formulate, represent and solve mathematical problems posed by Marian using the elastic bands creatively. Through Marian’s ongoing probing for proof and justification, her students were goaded and influenced to logically prove and articulate their conceptual understanding of angles.

Productive disposition was heightened through today’s activity. Marian’s students clearly had positive experiences with today’s math lesson, which ultimately functioned to strengthen and reinforce their capacity for mathematical understanding.

4.14 Vignette Six

March 9, 2016

Vignette Six will be devoted to Student Assessment, which Marian professes is a fundamental and integral component of her role in the classroom. When I walked into Marian’s class during her math block, the students were not working in their usual way; the sounds and scenery alerted me of the fact that something different was happening. There was neither
bustling nor excited chatter about a math problem among the students, to which I had become accustomed. Rather, the students were all seated quietly at their desks writing what appeared to be a test.

I spotted Marian, who was sitting at her desk looking over papers. I gingerly walked over, so as to avoid disturbing Marian’s students who seemed deeply focused on writing their test. I whispered into Marian’s ear that I recognized her students were writing a test and that I would return the following week. Marian smiled, and speaking softly, she informed me that her students were writing a “Show Me What You Know” quiz. Intrigued, I asked if she could explain what that means. Marian leaned back, getting comfortable in her chair, implying to me that she was at ease during this time to describe the meaning of this quiz. She starts, “You see, I give this quiz after every unit, which is every 2-3 weeks”. Before she can complete her description, a student was found standing in front of Marian with his quiz in hand. I took a step back and observed.

Marian placed the quiz onto her desk and quickly marked it in front of the student, providing him with immediate feedback. Marian simply remarked to the student, “Good. You got it!” Before Marian could return her attention back to me to complete her explanation, a small line-up began to grow adjacent to Marian’s desk. I wanted to catch what was happening, but I withdrew myself to allow Marian to focus on her students.

I decided to sit next to one of Marian’s students who was looking at his ruler and writing tentatively on his quiz sheet. I gently asked him what quiz he was writing. He looked up, smiled, and offered, “It’s a ‘See if You’ve been Paying Attention in Class’ quiz”. We both giggled. I thanked him and removed myself so he could have his quiet space back.

I sat on the sidelines, attempting to listen in on the one-to-one conversations Marian was having with each of her students. For those students who were on the right track, Marian quickly
checked their work and sent them off to read a book at their desks. For those students with errors, Marian isolated for them their misunderstandings. Without correcting them, she instead asked pointed questions so they could discover where they had gone wrong. The students were then sent back to make another attempt.

Later, Marian gave me samples of the written quiz to photocopy. As I looked at one quiz, I saw that the topic was ‘area’. Marian’s class was working on measurement. The quiz looked more like a test. It was on double-sided paper that was taken from a resource book, *Leaps and Bounds toward Math Understanding* (Small, 2011), which Marian explained is a diagnostic tool.

I mentioned to Marian that I had observed one of her students using a ruler while he was writing his quiz. As if having read my mind, Marian apprised me that she does not view this as cheating. Marian draws on The Big Bang Theory, one of her favourite TV series, to illustrate her point. Marian explains to me that The Big Bang Theory is an American sitcom, which primarily centres its humorous plot around the lives of a group of geeky scientists. Marian enthuses about how Sheldon, one of the genius protagonists, always uses models to help him solve seemingly impenetrable scientific/mathematical problems. Marian expresses her excitement over this; she believes this popular show will help validate the notion that manipulatives should be used by everyone. Marian then returns to my earlier question, and emphasizes that she always allows her students to use supportive manipulatives and tools, such as rulers and blocks, during quizzes and tests.

With a multitude of questions still on my mind, I was very curious to know if Marian always marked each of her students’ quizzes in front of them. Despite having witnessed this process, almost disbelievingly, I asked Marian how she has the time to evaluate and provide descriptive feedback for all 31 of her students on the spot. She responds:
I do a quick run through with those questions that reveal whether or not the kids understand the concept/skill. I believe in giving immediate feedback. I target each student so they can understand exactly what they need to understand. By meeting them on-by-one, I can figure out their thinking and what they are trying to do. (Marian Interview, March 9, 2016)

During a subsequent discussion Marian and I had about assessments, Marian made it clear to me that she uses all forms of assessments: assessment “for” learning, assessment “as” learning, and assessment “of” learning, which are found in the Growing Success document (Ontario Ministry of Education, 2010). We both grumbled about the ambiguity of the new phrases adopted, and expressed how we preferred the formerly used terms: diagnostic, formative, and summative assessments, as these descriptors were precise and unequivocal.

Marian explains that she uses all forms of assessments, and that she has established a practice of daily collecting formative assessments or ‘assessment as learning’ from her students. She then mentions, “My kids know how I always walk around and they know I am assessing” (Marian Interview, March 9, 2016). I too have come to recognize and understand this as being part and parcel of Marian’s daily praxis in her mathematics class.

I then asked Marian how she plans her lessons/topics. Marian responded, “I have an overall view of what I am doing. I usually spend about 1-2 months on one strand” (Marian Interview, March 9, 2016). I take this opportunity to inquire about how Marian plans for the school year. She explains:

We have 5 strands and each strand has 2 sub-sections. That is 10 months, give or take. I spend a lot of time on Number Sense. Only because I front-load the year with Number Sense, because once they get number sense, when it is time to do fractions, they are the same strategies. Fractions, the concepts are different, but the strategies are the same, like adding fractions and comparing fractions. So, when I know they understand these strategies, I can go through it quickly. Same thing with measurement because it is adding and multiplying. (Marian Interview, March 9, 2016)
Below is a sample of questions from a “Show me What you Know” quiz, which was created for the Number Sense unit. These questions are ones that Marian has created on her own; unlike the “area” quiz, these questions were not taken from a resource book:

1) Mario needs $263 to buy running shoes. He has saved $128 so far. How much more must he save?

2) Subtract 67 from 712 in two different ways. Tell which way you like better and why.

3) Tina wrote this to subtract 3824 from 5000.

\[
\begin{align*}
5000 & = 4999 + 1 \\
-3824 & = 3824 \\
1175 + 1 & = 1176
\end{align*}
\]

Why do you think Tina did that? Did her strategy work? Explain your thinking.

4) What are the 3 meanings of subtraction?

5) Calculate using any strategy you like. Marian then includes 5 subtraction questions.

In looking at the strategies Marian’s students have used to answer the questions, the number line as well as the traditional algorithm were used most frequently. Some of her students scored very high on the quiz, while others clearly demonstrated their lack of understanding and skill development with subtraction. Marian explains that the results from such quizzes are what allow her to determine where she needs to take her upcoming lessons next.

4.14.1 Formative Assessments

In perusing Marian’s formative assessments, the papers inside her clipboard that she writes on when circulating around the class, I readily noticed three things: 1) Marian’s notes are very simple and brief, 2) She concisely identifies what strategies students are using, and 3) Marian writes simple descriptive notes about her students.

Here are some examples of what Marian has written on various assessments: “Ralph - stuck, drawing”; “Ana is ready to move to groups 7 x 14”; “Mohammed mixed up how to cover
surface area.” Then, on another sheet, Marian lists the names of all the students who have struggled.

Marian uses these notes to guide her lessons. The jot-notes help Marian to decide what the learning focus will be for that day or the following day. When circulating, Marian always records the various strategies her students have come up with on their own. Using her notes, she intentionally selects specific students to present their thinking processes - so as to make the invisible become visible.

4.15 Case of Marian’s Students

Background to Student Interviews

To shed light onto Marian’s IBL practices and methods in mathematics, I interviewed her students. In deciding which students to privately interview, I needed to ensure that two conditions be met for my selection: 1) there exist equal representation/sampling of student abilities and 2) the students have returned to me their signed consent forms. Through employing these criteria, I was able to interview fifteen of Marian’s students. All interviews took place during non-instructional times, such as during the morning and afternoon recesses, and the interviews typically did not exceed 10 minutes in length. On two separate occasions, I interviewed two students at the same time, in one sitting. During these times, I ensured that the two students were of the same or similar mathematical abilities.

All of Marian’s students were apprised of the fact that their teacher would not have any knowledge of the information they disclosed to me during the interviews. The students were further informed that pseudonyms would be used in place of their real names. Most of the students felt self-conscious about selecting their own pseudonym and therefore asked that I pick one for them.
4.15.1 Student Experiences in Grade 4 Math

In order to alleviate possible pressure Marian’s students may feel regarding their need to express their experiences with math in Grade 5, the first question with which I opened up the interview for all of her students was, “Did you enjoy math in Grade 4?” The responses ranged from, “Not at all”, to “A lot”. Five out of the fifteen students expressed enjoyment of math in grade 4. Ten of Marian’s students admitted to not really enjoying math in Grade 4. The comments ranged from an unequivocal expression of loathing, to, “I did not like it that much”. When I inquired about why some of the students did not enjoy math in Grade 4, one student named Masoor responded, “Well, let me say that in Grade 4 it was ok, but I was not into it” (Student Interview, December 15, 2015). Another student named Veena, one whom Marian identified as needing to be in the HSP class, stated, “Last year, I felt like everything was so hard” (Student Interview, January 15, 2016).

4.15.2 Student Experiences in Grade 5 Math

All of the students provided positive responses regarding their experiences with math in Grade 5. One student named Tina, responds, “I love it and how Mrs. Marian teaches it. It is very interesting and it makes people want to learn more about it”. Tina elaborates when I ask her to explain what it is that Marian does to make math more interesting, “When she explains a strategy, she does it so everyone understands it, and they know what to do. Like when a teacher says to do something, you have to do it, but she doesn’t say it like that. She explains why you do that step” (Student Interview, January 8, 2016).

A student named Jason, who joined Mapleview in Grade 5, arrived at the beginning of the school year with a lengthy student file outlining his behavioural issues. Jason boldly states that math is his favourite subject. He explains why he enjoys math in Grade 5, “It is because
Mrs. Marian teaches so you understand it”. When I asked Jason what Marian does specifically so that he understands math better, he replies, “She pushes us to understand it. She makes us help each other and stuff”. Then Jason concludes by stating, “Math is soothing for me” (Student Interview, December 15, 2015).

Stephanie, a pleasant and amiable student, expresses that she has become a better math student compared to the past. When I asked Stephanie what Marian has done to help her become a better math student, she states, “Mrs. Marian teaches us one thing a lot and we get it in our head. And she supports us a lot”. I asked Stephanie how Marian supports her learning. She replies, “Some teachers, when you have a problem, they just give you the answer. But Mrs. Marian actually makes us struggle”. Stephanie then proceeds to talk about how Marian gives a lot of problems for the students to solve in class. When I questioned how Stephanie feels about problem solving, she responds, “It feels good because sometimes she gives us problems about her life. Like what happened last night, and about cookies, and about her friends - about real life” (Student Interview, June 22, 2016).

I interviewed one of Marian’s top math students named Bob. Bob gleams and asserts, “I enjoy everything about math in Grade 5”. Bob then exclaims, “Mrs. Marian is a great mathematician!” (Student Interview, May 26, 2016).

I interviewed Sammie, an HSP student, who is very timid and soft-spoken. Sammie shares why she enjoys Grade 5 math better, “It is more funner and we get to learn more strategies”. I say to Sammie, “But I heard that Mrs. Marian does not give you the answers. She makes you guys struggle. Is that hard?” Sammie agrees that it is hard. Then I question Sammie, “Do you wish Mrs. Marian would stop making you struggle?” Sammie immediately replies, “No”. “Why?” I curiously question her. Sammie offers, “Cuz it helps our learning much better.
and it helps us when we have a supply teacher. We can do it on our own”. I then ask Sammie how she believes Mrs. Marian has helped her to become a better math student. Sammie responds as follows, “When I was asking her (Marian) for help, she told me to try it on my own. And she told me that I could use the blocks. And then, when I did it, I would get the answer on my own” (Student Interview, June 24, 2016).

4.15.3 Comparing Grade 4 and Grade 5 Math

Veena, who earlier was heard sharing how she felt that everything was so hard in math the previous year, then confides, “Last year, the teacher only focused on the students who understood how to do math well. But this year, Mrs. Marian, if someone does not get it like me, she asks me the question, and she makes me answer it”. I inquire if this makes Veena feel uncomfortable. Veena responds, “No, because it makes me understand it more” (Student Interview, January 15, 2016).

Harpreet, a student whom Marian describes as being an academically average student, was eager to be interviewed. Marian earlier shared how Harpreet has recently made significant strides in mathematics. Harpreet enthusiastically shares that she enjoys math in Grade 5. When I inquired about Marian’s teaching methods, Harpreet offers, “Mrs. Marian is a good math teacher because she doesn’t tell us the answers. She makes us try very hard and struggle. And after she teaches us a lesson, she doesn’t tell us anything”. When I asked Harpreet how Grade 4 and Grade 5 math differ, she replies, “In Grade 4, I did not really like math. I learned strategies, but in Grade 5, math is one of my favourite subjects. I learned new strategies that I did not know before. Before, I always did the traditional algorithm, and now we learn lots more”. When I asked Harpreet if she felt confident as a math student, she responds, “Yes, I feel much more
confident this year. I didn’t like math that much before, but now I like math a lot” (Student Interview, May 26, 2016).

Mike, a gregarious and self-confident student, explains how Grade 4 and 5 math are different, “It is different because Mrs. Marian has more strategies. We are learning different ways of doing multiplication and division. We’re alway trying new strategies. If they work, we use them. If they don’t work, we don’t use them, and we try new strategies almost everyday”. I then ask Mike if he enjoys consolidation time, to which he responds, “I think it’s very useful because I see people using strategies that I couldn’t even have come up with and that’s helpful because I could use those strategies next time when I’m doing that type of question”. When I asked Mike to describe how Mrs. Marian helps him understand math, he explains, “I think she asks students to figure it out - how to do the steps one by one. But, in Grade 4, only one student was picked and that student did everything. But Mrs. Marian picks a lot of students and they do different parts” (Student Interview, December 11, 2015).

Alison, a student who always has a lovely smile on her face, explains to me how Grade 5 math is easier than Grade 4 math. When pressed to explain why, Alison shares, “It’s because we have a lot more strategies than in Grade 4. In Grade 4, we only knew the traditional algorithm, and sometimes we wouldn’t even understand that”. When I inquired if Grade 5 math is easier because Marian gives easy work, Alison quickly objects, “No. It’s because I understand it. Mrs. Marian gives us hard work, but we know the strategies”. I asked Alison to explain how Marian helps students to understand math. She explains, “Mrs. Marian tells us to use whatever strategy is easy for us, to just do it. She tells us if we struggle with any strategy, then don’t use it. She tells us to use another one” (Student Interview, June 22, 2016).
Similar to Alison’s response regarding understanding math better in Grade 5, Annette shares, “I did not really understand math in Grade 4, but in Grade 5 I understand it more”. I then asked Annette if she feels successful in math. Annette quickly exclaims, “Yes!” When I asked her how she knows when she is successful in math, Annette states, “Because now I get every homework question that we get right”. I asked Annette to tell me how this makes her feel. Annette responds, “It actually feels good because by the time we get the question to do on our own, I actually know how to do it on my own”.

Marian informs me that one of her HSP students, Kerry, really hated math. In my interview with Kerry, her countenance brightened up as she shared about her experiences with math in Grade 5, “When I first met Mrs. Marian, she taught me so much more than in Grade 3 and Grade 4”. I asked Kerry, “What does Mrs. Marian do differently to make you learn math better?” She responds, “Sometimes she gives kids harder work to do and then she says if I want to try then I can try so that I can get better at it”. I asked Kerry, “What is your favourite thing about math”. She gleefully replies, “It’s times tables because I learned my own strategy” (Student Interview, June 27, 2016)

I took an opportunity to interview Mike, a student I had interviewed earlier. I wanted to see if anything, since our earlier interview in December, had changed with his experiences in Marian’s math class. The following is an excerpted portion of our conversation:

**Interviewer:** I have interviewed you before. Is there anything new that you want to share with me about math in Grade 5?

**Mike:** That I have learned a lot of new strategies and I feel like I am ready now for Grade 6.

**Interviewer:** How do you know that you are a better math student this year?
Mike: Last year in Grade 4, like when we were doing division, for example, I was not that good in it because I didn’t understand it. But this year, since we know more strategies, we can try different ways to get the right answers.

Interviewer: How do you learn these strategies?

Mike: Mrs. Marian, she tries to make us guess on without telling us the answer of the strategy. And then somehow, some of the students come up with the answer and then we just learn on from there. (Student Interview, June 21, 2016)

I sought many opportunities to interview Ralph, an HSP student whom Marian describes as being very diligent and focused in math. This was positive news given Ralph’s ongoing, unfavourable reputation within the school for his unruly and often disruptive behaviour out in the field. It was the very last day of school that I found a small window to interview Ralph - right before lunchtime. This was due to the fact that throughout the year, at the sound of the bell, Ralph was either the first to run out of his class to play soccer, or he was held behind to serve detention with one teacher or another. Below is my brief interview with Ralph:

Interviewer: Did you like math last year in Grade 4?

Ralph: No

Interviewer: Can you tell me why?

Ralph: Because I didn’t really learn it last year. I didn’t really know how to do it, but now I do.

Interviewer: What do you think you have improved the most in math?

Ralph: Strategies like splitting. (Student Interview, June 28, 2016)

A huge smile emerged from my face when Ralph said ‘splitting’, and I found myself having to suppress my laughter. Here, it is fitting to share a telling anecdote about Ralph and the splitting strategy, which Marian, a few days earlier, related to me:

With the school year coming to an imminent close, Marian asked her students to assist her with removing and discarding all of the co-created anchor charts from around the class
Marian’s walls were filled, corner-to-corner, with math anchor charts. Marian describes how unexpectedly, her students wildly protested and pleaded with her that she not discard the math charts - they wanted to take them home. Marian expresses to me her deep shock and astonishment that her students seemed sincere in wanting to keep the charts, which had various student-generated math strategies written on them. Then tossing her head back, Marian chuckles, while recounting how she almost fell over when she saw Ralph forcefully running to the front, pushing his way through his peers, and making a loud and authoritative claim for the ‘splitting’ strategy. Marian gives me a wry smile, then states, “You know, splitting is Ralph’s favourite strategy”.

The significance and weight of this anecdote is seen when one understands that Ralph, a student who is functioning three years behind his Grade 5 peers, has earned a reputation within the school as being a troublesome student who does not care about his learning. Throughout his educational career at Mapleview, Ralph has often been found in the principal’s office for various misdemeanors.

4.16 Student Questionnaire

Background to Student Questionnaire

The majority of Marian’s students completed a double-sided questionnaire regarding their experiences with mathematics (Appendix F). In early December, during my preparation time, I distributed the questionnaire to Marian’s students. I allowed Marian to know that she could use this 40-minute block to plan, make photocopies or simply take a break. Marian stayed inside the classroom working on her plans. Subsequent to reviewing all of the student questionnaires, I recognized that the results pertaining to Grade 5 math were very positive; this occurred despite my telling the students that the information they provided would remain strictly anonymous.
Nonetheless, being cognizant of the fact that Marian was present inside of the classroom while her students were filling out the questionnaire, I began to experience some disquiet within me. I agonized about the tainted validity of the data results. Consequently, I determined to re-administer the questionnaire at a much later date. In early June, I believed that sufficient time had passed for me to re-administer the questionnaire. This lapse of time, I hoped, would provide me with two things: 1) richer and more substantive data and 2) objective results, which this time, would not be influenced by Marian’s presence. I candidly shared with Marian my need for her to leave the room during the time her students filled out the questionnaire. She conceded, acknowledging she understood the importance that she be absent during this time.

4.16.1 Responses to Student Questionnaire

With or without Marian’s presence during the administration of the questionnaire, no significant change in information occurred. The two questionnaires bore the same content and sentiments. The only difference gleaned between the two questionnaires from my data analysis was that the second questionnaire contained more elaborate details. Additionally, the second questionnaire provided more glowing comments regarding the students’ experiences with math in Marian’s class. Thus, in reporting the analyzed data, I will incorporate responses from both questionnaires.

There were commonalities in responses amongst the students regarding which math activities were deemed most enjoyable. Here are some sample tasks/activities Marian’s students listed: Strings, renaming, showing my work on the white board and chalkboard, playing with fraction pieces, using blocks, using cubes, working in groups, making number lines, doing interesting ways to solve problems, and splitting. Most of Marian’s students stated that they enjoyed word problems.
The questionnaire posed many similar questions to those asked during the student interviews. I assumed that Marian’s students would be more forthcoming in providing candid written responses, than they would providing oral responses in my presence. The common thread emerging from the questionnaire was the great distinction of experiences between Grade 4 and Grade 5 math. What is detailed below are responses that either reinforce or add to the data collected from the student interviews.

One student responds in his/her questionnaire, “Math was hard in Grade 4. In Grade 5, it’s more understandable”. The same student states, “I feel more confident now because math is more understandable” (Questionnaire, December, 2015).

Another student writes: “In Grade 4, math was done on worksheets and we did math from a book. In Grade 5, we write our math questions in our math books” (Questionnaire, June, 2016).

“I dislike nothing in math this year because everything is so easy and fun”. This same student states in a later question, “In Grade 4, I felt like I knew what to do but I didn’t understand things. I didn’t know why to do them” (Questionnaire, June, 2016).

Regarding Grade 4, a different student writes, “I felt confused because last year my teacher never taught me that good strategies like Mrs. Marian. Also, last year it wasn’t fun as math is today” (Questionnaire, December, 2015).

One student created a T-Chart to explain the differences between Grade 4 and Grade 5 math:
Another student also created a similar T-Chart:

<table>
<thead>
<tr>
<th>Grade 4</th>
<th>Grade 5</th>
</tr>
</thead>
</table>
| • Not that much fun  
• Learned less strategies  
• Nervous  
• Boring  
• Understood a little | • A lot of fun  
• Learned more strategies  
• Confident  
• Exciting  
• Understand a lot |

Many of the students repeatedly state how math is fun now and how they understand it. A student transparently confides about math in Grade 4, “I felt so bad. I thought I would get an F+ in Grade 4 because I could not understand it”. This student then says, “I feel ready for Grade 6 math with Mrs. Marian” (Questionnaire, June, 2016).

Again, with regards to Grade 4 and 5 math, a student claims, “It’s different because in Grade 5, I know what I’m doing and in Grade 4 I didn’t”. This student then writes, “That’s the most important thing for me, to know and understand what I’m doing” (Questionnaire, June, 2016).

A student writes about his/her experiences in Grade 4 math: “In Grade 4, I felt very annoyed and discouraged about math because it was in a textbook. And it wasn’t clear like my Grade 5 teacher teaches” (Questionnaire, June, 2016). A student shares about what he/she enjoys about Grade 5 math: “That you can use so many ways to answer just one question. You can
rename”. This student on his/her paper creates a question and shows multiple ways of solving it (Questionnaire, June, 2016).

To the question, “Do you feel confident as a math student?”, a student responds, “I feel confident because I keep on trying and I like math. I also don’t give up anymore” (Questionnaire, June, 2016). A student dramatically declares, “In Grade 4, I hated math. I would want to kill myself. I was always frustrated”. Then this very student proceeds to share how he/she enjoys math today (Questionnaire, June, 2016). Another student writes, “To be honest, math was my least favourite subject because I didn’t think it was important. When I came to Mrs. Marian’s class, everything was clearer and much more fun!” (Questionnaire, June, 2016). Finally, a student responds to the question, “Do you feel confident as a math student?” by answering, “Oh ho 110%. Mrs. Marian just makes it fun, easy, and she showed us all the enjoyment there is in math!” (Questionnaire, June, 2016).

4.17 Case of Genie Kim

Arguably, in life, it is not uncommon for people to find that their expectations have been met with varying degrees of disappointment or simply found unfulfilled. This was the skeptical state of mind I cultivated just prior to observing Marian: I sought to lower my expectations of what I would see, lest I become dismayed. Thus, I stubbornly made the decision to stave off any excitement despite the fact that my initial interviews with Marian sparked great enthusiasm within me. Early on, I had little doubt that I would witness everything Marian related to me through my interviews with her; all that I had learned about IBL was echoed by Marian, which she articulated with admirable ease, self-assurance and knowledge. Despite these positive sentiments, as time passed, I reminded myself not to become too excited about observing
Marian’s lessons, in order to circumvent possible disappointment. Hence, I exercised restraint and tempered myself from anticipating more than might be delivered.

A strange mix of eagerness, coupled with a deep uneasiness, progressively grew within me just days before my first classroom visit. To explain, I had entered upon my research with the express purpose of identifying Marian’s key IBL methods and strategies; however, I began to question whether Marian’s pedagogical practices were, in fact, consonant with what I had researched pertaining to the central tenets underlying inquiry-based learning. What if, unlike Marian’s well-versed responses vis-a-vis her IBL mathematical pedagogy, her style of teaching turned out to be at variance with IBL? I had committed a tremendous amount of reading and writing specifically about IBL prior to the data collection component of my research, and I suddenly began to feel apprehensive, and pondered what I would actually see during my observations.

The awaited day finally arrived. With my clipboard and recorder in hand, I would soon discover what was to transpire. To my relief and delight, I experienced feelings of amazement, incredulity, and gratitude, which pleasingly lasted throughout my year-long classroom visits. The idiom, ‘walking the talk’ became unmistakably manifest during all of my observations. All that Marian had shared during our earlier interviews clearly dovetailed with her practices inside her mathematics classroom. As well, her philosophy and practices have been revealed to align with the copious literature on IBL. There was an abundance of things I witnessed, which I was unprepared for and could not believe. Although my plate was full with what seemed to be increasing responsibilities in my own classroom and personal life, I always anticipated each upcoming visit into Marian’s classroom.
4.17.1 My Early Misunderstanding of IBL

Previous to my research of IBL in mathematics, I held a deep-seated aversion and distaste for anything inquiry-based. Through the ongoing professional development I received, my understanding of IBL became formulated as follows: The Three-Part math lesson is to be a central, structural element of the IBL process. First, the teacher is to prime his/her students’ thinking in preparation for the day’s problem. Subsequently, the teacher poses the mathematical problem to the entire class. Students then work in small groups, pairs, or individually to solve the problem at hand by recording their mathematical thinking. The teacher later gathers students together as a whole class to present their work. Herein, through the sharing and discussing of strategies, ‘consolidation’ of learning is to occur.

Frankly, I did not understand how this process would or could promote mathematical learning or skill development. My colleagues and I scathingly criticized IBL methods. We saw it as a foolhardy process that led to wasted time, classroom management issues, and the promotion of laziness among teachers. Self-righteously, I scoffed at what I believed was a harmful reform pedagogy. This personal loathing of IBL frankly stemmed from my own negative experiences. In teacher’s college, as well as in the many workshops I attended as a certified teacher, I consistently found that this method of instruction left me feeling short-changed and cheated. In these learning sessions, the teachers were expected to generate and produce all of the work (answers), with the instructors only providing vague and ambiguous responses to our questions. I remember leaving many workshops exasperated, grumbling with other attendees about the egregious use of money, given the lack of new and meaningful learning we acquired. During one of my Master’s courses, I remember a classmate vociferously expressing her indignation with the fact that we had paid a lot of money to have nothing taught nor learned. Sadly, after carrying out
multiple presentations to only be given vague or meaningless descriptive feedback, I found myself wholeheartedly agreeing with this peer.

Despite my seemingly justified reasons for detesting IBL, in wanting to be fair, I put IBL to the test with my class on several occasions. I approached it with a genuine desire to figure out whether, in fact, IBL possessed its touted merits in the classroom. My students appeared to enjoy the process, but as a teacher, I felt irresponsible in implementing IBL methods. It appeared to me that my students were not truly learning. The outcome from my inquiry-based lessons seemed to mirror my own abysmal experiences with IBL, with the only difference being that my students seemed incognizant of its lamentable effects on their learning. Again, it was not until I was able to see Marian in action that I came to understand my own failure to apprehend what IBL truly entailed, and what, if implemented properly, it could potentially reap for students.

4.17.2 A Paradigm Shift

Inquiry-based learning had been an anathema to me for two main reasons: 1) my own unsatisfactory experiences with IBL as a ‘learner’, and 2) my failure to understand what IBL really necessitated. Observing Marian weekly provided me with a whole new and enlightened perspective. To begin, I did not understand the critical role of the “guide on the side”. In learning that teachers are to adopt the stance of “facilitator”, I viewed the role as being a passive one. This was by virtue of the ongoing emphasis given by mathematics instructors that teachers are to encourage students to do the work while providing minimal instruction or guidance. Little did I conceive of the indispensable function teachers play in two very specific areas: 1) teachers’ intentional development of mathematical tasks and 2) teachers’ intentional questioning of their students’ thinking. “Intentional” clearly is the operative word. A critical turn in my own thinking occurred as I observed Marian construct well thought-out tasks and witnessed how she
questioned her students’ thinking, thereby eliciting their mental strategies. When I had run IBL lessons in my own class, I did ask my students questions, but clearly, the quality of my questions contrasted significantly with Marian’s.

4.17.3 A Renewed Perspective and Motivation

I was powerfully impacted by Marian. I was inspired and excited to teach mathematics in Marian’s commanding and effective way. In watching this expert teacher, I firsthand witnessed her students’ ongoing excitement with mathematics and readily grasped that IBL could positively influence students’ mathematical proficiency. After a few weeks of observing Marian in action, I became restless and felt a sense of frenzied excitement, as well as guilt-laden for neglecting to implement the same compelling teaching approach with my own students. I asked Marian an assortment of questions in the hopes of mimicking her IBL lessons. She kindly answered my questions and offered me several of her resources, which I cursorily skimmed. I felt equipped, armed, and ready to imitate what I assumed would be a straightforward replication of Marian’s lessons. I felt confident as I had been assiduous in studying and recording Marian’s methods and techniques for the past few weeks.

4.17.4 My First Attempts with IBL in Mathematics

My IBL lesson with my students had come and gone in a way that I did not anticipate. During the first part of my lesson, I remember feeling deeply jubilant. I carefully parroted Marian to the best of my ability. My students, in their typically excitable way, were quick to immerse themselves in solving their assigned task(s).

A feeling of deep satisfaction overwhelmed me as I circulated the room, up until I was beckoned by a student who was working independently. This mathematically-gifted student asked me to look at his work and confirm whether he was on the right track. I was highly
discomfited: he asked me a question for which I could not provide an informed response. Smiling outwardly, I deflected by speedily replying that he needed to verify this on his own.

The IBL method of redirecting questions back to students saved me, but only for that moment. My heart pounded - I had no idea what this student had done and whether he was correct or not. The time arrived when this student was to present his strategy. During this student’s presentation, the entire class looked as confused as I had felt. Not understanding his convoluted mathematical thinking, I congratulated him, and without skipping a beat, I called upon a different group of students to present their strategy. I concluded the lesson by showing the class one of the strategies I had earlier learned from observing Marian. The students looked impressed. However, it became evident to me that I gravely underestimated the nuances involved in using IBL in mathematics.

4.17.5 Personal Reflections and a Second Attempt with IBL

I felt a sense of defeat and senselessness about my lesson. I recognized immediately that I lacked mathematical knowledge to proceed with confidence and effectiveness using IBL methods. I was pensive and bleakly wondered if I would be able to reconcile my weak mathematical knowledge with my desire to utilize IBL methods. I went back into Marian’s class to do some more observing - this time, with an altered lens. I noticed Marian’s matter-of-fact honesty with her students (and with me) in admitting that she did not know all of the answers. I respected Marian’s transparency, and I reminded myself to tread likewise. I decided to begin the next math lesson afresh with a new commitment to being undisguised and candid with my own students - this helped to remove a huge psychological burden from me.

The opportunity arrived for me to redress my earlier evasive behaviour. With fresh resolve, I presented a new mathematical task. My students worked individually, in pairs, and in
small groups to tackle the problem. While circulating around the room, incredulously, I experienced the same feeling of helplessness. I could not understand what a number of my students were doing. I was afraid to sort their strategies, which Marian had done so easily in her class. There were only a few work samples I felt had made any sense to me. How could I be transparent when my students looked to me for direction and leadership? It was not a case of failing to understand “one” strategy, but rather “numerous” strategies. Not wanting to disappoint my students nor humiliate myself, I concluded that I was ill-prepared to expose my alarming ignorance.

During consolidation time, I was having a hard time understanding my students’ strategies, particularly those students who were weak in math. I dealt with my confusion and uncertainty just as I had done a few days earlier, by simply thanking each group and calling upon the next group. My math lesson once again was grossly misspent. To circumvent wasting more of my students’ learning time, I decided to revert to my tried and true method of teaching mathematics – explicitly teaching them what I knew how to do.

4.17.6 Back to Data Collection and a Forthright Discussion with Marian

I decided that I needed to put any thoughts of implementing Marian’s IBL strategies on the back-burner. After licking my wounds, I decided to place my focus on simply observing Marian and collecting data for my study. Nevertheless, after each class observation, I could not suppress my inspiration to copy Marian’s effective methods.

I finally approached Marian and let her know about my personal struggles. She smiled, reassuringly, and reminded me that she had been formally trained for an extensive period and had many years of experience using IBL methods in mathematics under her belt. Marian then offered me advice that helped to set me on a whole new trajectory.
Marian advised me to do something that was of critical importance: start with basic ‘number sense’. Marian clearly articulated to me the power of having students understand numbers and their flexibility. I soon discovered that I had jumped the foundational steps needed for my students to experience engagement and proficiency in mathematics. Armed with this knowledge, it became evident that I had a lot of homework to do.

4.17.7 Lunchroom Musings

During lunch with some of my colleagues, a variety of animated conversations took place; I felt ambivalent about changing the light-hearted mood to one of ‘shop-talk’. However, I could not resist. I asked Marian about the questions she had used with her students when teaching the ‘moving’ strategy for addition. A marvelous discussion ensued with everyone around the table actively engaged. My sense of inadequacy greatly diminished as I noticed how my colleagues, whom I respected, sounded just as lost as I was feeling. Our subsequent lunch gatherings became ones where we would anticipate Marian to provide us with a new math lesson/insight. I sat back one noon hour and smiled at the thought of how we were privileged recipients of such wonderful learning. I was further excited and relieved about the fact that my friends/colleagues possessed the same kinds of legitimate yet vacuous questions as I did.

A natural question arose from one of the teachers: Why have we not been given professional development on these important knowledge/skills? Marian’s response surprised all of us. The Ministry of Education, according to Marian, hired people who were not math experts to train teachers in mathematics. An “aha” moment occurred for me. This explained so much about the ineffective training teachers were given. Marian groused about the fact that Collaborative Inquiry Learning for Mathematics (CILM), a series of mathematics workshops that
were offered to teachers in the past, would continue - again instructed by individuals who lacked proficiency in mathematics.

4.17.8 Another Attempt at IBL in Mathematics

With the many informal math lessons (during lunch) I was fortunate to receive, coupled with my direct observations of Marian in action, I again felt primed and prepared to make another attempt to test out Marian’s IBL techniques with my own students. This time, I approached my lesson differently. I did not throw out a random problem for my students to solve, one in which my students and I were unprepared to confront.

At the end of this lesson, I was stupefied. Throughout the lesson, I felt that I was in control; I was able to figure out what my students were aiming to convey as they stumbled to articulate their mathematical thinking. I became excited when I recognized the strategies my students presented. In my state of exhilaration, I interjected while my students were speaking. Upon reflection, I realized I did this a little too often. Despite my developing and increasing grasp of IBL methods, I was confronted with a whole new challenge, one which I never envisioned occurring.

4.17.9 A Creature of Habit

Upon further reflection, I became aware of my unwitting role in undermining the very essence of inquiry-based methods, which gives it its inherent value: allowing students, through exploration and investigation, the space to meaningfully construct their own learning. Despite all of the spectacular things I learned about IBL, I became cognizant of the fact that I am an unrelenting creature of habit. To explain, notwithstanding the copious opportunities I had to observe Marian beautifully execute IBL in her mathematics class, which I wholeheartedly set my
mind to imitate, I realized that I seemed to unconsciously resort to my customary style/methods of delivering mathematics: teacher-directed.

I have established the new practice of having my students solve the mathematical task/problem on their own, without any assistance from me. This is a tremendous improvement for me, given that, in the past, I would enthusiastically demonstrate for my students the most efficient method of arriving at the answer and soon thereafter have my students practice and hone “my” given strategy. Nonetheless, an insidious roadblock sets in, which sabotages my efforts to experience further progress. Once my students are called up to the board to show the process of their thinking to the class, an irrepressible habit kicks in. I cannot believe how difficult it is to restrain and suppress my propensity to take control when I feel that a student has inadequately expressed his/her thinking.

Whenever students are finished explaining their strategy, while leaving significant gaps in their explanation, I find myself jumping in and eagerly taking over. This happens before I can even stop myself. I am fully aware that I need to allow the student who is presenting his/her strategy an opportunity to clarify areas that have been vaguely expressed, or better yet, have their peers ask for clarification. However, instead, I find myself thanking the student and sending him/her to the carpet as I re-articulate what I believe the student meant to convey. At first, I did not apprehend how unsound this inclination of mine was, until I continued my observation of Marian. Notwithstanding the noteworthy evolution taking shape with my IBL pedagogy, it soon became evident to me that I still have a long way to go before I can successfully implement IBL with my students. Despite these great challenges facing me, I am determined to press on, with a tenacious resolve and confidence.
Chapter Five: Interpretation and Discussion of Findings

5.1 Introduction

This chapter presents a summary of my study and highlights the significant conclusions drawn from the data presented in Chapter Four. A discussion of the research will occur in four parts. First, I will discuss the case study in relation to my two research questions. I explore how the findings from my case study align with the key components of the Ten Dimensions framework. While situating my findings within McDougall’s (2004) framework, I describe the common themes and major findings that have emerged from my case study. Moreover, mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001) will be addressed in relation to Marian’s IBL practices, which serve to promote her students’ efficacy in mathematics. Implications of these findings and suggestions for future research are considered. Finally, limitations of my study are addressed.

5.2 Research Questions

The guiding research questions for this study, as presented in Chapter One, are as follows:

1) How does an experienced Grade 5 teacher in IBL implement inquiry-based learning strategies/practices in her classroom to promote mathematical understanding and proficiency among her students?

2) What are some challenges teachers of Grades 4 to 6 may face when implementing IBL in their mathematics class?

Each research question will be discussed in relation to the findings from my observations, interviews with Marian, interviews with her students, and my own personal reflections. This will
concurrently be linked to existing literature on inquiry-based learning. Additionally, I will situate my research questions within the Ten Dimensions of Mathematics Education; this conceptual framework becomes practical and useful for understanding the effectiveness of my colleague’s implementation of IBL practices within her mathematics class.

5.3 Discussion of Each Research Question

5.3.1 Question 1: How does an experienced Grade 5 teacher in IBL implement inquiry-based learning strategies/practices in her classroom to promote mathematical understanding among her students?

There are both explicit and subtle inquiry-based practices and skills teachers must employ to effectively develop and advance their students’ mathematical understanding and proficiency (Barrow, 2006). From my investigation, it becomes apparent that Marian has, at her disposal, a medley of refined techniques and strategies that she regularly draws upon and executes when teaching mathematics to her Grade 5 students.

I shall enumerate and detail the key IBL strategies/techniques Marian implements to promote mathematical understanding among her students by situating them within the Ten Dimensions conceptual framework. The Ten Dimensions of Mathematics Education (McDougall, 2004) are highly fitting for elucidating and evaluating Marian’s key strategies, as each dimension is synonymous with the essential components required for improving teachers’ instructional repertoire in a reform classroom.

I will highlight six of the ten dimensions, which were found to be central in Marian’s IBL repertory: 1) Constructing Knowledge, 2) Students’ Mathematics Communication, 3) Student Tasks, 4) Manipulatives, 5) Teacher Attitude and Comfort with Mathematics, and 6) Meeting Individual Needs. These six dimensions will serve as major themes/categories to
illuminate how Marian implements her IBL techniques/strategies to increase her students’ understanding and achievement in mathematics.

I have chosen to tease apart and categorize Marian’s strategies into constituent parts, notwithstanding the fact that many of her IBL practices naturally blend and overlap. By discretely classifying those practices that inherently should be conflated, I hope to clearly isolate and highlight Marian’s key methods/strategies that promote mathematical understanding among her students.

5.3.1.1 Constructing Knowledge

McDougall (2004) states that student knowledge should be constructed through the appropriate use of questioning techniques, ones that operate to deepen students’ mathematical thinking. In keeping with McDougall’s suggestion, at the heart of Marian’s IBL practice is her questioning techniques. In addition to her questioning techniques, Marian utilizes two other key inquiry-based instructional strategies to help construct her students’ mathematical knowledge: making her students’ invisible learning become visible, and encouraging her students to ‘struggle’. ‘Repeated practice’ is another key strategy Marian utilizes to help construct her students’ mathematical knowledge; however, it needs to be mentioned here that ‘repeated practice’ is not inquiry-based in form nor function. These four subthemes will serve to illustrate the means through which Marian constructs mathematical knowledge and understanding among her students.

5.3.1.1.1 Questioning Techniques

Evidence from this study reveals that questioning is a key technique and strategy central to Marian’s IBL pedagogy in mathematics. Marian asks meaningful questions daily in her mathematics class. Marian expresses that her systematic custom of questioning all aspects of her
students’ thinking has important implications for mathematical understanding and achievement. Marian uses her questions strategically to elicit from her students their unformed ideas that require structure and formulation. For example, Marian presses for her students to explain each thought process involved in their solution method. She poses strategic questions to identify her students’ starting points, and from there, she proceeds to map out their invisible understandings. Marian utters, “I know it sounds crazy, but it is based on where the children lead me”. Marian’s questioning practice bears resemblance to Watt and Coyler’s (2014) definition of ‘blended inquiry’, where she skillfully offers a balance and flexibility between teacher and student direction.

Marian elicits varied and sophisticated mathematical thinking from her students through her meticulous questioning techniques. Marian shares that she often needs to ask intentional and deliberate questions to elicit what she is looking for. Through her questioning, Marian always probes her students to consider whether a better and possibly more efficient way exists to arrive at the answer. Marian believes the more questions one asks, the deeper students can delve into the mathematics, and more mathematical understanding is acquired by her students. Marian customizes, constructs, and scaffolds her questions, which function to guide and assist her students’ mathematical thinking. Marian’s questioning practices and tactics find agreement with Sanchez’s (2013) work. He speaks at length about the important role of questioning in promoting students’ mathematical understanding.

Marian subtly, through her questioning techniques, scaffolds for her students so they are provided with enough support to move forward. Marian’s scaffolding technique is consistent with Henningsen and Stein’s (1997) suggestion that teachers should scaffold for their students by
asking meaningful questions. These authors argue that the primary goal of scaffolding is to help fuel students’ development of conceptual understanding.

Henningsen and Stein (1997) insist that, when meaningful questions are asked, teachers must hold back from providing a solution method for their students. Marian’s pedagogy is consistent with Henningsen and Stein’s (1997) recommendations. Marian emphasizes that it should never be about teachers showing or telling students how to do something, but rather it must be about letting students figure out their own way. Marian stresses, “The more you show them, the less it becomes theirs and they do not own it, so they will not remember it”.

Marian asks a number of different types of questions to prime her students for learning. Marian is steadfast and staunch in both her conviction and practice that questions, whenever feasible, should be ‘open’ in nature. Marian defends her firm belief that all teachers should regularly use open-ended questions, by offering this key explanation: open-ended questions enable multiple access points. Marian argues that these assorted methods of tackling a problem, serve to engage and invite students of all abilities to participate in the solution process. Marian believes that these multiple solutions become the fundamental springboard for the development of students’ mathematical understanding.

Marian describes, “My questions are so open that anyone in the class can answer them. It is not a question that anyone can do in a few seconds. It is a question that requires them to think and reason”. This is analogous with Sanchez’s (2013) argument that open-ended questions lead to conceptual understanding. Sanchez further purports that open-ended questions hold the potential to expose students’ misconceptions.

Marian is disinclined to give her students answers when solicited by her struggling students. Marian’s students are cognizant that their teacher will not provide them with an answer,
even when repeatedly requested. Rather, Marian’s students are well versed in knowing that their teacher will wait, very patiently, until an answer proceeds from their lips, even if it requires for them to wait until the following day.

Marian is tenacious in her practice of soliciting her students’ thinking through the skillful questions she poses. When probed about what she does when students show errors in their understanding, Marian explains that she never tells students they are incorrect; rather, she will ask, “What do you think will happen if we do this?” She then elaborates, “You almost have to anticipate what the stumbling blocks will be so you know what questions to ask”. Marian’s practice is in line with Sanchez’s (2013) argument that teachers should be discouraged from providing solution methods for their students.

Through her ceaseless questioning, Marian enables her students to formulate their own answers. She expresses that, the more questions one asks, the deeper one delves, and the more students understand. According to Marian, teachers typically relent and provide answers to their students when a reply is not forthcoming. Marian is unwavering in her approach: she poses a question and merely waits upon her students. To the novice observer, it may appear that Marian’s ‘wait-time’ holds some magical elixir enabling her students to supernaturally arrive at the answer. Yet, there is more to this ‘question-and-wait’ process that might easily escape the uninitiated eye: Marian’s secret lies in her ability to ask incisive, thought-provoking questions.

Similar to Scardamalia’s (2002) delineation of inquiry-based learning, Marian’s primary method of constructing mathematical knowledge is to place her students’ questions, ideas and observations at the centre of their learning experiences. Marian regularly asserts that her students’ queries always guide her next steps. Marian believes that the fundamental role of the teacher is that of an inquisitioner; she likewise asserts that students must continually practice the
art of questioning their teachers, peers, in addition their own thinking. The ideal scenario for Marian is to witness students engage in student-led mutual questioning/debates, unprompted by the teacher. For this to occur, Marian insists, teachers must provide substantial modelling for their students – how to professionally and respectfully question one another’s thinking. This ongoing and persistent questioning cycle, Marian contends, is what permits students’ amorphous, unformed understanding(s) to become visible.

5.3.1.1.2 Making the Invisible become Visible

Notwithstanding the potent influence inherent in questioning, the results from this study suggest that questioning is an insufficient strategy to effectively construct students’ mathematical knowledge. Questioning must be complemented by a technique that is easily overlooked: recording students’ thinking. In doing so, Marian argues students’ invisible thinking becomes visible, thus helping to make explicit their conceptual understanding.

Making the ‘invisible’ become visible is Marian’s principal modus operandi when seeking to construct her students’ mathematical knowledge. To draw out her students’ invisible mathematical thinking, Marian asks a series of pointed questions, which she records on the board for everyone to see. Marian believes that this process of recording her students’ invisible thinking and making it visible, helps her students build their mathematical knowledge and understanding. Therefore, Marian’s practice is to always model for her students the process of recording their mathematical thinking to make it discernible and manifest.

Some common questions habitually heard emanating from Marian include, “Is there another way to do this?” “What is the question asking you?” “What are you thinking here?” “Show me, what were you doing here?” Through this complementary questioning and recording process, always operating in tandem, Marian forms and fashions her students’ invisible learning,
thus making it visible. Embedded within this process, we frequently see Marian’s students ‘struggling’ to develop mathematical understanding.

5.3.1.1.3 Encouraging Students to Struggle

The notion of permitting students to “struggle” in their learning for extensive periods, especially those who seem profoundly confused and hapless, may seem to be pedagogically counterintuitive – perhaps even harmful. Yet, this is a significant part of Marian’s strategy to promote mathematical understanding, which she evenhandedly imposes on all her students. Without foreknowledge of this unconventional practice, I was frankly troubled when I observed this event firsthand. During my initial observation of Marian’s questioning and recording sequence, I was startled with how Marian did not relent when a student, Aisha, could not articulate her thinking to the class community during consolidation time. Marian was uncompromising as she questioned Aisha and waited. Just as bewilderingly, Aisha’s classmates seemed to know they were not to intervene. Aisha appeared to have a blank look on her face, yet Marian persisted by alternating between waiting and tweaking her questions.

Initially, I was aghast and desperate to ‘save’ this child. However, to my surprise and relief, Aisha eventually offered a response that adequately satisfied Marian. Later, I raised this topic with Marian. Her response placated me. The process of struggling and wrestling, according to Marian, is what compels students to actively make sense of their learning.

Marian’s practice finds support from Hiebert and Stigler’s (2004) study. These authors conclude that teachers must learn how to avoid intervening and giving answers to students, which was commonly found among American teachers. Instead, these researchers encourage teachers to provide students with opportunities to think deeply about mathematical concepts without their intervention.
Accountability is vital to Marian. Marian expresses her concern with how students, who need remediation, have become accustomed to a learned helplessness. Marian believes this learned helplessness is nurtured by teachers who, in wanting to be sensitive, unwittingly become complicit in cementing inadequate skills to their failing math students. These well-intentioned teachers, seeking to be sympathetic to their struggling students, neglect to challenge them or hold them to account. I soon discovered my inadvertent culpability in fostering a learned helplessness among my own remedial students. All of Marian’s students understand the ‘struggling’ process is connected to their learning. In like manner, Boaler (2015) discusses the importance of struggling to the learning process. She references emerging literature surrounding the brain and argues that when one struggles through challenging and engaging conceptual problems, the brain grows. Furthermore, Boaler maintains that teachers and students must adopt the mindset that the process of struggling and making mistakes is productive to one’s learning.

The interview responses from Marian’s students corroborate the value and effectiveness of her seemingly unconventional strategy: compelling her students to struggle. One student, Jason, responds to a question asking him what his teacher does specifically to help him learn math better by stating, “She pushes us to understand it”. Responding to the same question, another student, Stephanie, comments, “Some teachers, when you have a problem, they just give you the answer. But Mrs. Marian actually makes us struggle”. The response of a timid HSP student, Sammie, validates Marian’s exacting practice.

In my interview with Sammie, I inquire if she finds it hard when Mrs. Marian makes her struggle. Sammie confesses that struggling is hard. When I asked Sammie if she wishes Marian would stop this practice, she immediately replies, “No”. I questioned Sammie about her
unexpected reply, to which she offered, “Cuz, it helps our learning much better, and it helps us when we have a supply teacher. We can do it on our own”.

Similarly, a student named Veena, who admitted to struggling all throughout Grade 4 math, shared how in spite of feeling uncomfortable when Marian makes her struggle, that she appreciates it. Veena declares, “If someone doesn’t get it like me, she (Marian) asks me the question, and she makes me answer it. It makes me understand it more”.

During the incipient stages of my research, I naively believed that I could simply imitate and replicate Marian’s IBL methods in my own math class. Although, I determined that I would bypass only the ‘struggle’ piece. I privately questioned why the kids needed to struggle. With the passage of time, I have come to discover the importance of having students wrestle and struggle to make sense of the math. I have also come to realize that ‘mimicking’ Marian is much easier said than done. This will be addressed at length during the discussion of my second research question.

5.3.1.4 Repetition and Repeated Practice

Once Marian makes her students’ mathematical thinking visible, through the continuous use of her penetrating questions, Marian is habitually observed to reiterate the newly constructed knowledge for her students. Marian consistently reiterates, in concrete terms, what the learning is, only after her students have discovered and identified it on their own.

The process of repetition may appear antithetical to those who staunchly uphold the inquiry-based learning philosophy. Yet, Marian boldly declares that ‘repetition’ and ‘practice’ of procedural knowledge are requisite vehicles for deep mathematical understanding to occur. Marian explains, “The more (students) are practicing, the more it is transferring over to new
situations. Transferring is the application”. Marian explains that, unless students can transfer or apply their thinking elsewhere, understanding has not occurred.

Hiebert and Stigler’s (2004) study lends support to Marian’s practice of encouraging ‘repeated practice’. These authors suggest that a balance is needed where time must be concurrently spent on practicing basic skills along with developing conceptual understanding. This also aligns with McDougall’s (2004) suggestion that teachers need to incorporate tasks that are of two varieties: procedurally-based and conceptually-based.

Marian explains that students have different thresholds for learning, with some becoming enlightened to a concept much later than others. This becomes Marian’s rationale for having her students practice and repeat their learning. Marian shares that, what she does on day one, she will do again on day two. She argues that it is about continually building, incrementally, in order to have the majority of her students develop conceptual understanding.

Automaticity is one of Marian’s key objectives when teaching mathematics; she emphasizes this point by stretching her words, “It really is through practice. Even my students know this. They know that once they have learned something, now comes the practice.” Marian underscores the point that her students will always practice until it becomes automatic. Marian defends her practice by arguing, “It almost looks like rote memorization, but it is not. It is a proficiency that comes through understanding: understanding at the conceptual level, and at the foundational level”.

Having Marian express this helped me to overcome my deep-rooted misconception about IBL. I assumed that IBL methods were restricted to exploration and the tackling of new problems, without opportunities embedded for practice. Marian explains that the process of repetition and practice lends itself to ‘pushing’ one’s students so they do not remain stagnant in
their learning. She contends that providing extensive practice, both written and oral, allows her to teach for ‘depth’ instead of ‘breadth’. Marian believes repetition cements students’ mathematical understanding, thus, expanding their potential.

5.3.1.2 Students’ Mathematics Communication

Marian believes that student communication is a cornerstone of mathematical learning; she argues that communication must be actively guided and practiced in a purposeful way. Communication for Marian comes in three main forms: 1) asking meaningful questions, 2) offering critical feedback to one’s peers, and 3) proving and defending one’s mathematical solution(s).

Marian contends that students must be educated in how they communicate with one another. What teachers might be presumptuous about concerning their students’ communication skills, Marian is not. A case in point is how Marian always explicitly models for her students a skill that teachers often take for granted: how to ask for clarification when understanding is vague. By asking questions such as, “Can you repeat what Ayub said in your own words?” or “Explain, what does he mean?” student understanding is verified and achieved. Marian’s practice aligns with McDougall’s (2004) belief that the construction of mathematical ideas arises through student communication. Similarly, Marks-Krpan (2013) contends that communication is a critical component to learning, as the process of discussing mathematics is what helps crystallize understanding.

For Marian, rich mathematical discussions occur during consolidation/congress time. Marian believes this is where she is able to understand and correct her students’ faulty understanding. Marian discusses how during congress, students’ misunderstandings come to the fore. She explains, “When we congregate, that is where you pick up all the misconceptions”.

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Marian believes mathematical discussions should ideally stem from the students. She argues that mathematical dialogue should primarily occur among students within the class, where questioning one’s peers becomes routine and commonplace. Marian argues that embedded within students’ mathematical discourse, is the need to be critical of one another’s thinking. Hence, Marian is explicit in modelling for her students, the methods of critically challenging one’s classmates. My observations of Marian’s students offer proof regarding her contention. Marian’s students are not shy about critically challenging their peers – nor are they concerned about leaving their teacher unscathed.

‘Justifying’ and ‘proving’ one’s mathematical solution is another one of Marian’s teaching strategies. A significant part of Marian’s lessons involves her students having to persuade, prove, and justify their mathematical positions to their class community. Marian emphasizes that, when her students ask her about the validity and soundness of their answers, she facetiously replies that she is uncertain. Marian then cajoles them, “Let us try it”. Marian feigns an ignorant stance, as this is her clandestine technique to coax her students into proving and justifying the accuracy of their own answers. Marian explains, “I want the student to be the one to prove it to the other students in the class. Then you hear the students exclaim that it works”.

This practice corresponds with Baumert et al.’s (2010) assertion that teachers should not declare the accuracy or inaccuracy of student responses. Rather, they suggest teachers must challenge their students to evaluate the validity of their solutions through engaging in peer discussions.

Marian describes how, by having students prove and justify their understanding to one another, it forces them to truly understand the math. Marian continually pushes her students to prove and justify their positions; her students are always directed to convince their peers. Marian
declares, “I am always pushing for the thinking, the thinking”. Marian believes that math must make sense, and she argues that the process of verbally proving and justifying the solutions to one’s peers is what increases the likelihood that students will acquire deep mathematical understanding.

5.3.1.2.1 Student Tasks

There is much research surrounding the pivotal role of ‘student tasks’ and its direct relationship to student learning. Hiebert et al. (2005) claim that, when teachers effectively use mathematical tasks, an increase in student achievement is witnessed. Consistent with this assertion, Marian’s students were consistently found to exude confidence and competence while they effectively engaged with various mathematical tasks. Maaß and Doorman (2013) argue that IBL supports the development of important competencies such as problem-solving and critical thinking skills, which are foundational to students’ success in mathematics.

We see how Marian increases her students’ understanding in mathematics through her systematic practice of incorporating various open-ended tasks, requiring her students to use their critical thinking skills. A noteworthy distinction was witnessed between Marian’s students’ abilities and my own students’ mathematical capacity during our joint lesson: Marian’s students were observed to be in greater command of the problem-solving task, in comparison to my own students.

Implementing engaging tasks, with the right level of challenge, is a fundamental skill that seems effortless for Marian. During the joint lesson, both groups of students appeared highly engaged and motivated with the challenging problem/task presented by Marian. This is consistent with Raphael, Pressley, and Mohan’s (2008) findings that high-level tasks have an ability to sustain students’ engagement while providing maximum academic instruction.
Marian is averse to having herself or others implement task-specific activities that are ‘closed’ in nature (offering only a single solution method). Marian desires for her students to discover that the varied strategies they generate can be used almost anywhere. Marian wants her students to understand that the strategies they have unearthed can move from one concept to another concept. Thus, Marian’s well-fashioned tasks serve her objective of enabling her students to discover how their wide-ranging strategies and concepts are transferable into new contexts. Marian equates transferability of skills and concepts with understanding.

Marian’s intentional design and implementation of rich mathematical tasks negate her need to heed Doyle’s (1988) cautionary words. Doyle warns that, notwithstanding a mathematical task appearing to be rich in content, teachers’ improper implementation can largely nullify its richness by transforming it into a low-level task. Marian’s proficiency with mathematics enables her to administer rich tasks that are appropriate for garnering deep new learning from her students.

Marian’s confidence with mathematics also enables her to evade Gainsburg’s (2008) findings related to teachers’ task implementation. Gainsburg proposes that teachers experience uneasiness about over-challenging their students and opt more often to under-challenge them. In my practice, I am guilty of Gainsburg’s indictment, as I often ‘water-down’ my mathematics program for fear that my students will become overwhelmed and disengaged. Marian, contrariwise, appears to possess no such reservations or fears. She administers various levels of challenges, trusting and believing that her students will discover their own solution method, which they will then be responsible for communicating to the whole class.

Marian’s method of selecting learning tasks is very straightforward. She looks at her students’ work to find her inspiration for the upcoming lesson(s). When Marian notices similar
errors, especially among her struggling students, she determines where to place her focus, and forthwith contrives rich and engaging tasks that will help eliminate or at least mitigate these common errors.

Marian selects open-ended tasks that enable a variety of strategies to be utilized. Marian is tenacious and unwavering in her belief that the solutions must come from the students themselves. She always reiterates to her students that there are multiple strategies and various ways to arrive at an answer. Through Marian’s regular practice of using open-ended, problem-solving tasks, she effectively creates ‘rich’ tasks that permit multiple entry-points and deep mathematical learning to occur. This parallels Doyle’s (1988) recommendation and description of the benefits of high cognitive-demand tasks, which allow participation from students of varying abilities. With these open-ended tasks, which frequently involve the use of manipulatives, Marian always invites a multitude of student-generated solutions.

5.3.1.2.2 Manipulatives

McDougall (2004) discusses the importance of using manipulatives and technology to aid students’ mathematical understanding. Marian often embeds technology into her students’ mathematical learning experiences. She shares, “I am always looking up new educational apps and activities to engage my students”. Consistent with McDougall’s recommendation, Marian’s lessons are consistently supported by hands-on activities, whether they come in the form of technology or in the form of tangible objects to be manipulated.

Marian maintains that hands-on manipulatives become an inviting entry point for learners of all abilities. She further explains that the process of manipulating such tools in an explorative and investigative manner serves powerfully to increase students’ conceptual understanding. Hence, Marian always starts at the concrete level, where each of her students has, at his or her
fingertips, a manipulative, model, or a tool. Marian seeks to craft a natural progression, from the concrete, to the semi-concrete (where students are making models), and then to the abstract. This sequence, Marian contends, enables her students to experience confidence and control over the inquiry process of learning.

Additionally, Marian explains that children need to undergo this progression, from concrete to abstract, before they can gain conceptual understanding, which facilitates the learning to become their own. Marian concludes that all students need the ‘concrete’, not just our struggling learners. She proposes that these tangible manipulatives help to stimulate and assist with students’ conceptual understanding.

5.3.1.2.3 Assessments

Marian’s ongoing assessments always inform her practice. She is keen on ‘formative’ assessments. Whenever students are engaged with a task, Marian regularly circulates around the room to gauge (assess) whether her students demonstrate understanding. She frequently administers quizzes as an informal ‘check-up’ to determine if her students possess any misunderstandings and errors in their thinking. Marian explains that her formative assessments allow her to provide immediate feedback to her students.

When identifying errors that arise from student assessments, Marian is loath to directly giving answers when providing feedback to her students. Instead, she prompts her students to discover where they may have taken the wrong path. Marian redirects her students by offering guiding statements, such as, “Maybe you need to check something”, “I don’t know that I quite follow you”, or “I follow you all the way up to here, but I do not follow you from this point”.

Marian’s formative assessments serve as teaching opportunities to help correct her students’ misunderstandings. Using her assessments, Marian targets her lessons to address
individual needs. Marian believes this ongoing practice of providing immediate feedback helps to ensure the accuracy of her students’ understanding.

5.3.1.2.4 Teacher Attitude and Comfort with Mathematics

McDougall (2004) states that teachers’ attitudes and comfort with mathematics influence students’ learning. This observation is consistent with the results from this study. Marian’s robust mathematical content knowledge (MCK), along with her positive disposition towards mathematics, are reflected in her students’ confidence and favourable sentiments about mathematics. The overwhelmingly positive responses rendered in both the student interviews and questionnaires, from whom ethics consent was provided, give credence to Marian’s positive influence on her students’ opinions and attitudes towards mathematics.

A sharp contrast and disparity of experiences and sentiments were noted regarding Marian’s students’ experiences with math in Grade 4, compared to their experiences with math in Grade 5. All of Marian’s students expressed positive feelings about math in Grade 5, with many sharing that they did not enjoy math in Grade 4.

Marian’s comfort and strong mathematical abilities enable her to take risks with her students, and she encourages her students to do likewise. At no point does Marian appear frazzled or uncertain when her students pose her with questions that stray from her scope of comfort.

Marian, in most circumstances, admits to having foreknowledge of the kinds of strategies her students may generate from the tasks she presents. If the expected strategy(s) is not forthcoming, Marian will, by way of her intentional use of numbers, create a situation that propels her students to figure out the strategy she wants them to discover.
Dewey (1938) maintains that teachers need to trust in their knowledge and skills so they can appropriately nurture a sense of ‘inquiry’ for their students. As espoused by Dewey, Marian’s ostensible facility and self-assurance with mathematics, is manifested in her students’ confidence and eagerness to engage in mathematical activities.

5.3.1.2.5 Meeting Individual Needs

In any given classroom, teachers will find that their students’ needs are immense and varied. How does one meet a diversity of individual needs that are so vast? Marian states that, when differentiating instruction, one must do so using the overall expectations that are found within the curriculum documents. The overall expectations, according to Marian, are what permit the ‘big picture’ to be seen. This enables teachers to adequately teach within each child’s zone of proximal development. Marian argues that this method mitigates student frustration, while permitting sufficient mathematical challenge.

It was noted in Chapter Four how Marian’s HSP students appeared, “complacent, comfortable, and satisfied members within the class”. As each student’s assorted and wide-ranging needs were met through Marian’s differentiated tasks, her students were regularly observed to be actively engaged in every inquiry problem presented to them. The foregoing are just a few examples of how Marian designs her reform-based program to meet her students’ individual needs.

5.3.1.2.6 Mathematical Proficiency

The National Research Council (2000) published a report identifying the importance of ‘mathematical proficiency’ for anyone to experience success in mathematics. Kilpatrick, Swafford and Findell (2001) define mathematical proficiency as encompassing five strands: 1) conceptual understanding, 2) procedural fluency, 3) strategic competence, 4) adaptive reasoning,
and 5) productive disposition. According to these researchers, when all five dimensions work together interdependently, mathematical proficiency is developed. Marian’s execution of IBL methods in her mathematics classroom enables her to achieve each of these five dimensions.

First, Marian’s skillful questioning techniques help guide her students in their development of conceptual understanding. Marian’s astute and well thought-out questions help scaffold for her students various understandings of mathematical concepts, operations and relations.

Through incorporating ample opportunities for repeated practice, Marian ensures her students have achieved procedural fluency. Extensive repetition permits Marian’s students to demonstrate flexibility, accuracy, and efficiency in carrying out various mathematical skills.

Next, in Marian’s practice of making ‘struggling’ an obligatory process in her students’ experiences with mathematics, she develops their strategic competence. Through compelling her students to engage in the continuous exercise of struggling to make sense of the math, Marian increases her students’ ability to formulate, represent and solve mathematical problems.

Marian strengthens her students’ adaptive reasoning skills through her insistence that students engage in ongoing mathematical discourse and debate. Thus, this dialogic process, which Marian makes mandatory, increases her students’ capacity for logical thought, reflection, explanation, and justification.

Lastly, Marian promotes her students’ productive disposition towards mathematics through encouraging her students’ regular use of mathematical manipulatives in an exploratory and investigative manner. Throughout my observations and interviews with Marian’s students, it was evident that her entire class possessed positive attitudes about mathematics as they engaged in various inquiry-based learning activities.
Each dimension is an aspect of a complex whole. When synchronized, they address all the critical components necessary for mathematical understanding: knowledge, skills, abilities and beliefs. Using various IBL devices, Marian integrates all five dimensions daily to promote her students’ development of mathematical proficiency.

5.3.1.2.7 Summary

This chapter focuses on how Marian meets the needs of her students. This was demonstrated in how Marian differentially constructs her students’ knowledge, and how she identifies practical challenges teachers may confront as they seek to implement IBL practices in mathematics that address all of their students’ needs. Marian focuses her pedagogy on a specific set of techniques/strategies. These include matters related to constructing knowledge, student communication, student tasks, manipulatives, assessments, and meeting individual needs. They parallel both the Ten Dimensions of Mathematics Education (McDougall, 2004) and the Five Strands of Mathematical Proficiency (Kilpatrick, Swafford & Findell, 2001).

The facility with which Marian utilizes these multidimensional strategies speaks resoundingly of her competence and expertise with mathematics. The idea of teachers’ mathematical aptitude will pervade and undergird the bulk of the discussion for my second research question.

5.3.2 Question 2: What are some challenges teachers of Grades 4 to 6 may face when implementing IBL practices in their mathematics class?

The value and benefits of teaching mathematics through the implementation of inquiry-based methods are copious and varied (Barrow, 2006; Capaldi, 2015; Engeln, Euler & Maas, 2013; Hattie, 2009; Maas & Doorman, 2013; Scardamalia. 2002; Watt & Coyler, 2014). Despite the many touted merits of using Inquiry-Based Learning (IBL) in mathematics, various
constraints exist in its implementation. Through analyzing my interviews and observations of Marian, interviews of her students, and my own personal reflections, it has become apparent that there are specific barriers/challenges teachers may confront when seeking to effectively implement IBL practices within their own mathematics class. These challenges are manifested in the following areas of need: 1) time, 2) professional support, and 3) teachers’ mathematical content knowledge.

5.3.2.1 Time

Time, or the lack thereof, appears to be a ubiquitous source of angst and concern for many teachers. The process of effectively implementing IBL practices in mathematics, due to its constructivist nature, necessitates considerable time. Time must be devoted for students to engage in exploration, discovery, as well as the all-important process of asking meaningful questions. Through Marian’s example, we recognize that the pedagogy involved in constructing students’ knowledge, demands a great amount of time.

In this study, ‘time’ is repeatedly alluded to by Marian, whether directly or indirectly, as being the primary culprit that interferes with effective IBL implementation in mathematics. Marian repeatedly references time when she complains about the inescapable challenges teachers face when approaching their lessons through the medium of inquiry. Marian indicates that the vast curricular demands, coupled with the limitations of time, force teachers to forego essential elements critical to the inquiry process. The process of asking questions, exploring, seeking clarification through investigation, and debating with one’s peers, often become short-changed due to the limitations of time.

Marian periodically laments how the overwhelming expansiveness of the Ontario Mathematics curriculum denies teachers the time required to implement a rich inquiry-based
learning program for their students. With the mammoth amount of content demanding coverage, Marian grieves, “We forget the child. It just becomes about how the child has to be ‘shown’ the curriculum”. Marian’s grumblings find validity in the work of Cohen and Barnes (1993). These authors state that educators often underestimate two critical factors when they attempt to teach using a new method of pedagogy: 1) the extraordinary amount of time that is required and 2) the abundance of learning that is involved in learning how to teach in a different way.

Additionally, Deters (2005) gives strong corroboration to Marian’s contention about the insufficiency of time. Deters remarks that, in instances where teachers favourably view IBL practices, the lack of school hours within a given day is offered as a major barrier to its implementation.

Limited time is a perpetual problem for elementary school teachers, as on any given day, they have numerous subject areas they need to responsibly teach. Marian’s experiences parallel with my own, in that we both feel like we are always having to play ‘catch-up’. We are frequently found in the frustrating predicament of having to rush through mathematical materials simply to cover requisite curriculum expectations.

The second challenge associated with time relates to finding time to seek out a support system where guidance is provided for the proper implementation of IBL practices. Despite having an expert mathematics teacher positioned perfectly in terms of easy accessibility, I lack the time to meet with Marian to discuss questions and concerns about my own IBL practices. Being ‘constantly’ busy and lacking time to engage Marian in meaningful dialogue, often causes me regret. Time needs to be actively and intentionally carved out, which can be done; however, in trying to be mindful of Marian’s time, I find myself sheepishly refraining from asking her too many questions.
5.3.2.2 Lack of Professional Support & Teacher Beliefs as a Constraint

There exists a practical challenge that, from the outset, can hinder IBL practices from gaining entrance into the classroom. It first occurs at the teacher level. Teachers’ knowledge and beliefs have a major impact on their mathematical teaching practices (Keys & Bryan, 2001). Marian decries how many teachers, due to their faulty understanding and stubborn adherence to traditional methods, reject reform approaches in their mathematics classroom.

Likewise, Engeln, Euler and Maass (2013) declare, “One of the main stumbling blocks for implementing IBL in the classroom is teachers’ beliefs about teaching and learning”. I happen to be a case in point. Due to my erroneous and misguided beliefs surrounding IBL practices, I was averse to entertaining anything that remotely resembled inquiry-based methods. With credit owed to Marian, my thinking has been transformed. Once teachers come to experience an evolution and transformation in mindset, like myself, what is left wanting is professional support that will ensure IBL’s proper implementation.

Inquiry-based practices require an amalgam of techniques and skills that must be delicately maneuvered and executed to reap its desired effects. Marian maintains that developing skills requisite for the effective use of reform-based methods demand long-term, ongoing support.

Marian reminds me of the arduous and lengthy process she underwent to eventually arrive at her current level of mathematical competence. Marian’s involvement with Collaborative Inquiry Learning for Mathematics (CILM), additional qualification courses in mathematics, and her extensive training as a math coach have enabled her to increase her capacity and confidence as a reform math teacher.
Dorier and Garcia (2013) assert that is it more than just teachers’ beliefs and practices about IBL which need to be changed; they contend that a comprehensive makeover needs to occur at the institutional level so that teachers will begin to develop an appreciation and understanding for the need of IBL methods in mathematics. It becomes appropriate here to discuss teachers’ content knowledge, which is likely the most significant and comprehensive ‘challenge’ for teachers who may want to implement IBL methods for the benefit of their students’ mathematical achievement.

5.3.2.3 Teachers’ Mathematical Content Knowledge (MCK)

A variety of influences serve as obstacles to impeding the effective instruction of mathematics through IBL means. The foremost challenge is the need for teachers to possess deep mathematical content knowledge. This was confirmed early on by Marian, when I asked her a recurring question: How does an average teacher like myself become effective in utilizing IBL techniques like her? Marian replies, “It requires a great amount of background knowledge”.

In my discussion of the compulsory need for teachers’ mathematical content knowledge (MCK), I will make recurring reference to McDougall’s (2004) conceptual framework. Through highlighting the elements within McDougall’s dimensions, I hope to shed light on various challenges that inevitably emerge in the absence of teacher content knowledge. In the hopes of clearly exposing the potential challenges teachers may confront if they lack the necessary MCK, I shall evoke elements from two specific dimensions: 1) teachers’ attitude and comfort with mathematics and 2) student tasks.

5.3.2.3.1 Teachers’ Attitude and Comfort with Mathematics

Marian states that teachers are not exempt from their need to actively engage in deep and active learning of the mathematics themselves. Marian admits that the process of learning for
teachers is not easy; she reminds me of its difficulty by reiterating that deep mathematical understanding is required before teachers can effectively support the construction of students’ conceptual understanding. To underscore this, Marian confesses, “The biggest obstacle for me was I did not really understand the math thoroughly and deeply; I understood it from the rote perspective”. Marian gives extra clarification, “I pose a question and say to the kids to answer it any way they want, but as a teacher, I did not know what I was looking for. All I was looking for was the traditional algorithm, because that made sense to me”. Marian’s past experiences of being limited in ‘deep’ and ‘thorough’ mathematical knowledge, happens to be my current and very real constraining obstacle today. I do not know how to respond to my students’ divergent and innovative methods of solving mathematical problems.

Marian stresses the need for teachers’ MCK by rhetorically asking, how will teachers elicit ideas from their students to strengthen their conceptual understanding if they do not know what they are looking for. Marian’s statements find corroboration in various studies demonstrating the significant relationship between teachers’ mathematical content knowledge and the resulting positive outcomes for their students (Baumert et al., 2010, Hill et al., 2005, Merrie et al., 2008).

Marian discusses how teachers need to instruct their class through problem-based learning, where students must be allowed to struggle and figure their own way out. However, Marian declares that the only way to help these students is for teachers to have a deep understanding of the math themselves.

Marian laments how teachers often protest about IBL not working when, in fact, it is due to their lack of background knowledge. She further notes how teachers’ lack of MCK puts a
glass ceiling on students’ capabilities. She illustrates this by speaking of the experiences she had coaching middle school teachers:

When (teachers) are not comfortable teaching math, they become poor teachers. You see that when a child’s answer deviates from what is in the textbook, they cannot help the child. It could be a simple misunderstanding, but they just say, ‘Wrong, you are wrong’. Meanwhile, the child really gets it.

Marian’s words mirror the existing literature advocating for teachers to develop subject matter competency; these studies demonstrate such knowledge positively influences students’ mathematical achievement (Baumert et al., 2010; Hill et al., 2005; Merrie et al., 2008).

Marian relates to me, “Teachers need to understand that they must set up the scenario where students do not have a choice to but think in a certain way. You are intentionally setting it up – picking the numbers that will force students to see it”. Marian helped me to discover that there can be a very large disconnect between theoretically understanding IBL pedagogy and putting it into practice.

Open-ended questions are Marian’s primary method of drawing out the mathematical thinking from her students. She explains to me the many benefits of using open-ended questions to develop students’ conceptual understanding in mathematics. I am striving to overcome my trepidation of using open-ended questions, an IBL mainstay; yet, they are gradually and progressively making a recurring appearance in my mathematics class. This is due to my deficient mathematical knowledge, which results in my deep-seated apprehension and reluctance to encourage my students to provide multiple solution strategies. In an effort to be “in control”, I am inclined to avoid situations, i.e., unwieldy solution paths, that will cause my students to question my mathematical knowledge and abilities. Conversely, Marian is skillful and comfortable in responding to her students’ questions, no matter the unconventional path any
student takes in his or her thinking. Correspondingly, her students exude great confidence and command of their mathematical abilities.

Another example of how conceptual understanding can be impaired when teachers’ mathematical knowledge is deficient becomes apparent through Marian’s own example. Despite being an expert math teacher, Marian admits to me, “The most difficult part of the lesson is when the students bring their papers up and I have to sort and cluster them into categories”. Contrary to her disclosure, Marian always appeared completely confident and in her element whenever she sorted and classified her students’ work. The importance of knowing how to effectively ‘categorize’ and ‘cluster’ students’ work must be explained here.

In the vignettes from Chapter Four, we became acquainted with Marian’s use of the Three-Part lesson. The most important element of this lesson approach is the consolidation component. Marian describes how this is the stage where conceptual understanding and construction of student knowledge is developed and honed. The clustered and categorized samples of students’ work serve as the raw materials through which discussions, questioning, justifying, proving, among other critical aspects of eliciting students’ understanding and thinking occur. If an experienced math coach like Marian struggles with this indispensable and critical skill, what hope is there for the rest of us generalist elementary school mathematics teachers?

Through the categorized samples of students’ work, Marian then probes her students to discover important patterns and mathematical rules. Her ability to do this always impressed and mystified me. I was always bewildered with Marian’s students’ ability to make mathematical connections that were not apparent to me. Marian was always adamant when she expressed, “You have to trust that someone in the class will see the pattern or rule. You do not tell them.
You must be intentional with your numbers”. Sadly, whenever I went back to my own classroom to practice this ‘intentional’ use of numbers, I stumbled and faltered and experienced dejection.

The foregoing contrasting examples between Marian and myself (of which there are many more) find support from Schoenfeld (2002). Schoenfeld declares that, when teachers are provided with long-term opportunities to develop their own skills and understanding, their students’ mathematical performance will improve significantly.

5.3.2.3.2 Student Tasks

When I probed Marian about identifying some challenges she encounters with teaching mathematics through IBL methods, she quickly admits that one of her greatest challenges is finding the ‘right’ task/problem. She explains this by itemizing the exhaustive criteria necessary for the creation of meaningful, inquiry-based tasks that promote mathematical understanding and achievement. She asserts teachers must find or create IBL questions that incorporate five of the following characteristics: 1) appropriate depth, 2) openness- as it relates to accessibility, 3) curricular-relevance, 4) an ability to engage all students and 5) extensive breadth to permit differentiation. Subsequent to listing the complex set of conditions, Marian concedes that she sometimes feels exasperated and unable to adequately meet these exacting requirements. ‘Curricular relevance’ and ‘student engagement’ are the only two elements I actively consider when constructing mathematical tasks for my students. Admittedly, I tend to avoid tasks that have depth, breadth and openness. I recognize that implementing such tasks is easy. However, having to deal with the aftermath, where students bring forward their work for class discussion, is where I am left paralyzed.

Consistent with Marian’s assertions, Stein et al. (1996) declare that task implementation plays a critical role in either supporting or inhibiting students’ learning of mathematics. This is
disconcerting given that many teachers are unaware of the complexities inherent in task selection/implementation. I have encountered various challenges with both selecting and implementing mathematical tasks with my Grade 5 students. Whenever I assign a task for which I am not well-informed, I find myself quickly reverting to my tried and true repertoire of questions. I am disposed to selecting tasks that will lead my students to using a traditional solution path with which I am familiar.

As well, I ensure these tasks will safely eliminate my need to respond to divergent thinking patterns. Palmer (2002) signals a warning bell concerning my default mechanism to the traditional approach. Palmer notes that students earned significantly higher grades when they were instructed through the inquiry-based learning approach compared to students who were taught via traditional methods.

From the standpoint of a teacher who is bereft of deep mathematical content knowledge, the idea of properly ensuring the potency and effectiveness of student mathematical tasks becomes a serious challenge. This difficulty becomes more pronounced when one understands the generalist teachers’ dilemma: Their need to increase students’ mathematical understanding and achievement through the process of eliciting, via rich tasks, a range of mathematical thinking processes, with which they are superficially familiar.

5.3.2.3.3 Summary

The foregoing reasons, which are coupled with concrete examples from my findings, as well as supported by current literature, consistently point to the same conclusion: Teachers’ mathematical content knowledge (MCK) is paramount for student achievement. This notion is reinforced when seen from two significant queries: 1) How can teachers effectively respond to and evaluate students’ unconventional mathematical responses or encourage multiple solution
paths without the necessary MCK? and 2) How can teachers appropriately select rich tasks that help promote mathematical understanding and proficiency without possessing deep MCK? In comparing my pedagogy with Marian’s, the critical importance of MCK becomes accentuated.

5.4 Pedagogies Compared: Marian and Genie

At this juncture, I will further elaborate upon the disparities between Marian’s pedagogy (inquiry-based) to my approach (traditional). In revisiting the differences addressed earlier, I hope to further underscore the notion that teachers’ MCK is of chief importance to promoting students’ mathematical understanding and proficiency.

Marian’s teaching methods and my own could only be described as a tale of stark contrasts. To clearly showcase the extent to which Marian and I vastly differ in our respective approaches to teaching mathematics, I shall present specific examples. In reflectively pitting my practice against Marian’s, glaring differences have been brought to my attention. Before I address these striking differences, I need to lay bare a succinct overview of my former teaching practices prior to being influenced by Marian’s reform-based mathematical pedagogy.

My former pedagogical approach could be viewed as teacher-directed and traditional. The tasks I imposed on my students were ones that invited a single method to arriving at an accurate solution, which I hoped to strengthen and reinforce through procedural practice. Skill development was my priority. I believed that strong procedural knowledge and automaticity with basic mathematical skills, in due time, would help lay the necessary groundwork for my students’ higher-order conceptual understanding. Having numerous opportunities to observe Marian, as well as being able to engage in meaningful discussions and debates with her, helped to reorient my philosophy and practice. Despite my new learning, there still exist great differences between Marian’s pedagogy and my own.
To illustrate, before I send my students off to work in groups, I tend to spend a considerable amount of time on ‘teacher talk’, eager to answer any, and all questions, believing that, in doing so, I am supporting my students’ understanding of the task(s) at hand. Marian, conversely, quickly, yet methodically reviews her students’ prior knowledge, and then she releases her students to attack their tasks. She provides minimal support to her students. Marian reminds me that it is for students to ‘discover’ and not for the teachers to ‘tell’.

Marian is never seen to directly answer her students’ queries. She always redirects the question back to the inquiring student. She allows her students to struggle through a question. I, on the other hand, wrestle intensely not to offer answers when solicited. The process of struggling, according to Marian, is where students’ conceptual understanding becomes stretched, challenged and actualized. It is a challenge for me to identify appropriate questions that will help guide and redirect my students who are ‘struggling’. Hence, I take the easy route: I give them the answer.

Marian draws out from her students the strategies she is looking for. I tend to teach my students the strategy directly, especially when I feel unsure about how to guide them so they generate it on their own. During consolidation time, when students present their work, I tend to take over, believing that my voice will clarify learning for the students. Also, I feel it permits me to securely focus on those strategies for which I have adequate command and knowledge.

Marian allows all students to explain their thinking to the class. She interjects only to ask her students questions that will help redirect and clarify their thinking. Furthermore, Marian’s practice is to ask the class if what the presenter has shared is clear to them. If the class expresses confusion, Marian asks the presenting student to rephrase his/her thinking to clarify what may
have been ambiguously communicated. As mentioned earlier, I often rephrase statements on behalf of my students.

There is an area in my pedagogy where I feel a continuous sense of shame and guilt: My deliberate underuse/provision of manipulatives for my students to engage with in their mathematical explorations. Again, my lack of mathematical understanding and competence inhibits my willingness to present these tangible tools, which I feel unable to meaningfully ‘manipulate’ myself. Mathematical manipulatives, which are concrete objects that serve to foster students’ conceptual understanding, appear abstract, perplexing, and burdensome to me. Thus, in my class, they often remain tucked away in bins, collecting dust. Marian, on the other hand, is a firm advocate for the daily, ongoing use of manipulatives. She explicitly and implicitly models for her students their appropriate use, which helps to guide and reinforce her students’ mathematical understanding.

Another example illustrating the distinct contrast between my own practices compared to Marian’s practices further demonstrates the importance of teacher knowledge. Instead of accepting inaccurate conjectures from her students, Marian always presses for more reasonable answers. This contrasts with my own practice, which I discovered upon subsequent reflection; I often deny opportunities to push my students’ thinking and explanations to higher levels due to my limited understanding of the math.

A challenge I consistently experience is finding the right questions to ask my students to help elicit their mathematical thinking. I frequently made attempts to use Marian’s questioning techniques to direct my students’ thinking. However, I was always stumped in figuring out what questions to pose. This was due to my inadequate understanding of the intricacies behind the
math; hence, I was always left feeling frustrated with my students and disappointed with myself. Marian had addressed this potential problem with me during one of my interviews with her.

These are just a few obvious differences I have come to notice. Many more exist, but for the sake of brevity, I have chosen to limit the examples outlined. In the following, I will describe the challenges I still encounter, despite my ‘reformed’ pedagogical understanding and desire to teach using inquiry-based methods.

5.5 Further Reflections

Engaging in personal reflection has become a common practice for me during my investigation of Marian. Through my research, I have come to discover important IBL practices/techniques that are effective in developing students’ mathematical proficiency: questioning techniques, encouraging students to struggle, task implementation, student communication, use of manipulatives, and assessments. It appears an umbrella term that could appropriately describe the above skills and practices is a subtheme discussed earlier: ‘making the invisible visible’.

At the end of my analysis, from distilling the various key themes, it has become apparent to me that all these skills and techniques are anchored and manifested through the teacher’s mathematical content knowledge. I acknowledge that I will require ongoing professional development and support from Marian to hone various inquiry-based techniques necessary to promote my students’ mathematical proficiency. How did I come to this discovery? It now becomes appropriate for me to briefly share my personal reflections pertaining to my current struggles today.
5.5.1 Current Struggles with IBL

I am aware of the areas in which I need to make changes as a reform mathematics teacher; however, I feel as if I cannot suppress my deeply ingrained habits. It is a very strange experience. Although I genuinely desire to mimic all aspects of Marian’s pedagogy by having my students become active agents in the construction of their knowledge, I find myself reverting to my old traditional ways. To illustrate, while I was trying to elicit the “splitting strategy” from my students, I found that I had to bite my tongue as I felt an intense temptation to simply teach my students the strategy.

During consolidation time, I unconsciously proceeded with my ‘teacher-directed’ custom of ‘telling’ my students how to solve mathematical problems. When self-awareness kicked in, I quickly stopped myself. After taking a deep breath, I asked my class if anyone wanted to present their strategy. One of my typically reserved students came up to the board and showed the class the splitting strategy, which to my surprise, she had generated on her own. Witnessing this student’s pride and pleasure, I reminded myself of the need to adhere to Marian’s admonition: let it come from the kids. Marian was right – students will come up with the strategies on their own, and the positive impact is definitely worth it.

Later, I reflected on why I have been feeling such a strong compulsion to take over. I generated four explanations: 1) When students present their work, the lesson moves very slowly as students typically speak in a manner that requires constant repetition, clarifying, and rephrasing; 2) Time: It usually takes twice the time, if not more, to have students present a strategy or concept that I could very well have taught more efficiently, 3) I do not trust that my students will present their thinking effectively, so that their peers will learn and gain new
understanding, and 4) I am nervous students will address something for which I have no knowledge.

I have concluded through my discussions and observations of Marian that the pros of permitting the inquiry process to reign outweighs the cons. I have witnessed it with Marian’s students and with my own. Yet, I admit I struggle greatly with implementing IBL as it is meant to be implemented. The term ‘inquiry’ denotes and connotes questioning and investigation. Yet, in my attempts with IBL, I have a difficult time allowing true inquiry-learning to occur. When I feel uncertain about student-generated responses, I am inclined to relapse and return to my teacher-directed ways. Despite my many observations and conversations with Marian throughout the school year, I still find myself ill-equipped to effectively implement IBL methods in my own classroom.

In the final analysis, I have come to recognize that the source of my struggles stems from my lack of mathematical content knowledge. I have admitted to this earlier, yet, I believe that it warrants repeating. Thinking about the monumental task of gaining sufficient content knowledge often overwhelms me, and at moments, makes me feel unable to move forward. Here, I am happy to think about the wise counsel offered by Hiebert and Stigler (2004), “Teaching can only change the way cultures change: gradually, steadily, over time as small changes are made in the daily and weekly routines of teaching” (p. 13). These researchers expressed this in the context of improving teaching practices. They further exhort teachers to slow down and to carefully examine, through the process of reflection, those targeted areas requiring change/improvement. This is the stage in which I currently find myself: reflecting and taking things day by day. ‘Approaching change slowly’ is the operative phrase I need to embrace.
“Aha” moments are to be sweetly cherished and never taken for granted, for they are few and far between. I was very blessed to experience many during my visits into Marian’s classroom. I often felt like a first-year teacher while observing Marian; I was constantly unearthing novel insights that filled me with fascination and an inspiration to mimic her. Early on, I was unprepared to ‘let go’ of my misunderstandings of IBL, and trust my students to actively construct their knowledge. However, a transformation in my belief system has occurred. Now, a new challenge awaits me: I need to develop my own mathematical proficiency, by building and strengthening my mathematical content knowledge.

5.6 Major Findings

Earlier in my paper, I stated, “There was an abundance of things I witnessed, which I was unprepared for and could not believe”. Here, I would like to briefly qualify this hyperbolized statement with concrete examples of Marian’s success in IBL, which serve as the major findings within my study:

1) Marian constructs/implements rich, open-ended inquiry-based tasks that permit multiple-entry points. These tasks, which embed the use of manipulatives, possess the capacity to invite a multitude of student-generated solutions. Marian ensures these tasks have the right level of difficulty to engage and challenge learners of varying abilities;

2) Marian constructs, customizes and poses questions to guide and scaffold her students’ mathematical thinking. To effectively know what kinds of questions to ask, Marian anticipates her students’ responses and possible stumbling blocks that accompany inquiry-based problems. These questions, which are deliberate and intentional in nature, help to elicit and develop her students’ thinking and conceptual understanding;
3) Marian makes the invisible become visible through the hands-on process of discovery and experiential learning. Marian models for her students the process of recording their invisible mathematical thinking to make it discernible and visible;

4) Marian provides experiences for her students to productively struggle and wrestle with the mathematics. During this ‘struggle’ time, Marian provides sufficient ‘wait time’ and allows her students opportunities to work through a problem without interfering;

5) Marian incorporates opportunities for her students to have extensive practice of procedural knowledge. ‘Repetition’ that is regularly embedded in her students’ learning experiences become a potent vehicle for deep mathematical understanding to occur. Repeated practice increases the likelihood that students’ understanding and skills are consolidated and therefore can be applied into new situations and contexts;

6) Marian actively guides her students in how to communicate their mathematical understanding with one another. She encourages her students to justify, prove and persuade their mathematical solutions to one another. This dialogic process increases the likelihood that students will acquire deep mathematical understanding;

7) Marian possesses deep mathematical content knowledge that enables her to effectively put into practice the foregoing inquiry-based methods, which all serve to facilitate and promote the construction of students’ mathematical understanding and proficiency; and

8) Marian incorporates Kilpatrick, Swafford, and Findell’s (2001) five dimensions of mathematical proficiency into her IBL mathematics pedagogy. All five dimensions, when operating in concert, increase the likelihood for students’ mathematical proficiency to be developed.
5.7 Future Research

This study reveals various key IBL techniques and practices that teachers can utilize to increase students’ understanding and achievement in mathematics, such as through their questioning techniques, encouraging student communication, and appropriately using manipulatives and assessments. Moreover, challenges to successfully implementing IBL methods in elementary classrooms have also been illuminated, specifically, the need for teachers to possess deep mathematical content knowledge. Marian, my case study, repeatedly pointed to the need for teachers’ deep mathematical background knowledge in order to implement IBL methods effectively. This has strong implications for pre-service and in-serve professional development. Both frameworks, Mathematical Proficiency and the Ten Dimensions, undergird and support the effective implementation of IBL practices for the improvement of mathematical understanding and proficiency among students; consequently, professional development programs would be judicious in incorporating and emphasizing these dimensions. Additionally, elementary school teachers, who predominantly tend to be subject generalists, need to actively commit to developing their mathematical content knowledge, through taking various math related courses, workshops, and engaging in personal readings.

Despite there being many indispensable IBL strategies gleaned from my investigation of Marian, further research is needed to develop a more comprehensive understanding of important IBL practices required to increase students’ understanding and proficiency in mathematics.

Teachers’ need for deep mathematical content knowledge was shown to be a potent indicator for student achievement in mathematics (Baumert et al., 2010, Hill et al., 2005, Merrie et al., 2008). Armed with this finding, a variety of practical questions arise. First, how deep must this mathematical content knowledge be for teachers to successfully support their students’
mathematical learning? It would be compelling and of great value to conduct a large-scale comparative study, with a diverse group of teachers having varying levels of MCK, who are given ongoing professional support with mathematics and IBL practices. This would be similar to the PRIMAS project (PRIMAS, 2010), but with specific attention focused on buttressing background mathematical knowledge for teachers. What would the implications be for those teachers who have a profound deficit in MCK? Should they be discouraged from using IBL practices in their classrooms?

Additionally, what processes need to be put in place to ensure that teachers, both at the preservice and in-service levels, acquire the necessary mathematical content knowledge (MCK) to help increase their students’ proficiency in mathematics? Unavoidable costs are associated with professional development (PD). Thus, additional research can determine how to practically isolate and prioritize key skills/knowledge for which teachers need to be coached, thus maximizing PD sessions aimed at promoting students’ mathematical proficiency.

Although there is much to learn from the case of Marian, further research is needed to develop a more comprehensive understanding, specifically related to two principal areas that pertain to inquiry-based teaching methods: 1) What additional IBL techniques and strategies, that may have been overlooked and not included in this research, must teachers be aware of, in order to promote mathematical proficiency among their students? and 2) what constitutes ‘adequate’ or ‘deep’ MCK that is sufficient for elementary school teachers to promote proficiency and achievement in their students, when using IBL methods?

5.8 Limitations

My study was exploratory in nature. It aimed to identify and isolate IBL techniques and strategies teachers can implement to effectively increase their students’ mathematical
understanding. Also, this study investigated the various challenges that may impede teachers’ implementation of such practices. This study did not formally test the factors that support and hinder effective IBL implementation. My conclusions were anecdotal in nature, wherein I discussed my subjective challenges and presented them as supporting evidence. A more comprehensive and large-scale study needs to be conducted to verify the conclusions offered.

I did not evaluate data from Marian’s students’ formal assessments to conclusively measure their mathematical growth or lack thereof. My conclusions were based solely on my subjective observations. It was my intention to have Marian and myself administer an identical unit test to our respective students to determine the measurable extent to which our students’ achievement outcomes differed, given our divergent pedagogical approaches. However, insufficient time disallowed me from conducting such an assessment.

Hill et al. (2008) state, “Large-scale educational production function studies never peer inside classrooms to compare the practice of higher-knowledge and lower-knowledge teachers” (p. 431). I have had the privilege of doing exactly this: Comparing a higher-knowledge teacher (Marian), and contrasting her specific practices in relation to a lower-knowledge teacher (myself) to assess the differences. In this chapter, I outlined various skills/qualities that can lead to such differences. However, due to the limited sample size (single case study), making generalizations becomes limited. Future studies involving more than one expert, reform-based mathematics teacher would provide a richer and more comprehensive understanding of whether the stated IBL strategies and skills are in fact key to promoting students’ proficiency in mathematics. Similarly, it is debatable whether the findings from this study are transferable to other classroom contexts. Thus, a focus on other expert mathematics teachers whose mathematical pedagogy is IBL centered, may or may not be found to reveal similar findings.
5.9 Concluding Thoughts

My final and all-encompassing conclusion: At the heart of my learning is that teachers’ MCK appears to be the non-negotiable bedrock for effective IBL instruction. MCK provides a stable foundation, helping teachers avert challenges that accompany those who lack the necessary mathematical understanding. Those possessing deep mathematical knowledge are well-equipped to flexibly run a rigorous and rich mathematics program within their classroom. The presence of deep MCK does not negate teachers’ need for pedagogical content knowledge (PCK) as was discussed by Grossman, 1995, Sherin, 1996, and Shulman, 1987.

Do my findings imply that all teachers need to be mathematical experts before implementing mathematics through the IBL approach? No. Research by Grossman, (1995), Sherin, (1996), and Shulman, (1987) reveal that MCK and PCK are both important. However, these authors did find that mathematical content knowledge trumped pedagogical content knowledge when it came to increasing students’ mathematical achievement. In the final analysis, teachers must acknowledge and recognize areas in which they need to strengthen or deepen their mathematical understanding and knowledge.

Boaler (2015) asserts, “Mathematics, more than any other subject, has the power to crush students’ confidence” (p.xvii). As a result, we see the urgent need to understand the kinds of strategies/tasks teachers must use to broadly and positively impact students’ confidence and ability to learn mathematics. I hope the results of my study have served to shed light on some specific IBL strategies/tasks that support students’ learning of mathematics.

To return to the earlier ‘gatekeeper’ metaphor, mathematics holds the potential to unlock for students a future filled with wonder and possibility, or conversely to prematurely shut doors to a promising future. It was in the hopes of attaining the former that this research has been
conducted. Hence, ongoing research becomes necessary in order to ensure the acquisition of teachers’ deep mathematical skills and knowledge in mathematics. This must be achieved with the sole purpose of dismantling the metaphorical ‘gate’, and instead, pushing open doors/gates so all students, newly equipped and proficient in mathematics, can stride into their futures, unimpeded, with confidence.
References


PRIMAS (2010). Promoting Inquiry in Mathematics And Science Education Across Europe. Freiburg, Germany. primas-project-eu.


Appendix A
Informed Consent Form (Colleague)

(On OISE/UT letterhead)

(Date)

Dear (Name of teacher involved in my case study),

I am very excited and grateful that you have provided verbal consent to participate in my study. As you are aware, I am currently a third year PhD student at the Ontario Institute for Studies in Education, University of Toronto (OISE/UT). It is with deep interest and eager anticipation that I undertake this research to investigate your pedagogical practices in mathematics, focusing on how you implement the inquiry-based learning (IBL) approach to promote mathematical understanding and capacity among your students.

During the summer, I had a brief opportunity to convey to you how you had wonderfully confounded and challenged my misguided and prejudiced assumptions about IBL. Three events precipitated my interest in re-evaluating IBL: 1) As an expert instructional leader and coach, your PD sessions on IBL have profoundly provoked my thinking, 2) Your former students’ genuine enthusiasm for mathematics incited me to reconsider the merits of IBL, and 3) The commanding displays on your walls have provided concrete evidence of the rich and meaningful learning occurring within your mathematics classes. These compelling experiences have prompted me to investigate you as my ‘unique’ case study, specifically as you engage with IBL. Through my observations and interviews, I look forward to re-assessing IBL with a renewed lens, one that is broad-minded and devoid of presumptions. I believe that in having you share your knowledge and experiences related to IBL, it will be of undoubted value and worth to not only myself, but also to the educational community at large.

My research will principally investigate three queries as they pertain to your teaching of mathematics:

1) How do you implement inquiry-based learning strategies/practices in your classroom to promote the learning of mathematics among your grade 5 students?
2) What process(es) do you use to construct rich and meaningful mathematical tasks that help develop your students mathematical understanding and skills?
3) How do inquiry-based strategies/methods/tasks help develop self-efficacy among your students?

The collection of data for my thesis will consist of three sources: 1) Interview with you throughout the school year, 2) interviews with your students regarding their experiences with IBL in your class, and 3) Seven to ten classroom observations during your mathematics blocks. All interviews will be audio-recorded, which I will transcribe at a later time. A video-recording
may take place during your instruction of mathematics, however, it will only include images of you and not of your students.

I would like to invite you to participate in my research as my case study. Upon your consent, I will: 1) access for you, at a later time, the information you provide through the interviews, 2) analyze the data for insights regarding effective IBL practices in mathematics, and 3) report the findings to you in both written and oral forms.

In all my written works, oral presentations, and publications, your name will not be used. Information regarding your identity will remain confidential, and any personally identifiable information will be omitted. Please note that should you change your mind, you are free to withdraw from this study at any time. It is also important that you are aware of your rights to decline any specific questions posed. I will destroy my copy of the collected data 5 years subsequent to the publication of findings. There are no known risks or benefits as a result of your participation in this study.

The External Research Review Committee of the TDSB has granted approval for this study. Additionally, this study has been approved by the University of Toronto Ethics Office. If you wish to obtain more information regarding the University of Toronto ethics approval process, please send your inquiries to: UT Office of Research Ethics, 12 Queen’s Park Crescent, McMurrich Building, 2nd floor, M5S 1S8, Toronto. Phone: 416-946-3273. You may also contact me at (416) 395-2480. I would be very happy to respond to any questions you may have concerning my research.

If you agree to participate in my study, please sign the attached form. A second copy is provided for your records. Thank you greatly for your assistance.

Yours sincerely,

Genie Kim
Thesis supervisor: Dr. Douglas McDougall
OISE/University of Toronto
doug.mcdougall@utoronto.ca
Phone number: (416) 978-0056

Consent Form

I acknowledge that the topic of this research has been explained to me. Any questions that I may have pertaining to this project, I have asked and they have been answered to my satisfaction. I understand that I can withdraw from this project at any time without penalty.

I have read the letter provided to me by Genie Kim and agree to participate in this study, and I provide information for the purposes described.
Signature:____________________________

Name (Printed):________________________

Date:________________________________
Appendix B
Informed Consent Form (Students’ Parents)

(On OISE/UT letterhead)

(Date)

Dear Parents/Guardian,

I am Ms Kim, a grade 5 teacher at your child’s school. I am currently undertaking graduate work at the Ontario Institute for Studies in Education, University of Toronto (OISE/UT). I am conducting a research project on Inquiry-Based Learning (IBL) in mathematics, and I would like to invite your child to participate in my study. The purpose of my research is to explore how your child’s teacher implements inquiry-based learning strategies and tasks to help support the learning of mathematics for students.

I would like to include your child in my study so I can develop a better understanding of how grade 5 students respond to various inquiry-based mathematical strategies and tasks. My investigation will not alter nor change anything that your child’s teacher does in her classroom; all strategies and tasks align and conform to the overall and specific expectations outlined within the Ontario mathematics curriculum. The objective of my study is to observe your child’s teacher use inquiry-based learning strategies and methods that are already at her disposal.

Students who participate in my research will have an opportunity to collaborate with me in investigating how their teacher uses inquiry-based learning in mathematics. Their contribution in this study will be twofold during the 2015-2016 school year: 1) to respond to a questionnaire (one page in length) concerning their thoughts and experiences related to mathematics, and 2) to provide journal responses that will help shed light on their experiences with the various inquiry-based learning instructional tasks implemented, which will take about 10 minutes in length. There will be an opportunity for some of the students to be interviewed by me during the study. The interview will involve students sharing their perspectives and attitudes regarding their experiences with various IBL mathematical tasks and the impact these tasks may have on their learning. The interviews will be audio-recorded and will take place at Gracedale P.S. during recess. Each student will be interviewed 5-7 times in total. Instruction time will not be used to interview the students. Interviews will only take place outside of instructional time. If your child is selected to be interviewed, his or her name or any information gained will not be used that may identify your child in any written work, oral presentation, or publications. The information will remain strictly confidential.

Participation in this study is entirely voluntary, and all information obtained will be used for research purposes only. Your child is free to change his or her mind and withdraw from the research at any time during the study, even after you have provided consent for him or her to participate. If your child is asked to be interviewed, he or she may decline to answer any or all questions posed. All audio recordings, as well as notes collected for the research, will be
destroyed five years after the data has been collected. There are no known risks associated with having your child participate in this study. The potential benefits of having your child participate in this study are as follows: Your child will have an opportunity to develop both critical thinking skills and oral communication skills. Additionally, your child will learn how to become a self-reflective learner who understands how to take ownership of his/her education. Moreover, your child will help to contribute to the body of knowledge surrounding teaching methods that may help improve students’ ability to learn mathematics.

The External Research Review Committee of the TDSB has granted approval for this study. The school Principal has also given permission for this study to be carried out in your child’s classroom. Additionally, this study has been approved by the University of Toronto Ethics Office. If you wish to obtain more information regarding the University of Toronto ethics approval process, please send your inquiries to: UT Office of Research Ethics, 12 Queen’s Park Crescent, McMurrich Building, 2nd floor, M5S 1S8, Toronto. Phone: 416-946-3273

If you have any questions or would like clarification regarding my research, I would be happy to have you contact me at (416) 395-2480. It would be my pleasure to discuss the details of my study with you.

Yours sincerely,

Ms Kim
OISE/University of Toronto
Thesis supervisor: Dr. Douglas McDougall.
Reply Slip

Student Name: _______________________

I have read and understand the letter provided to me by Ms Kim, and the following option has been chosen for my child:

I____________________ give permission for my child to participate in all aspects of this research, which includes data collection through a questionnaire, journal entries and possibly through an interview with Ms Kim.

I____________________ give permission for my child to only participate in responding to the questionnaire and the journal entry component of the research; I do not provide consent for my child to be interviewed at any time during the study.

I____________________ give permission for my child to participate in the interview portion of the research only.

I____________________ do not give permission for my child to participate in any part of the research project.

Parent’s Signature: _______________________

Parent’s Name: _______________________
       (Printed)

Date: ________________________________
Appendix C
Invitation Letter to School Administrator

(On OISE/UT letterhead)

(Date)

Dear (Name of Administrator),

Thank you for taking your time to read this letter of invitation. As you are aware, I am currently undertaking doctoral studies at the Ontario Institute for Studies in Education, University of Toronto (OISE/UT). I am conducting a research project on Inquiry-Based Learning (IBL) in mathematics, and I would like to request your permission to conduct my study at Gracedale Public School. It is with deep interest and eager anticipation that I embark on this research to investigate Ms Kunka’s pedagogical practices in mathematics, focusing on how she implements the inquiry-based learning approach to promote mathematical understanding and capacity among her Grade 5 students.

My investigation will not alter nor change anything that occurs in Ms Kunka’s classroom; all strategies and tasks utilized by Ms Kunka will conform to her own pedagogical style and will not undergo any changes in content nor praxis. The objective of my study is to simply observe Ms Kunka use inquiry-based learning strategies and methods that are already at her disposal. I hope to observe 7-10 of Ms Kunka’s mathematics lessons, for a duration of 40 minutes each.

Students who participate in my research will have an opportunity to contribute to my research by sharing their perspectives on how their teacher uses the inquiry-based learning approach in mathematics. Their participation in this study will be twofold: 1) to respond to a questionnaire (one page in length) concerning their thoughts and experiences related to mathematics, and 2) to provide journal responses that will help shed light on their experiences with the various inquiry-based learning instructional tasks implemented, which will take 10 minutes in length. There will be an opportunity for some of the students to be interviewed by me during the study. The interview will involve students sharing their perspectives and attitudes regarding their experiences with various IBL mathematical tasks and the impact these tasks may have on their learning. The interviews will be audio-recorded and will take place at Gracedale P.S. during recess. Each student will be interviewed 5-7 times in total. Instruction time will not be used to interview the students. Interviews will only take place outside of instructional time. A video-recording may occur of Ms Kunka’s instructional practices. In such an event, only Ms Kunka will be video-recorded and not her students. Each participating student’s identity and any information gained will remain strictly confidential.

Participation in this study is entirely voluntary, and all information obtained will be used for research purposes only. Each student is free to change his or her mind and withdraw from the research at any time during the study, even after consent has been obtained. If a student is selected to be interviewed, he or she may decline to answer any or all questions posed. All audiovisual recordings, as well as notes collected for the research, will be destroyed five years subsequent to the collection of data. There are no known risks associated with having students
participate in this study. The potential benefits of having students participate in this study are as follows: Participating students will have an opportunity to develop both critical thinking skills and oral communication skills. Additionally, the participants will learn how to become self-reflective learners who understand how to take ownership of their academic life.

I am hopeful that the results of this study will contribute to the body of knowledge surrounding teaching methods utilized by educators, such that students’ ability to learn mathematics will be vastly improved.

Herein, I shall provide some context in order for you to understand what has led to the inspiration and purpose of my study. Ms Kunka has wonderfully confounded and challenged my misguided and prejudiced assumptions about IBL. Three events precipitated my interest in re-evaluating IBL: 1) As an expert instructional leader and coach, Ms Kunka’s PD sessions on IBL have profoundly provoked my thinking, 2) Ms Kunka’s former students’ genuine enthusiasm for mathematics incited me to reconsider the merits of IBL, and 3) The commanding displays on Ms Kunka’s walls have provided concrete evidence of the rich and meaningful learning occurring within her mathematics classes. These compelling experiences have prompted me to investigate Ms Kunka as my ‘unique’ case study, specifically as she engages with IBL. Through my observations and interviews, I look forward to re-assessing IBL with a renewed lens, one that is broad-minded and devoid of presumptions. I believe that in having Ms Kunka share her knowledge and experiences related to IBL, it will be of undoubted value and worth to not only myself, but also to the educational community at large.

This letter is to formally request your permission to conduct my study at Gracedale Public School. Upon your consent, I will ensure to provide for you a report of the findings in both written and oral forms at the conclusion of my study.

The External Research Review Committee of the TDSB has granted approval of this study. This study has also been approved by the University of Toronto Ethics Office. If you wish to obtain more information regarding the University of Toronto ethics approval process, please send your inquiries to: UT Office of Research Ethics, 12 Queen’s Park Crescent, McMurrich Building, 2nd floor, M5S 1S8, Toronto. Phone: 416-946-3273

If you agree to having me conduct my research at Gracedale Public School, please sign the attached form. A second copy is provided for your records. Thank you greatly for your assistance.

Yours sincerely,

Genie Kim
Thesis supervisor: Dr. Douglas McDougall
OISE/University of Toronto
doug.mcdougall@utoronto.ca
Phone number: (416) 978-0056
Consent Form

I acknowledge that the topic of this research has been explained to me. Any questions that I may have pertaining to this project, I have asked and they have been answered to my satisfaction. I understand that I can withdraw Gracedale Public School’s participation from this project at any time without penalty.

I have read the letter provided to me by Genie Kim and agree to permit her to conduct her study at Gracedale Public School.

   Signature:____________________________

   Name (Printed):________________________

   Date:________________________________
Appendix D
Interview Questions (Participant-Marion)

Background Questions

1) Where did you attend teacher’s college?
2) In university, what was your degree of specialization?
3) How many years have you been teaching?
4) How did you become a math coach and instructional leader?
5) Please briefly describe your roles as math coach/instructional leader.
6) How confident did you feel about mathematics prior to becoming a coach/instructional leader?
7) How confident do you feel today as a grade 5 teacher who will be teaching the grade 5 mathematics curriculum?
8) What is your philosophy about teaching mathematics?

Questions Regarding Inquiry-Based Learning related to Teaching and Teachers

1) What does ‘inquiry-based learning’ (IBL) in mathematics mean to you?
2) How long did it take for you to become comfortable with IBL in mathematics?
3) What are the benefits you see in teachers using IBL in mathematics?
4) What are some shortcomings that you see with the IBL approach?
5) What are some difficulties/challenges you encountered with teachers when you worked as a math coach/instructional leader?
6) What were some successes you experienced with teachers when you instructed them about IBL practices in mathematics?
7) What are the main misconceptions you believe teachers have about using the IBL approach in mathematics?
8) From your perspective, how widely accepted/implemented is the IBL approach among junior teachers when they teach mathematics?
9) Do you believe that teachers today implement IBL practices appropriately/effectively?
10) What do you believe are some barriers for teachers vis-a-vis IBL in mathematics?
11) What do you believe needs to happen to increase teachers’ confidence and competence with IBL in mathematics?

Questions Regarding IBL related to Students’ Learning

12) How would you describe your goals for IBL in mathematics for your class this year?
13) How do you believe IBL impacts students’ ability to learn mathematics?
14) How do you implement IBL practices in mathematics in order to help support your students’ success in mathematics?
15) What challenges/barriers have you faced when trying to implement IBL practices/tasks in mathematics with your students?
16) What successes have you witnessed related to your students’ achievement with IBL in mathematics?
17) What is the general attitude of your students regarding IBL in mathematics?
Appendix E
Pre & Post Interview Questions for Participant (Before and after her Math Lessons)

(All pre & post interviews will always be prefaced with the statement that all questions pertain to inquiry-based learning practices).

Questions prior to lesson observation:

1) What are your learning goals for today’s lesson?
2) What IBL strategies will you utilize in order to help students achieve understanding?
3) Why have you chosen those IBL strategies?
4) What IBL tasks have you prepared for your students today?
5) How do you believe these tasks will help your students’ skills and understanding in mathematics?
6) How do you believe your students will respond to today’s lesson?

Questions following lesson observation:

1) What was/were some of the teaching/learning success(es) for this lesson?
2) What was/were some of the teaching/learning challenge(s) for this lesson?
3) From your perspective, how did your students respond to the lesson?
4) If you were to teach this lesson again, would you do anything differently?
Appendix F
Student Questionnaire

Please elaborate as much as possible by providing specific examples, thoughts, and feelings to the questions below:

1) What do you enjoy about school?
2) What are your favourite subjects?
3) What is your least favourite subject?
4) What do you enjoy about math?
5) Is there anything you dislike about math?
6) What kinds of activities in math do you enjoy most?
7) Are there any kinds of activities in math that you do not enjoy?
8) What did you enjoy or not enjoy about math in grade 4?
9) Do you feel confident as a math student?
10) What do you hope your teacher will do this year to make your experience in math more enjoyable?
Appendix G
Sample Journal Questions

Contingent on the nature of the mathematical activity, I will select one of the following questions below for students to provide a response in their journals:

• Please describe how you felt about today’s math lesson by providing specific examples related to your thoughts, feelings, and experiences.

• What did you like or dislike most about today’s math lesson? Please justify your answer by providing specific examples related to your thoughts, feelings, and experiences.

• How did today’s math tasks help you to understand the math concept/skill better? Please justify your answer by providing specific examples related to your thoughts, feelings, and experiences.

• Which math task used by your teacher would you recommend I use with my future grade 5 students? Please justify your choice by providing specific examples related to your thoughts, feelings, and experiences.
Appendix H
Student Interview Questions

The intent of the interview questions will be to glean more elaborate details from students’ journal entries (Appendix F). Consequently, the questions I pose will involve, in large measure, myself asking, “What did you mean when you said that the task made you feel...?” Creswell’s (2007) interview protocol will be used.

Below are a few sample questions I will ask during my interview with students:

Time of interview: __________
Date: __________
Place: __________
Interviewer: __________
Interviewee: __________ (Pseudonym will be used)
Description of instructional task:

1) How did you feel about this week’s math activities?
2) What did you enjoy about your teacher’s math lesson?
3) Did you find the math tasks to be very challenging, somewhat challenging, or too easy?
4) What did your teacher do in her lesson that helped you to understand the math skill/concept better?
5) Did the math task(s) help you feel comfortable with learning mathematics?
6) How do you feel as a math student today?
7) How do you feel about problem-solving in mathematics?
8) Do you prefer to solve math-problems by yourself or with partner(s)?

At the conclusion of all interviews, I will thank the participants for participating in the interview. I will assure them of the confidentiality of their responses as well as potential future interview responses.