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Experimental study on two-oscillating grid turbulence with viscoelastic fluids based on PIV

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Abstract: In this paper, in order to study the viscoelastic effect on isotropic turbulence without wall effect, two-oscillating grid turbulence is built to investigate this phenomenon using Particle Image Velocimetry. In the experiments, the classical drag-reducing additives are chosen-polyacrylamide (PAM) and cetyltrimethyl ammonium chloride (CTAC), which have shown remarkable drag-reducing effect in wall-bounded turbulent flows. The results show that the existence of drag-reducing additives makes velocity field more anisotropic and reduces turbulent kinetic energy. We propose an intuitive and natural definition for a reduction rate of turbulent kinetic energy to show viscoelastic effect. It suggests that there exists a critical concentration for the reduction rate of turbulent kinetic energy in CTAC solution case. Also, the small-scale vortex structures are inhibited, which suggests the drag-reducing mechanism in grid turbulence without wall effect.

Keywords: grid turbulence, viscoelastic fluids, drag reduction, vortex structures, particle image velocimetry

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1. Introduction

In 1949, Toms firstly reported that when adding a small amount of polymer additives into turbulent flows, it could cause remarkable turbulent drag reduction (DR) [1]. This phenomenon is called as Toms' effect or DR effect by additives and has great potential in industrial applications, such as in transportation of crude oil, in a district heating/cooling system, and so on. Since Toms' effect, there are many theoretical, experimental and numerical studies to focus the turbulent characteristics and physical mechanism of DR with additives in wall-bounded flows, such as channel flow, pipe flow and so on [2-10]. Now, two kinds of classical drag-reducing additives have been usually used to induce turbulent DR: (1) polymer with long-chain molecular structures, such as, polyacrylamide (PAM) and polyethylene oxide (PEO); (2) surfactant, for example, cetyltrimethylammonium chloride (CTAC) and Ethoquad. The results showed that the drag-reducing additives (polymer and surfactant) have enormous DR effect, i.e., DR can be reached up to 80% [11,12].

Although, there are extensive studies on wall-bounded turbulent flows with drag-reducing additives, the DR mechanism remains poorly clear. This is due to the fact the inhomogeneous nature causes to difficultly study the close interaction between molecular structures of additives and turbulent structures. Therefore, considering removing the inhomogeneity resulting from wall, homogeneous isotropic turbulence (HIT), bulk turbulence (BK) and grid turbulence (GT) can be easier to investigate the above problem. Actu-

ally, HIT, BK or GT does not change momentum fluxes but change energy cascading from large scales to small scales. In other words, to focus on HIT, BK or GT with drag-reducing additives are extremely important towards understanding the drag-reducing characteristics and the interactions between molecular structures of additives and turbulent structures.

Indeed, there are many studies focusing on HIT or GT with drag-reducing additives (polymer or surfactant) from theories, experiments and numerical simulations. In experimental aspect, McComb et al. [13] found that in grid turbulence with polymer solution, high-frequency components of turbulent kinetic energy show a noticeable attenuation especially at the case with larger concentrations. Doorn et al. [14] reported that compared to Newtonian fluid case, the decay rate of mean square velocity for longitudinal and transverse components is remarkably reduced and dissipation rate is smaller due to the presence of polymers. Also, small-scale structures are inhibited owing to an elastic absorption on those scales so as to cause a truncation of energy cascade. Recently, Vonlanthen and Monkewitz [15] experimentally studied grid turbulence with polyethylene oxide (PEO) solution at three concentrations of 25, 50 and 100ppm. The results showed that the energy spectrum changes abruptly from Kolmogorov $\kappa^{-5/3}$ inertial range to a κ^{-3} elastic range at time-dependent Lumley scale. The above experimental studies of GT with dilute polymer solution clearly showed that DR can be also observed even in situations where the wall plays no apparent role. In numerical aspects, Cai et al. investigated the polymer effect on decaying homogeneous isotropic turbulence (DHIT) by direct

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numerical simulation (DNS) [16] and observed a remarkable inhibition of vortex structures in DHIT with polymers [17]. Li et al. [18] studied the polymer effect on turbulent energy cascading in forced homogeneous isotropic turbulence (FHIT) using DNS. It was found that turbulent energy cascading in FHIT was greatly modified by polymers. Wang et al. carried out large-eddy simulation (LES) coupling with temporal approximate deconvolution model (TADM) for FHIT with polymers. The results showed that small-scale intermittency in FHIT was inhibited caused by polymers. Recently, Li et al. proposed a mixed subgrid-scale model coupling with coherent structure and temporal approximate deconvolution to simulate FHIT with polymers.

From the previous studies, it is found that all studies focus on the viscoelastic effect on decaying grid turbulence from experimental viewpoint. And few studies consider FHIT from numerical viewpoint. However, how the drag-reducing characteristics of FHIT with additives is still not clear. In our knowledge, this is the first time to experimentally study the viscoelastic effect on FHIT. In this paper, we carried out two-oscillating grid turbulence to realize forcing grid turbulence and investigate the viscoelastic effect on flows without wall effect using Particle Image Velocimetry (PIV).

2 Experimental setup and particle image velocimetry

2.1 Two-oscillating grid system

Usually, a grid located in channel flow [13, 15] or a towed grid [14] have been utilized to realize decaying isotropic turbulence. In order to realize forced isotropic turbulence, both grids are used and attached to two eccentric gears, which are driven by converter motor, as shown in Figure 1. And, both grids are kept synchronous and to oscillate in opposite directions. Therefore, there exists roughly zero-mean velocity turbulent flow in the center area between both grids. And this area is approximate to isotropic turbulence. We performed all experiments in a $28 \times 28 \times 75 \text{ cm}^3$ plexiglass tank filled with the fluid, as shown in Figure 1. Meanwhile, the experimental system should be reasonably designed and meet some requirements, as shown in Figure 1: (1) grid with a square mesh size $M=3\text{cm}$, and the characteristic scale for rod with $d=6\text{mm}$ so for $M/d=5$ ($4 < M/d < 6$); (2) grid solidity $\sigma = d/M(2-d/M) = 36\%$, which satisfies the condition proposed by Corrsin that the solidity should be less than 40% [20]; (3) the distance (Z_1) between free surface and top grid was designed as 15cm, meeting $Z_1/M > 2.5$ [21]; (4) the distance (Z_0) between bottom wall and bottom grid was designed as 35cm, also meeting $Z_0/M > 2.5$ [21]; (5) the distance (H) between top grid and bottom grid was considered as 16cm~18cm, meeting

$H/M > 3.0$ [21]. The oscillating amplitude $S=2\text{cm}$ and grid oscillating frequency for $f=5\text{Hz}$, 7.5Hz and 10Hz were chosen in our experiments. The area (A) in the central region between two-oscillating grids was chosen to measure, as shown in Figure 1. PAM and CTAC, both as turbulent drag-reducing additives, are two types of popular additives in the researches of turbulent drag reduction in the wall-bounded flows, e.g., channel flow [3,6]. However, the mechanisms to induce the elastic property of these solutions, which is indispensable to the turbulent drag reduction, are different from each other. The elasticity of PAM solution is due to the existence of long-chain molecular structures while shear induced micelle structures (SIS) are responsible for CTAC solution. In the wall-bounded flows, though with different structures, these two types of additives can contribute equivalent drag-reducing effects even at very low concentration, i.e., 25ppm. The discussion of HIT with viscoelastic fluids can promote the understanding of turbulent drag reduction for these two types of additives in the turbulent flow not only without wall effect but also with wall effect. So in this paper, we chose PAM (with molecular weight of 1.8×10^7 g/mol, produced by Shanghai Huiye ChemE Tech. Ltd., China) and CTAC (with molecular weight of 320 g/mol, produced by Aladdin, China). In order to form steady shear-induced structures (SIS) in CTAC solution, we add sodium salicylate (NaSal, with molecular weight of 160.1 g/mol) with the same weight concentration as that of CTAC into the solution so as to provide counterions. The concentrations of drag-reducing solution were chosen as 25ppm and 50ppm. The working fluid temperature was $21 \pm 1^\circ\text{C}$. Reynolds number is defined as: $Re_M = 2\pi SM/v$, here v is the solvent viscosity. Due to small amount of additives added into solution, the density of solution is almost no change, as the same with that of distilled water. But, the present of drag-reducing additives can remarkably change the solution viscosity. So we used a stress-controlled rotational rheometer (Kinexus Pro, Malvern instruments, UK) to measure the viscosities for all cases, as shown in Figure 2. The results show that the viscosity of 25ppm CTAC/NaSal solution is approximate to that of distilled water, and the viscosity of 50ppm CTAC/NaSal solution does not increase much, which is similar with that in Ref. [22]. Meanwhile, the viscosity of 25ppm PAM solution does not enhance much compared with that of distilled water, but the viscosity of 50ppm PAM solution increases remarkably. However, in order to make comparison among these three cases, the viscosity of drag-reducing solution was still chosen as that of distilled water. So in our experiments, the Reynolds numbers are 1.8×10^4 , 2.8×10^4 and 3.6×10^4 corresponding to three different grid-oscillating frequencies.

Figure 1 The schematic diagram of the total apparatus, two-oscillating grid system and oscillating grid.

Figure 2 The change trend for shear viscosity vs. shear rate for distilled water, PAM solutions and CTAC/NaSal solutions

2.2 Particle image velocimetry system

A standard two-dimensional (2D) two-component PIV system was utilized to obtain velocity field in two-oscillating grid turbulence. Some parameters of the key components in the PIV system are introduced as follows: the double-pulsed Nd-YAG (YAG-yttrium aluminum garnet) lasers with an output of 200mJ/pulse and the trigger rate of 5 Hz; the CCD camera (FlowSense 4M EO Model81C92) with a resolution of 2048×2048 pixels and the recorded time between image 1 and image 2 with 800μs~4000μs for different cases; the seeding particles were hollow glass spheres with typical diameter of the order $\leq 20\mu\text{m}$, and each interro-

gation area had rough 10 particles; image processing software was Dynamic Studio (ver.3.20). Measurements were performed for water, PAM solution and CTAC solution flows at all oscillating frequencies. The PIV image covered an area of about $x \times y = 66 \times 66 \text{ mm}^2$. The interrogation area was set to be 32×32 pixels (with 50% overlap in each direction). The spacing between adjacent vectors in each direction was around $\Delta x = 0.50 \text{ mm}$ and $\Delta y = 0.50 \text{ mm}$. The Kolmogorov scale was estimated ($\eta \sim (v^3/\varepsilon)^{1/4}$) about 0.20~0.50mm, so small-scale structures can be well captured. Totally 1000 realizations of instantaneous velocity field for each run (water, PAM solution and CTAC solution flows) have been obtained from PIV images (2000 double-exposed PIV photographs). It was validated to remove erroneous velocity vectors, which might be detected incorrectly during interrogation owing to random noise in the correlation function [6]. For the Newtonian fluid and PAM solution flows, the observed erroneous velocity vectors were less than 2%, but for CTAC solution flows, air bubbles were much easier to be drawn into the solution, which affected the PIV measurement, especially for cases at high Reynolds number. The results showed that the observed erroneous velocity vectors were less than 6% in the CTAC solution. So a median filter with radius 50 pixels was sought to search erroneous velocity vectors. When erroneous velocity vectors were removed, empty data holes occurred in the field. Then, the filling holes in the vector field by interpolating neighboring velocity vectors. The above procedures were accomplished by Dantec Dynamic Studio (ver.3.20).

3 Results and discussion

3.1 Isotropic turbulent degree

As is known, HIT is the simplest class of turbulent flows, which is amenable to theoretical analysis and provides an essential building block to the understanding of more complex inhomogeneous flows. However, in practical point of view, HIT is hypothetical because no actual flows can satisfy the conditions of HIT. So in experiments, the best one can do is to generate the flows in which the conditions for HIT are more or less approximate. Some requirements for HIT are: $\langle u^2 \rangle = \langle v^2 \rangle = \langle w^2 \rangle$, $\langle uv \rangle = \langle uw \rangle = \langle vw \rangle = 0$ and $\langle u^3 \rangle = \langle v^3 \rangle = \langle w^3 \rangle = 0$, where u , v and w are fluctuating velocities in x , y and z directions.

Firstly, we investigate the above conditions for HIT. In our experiments, we only get the two-dimensional velocity (u , v). So here the results show that $\langle uv \rangle \approx 0$ and $\langle u^3 \rangle \approx \langle v^3 \rangle \approx 0$ for all cases. Then the flow isotropy ($\langle u^2 \rangle^{1/2} / \langle v^2 \rangle^{1/2}$) for all cases are listed in Table 1.

Table 1 Flow isotropy for two-oscillating grid turbulence for all cases

Case	Concentration (ppm)	Flow isotropy ($\langle u^2 \rangle^{1/2} / \langle v^2 \rangle^{1/2}$)		
		5Hz	7.5Hz	10Hz
Water	/	0.93	0.91	0.95
	25	0.67	0.84	0.76
PAM solution	50	0.79	0.98	0.84
	25	0.93	0.93	0.91
CTAC solution	50	0.96	0.98	0.96

From Table 1, for the Newtonian fluid case, the flow isotropy is 0.91~0.95, suggesting that isotropic turbulence can be approximately realized using this experimental setup. However, it shows more anisotropic in PAM solution case than that in the Newtonian fluid and CTAC solution cases. From this point, it suggests that the flows in PAM solution cases should show different trends. It will be discussed in detail later.

3.2 Distribution of velocity field

Firstly, we show the instantaneous velocity field based on PIV measurement at grid-oscillating frequency $f=10\text{Hz}$ for the Newtonian fluid, PAM solution and CTAC solution cases in Figure 3. It can be clearly seen that the distribution of velocity field in 50ppm PAM solution and 50ppm CTAC solution cases is more uniform than that in the Newtonian fluid case. Meanwhile, the small-scale vortex structures are inhibited remarkably in viscoelastic fluid cases (however, as shown in Figure 3(a), there exist many small-scale vortex structures in the Newtonian fluid case). However, the distribution of velocity field in 25ppm CTAC solution case is different from that in 25ppm PAM solution case, i.e., the small-scales structures continue to exist in 25ppm CTAC solution case which is similar with that in the Newtonian fluid case, but are far less apparent in 25ppm PAM solution case. From this point, it suggests that for PAM solution case, the influence mechanism on velocity field is different from that for CTAC solution case. In order to further elaborate the viscoelastic effect on the small-scale vortex structures, the vortex vector field is investigated. In this paper, we only obtain two-dimensional velocity field (u, v), so only vortex vector ω_z is calculated as $\omega_z = \partial v / \partial x - \partial u / \partial y$, as shown in Figure 4. It is clearly seen that in viscoelastic fluid cases, the large vortex vector is remarkably inhibited compared to that in the Newtonian fluid case, except for 25ppm CTAC solution case. These above phenomena also appear in the cases at $f=5\text{Hz}$ and $f=7.5\text{Hz}$. Thereinafter, we can deeply find the difference between PAM solution case and CTAC solution case.

(a) Newtonian fluid case

(b) 25ppm PAM solution case

(c) 50ppm PAM solution case

(e) 50ppm CTAC solution case

Figure 4 The instantaneous vortex vector field based on PIV measurement at grid-oscillating frequency $f=10\text{Hz}$.

3.3 Turbulent kinetic energy

In homogeneous isotropic turbulence (HIT), there only exists fluctuation velocity. The main energy is turbulent kinetic energy, which represents the flow strength in HIT. So turbulent kinetic energy is investigated here. Figure 5 shows the spatial distribution of statistically-averaged turbulent kinetic energy ($\xi = \frac{1}{2} \overline{u_i u_i}$, meaning time average

for each spatial point). Here, we only show the statistical distribution of turbulent kinetic energy at grid frequency $f=5\text{Hz}$ for Newtonian fluid, 50ppm PAM solution and 50ppm CTAC solution cases. It can be seen that for the Newtonian fluid case, most flow areas contain much larger turbulent kinetic energy than those in viscoelastic fluid cases. But the statistical distribution of turbulent kinetic energy in PAM solution case is much weaker than that in CTAC solution case. In other words, the existence of viscoelastic fluid makes the flow have much smaller turbulent kinetic energy and the flow in PAM solution case is different from that in CTAC solution case.

(a) the Newtonian fluid case

(b) 50ppm PAM solution case

50ppm CTAC solution case

Figure 5 Spatial distribution of statistically-averaged turbulent kinetic energy at grid-oscillating frequency $f=5\text{Hz}$.

In order to investigate the statistical characteristics of turbulent kinetic energy ($\xi = \frac{1}{2} \langle u_i u_i \rangle_{x,t}$, meaning en-

semble average), we show the change of turbulent kinetic energy with grid-oscillating frequency and solution concentration in Figure 6. From Figure 6(a), it can be clearly seen that for PAM solution case, the turbulent kinetic energy increases with the increasing grid-oscillating frequency. And the turbulent kinetic energy decreases with the increase of solution concentration, but it is not remarkable. From Figure 6(b), it is found that the turbulent kinetic energy for 25ppm CTAC solution case is almost the same as that in the Newtonian fluid case. Similarly, as the CTAC solution concentration increases, the turbulent kinetic energy increases at the same grid-oscillating frequency. It suggests that there exists a critical concentration for CTAC solution to show the viscoelastic effect. Based on the statistical analysis for turbulent kinetic energy, it is found that the turbulent kinetic energy decreases due to the addition of drag-reducing additives.

Figure 6 Statistical results of turbulent kinetic energy at three different grid-oscillating frequencies.

3.4 Reduction Rate of Turbulent kinetic energy

From Section 2.1, it is clearly shown that the existence of drag-reducing additives reduces the turbulent kinetic energy. This phenomenon is similar with that in wall-bound drag-reducing flows. Therefore, based on the decrease of the turbulent kinetic energy, we propose an intuitive and natural definition for the reduction rate of turbulent kinetic energy occurred in two-oscillating grid turbulence (i.e., FHIT), as shown here:

$$\text{RTKE}(\%) = \frac{\xi_{eq}^{[N]} - \xi_{eq}^{[V]}}{\xi_{eq}^{[N]}} \times 100\% \quad (1)$$

where ξ_{eq} represents the turbulent kinetic energy at the statistical equilibrium state for each case, respectively. Here, superscripts N and V represent the Newtonian fluid and the viscoelastic fluid cases, respectively.

Figure 7 The reduction rate of turbulent kinetic energy at three different grid-oscillating frequencies.

Therefore, based on Eq. (1), the reduction rate of turbulent kinetic energy for PAM solution and CTAC solution cases at different solution concentrations and grid frequencies are shown in Figure 7. From Figure 7(a), it shows the remarkable drag-reducing effect for PAM solution cases: the reduction rate of turbulent kinetic energy increases with the increase of solution concentration and the maximum reduction rate reaches approximately 65%. For CTAC solution cases as shown in Figure 7(b), the reduction rate shows the different trend compared to that in PAM solution cases. For 25ppm CTAC solution case, it shows only 10% reduction rate at $f=5\text{Hz}$, but shows no reduction effect at $f=7.5\text{Hz}$ and 10Hz . Compared to 25ppm PAM solution case (the maximum reduction rate around 60% and the minimum reduction rate over 35%), there exists almost no reduction effect for 25ppm CTAC solution case. It suggests that there exists a critical concentration for initial reduction of turbulent kinetic energy in CTAC solution case. So the drag-reducing mechanisms for PAM solution cases and CTAC solution cases are also different in HIT without wall effect. As is known, the drag-reducing effect for CTAC solution cases is caused by the formation of shear-induced structure (SIS) of

CTAC micelles, while it is caused by long-chain polymer molecular structures in PAM solution cases. Therefore, in 25ppm CTAC solution case, it is difficult to form SIS due to the low concentration without wall effect so that the reduction effect is not remarkable. With the increase of CTAC solution concentration, the reduction rate of turbulent kinetic energy increases due to the formation of SIS. Therefore, based on the above analysis, it suggests that there also exists the reduction rate of turbulent kinetic energy in two-oscillating grid turbulence without wall effect due to the addition of drag-reducing additives.

4 Conclusions

In this paper, in order to investigate the viscoelastic effect on the flow without wall effect, we built two-oscillating grid turbulence so as to realize approximate isotropic turbulence and analyze the turbulent characteristics based on PIV experimental data. Some important conclusions are drawn as follows:

(1) The small-scale vortex structures are inhibited due to the existence of viscoelastic fluid. But compared to CTAC solution case, the inhibition effect is different in PAM solution cases due to different drag-reducing structures, i.e., long-chain molecular structures for PAM and SIS for CTAC.

(2) For viscoelastic fluid cases, the turbulent kinetic energy increases with the increase of grid-oscillating frequency and decreases with the increase of solution concentration. Based on the statistical analysis of turbulent kinetic energy, the results suggest that the turbulent kinetic energy decreases due to the addition of drag-reducing additives.

(3) An intuitive and natural definition for the reduction rate of turbulent kinetic energy is proposed and the results show that the drag-reducing effect also appears in two-oscillating grid turbulence without wall effect. But for 25ppm CTAC solution case, it shows almost no reduction effect. This phenomenon can be explained due to the fact that it is difficult to form SIS without wall effect in so low concentration solution case.

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