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A Semi-analytical Approach for Time-dependent Load-settlement Response of a Jacked Pile in Clay Strata

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**Abstract:** The mechanical behaviour of the soil around a jacked pile changes significantly during pile installation and subsequent consolidation. Hence an axially loaded jacked pile exhibits apparently time-dependent bearing performance after pile installation. This paper presents a semi-analytical approach to predict the time-dependent bearing performance of an axially loaded jacked pile in saturated clay strata. The effects of pile installation and subsequent consolidation on the changes in mechanical properties of the surrounding soil are modeled by the cavity expansion theory and the radial consolidation theory, respectively. An exponential function based load-transfer \((t-z)\) curve is employed to describe the nonlinear behaviour of the pile-soil interface during pile loading. The evolutions of the three dimensional strength and the shear modulus of the surrounding soil are subsequently incorporated into the two model parameters of the proposed \(t-z\) curve to capture the time-dependent pile-soil interaction behaviour. The time-dependent elastic response of the soil outside the pile-soil interface is also considered in the proposed approach. With the proposed load-transfer curve, an incremental algorithm and a corresponding computational code are developed for assessing the time-dependent load-settlement response of a jacked pile. To verify the proposed semi-analytical approach, predictions of the time-dependent load-settlement curves are compared with the measured values from pile tests at two sites. The good agreement shows that the time-dependent bearing performance can be reasonably predicted by the proposed approach.

**Key words:** jacked pile, time-dependent, bearing performance, load-transfer, nonlinear, pile-soil interaction
Introduction

When a pile is jacked into saturated clay strata, large deformations, large strains, and high excess pore water pressures are generated in the soil around the pile, which leads to remolding effects on the surrounding soil (Yang and Liang 2006; Abu-Farsakh et al. 2015). After pile installation, the surrounding disturbed soil consolidates with dissipation of the excess pore water pressures. As a result, the effective stress and the stress state related mechanical behaviours, such as the strength and the stiffness, of the disturbed soil recover and increase with the passage of time (Guo 2000). Because the load-settlement response of an axially loaded pile heavily depends on the stiffness and strength of the surrounding soil, a jacked pile in clay strata generally exhibits apparently time-dependent bearing performance after pile installation (referred to as “setup effect”). If the setup effect is properly considered, economical design of jacked piles can be achieved by reducing the pile length and diameter as well as the number of piles. However, most current design approaches approximately evaluate the load-settlement response of a jacked pile in clay from in-situ soil properties by ignoring setup effect, which may significantly underestimate the bearing performance of a jacked pile (Yang and Liang 2006). Although some design approaches determine the load-settlement response of a jacked pile in clay from pile loading tests, the bearing performance could still be underestimated if the pile loading test is conducted before full consolidation of the surrounding soils (Haque et al. 2014).

To date, many research efforts, ranging from field tests to theoretical studies, have
been performed to investigate the setup effect of a jacked pile in clay (e.g., Skov and Denver 1988; Lee et al. 2010; Ng et al. 2013a; Ng et al. 2013b; Basu et al. 2014). Although these studies presented various approaches for assessing the time-dependent bearing capacity of jacked piles in clayey soils, most of them only focused on the ultimate bearing capacity of the pile. It is well known that the bearing performance of a pile can be well reflected by the load-settlement response; while the ultimate bearing capacity is just one aspect of the bearing performance. Nonetheless, only a few of previous studies involved the time-dependent load-settlement response of a jacked pile in clay strata (Abu-Farsakh et al. 2015).

The current approaches for the pile load-settlement analysis can be generally divided into three broad categories (Ashour et al. 2010): (1) load-transfer methods; (2) boundary element methods; and (3) finite element methods. Although the finite element method can consider many complicated factors, such as the effects of pile installation and subsequent consolidation, its application is limited in practice due to its high computational requirements (Zhang et al. 2010). Compared with the latter two methods, the load-transfer method is more attractive in analyzing the load-settlement response for piles because of its simplicity and capability of incorporating nonlinear soil behaviour (Guo and Randolph 1997). The load-transfer method quantifies interaction between pile and soil through a series of independent springs distributed along the pile shaft and at the pile base. The key of this method is to develop a rational $t$-$z$ curve to model the pile-soil interaction behaviour. Generally, the function for the $t$-$z$ curve can be determined by either empirical approach or theoretical
analysis (Zhu and Chang 2002). Over the past few decades, the load-transfer method has been greatly improved by incorporating the soil stress history, the nonlinear soil behaviour, and the pile installation effects, etc., into the $t-z$ curve (e.g., Ashour et al. 2010; Roberts and Misra 2009; Nanda and Patra 2014; Sheil and McCabe 2016). Recently, Wang et al. (2012) further developed a load-transfer method by considering the elastic response of the soil outside the pile-soil interface. However, the evolution of the mechanical behaviour of the surrounding soil has not yet been incorporated into the $t-z$ curve to consider the time-dependent pile-soil interaction behaviour. Therefore, a general analytical approach for evaluating the time-dependent load-settlement response of a jacked pile is still not currently available.

The primary objective of this paper is to present a nonlinear $t-z$ curve which can properly describe the time-dependent pile-soil interaction behaviour. The ultimate desire is to develop a semi-analytical method to estimate the time-dependent load-settlement response of a jacked pile in clay strata. For this, the evolution of the mechanical behaviour of the surrounding soil is incorporated into an exponential function based nonlinear $t-z$ curve to account for the time-dependent pile-soil interaction behaviour. The in-situ stress history of the natural clay, the pile installation effect, and the three dimensional strength property of the surrounding soil, all of which have pronounced effects on the bearing performance of a jacked pile, are also reasonably incorporated into the proposed $t-z$ curve. An incremental algorithm is then proposed for capturing the time-dependent bearing performance of a jacked pile in clay strata. The validity of the proposed approach is examined by comparing the
predicted load-settlement curves with the data obtained from the pile field tests at two sites. The present approach provides a more rational way to predict the time-dependent load-settlement response of a jacked pile in clay strata, which has a great significance in economical design of jacked piles.

**Evolution of soil mechanical properties**

*Changes from pile installation effects*

The mechanical behaviour of soils is highly associated with the stress history and the current stress state. Generally, changes caused by installation of a jacked pile are more complex than changes caused by construction of a bored pile. Installation of a jacked pile completely destructures the soil around the pile, creating a large amount of excess pore water pressures (Zhu and Chang 2002). As a result, both the stress history and the stress state of the surrounding soil change significantly during pile instillation. There are two analytical approaches available for evaluating the changes in stress state caused by pile installation: the strain path method (SPM) and the cavity expansion method (CEM). Compared with the SPM, although the CEM is a one-dimensional approach, it has the advantage that a closed-form solution is likely to model the stress changes during pile installation, which facilitates the analysis procedure for the subsequent consolidation (Sheil and McCabe 2016). Moreover, many previous studies demonstrated that the cavity expansion theory can evaluate the stress state of the surrounding soil with sufficient accuracy (Guo 2000; Randolph 2003). Therefore, the cavity expansion theory is adopted in this study to
approximately assess the stress state of the surrounding soil during pile installation.

In the CEM method, pile installation is simulated by the expansion of a spherical cavity at the pile tip and a cylindrical cavity around the pile shaft from zero initial radii to the pile radius, \( r_0 \). During this process, a plastic region with radius, \( r_p \), and excess pore water pressures are formed in the surrounding clayey soil. So far, numerous solutions have been proposed to the undrained expansion of a cavity in clayey soils. The Cam-clay models based solutions appear to have advantages over others because the effect of the in-situ anisotropic stress, the stress history, and the large deformation on the expansion response can be reasonably incorporated in the solutions (Yu 2000; Cao et al. 2001; Chen and Abousleiman 2012). Following the \( K_0 \)-consolidated anisotropic modified Cam-clay model based solution for cylindrical cavity expansion (Li et al. 2016) and the modified Cam-clay (MCC) model based solution for spherical cavity expansion (Cao et al. 2001), the excess pore water pressures, \( u_{e,0} \), and the mean effective stress, \( p'_{cs} \), around the pile shaft and tip can be given as follows

\[
\begin{align*}
    u_{e,0} &= p'_{0} \left( \frac{3K_0}{1 + 2K_0} + \frac{mn'_{p}}{\sqrt{3}m} \right) + p'_{0} \left( \sqrt{3} \ln \frac{r}{r_0} - \frac{\xi - 2 \sqrt{\frac{3}{3}\xi} + \frac{\sqrt{4M^2 - 3\xi^2}}{6} - 1}{r_0} \right) \\
    p'_{cs} &= p'_{0} \left( \frac{OCR}{2} \right)^m
\end{align*}
\]

(1)

(2)

where \( m=1 \) and \( m=2 \) indicate the cylindrical case (Li et al. 2016) and the spherical case (Cao et al. 2001), respectively. The plus sign is taken when \( K_0 \leq 1 \); conversely, the minus sign is taken when \( K_0 > 1 \). \( K_0 \) is the coefficient of earth pressure at rest. \( M = 6\sin\phi'/(3 - \sin\phi') \) is the slope of the critical state line. \( \phi' \) is the effective
internal friction angle. \( p'_0 \) is the in-situ mean effective stress. \( \eta'_p = M^* \sqrt{\text{OCR} - 1} \) is the relative stress ratio at the elastic-plastic (EP) boundary. OCR is the overconsolidation ratio. \( M^* = \sqrt{M^2 - \eta_0^2} \) is the relative stress ratio at the critical state. \( \eta_0 = \left[ 3(1 - K_0) / 2K_0 + 1 \right] \) is the initial stress ratio. \( \Lambda = 1 - \kappa / \lambda \) is the plastic volumetric strain ratio. \( \kappa \) and \( \lambda \) are the slopes of the swelling line and the loading line, respectively. \( \varsigma \) is a parameter to simplify the expressions. \( r_p \) is the radius of the plastic region around the cavity, which can be determined by the volume conservation condition and the yield function of MCC model by ignoring higher order term of \( (\sigma'_p - \sigma'_{h0})/G_0 \). The expressions of \( r_p \) and \( \varsigma \) are as follows

\[
\frac{\left( \frac{r_p}{r_0} \right)^{m+1}}{2mG_0} = \frac{2mG_0}{(m + 1)(\sigma'_p - \sigma'_{h0})}
\]

\[
\varsigma = \frac{2\sqrt{3\left[ M^2(2K_0 + 1)^2 - 9(1 - K_0)^2 \right]}}{3(2K_0 + 1)}
\]

where \( \sigma'_p = \sigma'_{h0} + m\sigma'_0 p^*_p \sqrt{3m} \) is the radial effective stress at the EP boundary. \( \sigma'_{h0} \) is the in-situ horizontal effective stress. The in-situ shear modulus, \( G_0 \), is defined as

\[
G_0 = \frac{3(1 - 2\nu')\nu p'_0}{2(1 + \nu')(\kappa')}
\]

where \( \nu \) is the specific volume; and \( \nu' \) is effective Poisson’s ratio.

Because pile installation not only changes the stress state of the surrounding soil but also erases the in-situ stress history, i.e., the OCR of the surrounding soil is equal to unity after pile installation regardless of the initial value of OCR (Randolph and Wroth 1981). Hence, the surrounding remoulded soil exhibits the mechanical behaviour similar to the normally consolidated clay after pile installation. Based on
the MCC model, the undrained shear strength of the surrounding soil before and after pile installation, \( s_{uc,0} \) and \( s_{uc,t} \), can be expressed in terms of triaxial compression, respectively, as follows

\[
\begin{align*}
\ s_{uc,0} &= \frac{1}{2} M_p' \left( \frac{OCR}{2} \right)^\lambda \\
\ s_{uc,t} &= \frac{1}{2} M_p' \left( \frac{1}{2} \right)^\lambda
\end{align*}
\]

where \( p'_t \) is the mean effective stress of the remoulded soil after pile installation and will be determined in the following section.

Changes from subsequent consolidation

After pile installation, the disturbed soil around the pile consolidates with radial dissipation of the excess pore water pressures, as shown in Fig. 1. The surrounding disturbed soil experiences an increase in the effective stress during consolidation, which leads to the increase in the strength and the stiffness of the soil. Because the surrounding soil experiences a process like slight unloading and the corresponding strain is relatively small during consolidation, it can be approximately assumed that the soil deforms elastically (Guo 2000). Based on the assumption that the soil deforms elastically, Randolph and Wroth (1979) presented a closed-form analytical solution for the radial consolidation of the soil around a jacked pile. The expression of the excess pore water pressures at any time during consolidation can be given as (Randolph and Wroth 1979)

\[
\begin{align*}
\ u_{p,t} &= \sum_{n=1}^{\infty} C_n \left[ J_0 (\lambda_n r) + C_{2n} Y_0 (\lambda_n r) \right] e^{-\lambda_n^2 C_t}
\end{align*}
\]

where \( J_0 \) and \( Y_0 \) are zero-order Bessel functions of the first and second kind.
respectively; \( C_h = 2k_h(1-v')G/\gamma_w(1-2v') \) is the coefficient of consolidation for radial horizontal drainage; \( k_h \) is the horizontal coefficient of permeability; and \( \gamma_w \) is the unit weight of water. The integration constants \( C_{1n} \) and \( C_{2n} \), and the eigenvalues of the Bessel function \( \lambda_n \) can be determined by the following equations

\[
C_{2n} = -\frac{J_0(\lambda_n R)}{Y_0(\lambda_n R)} \tag{9}
\]

\[
C_{1n} = \frac{\int_0^R u_{e0} \left[ J_0(\lambda_n r) - \frac{J_0(\lambda_n R)}{Y_0(\lambda_n R)} Y_0(\lambda_n r) \right] rdr}{2\int_0^R \left[ J_0(\lambda_n r) - \frac{J_0(\lambda_n R)}{Y_0(\lambda_n R)} Y_0(\lambda_n r) \right]^2 rdr} \tag{10}
\]

\[
Y_0(\lambda_n R)J_1(\lambda_n r_0) - J_0(\lambda_n R)Y_1(\lambda_n r_0) = 0 \tag{11}
\]

where \( R \) is a radial distance beyond which the excess pore water pressures can be neglected, and can be empirically taken as 5-10 times \( r_p \) (Guo 2000). Generally, there are numerous constants \( C_{1n}, C_{2n} \) and \( \lambda_n \) that can satisfy Eq. (8) because the Bessel functions are oscillating functions. However, in practical calculating, the first 50 items can yield sufficient accuracy.

With Eq. (8), the average consolidation degree of the surrounding soil, \( \bar{U}_t \), can be given as

\[
\bar{U}_t = 1 - \frac{\int_0^R u_{e0}rdr}{\int_0^R u_{e0}rdr} \tag{12}
\]

During pile installation, a critical state region is developed around the pile due to the severe squeezing and shearing effects. For soils at the critical state, the mean effective stress \( p'_cs \) keeps unchanged regardless of further shearing and squeezing. Hence, after pile installation, the mean effective stress \( p'_t \) increases from \( p'_cs \) with
dissipation of the excess pore water pressures. However, because of the stiffness discrepancy of the soil around the pile, the increase of the effective stress does not strictly match the dissipation of excess pore water pressures and the mean total stress $p_i$ exhibits relaxation effects during consolidation. These effects can be properly modeled by Biot’s coupled consolidation theory along with the nonlinear stress-strain behaviour of soil. Nonetheless, if these effects are taken into account, it is extremely complex to derive a closed-form solution for consolidation of the soil around pile. To facilitate the application of the proposed approach, Terzaghi’s radial consolidation theory and an effective stress transfer parameter $\chi$ (Randolph and Wroth 1979) are adopted to approximately evaluate the mean effective stress after pile installation. Based on the principle of effective stress, the mean effective stress after pile installation can be given as follows

$$p_i' = p_{c,s} + \chi \left( u_{e,0} - u_{e,t} \right)$$  \hspace{1cm} (13)

where $\chi$ is the aforementioned effective stress transfer parameter under the plane strain condition, defined as $\chi = 1 + v' / \left[ 3 (1 - v') \right]$.

With Eq. (13), the change in the mean effective stress after pile installation can be quantified using a dimensionless factor, named the mean effective stress ratio ($ESR$), as follows

$$ESR = \frac{p_i'}{p_0'}$$  \hspace{1cm} (14)

Making use of Eqs. (5), (7) and (14), the undrained shear strength $S_{u,c,t}$ in terms of triaxial compression and the shear modulus, $G_t$, of the surrounding disturbed soil at any time after pile installation can be approximately determined as
\[ s_{utc,t} = s_{utc,0} \frac{ESR}{(OCR)^\lambda} \]  

(15)

\[ G_t = G_0 ESR \frac{\nu_t}{\nu_0} \approx G_0 ESR \]  

(16)

where \( \nu_t \) is the time-dependent specific volume of the surrounding soil after pile installation. However, if the change of \( \nu_t/\nu_0 \) is considered, it is extremely difficult to obtain an analytical solution for the elastic soil displacement outside the shear band later in the paper. Since the ratio \( \nu_t/\nu_0 \) of a typical clayey soil after full consolidation is in the range of 0.93-0.95 (Basu et al. 2013), which has a nearly negligible effect on the change in the shear modulus, the effect of the change in the specific volume on shear modulus is neglected in this study to facilitate the following analysis work.

**Time-dependent load-transfer curve**

**Nonlinear t-z curve for pile-soil interaction**

During pile loading, the total displacement of the pile at a given depth consists of the relative displacement at the pile-soil interface, the displacement of the shear band as well as the displacement of the soil outside the shear band, as shown in Fig. 2. Investigations by means of the in-situ test (Caputo and Viggiani 1984), the finite element analysis (Trochanis et al. 1991), and the theoretical study (Wang et al. 2012) show that the total displacement is primarily the nonlinear displacements developed at the pile-soil interface and in the shear band adjacent to the pile shaft, whereas the displacement of the bulk of the soil outside the narrow shear band is relatively small.
and largely elastic. Because the shear band forms in a fairly narrow zone adjacent to the pile shaft and its actual thickness is very difficult to determine, it can be approximated and simplified as a displacement discontinuity with no physical size in practice (Lee and Xiao 2001). Based on this approximation, the nonlinear displacements developed at the pile-soil interface and in the shear band can be evaluated by a single nonlinear \( t-z \) curve. According to the results of laboratory and field tests (Zhang et al. 2014), the nonlinear relation between the shear stress and the relative displacement at the pile-soil interface can be well described by a hyperbolic function or an exponential function. In this study, an exponential function based load-transfer curve, as shown in Fig. 3, is adopted to model the behaviour of the pile-soil interface. The expression of the exponential function takes the following form

\[
\tau_{s,z} = a_{s,z} \left( 1 - e^{-b_{s,z}W_{p,z}} \right)
\]  

(17)

where \( \tau_{s,z} \) and \( W_{p,z} \) are the mobilized shear stress and the corresponding pile-soil relative displacement at depth \( z \), respectively. \( a_{s,z} \) and \( b_{s,z} \) are the model parameters and \( a_{s,z} \) is the asymptote of the load-transfer curve; the product of \( a_{s,z} \) and \( b_{s,z} \) represents the initial stiffness of the load-transfer curve, as shown in Fig. 3.

Taking the derivative of Eq. (17) with respect to \( W_{p,z} \), the tangent stiffness of perimeter of the pile-soil interface, \( K_{sp,z} \), during loading can be written as

\[
K_{sp,z} = 2\pi r_0 a_{s,z} b_{s,z} e^{-b_{s,z}W_{p,z}}
\]  

(18)

The degradation of the stiffness at the pile-soil interface with the increase of shear displacement, which is commonly observed from pile loading tests, can be well
represented by Eq. (18). Fig. 4 shows sample stiffness degradation curves, presented as the variation of the normalized tangential stiffness, $K_{q,p,z}/K_{q,p,z}^{\text{max}}$, with the normalized shear displacement, $W_{q,p,z}/W_{q,p,z}^{\text{max}}$, generated by Eq. (18). It can be concluded from Fig. 4 that the parameter $b_{s,z}$ not only controls the magnitude of degradation, but also indicates the rate of degradation.

### 3.2 Displacement of the soil outside the pile-soil interface

Since the soil outside the shear band remains largely elastic, the corresponding displacement is evaluated based on the procedure proposed by Randolph and Wroth (1978) for analyzing the elastic displacement of the soil:

$$
W_{so,z} = -\tau_z r_0 \int_{r_m}^{r} \frac{1}{G_{r,z}} dr
$$

(19)

where $\tau_z = dP_z / 2\pi r_0$ is the local shaft stress. $dP_z$ is the axial force increments along the pile shaft at depth $z$. $G_{r,z}$ is the shear modulus of the surrounding remolded soil along the radial direction at depth $z$. $r_m = 2.5L\rho(1-\nu')$ is the limiting radius beyond which the shear stress induced by the pile becomes negligible. $\rho$ is the ratio of soil shear modulus at middepth to that of the pile tip.

Substituting Eq. (16) into Eq. (19), the elastic soil displacement, considering the pile installation effects, of the surrounding soil can be rewritten as follows

$$
W_{so,z} = -\frac{dP_z}{2\pi G_{0,z}} \int_{r_m}^{r} \frac{1}{(A-B\ln r)r} dr
$$

(20)

where $G_{0,z}$ is the in-situ shear stress at depth $z$. $A$ and $B$ are the parameters to simplify the expression. The expressions of $A$ and $B$ are as follows

**Note:** The expressions for $A$ and $B$ are not provided in the given text.
According to the relation between the plastic radius \( r_p \) induced by pile installation and the maximum influence radius \( r_m \) of the pile during loading, Eq. (20) can be integrated in the following two cases:

1) If the plastic radius \( r_p \) is larger than the maximum influence radius \( r_m \), i.e., \( r_p > r_m \), integrating Eq. (20) yields

\[
W_{se,z} = \frac{dP_z}{2\pi B G_{0,z}} \ln \frac{A - B \ln r_0}{A - B \ln r_m} \tag{23a}
\]

2) Oppositely, when the plastic radius \( r_p \) is less than the maximum influence radius \( r_m \), i.e., \( r_p \leq r_m \), Eq. (20) can be integrated to give

\[
W_{se,z} = \frac{dP_z}{2\pi B G_{0,z}} \ln \left[ \frac{A - B \ln r_0}{A - B \ln r_m} \left( \frac{r_m}{r_p} \right)^a \right] \tag{23b}
\]

It should be noted that the surrounding soil is assumed to deform elastically during consolidation, and hence \( r_p \) in Eqs. (23a) and (23b) is only used to determine the range in which the stress state of the soil is changed by pile installation.

From Eq. (23), the elastic stiffness of the soil around per unit length of pile shaft, \( K_{se,z} \), can be written as follows

\[
K_{se,z} = \begin{cases} 
\frac{2\pi B G_{0,z}}{\ln \frac{A - B \ln r_0}{A - B \ln r_m}} & r_p > r_m \\
\frac{2\pi B G_{0,z}}{\ln \left[ \frac{A - B \ln r_0}{A - B \ln r_m} \left( \frac{r_m}{r_p} \right)^a \right]} & r_p \leq r_m 
\end{cases} \tag{24}
\]
As shown in Fig. 2, the total shaft displacement, $W_{s,z}$, is the sum of the pile-soil relative displacement $W_{sp,z}$ and the surrounding elastic displacement $W_{se,z}$, i.e.,

$$W_{s,z} = W_{sp,z} + W_{se,z} \quad (25)$$

Based on Eq. (25), the generalized stiffness per unit length of pile shaft, $K_{s,z}$, defined as the force per displacement, can be given as

$$K_{s,z} = \frac{K_{sp,z}K_{se,z}}{K_{sp,z} + K_{se,z}} \quad (26)$$

**Load-transfer curve for pile base**

Results from the load tests on instrumented piles (Zhang et al. 2014) indicated that the nonlinear load-displacement relationship developed at the pile base was also conformed to a hyperbolic function or an exponential function. Similar to the proposed load-transfer curve for pile shaft, the relation between the mobilized pile base resistance, $q_b$, and the corresponding displacement, $W_b$, can be expressed by an exponential function as follows

$$q_b = a_b \left(1 - e^{-b_b W_b}\right) \quad (27)$$

where the model parameter $a_b$ can be taken as the ultimate pile base resistance; the parameter $b_b$ controls the magnitude and the rate of stiffness degradation at the pile base; and the product of $a_b$ and $b_b$ represents the initial stiffness of the soil at the pile base, as shown in Fig.5.

With Eq. (27), the tangential stiffness of the pile base during loading can be given as

$$K_b = A_p a_b b_b e^{-b_b W_b} \quad (28)$$
where \( A_p = \pi r_0^2 \) is the cross-sectional area of the pile.

**Determinations of the values of model parameters**

Generally, the values of the model parameters can be obtained experimentally either from the back-analysis of field load test results or from laboratory experimental tests (Zhang et al. 2010). However, these experiment methods are not only time-consuming, but also incapable of incorporating the evolution of the mechanical behaviour of the soil around the pile. Hence, the experiment methods have great limitations. In the following, a general theoretical approach is proposed to determine the time-dependent ultimate pile shaft and base resistances.

As mentioned previously, the model parameters \( a_{sz} \) and \( b_a \) are the asymptote of the load-transfer curves for the pile shaft and the pile base, respectively. Hence, the parameters \( a_{sz} \) and \( b_a \) can be determined as follows

\[
a_{sz} = \tau_{su,z} \quad (29)
\]
\[
b_a = q_{hu} \quad (30)
\]

where \( \tau_{su,z} \) and \( q_{hu} \) are the ultimate pile shaft resistance at depth \( z \) and the ultimate pile base resistance, respectively.

Fig. 6 shows the most likely stress state at failure along with the corresponding Mohr’s stress circle for a soil element adjacent to the pile shaft during loading of the pile as shown in Fig. 2. Based on the shearing pattern of the soil element during pile loading, the radius of the Mohr’s stress circle is defined as the undrained shear strength of the soil under the plane strain condition, \( S_{qun,z} \). As discussed before, the soil adjacent to the pile can be approximately taken as normally consolidated clay.
after pile installation. For the normally consolidated clay, the cohesion intercept can be ignored. Thus, the failure-envelope in Fig. 6 passes through the origin of coordinates. After pile installation, the radial effective stress becomes the largest component among the three normal stresses ($\sigma'_r, \sigma'_\theta, \sigma'_z$). Hence, the failure envelope is most likely to be tangent to the effective stress circle at point B, while the corresponding stress state at the pile-soil interface is located at point A. From the simple geometry shown in Fig. 6, the ultimate shaft resistance can be expressed as

$$\tau_{su,z} = s_{up,z} \cos \phi'$$  \hspace{1cm} (31)

The three-dimensional strength of soil under arbitrary shear condition can be properly modeled by the SMP or Lade’s criteria (Matsuoka and Sun 2006). Based on the stress transformed method proposed by Matsuoka et al. (1999) and the SMP criterion, the undrained shear strength under the plane strain condition can be evaluated by the undrained shear strength under the triaxial compression as

$$s_{ups} = \frac{3\sqrt{3}\sin \phi_f}{2M \cos \psi_f \sqrt{2 + \sin^2 \phi_f}} s_{ute}$$  \hspace{1cm} (32)

where $\phi_f$ and $\psi_f$ are the stress transformed parameters, which are given as follows

$$\sin \phi_f = \frac{\sqrt{2}M}{\sqrt{9 + 3M}}$$  \hspace{1cm} (33)

$$\psi_f = \frac{1}{3} \cos^{-1}\left\{-\left(\frac{3}{2 + \sin^2 \phi_f}\right)^{3/2} \sin \phi_f \cos 3\theta\right\}$$  \hspace{1cm} (34)

where Lode’s stress angle $\theta$ is equal to $\pi/6$ under the plane strain condition.

Combining Eqs. (15), (29), (31) and (32), the model parameter $a_{sz}$ can be finally
determined as
\[ a_{s,z} = \frac{3\sqrt{3}\sin \varphi \cos \varphi'}{2M \cos \varphi' \sqrt{2 + \sin^2 \varphi'}} \frac{ESR}{OCR} s_{utc,0} \]  

(35)

For a pile in clayey soils, the contribution of tip resistance to total pile capacity is quite small compared with the resistance developed along the pile shaft because of the mobilization of skin friction. Hence, it is sufficient to evaluate the ultimate end resistance \( q_{bu} \) by the following equation (Saldivar and Jardine 2005)

\[ q_{bu} = N_c s_{utc} \]  

(36)

where \( N_c \) is the end-bearing capacity factor and usually taken as 9.0 for clayey soils.

From Eqs. (15) and (30) along with Eq. (36), the model parameter \( a_b \) can be given as

\[ a_b = N_c \frac{ESR}{OCR} s_{utc,0} \]  

(37)

According to the initial condition, the initial tangential stiffness of the load-transfer curve for the pile shaft should be equal to the elastic stiffness of the surrounding soil. Hence, the model parameter \( b_{s,z} \) can be determined from Eqs. (18) and (24) as follows

\[ b_{s,z} = \begin{cases} 
\frac{BG_{0,z}}{a_{s,z} r_0 \ln \frac{A-B \ln r_p}{A-B \ln r_m}} & r_p > r_m \\
\frac{BG_{0,z}}{a_{s,z} r_0 \ln \left[ \frac{A-B \ln r_p}{A-B \ln r_m} \left( \frac{r_m}{r_p} \right)^g \right]} & r_p < r_m 
\end{cases} \]  

(38)

From the simple method of Randolph and Wroth (1978), the elastic response of the pile base can be evaluated by a rigid circular disc acting on the surface of a
homogeneous elastic stratum. The corresponding elastic stiffness of the pile base can be written as

\[ K_{eb} = \frac{4r_0 G_0}{1-v'^2} \]  \hspace{1cm} (39)

Introducing Eq. (16) into Eq. (39), the elastic stiffness of the pile base considering the pile installation effects can then be rewritten as

\[ K_{eb} = \frac{4r_0 G_0}{1-v'^2} ESR \]  \hspace{1cm} (40)

Based on the definition of the product of \( a_b \) and \( b_b \) for the load-transfer curve, the parameter \( b_b \) can be determined by combining Eqs. (28) and (40) as

\[ b_b = \frac{4G_0 ESR}{\pi r_0 a_b (1-v'^2)} \]  \hspace{1cm} (41)

**Prediction of time-dependent bearing performance**

**Incremental algorithm for load-settlement analysis**

Based on the proposed load-transfer curves for the pile shaft and the pile base, the time-dependent bearing performance of a jacked pile in clay strata can be evaluated by the load transfer method. As shown in Fig. 7, the load transform method models the pile as a series of elements supported by discrete nonlinear springs. The bearing behaviours of the pile shaft and the pile base are represented by the springs distributed along the pile shaft and at the pile base, respectively. Here, the pile is divided equally into \( n \) segments, with the length of each part \( L_n = L/n \). The discrete segments are numbered from the pile top to the pile base. The schematic diagram of this discretization is shown in Fig. 7. When \( n \) is large enough, the thickness of each
segment is so small that a linear variation can be assumed for the distribution of the axial force in each segment to facilitate analyses.

In this study, the original iterative procedure proposed by Coyle and Reese (1996) for the load-transfer method is modified and developed to accommodate the incremental algorithm. The present assessment procedure assumes a series of small displacement increments, $\delta W_{n,b}$, at the pile base. For each pile base displacement increment, the corresponding pile base load increment, $\delta P_{n,b}$, at a given time after pile installation can be calculated as follows

$$\delta P_{n,b} = K_p \delta W_{n,b}$$ (42)

Assuming that the displacement increment at the middle height of segment $n$, $\delta W_{sn,m}$, is equal to the current pile base displacement increment $\delta W_{n,b}$, the corresponding axial force increment at the top of segment $n$, $\delta P_{n,t}$, can be obtained as

$$\delta P_{n,t} = K_{sn,m} \delta W_{sn,m} L_n + \delta P_{n,b}$$ (43)

where $K_{sn,m}$ is the pile shaft stiffness at the middle height of the segment $n$. It should be noted that the required displacement $W_{spn,m}$ and $W_{n,b}$ in Eqs. (18) and (28) used to determine the tangential stiffness $K_{sn,m}$ and $K_b$ are equal to zero for the first displacement increment.

Following the assumption of the linear variation of the load distribution along the segment, the elastic deformation increment at the midpoint of segment $n$, $\delta W_{en,m}$, can be calculated as

$$\delta W_{en,m} = \frac{(\delta P_{n,m} + \delta P_{n,b}) L_n}{2E_p A_p}$$ (44)

where $E_p$ is Young’s modulus for the pile material. $\delta P_{n,m} = (\delta P_{n,b} + \delta P_{n,t}) / 2$ is the
axial force increment at the midpoint of segment \( n \).

With the assumed pile base displacement increment \( \delta W_{n,b} \) and the elastic deformation increment \( \delta W_{e,n,m} \), the updated midpoint displacement increment of segment \( n \), \( \delta W'_{m,n} \), can then be expressed as

\[
\delta W'_{m,n} = \delta W_{e,n,m} + \delta W_{n,b}
\]  

(45)

When the difference between the two displacement increments \( \delta W'_{m,n} \) and \( \delta W_{m,n} \) is greater than a specified tolerance, the above procedure will continue to repeat with the new displacement increment \( \delta W'_{m,n} \) until the convergence is achieved. The displacement increment at the top of segment \( n \), \( \delta W_{n,t} \), can be finally obtained as

\[
\delta W_{n,t} = \frac{(\delta P_{n,t} + \delta P_{n,b}) L_n}{2E_p A_p} + \delta W_{n,b}
\]  

(46)

From Eq. (17), the pile-soil relative displacement increment at the midpoint of segment \( n \), \( \delta W_{s,m,n} \), can be derived as

\[
\delta W_{s,m,n} = -\ln \left( 1 - \frac{K_{s,m} \delta W_{s,m,n}}{2\pi K a_{s,n,m}} \right)
\]  

(47)

where \( a_{s,n,m} \) and \( b_{s,n,m} \) are the model parameters of the load-transfer curve at the midpoint of segment \( n \), respectively.

Consecutively, taking the load and displacement increments at the top of each segment as the load and displacement increments at the base of the upper segment, the load and displacement increments of each segment can be obtained by repeating the above procedure from segment \( n \) to segment 1.

For successive pile base displacement increments, a series of corresponding load
and displacement increments of each segment can be obtained by conducting the above procedure. In each round of calculating, the displacement increments at the middle height of the each segment and the pile base are accumulated to determine the tangential stiffness $K_{su,m}$ and $K_b$ for the next round. The pile head load, $P$, and displacement, $W_1$, will be the accumulated load and displacement increments at the top of segment 1, respectively.

**Computational scheme**

Based on the proposed incremental algorithm, the corresponding computational flow chart, as shown in Fig.8, is drew for the general framework of the developed technique. Following the flow chart, a MATLAB software based computer code is developed to capture the time-dependent bearing performance of a jacked pile in clay strata. The present approach only requires a few seconds to predict the load-settlement curve of a pile at a given time after pile installation, which demonstrates the present approach is quite economical and efficient.

**Verification of proposed approach**

The proposed approach and procedure are verified by comparing the predicted results with the time-dependent load-settlement responses of jacked piles in (1) Shanghai soft clay strata; and (2) Quebec Champlain clay.

**Case 1: Pile in Shanghai soft clay strata**

The pile loading tests chosen for comparison were conducted by authors at Zhoupu town, Pudong District of Shanghai. The site investigation and the field boring log
show that the soil strata mainly consist of soft clay, clayey silt, mucky clay and silty clay at different depths, as shown in Fig. 9. The soils are normally consolidated with OCRs in the order of 1.1. The groundwater table lies at a depth of 1.3 m below the ground surface. The soil properties, from comprehensive laboratory testing on high quality Shelby tube samples retrieved from the boreholes, are summarized in Table 1.

Three concrete piles, each with diameter of 0.35 m, were jacked to the embedment depth of 30 m. The elastic modulus $E_p$ of the concrete pile is equal to 40 GPa. The static loading test was conducted on the three test piles, TP1, TP2 and TP3, at 7 days, 25 days and 65 days after pile installation, respectively. The reloading effects on the load-settlement response were effectively avoided in the designed pile testing program because each pile was tested only once. The quick test procedure suggested by ASTM D1143 was adopted for the static loading tests. Based on the quick test procedure, the loads were applied step-by-step with an increment of 5% of the anticipated failure load. During each load interval, the settlement at the pile head was recorded after the load had been applied and maintained for a time interval of 5 min.

Using the soil properties listed in Table 1, the proposed approach is applied to predict the load–settlement responses of the test piles. In the calculation, the values of the effective Poison’s ratio $\nu'$ of the surrounding soils are assumed to be 0.35. A typical value of 0.8 is assumed for $\Lambda$, because the parameter $\Lambda$ is essentially a constant, with an average value of 0.8 for natural intact clays under the plane strain shearing condition (Cao et al. 2001). The values of $K_0$ are estimated by the empirical formula suggested by Manyne and Kulhawy (1982) as follows
Fig. 10 shows the comparisons between the measured results from the loading tests and the predicted load-displacement curves. As seen in the figure, the predicted load-transfer curves agree well with the field observations. The good agreements demonstrate that the proposed semi-analytical approach can yield reasonable prediction for the time-dependent load-settlement responses of the test piles. It is also interesting to note that both the stiffness and the ultimate bearing capacity of the pile increase substantially with time, but the increase magnitude and the increase rate decrease with time after pile installation. This is because the hydraulic gradient of the excess pore water pressures decreases with consolidation time, which results in a faster increase in the strength and the shear modulus of the surrounding soils immediately after pile installation and followed by a slower rate in the increase. This phenomenon was also observed in the field test by Roy et al. (1981).

**Case 2: Pile in Quebec Champlain clay**

Konrad and Roy (1987) performed static loading tests on a jacked pile at various times after pile installation to investigate the time-dependent bearing performance of jacked piles in clay. The pile test site is located at St-Alban, 80 km west of Quebec City. In the test, a closed-ended steel pile with diameter of 0.22 m was firstly jacked vertically to a depth of 7.6 m below the ground surface. The static load tests were then conducted on the pile at time intervals of 4 days, 8 days and 33 days after completion of installation. During the process of pile loading, the loads were applied step-by-step with a small increment of 6.67 kN until pile failure. Each load was maintained for a
period of 15 min.

The soil within the embedment depth of the test pile is soft silty marine clay. The groundwater table was located at 0.4 m below the ground surface. The clay deposit is slightly overconsolidated with an OCR of about 2.1. The coefficient of earth pressure at rest $K_0$ is about 0.63. The ratio of the shear modulus over the mean effective stress, $G/p_0'$, is in the order of 90. The effective internal friction angle $\phi'$, in terms of triaxial compression tests, ranges from 27° to 30°. From depths of 2.0 m-6 m, the in-situ effective vertical stress, $\sigma_{z0}'$, increases linearly from 16 kPa to 36 kPa. The horizontal coefficient of consolidation $C_h$ is approximately equal to $3 \times 10^{-7}$ m²/s.

Taking the median value of $\phi' = 28^\circ$ and typical values of $\Lambda = 0.75$ and $\nu' = 0.3$, the time-dependent load-settlement responses of the test pile are predicted with the parameters of the St-Alban clay. Fig. 11 shows the load-settlement curves obtained from the proposed approach along with the results derived from the field measurements. This figure demonstrates satisfactory agreements between the predicted and field-measured results, although a stiffer and stronger axial response is predicted for the pile at 4 days after pile installation. The reason is properly that the recovery of the collapsed soil structures and the bonds between soil particles (referred to as thixotropy) is not completed in a short time interval after pile installation. Although the thixotropic effects are not incorporated in the present approach because of the extreme complexity, the proposed semi-analytical approach can yield reasonable predictions for load-settlement responses of a jacked pile in clay strata beyond certain time after pile installation, especially for the long-term load-settlement.
behaviour.

Conclusions

In this paper, a semi-analytical approach has been developed to evaluate the time-dependent load-settlement response of a jacked pile in clay strata. This approach adopts the exponential function-based load-transfer curves to describe the nonlinear pile-soil interaction behaviour. The evolution of the mechanical behaviour of the surrounding soil, which is the primary cause for the time-dependent load-settlement response, is properly incorporated in the proposed nonlinear load-transfer curves. An incremental algorithm-based iterative procedure is developed to enable the time-dependent load-settlement response to be determined using the presented load-transfer curves. The proposed technique is implemented by a highly efficient computer program based on the MATLAB software. Case studies of pile loading tests at two different sites indicate that the proposed procedure can be applied to predict the time-dependent load-settlement responses with reasonable accuracy.

The input parameters required for the proposed approach are the pile properties and the in-situ soil profile and properties. These parameters can be easily determined from in situ tests or laboratory experiments. The thixotropy is not considered in the present approach, thus the proposed approach may yield a stronger axial response at a short time interval after pile installation. It is worth noting that the present approach can be modified to incorporate the pile installation effects when assessing the load-settlement response of pile groups.
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References


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Figure captions

Fig. 1. Schematic diagram of radial consolidation of soils around a jacked pile

Fig. 2. Displacements of pile and surrounding soils and stress state of a soil element adjacent to pile shaft during pile loading

Fig. 3. Exponential function based load-transfer curve for pile shaft

Fig. 4. Effect of parameter $b_{s,z}$ on modulus degradation

Fig. 5. Exponential function based load-transfer curve for pile base

Fig. 6. Failure stress state of soil element in the $\sigma' - \tau$ plane

Fig. 7. Analytical model for pile-soil load transfer

Fig. 8. Computational flow chart for present incremental algorithm

Fig. 9. Soil profile at Shanghai test site

Fig. 10. Measured and predicted time-dependent load-settlement behaviours at Shanghai test site

Fig. 11. Measured and predicted time-dependent load-settlement behaviours at Quebec test site
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Input Data
Soil profile and properties: \( \gamma' \), OCR, \( K_\infty \), \( \beta \), \( \eta \), \( k_b \)
Pile properties: \( L \), \( r_0 \), \( E_p \), \( A_p \)
Time after pile installation: \( t \)

Divide the pile into \( n \) segments of length \( L_n \). Number the segments from the top to base. Determine model parameters \( a_{si,m} \), \( b_{si,m} \), \( a_b \), \( b_b \) at the given time \( t \)

Determine \( K_{si,m} \) and \( K_b \) using Eqs. (26) and (28), respectively. Initially, \( W_{si,0} \), \( W_{b,0} \)

Assume a small base displacement increment \( \delta W_{n,b} \). Use Eq. (42) to compute \( \delta P_{n,b} \)

Segment \( i = n \ldots 1 \)

Assume \( \delta W_{si,m} = \delta W_{si,b} \), compute \( \delta P_{si} \) using Eq. (42). For the first trial, assume \( \delta W_{si,m} = \delta W_{si,b} \)

Compute \( \delta W_{si,m} \) using Eq. (44). Use Eq. (45) to compute \( \delta W_{si,m} \)

No

\[ \delta W_{si,m} = \delta W_{si,t} < 10^{-4} \]  

Yes

Compute \( \delta W_{si,t} \) using Eq. (46). Compute \( \delta W_{spi,m} \) using Eq. (47)

Store \( \delta W_{spi,m} \), \( \delta P_{spi,m} \)
Take \( \delta W_{si,1,b} = \delta W_{si,t} \), \( \delta P_{si,1,b} = \delta P_{si,t} \)

Store \( \delta W_{1,t}, \delta P_{1,t} \)
Update \( W_{1,t}, P_{1,t} \) and \( W_{spi,m} \), \( W_{spi}, \delta W_{1,t}, \delta P_{1,t} \)

\[ P_{1,t} = P_{1,t} + \delta P_{1,t}, W_{spi,m} = W_{spi,m} + \delta W_{spi,m} \]

Output result
Load-settlement curve of a jacked pile at a given time after pile installation

Fig. 8. Computational flow chart for present incremental algorithm
Fig. 9. Soil profile at Shanghai test site
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Shanghai test site
Fig. 11. Measured and predicted time-dependent load-settlement behaviours at Quebec test site
### Table 1. Profile and properties of the soil strata at Shanghai test site

<table>
<thead>
<tr>
<th>Soil type</th>
<th>Layer thickness $h$ (m)</th>
<th>$\gamma'$ (kN/m$^3$)</th>
<th>$e_0$</th>
<th>$\phi'$ (°)</th>
<th>$E_s$ (MPa)</th>
<th>$k_h$ ($\times 10^{-7}$ cm/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clay</td>
<td>1.75</td>
<td>8.9</td>
<td>1.052</td>
<td>31.7</td>
<td>4.46</td>
<td>2.30</td>
</tr>
<tr>
<td>Clayey silt</td>
<td>2.21</td>
<td>7.6</td>
<td>1.187</td>
<td>27.5</td>
<td>3.67</td>
<td>11.8</td>
</tr>
<tr>
<td>Mucky clay</td>
<td>10.21</td>
<td>7.4</td>
<td>1.219</td>
<td>28.4</td>
<td>3.39</td>
<td>2.13</td>
</tr>
<tr>
<td>Silty clay</td>
<td>8.93</td>
<td>7.8</td>
<td>1.065</td>
<td>31.9</td>
<td>4.07</td>
<td>14.2</td>
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</table>