Fostering Students' Mathematics Communication in Measurement: A Grade 3 Teacher's Perceptions and Practices

by

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Abstract

This study explores and describes the teaching perceptions and practices of a Grade 3 teacher as she attempts to foster students’ mathematics communication skills in the content strand of measurement. For this inquiry, the qualitative data collected were: 1) semi-structured teacher interviews, and 2) classroom observations. The categories of program planning, learning environment, mathematics tasks, knowledge construction, technology, manipulatives, and assessment were identified and presented.

The data for this case study were first organized, analyzed, and interpreted within sub-units of lesson vignettes. Afterwards, the identified teaching perceptions and practices from the lesson vignettes were consolidated, and presented as a “whole” case. The sub-units and whole-case analysis identified five major findings related to the topic of fostering students’ mathematics communication in the classroom. Summarized, these are:

1) Teachers need to create opportunities for students to productively struggle with increasingly abstract concepts. The appropriate selection of mathematics tasks, the discussions on mathematical misconceptions, and the posing of effective questions may guide and encourage students as they express their understanding and misconceptions.

2) Teachers need to foster students’ self-monitoring and self-assessment skills, as students
communicate their emerging mathematical thoughts.

3) Teachers should strive to provide students with opportunities to learn to reason and justify their perspectives. Such opportunities can occur as students read, write, draw, and/or listen to recently learned mathematical concepts.

4) Teachers should intentionally define and explore various meanings and usages of subject-specific and unit-specific vocabularies. The learning of increasingly abstract ideas (e.g. area and perimeter) requires the acquisitions of new vocabularies, so that meanings can be more clearly articulated by the students.

5) When teaching and learning the topic of measurement, various education components such as program planning, learning environment, mathematics tasks, constructing knowledge, technology and manipulatives, and assessment are often intricately and purposefully linked with one another.
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1.1 Introduction

The importance of mathematics communication to the learning of mathematics has been continuously noted in numerous studies (e.g. Cai, Jakabcsin, & Lane, 1996; Perry, 2001; Tabach & Nachlieli, 2016; Uptegrove, 2015; Viseu & Oliveira, 2012; Walshaw & Anthony, 2008; White, 2003). As Tabach and Nachlieli (2016) observed, in current classroom communities, “communication and discourse are amongst the main foci of educational studies” (p. 299). Likewise, through a review of literature, Viseu and Oliveira (2012) concluded that “mathematics communication is essential to enabling students to understand about processes, discussions, and decisions that are made” (p. 288).

While there are many suggestions for the learning of mathematics communication, challenges in fostering verbal competencies and/or written proficiencies in mathematics students are still experienced (Franke et al., 2009; Hicks, 1998; Stein et al., 2008; Thompson & Chappell, 2007; Wilcox & Monroe, 2011). Secondly, amongst the mathematics content topics, studies have shown that measurement ideas may be difficult for students to understand and articulate (Battista, 2007; Chappell & Thompson, 1999; Kamii, 2006). For some students, the struggle to express understanding in the topics of area and perimeter may persist, affecting students in the primary grades, as well as the junior grades (Chappell & Thompson, 1999). As such, for this study, I intend to explore and describe some implementation strategies and practices used by an elementary teacher in the fostering of her students' mathematics communication skills within the measurement content strand.

Teacher perceptions are examined, as teachers' attitudes and beliefs have shown to affect
their practices (Applefield, Huber, & Moallem, 2000/2001; Bruce & Ross, 2008; NCTM, 2014). Likewise, teacher practices are examined because they have shown to greatly influence students' mathematical learning experiences (Brendefur & Frykholm, 2000; Hiebert et al., 2003; NCTM, 2000; Tyminski et al., 2014). I chose to focus on Grade 3, as it is one of the formative years where students encounter shifts and transitions in learning engagements (NCTM, 2000), mathematical strands focuses (Teaching and Learning Mathematics, 2004; NCTM, 2000), and mathematical proficiency expectations (Teaching and Learning Mathematics, 2004).

Specifically, within the topics of area and perimeter, the ideas introduced in Grade 3 become increasingly more abstract, and include the uses of standard units, in addition to non-standard units (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005).

In this chapter, I provide information regarding the research context, the purpose of the study, the guiding research question, the significance of the study, the background of the researcher, and the limitations of the study.

1.2 Research Context

In the recent few decades, mathematics education reform movements have given significant attention to the roles of classroom discussion and verbal communication (Cobb et al., 1997; Hintz & Tyson, 2015; Stein et al., 2008). With regards to the fostering of students' written mathematics communication skills, Van Dyke, Malloy, and Stallings (2014) highlighted the continuous encouragements from both the education research field and the teaching profession:

For the past 25 years, researchers and professional organizations have recommended that teachers have students write about mathematics in the classroom. In the United States, in the 1989 National Council of Teachers of Mathematics publication Curriculum and Evaluation Standards for School Mathematics, learning to communicate mathematically was one of five major goals and was deemed an important component for deepening student understanding. In 1992, the Mathematical Association of America (MAA)
published a book in the MAA notes series entitled *Using Writing to Teach Mathematics* (Sterrett, 1990). It is reported that in South African, Australia, England, and Wales, researchers and professional organizations advocate having students write in the mathematics classrooms as a means to promote deeper understanding (Ntenza, 2006). (p. 371)

Morgan et al. (2014) likewise concluded that “the pivotal role played by language in the learning, teaching, and doing of mathematics is increasingly being acknowledged by researchers in mathematics education” (p. 843). Given the topic's prominence in reform literature, the key roles of teachers in discourse-focused classrooms are highlighted by Brendefur and Frykholm (2000). These teachers' roles include: 1) an awareness of “teachers’ conceptions of communication as a vehicle for developing learners' mathematical understanding”, and 2) an understanding of ways to “help teachers develop practices that foster mathematics communication” (Brendefur & Frykholm, 2000, p. 125).

While mathematics communication has been identified as an essential component to mathematics learning, there are many barriers to its development. Anthony and Walshaw (2002) studied elementary school students, and observed that many students displayed difficulties in explaining their mathematical ideas. Sharing their thinking with others was challenging for the students, as some of them lacked the knowledge and skills to do so successfully (Anthony & Walshaw, 2002). Another identified barrier to students' attainment of mathematics communication is classroom cultures that do not value mathematics discourse (Engle & Conant, 2002; Pierson Bishop & Whitacre, 2010).

Students do not naturally talk about mathematics, and high quality mathematics discussions that enhance students' mathematical understanding do not seamlessly occur (Cobb, Wood, & Yackel, 1993; Thompson & Chappell, 2007; Walshaw & Anthony, 2008). As such,
teachers need to create discourse opportunities, and guide students in meaningful explorations (Cobb, Wood, & Yackel, 1993; Sfard & Kieran, 2001; Stein et al., 2008). If teachers desire students to become more proficient in their abilities to communicate mathematically, they ought to consistently provide opportunities for students to practise the needed skills (Thompson & Chappell, 2007). Unfortunately, teachers often find it challenging to make their classrooms discourse-integrated (Franke et al., 2009; Hicks, 1998). With regards to the fostering of students' verbal communication, Stein et al. (2008) observed:

> Since the advent of more student-centered, inquiry-based forms of instructional practice, teachers have struggled with how to orchestrate discussions in ways that both engage students' sense-making in authentic ways and move the class as a whole toward the development of important and worthwhile ideas in the discipline. (p. 332)

In addition, a common misunderstanding regarding classroom discourse is summarized by Stein (2007):

> There is a misconception that the shift toward the use of classroom discourse in teaching mathematics means that the teacher simply presents the problem and then stands aside while students discuss and solve it (Chazan & Ball, 1995). The teacher's instructional role is perceived as “don't tell the answer”. This perception severely underrates the complexity of the teacher's role in classroom discourse (Chazen & Ball, 1995). (p. 286)

Likewise, studies have shown that students may also experience challenges when attempting to express their mathematical thoughts through written means (Koch, 1993; Stonewater, 2002). While teachers agree that the integration of writing with science or writing with social science is easier, the integration of writing with mathematics is often less attempted and less conceived (Altieri, 2010; Applebee & Langer, 2006; Wilcox & Monroe, 2011). Moreover, Wilcox and Monroe (2011) shared their perspectives of literature on mathematics and writing:

> That we see so few examples of the integration of writing and mathematics in education
literature seems surprising, considering that the mathematics education community has affirmed the importance of such integration for many years. As early as 1989, the National Council of Teachers of Mathematics (NCTM) identified learning to communicate mathematically as a major goal for students. (p. 521)

1.3 Purpose of the Study

For this study, my goal is to explore and describe the teaching perceptions, practices, and strategies of an elementary teacher of mathematics, as she attempts to foster students' mathematics communication skills in their learning of measurement ideas. The data collected for this inquiry includes semi-structured teacher interviews and classroom observations.

1.4 Statement of the Problem

Knowing that teachers' attitudes can affect their pedagogical choices and practices (Applefield, Huber, & Moallem, 2000/2001; Bruce & Ross, 2008; NCTM, 2014), my case study inquiry is guided by the following research question: What teaching perceptions, practices, and strategies are utilized in the fostering of Grade 3 students' mathematics communication skills, in the content strand of measurement?

1.5 Significance of the Study

Through this study, I hope to better understand the strategies used in the fostering of mathematics communication skills in elementary students, specifically for the measurement content strand. The interview responses and classroom observations will shed light on the similarities and differences between a teacher’s perceptions and practices, along with their alignments to recommendations from relevant scholarly literature. In addition, the sub-unit vignette analysis in combination with the holistic case analysis of a Grade 3 teacher participant's implementation decisions and strategies will provide rich, descriptive stories.

Through her participation in this study, I believe that my Grade 3 teacher participant may
have more opportunities to reflect upon her teaching strategies and practices, and become more aware of her perceptions and approaches. Hopefully, this experience may encourage her to continuously reflect upon such practices, and seek additional professional learning opportunities. Beyond the teacher participant, the findings from this study may also be beneficial to educators and mathematics consultants, as they seek to foster and nurture students' mathematics communication skills.

1.6 Background of the Researcher

My interests in the topic of mathematics communication can be traced to my experiences as a learner and an educator. As a student, mathematics has always been amongst my favourite subjects. During my early elementary years, I experienced the learning of mathematics in different schools, under various teachers, and amongst various classmates. In one classroom, my experience of school mathematics comprised mostly of instructional periods that were lecture-styled. During that academic year, a typical mathematics lesson had the structure of my teacher: 1) taking up homework from the previous night, 2) introducing new mathematics concepts, 3) providing a few examples, and 4) assigning independent seat-work that rehearsed the newly taught concepts and skills. Yet, I still found the learning of mathematics to be delightful. I believe this stemmed from an intrinsic enjoyment that I derive from manipulating numbers and identifying patterns.

However, in a subsequent academic year, I was introduced to mathematics learning that, in many ways, differed from my previous experience. One memory, which I recall vividly, is the way my mathematics teacher introduced the concept of multiplication. Even though my parents had previously taught me the multiplication table at home, the activities that we completed in
class fostered a deeper understanding. In class, we sorted beans into groups, and glued the beans onto large poster boards. We decorated these displays, and transformed them into colourful art works that adorned the classroom walls. We also had several students presented their posters to the class, and explained the mathematics behind the grouping of the beans. Previously accustomed to expressing my mathematical understanding primarily through pencil and paper (e.g. worksheets, quizzes, tests), I was pleasantly surprised by the variety of mathematics expressions.

The intrigue with communicating mathematical concepts through different means has remained through the years, and is still present. As a teacher candidate in the Bachelor of Education program, I desired for my students to understand the mathematical concepts, and to be able to communicate that understanding. I encouraged this by modelling and scaffolding the speaking, writing, and drawing of mathematical ideas and processes. As a mathematics tutor for students in the elementary and high school grades, I encouraged my students to think-aloud their problem solving strategies and processes. While I believe I have experienced some successes in helping students gain better mathematical understanding and proficiencies, I continuously reflect on my teaching approaches and my implementation strategies, such that I may become a better educator.

As a graduate student, I had the opportunity of being a research assistant for the Elementary Teacher Learning Initiative (2012-2016). It was a privilege to be part of a team that was committed to increasing elementary teachers’ confidence and efficacy in mathematics instruction. Through the partnership, I was also introduced to the Ten Dimensions of Mathematics Education (McDougall, 2004) framework, which identified essential components to
the teaching and learning of mathematics. Given my interest in the fostering of mathematics communication, I wanted to explore how various teaching and learning components may contribute to its development.

1.7 Limitations of the Study

While this study may provide rich descriptions and new insights to the teacher participant, mathematics educators, and mathematics consultants, some plausible limitations need to be noted in this section. First, the findings for this study are drawn from the interviews and classroom observations of one mathematics teacher participant. As each teacher possesses his or her distinct combinations of education background, teaching perceptions, and teaching strengths, the findings from this study may not be generalizable to every teacher of mathematics. Yet, while the sample size may be small, it still serves to provide in-depth, rich illustrations of classroom practices, from a teacher participant who: 1) possesses knowledge of and experience in teaching mathematics, and 2) expresses interests in continuous professional learning.

Second, this study focuses on one mathematics teacher’s perceptions and practices, and does not take into consideration students’ resulting mathematics performances and achievements. Undoubtedly, students' perspectives and responses are crucial, but data regarding those aspects are not collected or analyzed in this study, as they are beyond the scope of this research.

1.8 Plan of the Thesis

This thesis consists of five chapters. In Chapter One, I provide an outline of my study by describing the research context, the purpose of the study, the guiding research question, the significance of the study, the background of the researcher, and the limitations of the study. In Chapter Two, I highlight relevant scholarly literature pertaining to: 1) mathematics
communication, 2) mathematics learners, 3) teachers' perceptions, and 4) teachers' practices. I also connect learner-centered curriculum theories (constructivism and sociocultural) to the learning of mathematics and the fostering of mathematics communication skills.

In Chapter Three, I describe and justify a suitable method for examining this study. Details regarding the research design and research context are provided. Data collection methods (semi-structured interviews and classroom observations) and data analysis procedures are justified.

In Chapter Four, findings from the data collected are analyzed, organized, and presented. The chapter begins with the education background, teaching experiences, and education goals of Sophie, a Grade 3 teacher participant selected from the Elementary Teacher Learning Initiative. Next, detailed categorical analysis of six lesson vignettes are described. Finally, in Chapter Five, the categories from the lesson vignettes depicted in Chapter Four are integrated and summarized. This summary also connects current findings with existing, relevant, scholarly literature. The chapter concludes with identifications of research limitations, and suggestions for future research studies in the area of mathematics communication development.
Chapter Two: Literature Review

2.1 Introduction

This chapter begins with a description of prominent curriculum and learning theories in the field of mathematics education research. Specifically, learner-centered curriculum theory, constructivist learning theory, and sociocultural learning theory are described, with an analysis of their influences in mathematics teaching standards (e.g. NCTM, 2000, 2014) and mathematics curriculum document (e.g. *Ontario Mathematics Curriculum, Grades 1-8*, rev. 2005). After presenting the theoretical lens, related literature surrounding the fostering of mathematics communication skills are presented.

Both scholarly literature and professional literature are embedded into relevant sub-topics of focus, as this may provide new insights and complementing ideas. Often written by researchers and experts in the field, most scholarly literature aim to expand our current understanding of the field through empirical research, rigorous study, and systematic analysis. Professional literature and policy documents may be written for and read by practitioners. Through synthesizing the two realms of literature, I hope to present the voices that are expressed in the research community, as well as the ones that are conveyed in the practitioner community (all the while being aware that one may be a member of both communities, as the two communities are not distinct).

First, mathematics communication is defined as a construct, and situated within the goals of mathematics learning and teaching. Second, measurement as a mathematics content strand is explored, and situated within the organization of the elementary curriculum. Third, elementary mathematics learners' needs are described. Next, I present scholarly literature regarding: 1) the
influences of mathematics teachers' perspectives on their practices, 2) the effects of teachers' practices on students' mathematics attitudes and achievements, and 3) some specific ways in which students' communication skills can be fostered in the classrooms. The strategies are organized into six components: 1) program planning, 2) learning environment, 3) mathematics tasks, 4) constructing knowledge, 5) technology and manipulatives, and 6) assessment.

2.2 Curriculum Theories and Mathematics Education

2.2.1 Theory and Curriculum as Constructs

Beauchamp (1982) suggested that a deeper understanding of “curriculum theory” should begin by delving into the concept of “theory”. What is theory? Some have referenced the Greek root “theorein”, which means “to see” or “to perceive” (Mason, 2011). Rose (1953) offered the following detailed definition: “A theory may be defined as an integrated body of definitions, assumptions, and general propositions covering a given subject matter from which a comprehensive and consistent set of specific and testable hypotheses can be deduced logically” (p. 52). The purposes of “theory” include: 1) organizing the research design (the questions to ask, the data to collect, the analysis to occur, the interpretations to make), 2) providing a means to discern, and 3) encouraging a discourse (Mason, 2011). “At their best, theories provide a complete weltanshauung, a way of being as an enquirer” (Mason, 2011, p. 2485).

Similarly, a more comprehensive understanding of “curriculum theory” should include examining the notions of “curriculum” (Egan, 2003). What is curriculum? Here, I share the sentiments expressed by Bruner (1996), in that “a curriculum is like an animated conversation on a topic that can never be fully defined” (p. 116). To start, the Latin root “currere”, which means “a running” or “a course”, has often been referenced (Egan, 2003; Kirylo, 2010; Pinar & Grumet,
1976). Beauchamp (1982) identified three usages of the word: 1) curriculum as a document (for the attainment of specific outcomes), 2) curriculum as a system (for development, organization, and evaluation of materials), and 3) curriculum as a field of study.

Over time, “curriculum” has gradually shifted from its Latin metaphorical beginnings to inquiries on learning and teaching (Egan, 2003). The increased focus on students and teachers is reflected in Schwab's (1983) definition of curriculum:

Curriculum is what is successfully conveyed to differing degrees to different students, by committed teachers using appropriate materials and actions, of legitimated bodies of knowledge, skill, taste, and propensity to act and react, which are chosen for instruction after serious reflection and communal decision by representatives of those involved in the teaching of a specified group of students who are known to the decision makers. (p. 240)

### 2.2.2 Definitions, Purposes, and Types of Curriculum Theories

Pinar (2012) defined “curriculum theory” as “the scholarly effort to understand the curriculum, conceived here as 'complicated conversation’” (p. 1). Ideally, curriculum design (educational content, organization, and strategies) is correlated with the underlying curriculum theories. The purposes of “curriculum theory” include: 1) the organization of thoughts and works in the curriculum field, and 2) the improvement of conversations amongst scholars within the field (Beauchamp, 1982).

When examining curriculum theory and curriculum design, fundamental questions regarding worth and significance arise (Dillon, 2009; Pinar, 2012; Schubert, 2009; Spencer, 1860; Tyler, 1950). Such questions include “What knowledge is of most worth?” (Pinar, 2012, p. 2) and “What do we need in order to live a good and fruitful life that grows with and not against our natural and human surroundings?” (Schubert, 2009, p. 27).

Dillon (2009) identified seven broad categories of curriculum questions: 1) who, 2)
whom, 3) what, 4) where and when, 5) why, 6) how, and 7) what results. Schubert (2009) expanded the idea of worth to further include the foci of need, experience, and wonder. He posed the question: “What is worth knowing, experiencing, doing, needing, being, becoming, overcoming, sharing, and contributing?” (Schubert, 2009, p. 26). Further, key questions in curriculum need to be negotiated in light of many perspectives, including (but not limited to) economics, environments, experiences, history, identity, social conditions, and politics (Petrina, 2004).

Through a review of literature on curriculum studies, some of the prominent curriculum theories can be categorized into: 1) content-centered (e.g. Eric Hirsch, 2010), 2) learner-centered (e.g. John Dewey, 1902), and 3) social-efficacy-focused (e.g. Ralph Tyler, 1950). Each one of these theories is supported by underlying ontological and epistemological assumptions. For my study, I focused on the perspectives of learner-centered curriculum theory and their influences on the Ontario mathematics curriculum.

2.2.3 Learner-Centered Curriculum Theory

Learner-centered curriculum theory emerged from the combined contributions in the fields of student development and learning development (Cleveland-Innes & Emes, 2005). A learner-centered curriculum focuses on students' learning experiences, and places great emphasis on providing students with learning choices (Cleveland-Innes & Emes, 2005). Through this approach, it hopes to: 1) encourage students to actively participate in creating their personal learning experiences, 2) provide diverse opportunities to develop, experience, and express the skills and knowledge attained, and 3) aid students to become continuous learners - in both formal and informal education environments (Cleveland-Innes & Emes, 2005).
Often, the perspectives of learner-centered curriculum theory are traced to the viewpoints expressed by education reformer John Dewey (Ball, Hill, & Bass, 2005; Cleveland-Innes & Emes, 2005; Cobb, Yackel, & Wood, 1992; Deng, 2007; Noddings, 1998). Noddings (1998) noted that “[the] curriculum, for Dewey, is not a body of material established before instruction. Instead, it is the material gathered, used, and constructed during instruction and inquiry” (p. 37). The curriculum is also viewed by Dewey as a vital component in the reformation of education (Deng, 2007). In comparison to the traditional curriculum, Dewey believed it should be constructed with the children's experiences, habits, interests, and reflections as priorities (Noddings, 1998; Villanueva, 2008).

With these perspectives on curriculum and student learning, Dewey suggested that teachers should provide learning opportunities in light of students' prior experiences (Noddings, 1998). In addition, teachers should select activities by envisioning the potential experiences the students may encounter through participating in the tasks (Ball, Hill, & Bass, 2005; Noddings, 1998). The latter considerations are important, as “every experience affects for better or worse the attitudes which help decide the quality of further experiences (Dewey, 1938, p. 38)” (Kanu & Glor, 2006, p. 107).

2.2.3.1 Learner-Centered Curriculum Theory in Mathematics Education

Ball, Hill, and Bass (2005) suggested that the learning of mathematics can be approached through a learner-centered framework. Referring to Dewey (1902) and his concept of “psychologizing” the subject matter, Ball, Hill, and Bass (2005) noted that new mathematical ideas should be viewed from the learners' perspectives, with an awareness of possible experiences when encountering the mathematical concepts for the first time.
Within the National Council of Teachers of Mathematics (2000, 2014), learner-centered perspectives are strongly noticeable. Students' prior knowledge is evidently crucial, as one of the five Process Standards is “connection” (NCTM, 2000). This strand suggests that the connection of mathematical ideas should include “linking new ideas to related ideas considered previously” (NCTM, 2000, p. 200). Likewise, effective teaching of mathematics is described to include understanding students' existing mathematical knowledge, and designing lessons that connect new ideas so students can “respond to, and build on, this knowledge” (NCTM, 2000, p. 27). These ideas are also reflected in the more recent NCTM publication (2014), which encourages teachers of mathematics to select mathematics tasks with students' prior knowledge and experiences as guide, and to envision students' plausible struggles and misconceptions.

Similarly, the Ontario Mathematics Curriculum, Grades 1-8, (rev. 2005) notes that “effective instructional approaches and learning activities draw on students' prior knowledge, capture their interest, and encourage meaningful practice both inside and outside the classroom” (p. 25). Thus, these publications and curriculum document place strong emphasis on teachers' considerations of students' prior knowledge, experiences, and interests. When applied to the development of students' mathematics communication skills, teachers can assess students' prior learning opportunities, and build upon students' existing mathematics vocabularies.

2.2.4 Constructivist Theory

“Constructivism is an epistemological view of knowledge acquisition emphasizing knowledge construction rather than knowledge transmission and the recoding of information conveyed by others” (Applefield, Huber, & Moallem, 2000/2001, p. 37). Viewed by Radford (2008) as a learner-centered theory, constructivist theory has become progressively more
prominent within the field of education in the last two decades of the 20th century (Applefield, Huber, & Moallem, 2000/2001). The influence of cognitive constructivism, sometimes also referred to as psychological constructivism, has not only been witnessed in North America, but also in many parts of the world (Radford, 2008). Though emerged from the field of cognitive science, constructivist ideas can be traced to the perspectives of John Dewey, William James, Jean Piaget, and Lev Vygotsky (Applefield, Huber, & Moallem, 2000/2001; Terwel, 1999).

A prominent voice in cognitive constructivism is Jean Piaget (Anderson, Reder, & Simon, 2000; Applefield, Huber, & Moallem, 2000/2001). Piaget (1936) identified four stages of cognitive development, which are: 1) Sensorimotor, 2) Pre-operational, 3) Concrete Operational, and 4) Formal Operational. The creations of meaningful learning experiences should consider the learners' stages of cognitive development. Likewise, a prominent voice in social constructivism is Lev Vygotsky (Applefield, Huber, & Moallem, 2000/2001; Terwel, 1999). Vygotsky (1978) pondered the relations between learning and development for school-aged children. After reviewing and summarizing three major theoretical positions, he suggested a learning framework which highlighted the social nature of knowledge acquisition and construction (Vygotsky, 1978). He expressed:

We propose that an essential feature of learning is that it creates the zone of proximal development; that is, learning awakens a variety of internal developmental processes that are able to operate only when the child is interacting with people in his environment and in cooperation with his peers. Once these processes are internalized, they become part of the child's independent development achievement. (Vygotsky, 1978, p. 90)

With these perspectives on learning, constructivist teachers should welcome opportunities and strategies for students to construct their own knowledge and understanding (Applefield, Huber, & Moallem, 2000/2001). An essential element is the “negotiation of meaning”, where
teachers influence students' reasoning without hindering students' self-determination (Radford, 2008). Terwel (1999) noted that constructivist ideas have essential potentials to offer the field of curriculum theory. With a more detailed understanding of students' learning processes, questions regarding curriculum development and curriculum implementation can be further explored (Terwel, 1999).

### 2.2.4.1 Constructivist Theory in Mathematics Education

Within mathematics education, significant ideas and insights have resulted from the “cognitive revolution”, which began in the 1960s (Anderson, Reder, & Simon, 2000). In particular, constructivist ideas and constructivist teaching approaches have been widely accepted in the science and mathematics disciplines (Applefield, Huber, & Moallem, 2000/2001). Given the amount of materials written from this theory's perspectives, Ball and Bass (2003) considered constructivism to be “one of the most influential- and most multiply interpreted- ideas in mathematics education” (p. 29).

When envisioning a constructivist mathematics learning environment based on ideas expressed by Bauersfeld, Krummheuer, and Voigt (1988), Cobb, Yackel, and Wood (1992) painted the following image:

Learning would be viewed as an active, constructive process in which students attempt to resolve problems that arise as they participate in the mathematical practices of the classroom. Such a view emphasizes that the learning-teaching process is interactive in nature and involves the implicit and explicit negotiation of mathematical meanings. In the course of these negotiations, the teacher and students elaborate the taken-as-shared mathematical reality that constitutes the basis for their ongoing communication (Bauersfeld, Krummheuer, & Voigt, 1988). (p. 10)

Constructivist theory suggests that one goal of mathematics instruction is the building and remodelling of more elaborate, more powerful, and more abstract mental structures (Cobb,
Thus, “one of the teacher’s primary responsibilities is to facilitate profound cognitive restructuring and conceptual reorganizations” (Cobb, 1988, p. 89). Such roles require teachers to have: 1) in-depth understanding of the subject matter, 2) awareness of students' mathematical concept construction, and 3) hypothesis of plausible alternative or erroneous mathematical concept constructions (Cobb, 1988).

Constructivist ideas can easily be found in NCTM’s *Principles to Actions* (2014). From the field of cognitive science, results and discussions regarding mathematics education are acknowledged. As such, “learning is viewed as an active process”, where students' unique experiences, coupled with their interactions with peers and teachers, contribute to their knowledge construction (NCTM, 2014, p. 8). Likewise, the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005) notes the importance of providing students with opportunities to investigate mathematical ideas independently.

Specific to the fostering of mathematics communication, negotiations of shared meaning are most successful when: 1) respect is mutually and equally shared amongst all group members, 2) teachers do not “teacher-talk”, and 3) mathematically rich tasks requiring divergent thinking are implemented (Perry, 2001). Being “confronted” with relevant and complex tasks was also noted by Applefield, Huber, and Moallem (2000/2001) as important in constructivist mathematics classrooms. The implementation of difficult mathematics tasks emphasizes teachers' guidance as an essential component in the development of needed skills (Applefield, Huber, & Moallem, 2000/2001; Jackson et al., 2013; Moschkovich, 2015). In selecting, creating, and/or implementing effective mathematics tasks, considerations should be given to multiple factors. Such factors may include students' culture, experiences, interests, language, and learning
preferences (Cross et al., 2012; Jackson et al., 2013; Kisker et al., 2012; Olteanu, 2015).

2.2.5 Sociocultural Theory

Sociocultural theory has enjoyed broad circles of influences, having contributed to several psychology fields (e.g. cross-cultural, developmental, educational), and numerous geographical locations (e.g. Asia, Australia, Europe, North and South America) (Forman, 2003). This prominent theory can be traced to the perspectives of Jerome Bruner and Lev Vygotsky (Radford, 2008; Scott, 2008). Jerome Bruner emphasized the importance of social and cultural elements in the curriculum, while Lev Vygotsky acknowledged that knowledge has components that are socially formed (Scott, 2008). Both psychologists believed that society and culture are prominent elements in pedagogical considerations (Scott, 2008).

From a sociocultural perspective, learning takes place within communities, where the social contexts play significant roles (Applefield, Huber, & Moallem, 2000/2001; Moschkovich, 2015). Vygotsky coined the phrase “zone of proximal development”, which is “the gap between the novice and the expert” (Scott, 2008, p. 87), or “the difference between assisted and unassisted performance” (Forman, 2003, p. 334). Specifically, this is the capacity of learning between what a learner can accomplish individually, and what he or she can accomplish with the assistance of a more advanced peer (Vygotsky, 1978).

In such cases, learning most likely occurs at the “edges” of students’ existing knowledge. Therefore, scaffolding can assist students to persevere beyond what they can accomplish individually (Radford, 2008). Other forms of learning support that may reflect scaffolding approaches include peer tutoring, cooperative learning, and learning communities (Applefield, Huber, & Moallem, 2000/2001; Brown, 1994; Moschkovich, 2015; Rogoff, 1998). With these
perspectives on learning, the roles of sociocultural-minded teachers include: 1) guiding students in their attempts of new learning experiences (also referred to as scaffolding), and 2) providing opportunities for learners to collaborate (Radford, 2008; Scott, 2008).

### 2.2.5.1 Sociocultural Theory in Mathematics Education

Sociocultural ideas were introduced into mathematics education around the early 1990s (Radford, 2008). Even recently, Vygotsky's perspective on learning is well-utilized as a theoretical framework for numerous mathematics education studies (e.g. Herbel-Eisenmann et al., 2015; Hintz & Tyson, 2015; Imm & Stylianou, 2012; Lee & Johnston-Wilder, 2013; Purdum-Cassidy et al., 2015). Much of the sociocultural insights and influences have been in the realms of classroom discourse, classroom perception, and the interplay of culture and cognition (Radford, 2008). Forman (2003) identified three aspects of Vygotsky's sociocultural theory that contribute to the discussions of mathematics education reforms. These are: 1) to view learning as a dynamic process, thus changing the ways in which mathematics research designs are formulated, 2) to create learning opportunities within communities of practice (Lave & Wenger, 1991), and 3) to elevate and recognize the importance of social and cultural elements in the learning of mathematics (Forman, 2003, p. 335).

Forman (2003) envisioned that “new developments within the sociocultural theory, as well as changes in mathematics instruction due to the influence of NCTM Standards documents suggest that numerous fruitful future connections can be made between this theory and educational practice” (p. 333). Indeed, NCTM (2000) discusses the importance of mathematics learning and teaching within a community. In the junior years (grades 3 to 5), the discovery of mathematical knowledge within a welcoming community of learners is reinforced and
recommended. Similarly, the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005) suggests providing students with cooperative learning opportunities alongside individual work, and teacher scaffolding alongside independent explorations.

Sociocultural perspectives emphasize the collective nature in the fostering of mathematics communication (Maheux & Roth, 2014). Through communicating, reasoning, and defending arguments, students participate in and contribute to their communities (Rasmussen & Marrongelle, 2006). The learning of mathematics can be viewed as community-embedded activities, and classroom discourse as interactions mutually co-created by the teacher and the students (Walshaw & Anthony, 2008). Maheux and Roth (2014) further described this co-ownership:

> From the perspective of the conversation as *social* phenomenon, question/answer, invitation/acceptance, invitation/rejection, instruction/following, etc., always come in pairs, where each part is a part only in relation to the other part: a question is a question because there is an answer, and an answer is an answer because there is a question (Roth & Gardener, 2012). Thus, although the teacher produces the sounds that we hear as words forming questions, these have to be designed for the students, in whose ears they resound to exist as such. The sounds, therefore, are the students' as much as they are the teacher's. (p. 516)

### 2.2.6 Summary

Mathematics curriculum and mathematics education are embedded with learner-centered perspectives. In both constructivist theory and sociocultural theory, the students' learning experiences and learning potentials are of great considerations. Yet, at the same time, “no single theory can provide an adequate foundation for the design of curricula” (Terwel, 1999, p. 197). Thus, multiple frameworks, theories, and viewpoints should be inter-weaved in the creations of meaningful curricula (Terwel, 1999).

Within mathematics education, the French word “bricolage”, defined as cherry-picking,
refers to the selection of relevant ideas from multiple theoretical and philosophical frameworks (Lester, 2005; Mason, 2011). As suggested by Radford (2008), “the investigation of integration of theories and their differentiation is likely to lead to a better understanding of theories and richer solutions to practical and theoretical problems surrounding the teaching and learning of mathematics” (p. 14).

While theories can be “compared and contrasted”, they can also be “coordinated and integrated” (Prediger, Bikner-Ahsbahs, & Arzarello, 2008; Radford, 2008). One notable mathematics education researcher who promoted the complementary approaches to learning theories is Paul Cobb (1988, 1994). Cobb (1994) noted the prominence of these two viewpoints in American education research development, and suggested that “each of the two perspectives, the sociocultural and the constructivist, tells half a good story, and each can be used to complement the other” (p. 17).

2.3 The Goals of Mathematics Education

Mathematics, from the perspective of Dutch mathematician Hans Freudenthal, is fundamentally a human activity (Gravemeijer & Terwel, 2000; Rasmussen & Marrongelle, 2006). The subject should be pursued continuously, and viewed as a process rather than a product. The word “mathematizing” was coined to describe this approach (Gravemeijer & Terwel, 2000). The teaching of mathematics can thus be described as “responding to student activity, listening to student activity, notating student activity, learning from student activity and so on. In this sense, mathematics teaching is a human activity about human (e.g. student) activity” (Rasmussen & Marrongelle, 2006, p. 414).

The Ontario Mathematics Curriculum, Grades 1-8 (rev. 2005) believes “the study of
mathematics equips students with knowledge, skills, and habits of mind that are essential for successful and rewarding participation in such a society” (p. 3). This can be achieved through engagements with mathematical experiences that are practical, relevant, and appropriately challenging (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005). Furthermore, the National Council of Teachers of Mathematics (2000) identifies the fostering of autonomous learners as one of the main goals of successful mathematics programs. This can be achieved by providing students with opportunities to establish their learning goals and monitor their own growths (NCTM, 2000). In both documents, mathematics content strands (e.g. geometry, measurement, number sense) and mathematics processes (e.g. communication, problem solving, reasoning and proof) are described in detail. Both mathematics content and mathematics processes are intricately essential to the success of mathematics learning, and a “false content-process dichotomy” should be avoided (Case, 2005).

In this section, I intend to describe: 1) the benefits of verbal and written communication for the teaching and learning of mathematics, as expressed by the mathematics education research community, 2) the promotions of mathematics communication development within mathematics principles and standards framework (e.g. NCTM, 2000), and mathematics curriculum document (e.g. Ontario Mathematics Curriculum, Grades 1-8, rev. 2005), and 3) the similarities and differences in the organization of mathematics communication skill among these standards and curriculum documents. The Principles and Standards for School Mathematics (NCTM, 2000) and the Ontario Mathematics Curriculum, Grades 1-8 (rev. 2005) are chosen for this analysis, as these documents are well-used within the Ontario mathematics education programs. Thus, they are familiar to the teacher participants in the Elementary Teacher Learning
2.3.1 Verbal and Written Mathematics Communication

Lutzer (2005) defined mathematical literacy as “being able to communicate and understand ideas written in the language of mathematics” (p. 6). For education researchers Santos and Semana (2015), mathematics communication is perceived to include “interpretation, justification, and the use of representations” (p. 67). Mathematical thinking is often linked with mathematics communication, and the expressions of mathematical understanding and misunderstandings can be accomplished through various means (Kostos & Shin, 2010; Lampert, Rittenhouse, & Crumbaugh, 1996; NCTM, 2000; Rasmussen & Marrongelle, 2006). Within the mathematics classrooms, these forms of communication may include speaking, listening, reading, and writing (Marks Krpan, 2013; Thompson & Chappell, 2007). Amongst educators and researchers, the importance of mathematics communication to the learning of mathematics is generally agreed upon (e.g. Cai, Jakabcsin, & Lane, 1996; Diez-Palomar & Olive, 2015; Hintz & Tyson, 2015; Ing et al., 2015; Kostos & Shin, 2010; Perry, 2001). Below, verbal and written communication are defined, and some benefits of these skills for the learning of mathematics are described.

Amongst the various methods of mathematics communication, Thompson and Chappell (2007) suggested that verbal communication (e.g. speaking and listening) may perhaps be the “most natural form” for students to express their emerging ideas. Walshaw and Anthony (2008) viewed mathematics talks and mathematics discussions as “acceptable”, “essential”, and the “defining features of a quality mathematical experience” (p. 516). Reviews of scholarly literature have shown that a substantial amount of studies are in support of the diverse benefits of
mathematics dialogues and discussions in the classrooms (e.g. Tyminski et al., 2014; Walshaw & Anthony, 2008). Specifically, by consistently discussing with and listening to one another, students may: 1) develop more expertise and ease in linking mathematical ideas (Marks Krpan, 2013), 2) co-create and establish learning environments that are supportive of mathematical inquiries (Diez-Palomar & Olive, 2015), and 3) become more accurate in assessing own mathematical understandings and misconceptions (Forman et al., 1998; Thompson & Chappell, 2007).

Mathematical ideas can also be communicated through written means, through various forms. Journals, logs, and daily diaries are some of the more frequently used methods for students to record their mathematical procedures, solutions, and reflections (Thompson & Chappell, 2007). Writing is viewed as a process that assists in developing both communication skills and mathematics proficiencies (Kostos & Shin, 2010). Communication skills are fostered in that, through writing, students: 1) practise explaining their ideas logically and coherently (Haltiwanger & Simpson, 2013; Rose, 1989), 2) develop an awareness of the intended audience (e.g. teachers, parents, fellow students) (Haltiwanger & Simpson, 2013) and 3) assess own communication abilities through the processes of reviewing and revising (Koch, 1993; McCormick, 2010). Mathematics abilities are developed in that, through writing, students: 1) practise the correct uses of mathematics vocabularies (Kostos & Shin, 2010), 2) facilitate their mathematical understanding and reasoning (Kostos & Shin, 2010; Pugalee, 2001; Rose, 1989) and 3) interconnect mathematical concepts (Haltiwanger & Simpson, 2013; NCTM, 2000).

2.3.2 Mathematics Communication in Standards and Documents

For the past few decades, the National Council of Teachers of Mathematics (1989, 2000,
2014) has emphasized the importance of mathematics communication. In the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), the development of communication skills was recommended as a standard at each grade level. Organized into five Content Standards and five Process Standards, the *Principles and Standards for School Mathematics* (NCTM, 2000) presents the knowledge and skills students should acquire as they progress through mathematics programs. Mathematics communication is included as one of the five mathematics process standards described in the *Standards* (NCTM, 2000). Acknowledged as “an essential part of mathematics and mathematics education”, communication is defined as “a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment” (NCTM, 2000, p. 60).

Similarly, the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005) emphasizes communication as “an essential process in learning mathematics” (p. 17). With the development of mathematics communication skills, students can better express their mathematical understanding, more confidently articulate their mathematical reasoning and arguments, and more capable of reflecting upon their ideas (*Ontario Mathematics Curriculum, Grades 1-8*, rev. 2005). Organized into five mathematics content strands and seven mathematics processes, the curriculum document highlights the facts, procedures, and skills that balanced mathematics programs should present. Included as one of the seven mathematics processes, mathematics communication should be viewed and taught as interconnected with the other processes.

Reflecting upon the organization of the process standards identified and presented in NCTM (2000), Perry (2001) notes that the communication standard only focused mainly on the verbal and written expressions of mathematical ideas. The pictorial and graphical expressions of
mathematical ideas are allotted into the representation standard. When examining the
organization of mathematics processes in the *Ontario Mathematics Curriculum, Grades 1-8* (rev.
2005), the communication process includes pictorial, graphical, verbal, and written expressions,
though an overlap of these descriptions can be found in the representation process. While the
organization of the content knowledge and process skills may differ slightly, both the *Principles
and Standards for School Mathematics* (NCTM, 2000) and the *Ontario Mathematics
Curriculum, Grades 1-8* (rev. 2005) successfully portrayed the essential need to practise and
acquire mathematics communication skills at all grade levels of mathematics learning.

### 2.4 Elementary Mathematics Learners

The elementary education years are often of great influences to students (Kilpatrick &
Swafford, 2002; *Teaching and Learning Mathematics*, 2004). During that season, students’
learning experiences may contribute to the fostering or hindering of: 1) foundational
mathematics knowledge, 2) mathematical processes developments, and 3) positive attitudes and
interests towards the subject (Erdogan & Sengul, 2014; Jameson, 2014; Kilpatrick & Swafford,

With regards to foundational mathematics knowledge and specific mathematics content,
as students advance through the elementary grades, it is accompanied by a growing focus on the
topics of algebra and probability (NCTM, 2000; *Teaching and Learning Mathematics*, 2004). In
Ontario, this gradual mathematics content shift occurs around the transition from the primary
division (grades 1 to 3) to the junior division (grades 4 to 6) (*Teaching and Learning
Mathematics*, 2004). In every grade, students should be encouraged and supported in their
explorations of increasingly abstract mathematical ideas. One strategy is to give students time
and opportunities to meaningfully connect newly introduced mathematics ideas to existing mathematics knowledge (Boaler & Humphreys, 2005; Franke, Kazemi, & Battey, 2007).

Measurement ideas are essential, as it is “an important set of concepts that are fundamental for students to competently navigate and work with objects and structures within their physical environment” (Lee & Francis, 2016, p. 218). While the understanding of measurement concepts and the development of measurement abilities are important to mathematics learners in all grades (Battista, 2007; Lee & Francis, 2016; NCTM, 2000; Ontario Mathematics Curriculum, Grades 1-8, rev. 2005), the early elementary years provide special opportunities for building fundamental skills (NCTM, 2000; Guide to Effective Instruction in Mathematics: Kindergarten to Grade 3 Measurement, 2007). The Guide to Effective Instruction in Mathematics (Kindergarten to Grade 3 Measurement, 2007) expressed this significance:

Learning opportunities in the primary grades help [the students] further develop concepts about what can be measured and about how to measure. These learning experiences occur not only in the mathematics program, but throughout the school day in various subject areas. (p. 4)

Unfortunately, as students progress through the grades, their overall levels of academic engagement tend to decrease (Fullan, 1991). Regarding students’ attitudes towards mathematics, NCTM (2000) states that, generally, “most students enter grade 3 with enthusiasm for, and interest in, learning mathematics” (p. 143). However, studies have also shown that students can develop anxieties and aversions towards the subject during the early years (Arslan, Yavuz, & Deringol-Karatas, 2014; Erdogan & Sengul, 2014; Jameson, 2014). If students experience mathematics as narrow, recitation-focused, and procedure-driven, their appreciation of the subject and their willingness to explore it may lessen (Stodolsky, 1985; Toluk Ucar et al., 2010). At the same time, students may view the subject in different light if they are exposed to learning
environments that: 1) channel positive attitudes towards the subject, and 2) promote high self-efficacies for learners (Ashcraft & Krause, 2007; Meece, Wigfield, & Eccles, 1990; Teaching and Learning Mathematics, 2004).

2.4.1 Mathematics Communication in Elementary Classrooms

While the development of mathematics communication skills is deemed important for students of all grades, the elementary school years provide unique opportunities. Copley (2000) recognizes the learning of mathematics in the early grades often involves communication skills such as talking, discussing, and listening. Likewise, in the primary and junior years, these skills should be continuously reinforced and practised (NCTM, 2000). Brendefur and Frykholm (2000) believe students' communication of mathematical ideas leads to greater mathematical understanding, and the Teaching and Learning of Mathematics (2004) notes the importance of developing mathematical understanding and mathematics communication during these years.

NCTM (2000) suggests that, from grades three to five, previously learned and newly learned mathematics vocabularies should be reviewed and practised, such that they become regularly used by students. In addition, communication opportunities (e.g. posing questions, explaining ideas, and justifying reasons) should be experienced frequently. As students progress through the elementary grades, they should also take on growing responsibilities in creating classroom communities that encourage mathematical dialogues (NCTM, 2000).

2.5 Attitudes and Practices of Teachers

“One of the main purposes of mathematics instruction is to help students think mathematically (Schoenfeld, 1988)” (Ding et al., 2007, p. 172). Ball, Hill, and Bass (2005) acknowledge the importance of “strong standards and quality curriculum”, but also point out that
“no curriculum teaches itself, and standards do not operate independently of professionals’ use of them” (p. 14). As such, skilled teachers are vital to the implementation of good curriculum.

Undoubtedly, teachers' classroom practices greatly influence students' mathematics learning experiences, since “mathematics teachers ultimately control the range of mathematical ideas made available to their students” (NCTM, 2000, p. 374). Based on personal teaching experiences and research findings, Taylor (1991) notes that educators' instructional decisions can also reinforce students' positive attitudes towards mathematics. Studies that examined teachers’ questions posing (e.g. Franke, Kazemi, & Battey, 2007; Stein et al., 2008), and teachers’ mathematics moments utilizations (e.g. Drageset, 2014; Stein et al., 2008) also yielded similar findings, as they affirmed the significance of teachers’ practices in creating and maintaining high quality learning environments.

While teaching practices are connected to students' learning experiences, teachers' attitudes and beliefs have also shown to affect teachers' practices (Applefield, Huber, & Moallem, 2000/2001; Brendefur & Frykholm, 2000; Bruce & Ross, 2008; Charalambous, 2015; NCTM, 2014; Rahal & Melvin, 1998). “Teachers' beliefs about teaching are important to their definitions of themselves as teachers”, and many beliefs are often formed early on when they themselves were students (Kennedy, 1997, p. 7). Teachers' personal beliefs regarding education philosophies and learning theories have influences on their instructional decisions (Applefield, Huber, & Moallem, 2000/2001). Principles to Actions (NCTM, 2014) contrasts productive and unproductive teacher beliefs, emphasizing the hindrances of negative beliefs to successful implementations.

Bandura (1997) described self-efficacy as a person’s beliefs regarding his or her ability to
adequately perform or complete challenging tasks. Related to teacher beliefs, teachers’ self-efficacy focuses specifically on perceptions of capabilities in teaching subject matters to students (Holzberger, Philipp, & Kunter, 2013). In a study on teacher self-efficacy, Bruce and Ross (2008) concluded that teachers are more likely to implement new teaching strategies, given that they perceive them as having positive contributions to students' learning. Given that increases in teachers’ self-efficacy can occur over an academic year for beginning and experienced teachers, teachers may benefit from more opportunities to nurture, reflect, and adjust personal perspectives on teaching capabilities (Holzberger, Philipp, & Kunter, 2013).

2.5.1 Teaching Attitudes and Practices that Foster Mathematics Communication

The National Council of Teachers of Mathematics and the Ontario Ministry of Education have identified, through their standards and documents, the importance of teachers' practices and implementations to the mathematics communication opportunities students receive. Verbal, visual, and written communication opportunities for students to learn mathematics are often provided by effective mathematics teachers (NCTM 2000, 2014). Likewise, the Ontario Mathematics Curriculum, Grades 1-8 (rev. 2005) notes that “effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class” (p. 25).

As mathematics communication is a significant component in standards-based classrooms, the study of teachers' conceptions of mathematics communication is important (Brendefur & Frykholm, 2000; Rahal & Melvin, 1998; Wood, Williams, & McNeal, 2006). Brendefur and Frykholm (2000) examined two beginning teachers' ideas and approaches to the promotion of mathematics communication, and focused on four forms of classroom communications: 1) uni-directional, 2) contributive, 3) reflective, and 4) instructive. Their two
participants contrasted in their selections of communication forms, with one more persistent in
the uses of uni-directional/teacher-directed instruction, while the other more open to the
explorations of various implementation forms. While Brendefur and Frykholm (2000) focused
their study on beginning teachers, the understanding of teachers' conceptions and practices in
fostering mathematics communication in all spectrums of teaching experiences may be valuable.

Successful mathematics programs consist of many inter-related components. For this
study, I utilized the conceptual framework of the Ten Dimensions of Mathematics Education
(McDougall, 2004; Ross et al., 2003) to categorically present strategies and practices that foster
students' mathematics communication. Specifically, beneficial teaching strategies, effective
activities, and useful tools will be highlighted. The Ten Dimensions of Mathematics Education
framework is selected, as it was created with standards-based mathematics visions and teaching
practices in mind. It is also a conceptual framework that the Elementary Teacher Learning
Initiative utilized. Hence, the teacher participant would already be familiar with the structure.
The mathematics teaching components described below are: 1) program planning (Dimension
One), 2) learning environment (Dimension Three), 3) mathematics tasks (Dimension Four), 4)
constructing knowledge (Dimension Five), 5) technology and manipulatives (Dimension Seven),
and 6) assessment (Dimension Nine).

2.5.1.1 Program Planning

Program planning focuses on the various types of lesson plans (e.g. daily, unit, long-
range) that teachers and teaching teams create and implement. Effective and articulate program
plans are vital, as the organizations of instructional times greatly influence the quality of
students’ mathematics learning (Chapin, O’Connor, & Anderson, 2009; Haneda, 2004; Stein et
al., 2000). Unfortunately, mathematics programs that are poorly organized may result in unnecessary content duplications across grades (NCTM, 2000). Some of the identified characteristics of effective mathematics programs include: 1) coherence and continuity in the presentations of mathematical content (NCTM, 2000), 2) emphasis on key mathematical ideas (Early Math Strategy, 2003; NCTM, 2000, 2014), and 3) transparency of learning goals communicated to students (Chappuis & Stiggins, 2002; Early Math Strategy, 2003; NCTM, 2014).

The designing and implementing of high quality mathematics programs require considerations for numerous instructional and pedagogical factors (Henning et al., 2012; McDougall, 2004; Stein, Grover, & Henningsen, 1996). When Henning et al. (2012) explored an elementary teacher’s instructional design for a grade seven mathematics program, four elements were examined in detail. These elements are: 1) the lesson sequence within the mathematics unit, 2) the learning goals for the lesson, 3) the types of instructional tasks, and 4) the assessment methods (Henning et al., 2012). Similarly, the Ontario Ministry of Education’s Guide to Effective Instruction in Mathematics (Volume One: Foundations of Mathematics Education, 2006) iterates the importance of program planning, and prompts teachers to make program organization decisions with considerations for various elements that affect learning and teaching. Such factors include “students’ prior knowledge, learning and cultural needs, and the appropriate uses of available resources” (Guide to Effective Instruction in Mathematics, Volume One: Foundations of Mathematics Education, 2006, p. 47).

When planning mathematics programs for students’ successes, one instructional approach for teachers to consider is cross-curricular and integrated learning. The benefits for combining
two or more subjects into one lesson include: 1) increased motivations and opportunities for mathematics dialogues (Young & Marroquin, 2006), 2) more relevant, comprehensive understanding and applications of the subjects (Michelsen & Sriraman, 2009; Riordain, Johnston, & Walshe, 2016), and 3) transferences of interests from one subject to another (Michelsen & Sriraman, 2009). The Ontario Language Curriculum, Grades 1-8 (rev. 2006) likewise encourages cross-curricular planning and learning:

Students need well-developed language skills to succeed in all subject areas. The development of skills and knowledge in language is often enhanced by learning in other subject areas . . . In cross-curricular learning, students are provided with opportunities to learn and use related content and/or skills in two or more subjects. (Ontario Language Curriculum, Grades 1-8, rev. 2006, p. 23)

The document highlights the connections between numeracy skills in mathematics and inquiry/research skills in all subject areas. Within the primary grades, students begin to pose meaningful questions, identify relevant information, and explore effective strategies. The Ontario Language Curriculum, Grades 1-8 (rev. 2006) further provides an example, which links the teaching of mathematics with the fostering of language. It states: “In mathematics, students learn to identify the relevant information in a word problem in order to clarify what is being asked” (Ontario Language Curriculum, Grades 1-8, rev. 2006, p. 23). To guide teachers in their organizations of mathematics programs, NCTM (2000) suggested that measurement concepts be comparatively more emphasized in the earlier years (kindergarten to Grade 6), and gradually less emphasized in the upper years (Grades 7 to 12).

Given that students do not engage in mathematics discussions naturally (Cobb, Wood, & Yackel, 1993; Thompson & Chappell, 2007; Walshaw & Anthony, 2008), nor do they encounter mathematics writing with ease (Koch, 1993; Stonewater, 2002), mathematics programs need to
be intentional in their integrations and balances of verbal and written practices (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005; Walshaw & Anthony, 2008). “The activities that teachers plan, and the sorts of mathematical discussions that take place around those activities, are crucially important to learning” (Walshaw & Anthony, 2008, p. 539). Indeed, students should have opportunities to explore a range of verbal (e.g. discussions, debates, presentations) and written (e.g. journals) strategies (Walshaw & Anthony, 2008).

2.5.1.2 Learning Environment

Learning environments include both the physical classroom organizations, along with the perceptions of classroom atmospheres. Intellectually stimulating and communication focused classrooms may include visual representations that encourage mathematics discourses (Arcavi, 2003; Lee & Herner-Patnode, 2007; Marks Krpan, 2013; Stylianou & Silver, 2004; Warshauer, 2015). Some examples of visual representations include: 1) word bank charts with essential vocabularies, and 2) posters with instructions and reminders for newly learned skills and processes (Furner, Yahya, & Duffy, 2005; Lee & Herner-Patnode, 2007; Marks Krpan, 2013).

NCTM (2000) believes the formation of “an intellectual environment where serious mathematical thinking is the norm” is an endeavour that teachers should continuously attend to (p. 18).

2.5.1.2.1 Safe Learning Environment

Within standards-based mathematics education, establishing and maintaining safe learning environments are well emphasized (Brendefur & Frykholm, 2000; McDougall, 2004; NCTM, 2000, 2014). For the fostering of mathematics communication, a safe and empowering environment is one where: 1) supportive statements are used by teachers to encourage discourse
participation from all students (Stein, 2007), 2) understanding of mathematics concepts is valued over the correct answers (Stein, 2007; White, 2003), 3) meanings are negotiated amongst teachers and peers (e.g. no “teacher-talk” or lecturing) (Lampert, Rittenhouse, & Crumbaugh, 1996; Perry, 2001), and 4) mathematical risks are encouraged (NCTM, 2000; Warshauer, 2015).

White (2003) analyzed two third-grade teachers' classroom discourse practices, and affirmed the importance of safe learning environments. From their study, they identified four emerging classroom discourse practices that encourages participation. The reoccurring themes were: 1) the importance of students' sharing, 2) the explorations of students' responses, 3) the incorporations of students' previous knowledge, and 4) the encouragements for discussions amongst students (White, 2003). In addition, Nuhrenborger and Steinbring (2009) observed mathematics interactions among students, teachers, and teacher-students in grade one and grade two classrooms. They noticed and described the influences of social environments on mathematics dialogues:

> Each social setting creates a new context within which, on the one hand, different ways of behavior and interaction come into their own and, on the other hand, mathematical knowledge is negotiated and mathematical teaching-learning processes are initiated in different ways. (Nuhrenborger & Steinbring, 2009, p. 127)

In safe learning environments, teachers perceive students' mathematical struggles and mistakes as valuable opportunities for explorations (Ding, Piccolo, & Kulm, 2007; Lampert, Rittenhouse, & Crumbaugh, 1996; NCTM, 2014; Warshauer, 2015). Given that viewpoint, teachers would often communicate that mistakes and struggles are natural and necessary components in the learning of new mathematical ideas and processes (Ding, Piccolo, & Kulm, 2007). Rather than inadequacies to be avoided, mathematical mistakes are worthwhile opportunities for new discoveries (NCTM, 2000; White, 2003). Students should also be given
regular occasions, individually and collectively, to wrestle with mathematical misconceptions (NCTM, 2014). Warshauer (2015) described such setting:

> While students’ struggle may arise in a wide spectrum of classroom environments, studies suggest that settings that are risk-free, where students can externalize their struggle and where consequences of “wrong” answers are not seen as failures but rather opportunities to explore, grow, and learn to serve better support and motivate students to persist. (p. 377)

> When students express opposing ideas or encounter contradicting facts, they need to learn how to respectfully navigate through these scenarios. With regards to mathematics argumentation, Lampert, Rittenhouse, and Crumbaugh (1996) highlighted the teacher’s role in guiding students on “how to disagree respectfully, how to express the evidence for their disagreement” (p. 734). One suggested way for fostering appropriate mathematical arguments is through encouraging students to approach the disagreements from the perspectives of their opposing peers (Lampert, Rittenhouse, & Crumbaugh, 1996). As an example to highlight several aspects of mathematics talk, Lampert (1998) described an incident in her fifth-grade class. While working collaboratively on a mathematics task, her students were confused with the “rules of the game”. Rather than simply providing her students with the correct information, Lampert encouraged them to: 1) present their various suggestions and positions, 2) pose questions to one another, and 3) request further clarifications.

### 2.5.1.2.2 Collaborative Opportunities

An effective mathematics program should provide students with a balance of diverse ways to learn - individually, in small groups, and with the whole class (Ding, Piccolo, & Kulm, 2007; NCTM, 2014; *Ontario Mathematics Curriculum, Grades 1-8*, rev. 2005). Nuhrenborger and Steinbring (2009) found students’ conversations among peers to be qualitatively different
from their conversations alongside teachers. As such, Nuhrenborger and Steinbring (2009) suggested communication opportunities with diverse audiences, through the regular inclusions of two-student dialogues, small group conversations, and large group discussions. Through these arrangements, students may learn to converse mathematics with different audiences, in various social settings.

Ding, Piccolo, and Kulm's (2007) investigation on cooperative learning also suggests that students' mathematical thinking and cognitive performance improved through a balance of individual work and group learning opportunities. Cooperative learning develops students' abilities to communicate mathematically with one another (Ding, Piccolo, & Kulm, 2007; Gillies & Boyle, 2010; Marks Krpan, 2013). From a constructivist perspective, the interactions generated in cooperative learning environments promote learners' formulations of ideas and concepts (Rasmussen & Marrongelle, 2006). Even though cooperative learning classrooms may be less quiet or orderly when compared with “traditionally conceived” classrooms, they may encourage more dynamic interactions between learners and their environments (Applefield, Huber, & Moallem, 2000/2001). Some of the teachers' roles in fostering a cooperative learning community include: 1) thoughtful grouping of students, 2) intentional selection of tasks and activities, 3) sufficient monitoring of group work, and 4) effective assessment of mathematics achievements and cooperation abilities (Bettenhausen, 2002).

2.5.1.3 Mathematics Tasks

Defined as “exercises, activities, or problems that focus students' attention on a particular mathematics idea” (Boston & Smith, 2011, p. 965), mathematics tasks are frequently implemented in lessons. As the TIMSS 1999 Video Study of the seven participating countries
discovered, at the minimum level, 80% of mathematics class times were devoted to working on problems (Hiebert et al., 2003). Numerous studies have also identified the importance of mathematics tasks to students’ learning (e.g. Clarke & Roche, 2010; Cole & Brown, 2013; Hiebert & Wearne, 2003; Kilpatrick & Swafford, 2002).

Since mathematics tasks vary in types and complexities, the selection of appropriately challenging tasks is an essential instructional decision that teachers make (Barrett et al., 2011; Lappan & Briars, 1995; Lee & Francis, 2016; Mills, 1995; NCTM, 2000, 2014; Olteanu, 2015). The importance of teachers’ task selections is reflected in Olteanu’s (2015) study, which examined tasks construction and the promotion of classroom communication. Olteanu (2015) concluded that “the notion of effective communication is important in this study because through and around tasks, teachers and students communicate and learn mathematical ideas” (p. 253).

When selecting mathematics tasks, two factors to consider are: 1) student engagements and, 2) cognitive demands (Kennedy, 2005; NCTM, 2014). Tasks with lower cognitive demand tend to emphasize the uses of steps, formulas, and algorithms, without thorough understandings of the connections with meaning (NCTM, 2014). On the contrary, appropriately challenging tasks and cognitively appropriate tasks can be used to motivate students to approach problems with confidence, to explore ideas with flexibility and creativity, and to meet struggles with perseverance (NCTM, 2000; Smith et al., 2009; Warshauer, 2015). Allowing students to be “confronted” with relevant, complex tasks is aligned with constructivist perspectives on learning and instruction, in that teachers’ guidance is an essential component in the development of needed skills (Applefield, Huber, & Moallem, 2000/2001). Barrett et al. (2011) described the selections and creations of measurement tasks that foster elementary students’ conceptual
understanding of quantities and units comparisons. They demonstrated the selection of tasks based on: 1) the emphasis on overarching mathematical concepts (e.g. “big ideas”), 2) the alignments with the learning goals, and 3) the possibilities for multiple representations (Barrett et al., 2011).

In addition, the ways in which mathematics tasks are implemented are as crucial to students' learning as the selected tasks themselves (Ball & Bass, 2003). Well-selected and thoughtfully implemented tasks often play multiple purposes (NCTM, 2000). They have potentials to: 1) encourage students' explorations of multiple strategies (NCTM, 2000; White, 2003), 2) increase students' mathematical discoveries (Calleja, 2013; NCTM, 2000; Stein et al., 2008), and 3) foster students' practices of mathematical expressions (Calleja, 2013; Stein et al., 2008).

### 2.5.1.3.1 Tasks that Promote Mathematics Communication

Appropriate tasks can provide opportunities for mathematics discussions (NCTM, 2000; *Ontario Mathematics Curriculum, Grades 1-8*, rev. 2005). In general, “students need to work with mathematical tasks that are worthwhile topics of discussion” (NCTM, 2000, p. 60). Open-ended, multiple solutions tasks can be utilized to encourage students' dialogues on mathematical strategies and mathematical thinking (Cai, Jakabcsin, & Lane, 1996; Hino, 2015; Viseu & Oliveira, 2012). Multiple solutions tasks that are cognitively challenging provide students with opportunities to compare and debate strategies of varying complexities (Cai, Jakabcsin, & Lane, 1996). This process serves to enhance students' mathematical thinking, mathematical reasoning, and mathematics communication (Cai, Jakabcsin, & Lane, 1996).

The potentials of mathematics tasks are influenced by how they are introduced and
structured within mathematics lessons (Jackson et al., 2013; Moschkovich, 2015; Perry, 2001). Prior to whole class discussions on mathematics tasks, opportunities to work with the tasks alone or in small groups may be beneficial, as explanations of strategies and answers can be polished prior to the “showcase” (Marks Krpan, 2013; Perry, 2001). Furner, Yahya, and Duffy (2005) also suggested encouraging students to practise “thinking aloud” when attempting word problems. In doing so, students would verbalize their metacognitive strategies, and many times, self-identify and self-correct their own mistakes. In addition, teachers could more easily discover and pinpoint students' misunderstandings (Furner, Yahya, & Duffy, 2005).

With regards to writing tasks, Lutzer (2005) noticed that “teachers are accustomed to providing students with exercises that guide their study of technique and understanding, but these are very different than exercises that target students' ability to read and write ideas in the language of mathematics” (p. 3). In “writing-to-learn”, students participate in short and informal writing tasks (Capacity Building Series, March 2012). Similar to the verbal think aloud (Furner, Yahya, & Duffy, 2005), “writing-to-learn” provides students with opportunities to freely display their emerging thoughts, even when those ideas are lacking in organization or coherence (Capacity Building Series, March 2012).

While writing may be completed individually, it can also be approached collaboratively. Koch (1993) implemented group writing assignments, where problem solving, mathematical thinking, and mathematics communication can be developed. “Writing for Reasoning” encouraged students to work in pairs or small groups, following specific writing procedures in the process of solving mathematical problems (Koch, 1993).

Kostos and Shin (2010) examined the uses of journal writing as tasks that benefit
students' mathematical thinking, reasoning, and communication. Through journaling, students: 1) practise the correct uses of mathematics vocabularies, 2) reflect and express their personal ideas, experiences, and goals, and 3) assess and record their personal learning journeys (Kostos & Shin, 2010). Similar benefits of journal writing for the fostering of written communication skills are also identified and supported in other studies (e.g. Lee & Herner-Patnode, 2007; Marks Krpan, 2013; Santos & Semana, 2015; Thompson & Chappell, 2007).

2.5.1.4 Constructing Knowledge

The refining of students' conceptual understanding should be emphasized along with the rehearsing of mathematical procedures (Franke et al., 2009; NCTM, 2000, 2014). In the construction of mathematical knowledge and understanding, teachers can: 1) highlight the connections between concepts and procedures, and 2) emphasize the relevance of mathematics to the world outside of the classrooms, 3) model multiple ways of representing mathematical ideas, and 4) ask effective questions that promote critical thinking (Bostiga et al., 2016; Ontario Mathematics Curriculum, Grades 1-8, rev. 2005; Purdum-Cassidy et al., 2015).

2.5.1.4.1 Multiple Representations

The demonstration of mathematical knowledge through multiple representations is seen as beneficial for the development of mathematical understanding (Marks Krpan, 2013; NCTM, 2000). “When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities (Fuson, Kalchman, & Bransford, 2005; Lesh, Post, & Behr, 1987)” (NCTM, 2014, p. 24). The representations that teachers model and students attempt may include: 1) concrete portrayals (e.g. manipulatives), 2) verbal depictions (e.g. words), and 3) visual
illustrations (e.g. charts, diagrams, graphs, pictures).

Specifically, as students progress from the primary grades to the junior grades, they should build upon representation methods previously learned, while expanding their abilities to express understanding through words, graphs, pictures, and models (NCTM, 2000). Teachers can promote multiple representations and knowledge constructions amongst students by modelling the approaches, and scaffolding the explorations of different methods (Moschkovich, 2015; NCTM, 2014). Teachers can also guide the sharing of problem solving strategies and solutions justifications amongst students, by facilitating and guiding the discussions (NCTM, 2014).

2.5.1.4.2 Questioning Techniques

In promoting productive mathematics discussions, teachers should steer students' attention to process development rather than product attainment (Walshaw & Anthony, 2008). Through effective question posing, teachers can foster students' mathematics communication (Koizumi, 2013; Kostos & Shin, 2010; Purdum-Cassidy et al., 2015). Mason (2000) acknowledged the vital role of question posing in the exploration of mathematical ideas, and deemed this area important for further investigations. Unfortunately, Mason (2000) also noticed that the questions posed by teachers “rarely reflect the process of mathematical exploration” (p. 98). Brendefur and Frykholm (2000) concluded that, while questions play important roles in students’ learning, teachers’ questions differ greatly in quality and construct.

Rather than quizzes that fish for correct responses, effective questions posed to students should encourage: 1) focuses on key mathematical ideas (Applefield, Huber, & Moallem, 2000/2001; Walshaw & Anthony, 2008), 2) verifications of correct and incorrect responses (Early Math Strategy, 2003; NCTM, 2014; Teaching and Learning Mathematics, 2004), and 3)
engagements of multiple voices and perspectives in the community (Walshaw & Anthony, 2008). A variety of questions should also be asked, as this helps students make elaborate connections between what they know and what they are experiencing (Cooke & Buchholz, 2005).

As students respond, teachers should listen attentively, such that they can further guide the directions of the dialogues and the types of follow-up questions posed (NCTM, 2014; Walshaw & Anthony, 2008). In times when discrepancies arise, teachers should assist students in formulating questions that: 1) express disagreements, and 2) request further reasoning and justifications (Lampert, Rittenhouse, & Crumbaugh, 1996; NCTM, 2000). Likewise, the art of question posing should be practised by students (Mason, 2000; NCTM, 2000). When engaging in mathematics discussions, students should listen attentively to their teachers and their peers, while processing their mathematical understanding. Through time, students should become increasingly proficient in asking questions of each other, and of their teacher (Mason, 2000). Lastly, adequate wait times should be given to students after the posing of questions, so that responses can be better formulated by students (Applefield, Huber, & Moallem, 2000/2001; NCTM, 2014).

2.5.1.5 Technology and Manipulatives

2.5.1.5.1 Technology

The incorporation of technology into mathematics programs is essential, as technology has become a prominent component in many students' current lifestyles and future career paths (diSessa, 2000; NCTM, 2014). Technological advances have altered the ways in which our society communicates and responds. Morgan et al. (2014) described some of these changes:

In a world in which new communication technologies provide new opportunities for interaction, it seems important to develop our understanding of how technologies may
affect pedagogic and mathematical communications... Of course, new technologies are changing our ways of communicating, not only introducing new semiotic resources, notably dynamic, manipulable, and multiply linked representations (Yerushalmy, 2005), but also new forms of human interaction, both asynchronous as is generally the case through email, discussion boards, blogs, podcasts, etc., and potentially synchronous as in chat rooms, instant messaging, video conferencing. (p. 848)

Within the realm of mathematics education, Misfieldt, Andresen, and Lee (2014) identified the topic of technology to be an “expanding and diverse field” (p. 66).

Incorporating technology into the teaching and learning of mathematics may yield several benefits. First, the use of technology can contribute positively to students' learning, in that it introduces students to new and different ways of representing and investigating mathematics (Attard, 2013; Falcade, Laborde, & Mariotti, 2007; NCTM, 2000, 2014). As expressed in the Early Math Strategy (2003), “technology changes the mathematics that students do and the way that students do mathematics” (p. 29). Specific to the fostering of mathematics communication, technology often allows students to demonstrate and express understanding that would otherwise be difficult to capture (NCTM, 2000). To establish mathematics investigations and mathematics communication as integral parts of the elementary program, one suggestion from the Early Math Strategy (2003) is frequent uses of technological tools within the classrooms. Effective technological devices may include those that are mathematical (e.g. calculators, mathematical software), and those that are non-mathematical (e.g. presentation software, communication software) (Attard, 2013; Cohen & Hollebrands, 2011; NCTM, 2014).

Second, the use of technology in the classrooms can enrich teachers’ instructional approaches (NCTM 2000, 2014). It affects what can be taught, in that mathematical ideas can be explored, explained, represented, and visualized differently (NCTM, 2000). The use of technology in classrooms also influences decisions on how mathematics can be taught (Early
Math Strategy, 2003; NCTM, 2000, 2014). For example, with the aid of technological devices (e.g. calculators, graphing software), less time can be spent on routine and procedural mathematics tasks, with more time devoted to thinking, developing, and communicating concepts.

As an introduction to a special issue on mathematics education and scaffold, Bakker, Smit, and Wegerif (2015) expressed that “more and more scholars started to characterize artefacts, including computer software, as scaffolds (e.g. Guzdial, 1994)” (p. 1050). Yet, they also cautioned that, while technological scaffolds may support students’ mathematics learning, teachers still play significant roles in the selection and implementation of technological tools. Clements and Sarama (2002) likewise hold a similar perspective, in that they advocated for teachers’ careful guidance of students’ self-directed technological explorations.

2.5.1.5.2 Manipulatives

Another component to consider is the uses of manipulatives, or concrete materials, in mathematics classes. As Golafshani (2013) identified, the incorporation of physical objects “is actually based on traditional techniques, using beans or counters, which have been replaced by more advanced items such as linking cubes, fraction circles, and technologies that are used in today’s mathematics classrooms” (p. 139). Effectively used, manipulatives can: 1) represent and model abstract mathematics ideas in more concrete ways (Ryve, Nilsson, & Pettersson, 2013; Thompson, 2002), 2) assist students in communicating their processes (Murata & Kattubadi, 2012; Ryve, Nilsson, & Pettersson, 2013; Sfard, 2008), and 3) provide teachers with valuable insights into students' mathematical thinking and misunderstanding (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005).
Sarama and Clements (2009) found that physical manipulatives and virtual manipulatives demonstrate similar levels of effectiveness. As such, both forms of resources should be readily available and easily accessible to students. The *Guide to Effective Instruction in Mathematics* (Volume Three: Classroom Resources and Management, 2006) recommends storing the manipulatives in classrooms, within the students’ reach. That way, as students become more familiar with the various types of concrete materials, they may proceed to select and utilize the visual objects without assistance.

### 2.5.1.6 Assessment

Assessment is an essential component in successful learning environments (Chappuis & Stiggins, 2002) and excellent mathematics programs (NCTM, 2014). Described as a continuous process of gathering student information through various sources and various times (McDougall, 2004; NCTM, 2014), assessment primarily serves to “inform and improve the teaching and learning of mathematics (NCTM, 2014, p. 91). This goal is also reflected in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005), as it recognizes that assessment and evaluation “serves to guide teachers in adapting curriculum and instruction approaches to students' needs and in assessing the overall effectiveness of programs and classroom practices” (p. 18). Within the mathematics classrooms, teachers may use assessment to: 1) monitor students' progresses, 2) inform instructional decisions, 3) evaluate students' achievements, and 4) evaluate mathematics programs (NCTM, 2014). For the assessment of group discussions and written responses, Cai, Jakabcsin, and Lane (1996) proposed the analysis of two components: 1) the quality and clarity of students' communication, and 2) the correctness of the ideas expressed.

To assist teachers in their assessment practices, the “Achievement Chart” in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005) provides a framework for evaluating students' understanding and mastery of mathematical concepts.
Mathematics Curriculum, Grades 1-8 (rev. 2005) highlights four categories of focus (knowledge and understanding, application, communication, and thinking), and four levels of attainment (level 1- limited effectiveness, level 2- some effectiveness, level 3- considerable effectiveness, level 4- high degree of effectiveness). This framework is provided, with goals of incorporating all curriculum expectations, assisting teachers in providing valuable feedback, and guiding programming and planning decisions (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005).

As students are the central focus of assessment, it is recommended that students also learn to monitor their own progresses through feedbacks and reflections (Chappuis & Stiggins, 2002; NCTM, 2014; Smith et al., 2009; White, 2003; Vazquez, 2008). When describing the importance of students' roles in classroom assessment, Chappuis and Stiggins (2002) expressed:

> We tend to think of students as passive participants in assessment rather than engaged users of the information that assessment can produce. What we should be asking is, “How can students use assessment to take responsibility for and improve their own learning?” Student involvement in assessment doesn't mean that students control decisions regarding what will or won't be learned or tested. It doesn't mean that they assign their own grades. Instead, student involvement means that students learn to use assessment information to manage their own learning so that they understand how they learn best, know exactly where they are in relation to the defined learning targets, and plan and take the next steps in their learning. Students engage in the assessment for learning process when they use assessment information to set goals, make learning decisions related to their own improvement, develop an understanding of what quality work looks like, self-assess, and communicate their status and progress toward established learning goals. (p. 40)

In order to assist students in becoming better self-assessors, they should be invited to plan aspects of learning (e.g. goals setting, data analyzing, and lesson program designing) (Clarke, 2001). Other ways that teachers can motivate students in becoming better self-assessors include: 1) providing descriptive feedback about current progresses, 2) identifying specific areas of strengths and weaknesses, and 3) guiding students to set realistic, attainable, personal goals.
2.5.1.6.1 Assessing Mathematics Communication

Vazquez (2008) expressed the importance of knowing students’ mathematical processes, stating: “I realized that unless I could tell how a student arrived at a solution, even a correct solution, I had little chance of assessing whether or not he or she truly understand the concept being studied” (p. 17). The assessment of students' abilities to communicate mathematically is seen as an essential and integral component to the teaching and learning of mathematics (Cai, Jakabcsin, & Lane, 1996; NCTM, 1989, 1995). NCTM (1989) described this endeavour:

The assessment of students' ability to communicate mathematics should provide evidence that they can: express mathematical ideas by speaking, writing, demonstrating, and depicting them visually; understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms; use mathematical vocabulary, notation, and structure to represent ideas, describe relationships, and model situations. (p. 214)

Studies have highlighted various pedagogical considerations that can increase the quality and quantity of mathematics communication assessment in the classrooms (Cai, Jakabcsin, & Lane, 1996; Marks Krpan, 2013; NCTM, 2014). First, mathematics programs and daily lessons can be strategically planned, with regular “check-in” points that provide students with opportunities to practise communicating and conveying their thinking (Marks Krpan, 2013; NCTM, 2014). Second, mathematics tasks can be selected and implemented in ways that would encourage students in sharing strategies and justifying responses (Cai, Jakabcsin, & Lane, 1996; Jackson et al., 2013; Olteanu, 2015; Viseu & Oliveira, 2012). Specifically, Cai, Jakabcsin, and Lane (1996) recommended the uses of open-ended mathematics tasks for assessment purposes, since they have the potentials of providing students with numerous opportunities to communicate, think, and reason.
2.5.2 Summary

Successful mathematics programs tend to incorporate many inter-related components (Ghousseini & Herbst, 2016; Henning et al., 2012; Hillen & Smith, 2007; McDougall, 2004). Here, six of the identified components (program planning, learning environment, mathematics tasks, constructing knowledge, technology and manipulatives, and assessment) from the Ten Dimensions of Mathematics Education (McDougall, 2004) are highlighted. Though organized differently, these central ideas are also resonated in the “Guiding Principles and Mathematics Teaching Practices” in NCTM (2014). While presented separately, it is important to remember that these elements are not discrete (McDougall, 2004). Often, the successful implementation of one component (e.g. student tasks) may involve the inclusions of other components (e.g. constructing knowledge, technology and manipulatives).

2.6 Conclusion

This chapter begins with an examination of curriculum theory, as theories often embody specific definitions and assumptions (Rose, 1953). The influences of learner-centered theories (constructivism and sociocultural) within the fields of education and mathematics education are traced and highlighted. Next, the goals of mathematics education are identified, and the importance of mathematics communication skills for the attainment of these goals is described. This is followed by defining the key roles of teachers, and illustrating some of the specific practices used in fostering mathematics communication skills.

While it is not the focus of this study, it is also important to acknowledge the roles and responsibilities that students, parents, and principals have in creating learning communities that foster mathematics successes. Based on the suggestions from the Ontario Mathematics...
Curriculum, Grades 1-8 (rev. 2005), some of these responsibilities include:

1. Students taking ownerships of their individual progresses in mathematics learning. This requires a willingness to commit to consistent efforts.

2. Parents and guardians becoming familiar with the mathematics curriculum, and participating in their children’s learning through partnerships with their schools. These opportunities may include school events (e.g. parent-teacher interviews, numeracy nights) and home reinforcements (e.g. encouraging homework completions, mathematics discussions).

3. Principals working collaboratively with teachers and parents in creating learning environments that are conducive to success.

Indeed, the success of mathematics programs in general require many thoughtful minds and caring hearts.
Chapter Three: Methodology

3.1 Introduction

“Methodology concerns the process through which we construct scientific knowledge. It is the description, explanation, and justification of research methods (Kaplan, 1964)” (Heck, 2006, p. 373). The three major research paradigms, as negotiated through various debates, are quantitative research, qualitative research, and mixed methods research (Johnson, Onwuegbuzie, & Turner, 2007). Quantitative research mainly presents broad scopes and overall patterns, and strives for maximal generalizations (Mahoney & Goertz, 2006).

In comparison, qualitative research primarily aims for narrower scopes and deeper understandings (Creswell, 2007, 2014; Mahoney & Goertz, 2006). Lastly, mixed methods research typically combines the traditions of quantitative and qualitative research, and “often will provide informative, complete, balanced, and useful research results” (Johnson, Onwuegbuzie, & Turner, 2007, p. 129). Rather than viewing the three research paradigms as distinct, Creswell (2014), in agreement with Newman and Benz (1998), suggests adapting a “continuum” perspective. This stance places qualitative and quantitative approaches at opposite ends of the continuum, with mixed methods approaches in the middle of the continuum (Creswell, 2014).

Methodology selection is intricately linked to the specifics of the disciplines of study (Lincoln & Guba, 2000). Indeed, the ways through which an inquiry is examined should be carefully considered in light of their suitability for the intended topic (Creswell, 2014; Merriam, 1998; Toulmin, 2001). My qualitative research inquiry was approached through a descriptive, embedded case study design (Yin, 1994). I collected two sources of data: 1) teacher interviews, and 2) classroom observations. The data was analyzed holistically and thematically, where the
teacher participant's expressed perceptions and implemented strategies in the fostering of Grade 3 students' mathematics communication skills were highlighted and described.

3.2 Research Design

Research design refers to the action plan, the processes, and the steps in conducting an inquiry (Berg, 2001; Heck, 2006; Yin, 1994). Within mathematics education research, “qualitative research methodologies have become not only acceptable but also predominant” (Zazkis & Hazzan, 1999, p. 429). Five prominent approaches for qualitative inquiry, as identified and described by Creswell (2007), are: 1) case studies, 2) ethnography, 3) grounded theory, 4) narrative, and 5) phenomenology. The selection of a suitable approach should be based on the intents and purposes of the topic investigated (Creswell, 2007; Feuer, Towne, & Shavelson, 2002).

My selection of a qualitative case study design for my inquiry was informed by: 1) the alignments between research question and research design, 2) the nature of the research question, and 3) the strengths of the research method. Case study research aims to collect different forms of data in order to assemble the fullest possible representations of the inquiry (Creswell, 2007; Heck, 2006; Merriam, 1998; Yin, 1994). In comparison to other forms of research design, it is most suitable for exploring: 1) contemporary phenomenon, 2) “how” and “why” questions of inquiry, and 3) participant's unaltered behaviours (Yin, 1994). Frequently used within the field of education research (Merriam, 1998), the strengths of case study research include: 1) cross-comparisons and corroborations from different sources (Creswell, 2007; Heck, 2006; Yin 1994), 2) “thick” descriptions of events (Creswell, 2007; Merriam, 1998) and 3) in-depth, holistic understanding of the case (Merriam, 1998; Yin 1994). Through the qualitative case study design,
my goal is to accumulate rich descriptions, capture reoccurring ideas, and highlight implementation struggles and successes as experienced by elementary teachers when they attempt to foster their students' mathematics communication skills.

3.2.1 Research Context

When conducting case study research, it is essential to consider and describe the environment where participant selection and data collection occurred. This contemplation is important, given the issues examined through the case study method are often intricately connected with and inseparable from their surroundings (Merriam, 1998; Yin, 1994). As Yin (1994) states: “A case study is an empirical inquiry that investigates a contemporary phenomenon within its real-life context, especially when the boundaries between phenomenon and context are not clearly evident” (p. 13).

For my study, the selection of an elementary teacher participant took place within a teacher professional learning initiative. The Elementary Teacher Learning Initiative was a partnership between a post-secondary institution and six elementary schools from a public school board in Southern Ontario. During the 2014-2015 academic year, the Elementary Teacher Learning Initiative focused on mathematics teaching and learning in grades three through six. The main goals of the project included: 1) examining strategies for increasing teachers' confidence and efficacy in mathematics instruction, and 2) investigating collaborative inquiry as a form of professional development. With the Ten Dimensions of Mathematics Education (McDougall, 2004) as a conceptual framework, participating principals, vice-principals, and teachers discussed and identified their areas of focus and improvement within the teaching and learning of mathematics.
The project included four full-day professional learning sessions at the post-secondary institution, with workshops aligned to the schools' identified areas of focus (e.g. assessment, manipulatives, mathematics tasks, technology). For every professional learning session, time was set aside for teachers to meaningfully collaborate and co-plan within and amongst school teams. Each participating school also formed a mathematics implementation team, which included the principal or vice-principal, along with four to seven elementary school teachers. The Elementary Teacher Learning Initiative was chosen as the context for case study participant selection because these elementary teachers were already engaging in professional learning initiative that explored new strategies for the teaching and learning of mathematics. As such, they may be more welcoming to continuous experimentation of various teaching strategies, and more consistent in their reflections on teaching practices.

3.3 Case Study Participant

Traditionally, probability sampling and purposive sampling are two prominent ways of participant selection within social science research (Collins, 2010; Creswell, 2008; Merriam, 1998; Teddlie & Yu, 2007). Probability sampling is more commonly used for inquiries that are quantitative in nature, with a goal of maximal generalizations to the populations through the selection of unbiased samples (Thomas, 2006b). On the other hand, purposive sampling is more commonly used for inquiries that are qualitative in nature, with a goal of in-depth understandings and explorations (Creswell, 2008; Merriam, 1998). Yin (1994) considered three rationales for conducting single case study research (testing a well-formulated theory, understanding a unique or extreme case, observing a revelatory case which was previously inaccessible), and suggested recruiting participants whom align with the purpose of the inquiry.
As part of the research component of the Elementary Teacher Learning Initiative, the administrator and teacher participants each completed a self-assessment survey during the first professional learning day at the post-secondary institution. The Likert-scaled survey focuses on participants’ attitudes and beliefs regarding their teaching of mathematics. In addition, one-to-one introductory semi-structured interviews were conducted, audio-taped, and transcribed by the research team. The interview questions can be categorized into four main areas of interests: 1) teaching background, 2) visions of successes, 3) goals in mathematics, and 4) perspectives regarding the teaching and learning of mathematics communication skills (Appendix A).

To further understand elementary mathematics teachers' strategies for fostering mathematics communication skills, some teacher participants who took part in the Elementary Teacher Learning Initiative were invited for additional interviews and classroom observations. As stated by Merriam (1998): “To begin purposive sampling, you must first determine what selection criteria are essential in choosing the people or sites to be studied” (p. 61). My criteria for participant selection was based on the combination of: 1) teacher background and experience, 2) expressed understanding of mathematics communication skills, and 3) expressed interest in learning more about the teaching and learning of mathematics communication. By further studying a subgroup of teachers in detail, I also applied the nested sampling technique, sometimes known as the within-case sampling technique (Collins, 2010). This strategy is suitable for my study, as it may: 1) provide further descriptions of confirming, diverging, or unique cases, 2) advance reflections on emerging theories, and 3) encourage an iterative process of data collection and data analysis (Collins, 2010; Miles & Huberman, 1994).

Using the interview data sources from the Elementary Teacher Learning Initiative, I
created teacher profiles which included: 1) number of years in the teaching profession, 2) number of years teaching in the current grade, and 3) expressed perspectives and interests regarding students’ development in mathematics communication skills. During one professional learning day at the post-secondary institution, I approached potential teacher participants individually, provided a summary of the purpose of my study, and invited their participation. I received expressed interest from five teacher participants, and proceeded to contact them by email. The email: 1) described in more details the goals of my study, 2) predicted the level of time commitment for participation in research (teacher interviews and classroom observations), and 3) extended an invitation to confirm their interest through email reply.

Upon receiving the email, two teachers responded to express their difficulties in scheduling the classroom observations and teacher interviews, given their existing commitments for the remaining school term. I emailed to thank them for their responses, and to wish them the best in their current teaching endeavours. Two other teachers, Sophie and Walter, confirmed further interest in my study. After meeting up individually with Sophie and Walter to explain and to sign the formal letters of consent (Appendix C), we proceeded to plan classroom observation times, choosing days which they believed to be suitable for their students and themselves.

After three classroom observations with Walter, he notified me that the boys’ sports team, which he was coaching, made it into the tournaments. As this was a big accomplishment and a rare occurrence for the school, he wanted to dedicate more time and energy to this event. He expressed that while he wanted to continue with the classroom observations and teacher interviews, he was unable to find the time that term to do so. I thanked him for his time, and informed him once again that his participation in my study is voluntary, and he can withdraw
anytime during the study. Sophie, the remaining teacher participant for my study, completed five days of classroom observations, and two monthly teacher interviews. Sophie’s perspectives, practices, and reflections regarding the fostering of students’ mathematics communication skills will be described in detail in Chapter Four.

3.4 Data Collection

Data collection concerns the types of information researchers attain for their inquiry (Heck, 2006; Johnson & Turner, 2003). While there are many varieties of data forms and data collection strategies, Creswell (2007) identified observations, interviews, documents, and audiovisual materials as the four basic types of information, and Johnson and Turner (2003) listed questionnaires, interviews, focus groups, tests, observations, and secondary data as the six major data collection methods.

Likewise, data for case study research can be collected from numerous sources (Merriam, 1998; Yin, 1994). Yin (1994) suggested the prominent data sources for case study inquiry include: 1) documentation, 2) archival records, 3) interviews, 4) direct observations, 5) participant-observations, and 6) physical artifacts. As all data collection methods have their unique strengths and weaknesses, they should ideally be selected with “complementary strengths” and “non-overlapping weaknesses” in mind (Brewer & Hunter, 1989; Johnson & Turner, 2003; Tashakkori & Teddlie, 1998).

For my case study, I collected data from two different sources: 1) semi-structured teacher interviews, and 2) classroom observations. Yin (1994) encouraged the use of multiple data sources when conducting case study research, as this “allows an investigator to address a broader range of historical, attitudinal, and behavioral issues” (p. 92). The teacher interviews and
classroom observations took place during the 2014-2015 academic year, from March 2015 to May 2015, inclusive. Following the suggestion from Merriam (1998), my data was collected through a recursive process, where the classroom observations and the teacher interviews occurred interactively. This strategy may contribute to more breadth and depth among the data collected, and may also result in more holistic analysis and descriptions (Merriam, 1998).

3.4.1 Teacher Interviews

Increasingly, interview has been a prevalent method of information collection for both qualitative and quantitative researchers (Fontana & Frey, 2000; Merriam, 1998). Within the field of mathematics education research, qualitative interview has been gaining popularity as one of the main data collection tools since the mid-1970s (Zazkis & Hazzan, 1999). Specifically, interviews are prominently used in case study research (Heck, 2006; Merriam, 1998; Yin, 1994). As Yin (1994) notes:

> Overall, interviews are an essential source of case study evidence, because most case studies are about human affairs. These human affairs should be reported and interpreted through the eyes of specific interviewees, and well-informed respondents can provide important insights into a situation. (p. 85)

Interview, as a data collection method, has its strengths and weaknesses (Creswell, 2008; Johnson & Turner, 2003; Yin, 1994). Interviews may provide in-depth, insightful information (Creswell, 2008; Glesne, 1999; Johnson & Turner, 2003; Yin, 1994) and the choice for interviewers to probe for clarity (Johnson & Turner, 2003; Yin, 1994). Yet, the responses received may be “filtered”, in that the participants may provide information that they may perceive the researchers want to hear (e.g. social desirability) (Creswell, 2008; Yin, 1994). Likewise, researchers may “filter” the participants' information, focusing on the details that are in alignment with their perspectives (Creswell, 2008). As such, interview responses should be
corroborated and triangulated with information gained from other data sources (Yin, 1994).

While there are numerous forms of interviews, they are commonly categorized as: 1) structured (pre-planned, specified questions, repeated amongst participants), 2) semi-structured (guiding questions, with the flexibilities for asking participants for further explanations and clarifications), and 3) unstructured (open-ended questions emerging through dialogues with the participants) (Fontana & Frey, 2000; Glesne, 1999; Merriam, 1998; Yin, 1994). Interviews can also be conducted individually (e.g. one-to-one) and in groups (e.g. focus group) (Fontana & Frey, 2000). The selection amongst interview types should be guided by the strengths and weaknesses of each form, with a well-grounded knowledge of the research goals and research settings (Fontana & Frey, 2000).

As part of the Elementary Teacher Learning Initiative, administrator and teacher participants were individually interviewed by the post-secondary institution's research team. Sophie’s introductory interview, which I conducted, audio-recorded, and transcribed, was used with participant consent for my current study. This face-to-face semi-structured interview was approximately 45-minutes in length, and focused on the participant’s: 1) education and teaching background, 2) visions of success and goals in mathematics, 3) challenging teaching circumstances, 4) fostering of mathematics communication skills, and 5) school support (Appendix A). A semi-structured approach was chosen, as this method is guided by the pre-established interview questions, while it provided the flexibility for further clarifications. Though more time consuming and costly when compared with group interviews and focus groups, one-to-one individual interviews provide opportunities for in-depth responses (Creswell, 2008). Through the interview, more detailed perspectives of the participant's beliefs and goals may be
Aside from the initial interview, two semi-structured interviews were also scheduled and conducted. Following Yin’s (1994) advice, these interviews were planned based on the schedule and availability of the interviewee. One interview occurred in April 2015, and another one in May 2015. A semi-structured format, where a pre-determined list of questions or topics, guided the conversation. This “allows the researcher to respond to the situation at hand, to the emerging worldview of the respondent, and to new ideas on the topic” (Merriam, 1998, p. 74).

With each interview approximately 35-minutes in length, these conversations were guided by a list of general questions, and focused on: 1) identified learning goals for the mathematics lesson, 2) perceived strategies used to develop students’ mathematics communication skills, and 3) perceived successes and challenges regarding various aspects of the mathematics lesson (Appendix B). Dependent on the responses to the initial questions, probes that followed up on the emerging and surfacing ideas were included in the dialogues (Merriam, 1998).

The interviews conducted in April and May primarily served to provide Sophie with more moments to reflect on her mathematics lessons and to comment on her pedagogical approaches, during the months when classroom observations concurrently occurred. All of the interviews were audio-recorded and transcribed verbatim. Sophie was given the opportunity to read over the transcripts, clarify her expressed thoughts, and provide any additional insights. She did not choose to participate in verifying the transcripts for the interviews.

### 3.4.2 Classroom Observations

Sometimes viewed as “the fundamental base of all research methods” (Adler & Adler,
1994, p. 389), observation is a prevalent method of data collection in the social and behavioural sciences (Angrosino & Mays de Perez, 2000; Merriam, 1998). This form of data collection involves gathering first-hand information by: 1) being in research sites and settings, 2) noticing the participants' dialogues and actions, and 3) recording first-hand information (Creswell, 2008). Observations are ideal sources of data when the phenomenon can be observed first hand, and when the situation benefits from fresh perspectives (Merriam, 1998).

Observations are also great for the triangulation of collected data - in witnessing whether participants do what they claim they do (Heck, 2006; Johnson & Turner, 2003). At the same time, observations risk being biased, as researchers may selectively perceive confirming aspects of the environment, or participants may be reactive to being noticed (Johnson & Turner, 2003). Being mindful of this limitation, Merriam (1998) suggested combining the firsthand accounts of observations with further interviews and document analysis, in hopes of attaining a more accurate and holistic interpretation.

During the months of April and May in 2015, I conducted classroom observations in Sophie’s grade three class. With regards to the number of visits for observations, Merriam (1998) suggested considering: 1) the time available, 2) the resources available, and 3) the amount of new information gained from additional observations. At some point, information saturation is attained, when no new theoretical development insight or information is gained from additional participants or further data collection (Collins, 2010; Glaser & Strauss, 1967). With an awareness of these factors, a total of five classroom observations were scheduled for my case study.

The classroom observation days were selected by Sophie, based on what she perceived as the most suitable times for herself and her students. During each observation day, I was present

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for the instructional periods that were designated for the learning of mathematics (e.g. numeracy blocks, mathematics instructional periods). For Sophie’s class, the teaching and learning of mathematics often occurred in the morning instructional periods. In addition, since Sophie was interested in the learning of mathematics throughout the instructional day, she often invited me to stay for the entire school day. Such opportunities allowed me to observe the integration of mathematics ideas, and the promotion of mathematics communication skills, as students engaged with language arts topics, science units, and media studies. As such, the number of instructional periods observed during the five classroom observation days ranged from four (Vignette Three, Vignette Four, and Vignette Five) to five (Vignette One, and Vignette Two) per day, dependent on the availability of participant and researcher.

Observation was chosen as the second means of data collection, as this method “allows one to directly see what people do without having to rely on what they say they do” (Johnson & Turner, 2003, p. 315). Interview data, my other source of information, is heavily reliant on self-report (Creswell, 2008; Yin, 1994). Amongst other things, the classroom observations served to shed light on the connections between teacher's backgrounds, perceptions, and classroom practices.

For each classroom observation session, extensive fieldnote was continuously recorded regarding the teacher's uses of strategies in fostering students' mathematics communication skills. As suggested by Creswell (2008) and Glesne (1999), both descriptive fieldnotes (e.g. activities, events) and reflective/analytic fieldnotes (e.g. insights, themes) were recorded. As a way to guide the observation sessions, implementation ideas that emerged from the literature review and the teachers’ interview responses for the Elementary Teacher Learning Initiative
served as prompts and guidelines for what I would observe and record. My role in these observations was the non-participant, complete observer, where observations were done “from the outside” (Creswell, 2007; Merriam, 1998).

During the classroom observation days, I also had brief, unstructured dialogues with Sophie. These informal conversations primarily served to quickly verify perceived emerging ideas. This step was implemented, as observations can lead to a “trail of understandings” that is inferred by the researcher (Glesne, 1999, p. 69). Yet, to truly “get the actor's explanations”, interviews are required (Glesne, 1999, p. 69).

3.5 Data Analysis

Data analysis, the procedures used in the interpretation of collected information, may involve examination, categorization, and consolidation of data (Merriam, 1998; Yin, 1994). Yin (1994) noted that, ultimately and ideally, the process of data analysis should aim “to treat evidence fairly, to produce compelling analytic conclusions, and to rule out alternative interpretations” (p. 103). He further suggested having a “general analytic strategy” when preparing for the analysis of case study research (Yin, 1994, p. 103). For my study, the process of data analysis was done iteratively and concurrently with the processes of data collection and report writing. This practice is in agreement with the belief that the three steps of research (data collection, data analysis, and report writing) are interconnected (Creswell, 2008; Merriam, 1998).

As Merriam (1998) cautioned: “Without ongoing analysis, the data can be unfocused, repetitious, and overwhelming in the sheer volume of material that needs to be processed. Data that have been analyzed while being collected are both parsimonious and illuminating” (p. 162).

For my research inquiry, the qualitative interview data was first transcribed, then
reviewed for accuracy. The transcripts and classroom observation fieldnotes were analyzed using nVivo10 software, guided by the inductive approach (Thomas, 2006a). The inductive approach to data analysis, a common method for qualitative data, requires careful readings of the data in the identification of key concepts, themes, and models (Thomas, 2006a). In initial coding, an openness to explorations of possible theoretical explanations is important (Charmaz, 2006). Detailed word-by-word, line-by-line, and incident-by-incident coding is suggested so that the theories developed will fit more closely with the data (Charmaz, 2006).

In the initial analysis of the data, I read through the interview transcripts and observation fieldnotes several times to develop a general sense of the key ideas. This step utilized the open-coding process, where the data goes through “constant comparisons” (Creswell, 2007). After getting a sense of some possible categories and over-arching ideas, categories and sub-categories were created as nodes in nVivo10. Because the data was collected and analyzed concurrently, one benefit is the possibility of collecting future data guided by the initial data (Charmaz, 2006).

Aside from examining the data through an iterative and continuous process (Creswell, 2008; Merriam, 1998), the interpretation of case study information should also consider the “units of analysis” (Yin, 1994). When sub-units within a case study are identified for exploration and interpretation, the research approach is known as “an embedded case study design” (Yin, 1994). As part of the data analysis and data interpretation, I further identified six lesson vignettes as my case study sub-units, and explored them in more detail to highlight the categories displayed. For these within-lesson analysis, the observation fieldnotes were read several times, in order to get a general sense for each lesson's implementation categories. The classroom observation notes were then examined with the teacher participant's interview responses.
While the identification of units within a case may be desirable and beneficial, Yin (1994) also promoted the returning to a larger, more holistic level of analysis. He highlighted the need for both sub-unit analysis and holistic analysis:

In short, where a genuine case study is involved, any analysis of the embedded units is done within each case (and not pooled). In addition, this analysis cannot be the sole analysis, but must be augmented by some other analytic technique at the level of the “whole” case, such as pattern-matching, explanation-building, time-series, or program logic models. (Yin, 1994, p. 120)

As such, I also conducted across-lesson analysis for my case study. For across-lesson analysis, the observation notes were analyzed amongst the six lesson vignettes. The implementations of similar and different learning strategies were noted.

3.6 Validity Issues

Schoenfeld (2000) described a difference between validity in mathematics and validity in the social sciences:

Here we find one of the major differences between mathematics and the social sciences. In mathematics one compelling line of argument (a proof) is enough: validity is established. In education and the social sciences we are generally in the business of looking for compelling evidence. (p. 648)

Various words have been proposed as synonyms for validity. Establishing validity can be seen as similar to promoting trustworthiness, credibility, quality, integrity, and “goodness” (Dellinger & Leech, 2007; Johnson & Turner, 2003; Lincoln, 1995; Thomas, 2006a). Yin (1994) proposed considering construct validity, internal validity, external validity, and reliability while designing and implementing various types of case study research.

Underlying these concepts is the goal of obtaining meaningful data and drawing interpretations that are “natural, practical, and useful, or pragmatic” (Dellinger & Leech, 2007, p. 329). Throughout each stage of the study (research design, data collection, data analysis,
interpretation), the issue of validity should be examined (Johnson & Turner, 2003; Yin, 1994). Here, I address some validity considerations in my data collection and my data analysis procedures.

Fontana and Frey (2000) noted that “human beings are complex, and their lives are ever changing; the more methods we use to study them, the better our chances to gain some understanding of how they construct their lives and the stories they tell us about them” (p. 668). Interpretive validity concerns the accuracy of the findings and results, and how they reflect the thoughts and perspectives of research participants (Johnson & Turner, 2003; Lincoln & Guba, 2000). Triangulation is one strategy for obtaining greater interpretive validity, as different sources may contribute to more accurate conclusions (Creswell, 2008; Merriam, 1998; Yin, 1994).

The uses of multiple data sources align with the idea of data triangulation, a strategy used in the validation process for clearer measurements, better data, and enhanced interpretations (Creswell, 2008; Johnson & Turner, 2003; Mathison, 1988; Yin 1994). In using both teacher interview data and classroom observation data, I hope that more facets of the phenomenon will emerge, and that the weaknesses in one data collection method (e.g. the reliance of self-report in interviews) will be compensated by the strengths in another (e.g. the first-handedness of classroom observations).

3.7 Ethical Considerations

Ethical approaches in research are essential, and some guidelines were suggested by numerous researchers (e.g. Creswell, 2008; Fontana & Frey, 2000; Glesne, 1999). At every stage of the research, decisions regarding ethics need to be made. Consolidating these guidelines, the
informed consent forms presented to the participants at the start of the study should include information on: 1) the purpose of the study, 2) the time required, 3) the types of data collected, 4) the ways in which the data and results will be used, 5) the voluntary nature of participation (and the choice to stop participation at any point during the study), and 6) any possible harm to their well-being (Creswell, 2008; Fontana & Frey, 2000; Glesne, 1999). During data collection, the confidentiality of participants, sites of study, and data should be kept (Creswell, 2008; Fontana & Frey, 2000).

Before the commencement of my study, I sought the approval of the Office of Research Ethics of the University of Toronto. Participants from the Elementary Teacher Learning Initiative, who were selected based on criterions, and who had verbally expressed interests in my study, were recruited through email. Prior to the commencement of the study, I presented the teacher participants with formal letters of consent (Appendix C). After confirming participation through signed letters of consent, the participants were reminded that they could take part in as much of the study as they wished to. This also included the option of stopping the study at any stage of the research. To ensure anonymity, pseudonyms were used for the names of the teacher participants and the schools. In addition, details regarding the school boards and the school locations were omitted, as to protect confidentiality.
Chapter Four: Findings

4.1 Introduction

This chapter describes the teaching perspectives, practices, and reflections of Sophie, a Grade 3 teacher participant selected from the Elementary Teacher Learning Initiative. First, depictions of Maple School are presented, and highlights of the learning and teaching environment are given. This information is included such that a clearer comprehension of the school context may be acquired. Next, the education background, teaching experiences, and education goals of Sophie are described. Afterwards, six lesson vignettes from the classroom observations are narrated, with detailed categorical analysis following each vignette. The first five vignettes depict the learning of measurement ideas (perimeter and area). The last vignette describes the reviewing of numeric operations, skills that Sophie believes to be helpful for the demonstrations of measurement concepts. Each lesson vignette concludes with a summary that often includes Sophie’s reflections on the teaching and learning that occurred.

4.2 School Context

Located in a metropolitan city in Southern Ontario, Maple School is a kindergarten to Grade 8 co-educational day school. With approximately 500 students and over 50 languages spoken, it is an environment where rich ethnic diversity is valued and celebrated. Some of the academics-related programs offered by the school include: 1) special education program classes for students with specific needs, and 2) an Extended French Program (several designated subjects instructed in French) for admitted students from Grade 4 to Grade 8.

Acknowledging that healthy and nutritious food plays an important role in academic success, the school encourages students to adopt healthy eating habits. This message is
communicated through food literacy education, and practised through the school's breakfast and snack programs. Each school day, the breakfast program serves hot, nutritious meals, and the snack program provides healthy refreshments to students.

The public school board that Maple School is affiliated with places great emphasis on: 1) establishing and maintaining safe, positive, and nurturing learning environments, and 2) fostering meaningful partnerships between communities, families, schools, staffs, and students. Specifically, Maple School's mission statement highlights their goal of supporting all students in becoming responsible members of the society. As such, the school aims to assist every student in developing and acquiring the necessary values, knowledge, and skills.

Maple School strives to establish an environment where students cultivate and practise cooperation, empathy, fairness, honesty, integrity, kindness, perseverance, respect, responsibility, and teamwork. These positive traits are reinforced through the school board's monthly character development themes. To further encourage students, monthly award assemblies are held in the school gymnasium to recognize students who demonstrated exceptional character development.

Maple School highly values professional learning opportunities for their staff. As part of the school board's Science, Technology, Engineering, and Math (S.T.E.M.) Education initiative, the administrators and teacher leaders at the school take part in focused collaborative learning opportunities. Using an inquiry-based process, the S.T.E.M. Education program equips teacher participants to meaningfully engage students through inter-disciplinary learning.

In addition, during the 2014-2015 academic year, a mathematics implementation team from Maple School participated in the Elementary Teacher Learning Initiative at a post-secondary institution. The mathematics implementation team members consisted of six teachers
of the junior grades (Grades 3 to 6), and the vice-principal of the school. With four full-day in-service professional learning sessions on mathematics teaching strategies, this partnership between a post-secondary institution and six mathematics implementation teams from local elementary schools aimed to increase generalist teachers' confidence and competence in mathematics instruction.

4.3.  **The Case of Sophie**

4.3.1 **Education Background and Teaching Experience**

Sophie completed a Bachelor of Science, a Master of Science, and a Bachelor of Education. She has ten years of teaching experiences in Southern Ontario, and was a substitute teacher for her first teaching year in the province. Sophie recounts how, during that year, she had many opportunities teaching various grades and various subjects, in numerous public schools within the same school board. The following nine years of teaching were at Maple School, where she taught special education, Grade 2, Grades 2 and 3 combined class, Grade 3, and Grades 4 and 5 combined class. Currently, she is teaching Grade 3 for the fifth consecutive year. Her teaching responsibilities include all of the Grade 3 subjects, with the exceptions of drama, health and physical education, and library.

Sophie is very dedicated to sharing what she learned with her colleagues. She is currently the Mentor Teacher for the Primary Division at her school, and has been the Leading Instructor for the third consecutive year. Teacher leaders who hold the roles of Mentor Teacher are given additional opportunities to develop effective leadership skills (e.g. equity and inclusion practices, learning community support). These skills may be developed through e-learning, workshops, and/or presentations. The Leading Instructor attends professional development sessions, and
shares best practices with the administrators, and the teachers within his/her school. Sophie feels that these roles provide her with additional opportunities to attend monthly workshops, where various teaching initiatives are introduced and examined. She sees the value of these workshops, and their alignments to current education goals for the school board and the province.

Sophie identifies Primary Division staff meetings, inter-division staff meetings, and Professional Learning Community gatherings as opportune times for sharing with her colleagues. After attending professional learning workshops, she looks forward to sharing and exploring new teaching insights with teachers at her school. During our interviews, she reflects on these opportunities:

Being the Leading Instructor for my school, I get more exposures to what is going on [with the learning initiatives]. I am the one who gets [the initiative idea, the workshop activity], and I bring it to the school. I share it with my colleagues. It does work out, so everybody is on board. (Sophie interview, March 9, 2015)

Sophie further describes the sharing of novel ideas, and the implementations of teaching strategies following her professional learning endeavours:

The last two years, I am the Leading Instructor. It is a lead position. You are kind of the lead teacher in your school. You go to those [professional development] modules, or the coach comes to you. Then, whatever I learn, I have to share it with my colleagues. If not, at least I have to practise it in my class, so that whatever I learn, I can continue. And then I build more, add my own personality and my own style. (Sophie interview, May 26, 2015)

4.3.2 Goals for Mathematics Teaching and Learning

When inquired about the provincial ministry's vision for mathematics, Sophie responds:

I think [the school's mathematics education goal] is all aligned [with the provincial ministry's vision]. The province is concerned about the EQAO scores, but that should not be the only concern, I think. That is just one thing. We are preparing our kids for their better futures, and math is very important in our daily lives. Since morning you wake up until you go to bed, math is all around. And every time, every moment, you need math one or the other way. (Sophie interview, March 9, 2015)
Sophie believes that “students are the core of education” (Sophie interview, March 9, 2015). As such, her over-arching goal in education is to understand students' needs, and then design and implement her programs accordingly. She feels lucky that the staff at her school and the parents of her students have similar learner-centered education perspectives. She expresses these beliefs, and elaborates on the perceived benefits:

I think I am lucky that way . . . . We [my colleagues and I] all believe that every child can succeed. Our students really have potentials that we need to work to develop more. And the parents are on board too. (Sophie interview, March 9, 2015)

Sophie's intention in assisting her students achieve their full potentials is also evident when she describes her goals for the teaching and learning of mathematics. From interactions with parents and students, Sophie notices negative perceptions of the subject. The parents’ and students’ anxious ideas about mathematics often stemmed from their negative prior learning experiences. Sophie advocates for changes in mindsets, and urges parents and students alike to view mathematics learning as an enjoyable and adventurous journey. She voices these sentiments:

I believe everyone can learn math. If we try, if everyone tries. They [the parents and students] should not have the mindsets that, “Oh, math is difficult” . . . . So I tell them not to generalize and not to say that, and believe that, yes, everyone can learn math and there are different ways of learning math. Not just one way of learning or showing or working out math. That is my goal. That everyone should understand that it is possible. That we can do it. (Sophie interview, March 9, 2015)

Sophie recognizes the important roles parents play in their children's successes. She describes the community of parents whom she encounters as “quite enthusiastic” (Sophie interview, March 9, 2015). When asked about ways in which she communicates with the parents of her students, Sophie responds:
Newsletters. My way of communication every day is the agenda, which marks [the student's] behaviour, my notes, and stuff like that. And if parents have a concern, they do write a note in the agenda, or they call me. And that works. I guess most of their parents, they drop off the kids and they pick up the kids [themselves]. So everyday, when I am on [yard] duty, we just connect. So any moment, any time, just on the go. That works too. (Sophie interview, March 9, 2015)

In addition, Sophie believes “helping [students] build up their confidence” is one method for achieving various forms of success (Sophie interview, March 9, 2015). She attempts to do so by “providing them with various activities, various things that expose them to different aspects of learning and make them feel successful” (Sophie interview, March 9, 2015). When asked about her students' motivation levels for mathematics communication, Sophie shares that they are often excited and motivated to share. She believes several factors contribute to her students' confidence and eagerness in sharing. These include: 1) patience with the learning journey, 2) emphasis on the thinking processes, and 3) comfort with mathematical struggles. Sophie expresses:

They need to see the connections, and [seeing the connections] do help them. Because now, they are kind of confident. That [they may say], “It is okay if I am not able to do it before, or if my answer turns out wrong. But still, I have done some thinking”. (Sophie interview, March 9, 2015)

Aside from helping students and parents adapt favourable mindsets towards the learning of mathematics, Sophie also acknowledges the importance of refining her teaching practices in order to achieve her goals in mathematics education. Reflecting on her scores from the *Attitudes and Beliefs to Teaching Math Survey* (McDougall, 2004; Ross et al., 2003), she identifies two components in mathematics instruction as areas for personal growth. For this academic year, those areas are: 1) constructing students' mathematical knowledge (Dimension 5), and 2) using manipulatives and technology (Dimension 7). Knowledge construction is emphasized, as Sophie
senses her students struggle with knowledge and concept retention. Sophie describes her observations: “When they learn something, retaining that part. Maybe at this age, I am not sure, is a challenge” (Sophie interview, March 9, 2015).

Given her school's recent acquisitions of iPad minis and other technological resources for students in the Primary Division, Sophie wants to effectively utilize these new devices in her teaching. The numeracy team at the school is also in the process of doing inventory of math manipulatives within the classrooms, and gathering mathematics resources to set up a math room. As such, Sophie selects manipulatives and technology as her second mathematics education dimension of focus. She confirms her selection of this mathematics teaching dimension: “Especially myself, using technology is one thing that I am going to work on this year. Using technology for mathematics” (Sophie interview, March 9, 2015). At the same time, Sophie draws attention to the interconnectedness of all mathematics education dimensions. She expresses:

But I think, even if you go for all of [the mathematics education dimensions]. I mean, it all ties in. You cannot separate them at all. If you do not have a rich task, it is not going to help construct knowledge, and it is not going to help them communicate better. It has to be blended in. (Sophie interview, March 9, 2015)

4.3.3 Goals for Fostering Mathematics Communication

Sophie believes that mathematics communication encompasses a mixture of forms, often involving verbal, written, and visual representations. “It is changing. I do not believe that all those traditional ways are the only ways that you can show mathematics communication,” she observes (Sophie interview, March 9, 2015). Sophie encourages her students to communicate their mathematical understanding through various means. She notes how she often models appropriate strategies, and regularly reminds her students to use different forms of expressions.
Formulating the message she wants her students to receive, Sophie expresses:

I can show it all different ways: numbers, words, and pictures. This is how I can explain. But I kind of stress that, since this is math, do not just use words. Because if there are numbers, kind of get in numbers too. So I am trying to blend in everything. (Sophie interview, March 9, 2015)

Sophie acknowledges that being able to communicate mathematical thinking is “really important” for students in Grade 3 (Sophie interview, March 9, 2015). Yet, she notices that many of her students still require some strengthening in this area, as they tend to provide solutions without justifications. Sophie summarizes these thoughts:

I think [mathematics communication] is very important. Yes, they can do it [solve the mathematics tasks], but it [mathematics communication] is necessary. It is building their language skills. It is a kind of language too. It is really important that they are able to explain. So basically, it develops critical thinking. [The students should be able to say], “Okay, I have done this and there is a way of showing that my thinking is right too.” (Sophie interview, March 9, 2015)

Being purposeful about fostering her students' mathematics communication skills, Sophie examines the Grade 3 mathematics curriculum expectations and notes the ones pertaining to communication. These aspects are intentionally inserted into the long-range plan, and regularly visited in daily lesson plans. Rather than teaching these skills independently, Sophie strives to create holistic learning experiences through subject integrations and mathematics content strands combinations. With regards to this, she shares:

Yes, I look at the long-range plan and it trickles down to my daily plans. But I also try to look at all those cross curricular, cross strands. I do not have to teach it all separate, separate, separate . . . If I teach comparison in language, I carry it over to social studies, or science, or math, and everything. So they know how to use it, and then I get my marks from there, too. Because it is cross curricular, right? I do not have to teach everything separate. (Sophie interview, May 26, 2015)

4.3.4 Class Context

At Maple School, students from Kindergarten to Grade 6 follow a timetable where the
school hours are organized into eight instructional periods. Each instructional period ranges from 30 to 40 minutes, and each school day includes an hour-long lunch, a 15-minute recess in the morning, and a 10-minute recess in the afternoon. Table 1 shows more details regarding how the instructional hours are organized. Asked to comment on this time tabling arrangement, Sophie describes the school’s prioritization of literacy and numeracy blocks. When possible, literacy and numeracy lessons are scheduled in the morning, preferably with two continuous periods for each subject.

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<td>8:45 AM</td>
<td>Bell</td>
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<td>8:45 AM- 8:50 AM</td>
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<td>8:50 AM- 9:30 AM</td>
<td>Period 1</td>
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<td>Period 2</td>
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<td>10:10 AM- 10:25 AM</td>
<td>Recess</td>
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<td>10:25 AM- 11:05 AM</td>
<td>Period 3</td>
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<td>11:05 AM- 11:35 AM</td>
<td>Period 4</td>
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<td>11:35 AM- 12:30 PM</td>
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<td>12:30 PM- 12:35 PM</td>
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<td>12:35 PM- 1:15 PM</td>
<td>Period 5</td>
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<td>1:15 PM- 1:55 PM</td>
<td>Period 6</td>
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<td>Recess</td>
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<td>2:05 PM- 2:45 PM</td>
<td>Period 7</td>
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<td>2:45 PM- 3:15 PM</td>
<td>Period 8</td>
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<tr>
<td>3:15 PM</td>
<td>Dismissal</td>
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Table 1: Instructional hours for Kindergarten to Grade 6 at Maple School

Sophie believes this organization gives students more time to explore and consolidate concepts. Sophie says: “Our time table is always built in such a way that we have enough time to have those [numeracy and literacy] blocks uninterrupted” (Sophie interview, May 26, 2015).
addition, the administration team also considers the time tabling for teachers of common grades, being mindful of maximizing common planning times. With regards to these purposeful collaborative opportunities, Sophie comments: “Yes, the time tabling is great. We are given common planning time once a week, and that helps. To sit down and do it” (Sophie interview, May 26, 2015).

Some classrooms at Maple School follow the window-less, door-less, open-concept design. Sophie teaches in one of these rooms, with the adjacent classroom being a Grades 2 and 3 combined class. When asked to comment on the benefits and disadvantages of this arrangement, she says:

Pro, it is open. It is open, open. But then cons. Sometimes it gets noisy, because you do not even have doors to close, and it is all open. So maybe, if I am loud, I am disturbing the next class. If my students are noisy and the teacher next door is doing some quiet lessons, we bother them. Happens to us too. That is one thing about open concept that I do not like . . . I see more cons than pros. I do not see this as beneficial. (Sophie interview, April 23, 2015)

Inside Sophie's classroom, student tables are arranged into six groups, with each group consisting of three or four tables. Sophie describes her reasoning for this student seating plan:

Basically, I try to pair students in such a way that it is a heterogeneous group. It is not a homogeneous group, because I see there are some students who really need help. Some students who are already ahead of them. It helps though, but not necessarily. I keep changing the groups. Sometimes it is groups of four, sometimes it is in pairs. It is just that I want them to work in groups, and that set up helps managing space too. That is why. (Sophie interview, April 23, 2015)

The teacher's table is located at the front of the classroom, beside the chalkboard. A round conference table is positioned at the back of the classroom. Displayed on the three classroom walls are: 1) student-generated character development goals (conflict resolution strategies, cooperative group rules, healthy habits suggestions, ideal classroom descriptions), 2) subject-
specific prompts and success criteria (mathematics vocabularies, procedural writing learning
goals, punctuations guidelines), and 3) student works from multiple subjects (language,
mathematics, science).

With regards to class population and distribution, Sophie's current Grade 3 class consists of 22 students. Of these students, five are assigned Individualized Education Program (IEP), and one is an English Language Learner (ELL).

4.4 Vignette One

*Monday, April 20th, 2015*

For the first classroom observation day, I arrived at Maple School at 9:30 am for Period 2, and stayed until school dismissal at 3:15 pm. During those instructional hours, I was in Sophie's classroom during Periods 2, 4, 6, 7, and 8, where I observed her teach students language, mathematics, and science. For Period 3, the students had health and physical education class with another teacher, and Sophie was scheduled for a preparation period. Sophie and I chatted casually, as she gathered and organized materials for upcoming lessons and assessments. For Period 5, Sophie's students were in the school library for a scheduled session with the school librarian. I spent those moments in the staff room, as Sophie had a meeting to attend.

In Sophie's class that day was another elementary teacher from a nearby public school. She was temporarily assigned to Maple School, as her school was undergoing building renovations. Below, I will describe the Period 4 and Period 6 mathematics classes, both of which were taught by Sophie.

*11:07 am - Mathematics*

Sophie began Period 4 mathematics class by asking her students to clear their table tops
of distractions. She then summarized the previous mathematics lesson, and described how they explored covering simple shapes with virtual manipulatives. As they recalled that online activity in the computer lab, some students responded with nods. “That was fun!” blurted a student. Following that recap, Sophie explained today's task, which involved covering irregular shapes using a larger variety of concrete manipulatives. Sophie went on to connect today's learning to an upcoming mathematics lesson. In two days, the concepts from this class will be further explored on the iPad minis. For that lesson, they will attempt to cover shapes using square grids.

When the brief introduction concluded, students were each given worksheets with descriptions and instructions for three mathematics tasks. After reading the first task, some students located a box of pattern blocks, which was conveniently placed at the front of the classroom, next to the document camera. Students proceeded to select a variety of pattern block shapes, ones which they deemed useful.

At the same time, a few students decided to attempt this task without concrete pattern blocks. Instead, they used pencils and rulers to draw out the corresponding shapes. As the students worked quietly on the task, Sophie invited a number of students to the conference table, where they read the task prompt together. Guided by Sophie's questions, the students paraphrased the aim of the task, and brainstormed ways to achieve a correct solution. When the students at the conference table identified some problem solving approaches they wanted to attempt for the task, they went back to their assigned desks to explore these strategies.

Sophie then walked around the class, and asked students to describe their approaches for the mathematics task. Sophie posed questions that required her students to think about: 1) their knowledge of topics that are related or relevant to the task, 2) the description of the task, and 3)
the materials they would need to attempt the task. For students who displayed incorrect ideas or incomplete solutions, Sophie provided them with helpful hints. These responses served to guide her students through the process of self-identifying flaws in their current approaches. At the same time, for students who arrived at a correct solution, Sophie motivated them to explore alternative strategies. Throughout the process, she was tremendously encouraging. “Just try your best,” she responded, when students expressed the task was challenging.

With the instructional period concluding, a good portion of the students made significant progresses towards their solutions. Moments before the lunch bell rang, students placed their mathematics worksheets into their subject folders, returned the pattern blocks, and lined up to proceed to the room where they would eat their food.

1:20 pm-Mathematics

The Period 6 mathematics class started with a large group discussion on the mathematics task explored in Period 4. Sophie placed an assortment of pattern blocks on each student's table. These included: a hexagon, a rhombus, a square, a trapezoid, and a triangle. “Please put the blocks away,” Sophie reminded, when a few students were distracted by the colourful objects. With the mathematics task projected through the document camera, Sophie picked up a green triangle and placed it at the top of the irregular shape. She asked students to express how they covered the shape using pattern blocks. Several students' hands went up, and Sophie selected one student to demonstrate and explain her strategy. Placing the pattern blocks on the document camera, the student verbalized the process of covering the shape with these manipulatives. Sophie encouraged her to use mathematics vocabularies (e.g. “surface area”, names of geometric shapes) as the steps were described. When the student finished explaining her process, Sophie
complimented her approach.

Next, Sophie turned to her class and asked if they know of other methods. After a moment, three more hands from the class went up. Another student was selected, and went to the front of the class to demonstrate his approach. However, midway through the sharing, he was unable to recall the rest of the procedures. Sophie waited for a moment, before inviting another student to the front of the class. Together, they explained the forthcoming steps. Sophie then wondered if it was possible to approach this mathematics task without the concrete pattern blocks. She used a pencil and ruler, and drew shapes that corresponded with the pattern blocks. During this time, students were asked to envision the relationships between different shapes. This moment turned out to be a preliminary exploration of surface area, geometry, and ratio.

For both the independent work and the large group sharing, Sophie encouraged her students to persist in their problem solving attempts. “Are we missing something?” hinted Sophie, when students struggled to arrive at a solution. “Detective heads and detective eyes,” she suggested, during several instances when her students seemed stuck and unable to connect different aspects of the mathematics task. “It is okay to struggle, but it is not okay to give up,” Sophie would say, when students seemed discouraged. With the remainder of the period, students were given two similar but more challenging mathematics tasks to explore. They could choose to either attempt these tasks independently, or with a peer.

4.4.1 Program Planning

When asked about the learning goal for today's mathematics lesson, Sophie explains how she wants students to explore the relationships between shapes of varying sizes. Specifically, she wants her students to examine how several shapes with smaller areas (the pattern blocks) fitted
into shapes with larger areas (the irregular shapes featured on the mathematics worksheets). With
regards to mathematics communication, Sophie hopes to provide her students with opportunities
to express, through verbal and visual means, their growing understanding of area comparisons.

In this lesson, verbal communication of mathematics knowledge was practised during the
group discussions and individual conversations (with Sophie or with peers, during independent
work). In addition, visual representations of mathematical understanding were fostered through
the drawing of shapes and the arranging of pattern blocks.

Based on my observation of the mathematics classes and my conversations with Sophie, I
identified some of the Grade 3 expectations from the *Ontario Mathematics Curriculum, Grades
1-8* (rev. 2005) that were explored during this lesson. They include:

**Specific Expectations in Geometry and Spatial Sense**

- “Identify and compare various polygons (i.e., triangles, quadrilaterals, pentagons,
  hexagons, heptagons, octagons) . . .” (p. 59)

- “Solve problems requiring the greatest or least number of two-dimensional shapes (e.g.,
  pattern blocks) needed to compose a larger shape in a variety of ways (e.g., to cover an
  outline puzzle)” (p. 59)

I also analyzed the content of this lesson and identified additional mathematics expectations that
this lesson may scaffold for. These expectations include:

**Specific Expectations in Measurement**

- “Draw items using a ruler, given specific lengths in centimetres” (p. 57)

- “Describe, through investigation using grid paper, the relationship between the size of a
  unit of area and the number of units needed to cover a surface” (p. 58)

Sophie acknowledges flexibility as an important component to program planning and
lesson design. Sophie confides: “I do not usually plan my lesson strictly, because when teaching
mathematics, you have to be flexible” (Sophie interview, April 23, 2015). She understands that the time needed to learn a mathematical concept varies from student to student. In addition, different learning strategies are needed in order to reach all students. As such, Sophie designs her lessons with students' varying needs in mind. She explains:

Differentiation. Who needs more [time, assistance, accommodation], who needs less . . . . So, basically, depending on my students' needs. To get the lesson done and to achieve the learning goals. Make sure that everybody gets it. I use those different learning strategies [more time, group work, accommodation, and differentiation]. (Sophie interview, April 23, 2015)

For this mathematics lesson, Sophie considered students' prior mathematics knowledge. Sophie recounts how, in the past, her students may have developed relevant mathematics skills through: 1) opportunities to explore concrete and virtual pattern blocks, and 2) activities that involved covering one shape with another shape. Regarding her students' prior mathematics knowledge and their preparedness for today's lesson, Sophie comments:

They knew what pattern blocks are. They knew how to cover the shapes with the pattern blocks. But application of that knowledge . . . . But the relationship between the different pattern blocks, how they are related. So that piece was missing. It was a challenge. (Sophie interview, April 23, 2015)

Believing that students should be aware of the organized continuations and the deliberate extensions of lesson content, Sophie began the class by refreshing her students’ memories of the activity (covering shapes using virtual manipulatives) explored in the previous mathematics class. Sophie also sees today's lesson as a preparation for subsequent lessons on perimeters of shapes. At the beginning of class, she verbalized these links by briefly describing to her students the activities (covering shapes using square grids) they will attempt in upcoming mathematics classes.
4.4.2 Learning Environment

Sophie believes it is crucial to build and maintain a learning environment that respects all individuals. Students in her class are constantly reminded of this goal, as there is an abundance of displays on the classroom walls (e.g. self-regulation posters, conflict resolution charts, class pledges). In today’s classroom observation, there were moments when students disengaged (e.g. fidgeting with manipulatives, not staying on task during independent work) or disputed (e.g. minor verbal disagreements with fellow classmates). Sophie responded to such situations by pointing to the conflict resolution chart on the wall, and simply mentioning “self-regulation”. After that reminder, students often corrected their behaviours and resumed their assigned task. I inquired Sophie about this approach:

**Interviewer:** You mentioned self-regulation a few times during your teaching. How was that taught to the students? What was [teaching this process] like?

**Sophie:** We have Future ACES focus at this school, and the character traits [education]. From the beginning of the year, we follow the [character education] units that were given to us. That also focused on conflict management, conflict resolution. We have been talking about all of the character traits that are important to us. When it comes to character trait... Like self-regulation, you need to know what your emotions are, have empathy, and then act accordingly. When it comes to behaviour, I am just expecting them to manage their own behaviour, [know] whether it is right or wrong and whether they need to stop. Basically regulating or controlling their own behaviour, that is what. I think they are now used to it. So I am using that word. (Sophie interview, April 23, 2015)

Sophie describes how she and her students spent a few weeks at the beginning of the academic year discussing and envisioning what a safe classroom looks, sounds, and feels like. They collectively created classroom learning agreements, and suggested workable classroom conflict resolutions for various potential dispute scenarios. Through these and other activities, students were introduced to the idea of self-regulation- how the concept is defined, and how this approach is practised. Sophie feels that, by this point in the academic year, her students only
need slight cues when reminders are required. The messages broadcasted from the class posters suggest that self-regulation may contribute to a more positive learning environment. The drawings and statements displayed advocate for continuous practices of self-regulation, through developing heightened awareness of one's internal states (e.g. emotions) and external states (e.g. interactions with others). Thus, over time, students may become better at controlling and changing negative emotions and situations.

Sophie’s view of mathematical struggles and misconceptions are important to the fostering of communication skills. She carries positive perspectives regarding the two, and often envisions them as paths that could potentially lead to greater understanding. Sophie's perceptions were displayed in today's lesson, in her responses to students’ struggles. When students expressed difficulties with the task, Sophie calmly reminded them that “it is okay to struggle”. At the same time, she encouraged them to persevere in their attempts. “It is not okay to give up,” she said. Often, she guided struggling students by posing questions. These questions allowed students to generate strategies they could then explore.

With Sophie reinforcing these views and modelling effective approaches to mathematical struggles, students are also equally encouraging to one another. During the large group sharing, a student who volunteered to discuss his strategy had difficulties voicing his ideas. His classmates were patient when he required more time to think about ways to communicate his thoughts. The class was also enthusiastic in lending their assistance when Sophie asked if an additional student could go to the front of the class to continue the sharing with him. By creating and fostering an environment that welcomes struggles, students may feel encouraged to present ideas, even when such ideas are incomplete or unrefined.
Sophie believes the mathematics tasks are challenging and cognitively demanding for her students. Thus, she tried to encourage them by making the activities more appealing. During this lesson, she often equated the first task to a mystery, and invited her students to envision themselves as detectives working on a case. I was intrigued by this approach, and inquired Sophie further during our follow-up interview. An excerpt of our conversation goes as such:

**Interviewer:** You like to use the term “detective eyes and detective heads”. Are [the students] familiar with the terms as well?

**Sophie:** Well, not that much. Just for that specific problem. I felt like using it. You have to use detective eyes. Even though you do not see [the solution], you need to try to think about it and figure it out. Put the pieces together and think that, “Oh, this must have happened. Or this must be the thing”. So that is why I used those words, but I do not really use them often.

**Interviewer:** Not often. Okay.

**Sophie:** Maybe I felt like using it that day, and it went well with [the mathematics task]. Maybe I will use it frequently now.

**Interviewer:** It made it seemed like a game. *Laughs*

**Sophie:** Yes, I know. *Laughs* (Sophie interview, April 23, 2015)

Sophie's students seemed to respond well to this game-like approach to problem solving. They playfully scrutinized the shape's outline and the pattern blocks, inverting and rotating the manipulatives. They were also eager to share the “hints” and “clues” they discovered along the way.

### 4.4.3 Mathematics Tasks

For today's mathematics classes, the tasks required students to cover the areas of shapes with concrete manipulatives. Students were given ample of time to work on the first task independently. During those moments, Sophie observed their uses of strategies and engaged her
students in mathematical discussions. Sophie selected this task because it reviews some of the skills introduced in the previous mathematics class (e.g. covering shapes using virtual manipulatives). At the same time, the task prepares some of the needed skills for the upcoming class (e.g. covering shapes using square grids). Compared to what the students have previous experienced, Sophie believes this task is more cognitively challenging, since a greater variety of pattern blocks are required. Yet, it is less abstract than the upcoming lesson, as that activity will involve looking at grids and square units.

Sophie attempts to find mathematics tasks that are appropriately difficult for her students’ stage of development. She notices tasks that are too challenging or too different from what her students are accustomed to may trigger some levels of anxiety. Reflecting on the three tasks chosen for today's mathematics class, Sophie thinks they are a bit on the challenging side. For the second task, she feels that the multiple components (Part A, Part B, etc.) make it more difficult for her students to identify and focus on the procedures that are required.

In addition, today's tasks require some new ideas of application, and not simply a regurgitation of facts. She says: “But the [second] question that we did. That was challenging because that was three or four parts of the question. There was basically [the need for] application. Not just knowledge and understanding” (Sophie interview, April 23, 2015). Sophie feels that the difficulty of the tasks resulted in more class time needed for students to adequately formulate their strategies.

4.4.4 Constructing Knowledge

Reflecting on the lesson, Sophie believes that her students struggled with transitioning from knowing to understanding, from understanding to experiencing, and from experiencing to
communicating. She describes this observation: “Transferring knowledge, understanding, and thinking into applying. Communication and application. [For students], seeing the relationship between what was taught [in previous mathematics classes] and what they needed to apply to was a kind of challenge” (Sophie interview, April 23, 2015). Sophie suggests that a way to deepen students' awareness of these connections is through demonstrating and scaffolding this progression. During today's class, she approached this challenge by modelling ways to identify and “break the information down into chunks” so that the content could be more easily analyzed and processed (Sophie interview, April 23, 2015).

After the objective of the inquiry (discovering strategies to cover an area using pattern blocks) was identified, previous knowledge that was deemed useful for this task was gathered. Sophie concludes: “That is the challenge. They know how to solve word problems. They know what problem solving is. But going back to what they know, making connections, thinking, communicating, and applying the knowledge was not happening” (Sophie interview, April 23, 2015).

Sophie recognizes strategic questioning techniques as another method that assists students in connecting their mathematical ideas. During our initial interview, Sophie shares her beliefs that good questioning techniques may result in better knowledge building for students. She suggests: “So good questioning. The better you are at questioning, it is helping them to use their knowledge, build on their knowledge, and give them more knowledge” (Sophie interview, March 9, 2015). Specific to this lesson, she identifies her utilization of this strategy as an essential component in fostering her students' mathematics skills. The interview transcript excerpt below provides more detail regarding Sophie's thoughts on this technique:
Interviewer: What strategies did you use in that lesson to develop students' mathematics skills? [You mentioned] you used a task that was challenging.

Sophie: And questioning. So I used the questioning technique. Meaningful questions, relevant questions. The questions that take them back to thinking. Help them to think forward. Make some connections. So questioning was the main thing. To see the relationship, to bring them to the expectations, to bring them to the end of the answer. We kind of had to go step by step and basically look at the connection, applying the knowledge that they have learned. (Sophie interview, April 23, 2015)

During lesson observation, I noticed Sophie devoted ample of time paraphrasing with her students the objectives of the mathematics tasks. In our interview, I expressed my interest in knowing more about this approach for the fostering of mathematics communication. She then provides the following explanations:

Interviewer: Yes. I also find that for your lesson, that maybe for this grade or for your class, paraphrasing is something that you emphasize.

Sophie: Yes.

Interviewer: Taking what is written and asking them to reword it. Do you find that [paraphrasing] is an important element? For mathematics and also for communication?

Sophie: That is. Otherwise, they may not be able to communicate if we do not give them all the different vocabulary that they need. I know, math vocabulary. If it is geometry, they need to call it a specific name. But still, there are different ways of explaining. It is not one set way of explaining or communicating your ideas. You can write it this way or you can write it that way. But as far as you are explaining your thinking, showing your work, proving your point, that is good enough. (Sophie interview, April 23, 2015)

For the introduction of the first task, Sophie posed questions regarding its description. She inquired about what they knew, and invited students to search their memories for relevant knowledge that could be utilized. She continued with questions about the task, and prompted students to an in-depth examination of the current activity. Next, she asked about the materials and skills required, and suggested a connection between the goal of the task and the usage of previous mathematics knowledge.
Guided by these questions, Sophie's students reworded and redefined task-related vocabularies. This process of paraphrasing was another opportunity where her students would encounter and utilize new vocabularies (the ones featured in the task) in combination with their existing mathematics knowledge (the ideas resurfaced when engaging with Sophie's probing questions).

As students proceeded to solve the mathematics task, Sophie asked questions that encouraged students to identify and communicate what they understood, as well as what they found difficult. A question like “How did you cover the shape with pattern blocks?” resulted in students demonstrating visually and verbally their strategies. “Are we missing something?” hinted the strategies or the procedures used require further examination. Sophie asked these types of questions, as she realizes their potentials in assisting students to self-assess approaches and self-identify areas that required modifications.

4.4.5 Technology and Manipulatives

Sophie expresses: “I am not a big fan of manipulatives. But then when I need them, I would use them” (Sophie interview, May 26, 2015). Within the Ten Dimensions of Mathematics Education (McDougall, 2004), Sophie selects this dimension as one of her two foci for personal growth. Reflecting on her teaching practices, she notes:

I think that is one area that I need to work more on. I need to build more knowledge about different manipulatives and how to blend that into math lessons. Making it more, using it more. That is one of my weaknesses, and I think I need to come out of it and use it more. (Sophie interview, May 26, 2015)

She believes there should be a balance, where students have ample opportunities for displaying their mathematical understanding through concrete and abstract representations. Often, Sophie selects and incorporates manipulatives use “depending on [her students'] needs”
(Sophie interview, May 26, 2015). She mentions: “I am trying to use them for differentiating, like for my HSP [Home School Program] students, and students who need them. Students who are visual. Trying to balance it that way” (Sophie interview, March 9, 2015). When asked how she supports success in mathematics, Sophie states that her approach to manipulatives use is contributing to that goal. She explains:

I do show them that, “Ok, this is a manipulative. You can use it here. But if you really, really think that you do not need it, you can show it some other way. Do it”. So [I] give them freedom of what they want to do, because sometimes I think that manipulatives distract them too. Not all the time that it is really useful. (Sophie interview, March 9, 2015)

Prior to this class, students had exposures to different forms of concrete and virtual manipulatives. Sophie recounts that the concrete manipulatives explored in the past months included base-ten blocks, cards, counters, money, and pattern blocks. She also notes how virtual manipulatives were likewise introduced when students explored mathematics games provided through the school board's online resources. Bins of concrete manipulatives are stored neatly on the classroom shelves, fully accessible to students at all times. Whenever students require materials (e.g. rulers, calculators, manipulatives) for solving mathematics tasks, the students can easily locate the needed resources and proceed to acquire and use them. Sophie believes that having students identify and select the tools and materials they require is part of the process of becoming better problem solvers. In her class, students are often given the liberty and expectation of identifying and locating the needed items.

Alongside learning to select the appropriate concrete materials, Sophie also educates students in the appropriate maintenance and uses of manipulatives. A system for organizing and storing manipulatives is in place, and students are instructed to return materials to their
designated locations. Foreseeing that visual objects may distract and lead students off task, when needed, Sophie reminds her students to set aside their manipulatives during class discussions. When students are tempted to fidget and play with the pattern blocks, Sophie mentions “self-regulation” and the class would quickly readjust their behaviours.

4.4.6 Assessment

Sophie believes that a variety of assessment methods should be utilized to accurately capture students' mathematical understanding, and tangibly support mathematics success. She lists some of her methods: “So for assessment, it is not just tests. My observations, conferences, one-on-one, and stuff like that” (Sophie interview, March 9, 2015). While she does give students pencil and paper tests, she incorporates different formats, including “multiple choice and all variety of things too” (Sophie interview, March 9, 2015). She also makes it known to students that they can select any necessary tools to assist them in the completion of tests and projects. For today's lesson, formative assessment took place during: 1) her conversations with students as they worked independently, 2) the small group dialogues at the conference table, and 3) the whole class discussions and presentations. In all three arrangements, questioning techniques were used to explore students' attainment of the learning goal.

During individual work, Sophie walked around the classroom and chatted with students. The questions she asked required an understanding of the task (e.g. “What is the question asking you?”), a recount of previously learned mathematical facts (e.g. “Can we make a trapezoid out of triangles?”), or an extrapolation (e.g. “If you take this line out, what shape would you get?”). The types of questions asked were largely dependent on the students' abilities and their progresses on the task. At the conference table, the selected students were ones who Sophie felt required more
scaffolding. Questions posed to these students were mostly ones that aimed to guide the students to a plausible strategy. As students responded to Sophie's questions, she assessed whether they understood the mathematics task adequately. When a student demonstrated a suitable degree of understanding, she encouraged him or her to return to his or her table to attempt the task independently.

Some of the questions asked by Sophie were used to elicit instantaneous feedback from students. Such included “How many of you are confused?” and “Will you be able to complete a question like this one?” Most of these questions could be responded to by the raising of hands. Through posing various forms of questions, Sophie feels she gained a better sense of students' achievement of today's learning goal. Of course, the verbal discussions were but one quick form of feedback. Since students recorded their thinking processes and task solutions on the worksheets, Sophie would also collect their mathematics folders and further analyze her students’ written communication skills and mathematics content understanding. When asked to recount what went well in today's lesson, Sophie notes the exchanges of questions and responses between herself and her students as a personal indicator of success. She recalls:

[The students] were trying. Trying to figure out what it was really . . . . They were at least thinking. And when I was questioning them, they were asking me some questions back. “Why?” “And how?” “It is not matching?” “How can I do this?” (Sophie interview, April 23, 2015)

4.4.7 Summary

Reflecting upon this mathematics lesson, Sophie thinks her students are “kind of able to see the connection [with] what they did [in mathematics] the day before” (Sophie interview, April 23, 2015). She feels that, as a lesson, it is “pretty challenging” (Sophie interview, April 23, 2015). She concludes: “Some of them reach half way [for the first mathematics task], then we
had to do it together. But it worked out pretty well” (Sophie interview, April 23, 2015). Asked what she would have done differently were she to teach the lesson again, Sophie notes the difficulty of the mathematics tasks and proposes to first attempt one or two similar but simpler tasks. This, she believes, would have provided a better scaffold. Regarding this approach, she comments: “Maybe that would give them more clues, or more background knowledge, and how to transfer this into the question that we had” (Sophie interview, April 23, 2015).

Specific to the fostering of mathematics communication skills within this lesson, Sophie once again emphasizes the need for variety. She reinforces that students showing and communicating their thinking are essential to their learning. Sophie emphasizes: “Yes, they have to show their work” (Sophie interview, April 23, 2015). She explains that, when students “figured out the answer in their heads” without explicitly demonstrating what they have done in visual, verbal, or written forms, their mathematical understanding and preferred learning styles are less transparent. She elaborates on the disadvantages of solutions without explanations: “It is like you may not get to know how different kids learn in different ways. Or how they come up with different ways to solve a question or solve a word problem” (Sophie interview, April 23, 2015). Sophie is glad that the chosen mathematics tasks provided students with opportunities to communicate their understanding in different forms. Paired with her uses of questioning techniques, additional opportunities to communicate mathematical knowledge were also created.

4.5 Vignette Two

Wednesday, April 22, 2015

I arrived at the school before morning bell at 8:45 am, and stayed until 1:55 pm, the end of Period 6. The subjects of instruction for the morning included language, health and physical
education, mathematics, and technology. The teacher from the neighbouring elementary school continued to assist in the school. Each day, her contributions to the classes were outlined and assigned by the administrators. Today, she taught language to Sophie's class during Period 1.

In the afternoon, a numeracy coach from the school board visited Sophie's class to do some co-teaching during Periods 5 and 6. For those two instructional periods, the students explored the Explain Everything App on iPad minis. The mathematics task the students investigated required them to estimate and calculate the number of grids needed to cover a shape. After exploring the strategies used, the students communicated their approaches through visual (labelled diagrams), audio (voice recordings), and/or written (summary sentences) forms.

Aside from the health and physical education class, which was taught by another teacher in the school, I observed Sophie's students interact with all other classes and subjects taught to them today. Below, I will describe the Period 2 technology lesson and the Period 4 mathematics lesson, which were both taught by Sophie.

9:35 am - Technology

A few months ago, Maple School purchased one class set of iPad minis for students in the Primary Division. While Sophie's students had some previous exposures to this technological equipment, she feels that spending Period 2 exploring the Explain Everything iPad app may contribute positively to the class's learning experiences in the afternoon. When the iPad minis arrived to the classroom in their storage carts, Sophie distributed one iPad mini to each student. Next, the students were guided through the procedures of: 1) turning on the iPad minis, 2) typing in their student identifications and passwords for access to the school's wifi, and 3) exploring the Explain Everything iPad app. Sophie modelled the steps and projected the screen images through
a document camera. At the same time, students assisted one another.

Unfortunately, connecting to the school's wifi proved to be challenging, partly due to students' struggles with sign in, and mostly due to a weak and slow signal. After a while, several students still had difficulties signing into their student accounts. At this point, Sophie selected an electronic audio book, displayed it through the projector screen, and assigned a student to monitor the pace of the reading and the page turning. Thus, the students read Everyone Uses Math by Brian Sargent, a non-fiction children's book that showcases the uses of mathematics in various professions. Meanwhile, Sophie helped the remaining students with their iPad issues.

When the technological struggles were resolved, the class spent a brief moment exploring the features in the Explain Everything app. For the remaining time in this period, students learned to: 1) create a new document, 2) take a photo of a mathematics task, 3) paste the image of the task onto a blank page, 4) use the writing and recording functions, and 5) save the document for the numeracy workshop in the afternoon.

11:12 am - Mathematics

Returning from the health and physical education class in the gymnasium, students settled into their assigned seats. Sophie asked them to take out their Math Makes Sense textbooks from their desk drawers and turn to the specified page. When they found the corresponding task, Sophie read the information in the “Connect” section, pausing often to paraphrase the sentences. The question described a bulletin board, where either 60 small squares or 15 large squares could be used to cover the entire surface. While visual depictions were provided in the textbook, Sophie also drew her students' attention to an actual bulletin board in the classroom. With smaller tiles and bigger tiles in hand, she asked students to visualize what the task was
describing. “Is there a difference in size? Which grid would I need more?” she asked. A student was selected to go up to the bulletin board and show how the paper tiles can be arranged. After her explanation and demonstration, Sophie asked the class: “She says the smaller one. Does anyone agree with her? Who can explain why they agree with her?” Many hands were raised. Another student was selected to respond. Sophie paraphrased his explanation, and emphasized the uses of mathematics vocabularies (e.g. area, perimeter).

Sophie then linked the current “Connect” ideas with mathematics concepts from previous lessons. Together, she and her students recalled the manipulative tasks described in Vignette One. With multiple possible solutions, the tasks used in the mathematics lessons described in Vignette One illustrated how different types and quantities of pattern blocks could be used to cover the same irregular shape. This is because the pattern block shapes differ in areas (e.g. the yellow hexagon is twice the area of the red trapezoid). Emphasizing this idea in today's task, Sophie held up two different sized paper tiles and asked students if they were the same in area. A student answered no, and explained that one was bigger than the other. Sophie agreed, then asked the class if the size of the bulletin board changed. Throughout this discussion, students nodded in agreement, and appeared to understand that different number of tiles are needed to cover a bulletin board, given that the tiles' sizes are unalike.

Next, Sophie presented another activity from the Math Makes Sense textbook to highlight the significance of different tile sizes. Taken from the “Practice” section of the textbook lesson, students had to vote whether “large tiles” or “small tiles” should be used to cover: 1) the classroom floor, 2) a shoe box, 3) a book, and 4) a playground. Sophie also selected students to share their selection rationale. At the end of this activity, one student commented that the words
“large tiles” and “small tiles” were confusing, as actual sizes were not provided. Sophie applauded that thinking, and agreed that this was something that should be considered.

Lastly, Sophie introduced Jen and Madhu from the “123” component in the textbook. These two characters were trying to cover a bulletin board with pieces of paper tiles. Jen decided that 8 tiles were enough, while Madhu concluded that 50 tiles were needed. Students were then asked to determine who was correct. Drawing their attention to the concepts presented in the “Connect” section, Sophie told her students to first share their insights with their table group members. Her students were instructed to discuss with their neighbours how they would go about resolving the supposed disagreement. She reminded her students that this month's character education theme is cooperation, and that they should work alongside one another harmoniously. Sophie walked around the classroom, and listened in on various conversations that her students were having.

“So who is right? Who says Jen is right? Who says Madhu?” Sophie asked. Students responded by raising their hands. “Both right?” More hands went up. Sophie then asked students to explain their positions. The class concluded that both Jen and Madhu could be correct, as the number of tiles needed to cover the bulletin board was dependent on the areas of the tiles.

4.5.1 Program Planning

Asked about the learning goal for today's mathematics encounters, Sophie responds that “the learning goal was to figure out the areas of shapes using grid paper . . . . Find the areas and compare the areas. Using different shapes. Using the grid paper” (Sophie interview, April 23, 2015). She elaborates that this learning goal encompasses several sub-goals. These include: 1) understanding conceptually the square as a unit, 2) knowing the definition of area, 3) applying
the understanding of square units and areas, by counting the units to arrive at measurements for
the areas, and 4) comparing the areas of different shapes. She speaks of the mathematics lessons
from Vignette One as preparations for students to explore the mathematical concepts and skills
presented in this lesson. The concrete experience of covering a shape with pattern blocks bridges
beautifully into the usage of squares as units of measurement for areas.

Sophie also wants students to demonstrate the learning goal through the Explain
Everything iPad app. With this in mind, Sophie plans the morning technology and mathematics
classes partially as preparation for the numeracy and technology session in the afternoon. For the
technology class in Period 2, the learning goal is to attain basic competence in using the Explain
Everything app. For mathematics class in Period 4, the learning goals are: 1) to understand the
usage of square grids in “covering” an area, and 2) to realize that the square units (e.g. tiles) can
vary in sizes.

Given my observations of Sophie's mathematics class and my subsequent conversations
with Sophie, I identified some of the Grade 3 expectations from the Ontario Mathematics
Curriculum, Grades 1-8 (rev. 2005) that were incorporated into this lesson. Below is a summary
of these expectations:

Specific Expectations in Measurement

- “Estimate, measure, and record area” (p. 57)

- “Compare and order various shapes by area, using congruent shapes (e.g., from a set of
  pattern block or Power Polygons) and grid paper for measuring” (p. 58)

- “Describe, through investigation using grid paper, the relationship between the size of a
  unit of area and the number of units needed to cover a surface” (p. 58)

In addition, within the Ontario Science and Technology Curriculum, Grades 1-8 (rev.
specific expectations with regards to “developing investigation and communication skills” (pp. 71, 74, 77, 80) are meant to be embedded into every topic in the Grade 3 science curriculum. A specific expectation that was explored in today's lessons is “[using] a variety of forms (e.g., oral, written, graphic, multimedia) to communicate with different audiences and for a variety of purposes” (p. 71). During today's mathematics lesson and numeracy session, it is seen that the fostering of investigation and communication skills as recommended in the *Ontario Science and Technology Curriculum, Grades 1-8* (rev. 2007) can be applied and practised within many topics and subjects.

### 4.5.2 Learning Environment

The staff at Maple School enjoys multiple professional learning initiatives and collaborative teaching opportunities. When asked about the development of the vision for the school, Sophie emphasizes how the staff at the school “is always ready” and “has growth mindset” (Sophie interview, March 26, 2015). Throughout the year, teachers are encouraged to attend workshops that are held outside of the school. Earlier, Sophie attended workshops and learned about a variety of iPad apps. Upon returning to the school, Sophie shared these new knowledge and resources with the Primary Division teachers during Professional Learning Community meetings.

In addition to the staff seeking professional improvements through workshop attendances and Professional Learning Community meetings, education experts also visit their classrooms. These co-teaching sessions allow for interactions between coaches, teachers, and students - all within the familiar environment of the elementary school. In today's afternoon session, we had a glimpse of this occurrence. Sophie recounts how, prior to this lesson, she and the numeracy
coach met to discuss the class context, the topic of focus within the subject of mathematics, and the students' mathematics abilities. The incorporation of technology into the teaching of mathematics is a strategy that Sophie wants to explore. Thus, she made known of this interest to the numeracy coach. Afterwards, they co-planned the mathematics lesson, specified the learning goal, and incorporated the uses of the school's iPad minis.

4.5.3 Mathematics Tasks

During the Period 4 mathematics class, Sophie utilized some questions and activities from the textbook series *Math Makes Sense - Grade 3* (2004). A resource published by Pearson Addison Wesley, *Math Makes Sense - Grade 3* is organized into units (11 for this grade), and within each unit, lessons (ranges from 5 to 14 per unit). Each lesson contains different elements that assist students in their mathematical knowledge attainment. These features include:

- **Explore** - investigate an idea, either individually or with a partner
- **Show and Share** - demonstrate and explain to a partner
- **Connect** - join the topics investigated in “Explore” with other mathematical ideas
- **Numbers Every Day** - develop numerical fluency
- **Practice** - complete similar questions to solidify the concepts introduced
- **123** - use various means (e.g. pictures, words, numbers) to show understanding
- **Reflect** - identify the overarching mathematical ideas in the lesson

For this lesson, Sophie incorporated the “Connect”, “Practice”, and “123” features. Taken from the lesson on “Measuring Area in Square Unit”, within the unit of “Length, Perimeter, and Area”, the “Connect” section introduces the concept that tiles of varying sizes can be used to cover surfaces. This idea can be seen as preliminary to the concept of varying units of
measurements used to report areas. Next, the “Practice” section gives students an opportunity to evaluate the practicality of measuring with large and small squares. Finally, the “123” section sums up the concepts explored in the “Connect” and “Practice” segments, and illustrates once again considerations for varying units of measurements.

Although these textbook activities could have been done independently as seatwork or homework, Sophie facilitated them in ways that promoted more verbal communication opportunities amongst her students. Some of the strategies she used include: 1) reading the task descriptions and paraphrasing the objectives (for “Connect” and “123”), 2) casting votes on solution options, then asking for justifications for the selections (for “Practice”), and 3) discussing in table groups, then sharing with the class (for “123”). The textbook tasks utilized are also rich in discussion possibilities. The “Practice” and “123” activities both: 1) have more than one correct solution, 2) require explanations and justifications of responses, and 3) can be expressed in different forms (verbal, visual, written). Rather than incorporating all features from the textbook lesson, Sophie selected three segments that she then connected to form one encompassing idea (the uses of different units to measure an object's area). As a result of having fewer activities, more time was allotted to exploring and discussing each task.

4.5.4 Constructing Knowledge

As seen in Vignette One, Sophie places great emphasis on connecting students' previous experiences and current knowledge. Having selected constructing knowledge as an area for personal growth in the teaching of mathematics, Sophie sees that “it ties into mathematics communication and good questioning techniques” (Sophie interview, March 26, 2015). From her unit organization, she seems to consider with care the relationship between students' previous
knowledge and current acquisitions. This is demonstrated through her deliberate connections between the current mathematics class and the one described in Vignette One.

During this lesson, Sophie often invited her students to actively discover similarities between their current mathematics topics and their previous learning experiences. As the class began the activity, she encouraged them to read the textbook and become more aware of the connections between this lesson and the previous ones. She reminded them of their experiences with covering shapes using concrete pattern blocks, a task which she believes can be linked to the more abstract visualization of square grids “covering” an area. Within today's lessons, the skills introduced this morning also acted as scaffolds for the technology and numeracy session in the afternoon. The mathematics lesson in Period 4 demonstrated the use of squares as a unit of measurement. The mathematics discussions about square tiles with varying sizes may also prepare for future lessons that explore the idea of different units of measurements (e.g. centimetre square, metre square).

When asked what strategies she used to achieve the learning goal for today's mathematics class, Sophie identifies modelling and questioning as her main tools. Sophie believes that “questioning for any teaching, and questioning technique, really helps [the students] get going” (Sophie interview, May 26, 2015). She recounts her demonstration of covering a bulletin board using paper tiles in Period 4 mathematics class as one example of modelling and representing. She comments on her approach: “Demonstrating how to do it. Questioning, and making connection to the symmetry [lesson] that we did [earlier this academic year]” (Sophie interview, April 23, 2015).

Other benefits of modelling and questioning which Sophie perceives and recalls include:
1) “developing their own understanding of the task” (e.g. knowing the objective), 2) “making them understand why they are doing it” (e.g. discovering the rationale), and 3) “helping them think” (e.g. reasoning and justifying) (Sophie interview, May 26, 2015). Sophie suggests that both the techniques for questioning and the types of questions asked are important factors. Rather than distributing the solutions to students, Sophie wants her questions to guide students through the knowledge discovery process. She elaborates:

The questions should be probing . . . . It is their learning, just facilitated by me. They are learning on their own. I am just there to guide them. I just do not go and teach. I want them to learn it themselves. So that, I think, for questioning technique, the way you ask questions, the way you help them think and see the big picture, see the correlation, see the connection between different strands and all those stuff, really helps. For any teaching, questioning is important. (Sophie interview, May 26, 2015)

As I observed today's teaching, I noticed Sophie frequently posed both closed questions (e.g. “yes or no”, “this or that”) and open-ended questions. There were several instances when closed questions were used to receive quick responses from students on their learning progresses. In this lesson, such questions included: 1) “Do we remember, the other day, we did covering the shapes?”, and 2) “Does anyone agree with her [response]?”. The first question was posed to the class at the beginning of the lesson, and students responded with raising their hands, nodding or shaking their heads, or saying “yes” or “no”. The second question was posed after a student shared a strategy for solving the task. The same chorus of responses were elicited. Sophie notes that such responses were used as coarse estimates of how her students were responding to the new materials. She also suggests that these brief moments of assessments may be used to help guide some instructional decisions as the lesson unfolded.

Other closed questions were used to steer and guide students' thinking. In this lesson, these questions included: 1) “Has the size of the bulletin board change? Is it the same area?”, 2)
“Is there a difference in size [for the grids]?”, and 3) “Which grid would I need more?”. The responses to the first two questions were “yes” or “no”, while the choices to the third question were “large tile” or “small tile”. Even though “this or that” questions may appear to lack potentials for highly detailed and thorough responses, Sophie used them as starting points to encourage students to participate, during times when her students may not have reached the point of confidently sharing their strategies.

As expected, open-ended questions were also incorporated into the lesson. Examples of such questions from today’s mathematics class included: 1) asking for justifications on why they are in agreement about a solution (e.g. “Who can explain why they agreed with her?”), and 2) inviting explanations for why they think both Madhu and Jen are correct. These questions were asked after students responded to the “this or that” questions. As such, the closed questions encouraged students to select their positions, and the open-ended questions requested rationales for such selections. It seemed that this approach to questioning may allow Sophie's student to build up their confidences in mathematic communication, as they progressed from picking amongst two options, to justifying their choices.

4.5.5 Technology and Manipulatives

The use of iPad minis was an essential part of today's lessons. A portion of the morning was spent getting acquainted with these devices, while the afternoon was the application and demonstration of the acquired skills. Sophie recounts that the iPad minis were purchased a few months ago, and the students have had several opportunities to use them in classes. She mentions: “iPads. Yes, this is the first year” (Sophie interview, May 26, 2015). She believes that as the students and the staff in the Primary Division continue to explore these devices in the
coming years, they will discover many more ways to connect technology with the learning of mathematics. Sophie envisions this prospect: “I think next year will be better for planning. And using it more for teaching as well” (Sophie interview, May 26, 2015).

Sophie recounts how, when she discussed her professional learning goals with the numeracy coach, Sophie made known her wishes to further explore the uses of iPads minis for the teaching of mathematics. From there onwards, they co-planned the lesson, identified the learning goals, selected the mathematics task, and embedded the uses of technology. The Explain Everything Interactive Whiteboard app was chosen, since it has many functions which enable students to practise multiple forms of mathematics communication. According to Explain Everything Interactive Whiteboard app's website, over two million people across the globe have used this app. The website also mentions several special features which make this tool fitting for facilitating explanations. Such features include: 1) a laser pointer that directs attention to important written or visual contents, 2) a recording function that captures sound, and 3) a sharing and exporting method that is convenient and instantaneous.

During the afternoon, as students worked on the assigned mathematics task through the iPad minis, they had the freedom to choose how they preferred to express their thinking. Many students began by indicating their counting approaches. Some of these strategies included sorting the square grids in groups, tallying the number of square grids individually, and writing the numbers in the square grids. Other students used number sentences to express their calculations. Their mathematical reasoning and solutions were then typed up using the keyboard function, or written up using the drawing device. For some students who completed these components efficiently, they proceeded to record audio explanations of their problem solving procedures for
this task. Towards the end of the lesson, one of the students was invited to share his recorded explanation with the class. This gave students who were unable to complete a recording during the allotted time a glimpse of what they could do in subsequent learning opportunities. As such, visual, written, and verbal forms of communication could be practised, recorded, and shared through the Explain Everything app.

When Sophie reflects upon today's lessons, she is pleased with how her students utilized the iPad minis and the Explain Everything app in their learning of mathematics. She says:

Yes, [the students] know how to find the area for sure. Now this question, [the mathematics task], was difficult using iPad. Drawing the lines on the iPad using a finger. That was a challenge. But then, on the other hand, they got that they have to compare it, yes? They can use the technology pretty well. They were able to use that app that we introduced. They were all able to get the picture, get going on Explain Everything. They were all able to write using that app and they learned how to record too. (Sophie interview, April 23, 2015)

4.5.6 Assessment

When we discussed the benefits of using the Explain Everything app, Sophie also notes its potential for assessment purposes. Specifically, through this device, verbal communication can be easily recorded, archived, and re-visited. Sophie explains:

Basically, this app and the things that we do is for pedagogical assessment. To keep the assessment in records. It helps me to review it, go back, and see what the kid is thinking. How the kid is able to explain their thinking using the math vocabulary and stuff like that. It is an assessment tool. Teaching, learning, as well as an assessment tool. That is like to try to see how much they have learned. (Sophie interview, April 23, 2015)

When asked to comment more about the benefits of the iPad and the Explain Everything app for assessing students' mathematics communication, Sophie notes the comfort that students have with technology. She expresses:

[The iPad] is like a pedagogical assessment that you have documentations. The traditional tests, not everyone is comfortable with that. This, [the Explain Everything app
documentation], gives my observations, their recorded communication, their work, the way they share. Everything gives me an idea of where they are, what they can do, and where I can go from there. (Sophie interview, May 26, 2015)

Indeed, verbal communication may be practised and assessed in various forms. As observed in Sophie's class, verbal communication assessment may occur during: 1) student-teacher dialogues during independent work, 2) teacher observations during inquiry-based group work, 3) student-teacher conferences, and 4) student presentations. At the same time, capturing these moments for records and reflections may be difficult. As such, the record function from Explain Everything app is highly valuable, in that it is one of several devices that can capture and store students' verbal communication. As Sophie mentions, these data can then be used to track students' progresses. These excerpts and recordings of students' explanations can then be assessed thoroughly and repeatedly.

### 4.5.7 Summary

As Sophie reflects on her teaching today, she notes that most students attempted the tasks for the mathematics lesson and the iPad session. Students engaged in dialogues with her, with the numeracy coach, and with their peers. The questions posed by the students suggested they were thinking deeply about the concepts presented. Sophie acknowledges that the task chosen for the iPad activity is difficult to tackle on the technological device. If given the opportunity to redo this lesson, she would have presented the same task, but scaffold it more.

Since Sophie identifies how some of the challenges with the task originate from students having difficulties drawing square grids on the iPad minis, she suggests to first do the drawings on the worksheets, and then take images of them. “No, I would not change the task, but I would use paper and pen, and they can use ruler to draw the straight lines,” Sophie describes (Sophie
interview, April 23, 2015). Afterwards, students would continue onwards with completing the task on the iPads and the Explain Everything app. That way, more time may be spent on recording and writing their explanations. She concludes:

Figure out the answer, and then take a picture. Record their answers, and then I can have a video of them talking and all that stuff. Then they can add their text box to explain their writing, explain their communication. Explain their thinking. So that is what I am going to change. Because this kind of question with iPad was difficult. (Sophie interview, April 23, 2015)

4.6 Vignette Three

Friday, April 24, 2015

I arrived at Maple School at 8:40 am, and remained there until 11:35 am. During those hours in Sophie's class, I heard the morning announcements through the public address system, and observed the numeracy routine, language lesson, and mathematics lesson. Once the students entered and settled into their classrooms, the national anthem was played through the public address system. Following the anthem, two students spoke through the microphone and updated the staff and students on upcoming events: music practices, sports tournaments, and student council meetings. The announcements ended with an uplifting statement: “Do not be somebody else. Be yourself”. Afterwards, Sophie began the learning day with the Numeracy Routine, a math of the day task that will be described in Vignette Six. In this section, I will recount in detail the numeracy block, which occurred in Periods 3 and 4.

10:30 am- Mathematics

Sophie started the mathematics class by holding up a piece of grid paper, and asking students how it is connected to recent mathematics lessons. Students recalled counting paper tiles for the Math Makes Sense bulletin board task, and counting grids for the iPad task. Both of these
lessons occurred during the observation day described in Vignette Two. Written on the chalkboard was the instruction for today's warm-up activity: “Draw three shapes/figures that have an area of 24 square units”. Connecting this with the language lesson earlier today, Sophie posed questions about the goal, the materials needed, and the steps involved in solving this problem. She told her students that they would have ten minutes to work on the task independently, after which they would have opportunities to share their solutions with the class.

As Sophie walked around the classroom to check her students' progresses, she continually encouraged them in their efforts. “It is okay to struggle. Try,” she said. Sophie also used these moments to reinforce mathematical knowledge and vocabularies. She wanted her students to be creative with their solutions, drawing attention to the variety of characteristics (e.g. regular and irregular shapes) that could fulfill the task expectation. With the document camera set up, Sophie invited students to come to the front of the classroom and share their solutions. Three students each drew one shape that fulfilled the indicated measurement requirement. Sophie then asked the class to examine why these shapes meet the task description. Students shared how they checked these proposed answers. The strategies they used include counting the grids systematically (e.g. by rows or columns), and writing numbers in the grids. Sophie concluded this discussion by noting that, in addition to the various methods for verification, there are also numerous correct solutions for this task.

For the second mathematics task, Sophie intended to utilize the Explain Everything iPad app. She distributed the iPad minis and instructed her students to sign in and take a photo of the mathematics question. Afterwards, they were to complete the task on the iPads minis and show, through diagrams, written sentences, and audio recordings, the steps they took to reach the
solution. Unfortunately, due to weak wifi signal, the class was unable to log on. After attempting for approximately ten minutes, Sophie told her students to complete the task on the worksheets instead. She reassured her disappointed students that they will have other opportunities to use the Explain Everything app for learning.

Looking at the paper version of the activity, Sophie suggested they try the procedural problem solving approach. After using these questions during language class this morning, the students seemed familiar with the process. Sophie first inquired about the goal of the task. The description for the task was then read aloud by Sophie, with frequent pauses for students to underline important information, and evaluate the relations between these concepts. A student highlighted “three schoolyards” as important, since this quantified the areas that needed to be calculated. Another student drew attention to the legend, which accounted that each square grid is a metre squared.

“Which schoolyard has the greatest area?” Sophie continued to read. A student noted “greatest” as an important word, since it meant that comparisons needed to be done. Sophie complimented that observation, and reiterated the comparative nature of the vocabulary “greatest”. Sophie then asked about the steps required. The class agreed to first find the areas of the schoolyards, and then compare them to identify the one with the greatest area. Next, Sophie inquired about the materials that they would need. With past mathematics activities as guides, students responded that pencils and rulers were the only necessary instruments.

Having an understanding of the goal, the procedures, and the materials needed, Sophie instructed her students to work in small groups. The steps and solution were later shared with the whole class. Sophie concluded the lesson with a discussion on misconceptions and mistakes. She
asked students to identify some issues that they should be more aware of when attempting today's task. A student recounted how her table group initially counted the areas of the schools instead of the schoolyards. Sophie agreed that such mistake was made in her group, and emphasized the need to read instructions carefully.

4.6.1 Program Planning

When asked to identify the learning goal for the mathematics lesson, Sophie says:

The learning goal was to look at using the grids to cover an area. Looking at the relationships of how the squares, the grid paper, and the area are related . . . . Like the bigger the size [of the tile], the less the number, and so on. To see that relationship and basically get the concept of area on a grid paper. (Sophie interview, May 26, 2015)

This lesson topic is a continuation of the ideas presented during the mathematics lesson in Vignette Two, which looked at surface areas being covered with different quantities and sizes of tiles. Now, students had to expand their understanding from identifying different shapes and different tiles, to creating different two-dimensional shapes that had the same areas.

From my observations of Sophie's mathematics class and my discussions with Sophie, I noted some of the Grade 3 expectations from the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005) that were explored during this lesson. These expectations include:

Specific Expectations in Measurement

- “Estimate, measure (i.e., using centimetre grid paper, arrays), and record area” (p. 57)
- “Describe, through investigation using grid paper, the relationship between the size of a unit of area and the number of units needed to cover a surface” (p. 58)

4.6.1.1 Subjects Integration

As Sophie plans her lessons, she emphasizes the importance of engaging with multiple subjects and various mathematics content strands. She expresses her thought: “I do not believe
that math can be taught in isolation, like this unit and that unit. It is always cross strands too” (Sophie interview, April 23, 2015). Sophie describes her approach to mathematics content strands integrations:

You are following the [provincial] curriculum [document], you are trying to see the cross strands links, how one strand ties into another. Not teaching it in separation, but showing kids that it is all linked, and you can do it. I think [the staff at school] are very good at doing that, and it is working out. (Sophie interview, March 9, 2015)

Recently, she linked mathematics with language when students reviewed their knowledge of money through poetry writing. Sophie recalls this activity: “We did a money unit before, and they wrote poems on money. Money matters, and stuff like that” (Sophie interview, April 23, 2015). In upcoming classes, Sophie plans to investigate the topic of plants through combining the subjects of science and mathematics. She envisions utilizing some tasks which: 1) explore areas of outdoor gardens (connection to the “measurement” mathematics content strand), and/or 2) represent the variety, quantity, and diversity of plants (connection to the “data management” mathematics content strand).

Another way which Sophie incorporates mathematics with other subjects is through the use of picture books. She notes: “We do have some math focus books. [The school librarian] in the library, she displayed some. We have some given to [the teachers, for their classes], a few years ago. I have a book that talks about probability” (Sophie interview, May 26, 2015). When describing children's picture books, Sophie says they give her students “visual clues and ideas about how mathematics is all around us” (Sophie interview, May 26, 2015). As described in Vignette Two, students listened and read the audio book *Everyone Uses Math* (Sargent, 2005) from the Rookie Read-About Math series. Other times, Sophie selects general, non-subject specific picture books and embeds mathematics into them. She provides an example of a recent
Whenever I am reading any book, like on urban and rural communities [for social studies], I am looking for different shapes that were used in the communities. All different kinds of structures and stuff like that. So it is combined with science, social studies, and math . . . . Basically, everything that I teach, I make them see the big picture that it is math, it is around us. It is really important for us to know all this and learn all this. (Sophie interview, May 26, 2015)

Sophie's integrations of subjects within lessons reinforce her stated belief that mathematics should not be taught in isolation. During today’s language class, activities from previous mathematics lessons (e.g. tiling, covering shapes) served as topics used for practising procedural explanations. Likewise, during mathematics class, the success criteria and the steps for procedural writing introduced in the language class were equated to the inquiry model and the procedures in problem solving. The similarities amongst these two models (e.g. questioning techniques, guiding processes) were highlighted by Sophie. The familiarity and continuity of these approaches may have motivated students to participate more in the discussions, as they could draw upon their experiences and skills from previous diverse learning environments.

4.6.1.2 Three-Part Lesson Plan

Sophie identifies the three-part lesson plan as a template that helps her to thoughtfully consider the inclusion of different forms of communication during the class. During our initial interview for the Elementary Teacher Learning Initiative, I asked Sophie: “How do you create a classroom that fosters those types of communication? Both verbally, and also in written form?” Sophie responses: “Three-part mathematics lesson. The consolidation part is the most important part [for fostering mathematics communication]” (Sophie interview, March 9, 2015). Sophie discusses her general approach to planning three-part lessons:

I try to choose word problems that are kind of related to real life situations . . . The
“before” part is always something that they know from before. It is a quick activity that I do, whether using some manipulatives or not. Something that brings their knowledge of what they know from before. The “during” part is always working on a question in pairs or in groups. They show what are the strategies that they use. Then we discuss them together. Or a gallery walk, or maybe a math congress, or stuff like that. Then they present. All the students in my class can talk. They are out of their shells, and they know it is okay to say something wrong . . . . It is okay to speak up, or it is okay to share your way. Then the “consolidation” connects back to the learning goals and success criteria and all those things. I see it really makes sense to them. The way they are learning. It is all linked. And then some follow up activities. (Sophie interview, May 26, 2015)

While Sophie describes this template, she also emphasizes the need for differentiation and flexibility. She constantly assesses her students' understanding during the lessons. She states how, depending on the quality and correctness of students' responses to her questions and the activities, the lesson may need to be adjusted. She shares her thoughts: “It is not necessary I always do three-part lessons. I plan it according to my students' needs. I know that if this is not going to work out, I take more time to teach the basics too. I go back to the basics, because they are important too” (Sophie interview, May 26, 2015).

When asked what strategy was used in today's lesson to promote students' mathematics communication, Sophie once again notes the structure of the lesson, which she refers to as “a kind of three-part lesson, but not a planned three-part” (Sophie interview, May 26, 2015). As the “before” or “getting started” component, she identifies the review of and the discussion on the tiling activity in the *Math Makes Sense* textbook. This is an activity that was completed during the mathematics lesson described in Vignette Two. Here, students were invited to verbally express what they have previously learned. The “during” or “working on it” segment involved solving the two mathematics tasks independently and in groups. This time afforded students to share their ideas with peers, while Sophie observed their interactions. Where needed, she would contribute to their discussion groups’ work by asking probing questions, reviewing strategies, or
suggesting mathematics vocabularies. The “after” or “consolidation and practice” portion was the sharing of strategies, misconceptions, and mistakes. Each component of the three-part lesson plan provided students with opportunities to inquire, discuss, and review.

### 4.6.2 Learning Environment

As Sophie notices, the structure of the three-part lesson often provides opportunities for students to engage in mathematics discussions, both with table group members, and with the entire class. A learning environment where students feel safe and encouraged is beneficial to all components of the three-part lesson plan. When asked about her students' levels of comfort in sharing and communicating mathematical ideas, Sophie observes that “they feel excited when they get to share” (Sophie interview, March 9, 2015). She attributes this joy of sharing to an accepting attitude towards mathematical mistakes. Sophie believes that her students, aware that the primary focus of mathematics learning is not the display of flawless knowledge, are more willing to take mathematical risks. Sophie summarizes this idea:

> Of course, there are times when they have not done [the mathematics tasks] right, and they kind of know. But then I always say that when it is math, I do not think [it is about] right and wrong answers right now, because you are learning . . . . And if you are stuck, I am there to help you. (Sophie interview, March 9, 2015)

Within today's mathematics lesson, Sophie often encouraged students when they struggled with the mathematics tasks. She reminded them that it is acceptable and beneficial to find the tasks demanding. At times, students struggled with verbalizing their mathematical ideas. During those moments of silences and pauses, Sophie patiently waited for her students to gather their thoughts. In addition, she asked students to verbalize what they found difficult, and guided them through the problem solving process by asking questions that prompted their thinking.

To create an environment that supports students in their mathematical struggles, Sophie
believes that the three-part lesson plan needs to be approached with flexibility. The allotment times for each of the three-part lesson plan segments should be adjusted as the lesson progresses. Sophie states: “We always have some three-part math lessons, and I change them as I need to” (Sophie interview, March 9, 2015). While it may be good to have a thoughtful and detailed lesson plan, being attentive to students' needs as the lesson progresses and making necessary changes are vital to students' experiences of the lesson. Sophie suggests that, sometimes, an attentiveness to students' displayed needs may result in altering the lesson plan, and embedding a review of mathematics contents from previous grades. Other times, such adjustments mean providing extra time for students to explore. Sophie comments: “If I need to teach something. One step at a time. Some problems. Some daily skills. If they need to, I do it. I take my time. I do not just rush through. To just touch bases on things” (Sophie interview, March 9, 2015).

This flexible approach to lesson planning was demonstrated in today's mathematics class, as Sophie moved fluidly from “getting started”, to “working on it”, to “consolidation and practice”. Though Sophie had estimated time allotments for each component, she spent as much time as appeared to be necessary for students to feel ready for the upcoming task. In comparison to time suggestions from three-part lesson plan templates, Sophie spent less time in the “getting started” segment, and more on the “consolidation and practice” portion. Observing her interactions with students during the class, I sensed that her continuous assessment of students’ progress informed many of her lesson delivery decisions today.

4.6.3 Mathematics Tasks

For today's mathematics lesson, the first task required students to create three two-dimensional figures, each with an area of 24 square units. This task made use of some skills
practised in recent mathematics classes. After her students had adequate time for exploring, Sophie selected three students to share their solutions. Each student proceeded to create a different figure. Sophie then asked her class follow-up questions, and invited them to justify how each solution fulfilled the required characteristic. By having more than one correct solution to a mathematics task, similarities and differences amongst the solutions were explored and expressed.

The second task attempted in today's mathematics lesson involved three schoolyards. This question prompted the calculations of respective areas, followed by comparisons amongst the schoolyards to identify the one with the largest area. The procedures used for this task were similar to the ones explored during the iPad session with the numeracy coach (Vignette Two). During several occasions in today's mathematics lesson, Sophie reminded her students the commonalities between the current tasks and the previous ones. Students would likewise suggest strategies that were previously used (e.g. the methods for counting the grids, the importance of certain mathematics vocabularies).

4.6.4 Constructing Knowledge

Aside from the three-part lesson plan, another strategy that Sophie believes helped foster students’ mathematics communication during this class was their explicit awareness of the problem solving strategies used. Sophie lists some of these strategies:

Strategies like reading the questions, trying to understand what is the question asking, underlining the important ideas, and then planning. Problem solving. All the steps of problem solving were focused [during this lesson]. Any math that we do has that focus. (Sophie interview, May 26, 2015)

These inquiry steps are also linked to the procedural writing processes explored in today's language class. Modelling, practising, and having structured approaches to problem solving may
help students anticipate the upcoming questions and prepare for plausible responses. For students, this approach may also reinforce a sense of familiarity, as they could connect their current learning to previous experiences.

During today's lesson, in addition to sharing their strategies, students also practised communicating their struggles and misconceptions during the “consolidation and practice” component of the three-part lesson plan. The act of verbalizing and sharing the challenges experienced may reinforce to the class Sophie's attitude that mistakes and struggles are anticipated components to mathematics learning. Sophie believes that being able to identify and verbalize “what worked” and “what did not work” in their problem solving approaches may increase students' abilities to evaluate future mathematics strategies and responses.

For the two mathematics activities explored in today’s lesson, students identified their tendencies to misread or misunderstand the question prompts as one area of weakness. Instead of finding the areas of the schoolyards, some students calculated the areas of the schools. Sophie agreed that this was a problem experienced by several students, and asked how they could have avoided it. Together, they came up with the strategies of reading the sentences more than once, reading the sentences slowly, and underlining the key words. Sophie also suggested that the questions used for procedural writing (What is the goal? What are the materials needed? What are the steps for solving this problem?) may be helpful for organizing thoughts and minimizing reading errors.

During the classroom observation, I noticed that students were focused on identifying, and comfortable in sharing where they struggled, and on what they made errors. I discussed this with Sophie, and she agreed that the students were comfortable with making mathematical
mistakes and talking about struggles. She shares that this attitude towards learning was fostered throughout the academic year. Several strategies that she utilized for encouraging students to embrace and to discuss mathematical struggles include: 1) building a community which encourages honesty, respect, and courage, 2) modelling approaches to struggles and resolutions, and 3) providing positive feedback to students when they engage in honest sharing and thoughtful reflections. Sophie believes many of her students’ abilities to communicate struggles and errors have improved (both in motivation and frequency) as they continue to foster this practice.

4.6.5 Assessment

Sophie outlines her general approaches to assessment practices in mathematics. She states: “So basically, I start with the achievement chart and I look at all those [success criteria]” (Sophie interview, March 9, 2015). Afterwards, she focuses on the category of mathematics communication, and how that skill can be blended with the teaching and the assessing of knowledge, understanding, and thinking. She believes knowledge and understanding are essential, and those elements contribute to the ability to effectively communicate. She expresses these thoughts: “How can I blend in communication and thinking and knowledge? Because if you do not have knowledge and understanding, I do not think you can think about it. And once you have all those, you are able to communicate” (Sophie interview, March 9, 2015). Sophie notes the usefulness of the three-part lesson plan in assessing students’ mathematics communication abilities.

Specifically, she sees the “consolidation and practice” times as important for “bringing [the students] towards learning about mathematics communication” (Sophie interview, March 9,
2015). Sophie describes her explorations of mathematics communication assessment potentials within the three-part lesson plan:

> During the [“during” or “working on it”] part of the three-part math lesson, I do observations as the students are talking. Going around. And then the “consolidation” part, it can be gallery walk or something. I observe my students. Sometimes, as they are working, [I do] one on one conference also. If I have given them a problem solving [task], or maybe some individual work, I sit with them, I ask them questions. Try to get their mathematical thinking going. (Sophie interview, March 9, 2015)

Sophie identifies different times within the three-part lesson plan organization when she would glean information about her students’ learning:

> So yea, to check their basic skills, maybe sometimes I do a quick quiz and stuff like that. But mostly, the three-part framework. When I go around, when they do gallery walk, or when they are commenting on others' work, or when they are sharing their work. It gives me an idea that they know what they learned. Or need to do something more. So basically, the assessment, as [the lesson] is going. Daily observations, the routines, how they are doing it, and stuff like that. (Sophie interview, May 26, 2015).

In today's mathematics lesson, Sophie indeed followed her described strategies and had meaningful mathematics discussions with her students during the “working on it” segment. When her students were conjuring solutions for the first task, she listened attentively to their group discussions, and posed questions to individual students such that the inquiries would “get their mathematical thinking going” (Sophie interview, March 9, 2015). These were all moments when Sophie would assess various aspects of students' mathematics communication abilities. For the “consolidation and practice” portion of today's three-part lesson plan, Sophie did not utilize a cooperative learning technique (e.g. gallery walk). Instead, she invited students to simply share their processes: 1) visually through their images projected from the document camera, and 2) verbally through their spoken explanations.

Sophie also used this time to informally assess students' progresses in mathematics
communication. Based on her perceptions of students' learning successes through these frequent assessments, Sophie adjusted her immediate teaching approaches (e.g. time allotments for tasks, types of questions posed, vocabulary reviews). Sophie mentions that these frequent informal assessments also guided instructional decisions for long term lesson planning (e.g. mathematics task selections, cooperative learning approaches, summative assessment times).

4.6.6 Summary

When asked to reflect upon her teaching successes for today's mathematics lesson, Sophie believes the lesson went “pretty well” (Sophie interview, May 26, 2015). From her general perceptions and informal assessments of her students' discussions and sharing, Sophie feels that “[the students] kind of really understood the relationship between the size and the shape we used to cover the area” (Sophie interview, May 26, 2015). Sophie recounts how, as she feels her students demonstrated good grasps of today's learning goal, she proceeded to explore a similar but more challenging task in a subsequent mathematics lesson. This incident took place on a day when I was not present for classroom observations. Sophie recalls that lesson: “To follow up, I gave them a test. Kind of like an assessment task” (Sophie interview, May 26, 2015).

For the follow-up task, students had to calculate and compare the areas of the schools instead of the schoolyards. Sophie recounts most students were able to conjure accurate measurements, but a good number of students mistakenly produced the solutions for the schoolyards instead. Sophie describes her students’ error: “They forgot that [the question] asked them about the schools. They had that question, [the second task from Vignette Three], in their minds. So they did not read the word 'school' . . . They got confused with the school and the schoolyard” (Sophie interview, May 26, 2015).
Based on her informal assessment of the follow-up lesson, Sophie decides that most of her students “were able to do it, [to find the area using grid paper]” (Sophie interview, May 26, 2015). While she expresses that she is “glad that they know [how to calculate areas]”, she realizes that attentiveness to instruction is a skill that many of her students still require continuous reminders and practices (Sophie interview, May 26, 2015).

4.7 Vignette Four

_Tuesday, April 28, 2015_

I was in Sophie's class from 8:45 am to 11:35 am. The school day commenced with the playing of the national anthem and the announcing of school events, both through the public address system. Following that, Sophie's students completed the numeracy routine, a patterning exercise that was attempted as a class. This task will be described in detail in Vignette Six. The subjects for the morning included a literacy block for Periods 1 and 2, media class for Period 3, and mathematics class for Period 4.

_10:25 am- Media_

When students returned from morning recess, Sophie announced that Period 3 will be spent in the computer lab. She introduced the online task, and projected her laptop screen through the document camera. She explained to her students that once they entered the computer lab, they were to: 1) log on to their computers using their student identifications, 2) go to their school board's virtual library, 3) select the game “Math 3 Under the Sea!” in the mathematics resources section, and 4) focus on the topic of “Perimeter” within the game. “What are we learning in math class now?” Sophie asked. Students responded with “counting grids”, “covering shapes”, and “area”. Sophie acknowledged these mathematics topics. She suggested that, while
they play “Math 3 Under the Sea!”

With the instructions clearly articulated, Sophie led her class to the computer lab, an open-concept room with desktop computers lined along the three walls. Her students each selected a computer and followed the procedures Sophie provided. During log in, it became apparent that several computers were in need of repair or updates. To save time, six students formed groups of twos, and each pair shared one computer. Students proceeded to play the online game and select the topic of perimeter as their focus. With colourful, lively animation, and joyous, upbeat soundtrack, the students seemed captivated with this resource. Some features of the game include: 1) explanations of mathematical concepts, 2) definitions for vocabularies, 3) scaffolds in the forms of suggestions and hints, and 4) feedback for correct and incorrect responses.

Sophie walked around the classroom, initially assisting students with logging in to the computers. When all students were logged in and playing, Sophie conversed with them about their approaches to this game. As the period progressed, more students wanted to work in groups of twos. Some decided to abandon their own computers and join their peers. Others sat adjacent to one another. Though they kept individual computers, they conversed about the task options before making identical selections. “Can I work with you guys?” a student asked, as he walked up to two students who were engaged in a conversation about possible solutions. A myriad of strategies for solving these questions were displayed by the students. These included: 1) paraphrasing the written prompts, 2) questioning one another, 3) counting with their fingers, 4) tracing the shapes with their fingers placed on the monitors, and 5) utilizing strategies learned in
numeracy routines. Upon arriving at a correct response, a student enthusiastically said, “I got it right! I got it right!” With five minutes remaining in this instructional period, Sophie asked her students to log out of their computers and make their ways back to their homeroom.

11:07 am- Mathematics

As the class re-entered their homeroom, Sophie distributed to each of her student a worksheet with one mathematics question. The task displayed a shaded polygon, with the center portion of the shape unshaded (or “cut out”). She invited her students to spend a moment, to look at the question, and to think of ways for solving it. Sophie then provided approximately three minutes for students to familiarize themselves with task expectations. Afterwards, the instruction for the task was read aloud. At the end of each sentence, Sophie paused and invited students to put it in their own words. When they were done paraphrasing each sentence, they reread the instruction, this time noting key words for underlining. A student suggested “remaining” qualified as a key vocabulary. Sophie agreed, and asked the class to recall instances when they used the word.

Recounting those experiences, students defined the word to be similar to “the part that is still there”, and “the leftovers”. Sophie emphasized how the word “remaining” gives the impression that they needed to make comparisons. They had to determine how much they had before, and how much they have currently. Applying this concept to the task, she took out a piece of construction paper that had the same shape as the one depicted in the task. She then cut out the center portion of the shape. From this, Sophie illustrated that the current shape was what “remained” after the center portion was eliminated.

The class then did a “think-pair-share” cooperative learning exercise. As they were
previously given individual time to devise a plan, and large group time to paraphrase the instruction, Sophie told her students they only had two minutes to discuss with their elbow partner. Knowing her students were eager to find the numeric solution, she specified that the talks should be limited to strategies, and that a time for calculations will be allotted after the large group sharing. With a fixed time limitation for discussion, students chatted away instantly.

When the time was up, Sophie asked for the class’s attention. Following the questioning structure from procedural writing, Sophie inquired about the goal of the mathematics question. A student responded that they needed to find the remaining area of the shape. Next, she asked about the tools that they would require for this task. As the shape did not have grids, a student pointed out that a ruler for drawing the grids would be helpful. Lastly, Sophie wanted to know the steps and strategies they devised during “think-pair-share”. She invited each group to share a good strategy with the class.

A group considered counting the grids for the whole rectangle, then deducting the grids from the cut out portion. Sophie asked if there are other groups that used this strategy. Several groups said they used the same strategy, except they varied on minor approaches (e.g. count the center cut out first). As a class, they paraphrased this approach, and discussed why it would be workable. Some groups proposed estimating and approximating an answer before performing detailed counting. One group looked at the overall size of the rectangle, estimated an approximate amount, and used it as a benchmark. Another group noted the multiple choice format of the task, and decided to apply the process of elimination.

Over the course of eight minutes, every group had an opportunity to share their strategy. Sophie commented how the class collectively discovered different ways. She also identified
some of the similarities and differences between the suggested strategies. As a class, they solved the task by a combination of methods, which included: 1) estimating, 2) counting, 3) subtracting, and 4) verifying. Different students were invited to the front of the class to demonstrate and communicate their processes. When students encountered difficulties verbally expressing their steps, the class was patient and encouraging. When errors were made, other students would volunteer to go to the front of the class and assist in the corrections. Sophie constantly reminded students that the focus should not be on getting the correct answer. “We need to figure out where we made the mistake,” Sophie urged.

Towards the end of the lesson, Sophie once again invited students to share methods for minimizing mistakes in mathematics. Some of the strategies mentioned were: 1) reading instructions carefully and thoroughly, 2) understanding the question and the goal of the problem, 3) underlining important vocabularies, 4) making a plan or a list of steps, and 5) being patient and careful with the procedures. They discussed how, for this task, they were able to apply these strategies, in that they: 1) read the instruction multiple times, 2) paraphrased the ideas and discussed with peers, 3) came up with alternative words to describe the vocabularies, 4) discussed in pairs and in large group the strategies and steps, and 5) estimated, calculated, and checked their answer.

4.7.1 Program Planning

When describing today's mathematics lesson, Sophie states the learning goal for students was once again identifying and utilizing strategies to calculate the areas of shapes using the given information (dimensions and grids). Sophie notes how, while this lesson utilized the skills and concepts from previous mathematics classes (Vignette Two and Vignette Three), it also
presented additional challenges. Sophie voices these differences: “But how to find the area of the frame? Or the area of the part that is not in the shaded shape?” (Sophie interview, May 26, 2015).

Sophie also wants this lesson to highlight the practical uses of area measurements in everyday environments. She lists some potential connections between this “frame” shape and everyday objects. These included “the area of the wall excluding the window” and the area of the school field excluding the adventure playground (Sophie interview, May 26, 2015). Sophie envisions this lesson as an introduction for students to form “real life connection, and talk about all that” (Sophie interview, May 26, 2015). While this lesson did not include a lengthy discussion on practical uses of area calculations, Sophie foresees a good segue into that direction in subsequent lessons. She reflects: “I wanted them to feel that out. It went pretty good” (Sophie interview, May 26, 2015).

In connection to the Grade 3 curriculum expectations listed in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005), I observed the following criteria being introduced and reinforced during this class:

**Specific Expectations for Measurement**

- “Estimate, measure, and record length, height, and distance, using standard units (i.e., centimetre, metre, kilometre)” (p. 57)

- “Draw items using a ruler, given specific lengths in centimetres” (p. 57)

- “Estimate, measure, and record the perimeter of two-dimensional shapes, through investigation using standard units” (p. 57)

Similar to Vignette Three, for today’s lesson, Sophie points to her uses of the three-part lesson plan in fostering her students' mathematics communication. She identifies the “before” or “getting started” segment as the moments spent asking her students questions, and prompting
them to think of plausible problem solving strategies. She notes that “these [paraphrasing the statements, defining the words] were like the 'before' part of the three-part lesson, where [she] had questions about a rectangle, and a missing square from that rectangle” (Sophie interview, May 26, 2015). Sophie believes the time spent in the “before” segment prepared students to more readily communicate with their peers in the “during” or “working on it” segment of the lesson.

Sophie states that she is responsible for teaching her Grade 3 students every required subject, with the exceptions of drama, health and physical education, and library. One possible benefit of this arrangement is gaining more comprehensive perspectives of each student's learning. She is aware of the units and topics that her students are concurrently exploring, across multiple subject areas. She may also be more acquainted with the strengths, struggles, and learning styles of each student, across different subject areas.

During our informal discussions, Sophie notes that she considers some of these facets when planning the programs within and across subjects. Being aware of students' progresses in their mathematics learning, Sophie utilizes today's media class to further reinforce the mathematics content explored. In Period 3 media class, students expanded their exposures to mathematical ideas, as they simultaneous practised computer skills and mathematics content knowledge. By combining, overlapping, and teaching multiple subjects simultaneously, Sophie feels that students may receive more instructional times for each subject.

While Sophie mentions that, at the Grade 3 level, “it is a good chunk of time that is given to mathematics and literacy”, the cross-curricular and cross-strands approaches increase the allotted time (Sophie interview, May 26, 2015). She explains: “For me, yes, I do math for two periods. But not necessary it will be just two periods. You can go further, or you can bring in
social studies there and try to work it out” (Sophie interview, May 26, 2015). From her recounts of combining mathematics with social studies (e.g. using picture books), and integrating mathematics with science (e.g. using tasks that focused on areas of gardens and plants), subject integration may also provide students with opportunities to envision and understand mathematics within different subject domains.

4.7.2 Learning Environment

4.7.2.1 Computer and Media Room

Sandwiched between two homeroom classes is the computer room. Just down the hall from Sophie's classroom, this open-concept space has three walls covered with bulletin boards and chalkboards. Displayed on these boards were posters related to computer uses and media literacy. The room is equipped with approximately two-dozen desktop computers, which are lined along the three walls. Students would sit facing the wall when they use these devices.

While observing this lesson, I became aware of some of the possible benefits and disadvantages to this set up. One benefit to having the computers set up in such a way is that it may allow for better observations of all students' online activities from the center of the classroom. Knowing that their online activities can be easily observed may help prevent students from wandering off task. Also, due to this arrangement, students' struggles with technical issues (e.g. computers not working, log in problems), instructional issues (e.g. what they were supposed to be accomplishing), or task issues (e.g. the skills needed to complete the tasks) may be more easily noticed and promptly addressed. During today's lesson, from the center of the room, Sophie was quickly aware of students' various struggles, and proceeded to attend to those needs.

At the same time, while this set up may encourage students to work more diligently, it
may potentially promote independent work and diminish collaborative efforts. Sophie thoughtfully arranges her homeroom in table groups, such that her students may engage in more group work and discussions. A contrast to the set up in Sophie's class, students working in this space do not belong in table groups. Yet, from this observed lesson, students who wished to work in pairs or small groups initiated conversations with those who sat beside them. When Sophie provided the instructions for this lesson, she did not specify whether students were to complete the computer tasks independently, in pairs, or in small groups. Students were given the liberty to select their preferences. As such, some students worked independently, while others worked with peers. There were also those who oscillated between individual work and group work. The ways in which students collaborated were also based on preferences of individual groups. Some students would share one computer, debating with and listening to one another, before selecting one answer choice that the group agreed on. Other groups would work on separate computers, discussing with one another throughout the sequences.

From the classroom observations, I noticed students' willingness to assist one another. This quality was especially evident during this class, when students helped one another: 1) log in to the computers, 2) understand the lesson instructions, and 3) accomplish the tasks in the mathematics game. During an interview, I mentioned to Sophie my awareness of her students' general alacrity in helping one another. She agrees that her students are indeed very keen on assisting their peers. She identifies some ways of building such a community:

Some really good students are always eager to help, and they come out and help. Maybe the tone of the classroom that I have set up at the beginning of the year. That is why it is helping. We do not have name calling and bullying because someone is not able to do something. That is really the thing. (Sophie interview, April 23, 2015)

She acknowledges that building a fostering community requires intentionality and time. Sophie
describes the sense of inclusiveness was something that they “progressed into” (Sophie interview, April 23, 2015). She recounts this observation: “It was not like that at the beginning of [the academic year]. It took two months to get this thing started” (Sophie interview, April 23, 2015).

4.7.2.2 Cooperative Learning Strategies

Sophie shares her thoughts regarding collaborative work amongst students: “I believe that it should be an interactive class, where kids have their say. They kind of need to know why they are learning things” (Sophie interview, May 26, 2015). She feels that students should be given opportunities to communicate mathematics with different audiences. As such, opportunities to converse with peers are unique and beneficial. She elaborates on this approach: “Let the students figure out themselves. When they talk to each other, different kids have different ideas. Then they kind of get it in a better way. Hearing it from their peers make a big difference” (Sophie interview, May 26, 2015).

She finds think-pair-share to be an effective way for students to dialogue with one another. As observed during the lesson, this approach invited students to: 1) independently ponder plausible solutions, 2) discuss their thoughts with a peer, and 3) present their findings to the class. Sophie attributes several factors that contribute to her students' abilities to converse with peers, and their willingness to learn collaboratively. These possible features include: 1) students' training in previous grades, 2) teachers' professional development on program planning, and 3) activities that aimed at class building. Sophie describes these features:

I know that, coming from Grade 2, [the students] are used to [talking in pairs]. Because our Grade 2 teachers are amazing and they do [think-pair-share] with them. I think with the three-part math lesson delivery being consistent in most of the grades now. I think they are getting used to that more. And from September, we started the class building
activities. First week itself. And they always get the chance to work in groups, in pairs. (Sophie interview, May 26, 2015)

In our initial interview, Sophie lists and describes several forms of cooperative learning activities her students explored during this academic year. She says:

We will do bansho, gallery walk, math congress, and stuff like that. I am a big fan of the math congress and gallery walk because they give [the students] chances to look at other people's work. To share their own thinking, learn different ways of expressing mathematical thinking. “Oh, I can do it this way too”. (Sophie interview, March 9, 2015)

In today's mathematics lesson, Sophie asked her students to discuss problem solving methods through the “think-pair-share” cooperative learning technique. Sophie placed a time limit of two minutes for discussions with table partners, and her students were encouraged to use that time wisely. During the table groups sharing, students were gesturing, scribbling, and chatting continuously. To encourage accountability during table groups sharing, Sophie told students that everyone in the group should be able to share in front of the class afterwards. She reasons the necessity of such instruction for her students:

When they share [with the class], it is not just one partner presenting. I always tell them that everybody should know what they are doing. I give them probing questions. I ask them questions that kind of get me to know which kid, where they are. Whether they understood it or not. And then it [guides] me for further teaching too. (Sophie interview, May 26, 2015).

4.7.3 Mathematics Tasks

4.7.3.1 Math 3 Under the Sea!

According to their website’s acknowledgements, “Math 3 Under the Sea!” was copyrighted in 2006. It is a resource developed by the staff and contractors from Alberta Education, under the guidance of their Learning Technologies Branch. Involved in the various stages of product development were consultants and teachers from: 1) Calgary Separate School
District, 2) Calgary Board of Education and Learning, and 3) Skills Television of Alberta Limited. Through the school board's virtual library, staff and students at Maple School can access this online resource.

The game takes place in an oceanic environment, with sea creatures as trusty navigators. The adventures are organized by mathematics topics, covering all five content strands in the Ontario Mathematics Curriculum, Grades 1-8 (rev. 2005). The “Perimeter” lesson was selected as today's focus, with “Measuring and Recording Perimeter” and “Constructing Shapes” as sub-topics. The “Measuring and Recording Perimeter” section contains four activities, each one hidden within a magic crystal. Students can choose to commence and continue in any order they desire. These mathematics tasks include: A) using the diagram and the stated measurements for all sides, find the perimeter of the given quadrilateral, B) using the diagram and knowledge of the given quadrilateral’s properties, find the perimeter, C) using the diagram and the virtual ruler, measure and calculate the perimeter of an equilateral triangle, and D) given the measurements of two identical virtual strings, compare and determine the perimeters of shapes they create.

The “Constructing Shapes” section contains three activities that look at: E) creating on virtual grid paper a shape with specified dimensions, where perimeter measurements are given each time the horizontal and vertical borders are adjusted, F) creating on virtual grid paper a shape with specified dimensions, where perimeter measurements are given based on the number of grids highlighted, and G) changing the perimeter of a triangle to the desired measurement by adjusting one vertex.

From observing students' engagements with the perimeter tasks, I noticed how some of the game's features may promote students' mathematics communication. First, the tasks include
mathematics vocabularies, which are defined in ways that are easy for students in Grade 3 to understand. “I will give you feedback when you answer a question. Move your mouse over a pink word to see what it means,” a pink starfish says. For the “Perimeter” lesson, the words “perimeter” and “equilateral” are defined. Since these words were previously defined by Sophie during mathematics lessons, further exposures to these vocabularies may remind students of their uses.

Second, for students who prefer or require additional assistance, each mathematics task also comes with a video lesson that reviews the needed mathematical content knowledge. The video lessons are thorough, with visual representations and audio explanations that are suited for this grade level. The video lessons also serve as great models for how students could visually and verbally communicate their mathematical ideas. Several of Sophie's students made use of this feature, and clicked on the video lessons for reviews and hints.

4.7.3.2 Mathematics Class

For today's mathematics lesson, Sophie decided to look at the area of a cut-out shape because she feels it connects to and extends from what the class previously worked on. She explains her selection: “I just wanted them to have an idea [of] when you cut things out” (Sophie interview, May 26, 2015). She also expresses the difference between today's task and the previous ones:

It was on the grid paper. But the grids in the shape, the shaded area, were not given. So [the students] have to see, they have to make those grids. They have to draw the lines and make those grids, and then count them. (Sophie interview, May 26, 2015)

As this task is similar to previous ones, Sophie and her students often referenced some of the former approaches and attempts. Being a multiple choice task, the provided options also became
a part of the class discussions. Together, Sophie and her students explored the multiple choice options and practised estimations through the process of elimination, evaluations through comparisons, and justifications for option selected.

4.7.4 Constructing Knowledge

4.7.4.1 Math 3 Under the Sea!

Within this online game, a sea creature character serves as a guide for students. “I will help if you click on me,” says the clam. Clicking on the image of the clam prompts a brief video lesson that teaches and reviews needed mathematical content for the corresponding mathematics task. The video lessons are also automatically shown when the student accumulate three incorrect responses. The video lessons use easy to follow visual representations, grade appropriate vocabulary, and an organized model for procedural explanations. This could serve as another instance when students may acquaint with examples of clear, concise mathematics explanations. The video lessons also connect well with the procedural structure that Sophie was teaching and reinforcing in recent language and mathematics classes.

For the first two incorrect responses, the clam character provides students with suggestions, hints, and encouragements. These feedbacks include:

1. Vocabulary Definitions

“Incorrect. Remember the perimeter is the total distance around the shape. Try again.”

2) Shapes Properties Descriptions

“Incorrect. The opposite sides on the shape have the same length. Try again.”

“Incorrect. If one side [of a square] is 30 cm, then all the sides are the same length. Add the lengths of all the sides to find the perimeter.”
3) Procedural Suggestions

“Incorrect. Add all the lengths of the sides of the shape to find its perimeter. Try again.”

4) Differences Between Given Response and Correct Solution

“Incorrect. Your shape has a perimeter of 20 cm. make a shape that has a perimeter of 22 cm. Try again.”

The descriptive feedbacks attempt to draw students to plausible causes of errors (incorrect vocabulary understanding, incomplete shapes properties knowledge, inaccurate numerical calculations). They prompt students to examine their understanding, applications, and procedures.

This practice is aligned with the mathematical misconceptions discussions Sophie has with her students during the “consolidation” segments of the three-part mathematics lessons. In addition, the clam character also provides justifications when the correct responses are inputted (e.g. “Correct. The perimeter of the 2\textsuperscript{nd} shape is the same as the perimeter of the 1\textsuperscript{st} shape because the same length of string was used to make both shapes.”). This approach also reinforces the class’s practice of reflecting, evaluating, and justifying all mathematics solutions.

4.7.4.2 Mathematics Class

Similar to Vignette Three, Sophie concluded today’s mathematics lesson with a group discussion on mistakes and misconceptions. The students willingly listed common mishaps (not following instructions carefully, not understanding the goal of the task, and not having a workable plan for problem solving). Together, they formulated strategies that could combat these likely errors (reading the instructions multiple times, underlining important words, checking the solutions through multiple strategies). Though communicating what one understands is

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important, Sophie also places great emphasis on communicating what one struggles with. By devoting class time to regularly examine one's misunderstandings, Sophie hopes to encourage students to routinely share their correct and incorrect approaches. This may also reinforce Sophie's belief that mathematical mistakes are indeed welcomed potentials for learning.

4.7.5 Summary

Reflecting upon the mathematics lesson, Sophie feels that “it went pretty well” (Sophie interview, May 26, 2015). She concludes that most of her students: 1) demonstrated understanding of the objective of the mathematics task (calculate the shaded area of an irregular shape), 2) identified the challenge (missing grids for counting the area), and 3) suggested plausible strategies and solutions (ideas explored during “think-pair-share” discussion). In terms of the learning goal and the mathematics task, Sophie notices that her students were able to achieve them and that “they were able to do it” (Sophie interview, May 26, 2015).

From Sophie's summary of the lesson, it seems she considers it to be exploratory and practical in nature. It is exploratory, in that students are looking at previously learned ideas in new ways (different shape, methods, task format). It is also practical, in that the shape resembles real life objects that students are easily exposed to (windows in the school’s building, structures in the school’s playground). As such, Sophie hopes her students may see clearer connections between mathematics and their everyday environments. Sophie suggests that in subsequent lessons, they could explore the uses of legends, and the units of measurements for areas. She describes the uses of legends in these upcoming lessons: “If you have the legend, if you have the key that tells what one square means, they can measure the area and square units” (Sophie interview, May 26, 2015).
4.8 Vignette Five

Friday, May 1, 2015

I arrived at Maple School at 9:20 am and stayed until 1:55 pm. I was in Sophie’s classroom for the morning, and observed the Periods 1 and 2 literacy block, and the Periods 3 and 4 numeracy block. I spent lunch time and the afternoon in the staff room, as Sophie and I had previously arranged to do a semi-structured interview during her preparation period. However, as there were changes to her timetable arrangement that day, Sophie was unable to find time to meet up for the discussion. The interview session was re-scheduled to take place at the school on another day.

10:28 am - Mathematics

Prior to the start of the lesson, Sophie instructed her students to clear their desktops of distractive items. Next, each student received a Mathematics Booklet, which contained ten multiple choice tasks. These mathematics questions were organized such that they got progressively more challenging. The package was used as a review of the mathematics concepts acquired during recent lessons. Sophie commenced by asking her class to brainstorm recent mathematics topics encountered. “Perimeter”, “area”, “grid counting”, and “covering surfaces” were some of the vocabularies and activities students recalled. Sophie hinted that they may want to keep those words and skills in mind when they attempt the questions in this booklet.

For the first task, Sophie once again modelled the process of problem solving. While her students studied the diagram, she posed questions regarding the objectives of the task, and the choices available. The students read the text, underlined the key words, and identified the goal. They were then given time to individually solve the question. During that time, Sophie walked
around the room and assessed the strategies used by her students. She commented that many of them were completing the task correctly. As Sophie continued to observe her class, she noticed a few students seemed confused with the task. She approached them and guided them through the problem solving procedures. Together, they reread the statements, and reworded the goal.

When majority of the students selected a multiple choice option, Sophie held a class vote. For each of the four options, Sophie asked students to raise their hands if they were in agreement with the suggested answer. Next, she selected one student to explain why he or she supported the choice, and what steps were taken to arrive at that conclusion. Sophie then asked the class what they thought of the response, and if they had additional thoughts. Following that, she selected a student to explain why he or she did not choose that response option. While the students’ verbal explanations were at times difficult to follow due to pauses (e.g. searching for the right vocabulary) or incoherencies (e.g. lack of chronology), the class was patient and quietly waited during those moments.

For the second task, Sophie invited her students to “tell [her] what to do”. Students took the initiative of solving the problem, while Sophie observed their methods. Students were quick to apply the problem solving model. As a class, they identified the need to read the question, underline the key words, define the meanings of the words, and paraphrase the task. They were able to complete these steps collaboratively, and with only a little support from Sophie. As they tried to define the words “gap” and “overlap”, Sophie sensed that some students required concrete examples to understand the definitions more thoroughly. She suggested her students to utilize tables in their classroom and create scenarios that reflect the vocabularies. Students moved desks apart to demonstrate the idea of a “gap”, and stacked the edges of the desks to show
an “overlap”. These concepts were then reapplied to the mathematics task, which cautioned against overlapping and leaving a gap when arranging the manipulatives.

At that moment, Sophie suggested she lend them more assistance with the problem solving processes. She first demonstrated a wrong method of arranging the manipulatives. “You cannot do that!” responded a student. “Why not?” Sophie asked, as she pretended to be surprised. The student then reasoned that it produced a gap in the arrangement, and the instruction warned against that. Sophie demonstrated several other common mistakes. Each of these suggestions resulted in students strongly disproving the method. “You have to do some upside down [rotated]!” “You can only use one shape!” Sophie acted surprised each time, and asked them to elaborate on why they would disagree with what she was doing. She then invited a student to demonstrate how he would go about solving the question. While one student explained, several other students chimed in to provide further clarifications. The task was completed, a multiple choice option was selected, and the process was described. At this point, one student smiled and said to Sophie: “Are we teaching you?”

For the remainder of the period, students worked on the Mathematics Booklet in table groups. Rulers and manipulatives were placed at the front of the classroom, readily available for students should they choose to use them. Sophie went around the classroom, and asked the table groups regarding the strategies they used. She asked a student to express how drawing a diagram helped him. She complimented students on their manipulatives uses, their awareness of materials that would be useful for problem solving, and their choices of strategies. As she wanted her students to demonstrate more written communication, Sophie reminded them to provide words, numbers, and/or pictures in support of their selected multiple choice options.
4.8.1 Program Planning

When we discussed the purpose of today's mathematics lesson, Sophie identified it as a review session, where a series of recently learned materials were presented simultaneously. Sophie notes: “I think that [the tasks in the Mathematics Booklet] were all follow-up activities. Because I taught them area, area on the grid paper, perimeter, and all that” (Sophie interview, May 26, 2015). Sophie reflects on the successes, struggles, and progresses of previous mathematics lessons, and concludes that a review of strategies and mathematics content knowledge now would be suitable. She reasons: “Because I know that even after three-part math lesson [from Vignette Three], and all the strategies and all the stuff they reviewed [in Vignette Three and Vignette Four], they need to see how someone would solve the questions” (Sophie interview, May 26, 2015).

Of the specific expectations for Grade 3 listed in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005), I noticed that the focus for today's lesson include:

**Specific Expectations for Measurement**

- “Draw items using a ruler, given specific lengths in centimetres” (p. 57)

- “Estimate, measure, and record the perimeter of two-dimensional shapes, through investigation using standard units” (p. 57)

- “Estimate, measure, and record area” (p. 57)

**Specific Expectations for Geometry and Spatial Sense**

- “Identify congruent two-dimensional shapes by manipulating and matching concrete materials (e.g., by translating, reflecting, or rotating pattern blocks” (p. 59)

- “Identify flips, slides, and turns, through investigation using concrete materials and physical motion...” (p. 60)

Within this lesson, students' mathematics communication skills were practised during the
group work and class discussions portions of the class. Sophie comments on her approaches to the multiple choice tasks: “It was not individual work. It was whole class doing it one thing at a time” (Sophie interview, May 26, 2015).

### 4.8.2 Mathematics Tasks

Sophie identifies this lesson as a “follow-up” class, in which recent mathematics content and problem solving strategies were reviewed. The Mathematics Booklet used for this lesson contains ten multiple choice questions, and comprises of Primary mathematics EQAO tasks from previous years. Even though students could have completed the Mathematics Booklets individually, Sophie feels that discussing the tasks and the strategies as a class would entail a richer learning experience. She reflects upon this approach: “All the problem solving strategies [were] reviewed. And I was doing it. It was not individual work. It was the whole class doing it one thing at a time” (Sophie interview, May 26, 2015). Her rationale for attempting these tasks as a whole class is for students to be exposed to different ways of approaching and solving the tasks. Through this process, Sophie hopes that students may also be exposed to incorrect methods or solutions, which would prompt them to discuss, debate, and justify their reasoning. Sophie elaborates: “And not [done in] a perfect way. They need to see it can be done this way. And if they have a different way, they suggest that too. That was the goal” (Sophie interview, May 26, 2015).

### 4.8.3 Constructing Knowledge

For today’s lesson, Sophie reversed the roles of the “teacher” and the “learner”. During the second mathematics task, she purposely demonstrated some common mistakes. Her students quickly caught her errors, reasoned with her, and explained to her and the class why she was
incorrect. After several similar instances, her students conjured an acceptable solution. Through visual representations and verbal explanations, her students conveyed how the proposed solution met the requirements of the task prompt. With Sophie's guidance, they also expressed how their solution differ from the erroneous ones which Sophie suggested. The students seemed to find this process amusing, as one student jokingly asked Sophie if they were the ones teaching her instead.

At the follow-up interview, I conversed with Sophie about the use of this approach in developing students' mathematics communication. Here is an excerpt from that dialogue:

**Interviewer:** I like how you not just looked at the right way to do it, but you also demonstrated the wrong ways and have them explain why those are the wrong ways to cover the shape.

**Sophie:** Because they need to know it is okay to make mistakes. Again, we can always go back and check our work. And if there is a mistake, there is always a chance to correct it. So I want them to learn from the mistakes too.

**Interviewer:** So not just have them talk about why it is right, but also why it is not right.

**Sophie:** Sometimes I make mistakes on purpose. I make mistakes and I am like, “Perfect”. And I cross question them when they say that, “No, it is not right”. Or when I say, “No, it is wrong”. Or I say it is right and they argue wrong, and they have to prove that I am wrong. So that also helps them to learn to think more.

**Interviewer:** That is important.

**Sophie:** Critical thinking.

**Interviewer:** Knowing it is the wrong way to do something, and why. Have them explain that. (Sophie interview, May 26, 2015)

**4.8.4 Technology and Manipulatives**

During this mathematics lesson, Sophie utilized the document camera, manipulatives, and concrete materials to assist students' learning. Some possible benefits of the document camera
(e.g. visual representations, collaborative discussions) were observed in this scenario. For the second mathematics task, Sophie displayed erroneous solutions on the document camera. Students reacted to these projected images, as they eagerly explained and demonstrated to the class why her approaches were inadequate. With the use of a document camera, Sophie and her students may communicate their strategies simultaneously through words (verbal) and visual displays (numbers, diagrams, and/or manipulatives).

Aside from traditional manipulatives, when favourable, Sophie also utilizes concrete materials found around the classroom. Previously in Vignette Three, Sophie explained the textbook task of covering a bulletin board with a demonstration on their classroom's bulletin board. For this lesson, Sophie and her students used desks to re-create displays that corresponded with mathematics vocabularies (e.g. overlap, gap).

4.8.5 Assessment

Sophie discusses how she uses multiple choice mathematics tasks to assess students’ mathematics knowledge and communication. These tasks are explored during mathematics lessons, and included in her weekly Homework Booklets. She recounts how, for the past two months, she assigns four pages of literacy and numeracy exercises for students to complete at their leisure (e.g. during class times, lunch times, and/or at home). When the booklets are completed, Sophie assesses the written responses and determines suitable tasks for upcoming weeks.

Throughout the school days, Sophie searches for opportunities when she can individually dialogue with students regarding their Homework Booklet solutions. During the days when I observed Sophie's teaching, I witnessed and heard several of these exchanges (e.g. on days
described in Vignette One and Vignette Five). Sophie used these brief conversations at the round table to: 1) clarify students' written responses, 2) extend students' suggested ideas, and 3) attempt collaboratively tasks which students deemed too difficult. The combination of written and verbal responses may also provide a fuller sense of students' mathematical understanding.

### 4.8.6 Summary

When reflecting on the lesson, Sophie thinks she reviewed recent mathematics learning through integrated ways. She notes that the mathematics contents were examined through tactile means. She describes the approach: “So some hands-on and stuff like that. Because for area and covering the shapes, we have done them with geometric pattern blocks. So they can kind of know what it is” (Sophie interview, May 26, 2015). At the same time, there was new learning involved, even as some of the mathematics tasks built upon previously attempted ones. She notes that a “grid paper and squares [task]. It was something new” (Sophie interview, May 26, 2015). Sophie believes that the second mathematics task could prepare students for the learning of geometric reflection and transformation.

While she did not explicitly use those vocabularies because she “did not want to bring something new to [the students]”, she feels that this exposure will aid students to better understand and communicate their knowledge when the geometry lesson on tessellation is taught (Sophie interview, May 26, 2015). She says: “Now, when I teach them tessellation and all those stuff, like reflection and transformation in geometry, that will come back to them. It will make more sense to them” (Sophie interview, May 26, 2015).

Sophie feels an important part of today's review session is the group approach. Compared with assigning the entire Mathematics Booklet as independent work, Sophie believes that
reviewing and discussing the concepts and tasks collectively enhance the learning experience. She finds this to be especially true for the second mathematics task. She voices:

   Showing it, demonstrating it, modelling it. How to do it, how to find that shape, and how to fill that area using that shape. I think it is necessary because not necessarily [the students] have seen it, or everybody just gets it through exercise, [through] just one or two questions. (Sophie interview, May 26, 2015)

4.9 Vignette Six

   Numeracy Routines

   This final vignette captures the two numeracy routines which I observed in Sophie’s class. Both occurred after the morning school announcements, with the first routine being approximately 10 minutes, and the second routine being around 15 minutes.

   Numeracy Routine One- Friday, April 24, 2015

   After the morning announcement concluded, Sophie wrote down the numeracy exercise for the day: “688 + 73”. She invited students to demonstrate, in as many ways as possible, how the numeric operation exercise could be approached. The students, familiar with the procedures by this point in the academic year, either had their math journals displayed, or retrieved them from within their desk drawers. For the next five minutes, students worked quietly and diligently on the task. When they were satisfied with what they wrote, students exchanged their math journals and checked one another’s strategies and solutions. “You did this part wrong,” one student said to another. Students quietly explained their thinking to their partners, and pointed to different sections of their journals to highlight their reasoning.

   On several occasions, Sophie asked students about their progresses. She also reminded them that, once they are complete, they should exchange notebooks and check one another’s solutions. With majority of the students expressing they were done, Sophie asked her class to
share their strategies. One student suggested estimating the solution using “friendly numbers”. After exploring this strategy as a class, another student mentioned skip counting, starting from 688, and then counting by 10s, 5s and 1s until the correct sum was reached. Still, another student used addition, writing out the equation and solving it procedurally. Sophie complimented their uses of strategies. After recording all these methods on the chart paper, she asked the class to copy these strategies into their math journals.

Numeracy Routine Two- Tuesday, April 28, 2015

Following the morning announcements, students were once again presented with a numeracy question. Sophie introduced the task for today: “525 to 985”. As the prompt was in a different format than what they were more accustomed to, she suggested they attempt this together. Sophie first invited students to ponder scenarios where they may encounter the use of 525 to 985. After a short period of silence from her students, Sophie told a story which incorporated these numbers. The plot can be summarized as follow:

[Insert name of a student in Sophie's class] loves baseball and has a collection of 525 baseball cards. A good friend then gave her a giant box of baseball cards. Without counting how many cards were in that box, [the student] put her original collection of baseball cards with the new ones. Now, there are 985 together. How do we figure out how many cards her friend gave her?

The students listened intensely and nodded. Sophie suggested they use the technique of skip counting to solve this task. She asked her class to provide inputs for this method - where to start, where to stop, and what to do in between. Sophie wrote 525 on the left side of the chart paper, and 985 on the right side. She asked her students to consider the fastest way to count. They responded that 100s should be used. Sophie then wondered why 100s was chosen, rather than 10s. A student explained that counting by 10s would be time-consuming in this case, as the
distance between the numbers is big. When the distance is big, they count by bigger groups of numbers. Sophie nodded and proceeded to count by 100s. “525, 675, 725,” Sophie and the class counted, as she recorded the representation onto the chart paper. She drew their attention to the hundreds digits, where the 5 became 6, and the 6 became 7. She inquired about this pattern, and students raised their hands, eager to provide explanations.

“825, 925,” Sophie continued. At this point, students motioned for Sophie to pause the skip counting. Sophie asked if she was getting close to their desired number. A student proposed they should switch to counting by 10s. Sophie affirmed that was acceptable. Another student preferred they count by 25s. Sophie expressed she also liked that idea. As a class, they explored both counting patterns, first counting by 25s, then counting by 10s. “925, 950, 975,” the class chanted in unison. “925, 935, 945, 955, 965, 975, 985,” the class tried the other number grouping.

Afterwards, Sophie asked her class which counting groups they preferred more. The opinions and preferences were mixed, and Sophie concluded that they could count in groups of 100s, 25s, 10s, or 5s, as long as the number grouping selected is “friendly” for them. The class took a moment to record the skip counting strategy into their math journals. Sophie encouraged them to represent this strategy using counting groups of their choices.

Even though a solution to this numeracy task was conjured, Sophie wanted the class to explore alternative strategies. Sophie asked her students to ponder why having more than one way of solving a question is beneficial. Together, they stated that having more than one method is good, as that is useful for: 1) checking answers, 2) practising and reviewing different mathematical ideas (e.g. grouping, balancing, comparing), and 3) explaining to others. Sophie
explained that mathematical errors occur, sometimes due to anxiousness or inattentiveness. While taking the time to do mathematics exercises is important, having alternative methods to confirm the solution is also favourable.

A student suggested an alternative method for this task would involve simple arithmetic. Sophie referred once again to the baseball card narrative. Sophie described how, if they knew the total number of cards is 985, and they had 525 to begin with, they could put 525 cards in one pile and count the remaining ones. Subtraction was the chosen numerical operation. Sophie wrote “985 - 525” as the second method, and they calculated the difference. Students were then asked to write short summaries of this routine in their journals.

4.9.1 Program Planning

When asked about the incorporation of numeracy routines in her Grade 3 class, Sophie recounts: “Actually, I have done it for a few years now” (Sophie interview, May 26, 2015). This practice commenced after she reflected on her own approaches to numeric operations. Sophie realizes she enjoys the flexibility that comes with utilizing multiple strategies, and believes her students should likewise practise numeric calculations using various methods. She describes:

Because I am into mathematics so much. Multiplication and all that I do it in my head. Or different ways of multiplying, tens by tens, hundreds by hundreds. Adding and doing mathematics in all different ways. I am doing the skip counting and all. (Sophie interview, May 26, 2015)

Since then, she frequently starts the school day with a numerical task for her students to tackle. It was not until this academic year when Maple School was involved in a numeracy initiative that Sophie became aware that her approach is actually very popular. She exclaims:

I did not know that it is called something! I have been doing it a few years, and then this year the school has an initiative called numeracy routines . . . . And then when I looked at the book, I went to the meetings, I was like, “I am already doing this”. So it feels good
that you are doing something that you think you invented, but it is already there. (Sophie interview, May 26, 2015)

Based on the two numeracy routines that I observed, these exercises aligned with several of the Grade 3 specific expectations in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005).

**Specific Expectations for Number Sense and Numeration**

- “Identify and represent the value of a digit in a number according to its position in the number” (p. 55)

- “Compose and decompose three-digit numbers into hundreds, tens, and ones in a variety of ways, using concrete materials” (p. 55)

- “Count forward by 1's, 2's, 5's, 10's, and 100's to 1000 from various starting points, and by 25's to 1000 starting from multiples of 25, using a variety of tools and strategies” (p. 55)

- “Solve problems involving the addition and subtraction of two-digit numbers, using a variety of mental strategies” (p. 56)

- “Use estimation when solving problems involving addition and subtraction, to help judge the reasonableness of a solution” (p. 56)

**Specific Expectations for Patterning and Algebra**

- “Identify and describe, through investigation, number patterns involving addition, subtraction, and multiplication, represented on a number line...” (p. 61)

- “Extend repeating, growing, and shrinking number patterns” (p. 61)

- “Create a number pattern involving addition or subtraction, given a pattern represented on a number line or a pattern rule expressed in words” (p. 61)

- “Determine, through investigation, the inverse relationship between addition and subtraction” (p. 61)

Since the numeracy routine tasks simultaneously serve as reviews of mathematical ideas learned in previous grades, I perceived the following specific expectations from Grade 2 mathematics
Specific Expectations for Number Sense and Numeration

- “Describe relationships between quantities by using whole-number addition and subtraction” (p. 44)

- “Solve problems involving the addition and subtraction of two-digit numbers, with and without regrouping, using concrete materials (e.g., base ten materials, counters), student-generated algorithms, and standard algorithms” (p. 44)

Specific Expectation for Patterning and Algebra

- “Demonstrate, through investigation, an understanding that a pattern results from repeating an operation (e.g., addition, subtraction) or making a repeated change to an attribute (e.g., colour, orientation)” (p. 49)

4.9.2 Learning Environment

Sophie strives to create and maintain an atmosphere where students feel continuously motivated, socially engaged, and cognitively challenged. Sophie feels that some students in her class may require moments at the beginning of the school days to focus on their learning opportunities. When Sophie and I conversed about her morning numeracy routines, she notes some of the motivational and social benefits. We agreed that one possible motivational benefit includes the providence of a thoughtful task at the beginning of the school day for students to focus their minds on and invest their time in. Most of the mathematics problems selected could be solved by her students with relatively ease. Thus, the task could be completed within a ten minutes time frame, and her students may feel that they have successfully accomplished an activity.

Another possible motivational benefit of numeracy routines is students' mindfulness of a predictable class structure. During the two days when I observed the class's morning numeracy routine sessions, I noticed that Sophie's students were comfortably aware of the tasks that would
occur after the morning announcements. After Sophie’s brief introduction of the activity, some of her students would proceed knowingly with the task (e.g. take out their mathematics journals). Thus, this routine and its predictability may motivate students to take initiatives in preparing for the learning that will occur throughout the day.

4.9.3 Mathematics Tasks

For the numeracy routines, Sophie utilizes mathematics tasks that would review and reinforce basic numerical operations. Compared to the ideas explored in their mathematics lessons, Sophie presents mathematics questions that tend to be more “simplistic”. As such, these activities primarily serve to rebuild previously learned skills, including skills that were taught in previous grades and skills that were introduced earlier in the academic year. Sophie believes fluency in basic numerical operations should be acquired and practised. She also notes that many of her students struggle with this aspect of mathematics. She recalls some of the challenges her students experienced: “Many students get distracted with [numerical procedures like] carrying or borrowing. They make mistakes” (Sophie interview, May 26, 2015).

Sophie believes that, as students become more fluent with numerical operations, they may also gain more confidence in expressing their knowledge. In Numeracy Routine One, students were busily jotting down their strategies into their mathematics journals, awaiting their turns to share their strategies with the class. Sophie expresses the ease with which students shared their ideas was something they “grew into”, as they have been practising regularly since the beginning of the academic year.

4.9.4 Constructing Knowledge

Even though the tasks used in numeracy routines may be less mathematically demanding,
Sophie utilizes them to emphasize the uses of multiple strategies and multiple representations. For the task used in Numeracy Routine One, three different representations were shared and explained. The first strategy involved estimating, ball-parking, and using friendly numbers. A student explained that easier, more familiar numbers can be used to estimate. Together, Sophie and the student demonstrated how the numbers 688 and 73 could be changed to 680 and 70.

The second strategy involved skip counting. An open number line was drawn, and Sophie inquired whether they should start counting from 688 or 73. After hearing a few responses, Sophie explained that both 688 and 73 could be used as the starting points (a possible preparation for the Commutative Property of addition), but practically, using the larger number as the starting point would be more time efficient. With 688 as the starting point, Sophie inquired her students regarding how they would increase it by 73. The class counted upwards by 10s, and Sophie recorded the increases of 10s on the open number line: “698, 708, 718, 728, 738, 748, 758”. Sophie then noted how the amount was growing by 10 with each skip, and how the tens digit was increasing by 1 each time. After skipping by 10s seven times, the class switched to counting by 1s, and concluded that the sum was 761.

With regards to perceived benefits of skip counting as a strategy, Sophie comments: “Because in algorithm, I have seen that students, especially at this age, they make mistakes adding, subtracting. Those carry on, borrowing, and all that stuff are kind of confusing. But when it is skip counting, they do it better” (Sophie interview, March 9, 2015). The third and final strategy involved arranging the numbers vertically and using traditional algorithmic procedures to find the sum.

For the task used in Numeracy Routine Two, Sophie engaged students in a more in-depth
examination of the relationships between patterning and number sense. She first transformed the number sentence into a story that her students may relate with. This was a strategy for making an unfamiliar mathematical format (525 to 985) seems more personal. Based on that story, Sophie and her students then used the skip counting method to determine the difference. Students suggested using number groups of various quantities (100s, 25s, 10s, 5s, and 1s). They explored a few of those options, and evaluated the benefits and disadvantages of each. Sophie was mindful in drawing connections to patterning, highlighting how: 1) the hundreds digit increased by 1 each time 100 was added, 2) the tens digit increased by 1 each time 10 was added, and 3) the hundreds digit increased by 1 each time four 25s were added (while also connecting that to the Canadian currency of quarters and loonies).

In both numeracy routines, multiple representations were displayed, and different mathematical content strands were integrated. Sophie senses her students' attention may be preoccupied by “the solutions” if “it is traditional algorithms” (Sophie interview, May 26, 2015). Rather than approaching the selected tasks through purely algorithmic procedures, Sophie wanted to extend beyond that focus. “Patterning, I tell them. Skip counting and patterning are everywhere, for every little thing that you do,” Sophie notes, when we discussed about her focus for numeracy routines (Sophie interview, May 26, 2015).

While various mathematical representations are practised, the students in Sophie's class had personal favourites. Given the differences in learning strengths, styles, and preferences of mathematics learners, multiple representations of mathematical ideas, calculations, and understanding may better facilitate some of the class dialogues. A glimpse of this was seen during my observation of Numeracy Routine One, when two students were peer assessing one
another's strategies. When confusion regarding one strategy arose, his peer pointed to the alternative strategy below and used it to clarify the first strategy.

4.9.5 Technology and Manipulatives

As one of the two dimensions of mathematics education that Sophie identifies for her personal growth this academic year, she incorporates several technological devices (e.g. document camera, iPad minis, computers) regularly into her mathematics program. Likewise, Sophie expresses an interest in being more versatile in her manipulatives uses. She recounts how, throughout the academic year, she models to her students the proper uses of various concrete and virtual manipulatives. Sophie also encourages students to evaluate their own learning needs, and to select and incorporate suitable manipulatives to convey understanding.

During our lessons reflections, Sophie speaks of her perceived benefits of technology and manipulatives to her students' mathematics understanding and communication. At the same time, Sophie also believes that providing students with diverse ways to learn and express mathematics is essential. Sophie wants students to have encounters where they can practise basic mathematics skills. Normally, numeracy routines in Sophie's class do not include the uses of technological devices or manipulatives. Sophie states: “For numeracy routines. For skip counting, that is fine. You do not need manipulatives for every little thing here” (Sophie interview, May 26, 2015).

Instead, Sophie prefers class discussions displayed on chart papers, and students' ideas and processes recorded in mathematics journals. Sophie suggests that the pencil-and-paper approach for numeracy routines may build up students' confidence and independence. Through this approach, Sophie wants her students to see mathematical connections in their everyday encounters. With a rationale and an example, she explains:
Because my goal is to prepare them for the outside world. If they are at a store, they are not going to have those fake moneys to count, how much change I am going to get back. They need to do it in their minds. That is where my numeracy routines and all those different techniques, different strategies, come in. I want them to be independent and use more of the mental math and more of something that they can just do without manipulatives. (Sophie interview, May 26, 2015)

4.9.6 Summary

Sophie feels that the numeracy routines are beneficial to her students' mathematics competencies and communication in several ways. First, she notices that, for some students, the strategies practised during these sessions are later demonstrated. She describes her observations: “Some of [the students] are really using [the patterning from numeracy routines] for every little thing that they do” (Sophie interview, May 26, 2015). To Sophie, seeing the skills developed in numeracy routines transferred to other aspects of mathematics learning is encouraging. Second, she feels that the routines help foster careful approaches to mathematical procedures.

Through consistent practices, discussions, and demonstrations, students may become more aware of the mathematical errors they produce. Together, Sophie and her students would identify ways to reduce mathematical carelessness (e.g. finding multiple strategies to solve a question, developing more numerical operation fluency). Sophie concludes: “My numeracy routines are really helping them” (Sophie Interview, May 26, 2015).

4.10 Sophie's Reflections

Sophie believes her passion for mathematics contributes greatly to her enjoyment of mathematics teaching. She feels her own mathematical foundations are well-formed, and identifies her “mathematical strength and knowledge” as components which help her plan better lessons (Sophie interview, March 9, 2015). Sophie believes her comfort with the subject also contributes to her willingness to experiment with different teaching ideas and techniques.
Adopting a growth mindset, she is eager to implement teaching strategies that she was introduced to during professional learning sessions. Sophie articulates in our initial interview: “I love teaching mathematics. With three-part math lessons, and all different numeracy routines” (Sophie interview, March 9, 2015).

Sophie acknowledges that moments spent planning the mathematics programs and mathematics lessons are important. At the same time, she also emphasizes the need for teachers and students to be flexible in their approaches to learning. She elaborates:

Sometimes as I am teaching, ideas come. Change. We do it. And I am glad my students are flexible too. That they go with whatever I plan. So, yes, I do plan some tasks beforehand. Some just come to me and we just do them, thinking that they are relevant, that this would make more sense. (Sophie interview, May 26, 2015)

With regards to planning mathematics teaching with a focus on developing students' communication, Sophie believes that “everything [e.g. group task, individual work, whole class approach] makes sense. And I think it is all interwoven. Everything is geared towards the focus” (Sophie interview, May 26, 2015).

Sophie points to the current learning environments for encouraging teachers in their explorations of new teaching methods. She lists some of the current initiatives occurring in her school regarding the teaching and learning of mathematics. These include: 1) three-part lesson plans, 2) practical perceptions of mathematics (S.T.E.M.-focused curriculum), and 3) technological devices (iPad minis). She observes that “most of the teachers [at Maple School], at least for the Primary grades” are experimenting with these teaching ideas and tools (Sophie interview, May 26, 2015). She also notes that some of these ideas are “really getting contagious now” (Sophie interview, May 26, 2015). Reflecting upon these new initiatives and their contributions to students' mathematical learning and communication, Sophie responds:
Takes time. But yea, I think [the school initiatives are] helping out [with applications of strategies in the classrooms]. And I am not sure if I will be the Leading Instructor next year or not. But then, even otherwise, whatever I have learned, I am going to continue. (Sophie interview, May 26, 2015)

When I asked Sophie to express her concluding thoughts on the fostering of mathematics communication at the Grade 3 level, she first notes that the development of mathematics communication skills is something that should be emphasized in all grades. When she envisions herself teaching mathematics in different grades, she proposes that many of the same strategies would still be applicable. She believes it ultimately comes down to several key aspects of teaching. She expresses:

I think there is nothing that Grade 1 would not be able to do it, [that] Grade 2 would not be able to do it. It is the way you plan it, the way you demonstrate it, the way you model it, the way you get it done. (Sophie interview, May 26, 2015)

She suggests that possessing positive mindsets and being resilient are essential. She concludes with this message:

I am a positive thinker. I see that [new teaching strategy] is there and we have to do it. Let us try it out. If we fail the first time, we eventually will get it. So I always believe that, yes, we can do it and do not give up easily. And I think all my students have moved up compared to the way they came to me. (Sophie interview, May 26, 2015)
Chapter Five: Discussion and Interpretation of Findings

5.1 Introduction

Based on the findings from the case study of Sophie, I attempt to answer the posed inquiry question by consolidating the selected categories of analysis for the lesson vignettes. This process of analyzing and reporting is also in alignment with the characteristics of a “genuine case study”, which goes beyond the analysis of the embedded sub-units, and into the analysis of the “whole” case (Yin, 1994). Following the combined interpretations of sub-units and whole case, insights gleaned from this study are also examined alongside relevant scholarly literature. This chapter concludes with a summary of the major research findings, and includes several suggestions for further research on this topic.

5.2 The Research Question

This study was conducted with the primary focus stemming from the research question presented in Chapter One. Specifically, the question is: “What teaching perceptions, practices, and strategies are utilized in the fostering of Grade 3 students' mathematics communication skills, in the content strand of measurement?” The identified categories related to this research question are individually examined, based on the findings from the case study of Sophie.

5.3 Cross-Lesson Vignettes Analysis

The cross-lesson vignettes summaries are organized and described, following the categories presented in Chapter Two. The six categories (program planning, learning environment, mathematics tasks, constructing knowledge, technology and manipulatives, assessment) are aligned with the dimensions found in the Ten Dimensions of Mathematics Education (McDougall, 2004).
5.3.1 Program Planning

5.3.1.1 Curriculum Perspectives

Sophie refers to the provincial mathematics curriculum documents to identify the key ideas and essential skills to include in her Grade 3 mathematics program. She considers the various learning goals, mathematics contents, and mathematics processes that students in Grade 3 are expected to attain by the end of the academic year. Sophie notes how, throughout the elementary grades, the development and assessment of mathematics communication skills are consistently emphasized in the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005).

Through her own teaching and learning experiences, she also witnessed the importance of mathematics communication to the learning of mathematics. Naturally, when she thoughtfully and purposefully aligns her mathematics lessons with the provincial vision of mathematics teaching and learning, the significance of mathematics communication development is also continuously highlighted. As Rahal and Melvin (1998) expressed regarding the development of students’ mathematics communication skills, “teachers need to ‘buy into’ the belief that the nature of discourse occurring in their classrooms will influence their students’ knowledge and understanding of mathematics” (p. 102).

The importance of teachers’ perceptions regarding learning theories and instructional practices have also been prevalently noted in previous studies (e.g. Applefield, Huber, & Moallem, 2000/2001; Bruce & Ross, 2008; Charalambous, 2015; Golafshani, 2013; Rahal & Melvin, 1998). Through a multiple-case study of three prospective teachers, Charalambous (2015) likewise concluded that the participants’ beliefs regarding the teaching and learning of mathematics informed instructional decisions. These decisions included the selections,
adaptations, and presentations of curriculum tasks and ideas.

With regards to teachers’ uses of curriculum materials, Sullivan et al. (2013) analyzed responses from surveys completed by Australian primary and secondary teacher participants. These data highlighted the themes of: 1) curriculum focus, 2) teachers’ interpretations and understandings of the curriculum, and 3) teachers’ choices and preferences in program planning. The findings suggest that the ideas presented in the mathematics curriculum are processed through the teachers’ perspectives, and implemented through the teachers’ professional judgements. As such, Sullivan et al. (2013) recommended more discussions within the mathematics education community, such that teachers can be supported in their interpretations and implementations of curriculum documents.

5.3.1.2 Planning Within Mathematics Lessons

Sophie expresses that students require adequate time to thoroughly and meaningfully learn mathematics content and mathematics processes. Given sufficient moments to unhurriedly explore mathematical activities and ponder mathematical ideas, students may feel more confident sharing and expressing their emerging thoughts. At Maple School, the administrative team strives to arrange and incorporate “numeracy blocks”, two consecutive instructional periods of mathematics, into the instructional timetable. During lesson observations, three of those days (Vignette One, Vignette Three, and Vignette Five) included two instructional periods dedicated to mathematics. Of those three days, two of them (Vignette Three and Vignette Five) had mathematics lessons scheduled consecutively.

Jackson et al. (2013) conducted classroom observations of mathematics teachers of middle-grades, and noted that “clearly, time is of the essence in classroom instruction” (p. 656).
Thus, they suggested that teachers should carefully assess which aspects of the mathematics lesson to invest time in. Similarly, while exploring productive mathematics struggles among middle school students, Warshauer (2015) observed that “some teachers chose to afford students time; to question, probe, clarify, interpret, or confirm students’ thinking; and to provide opportunities for discussion among classmates” (p. 393).

Recognizing the need for adequate time in the fostering of mathematical concepts and processes, the Ontario Ministry of Education’s *Guide to Effective Instruction in Mathematics* (Volume Three: Classroom Resources and Management, 2006) advised teachers and administrators to ensure “sufficient blocks of time everyday for mathematics” (p. 40). Likewise, in various studies, teachers’ organization of lesson times and wait times have also been identified as factors which contribute to the quantity and quality of students’ mathematics communication opportunities (Chapin, O’Connor, & Anderson, 2009; Haneda, 2004; Stein et al., 2000).

With regards to mathematics lesson organization, Sophie points to her implementation of the three-part lesson plan as a strategy that may foster her students' mathematics communication skills. In Vignette Three, Sophie's students engaged in discussions with their table group members during the “working on it” component of the three-part mathematics lesson. Afterwards, her students shared their table group's strategies, solutions, and misconceptions with the whole class during the “consolidation” segment. Since this lesson occurred within a 70-minutes numeracy block, Sophie's students had time to explore two related mathematics tasks, and engage in extended episodes of conversations with peers.

Sophie also implemented the three-part lesson plan during the mathematics lesson in Vignette Four. However, as the lesson was a 30-minutes instructional period, she took this into
consideration and selected one mathematics task for in-depth exploration with her students. Thus, Sophie mindfully allots the time spent on each segment of the three-part lesson template, in hopes of prioritizing quality conversations with and amongst her students. The mathematics discussion opportunities gained through effective uses of three-part lessons, while noticed in Sophie’s lessons, are also observed in other studies (e.g. Jackson et al., 2013; Marks Krpan, 2013; Warshauer, 2015).

5.3.1.3 Planning Beyond Mathematics Lessons

In addition to the numeracy blocks and the mathematics instructional periods, Sophie also incorporates numeracy routines and subject integrations to increase mathematics conversations and representations. By scheduling the numeracy routines (Vignette Six) at the beginning of the school days, rather than having them structured at the beginning of mathematics lessons, Sophie may reinforce her belief that mathematics learning should not be confined within a specific time frame. Similarly, the technique of including a daily mathematics exercise to promote student thinking and student discussion is recorded in a study conducted by White (2003). When observing two Grade 3 teachers’ strategies in fostering mathematics discourse among diverse students, White (2003) found that each day, “an early-bird mathematics problem” was made available to students as they entered the classroom (p. 41).

For subject integrations, the media class in Vignette Four provided Sophie's students with opportunities to review recently learned mathematical ideas, and to practise computer skills. In Vignette Three, the inquiry model introduced in language class was further explored in mathematics class. The concepts of area investigated in mathematics class were revisited as writing prompts in language class. In a study of secondary school mathematics and science
teachers that was conducted by Riordain, Johnston, and Walshe (2016), the integration of subjects was likewise perceived by teacher participants to: 1) benefit their students’ learning experiences, 2) assist their students in seeing connections and relevance among subjects, and 3) increase student engagement for both curricular subjects. Studies that explored the promotion of students’ measurement ideas also recommended: 1) the integration of science and mathematics (Selmer et al., 2016), and 2) the integration of language and mathematics (Lee & Francis, 2016; Moyer, 2001).

5.3.2 Learning Environment

5.3.2.1 Physical Environment

Sophie uses the walls within her homeroom class to display a multitude of materials related to her students' learning journey. Descriptions of an ideal classroom, suggestions for conflict resolutions, and a list of cooperative group engagement rules visually remind students of some of the agreed-upon guidelines for classroom interactions. Individual work and group assignments from various subjects (language, mathematics, science, social studies, and visual arts) highlight and celebrate the progresses and accomplishments of each student. General learning strategies (problem solving suggestions) and subject-specific prompts (vocabulary chart, procedural writing learning goals) remind students of previously learned materials, and scaffold them in their current learning endeavours.

In addition, student-drawn character education themed posters decorate the wall in the adjacent hallway, and reinforce the values of the class, the school, and the school board. This description is similar to the findings of Rahal and Melvin (1998) and Warshauer (2015), who recounted the incorporations of student creations in adorning the classrooms and the hallways.
When examining instructional strategies that promote mathematics discussions within the classroom, Rahal and Melvin (1998) concluded that the learning environment is one of several factors that may contribute to the types of discourse that occur.

Based on mathematics lessons observations in grades six and seven classrooms, Warshauer (2015) also recommended the utilizations of community building strategies. These guidelines assisted students in respectful discussion approaches, questioning techniques, and justification methods, and thus made the acknowledgment and communication of mathematics struggles easier for students.

In her homeroom, Sophie arranges her students' tables into groups of three or four. She believes that this layout, when paired with suitable activities, may provide her students with more opportunities to work collaboratively. Additionally, as students sit in their assigned table groups, they may become better acquainted with their peers and develop a greater sense of community. The importance of classroom organization for student learning has been noted in the Ontario Ministry of Education’s Guide to Effective Instruction in Mathematics (Volume Three: Classroom Resources and Management, 2006), which recommends set-ups that thoughtfully consider various interaction possibilities and purposes. The document suggests a whole class “meeting area”, along with set-ups of smaller groups. Through these physical classroom arrangements, students may: 1) gather and experience a sense of belonging, 2) build up their mathematics confidence, 3) communicate their mathematical thoughts, and 4) “experience mathematics as a social activity and strengthen relationship with their peers” (p. 12).

In contrast, the tables in the school computer lab are not arranged in groups. Instead, the tables and the computers are lined along the three walls, with students facing the walls when they
work on the computers. Yet, during the media class described in Vignette Four, students who wished to work collaboratively nonetheless initiated group work. Sophie’s views about mathematics teaching, along with the value she places in fostering students who take initiatives in their own learning, may have contributed to her students feeling the autonomy to make learning choices that align with their strengths and needs.

This perspective is supported by studies that expressed the importance of teachers’ perspectives and practices in molding students’ learning engagements and approaches (Michelsen & Sriraman, 2009; Stein, Remillard, & Smith, 2008; Taylor, 1991). In addition, the considerations of and flexibilities towards students’ preferred learning preferences are in alignment with student-centered learning theory (Cleveland-Innes & Emes, 2005; Dewey, 1938).

5.3.2.2 Social Environment

Sophie aims to establish a safe and welcoming classroom community with her students. She hopes to achieve this by reinforcing the importance of: 1) positive character traits developments, 2) improved self-regulation practices, and 3) appropriate conflict resolution approaches. Throughout this academic year, Sophie searches for suitable moments to discuss and model these approaches and practices with her students. During the observation days, some moments when she reinforced and re-established a positive learning atmosphere included: 1) encouraging self regulation practices when students encountered distractions or different perspectives (Vignette One), and 2) highlighting cooperation as the monthly character education theme prior to students' engagement in group work (Vignette Two). Mark Krpan (2013) and Warshauer (2015) reported similar findings, in that welcoming classroom environments may contribute to students’ participation and engagement in group discussions.
Sophie believes the learning of mathematics should involve struggles with new ideas and experimentations with unfamiliar strategies. She sees learning as a process, with mathematical misunderstandings as welcomed opportunities for increases in existing knowledge. She encourages her students to persevere in their mathematical struggles, and invites them to share their misconceptions with one another. Some strategies which Sophie uses to encourage her students in their perseverance with difficult mathematical encounters include: 1) turning challenging mathematics tasks into games (Vignette One), 2) acknowledging struggles as expected components in the learning process (Vignette One, Vignette Three), and 3) providing suitable assistances to students through scaffolding, paraphrasing, questions posing, and strategies brainstorming (Vignette One, Vignette Two, Vignette Three, Vignette Four, Vignette Five, and Vignette Six).

To convey the importance of learning from mathematical struggles and misconceptions, Sophie invites her students to: 1) identify and share with the class their mathematical mistakes for an attempted task (Vignette Three), 2) suggest and practise problem solving strategies that may minimize procedural errors (Vignette Four), and 3) discern and explain mathematical mistakes that were deliberately made by Sophie (Vignette Five).

During the observed three-part lessons (Vignette Three and Vignette Four), the communication of mistakes was incorporated into the “consolidation” segments. Sophie inquired her students' mathematical errors, and scaffolded them to the correct strategies and responses. This approach is aligned with the *Ontario Mathematics Curriculum, Grades 1-8* (rev. 2005), which recommends reflections on one’s mathematical progresses and struggles. Specifically, one of the seven mathematical processes highlighted in the curriculum is the ability to reflect
(Ontario Mathematics Curriculum, Grades 1-8, rev. 2005). Warshauer (2015), who investigated productive mathematics struggles among grades six and seven students, also reinforced the need for students to identify and communicate their misconceptions. Through verbally expressing their ideas in small group conversations and large group discussions, students were able to bring to light their misconceptions, and resolve confusions or carelessness (Warshauer, 2015).

Sophie arranges her students' desks into table groups in hopes that this organization may assist in promoting more discussions and collaborations amongst her students. During the observed mathematics lessons, there were several occasions when Sophie invited her students to collaborate with one or more of their table group members. In Vignette Two, Sophie asked her students to share, with their table group neighbours, some insights regarding two mathematical perspectives presented in the Math Makes Sense (2004) textbook. After the brief discussion, Sophie invited her students to communicate their positions with the class.

Similarly, in Vignette Four, Sophie used the “think-pair-share” cooperative learning technique to guide her students' mathematics problem explorations. First, students partnered with their table group neighbours and discussed plausible strategies for solving the posed mathematics task. Afterwards, each student pair presented their proposed problem solving approaches to the class. Having had moments to individually survey the mathematics task, followed by time to discuss their thoughts with peers, the students seemed more articulate and confident when they subsequently shared with the whole class.

In addition, Sophie embeds various collaboration and discussion opportunities when completing the daily numeracy routines (Vignette Six). For Numeracy Routine One, students engaged in mathematics dialogues with their peers when they exchanged mathematics journals
and assessed one another's strategies and solutions. Later, more sharing and conversations ensued when Sophie invited students to present their methods. For Numeracy Routine Two, the class engaged in meaningful large group mathematics discussions, as they explored ways to represent the numeric statement, state the possible strategies, and verify the solution. Sophie’s thoughtful organizations and balances of individual work and collaborative work are in agreement with Nuhrenborger and Steinbring’s (2009) recommendations, which support conversation opportunities with diverse groupings, in different social settings, and among different audiences.

5.3.3 Mathematics Tasks

5.3.3.1 Types of Mathematics Tasks

Sophie combines a variety of mathematics tasks within her numeracy program. During the observed lessons, some of the mathematics activities were selected and adapted from: 1) mathematics textbook (Vignette Two), 2) online resources accessed through the school board's virtual library (Vignette Four), and 3) provincial primary mathematics assessment tasks from previous years (Vignette Four, Vignette Five). Along with the selection of suitable mathematics tasks, the facilitation of these activities may also promote students' mathematics exploration and communication. Below, some of Sophie's mathematics tasks implementation strategies for multiple choice questions and mathematics journal writing prompts are further described.

Sophie uses mathematics tasks with multiple choice options to engage students in strategies sharing and solutions comparing. In Vignette Four, Sophie selected a multiple choice task for her students to engage in “think-pair-share”. Sophie drew her students' attention to all the multiple choice options, and invited her students to compare, reason, and justify their responses. She also looked at the distractor options, and encouraged her students to ponder why these
choices were incorrect or incomplete. The multiple choice format also promoted the uses of problem solving skills. These strategies included estimating, ball-parking, and eliminating.

In Vignette Five, Sophie gave each of her students a Mathematics Booklet that contained ten multiple choice questions. The Mathematics Booklet is organized such that the subsequent tasks are progressively more challenging. For the first mathematics task, Sophie aimed to increase her students' mathematics communication opportunities by: 1) asking prompting questions that followed the procedural practices familiar to the class (What is the question asking? What are the choices available?), 2) giving students time to independently form their responses, 3) holding a class vote based on the four multiple choice options, 4) discussing students' reasons for their selections, and 5) guiding the discussions when students had conflicting ideas (when students voted on different multiple choice options and were justifying their selections).

For the second multiple choice task, Sophie provided opportunities for students to share their mathematical thinking by: 1) inviting students to lead the problem solving procedures (ask and respond to inquiry questions, note and define vocabularies), 2) intentionally demonstrating incorrect strategies for task completion (showing common errors such as overlapping, spacing, and non-congruence), 3) requesting students to explain why those methods are erroneous, and 4) encouraging students to display their strategies through the document camera.

Through a review of scholarly literature, Santos and Semana (2015) identified justification as an essential element to students’ development of mathematics communication skills. They explained:

Justification is an important component of students’ mathematics communication. The types of justifications presented by the students can give information about their
understanding. However, students seem especially concerned with producing correct solutions (Sanchez & Sacristan, 2003) rather than with justifying their solutions. (Santos & Semana, 2015, p. 67).

Similarly, the Ontario Ministry of Education’s Guide to Effective Instruction in Mathematics (Volume Two: Problem Solving and Communication, 2006) suggests teachers to encourage students’ explanations and justification of mathematical outcomes, as “reasoning must be the major focus of students’ communication” (p. 57). Through the uses of multiple choice tasks, along with small group and large group discussions, Sophie provided her students with solutions which they could select, justify, and/or refute. Thus, through this process, students may practise the necessary skill of mathematical justification.

In addition, mathematics journals are regularly used by Sophie's students to practise communicating and sharing written ideas. For the numeracy routines described in Vignette Six, strategies (expressed in words, calculations, and diagrams) for solving the tasks were inscribed by students into these notebooks. Sometimes, Sophie assigns mathematics-related writing prompts for students to thoughtfully consider their positions and responses. For example, Sophie recalls how she recently invited her students to ponder and explain, through a short written response, “why a ruler is a very important tool”. At other times, she asked her students to consider which is their favourite manipulatives, and why.

While these prompts may not require much mathematical calculations or mathematical content knowledge, they nonetheless integrate literacy with numeracy, and promote the uses of words, sentences, and paragraphs. These benefits of mathematics journaling for the fostering of students’ written communication skills have also been identified by Lee and Herner-Patnode (2007), Marks Krpan (2013), Santos and Semana (2015), and Thompson and Chappell (2007).
5.3.3.2 Characteristics of Mathematics Tasks

While Sophie selects and implements mathematics tasks of varying formats (e.g. multiple choice questions, open-ended writing prompts), many of the chosen activities possess similar qualities. The mathematics tasks implemented during the observed lesson vignettes demonstrate an awareness of students' existing knowledge and previous experiences. For example, in completing the first mathematics task during Vignette Three, students may draw upon their knowledge and experiences in: 1) using manipulatives to cover an area (Vignette One), 2) using squares for the calculation of areas (Vignette Two), and 3) using a procedural approach when solving a problem (Vignette Three). During the two numeracy routine tasks described in Vignette Six, students may review and strengthen mathematical concepts that were introduced to them in previous grades. Cross et al. (2012) and Kisker et al. (2012) also found it important and necessary to consider students’ interests, previous knowledge, and learning preferences when planning, selecting, and implementing mathematics tasks.

Sophie utilizes mathematics tasks that can be explored and expressed through multiple strategies and various representations. For example, the mathematics tasks chosen for Vignette One provide flexibilities and choices, as students conveyed their understanding in multiple forms: 1) concrete representations (pattern blocks), 2) verbal representations (presentations, discussions with Sophie and peers), 3) visual representations (drawings, charts), and 4) written representations (words, sentences). To promote growths in all forms of representations, Sophie modelled the first mathematics task such that students had a detailed exemplar for how each form could be expressed.

Other mathematics tasks that were expressed through multiple representations (verbal,
visual, and written) include: 1) the “Practice” and “123” activities from *Math Makes Sense* (2004) described in Vignette Two, and 2) the numeracy routine exercises described in Vignette Six. Likewise, mathematics tasks from the observed lessons which were explored and solved through various strategies include: 1) the multiple choice questions in Vignette Four and Vignette Five (calculating, estimating, verifying), and 2) the numeracy routine exercises in Vignette Six (balancing, grouping, skip counting).

Having different forms of representations and expressions allow students to accomplish the tasks through a multitude of learning styles (audio, tactile, and/or visual). Hence, students may demonstrate their understanding through learning styles that are their strengths, as well as learning styles that are their areas of refinement. Similarly, White (2003) found that mathematics tasks that can be expressed through multiple means are useful in fostering mathematics discussions among diverse students in Grade 3, as the tasks could be explored through various means (e.g. independently and/or in small groups, hands-on and/or pencil and paper).

Some of the mathematics tasks Sophie introduced have more than one correct solution. For example, different quantities and various types of pattern blocks can be used to cover the outlined shapes (Vignette One), while numerous geometric figures can be drawn to fulfill the criteria of an area of 24 square units (Vignette Three). When exploring mathematics tasks with multiple solutions, Sophie encourages students to search for alternative answers when they arrived at one workable method. Having multiple solutions may also make group discussions more captivating, as students seemed more intrigued by strategies they have yet to discover. An interested class can further motivate student presenters to persevere in clearly articulating their mathematical ideas.
The benefits of open-ended, multiple solutions tasks for students’ development of mathematics communication skills have also been identified in the study by Viseu and Oliveira (2012). Furthermore, Hino (2015), who compared multiple solutions mathematics tasks with structured problem solving questions, found that the students seemed more engaged in the process of sharing strategies for multiple solutions tasks. Hino (2015) recounted some of the benefits of multiple solutions mathematics tasks:

First, [the students] developed their own ideas and methods of obtaining answers. The strategies they devised then formed the basis for how they addressed solutions proposed by their classmates. Second, the activity of individual problem solving helped them develop a sense of *ownership* of their own thinking. This sense of ownership seemed to trigger feelings of surprise or awareness of the similarities and differences between their work and that of others. Comparing multiple solutions also provided the students with opportunities to make personal meanings. (p. 138)

In their study of Grade 8 students of mathematics, Lee and Johnston-Wilder (2013) agreed with the result of a previous study done by Young-Loveridge et al. (2005), which demonstrated the encouragement of students’ voices is needed to increase mathematics communication in the classroom. Lee and Johnston-Wilder (2013) identified the traits of mathematics tasks as one factor that may promote discussions that incorporate multiple perspectives. Thus, they expressed: “More variety in mathematics lessons would be appreciated by the pupils”, as students “saw the value of working on more complex tasks, using a range of skills” (Lee & Johnston-Wilder, 2013, p. 172).

5.3.4 Constructing Knowledge

5.3.4.1 Mathematics Vocabulary

Sophie places great emphasis on the definitions of mathematics vocabularies, and the reinforcements of their uses. In her classroom, a large portion of the wall is occupied by a
vocabulary chart. Arranged alphabetically, this visual organizer displays the vocabularies learned throughout the academic year. The vocabulary chart includes words from all subjects, and are not exclusive to the subject of mathematics. Most of the mathematics vocabularies displayed highlight numerical operations (“addition”, “multiplication”, “subtraction”), tasks information (“key”, “legend”, “scale”), and problem solving strategies (“skip”, “strategy”).

During my lesson observations, I noticed, on several occasions, students looking to the displays on the walls for reminders. While students were conversing with peers, presenting in class, and writing in their mathematics journals, the words displayed around the classroom served as reminders for the need of proper vocabulary incorporation and expression. This organization is similar to the suggestions of Lee and Herner-Patnode (2007), who expressed the value of “word cards on the board” in assisting a diverse group of Grade 4 students to review, recall, and communicate their mathematical ideas. Likewise, during various activities in the mathematics class, Marks Krpan (2008) noticed students’ continuous references to the mathematics words on the classroom walls. This occurred when the students were reading, writing, and discussing mathematical ideas.

Sophie reinforced the uses of proper mathematics vocabularies in several instances. In Vignette One, while students described the mathematics tasks and problem solving steps, Sophie provided corresponding words, which students would then incorporate into their presentations. For example, when a student described the task and “the space that is being covered”, Sophie suggested the words “surface area”. Likewise, in Vignette Three, Sophie examined with her class the alignment of certain words with mathematics approaches. As they paraphrased the mathematics task prompt, the word “greatest” was defined and connected to a problem solving
procedure. Sophie noted that “greatest” often conveyed a need to compare.

Essential mathematics vocabularies in the unit of perimeter and area were reviewed during the media class in Vignette Four, as students played an online mathematics game. Afterwards, the students discussed the words in the task prompt during the mathematics lesson in Vignette Four. Sophie noted the word “remaining”, and asked for its meaning. Students drew illustrations from their experiences and previous encounters with the word. One student voiced how, in the context of meals, “remaining” is similar to “leftovers”. Sophie agreed, and asked the student to utilize that idea and connect it to the meaning within the context of the mathematics task. In Vignette Five, Sophie used students' desks to model the mathematics words “gap” and “overlap”. Students were asked to arrange the desks in their table groups to correspond with the given vocabulary. Reviewing and reinforcing students' mathematics vocabulary is important, as students require subject-specific words when conversing their mathematical ideas.

Mathematics vocabulary is an important component for articulate communication of measurement ideas (Guide to Effective Instruction in Mathematics: Kindergarten to Grade 3 Measurement, 2007). Within The Achievement Chart (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005), mathematics vocabulary is included in two of the four assessment categories. Students should be given opportunities to display their knowledge and uses of mathematics vocabularies, as words are considered subject-specific content. In addition, students should also have opportunities to develop the “use of conventions, vocabulary, and terminology of the discipline (e.g. terms, symbols) in oral, visual, and written forms” (Ontario Mathematics Curriculum, Grades 1-8, rev. 2005, p. 23). Others who found mathematics words and expressions to be important for students’ verbal and/or written communication include Lee and
Herner-Patnode (2007), Marks Krpan (2013), and Sfard (2008).

5.3.4.2 Modelling, Scaffolding, and Question Posing

Sophie perceives that her students tend to struggle with the transference of mathematical knowledge and understanding, to mathematical application and communication. To promote more coherent expressions of mathematical thoughts and mathematical reasoning, she encourages her students to systematically reflect upon their problem solving approaches and processes. In Vignette One, Sophie demonstrated and explained her approaches to comprehending a mathematics task (read the descriptions, underline the key ideas, paraphrase the objectives, and define the mathematics vocabularies). She also posed questions, which attempt to: 1) provide structured ways for students to approach the task, 2) emphasize important connections between previously learned knowledge and current applications, and 3) create more opportunities for students to verbalize their thoughts regarding the task. Similarly, in Vignette Two and Vignette Three, Sophie utilized modelling and questioning techniques to guide students in verbalizing and recounting their problem solving progressions.

The strategies that Sophie utilized in modelling and scaffolding her students’ verbal communication skills resemble the methods identified by Bostiga et al. (2016). When investigating teaching strategies which foster students’ written communication skills, Bostiga et al. (2016) concluded that teachers: 1) predicted students’ mathematical misconceptions, such that they can guide students’ reflection and self-evaluation processes, 2) modelled a systematic method (question reading, question paraphrasing, solutions sharing, and solutions evaluating) to scaffold both the verbal expressions and written ideas, and 3) provided examples of written responses with varying levels of sophistication (such that students could evaluate their own
levels of written communication abilities). Likewise, teachers’ uses of effective questions, and their positive effects on the quality of class discussions, have been acknowledged in various studies (e.g. Drageset, 2014; Koizumi, 2013; Purdum-Cassidy et al., 2015).

In addition to modelling and scaffolding mathematics explanations, Sophie also encourages her students to participate through reversing the roles of the “teacher” and “learner”. It seemed that the role reversal may have given Sophie's students more opportunities to: 1) discover other's faulty understandings, 2) identify the parts which they disagree with, 3) propose alternative solutions, and 4) justify these new perspectives. Sophie also mentions that these skills can be used for self-assessments, as students learn to review their solutions and evaluate their own mathematical correctness.

Various games that promote teachers’ and students’ role reversals are described in the Ontario Ministry of Education’s *Guide to Effective Instruction in Mathematics* (Volume Two: Problem Solving and Communication, 2006). Previously described by Clements and Callahan (1983), and recommended for students in the primary grades by *Guide to Effective Instruction in Mathematics*, “Catch the Mistake and Make It Right” is a game that introduces a puppet called Mr. Mixup. The character regularly makes mathematical mistakes, and students are given the roles of explaining to Mr. Mixup why his responses are incorrect, and then providing him with further guidance (*Guide to Effective Instruction in Mathematics*, Volume Two: Problem Solving and Communication, 2006).

For students in the junior grades, the *Guide to Effective Instruction in Mathematics* (Volume Two: Problem Solving and Communication, 2006) described and recommended “Prove It or Disprove It”, a game where the teacher makes either a correct or incorrect mathematical
statement, and the students need to identify its validity. If the students believe it is an accurate statement, they would proceed to justify this conclusion. However, if the students believe it is a false statement, they would need to adjust the conjecture such that it becomes truthful. Bostiga et al. (2016) similarly identified the learning potentials from: 1) creating prompts based on common mathematical misconceptions, and 2) probing students to discuss the conceptual flaws of the statements.

5.3.5 Technology and Manipulatives

5.3.5.1 Technology

For this section, Sophie’s uses of different technological devices for the fostering of students’ mathematics communication are described. The technologies included are: 1) document camera, 2) iPad, and 3) computer.

5.3.5.1.1 Document Camera

Within Sophie's classroom is a document camera, a device that magnifies and projects visual representations. Sophie's students are very familiar with this device, such that they could properly adjust and focus the projected images. While it was incorporated into numerous subjects, within mathematics lessons, the device was most often used for task introductions and strategy discussions (Vignette One, Vignette Three, Vignette Five). In Vignette One, the mathematics tasks were projected onto the screen, as Sophie introduced the activity and modelled her approaches (reading, underlining, paraphrasing). In Vignette Three, students illustrated and displayed geometric figures that conformed to the requirement of a 24 units square area. In Vignette Five, the multiple choice questions were shown, as the class voted for their solution choices, and as individuals presented their ideas and justifications.
The document camera assisted students in simultaneously representing their ideas visually and verbally. This may be beneficial for the presenters (as images were referenced as they spoke) and the observers (as different learning preferences were incorporated). In addition, during incidents when the presenter struggled with verbal explanations (Vignette One), other students could continue the demonstrations using the same images. Indeed, learning and communicating mathematical ideas through multiple representations and various learning styles have previously been advocated (e.g. Franke, Kazemi, & Battey, 2007; NCTM, 2000, 2014; Thompson & Chappell, 2007). The document camera projected a visual representation of the mathematical ideas, while the students provided a verbal representation of the same mathematical concepts. The benefits of combining visual displays and verbal explanations are also seen in classroom observations by Ryve, Nilsson, and Pettersson (2013), who investigated the uses of visual representations and mathematics vocabularies in effective student group discussions.

### 5.3.5.1.2 iPad

Maple School recently purchased a class set of iPad minis for uses in the primary division. Sophie had several opportunities during the academic year to incorporate these technological devices into her teaching. The iPad minis were used when students explored various functions within the Explain Everything app (Vignette Two). While engaging in a mathematics task, students drew, wrote, and audio recorded their problem solving techniques and procedures. Attard (2013) examined the uses of iPad in the primary classrooms, and also found that non-mathematics based apps, such as the Explain Everything app, can facilitate mathematics activities such that the task purposes and cognitive demands can be maintained. The study
similarly recognized the benefits of the recording function, where students could replay and listen to their own verbal explanations.

Beyond the primary years, Audi and Gouia-Zarrad (2013) highlighted the increasing practice of incorporating iPads into the teaching and learning of mathematics at all grade levels. In their study of engineering students in an introductory mathematics course, the use of android tablets “increases the interaction between the instructor and the students, rendering the class more dynamic and tailored in real time to the students’ needs” (Audi & Gouia-Zarrad, 2013, p. 54).

Initially, Sophie also intended to utilize the class set of iPad minis for the second mathematics task in Vignette Three. In the planned scenario, students would first calculate and compare the areas of the three schoolyards. Having arrived at a solution, each student would then capture an image of their task and proceed to demonstrate their understanding using the Explain Everything iPad app. As a review of and a continuation in the skills practised with the numeracy coach, students would draw diagrams, write sentences, and record verbal explanations. These documents would then be shared with the class, and saved for further analysis of students’ performances.

Due to challenges with the school's wifi, Sophie had to readjust her planned mathematics lesson. Other studies also recorded coming across similar challenges when technological devices were implemented in the classrooms (e.g. Aksan & Eryilmaz, 2011; Riordain, Johnston, & Walshe, 2016). As such, teachers may benefit from being flexible, and being open to adjust, adapt, and modify the lesson plans as required.

5.3.5.1.3 Computer

Sophie’s students have scheduled media classes on a regular, weekly basis. With
advanced booking, the school's computer room is also available to classes during other instructional periods. In Vignette Four, the uses of computers and online education game served as important aspects in mathematics learning. While engaging with the mathematics tasks in the “Perimeter” section of “Math 3 Under the Sea!”, students had occasions where the online game: 1) reviewed mathematics vocabularies and shape properties, 2) presented mathematical concepts audibly and visually through the video lessons, and 3) provided feedback and suggestions on solution selections. The affordances observed resemble some of the ones identified in other studies (e.g. Bakker, Smit, & Wegerif, 2015; Holton et al., 2001).

While there were enough computers for each student to work on one, unfortunately, not all computers were functioning properly. As a result, three pairs of students decided to share computers for the task, as trouble shooting the technological issues would have been very time consuming. Out of this unexpected circumstance came opportunities for students to collaborate and converse. With three pairs of students working together, other students also varied between working independently, in pairs, or in small groups, as the media lesson progressed. Similar to the incident where the class experienced wifi connection problems, this event reminds us that both teachers and students may need to be flexible and open to changes when utilizing technological devices for teaching and learning.

5.3.5.2 Manipulatives

Sophie guides her students in the proper maintenance, selection, and uses of beneficial tools and devices, during the explorations of mathematical concepts. She recognizes the benefits of manipulatives for engaging students of varying learning styles and preferences (e.g. auditory, tactile, visual), and representing mathematical concepts. Acknowledging the incorporation and
implementation of manipulatives as an area for personal growth in the teaching of mathematics,
Sophie continuously explores an assortment of concrete materials and attempts various
facilitation strategies.

Throughout the academic year, Sophie embeds various concrete and virtual objects into
the teaching of mathematics. In Vignette One, while scaffolding her students for the upcoming
lessons on area and perimeter, Sophie utilized different types of pattern blocks to demonstrate the
“covering” of larger polygons with smaller two-dimensional shapes. Incorporating the pattern
blocks into the activities may have eased the cognitive demand of the tasks, as students would
not need to mentally rotate shapes, nor estimate areas and proportions. When students had
difficulties putting into words what they were doing, they may still be able to produce visual
models of their solutions. Using the created models, Sophie guided her students in explaining
their visual representations.

At other times, having seen the visual representations, peers would volunteer and attempt
to articulate what the proposed strategies and steps involved. In these scenarios, the
manipulatives assisted students in conveying what they were thinking, even as they were
searching for the appropriate words to express these ideas in written and verbal forms. The
benefits of visual representations and visual mediators to the practise of mathematics
communication have been acknowledged in various studies (e.g. Murata & Kattubadi, 2012;
Ryve, Nilsson, & Pettersson, 2013; Sfard, 2008). From their classroom observations of Grade 6
students, Ryve, Nilsson, and Pettersson (2013) proposed the combination of manipulatives (e.g.
dice) and mathematics vocabularies assisted the students in communicating mathematical ideas
more effectively.
In addition, Sophie also utilized virtual manipulatives and concrete materials (the paper tiles in Vignette Two, and the desks in Vignette Five) in the practices and demonstrations of mathematical concepts. In the mathematics lesson prior to Vignette One, students completed an online mathematics task that explored the covering of a larger shape with smaller virtual shapes. In Vignette Three, Sophie showed two paper tiles of varying sizes to highlight and clarity the mathematics task described in the *Math Makes Sense* (2004) textbook. In Vignette Five, student desks were maneuvered to display corresponding mathematics vocabularies. Sophie’s use of familiar classroom materials in the explanation of mathematical concepts is actually a time-proved practice. The use of concrete objects for the learning of mathematics is a timeless method that is “actually based on traditional techniques” (Golafshani, 2013, p. 139).

5.3.5.3 Complementary Approaches

While Sophie is dedicated to incorporating technology and manipulatives into her mathematics program, she also believes that students should have occasions to explore and present mathematical ideas without concrete objects or technological devices. Sophie expresses that these instances may show students how they can engage in mathematical conversations without all the resources that they are accustomed to during mathematics lessons.

Without the concrete materials, computers, document cameras, and iPads, students would still have the needed curiosity, confidence, and skills to participate in mathematical explorations in everyday moments. As such, Sophie normally utilizes minimal technological devices and concrete materials for the daily numeracy routines. As observed in Vignette Six, Sophie’s students nonetheless communicated their mathematical knowledge effectively with pencil-and-paper during those numerical operation exercises.
5.3.6 Assessment

5.3.6.1 Verbal Communication

For Sophie, information regarding students' progresses often plays a role in time allotment for current and subsequent mathematics lessons. These programming decisions may include: 1) the amount of time spent on different lesson components (e.g. strategy discussion, lesson consolidation, activity extension), and 2) the program for upcoming mathematics lessons (e.g. content review, task selection, formal assessment). The use of continuous classroom conversations for the assessment of students’ mathematical understanding, and for the planning of current and future mathematics lessons, is recommended by the Guide to Effective Instruction in Mathematics (Volume Four: Assessment and Home Connections, 2006).

During the observed mathematics lessons, Sophie uses various methods to informally assess her students' verbal communication abilities and mathematical understanding. These moments included: 1) student-teacher dialogues during independent work (Vignette One, Vignette Three, Vignette Four, Vignette Five, and Vignette Six), 2) within-table members discussions during group work (Vignette Two, Vignette Three, Vignette Four, Vignette Five, and Vignette Six), 3) student-teacher conversations during round table conferences (Vignette One, and Vignette Five), and 4) whole class discussions during consolidation sessions (Vignette One, Vignette Two, Vignette Three, Vignette Four, Vignette Five, and Vignette Six).

Aspects of students' verbal communication skills were demonstrated during interactions, when students: 1) responded to questions (Vignette One, Vignette Two, Vignette Three, Vignette Four, Vignette Five, and Vignette Six), 2) voiced their opinions regarding Sophie's suggestions (Vignette Four, and Vignette Five), and 3) demonstrated and justified their multiple choice
selections (Vignette Four, and Vignette Five). Below, Sophie’s implementations of the three-part lesson plan, cooperative learning strategies, and the Explain Everything app for the assessment of students’ verbal communication skills are summarized.

Sophie utilizes the discussion opportunities within the three-part lesson plan to gain a clearer awareness of students' mathematical understanding and expressions. She identifies the three-part lesson plan as a method to assess students' mathematics communication abilities. In Vignette Three, the communication criteria informally assessed during the “working on it” segment include: 1) expressing and organizing mathematical ideas (e.g. when students explained strategies being attempted), 2) communicating with different audiences (e.g. with their peers and with their teacher), and 3) using effective vocabularies (e.g. when students described the characteristics and properties of shapes).

Funahashi and Hino (2014) described a four-phase lesson plan, where: 1) the students explore a mathematics task, while the teacher engages in individual conversations, 2) the students present their ideas to the class, while the teacher asks probing questions, 3) the teacher narrows in on one pertinent mathematical idea that was highlighted in the class discussions, while comparing it to other strategies, and 4) the teacher summarizes the results and mathematical approaches. Within this structure, the students have multiple opportunities to communicate their mathematical thinking. Hence, their mathematical thinking and reasoning can be revealed and assessed by the teacher (Funahashi & Hino, 2014). Warshauer’s (2015) study of productive mathematics struggle among Grades 6 and 7 students also found similar benefits of three-part lesson plans for the assessment of students' verbal communication and mathematical thinking.
In Vignette Four, students’ verbal mathematics communication could be informally assessed during the “think-pair-share” cooperative learning activity, a component which Sophie identified as the “during: working on it” segment of the three-part lesson. While students brainstormed with their table group partners, Sophie walked around the class, observed their interactions, and listened to their dialogues. At several points, Sophie joined in the conversations, and asked questions that prompted students to think deeper and with greater precision.

During the large group sharing, a segment that Sophie identified as the “after: consolidation and practice” portion of the three-part lesson, it was evident that Sophie made mental notes of her observations during the “think-pair-share” activity. As table groups shared their strategies, Sophie recalled some of the preceding conversations with students. She also highlighted the commonalities among the student-generated strategies. From the informal assessments during “think-pair-share”, Sophie may have gotten glimpses of students' mathematical thinking and understanding through their verbal expressions.

The potential of using groups discussions and cooperative learning strategies in the assessment of students’ mathematics communication skills and mathematics thinking have previously been proposed by the National Council of Teachers of Mathematics (2014) and Marks Krpan (2013). Marks Krpan (2013) also highlighted the potentials for “think-pair-share” in fostering students’ abilities to self-assess their mathematical ideas that were verbally expressed. She conveyed to teacher readers:

[Think-pair-share] cooperative structure is very effective as you can infuse it into any learning experience, and it does not take up a lot of time. By inviting your students to think and then discuss an idea with another classmate, you ensure that all students have an opportunity to reflect on their knowledge, discuss their mathematical thinking, and learn from others. (Marks Krpan, 2013, p. 11)
Third, in Vignette Two, Sophie employed the iPad minis and the Explain Everything app to record students' verbal and written communication. She expresses the benefits of audio and visual recording devices, as students' verbal and written expressions may be: 1) projected through a screen and shared with the class, 2) archived to further assess students' individual improvements and progresses, and 3) used to complement the pencil and paper mathematics assessments. The benefit of the recording function for the Explain Everything app was also identified by Attard (2013), who valued the possibility of archiving and re-examining students’ verbal expressions.

5.3.6.2 Written Communication

Students' ability to communicate mathematical ideas through written means is also an area of assessment. Sophie gleans information regarding students' progresses by collecting and reviewing students' individual responses to various in-class and at-home assignments. In Vignette One, Sophie requested students to include written responses (e.g. thinking processes, solution sentences, visual diagrams) when they complete the mathematics tasks worksheets. At the end of the lesson, the students placed their worksheets in individual mathematics folders. In Vignette Two, the Explain Everything app served as virtual “mathematics folders”, storing students' written and verbal expressions for continuous assessments and reflections.

Aspects of students' written communication abilities were demonstrated through their responses recorded in their Mathematics Booklets (Vignette Five) and Homework Booklets. Instead of simply indicating their multiple choice selections, Sophie encourages her students to include explanations and steps to justify their conclusions. With these additional information, Sophie can better decipher the thinking and reasoning behind correct options selections (e.g.
selections made randomly, estimations based on partial knowledge, or conclusions justified by understanding).

Multiple choice distractors are often created with common mathematical misconceptions and errors in mind. Therefore, students' incorrect selections, in addition to their written responses, may provide Sophie with more information regarding content areas that require further review. Sophie’s use of multiple choice tasks is similar to her approach with role reversal, where her students had to justify, through verbal and written means, their mathematical reasoning behind the selection. Thus, the ways in which Sophie selects, implements, and guides the mathematics tasks increased her students’ mathematics communication opportunities, and revealed her students’ mathematical understandings. Jackson et al. (2013), Olteanu (2015), and Viseu and Oliveira (2012) also found that task selections and task implementations may give teachers increased opportunities to assess students' mathematics communication and mathematics reasoning skills.

The mathematics journals that Sophie’s students record in serve to archive their written expressions. Mathematics journals have been identified as a method to assess students’ written communication and mathematics reasoning (Bostiga et al., 2016; Parker & Breyfogle, 2011; Pugalee, 2001). In a study on the utilization of argumentative mathematics journals, students wrote and justified their mathematical solutions (Bostiga et al., 2016). The authors stated: “Using the journals as a formative assessment, we were able to tailor our instruction. Initially, most students’ responses were basic, but grew in length and depth over time, as we addressed the quality of students’ writing” (Bostiga et al., 2016, p. 551). The use of mathematics journals for formative assessment, and the adjustments of instructional practices to increase students’
mathematics reasoning opportunities were also present in Sophie’s teaching of mathematics.

The process of regularly writing in journals may also give students opportunities to self-assess their clarity and organization, to refine and reorganize what they strive to convey. As they read over their written works, students may develop clearer awareness of their methods of expressions. In addition, the entries written in mathematics journals are sometimes shared with peers. During Numeracy Routine One, Sophie asked students to exchange mathematics journals with peers, and to carefully check one another's procedures. Here, the students practised peer assessing each other's mathematical reasoning. Students' responses during these interactions included: 1) nodding in agreement with the stated responses, 2) asking for further clarifications on written depictions, and 3) identifying mathematical errors. Sophie also occasionally collects students' mathematics journals and written entries, such that she may gain better ideas of her students' progresses. The fostering of self-assessment and peer assessment through verbal communication and/or written communication is also advocated by Smith et al. (2009), White (2003), and Vazquez (2008).

5.3.7 Summary

When attempting to foster students’ mathematics communication skills, a variety of teaching strategies were implemented into each mathematics learning opportunity. Different teaching strategies interacted with one another, such that, collectively, they enhanced students’ mathematics communication opportunities. The selection and integration of multiple teaching strategies are in agreement with the mathematics education conceptual framework created by McDougall (2004), which views the teaching strategies as non-discrete elements.

Likewise, Ghousseini and Herbst (2016), Henning et al. (2012), Hillen and Smith (2007),
and Morgan et al. (2014) point to the thoughtful integrations of teaching strategies that are often necessary in the improvement of students’ mathematics learning. To create this envisioned, balanced mathematics program, Marks Krpan (2013) says:

Educators often wonder how they can include student communication in an already full mathematics program. Engaging students in sharing their thinking and exploring mathematical concepts does not require adding something onto our mathematics program, but making changes to how we teach. A balanced mathematics program engages students in making discoveries about mathematical concepts on an ongoing basis. Communication, which encompasses making conjectures through to recording, discussing, and explaining observations and results, is a natural part of this learning process. (p. x)

With regards to the development of mathematics communication skills, Morgan et al. (2014) concludes: “In some sense, almost all studies involving language and communication in mathematics education also address other significant issues- learning, teaching, affect, identity, curriculum, assessment, etc.” (p. 850).

5.4 **Major Findings**

The case study of Sophie explores and describes some of the recurring teaching perceptions nurtured, practices ventured, and strategies implemented in the development of students’ mathematics communication skills, and measurement content knowledge. Based on this inquiry, five major findings related to the topic of mathematics communication and the development of measurement concepts in the classroom are summarized:

1) Teachers need to create opportunities for students to productively struggle with increasingly abstract concepts. The appropriate selections of mathematics tasks, the discussions on mathematical misconceptions, and the posing of effective questions may guide and encourage students as they express their understanding and misconceptions.

2) Teachers need to foster students' self-monitoring and self-assessment skills, as students
communicate their emerging mathematical thoughts.

3) Teachers should strive to provide students with opportunities to learn to reason and justify their perspectives. Such opportunities can occur as students read, write, draw, and/or listen to recently learned mathematical concepts.

4) Teachers should intentionally define and explore various meanings and usages of subject-specific and unit-specific vocabularies. The learning of increasingly abstract ideas (e.g. area and perimeter) requires the acquisitions of new vocabularies, so that meanings can be more clearly articulated by the students.

5) When teaching and learning the topic of measurement, various education components such as program planning, learning environment, mathematics tasks, constructing knowledge, technology and manipulatives, and assessment are often intricately and purposefully linked with one another.

5.5 Suggestions and Considerations

In light of the research findings on mathematics communication and measurement understanding, the following suggestions are proposed to teachers of mathematics, school administrators, and professional development leaders.

1) Professional development leaders may consider facilitating workshops that provide teachers with opportunities to grow in the identified teaching components (e.g. program planning, question posing, task implementing). More importantly, these workshops should assist teachers in recognizing the inter-connectedness of the teaching components within the teaching and learning of mathematics, communication, and measurement.

2) School administrators may support their staff by providing them with the needed
resources (e.g. planning time, manipulatives, technology, training), so that the teachers may feel more confident and prepared in their mathematics instruction.

3) Teachers of mathematics may peruse through the lesson vignettes as case studies, and glean insights about own teaching practices. It may also be helpful to utilize an education conceptual framework (e.g. McDougall, 2004), and identify one or two components to focus on for each academic year.

5.6 **Implications for Future Research**

While the case study of Sophie provided some responses to the proposed research question, nonetheless, further research into this area of mathematics teaching and learning may generate a more comprehensive understanding. First, as the findings for my study were derived from the interviews and classroom observations of one Grade 3 teacher, it may be of interest to explore the fostering of students’ mathematics communication skills by other teachers (e.g. varying teaching experiences and content expertise), in other grades (e.g. Junior Division, Intermediate Division), for other mathematics content strands (e.g. patterning and algebra), and in other education settings and geographical regions (e.g. other provinces or countries). Through this endeavour, cross case analysis and comparisons may be performed, and similarities and differences in teacher perspectives and strategies may surface. As a result, a more comprehensive depiction of this phenomenon may be presented.

My study focused primarily on Sophie's perceptions, strategies, and practices, as expressed during interviews and noticed during classroom observations. Though teachers’ education perceptions and classroom practices have significant influences on students’ learning experiences (Bruce & Ross, 2008; NCTM, 2000; Taylor, 1991), students’ perspectives are also
highly valuable in better understanding their preferred strategies for mathematics learning (Young-Loveridge et al., 2005). What teaching strategies, education tools, or learning approaches might students identify as beneficial to the fostering of their mathematics communication opportunities? What may students indicate as helpful in encouraging their participation, refinement, and engagement in various forms of mathematics expressions (verbal, written, visual)? Future studies on mathematics communication within the classrooms may include more of students’ points of views, possibly by collecting such information through surveys or interviews.

5.7 Conclusion

This research inquiry describes and displays some of the teaching strategies implemented in the fostering of Grade 3 students’ mathematics communication skills, during their learning of measurement ideas. Previous studies have noted students' struggles with mathematics communication (e.g. Anthony & Walshaw, 2002; Thompson & Chappell, 2007) and measurement ideas (e.g. Chappell & Thompson, 1999; Kamii, 2006). In response, this study highlights various strategies for the fostering of elementary students' mathematics communication skills, and captures the uses of these strategies within the learning of measurement content (e.g. perimeter and area). In agreement with Case (2005), mathematics processes (e.g. mathematics communication skills) and mathematics content topics (e.g. measurement concepts) were perceived intricately, and a dichotomy approach was minimized.

Further, the categories of program planning, learning environment, mathematics tasks, constructing knowledge, technology and manipulatives, and assessments were identified within each lesson vignettes (sub-units analysis), and across lesson vignettes (holistic, “whole” case
analysis). Considerations regarding the physical learning environments, the social components of learning, the perspectives and learning strengths of students, the topics of focus on communication, and the integration of teaching strategies were highlighted and summarized in the major findings.

Though the purpose of this case study research may not be generalizations of depicted implementation strategies into different classrooms, different grades, and different learners, it hopefully encourages teachers to take moments to ponder about their own teaching perspectives, their students’ learning strengths, and the environment in which they teach in. As Sophie confidently states: “It is the way you plan it, the way you demonstrate it, the way you model it, the way you get it done” (Sophie interview, May 26, 2015).
References


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Appendix A
School and District Improvement in Elementary Mathematics
Principal and Teacher Questions

Background questions
What is your name?
Where did you go to university? What is your degree of specialization?
Why did you become a teacher?
How many years have you been teaching?
Where did you teach before and what grades have you taught?
How long have you been here at this school?
What subjects and grades do you teach or what is your role in the school?

Versions of success
For you, what counts as success for students in this school?
What are your goals in education?
How widely accepted are your goals with other teachers in the school? Among parents?
How does your school improvement plan incorporate your goals for students?
How is the school improvement plan created in this school?

Challenging circumstances
What are the most challenging things (the barriers) for you as you go about your work in this school?
What are the most successful things for you as you go about your work in this school?
Do you think this school is different from other schools in its challenges?
How would you describe the community of parents with whom you work?
How has the school context changed over the past few years, and what changes are going on now?

Mathematics
How would you describe your goals in mathematics?
How widely accepted are these views in the school? Among the parents?
How would you describe the provincial ministry’s vision of mathematics?
How do you meet the mathematics goals of the province?
Which of the Ten Dimensions have you selected for your personal growth? Why did you select those dimensions?
Which of the Ten Dimensions have you selected for your school improvement plan? Why did you select those dimensions?
Fostering Mathematics Communication
How would you define mathematics communication?
How do you perceive the role of mathematics communication in your mathematics program?
What mathematics communication goals do you have for your class?
How do you create a classroom environment that fosters students' mathematics communication?
What are some of the challenges you've encountered when attempting to develop students' mathematics communication?
What are some of the successes you've encountered when attempting to develop students' mathematics communication?

School support
How do you create an environment, which supports success in mathematics?
What challenges (barriers) have you faced in trying to create a culture that supports student achievement in mathematics?
How do you work with staff and administration to develop the goals/vision of the school?
To develop mathematics improvement?
How were the issues resolved?

Overall
What are the programs that support success in mathematics outside of the classroom?
What do you think we should say in our report about how schools can be more effective in supporting mathematics improvement?
Do you have a mathematics implementation team? If so, what is their role and what do they do?
Appendix B
Guiding Questions for Lesson Observations

Questions prior lesson observation

1. What are your learning goals for today's lesson?
2. What strategies will you utilize to develop students' mathematics communication skills?
3. Why have you chosen these strategies?
4. How might students respond to the lesson (the activities, the strategies, etc.)?

Questions following lesson observation

1. What were some of the teaching/learning successes for this lesson?
2. What were some of the teaching/learning challenges for this lesson?
3. How did the students respond to the lesson (the activities, the strategies, etc.)?
4. Do you think they were practising mathematical communication?
5. If you were to teach this lesson again, what would you do differently?
Appendix C
Letter of Consent

Dear [name of teacher involved in the Elementary Teacher Learning Initiative],

My name is Mimi Kam and I am a third year PhD student at the Ontario Institute for Studies in Education, University of Toronto (OISE). I am interested in exploring the perceptions and practices of teachers of junior mathematics (Grades 3 to 6) in the fostering of mathematics communication skills. I believe the development of mathematics communication is an important component to the learning of mathematics. I also believe that teachers play significant roles in the development of such skills. As such, the sharing of your knowledge and experience with regards to this topic will be of great value.

My research design consists of two-phases, and the data for my study consists of three sources: 1) Attitudes and Practices of Teaching Math Survey, 2) Interviews (with questions on participant background, versions of success, challenging circumstances, mathematics teaching, mathematics communication, and school culture), and 3) Classroom Observations. Phase 1 of the research consists of analyzing and interpreting the Attitudes and Practices of Teaching Math Survey and the interview transcripts, which were collected as part of the Elementary Teacher Learning Initiative (2014-present). The Attitudes and Practices of Teaching Math Survey was completed during the professional learning session at the post-secondary institution, and the interview was conducted, audio-recorded, and transcribed by graduate assistants of the Elementary Teacher Learning Initiative. Phase 2 of my study involves classroom observations and further interviews with five teacher volunteers.

Thank you for previously consenting to participate in Phase 1 of my study. I would like to invite you to participate in Phase 2 of my study. Phase 2 consists of the following data collection:

- Classroom observations- I will observe your lessons every other week for the remainder of this term. If time permits, prior to your teaching, I will ask you to describe your plans and strategies for the lesson. During your teaching, I will take notes on your teaching strategies and the students' responses and reactions. After your lesson and if time permits, I will ask you to reflect upon the lesson.
- Final interview- This is a 30-minute semi-structured interview, where several questions regarding any additional insights on the teaching of mathematics communication. Reflecting upon the experience of participating in this study is also welcomed. This interview will be audio-taped and transcribed for analysis.
In all my written works, oral presentations, and publications, your name will not be used. While information regarding your identity (e.g. name, roles in school, teaching experiences) may be collected, your identity remains confidential and any personal identifiable information will be replaced with pseudonyms. Please note that, should you change your mind, you are free to withdraw from this study at any time. It is also important that you are aware of your rights to decline any of the specific questions. I will destroy my copy of the collected data 5 years after the publication of findings. This study has a minimal possibility of psychological and/or emotional risk.

If you agree to participate, please sign the attached form. A second copy is provided for your records. Thank you very much for your assistance.

Yours sincerely,
Mimi Kam
OISE/ University of Toronto
mimi.kam@mail.utoronto.ca

Thesis supervisor: Dr. Douglas McDougall
doug.mcdougall@utoronto.ca
Phone number: (416) 978-0056

Consent Form

I acknowledge that the topic of this research has been explained to me. Any questions that I may have pertaining to this project, I have asked and they have been answered to my satisfaction. I understand that I can withdraw from this project at any time without penalty.

I have read the letter provided to me by Mimi Kam and agree to participate in this study and provide information for the purpose described.

Signature: ________________________
Name (printed): _____________________
Date: __________________________