Application Of Real-Time Hybrid Simulation Method In Experimental Identification Of Gyromass Dampers

BY

REZA MIRZA HESSABI

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Reza Mirza Hessabi
Doctor of Philosophy
Department of Civil Engineering
University of Toronto
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Abstract

Structural control techniques have been extensively explored in the past decades, contributing to substantial safety improvements. Still, recent experience shows that the property damage resulting from even a moderate earthquake is significant. This dissertation studies the design and development of a new mechanical vibration absorber (i.e., gyromass damper or GMD) that uses gear assemblies to generate acceleration-proportional resisting forces. Similar to dynamic vibration absorbers such as tuned mass dampers, GMDs allow the introduction of control forces that are caused by inertial forces. However, unlike tuned mass dampers, they do not require a large additional physical mass. With the aim of incorporating GMDs in real structures, sample configurations are presented that are simple to build, and the effectiveness, robustness, and limitations of these devices for linear systems are investigated. This dissertation has two main foci regarding the application of GMDs for passive control of structures. The first is to improve the current state of the literature by characterizing and modelling the dynamic behaviour of GMDs. While the second is the application of real-time hybrid simulation method as a dynamic testing tool to experimentally examine the effects of GMDs on the performance of SDOF systems. The
numerical simulations and experimental results show that GMDs consistently enhances the performance of SDOF systems and it is possible to improve the efficiency of the control system further by employing braces or energy dissipation devices in parallel with the GMD.
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<tr>
<td>2DOF</td>
<td>Two-degree-of-freedom</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Stiffness ratio</td>
</tr>
<tr>
<td>$\beta_{am}$</td>
<td>Newmark’s method parameter</td>
</tr>
<tr>
<td>$\beta_{gmd}$</td>
<td>Frequency ratio of the system with the stand-alone GMD</td>
</tr>
<tr>
<td>$\beta_1$</td>
<td>Frequency ratio of the system with the GMD in MDOF systems (first mode)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Non-dimensional parameter for the equivalent damping of nonlinear GMDs</td>
</tr>
<tr>
<td>$\gamma_{am}$</td>
<td>Newmark’s method parameter</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>Amplitude ratio</td>
</tr>
<tr>
<td>$\Gamma_1$</td>
<td>Modal participation factor of the first mode</td>
</tr>
<tr>
<td>$\Delta A$</td>
<td>Amplitude error</td>
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<td>$\Delta t$</td>
<td>Time step</td>
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<td>$\theta_i$</td>
<td>Rotation of the $i$th gear</td>
</tr>
<tr>
<td>$\theta_{pc}$</td>
<td>Amplitude indicator</td>
</tr>
<tr>
<td>$\mu$</td>
<td>Equivalent mass ratio</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>Equivalent mass ratio of the first mode</td>
</tr>
<tr>
<td>$\mu_f$</td>
<td>Coefficient of friction</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Damping ratio of the SDOF system</td>
</tr>
<tr>
<td>$\zeta_{opt}$</td>
<td>Equivalent damping ratio for the GMD in MDOF systems (first mode)</td>
</tr>
<tr>
<td>$\zeta_{eq}$</td>
<td>Equivalent damping ratio of the system with the nonlinear GMD</td>
</tr>
<tr>
<td>$\zeta_{gmd}$</td>
<td>Damping ratio of the system with the stand-alone GMD</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Phase error</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>Mode shape of the first mode</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Phase angle of the command displacement</td>
</tr>
<tr>
<td>$\phi_m$</td>
<td>Phase angle of the measured displacement</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time delay</td>
</tr>
<tr>
<td>$\tau_{crit}$</td>
<td>Critical time delay in stability analysis</td>
</tr>
<tr>
<td>$\tau_{est}$</td>
<td>Estimated time delay</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Window size ratio for FDB error indicators</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Angular frequency of the signal</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>Angular loading frequency</td>
</tr>
<tr>
<td>$\omega_b$</td>
<td>Natural angular frequency of the GMD-brace system</td>
</tr>
<tr>
<td>$\omega_{crit}$</td>
<td>Critical angular frequency in the stability analysis</td>
</tr>
<tr>
<td>$\omega_{ngmd}$</td>
<td>Natural angular frequency of the system with the stand-alone GMD</td>
</tr>
<tr>
<td>$a_n$</td>
<td>Fourier series coefficients</td>
</tr>
<tr>
<td>$A_c$</td>
<td>Amplitude of the command/target displacement</td>
</tr>
<tr>
<td>$A_m$</td>
<td>Amplitude of the measured displacement</td>
</tr>
</tbody>
</table>
\( b \)  Equivalent mass of the GMD
\( b_n \)  Fourier series coefficients
\( c_{\text{an}} \)  Damping of the analytical substructure
\( c_{\text{eq}} \)  Equivalent viscous damping of a system with friction forces
\( c_{\text{exp}} \)  Damping of the experimental substructure
\( c_{\text{cmd}} \)  Viscous damping coefficient in a TMD system
\( C \)  Damping of the SDOF oscillator
\( \tilde{C} \)  Constraints matrix in PAEI’s method
\( \tilde{D} \)  Design matrix in PAEI’s method
\( E_D \)  Viscously dissipated energy in the energy balance equation
\( E_{\text{eq}} \)  Equivalently dissipated friction energy in the energy balance equation
\( E_{\text{in}} \)  Input energy in the energy balance equation
\( E_K \)  Kinetic energy in the energy balance equation
\( E_S \)  Strain energy in the energy balance equation
\( E_{\text{dgm}} \)  Dissipated energy of a GMD
\( E_{\text{error}} \)  Energy error in HSEM’s formula
\( E_{\text{input}} \)  Input energy in HSEM’s formula
\( E_{\text{strain}} \)  Strain energy in HSEM’s formula
\( f \)  Force in the GMD
\( f_d \)  Damper force
\( f_{i,j} \)  Contact force between the \( i \)th and \( j \)th gears
\( f_n \)  Friction force
\( F_0 \)  Amplitude of the harmonic loading
\( F_{f\text{max}} \)  Friction force coefficient
\( F_r \)  Friction force ratio
\( F_{\text{mes}} \)  Measured force from the experimental substructure
\( \tilde{f}_c \)  Dominant frequency of the command displacement
\( \tilde{f}_m \)  Dominant frequency of the measured displacement
\( j \)  Imaginary unit (equal to \( \sqrt{-1} \))
\( J_i \)  Mass moment of inertia of the \( i \)th gear
\( k_{\text{an}} \)  Stiffness of the analytical substructure
\( k_p \)  Lateral stiffness of V braces
\( k_{\text{cmd}} \)  Lateral stiffness of a TMD system
\( K \)  Stiffness of the SDOF oscillator
\( m_{\text{an}} \)  Mass of the analytical substructure
\( m_{gd} \)  Mass of the rotating disk in Saitoh’s model
\( m_{gi} \)  Mass of the \( i \)th gear
\( m_{\text{cmd}} \)  Mass of a TMD
\( M \)  Mass of the SDOF oscillator
\( M_w \)  Moment magnitude
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>Normal force in Coulomb damping model</td>
</tr>
<tr>
<td>$N_{cycle}$</td>
<td>Number of data points within each complete cycle of signal</td>
</tr>
<tr>
<td>$N_g$</td>
<td>Gear ratio in the compound gear</td>
</tr>
<tr>
<td>$N_{win}$</td>
<td>Number of data points within each selected window</td>
</tr>
<tr>
<td>$P_{eff}$</td>
<td>Effective force</td>
</tr>
<tr>
<td>$r$</td>
<td>Influence vector</td>
</tr>
<tr>
<td>$r_d$</td>
<td>Radius of the rotating disk in Saitoh’s model</td>
</tr>
<tr>
<td>$r_i$</td>
<td>Radius of the $i$th gear</td>
</tr>
<tr>
<td>$R_{jb}$</td>
<td>Joyner-Boore distance</td>
</tr>
<tr>
<td>$s$</td>
<td>Complex variable (Laplace transform)</td>
</tr>
<tr>
<td>$\mathbf{S}$</td>
<td>Scatter matrix in PAEI’s method</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$T$</td>
<td>Auxiliary parameter in the stability analysis</td>
</tr>
<tr>
<td>$T_D$</td>
<td>Length of the time window</td>
</tr>
<tr>
<td>$u$</td>
<td>Displacement</td>
</tr>
<tr>
<td>$\dot{u}$</td>
<td>Velocity</td>
</tr>
<tr>
<td>$\ddot{u}$</td>
<td>Acceleration</td>
</tr>
<tr>
<td>$u_{max}$</td>
<td>Maximum input displacement</td>
</tr>
<tr>
<td>$x_0$</td>
<td>Maximum dynamic displacement</td>
</tr>
<tr>
<td>$x_c$</td>
<td>Command/target displacement</td>
</tr>
<tr>
<td>$x_m$</td>
<td>Measured displacement</td>
</tr>
<tr>
<td>$(x_{st})_0$</td>
<td>Maximum static displacement</td>
</tr>
<tr>
<td>$X(s)$</td>
<td>Laplace transformation of the displacement</td>
</tr>
<tr>
<td>$X_g(s)$</td>
<td>Laplace transformation of the ground displacement</td>
</tr>
<tr>
<td>$Y(s)$</td>
<td>Laplace transformation of the measured displacement</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier transform</td>
</tr>
<tr>
<td>DMF</td>
<td>Dynamic magnification factor</td>
</tr>
<tr>
<td>DVA</td>
<td>Dynamic vibration absorber</td>
</tr>
<tr>
<td>EQ</td>
<td>Earthquake</td>
</tr>
<tr>
<td>EPA</td>
<td>Effective peak acceleration (for records with large effective peak acceleration)</td>
</tr>
<tr>
<td>EPV</td>
<td>Effective peak velocity (for records with large effective peak velocity)</td>
</tr>
<tr>
<td>FBD</td>
<td>Free-body-diagram</td>
</tr>
<tr>
<td>FDB</td>
<td>Frequency-domain based error indicators</td>
</tr>
<tr>
<td>FEI</td>
<td>Frequency evaluation index</td>
</tr>
<tr>
<td>FPGA</td>
<td>Field-programmable gate array</td>
</tr>
<tr>
<td>GMD</td>
<td>Gyromass damper</td>
</tr>
<tr>
<td>HSEM</td>
<td>Hybrid simulation error monitors</td>
</tr>
<tr>
<td>IV</td>
<td>Incremental velocity (for records with large incremental velocity)</td>
</tr>
<tr>
<td>ID</td>
<td>Incremental displacement (for records with large incremental displacement)</td>
</tr>
<tr>
<td>LVDT</td>
<td>Linear Variable Differential Transformer</td>
</tr>
<tr>
<td>MCE</td>
<td>Maximum tracking error</td>
</tr>
<tr>
<td>Abbreviation</td>
<td>Description</td>
</tr>
<tr>
<td>--------------</td>
<td>-------------</td>
</tr>
<tr>
<td>MDOF</td>
<td>Multi-degree-of-freedom</td>
</tr>
<tr>
<td>NRMSE</td>
<td>Normalized root mean square error</td>
</tr>
<tr>
<td>PAEI</td>
<td>Phase and amplitude error indices</td>
</tr>
<tr>
<td>PA</td>
<td>Peak acceleration (for records with large peak acceleration)</td>
</tr>
<tr>
<td>PD</td>
<td>Peak displacement (for records with large peak displacement)</td>
</tr>
<tr>
<td>PGA</td>
<td>Peak ground acceleration</td>
</tr>
<tr>
<td>PSD</td>
<td>Pseudo-dynamic test</td>
</tr>
<tr>
<td>PV</td>
<td>Peak velocity (for records with large peak velocity)</td>
</tr>
<tr>
<td>RMS</td>
<td>Root mean square</td>
</tr>
<tr>
<td>RTHS</td>
<td>Real-time hybrid simulation</td>
</tr>
<tr>
<td>SDOF</td>
<td>Single-degree-of-freedom</td>
</tr>
<tr>
<td>SSP</td>
<td>Synchronization subspace plot</td>
</tr>
<tr>
<td>TI</td>
<td>Tracking indicator</td>
</tr>
<tr>
<td>TID</td>
<td>Tuned inerter damper</td>
</tr>
<tr>
<td>TLD</td>
<td>Tuned liquid damper</td>
</tr>
<tr>
<td>TMD</td>
<td>Tuned mass damper</td>
</tr>
<tr>
<td>TMDI</td>
<td>Tuned mass damper inerter</td>
</tr>
<tr>
<td>TVMD</td>
<td>Tuned viscous mass damper</td>
</tr>
<tr>
<td>VD</td>
<td>Viscous damper</td>
</tr>
</tbody>
</table>
Chapter 1 . Introduction

Structural systems such as buildings and bridges are prone to vibrations due to dynamic forces like seismic or wind loading and failure of structures during extreme loading (e.g. major earthquakes) can cause significant loss of life and property damage. Conventionally, the primary objective of earthquake engineers is to prevent human injury and death during these catastrophic events by preventing the collapse of the structures. In accordance with this aim, the failure of some structural elements was tolerated so long as the safety of the occupants was ensured. Still, according to a report from the Centre for Disaster Management and Risk Reduction in Germany (CEDIM, 2012), only in 2012, earthquakes and their consequences, caused damage of $365 billion worldwide. This report also mentions that 2011 earthquakes and their aftereffects destroyed or damaged more than 1.7 million buildings globally. Among these, Japan had more than one million damaged buildings. Even in countries with more advanced seismic design codes and practices, which have led to reduced casualties, the annual loss due to earthquakes is staggering. For instance, the earthquake near Christchurch, New Zealand, in February 2011 caused $12 billion in losses (Murdoch & Fraser, 2011). In the United State, since the turn of the new century, the number of deaths from earthquakes have been limited to 2 (USGS, 2014). However, earthquake losses in the United States are reported to be $4.4 billion dollars annually (FEMA, 2002). The heavy financial burden on the nations to recover after earthquakes as well as advancements in computer modeling and engineering practice have increasingly encouraged the engineering community to aim beyond these requirements and design higher performance structures where the damage to the components of the structures are also significantly reduced.

Over the past few decades, a wide range of structural control techniques has been developed to enhance the dynamic performance of structures. Each of these structural control methods employs one or a combination of several different devices. Consequently, there has been a constant push for new devices and control strategies.

In general, current structural control systems can be categorized into three broad areas, namely, base isolation, passive energy dissipation, and active/semi-active control systems (Christopoulos...
Chapter 1. Introduction

& Filiatrault, 2006). Passive control techniques are perhaps the most widely employed as they require no external energy supply and are easier to set-up and maintain. Some of these systems such as various types of braces, viscous dampers, and viscoelastic dampers are widely accepted by the engineering community and have been used extensively in real structures while others such as tuned mass dampers (TMDs) and tuned liquid dampers (TLDs) have been used for special applications. Despite their effectiveness in enhancing the seismic response of the structures, each of these control strategies has its limitations, and thus there is room for improvement.

One of the possible methods that have been studied in the past to improve the dynamic response of structural systems is by modifying the inertia of structures. For decades, the effects of introducing Dynamic Vibration Absorbers (DVA) such as TMDs and TLDs to structures for wind engineering applications have been studied. However, these devices have some limitations and disadvantages under seismic loading that will be discussed in more detail in Chapter 2. For instance, they occupy a relatively large space and increase the input energy of the system. It can be advantageous to introduce a new type of compact mechanical device that does not suffer from the shortcomings of previous DVA devices and can be easily incorporated into the conventional structures to change the inertial terms of the equation of motion of the system and improve the performance. There is a considerable amount of research focused on the development and design of these types of dampers for mechanical vibrations. However, their application in civil engineering and especially structural dynamics is limited.

This dissertation has two main foci concerning the application of a new type of inertial damper, called Gyromass Damper (GMD), for passive control of conventional structures. The first is the identification and characterization of the behaviour of these mechanical devices, while the second is the application of Real-Time Hybrid Simulation (RTHS) method as a dynamic testing method to experimentally examine the effects of GMDs on the performance of Single-Degree-of-Freedom (SDOF) systems. The experimental results are then used to investigate the GMD effects on the seismic behaviour of structures. As such, the following four objectives can be listed as the principal objectives of this study:

- Introduction of new error assessment tools to monitor the accuracy of the test results and to ensure the stability of the RTHS experiments.
Chapter 1. Introduction

- Identification and modelling of the dynamic properties of stand-alone GMDs.
- Carrying out experiments with a GMD prototype using RTHS method.
- Investigation of the application of GMDs in buildings.

The following chapter presents a summary of the current literature for the inertial dampers and reviews some of the previous studies that have examined other relevant techniques in structural control. The comparisons show that one of the features that make inertial dampers different than many other types of dampers is that they employ gear trains to transform applied translational displacements to amplified rotations to generate rotational inertial forces. Hence, the amount of resisting force generated within these devices is proportional to the applied acceleration. As a particular type of inertial dampers, GMD is introduced in Chapter 2, and its components are introduced. Using first principles, two models are developed to describe the device behaviour. These models are later revisited in the next chapters, and the effects of friction forces are examined. Based on these models, the generated force in a GMD is rate-dependent, and thus, a proper dynamic testing technique should be employed to test this device.

In this study, RTHS is chosen as the dynamic testing method, and the theoretical aspects of the RTHS technique are discussed in Chapter 3. It is well-documented in the literature that since measured signals are fed back to the integration algorithm in RTHSs to generate the next step displacements, this testing technique is prone to error propagation. These errors may often jeopardize the stability of the simulations which makes it necessary to monitor and compensate for them. Thus in Chapter 3, after briefly discussing the theory behind RTHS and reviewing previous related studies. New sets of error assessment indicators are proposed, and through a stability analysis of the outer loop dynamics of the testing setup, stability limits are established, and the reliability of the RTHS results in this study is examined.

Chapter 4 describes the experimental testing and evaluation of two GMD prototype specimens. These dampers are evaluated under different experimental scenarios, generating a diverse database. The obtained database includes characterization tests and RTHS emphasizing the nonlinearities of the behaviour. Through several experiments with various loading scenarios, it is shown in this chapter that linear models cannot sufficiently predict the behaviour of GMDs. After analyzing the test results performed on two different prototype specimens with different values of
equivalent mass, models of Chapter 2 for describing the damper force are modified and the effects of the frictions forces are included. Using the modified model, RTHSs with a suite of six different earthquakes are carried out, and the experimental data are used to verify that the modified “lumped mass” model provides a fair representation of the device behaviour. Since it is shown that GMD’s reduce the equivalent damping of the system, to further improve the behaviour of these devices, a nonlinear viscous damper was introduced to the testing setup and the superior performance the hybrid control strategy was shown.

Based on the developed models of Chapter 2 and 4, MATLAB, Simulink, and OpenSees models are developed, and numerical studies are presented in Chapter 5 to numerically study different aspects of the dynamic behaviour of these devices. Using the information extracted in the time and frequency domain, and by employing various performance criteria such as energy terms, the effects of seismic control of structures with GMDs are investigated. With the objective of introducing GMDs into real structures, different configurations are studied, and the performance of these configurations is compared. In these configurations, the effects of the stiffness of the braces that connect the GMDs to the top floors are considered, and the behaviour of GMD systems with and without the braces is evaluated. The response of the uncontrolled SDOF is chosen as the reference case to assess the effects of a range of values of dynamic properties for each of the configuration components (i.e., equivalent mass of the GMD, the stiffness of the brace and equivalent viscous damping of the friction forces, and so forth). These effects are investigated in time and frequency domain. While this dissertation is primarily directed at the evaluation of the effects of GMDs in linear SDOF systems, some of the findings can be extended to MDOF systems. The impacts of the application of this control strategy on the response of MDOF structures as well as the effects of the ground motion selection on the behaviour of these systems are also discussed in this chapter.

Finally, in Chapter 6, conclusions of the presented work are summarized, and directions for future work are recommended. The presented study presents the theory and experimental results that are suitable for linear shear-story type systems under certain predefined loading scenarios. The presented models can illustrate many of the essential characteristics of the behaviour of these systems. However, before incorporation of these devices into real structures, further studies have to be carried out on 3-dimensional MDOF structures with nonlinearities to reveal the effects of imperfection sensitivity, load and structural uncertainties, and stability.
Chapter 2. Gyromass Dampers – Background and Modelling

One of the primary objectives of designing dampers is to improve the performance of the uncontrolled system under earthquakes and other possible dynamic loads. However, to ensure that the device provides the appropriate level of performance, the dynamic behaviour of the dampers should be characterized and understood accurately, and extensive numerical and experimental studies are required to achieve this objective. This dissertation investigates the potentials of improving the seismic performance of conventional structures by incorporating GMDs and this chapter, reviews the literature and provides a better understanding of the behaviour of GMDs. After the derivation of the equations for two different prototype specimens for characterizing GMDs, these devices are compared to some other passive control strategies (e.g. TMDs), and their advantages are emphasized.

2.1 Structural control systems

Structural control systems can be employed to enhance the response of structures to various types of dynamic loads such as earthquakes, winds, traffic, and other types of service loads. Based on their operational mechanisms, most of the structural control devices can be classified into four main groups: passive, active, semi-active, and hybrid, as presented in Fig. 2.1 (Saaed et al., 2015). Passive control techniques are perhaps the most widely employed as they are inherently stable, do not require any external energy to operate or structural response measurements and are simpler to design and construct (Christenson, 2001). In these systems, the designer pre-sets the fixed dynamic properties to achieve optimum performance for the intended application. The disadvantage of passive control systems is that for the predetermined dynamic parameters, the efficiency may not be optimal for other cases or other types of dynamic loadings. This is of particular importance because the energy dissipation mechanism is related only to the local response and is entirely dependent on the relative structure movement (Saaed et al., 2015). On the other hand, in active and semi-active systems, the reliance of the control systems on external power, the precise functioning of sophisticated equipment and the fact that for loads due to earthquakes much of this
equipment would need to stay dormant for a long time before being called upon, may not seem convenient. That is why passive control systems continue to be viewed as more practical and reliable than other systems and the problem with the non-optimal performance of the passive systems is usually addressed by assessing the robustness of the passive control device against the detuning effects and under numerous potential loading scenarios both numerically and experimentally.

Figure 2-1. Structural control system categorization (Saaed et al., 2015)
In this section, some of the typical passive control systems are introduced, and the application of the RTHS testing technique in studying each of these systems is discussed in the next section.

2.1.1 Seismic isolation

The possibility of uncoupling a building from the destructive impacts of the earthquake-induced ground motion has appealed to structural engineers for many years. The goal should be to reduce the accelerations in buildings to below the ground accelerations and to achieve this the building must be flexible. Since the flexibility of the primary structure causes numerous problems, the necessary flexibility can only be achieved at the foundation level by the use of base isolation. The technique of seismic isolation is now widely employed in many buildings in the world. For instance, several high-rise buildings have been recently constructed integrating base-isolation systems in Japan (Ariga et al., 2006). Schematics of two types of base isolation systems are shown in Fig. 2-2. Together with the elastomeric and lead-plug bearings, friction pendulum systems are among the most commonly implemented base isolation systems in buildings and bridges (Christopoulos et al., 2006).

![Figure 2-2. Schematic of (a) laminated-rubber bearing, and (b) lead-plug bearing base isolators](image)

A practical isolation system typically consists of three basic elements (Zhang et al., 2002): (1) a base isolation element with a relatively low lateral stiffness which increases the fundamental period of the system to decrease the acceleration response, (2) a supplemental damper or energy-dissipation device to limit the relative displacement across the isolation level, and (3) a component for providing rigidity under small, non-critical loads such as the wind and minor earthquakes. These elements allow the isolation system to partially absorb and reflect the seismic input energy before the transmission of the energy to the structure. The net effect is a reduction of energy dissipation demand on the structural system resulting in an increase in its serviceability.
However, the enhanced performance of the base isolated structures may be achieved at the expense of high base displacement level, which may lead to instability of the structure or impractical design for the base isolators. Thus, the practical implementation of a seismic isolation system represents a tradeoff between force reduction and increased displacement across the isolation system (Christopoulos et al., 2006). In addition, these systems are vulnerable to long-period ground motions, such as some famous earthquakes like Landers (1992) and Chi-Chi (1999), which are characterized by pulse-type wave shape, abundant long-period components, and high ratios of peak ground velocity to peak ground acceleration (Xiang et al., 2014).

2.1.2 Hysteretic devices

Hysteretic dampers dissipate the energy by a mechanism that is independent of loading rate and which can be divided into two groups: metallic dampers, which utilize the yielding of metals to dissipate the energy, and friction dampers, which generate heat by dry sliding friction (Constantinou et al., 1998). These dampers belong to the category of displacement-activated supplemental damping systems (Christopoulos et al., 2006). Metallic dampers take advantage of the hysteretic behaviour of metal when deformed into post-elastic range to dissipate energy while on the other hand, friction dampers dissipate the seismic energy by friction that develops at the interface between two solid bodies sliding relative to each other. Hysteretic dampers in general exhibit hysteretic behaviour that can be idealized by elastic-plastic load-displacement relationships. These types of dampers have been the topic of research interest at University of Toronto ((Gray, 2012) and (Guo, 2015)). In Fig. 2-3, a schematic of a metallic damper is shown. Since the energy input during an earthquake is relatively independent of the restoring force characteristics of the structural system, introduction of hysteretic dampers into the structure can effectively reduce the damage to the main frame (Inoue et al., 1998). A combination of the strength and stiffness of hysteretic dampers for optimizing the damping effect and minimizing the damage to the mainframe is a major design consideration (Christopoulos et al., 2006).

2.1.3 Viscous and viscoelastic devices

The displacement characteristics of viscous and viscoelastic devices depend on the frequency of the motion and relative velocity between the damper ends. There are various types of viscous and
viscoelastic dampers that utilize different materials and mechanisms to dissipate energy, but they commonly share one important property: the damping force in these devices is proportional to velocity or the powers of velocity. Solid viscoelastic dampers use copolymers or glassy substances that dissipate energy through shear deformation. On the other hand, in viscous dampers, the energy is dissipated by pushing a compressible fluid through an orifice from one chamber to another. This action creates a damping effect which turns into a resisting force. The force in a viscous damper and the displacements of the structure are 90 degrees out-of-phase which mean that the damping force does not increase the seismic loads for an equivalent degree of structural deformation. However, one of the main disadvantages of these devices is that it is hard to control the peak structural response in the early stages of loading, mainly due to the dependence of the damper’s resisting force on the velocity (Saaed et al., 2015). A schematic of a fluid viscous damper is shown in Fig. 2-4.

![Figure 2-3. Schematic of a typical metallic damper (Gray, 2012)](image)

A pioneering use of viscoelastic dampers in Civil Engineering structures is the application of 10,000 dampers installed in the twin towers of the World Trade Centre in New York in 1969 to mitigate the effects of wind loads (Mahmoodi et al., 1987). Since then these devices have been extensively studied, and their effects on the performance of building structures as well as the optimal placement of them for practical seismic building design have been investigated in the literature ((Nielsen et al., 1996) and (Makris, 1991)).
Chapter 2. Gyromass Dampers – Background and Modelling

Figure 2-4. Schematic of a typical fluid viscous damper (Haskell et al., 1996)

Fluid type dampers can be designed to behave as nonlinear viscous elements by adjusting their silicone oil and orifice characteristics. The main advantage of nonlinear viscous dampers with velocity exponents of less than one is that, in the event of a velocity spike, the force in the viscous damper is controlled to avoid overloading the damper or the bracing system which it is connected. On the other hand, viscous dampers with larger velocity exponents can be used as lock-up devices (Christopoulos et al., 2006). Since these devices can directly increase the damping of the system, in Chapter 4 a nonlinear viscous damper is employed in parallel to a GMD and the effectiveness of the hybrid system is investigated experimentally.

2.1.4 Dynamic vibration absorbers

With the introduction of the DVAs to an uncontrolled structure, the energy dissipation demand in the primary structure is reduced. This reduction is achieved by transferring (rather than directly dissipating) some of the vibrational energy to the absorber. This secondary system usually includes one or multiple mass, stiffness, and damping elements. With the objective of tuning the dynamic properties of the absorber to those of the primary structure, the elements of DVAs are designed.

The primary applications of the DVAs are for mitigation of wind loads. There are limitations to using these devices for seismic applications due to detuning that may occur as the primary structure yields, high damping level demand and an incapability to control higher modes in an efficient manner (Saaed et al., 2015). These devices can be mainly divided into tuned mass dampers (TMDs), and tuned liquid dampers (TLDs) and tuned liquid column dampers (TLCDs).
A TMD, in its simplest form, consists of an auxiliary mass-spring-dashpot system anchored or attached to the main structure. According to (Soong et al., 1997), the first structure in which a TMD was installed is the Centrepoint Tower in Sydney, Australia. However, since then TMDs have been used in many structures (Fig. 2-5). An extensive list of buildings and towers with TMD applications in chronological orders can be found in (Soto et al., 2013).

![Figure 2-5. TMD in the Taipei 101 tower](image1)

A traditional TMD is less efficient for broad-banded ground motions or ineffective for those narrow banded ground motions of which the predominant frequencies are not close to the primary structure natural frequency. Also, because of the large stroke lengths of TMDs, large space is required for TMDs which for practical applications may be challenging to accommodate (Xiang et al., 2014).

Being similar to a TMD in concept, the mass of a TLD imparts indirect damping to the system. As the primary structure oscillates, the fluid in the tank begins to slosh, applying inertial forces to the structure, out of phase with the structural motion, and thereby reducing the movement. Additionally, the direct damping mechanism in a TLD develops primarily by viscous actions along the boundary layer near the bottom surface and the sidewalls of the TLD tank and the sloshing motion and wave-breaking of the free surface layer of the liquid. By tuning the fundamental frequency of the damping device to the structure’s natural frequency, through a large amount of
sloshing and wave-breaking at the resonant frequencies of the combined TLD-structure system, a significant amount of energy can be dissipated.

The primary difference from a TMD is the amplitude dependent (non-linear) nature of the fluid response. Moreover, some of the drawbacks of TMD systems are not present in TLDs. Due to simple physical concepts on which the restoring force is provided in TLDs, no activation mechanism is required. These devices are actively studied at University of Toronto ((Mirza Hessabi et al., 2012) and (Ashasi-Sorkhabi, 2015)) (Fig. 2-6) and have also been used in many real structures (e.g., 1 King West Tower in Toronto (Cassolato, 2007)).

The mathematical theory involved inaccurately describing the motion of fluid in the container is quite complicated, and the associated uncertainties about the behaviour of these devices under complex types of dynamic loads such as seismic loads are still some of the disadvantages of TLDs. Due to these shortcomings, the application of these devices has been limited to wind loads.

**Figure 2-6.** Different behaviour regimes in a TLD: a) no wave-breaking, b) weak wave-breaking, and c) strong wave-breaking conditions (Mirza Hessabi et al., 2012)

### 2.1.5 Other energy dissipaters

In addition to the devices that have been mentioned in the previous sections, many other developed innovations can be classified as passive energy dissipation devices. Re-centering devices possess
an inherent re-centering capability due to a little residual deformation remaining after load removal. Pressurized fluid dampers (Constantinou et al., 1998) and self-centering energy dissipative braces (Erochko et al., 2013) are some of the examples for re-centering dampers. Other types of passive dampers can employ new materials. For instance, shape memory alloy (SMA) can be used in passive dampers. These metals have the ability to change status between martensitic and austenitic crystalline phases responding to reversible stress or temperature. These dampers can also provide a self-centering mechanism and are insensitive to environmental temperature changes (after being properly heat treated), have excellent fatigue resistance, are corrosion resistance, and are capable of producing large control forces (Saaed et al., 2015). However, there have been no practical applications for this type of material, except for research and experimental investigations.

While most of the other available passive control strategies and technologies that are shown in Fig. 2-1 are likely to have an increasingly important role in structural design (Fig. 2-7), the focus of the present study is on inertial dampers which are a particular type of passive control device that has been investigated recently.

![Figure 2-7. Implementation of passive control systems in North America for seismic applications (Soong et al., 2002)](chartimage.png)
2.2 Inertial dampers

There is a long history of development, design, modelling, analysis and application of various mechanical devices that use gear assemblies. However, their first practical application as a passive control system that uses the inertial forces of rotating masses was reported in the early 2000s by Smith (Smith, 2002), where the term “inertor” was used for them. They have since been used to suppress vibrations of optical tables (Wang et al., 2014) and cables (Lazar et al., 2014) and in Formula 1 racing car suspension systems, under the name of “J dampers” (Chen et al., 2009). In Fig. 2-8 cross section of a J-damper is shown where a threaded rod is connected to one suspension rocker and the casing to the other one. The threaded rod turns the flywheel which in turn spins in the bearings to reduce the vertical movement of the car. Since this damper is mounted between the suspension rockers, it is inactive in body roll and only operates in vertical movement or jounce, and that is why it is called a J-damper (Scarborough, 2008). Subsequently, similar dampers were developed and were used in motorcycles, trains, and regular cars (Smith et al., 2004), (Evangelou et al., 2007) and (Jiang et al., 2012)). In most of these studies, the dynamics of inertors was usually studied by using the electrical circuit analogy (Section 2.3.1). However, although there has been considerable research focused on using inertors in mechanical systems, their application in civil engineering (where they are called “inertial dampers”) and especially structural dynamics is limited.

![Diagram of J-damper](image)

**Figure 2-8.** A schematic of a J-damper in a Formula 1 racing car

A quick survey of existing research papers on the application of inertial dampers in civil engineering structures reveals that the first group of studies investigates inertial dampers in various structures from a purely analytical point of view. In these studies, without defining any particular
mechanical device, a theoretical element that generates acceleration-proportional resisting forces and is capable of producing high equivalent mass is considered and the effects of introducing this device in different structures are assessed.

Some researchers proposed design equations for the optimum design of inertial dampers in different structures. For example, Hu et al. (2015) considered an inertial damper as a part of a configuration to isolate an SDOF structure from the supporting foundation and used the fixed-point theory to derive an optimum value for the dynamic properties of their system. In another study, Asai et al. (2015) which proposed to install the inertial dampers with high damping ratios vertically between the outrigger and perimeter columns of high-rise buildings to achieve large energy dissipation. In their study, optimum design equations are suggested where the modal parameters of the high-rise buildings can be used to determine the equivalent mass ratio and damping ratio of the dampers.

Other researchers, analytically studied the effects of inertial dampers in different structures. In 2004, Saito et al. (2004) assessed the seismic response characteristics of a base-isolated structure to which an inertial damper was applied. Through the analytical study of a multi-lumped mass system, they reached the conclusion that an effective response control of the base isolated structure is possible with an equivalent mass ratio of around 0.10. They also found that the inertial dampers are most affected when they are used in a base isolated system with a considerably long natural period because the response is less sensitive to the effect of input earthquake motion.

Wang et al. (2007) considered three two-dimensional structural models, namely the rigid-body, flexible beam and portal frame models, and introduced inertial dampers to isolate these structures. Based on their simulation results, they found that inertial dampers were deemed effective in suppressing vibrations from both traffic and earthquake vibrations.

In a comprehensive study in 2012, Takewaki et al. (2012) investigated the fundamental mechanisms of seismic response reduction in MDOF building structures with inertial dampers. They found that the effects of inertial dampers in a structure can be modelled by introducing an influence coefficient vector to the right-hand side of the equations of motion. They also showed that, by removing one of the inertial dampers from a particular story in an MDOF building, the
dampers located above that story become ineffective in controlling the input accelerations. From numerical simulations on a 12-story model with inertial dampers under three different earthquake ground motions they observed that the inertial dampers were effective for the reduction of the maximum absolute horizontal acceleration of floors and the maximum base shear. Maximum horizontal displacement and horizontal velocity were also decreased to some extent. However, they noticed that some of the maximum interstory drifts might be increased.

In 2014, Lazar et al. (2014a) modelled inertial dampers inside an MDOF building and showed that vibration reduction can be achieved and that the TMD and inertial damper systems performances were almost identical when they were tuned at the same mass ratio. They also confirmed that the best structural response could be obtained with the inertial damper installed at the bottom story level, connected to the ground. Their design approach can also be used to design inertial dampers at different levels in an MDOF building. In another study (Lazar et al., 2014b), they examined the performance of four five-story structural systems to show the advantages brought by the inertial damper's capacity of generating extra apparent mass. They evaluated the performance of an uncontrolled structure, a structure having a TMD installed at the top, a third structure having an inertial damper installed at the bottom and a structure having a viscous damper installed at the bottom. Their analysis demonstrated that unlike TMDs that can only control the mode for which they are tuned for, inertial dampers could enhance the response by affecting the higher vibration modes. Moreover, they found that the inertial damper is most efficient when located at the bottom of the structure. This can be a potential advantage compared with TMD installation. In another study by the same group, a three-story building model with an inertial damper at the bottom was considered under a base excitation loading case (Zhang et al., 2016).

Marian and Giaralis (2014) proposed a novel structural control system called the tuned mass–damper–inerter (TMDI) (Fig. 2-9) and through numerical simulations concluded that the incorporation of the inertial damper in TMDI systems could improve the performance of the existing TMDs or alternatively, the inertial damper could be used in parallel to a TMD to reduce the need for large physical TMD masses.

In general, three types of inertial dampers have been proposed and tested experimentally: (1) hydraulic, (2) ball-screw and (3) rack and pinion type of devices (Papageorgiou et al., 2008).
Chapter 2. Gyromass Dampers – Background and Modelling

In the first group of inertial dampers, large resisting forces are generated through a hydraulic mechanism. The idea of a hydraulic inertial damper was proposed by (Smith, 2008), which suggested the use of a gear pump to convert the linear motions into rotational motion. In 2011, Wang et al. (Wang et al., 2011) proposed replacing the gear pump with a hydraulic motor and discussed the dynamics of the device and experimentally verified the derived equations. The schematic of their hydraulic inertial damper is shown in Fig. 2-10. The equivalent mass of a hydraulic inertial damper can be calculated from the following equation,

\[ b = I \times \left( \frac{A \dot{\theta}}{Q_{in}} \right)^2 \]  \hspace{1cm} (2-1)

where, \( I \) is the moment of inertia of the motor and the flywheel, \( A \) is the area of the piston, \( \dot{\theta} \) is the angular velocity of the motor and finally, \( Q_{in} \) is the input flow rate of the hydraulic cylinder.

Figure 2-9. A schematic of a TMDI (Marian and Giaralis, 2014)

Figure 2-10. The working principle of the hydraulic inertial damper (Wang et al., 2011)
In the second type, a ball-screw mechanism is used. This configuration is very similar to the original inerter proposed by Smith (Smith, 2002) (Fig. 2-11) and thus in comparison to the other two types, has been studied more extensively. In these devices, the rotary ball-screw converts the axial movement into rotary movement thereby the axial velocity is amplified and applied to a viscous material. In 2010, (Wang et al., 2010) studied the performance of one- and two-dimensional SDOF structures that were isolated with ball-screw inerters at the base level and based on their experimental results on a small-scale damper, concluded that these dampers are effective in suppressing vibrations from both traffic and earthquakes. In this paper, the equivalent mass of the screw-ball damper is related to the pitch of the screw $p$ (in units of m/rev) and the inertia of the flywheel $J_{fw}$,

$$b = J_{fw}(2\pi/p)^2$$

(2-2)

Based on this model they reported that the responses of theoretical and experimental models were similar at low-frequency ranges, but were different at higher frequency ranges. There also no discussion regarding the properties of the input earthquakes are included in the study which makes the generalization of the study findings to full-scale buildings under various types of earthquakes uncertain.

In Japan, a group of researchers (Ikago et al., 2012), combined the ball-screw mechanism with a viscous damper and called this type of inertia-based dampers “Tuned Viscous Mass Dampers (TVMDs)” (Fig. 2-12). They derived a closed-form optimum design equation for the TVMD.
vibration control system to minimize the peak amplitude of the resonance curve in undamped structures and verified their numerical simulation by conducting shake table tests.

Figure 2-12. A schematic of a TVMD (Asai et al., 2015)

More information about the development and verification of TVMDs can be found in other publications of these authors. For instance, (Watanabe et al., 2012) tested a prototype specimen with an apparent mass of 5400 t. Since this large equivalent mass can be problematic for the structure, in another experiment, (Ikenaga et al., 2012) used a shaking table to test a force restriction mechanism for the damper to limit the maximum damping force without deteriorating the maximum displacement reduction of the primary structure. A simple design method for TVMDs in MDOF structures is presented in (Ikago et al., 2012) and the real-life application of these devices in a steel building in Japan is demonstrated in (Sugimura et al., 2012).

In the third type, using a rack and assembly of gears, the relative translation of the terminals is transformed into the rotation of the gears (Papageorgiou et al., 2008). In this group of inertial dampers, the equivalent mass of the device can be easily adjusted and unlike TVMDs, there is no viscous damper component inside these dampers. The lack of the necessity of employing a viscous medium inside the damper leads to reduced maintenance requirements. In addition, the simple assembly of this type of inertial dampers makes them an effective alternative solution for the vibration control of building structures in various parts of the world (Mirza Hessabi et al., 2015a) and (Mirza Hessabi et al., 2015b). Unlike the other types, the performance of the rack and pinion inertial dampers has still not been well studied and thus the current study investigated this kind of
inertial dampers in more detail. At the time of writing this thesis, the only study on the application of rack and pinion type inertial dampers in buildings has been carried out by (Saitoh, 2012).

In his study, Saitoh uses purely numerical studies in the frequency domain and in the time domain with various types of earthquake waves to investigate the effectiveness of three different types of systems incorporating an inertial damper to mitigate the large displacements of a base isolated structure at the base level. They found that unless they use a control strategy in which viscous dampers and gyromass dampers were employed in a parallel configuration, the controlled systems still underwent large displacements when it was subjected to earthquake ground motions with long-period components. The device considered in the current study is similar to the damper in the study of (Saitoh, 2012) and hence the same terminology, i.e. “gyro-mass damper”, is used herein. In his study, Saitoh proposed a mechanical device that uses a rack and pinion mechanism and used the model of Section 2.3.2 to describe the behavior of that device. However, the study did not contain any experimental verification for the derived equations. In the following sections, a practical design is presented, and GMD prototypes are built, and tested. The lack of research on rack-and-pinion devices is addressed, and the behavior of these devices is studied, and the effects of various resisting force components are modelled.

Table 2-1 summarizes the relevant empirical studies on various types of inertial dampers. The survey of the available studies on inertial dampers reveals that, (1) none of the previous studies on developing an inertial damper for structural engineering applications look at the identification of these devices in detail and there is need for an experimental study to examine the effects of input frequencies and amplitude dependency of the response, the effectiveness of the inertial component and the size effects, and (2) in comparison to other types of inertial dampers, there is a lack of
experimental study on the development and modelling of rack and pinion type inertial damper. In the following sections, the behaviour of a particular type of rack and pinion inertial damper, i.e., GMD, is studied.

Table 2-1. Relevant experimental studies on various types of inertial dampers

<table>
<thead>
<tr>
<th>Study</th>
<th>Year</th>
<th>Inertial damper type</th>
<th>Equivalent mass $b$ (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Wang et al., 2011)</td>
<td>2011</td>
<td>Hydraulic</td>
<td>424</td>
</tr>
<tr>
<td>(Wang et al., 2008)</td>
<td>2008</td>
<td>Ball-screw</td>
<td>113</td>
</tr>
<tr>
<td>(Wang et al., 2010)</td>
<td>2010</td>
<td>Ball-screw</td>
<td>115</td>
</tr>
<tr>
<td>(Ikago et al., 2012)</td>
<td>2012</td>
<td>Ball-screw (TVMD)</td>
<td>350</td>
</tr>
<tr>
<td>(Watanabe et al., 2012)</td>
<td>2012</td>
<td>Ball-screw (TVMD)</td>
<td>$5400 \times 10^3^*$</td>
</tr>
<tr>
<td>(Nakamura et al., 2014)</td>
<td>2014</td>
<td>Ball-screw (EIMD)</td>
<td>$6000 \times 10^3^*$</td>
</tr>
<tr>
<td>(Papageorgiou et al., 2008)</td>
<td>2009</td>
<td>Rack and pinion</td>
<td>69 ~ 700</td>
</tr>
</tbody>
</table>

2.3 Modelling the GMDs: available models in the literature

2.3.1 Electrical-mechanical analogy

Inertial dampers were widely used in Mechanical Engineering applications before being introduced to Civil Engineering and therefore, many of the works on these dampers use the network analogy. The use of the force-current analogy or electrical-mechanical analogy allows the classical theorems in electrical engineering in terms of resistors, capacitors, and inductors to be translated directly into the mechanical context. The force-current analogy between mechanical and electrical networks has the following correspondences (Smith et al., 2004):

\[
\text{force} \leftrightarrow \text{current} \\
\text{velocity} \leftrightarrow \text{voltage} \\
\text{spring} \leftrightarrow \text{inductor} \\
\text{viscous damper} \leftrightarrow \text{resistor}
\]

If the resisting force in an inertial damper is assumed to be proportional to the applied accelerations or $f = b \ddot{u}$ (from Eq. 2-26), the inertial damper can be shown by a capacitor. Additionally, the

* Apparent equivalent mass
mass element can be taken as the analog of a capacitor with one terminal connected to ground. The circuit symbols and the correspondence mechanical components are shown in Fig. 2-14.

<table>
<thead>
<tr>
<th>Mechanical</th>
<th>Electrical</th>
</tr>
</thead>
<tbody>
<tr>
<td>( F )</td>
<td>( i )</td>
</tr>
<tr>
<td>( \frac{dF}{dt} = k(v_2 - v_1) ) spring</td>
<td>( \frac{di}{dt} = \frac{1}{L}(v_2 - v_1) ) inductor</td>
</tr>
<tr>
<td>( F = b \frac{d(v_2 - v_1)}{dt} ) inertial damper</td>
<td>( i = C \frac{d(v_2 - v_1)}{dt} ) capacitor</td>
</tr>
<tr>
<td>( F = c(v_2 - v_1) ) viscous damper</td>
<td>( i = \frac{1}{R}(v_2 - v_1) ) resistor</td>
</tr>
</tbody>
</table>

Figure 2-14. Circuit symbols and the correspondences between mechanical and electrical elements (Smith et al., 2004)

In Fig. 2-15 the application of these analogies in modelling an inertial damper with viscous damper and springs is shown. Although the force-current analogy does not provide any new information about the behaviour of the GMDs, it can be used as an alternate tool to study the performance of these devices in larger systems.

Figure 2-15. Mechanical network circuit diagram for inertial damper-viscous damper series connection with centering springs (Smith et al., 2004)
2.3.2 Modelling the friction forces

Modelling of dry friction has been the topic of active scientific research since Coulomb’s hypothesis (Coulomb, 1785). The significant effects of friction forces on the behaviour of inertial dampers have been mentioned in some of the studies in the literature (e.g. (Wang et al., 2008)). This will be confirmed in Chapter 4 where it is shown that the friction forces contribute to the total resisting forces significantly. Since, the primary objective of this study is to propose easy-to-use equations that can describe the behaviour of GMDs with an acceptable accuracy, one of the simplest models for characterizing friction forces, i.e., the dry friction model, is used in Chapter 4. This simplified approach gives the first approximation and allows the structural engineers to use the equations and design control systems that include GMDs. Still, for device assessment purposes and in the presence of necessary measurement instruments the existing friction forces can be described by more complicated models. More complicated friction models employ auxiliary state variables to determine the friction forces. The time evolution of these state variables is given by additional differential equations which results in more complex models that consider more complicated phenomena such as the memory effect (Fig 2-16).

For instance, Eq. 2-3 and 2-4, shows a form of the dynamic Dahl model (Bliman, 1992), where $F_f$ is the kinetic friction force, $\mu$ is the Coulomb’s friction coefficients, $z$ is the state variable and $k$ is the stiffness coefficient and should be measured.

![Figure 2-16](image_url)
There are several different techniques for modelling the dynamic friction forces and express the relationship between the friction force and the relative velocity. A review of some of the most commonly used models that describe the hysteretic effects of dry friction can be found in (Wojewoda et al., 2008).

2.3.3 Inclusion of the torsional stiffness of the gear shafts

Over the last decades, due to the high importance of gear components in industrial and mechanical machinery, researchers have been motivated to study various aspects of gears modelling such as vibration analysis and noise control, transmissions errors and stability analysis. Based on the application of the gear system, the objective of these studies differ. Various models have been developed in the literature to specifically investigate stress analysis such as bending and contact stresses, reduction of surface pitting and scoring, transmission efficiency, radiated noise, loads on the other machine elements of the system especially on bearings and their stability regions, reliability and fatigue life, etc. (Wang, 2003). Many of these studies consider complex finite elements method (FEM) models to evaluate the behaviour of high-speed gearing and noise and as such the findings of many of the previous studies on gear dynamics are not applicable to this study.

The models for gear dynamics usually include the flexibility of the other elements as well as the tooth compliance (Gregory et al., 1963). Specifically, the shaft torsional flexibility and the lateral flexibility of the bearings and shafts along the line of action are two important factors that can be modelled. For instance, for the large scale prototype in this study (shown in Fig. 2-23), the kinetic energy $T$ can be calculated as,

$$T = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} J_2 \left(\frac{r_1}{r_2}\right)^2 \dot{\theta}_2^2 + \frac{1}{2} J_3 \dot{\theta}_3^2 + \frac{1}{2} J_4 \left(\frac{r_3}{r_4}\right)^2 \dot{\theta}_4^2 + \frac{1}{2} J_5 \dot{\theta}_5^2$$  \hspace{1cm} (2-5)

The strain energy of the total structure is the sum of the strain energies of the two shafts. Their strain energies are (Donaldson, 2006),
Thus, the matrix equations of motion are,

\[
\begin{bmatrix}
J_1 + J_2 \left( \frac{r_1}{r_2} \right)^2 & 0 & 0 \\
0 & J_3 + J_4 \left( \frac{r_3}{r_4} \right)^2 & 0 \\
0 & 0 & J_5
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_1 \\
\ddot{\theta}_3 \\
\ddot{\theta}_5
\end{bmatrix}
+ \begin{bmatrix}
g_{1sh1} \left( \frac{r_1}{r_2} \right)^2 & -\frac{r_1}{r_2} & -\frac{r_3}{r_4} \\
-\frac{r_1}{r_2} & 1 + \frac{g_{2sh2}}{L_2} \left( \frac{r_2}{r_4} \right)^2 & 0 \\
-\frac{r_3}{r_4} & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\theta_1 \\
\theta_3 \\
\theta_5
\end{bmatrix}
= \begin{bmatrix}
-f \times r_1 \\
0 \\
0
\end{bmatrix}
\]

where, \( G \), \( J_{sh} \) and \( L \) are the modulus of rigidity (shear modulus), torsional constant and length of the shafts, respectively and other parameters are defined in Fig. 2-17.

\[U_1 = \frac{1}{2} \left[ \begin{array}{c} r_1 \theta_1 \\ r_2 \end{array} \right] \times \frac{g_{1sh1}}{L_1} \times \left[ \begin{array}{c} +1 \\ -1 \end{array} \right] \left[ \begin{array}{c} r_3 \theta_1 \\ \theta_3 \end{array} \right]
\]

\[U_2 = \frac{1}{2} \left[ \begin{array}{c} r_3 \theta_3 \\ r_4 \end{array} \right] \times \frac{g_{2sh2}}{L_2} \times \left[ \begin{array}{c} +1 \\ -1 \end{array} \right] \left[ \begin{array}{c} r_3 \theta_3 \\ \theta_5 \end{array} \right]
\]

\[\textbf{Figure 2-17. Torsional modelling of the gear shafts}\]
2.3.4 Comparison of GMDs with other passive control devices

Some of the common passive control strategies and devices were introduced in Section 2.1. Similar to base isolation systems, the introduction of a stand-alone GMD to an SDOF system can affect the natural period of the system and increase it. In addition, similar to base isolation system, the application of an energy dissipation component in parallel to GMDs is advantageous. However, unlike base isolation systems in which the excessive deformation of the isolation layer is problematic for the structural response, the gears in a GMD are fixed, and they spin freely around their axes. As it will be discussed in the presented study, GMD prototype specimens also exhibit hysteretic behaviour and the existence of friction forces in the specimens results in some energy dissipation. However, since the acceleration-proportional components of the resisting force in GMDs is considerably larger than the friction forces, GMDs will not be classified as hysteretic dampers. By introducing the acceleration-proportional resisting force, GMDs increase the inertial terms of the system and hence, it is common in the literature to draw a comparison between various types of DVAs and GMDs (i.e., inertial dampers in general) and several previous studies have shown the similarities between these two control systems (Lazar et al., 2014). However, it should be noted that there are several major differences between these two control strategies. TMDs require a relatively large mass and thus a large space for their installation. The equivalent mass of GMDs can be modified by changing the gear ratios and employing multiple compound gears. Thus, GMDs can provide a similar level of force while occupying a much smaller space. Moreover, by design, TMDs are in resonance with the primary structure, and they undergo large displacements which need to be accommodated. As mentioned before, the free spinning of the gears of the GMDs does not impose any such problems.

2.3.5 Saitoh’s model for rack and pinion dampers

In 2012, Saitoh used a model in which the rotational mass was placed at the end of the gear train. As it can be seen in Fig. 2-18, in the prototype that was considered in (Saitoh, 2012), a rotating disk was used as the rotational mass. It is assumed in (Saitoh, 2012) that the relative acceleration of the rack with respect to a fixed node is geometrically related to the rotational acceleration and consequently, the following relation between the external force $f$ and the relative acceleration $\ddot{u}$ can be obtained.
\[ f = b \ddot{u} \quad (2-9) \]

In Eq. 2-9, the proportionality constant \( b \) is called the *equivalent mass* and Saitoh assumes that it is proportional to the inverse of the \( r \) squared,

\[ b \propto \frac{J}{r^2} \quad (2-10) \]

where, \( r \) is the distance from the centre of the disk to the point where the rod is attached to the disk and \( J \) is the moment of inertia of the disk. Based on this assumption, Saitoh proposed the following equation for the equivalent mass of GMDs:

\[ b = \frac{J}{r^2} \left( N_g \right)^2 \quad (2-11) \]

where, \( N_g \) is the gear ratio of the compound disk gears and can be expressed as the ratio between the number of teeth of the disk gears. It should be mentioned that the model of Eq. 2-11 is similar to a previous model that was proposed by Smith (Smith, 2003) at a conference in Japan. However, in Saitoh’s model, the \( J \) term can account for the mass moment of inertia of any other rotational mass which is not a disk.

![Prototype mechanical system of gyromass](image)

*Figure 2-18. Prototype mechanical system of gyromass in (Saitoh, 2012)*

In Eq. 2-11, the equivalent mass of the device is simply expressed as the product of the gear amplification factor by the mass moment of inertia of the rotating mass divided by the radius of the first gear. This equation is very easy to use but has some limitations. Eq. 2-11 does not consider the effects of the middle gears, and it ignores the mass moment of inertia of any other gear in the
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gear train except the rotating disk. This assumption may lead to significant errors if the middle gears have relatively larger radii than the rotating disk or if there are more than three compound gears in the device. This limitation has been addressed in the next section where based on the Free-body-diagrams (FBDs) of the gears, new equations are developed. Also, note that the equivalent mass in this model is independent of the radii of the gears which is not true if various gears with different radii are used. This model will be numerically compared to other models in the next section and will be compared to the experimental results in Chapter 4.

2.4 Modelling the GMDs: prototype specimens in this study

GMDs use gear mechanisms to counteract dynamic forces by transforming the transitional motion into rotational one. Consequently, the dynamic properties of GMDs depend on the arrangement and dimensions of the physical components. In this study, two GMD prototypes are built and tested. The first prototype is a 25 cm × 6 cm × 8 cm device that weighs 1.495 kg. This prototype is shown in Fig. 2-19 and will be referred to as the “small-scale prototype.” The second prototype or the “larger-scale prototype is a 61 cm × 18 cm × 50 cm device which is made of steel plates and weighs 125.7 kg (Fig. 2-19). The experimental results for these two prototypes are presented in Chapter 4. In this section, more detail about each of these prototypes such as their properties and their equivalent mass is provided.

Figure 2-19. Small-scale GMD prototype
Chapter 2. Gyromass Dampers – Background and Modelling

2.4.1 Small-scale prototype

To explore the validity of the existing characteristic equations of GMDs from the literature, a small-scale prototype (Fig. 2-19) was built and tested. To reduce the self-weight of the prototype (a total physical mass equal to 1.495 kg), the components of the small-scale prototype were chosen from a VEX robotics catalog and all components were attached by nuts and screws.

2.4.1.1 Derivation of the equivalent mass equation for the small-scale prototype

In many ways, gears in rotating systems act similarly to levers in translating systems and FBDs of the gears can be used to determine the equivalent mass of the device. The side-view of the small-scale GMD prototype is shown in Fig. 2-20, where the GMD is subjected to a time-varying relative axial force \( f(t) \). This device consists of two simple (gears 1 and 6) and two compound (gears 3 and 5 and pinions 2 and 4) gears.

\[
\begin{align*}
\text{Figure 2-20. Side-view of the gears in the small-scale GMD prototype}
\end{align*}
\]

Similar to other gear mechanisms, the smaller gears in the gear train (i.e., gear 2 and gear 4) can be referred to as pinions. The forces acting on each of these gears are determined by considering the equilibrium between the gears in contact. Unlike the study of (Saitoh, 2012), the rotational inertias of the intermediate gears are also considered in the following derivation. FBDs of the gears are shown in Fig. 2-21. In these diagrams, contact forces are tangent to both gears and produce a torque that is equal to the radius times the force.
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Application of Real-Time Hybrid Simulation Method in Experimental Identification of GMDs

By neglecting the mass of the rack \( m_{\text{rack}} = 205 \text{ g} \) and using the FBDs in Fig. 2-21, the corresponding equations of motion of the first and last gears can be written as,

\[
J_1 \ddot{\theta}_1 - f_1 r_1 + f_{1-2} r_1 = 0 \tag{2-12}
\]

\[
-J_6 \ddot{\theta}_6 + f_{5-6} r_6 = 0 \tag{2-13}
\]

Since pinion 2 and gear 3 form a compound gear that rotates as a single object, Eq. 2-14 can be used to obtain the equation of motion of the compound gear:

\[
-J_2 \ddot{\theta}_2 - J_3 \ddot{\theta}_3 + f_{1-2} r_2 - f_{3-4} r_3 = 0 \tag{2-14}
\]

Similarly, Eq. 2-15 can be used for the second compound gear (i.e., pinion 4 and gear 5).

\[
J_4 \ddot{\theta}_4 + J_5 \ddot{\theta}_5 - f_{3-4} r_4 + f_{5-6} r_5 = 0 \tag{2-15}
\]

These equations should be rewritten to eliminate \( f_{1-2}, f_{3-4} \) and \( f_{5-6} \) terms. From Eq. 2-13:

\[
f_{5-6} = \left( \frac{l_6}{r_6} \right) \ddot{\theta}_6 \tag{2-16}
\]

Substituting \( f_{5-6} \) in Eq. 2-15 leads to,

\[
f_{3-4} = \left( \frac{l_6}{r_6} \right) \left( \frac{r_6}{r_4} \right) \ddot{\theta}_6 + \left( \frac{l_5}{r_6} \right) \ddot{\theta}_5 + \left( \frac{l_4}{r_4} \right) \ddot{\theta}_4 \tag{2-17}
\]

This procedure should be repeated to eliminate \( f_{1-2} \).

\[
f_{1-2} = \left( \frac{l_2}{r_2} \right) \ddot{\theta}_2 + \left( \frac{l_3}{r_2} \right) \ddot{\theta}_3 + \left( \frac{l_6}{r_6} \right) \left( \frac{r_6}{r_4} \right) \ddot{\theta}_6 + \left( \frac{l_5}{r_4} \right) \left( \frac{r_5}{r_2} \right) \ddot{\theta}_5 + \left( \frac{l_4}{r_4} \right) \left( \frac{r_4}{r_2} \right) \ddot{\theta}_4 \tag{2-18}
\]
Now, by substituting this term in Eq. 2-12, \( f_1 = f(t) \) can be computed,

\[
f_1 = \left( \frac{l_1}{r_1} \right) \ddot{\theta}_1 + \left( \frac{l_2}{r_2} \right) \ddot{\theta}_2 + \left( \frac{l_3}{r_3} \right) \ddot{\theta}_3 + \left( \frac{l_4}{r_4} \right) \ddot{\theta}_4 + \left( \frac{l_5}{r_5} \right) \ddot{\theta}_5 + \left( \frac{l_6}{r_6} \right) \ddot{\theta}_6 + \left( \frac{j_3}{r_3} \right) \left( r_3 \right) \ddot{\theta}_3 + \left( \frac{j_4}{r_4} \right) \left( r_4 \right) \ddot{\theta}_4 + \left( \frac{j_5}{r_5} \right) \left( r_5 \right) \ddot{\theta}_5 \]  

(2-19)

where, \( r_i \) and \( J_i \) denote the radius and mass moment of inertia of the \( i \)th gear, respectively. In the above equations of motion, an ideal efficiency is assumed, and the damping effects of gear friction or backlash are neglected. Since the rotation of the first gear is related to the applied acceleration, the following equation can be written:

\[
\ddot{u} = r_1 \ddot{\theta}_1 \rightarrow \ddot{\theta}_1 = \ddot{u} / r_1 
\]  

(2-20)

Now, the geometric relationship between the arc lengths of the gears should be used to express the rotation of each gear in terms of the applied acceleration:

\[
\ddot{\theta}_2 = \frac{r_1}{r_2} \ddot{\theta}_1 \rightarrow \ddot{\theta}_2 = \ddot{u} / r_2 
\]  

(2-21)

\[
\ddot{\theta}_3 = \ddot{\theta}_3 \text{ and } \ddot{\theta}_4 = \frac{r_3}{r_4} \ddot{\theta}_3 \rightarrow \ddot{\theta}_4 = \frac{r_3 r_1}{r_4 r_2} \ddot{\theta}_1 = \frac{r_3}{r_4 r_2} \ddot{u} 
\]  

(2-22)

\[
\ddot{\theta}_5 = \ddot{\theta}_5 \text{ and } \ddot{\theta}_6 = \frac{r_5}{r_6} \ddot{\theta}_5 \rightarrow \ddot{\theta}_6 = \frac{r_5 r_3 r_1}{r_6 r_4 r_2} \ddot{\theta}_1 = \frac{r_5 r_3}{r_6 r_4 r_2} \ddot{u} 
\]  

(2-23)

And since \( f = f_1 \), thus,

\[
f = \left( \frac{l_1}{r_1} \right) \left( \ddot{u} \right) + \left( \frac{l_2}{r_2} \right) \left( \ddot{u} \right) + \left( \frac{l_3}{r_3} \right) \left( \ddot{u} \right) + \left( \frac{l_4}{r_4} \right) \left( \frac{r_3}{r_4} \right) \left( \ddot{u} \right) + \left( \frac{l_5}{r_5} \right) \left( \frac{r_3 r_1}{r_4 r_2} \ddot{u} \right) + \left( \frac{l_6}{r_6} \right) \left( \frac{r_3 r_5}{r_6 r_4 r_2} \ddot{u} \right) + \left( \frac{j_3}{r_3} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_4}{r_4} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_5}{r_5} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_2}{r_2} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_3}{r_3} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_4}{r_4} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_5}{r_5} \right) \left( \ddot{\theta}_2 \right) + \left( \frac{j_6}{r_6} \right) \left( \ddot{\theta}_2 \right) \]  

(2-24)

After simplifying this equation, the following equation can be used to show the force-acceleration relationship in the GMD,

\[
f = \left\{ \left( \frac{l_1}{r_1} \right) + \left( \frac{l_2}{r_2} \right) + \left( \frac{l_3}{r_2 r_4} \right)^2 j_4 + \left( \frac{r_3}{r_4 r_2} \right)^2 j_5 + \left( \frac{r_3 r_5}{r_6 r_4 r_2} \right)^2 j_6 \right\} \ddot{u} 
\]  

(2-25)

The following equation can be used to summarize Eq. 2-25,

\[
f = b \ddot{u} 
\]  

(2-26)
Once again, parameter $b$ in Eq. 2-26 is the equivalent mass and for the considered GMD prototype is equal to,

$$b = \left( \frac{J_1}{r_1^2} \right) + \left( \frac{J_2}{r_2^2} \right) + \left( \frac{r_3}{r_2 r_4} \right)^2 J_4 + \left( \frac{r_3}{r_2 r_4} \right)^2 J_5 + \left( \frac{r_3 r_5}{r_6 r_4 r_2} \right)^2 J_6$$

(2-27)

Thus, the force generated in a GMD is proportional to the relative translational acceleration with $b$ as the proportionality constant.

### 2.4.1.2 Calculation of the equivalent mass of the small-scale prototype

Three different models are presented here to calculate the equivalent mass $b$ of the small-scale GMD prototype. The first model is based on Eq. 2-11 (Saitoh, 2012) and will be referred to as the “Saitoh’s model.” The second and third models are based on the FBD of the GMD gears and use Eq. 2-27 to calculate the value of the equivalent mass. However, the mass moment of inertia of the gears ($J_i$), is calculated differently in these two models. In the second model, gears are considered to be continuous disks (i.e., the “disk model”), whereas in the third model (i.e., the “lumped-mass model,” rotational mass of the gears is concentrated at the point of contact between the gears.

As it is shown in Fig. 2-20, the small-scale prototype consists of a rack, four gears, and two pinions. The diameters of the gears and pinions are 1.27 cm (0.5”) and 6.35 cm (2.5”), respectively. Each of the gears and pinions has the mass of 0.907 g and 25 g, respectively. The two compound gears in this prototype have a gear ratio that is equal to 5:1, this the equivalent amplification factor of the device is equal to $N_g = 5 \times 5 = 25$.

The first model that is used here to calculate $b$, is the Saitoh’s model. After substituting $J$ in Saitoh’s model with the mass moment of inertia of the last gear $r_6^2 m_{g_6}/2$, and $r$ with $r_1$, Eq. 2-5 can be rewritten as:

$$b = \frac{m_{g_6}}{2} \left( \frac{r_6}{r_1} \right)^2 \left( N_g \right)^2$$

(2-28)

which leads to an equivalent mass $b$ of 7.81 kg. This value is 312.5 times larger than the mass of the last rotating gear which shows that a significantly large equivalent mass can be produced with a small disk mass in a compact space.
The second model is the disk model. By assuming that the gears are solid homogeneous circular sections, mass moment of inertia of the gears can be expressed as \( J_1 = \left( \frac{1}{2} \right) m_{g1} r_1^2 \), \( J_2 = \left( \frac{1}{2} \right) m_{g2} r_2^2 \), \( J_3 = \left( \frac{1}{2} \right) m_{g3} r_3^2 \), \( J_4 = \left( \frac{1}{2} \right) m_{g4} r_4^2 \), \( J_5 = \left( \frac{1}{2} \right) m_{g5} r_5^2 \), and \( J_6 = \left( \frac{1}{2} \right) m_{g6} r_6^2 \). Substituting these values in Eq. 2-27 yields,

\[
b = \left( \frac{1}{2} \right) \left\{ m_{g1} + m_{g2} + \left( \frac{r_3}{r_2} \right)^2 m_{g3} + \left( \frac{r_4}{r_2} \right)^2 m_{g4} + \left( \frac{r_5 r_3}{r_2 r_4} \right)^2 m_{g5} + \left( \frac{r_5 r_3}{r_2 r_4} \right)^2 m_{g6} \right\} \quad (2-29)
\]

Now, the gear ratios of the compound gears are defined as \( N_{g_2} = r_3/r_2 \) and \( N_{g_3} = r_5/r_4 \). It can be shown that the number of teeth on a gear is proportional to the radius of its pitch circle, which means that the ratios of the number of teeth of the corresponding can also be used for the gear ratios interchangeably. Thus, by replacing the gear ratios with their respective expressions, the equivalent mass relationship in the disk model can be rearranged as Eq. 2-30,

\[
b = \left( \frac{1}{2} \right) \left\{ m_{g1} + m_{g2} + N_{g2}^2 \left( m_{g3} + m_{g4} \right) + N_{g3}^2 \left( m_{g5} + m_{g6} \right) \right\} \quad (2-30)
\]

where, \( m_{g_i} \) shows the mass of the \( i \)th gear. In the small-scale prototype, gears have the same radii (i.e., \( r_1 = r_3 = r_5 = r_6 \)) and also for pinions \( r_2 = r_4 \). Using this equation, the equivalent mass of the small-scale GMD can be calculated as follows,

\[
b = \left( \frac{1}{2} \right) \left\{ 25 + 0.91 + 5^2 (25 + 0.91) + (5 \times 5)^2 (25 + 25) \right\} \times 10^{-3} = 15.96 \text{ kg} \quad (2-31)
\]

It can be seen that in comparison to the total physical mass of the prototype which is 1.495 kg, the equivalent mass is significantly larger.

Finally, the equivalent mass of the small-scale prototype can be calculated from the lumped mass model. In this model, the rotational mass of the gears is lumped at the contact points between the gears. As a result, the equivalent mass of the prototype can be found from the following equation:

\[
b = m_{g1} + m_{g2} + \left( \frac{r_3}{r_2} \right)^2 m_{g3} + \left( \frac{r_4}{r_2} \right)^2 m_{g4} + \left( \frac{r_5 r_3}{r_2 r_4} \right)^2 m_{g5} + \left( \frac{r_5 r_3}{r_2 r_4} \right)^2 m_{g6} \quad (2-32)
\]

After substituting the corresponding values in Eq. 2-27, the equivalent mass \( b \) of the small-scale prototype can be calculated as 31.92 kg. It can be seen that \( b \) from the lumped mass model has the
largest value and is approximately four times larger than the equivalent mass from Saitoh’s model. The results for all three models are summarized in the first row of Table 2-2 (i.e., no extra mass).

![Table 2-2. Equivalent mass of the GMD prototype with various number of extra screws](image)

<table>
<thead>
<tr>
<th>Actual mass of the prototype (kg)</th>
<th>Equivalent mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Saitoh’s Model</td>
</tr>
<tr>
<td>No extra mass</td>
<td>1.495</td>
</tr>
<tr>
<td>2 extra screws</td>
<td>1.501</td>
</tr>
<tr>
<td>4 extra screws</td>
<td>1.508</td>
</tr>
<tr>
<td>8 extra screws</td>
<td>1.520</td>
</tr>
</tbody>
</table>

The discrepancy between the predicted values from the models in Table 2-2 can be attributed to their different underlying assumptions. In the Saitoh’s model, the effects of the rotational inertia of the intermediate gears have been neglected and hence the predicted values are smaller than the other two models. The disk and lumped-mass models also consider different formulas for calculating the mass moment of inertia of the rotating gears. The success of each of these models in predicting the measured equivalent masses will be investigated in the following sections.

To assess Eq. 2-28, 2-30, and 2-32 for different values of equivalent masses and to increase the equivalent mass of the prototype without changing the gears, small metal screws were attached to the last gear and the mass moment of inertia of the last gear was thereby increased. Each of these metal parts weighs 3.125 g, and as it is shown in Fig. 2-22, they are located at a distance of 1” (2.54 cm) away from the centre of the gear. The equivalent mass for GMD prototypes with 2, 4 and 8 extra screws obtained from each model are shown in Table 2-2.

2.4.2 Larger scale prototype

To explore the size effects and with the objective of developing larger prototypes that can be used in more practical structural systems, a larger scale prototype was built and tested. This device is shown in Fig. 2-23. Because of the modifications that will be discussed in Section 2.5, Eq. 2-25 can no longer be used to determine the equivalent mass of the device and a new formula has to be derived for this purpose.
Figure 2-22. Attachment of 4 extra screws to increase the b of the small-scale prototype

Figure 2-23. Top view of the larger scale prototype

2.4.2.1 Derivation of the equivalent mass equation for the larger scale prototype

The side-view of the prototype and free-body-diagram of one side of the device is shown in Fig. 2-24. Using the new FBD in this figure, Eq. 2-25 in Section 2.4.1.1 can be revisited to determine the equivalent mass of the prototype.

The total resisting force in the larger scale GMD prototype can be expressed as,

\[
f = 2 \times \left\{ \left( \frac{J_1}{r_1^2} \right) + \left( \frac{J_2}{r_2^2} \right) + \left( \frac{J_3}{r_3^2} \right) + \left( \frac{J_4}{r_{24}^2} \right)^2 J_4 + \left( \frac{J_5}{r_{24}^2} \right)^2 J_5 \right\} \ddot{u}
\]  

(2-33)
2.4.2.2 Calculation of the gear moments of inertia in the larger scale prototype

As it is shown in Fig. 2-25, the mass of the gears and the steel disk are not distributed uniformly, and also there are holes in these metal pieces. Hence, the application of the formula for solid metal disks to calculate the mass moment of inertia of these components will lead to inaccuracies. The procedure for calculating the exact $J$ value for one of the gears is explained in this section.

The larger gear on the left side of Fig. 2-25 is considered here. The diameter of this gear is 7.5” ($R = 19.05$ cm). This gear has five circular holes that are 1.5” ($r' = 3.81$ cm) in diameter. The centre of each of these holes is located at a 2” (5.08 cm) distance from the centre of the gear. Also, at the centre of the gear, there is a 0.75” ($r = 1.91$ cm) hole to connect the gear to the shaft. The gear mass was measured in the structural laboratory as $M = 1.245$ kg. This value can be written in terms of the material density and the thickness of the gear,

$$M = \pi \rho t \times (R^2 - 5r'^2 - r^2)$$

(2-34)
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Figure 2-25. Gears and metal disk used in the GMD prototype

On the other hand, the assumption of a solid circular section (i.e., the holes do not exist) leads to a new mass that can be calculated from the following equation:

\[ M_T = \pi \rho t R^2 \]  

(2-35)

After eliminating \( \pi \rho t \) term in Eq. 2-34 and 2-35, \( M_T \) can be expressed in terms of \( M \):

\[ M_T = \frac{R^2}{R^2 - 5r^2 - r^2} M = \frac{3.75^2}{3.75^2 - 5 \times 0.75^2 - 0.375^2} M = 1.266 M \]  

(2-36)

Similarly, the fictional mass of the middle hole (\( m \)) and the other holes (\( m' \)) can be found from the following equations:

\[ m = \pi \rho r^2 = \frac{r^2}{R^2 - 5r^2 - r^2} M = 0.013 M \]  

(2-37)

\[ m' = \pi \rho r'^2 = \frac{r'^2}{R^2 - 5r'^2 - r'^2} M = 0.051 M \]  

(2-38)

For the measured value of \( M = 1.245 \, kg \), \( M_T, M \) and \( m \) are 1.576 kg, 0.016 kg and 0.063 kg, respectively. Now, mass moment of inertia of the solid disk (\( J_T \)), middle hole (\( j \)) and the other holes (\( j' \)) can be calculated separately,

\[ J_T = 0.5 M_T R^2 = 7.15 \times 10^{-3} \, kg.m^2 \]  

(2-39)

\[ j = 0.5 m r^2 = 7.15 \times 10^{-7} \, kg.m^2 \]  

(2-40)

\[ j' = 0.5 m' r'^2 = 1.14 \times 10^{-5} \, kg.m^2 \]  

(2-41)

Finally, the total mass moment of inertia \( J \) of the gear can be found from Eq. 2-42,
\[ J = J_T - j - 5 \times (j' + m'd^2) = 6.28 \times 10^{-3} \text{ kg.m}^2 \] (2-42)

Note that the assumption of a solid disk (with no holes) will lead to a mass moment of inertia of,

\[ J_{solid} = 0.5 \, MR^2 = 5.65 \times 10^{-3} \text{ kg.m}^2 \] (2-43)

which is 10% lower than the actual value.

Using the same procedure, mass moment of inertia of the other GMD prototype components are calculated, and the corresponding values are shown in Table 2-3.

**Table 2-3.** Mass moment of inertia of the GMD prototype components

<table>
<thead>
<tr>
<th>Component</th>
<th>Radius (in)</th>
<th>Radius (cm)</th>
<th>Mass (kg)</th>
<th>( J ) (kg.m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.5&quot; diameter gear</td>
<td>0.75</td>
<td>1.9</td>
<td>0.080</td>
<td>1.81×10⁻⁵</td>
</tr>
<tr>
<td>7.0&quot; diameter gear</td>
<td>3.50</td>
<td>8.9</td>
<td>1.195</td>
<td>4.96×10⁻³</td>
</tr>
<tr>
<td>7.5&quot; diameter gear</td>
<td>3.75</td>
<td>9.5</td>
<td>1.245</td>
<td>6.28×10⁻³</td>
</tr>
<tr>
<td>Solid disk</td>
<td>3.94</td>
<td>10.0</td>
<td>3.165</td>
<td>1.58×10⁻²</td>
</tr>
</tbody>
</table>

2.4.2.3 Calculation of the equivalent mass of the larger scale prototype

To be able to perform experiments with different values of equivalent mass, the prototype is designed and built in such way that the gears can easily be changed without replacing the device or removing other components of the GMD (Fig. 2-26).
Fig. 2-26 shows that the shaft can be easily pushed out and the gears can be consequently replaced. In the experimental sections of this study, three main cases are considered. In the first case to reduce the amplification factor of the gear train and lower the equivalent mass of the prototype, gears 4 and 5 in Fig. 2-24 are removed (i.e., 2-shaft case). In the other two cases, the same configuration as shown in Fig. 2-24 is used where first, a 7.0” diameter gear is used as the rotational mass (i.e., 3-shaft with 7” gear) and then a solid steel disk with a diameter of 20 cm (Fig. 2-25) is used as the rotational mass (i.e., 3-shaft with disk). Similar to Section 2.4.1.2, three different models are considered in this section to calculate the value of the equivalent mass \( b \) of the larger scale prototype for each of these three cases.

The first model that is used here to calculate \( b \), is the Saitoh’s model. Note that the mass moment of inertia of the last rotating mass \( J \) in Saitoh’s model for the larger scale prototype should be determined using Table 2-3. Since two compound gears exist in this prototype, the Saitoh’s model for the larger scale prototype can be expressed as:

\[
\begin{align*}
    b &= J_5 \left( \frac{1}{r_1} \right)^2 \left( N_g \right)^4 \\
    \text{(2-44)}
\end{align*}
\]

In the disk model, it is assumed that the gears are circular sections and the mass moment of inertia of each of them can be found in Table 2-3. For instance, if the 7.0” diameter gear is used as the rotational mass, then the equivalent mass of the GMD prototype can be calculated as,

\[
\begin{align*}
    b &= 2 \times \left\{ \left( \frac{J_1}{r_1^2} \right) + \left( \frac{J_2}{r_2^2} \right) + \left( \frac{J_3}{r_2r_4} \right)^2 J_4 + \left( \frac{r_2}{r_2r_4} \right)^2 J_5 \right\} = 722 \text{ kg} \\
    \text{(2-45)}
\end{align*}
\]

Alternatively, by using the solid disk as the fifth gear, the equivalent mass of the damper can be increased to 2219 kg. Also, in the absence of the 4th and 5th gears, the equivalent mass of the device for the 2-shaft case becomes 38.6 kg, which is considerably lower than the other two cases.

Finally, by considering the lumped mass model, the rotational mass of the gears can be assumed to be lumped at the contact points between the gears. Table 2-4 summarizes the values of the equivalent mass \( b \) for the larger scale prototype obtained from each for these models.

Unlike the predictions in Table 2-3, for the larger scale prototype specimen, the predicted values of the equivalent mass \( b \) from the Saitoh’s model are closer to the values of the lumped mass
model. This is because of the particular configuration of the gears in this prototype. However, as it was shown in Table 2-3, this is not always necessarily the case. The values of $b$ from the linear models of Table 2-4 can be used to model an idealized GMD (e.g., the model in Appendix A).

<table>
<thead>
<tr>
<th>Actual mass of the prototype (kg)</th>
<th>Equivalent mass (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Saitoh’s Model</td>
</tr>
<tr>
<td>2-shaft</td>
<td>122.8</td>
</tr>
<tr>
<td>3-shaft with 7” gear</td>
<td>125.7</td>
</tr>
<tr>
<td>3-shaft with disk</td>
<td>129.5</td>
</tr>
</tbody>
</table>

### 2.5 Larger scale prototype

In Section 2.4.2 the larger scale prototype in this study was introduced and the equivalent mass of this specimen was calculated. In this section, further details about the properties of this prototype specimen are discussed.

#### 2.5.1 Design of the larger scale prototype

Several modifications have been applied to the design of the smaller prototype in Section 2.4.1 to improve the design. As it is shown in Fig. 2-19, all of the gears in the small-scale prototype were located on one side of the rack. It was found that the asymmetry in the design of the prototype may introduce undesirable torque. Thus, the new prototype was designed symmetrically, and two sets of gear trains were used on both sides of the rack (see the top-view in Fig. 2-23 and 2-27). Secondly, to reduce the sensitivity of the device performance to fabrication tolerances, instead of locating the gear shafts on one long line, a new layout was considered in which only three gear shafts in an L-shape configuration were used (see the side view in Fig. 2-27). The new design makes it easier to align the gear shafts precisely. The prototype was built to function under repetitive testing of the device, and because of its design, it is possible to replace the components easily and without disconnecting the damper from the actuator. Dimensions of the prototype are shown in Fig. 2-27.

The gears for the larger-scale prototype were selected from the available product catalog of Boston Gear. Similar to the small scale prototype, a gear ratio of 5:1 was chosen for the compound gears.
As a result, two 19.05 cm (7.5”) diameter gears, two 3.81 cm (1.5”) diameter pinions and a rational mass were used in the prototype (on each side of the rack). The rotational mass in the experiments of this section was chosen to be either a 17.78 cm (7”) diameter gear or a 20 cm diameter (0.5” thick) steel disk.

![Figure 2-27. Dimensions of the larger scale GMD prototype](image)

As a result of fatigue and wear, gear failure can occur due to tooth breakage (tooth stress) or surface failure (surface durability). Based on the tooth-beam stresses for static and dynamic conditions, various formulas and procedures can be used to determine the strength. According to the gear manufacturer of the gears for the larger scale prototype, Barth’s Revision to the Lewis Formula can be used to identify the gear strengths. The original formula was proposed by Lewis in 1893. The modified equation (Eq. 2-46) is satisfactory for gears at pitch circle velocities of up to 7.62 m/s (1500 fpm) but does not consider wear.

\[
W = \frac{SFY}{p} \times \left( \frac{600}{600 + V} \right) \quad (2-46)
\]
where, $W$ is the tooth load along the pitch line (lbs.), $S$ is the safe material stress (static) (psi) and can be determined from the available tables. $F$ is the face (in), $P$ is the diametral pitch (in) and $Y$ is the Lewis form factor (or tooth form factor). $Y$ is a function of the number of teeth, pressure angle and an involute depth of the gear and $V$ is the pitch line velocity (fpm). These parameters are shown in Fig. 2-28.

For the 7.5” gear in Fig. 2-27 (first gear), the face is 0.5” and with 120 teeth with 14-1/2° pressure angle, $Y$ could be found to be 0.37. The diametral pitch of the gear is also 16 so assuming a maximum pitch line velocity of 0.2 m/s (40 fpm), the allowable tooth load could be calculated to be 460 lbs (2.05 kN). There are two sets of gear trains on each side of the rack, so the total tooth load is around 4.10 kN. This load can be increased for lower velocities.

Based on this formula few recommendations could be made for designing the gears for a GMD:

- Since steel has a higher safe static stress ($S$), it is preferable to use steel instead of cast iron gears. Moreover, heat treating the steel gears and using steel alloys with higher carbon percentage lead to stronger gear teeth.

- A higher number of teeth and larger pressure angles result in higher strengths. Gears with lower diametral pitch (i.e. the ratio of the number of teeth to the pitch diameter) values have stronger teeth.
Chapter 2. Gyromass Dampers – Background and Modelling

- Finally, the strength of the gear can be increased by using wider gears.

The last component of the GMD prototype is the linear bearings which are necessary to allow the movement of the rack. For this purpose, 15mm linear guide rails and blocks were purchased from Anaheim Automation. These rails and blocks can resist up to 5.35 kN dynamic loads and up to 9.08 kN static loads and two of them were used underneath the moving rack. Special care was made to lubricate the ball bearings and consequently reduce the friction forces.

2.5.2 Physical mass of the larger scale prototype

One difference between TMDs and GMDs is that GMDs do not require to place an actual mass on top of the primary structure and consequently they do not increase the seismic effective forces of the system considerably. Nonetheless, the structure of GMDs should be robust enough to withstand the generated forces. To estimate the weight of the prototype which is used in this study, the components of this device are listed in Table 2-5 and the total weight of the device is calculated.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Item</th>
<th>Description</th>
<th>Unit Weight (kg)</th>
<th>Total Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>19.05 cm (7.5&quot;) diameter gear</td>
<td>Large gears</td>
<td>1.245</td>
<td>4.980</td>
</tr>
<tr>
<td>4</td>
<td>3.81 cm (1.5&quot;) diameter gear</td>
<td>Small gears</td>
<td>0.080</td>
<td>0.320</td>
</tr>
<tr>
<td>2</td>
<td>20 cm diameter (1.27 cm thk.)</td>
<td>Round metal disk</td>
<td>3.165</td>
<td>6.330</td>
</tr>
<tr>
<td>3</td>
<td>21 cm × 1.27 cm diameter</td>
<td>Gear shaft</td>
<td>0.209</td>
<td>0.626</td>
</tr>
<tr>
<td>2</td>
<td>1.5 cm × 61 cm × 50 cm</td>
<td>Side plates</td>
<td>35.914</td>
<td>71.828</td>
</tr>
<tr>
<td>1</td>
<td>1.5 cm × 61 cm × 19.05 cm</td>
<td>Base plate</td>
<td>13.683</td>
<td>13.683</td>
</tr>
<tr>
<td>1</td>
<td>1.5 cm × 61 cm × 15.24 cm (6&quot;)</td>
<td>Supporting plate for the rack</td>
<td>10.947</td>
<td>10.947</td>
</tr>
<tr>
<td>2</td>
<td>1.2 cm × 12 cm × 15 cm</td>
<td>Stiffener plates</td>
<td>1.696</td>
<td>3.391</td>
</tr>
<tr>
<td>1</td>
<td>1.5 cm × 15 cm × 15 cm</td>
<td>Connection plate</td>
<td>2.649</td>
<td>2.649</td>
</tr>
<tr>
<td>1</td>
<td>5.08 cm (2&quot;) × 61 cm (2&quot;)</td>
<td>Rack</td>
<td>1.930</td>
<td>1.930</td>
</tr>
<tr>
<td>1</td>
<td>1.5 cm × 61 cm × 11 cm</td>
<td>Plate connected to the rack</td>
<td>7.901</td>
<td>7.901</td>
</tr>
<tr>
<td>2</td>
<td>Rails and blocks (50 cm)</td>
<td>Linear bearing</td>
<td>0.544</td>
<td>1.088</td>
</tr>
</tbody>
</table>

As it is shown in Table 2-5, the total weight of the GMD prototype with an equivalent mass of 2219 kg is equal to 125.7 kg. In other words, the actual mass corresponds to 5.67% of the equivalent mass. The two large side plates consist more than 57% of the total weight of the device.
Since it was the first time that this prototype was built at University of Toronto structural laboratory, for safety reasons, dimensions were chosen to be larger, and steel was selected for most components of the damper. For future designs, it is possible to adjust the dimensions of the plates and optimize the geometry of the damper.

### 2.5.3 Cost of building the larger scale prototype

Cost is another important parameter that is calculated for the prototype. The rails and blocks for the linear bearings, as well as the gears, bushings, and the rack, were ordered from company catalogs and were assembled at the structural laboratory. The breakdown of the costs is shown in Table 2-6. As it is shown in this table, technician cost consists 57% of the total cost. Since the prototype in this study was built at the structural laboratory for the first time, the total time that technicians spend on building the device can be reduced significantly for the next prototypes or by mass producing this damper.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Item Description</th>
<th>Unit Price ($)</th>
<th>Total Price ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Rails and blocks (50 cm)</td>
<td>170.69</td>
<td>170.69</td>
</tr>
<tr>
<td>4</td>
<td>7.5 in diameter gear</td>
<td>148.77</td>
<td>595.08</td>
</tr>
<tr>
<td>4</td>
<td>1.5 in diameter gear</td>
<td>37.08</td>
<td>148.32</td>
</tr>
<tr>
<td>2</td>
<td>20 cm diameter (0.5 in thk)</td>
<td>22.51</td>
<td>45.03</td>
</tr>
<tr>
<td>1</td>
<td>2 in × 2 feet (5/16 thk)</td>
<td>325.00</td>
<td>325.00</td>
</tr>
<tr>
<td>6</td>
<td>Change gear bushing</td>
<td>47.67</td>
<td>286.02</td>
</tr>
<tr>
<td>≈ 110 kg</td>
<td>Plates for the body of the device (material cost)</td>
<td>---</td>
<td>785.18</td>
</tr>
<tr>
<td>100.5 hrs.</td>
<td>Technician cost - UofT structural laboratory</td>
<td>≈ 32.00</td>
<td>3,178.50</td>
</tr>
</tbody>
</table>

| **Total** | **5,534** |

---

**Application of Real-Time Hybrid Simulation Method in Experimental Identification of GMDs**

44
Experimental testing is an essential tool for understanding how structures respond to extreme events, thus allowing the design and construction of safer structures. Laboratory tests play an important role in earthquake engineering when the structure is difficult to model through numerical methods. Even when numerical models are available, their accuracy needs to be assessed using reliable experimental results. Hybrid simulation techniques with shake tables and dynamic actuators are among commonly used seismic testing methods (Lamarche et al., 2010). It has shown in the past that for rate-dependent devices, such as viscous and elastomeric dampers, conventional test methods such as pseudo-dynamic testing may be insufficient due to the scaling effect in time or in size (Carrion et al., 2009) and alternative testing techniques should be employed to ensure the reliability of the experimental results. Real-time hybrid testing is a variation of the pseudo-dynamic testing method where the imposed displacements and response analysis are executed in real time, thus allowing testing of systems with rate-dependent components. RTHS enables the testing of a large class of structural elements associated with vibration control, including passive, semi-active, and active control devices (e.g., base isolation and dampers), which are typically nonlinear and rate-dependent. The hybrid simulation part of this technique provides an attractive alternative for dynamic testing of structural systems, combining physical testing with numerical simulation. This approach allows the researchers to test only the parts of the structure of interest while the rest of the structure is modelled numerically. It thereby makes it possible to test new devices with complex behaviour at much lower costs and at a system level. The unique attribute of the hybrid simulation method is the versatile implementation of inertia forces and substructuring.

The real-time execution of the hybrid simulation testing technique during which physical experiments of large-scale structural components/assemblies are subjected to real-time loading while the computational simulations of the remaining structure are executed in parallel with the
necessary information transfer in between enables the researchers to achieve the replication of
dynamic behaviour of the structural system under study. The real-time execution of these tests
requires performance of all calculations, application of displacements, and acquisition of measured
forces, within a small increment of time (in the order of milliseconds). There are also some
inevitable time delays associated with the numerical calculations and the communication between
the computer and data acquisition systems. The computing time can become large, especially for
complex models with a large number of degree-of-freedoms or with a nonlinear response. Because
of these time delays, the force measured and fed back from the experiment does not correspond to
the desired position (it is measured before the actuator has reached its target position). Due to the
existence of these delays and lags and also nonlinearities in the dynamic actuators, there are two
important concerns that should be addressed for each RTHS. First, RTHS technique is vulnerable
to error propagation, which can affect the accuracy and even the stability of the entire experiment
(Horiuchi et al., 1999). Since it is not always possible to have accurate theoretical or shake table
results for assessment and comparison of the quality of the hybrid simulations, it is crucial to
employ suitable measures to establish the reliability of the experiments. Secondly, unless
appropriate compensation for actuator dynamics is implemented, stability problems are likely to
occur during the experiment. Thus, there is a need to monitor the errors during the simulations and
then compensate for them properly.

This chapter addresses the abovementioned issues and presents several error measures to evaluate
the response of an RTHS and then discusses the applications of these error measures in establishing
stability limits for the tests and in developing a new approach for error compensation for actuator
dynamics using a 2-degree-of-freedom (2DOF) compensator. Numerical and experimental results
are also used to show the effectiveness of the proposed methods for improving the performance of
RTHSs.

3.1 Categorization of the RTHS technique

According to (Mosalam et al., 2015), hybrid simulation technique can be categorized into three
main types: (1) Slow hybrid simulation (slow HS) in a discrete actuator configuration, (2) RTHS
in a discrete actuator configuration, and (3) RTHS in a shake table configuration. In the first
category (i.e. slow HS), the experimental substructure(s) is directly connected to actuator(s),
physical mass does not exist, and the test rate is slower than the computed velocity. Over the past four decades after the early studies of (Takanashi et al., 1975), a large number of studies have been conducted on this technique (e.g. (Nakashima et al., 1988), (Nakashima et al., 1999), (Horiuchi et al., 1999), (Igarashi et al., 2000), (Lee et al., 2007), (Mercan et al., 2009), (Karavasilis et al., 2011), (Shao et al., 2011), (Gao et al., 2013), (Zhu et al., 2016), and so forth). According to (Carrion et al., 2007), the idea of conducting a real-time hybrid experiment was introduced by (Hakuno et al., 1969). Early application of dynamic actuators and digital servo-mechanisms for RTHS can be found in (Nakashima et al., 1992). However, it was after the development of actuator-delay compensation methods by (Horiuchi et al., 1999) that the research on RTHS gained momentum. Rapid advancements in control methods and computing technologies increased the number of RTHS research in recent years and same testing technique is used in the current study. The third category uses shake tables to perform RTHS. In conventional shake table testing, the test specimen, as a physical model (at small or full scale) representing the prototype structure under investigation, contains all the structural dynamic properties such as mass, damping, and stiffness. Because of the limitation in the shake table’s capacity, typically reduced-scale specimens can be tested with shake tables. This makes it difficult to extrapolate the results to a full-scale prototype structure (Shao et al., 2011). However, when shake tables are used in RTHS, there is less restraint in the structural system that can be evaluated, ranging from a lumped-mass system to a distributed mass system (such as soil-structure system). Although conventional shake table testing is commonly used in many laboratories and there is considerable amount of experience on RTHS in the recent years, research and developments on the application of RTHS on a shaking table, is limited ((Mosalam et al., 2015), (Ashasi-Sorkhabi et al., 2014) and (Lee et al., 2007)). It has been shown that the inertia of the testing setup in shake table could result in poor tracking of accelerations (Nakata, 2010) which has led to the development of force-controlled actuators ((Xiaoyun et al., 2010) and (Sivaselvan et al., 2008). It will be shown in the current study that the acceleration errors in RTHS in a discrete actuator configuration with successful displacement tracking control are not significant.

### 3.2 Real-time hybrid simulation technique

In all of these methods, the structure to be tested is divided into a physical component (i.e., the experimental substructure) and a numerical model (which is called the numerical substructure).
Chapter 3. Real-Time Hybrid Simulation (RTHS) Technique

During the test, the dynamic response of the structure under the seismic load is calculated numerically on a computer using time-step integration of the equation of motion. The calculated displacements are then applied to the test specimen (experimental substructure) using actuators. The required forces to produce these displacements are measured and fed back to the computer to calculate the displacements for the next time step. These displacements are then imposed by actuators in a step-by-step manner at a real-time rate. The load applied to the specimen is predetermined, and no measurements of the specimen’s response are required to determine the next step loading command (except for controlling the fidelity of implementation).

![Figure 3-1. Hybrid simulation testing method](image)

An RTHS setup usually consists of dynamic actuator(s), data acquisition and a digital controller unit with a computer, and displacement (i.e., digital position encoder), force (i.e., load cell) and acceleration (i.e., accelerometer) measurement devices. The schematic of the test setup is displayed in Fig. 1. In the controller computer, there is a simulation core to run the time integration and state determination of the analytical substructure which is interfaced to the hardware by a real-time program. The user programmable computational/control platform used to conduct RTHS for the tests in the current study were developed at University of Toronto and details of the platform can be found in (Ashasi-Sorkhabi et al., 2014). The developed platform not only allows the user to implement user-defined control laws to control the experimental substructures, but also provides ample computational resources to run the integration algorithm and analytical substructure state determination in real-time. In this platform the need for SCRAMNet as the communication device between real-time and servo-control workstations has been eliminated which was a critical component in several former RTHS platforms. The successful implementation of RTHS, enables a wide range of parameters such as input frequency, mass, damping ratio, and so forth to be
experimentally investigated by adjusting the parameters in the analytical substructure. The experimental results for the RTHS are presented in more detail in Chapter 4.

### 3.3 RTHS method for testing passive control devices

This study is not the first time that RTHS has been used to test various active, semi-active and passive control devices. In 2000, Igarashi et al. (Igarashi et al., 2000) performed a full-scale RTHS of an idealized 2DOF structural system with a tuned mass damper (TMD) providing structural control. Lee et al. (Lee et al., 2007) evaluated the vibration control effect of a scaled TLD for a building structure using the RTHS method. They also provided a comparison between the RHST method and a standard shake table test and the results showed good agreement. Ashasi-Sorkhabi (Ashasi-Sorkhabi, 2015) and Malekghasemi et al. (Malekghasemi et al., 2015) performed other sets of experiments on TLDs and investigated the effects of these devices on MDOF structures.

To provide experimental validations for his proposed design approach Guo (Guo, 2015) used the RTHS platform and tested viscoelastic-plastic (VEP) dampers. Carrion et al. (Carrion et al., 2007) described a full-scale RTHT of a magnetorheological damper for semi-active control of a three-story steel framed structure. Asai et al. (Asai et al., 2013) used the RTHS tests to study the effectiveness of MR dampers in outrigger damping systems. Several other researchers have also used RTHS to test magnetorheological dampers (e.g., (Christenson et al., 2008), (Jiang et al., 2011), (Fujitani et al., 2008) and (Chae et al., 2013)).

The RTHS testing of structures with full-scale elastomeric dampers can also be found in (Mercan et al., 2009) and (Karavasilis et al., 2011). Wei and Dyke (Wei et al., 2013) used the RTHS technique to test the behaviour of a hysteretic structural system. Stavridis and Shing (Stavridis et al., 2010) performed a series of RTHSs on a three-story suspended zipper steel frame, where the zipper struts were designed to transfer unbalanced forces up to the story above when the V-bracing in the story below buckles.

### 3.4 Tracking error monitors

Several potential experimental and numerical error sources, truncation errors, communication delays and influence of internal actuator dynamics can lead to undesirable discrepancies between
the command and measured signals in RTHS (Christenson et al., 2014). To build confidence in the use of this testing technique and explore its accuracy, there is a need to understand the principle features that determine the fidelity and the success of RTHSs. Assessing the accuracy and reliability of an RTHs is a complex problem because each hybrid simulation is different. Although in the absence of standard error monitoring measures, it is possible to increase the reliability of the tests by minimizing the error sources in conducting hybrid simulations with experience and planning, it is challenging to compare different RTHS results performed at various testing facilities. The experimental errors should be understood and acknowledged and examined relative to the intended objectives of the particular hybrid simulation.

Thus, it is crucial to develop error monitors that are reliable; that can be used consistently for different testing setups; that can be implemented and executed in real-time; that can be used to study the stability of various simulations; and that can be compensated by using a proper error compensation algorithm. These errors in a hybrid simulation can be attributed to either the experimental or numerical components, and even in the connections/steps required to bring the two together. Systematic and random and systematic experimental errors related to accurate boundary condition implementation or system delay/lag are some of the common sources of the errors in RTHS. Based on a variety of assessment measures from the literature, the available error assessment measures can be categorized into two broad groups: (1) The measures that focus on the local level compatibility at the interface between the physical and numerical components, (2) Those that emphasize on the system-level or global (system-level) responses (considering partitioning effects, equipment compatibility and global stability). The first group is called local response assessment measures while the measures in the second category are called global response assessment measures (Fig. 3-2). The substructuring assumptions for partitioning of a system into two separate components (physical and numerical), can lead to errors. When the boundary conditions at the interface between these two components are not fully captured by the transfer function, global response assessment measures should be employed. For purposes of this study, as the numerical substructure is small and the behaviour of the boundary conditions at the interface are relatively well-known, the focus will be on local response assessment measures.

The chief emphasis of the local response assessment measures is on the evaluation and examination of the accuracy of the synchronization of the numerical and physical components.
Using different frequency-domain, time-domain, and energy-based techniques, analyses are carried out to measure the actuator tracking error (i.e. inability of the hydraulic actuators to attain the command displacements during the specified time interval).

**Figure 3-2. Assessment criteria categorization**

Fig. 3-3 illustrates the difference between command and measured displacements in a hybrid simulation. As it will be discussed in this section, this error can be caused by amplitude errors (undershoot/overshoot) or phase errors (lead/lag). In the presence of a reliable error assessment measure that can decouple the amplitude errors from the phase errors, the effects of each of these error sources can be studied and then compensated independently. For instance, (Mercan et al., 2007), independent of a particular integration algorithm, investigated the accuracy and stability characteristics of the outer loop dynamics of RTHS. They concluded that in comparison to other types of errors, the delay (i.e. phase lag) in the measured signals is more critical as it introduces additional energy into the system and causes instability in the outer loop dynamics. Consequently, it is crucial to develop error indicators that are capable of decoupling various types of tracking errors (i.e. phase and amplitude errors) and can be implemented and executed in real-time. The
latter enables the error indicators to be used online to monitor the actuator tracking as the simulation progresses.

**Figure 3-3.** Block diagram for a hybrid simulation (Mirza Hessabi et al., 2012)

Each hybrid simulation is different, and with planning and experience, the error sources may be minimized. In the presence of modern computers with high processing capabilities and with the application of robust integration algorithms the errors that are associated with the numerical components can be minimized. Values of the numerical errors are particularly small when the analytical substructure is an SDOF system with well-known mechanical properties. Common sources of experimental errors encountered in hybrid testing include: inaccurate displacement control of the hydraulic actuators, flexibility in the test setup and reaction frame, calibration errors in the instrumentation, noise generated in the instrumentation and analog to digital converters, precision errors due to limited range of the instrumentation, frictional force in the actuator connections, force relaxation, and strain-rate effects. With the careful instrumentation of the testing setup and accurate calibration of load cells errors in the measured forces due to measurement can be neglected. As such, the reliability of the test results obtained for the tests in this study mainly depends on the ability of the hydraulic actuators to accurately apply the target command displacements. It is important to note that depending on the intended objective of the particular experiment, even large errors may be acceptable. However, the experimental errors should be examined, and in some cases, proper error compensation techniques should be used to reduce the amount of these errors.
Chapter 3. Real-Time Hybrid Simulation (RTHS) Technique

In the next sections, after a brief introduction of the available error measures that have been developed by other researchers, three new error assessment measures are introduced that satisfy the abovementioned conditions. It is worth mentioning that despite being proposed as real-time error monitors, these indicators can also be employed as a post-processing tool to measure the accuracy of an RTHS after the completion of the test.

3.4.1 Error assessment measures developed by other researchers

Perhaps one of the most straightforward procedures for evaluating the accuracy of the synchronization of the numerical and physical components by measuring the actuator tracking error is by using the normalized root mean square in the experiment (NRMSE). This time domain measure lumps the errors due to the difference between the measured and command displacements in a single value that is defined by Eq. 3-1 (Christenson et al., 2014):

\[
NRMSE = \sqrt{\frac{\sum_{i=1}^{N} (x_m(i) - x_c(i))^2}{\sum_{i=1}^{N} x_c(i)^2}}
\]  

where, \(x_c\) and \(x_m\) represent the command and measured displacement, respectively. Also, in Eq. 3-1, \(N\) shows the total number of command/measured displacement steps.

Similarly, in the time domain a peak error in specified responses might be of interest. To quantify the Maximum Tracking Error (MCE), one can track the measured displacements comparing with the command displacements and calculate the single value of MCE for peak displacements (Guo et al., 2014):

\[
MCE = \left| \frac{x_{m_{\text{max}}} - x_{c_{\text{max}}}}{x_{m_{\text{max}}}} \right|
\]  

Unlike the NRMSE and MCE, the cross correlation between the target (command) and measured displacement can provide frequency-domain insight into the performance of actuator tracking. Cross-correlation quantifies the degree of similarity between two signals. The cross-correlation between the measured and command can also provide a reasonable estimate of the actuator time delay. The formula for the calculation of the cross-correlation, and the relationship between the estimated time delay \(\tau_{est}\) and the cross-correlation are given below (Christenson et al., 2014):

\[
[x_c * x_m](\tau) = \int_{-\infty}^{\infty} x_c(\tau) x_m(t - \tau) d\tau
\]

\[
\tau_{est} = \text{argmax}\{[x_c * x_m](\tau)\}
\]
Chapter 3. Real-Time Hybrid Simulation (RTHS) Technique

Eq. 3-4 provides an estimate of the time delay, and in a case of experiments with tracking errors that are varying with time, this single value may not be a reliable representation of the errors.

Guo et al. (2014a) proposed the Frequency Evaluation Index (FEI) method to evaluate the actuator tracking error in terms of the phase and amplitude errors. In their formulation equivalent frequency was used to estimate the time delay of the test. According to their study, FEI can be defined as Eq. 3-5,

$$FEI = \sum_{j=1}^{N} \left\{ \frac{\text{fft}(x_{mj})}{\text{fft}(x_{cj})} \times \frac{\| \text{fft}(x_{cj}) \|^l}{\sum_{l=1}^{N} \| \text{fft}(x_{cj}) \|^l} \right\}$$

where, \(l\) is an integer (i.e., 1, 2, 3, etc.). The relationship between the calculate value of FEI and estimated amplitude error \(\Delta A_{est}\) and phase errors \(\phi\) can be shown in Eq. 3-6 and 3-7:

$$\Delta A_{est} = \| FEI \|$$

$$\phi = \arctan[\text{Im}(FEI)/\text{Re}(FEI)]$$

Finally, using the equivalent frequency from Eq. 3-8, the delay of an experiment can be estimated from the Eq. 3-9,

$$f_{eq} = \sum_{j=1}^{N} \left\{ \frac{\| \text{fft}(x_{cj}) \|^l}{f_j} \right\}$$

$$\tau = -\frac{\phi}{2\pi f_{eq}}$$

Later in the same year, Guo et al. (2014b) proposed two decimation factors to improve the calculation efficiency of the FEI. It should be noted that FEI has a significant advantage over the previous three error indicators and that is its ability to decouple the amplitude and phase errors and determine each of these components of the tracking error from separate closed-form equation. However, like NRMSE, MSE and cross-correlation, FEI lumps the errors into a single value and cannot clearly show the changes of tracking error values throughout an experiment. In other words, FEI can be used for post-processing of RTHS results however, by definition it cannot be used as a real-time error indicator.
In addition to the error assessment measures reported above, there are two other types of indicators that qualitatively provide insight to the quality of the real-time test results. The first category is the energy-based indicators ((Mosqueda et al., 2007) and (Ahmadizadeh et al., 2009)), whereas the second type is based on the properties of synchronization subspace plots (SSPs) of command and measured displacements ((Wallace et al., 2005) and (Mercan et al., 2009)). Although the base of these two types of indicators is different, it can be seen that their behaviour and trends are comparable to each other (Chen et al., 2010). Both energy and SSP based indicators will be discussed in detail in the following sections, and their advantages and limitations will be pointed out.

In the derivation of energy-based error indicators, the cumulative growth of energy errors resulting from the actuator tracking error is used as a criterion for evaluating the accuracy of PSD test results. In this approach, the energy terms are computed individually using the structural properties, input forcing history, measured restoring forces, command, and measured displacements at each time step. The energy errors are then computed by considering the energy balance of the system. In the formulation of the energy errors by (Thewalt et al., 1994), a tangent stiffness matrix for the test structure is required, which may be difficult to obtain for nonlinear structures. To address this problem, Mosqueda et al. introduced Hybrid Simulation Error Monitors (HSEM), which do not require a tangent stiffness matrix (Mosqueda et al., 2007). In this approach, energy error ($E_{error}$) is defined as the difference between the energy that is actually dissipated by the experimental substructures and the energy dissipation that is observed in the integration algorithm. $E_{error}$ is then normalized to obtain the HSEM:

$$HSEM = \frac{E_{error}}{E_{input} + E_{strain}}$$

(3-10)

As can be seen in Eq. 3-10, the normalization takes place by dividing the energy error ($E_{error}$) by the sum of the input energy ($E_{input}$) and the maximum recoverable strain energy ($E_{strain}$). When the input energy is very small $E_{strain}$ can be estimated as follows:

$$E_{strain} = \frac{1}{2} x_0^T K_0 x_0$$

(3-11)
where, $K_0$ is the initial stiffness matrix of the test structure and $x_0$ is an experimental displacement vector which can be roughly selected as the yield displacement of the experimental substructure. Also, $E^{error}$ and $E^{input}$ are defined as follows,

$$E^{error} = \int (F_m)^T dx_m - \int (F_m)^T dx_c$$

(3-12)

$$E^{input} = \int (P_{eff})^T dx_c$$

(3-13)

where, $P_{eff}$ is the applied loading and $F_m$ is the measured force from the experiment.

Although, it has been shown (Mosqueda et al., 2007) that the displacement and force errors of a hybrid simulation can be limited by restricting the amount of HSEM, this error monitor is introduced to estimate errors associated with phase (i.e., lead/lag) and it does not address the amplitude errors. As such, in the presence of both amplitude and phase errors, this indicator lumps them together. Moreover, the maximum recoverable strain energy term used in the normalization requires an assumption for structural behaviour (e.g. elasto-plastic) and the extent of nonlinearities of the test. In other words, HSEM can be influenced by the structural model and ground motion considered. To assess the accuracy of a given test result and establish an acceptable threshold for HSEM, numerical simulations that determine a relationship between the accuracy measures and HSEM need to be carried out. As a result, HSEM can be considered as test structure/experiment specific, implying that the values of this indicator obtained from two different tests with considerably different command displacement histories and/or different test structures cannot directly be compared with each other.

There is also another group of error assessment measures that use a different tool to evaluate the values of tracking error in a simulation. These error monitors use a plot called a Synchronization Subspace Plot (SSP). SSPs have the measured signal (i.e., displacements) plotted against the commands, and are used in signal processing. In Fig. 3-4 (a), for one cycle of command displacement (referred to as command), the measured displacements are considered to have overshoot and undershoot amplitude error (indicated as overshoot and undershoot, respectively). In Fig. 3-4 (b), it can be seen that for the perfect tracking case (i.e., when the measured and command displacements are identical), the SSP is a straight line with 45° inclination. With overshoot amplitude error, the inclination of the SSP is greater than 45°, and similarly, with the
undershoot error it is less than 45°. Figure 3-4 (c) considers phase errors in the form of lead and lag as indicated. When there is a phase error in the measured displacement, the SSP is no longer a straight line, but it exhibits elliptical hysteresis. The hysteresis loops evolve clockwise in the presence of phase lead and they evolve counterclockwise when there is a phase lag in the measured displacement (see Fig. 3-4 (d)).

**Figure 3-4.** Sinusoidal actuator displacement time histories and associated SSPs (Mirza Hessabi et al., 2012)

Figure 3-5 shows the command and measured displacements from an RTHS test (Fig. 3-5 (a)), and the corresponding SSP (Fig. 3-5 (c)). As can be seen in the enlarged view in Fig. 3-5 (b) there are amplitude and phase errors in the measured displacements superimposed with the noise. The effect of the noise in the SSP can be observed better in the enlarged view in Fig. 3-5 (d).

As a tool for assessing the actuator tracking performance in hybrid simulation, SSPs were first introduced by (Wallace et al., 2005), where their evolutions were only visually observed. Using the areas enclosed by the SSPs and computing their major axis inclinations through principal
component analysis, (Mercan et al., 2009) proposed the Tracking Indicator ($TI$) and amplitude indicator ($\theta_{pc}$). To compute the $TI$ for a set of command and corresponding measured displacements, the area enclosed by the SSP is first computed. The area enclosed by the loop can numerically be calculated, where for an increment in command displacement, the enclosed area $Area(i + 1)$ at the end of the time step $i + 1$ is related to the area at the beginning of the time step $Area(i)$ by:

$$Area(i + 1) = Area(i) + \Delta_{area}^{i+1}$$ (3-14)

where,

$$\Delta_{area}^{i+1} = 0.5 (x_c^{i+1} + x_c^i)(x_m^{i+1} - x_m^i)$$ (3-15)

![Displacement time histories of real experiment and associated SSPs](image)

**Figure 3-5.** Displacement time histories of real experiment and associated SSPs (Mirza Hessabi et al., 2012)

Then ‘transpose area’ is computed by transposing the measured and command displacements in Eq. 3-14. This area is shown in Fig. 3-6. The tracking indicator is taken to be equal to half of the difference between the area and the transpose area:

$$TI(i + 1) = \frac{Area(i+1) - TransposeArea(i+1)}{2}$$ (3-16)
As pointed out earlier, for the perfect tracking case, the SSP is a straight line with a 45° slope. In the presence of only phase error this plot exhibits an elliptical hysteresis, where the inclination of the major axis is at 45°. When an amplitude error is introduced, the major axis of the ellipse will deviate from an inclination of 45°. This property was used by (Mercan et al., 2009) and by examining the angle of inclination of the major axis (for the cases when the phase error is present) or the straight line (in the absence of phase error) they proposed $\theta_{pc}$ to identify the existence and type of any amplitude error. Using the covariance of measured and command displacements, $\theta_{pc}$ is defined as shown in Eq. 3-17,

$$\theta_{pc} = \arctan\left\{ \frac{\text{cov}(x_c,x_m)}{\lambda_1 - \text{cov}(x_m,x_m)} \right\}$$

(3-17)

where,

$$\lambda_1 = \frac{\text{cov}(x_c,x_c) + \text{cov}(x_m,x_m) \pm \sqrt{(\text{cov}(x_c,x_c) - \text{cov}(x_m,x_m))^2 + 4 \left( \text{cov}(x_c,x_m) \right)^2}}{2}$$

(3-18)

Similar to HSEM, although they are useful in evaluating the test results, TI and $\theta_{pc}$ cannot determine the amplitude and phase errors separately and are affected by the amplitude of the command displacements. As such, they cannot serve as a standard assessment tool for the real-time test results. It is beneficial to have standard sets of indicators that do not suffer from the limitations of the previous error indicators. In the next three sections, three new sets of error indicators are introduced to address the need for standard and on-line error indicators.
3.4.2 Error assessment measures developed in this study

3.4.2.1 Phase and Amplitude Error Indices (PAEI)

In this section, a new set of indicators (i.e., PAEI) are introduced that similar to TI and \( \theta_{pc} \), use the SSPs of command and measured displacements. However, instead of computing the enclosed areas under the SSPs (as done in the formulation of TI), in the derivation of PAEI, the equations of ellipses as they evolve in the synchronization subspace are directly correlated to the phase and amplitude errors. Details for calculation of these error indices are explained in the following subsections.

3.4.2.1.1 Expression of errors in terms of ellipse coefficients

Considering a sinusoidal command signal and the corresponding measured signal with constant amplitude and phase error, closed-form relationships between the introduced error and ellipse parameters are derived as the ellipse evolves in the synchronization subspace. Eq. 3-19 and 3-20 represent the sinusoidal command \( x_c \) and measured \( x_m \) displacement signals as a function of time \( t \), where both have the same frequency \( \omega \), but different amplitudes \( A_c \) and \( A_m \), respectively, and a constant phase shift \( \phi \):

\[
x_c(t) = A_c \sin(\omega t) \tag{3-19}
\]

\[
x_m(t) = A_m \sin(\omega t + \phi) \tag{3-20}
\]

Eliminating \( \omega t \) terms, these two equations can be combined. Thereby, the measured displacement can be expressed in terms of the command displacement:

\[
x_m = A_m \sin \left[ \sin^{-1} \left( \frac{u_c}{A_c} \right) + \phi \right] \tag{3-21}
\]

With the use of trigonometric relationships, Eq. 3-21 can be rewritten as Eq. 3-22 and 3-23, and then simplified as Eq. 3-24 and 3-25:

\[
\left( \frac{x_m}{A_m} \right) = \sin \left[ \sin^{-1} \left( \frac{x_c}{A_c} \right) \right] \cos(\phi) + \cos \left[ \sin^{-1} \left( \frac{x_c}{A_c} \right) \right] \sin(\phi) \tag{3-22}
\]

\[
\left( \frac{x_m}{A_m} \right) = \left( \frac{x_c}{A_c} \right) \cos(\phi) + \cos \left[ \sin^{-1} \left( \frac{x_c}{A_c} \right) \right] \sin(\phi) \tag{3-23}
\]
\[
\left(\frac{x_m}{A_m}\right) = \left(\frac{x_c}{A_c}\right) \cos(\phi) + \left(\frac{1}{A_c}\right) \sin(\phi) \sqrt{A_c^2 - x_c^2}
\] (3-24)

\[
\left(\frac{A_c}{A_m}\right) x_m = x_c \cos(\phi) + \sqrt{A_c^2 - x_c^2} \sin(\phi)
\] (3-25)

By taking the square of both sides of Eq. 3-25, Eq. 3-26 can be obtained:

\[
x_c^2 - \left[\left(\frac{2A_c}{A_m}\right) \cos(\phi)\right] x_m x_c + \left(\frac{A_c}{A_m}\right)^2 x_m^2 - A_c^2 \sin^2(\phi) = 0
\] (3-26)

Comparing Eq. 3-26 to the standard equation of an ellipse given in Eq. 3-27 it can be seen:

\[
a x_c^2 + b x_m x_c + c x_m^2 + d x_c + e x_m + f = 0
\] (3-27)

\[
\begin{align*}
a &= 1 \\
b &= -2 \left(\frac{A_c}{A_m}\right) \cos(\phi) \\
c &= \left(\frac{A_c}{A_m}\right)^2 \\
d &= e = 0 \\
f &= -A_c^2 \sin^2(\phi)
\end{align*}
\] (3-28)

It is now possible to obtain closed-form expressions for the amplitude and phase errors in terms of the coefficient of the ellipse that appear in the synchronization subspace due to the error between the command and measured signals:

\[
\phi = \left|\cos^{-1}\left(\frac{b}{2\sqrt{c}}\right)\right|
\] (3-29)

\[
A_c = \sqrt{\left|\frac{f}{\sin^2(\phi)}\right|}
\] (3-30)

\[
A_m = \sqrt{\left|\frac{A_c^2}{c}\right|}
\] (3-31)

While Eq. 3-29 directly defines the phase error, the amplitude error needs to be determined by subtracting Eq. 3-31 from Eq. 3-30:

\[
\Delta A = A_c - A_m
\] (3-32)

For the amplitude error, the sign of Eq. 3-32 readily determines the type of the error; where a positive \(\Delta A\) is an indication of undershoot error, and a negative value represents overshoot. On the
other hand, the phase error determined by Eq. 3-29 is always a positive quantity. To distinguish a negative phase error (i.e., lag) from a positive one (i.e., lead) the proper sign needs to be introduced, which will be explained in subsection 3.3.2.1.5.

3.4.2.1.2 Fitting an ellipse to the SSPs

Given the equation of an ellipse that appears on the SSP, Eq. 3-27 through 3-30 can quantify the errors between the command and measured signals. As the next step, it is necessary to fit ellipses to the data in the SSP as they form with the addition of each pair of command and measured displacement data points.

Ellipse detection is a focus of research in the pattern recognition and computer vision fields, and the range and complexity of the various algorithms available in the literature are very broad. These techniques can mainly be separated into two groups: voting/clustering and optimization. Methods in the first group include Hough Transform-like techniques, fuzzy logic, competitive learning or the like (Zhang et al., 2005). Because of their computationally expensive, and iterative nature, these methods suffer from the shortcomings of high complexity or non-uniqueness of solutions, which may render them infeasible for some large problems. Alternatively, in the second group (i.e., optimization methods), ellipse fitting is performed by minimization of the sum of squared algebraic distances between the data points and the conic section.

In the original context of the application of these algorithms for ellipse detection, all of the data points would be available simultaneously to fit the ellipse (usually just once). On the other hand, in the current application, ellipse-fitting should be repeated when enough number of data points becomes available in the evolving SSP as the real-time test proceeds. Noting this distinction, it is apparent that the ellipse fitting technique suitable for this paper’s application should be computationally inexpensive. That is why, among the second group of methods, the direct least-squares fitting method proposed by (Fitzgibbon et al., 1999) is chosen for this study. This method is not only straightforward to apply, but it also guarantees the uniqueness of the solution through the introduction of a constraint condition.

Eq. 3-33 represents a general conic in the SSP by an implicit second-order polynomial:
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\[ F(A, X) = A.X = ax_c^2 + bx_m x_c + cx_m^2 + dx_c + ex_m + f = 0 \]  
(3-33)

where, \( F(A, X) \) is called the algebraic distance of a point \((x_c, x_m)\) to the conic \( F(A, X) = 0 \) and vectors of \( A \) and \( X \) can be defined as,

\[ A = [a \ b \ c \ d \ e \ f]^T \]  
(3-34)

\[ X = [x_c^2 \ x_m x_c \ x_m^2 \ x_c \ x_m \ 1] \]  
(3-35)

The nature of the conic in Eq. 3-33 depends on the value of \( b^2 - 4ac \); and this general conic is an elliptical conic only if:

\[ b^2 - 4ac < 0 \]  
(3-36)

The aim of least-square fitting is to determine the values of the coefficients that minimize the sum of the square of the errors or the sum of squared algebraic distances. Bookstein (Bookstein, 1979) showed that the least-square fitting problem under a constraint could be solved using generalized eigenvectors. Fitzgibbon et al. (Fitzgibbon et al., 1999) used this approach for ellipse-specific directed least-square fitting as the generalized eigenvector problem subject to the constraint described in Eq. 3-37:

\[ 2\bar{D}^T \bar{D}A - 2\lambda \bar{C}.A = 0 \rightarrow \bar{S}.A = \lambda \bar{C}.A \]  
(3-37)

where, the \( N \times 6 \) design matrix (\( \bar{D} \)) is defined as \([X_1 \ X_2 \ \ldots \ X_N]^T\) resulting in a scatter matrix (\( \bar{S} \)) of the size \( 6 \times 6 \) and a constraint matrix (\( \bar{C} \)) of the size \( 6 \times 6 \):

\[
\bar{C} = \begin{bmatrix}
0 & 0 & 2 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]  
(3-38)

\[ \bar{S} = \bar{D}^T \bar{D} \]  
(3-39)

The vector \( A \) that contains the coefficients of the ellipse fit (see Eq. 3-34) can thus be obtained by solving for the non-infinite and positive eigenvectors in Eq. 3-37.
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3.4.2.1.3 Transformation of the data points

As observed in Fig. 3-5 (c) and 3-5 (d), the presence of noise in the measured data causes the subspace plot exhibit a hysteretic response with distorted loops (instead of a smooth ellipse); which in turn can influence the effectiveness of the proposed procedure. To circumvent this problem, data points are scaled and centred before fitting of the ellipses, thus reducing the sensitivity to the noise. To lessen the possibility of numerical instabilities, scaling of the data points is also a standard procedure in image processing (Fitzgibbon et al., 1999).

The transformation for centering and normalizing data points can be summarized in Eq. 3-40 and 3-41:

\[
(X_c)_i = ((x_c)_i - x_{cc})/s_{uc} \quad (3-40)
\]

\[
(X_m)_i = ((x_m)_i - x_{mc})/s_{um} \quad (3-41)
\]

where, \((x_{cc}, x_{mc})\) is the coordinate of the centroid of the data points and \(N\) is the number of data points considered for ellipse fitting:

\[
x_{cc} = \frac{\sum_{i=1}^{N} (x_c)_i}{N} \quad (3-42)
\]

\[
x_{mc} = \frac{\sum_{i=1}^{N} (x_m)_i}{N} \quad (3-43)
\]

and \(s_{uc}\) and \(s_{um}\) are:

\[
s_{uc} = \frac{\max((x_c)_i) - \min((x_c)_i)}{2} \quad (3-44)
\]

\[
s_{um} = \frac{\max((x_m)_i) - \min((x_m)_i)}{2} \quad (3-45)
\]

The eigenvalue problem described in Eq. 3-37 is solved using the data points in the transformed coordinates (i.e., \(X_c\) and \(X_m\)); thus the coefficient of the ellipse in the transformed plane are obtained as \((A_T, B_T, C_T, D_T, E_T\) and \(F_T\)). It should be noted that the errors quantified by Eq. 3-29 through 3-32 are independent of the axis origin (i.e. centering with vertical or horizontal constant offsets has no effect on the results). However, they are affected by the size of the ellipse (i.e., major and minor axes lengths) and the inclination of the major axis. Hence, it is necessary to obtain the coefficient of the ellipse \((a, b, c, d, e \text{ and } f)\) in the original coordinate system (i.e. \((x_c, x_m)\)). This can be accomplished by Eq. 3-46 through 3-51 which are obtained by substituting the transformed
coordinates from Eq. 3-40 and 3-41 into the general equation of an ellipse and comparing the results with the coefficients of Eq. 3-34.

\[ a = \frac{A_T}{S_{uc}^2} \quad (3-46) \]
\[ b = \frac{B_T}{S_{uc}S_{um}} \quad (3-47) \]
\[ c = \frac{C_T}{S_{um}^2} \quad (3-48) \]
\[ d = \left(\frac{-2x_{cc}}{S_{uc}^2}\right)A_T + \left(\frac{-x_{mc}}{S_{uc}S_{um}}\right)B_T + \left(\frac{1}{S_{uc}}\right)D_T \quad (3-49) \]
\[ e = \left(\frac{-2x_{mc}}{S_{um}^2}\right)C_T + \left(\frac{-x_{cc}}{S_{uc}S_{um}}\right)B_T + \left(\frac{1}{S_{um}}\right)E_T \quad (3-50) \]
\[ f = \left(\frac{x_{cc}^2}{S_{uc}^2}\right)A_T + \left(\frac{x_{cc}x_{mc}}{S_{uc}S_{um}}\right)B_T + \left(\frac{x_{mc}^2}{S_{um}^2}\right)C_T + \left(\frac{-x_{cc}}{S_{uc}}\right)D_T + \left(\frac{-x_{mc}}{S_{um}}\right)E_T + F_T \quad (3-51) \]

### 3.4.2.1.4 Selection of the time window size

As emphasized earlier, in the current application, ellipse fitting procedure needs to be repeated as the SSP evolves with the addition of each data point from new command and measured signals. This requires a careful selection of the number of data points (called time window hereafter) that will be used in each ellipse fit to reveal the error between the command and measured signals. As can be seen in Fig. 3-7, using too short of a time window in fitting the ellipse may result in a poor fit, and consequently erroneous quantification of the error. On the other hand, the use of data points from a couple of complete hysteresis loops will lead to a fewer number of amplitude and phase errors being quantified and the results will also be averaged.

In this study, a moving time window approach is adopted, where for the \(i\)th step at time \(t_i\) (i.e., \((x_c)_i, (x_m)_i\)), in addition to this last point on the SSP, an appropriate number of previous data points from the time window. To determine a proper size for the time window such that the above limitations of having too few or too many data points do not occur, a process with double checks (one for convergence of the ellipse centres, the other for accumulated angles) is introduced. This process starts with a data subset with an initial number of data points. By looking at the sizes of the matrices in the eigenvalue problem in Eq. 3-37, it can be inferred that in order to determine the
six unknown ellipse coefficients, at least six data points are required. This determines the minimum number of data points to get the ellipse fitting process started in the very beginning. However, for computational efficiency, the use of a larger initial data set with 50 to 100 data points is suggested. Starting with a data subset, ellipse fitting process is applied as described previously. For each ellipse, the geometric centre and the angle accumulated between the first and the last data points are computed. A new data point to the SSP is introduced until the centre of the ellipse does not move appreciably with the addition of the next data point (i.e., the distance between two consecutive ellipse centres is less than a prescribed threshold). Afterward, the accumulated angle is compared against a minimum and a maximum angle limit. The number of data points is deemed adequate when they form a particular fraction of the ellipse. In this study, the minimum angle limit is set as 180°, and 360° is used as the maximum angle limit. This way, the time window used in each ellipse fit forms at least half an ellipse, and at most a complete ellipse.

![Figure 3-7. Fitting ellipses to too few data points (Mirza Hessabi et al., 2012)](image)

3.4.2.1.5 Determination of the sign of the phase error

The angle accumulated within each time window is used to assign the correct sign to the phase error determined by Eq. 3-29. The angle accumulated is computed in such a way that it is a positive quantity when the SSP evolves clockwise (e.g., phase lead behaviour) and is negative for the opposite evolution of the SSP (e.g., phase lag behaviour) for the time window considered. Thus, the sign of the accumulated angle readily determines the correct sign for the phase error.
3.4.2.1.6 Flowchart for calculation of PAEI

Fig. 3-8 introduces the flowchart that explains the implementation of the above-mentioned steps for PAEI. The list of the parameters in the order as they appear in the flowchart is provided in Table 3-1.

**Table 3-1. List of the parameters in the flowchart of Fig. 3-8**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
</table>
| MINA      | Minimum value for ANGL  
Suggested value is 180 degrees (to have half an ellipse) |
| MAXA      | Maximum value for ANGL  
Suggested value is 360 degrees (to have a full ellipse) |
| CENT      | Threshold for the geometric distance between the centres of two consecutive fitted ellipses  
A value between $10^{-4}$ to $10^{-6}$ mm is recommended |
| LENI      | The initial number of data points for the data subset  
Minimum value is 6 but for computational efficiency a value of 50 to 100 is recommended |
| FITE      | Iteration counter for the very first time window |
| STRT      | Index of the first data point in each data subset |
| END       | Index of the last data point in each data subset |
| CEND      | Geometric distance between the centres of two consecutive fitted ellipses |
| ANGL      | The angle accumulated between the first and last data point of a data subset |
| FINI      | Number of data points for which the convergence was obtained for the very first time window |
| TIMI      | Time index |
| NITE      | Iteration counter for each time window after the very first time window |
| NTOT      | Total number of data points in the global data set |

As can be seen in the flowchart, the very first time-window needs special attention. For this very first time window, the process starts with a predefined number of initial data points (i.e., LENI) followed by transformation, ellipse fitting, and inverse transformation processes.

For each iteration, the centre of the fitted ellipse is computed and a convergence check for the ellipse centre and limit checks for the accumulated angle are performed. When these checks are not satisfied, one more data point is added marching forward in time (i.e., the end index of the data subset array is modified).
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Figure 3-8. Flowchart for the implementation of PAEI (Mirza Hessabi et al., 2012)
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The steps are the same for all the consequent time windows; the only difference is that each new data point is added going backward in time (i.e., the start index of the data sub-array is modified). This way, the proposed PAEI can also be applied online for processing the data in real-time as the test progresses. Once each time window converges, the phase and amplitude errors are computed using the closed-form relationships presented in Eq. 3-29 and 3-32, and they are recorded.

The MATLAB script files for computing the values of PAEI as a post-processing tool for an RTHS are shown in Appendix B.

3.4.2.1.7 Numerical and experimental case studies for performance evaluation of PAEI

To evaluate the performance of PAEI, four different cases summarized in Table 3-2 are considered. As will be discussed in detail later, comparisons of tracking evaluation with HSEM, TI and PAEI are provided using numerical simulations, and also real experimental data (Case (iv)).

a) Case (i)

In Case (i) predefined displacement command (Eq. 3-19) and measured (Eq. 3-20) signals with known phase and amplitude errors are processed using PAEI.

\[
\begin{align*}
  x_c &= A_c \times \cos(3\pi t) \\
  x_m &= A_m \times \cos(3\pi [t + 15 \times \Delta t])
\end{align*}
\]  

(3-52)

(3-53)

where, \( t \) is time, \( \Delta t \) is the size of time steps and is equal to 1/1024 seconds (which is the clock speed of a typical digital controller), as mentioned before, \( x_c \) and \( x_m \) represents command and measured displacements, respectively. Also, \( A_c \) and \( A_m \) refer to the amplitudes of the command and measured displacements. In this case, \( A_c \) and \( A_m \) are assumed 1.5 and 2.0 mm, respectively. As can be seen from Eq. 3-19 and 3-20 (and also summarized in Table 3-2), there is a constant overshoot error of 0.5 mm in the measured displacement, and a time lead of 15 time steps. Note that, considering the frequency of the signals (i.e., \( \omega = 3\pi \)) this time lead corresponds to a phase lead of 0.1381 rad (i.e. \( \phi = \frac{3\pi \times 15}{1024} = 0.1381 \text{ rad} \).
### Table 3-2. Cases considered in the performance evaluation of the tracking indicators

<table>
<thead>
<tr>
<th>Case</th>
<th>Description</th>
<th>Input Force (kN)</th>
<th>Introduced time shift</th>
<th>Introduced amplitude error</th>
<th>Nonlinearity</th>
<th>Noise</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i)</td>
<td>Predefined displacement signals</td>
<td>-----</td>
<td>15 time steps (lead)</td>
<td>- 0.5 mm (overshoot)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>(ii-a)</td>
<td>Simulink model</td>
<td>$50 \times \sin (2\sqrt{2\pi} \times \frac{t}{T_n})$</td>
<td>5 time steps (lag)</td>
<td>10% overshoot (- 0.1 × $A_c$)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>(ii-b)</td>
<td>Simulink model</td>
<td>$100 \times \sin (2\sqrt{2\pi} \times \frac{t}{T_n})$</td>
<td>-5 time steps (lag)</td>
<td>0 &lt; $t$ &lt; 5s; 10% overshoot (- 0.1 × $A_c$) 5 &lt; $t$ &lt; 10s; 25% undershoot (0.25 × $A_c$)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(iii)</td>
<td>Simulink model</td>
<td>$100 \times \sin (2\sqrt{2\pi} \times \frac{t}{T_n})$</td>
<td>-5 time steps (lag)</td>
<td>0 &lt; $t$ &lt; 5s; 10% overshoot (- 0.1 × $A_c$) 5 &lt; $t$ &lt; 10s; 25% undershoot (0.25 × $A_c$)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>(iv)</td>
<td>Experimental data</td>
<td>Canoga Park earthquake ground acceleration</td>
<td>-----</td>
<td>0 &lt; $t$ &lt; 5s; 10% overshoot (- 0.1 × $A_c$) 5 &lt; $t$ &lt; 10s; 25% undershoot (0.25 × $A_c$)</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

Fig. 3-9 (a) shows the command and measured displacements for Case (i) for 6 seconds, where the corresponding SSP is provided in Fig. 3-9 (b). A closer look at the PAEI in Fig. 3-9(c) and 3-9(d) shows that the phase and amplitude errors are identified exactly.

![Fig. 3-9. Tracking Evaluation with PAEI for Case (i) (Mirza Hessabi et al., 2012)](image-url)
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It should be noted that Case (i) represents a highly ideal case, where the signals involved has a constant frequency, constant phase and amplitude errors and in the absence of any nonlinear or noise effects.

b) Case (ii)

In Case (ii), the command displacements (indicated as $u_c$ in Fig. 3-10) are generated by solving the equation of motion of a SDOF structure using the MATLAB Simulink model shown in Fig. 3-10.

![Simulink model used in Cases (ii) and (iii)](Mirza Hessabi et al., 2012)

The measured displacements (indicated as $u_m$ in Fig. 3-10) are then obtained by introducing an integer delay block from the Simulink library for the phase error, a multiplier (gain) block to introduce the amplitude error and a random number generator to consider the noise effects. As can be seen from Table 3-2, the integer delay block $z^{-5}$ provides a time lag of 5 time steps, where the time step size is set by the Simulink solver (in this case 0.001 sec.). Setting the gain value as 1.1, a 10% overshoot amplitude error is introduced in the measured displacement. The restoring force characteristics of the SDOF are captured in the embedded function with the name “State determination” where an elastic-perfectly plastic spring with strain hardening is programmed. By adjusting the value of the yield parameter this spring can be made to respond linearly as well. The numerical values of the parameters that are used in representing the SDOF system are given in Table 3-3.
As noted previously, HSEM requires the measured restoring force to evaluate tracking performance. In Case (ii) by using the Simulink model to solve the equation of motion with the state determination process, the restoring force becomes readily available which in turn makes it possible to have a comparison between HSEM, TI and PAEI. In Case (ii), the structure is assumed to be linear, there is no noise, and the above mentioned phase and amplitude errors are present in the measured displacements. The only difference between Case (ii-a) and Case (ii-b) is the amplitude of the forcing function. Since the amplitude of the applied force is twice as much in Case (ii-b) as in Case (ii-a), the resulting displacements in Case (ii-b) have twice larger amplitudes compared to Case (ii-a) (See Fig. 3-11 (a) and (b)). This is expected as a linear dynamic system is simulated under two different level of force with everything else being the same. Fig. 3-11 (c) and (d) show the resulting SSP’s where the latter has the same major axis inclination but is bigger as a result of the larger displacements involved. The restoring force-displacement plots provided in Fig. 3-11 (e) confirms that the structure behaves linearly elastic.

Although the same errors are present in both cases, the tracking evaluations performed by using HSEM and TI (Fig. 3-11 (f) and 3-11 (g)) erroneously indicate larger errors for Case (ii-b). This is attributed to the formulation of both of these methods. HSEM is based on the energy error between the hysteresis loops of measured restoring force and measured and desired displacements respectively (i.e., $E_{error}$). When the command displacement amplitudes increase, even if the error between the command and measured signals remains the same, the energy error increases (simply because the hysteresis loops involved get larger). Although the normalization terms (i.e., strain energy and input energy) are introduced in the denominator of HSEM, as Case (ii) reveals, HSEM results are still affected by the amplitude of the command displacements. Similarly, TI results are affected when the size of the SSPs change as a result of the change in the amplitude of the displacements involved (even though the errors between them remain unchanged).
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Figure 3-11. Tracking evaluation comparisons with PAEI, HSEM and TI for Case (ii) (Mirza Hessabi et al., 2012)

Figures 3-11 (i) through (l) show that PAEI do not suffer from the same limitation. Again, this is attributed to the formulation of the proposed indices: rather than the enclosed areas (either under the hysteresis loops or SSPs), PAEI directly uses the closed-form relationships between the ellipse coefficients and the phase and amplitude errors.
In Case (ii) the command displacements are obtained by solving the equation of motion of an underdamped SDOF system subject to a sinusoidal forcing function. As a result, the transient and steady state components of the resulting displacements will contain the forcing frequency and natural frequency of the structure. Considering these frequencies, the delay introduced in time domain (i.e., 5 time steps) translates into 0.03 rad, and 0.02 rad of phase lags, respectively. As can be seen in Fig. 3-11 (i) and (j), in both Case (ii-a) and Case (ii-b) the PEI is able to exhibit phase values around this range. Also, as the amplitude error in the measured displacements is introduced as a percentage (10%) of the command displacement in both cases, the overshoot errors are expected to roughly range from 0 to 1 mm. in Case (ii-a) and from 0 to 2 mm. in Case (ii-a). In Fig. 3-11 (k) and (l) the amplitude errors identified are in the expected range.

c) Case (iii)

Case (iii) studies the tracking evaluation in the presence of nonlinearity in the structural response, noise and a sudden change in error in the measured signal (in this case a change from overshoot to
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undershoot amplitude error). Simulink model presented in Fig. 3-10 is used where the errors provided in Table 3-2 are present in the measured signal.

Figure 3-12. Tracking evaluation comparisons with PAEI, HSEM and TI for Case (iii) (Mirza Hessabi et al., 2012)

As typical for the nonlinear dynamic response of the structures, the command displacement history provided in Fig. 3-12 (a) oscillates about a non-zero position after the yielding phenomenon takes place (see Fig. 3-12 (c)). Also, there is a sudden change in the amplitude error introduced; it
switches from overshoot to undershoot suddenly at 5 seconds. Although not very realistic (even if such a switch were to occur in a real experiment, it would do so gradually over a period of time), the change in the error, together with the displacement history that oscillates about a nonzero position result in SSPs that move around and rotate in the synchronization subspace (see Fig. 3-12 (b)). As such, Case (iii) is a challenging scenario for PAEI where the centering and scaling transformations introduced in the formulation will be put to test.

In Fig. 3-12 (d) and (e) both HSEM and TI are able to qualitatively indicate the presence of the existence of the time delay, where, as a result of the use of the transpose areas in its formulation, TI is less sensitive to the presence of the noise. Fig. 3-12 (g) and (h) show that PAEI successfully quantify the phase and amplitude errors. Note that in the results from all the indicators, there is a distortion around the time where the sudden switch from overshoot to undershoot takes place, which should be ignored.

d) Case (iv)

In Case (iv) tracking evaluation of the results of a real-time hybrid test is provided. The test was performed at the Lehigh University NEES RTMD facility and involved a SDOF steel moment resisting frame (MRF) as the nonlinear analytical substructure and a pair of elastomeric dampers (that was assumed to be installed on the web of the floor beams of the MRF) as the experimental substructure where the fundamental natural frequency of the test structure was found to be around 5.5 rad/sec (Mercan, 2007). Fig. 3-13 (a) displays the command and measured displacements for the first floor obtained from the real-time hybrid test, the corresponding SSP can be seen in Fig. 3-13(b). The experimental substructure exhibits nonlinear behaviour and the hysteresis loops are provided in Fig. 3-13(c). Tracking performance evaluation using HSEM (Fig. 3-13(d)) reveals that there is a time delay between the command and measured signals. Phase and amplitude errors are also quantified in Fig. 3-13 (e).

To verify the PAEI results enlarged time windows are investigated in Fig. 3-14. Fig. 3-14 (a) shows the enlarged view of command and measured displacements from Fig. 3-13 (a) around 14.35 sec where a lumped overshoot error of 0.07 mm can be observed. The enlarged view of AEI around 14.35 sec (Fig. 3-14 (b)) has an acceptable agreement with the observed amplitude error. Using the fundamental natural frequency of the test structure considered in Case (iv) (i.e., 5.5 rad/sec)
the delay observed in Fig. 3-14 (a) translates into a time delay of 0.002 sec (or $2\Delta t$), which is in good agreement with the results in Fig. 3-13 (a).

![Figure 3-13](image1.png)

**Figure 3-13.** HSEM and TI tracking evaluation for Case (iv) (Mirza Hessabi et al., 2012)

![Figure 3-14](image2.png)

**Figure 3-14.** Enlarged time windows to verify PAEI results for Case (iv) (Mirza Hessabi et al., 2012)

### 3.4.2.2 Simplified statistical error monitors

Since PAEI quantify the errors, they have the potential to be incorporated in the control law and to improve the tracking of the hydraulic actuators. However, the approach for computing PAEI
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(see Fig. 3-8) involves solving an extensive amount of large eigenvalue problems at each time step and therefore it is computationally costly. This is a major problem in RTHS and there is a need for simpler but faster methods to estimate the errors. In the following sections, the formulation and derivation of a new practical procedure with generalization capability are explained which uses simple statistical tools, to estimate the systematic errors in executing the displacement command signals (Mirza Hessabi et al., 2013).

3.4.2.2.1 Simplified statistical error monitors for a complete period

Once more Eq. 3-19 can be used to describe the command (target) displacements as a harmonic function of time with the amplitude and frequency of $A_c$ and $\omega$, respectively.

$$x_c = A_c \sin(\omega t)$$

However, a measured displacement signal ($x_m$) can be defined as another sinusoidal function of time ($t$) with the same frequency ($\omega$), but different amplitude ($A_m$) and a phase shift $\phi$ (Eq. 3-20).

$$x_m = A_m \sin(\omega t + \phi)$$

The objective here is to determine $\phi$ and amplitude errors (as shown in Eq. 3-54) independently and through closed-form equations.

$$\Gamma = \frac{A_c}{A_m}$$ (3-54)

To achieve the abovementioned objective, Root Mean Squares (RMS) of these functions will be used. By definition, RMS of a continuous function $x(t)$ over an arbitrary interval of $T_1 \leq t \leq T_2$ can be defined as,

$$f_{rms} = \sqrt{\frac{1}{\Delta T} \int_{0}^{\Delta T} [x(t)]^2 dt}$$ (3-55)

where, $T_2 = T_1 + \Delta T$. Similarly, RMS can be used for discrete functions. The RMS of $n$ different data points can be calculated using Eq. 3-56.

$$x_{rms} = \sqrt{\frac{1}{n} \left\{x_1^2 + x_2^2 + \cdots + x_n^2\right\}}$$ (3-56)

Substituting back Eq. 3-19 and 3-20 in Eq. 3-55,
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\[
(f_{rms})_c = \sqrt{\frac{1}{\Delta T} \int_0^{\Delta T} \{A_c \sin(\omega t)\}^2 dt} \quad (3-57)
\]

\[
(f_{rms})_m = \sqrt{\frac{1}{\Delta T} \int_0^{\Delta T} \{A_m \sin(\omega t + \phi)\}^2 dt} \quad (3-58)
\]

These equations can be simplified further by computing the integrals for a complete time period \(T\) of these harmonic functions (i.e. \(\Delta T = T\)).

\[
(f_{rms})_c = \frac{A_c}{\sqrt{2}} \quad (3-59)
\]

\[
(f_{rms})_m = \frac{A_m}{\sqrt{2}} \quad (3-60)
\]

It should be noted that in Eq. 3-60, the phase shift \(\phi\) is eliminated during the integration process.

Using the available data points for command and measured displacements and using Eq. 3-56, \((x_{rms})_c\) and \((x_{rms})_m\) can be calculated. By equating these values to the corresponding values from Eq. 3-59 and 3-60,

\[
A_c = \sqrt{2}(x_{rms})_c \quad (3-61)
\]

\[
A_m = \sqrt{2}(x_{rms})_m \quad (3-62)
\]

Thus, the amplitude ratio can be calculated using Eq. 3-63,

\[
\Gamma = \frac{(x_{rms})_c}{(x_{rms})_m} \quad (3-63)
\]

For the calculation of the phase error, first the error function should be defined as follows,

\[
E(t) = x_c(t) - x_m(t) \quad (3-64)
\]

Using Eq. 3-56, the error function RMS can be calculated over a complete time period \((\Delta T = T)\),

\[
(f_{rms})_E = \sqrt{\frac{1}{T} \int_0^T \{A_c \sin(\frac{2\pi t}{T}) - A_m \sin(\frac{2\pi t}{T} + \phi)\}^2 dt} \quad (3-65)
\]

where, \(\omega\) is replaced by \(2\pi/T\). Expanding Eq. 3-65,

\[
(f_{rms})_E = \sqrt{\frac{1}{T} \int_0^T A_c^2 \sin^2 \left(\frac{2\pi t}{T}\right) dt + \frac{1}{T} \int_0^T A_m^2 \sin^2 \left(\frac{2\pi t}{T} + \phi\right) dt - \frac{2}{T} \int_0^T A_c A_m \sin \left(\frac{2\pi t}{T}\right) \sin \left(\frac{2\pi t}{T} + \phi\right) dt} \quad (3-66)
\]
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The first two terms are already calculated in Eq. 3-57 through 3-60, therefore,

\[
(f_{rms})_E = \sqrt{\frac{1}{2}A_c^2 + \frac{1}{2}A_m^2 - A_cA_m\cos(\phi)} \tag{3-67}
\]

Once more, real data points can be used to evaluate the value of Eq. 3-67 by assuming \((f_{rms})_E = (x_{rms})_E\), which leads to,

\[
(x_{rms})_E^2 = \frac{1}{2}A_c^2 + \frac{1}{2}A_m^2 - A_cA_m\cos(\phi) \tag{3-68}
\]

Finally, by combining Equations 3-61, 3-62 and 3-68, the phase shift can be determined using Eq. 3-69,

\[
\phi = \arccos \left(\frac{(x_{rms})_E^2 - (x_{rms})_c^2 - (x_{rms})_m^2}{-2(x_{rms})_c(x_{rms})_m}\right) \tag{3-69}
\]

3.4.2.2.2 Application of SPP

The calculation described in the previous section assumes no sign for the phase error obtained from Eq. 3-69. Moreover, both Eq. 3-63 and 3-69 are valid for a complete period of \(T\). These two issues are addressed in this study by using the SSPs.

First, similar to the procedure that is described in Section 3.3.2.1.5, the sign of the accumulated angle can be used to assign the correct sign to the phase error determined by Eq. 3–69. Also, in order to ensure that the proposed error indicators are calculated for a complete period, a moving time window approach is adopted. This process starts with the data that represents the command and measured displacements at that specific time and moves backward in time until the limitation for the size of the window is met. This means that at each instance, data points from the known previous time steps are added to the window and the accumulated angle is calculated. This procedure continues until the value of the accumulated angle reaches \(2\pi\) (corresponding to a complete period). Consequently, the whole step is repeated for the next time steps until the last data point.
3.4.2.2.3 Comparison with TI

As mentioned in the previous sections, one of the most important drawbacks of some of the previous error indicators was that these indicators are being affected by the amplitude of the command displacements. As shown in Case (ii) of Section 3.3.2.1.7, the above statement is true for both the energy-based error monitors like HSEM and the SSP-based indicators like TI. A similar example is shown in Fig. 3-15, where the figures are plotted for two sets of predefined displacement command (Eq. 3-70) and measured (Eq. 3-71) signals.

\[ x_c = A_c \times \cos(3\pi t) \]  \hspace{1cm} (3-70)

\[ x_m = \left(\frac{A_c}{\gamma}\right) \times \cos(3\pi [t + 50 \times \Delta t]) \]  \hspace{1cm} (3-71)

where, \( t \) denoted the time, \( x_c \) and \( x_m \) are command and measured displacement, respectively. \( \Delta t \) is the size of time steps and is equal to 1/1024 sec (i.e., the clock speed of a typical digital controller). In addition, the amplitude error \( \gamma \) is constant and equal to 2/3.

\[ \gamma = \frac{2}{3} \]

Figure 3-15. The effect of command displacement amplitude on TI (Mirza Hessabi et al., 2013)
The only difference between the top and bottom figures in Fig. 3-15 is the value of the command displacement amplitude $A_c$. $A_c$ is equal to 1 mm for the top figures and 2 mm for the bottom ones. It should be noted that since the frequency of the signals is constant, the time delay could be converted to the phase error. In other words, the $50 \Delta t$ time is equal to a phase error of $\phi = 3 \pi \times (50/1024) = 0.46 \text{ rad} = 26.367 \text{ deg}$.

Fig. 3-15, clearly shows that although the predefined phase and amplitude errors are the same for both sets of command and measured displacements, the values of the corresponding TIs are not the same. Based on the definition of the TI, it can be shown that for any predefined scenario the slope of HSEM or TI is a function of the square of the command displacement. On the contrary, unlike the abovementioned indicators Fig. 3-16 shows that the proposed indicator in this study is not a function of command displacement amplitude and can estimate the phase and amplitude errors correctly for both of the signal sets.

**Figure 3-16.** The effect of command displacement amplitude on the simplified statistical error monitors (Mirza Hessabi et al., 2013)
3.4.2.2.4 Comparison with PAEI

Unlike the theoretical case shown in Fig. 3-16, during an earthquake the amplitude of the command signal is usually not constant over the time. In this section, a new case is introduced where the amplitude of the command displacement is changing linearly with time. This variation is shown by Eq. 3-72 and 3-73.

\[ x_c = \{0.1t\} \times (A_c \cos(3\pi t)) \]  
\[ x_m = \{0.1t\} \times \left(\left(\frac{A_c}{f}\right) \cos[3\pi \cdot (t + 50 \times \Delta t)]\right) \]

As it can be observed from Fig. 3-17, TI lumps the phase and amplitude errors, and the values fail to capture the constant phase and amplitude errors.

![Figure 3-17. Signals with time varying amplitudes (Mirza Hessabi et al., 2013)](image)

Tracking performance evaluation using PAEI and the proposed error indicators in this study reveal that the new set of indicators can estimate the errors with an acceptable accuracy. This comparison is shown in Fig. 3-18 where the dotted points show the results from the PAEI method and the solid lines represent the results of the proposed error estimator.

![Figure 3-18. PAEI and the proposed error estimator for signals with varying amplitudes (Mirza Hessabi et al., 2013)](image)
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It should be mentioned here that the proposed error estimator has the advantage of using only simple arithmetic and trigonometric operators where PAEI uses eigenvalue solvers in the algorithm.

As it can be seen in this section, the simplified statistical error monitors address some of the inadequacies of the previous indicators such as HSEM and TI. Although being comparatively less accurate, these equations alleviate a critical problem with PAEI, namely the necessity of using eigenvalue solvers. Due to their formulation, it should be emphasized here that the simplified statistical error monitors may be influenced by choice of ground motions. That is because the signals may change considerably during one cycle of 360 degrees of the SSP evolution. To address this shortcoming and to enhance the accuracy of the indicators and to increase the robustness of the error monitors, the simplified statistical error monitors were followed by a new set of error indicators that are introduced in Section 3.4.2.3.

3.4.2.3 Frequency-Domain Based (FDB) error indicators

In Section 3.3.2.1, PAEI were introduced as an improved set of SSP-based indicators that do not suffer from the same limitations as their predecessors. PAEI are able to successfully uncouple the phase and amplitude errors between command and measured signals and accurately quantify them. However, since it is necessary to solve a large eigenvalue problem at every time step, it is easier to use PAEI only to post-process the experiment results. As of yet, the computational time required prevents PAEI to become a set of online indicators for RTHS and hence this is an issue for incorporating them into the servo-hydraulic control law. The simplified statistical error monitors in Section 3.3.2.2 solve this issue at the cost of sacrificing the accuracy and robustness of the error monitors. That is why a new set of error indicators (i.e., FDB error indicators) are introduced in this section that are able to uncouple the phase (lead/lag) and amplitude (overshoot/undershoot) errors between command and measured displacements and quantify them. More importantly, unlike their predecessors, the implementation of these new indicators is not computationally expensive and therefore they can be executed in real-time and serve as online indicators. As such they can be incorporated into the servo-hydraulic control law to improve the tracking of the command displacements by the actuator, which in turn will result in more accurate RTHS results (Mirza Hessabi et al., 2016).
3.4.2.3.1 Formulation of the FDB error indicators

Similar to the procedure for the error monitors in Section 3.3.2.1 and 3.3.2.2, the derivation begins with considering harmonic functions of time that can describe the command (target) and measured displacements. As shown in Eq. 3-74 and 3-75, these functions have the same frequency of $\omega$ but their amplitudes and phase shifts are different:

$$x_c = A_c \sin(\omega t + \phi_c) \quad (3-74)$$
$$x_m = A_m \sin(\omega t + \phi_m) \quad (3-75)$$

where, $\phi_c$ and $\phi_m$ represent the phase shifts for the command and measured signals, respectively. An amplitude ratio is then defined as shown in Eq. 3-76 in terms of the ratio of the command over measured displacement amplitudes. Similarly, a phase error is defined in Eq. 3-77 based on the discrepancy of the phase shifts between the command and measured signals.

$$\Delta A = A_c / A_m \quad (3-76)$$
$$\phi = \phi_c - \phi_m \quad (3-77)$$

The FDB error indicators are developed using Discrete Fourier Transform (DFT). By definition, the DFT spectrum is a periodical spectrum and is discrete in the frequency domain. With a uniform time increment of $\Delta t$ and a time window of a finite length of $T_D$, the number of data points in time series approximation ($N$) can be defined as the ratio of $T_D / \Delta t$. The frequency coefficients for signal $x_i$ can be expressed as shown in Eq. 3-78. Note that $x_i$ can be either command or measured signal (Rajasekaran, 2009).

$$X_i(k) = \sum_{n=0}^{N-1} \{x_i(n)[\cos(2\pi n k/N) - j \times \sin(2\pi n k/N)]\} \quad (3-78)$$

and,

$$Im[X_i(e^{-2\pi n k/N})] = \sum_{n=0}^{N-1} \{x_i(n)\sin(2\pi n k/N)\} \quad (3-79)$$
$$Re[X_i(e^{-2\pi n k/N})] = \sum_{n=0}^{N-1} \{x_i(n)\cos(2\pi n k/N)\} \quad (3-80)$$

Amplitude of the two harmonics can be found as,

$$A_i^H = |X_i| = \sqrt{Re[X_i(e^{-2\pi n k j/N})]^2 + Im[X_i(e^{-2\pi n k j/N})]^2} \quad (3-81)$$
Similarly, the phase of $u_c$ and $u_m$ can be computed using Eq. 3-82:

$$
\phi_i^* = \arctan \left[ \frac{\text{Im}[X_i(e^{-2\pi n kj/N})]}{\text{Re}[X_i(e^{-2\pi n kj/N})]} \right]
$$

The values of the power spectrum around different frequency components can be computed by using Eq. 3-81 and 3-82. To obtain the FDB error indicators, first the DFT power spectra need to be calculated for the command and measured signals independently. Then, from each power spectrum, the frequency that corresponds to the largest amplitude is located (i.e. $\bar{f}_c$ and $\bar{f}_m$ for command and measured signals, respectively). As shown in Eq. 3-83, the ratio between the magnitudes of the power spectra at the frequencies of $\bar{f}_c$ and $\bar{f}_m$ can be used to determine the amplitude ratio,

$$
\Delta A = \frac{|U_c(\bar{f}_c)|}{|U_m(\bar{f}_m)|}
$$

Likewise, the phase error defined in Eq. 3-77 is found as the difference of the DFT phase spectrum values of the two signals at $\bar{f}_c$ and $\bar{f}_m$.

$$
\phi = \phi_c^*(\bar{f}_c) - \phi_m^*(\bar{f}_m)
$$

Positive and negative phase error values from Eq. 3-84 show phase lead and lag, respectively. Similarly, a value greater than one obtained from Eq. 3-83 is an undershoot error and a value less than one represents overshoot. Moreover, it should be emphasized that the closer $\Delta A$ and $\phi$ values be to 1 and 0, respectively, the more accurate is the actuator control and the more reliable are the RTHS results. In order to implement FDB error indicators as online indicators that assess the error as a function of time as the RTHS progresses, the DFT spectra are obtained using Fast Fourier Transform (FFT), and a windowing approach is employed. To reduce spectral leakage effects, windowing functions (e.g. Han, Hamming windows, etc.) can be used. Special care should be taken about the DC component of the frequency response. In this study before finding the location of $\bar{f}_c$ and $\bar{f}_m$, the component associated with the frequency of zero was removed from the power spectrum of command and measured signals. The MATLAB script files for computing the values of FDB indicators are shown in Appendix C.
It should be noted that there are some major differences between the FDB indicators developed in this study and the recently proposed frequency-domain-based parameter (i.e. FEI) by Guo et al. (Guo et al., 2014) that calculates phase and amplitude errors. First, FEI are proposed as a post-processing tool whereas with the introduction of the moving time windows in this study, FDB indicators can be used as an on-line assessment tool. The second key difference can be attributed to the way that these two methods interpret the frequency responses of the signals. A weighted averaging procedure is used for calculation of the FEI where an arbitrary number of dominant frequencies of the response are chosen. The values of the power of weights were also determined experimentally. Through this averaging procedure the frequency response of noise may contaminate the calculations. On the other hand, in this study it was found that only the dominant frequency of the frequency response of the command signal should be used in the calculation of the FDB tracking error indicators. A quantitative comparison between these two types of indicators is shown in Section 3.3.2.3.2

3.4.2.3.2 Effects of window size on the accuracy of the FDB error indicators

Fourier transforms are used in the formulation of the FDB error indicators. As explained previously, a moving time window is used for their online implementation. To prevent spectral leakage effects, a suitable windowing function should also be introduced. In this section, the effects of using different windowing functions together with the size of the windows are investigated. Predefined sinusoidal displacements with a known amplitude ratio of 2/3 and a time delay of 10 time-steps (that corresponds to 0.061 rad) are used.

To consider the effects of window size on the accuracy of the error indicators, the ratio $\chi$ is defined using Eq. 3-85:

$$\chi = \frac{N_{\text{win}}}{N_{\text{cycle}}}$$ (3-85)

where, $N_{\text{win}}$ is the number of data point within each selected window and $N_{\text{cycle}}$ is the number of data points within each complete cycle of the signal. This way, a value of 1 for $\chi$ means that selected time window contains one complete cycle of the signal being processed.
Using Eq. 3-83 and 3-84, and for different values of $\chi$, phase errors and amplitude ratios are computed and compared to the known values for these errors. The difference between the estimated and known error values are shown in Fig. 3-19.

From the plots in Fig. 3-19, it can be observed that the error in the estimation of the phase error is less than 2% when $\chi > 0.90$ and less than 10% when $\chi > 0.73$. A more important observation is that the phase error is more sensitive to the size of the selected window. To ensure the accuracy of the results, it is recommended here that size of the FFT signal should be larger than the fundamental period of the input signal.

**Figure 3-19.** Variation of the accuracy of FDB indicators with $\chi$ (Mirza Hessabi et al., 2016)

To study the effects of windowing functions, five different commonly used functions, namely rectangular, Hann, Hamming, Blackman and Gaussian functions are employed. In Table 3-4 maximum errors in the phase error and amplitude ratio estimation, computed using the known amplitude and phase error values, are reported for different windowing functions. Based on the results of this table, the best results could be obtained when Hamming windows are used.

**Table 3-4.** Maximum estimation error for different windowing techniques ($\chi = 0.73$) (Mirza Hessabi et al., 2016)

<table>
<thead>
<tr>
<th>Windowing Function</th>
<th>Amplitude Ratio</th>
<th>Phase Error (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular</td>
<td>1.95</td>
<td>36.32</td>
</tr>
<tr>
<td>Hamming</td>
<td>0.50</td>
<td>8.47</td>
</tr>
<tr>
<td>Hann</td>
<td>0.89</td>
<td>15.53</td>
</tr>
<tr>
<td>Blackman</td>
<td>2.49</td>
<td>47.83</td>
</tr>
<tr>
<td>Gaussian</td>
<td>1.11</td>
<td>19.61</td>
</tr>
</tbody>
</table>
3.4.2.3.3 Numerical and experimental case studies for performance evaluation of FDB error indicators

a) Case (i)

In this case, using the Simulink model of the inner loop, the effectiveness of the FDB error indicators in handling the signals with noise is studied. A comparison is made between the results obtained from the proposed FDB indicators and other available indicators (i.e., TI and PAEI) in order to evaluate the performance of the FDB error indicators. The predefined displacements are shown in Eq. 3-86 and 3-87:

\[
x_c = \cos(\pi t) + K \times \{\text{noise}\} \quad (3-86)
\]

\[
x_m = 2 \cos(\pi [t + 25/1024]) + K \times \{\text{noise}\} \quad (3-87)
\]

In Eq. 3-86 and 3-87, a random number generator with a Gaussian distribution is used to consider the noise effects where the random number changes between zero and one and the multiplier $K$ is assumed to be 0.10 mm. Time history and SSP of the $x_c$ and $x_m$ signals are shown in Fig. 3-20(a) and 3-20(b), respectively.

![Figure 3-20. Effectiveness of the FDB error indicators in dealing with noise: d) amplitude ratio and e) phase error of the signals (Mirza Hessabi et al., 2016)](image-url)
TI and estimated amplitude ratio and phase error are also shown in Fig. 3-20. Note that $\phi = 0 - \pi \times [25/1024] = -0.077$ and $\Delta A = 0.5$. By comparing the results in Fig. 3-20 (d) and 3-20 (e) with the known errors, it can be seen that despite the relatively high introduced noise level, FDB indicators can estimate the errors with an acceptable level of accuracy.

b) Case (ii)

To investigate the effects of the command displacement amplitudes on the accuracy of the error indicators, the amplitudes of the predefined command and measured displacements in Case (ii) are changed linearly with time. This variation is shown in Eq. 3-88 and 3-89:

$$x_c = \{0.1t\} \times 2.5 \sin(3\pi t + \phi_c) \quad (3-88)$$

$$x_m = \{0.1t\} \times 2.0 \sin(3\pi t + \phi_m) \quad (3-89)$$

where, $\phi_c$ and $\phi_m$ are 0.092 and 0.369 radians, respectively. Thus, the amplitude ratio for this set of command and measured signals is equal to 1.25. Similarly, the phase error for this case is equal to -0.277 rad or -15.82 deg. Fig. 3-21 shows time histories of command and measured displacements (Fig. 3-21 (a)), the corresponding synchronization subspace plot (Fig 3-21 (b)), TI (Fig. 3-21 (c)) and PAEI (Fig. 3-21 (d) and 3-21 (e)) for this case. FDB indicators are also plotted in Fig. 3-21 (f) and 3-21 (g). Since all the command displacements considered in Fig. 3-21 have the same frequency and a constant time delay, the phase between the command and measured displacements estimated by TI, PAEI and FDB error indicators should be constant and the same. However, as discussed above, TI for each case is changing, implying that the slope of the tracking indicator is affected by the amplitude of the command displacement.

Table 3-5 compares the averaged FDB error indicators (from Fig. 3-21 (f) and 3-21 (g)) with the FEI error indicators.

Table 3-5. Comparison between FDB and FEI error indicators and the exact solution (Case (ii))

(Mirza Hessabi et al., 2016)

<table>
<thead>
<tr>
<th></th>
<th>Known values</th>
<th>Averaged FDB indicators</th>
<th>FEI indicators</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amplitude Ratio</td>
<td>1.25</td>
<td>1.245</td>
<td>1.250</td>
</tr>
<tr>
<td>Phase Error (deg)</td>
<td>-15.82</td>
<td>-15.749</td>
<td>-15.871</td>
</tr>
</tbody>
</table>
The known values of the amplitude and phase errors are also provided in the table. FDB indicators are able to accurately quantify both the phase and amplitude errors and are in agreement with the values of the known errors and those measured by FEI.

Figure 3-21. Comparison between FDB and other error indicators for signals with time-varying amplitudes (Mirza Hessabi et al., 2016)

c) Case (iii)

In this case, in order to assess the accuracy of the FDB error indicators in quantifying the actuator tracking error, results from a real-time hybrid simulation are evaluated. The experiment was performed at the Lehigh University NEES RTMD facility and involved an SDOF steel moment.
resisting frame as the nonlinear analytical substructure and a pair of elastomeric dampers as the experimental substructure. This particular real-time hybrid simulation result corresponds to another experiment with the same test setup explained in Section 3.3.2.1.7 (Case (iv)).

Recorded measured and command displacements are evaluated using TI, $\theta_{pc}$ and HSEM. Fig. 3-22 (a) shows the command and measured displacements obtained from the real-time hybrid simulation and the enlarged windows of these signals around 19.53 sec and 23.35 sec are shown in Fig. 3-22 (c) and 3-22 (d), respectively. In addition, the experimental substructure exhibits nonlinear behaviour and the hysteresis loops are provided in Fig. 3-22 (b).

![Figure 3-22](image)

**Figure 3-22.** Command and measured displacement time histories and the corresponding hysteresis loop for Case (iii) (Mirza Hessabi et al., 2016)

An important property of both HSEM and TI is their ability to qualitatively determine the phase error type. The negative and positive global slopes of these plots correspond to phase lead and phase lag, respectively. This test is of particular importance because as shown in Fig. 3-23 (a) and 3-23 (b), the results of previous studies showed that the phase error switches from lead to lag to
lead during the experiment. This is consistent with the observations in Fig 3-23 (c) and 3-23 (d). The slope of TI does not provide any information about the amplitude errors.

Figure 3-23. Evaluation of the tracking errors using available methods: a) HSEM, b) TI and c) $\theta_{pc}$ (Mirza Hessabi et al., 2016)

Fig. 3-24 (a) and 3-24 (b) show the FDB error indicators for Case (iii). A closer look at Fig. 3-24 indicates that FDB error indicators successfully distinguish between lag and lead, and overshoot and undershoot errors. In other words, Fig. 3-24 (a) shows that sign of the FDB phase error indicator is positive for the first portion of the signal and it becomes negative between the 16.5 and 21 seconds and then finally it changes to positive after the 21st second. This is in agreement with the observations from TI and HSEM. The amplitude error in Fig. 3-24 (b) is also smaller than one which indicates an overshoot error. Moreover, using the fundamental natural frequency of the test structure considered in Case (iii) (i.e., 5.5 rad/s) the 0.002 sec time delay observed in Fig. 3-22 (c) translates into a -0.65 deg phase error, which is in good agreement with the results in Fig. 3-24 (a) at around 19.53 sec.

Figure 3-24. FDB error indicators for Case (iii) (Mirza Hessabi et al., 2016)
It is shown here that FDB error indicators can uncouple the phase and amplitude errors and can accurately quantify them. As such, their performance is superior in comparison to the previous indicators. In 2014, FDB error indicators were successfully implemented in real-time and incorporated into the control loop of an adaptive controller (i.e., 2DOF controller). More information about the incorporation of these indicators into the controller and their experimental validation could be found in Section 3.5.3.

### 3.5 Applications of the developed tracking error monitors

The proposed tracking error monitors of the previous sections (i.e., PAEI, simplified statistical error monitors and FDB error indicators) provide insight into the quality of the RTHS result. While these indicators present a method for estimating the tracking errors while a test is in progress, they can also be used to check the stability of a simulation and to compensate for the errors during an experiment. If the tracking errors and in particular time delays are not accounted for, instability of the feedback loop is likely to occur. By developing proper stability criteria for both the numerical and experimental components of the simulation, it is possible to ensure that the measured errors are within the acceptable range for a stable experiment. Using the solution for the delay differential equation of a testing setup with a GMD, the stability of these systems is examined in this section. In addition, since effective compensation for the measured errors is a key factor in obtaining reliable RTHS results, it is shown in the second part of this section that the indicators of Section 3.3 can be incorporated into a new two degree-of-freedom controller and develop closed-form equations to design an adaptive servo-hydraulic controller with improved tracking performance.

#### 3.5.1 Stability analysis of the outer loop dynamics in RTHS testing

According to (Nise, 2008), “a system is stable if every bounded input yields a bounded output.” In this section, a linearization procedure is used to establish the stability boundaries in the presence of a time delay in the restoring forces in the RTHS of SDOF systems equipped with GMDs. The derivation in this section can be used to determine the stability boundaries for the experimental substructure of this study.

The equation of motion for a system with a time delay in the measured force can be written as:
Eq. 3-90 is a delay differential equation that is a function of the current state of the system and the state of the system a fixed time $\tau$ ago. Since the inertial, damping and stiffness (i.e., $m_{an}$, $c_{an}$ and $k_{an}$) characteristics of the analytical substructure are specified analytically in the integration algorithm, it is assumed that no delay is associated with them. To illustrate the effects of the structural properties (mass, damping, and stiffness) on the delay-dependent stability boundary and the use of the characteristic equation to provide insight into the results of a stability analysis, RTHS testing of an SDOF analytical substructure with an experimental linear viscous damper (similar to what is shown in Fig. 4-49) is considered here. The equation of motion of the system with a linear viscous damper can be written as,

$$m_{an}\ddot{u}(t) + c_{an}\dot{u}(t) + k_{an}u(t) + c_{exp}\dot{u}(t - \tau) = P_{eff}(t)$$  \hspace{1cm} (3-91)

The characteristic equation of Eq. 3-91 can be shown by the following equation:

$$c(s, e^{-s\tau}) = m_{an}s^2 + c_{an}s + k_{an} + c_{exp} s e^{-s\tau}$$  \hspace{1cm} (3-92)

Now, for the stability analysis of the above system in the presence of time delay, the pseudodelay technique (Gu et al., 2003) is used. This method has previously been used by Mercan and Ricles to investigate the stability of RTHS’s with SDOF and MDOF analytical substructures (Mercan et al., 2007) and (Mercan et al., 2008). In this technique, instead of the linearization of the nonlinear term, an exact mapping of the terms expressing the delay in the restoring force is used to determine the stability limit. The pseudodelay technique uses a mapping technique that transforms the characteristic equation $c(s, e^{-s\tau})$ of the time delay system into a polynomial $q(T, s)$, and investigates the roots of $q(T, s)$ for imaginary axis (which is the boundary of stability on the complex plane) crossings. The transformation of the characteristic equation $c(s, e^{-s\tau})$ is performed by deploying an exact substitution for $e^{-s\tau}$ by Rekasius (Rekasius, 1980) given as,

$$e^{-s\tau} = \frac{1-sT}{1+sT}$$  \hspace{1cm} (3-93)

which is defined for $s = i\omega$, where $\omega \in R$. Eq. 3-93 is an exact relationship on the imaginary axis (Mercan et al., 2007) with the following mapping condition:
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\[ \tau = \frac{2}{\omega} [\tan^{-1}(\omega T) \pm k\pi], \quad k = 0, 1, 2, ... \] (3-94)

If at some value of \( T = T_{crit} \) for the case where \( 0 \leq T < \infty \) (where \( T \in R^+ \)), the polynomial \( q(s, \tau) \) has a pair of imaginary roots at \( s = \pm i\omega_{crit} \) for \( \omega_{crit} > 0 \), the corresponding time delay \( \tau_{crit} \) to cause an instability can be found by using the mapping condition expressed in Eq. 3-94:

\[ \tau_{crit} = \min \left\{ \frac{2}{\omega_{crit}} [\tan^{-1}(\omega_{crit} T_{crit})] \right\} \] (3-95)

Now, after substituting Eq. 3-93 into the characteristic equation of Eq. 3-91, the new characteristic equation \( c = (s, \frac{1-sT}{1+sT}) \) can be written as follows,

\[ c \left( s, \frac{1-sT}{1+sT} \right) = m_{an}s^2 + c_{an}s + k_{an} + c_{exp}s \frac{1-sT}{1+sT} \] (3-96)

Now the polynomial \( q(T, s) \) can be expressed as:

\[ q(s, \tau) = (1 + sT)^p \times c \left( s, \frac{1-sT}{1+sT} \right) \] (3-97)

Thus,

\[ q(s, \tau) = m_{an}Ts^3 + (m_{an} + (c_{an} - c_{exp})T)s^2 + (c_{an} + c_{exp} + k_{an}T)s + k_{an} \] (3-98)

To check the stability limits of this system, the Routh’s stability criterion method may be used. However, since Eq. 3-91 contains only a few terms, it is faster to use the alternative method of (Ogata, 2009). In this approach, \( s \) in the characteristic equation is replaced by \( j\omega \), and then both the real part and the imaginary part are set to zero, and then both equations are solved for \( \omega \) and \( T \). For the present system, the polynomial equation of Eq. 3-98, with \( s = j\omega \), is

\[ q(s, \tau) = \left[ (c_{an} + c_{exp} + k_{an}T)\omega - m_{an}T^3 \omega^3 \right] j + \left[ k_{an} - (m_{an} + (c_{an} - c_{exp})T)\omega^2 \right] \] (3-99)

Equating both the real and imaginary parts of this last equation to zero, respectively, the following equations can be obtained,
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\[ k_{an} - (m_{an} + (c_{an} - c_{exp}) T)\omega^2 = 0 \]  
(3-100)

\[ (c_{an} + c_{exp} + k_{an}T)\omega - m_{an}T\omega^3 = 0 \]  
(3-101)

from which,

\[
\left[ k_{an}(c_{an} - c_{exp})T^2_{crit} + (c_{an}^2 - c_{exp}^2)T_{crit} + m_{an}(c_{an} + c_{exp}) \right] \times \left\{ \frac{k_{an}}{m_{an} + (c_{an} - c_{exp})T_{crit}} \right\}^{0.5} = 0
\]  
(3-102)

\[ \omega_{crit} = \left\{ \frac{k_{an}}{m_{an} + (c_{an} - c_{exp})T_{crit}} \right\}^{0.5} \]  
(3-103)

For instance, for an SDOF system with a mass of 14,440 kg, a period of 1 sec, a damping ratio of 2% and a linear viscous damper with a damping coefficient of 20% of the critical damping of the SDOF system, \( T_{crit} \) and \( \omega_{crit} \) of 0.144 and 7.6567 rad/sec can be found, respectively. Substituting these values in Eq. 3-93 leads to a critical delay of 0.218 sec. For a time-step of 0.001 sec, the calculated delay corresponds to 218 time steps. Therefore, in order to have a stable RTHS for the SDOF system with the linear viscous damper, the calculated experimental delay that can be determined by using the error monitors of the previous sections should be limited to 218 time steps.

It should be noted that for other systems, a similar approach can be used and different values of stability limits can be found. For example, the critical delay for an SDOF system with an experimental spring in (Mercan et al., 2007) is found to be 6 time steps.

### 3.5.2 Stability analysis of RTHS testing of GMDs

As it will be shown in the next chapter, GMD forces can be described by an acceleration proportional term plus a friction force term. Substituting the resisting force from either the lumped mass model with friction or the disk model with friction into Eq. 3-90 leads to the following differential equation for a system with an SDOF analytical substructure and a GMD as the experimental substructure.

\[ m_{an}\ddot{u}(t) + c_{an}\dot{u}(t) + k_{an}u(t) + b\dot{u}(t - \tau) + F_{fmax}\text{sign}(\dot{u}(t - \tau)) = P_{eff}(t) \]  
(3-104)
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In Eq. 3-104, $F_{f_{\text{max}}}$ and $b$ are friction force coefficient and the equivalent mass, respectively. Now, the friction force term in Eq. 3-102 can be replaced by the equivalent viscous damping $c_{eq}$ (Chopra, 2011). Thus, the corresponding characteristic equation of the time delay system with the equivalent viscous damping can be presented in Eq. 3-103:

$$c(s, e^{-s\tau}) = m_{an}s^2 + c_{an}s + k_{an} + b s^2 e^{-s\tau} + c_{eq}s e^{-s\tau}$$ (3-105)

After substituting the delay terms from Eq. 3-93 into Eq. 3-105, the characteristic equation $c(s, e^{-s\tau})$ can be expressed as,

$$c\left(s, \frac{1 - sT}{1+s\tau}\right) = m_{an}s^2 + c_{an}s + k_{an} + b s^2 \frac{1 - sT}{1+s\tau} + c_{eq}s \frac{1 - sT}{1+s\tau}$$ (3-106)

which can be rearranged as Eq. 3-107:

$$c\left(s, \frac{1 - sT}{1+s\tau}\right) = \frac{m_{an}Ts^3 + m_{an}s^2 + c_{an}Ts^2 + c_{an}s + k_{an}Ts + k_{an} + b s^2 - bTs^3 + c_{eq}s - c_{eq}Ts^2}{1+s\tau}$$ (3-107)

Now the polynomial $q(T, s)$ for this characteristic equation can be expressed as:

$$q(s, \tau) = (1 + sT) \times c\left(s, \frac{1 - sT}{1+s\tau}\right)$$ (3-108)

Following the steps outlined above, the polynomial $q(s, \tau)$ for the RTHS testing of the GMD with an SODF analytical substructure can be shown by the following equation:

$$q(s, \tau) = (m_{an} - b)Ts^3 + (m_{an} + b + [c_{an} - c_{eq}]T)s^2 + (c_{an} + c_{eq} + k_{an}T)s + k_{an}$$ (3-109)

The stability of the system whose dynamics are represented by a polynomial $q(s, \tau)$ can be investigated by applying Routh’s stability criterion (Mercan et al., 2007). The Routh table for this polynomial is shown in Table 3-6. In order to create this table, rows of the table should be labeled with powers of $s$ from the highest power to $s^0$. Next, the coefficient of the highest power of $s$ should be written in the first column of the first row, and then every other coefficient should be listed horizontally in the first row. In the second row, starting with the next highest power of $s$, every coefficient that was skipped in the first row should be listed. The remaining entries are filled...
as follows. Each entry is a negative determinant of entries in the previous two rows divided by the entry in the first column directly above the calculated row. The left-hand column of the determinant is always the first column of the previous two rows (Nise, 2008).

Table 3-6. Completed Routh table for polynomial of Eq. 3-109

| \( s^3 \) | \( (m_{an} - b)T \) | \( c_{an} + c_{eq} + k_{an}T \) |
| \( s^2 \) | \( m_{an} + b + [c_{an} - c_{eq}]T \) | \( k_{an} \) |
| \( s^1 \) | \( A^* \) | 0 |
| \( s^0 \) | \( k_{an} \) | 0 |

According to Routh’s stability criterion, the number of sign changes in the first column of Routh’s array is equal to the number of unstable poles of the system. This column is shown in Table 3-6. Since \( m_{an} \) and \( b \) are physical quantities that are greater than zero. By definition, \( k_{an} \) is always positive so in order to check the satisfaction of all of the necessary and sufficient conditions for stability, the first row, second row and \( A^* \) terms should be shown to be also positive. Starting with the first row, it can be concluded that in order to have a positive \( (m_{an} - b)T \) term, the maximum value of the equivalent mass should always be limited to the mass of the analytical substructure. This means,

\[
 b < m_{an} \quad (3-110)
\]

Following the same logic, \( A^* \) in the third row should also be positive,

\[
 A^* = \frac{(m_{an} - b)T}{m_{an} + b + [c_{an} - c_{eq}]T} \frac{c_{an} + c_{eq} + k_{an}T}{k_{an}} \quad (3-111)
\]

Since the denominator is positive, the numerator should be positive too.

\[
 (c_{an} + c_{eq} + k_{an}T)(m_{an} + b + [c_{an} - c_{eq}]T) - (m_{an} - b)(k_{an})T > 0 \quad (3-112)
\]

Eq. 3-111 should be solved for the values of \( T \) that satisfy the inequality. In addition, after finding the value of \( T \), sign of the second row should be examined:

\[
 m_{an} + b + [c_{an} - c_{eq}]T > 0 \quad (3-113)
\]
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It can be shown that the equation in Eq. 3-112 is a third order polynomial of $T$. After solving Eq. 3-112 for the specified testing setup parameters, all of the three roots for $T$ can be calculated and the corresponding stability limits can be established.

Alternatively, one of the available tools to examine the inequalities of Eq. 3-112 and 3-113 and to illustrate the effects of the changes of each the system parameters on delay-dependent stability is the application of root locus plots (Ogata, 2009) to observe the migration of the roots of $q(s, \tau)$ in Eq. 3-109 as the delay is increased. A root locus plot portrays the variation in pole locations (i.e. the roots of Eq. 3-109) as one parameter is increased from zero to infinity. The instability occurs when a root crosses the imaginary axis into the right-hand side of the complex plane of the root locus plot. MATLAB’s `rlocus` command can be used to track the root locations of a given polynomial as one variable is increased from zero to infinity, provided that the polynomial is expressed in the following form:

$$1 + K \left[ \frac{\text{num}(s)}{\text{den}(s)} \right] = 0 \quad (3-114)$$

where, `num(s)` and `den(s)` are the two polynomials, and $K$ is the variable that is increased. In the usual application of the root locus method in a classical controller design, `num(s)` and `den(s)` are the numerator and denominator polynomials of the open-loop system transfer function and $K$ is the system gain. For the current application, $q(s, \tau)$ from Eq. 3-109 can be expressed in the form of Eq. 3-114 where $K$ is equal to $T$, $\text{num}(s) = (m_{an} - b)s^3 + (c_{an} - c_{eq})s^2 + k_{an}s$, and $\text{den}(s) = (m_{an} + b)s^2 + (c_{an} + c_{eq})s + k_{an}$.

Now in order to show the results, a new SDOF system is considered here which is similar to the system of Section 3.4.1 and has a mass of 14,440 kg, a period of 1 sec, an inherent damping ratio of 2% but has no supplemental viscous damper. When the friction term is neglected, for $b < m_{an}$ the system is always stable. In fact, Eq. 3-104 for $F_{fmax} = 0$ becomes the equation of motion of a stable SDOF system with a smaller mass. The root locus plot in Fig. 3-25 (for $b/m_{an} = 9.56 \%$) verifies this conclusion too. As it can be seen, the diagram does not intersect with the imaginary axis at any point. In other words, theoretically, the critical time delay $\tau_{crit}$ limit for the stability of an SDOF system with $b < m_{an}$ and $F_{fmax} = 0$ is equal to infinity.
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Figure 3-25. Root locus plots for the stability analysis of RTHS testing of GMDs \((F_{fmax} = 0)\)

With the increase of the friction force term \(F_{fmax}\), the critical limit for the time delay decreases. For instance, for the GMD used in this study with an equivalent mass \(b\) of 1379.9 kg, \(F_{fmax}\) is not equal to zero. As it will be shown in the next chapter, this parameter can be experimentally measured as 53.22 N. The equivalent damping of the system at the resonance frequency for the maximum displacement of 7.5 mm is equal to \(c_{eq} = 623.31\) N.s/m \((\zeta_{eq} = 0.31\%\)) which is less than the damping ratio for the analytical substructure (Section 5.1.2). These dynamic parameters can be substituted in Eq. 3-114. Plots of the root locus diagrams show that the imaginary axis is not crossed for these values. This means that since the friction forces are relatively small, the test is stable for different values of delay.

However, the root locus diagrams crossed the imaginary axis at two points when the friction force coefficient increases to 197 N. The enlarged view of the plot in Fig. 3-26 (b) shows that \(T_{crit}\) at the intersection of the root locus diagram with the imaginary axis is equal to 121. At this point \(\omega_{crit}\) is also 6.61 rad/sec. Substituting these values in Eq. 3-95 yields,

\[
\tau_{crit} = \min \left\{ \frac{2}{6.61} \left[ \tan^{-1}(6.61 \times 121) \right] \right\} = 0.475\ sec \quad (3-115)
\]
which is a relatively large time delay and with a well-tuned controller, it is possible to keep the actual time delay lower than this value. This is because the value of the friction force in comparison to the acceleration-proportional term is smaller and as a result the measured $F_{f_{\text{max}}}$ cannot cause instability concerns. With the increase of friction forces $F_{f_{\text{max}}}$ for the same analytical substructure, the critical time delay $\tau_{\text{crit}}$ decreases. For instance, for a friction force coefficient of 532 N (10 times larger than the measured maximum friction force), the $\tau_{\text{crit}}$ from the root locus analysis is equal to 0.149 sec (Fig. 3-26 (c)).

Figure 3-26. Root locus plots for the stability analysis of RTHS of GMDs with (a) $F_{f_{\text{max}}} = 53.2 \, N$, (b) $F_{f_{\text{max}}} = 2,182 \, N$ and (c) $F_{f_{\text{max}}} = 5,322 \, N$
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Figure 3-26 (Continued). Root locus plots for the stability analysis of RTHS of GMDs with (a) $F_{f_{\text{max}}} = 53.2 N$, (b) $F_{f_{\text{max}}} = 2,182 N$ and (c) $F_{f_{\text{max}}} = 5,322 N$

In summary, the stability analysis shows that for the experimental setup considered in the next chapter, the system is stable and the dynamics of the outer loop will not cause instability in the system when a delay in the measured restoring force occurs. However, it should be noted that for more complicated analytical substructures, when a direct step-by-step integration algorithm is employed, depending on the error propagation characteristics of the given integration algorithm, numerical instabilities may still occur when the magnitude of the amplitude error exceeds a certain value (Mercan et al., 2007) and for other experimental studies with the GMD prototype specimen, similar stability analyses should be performed.

3.5.3 Error compensation using the developed tracking error indicators

Another application of the proposed error indicators in Section 3.3.2.1 to 3.3.2.3 is to incorporate them into the servo-hydraulic control law. This is not a focus of this dissertation, however, due to the importance of this application, it is briefly discussed in this final Section of Chapter 3.

Since the implementation of the introduced error indicators in the Sections 3.3.2.3 (i.e., FDB error indicators) is not computationally expensive, they can be executed in real-time and serve as online indicators. As such they can be incorporated into the servo-hydraulic control law to improve the tracking of the command displacements by the single actuator system, which in turn will result in
more accurate RTHS results. To accomplish this, FDB error indicators are introduced into a 2DOF control system. 2DOF controllers have been used before (Carrion et al., 2009), but most of them were model-based and required accurate system identification before the tests. The new adaptive implementation of the 2DOF controller with FDB error indicators requires neither a model of the system dynamics identified a priori nor a set of controller constants to be adjusted ad hoc. Hence, the new 2DOF controller in this section does not suffer from the limitations of the previous 2DOF controllers and previous feed-forward controllers (Jung et al., 2007). The block diagram of the actuator compensation using 2DOF controllers is shown in Fig. 3-27.

In Fig. 3-27, the 2DOF controller (i.e., the yellow block) is added to the inner loop and can improve the performance of the existing RTHS controller (e.g., PID controller). In Appendix D, the derivation of the 2DOF controller is shown.

\[ b_1 = \sqrt{\frac{(1/\Delta A)^2(1+\omega^2)}{((\omega-tan(\phi)\omega^2)^2 + \omega^2}\right) \right) + \omega^2}} \]

\[ b_0 = \frac{\omega-tan(\phi)\omega^2}{\omega+tan(\phi)} b_1 \]
3.6 Summary

One of the central aspects of obtaining reliable RTHS results is the accuracy of the servo-hydraulic actuator in tracking the command displacements. In recent years, several tracking error indicators were developed to assess the accuracy of actuator control in hybrid simulations. In order to address the limitation of the previous indicators, in this chapter, three new sets of indicators (PAEI, simplified statistical error monitors, and FDB error indicators) are introduced. PAEI are derived based on the characteristics of the SSP of the command and measured displacements and FDB error indicators are formulated in the frequency domain. Both of these local response measures can uncouple and separately quantify the amplitude and phase errors. Moreover, both of these error indicators employ a moving window approach with proper windowing functions. This enables the FDB error indicators to be implemented and executed in real-time. As such, FDB indicators can be used to examine the stability of RTHS and to set the acceptable time delay delays. In addition, these indicators not only become online assessment tools that can assess the tracking performance of the hydraulic actuator as the RTHS progresses, but they can also be incorporated into the servo-hydraulic control law. Through several numerical simulations, the effects of the window size, windowing functions, noise, frequency, and amplitude characteristics of the signals processed were investigated through numerical simulations. It is shown that PAEI are able to quantify the errors, even when the test structure considered exhibit nonlinear behaviour and in the presence of noise. It is also shown that unlike energy-based measures and TI, PAEI and FDB error indicators results are unaffected by the amplitude of the command displacements. As such, the two discussed error measures can serve as standard indicators to compare the accuracy of the real-time test results which have considerably different command displacement histories. Moreover, the developed error indicators can be used to establish the stability boundaries in the presence of a time delay in the restoring forces in the RTHS. The stability analysis performed in this chapter for SDOF systems equipped with GMDs showed that, for the existing equivalent mass and friction forces, RTHS is stable. The performance of the introduced error indicators is once again investigated by processing the command and measured displacements from RTHS tests performed on the testing setups with GMDs in the next chapter.
Chapter 4. Experimental testing of gyromass dampers

This chapter describes the experimental testing and evaluation of two prototype gyromass dampers (GMDs). These dampers are evaluated under different experimental scenarios, generating a diverse database. The obtained database includes characterization tests and real-time hybrid simulations (RTHS) emphasizing on the effects of friction forces and the nonlinearities of the behaviour. To demonstrate the differences between the predictive models outlined in Chapter 2 with the experimental results, the GMD prototypes were subjected to a range of predefined displacement inputs with various frequencies and amplitudes. The collected data was used to obtain calibrated models of this device with different levels of complexity.

In general, the following cases were considered for the input displacements:

**Case 1** reveals the dynamic properties of a GMD when it is subjected to harmonic displacements. For this case, the applied command displacements may be a harmonic function which contains a single dominant frequency or may be a summation of several different harmonics.

**Case 2** illustrates the behaviour of a GMD as this device is subjected to a more complicated loading scenario such as chirp inputs ground acceleration emphasizing on the effects of input frequency on the performance of these dampers.

**Case 3** shows the performance of a GMD in an SDOF system that is subjected to several earthquakes ground motions. Each ground motion record is selected to represent a particular category of earthquakes with a certain attribute. This case will also be used in RTHS.

This chapter is split into five related sections. The two prototype GMDs that are tested in this chapter differ in terms of their size and the material of their components. The experimental study of the smaller prototype is shown in the first section. Next, in the two sections that follow experimental studies including characterization tests and RTHS of the larger prototype are
Chapter 4. Experimental Testing of Gyromass Dampers

presented, and the corresponding results are discussed. Finally, a fluid viscous damper (VD) is added to the GMD. After the characterization of the VD, the effect of the added damping on the performance of the control system is examined through RTHS.

4.1 Characterization of the small-scale prototype

To explore the validity of the existing characteristic equations of GMDs from the literature, a small-scale prototype was built and tested. This prototype is shown in Fig. 4-1. Some of the properties of this prototype can be found in Chapter 2. For instance, the equivalent mass of the prototype specimen is calculated in Section 2.5.1.

![Small-scale GMD prototype](image)

**Figure 4-1.** Small-scale GMD prototype

4.1.1 Testing setup

Fig. 4-2 shows the experimental setup for the small-scale prototype. The experiments were performed using a Quanser shaker II located at the University of Toronto. The shaker consists of a servo-motor driving a 12.7 mm lead screw. The lead screw drives a circulating ball nut, which is coupled to the 46×46 cm² table. The shake table itself slides on low friction linear bearings on two ground hardened shafts and has a ±7.6 cm stroke. The shake table consists of a top stage driven by a motor that allows it to achieve an acceleration of 2.5g when loaded with a 7.5 kg mass. Under
displacement control, the built-in controller is able to impose predefined (e.g., harmonic) displacements. The setup also has a 22.2 N (5 lb) load cell between the shake table and the GMD that can measure the forces. In addition, an external LVDT is used to measure the horizontal displacements of the rack, and these measurements are checked against the readings of the internal displacement encoders of the shake table.

The GMD prototype is fixed on the table, and the rack of the damper is connected to the load cell and is placed on greased sliders to minimize friction. Special attention was given to keep the horizontal alignment of the GMD.

4.1.2 Experimental results

The small-scale GMD prototype was first subjected to sinusoidal displacement inputs (i.e. Case 1) with a constant frequency of 1.0 Hz while the excitation peak-to-peak amplitude of the input was changed from 2.54 mm (0.1”) to 19.05 mm (0.75”).

In Fig. 4-3 measured forces for a GMD with 8 extra screws are compared with the predicted forces from the Saitoh’s model (Eq. 2-22), the disk model (Eq. 2-24) and the lumped mass model (Eq. 2-26). The applied sinusoidal displacement had the amplitude of 6.35 mm (0.25”) and frequency of 1.0 Hz, therefore, the applied acceleration is a sinusoid with the same frequency and an amplitude.
of \(0.0063 \times (2\pi \times 1.0)^2\) or 0.25 \(\text{m/s}^2\). This leads to the maximum predicted forces of 3.92 \(N\), 5.96 \(N\) and 11.92 \(N\), for the Saitoh’s model, the disk model, and the lumped mass model, respectively. All of these predicted values are smaller than the maximum measured force of 21.78 \(N\).

![Figure 4-3. Time histories of the measured and predicted forces (8 extra screws)](image)

Fig. 4.3 reveals that other than the difference between peak values, the function that describes the measured force values is not merely a product of \(b\) times the acceleration, which should have been a single harmonic. Instead, the measured force exhibits oscillations with multiple frequencies. When the experiments were repeated for different values of the equivalent masses, it can be seen in Fig. 4-4 that with the increase of the equivalent (effective) mass of the device, the maximum measured force in the device increases.

A closer look at Fig. 4-4 also shows that when subjected to the same input displacements, GMDs with different \(b\) values exhibit somewhat different behaviour. In general, for smaller values of equivalent mass, the response becomes flatter and when \(b\) increases, the peaks become more distinct. The shape of the measured force plots also changes when the amplitude of the applied displacements is varied. This is shown in Fig. 4-5, where for the same GMD with 8 extra screws \((b = 47.55 \text{ kg in the lumped mass model})\), force measurements are compared for input displacements with the amplitudes of 3.81 mm (0.15”) and 6.35 mm (0.25”).
Chapter 4. Experimental Testing of Gyromass Dampers

Figure 4-4. Time histories of the measured forces for GMDs with different $b$ values

Figure 4-5. Measured forces for sinusoidal displacements with different amplitudes

To further study the behaviour of the small-scale prototype the force-displacement, force-velocity and force-acceleration plots can be examined under sinusoidal excitation. Measured force-displacement plot and the corresponding force-velocity loops for the GMD with 8 extra screws subjected to a sinusoidal input displacement of $u = 6.35 \sin(2\pi t)$ mm are shown in Fig. 4-6.
Figure 4-6. (a) Force-displacement, (b) force-velocity of the GMD with 8 extra screws

Figure 4-7. Comparison between the measured force-acceleration of the GMD with 8 extra screws and the Saitoh’s, disk and lumped mass models (Amp. = 6.35 mm)

The Saitoh’s, disk and lumped mass models presented in Chapter 2 all predict a linear relationship between the acceleration and the measured force. However, Fig. 4-7 shows that despite the similar trends between the slope of the loop’s major axis of the hysteresis loop and the linear force-acceleration functions, a considerable energy dissipation exists. The enclosed area in this plot may be attributed to the presence of frictional forces and higher-order unmodelled dynamics. To further examine this, a larger scale prototype was built and tested. The results for the larger-scale prototype will later be presented in the following sections.
As observed in Fig. 4-5, the magnitude of force oscillations varies with the amplitude of a given displacement input with a constant frequency (e.g. 1 Hz). Increasing the equivalent mass of the device also results in larger enclosed areas under the displacement vs. force diagrams (Fig. 4-8). Note that due to the measurement capacity of the load cell used in these experiments, forces are capped at 22.2 N.

4.1.3 Friction forces effects

It has been mentioned in the literature that the performance of inertial dampers can be affected by nonlinearities. For instance, Wang and Su (Wang et al., 2008) investigated the effects of friction, backlash and the elastic effects on the behaviour of ball-screw ininers and Ikago et al. (Ikago et al., 2012) discussed the effects of friction on the behaviour of TVMDs. Smith and Wang (Smith et al., 2004) found that the experimental data matches better with the theoretical predictions when the effects of friction forces are considered. Friction is a complicated phenomenon arising at the contact of surfaces in many different systems. A quick review of the literature shows that the friction forces depend on several factors, including sliding speed, critical sliding distance, humidity, acceleration, normal load, surface preparation, temperature and, of course, material characteristics (Wojewoda et al., 2008). However, the friction force effects have not studied for rack and pinion type inertial dampers.

![Figure 4-8](image-url)  
**Figure 4-8.** Force-acceleration plots for GMDs with different equivalent mass values
In the first method to measure the friction, as suggested by Wang and Su (2008), gears are removed, and a low-frequency sinusoidal input displacement with the frequency of 0.1 Hz and amplitude of 10 mm is applied. As it is shown in Fig. 4-9, the friction force $f_f$ is almost a square wave with the direction opposite to the sign of the velocity and the amplitude of about 0.77 N. This force is relatively small and cannot entirely account for the observed difference between the measured forces and the predicted values from the linear model.

In the second method, the friction parameters are calculated indirectly and based on the force measurements. To develop a meaningful understanding of friction parameters, and to predict dynamic system's response and performance, a robust friction model must be employed. In this section, dry friction (Coulomb friction) is selected as a simplified model. The friction damping force can be expressed as,

$$f_f = F_{f_{max}} \times sign(\dot{x}) = \mu_f N \times sign(\dot{x})$$  \hspace{1cm} (4-1)

![Figure 4-9. Measured friction forces from the procedure suggested by Wand and Su (2008)](image)

When the device is subjected to the applied displacement of $x = A_f \times \sin(\omega_f t)$ with predefined amplitude and frequency of $A_f$ and $\omega_f$, respectively, the velocity term in Eq. 4-1 can be expressed by the following equation,

$$sign(\dot{x}) = sign[\cos(\omega_f t)] = sign[\cos(-\omega_f t)]$$  \hspace{1cm} (4-2)

A closer look at Eq. 4-2 shows that the following equation holds for all $t$ and $-t$ in the time domain and thus, the damping force generated by dry friction damping is an even function of time.

$$f_f(t) = f_f(-t)$$  \hspace{1cm} (4-3)
It can be shown that the Fourier series of a periodic even function includes only cosine terms.

\[
a_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \cos[n \omega_T t] \, dt = \frac{8}{T} \int_0^{T/4} f(t) \cos[n \omega_T t] \, dt
\]  

(4-4)

\[
b_n = \frac{2}{T} \int_{-T/2}^{T/2} f(t) \sin[n \omega_T t] \, dt = 0
\]  

(4-5)

Also, from Eq. 4-1 it can be seen that the mean value of the damping force is zero and as a result, \(a_0\) is zero. In the case of \(\omega_T = \omega_f\), the Fourier coefficient \(a_n\) from Eq. 4-4 can be calculated as,

\[
a_n = \frac{8}{T} \mu N \int_0^{T/4} \cos[n \omega_f t] \, dt = \frac{4 \mu N}{n \pi} \sin \left(\frac{n \pi}{2}\right)
\]  

(4-6)

Note in the above integration, \(\frac{T}{4} = \pi/(2 \omega_f)\). When \(n\) is an even number, the term \(\sin \left(\frac{n \pi}{2}\right)\) in Eq. 4-7 is equal to zero. Consequently, using Fourier series, Eq. 4-1 can be expressed by the following equation,

\[
f_f = \mu N \times sgn(\ddot{x}) = \frac{4 \mu N}{\pi} A_f \left\{ \cos(\omega_f t) - \frac{1}{3} \cos(3 \omega_f t) + \frac{1}{5} \cos(5 \omega_f t) - \cdots \right\}
\]  

(4-7)

To demonstrate the effects of each of the terms in Eq. 4-7, normalized friction damping forces and the corresponding Fourier series with only the first term \((a_1)\), the first four terms \((a_1 - a_4)\), and the first six terms \((a_1 - a_6)\) are shown in Fig. 4-10.

It can be observed in Fig. 4-10 that with the increase in the number of used Fourier terms, the truncated series tends to approach to the original function. However, the discontinuity of the function leads to the existence of an overshoot around the corners. This behaviour is referred to as the Gibbs phenomenon (Liang et al., 2012).

Now only with the first two terms, the force can be approximated by the following equation,

\[
f_f = \frac{4 \mu_f N}{\pi} A_f \left\{ \cos(\omega_f t) - \frac{1}{3} \cos(3 \omega_f t) \right\}
\]  

(4-8)

The value of \(\mu_f N\) in Eq. 4-8 should be measured for each device separately. Eq. 4-8 can be combined with the previous force-acceleration equation for the linear model to obtain a better predictive model.
Based on the measured values in this study, it was found that when the prototype is subjected to the sinusoidal displacement input of $u = A_f \sin(\omega_f t)$, Eq. 4-9 can be used to predict the maximum forces with an acceptable accuracy.

$$f_d = C_1 \times \left(\frac{A_f \omega_f^2}{d}\right) \sin(\omega_f t) + C_2 \times A_f \left\{ \cos(\omega_f t) - \frac{1}{3} \cos(3\omega_f t) \right\}$$

In Eq. 4-9, $C_1$ and $C_2$ are the coefficients of the inertial and friction force terms, respectively and $\phi$ is due to the presence of a phase difference between the applied displacement and the cosine terms. These three coefficients change with the frequency of the input displacements (i.e., $\omega_f$).

The results for a GMD prototype with 8 extra screws and an input displacement with the amplitude 6.35 mm (0.25") are shown in Fig. 4-11. Under these conditions, $C_1$, $C_2$ and $\phi$ were experimentally determined to be 52.3 kg, 3.2 N and $\frac{\pi}{4}$, respectively. It should be noted that the differences mainly occur when the applied displacements (and the associated accelerations) are relatively small. Moreover, the measured value is very close to the theoretical $b$ derived from the lumped mass model (i.e., 47.55 kg).
Chapter 4. Experimental Testing of Gyromass Dampers

Figure 4-11. Comparison between the measured forces and the predicted values from Eq. 4-9

Fig. 4-12 also shows the comparison between the hysteresis loops of the measured and predicted forces. Since the enclosed area of these diagrams is related to the energy dissipation of the prototype and its damping energy, this area can be chosen as a criterion to compare the hysteresis loops.

Figure 4-12. Comparison between hysteresis loops of the measured and predicted forces
Based on the results shown in Fig. 4-12, the enclosed area of the predicted force diagram obtained from Eq. 4-9 in one cycle is 0.286 $J$ which is only 0.046 $J$ higher than the corresponding parameter for the measured forces (i.e. 0.240 $J$). It can be observed from Fig. 4-10 that the use of only two terms in Eq. 4-9 may not be an accurate representation of the friction forces. As a result, it is shown in Fig. 4-12 that the accuracy of the force prediction of the model increases when a higher number of harmonic terms are considered. The agreement between the force predictions and measurements in Fig. 4-12 demonstrates that in comparison to the procedure proposed by (Wang et al., 2008), Eq. 4-9 provides a much better estimation of the measured forces. Thus, the same approach will be used for the characterization of the larger scale prototype in the following sections.

In summary, tests on the smaller scale prototype show that the linear models (Saitoh’s, disk or lumped mass models described in Section 2.4.1.2) are not sufficient to predict the measured forces accurately. The relationship between the amplitude of the applied accelerations and the resulting forces also show that to model the nonlinearities, there is a need to know the properties of the input displacements (e.g., frequency and amplitude). In order to study the effects of the friction forces in more detail and to consider the effects of other parameters such as input frequency and amplitude in a more general sense, a larger scale prototype is built and tested in the next sections.

### 4.2 Characterization of the larger scale prototype

With the aim of investigating the potential challenges in developing a full-scale prototype for practical application of GMDs in buildings, and to perform an analysis of the effects of the input variables on the GMD by employing Case 1 and Case 2 loading scenarios, a larger prototype was built and tested in the structural laboratory of the University of Toronto. This prototype, which will be referred to as the larger scale prototype, is made of steel plates and metal gears and can have an equivalent mass which is approximately 93 times larger than the equivalent mass of the small-scale prototype. This prototype is shown in Fig. 4-13 and more detail about the prototype can be found in Section 2.5.2.

#### 4.2.1 Testing setup

The experiments on the larger scale prototype were performed using a testing setup which consists of a hydraulic service manifold which could provide a continuous oil flow rate of 120 gpm and
regulates the oil pressure at 20.6 MPa (3000 psi). This manifold is connected to a hydraulic actuator with a stroke length of ±127 mm (±5 inch) and a maximum force capacity of 33 kN (±7,500 lbf) driven by an electro servo-valve with a flow capacity of 16.5 gpm. A similar actuator was also used to test the viscous damper. To measure the displacements, ±127 mm (±5 inch) built-in LVDTs were used and a load cell with a capacity of 50 kN (±12,500 lbf) mounted on each actuator was used to obtain the force feedback. An external K-Beam® accelerometer was also connected to the rack of the GMD and was used to measure the accelerations.

![Figure 4-13. Larger scale GMD prototype](image)

In order to control the actuators and conduct the tests, a user programmable computational/control platform that was previously developed and verified at the University of Toronto was used.
(Ashasi-Sorkhabi et al., 2014). A quad-core real-time processor in the National Instruments PXI system and an integrated Field Programmable Gate Array (FPGA) are the other key components that form the architecture of this platform. As explained in Chapter 3, accurate tracking of the command displacements by the actuators was ensured by monitoring the tracking errors using the PAEI and FDB error indicators. The testing setup is shown in Fig. 4-14.

![Experimental setup for the larger scale GMD](image)

**Figure 4-14.** Experimental setup for the larger scale GMD

### 4.2.2 Acceleration measurements

The hydraulic actuators used in the testing setup of Fig. 4-15 are displacement-controlled with proportional-integral-differential (PID) controllers. Some of the previous studies have shown that displacement control in hybrid simulations provides reasonable performance but may produce poor acceleration tracking in the time domain and may not provide adequate repeatability in generated accelerations (Stehman et al., 2013). Due to the inherent dynamics of the control system (i.e., servo hydraulic actuators) and its interaction with the test structures, often referred to as control-structure interaction, poor acceleration tracking can be an issue in RTHS with shake tables. Since the GMD is an acceleration-dependent device, the acceleration tracking of the actuators is examined in this section.

Using the external accelerometer and a displacement input of \( u = 0.015 \sin(f_0 \times 2\pi t) \), measured and command accelerations are compared in Fig. 4-16. To observe the effects of the input frequency on the accuracy of the measurements, tests are performed with \( f_0 \) equal to 0.6 Hz, 0.8 Hz and 1.0 Hz. The command acceleration is simply the second derivative of the command
displacement and can be shown by $\ddot{u} = -0.015 (f_0 \times 2\pi)^2 \times \sin(f_0 \times 2\pi t)$. In these plots, the blue and red lines show the command and measured accelerations, respectively.

As it can be seen in Fig. 4-16 the measured accelerations have an acceptable agreement with the command accelerations. The difference can be quantified by looking at the phase and amplitude error indicators between these two signals (Table 4-1). In Fig. 4-16 these indicators are shown for the displacement input with $f_0$ of 1.0 Hz. From this figure, a mean amplitude ratio of 1.047 which is less than 5% of the command acceleration amplitude. Moreover, there is a phase error of $-0.0547$ deg which for a constant frequency of 1.0 Hz can be expressed as 8.71 msec.

<table>
<thead>
<tr>
<th>$f_0$ (Hz)</th>
<th>Amplitude Ratio</th>
<th>Phase Error (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6</td>
<td>1.056</td>
<td>-0.1052</td>
</tr>
<tr>
<td>0.8</td>
<td>1.079</td>
<td>-0.0758</td>
</tr>
<tr>
<td>1.0</td>
<td>1.047</td>
<td>-0.0547</td>
</tr>
</tbody>
</table>

4.2.3 Experimental results

In order to characterize the larger scale prototype Case 1 and Case 2 loading scenarios were used. For cyclical experiments (i.e., Case 1), based on the peak-to-peak stroke distance, and the loading frequency with data sampling at 1000 Hz, the effects of the input parameters were studied. The
range of tests was adjusted given the limitations of the damper and the testing setup. For the cyclic tests with displacement input of \( u = A \times \sin(f_0 \times 2\pi t) \), frequencies of \( f_0 = 0.4 \) Hz to 2.0 Hz were considered. There were several cyclic loading scenarios which contained more than one sinusoidal function. These loading scenarios are summarized in Table 4-2.

**Figure 4-16.** FDB error indicators for measured and command accelerations \( (f_0 = 1.0 \text{ Hz}) \)

Fig. 4-17 shows the effect of the variation of measured forces with the change of the frequency of the input displacements. In all of these figures, gears 4 and 5 are removed and therefore, the device has a relatively small equivalent mass \( b \) from the lumped mass model equal to 68.9 kg. The measured forces exhibit zigzag patterns with distinct peaks. Also, it should be noted that the measured forces for this device with a small equivalent mass are not symmetric when the input frequency is low.

**Table 4-2.** Considered loading cases for the experimental tests

<table>
<thead>
<tr>
<th>Loading Case</th>
<th>Descriptive Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case 1.2.1.</td>
<td>( u = 3 \times (\sin(0.4 \times 2\pi t) + \sin(1.2 \times 2\pi t)) )</td>
</tr>
<tr>
<td>Case 1.2.2.</td>
<td>( u = 3 \times (\sin(0.4 \times 2\pi t) + 0.25 \times \sin(1.2 \times 2\pi t)) )</td>
</tr>
<tr>
<td>Case 1.2.3.</td>
<td>( u = 3 \times (\sin(0.4 \times 2\pi t) + 0.50 \times \sin(0.8 \times 2\pi t)) )</td>
</tr>
<tr>
<td>Case 1.2.4.</td>
<td>( u = 3 \times \sin(0.4 \times 2\pi t) + 0.5 \times \sin(0.8 \times 2\pi t) + 0.5 \sin(1.2 \times 2\pi t) )</td>
</tr>
<tr>
<td>Case 1.2.5.</td>
<td>( u = 3 \sin(0.4 \times 2\pi t) + \sin(2\pi t) + \sin(1.6 \times 2\pi t) )</td>
</tr>
</tbody>
</table>
Figure 4-17. Force time histories of a GMD under sinusoidal input displacements \((b = 68.9 \text{ kg})\)

In Fig. 4-18, the equivalent mass of the GMD is increased by adding the 7” diameter gear (as shown in Section 2.5) to the third gear shaft and employing it as the rotational mass of the device. This change leads to an equivalent mass of 1379.9 kg (from the lumped mass model or 722 kg...
from the disk model). In addition to the variation of the frequency of the input displacement, the amplitude of the displacements is also changed in Fig. 4-18. A comparison between the force time histories of Fig. 4-18 and Fig. 4-19 reveals that with the increase of the equivalent mass, the pattern changes too. In all of the plots in Fig. 4-18, in addition to two main peaks in each cycle, two smaller local maxima between the main peaks are recognizable. This is more evident for higher frequencies and amplitudes and is very similar to the observations for the smaller prototype in Section 4.1.

![Figure 4-18. Force time histories of a GMD under sinusoidal input displacements (b = 1380 kg)](image-url)
Finally, the solid disk is used on the third shaft as the rotational mass to increase the equivalent mass of the device to 4452.6 kg (lumped equivalent mass). This value is significantly larger than the equivalent mass of the device in Fig. 4.17. It can be seen that for lower frequencies in addition to the main two peaks of the measured force, two local maxima could be found which correspond to harmonic functions with higher frequencies. However, this pattern changes as the frequency increases.

**Figure 4-18 (Continued).** Force time histories of a GMD under sinusoidal input displacements ($b = 1379.9$ kg)

**Figure 4-19.** Force time histories of a GMD under sinusoidal input displacements ($b = 4453$ kg)
Chapter 4. Experimental Testing of Gyromass Dampers

Figure 4-19 (Continued). Force time histories of a GMD under sinusoidal input displacements

\( b = 4453 \text{ kg} \)

To compare the results of Fig. 4-17, Fig. 4-18 and Fig. 4-19 with the predictions of the linear models, maximum measured forces are obtained from these figures and are plotted against the frequency of the input displacement.

Figure 4-20. Variation of maximum measured force with the input frequency (a) \( b = 68.9 \text{ kg} \), (b) \( b = 4452.6 \text{ kg} \), (c) \( b = 1379.9 \text{ kg (Amp. 7.5 mm)} \), and (d) \( b = 1379.9 \text{ kg (Amp. 15 mm)} \)
From Fig. 4-20, it can be concluded that the linear models underestimate the maximum measured forces. Also, Fig. 4-20 (c) and (d) show that by increasing the amplitude of the displacement input by a factor of two, the value of the maximum measured displacement does not double. This is also in contrast with the predictions of the linear models.

Another useful way of examining the force measurements is by plotting the hysteresis loops of these experiments. These loops are plotted for experiments on GMDs with different equivalent mass values and are shown in Fig. 4-22 to 4-24.

![Hysteresis Loop Images](image)

**Figure 4-21.** Hysteresis loop for a GMD under sinusoidal input displacements ($b = 68.9$ kg)

(Tension forces are positive)
Figure 4-22 (Continued). Hysteresis loop for a GMD under sinusoidal input displacements

\( b = 68.9 \text{ kg} \) (Tension forces are positive)

\( b = 1379.9 \text{ kg} \)

**Figure 4-22.** Hysteresis loop for a GMD under sinusoidal input displacements \( b = 1379.9 \text{ kg} \)
Comparison between the hysteresis curves of Fig. 4-22 to Fig. 4-24 shows that with the increase of the equivalent mass of the GMD, the hysteresis curves become smoother and form distinct geometrical shapes. Furthermore, it is apparent that with the increase in the frequency or amplitude of the input displacement, the enclosed area of the diagrams increases which results in fatter hysteresis loops. This trend is particularly evident in Fig. 4-23.

**Figure 4-23.** Hysteresis loop for a GMD under sinusoidal displacements \((b = 1379.9 \text{ kg})\)

**Figure 4-24.** Hysteresis loop for a GMD under sinusoidal input displacements \((b = 4452.6 \text{ kg})\)
4.3 Modelling and identification of the larger scale prototype

The measured force-displacements hysteresis loops under cyclic loadings of Fig. 4-22 to 4-24 are repeatable and stable over several loading cycles. Similar to the observations of Section 4.1.3 and based on these figures, the hysteretic behaviour of a GMD can be characterized by two main parameters: the enclosed area of the diagrams ($E_{dgm}$) and the slope of the hystereses major axis ($b$). $E_{dgm}$ is closely related to the existing friction forces and larger values of $F_{fmax}$ result in greater $E_{dgm}$ values. However, in the absence of friction forces, the hysteresis diagram becomes a straight line, and the slope of this line is equal to $b$ (Fig. 4-25). Thus, the governing equations for the GMD force $f_d$ predicted by the present model can be summarized in Eq. 4-10:

$$f_d = b \times \dot{u} + F_{fmax} \times \text{sign}(\dot{u})$$  \hspace{1cm} (4-10)

It should be noted that if $b$ is derived from the mechanical representation of the device and the friction forces are ignored, Eq. 4-10 becomes the linear model of Section 2.4.
A two-step procedure is proposed here to determine the numerical values of these parameters for the prototype specimen for any force-displacement hysteresis:

**Step (1) – Determining the friction force term coefficient of Eq. 4-10:** In order to calculate the enclosed area of the hysteresis loops ($E_{dgmD}$). Since the performed experiments in this study are displacement-controlled tests, the limits of the applied input displacements are known ($\pm u_{max}$), a closed-form equation can be used to use the enclosed area of the plots and find the value of $F_{fmax}$. Using the formula for the area of a parallelogram, Eq. 4-24 can be used to determine $F_{fmax}$:

$$F_{fmax} = \frac{E_{dgmD}}{4u_{max}}$$  

(4-11)

**Step (2) – Calculating the acceleration term coefficient, $b$, of Eq. 4-10:** To find the value of $\alpha_b$, the principal slope of the hysteresis diagram should be calculated. Alternatively, $b$ can be found from Eq. 4-12,

$$b = \max(f_d - F_{fmax} \times \text{sign}(\ddot{u})) / \max(\ddot{u})$$  

(4-12)

In Eq. 4-11, the hysteresis loops of Fig. 4-22 to Fig. 4-24 can be used to numerically compute the dissipated energy of the device. The energy dissipated by the larger scale GMD in each cycle,
Chapter 4. Experimental Testing of Gyromass Dampers

$E_{dgm\text{d}}$, is the area under the force-displacement diagrams and can be calculated from Eq. 4-13 (Christopoulos et al., 2006):

$$E_{dgm\text{d}} = \int_0^T F_{\text{mes}}(t)\ddot{u}(t)dt$$

(4-13)

where, $F_{\text{mes}}$ denotes the measured forces in each experiment. The calculated $E_{dgm\text{d}}$ values for different input frequencies and amplitudes and various equivalent mass values are shown in Fig. 4-26.

![Dissipated energies of the larger scale prototype with, (a) $b = 68.9$ kg, (b) $b = 4452.6$ kg and (c) $b = 1379.9$ kg](image)

Figure 4-26. Dissipated energies of the larger scale prototype with, (a) $b = 68.9$ kg, (b) $b = 4452.6$ kg and (c) $b = 1379.9$ kg

From Fig. 4-26 it can be observed that $E_{dgm\text{d}}$ increases with a higher rate for greater frequencies of the input displacement. The variation of $F_{f\text{max}}$ with the input frequencies is also shown in Fig. 4-27 where the $F_{f\text{max}}$ values are calculated from Eq. 4-11 and are plotted against $\omega_f^2$. It is clear that natural logarithm functions can describe the relationship between $F_{f\text{max}}$ and $\omega_f^2$. The parameters of these functions are obtained by using a curve fitting toolbox in MATLAB.
Figure 4-27. Variation of $F_{f_{\text{max}}}$ with the frequency of the input displacement for the larger scale prototype with, (a) $b = 68.9$ kg, (b) $b = 4452.6$ kg and (c) $b = 1379.9$ kg.

Since the curve fitting parameters for experiments with different input displacement amplitudes are very close, from Fig. 4-27 (c), it can be concluded that $F_{f_{\text{max}}}$ does not depend on the input displacement amplitude. In addition, a comparison between the obtained parameters for Fig. 4-27 (b) and Fig. 4-27 (c), shows that the relation between $F_{f_{\text{max}}}$ and $\omega_f^2$ is relatively similar for both values of $b = 1379.9$ kg and $b = 4452.6$ kg. It should be noted that in both cases, three gear shafts are used. On the other hand, the curve fitting parameters in Fig. 4-27 (a), where the prototype specimen has two gear shafts, are different. Based on these curve fitting equations, a new model can be proposed to describe the behaviour of the larger scale prototype which can improve the accuracy of the linear model in predicting the GMD forces. The equations of the new model, which will be called the “nonlinear model” from this point forward, are shown in Eq. 4-14 and 4-15,

For two gear shafts: 
$$f_d = b\ddot{u} + \left[\exp\left(\frac{\omega_f^2 + 461}{133}\right)\right] \times \text{sign}(\ddot{u})$$  \hspace{1cm} (4-14)

For three gear shafts: 
$$f_d = b\ddot{u} + \left[\exp\left(\frac{\omega_f^2 + 155}{39}\right)\right] \times \text{sign}(\ddot{u})$$  \hspace{1cm} (4-15)
Chapter 4. Experimental Testing of Gyromass Dampers

It should be emphasized here that the coefficient of the acceleration term \( b \) is unaffected by the excitation frequency, as the parameter is equal to \( b \) from the *lumped mass model* for both cases. In Fig. 4-28 and 4-29, the success of the proposed Nonlinear Model in predicting the GMD forces are shown:

![Figure 4-28. Comparison between the time histories of the measured and predicted forces (1 Hz) (Nonlinear Model of Eq. 4-14/4-15), (a) \( b = 4452.6 \) kg, (b) \( b = 1379.9 \) kg (Amp. = 15 mm)](image)

Eq. 4-14 and 4-15 clearly show that the force in a GMD is frequency-dependent. In other words, the GMD force changes with the frequency of the applied displacement. This creates no problem when the applied displacement has a single known excitation frequency. However, from the design perspective, the characteristic equations of Eq. 4-14 and 4-15 are not practical when the input displacements contain multiple excitation frequencies, or the excitation frequency is unknown.

![Figure 4-29. Comparison between the force measurements and predictions from the Nonlinear Model of Eq. 4-27 and 4-28 (1 Hz), (a) \( b = 4452.6 \) kg, (b) \( b = 1379.9 \) kg (Amp. = 15 mm)](image)

Consequently, a simplification can be made here by assuming that the input frequency is equal to zero. This assumption is conservative because it leads to the least value of energy dissipation from Eq. 4-14 and 4-15. This assumption introduces some error but at the same time, makes it possible
to use Eq. 4-14 and 4-15 (which will be called the “Simplified Nonlinear Model”), for design purposes or for the analysis of systems subjected to random excitation.

\[
\begin{align*}
\text{For two gear shafts: } f_d &= b \ddot{u} + 32.01 \times \text{sign}(\dot{u}) \quad (4-16) \\
\text{For three gear shafts: } f_d &= b \ddot{u} + 53.22 \times \text{sign}(\dot{u}) \quad (4-17)
\end{align*}
\]

To assess the success of the Simplified Nonlinear Model, more complex loading scenarios are applied to the larger prototype and the resulting forces are compared to the predicted forces from the above equations. The results are plotted in Fig. 4-30 to 4-34 where the value of the equivalent mass \( b \) in all of these figures is equal to 1379.9 kg. In these figures, tension forces are positive.

![Graphs showing force and displacement over time and velocity vs. force diagrams](image)

**Figure 4-30.** Larger prototype specimen subjected to the load case of 1.2.1 from Table 4-2

Since the second terms of Eq. 4-16 and 4-17 are functions of velocity, in addition to the hysteresis loops and force and displacement time histories, the velocity vs. force diagrams are also compared. The velocity vs. force diagrams for all cases show that the simplified nonlinear model successfully predicts the outer boundaries of the measured diagrams. Also, although the measured and predicted forces may differ at some local peaks, the general trends are the same and the simplified model predicts the main peaks effectively. Finally, although the enclosed areas of the predicted loops are smaller than the measured ones (due to the conservative assumptions of Eq. 4-16 and 4-17), the
The overall shape of the hysteresis curves resemble the hysteresis of the measured diagrams. Since similar observations were made for the other GMD specimens with the equivalent mass of 68.9 kg and 4452.6 kg, Eq. 16 and Eq. 17 will be used in the following sections to model the GMD device.

**Figure 4-31.** Larger prototype specimen subjected to the load case of 1.2.2 from Table 4-2

**Figure 4-32.** Larger prototype specimen subjected to the load case of 1.2.3 from Table 4-2
4.4 RTHS of the larger scale prototype

One of the main issues in the damper design process is to assess the damper requirements and constraints within the earthquake engineering context. Unlike the load cases of Section 4.3, the
actual seismic loads due to major earthquakes contain significant uncertainties and various earthquake ground motions have different durations, frequency contents or amplitudes. Consequently, the cyclic tests of Section 4.3 may not be sufficient to fully examine the performance of a GMD. Besides, since the GMD is installed inside a structure, the performance of the device is also influenced by the dynamic properties of the uncontrolled system. The rate-dependency of GMDs also make it necessary to use a testing method that can be carried out in real time. Considering all of these factors, RTHS is chosen to study the seismic performance of structures by modelling an SDOF system as the analytical substructure in computer and testing the larger scale prototype specimen as the experimental substructure. A schematic of the substructures for the RTHS of this section is shown in Fig. 4-35.

**Figure 4-35. Schematic of the substructures for the RTHS**

### 4.4.1 Tracking Performance Assessment of the RTHS

As has been pointed out in the previous chapter, to ensure stability and accuracy of RTHS tests, the servo-hydraulic system needs to impose the command displacements to the experimental substructure as accurately as possible. It has been shown before (Mercan et al., 2007) that in RTHS, the effects of a time delay (i.e., phase error) in the measurements is much more detrimental compared to an amplitude error. In this section, the error monitors developed in Chapter 3 are used to decouple the amplitude and phase errors and compute each of them through closed-form equations. Based on the stability analysis of Chapter 3, for the testing setup considered in this study
with a relatively small friction force ratio, the stability of the RTHS is not sensitive to the experimental phase error. However, the value of this parameter can be used to compare the accuracy of the experiments with other similar experiments in future.

For an RTHS with cyclic ground accelerations, the time histories of the command and measured displacements are shown in Fig. 4-36 (a) and enlarged windows of these displacements are shown in Fig. 4-36 (b). Hysteresis loop of the measured force is also shown in Fig. 4-36 (c). Also, to visually evaluate the tracking of the command displacements, the synchronization subspace plots introduced are shown in Fig. 4-36 (d), where the actual (measured) response is plotted against the desired (command) response. As shown in Fig. 4-36 (d) due to the adequate control of the actuators, the major axis slope of the SSP is very close to 45°, and the enclosed area of the plot is relatively small. This can be checked numerically, by using the PAEI indictors. The corresponding phase errors and amplitude ratios are shown in Fig. 4-36 (e) and (f), respectively.

As it can be seen in these figures, the average amplitude ratio is 0.992, and the maximum phase error is equal to -5.131 deg. The negative sign shows that the measured displacements are lagging behind the command displacements. Finally, in Fig. 4-36 (g) and (h), the FBD indicators are used to quantify the phase error and amplitude ratios. The average value of the amplitude ratio is equal to 0.983 and the average phase error is equal to -0.070 deg which is relatively small. Since the input frequency of the command displacement is constant and is equal to 1.0 Hz, it is possible to compute the delay in terms of msec. The time delay is equal to 1.23 msec which is less than two time steps. From the stability analysis of Chapter 3, it was concluded that for small time delays such as the above values, RTHS is stable.

The calculated values of phase and amplitude errors can also be employed to compare the results of each simulation to other tests. For example, in 2014, Guo et al. tested a viscous damper with a load capacity of 180 kN and a stroke limit of 150 mm, at the Key Laboratory of Concrete and Prestressed Concrete Structures of the Ministry of Education at Southeast University (Guo et al., 2014). According to their results, for real-time tests with a predefined sinusoidal signal with an amplitude of 15 mm and frequency of 1 Hz, the amplitude ratio and phase errors were measured to be 0.983 and -8.537 deg (-0.149 rad), respectively.
4.4.2 RTHS under sinusoidal input ground accelerations

To experimentally obtain the transfer function for the SDOF system equipped with the stand-alone GMD, a numerical substructure is chosen which has a mass of 65,475 kg, a natural period of 1.0 s and a damping ratio of 1.5%. The frequency response of this system is numerically obtained and
is shown with the solid line in Fig. 4-37. Then, the stand-alone larger scale GMD prototype specimen is considered as the experimental substructure and a set of six RTHS tests with sinusoidal inputs and input frequencies of 0.4 Hz, 0.6 Hz, 0.8 Hz, 1.0 Hz, 1.2 Hz, and 1.4 Hz have carried out. The results from the steady-state response of the controlled system are shown with the * marks on the diagram. It can be observed that the introduction of the GMD to the SDOF system, consistently reduces the maximum displacement of the system.

![Graph](image)

**Figure 4-37.** Experimentally obtained frequency response of the SDOF with a GMD

### 4.4.3 RTHS under chirp input ground accelerations

Performing the tests with RTHS method makes it possible to test the device under two new loading scenarios. In addition to previous loading cases of Table 4-2, a chirp signal is used in this section as the input ground excitation. The signal frequency changes from 0.5 Hz to 2 Hz within 60 sec. The chirp signal is illustrated in Fig. 4-38.

Results for the chirp signal are shown in Fig. 4-39, where demonstrate the comparison between the displacement, enlarged windows of the plot are chosen and shown in Fig. 4-39 (b) to Fig. 4-39 (e). To compare the simplified nonlinear model with a linear model, disk model is shown in these plots.
As it is apparent from Fig. 4-39 (a) the period of vibration of the uncontrolled SDOF oscillator is equal to 1.0 s. The analytical substructure has a damping ratio of 2%, and mass of the primary system is 1.44 t. This corresponds to an equivalent mass ratio of 0.5%. The results in Fig. 4-39 (a) shows that the introduction of the GMD with $b = 1379.9$ kg, reduces the maximum displacements
from 14.23 mm to 13.76 mm. As it will be shown in the next sections, higher reductions can be observed for larger equivalent mass ratios.

A closer look at Fig. 4-39 (a) reveals that the response can be divided into four different regions. For lower input frequencies (from 0.5 Hz to approximately 0.9 Hz), the GMD is less effective and the response of the uncontrolled and controlled systems are close (Fig. 4-39 (b)). Moreover, the predictions from the linear model and the simplified nonlinear models are close. However, this trend changes when the input frequency of the chirp signal approaches the vibration period of the system. As it is shown in Fig. 4-39 (c), at resonance frequency the GMD reduces the maximum displacement effectively, and the simplified nonlinear model provides an acceptable prediction for the measured displacements. By increasing the input frequency of the chirp signal, the GMD still reduces the response of the system, but the accuracy of the nonlinear model reduces (Fig. 4-39 (d)). Finally, for higher frequencies (higher than 1.5 Hz), the effectiveness of the GMD reduces. On the other hand, for higher frequencies, the simplified nonlinear model predicts the measured forces with an acceptable accuracy.

![Figure 4-40. Force results for the RTHS with chirp input ground acceleration](image)

Fig. 4-40 shows the comparison between the predicted forces and the measured forces from the experiment. To show the plots better, the middle portion of the diagram around the resonance
frequency is enlarged in Fig. 4-40 (b). The simplified nonlinear model predicts the peak values of the measured forces accurately, and the linear model underestimates the measured forces throughout the experiment.

4.4.4 RTHS of the prototype specimen subjected to earthquakes

A suite of six different earthquake ground motions is used in this section to test the controlled system. The earthquake ground motion records are selected from the PEER-NGA database (Pacific Earthquake Engineering Research Center, 2015), and time history analyses are performed to numerically find the response of the uncontrolled SODF oscillator. The procedure is repeated for the systems with GMDs, and the Simplified Nonlinear Model is used to predict the resulting force and displacements. These earthquakes are selected here because each of them has a particular attribute. For example, EQ2 has a large peak displacement and large incremental displacement whereas EQ5 has a large peak acceleration. These attributes are categorized in (Naeim et al., 1996) and (Liang et al., 2011). Details of these ground motions are reported in Appendix E. Same ground motion records are then used in an RTHS of an SDOF oscillator with an equivalent mass ratio and damping ratio of 1% and 2%, respectively. The period of vibration of the SDOF system is selected to be 1.0 sec. The experimental substructure of the system also contains a GMD with an equivalent mass of 1379.9 kg. The ground motion records are scaled to limit maximum displacement of the uncontrolled SDOF system ($x_{0,max}$) to 20 mm and at the same time, limit the resulting forces in the GMD to 3.5 kN. List of the scaled peak ground acceleration for each of these earthquakes is shown in Table 4-3.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Scaled PGA (g)</th>
<th>($x_{0,max}$) (mm)</th>
<th>($x_{mes,max}$) (mm)</th>
<th>($x_{pred,max}$) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQ1 El Centro – Imp Vall Irr Dist</td>
<td>0.041</td>
<td>21.82</td>
<td>18.66</td>
<td>17.26</td>
</tr>
<tr>
<td>EQ2 El Centro - Array #6</td>
<td>0.031</td>
<td>17.25</td>
<td>14.37</td>
<td>12.92</td>
</tr>
<tr>
<td>EQ3 El Centro - Array #8</td>
<td>0.092</td>
<td>15.60</td>
<td>14.15</td>
<td>13.43</td>
</tr>
<tr>
<td>EQ4 Corralitos - Eureka Canyon Rd.</td>
<td>0.041</td>
<td>13.12</td>
<td>11.18</td>
<td>10.13</td>
</tr>
<tr>
<td>EQ5 Santa Monica - City Hall Grounds</td>
<td>0.102</td>
<td>10.14</td>
<td>10.14</td>
<td>9.70</td>
</tr>
<tr>
<td>EQ6 Tarzana - Cedar Hill Nursery A.</td>
<td>0.102</td>
<td>9.86</td>
<td>10.61</td>
<td>11.29</td>
</tr>
</tbody>
</table>
Figure 4-41 to Fig. 4-46 show the scaled accelerogram, the time-histories of displacement and dissipated energy for controlled and uncontrolled systems along with their force–displacement responses. Note that the simplified nonlinear model is used to predict the responses numerically.

**Figure 4-41.** Results for the RTHS of the SDOF system under EQ1 record

**Figure 4-42.** Results for the RTHS of the SDOF system under EQ2 record
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Figure 4.41 (Continued). Results for the RTHS of the SDOF system under EQ2 record.

Figure 4.43. Results for the RTHS of the SDOF system under EQ3 record.
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Figure 4-44. Results for the RTHS of the SDOF system under EQ4 record

Figure 4-45. Results for the RTHS of the SDOF system under EQ5 record
Figure 4-44 (Continued). Results for the RTHS of the SDOF system under EQ5 record

Figure 4-46. Results for the RTHS of the SDOF system under EQ6 record
A closer look at Fig. 4-41 to Fig. 4-46 reveals that in general the addition of the GMD to the uncontrolled SDOF system leads to a reduction of the maximum displacement. The values of the maximum displacements in the controlled and uncontrolled systems are compared in Table 4-3. Even for an earthquake like EQ5, where the effect of the GMD on the maximum displacement is negligible, it can be seen that the RMS of the displacements reduces considerably. Also, part (c) of these figures shows that the simplified nonlinear model, follows the trend of the measured displacements well. However, it should also be noted that in the first few cycles after the occurrence of the maximum displacements, the accuracy of the predictive model reduces. According to Table 4-3, for the considered earthquake ground motions, the highest error of the simplified nonlinear model for predicting the peak displacements is limited to 10% (i.e., for EQ1).

### 4.4.5 Analysis of the results using wavelet transforms

From Section 4.2.3 it is clear that the force–displacement relationship of GMDs exhibits hysteric nonlinear characteristics. The GMD forces are frequency dependent (Eq. 4-14 and 4-15). As such the strength and stiffness of a system that includes a GMD changes under an intensive earthquake. Consequently, the equivalent frequency of the controlled system as well as the forces in the GMD are time variant, and the variation of these parameters is associated with the frequencies of the applied ground acceleration. To study the behaviour of nonlinear systems in both time and frequency domains, as a reliable tool, wavelet transforms have been employed extensively by researchers (Li et al., 2008). In this section, the time-frequency response of an SDOF oscillator incorporated with GMD devices is addressed using the wavelet transform method and the governing frequencies of the time-varying frequency response of the nonlinear system are studied. Unlike Fourier transform, continuous wavelet transforms can construct a frequency-time representation of a signal that offers a good time and frequency localization. The integral in Eq. 4-18 can be used to express the continuous wavelet transformation of a signal \( x(t) \) at a scale \( \tilde{a} > 0 \) and a translational value \( \tilde{b} \):

\[
W_\Psi(\tilde{a}, \tilde{b}) = \frac{1}{\sqrt{\tilde{a}}} \int_{-\infty}^{+\infty} x(t) \Psi^* \left( \frac{t-\tilde{b}}{\tilde{a}} \right) dt
\]  

(4-18)

where, \( \tilde{b} \) is the parameter localizing the wavelet function in the time domain, \( \tilde{a} \) is the dilation parameter defining the analytical window stretching and \( \Psi^* \) is the complex conjugate of the basic
wavelet function. This means, $\tilde{b}$ is a time parameter and $\tilde{a}$ is related to frequency. The Gaussian wavelet function $\Psi$, is used here to obtain the structural time–frequency response. Signal Processing Toolbox™ of MATLAB is used to obtain the scalograms of the force measurements from each earthquake. Using the `scal2freq` command in MATLAB, the Fig. 4-47 is generated and can be used to convert the scales of the scalogram to the corresponding frequencies. To show the graph better, only the first 512 scales are shown on the horizontal axis.

![Graph showing frequency vs. scale](image)

**Figure 4-47.** Correspondence table of scales and frequency for the Gaussian wavelet transform

Fig. 4-48 (a) to (f) show the scalograms of the measured applied displacements to the GMD during each RTHS test for earthquakes EQ1 to EQ6. The governing frequencies of each of these responses are found and summarized in Table 4-4.

![Scalogram of measured displacement](image)

**Figure 4-48.** Wavelet transform scalogram of the RTHS with earthquake records
Figure 4-47 (Continued). Wavelet transform scalogram of the RTHS with earthquake records.
Figure 4-47 (Continued). Wavelet transform scalogram of the RTHS with earthquake records

Table 4-4. Governing frequencies in RTHS of structures under various earthquake records

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Governing Frequency (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>EQ1 El Centro – Imp Vall Irr Dist</td>
<td>1.073</td>
</tr>
<tr>
<td>EQ2 El Centro - Array #6</td>
<td>1.096</td>
</tr>
<tr>
<td>EQ3 El Centro - Array #8</td>
<td>0.746</td>
</tr>
<tr>
<td>EQ4 Corralitos - Eureka Canyon Rd.</td>
<td>1.241</td>
</tr>
<tr>
<td>EQ5 Santa Monica - City Hall Grounds</td>
<td>0.969</td>
</tr>
<tr>
<td>EQ6 Tarzana - Cedar Hill Nursery A.</td>
<td>0.940</td>
</tr>
</tbody>
</table>
From Table 4-4 it can be observed that although the frequency of vibration of the uncontrolled SDOF oscillator is equal to 1.0 sec, depending on the characteristics of the applied earthquake, the maximum frequency response occurs at different frequencies. This observation justifies the application of the simplified nonlinear model because due to complexities of the response to random vibration signals, selecting a certain design frequency is not straightforward and a design equation independent of the input frequency is advantageous.

**4.4.6 Effects of the equivalent mass ratio on the effectiveness of the device**

To this point, experimental results have been shown for different systems with constant values of equivalent mass ratios. To further investigate the effects of this dimensionless parameter on the response of the controlled structure, and to provide some insight in the seismic response of these systems over time, a series of structural systems were subjected to cyclic sinusoidal ground accelerations (with a frequency of 1 Hz). All systems considered have an initial period of 1.0 s and a damping ratio of 2%. In these tests, two GMD prototype specimens with equivalent masses of 1379.9 kg and 4452.6 kg were used, and the mass of the uncontrolled SDOF oscillator was changed to obtain different equivalent mass ratios. The resulting displacement time histories for the measured displacements were compared to the response of the uncontrolled system and the results are reported in Fig. 4-49. The ratio between the maximum displacement of the system with a GMD over the maximum displacement of the original uncontrolled SDOF system for various equivalent mass ratios is illustrated in this figure.

Fig. 4-49 shows the effectiveness of the GMD in enhancing the performance of the uncontrolled SDOF system by reducing the maximum displacements increases when larger values of the equivalent mass ratio are selected. It should be noted that this figure is only valid for a predefined sinusoidal input with an input frequency of 1.0 Hz and for any other loading scenarios, similar diagrams should be obtained. However, the general trend is shown in Fig. 4-49. Also, larger values of the equivalent mass also result in higher damper forces which in turn may require stronger and more expensive design of the device.
4.5 Experimental results on a GMD with added viscous damping

As it has been highlighted in Chapter 2, there are various techniques and devices for seismic control of the structure. When properly designed, each of these devices enhances the performance by modifying the dynamic characteristics of the structure. Despite the additional cost of the installation of these devices, their strategic use is cost-effective. The extra expense can often be justified by the need to meet the structural design goals. One of the common approaches in the literature is to use hybrid control systems. In these hybrid systems, the shortcomings of one of the techniques can be resolved by combining it with another control method and consequently improve the overall performance of the system. For instance, viscous fluid dampers, magnetorheological dampers or other dynamic systems have been added to base isolation systems to reduce the undesired displacements at the base isolation level (e.g. (Yoshioka et al., 2002) and (Mirza Hessabi et al., 2014)). In this section, a viscous fluid damper is added to a GMD to improve the performance of both of these devices.

As shown in Eq. 4-10, the only source of energy dissipation in the GMD is friction. However, the friction force term is frequency dependent and should be determined for each GMD device.
independently. By neglecting this term and thus using the linear model, the equation of motion of an SDOF system with a GMD can be written as Eq. 4-20:

\[ \ddot{u} + \left( \frac{c}{(M+\alpha b)} \right) \dot{u} + \left( \frac{k}{(M+\alpha b)} \right) u = -\left( \frac{M}{(M+\alpha b)} \right) \ddot{x}_g \]  

(4-20)

A comparison of this equation, with the equation of motion of an uncontrolled SDOF system reveals that the damping of the controlled system has reduced by \( \frac{c}{(M+\alpha b)} \). The \( \alpha b \) term is positive, so the \( \frac{c}{(M+\alpha b)} \) ratio is always smaller than \( \frac{c}{M} \) for an uncontrolled SDOF system. This means that the ability of the system for dissipating the energy imparted by earthquake excitations, which tends to induce higher dynamic response with increased potential for structural and non-structural damage, reduces with the application of a GMD. Although this statement should be reevaluated for nonlinear GMD models, it is advantageous to increase the capacity of GMD for energy dissipation.

Any energy dissipative device can be added to GMDs to enhance their performance. As an example, in this study, a viscous fluid damper is selected for this objective. The schematic of the resulting hybrid control system is shown in Fig. 4-50. It should be noted the braces that transfer the forces from the structure to the dampers could be assumed to be infinitely stiff.

![Figure 4-50. An SDOF system with a GMD and a viscous fluid damper](image)

Viscous fluid dampers, working on the premise of energy dissipation, are common among the passive control devices. In these devices, the generated damper forces are “out-of-phase” with the forces generated by the structural system. The energy dissipated in a viscous damper is proportional to the damping constant and the excitation frequency and is proportional to a power of the displacement amplitude. Thus in the first step, it is necessary to determine the damping constant and velocity exponent of the viscous damper consider in this study.
4.5.1 Viscous damper

The custom-built viscous damper used here was built by Taylor Devices Inc. and was used and tested in the structural laboratory of the University of Waterloo in the past. The unit weight of the damper is 22.7 kg and it operates with an inert silicone fluid with an operating ambient temperature range of -23°C to 54°C. The damper stroke is a total of 178 mm (7"), with a total length at full compression of 692 mm and a total length at full extension of 870 mm.

Despite shop drawings listing the maximum allowable force to be approximately 31 kN, program interlocks were set to 25 kN for safety purposes. The viscous damper also has an orifice that can be adjusted via the use of a mechanical valve. This ability to control the damping coefficient makes it possible to use the damper for active control of structures. However, as this is not the main focus of this study, all of the tests in the following sections are performed on a viscous damper with a fully-closed valve. The experimental setup for testing the damper is shown in Fig. 4-51.

Custom clevises were machined to fit the specifications of the piston end of the damper for a slip-free fit. Special aluminum plates were designed and machined to connect the device to the base plates and to hold it horizontally at the same elevation as the actuator head.

Figure 4-51. Testing setup for the viscous damper
4.5.2 Characterization and identification of the viscous damper

In this section, characterization tests are conducted on the viscous fluid damper to develop an experimental and analytical understanding of the relationship between the relative velocity of the end nodes and the resulting damper forces. Through a series of set frequency and displacement-controlled tests, the damper is tested and a proper force-velocity relationship is fitted. Fig. 4.52 shows the results of a cyclic test with an input displacement of 15 mm and a frequency of 1.0 Hz. Fig. 4-52 (b) indicates that the force time history cannot be described by a cosine function. This means that the viscous damper is not linear and the velocity exponent should be found for the nonlinear damper. In Fig. 4-52 (c), the hysteresis loop of the device is shown. From this figure, the enclosed area of the diagram for each cycle is equal to 277.40 N.m. This value can be compared to 9.27 N.m of the larger-scale prototype with the equivalent mass of 1379.9 kg in Section 4.3. Finally, in Fig. 4-52 (d), the force-velocity relationship is illustrated. Once more, it can be observed that the connection between these two parameters is not linear and the viscous fluid damper is nonlinear.

![Figure 4-52. Viscous damper under the subjected to the harmonic input displacements](image)

To identify the parameters of the damper, 18 cyclic characteristic tests have been carried out with the input frequencies of 0.1, 0.2, 0.6, 1.0, 1.4 and 2.0 Hz and the amplitudes of 7.5 mm, 15 mm...
Chapter 4. Experimental Testing of Gyromass Dampers

and 22.5 mm and the maximum forces and velocities are recorded. The results are shown in Fig. 4-53. Using the curve fitting toolbox of MATLAB, Eq. 4-21 is fitted to the data measurements.

\[ f = 505 \dot{u}^{1.8} \]  

(4-21)

The manufacturer shop drawings specify that with the valve closed, the force is proportional to the square of the velocity. The manufacturer also provides a value of 22.73 kN.s/m for the damping constant which is considerably smaller than 505 kN.s/m which was obtained in Eq. 4-21. However, the identified parameters are comparable to the values that were measured at University of Waterloo (i.e., the damping coefficient of 426.39 kN.s/m and velocity exponent of 1.968).

Figure 4-53. Peak damper force vs. peak velocity for the tested viscous damper

Figure 4-54. Results for the viscous damper under a cyclic displacement (Case 1.2.1)
The success of the prediction model is evaluated in Fig. 4-54, where the cyclic input displacement of Fig. 4-54(a) is applied to the damper, and the forces are measured. This figure shows that there is a satisfactory agreement between the model predictions and the force measurements.

4.5.3 RTHS of the system with a GMD and an added viscous damping

After the identification of the damper parameter, it is possible to use the experimental testing setup shown in Fig. 4-55 and test the system of Fig. 4-50. The same set of earthquake ground motions as the ones that were used in Section 4.4.3 are simultaneously applied to both dampers, and the resulting forces are measured. The dynamic properties of the primary structure are introduced in Section 4.4.3 and the properties of the earthquakes, and their scaling factors are listed in Table 4-3. The results of the experiment with the EQ1 record are shown in Fig. 4-60.

![Figure 4-55. Testing setup for the system with a GMD and a viscous damper](image)

Fig. 4-56 (a) compares the response of the uncontrolled SDOF system with the corresponding values when the system is equipped with both the GMD and the viscous damper. As it is shown in this figure, the effect of the hybrid control system on the displacements of the SDOF oscillator is significant. For further comparison of the hysteretic behaviour of the hybrid system and its energy
dissipation during the earthquake, Fig. 4-56 (b) and (c) are plotted. Note that in comparison with Fig. 4-40 (e), the maximum dissipated energy for the hybrid control system is 2.5 times higher than the GMD-only system. To compare the effects of the hybrid control strategy, in Fig. 4-56 (d), the displacement time-histories of Fig. 4-56 (a) are enlarged, and the responses from all systems are compared. Note that the performance of the GMD-only and viscous damper-only systems are very similar, but when these systems are introduced at the same time, their effectiveness increases considerably. This is better shown when the displacement time-histories are enlarged around the maximum displacement of the structure.

After repeating the experiments for the uncontrolled SDOF system, the system with only a GMD, the system with only a viscous damper and the system with a combination of a GMD and a viscous damper, their performances are compared in Table 4-5 and Table 4-6.

![Figure 4-56. RTHS results for the system with a GMD and a viscous damper (subjected to EQ1)](image-url)
From Fig. 4-55 and Table 4-5 and Table 4-6 it can be observed that the performance of the system with a GMD and a viscous damper is superior to the performance of the other systems. Thus, the viscous damper (or any other energy dissipative device) can be employed in parallel to GMDs to improve the effectiveness of both control strategies.

### 4.6 Summary

Through several experiments for various loading cases, it was shown in this chapter that linear models cannot sufficiently predict the behaviour of GMDs. After analyzing the test results performed on two different prototype specimens with various values of equivalent mass, the parameters of a nonlinear model for describing the damper force were determined, and the effects of the frictions forces were illustrated. Utilizing the nonlinear model, RTHS’s with a suite of six different earthquakes were carried out and the experimental data verified the proposed model. Also, to further improve the behaviour of these devices, a nonlinear viscous damper was introduced to the testing setup and the superior performance the hybrid control strategy was shown. Based on the developed models of this chapter, numerical studies will be presented in the next chapter which provides more details about the seismic performance of these devices.
Chapter 5 Incorporation of GMDs into SDOF and MDOF structures

After investigating the dynamic characteristics of GMDs experimentally, the next step involves the study of the impact of introducing GMDs into SDOF and MDOF structures on their seismic performances. Unlike the other types of inertia-based devices such as TVMDs, to utilize the proposed GMD, there is no necessity to use it inside a viscous damper. Moreover, the application of rack and pinion mechanism in GMDs makes it easier to build these devices and modify their dynamic properties. The potential of using a GMD as a structural control device is promising. With the objective of introducing a GMD into structures and reducing energy dissipation demand on the primary structural members, numerical studies should be carried out. This reduction is accomplished by transferring some of the structural energy to the rotating masses of the GMD.

This chapter starts with a brief description of the theory of vibration in GMD-SDOF systems subjected to harmonic base excitation. Harmonic ground motion is discussed next and various cases including a stand-alone GMD attached to a damped SDOF system, and a GMD in parallel with a viscous damper and in series with a bracing system in an SDOF system are considered. Using a suite of ground motion records selected from 99 historical earthquakes, time history responses for SDOF systems with designed GMD and subjected to harmonic and seismic excitations are studied. The natural period of the original structures is also changed to have a better understanding of the performance of these control systems. The numerical studies are then repeated for MDOF systems where the GMD is used to modify the vibrations of the structure. Lastly, a simplified study for placement of the GMD in MDOF systems is presented.

5.1 Incorporating GMDs into SDOF systems

5.1.1 Structural control with a linear stand-alone GMD

To obtain the equation of motion for an SDOF system with a stand-alone GMD (Fig. 5-1), the equation of motion of an SDOF system with a known mass $M$, stiffness $K$ and inherent damping
Chapter 5. Application of GMDs in Building Structures

C (and damping ratio of $\xi$) (as shown in Eq. 5-1) needs to be modified. Starting with the simpler case of an ideal GMD, which can be characterized by a linear model (Eq. 2-3), the force developed in a GMD can be assumed to be proportional to the relative acceleration applied at the end nodes. This means that the equation of the motion for a system equipped with a GMD can be modified as Eq. 5-2,

$$M \ddot{x} + C \dot{x} + Kx = -M \ddot{g} \quad (5-1)$$

$$(M + b) \ddot{x} + C \dot{x} + Kx = -M \ddot{g} \quad (5-2)$$

By dividing both sides of Eq. 5-1 and 5-2 by the coefficients of the inertial forces, the equations of motion can be rewritten as,

$$\ddot{x} + \left[\frac{C}{M}\right] \dot{x} + \left[\frac{K}{M}\right] x = -\left[\frac{M}{M}\right] \ddot{g} \quad (5-3)$$

$$\ddot{x} + \left[\frac{C}{M+b}\right] \dot{x} + \left[\frac{K}{M+b}\right] x = -\left[\frac{M}{M+b}\right] \ddot{g} \quad (5-4)$$

For this system, the equivalent mass ratio $\mu$ can be defined as,

$$\mu = \frac{b}{M} \quad (5-5)$$

The purpose of adding the stand-alone GMD is to increase the inertia of the structure without increasing the effective load of the system when the structure is subjected to a particular excitation. Since both of the differential equations of Eq. 5-3 and 5-4 are normalized and have the same
coefficients for the acceleration term on the left-hand side of the equation, a comparison between them shows the effects of the incorporation of the stand-alone GMD into the SDOF system. The comparison reveals that the natural period of vibration of the SDOF system with the stand-alone GMD increases by \(\sqrt{(M + b)/M}\) or using the definition of the equivalent mass ratio, by \(\sqrt{1 + \mu}\). This means that for an equivalent mass ratio of 7.15%, the natural period of the modified SDOF system increases by 3.51%. Furthermore, comparison of the other dynamic parameters of the equation of motion of the system with a linear GMD with the corresponding values of the original system also shows that the damping ratio of the SDOF system with the GMD decreases by \(\sqrt{M/(M + b)}\). For a system with \(\mu = 7.15\)%, the reduction of the damping ratio is equal to 3.39%. The effective force on the right-hand side of the equation of motion of the original SDOF system also reduces by \(M/(M + b)\). This corresponds to an effective force reduction of 6.67% for a system with \(\mu = 7.15\)%.

If designed properly, these effects could lead to an enhanced seismic performance of the controlled structures.

### 5.1.2 Structural control with a nonlinear stand-alone GMD

The experimental results in Chapter 4 show that a linear model may not be sufficient to characterize the dynamic behaviour of a GMD fully. It was also observed that the friction force was small compared to external loading or restoring force resulting from initial motion which ensures that the input energy is dissipated by slip motion of the damper element without stop motion. The addition of the Coulomb damping force of \(F_{f\max}\) changes the equation of motion to,

\[
(M + b)\ddot{x} + C\dot{x} + Kx + F_{f\max} \text{sign}(\dot{x}) = -M\ddot{x}_g
\]

(5-6)

\(\text{sign}(\dot{x})\) is the signum function which is defined as \(-1\), 0 and 1, respectively provided that \(\dot{x} < 0\), \(\dot{x} = 0\) and \(\dot{x} > 0\).

Closed-form solution of Eq. 5-6 is dependent on the form of the external loading. It is impossible to obtain an analytical solution for a randomly excited loading such as an earthquake. As an alternative, using the approach proposed by (Min et al., 2010), using energy balance equation, a closed-form solution for the dynamic magnification factor (DMF) for a steady-state response of the SDOF system and the associated equivalent viscous damping ratio is derived here at the natural
frequency. As a result, the derivation of this section is only valid for steady-state response in the frequency domain by considering the response at resonance without taking into account free and transient vibrations. For more general scenarios, the study of (Seong et al., 2012) should be consulted, and new solutions should be derived.

**Figure 5-2.** Model of an SDOF system with a nonlinear stand-alone GMD

According to Chopra (Chopra, 2011), by equating the dissipated energy by a friction damper with the energy dissipated by viscous damping for one cycle, a friction damping force can be replaced by an equivalent viscous damping force. As such, Coulomb damping force term in Eq. 5-6 can be replaced by an equivalent viscous damping subjected to a harmonic force. The resulting equation of motion can be represented as,

\[
(M + b)\ddot{x} + (C + c_{eq})\dot{x} + Kx = (-M\ddot{x}_g)\sin(\omega_0 t) = F_0 \sin(\omega_0 t)
\]

where, \(c_{eq}\), \(F_0\), and \(\omega_0\) are the equivalent viscous damping constant, amplitude of harmonic loading, and angular loading frequency, respectively. Eq. 5-6 can be transformed into an energy balance equation by multiplying a differential displacement and integrating over the entire displacement (Christopoulos et al., 2006):

\[
E_K + E_D + E_{eq} + E_S = E_{in}
\]
where, $E_K$, $E_D$, $E_{eq}$, and $E_S$ are the kinetic energy, viscously dissipated energy, equivalently dissipated friction energy, and strain energies, respectively. Moreover, $E_{in}$ is the input energy from the external harmonic loading. Since changes in kinetic and strain energies over one cycle are zero for the steady-state response, the sum of $E_D$ and $E_{eq}$ over one cycle is equal to the input energy, $E_{in}$, which yields,

$$\pi \left( C + c_{eq} \right) \omega_0 x_0^2 = \pi F_0 x_0 \sin(\varphi)$$  \hspace{1cm} (5-8)$$

where, $x_0$ is the amplitude of the maximum dynamic displacement and $\varphi$ is the phase angle. After computing the phase angle and substituting it in the DMF of an underdamped system under harmonic loading, the following equation can be derived (Min et al., 2010) to express the DMF of the system of Eq. 5-6,

$$DMF = \frac{x_0}{(\alpha_{st})_0} = \frac{-\left(\frac{8}{\pi}\right) \beta_{gmd} \xi_{gmd} F_r + \left[\alpha^2 + 4 \beta_{gmd} \xi_{gmd} \left(\frac{8}{\pi} \alpha F_r\right)\right]^{\frac{1}{2}}}{a^2 + \left(2 \beta_{gmd} \xi_{gmd}\right)^2}$$  \hspace{1cm} (5-9)$$

where,

$$\beta_{gmd} = \frac{\omega_{n_{gmd}}}{\omega_0}$$  \hspace{1cm} (5-10)$$

$$\omega_{n_{gmd}} = K / (b + M)$$  \hspace{1cm} (5-11)$$

$$\xi_{gmd} = C / \left[2(b + M)\omega_{n_{gmd}}\right]$$  \hspace{1cm} (5-12)$$

$$\alpha = 1 - \beta_{gmd}^2$$  \hspace{1cm} (5-13)$$

$$F_r = \frac{F_{f\max}}{F_0}$$  \hspace{1cm} (5-14)$$

Also, since the numerator of Eq. 5-9 should be positive, $F_r < \pi/4$.

The DMF is narrow banded, and its peak occurs at the natural frequency. Thus the value of the equivalent damping of the system can be approximated from Eq. 5-8 by determining DMF in Eq. 5-9 at $\beta_{gmd} = 1$:

$$\xi_{eq} = \frac{F_r}{\left(\frac{8}{\pi}\right) - F_r} \xi_{gmd}$$  \hspace{1cm} (5-15)$$
Eq. 5-15 can be used to estimate the equivalent viscous damping ratio of the system with a nonlinear GMD. As shown in Fig. 4-18 in Chapter 4, it was found that the measured value of $F_{f\text{max}}$ under a harmonic displacement with $f_0 = 1\ Hz$ and maximum displacement amplitude of 7.5 mm is 53.22 N. Also, based on the measurements $F_0 = 500\ N$ which leads to a friction force ratio $F_r$, that is equal to 0.106. Using the obtained values, the equivalent viscous damping of the system can be calculated as,

$$\xi_{eq} = \frac{0.106}{\left(\frac{\pi}{4}\right) - 0.106} \xi_{gmd} = 0.157 \xi_{gmd}$$ (5-16)

which is considerably smaller than the inherent damping of the system. This means that for an SDOF system with an inherent viscous damping of 2%, the equivalent viscous damping of the GMD is equal to 0.31%.

The effects of the variation of the friction force ratio $F_r$ on the response of the SDOF system is shown in Fig. 5-3 where the system is subjected to a harmonic input displacement at the base with the amplitude of 7.5 mm and $f_0 = 1\ Hz$. As expected, the inclusion of the friction forces changes the response of the system and reduces the maximum SDOF displacements. Note that $F_r = 0.10$ corresponds to the experimental setup of Chapter 4 and $F_r = \pi/4$ is the theoretical upper limit for the friction ratio.

![Figure 5-3. Displacements of the structure with various friction force ratios ($\mu = 0.1$)](image-url)
5.1.3 Structural control with a linear GMD and braces

One of the main issues for the incorporation of the GMDs in structures is the placement of these devices in building systems. Due to geometric limitations and alignment requirements, it would be challenging, if not impossible, to design a GMD that spans along the height of a story diagonally to work based on the relative floor accelerations. Instead other configurations, such as the V brace shown in Fig. 5-4 should be considered to attach them. The braces serve as extenders and will transfer the displacement and acceleration of the main mass to the end terminal of the GMD.

The lateral stiffness of the braces $k_b$, should be accounted for and after the addition of the braces, the system equation of motion should be modified. Starting with the equation of motion of this system in the time domain, Eq. 5-17 can be written to describe the GMD-brace system,

$$
\begin{bmatrix}
M & 0 \\
0 & b
\end{bmatrix}
\begin{bmatrix}
\ddot{x} \\
\ddot{u}
\end{bmatrix}
+
\begin{bmatrix}
0 & 1 \\
0 & 0
\end{bmatrix}
\begin{bmatrix}
\dot{x} \\
\dot{u}
\end{bmatrix}
+
\begin{bmatrix}
K + k_b & -k_b \\
-k_b & k_b
\end{bmatrix}
\begin{bmatrix}
x \\
u
\end{bmatrix}
=
-\begin{bmatrix}
M \\
0
\end{bmatrix}
\ddot{x}_g
$$

(5-17)

where in Eq. 5-17 and the following equations of this section, $x$ and its derivatives denotes the displacement, velocity and acceleration of the SDOF oscillator and $u$ and its derivatives represents the corresponding parameters for the degree-of-freedom at the node that connects the GMD to the braces. In Eq. 5-17, the linear model is used to describe the behaviour of the GMD.

![Figure 5-4. Model of an SDOF structure with a GMD-brace system](image)

A closer look at Eq. 5-17 reveals that this equation is similar to the equation of motion of a 2DOF system with an additional stiffness element. To study the dynamic behaviour of the system shown in Fig. 5-4, the transfer function $T_{GMD-\text{brace}}$ can be written in the frequency domain as:

$$
T_{GMD-\text{brace}}(j\omega) = \frac{X(j\omega)}{\ddot{x}_g(j\omega)} = -M\omega^2 \left[-M\omega^2 + C j\omega + \left(\frac{1}{k_b} + \frac{1}{-b\omega^2}\right)^{-1}\right]^{-1}
$$

(5-18)
where, \( j = \sqrt{-1} \). Note that since \( k_b \) is the horizontal stiffness of the braces, brace geometry should be taken into account when this parameter is calculated. At this stage, in addition to the equivalent mass ratio which was defined in Section 5.1.1, another dimensionless parameter should be defined to describe the system in Fig. 5-4. The ratio of the brace stiffness to that of the structure is defined by \( \beta = \frac{k_b}{K} \). Fig. 5-5 shows the magnitude of the transfer function \( T_{GMD-brace} \) for different frequencies.

![Frequency response of the SDOF with a GMD-brace system](image)

**Figure 5-5.** Frequency response of the SDOF with a GMD-brace system

The addition of the GMD-brace system separates the resonant peak of the original SDOF system into two distinct frequencies. For each value of \( \beta \), the higher-frequency resonant peak of the controlled system is consistently larger than the resonant peak of the original system. As a result, in comparison to the performance of the original SDOF system, the higher frequency peak of the GMD-brace system is continuously increasing. This can be particularly critical when the governing frequency of the applied load coincides with the higher-frequency resonant peak of the GMD-brace system.

### 5.1.4 Structural control with a nonlinear GMD and braces

For a more realistic modelling of the GMD, the effects of the friction forces should be considered. The new model is shown in Fig. 5-6.
The governing equations of motion are given by,

\[ M\ddot{x} + C\dot{x} + Kx + f_d = -M\ddot{x}_g \]  \hspace{1cm} (5-19)

where,

\[ f_d = k_b (x - u) = b\ddot{u} + F_{\text{fmax}}\text{sign}(\dot{u}) \]  \hspace{1cm} (5-20)

According to the relation in Eq. 5-15, the friction force can be substituted by the equivalent viscous damper. Thus, Eq. 5-20 can be rewritten as below,

\[ f_d = k_b (x - u) = b\ddot{u} + c_{eq}\dot{u} \]  \hspace{1cm} (5-21)

After, rearranging the terms in Eq. 5-19 and 5-21, the following equations of motion can be written for the system of Fig. 5-6 (a),

\[ M\ddot{x} + C\dot{x} + Kx + k_b (x - u) = -M\ddot{x}_g \]  \hspace{1cm} (5-22)

\[ b\ddot{u} + c_{eq}\dot{u} + k_b (u - x) = 0 \]  \hspace{1cm} (5-23)

Eq. 5-22 and 5-23 can be summarized as,

\[
\begin{bmatrix} M & 0 \\ 0 & b \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{u} \end{bmatrix} + \begin{bmatrix} C & 0 \\ 0 & c_{eq} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} + \begin{bmatrix} K + k_b & -k_b \\ -k_b & k_b \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} = - \begin{bmatrix} M \\ 0 \end{bmatrix} \ddot{x}_g
\]

Eq. 5-24 has similarities with the equations of motion of an SDOF system with a TMD. This is very similar to the model that was originally presented by Saitoh (Saitoh, 2012), where he called it a Model II system and investigated the effectiveness of adding supplemental viscous dampers to
linear GMDs for a different target structure (i.e., a base-isolated system). Using the similarities between SDOF-TMD models and SDOF-GMD-brace systems, Lazar et al. (2014) and Giaralis et al. (2015) also proposed new models, which are called TID and TMDI, respectively. These two studies as well as the study by Takewaki et al. (2012) considered a viscous damper in parallel with the braces. For the same reasons that were explained for stand-alone GMDs (i.e., geometric limitations and alignment issues), this configuration would not be practical in real applications. In another study, Ikago et al. (2012), proposed a model in which the inertial component was arranged in series with the braces and in parallel with a viscous damper. However, the inertial component in their model is a TVMD (and not a GMD), where a ball-screw mechanism is added to a conventional linear viscous damper. Despite the similarities between GMDs and TVMDs, the main advantage of using GMDs is that the supplemental energy dissipation component is not built-in to the device and can be attached as an external component. This will allow the design engineers to select from a wide range of available energy dissipation devices to use in parallel with the GMD. In this case, where the friction forces are considered, the GMD can be employed without any supplemental energy dissipative device. In addition, it should be noted that in both of the studies by Ikago et al. and Lazar et al., damping of the primary structure was neglected which is not the case in Fig. 5-6.

In Eq. 5-24, like the mass matrix, the damping matrix is diagonal. To have a mass-proportional classical damping matrix, the following condition should be satisfied:

\[ F_r = \frac{\pi \mu}{4(1+\mu)} \]  \hspace{1cm} (5-25)

Since \( b \) and \( F_{f_{\text{max}}} \) are experimentally measured parameters for each device, Eq. 5-25 may not be necessarily satisfied and thus, the damping matrix in Eq. 5-24 can be a non-classical damping matrix.

To use the modal superposition method and solve this equation, appropriate approaches such as the pseudo-force method for dynamic analysis of systems with non-proportional damping can be used (Claret et al., 1991). In this method, the damping matrix can be expressed as the summation of a classical damping matrix and another matrix which contains the off-diagonal elements. By
moving the off-diagonal part to the right side of Eq. 5-24 as a pseudo force, the left side becomes decoupled and can be solved by iteration.

Alternatively, by using the relative displacements, the transfer function approach can be employed directly to find the frequency response of the SDOF-GMD-brace systems. This transfer function, named as \( T \), can be written in the frequency domain as:

\[
\frac{X(j\omega)}{X_g(j\omega)} = -M\omega^2 \left[ -M\omega^2 + Cj\omega + K + \left( \frac{1}{k_b} + \frac{1}{c_{eq} j\omega - b\omega^2} \right)^{-1} \right]^{-1} \tag{5-26}
\]

A non-dimensional parameter \( \gamma \) is defined as the ratio of the equivalent viscous coefficient due to the friction forces over the corresponding damping coefficient of the original structure \( (\gamma = \frac{c_{eq}}{c}) \). Using \( \mu, \beta \) (which were defined previously) and \( \gamma \), Equation 5-26 can be rearranged as follows,

\[
T(j\omega) = \frac{\mu \omega^4 - \beta \omega_n^2 \omega^2 - 2 \omega \omega_n \omega^3}{\mu \omega^4 - 2 \zeta (\mu + \gamma) \omega_n \omega^3 j - (4 \zeta^2 \gamma + \mu + \beta + 2 \mu \omega_n^2 + 2 \zeta (\beta + \gamma) \omega_n^2) j + \beta \omega_n^4} \tag{5-27}
\]

which can be rearranged as the following equation,

\[
T(j\omega) = \frac{\{ \mu \left( \frac{\omega}{\omega_n} \right)^4 - \beta \left( \frac{\omega}{\omega_n} \right)^2 \} - 2 \zeta \gamma \left( \frac{\omega}{\omega_n} \right)^3 j}{\{ \mu \left( \frac{\omega}{\omega_n} \right)^4 - (4 \zeta^2 \gamma + \mu + \beta + \beta \omega_n^2 + 2 \zeta (\beta + \gamma) \omega_n^2) j + \beta \omega_n^4 \} j} \tag{5-28}
\]

After some further manipulations, Eq. (5-28) can be expressed as,

\[
T(j\omega) = \frac{-\left( \frac{\omega}{\omega_n} \right)^2}{\left[ 1 - \left( \frac{\omega}{\omega_n} \right)^2 \right] + 2 \zeta \left( \frac{\omega}{\omega_n} \right) j + G(\mu, \beta, \gamma, \omega, \omega_n)} \tag{5-29}
\]

where,

\[
G(\mu, \beta, \gamma, \omega, \omega_n) = \frac{\mu \beta \left( \frac{\omega}{\omega_n} \right)^2}{\mu \left( \frac{\omega}{\omega_n} \right)^4 - \beta - 2 \zeta \gamma \left( \frac{\omega}{\omega_n} \right) j} \tag{5-30}
\]

Note that when \( \mu = 0 \) or \( \beta = 0 \), \( G(\mu, \beta, \gamma, \omega, \omega_n) \) becomes zero, and the transfer function becomes that of the original SDOF system. This equation shows that the performance of an SDOF-GMD-brace system is influenced by the selection of the lateral brace stiffness \( k_b \). Variation of the parameters \( \mu, \beta \) and \( \gamma \) results in the modification of the dynamic response of the system. Thus, a proper selection of a combination of these variables needs to be made during the design of the control system.
In order to address this issue and select an optimum set of parameters, Ikago et al. (2012) used the fixed-point method and derived a closed-form solution for the design of a structure subjected to harmonic excitation. Based on their study, the optimal response can be obtained when the equivalent damping ratio $\xi_{eq}$ is calculated from Eq. 5-31,

$$\xi_{eq} = \frac{\mu}{2} \times \frac{3\mu}{\sqrt{(1-\mu)(2-\mu)}}$$

(5-31)

From Eq. 5-24, $\xi_{eq}$ is also a function of $F_r$. After combining Eq. 5-24 and Eq. 5-31, the equivalent mass ratio of the GMD can be expressed in terms of the friction force ratio $F_r$,

$$\mu^3 - \left(\frac{\xi^2}{3}\right) \left(\frac{F_r}{\pi/4 - F_r}\right)^2 [4\mu^2 - 12\mu + 8] = 0$$

(5-32)

After choosing a reasonable value for the $F_r$ of the device, optimal $\mu$ can be found by solving Eq. 5-32. The variation of $\mu$ for different values of $F_r$ is shown in Fig. 5-7.

For instance, from Chapter 4, it was found that $F_r = 0.106$. This corresponds to an equivalent mass ratio of $\mu = 7.15\%$ for a system with inherent damping of $\xi = 2\%$. This value is slightly larger than the commonly used value of the equivalent mass ratio in TMDs. Now, using the value of the optimal equivalent mass, the lateral stiffness of the braces can be determined from Eq. 5-33,

Figure 5-7. Variation of the optimal equivalent mass ratio for different $F_r$ values
\[ \beta = \frac{k_b}{K} = \frac{\mu}{1-\mu} \]  

(5-33)

From Eq. 5-33 it can be shown that \( \omega_b = \omega_n \sqrt{\frac{\beta}{\mu}} = \omega_n \frac{1}{\sqrt{1-\mu}} \), where \( \omega_b = \sqrt{\frac{k_b}{b}} \).

It can be shown that even for non-optimal values of \( \mu \), GMDs can improve the performance of the SDOF system subjected to harmonic loads. In Fig. 5-8, the frequency response of SDOF systems with and without GMDs are compared for different values of \( \mu \). Two main observations can be made by looking at Fig. 5-7. First, even for smaller values of \( \mu \), a significant reduction of the peak response can be obtained. As it can be seen in this figure, although increasing the values of \( \mu \) leads to lower vibration amplitudes, the rate of this decrease reduces for the effective mass ratios larger than of 5%. Secondly, since Ikago et al. assumed that the original uncontrolled system is undamped the separated peaks does not have the same magnitudes but the difference is relatively small.

![Figure 5-8. Frequency response of the main mass for various values of μ (F₀ = 0.106)](image)

To further examine the sensitivity of the response of an SDOF-GMD system to the variation of its design parameters, the performance of an example SDOF system (shown in Fig. 5-9) is investigated in this section. The portal frame is chosen based on the properties of the larger scale prototype (Fig. 5-9). The main mass of the frame is equal to 19.3 t (i.e., \( \mu = 7.15\% \)). The supporting columns have an \( EI \) equal to 370 kN.m² (corresponding to a period of vibration of 1.53 sec).
(Christopoulos et al., 2006). System identification tests on the frame indicated an inherent damping ratio of 2% of critical.

From Fig. 5-8, it can be observed that the equivalent mass ratio ($\mu$) has the maximum effects on the behaviour of the system when the primary structure is in resonance with the applied ground motion. Also, with the increase of this ratio (i.e., $\mu$), peak accelerations and displacements of the SDOF system reduce. However, the effects of $\beta$ and $\gamma$ on the performance of the system is more complex. For a GMD-brace system, Eq. 5-31 to 5-33 can be employed to determine the optimum $\beta$ and $\gamma$. In Fig. 5-10 and 5-11, a fixed value of 0.0715 is chosen for $\mu$ and parameters $\beta$, and $\gamma$ are changed. The normalized maximum displacements for a harmonic excitation with an input frequency that is equal to the natural period of the primary system are shown in Fig. 5-10 (a) with $\beta$ plotted on the x-axis, $\gamma$ on the y-axis and the ratio of the maximum displacement of the SDOF-GMD-brace system over the maximum displacement of the uncontrolled system on the z-axis. When $\beta$ and $\gamma$ are zero, the ratio becomes one. The contours of the relations between these parameters are shown in Fig. 5-11.

From these figures, it can be concluded that the combination of $\beta$ and $\gamma$ can result in different system responses and it emphasizes the necessity for the appropriate selection of a combination of these variables during the design procedure. It should be noted that the filled points in Fig. 5-11 show the selected optimum values from Eq. 5-31 to 5-33. As it can be seen in these figures, the designed values are located within the regions with low values. However, these optimal values
obtained from Eq. 5-31 to 5-33 are closer to the actual optimal response when the system has a lower frequency. In addition, it can be observed that the GMD-brace system can reduce the maximum displacement of the uncontrolled SDOF system by around 70% (at the frequency ratio of 1). Time history analysis results of structures with and without the nonlinear GMD-brace system subjected to a harmonic ground motion can be used to show the effectiveness of this control strategy (Fig. 5-12).

Figure 5-10. Effects of $\beta$ and $\gamma$ on the maximum displacement of the SDOF-GMD-brace system

Figure 5-11. Contours for the normalized maximum displacements of Fig. 5-10: (a) $T_n = 0.5$ sec, (b) $T_n = 1.53$ sec, and (c) $T_n = 2.5$ sec ($\mu = 0.0715$)
As it is shown in Fig. 5-6(b), the GMD is consisted of two components: the friction force component and the inertial component. To examine the effects of the friction force component independently, Fig. 5-8 has been reproduced for the same values of $\beta$ and $\gamma$ but for $\mu = 0$ (i.e., $b = 0$). The results are shown in Fig. 5-12 and 5-13 ($T_n = 1.00$ sec). From these figures it can be observed. First, the control strategy is less effective in the absence of the inertial component. For
instance, the normalized maximum displacement in Fig. 5-13 for $\beta$ and $\gamma$ of 0.08 and 0.6 is equal to 0.78 which is significantly smaller than 0.3 from Fig. 5-11. Secondly, the variation of the performance criterion has a different pattern when the inertial component is removed.

**Figure 5-12.** Effects of $\beta$ and $\gamma$ on the maximum displacement of the SDOF-GMD-brace system ($\mu = 0$)

**Figure 5-13.** Contours for the normalized maximum displacements of Fig. 5-12 ($\mu = 0$)
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5.2 Seismic performance of systems with GMDs

5.2.1 Seismic performance

To further study the performance of a GMD-brace system and to provide some insight into their seismic response over time, the response of the system should be obtained from numerical simulations by using a properly chosen group of earthquake records.

There are several methods in the literature for selecting and scaling the earthquake ground motion records. Obviously due to the random nature of earthquakes, by using a group of earthquake records, different results may be obtained. As a result, a specific group of 99 earthquake records which are introduced in (Liang et al., 2012) are used in this section. These reference historical earthquake records are typical ground motion records that are commonly used in the seismic analysis (Naeim et al., 1996). The ground motions are selected from the PEER-NGA database (Pacific Earthquake Engineering Research Center, 2015) and the USGS database (USGS, 2014). Details of these ground motions are reported in Table E.1, where the main attributes of these specific ground motions are mentioned (Liang et al., 2011).

![Response of the original and controlled SDOF systems](image)

**Figure 5-14.** Response of the original and controlled SDOF systems

The elastic response spectra of the unscaled records are shown in Fig. 5-15 where, the average spectrum for these records is shown with a solid black line. As it can be seen in this figure, the considered ground motion records cover a broad range of earthquakes with different properties.
Moreover, from the scatter plot of the peak ground acceleration (PGA) and the average period of the records (shown in Fig. 5-16), it can be seen that the selected records cover a wide range of earthquakes of with several PGA and average periods.

To investigate the effects of the uncertainties of the applied earthquake ground motions, for each ground motion of Table E-1, time history analyses are performed on four systems separately: (1)
the uncontrolled SDOF system shown in Fig. 5-9, (2) an SDOF structure with a stand-alone GMD (linear model), (3) another SDOF structure with a stand-alone GMD (nonlinear model), and 4) an SDOF structure controlled with a nonlinear GMD-brace system. In Fig. 5-17, time histories of displacement and acceleration responses are shown for these systems when they are subjected to the El Centro earthquake (i.e., record no. 1 in Table E-1).

![Graph showing responses of original and controlled SDOF systems](image)

**Figure 5-17.** Response of the original and controlled SDOF systems, (a) displacements, (b) acceleration (El Centro earthquake)
In Fig. 5-17, the dashed line shows the results for the uncontrolled system and the solid lines shows the improved responses in the controlled systems. Although these figures are plotted for a sample earthquake record and the effects of each of these control systems changes for other earthquake records, they reveal the drastic reduction of the root mean square of the accelerations and displacements for the configuration where GMD-brace systems are employed.

A better representation of the problem is shown in Fig. 5-18 where for all of the 99 ground motion records, the maximum displacements of the three controlled systems are normalized by dividing them by the corresponding maximum displacement of the main uncontrolled system and plotted against the normalized maximum accelerations. Since the objective of introducing these control systems to the primary system is to reduce the displacements and accelerations, the desirable region in these plots is inside the black square where, both the normalized maximum displacements and normalized accelerations are less than one. As it can be seen in this figure, the nonlinear GMD-brace system has the best performance among the considered controlled systems. The incorporation of the GMD-brace system is more effective for reducing the maximum displacements as for the majority of records, the normalized value of the maximum displacements is less than unity. However, the maximum acceleration is unaffected (or greater than one) for many records. This is because the largest resisting forces system are produced after it undergoes the maximum accelerations. It should be noted that the controlled systems may experience worse responses when they are equipped with stand-alone GMDs. This applies in particular to earthquake records that have a broad frequency range.

This procedure is repeated in Fig. 5-19 for the root mean square (RMS) of the responses. For each point on these figures, for each controlled system, first, the RMS of every data point in the displacement or acceleration time histories are calculated and then the obtained values are divided by the RMS of the corresponding displacements or accelerations of the uncontrolled system. Once more, the black square shows the limits of the ideal response. When a point is located in the square, it means that the associated system has reduced both the RMS of the accelerations and the RMS of the displacements. This can be used as an indicator of the performance of the control systems in average. A closer look at this plot reveals that the GMD-brace system is more efficient in reducing the RMS values and except one earthquake, it successfully improves the performance of
the primary system when the SDOF structure is subjected to the considered earthquake ground motions.

**Figure 5-18.** Performance of the controlled systems subjected to 99 earthquake ground motions of Table E-1 (maximum responses)

**Figure 5-19.** Performance of the controlled systems subjected to 99 earthquake ground motions of Table E-1 (RMS of the responses)
To examine the effects of the earthquake parameters on the performance of these devices, normalized maximum and RMS of the displacements and accelerations of the GMD-brace systems are plotted against the moment magnitude ($M_w$) and the Joyner-Boore distance $R_{jb}$ (i.e., the distance to the vertical projection of the fault to the earth's surface) (Fig. 5-20).

**Figure 5-20.** Effects of the earthquake parameters on the performance of the GMD-brace systems subjected to 99 earthquake ground motions of Table E-1
A closer look at these plots reveals that on average, the earthquake magnitude and the distance from the fault do not affect the performance significantly. This conclusion is based on the records of Table E-1 and to generalize the finding, a larger number of earthquake ground motion records need to be assessed in future. In Table E-1, the earthquake ground motions are also categorized based on their main attributes. The definition of each of these attributes can be found in Appendix E. In Table 5-1, the normalized displacements and accelerations of the GMD-brace systems are averaged for each category and then the averaged values are compared. As it can be observed in this table, the GMD-brace system is most effective in reducing the maximum displacements and accelerations when the structure is subjected to earthquake records with long durations (e.g. El Centro – Imperial Valley). On the other hand, this control strategy is least effective when the system is subjected to pulse-like ground motions.

Table 5-1. Performance of the GMD-brace system subjected to 99 earthquake ground motions with various attributes

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Averaged Normalized Displacement</th>
<th>Averaged Normalized Acceleration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max.</td>
<td>RMS</td>
</tr>
<tr>
<td>Duration</td>
<td>0.57</td>
<td>0.38</td>
</tr>
<tr>
<td>PA</td>
<td>0.69</td>
<td>0.28</td>
</tr>
<tr>
<td>PV</td>
<td>0.70</td>
<td>0.28</td>
</tr>
<tr>
<td>PD</td>
<td>0.67</td>
<td>0.26</td>
</tr>
<tr>
<td>IV</td>
<td>0.72</td>
<td>0.28</td>
</tr>
<tr>
<td>ID</td>
<td>0.66</td>
<td>0.27</td>
</tr>
<tr>
<td>EPA</td>
<td>0.66</td>
<td>0.26</td>
</tr>
<tr>
<td>EPV</td>
<td>0.72</td>
<td>0.27</td>
</tr>
<tr>
<td>---</td>
<td>0.59</td>
<td>0.28</td>
</tr>
</tbody>
</table>

The above observations were made for the SDOF system of Fig. 5-9 which has a natural period of 1.53 sec. To examine the impacts of the natural period of the original uncontrolled system on the effectiveness of these systems, the spectra for each of these parameters are generated by changing the natural period of vibration of the original SDOF system and plotting the corresponding maximum value of the parameters. Using this procedure, for a certain ground motion, the performance criteria can be plotted as a function of the elastic natural period of the original SDOF system. For instance, these figures are plotted for the record no. 1 from Table E-1 in Fig. 5-21 and
Fig. 5-22. The first objective is to find the range of the structural period for which the response is sensitive to the applied ground motion. From these figures, the effects of the equivalent mass terms on elongating the natural period of the structures with linear and nonlinear GMDs are evident. A closer look at Fig. 5-21 reveals that for structures with a natural period of vibration of greater than 1.5 s, the maximum accelerations are relatively unaffected by the addition of the control system. Moreover, when the primary structure is equipped with the GMD-brace system, the control technique reduces the maximum and RMS of the responses consistently. For the particular earthquake record that has been considered in these figures, the nonlinear stand-alone GMD is also effective in reducing the accelerations when the natural period is less than 0.7 s.

**Figure 5-21.** Maximum response of the original and controlled systems for primary structures with different periods (El Centro earthquake)

The plots in Fig. 5-21 and 5-22 are only valid to the considered earthquake ground motion. These figures are used in this section to demonstrate the procedure. However, to study the effects of the variation of the primary structure natural period on the performance of these systems, the same procedure was repeated for all 99 earthquake records, and average values of the parameters were calculated for each period. The results are shown in Fig. 5-23 and 5-24.
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Figure 5-22. RMS of the response of the original and controlled systems for primary structures with different periods (El Centro earthquake)

The results from Fig. 5-23 and 5-24 confirm that on average the application of ideal GMDs (linear models) is ineffective for controlling the displacements of the SDOF system and can slightly reduce the accelerations. On the other hand, for primary systems with a natural period less than approximately 0.7 sec, stand-alone GMDs with friction (nonlinear models) can decrease the accelerations significantly.

Figure 5-23. Averaged max. responses of the systems with different primary structure periods
Finally, the introduction of the GMDs in parallel with braces enhance the performance of the system by reducing the displacements consistently. This improvement is more evident for displacement RMS values. Finally, for primary systems with long natural periods (e.g. $T_n > 1.5$ sec), the application of these methods is ineffective for reducing the accelerations.

![Figure 5-24](image.png)

**Figure 5-24.** Averaged RMS of the responses of the systems with different primary structure periods

### 5.2.2 A numerical example for the SDOF system

The objective of the numerical example in this section is to evaluate the design parameters of various control techniques, including the GMD system, for the seismic retrofit of an SDOF system. This approach is similar to (Mirza Hessabi et al., 2016). As discussed in the previous section, the performance criteria such as maximum displacement and acceleration for each specific control system depends on the properties of the uncontrolled structure and the applied ground motion. However, this example can be used to qualitatively compare the size of each component for these control systems.

Once again, the portal frame shown in Fig. 5-9 and the El Centro ground motion record are used here. The stiffness of the columns is increased to obtain a natural period of 1.0 sec for the structure.
Under the El Centro earthquake ground motion, the maximum displacement of the primary system was determined to be 150 mm. The equivalent mass of the GMDs is found from Eq. 5-32 and similar to previous analyses is chosen to be 7.15%. The value of $F_{f,\text{max}}$ is also measured in the previous chapter. $c_b$ and $k_b$ for the GMD-brace system are determined from Eq. 5-24 and 5-33, and are the optimal values for this system. The stiffness and viscous damping terms for the TMD with an actual mass of 1.38 t are found by using the following equations. These design equations are obtained for random base acceleration loading where the optimization criterion is selected to be the minimum root mean square value of relative displacement of the primary structure. More details about these design equations can be found in (Christopoulos et al., 2006).

$$k_{tdm} = \left(\frac{k}{m}\right) \left(\frac{1-\mu}{(1+\mu)^2}\right) \times m_{tdm}$$  \hspace{1cm} (5-34)

$$c_{tdm} = 2 \times m_{tdm} \times \sqrt{\frac{k_{tdm}}{m_{tdm}}} \times \sqrt{\frac{\mu(1-\mu)}{4(1+\mu)(1-\mu^2)}}$$  \hspace{1cm} (5-35)

Finally, the values of the stiffness and damping terms for stand-alone viscous damper and brace were obtained by trial and error for a target displacement of 84 mm (displacement of the system with the GMD-brace system).

It should be emphasized that the design values shown in Table 5-2 change for a different primary structure under a different input ground motion and the objective here is not to compare the responses quantitatively. Instead, a quick look at the table qualitatively shows that the required stiffness of the braces for the GMD-brace system is about 15 times less than the system with only a stand-alone brace. The GMD-brace system can achieve the same level of performance as the TMD without the need for the viscous damper. Due to the high price of viscous dampers, this is an important difference economically. More importantly, there is no need for placing an actual physical mass of 1.38 t on the structure.

### 5.3 Incorporating GMDs into MDOF systems

In addition to the differences between TMDs and GMDs in GMD-brace systems that were mentioned in the previous sections, there is two main differences when GMDs are incorporated into MDOF structures: (a) TMDs work based on the absolute values of accelerations at each floor.
That is the reason why engineers design the TMDs to be placed on top of the structures, where the absolute accelerations are usually the largest. On the other hand, GMDs work on the relative values of accelerations between their ends, (b) as it is shown in Fig. 5-25, the equivalent viscous damping component is arranged differently these two system resulting in different system dynamics. In TMDs, the viscous damping element is in series with the secondary mass whereas in GMD-brace systems the equivalent viscous damping component is in parallel with the GMD. As a result, the design equations for inertial dampers that make comparisons between these two different strategies may not be accurate (e.g. (Takewaki et al., 2012) or (Lazar et al., 2014)). Especially, since these studies are purely numerical, the reliability of them has to be tested in future.

Table 5-2. Details of the considered control techniques

<table>
<thead>
<tr>
<th>Control Technique</th>
<th>Design Parameter</th>
<th>Maximum Displacement (mm)</th>
<th>Maximum Acceleration (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original SDOF System</td>
<td>---</td>
<td>150</td>
<td>0.81</td>
</tr>
<tr>
<td>Only a viscous damper</td>
<td>$c^* = 18.1 kN.s/m$</td>
<td>84</td>
<td>0.50</td>
</tr>
<tr>
<td>Only a brace</td>
<td>$k^* = 937.16 kN/m$</td>
<td>84</td>
<td>0.81</td>
</tr>
<tr>
<td>A TMD</td>
<td>$m_{td} = 1.38 ton$</td>
<td>86</td>
<td>0.50</td>
</tr>
<tr>
<td>Only a GMD (linear)</td>
<td>$b = 1.38 ton$</td>
<td>142</td>
<td>0.72</td>
</tr>
<tr>
<td>Only a GMD (nonlinear)</td>
<td>$F_{fmax} = 57.33 kN$</td>
<td>133</td>
<td>0.71</td>
</tr>
<tr>
<td>A GMD-brace System (nonlinear)</td>
<td>$b = 1.38 ton$</td>
<td>84</td>
<td>0.52</td>
</tr>
</tbody>
</table>

To this date, the only studies in the literature that consider the model in Fig. 25(a) for designing GMDs for MDOF systems are carried out by Ikago et al. (Ikago et al., 2011) and (Sugimura et al., 2012). The solution of the equations of motion of these systems require complex-valued eigenvalue analysis and numerical optimization method (Ikago et al. 2011a, b). Consider the following $N$ degree of freedom structures in Fig. 25(a). The equations of motion for this system can be written as follows,
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\[ M_1(\ddot{x}_g + \ddot{x}_1) + C_1(\dot{x}_1) + C_2(\dot{x}_1 - \ddot{x}_2) + K_1(x_1) + K_2(x_1 - x_2) + k_{b1}(u_1) - k_{b2}(u_2 - x_1) = 0 \]  
(5-36)

\[ M_2(\ddot{x}_g + \ddot{x}_2) + C_2(\dot{x}_2 - \ddot{x}_1) + C_3(\dot{x}_2 - \ddot{x}_3) + K_2(x_2 - x_1) + K_3(x_2 - x_3) + k_{b2}(u_2 - x_1) - k_{b3}(u_3 - x_2) = 0 \]  
(5-37)

\[ M_l(\ddot{x}_g + \ddot{x}_l) + C_l(\dot{x}_l - \ddot{x}_{l-1}) + C_{l+1}(\dot{x}_l - \ddot{x}_{l+1}) + K_l(x_l - x_{l-1}) + K_{l+1}(x_l - x_{l+1}) + k_{bl}(u_l - x_{l-1}) - k_{bl+1}(u_{l+1} - x_l) = 0 \]  
(5-38)

\[ M_N(\ddot{x}_g + \ddot{x}_N) + C_N(\dot{x}_N - \ddot{x}_{N-1}) + K_N(x_N - x_{N-1}) + k_{bN}(u_N - x_{N-1}) = 0 \]  
(5-39)

\[ b_1(\dddot{u}_1) + c_{eq1}(\dddot{u}_1) + k_{b1}(\dot{x}_1 - u_1) = 0 \]  
(5-40)

\[ b_l(\dddot{u}_l - \dddot{x}_{l-1}) + c_{eql}(\dddot{u}_l - \dddot{x}_{l-1}) + k_{bl}(\dot{x}_l - u_l) = 0 \]  
(5-41)

\[ b_N(\dddot{u}_N - \dddot{x}_{N-1}) + c_{eqN}(\dddot{u}_N - \dddot{x}_{N-1}) + k_{bN}(\dot{x}_N - u_N) = 0 \]  
(5-42)

which results in a system of 2N equations and 2N unknowns.

As it can be seen in these equations, in general, this system has a non-classical damping and therefore, conventional modal superposition methods cannot be utilized to solve the differential equations. In this section, Simulink models were build and utilized to analyze these systems.

Since the chosen properties of the MDOF affects the response of the control systems significantly, an example is shown here to simulate the performance of the tested larger scale prototype of Chapter 4 in a sample 3DOF system. The considered 3DOF shear-story building has equal masses of 19.3 t at each floor (i.e., similar to the mass of the SDOF oscillator in Section 5.2.2). Columns have identical stiffness coefficients which are equal to 12,500 kN/m. This results in natural angular frequencies of 11.33 rad/s, 31.73 rad/s and 45.86 rad/s. Rayleigh damping matrix is calculated in which the damping ratio of 2% is considered for the first and second modes. The first mode shape of the structure is also found to be \([0.445, 0.802, 1.000]^T\).

To design the GMD, the same value of \(F_{fmax} = 57.22 \text{ kN}\) found in Chapter 4 is considered. Using Eq. 5-15 for the corresponding friction force ratio of 0.106, the equivalent viscous damping ratio for the GMD was determined as 0.31%. To design the other properties of the GMD, design equations from Sugimura et al. (Sugimura et al., 2012) are used. Based on their proposed equations...
for MDOF structures, Eq. 5-43 can be used for each mode to relate the value of the equivalent mass ratio to the equivalent viscous damping ratio of the GMD in the same mode (e.g. first mode):

$$\xi_{1}^{opt} = \left(\frac{1}{4}\right) \sqrt{3(1 - \sqrt{1 - 4\mu_1})}$$  \hspace{1cm} (5-43)

Figure 5-25. Comparison between the models of a 3DOF system with TMDs and GMD-brace systems

Setting Eq. 5-43 to 0.31% and choosing the first mode of vibration for the design, leads to an equivalent mass ratio of 0.26%. It can be noted that this value is significantly smaller than the
equivalent mass ratio for the SDOF system in the previous section. Since the equivalent mass ratio is defined for each mode, the following equations should be employed to find the equivalent mass \( b \) of the GMDs for each floor:

\[
\mu_1 = \frac{\bar{B}_1}{\bar{M}_1} \tag{5-44}
\]

where,

\[
\bar{M}_1 = \sum_{i=1}^{N} \left\{ M_i \left( \Gamma_1^i \phi_1^i \right)^2 \right\} \tag{5-45}
\]

\[
\bar{B}_1 = b_1 \left( \Gamma_1^1 \phi_1^1 \right)^2 + \sum_{i=2}^{N} \left\{ b_i \left( \Gamma_1^i \left[ \phi_1^i - \phi_1^{(i-1)} \right] \right)^2 \right\} \tag{5-46}
\]

and \( \phi_1 \) show the mode shape for the first mode and \( \Gamma_1 \) is the modal participation factor for the first mode. This parameter is defined as,

\[
\Gamma_1 = \frac{\phi_1^T[M]r}{\phi_1^T[M]\phi_1} \tag{5-47}
\]

In Eq. 5-47, \( r \) shows the influence vector of the structure.

Using an iterative procedure for finding the value of \( b \) from Eq. 5-44, the equivalent mass of the typical GMDs is determined to be 250 kg for the particular 3DOF structure considered in this section. The frequency ratio of the braces in the GMD-brace system can also be calculated from the following equation from (Sugimura et al., 2012):

\[
\beta_1 = \left( \frac{1}{4\mu_1} \right) \left[ 1 - \sqrt{1 - 4\mu_1} \right] \tag{5-48}
\]

which can be used to determine the typical stiffness of these braces,

\[
k_{bi} = b_i (\beta_1 \omega_1)^2 \tag{5-49}
\]

Substituting the first natural frequency of 11.3260 rad/sec in the above equation leads to \( k_b \) of 32.15 kN/m. The designed system is subjected to harmonic loads and the effectiveness of the stand-alone GMD (linear and nonlinear modes) and the GMD-brace system in reducing the maximum displacements and accelerations are shown in Fig. 5-26 and Fig. 5-27.
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**Figure 5-26.** Frequency response for different control systems (Maximum displacements)

**Figure 5-27.** Frequency response for different control systems (Maximum accelerations)
The enlarged windows of the plots in Fig. 5-27 for the first-floor accelerations are shown in Fig. 5-28 where the maximum accelerations of each of the control systems around the natural frequencies are compared to the peak acceleration of the uncontrolled MDOF system. A closer look at the responses around the first mode frequency shows that the control strategy with stand-alone GMDs is ineffective, whereas the GMD-brace system successfully reduces the peak accelerations. Still, from the difference between the separated peaks and the trough, it can be concluded that more research can be done in future to improve the design equations of 5-43 to 5-48 and obtain closer-to-optimal responses.

**Figure 5-28.** Enlarged frequency response of 1st floor for the maximum acceleration ($F_r = 0.106$)
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Figure 5-25 (Continued). Enlarged frequency response of 1st floor for the maximum acceleration ($F_r = 0.106$)

Also, from the diagrams for the higher mode frequencies, it can be seen that the GMD-brace system does not affect the response of the primary structure significantly. The stand-alone systems do not change the values of the peak responses, and they only shift the frequency at which the maximum accelerations occur.

Figure 5-29. Response of the original and controlled MDOF systems, (a) displacements, (b) acceleration (El Centro earthquake) ($F_r = 0.106$)

When the system is subjected to seismic loads, the performance of the control systems highly depends on the characteristic of the input ground motion as well the dynamic properties of the primary structure. Moreover, since the original structure is an MDOF system, depending on the design of the system, higher mode effects can play a major role in the responses. Moreover, as it
is shown in Fig. 5-29 maximum displacement or acceleration values may not be reliable performance indicators for the behaviour of these systems because for some regions (e.g. around the maximum peaks), they efficiently improve the response whereas for some other parts they make it worse.

**Figure 5-26 (Continued).** Response of the original and controlled MDOF systems, (a) displacements, (b) acceleration (El Centro earthquake) ($F_r = 0.106$)

**Figure 5-30.** Performance of the controlled systems subjected to 99 earthquake ground motions of Table E-1 (maximum responses for the MDOF primary structure)
Figure 5-31. Performance of the controlled systems subjected to 99 earthquake ground motions of Table E-1 (RMS of the responses for the MDOF primary structure)

The results for the maximum and RMS values of the displacements and acceleration responses of the sample MDOF system are plotted in Fig. 5-29. As it can be seen in these figures, the performance of the designed GMD-brace systems is not as good as their performance for the SDOF structures.

It should be emphasized here that the results of Fig. 5-30 and 5-31 do not necessarily mean that GMDs are ineffective for MDOF system. However, they underline the need for further research on the optimal design of these systems. This is particularly important for low damping GMD-brace systems like the system in this study because the design equations of Ikago et al. (Ikago et al., 2011) and Sugimura et al. (Sugimura et al., 2012) were developed for TVMDs which by design have large viscous damping ratios.
To address the need for a comprehensive study on the performance of the rack and pinion inertial dampers, in this dissertation, GMDs were numerically and experimentally modelled, and the potential benefits of their incorporation into structures as passive control systems were evaluated. Using first principles, two models were proposed to calculate the resisting force of a GMD and to determine its equivalent mass $b$. These new models make it possible to use the free-body-diagram of the device and calculate the values of $b$ with an acceptable level of accuracy. To verify these models, a small-scale prototype was built and tested. The experimental results indicated that the friction forces in these devices play an important role and larger prototypes should be tested to investigate the size effects on the contribution of friction forces to the resisting forces. The characterization tests on the larger-scale prototype confirmed the findings of the experiments on the smaller prototype specimen and showed the dependency of the resisting forces on the applied relative velocity and acceleration at GMD’s end terminals. Due to the rate-dependency of the forces, a proper dynamic testing technique, namely RTHS, was chosen to study its effects in an SDOF system and examine the effects of the input amplitude and frequency on the effectiveness of the device. The hybrid simulation made it possible to introduce several different mass, stiffness and damping terms to the analytical SDOF systems and conduct several experiments on these systems with large masses (i.e., tens of tons) or with different natural frequencies without the need to use these elements physically. Three different sets of error indicators were then developed in this study that could be used as standard RTHS error monitors. It was shown that these error measures could decouple various sources of errors and are not test- or structure-specific. These error indicators were used to assess the stability of the tests. The analysis of the RTHS test results performed on the large scale prototype specimen with various values of equivalent mass and subjected to harmonic, chirp and earthquake excitation inputs, showed that the linear models for describing the damper force are not accurate. RTHS’s with a suite of six different earthquakes were carried out and the experimental data verified that the inclusion of the friction forces in the simplified nonlinear model leads to a solid representation of the device behaviour. The RTHS results also showed that the introduction of the GMD to the SDOF system consistently enhances
the performance of the system. It was also demonstrated that it is possible to improve the efficiency of the control system further by employing an energy dissipation device in parallel with the GMD. The identified values of the dynamic properties of the GMDs from the experiments of Chapter 4 were then used to study the impact of the use of GMDs in SDOF and MDOF structures. It was shown that stand-alone linear GMDs elongate the fundamental vibrational period of the structures and however, at the same time, they reduce the equivalent damping of the system. To address this drawback and to provide a more practical solution for incorporating GMDs into the real structure, these dampers were used in parallel with V-braces. The application of GMDs in GMD-brace configurations leads to new equations of motion that resemble the equations of motion of TMDs. Similar to TMDs, GMD-brace control systems are efficient in reducing the response of the primary structure around the resonance frequency. To investigate the effects of the uncertainties of the applied earthquake ground motions, a large set of 99 historical ground motion records were used and time history analyses were performed. The results of the simulations indicated that the nonlinear stand-alone GMD could reduce the accelerations for primary structures with short natural periods. The application of nonlinear GMDs-brace systems can also effectively improve the seismic performance of the primary structure in general.

Some of the other key findings of this study are listed below:

1) The available models in the literature for estimating the equivalent mass of a GMD are simplistic and cannot predict this parameter consistently. The proposed models in this study, on the other hand, account for the dynamic effects of the intermediate gears and makes it available to calculate $b$ for different gear configurations in GMDs. In particular, the predicted resisting forces from the “lumped mass model” agree well with the experimental results. It is shown in these models that the equivalent mass of a GMD can be adjusted by changing the arrangement of the gears and pinions as well as the properties of the gears. For instance, a prototype was built and tested in this study that had a constant physical mass of 125.7 kg but depending on the arrangement of its gears could produce equivalent masses $b$ of 68 kg, 1380 kg, and 4452 kg. Some types of the available dampers have fixed parameters and are typically tuned to a particular disturbance or excitation frequency. However, the design of the GMD allows changing the value of $b$ relatively easily.
2) The characterization tests on two GMD prototype specimens with different size and equivalent masses verified the accuracy of the lumped mass model in predicting the value of the equivalent mass of the tested GMD prototypes. This was confirmed by comparing the major axis slopes of the hysteresis loops with the predicted values. The results also showed that the resisting force in GMDs is amplitude and frequency dependent and that the effects of friction forces have to be considered. Due to the inherent randomness of the seismic loads, the dependency of the friction forces and as a result the resisting forces to the input parameter makes it challenging to design the GMDs for earthquakes. Although it was shown that these forces could be accurately predicted by complicated mathematical models, simplifying assumptions were made to obtain simpler models that can be used for design purposes. As a result, a simplified nonlinear model was proposed in which a conservative constant friction force was determined for each GMD prototype which does not change for various values of input frequency and amplitude. Different earthquakes with various attributes were used in RTHS on the SDOF systems with the GMD prototype, and it was demonstrated that the simplified nonlinear model could reliably predict the behaviour of the GMD devices. Despite the observed errors after peak responses of the structures, the simplified nonlinear model is simple to use and can estimate the hysteresis loops of the GMDs relatively well.

3) It was shown through experiments that with the addition of an energy dissipation element to a GMD, it is possible to enhance the performance of the control system and obtain a response that is superior to the response of the control with only the energy dissipation device. The advantage of GMDs over other inertial dampers such as TVMDs is that the energy dissipation component does not have to be inside the damper and be used in parallel with the GMD. The external application of the energy dissipation device makes it possible to increase the number of available options for this component.

4) Although stand-alone GMDs have the potential to enhance the seismic performance of structures, due to geometric limitations and alignment issues, they could not be introduced into structures as stand-alone devices. Their installation through braces was shown to be problematic for ideal GMDs with no friction forces. However, the introduction of nonlinear models in GMD-brace-SDOF system was shown to be effective. Optimal design equations for these systems were re-defined, and it has been demonstrated that these optimally designed
systems can improve the behaviour of the uncontrolled primary structure by up to 70% when the SDOF system is subjected to harmonic loads.

5) The seismic performance of GMD-brace systems was studied by performing time history analysis for 99 different historical earthquake ground motion records, and these control systems can improve the seismic performance of the primary structure in general.

6) Numerical and experimental results showed that the developed tracking error indicators could successfully quantify the RTHS errors. These error indicators could be reliably used for real-time assessment of RTHS and can also be used as a post-processing tool for the evaluation of the accuracy of the test results. These error indicators were employed together with the results of the stability analysis for the testing setup, and it was demonstrated that for the expected level of friction forces, the RTHS was stable.

To build larger GMD devices that can produce larger resisting forces, wider gears with a larger number of teeth and larger pressure angles should be used. In addition, it is preferable to use the heat treated steel to have stronger gear teeth.

For future studies, the effectiveness of GMDs in MDOF systems need to be explored in more detail, and proper optimization techniques can be employed to find the optimal distribution of these devices over the height of the structure. RTHS can also be used for these systems to experimentally compare various design methods and investigate the accuracy of modelling assumptions and the effects of this control system on higher modes. Also, different energy dissipation devices can be used in parallel with the GMDs to improve their effectiveness. These devices can also be used for wind control application to assure occupants’ comfort and their beneficial effects in reducing wind-induced vibrations under extreme wind loads can be evaluated.

GMDs in this study have been considered as passive control devices. However, by incorporating sensors and feedback systems into the damper, GMDs can, in principle, be converted to semi-active or active systems that use the rotation of the last gears to control the amplified inertial forces and produce more efficient resisting forces than possible in a purely passive mode. Finally, the performance of GMDs in nonlinear structures should be assessed, and the efficiency and robustness of these devices should be examined.


Ashasi-Sorkhabi A Implementation, verification and application of real-time hybrid simulation [Book]. - Toronto, Canada : PhD dissertation, Civil Engineering Department, University of Toronto, 2015.


References


References


A Modelling the acceleration-proportional GMD in OpenSees

While it is crucial to carry out various experiments on GMDs to establish a solid theoretical background for these new devices, their effectiveness in any experimental setting needs to be investigated by using different numerical simulation techniques. In the early stages of this study and to establish a simulation framework for RTHS testing setup, a new model was developed on the OpenSees platform to represent the behaviour of a GMD.

OpenSees (Open System for Earthquake Engineering Simulation) (McKenna et al., 2000), is an object-oriented, open source software application. Running simulations on OpenSees not only saves strenuous efforts of simultaneously solving large systems of differential equations, but also makes it possible to numerically model a GMD with together with a combination of other materials and elements, such as non-linear material models, with relative ease. In this section, using C++ programming language, a new “uniaxial material” is defined (as defined in the OpenSees Manual) that can represent the liner acceleration-proportional models for GMDs.

A.1 Modelling the GMD material in OpenSees

The developed OpenSees model for a GMD in this appendix is based on Eq. A-1 in which the resisting GMD force $f$, is proportional to the applied end accelerations $\ddot{u}$:

$$ f(t) = b \times \ddot{u}(t) $$

(A-1)

where, $b$ is the equivalent mass and can be determined from any of the linear models in Chapter 2. As it can be seen in this equation, the effects of friction forces on the behaviour of the device are neglected in those models.

Before the development of this OpenSees material for GMD devices, unlike displacements and velocities, there was no built-in function in OpenSees to obtain the local acceleration values at the
ends of different elements. To address this problem, at each time step, the approximation from the Newmark-Beta was used to calculate these accelerations. These equations can be summarized by the following expressions (Chopra, 2011):

\[
\dot{u}_{i+1} = \dot{u}_i + [(1 - \gamma_{nm})\Delta t] \ddot{u}_i + (\gamma_{nm}\Delta t)\ddot{u}_{i+1} 
\]

\[
u_{i+1} = u_i + \Delta t \ddot{u}_i + [(0.5 - \beta_{nm})\Delta t^2] \dot{u}_i + (\beta_{nm}\Delta t^2) \dddot{u}_{i+1} 
\]

where, \(\Delta t\) is the integration time step; \(u_i, \dot{u}_i, \ddot{u}_i\) are the displacement, velocity, and acceleration of the current step and \(u_{i+1}, \dot{u}_{i+1}, \ddot{u}_{i+1}\) are the displacement, velocity, and acceleration of the next step. To obtain an explicit expression for acceleration, a non-iterative formulation using incremental quantities are used, which can be defined as:

\[
\Delta u_i = u_{i+1} - u_i 
\]

\[
\Delta \dot{u}_i = \dot{u}_{i+1} - \dot{u}_i 
\]

\[
\Delta \ddot{u}_i = \ddot{u}_{i+1} - \ddot{u}_i 
\]

Substituting Eq. A-4, A-5 and A-6 into Eq. A-2 and A-3 and solving for \(\Delta \dot{u}_i\) and \(\Delta \ddot{u}_i\), the incremental velocity and acceleration can be expressed as:

\[
\Delta \dot{u}_i = (\gamma_{nm}/\beta_{nm}\Delta t)\Delta u_i - (\gamma_{nm}/\beta_{nm}) \dot{u}_i + \Delta t[1 - (\gamma_{nm}/2\beta_{nm})] \ddot{u}_i 
\]

\[
\Delta \ddot{u}_i = (1/\beta_{nm}\Delta t^2)\Delta u_i - (1/\beta_{nm}\Delta t) \dot{u}_i - (1/2\beta_{nm}) \ddot{u}_i 
\]

Using these expressions, a new material was created in OpenSees. For every introduced equivalent mass values for this material in the corresponding Tcl file, forces are calculated that are consistent with the device force definition in Eq. A-1. Next section outlines the most important sections of the developed C++ source code that is developed for the GMD material.

### A.2 Source code

The new material representing the GMD is called “GyroMassMaterial” and is defined in the header file under the class UniaxialMaterial. A short description of each is included after the “//” (comment) symbol and should be fairly self-explanatory.
class GyroMassMaterial : public UniaxialMaterial
{
    public:
        ...
        // function that models the Newmark approximation of acceleration
        int newmark(double gama, double beta, double dt, double trialStrain, double commitStrain, double &trialStrainRate, double commitStrainRate, double &trialAccel, double commitAccel);
        ...
    protected:
    private:
        double trialStrain; // displacement at next step
        double trialStrainRate; // velocity at next step
        double trialAccel; // acceleration at next step
        double trialStress; // inertial force at next step
        double b; // equivalent mass provided for the GMD
        double currentTime; // time at the next step
        double commitStrain; // displacement at the current step
        double commitStrainRate; // velocity at current step
        double commitAccel; // acceleration at current step
        double commitStress; // inertial force at current step
        double commitTime; // time at the current step
};

The setTrialStrain method defines how the solution algorithm is updated. The inertial force is defined in this method as “trialStress = b*trialAccel”, following the definition of Eq. A-1.

int GyroMassMaterial::setTrialStrain(double strain, double strainRate)
{
    double dt;
    currentTime = ops_TheActiveDomain->getCurrentTime();
    dt = currentTime - commitTime;
    trialStrain = strain;
    newmark(0.5, 0.25, dt, trialStrain, commitStrain, trialStrainRate, commitStrainRate, trialAccel, commitAccel);
    trialStress = b*trialAccel;
    return 0;
}
Then the `getDampTangent` section returns the damping tangent of the material. Determining the damping tangent for a GMD is based on a trial and error process.

```cpp
double GyroMassMaterial::getDampTangent(void) {
    double dt;
    currentTime = ops_TheActiveDomain->getCurrentTime();
    dt = currentTime - commitTime;
    return 2*b/dt;
}
```

Finally, Eq. A-5 and A-6 are used to gives an approximation of the velocity and acceleration of the next step:

```cpp
int GyroMassMaterial::newmark(double gama, double beta, double dt, double trialStrain, double commitStrain, double &trialStrainRate, double commitStrainRate, double &trialAccel, double commitAccel) {
    double dDis, dVel, dAccel = 0;
    dDis = trialStrain - commitStrain;
    dVel = gama * dDis / (beta*dt) - gama*commitStrainRate / beta + commitAccel*dt*(1 - gama / (2 * beta));
    dAccel = dDis / (beta*pow(dt, 2)) - commitStrainRate / (beta*dt) - commitAccel / (2 * beta);
    trialStrainRate = commitStrainRate + dVel;
    trialAccel = commitAccel + dAccel;
    return 0;
}
```

### A.3 Verification of the model

The results from the simulations with the new material are compared with outputs from Simulink models under harmonic and seismic loads to verify the accuracy of the new OpenSees material.

Fig. A.1 shows the time-displacement response of an SDOF system with GMD under a sinusoidal input using both the OpenSees model (dashed black line) and Simulink model (blue line). It evident that both align closely with each other. The response of the system without the GMD is also shown.
in the red curve. It can be observed that the GMD indeed has the effect of reducing the amplitude of the displacement response.

Fig. A-2 also illustrates the time-displacement response of an SDOF system with a GMD under the El Centro ground acceleration using both the OpenSees model (dashed black line) and the Simulink model (blue line). As it can be seen in this figure, both curves are also in close agreement.

![Graph 1](image1.png)

**Figure A-1.** Response of a SDOF system with/without a GMD under a harmonic input
Appendix A. Modelling the acceleration-proportional GMD in OpenSees

Figure A-2. Response of the SDOF system with a GMD under El Centro ground motion

A.4 Summary

In this appendix, details of creating a new OpenSees material for modelling acceleration-proportional GMDs is explained, and the associated C++ script file is shown. It should be emphasized that due to the simplicity of the Simulink model, most of the numerical simulations in this study have been carried out with the Simulink model. However, this new material in OpenSees...
has the potential to be used in future works. Furthermore, improvements to the script code should be explored in future. The current material is only compatible with Newmark’s constant average stepping methods. Therefore, incorporating more efficient approaches for estimating the current element accelerations as well as better ways of determining the damping tangent should be the primary focus of future studies on this new OpenSees material.
The computer code utilized for calculation of PAEI error indicators introduced in Chapter 3 is written in MATLAB. To demonstrate different parts of the MATLAB code better, the corresponding script files for the determination of PAEI as a *post-processing* tool are introduced in this appendix. The script files are commented in a way, which should make clear to anyone familiar with the MATLAB operating language what process was employed to analyze data. However, it should not be expected to copy-paste these files into MATLAB and have them work as-is and depending on the application, each parameter of Table 3-1 should be defined properly.

- **Main.m**: As it is indicated in the title of this function, this is the main file that reads the raw input command and measured data, calls other subroutines and returns the corresponding phase and amplitude error results. In this function, first, the main parameters of Table 3-1 such as CENT, and thresholds for selecting a proper number of data points in each time window (i.e., LENI, MINA and MAXA) are defined. Then, as it is shown in Fig. 3-8, the *FirstNum.m* subroutine should be run for the very first time window to determine FINI. After that, the procedure is repeated for the rest of the data points. In each iteration, the parameters of the fitted ellipse are determined in the *FitEllipse.m* subroutine. After finalizing the size of each time window, the *FitEllipse.m* subroutine is called for the last time for each time window, and the coefficients are sent to the *ErrorIndices.m* subroutine. In the *ErrorIndices.m* subroutine, the phase and amplitude error values are determined. Based on the definitions of Section 3.3.2.1.5 and using the *Angle.m* subfunction, the correct sign of the phase error index is assigned in the *Main.m* subroutine. Finally, the calculated values of PAEI are assigned to the index of the data point at the middle of the time window and are stored in a matrix.

- **FirstNum.m**: This subroutine returns the number of data points which are necessary for the very first moving time-window (i.e., FINI). Unlike the main part of the algorithm where the
Appendix B. MATLAB code for the calculation of PAEI

counter starts with the last point and counts backward, in this subfunction, the counter starts from 1 and keeps increasing the number of the data points until the convergence criteria are met.

- **FitEllipse.m:** As explained in Section 3.3.2.1.3, the raw input data is scaled and centred in this file and then using the least square error method, the coefficient of the fitted ellipse (Eq. 3-44 to Eq. 3-49) as well as the location of the centre of the ellipse and its radii are determined.

- **ErrorIndices.m:** Based on the values of the fitted ellipse, this subroutine uses the Eq. 3-27 and 3-30 and evaluates the phase and amplitude error indices.

- **Angle.m:** The total angle between the first and last point of each time window is calculated in this subroutine, and special care is made to remove the spurious $2\pi$ phase shifts.

**B.1 Main.m**

```matlab
function Z = Main(dc,dm)
% "PAEI: Phase and Amplitude Error Indices"
% -----------------------------------------

% Clearing Figures and Windows
clc; clf; clear all; close all;

X = dc; % Command/Target displacements in mm
Y = dm; % Measured displacements in mm
timestep = 1/1024; % NTOT
N = length(X);
t = 0:timestep:N*timestep;
Filter = 0; % Set it to 1 for filtering the data

% Filtering the data
if (Filter == 1)
    [a,b] = butter(4,60/1000);
    xfil = filter(a,b,X);
    yfil = filter(a,b,Y);
    X = xfil; Y = yfil;
end

% Convergence criteria

FN = FirstNum(X,Y); % Calling FirstNum.m function
localerr = 1024;
p = min(localerr,FN); % FINI
```

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totalerror = 10^(-6); % CENT
minimumangle = 180; % MINA (in deg)
maximumangle = 360; % MAXA (in deg)
initialN = 50; % LENI
NextInterval = 1;
% Main Loop
k = 0;
while (p < N)
    for i = 1:initialN
        x(i) = X(p-i);
        y(i) = Y(p-i);
    end
    error = 1;
    j = 1;
    while (error>totalerror)
        if ((p-initialN-j)<1)
            break;
        end
        x(initialN+j) = X(p-initialN-j);
        y(initialN+j) = Y(p-initialN-j);
        FitEl = FitEllipse(x,y);
        param = FitEl(2,:);
        if (j==1)
            dist0 = sqrt(xcen^2+ycen^2);
        end
        xcen = param(1);
        ycen = param(2);
        dist = sqrt(xcen^2+ycen^2);
        if (j==1)
            error = abs(dist0-dist);
        end
        j = j+1;
        if (j > localerr)
            break;
        end
    end
    teta = 0;
    while (abs(teta) < minimumangle)
        teta = Angle(x,y,xcen,ycen);
        if ((p-initialN-j)<1)
            break;
        end
        x(initialN+j) = X(p-initialN-j);
        y(initialN+j) = Y(p-initialN-j);
        j = j+1;
        if (abs(teta) > maximumangle)
            break;
        end
        if (j > localerr)
            break;
        end
    end

    % Determining the Sign of the Phase Error
    if (teta>0)
        Phasesign = 1;
    end
end
elseif (teta < 0)
    Phasesign = -1;
elseif (teta == 0)
    Phasesign = 0;
end

% Finializing the Fitting
FitEl = FitEllipse(x, y);
coef = FitEl(1,:);

% Determining Phase and Amplitude Error Indices
EI = ErrorIndices(coef, Phasesign); % Calling ErrorIndices.m function
PE = EI(1);
AE = EI(2);
k = k+1;
solution(k,:) = [p, AE, PE, teta, j];
p = p+NextInterval;
p;
clear x; clear y;
end

for i=1:k
    mid = floor((2*solution(i,1)-solution(i,5))/2);
    msolution(i,:) = [mid, solution(i,2), solution(i,3)];
end

midsolution = sortrows(msolution);

Z = midsolution;

### B.2 FirstNum.m

```matlab
function Z = FirstNum(X,Y)
    N = length(X);
    initial = 50;

    for i = 1:initialN
        x(i) = X(i);
        y(i) = Y(i);
    end
    error = 1;
    j = 1;
    while (error>(0.5*10^(-6)))
        if ((initialN+j)>N)
            break;
        end
        x(initialN+j) = X(initialN+j);
        y(initialN+j) = Y(initialN+j);
        xfe = x; yfe = y;
        FitEl = FitEllipse(xfe, yfe);
        Param = FitEl(2,:);
        if (j~=1)
            dist0=sqrt(xcen^2+ycen^2);
        end
    end
end
```
end
xcen = param(1);
ycen = param(2);
dist = sqrt(xcen^2+ycen^2);
if (j~=1)
    error = abs(dist0-dist);
end
j=j+1;
end
teta=0;
while (abs(teta)<180)
teta = Angle(x,y,xcen,ycen);
if ((initialN+j)>N)
    break;
end
x(initialN+j) = X(initialN+j);
y(initialN+j) = Y(initialN+j);
j = j+1;
if (abs(teta)>360)
    break;
end
end
FINI = initialN+j;

Z = FINI;

B.3 FitEllipse.m

function Z = FitEllipse(xfe,yfe)
% 1. Normalizing data
mx = mean(xfe);
my = mean(yfe);
sx = (max(xfe)-min(xfe))/2;
sy = (max(yfe)-min(yfe))/2;
xf = (xfe-mx)/sx;
yf = (yfe-my)/sy;

% 2. Converting them to column vectors
xf = xf(:);
yf = yf(:);

% 3. Building design matrix
D = [xf.*xf  xf.*yf  yf.*yf  xf  yf  ones(size(xf))];

% 4. Building scatter matrix
S = D'*D;

% 5. Building 6x6 constraint matrix
C = zeros(6); C(1,3) = -2; C(2,2) = 1; C(3,1) = -2;

% 6. Solving eigensystem
% Break into blocks
    tmpA = S(1:3,1:3);
tmpB = S(1:3,4:6);

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```matlab
tmpC = S(4:6,4:6);
tmpD = C(1:3,1:3);
tmpE = inv(tmpC)*tmpB';
[evec_x, eval_x] = eig(inv(tmpD) * (tmpA - tmpB*tmpE));

% Find the positive (as det(tmpD) < 0) eigenvalue
I = real(diag(eval_x)) < 1e-8 & ~isinf(diag(eval_x));
% Extract eigenvector corresponding to negative eigenvalue
A = real(evec_x(:,I));
% Recover the bottom half...
evec_y = -tmpE * A;
A = [A; evec_y];

% 7- Un-normalizing
par = [
    A(1)*sy*sy, ...
    A(2)*sx*sy, ...
    A(3)*sx*sx, ...
    -2*A(1)*sy*sy*mx - A(2)*sx*sy*my + A(4)*sx*sy*sy, ...
    -A(2)*sx*sy*mx - 2*A(3)*sx*sy*my + A(5)*sx*sx*sy, ...
    A(1)*sy*sy*mx*mx + A(2)*sx*sy*mx*my + A(3)*sx*sx*my*my ...
    - A(4)*sx*sy*sy*mx - A(5)*sx*sx*sy*my ...
    + A(6)*sx*sx*sy*sy ...
]';

% 8- Limiting the answer to a unique solution
c = 1 / par(1);
coef(1,:) = c.*par;

% 9- Computing the properties of the fitted ellipse
thetarad = 0.5*atan2(par(2),par(1) - par(3));
cost = cos(thetarad);
sint = sin(thetarad);
sin_squared = sint.*sint;
cos_squared = cost.*cost;
cos_sin = sint.*cost;

Ao = par(6);
Au = par(4) .* cost + par(5) .* sint;
Av = - par(4) .* sint + par(5) .* cost;
Auu = par(1).*cos_squared+par(3).* sin_squared+par(2).*cos_sin;
Avv = par(1).* sin_squared+par(3).* cos_squared-par(2).* cos_sin;

tuCentre = - Au./(2.*Auu);
tvCentre = - Av./(2.*Avv);
wCentre = Ao - Auu.*tuCentre.*tuCentre - Avv.*tvCentre.*tvCentre;
uCentre = tuCentre .* cost - tvCentre .* sint;
vCentre = tuCentre .* sint + tvCentre .* cost;
Ru = -wCentre./Au;
Rv = -wCentre./Avv;
Ru = sqrt(abs(Ru)).*sign(Ru);
Rv = sqrt(abs(Rv)).*sign(Rv);

coef(2,:) = [uCentre, vCentre, Ru, Rv, thetarad, 0];
```

Application of Real-Time Hybrid Simulation Method in Experimental Identification of GMDs
Appendix B. MATLAB code for the calculation of PAEI

Z = coef;

B.4 ErrorIndices.m

function Z = ErrorIndices(coef,phasesign)
    % 1. Selecting the important coefficients of the fitted ellipse
    bco = coef(2);
    cco = coef(3);
    fco = coef(6);
    % 2- Computing the Phase and Amplitude Error Indices
    phi = abs(acos(-bco/(2*(cco)^0.5)));
    Ac = (abs(-fco/(sin(phi))^2))^0.5;
    Am = (abs(Ac^2/cco))^0.5;
    if (phasesign<0)
        PE = -phi;
    else
        PE = phi;
    end
    AE = Ac-Am;
    Z = [PE,AE];

B.5 Angle.m

function Z = Angle(x, y, xcen, ycen)
    N=length(x);
    xs =(x-xcen);
    ys =(y-ycen);
    % 2. Converting them to column vectors
    xs = xs(:);
    ys = ys(:);
    % 3. Determining the quadrants of the points
    for i=1:N
        if (xs(i)>0)
            if (ys(i)>0)
                qua(i)=1;
            elseif (ys(i)<0)
                qua(i)=4;
            end
        elseif (xs(i)<0)
            if (ys(i)>0)
                qua(i)=2;
            elseif (ys(i)<0)
                qua(i)=3;
            end
        end
    end
    if (xs(i)==0)
        if (ys(i)>0)
Appendix B. MATLAB code for the calculation of PAEI

```matlab
qua(i)=1;
elseif (ys(i)<0)
    qua(i)=4;
end
end
if (ys(i)==0)
    if (xs(i)>0)
        qua(i)=1;
    elseif (xs(i)<0)
        qua(i)=2;
    elseif (xs(i)==0)
        qua(i)=0;
    end
end
end
% 4. Determining the angel between the x axis and each point
for i=1:N
    if qua(i)==0
        teta0(i)=0;
        teta(i)=0;
    else
        teta0(i)=atan(ys(i)/xs(i));
    end
    if ((qua(i)==2)||(qua(i)==3))
        teta(i)=pi+teta0(i);
    elseif (qua(i)==4)
        teta(i)=teta0(i)+2*pi;
    elseif (qua(i)==1)
        teta(i)=teta0(i);
    end
end
teta=(180/pi)*teta;

% 5. Determining the angel between two consequent points
for i=1:(N-1)
    if {(qua(i)==2)||(qua(i)==3)}
        Dteta(i+1)=teta(i+1)-(teta(i)-(360));
    elseif (qua(i)==4)
        teta(i)=teta0(i)+2*pi;
    elseif (qua(i)==1)
        teta(i)=teta0(i);
    end
end
% 6. Calculating the summation of the angles
toteta(1) = 0;
for i=2:N
    toteta(i)=Dteta(i)+toteta(i-1);
end
Z= toteta(N);
```
Appendix C. MATLAB code for the calculation of FDB error indicators

C MATLAB code for the calculation of FDB error indicators

Similar to the PAEI error indicators in Appendix B, the calculation of the FDB error indicators was carried out in MATLAB. Once more, to demonstrate different parts of the MATLAB code better, the corresponding script files for the determination of FDB as a post-processing tool are introduced in this appendix. It should be noted that these error indicators have also been implemented in LabVIEW and details of their implementation in RTHS can be found in (Mirza Hessabi et al., 2016) and (Ashasi-Sorkhabi et al., 2014).

- **Main.m:** In comparison to PAEI error indices, the main function for the computation of FDB error indicators is much shorter. In this function, raw input command and measured data are read, and the other subfunction (i.e., *FFT_func.m*) is called. Similar to PAEI, a moving time-window is used for the calculation of these indicators. As a result, the first step in the *Main.m* function is to set a proper value for the size of the time window. Because of the time windowing technique and to prevent spectral leakage effects, a suitable windowing function should also be introduced. In this study, Hamming windows are pre-multiplied by both the measured and command displacements in each time window. After the preparation of the input data, FDB error indicators are determined in the *FFT_func.m* subroutine. Proper care is made to remove the spurious $2\pi$ phase shifts. Similar to the code in Appendix B, the calculated values of the error monitors are assigned to the index of the data point in the middle of the corresponding time window. The procedure continues until the last data point is reached.

- **FFT_func.m:** In this subroutine, first the data points are centred and then the FFT of each signal is computed. Based on the maximum absolute value of each of these FFT functions and the corresponding frequency, Eq. 3-81 and Eq. 3-82 are used and FDB error indicators are determined.
Appendix C. MATLAB code for the calculation of FDB error indicators

C.1 Main.m

function Z = Main(dc, dm)
    % "FDB Error Indicators
    % -----------------------------------------
    % Clearing Figures and Windows
    clc; clf; clear all; close all;

    X = dc(1,:); % Command/Target displacements in mm
    Y = dm(1,:); % Measured displacements in mm

    Ts = 1/1024; % Time step
    N = length(X);
    totime = N*Ts; % Total time
    t = 0:Ts:(totime-Ts);

    windowsize = 1024;

    L = windowsize+1;
    resulth = zeros(length(t),3);

    for i=1:length(t)
        if i > windowsize
            x = dc(i-windowsize:i);
            y = dm(i-windowsize:i);
            Xh = x'.*hamming(L); % Using the Hamming window
            Yh = y'.*hamming(L);
            temp = FFT_func(Xh,Yh); % Calling the FFT_func.m subroutine
            resulth(i,1) = t(i);
            resulth(i,2) = temp(1); % Amplitude error
            resulth(i,3) = temp(2);
            if temp(2)>=0
                if temp(2)>(1.5*pi()) && temp(2)<(2*pi())
                    resulth(i,3) = temp(2)-2*pi(); % Phase error
                end
            elseif temp(2)<0
                if temp(2)<(-1.5*pi()) && temp(2)>(-2*pi())
                    resulth(i,3) = temp(2)+2*pi(); % Phase error
                end
            end
        end
    end
    clear x y Xh Yh temp
    disp(i)
end

Z(:,1) = resulth(:,1);
Z(:,2) = resulth(:,2);
Z(:,3) = resulth(:,3);

Z = midsolution;
C.2 FFT_func.m

```matlab
function Z = fft_func(x,y)
    % Centre the data points
    x = x - mean(x);
    y = y - mean(y);

    % take the FFT
    X = fft(x);
    Y = fft(y);

    % Determine the max value and max point.
    [mag_x idx_x] = max(abs(X));
    [mag_y idx_y] = max(abs(Y));

    % determine the phase difference at the maximum point.
    px = angle(X(idx_x));  % in radians
    py = angle(Y(idx_x));
    phs_error = px - py;
    % determine the amplitude scaling
    amp_error = mag_x/mag_y;

    Z = [amp_error, phs_error, px, py];
```

Appendix C. MATLAB code for the calculation of FDB error indicators
Appendix D

Adaptive formulation of the 2DOF controller

In Chapter 3, a new set of tracking error indicators (i.e., FDB error indicators) are introduced which are able to uncouple the phase (lead/lag) and amplitude (overshoot/undershoot) errors between command and measured displacements and quantify them. Unlike their predecessors, the implementation of these new indicators is not computationally expensive and therefore they can be executed in real-time and serve as online indicators. As such they can be incorporated into the servo-hydraulic control law to improve the tracking of the command displacements by the single actuator system, which in turn will result in more accurate RTHS results. To accomplish this, FDB error indicators are introduced into a new 2DOF control system (Section 3.4.3).

The block diagram of RTHS that includes the adaptive 2DOF controller developed in this study is shown in Fig. D-1. In the outer loop, the equations of motion are solved by a step by step integration algorithm, and state determination computations are carried out for the analytical substructure. In the inner loop, a feedback controller imposes the command displacements during the specified time interval.

![Figure D-1. Block diagram of the actuator compensation using 2DOF controllers](image)

These two loops exist in the standard implementation of RTHS. Depending on the type (i.e., lead/lag/overshoot/undershoot) and magnitude of the tracking errors from the inner loop which are identified online by the FDB indicators at every moving time window, the resulting real-time
tuned, adaptive lead-lag compensator can take the necessary corrective actions and can improve the tracking of the command displacements by the hydraulic actuator. This is introduced in the adaptive loop in Fig. 1. The coefficients of the first order transfer function labeled as "Adaptive Compensator" in Fig. 1 are determined in such a way to eliminate the identified errors from the inner loop; in other words, to introduce $-\phi$ and $A^{-1}$.

A typical lead-lag compensator which is used in the 2DOF controller is shown in Fig. D-2. $X(s)$ and $Y(s)$ indicate Laplace transformations of the input and output of the system and $a_0$, $a_1$, $b_0$ and $b_1$ are the parameters of the controller and should be determined to provide the necessary performance.

![Block diagram of a lead-lag controllers](image)

**Figure D-2.** Block diagram of a lead-lag controllers

The block diagram shown in Fig. D-2, can be described by Eq. D-1 in the time domain:

$$a_0y(t) + a_1\frac{dy(t)}{dt} = b_0x(t) + b_1\frac{dx(t)}{dt} \quad (D-1)$$

Since the first order transfer function shown in Fig. 2 is linear, when the input $x(t)$ in time domain is sinusoidal as described in Eq. D-2 with amplitude $A_c$ and frequency $\omega$, the output $y(t)$ in steady state will be another sinusoid with the same frequency but a different amplitude $A_m$ and a phase shift $\phi$:

$$x(t) = A_c \sin(\omega t) \quad (D-2)$$
$$y(t) = A_m \sin(\omega t + \phi) \quad (D-3)$$

thus,

$$\frac{dx(t)}{dt} = A_c \omega \cos(\omega t) \quad (D-4)$$
$$\frac{dy(t)}{dt} = A_m \omega \cos(\omega t + \phi) \quad (D-5)$$
Appendix D. Adaptive formulation of the 2DOF controller

Upon substitution of Eq. D-2 to D-5 in Eq. D-1, the latter equation can be replaced by:

\[
[a_0 A_m \cos(\phi) - a_1 A_m \omega \sin(\phi)] \sin(\omega t) + [a_0 A_m \sin(\phi) + a_1 A_m \omega \cos(\phi)] \cos(\omega t) = b_0 A_c \sin(\omega t) + b_1 A_c \omega \cos(\omega t)
\] (D-6)

which yields,

\[
a_0 \cos(\phi) - a_1 \omega \sin(\phi) = \Delta b_0
\] (D-7)

\[
a_1 \omega \cos(\phi) + a_0 \sin(\phi) = \Delta b_1 \omega
\] (D-8)

where, \(\Delta A\) is defined by Eq. 3-74. This set of two equations and two unknowns can be solved for \((\Delta A) \cos(\phi)\) and \((\Delta A) \sin(\phi)\). The solution is shown in Eq. D-9 and D-10:

\[
\sin(\phi) = \frac{(a_0 b_1 - a_1 b_0)}{a_0^2 + \omega^2 a_1^2} \omega \Delta A
\] (D-9)

\[
\cos(\phi) = \frac{a_0 b_0 + \omega^2 a_1 b_1}{a_0^2 + \omega^2 a_1^2} \Delta A
\] (D-10)

By dividing Eq. D-9 and D-10, \(\phi\) can be expressed in terms of \(a_0, a_1, b_0, b_1\) and \(\omega\).

\[
\tan(\phi) = \frac{(a_0 b_1 - a_1 b_0)}{a_0 b_0 + \omega^2 a_1 b_1} \omega
\] (D-11)

It can be shown that,

\[
\Delta A = (a_0^2 + \omega^2 a_1^2) \times \{(a_0 b_0 + \omega^2 a_1 b_1)^2 + \omega^2 (a_0 b_1 - a_1 b_0)^2\}^{-1/2}
\] (D-12)

Using the Brahmagupta–Fibonacci identity, \(\Delta A\) can be simplified as,

\[
\Delta A = \sqrt{a_0^2 + \omega^2 a_1^2} / \sqrt{b_0^2 + \omega^2 b_1^2}
\] (D-13)

As can be seen, Eq. D-11 and D-13 establish a relationship between the coefficients of the lead/lag compensator and the amplitude ratio and phase shift specified at a frequency of \(\omega\). The adaptive formulation of the 2DOF controller developed in this appendix first uses the FDB error indicators to quantify the \(\Delta A\) and \(\phi\) between the command and measured signals from the inner loop.
Appendix D. Adaptive formulation of the 2DOF controller

(identified in Fig. D-1). The coefficients of the lead-lag compensator are determined in such a way to eliminate the detected errors from the inner loop; in other words, to introduce $-\phi$ and $\Delta A^{-1}$.

For any dynamic system to be stable the roots of the denominator polynomial of the transfer function (i.e., system poles) need to be on the left-hand side of the complex plane. As such, $a_0$ and $a_1$ are set to unity to ensure the stability of the adaptive compensator. In lead/lag compensator design, the relative location of the zeros (the roots of the numerator polynomial of the transfer function) and poles determine the lead/ lag characteristics. After fixing the pole location of the first order adaptive compensator, the zero location is determined from Eq. D-14 and Eq. D-15. Thus, the resulting adaptive compensator can provide the necessary lead/lag compensation as specified by $\phi$ and $\Delta A$ by the FDB error indicators:

\[
b_1 = \sqrt{\frac{(1/\Delta A)^2 (1+\omega^2)}{\left(\frac{\omega-\tan(\phi)\omega}{\omega+\tan(\phi)}\right)^2 + \omega^2}} \quad (D-14)
\]

and,

\[
b_0 = \frac{(\omega-\tan(\phi)\omega^2)}{\omega+\tan(\phi)} b_1 \quad (D-15)
\]

For each time window, a phase error and an amplitude ratio are estimated and based on their values adaptive compensator parameters are updated as described above.

More detail about the implementation of the 2DOF controller as well as numerical and experimental studies on the effectiveness of this controller can be found in (Mirza Hessabi et al., 2016).
Appendix E

Ground motions used in Analysis

A set of 99 typical ground motion records which are commonly used in earthquake engineering studies is used in this study. The application of these records are suggested in (Naeim et al., 1996) and (Liang et al., 2011), and Lee et al. (2007) used them to investigate the seismic performance of base isolation bearings.

Table E-1, shows the information about each of these records such as their particular attributes. In this table, PD, PV, and PA stand for large peak displacement, velocity, and acceleration, respectively. Also, large incremental velocity, and displacement are shown with IV and ID. By definition, the area under an acceleration pulse represents an incremental velocity and the area under a velocity pulse represents the incremental displacements. These two parameters can be used to show the existence of large velocity of displacement pulses in the accelerograms. Finally, EPA and EPV are used to describe earthquake ground motion records with high peak acceleration or velocities. If a record does not have any particular attribute, an asterisk is used in the last column.

Table E-1. Ground motion records used in this study

<table>
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<tr>
<th>No.</th>
<th>Year</th>
<th>Earthquake</th>
<th>Station</th>
<th>Deg</th>
<th>Attribute</th>
</tr>
</thead>
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<td>El Centro - Imp Vall Irr Dist</td>
<td>180</td>
<td>Duration</td>
</tr>
<tr>
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<td>El Centro - Imp Vall Irr Dist</td>
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<td>3</td>
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Table E-1 (Continued). Ground motion records used in this study

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### Table E-1 (Continued). Ground motion records used in this study

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### Table E-1 (Continued). Ground motion records used in this study

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