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Transient flows past arrays of yawed finite prisms

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Abstract

Transient flows past arrays of yawed finite length prisms in tandem and staggered arrangements are simulated. Large Eddy Simulations (LES) method is employed to characterize spatial and temporal characteristics of flows in the vicinity of prisms. Drag coefficients of prisms are determined for different spacing and yaw angle of prisms. Three-dimensional effects are investigated for different spacing of prisms in a tandem array. Drag coefficient of downstream prism is significantly lower compared to that of upstream prism when prisms are tightly spaced. Drag coefficient of downstream prism predicted by simulations in three-dimensional geometry is recovered fully as spacing approaches ten times of the height of the prism while drag coefficient predicted by simulations in two-dimensional geometry is only about 30% of the upstream prism even when two prisms are separated by twenty times the height of the prism. The wake flow patterns do not influence the drag coefficient strongly for the staggered arrays of yawed prisms. Prisms can be placed in a tighter arrangement without a significant penalty of drag reduction. Mathematical models and numerical methods employed are validated by comparing simulation results against experimental results. Drag exerted on these prisms can directly be related to power generation by devices containing translating blades of marine-current applications. This study demonstrates that arrangement, spacing and yaw angle can be used to design and optimize energy harvesting devices and other offshore structures.

Keywords

large eddy simulations; drag coefficient; tandem and staggered arrays; yawed prisms; turbulent wake flows

1. Introduction

Flows past multiple bluff bodies are encountered in many engineering applications, such as chimney stacks, buildings, bridges, and offshore structures. Offshore structures like marine current energy harvesting devices are often placed in multiple units with an array configuration. Turbulent wake flows generated by upstream units will increase loading of downstream units and thus can compromise the durability of structures. Understanding the solid-fluid interactions in flows past structures is important to design and optimize marine current devices.

Flows past a single infinite prism have been well studied both experimentally and numerically [1-9]. When multiple prisms are in the field with close proximity of each other, flow structures become more complex due to the interactions between wake flows and prisms. Prisms situated downstream interfere with the wake flow structure induced by upstream prisms. This interaction has profound influence on the drag and the lift forces exerted on prisms. Flows past multiple prisms in tandem arrangement are classical flow problems in engineering applications and have been studied by many investigators [6,10-12].

It is well-documented that flow patterns are strongly dependent on the aspect ratio, G/D , where G is the gap between two successive infinite prisms and D is the diameter of the prism. Different flow regimes are encountered depending on values of G/D . Three types of wake behavior are classified [13-15]. At low values of G/D , the vortex shedding from upstream prism is suppressed. Two prisms act like a single body since the shear layer separated from upstream prism wraps around the downstream prism. At intermediate values of G/D , the vortex shedding from downstream prism synchronized with shedding from the upstream prism. At large values of G/D , the Karman vortex street generated from both upstream and downstream prisms does hardly interfere with each other. As described in Auteri et al. [15], critical values of G/D can be

identified as wake flow transitions from one pattern to another. Drag and lift coefficient and Strouhal number display rapid changes at around the critical values of G/D , as reported in Sakamoto et al. [6] and Liu and Chen [10]. Several investigators have reported hydrodynamic loads in flows past arrays of tandem cylinders for various values of G/D . An extensive review on this topic is given by Sumner [13]. For low and intermediate values of G/D , C_D of the upstream cylinder in an array is lower than C_D of the single cylinder at the same Re . C_D of the upstream cylinder decreases as G/D is increased. At values of G/D near criticality, C_D of the upstream prism increases rapidly. Sudden increase in C_D coincides with wake flow pattern changes induced by the Karman vortex shedding. Vortex shedding set off from both upstream and downstream prisms for G/D around the critical value. As G/D is increased beyond the critical value, C_D gradually approaches the value of C_D in flows past a single prism. Generally speaking, C_D of the downstream prism is lower than that of the upstream prism due to wake effects [12].

Aforementioned studies consider prisms whose axes are perpendicular to the oncoming flow. Flows past an infinite yawed prism are also well-documented. Shirakashi et al. [16] conducted experiments in flows past a yawed cylinder, and reported that the vortex shedding become irregular and the vortex shedding frequency is lower for the yawed cylinder compare to that for the perpendicular cylinder. Numerical investigations for flows past an inclined cylinder reported by Yeo and Jones [17] concluded the swirling flow around the cylinder is responsible for generating low-frequency loadings. Zhao et al. [18] performed direct numerical simulation to study the effects of the yaw angle on wake structures, vortex shedding frequencies, and hydrodynamic forces. They have documented that vortices shed from the cylinder propagate along the axis of the cylinder axis while they are convected downstream. They also reported that the drag coefficient increases gradually as the incline angle is increased. The yaw angle in these

studies is defined as the angle between the oncoming flow direction and the direction perpendicular to the axis of the prism. Jordan [19] investigated flow past yawed cylinders at the angle of 45° and 60° as referenced to the flow direction. Jordan [19] characterized the flow transition to turbulence in the wake of yawed cylinders.

Despite of extensive studies in flow past infinite prisms (cylinders), many of actual structures should be modeled as a finite rectangular prism with free ends. Several investigators studied flows past a finite length prism or a cylinder with one free end [20,21]. Sumner [22], recently reported a comprehensive review on the subject. To date, there are not many investigations considering flows past multiple bluff bodies with two free ends.

Flow patterns in the wake of finite prisms with free ends are fully three-dimensional. The wake flow field is characterized by both the alternating Karman vortex shedding that occurs along the prism, and the tip vortices formed at free ends [23]. Wang et al. [20] experimentally studied interactions between the boundary layer and the wake of wall-mounted finite square prism. They concluded that highly three-dimensional flow patterns around a finite square prism induced by the Karman vortex along the surface of the prism and tip and base vortices within the wake. Rostamy et al. [24] conducted experiments for finite square prisms of various length to diameter (L/D) ratios. Their results showed aspect ratio has profound influence in the velocity field around free ends. The wake is dominated by tip vortices at lower values of L/D , while for higher values of L/D , the Karman vortex shedding from the prism sides dominates the wake flow pattern.

The drag coefficient of finite length cylinder for different values of L/D has been reported extensively. C_D is decreased as L/D is decreased [21,25-27]. Zdravkovich et al. [25] carried out

experiments on a finite length circular cylinder with two free ends for various values of L/D . It is observed that C_D decreases with decreasing L/D . Inflow around cylinder ends causes an increase of pressure at the back side of the cylinder, which in turn reduces the drag coefficient. Strouhal number (St) varies with L/D . Vortex shedding behind the prism becomes more irregular due to the presence of unstable tip vortices. Wang and Zhou [27] experimentally studied the near wake of a finite length square cylinder. They reported that the spatial wake configuration is dependent on the interaction between the Karman vortex shedding (asymmetric flow pattern) and the tip vortices (symmetric flow pattern). As L/D ratio is decreased, tip vortices dominate wake flow patterns. Symmetrical vortex patterns correspond to weaker stresses, which lead to lower C_D compared against infinitely long cylinders. They concluded that the presence of tip vortices contributes to the decrease of C_D monotonically with a decrease of L/D . There are only few studies for flows past finite yawed prisms. As reported by Hu et al. [28], the flow field in the vicinity of a finite prism differs from that of an infinite prism. LES simulations presented by Hu et al. [28] revealed that the drag coefficient of the finite yawed prism decreased as the yaw angle (as specified an angle from the vertical axis) is increased. Down-wash and up-wash flows behind the yawed cylinder resulted in downward and upward axial flows. Vortex pairs at the free end also cause a decrease in the shedding frequency.

The present study focuses on the characterization of transient flows past arrays of yawed finite prisms in different arrangements. Simulations employing the LES model are carried out to study wake flow patterns and their influence on drag coefficients in flows past a single and multiple finite length prisms with two free ends. Drag coefficient of prisms in various arrays is presented. The aspect ratio, G/D , and the yaw angle are varied and their influence on the flow structures is presented. To the best knowledge of present authors, flows past arrays of yawed

finite prisms with tandem and staggered arrangements have not been considered previously. The present study can aid designing and optimizing marine current energy harvesting devices by characterizing wake flow patterns and their influence on the forces exerted on structures.

NOMENCLATURE

A	projection area	Δt	time step size
C_D	drag coefficient	u_i, u_j	velocity
C_L	lift coefficient	$\bar{u}_i, \bar{u}_j$	resolved scale velocity vector
C_p	pressure coefficient	U_∞	free stream velocity
C_w	WALE constant	V	volume of cell
D	prism height	w	prism width
d	tip to the centerline distance	ω	vorticity magnitude
F_D	drag force	x_i, x_j	position vector
F_L	lift force	y	distance to the nearest wall
f	vortex shedding frequency		
G	spacing distance		
h	square cylinder height		
L	prism length		
L_s	SGS mixing length		
m_1, m_2, m_3	number of cells		
$\bar{p}$	resolved pressure		
p_∞	reference pressure		
Re	Reynolds number		
SGS	subgrid scale		
St	Strouhal number		
$\bar{S}_{ij}$	rate-of-strain tensor		
t	time		

GREEK SYMBOLS

λ	non-dimensional time
$\Delta\lambda$	non-dimensional time step
θ	yaw angle
δ_{ij}	Kronecker delta
μ	dynamic viscosity
ν	kinematic viscosity
ν_t	SGS eddy viscosity
ρ	Density
κ	von Karman constant
τ_{ij}	subgrid stress tensor

2. Mathematical Model

Large eddy simulation approach applies a spatial filtering to the Navier-Stokes equation. The large-scale eddies are resolved numerically and the smaller-scale eddies – eddies that are smaller than the grid size - are not resolved but are approximated. Resolving the large-scale

127 motions enables LES to predict instantaneous flow characteristics and to resolve turbulent flow
128 structures.

129 Time and spatial averaged Navier-Stokes equations with low pass filtering yield

$$130 \quad \frac{\partial \bar{u}_j}{\partial x_j} = 0 \quad (1)$$

$$131 \quad \frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_j \bar{u}_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \nu \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} \quad (2)$$

132 where $\bar{u}_j = u_j - \dot{u}_j$ represents the filtered velocity vector and the over bar denotes filtering
133 operator; ρ is the fluid density; and $\bar{p}$ is the resolved pressure.

134 The subgrid shear stress $\tau_{ij} = \overline{u_i u_j} - \bar{u}_i \bar{u}_j$ is calculated using the subgrid-scale (SGS)
135 models. Employing the Boussinesq hypothesis the SGS turbulent stress is computed from

$$136 \quad \tau_{ij} - \frac{1}{3} \tau_{kk} \delta_{ij} = -2\nu_t \bar{S}_{ij} \quad (3)$$

137 where δ_{ij} is the Kronecker delta; ν_t is the SGS eddy viscosity; and $\bar{S}_{ij} = \frac{1}{2} \left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)$ is the
138 rate-of-strain tensor.

139 The wall-adapting local eddy-viscosity (WALE) method is used to model the SGS eddy-
140 viscosity [29]:

$$141 \quad \nu_t = L_s^2 \frac{(S_{ij}^d S_{ij}^d)^{3/2}}{(\bar{S}_{ij} \bar{S}_{ij})^{5/2} + (S_{ij}^d S_{ij}^d)^{5/4}} \quad (4)$$

$$142 \quad S_{ij}^d = \frac{1}{2} \left(\left(\frac{\partial \bar{u}_i}{\partial x_j} \right)^2 + \left(\frac{\partial \bar{u}_j}{\partial x_i} \right)^2 \right) - \frac{1}{3} \delta_{ij} \left(\frac{\partial \bar{u}_k}{\partial x_k} \right)^2 \quad (5)$$

$$143 \quad L_s = \min(\kappa y, C_w V^{1/3}) \quad (6)$$

Here L_s is the mixing length for subgrid scales; κ is the von Karman constant; y is the shortest distance to the wall; the WALE constant C_w is 0.325; and V is the mesh cell volume. Details of the WALE model are documented by Nicoud and Ducros [29].

3. Computational domains

LES simulations for flows past prisms are conducted at $Re = 50,000$. Reynolds number is defined as $Re = \rho U_\infty D / \mu$, where U_∞ is the free stream velocity, D is the prism height, and μ is the fluid dynamic viscosity. The schematic of the three-dimensional computational domain is shown in Fig. 1(a)-Fig. 1(c) while the top view (x - z plane) are shown in Fig. 1(d)-Fig. 1(e). Fig. 1(a)-Fig. 1(d) shows prisms that are arranged in a tandem array and situated perpendicular to the oncoming flow. Fig. 1(c)-Fig. 1(e) shows prisms that are arranged in a staggered array and placed against the oncoming flow with a yaw angle, θ . The yaw angle in the present study is defined as the angle between the axis of prism and the oncoming flow direction. The geometry of a single prism is depicted in Fig. 1(b). The length, height, and width of the prism are L , D , and w , respectively. Spacing between two successive prisms, G , and the yaw angle, θ , are illustrated in Fig. 1. For staggered arrays, distance from prism tip to the centerline is defined as e . Simulations for flows past staggered arrays of prisms are conducted for fixed value of e . Prisms in tandem arrays are labeled as Pi while prisms in staggered arrays are labeled as Ai. Prisms are numbered in an order from upstream to downstream. Values of physical properties and geometric and flow parameters used in the present simulations are listed in Table 1. Table 2 lists different prism array configurations. Each configuration has different spacing G , yaw angle, θ , and number of prisms in the arrangement.

Fig. 1. Schematic diagram of the prism arrays (a) in a tandem arrangement (c) in a staggered arrangement (d) in a tandem arrangement illustrated at $y = 0$ plane (e) in a staggered arrangement illustrated at $y = 0$ plane (b) Schematic of a single prism geometry.

Table 1. Parameters used in the present study.

Parameter Values					
D	0.1	[m]	U_{∞}	0.5	[m s ⁻¹]
e	0.2	[m]	w	0.01	[m]
L	1	[m]	ρ	1000	[kg m ⁻³]
Re	50,000	[-]	μ	0.001	[kg m ⁻¹ s ⁻¹]

Table 2. Various array arrangements of prisms.

	Spacing	Angle	Prism label							
	G	θ	P1	P2	A1	A2	A3	A4	A5	A6
Single prism in different angles										
Case 1	-	45°	-	-	✓	-	-	-	-	-
Case 2	-	60°	-	-	✓	-	-	-	-	-
Case 3	-	90°	-	-	✓	-	-	-	-	-

Two tandem prisms with different gaps									
Case 4	5D	90°	✓	✓	-	-	-	-	-
Case 5	7D	90°	✓	✓	-	-	-	-	-
Case 6	10D	90°	✓	✓	-	-	-	-	-
Case 7	20D	90°	✓	✓	-	-	-	-	-
Multiple staggered prisms with different gaps and angles									
Case 8	10D	45°	-	-	✓	✓	-	-	-
Case 9	10D	60°	-	-	✓	✓	-	-	-
Case 10	10D	45°	-	-	✓	✓	✓	✓	-
Case 11	2.5D	45°	-	-	✓	✓	✓	✓	-
Case 12	10D	60°	-	-	✓	✓	✓	✓	-
Case 13	5D	45°	-	-	✓	✓	✓	✓	-
Case 14	10D	45°	-	-	✓	✓	✓	✓	✓

The size of the computational domain varies among cases based on the configuration. For examples: dimensions of the domain for case 5 is $52.5D \times 16D \times 16D$ while dimensions for case 11 is $60D \times 16D \times 30D$ in the stream-wise, the cross stream, and the span-wise direction, respectively. For all cases, the length of the inlet region - distance from the inlet to the closest upstream prism – is selected as $12D$ and the distance from the outlet to the closest downstream prism is selected as $30D$ to minimize the influence of the imposed inlet and the outlet velocity boundary conditions on the flow field near prisms.

4. Numerical Methods, Convergence and Validation Studies

ANSYS FLUENT commercial software is utilized to conduct simulations. The semi-implicit method for pressure-linked equations (SIMPLE) solution algorithm that fully couples the velocity and the pressure field is employed to solve the mathematical model. The bounded central differencing and bounded second order implicit scheme are used for discretizing momentum equation and time, respectively. Detailed descriptions of numerical schemes and simulation tools are provided in ANSYS theory guide and ANSYS user guide [30,31].

The boundary conditions for simulations in ANSYS FLUENT are as follows. Mass flow rate is prescribed at the inlet while zero gauge pressure is applied at the outlet. No-slip and no-penetration boundary conditions are imposed along surfaces of prisms and top and bottom surfaces of the channel. Symmetry boundary conditions are imposed on the side surfaces of the computational domain.

OpenFOAM, an open source CFD software package, is used here to validate the numerical model implemented in ANSYS FLUENT. Flow geometry depicted in case 3 and case 5 are considered for validation study. The “pimpleFoam” solver that combines the Pressure-Implicit with Splitting of Operators (PISO) and SIMPLE algorithms are employed. An adjustable time step function with a constraint in Courant number in pimpleFoam solver is employed to enhance numerical stability. Details of the methods are documented by Greenshields [32].

Spatial and Temporal Convergence

Mesh structures for cases 5 and 11 are shown in Fig. 2. Finer mesh is used in regions near the surface of prisms to capture boundary layer separations and to resolve near wake flow patterns. Mesh becomes gradually coarser away from prisms, as depicted in Fig. 2. A mesh optimization study is conducted for case 5. Meshes of different density generated are: 1.2×10^6 elements (m_1), 2.4×10^6 elements (m_2), and 5.7×10^6 elements (m_3). The main difference between mesh m_1 and mesh m_2 is an increase of number of cells in the region near prisms. The average value of drag coefficient for each prism in the array is calculated for m_1 , m_2 and m_3 . Drag coefficient is defined by $C_D = F_D / \left(\frac{1}{2} \rho U_\infty^2 A \right)$, where F_D is the drag force exerted by fluid on the prism and A is the area of the prism projected perpendicular to the oncoming flow. Non-

dimensional time and non-dimensional time step are defined as $\lambda = tU_\infty/D$ and $\Delta\lambda = \Delta tU_\infty/D$, respectively.

Fig. 2. Examples of mesh around prisms for (a) case 5 and (b) case 11. Images are taken at $y = 0$ plane.

The drag coefficient calculated by using FLUENT and OpenFOAM solver for case 5 is shown in Fig. 3 as a function of time. $\Delta\lambda = 0.004$ is used for simulations conducted with FLUENT while adjustable time step is used for simulations conducted with OpenFOAM. Graphs on the left column denote drag coefficient of the upstream prism P1 and graphs on the right column denote drag coefficient of the downstream prism P2. Rows from top to bottom denote time signature of drag coefficient obtained using m_1 , m_2 and m_3 mesh, respectively. The last two rows show drag coefficients predicted by OpenFOAM solver using m_1 and m_3 mesh density. Presence of irregular fluctuations in drag coefficient signal, as shown in Fig. 3(a) and Fig. 3(d), indicates alternating flow patterns in the wake region. Similar characteristics have been reported

in flows past a normal prism [5,33]. In order to demonstrate if these flow patterns alternate periodically or irregularly simulations needs to be carried out with much larger flow times. Due to limited computational resources present simulations are halted at $\lambda = 700$. High frequency fluctuations are detected in the signature of C_D for P2. This is due to the fact that oncoming flows for P2 are highly turbulent wake flows induced by the presence of P1 in the field.

Fig. 3. Drag coefficient as a function of time obtained with mesh (a) m_1 , (b) m_2 , and (c) m_3 by using FLUENT and obtained with (d) m_1 and (e) m_3 by using OpenFOAM.

Time averaged values of drag coefficient, $\overline{C_D}$, and standard deviation of fluctuations predicted by FLUENT and OpenFOAM are listed in Table 3. Time steps and mesh density used are also listed in Table 3. Strouhal number is determined by performing fast Fourier Transform

to the time signature of the lift coefficient of P1 and P2. Strouhal number predicted using m_1 , m_2 and m_3 are around 0.096; implying that time signature of hydrodynamic loadings is not sensitive to the mesh density. Strouhal number is defined by $St = fD/U_\infty$, where f is the frequency of fluctuating lift coefficient, C_L . Lift coefficient is defined by $C_L = F_L / \left(\frac{1}{2} \rho U_\infty^2 A \right)$, where F_L is the lift force exerted by fluid on the prism. The lift force acting on finite prisms has more complicated dynamics when compared to that on infinite prisms. Smaller amplitude fluctuations are observed in the time signature of drag coefficient for P1 compared to that for P2. That is attributed to the turbulent nature of induced wake flows as oncoming flows for P2 (see Fig. 3). Time averaged value of C_D for m_1 , m_2 and m_3 are similar, as listed in Table 3. Drag coefficient for m_1 are also very close at $\Delta\lambda = 0.004$ and at $\Delta\lambda = 0.012$. Due to the irregular dynamics rms values listed in Table 3 are relatively large, especially for P2 prism. It is demonstrated here that grid m_1 and time step of 0.004 are sufficient to ensure spatial and temporal convergence. Results presented in the present paper are obtained using mesh m_1 and $\Delta\lambda = 0.004$. Stronger claim for spatial and temporal convergence could be made by refining the mesh further and using smaller time step. The level of computation and available computational resources make that to be a challenging task.

Table 3. Mean drag coefficient and rms value of P1 and P2.

Case No.	Mesh Size	Solver	Time step $\Delta\lambda$	P1		P2	
				$\bar{C}_D$	C_{Drms}	$\bar{C}_D$	C_{Drms}
5	m_1	FLUENT	0.004	1.46	0.06	1.12	0.30
5	m_1	OpenFOAM	0.01	1.54	0.09	1.23	0.30
5	m_2	FLUENT	0.004	1.48	0.04	1.13	0.20
5	m_3	FLUENT	0.004	1.48	0.04	1.26	0.26
5	m_1	FLUENT	0.012	1.49	0.04	1.22	0.21
5	m_3	OpenFOAM	0.0025	1.45	0.03	1.17	0.27
3	m_1	FLUENT	0.004	1.51	0.04	-	-
3	m_1	OpenFOAM	0.007	1.47	0.04	-	-

Table 4 lists the maximum value of Courant number and y^+ values of P1 and P2 for test cases. Courant number, $C = u\Delta t/\Delta x$, is a non-dimensional stability indicator that shows if solution satisfies the Courant-Freidrichs-Lewy (CFL) condition of $C \leq 1$. This condition ensures the numerical stability of simulations. Courant number in all test cases is bounded by 1.67 and the maximum value is only observed at few cells in the vicinity of prisms. y^+ value of P1 and P2 for m_1 , m_2 , and m_3 in case 5 are ranging from 8 to 38. Smaller value of y^+ indicates a better quality of mesh. Mean drag coefficient of both prisms predicted using m_1 and m_3 mesh density are very similar. Even though m_3 provides a better quality of mesh, grid m_1 is selected to conduct simulations here to reduce computational cost.

Table 4. Maximum value of Courant number and y^+ of P1 and P2.

Case No.	Mesh Size	Solver	Time step Δt	C_{max}	P1	P2
					y^+	y^+
5	m_1	FLUENT	0.004	0.34	30.03	33.65
5	m_1	OpenFOAM	0.01	0.99	22.91	28.02
5	m_2	FLUENT	0.004	0.79	10.09	26.07
5	m_3	FLUENT	0.004	1.67	8.47	8.04
5	m_3	OpenFOAM	0.0025	0.99	7.98	7.63
5	m_1	FLUENT	0.012	1.11	31.95	38.01
3	m_1	FLUENT	0.004	0.57	10.63	-
3	m_1	OpenFOAM	0.007	0.99	8.97	-

Flow patterns and drag coefficients predicted by FLUENT and OpenFOAM solver for case 3 and case 5 are compared. Isosurfaces of vorticity, contours of vorticity and velocity are depicted in Fig. 4 for flows past two finite tandem prisms (case 5). Isosurfaces of vorticity are presented at the Q-criterion of 0.01. The Q-criterion is the second invariant of the velocity gradient tensor that describe the vortex core regions (see Ref [34] for detailed description). The Q-criterion is calculated by $Q = -0.5(\|S\|^2 - \|\Omega\|^2)$, where S and Ω are the strain and the rotation tensor, respectively. Isosurfaces of the Q-criterion is colored with the normalized

vorticity magnitude, $\omega D/U_\infty$, where ω is the vorticity magnitude. Images on the left column denote flow patterns predicted by FLUENT solver while images on the right column denote flow field predicted by OpenFOAM solver. Flow characteristics obtained by both solvers are very similar. Similar results are obtained for flow past a single prism (images not shown here). The time signature in drag coefficient predicted by both solver is also very similar, as shown in Fig. 3. Drag coefficients predicted by FLUENT and OpenFOAM differ less than 6% for P1 and less than 9% for P2 for case 5. The difference in the drag coefficient is less than 3% for case 3. Flow field and hydrodynamic loading predicted by each solver is nearly the same, validating the numerical methods employed here.

Fig. 4. (a,b) Isosurface of vorticity at $Q = 0.01$, (c,d) contours of vorticity, and (e,f) contours of velocity acquired at $y = 0$ plane for case 5. Results are obtained by using (a,c,e) ANSYS FLUENT and by using (b,d,f) OpenFOAM solver.

Validation study

A simulation is conducted for flows past two tandem square cross-sectioned prisms in order to validate mathematical models and numerical methods employed. Results are compared against experimental results reported by Liu and Chen [10]. The schematic of the computational domain used in the validation study is depicted in Fig. 5(a) with the side view shown in Fig. 5(b).

The upstream and downstream prisms are labeled as S1 and S2, respectively. The origin of the computational domain is located at the center of S1. The height of cylinders is defined as h , where h is 0.2m. The spacing between cylinders from center to center is $4.25h$. Inlet is located at $18h$ upstream of origin and the outlet is located at $37h$ downstream of the origin. The height of the computational domain is $15.25h$ and the length in the span-wise direction is $10h$. Reynolds number is set to 2700.

Fig. 5. Schematic diagram of flows past two tandem square cross-sectioned prisms in (a) a three-dimensional view (b) a side view.

Drag and lift coefficients as a function of time and the pressure coefficient along the surface of the prisms are depicted in Fig. 6. The hysteresis phenomenon induced by flow transition is observed at this spacing of prisms, as reported by Liu and Cheng [10]. Hydrodynamic loadings on prisms changes drastically as flow transition occurs. Flow characteristics referred to as Mode I and Mode II are observed at different stages. Mode I is only observed at the early stage of the flow while Mode II becomes persistent after the onset of flow

transition. The vorticity contours acquired at $y = 0$ plane demonstrate both flow patterns associated with the Mode I and the Mode II, as depicted in Fig. 7. Similar to the observations, flow patterns associated with the Mode I occurs at the earlier stage then the flow pattern switch to Mode II following the flow transition. Flow patterns predicted by the present study match qualitatively with those reported in Ref [10] for each mode. Drag coefficient of S1 prism for the flow Mode I is obtained as 1.67 while it is 2.07 for the flow Mode II. Drag coefficient of S2 prism for the flow Mode I and II are -0.03 and 0.8, respectively. These values match well with those documented in Ref [10]. Drag coefficient values reported in Liu and Cheng [10] for S1 prism is around 1.6 for the flow Mode I and 1.9~2.1 for the flow Mode II. C_D of S2 is around -0.4 for the flow Mode I and 0.7 for the flow Mode II. Slight deviations in drag coefficient for S2 prism can be attributed to the fact that finer mesh might be needed to determine complex unsteady wake flow structures near S2 prism, see Fig. 7. Lift coefficients of both prisms transition to larger amplitude fluctuations as flow transition from the Mode I to the Mode II.

The pressure coefficient, C_p , is defined by $C_p = (p - p_\infty) / \left(\frac{1}{2} \rho U_\infty^2 \right)$, where p_∞ is zero gauge pressure. The pressure is significantly greater along the front face of the prism S1 and it decreases rapidly to a lower value at the side and back faces for both flow Mode I and II. The distribution of mean pressure coefficient predicted here along the surfaces of S1 and S2 are in good agreement with experimental results reported in Ref [10]. Overall, the hysteresis phenomenon, hydrodynamic loadings on prisms, and flow structures observed by Liu and Cheng [10] are predicted by the current study; validating mathematical models and numerical methods employed.

Fig. 6. (a,b) Drag coefficient and (c,d) lift coefficient as a function of time, and (e,f) distribution of pressure coefficient along surfaces of prisms. Force coefficients are calculated for (a,c,e) S1 prism and for (b,d,e) S2 prism.

Fig. 7. Instantaneous vorticity contour acquired at $y = 0$ plane for (a) flow Mode I at 6.2 sec and for (b) flow Mode II at 16.6 sec.

5. Results and Discussions

5.1. Flows past an infinite and a finite prism

The drag coefficient of a finite prism is vastly different compared to that of an infinitely long prism, as documented by several investigators. It is also well-documented that drag coefficient of a finite prism strongly depend on the aspect ratio L/D . Transient simulations in three-dimensional geometry are conducted for flows past an infinitely long prism. Drag coefficient measured by Fage and Johansen [3] is 2.13 and predicted by Najjar and Vanka [4] and Tian et al. [5] is 2.26 and 2.30, respectively. The drag coefficient predicted by the current study is 2.22, which agrees well with those documented by previous investigators. As a base case (case 3), flows past a finite length prism with L/D of 10 are simulated. The mean drag coefficient is calculated to be 1.5, which is 32% smaller compared to that of an infinitely long prism. The tip vortices spawned from the prism's free ends make the flow highly three-dimensional. Fluid past

free ends and wrap around the back of the prism. Such flow alters the pressure field and leads to a reduction in drag coefficient. Isosurface of vorticity at $Q = 0.01$, and the vorticity contour in flows past a finite and an infinite prism are depicted in Fig. 8. Drag coefficient determined for case 3 is used as a reference in flows past tandem prisms.

Fig. 8. (a,b) Isosurface of vorticity at $Q = 0.01$ and (c,d) contours of instantaneous vorticity at $y = 0$ plane. Images are acquired for (a,c) a finite prism (b,d) an infinite prism.

5.2. Flows past prisms in tandem arrays

Cases 4-7 represent two tandem prisms in arrays with different spacing G/D . Simulations in two-dimensional and three-dimensional geometries are conducted for the same G/D , and results are shown in Fig. 9. The x - y plane is used in simulations for a two-dimensional geometry. Drag coefficient predicted in three-dimensional flows differ drastically compared to those predicted in two-dimensional flows. Solid symbols denote drag coefficient for P1 while hollow symbols denote drag coefficient for P2. Circles designate drag coefficients predicted by two-dimensional transient LES simulations and squares designate drag coefficients predicted by three-dimensional simulations. It is documented that two-dimensional flow models over predict drag coefficient [1,4]. The drag coefficient of downstream prism, P2, is lower than that of P1. Intense turbulent wake flows induced by P1 influence the drag coefficient of P2 strongly for low values of G/D . As G/D is increased the influence of wake decreases and drag coefficient of downstream prism increases. Drag coefficient of both prisms predicted by three-dimensional simulations become nearly the same for G/D of 10. On the other hand, drag coefficient predicted by two-dimensional simulations for downstream prism rebound very little even for $G/D = 20$, as depicted in Fig. 9. It is clear that there is a flow transition occurring as G/D is varied. The critical value of G/D for the onset of flow transition is around 7. The flow transition manifests itself by rapid rebound in drag coefficient of the downstream prism P2. Obviously, two-dimensional flow modeling fails to capture such flow transitions or it predicts the flow transition at values of G/D much greater than 20. Characteristics of flow transition can be identified better by examining flow structures near prisms.

Fig. 9. Mean drag coefficient of P1 and P2 predicted by two- and three-dimensional simulations for various values of G/D .

Isosurface of vorticity, based on Q-criterion, and contours of vorticity in flows past two tandem prisms with different spacing are depicted in Fig. 10. Contours of magnitude of vorticity are depicted at $y = 0$ plane for $G/D = 5, 7, 10$ and 20 . Streamlines originating from upstream of P1 are superimposed on images to better understand flow structures near prisms. Black lines in Fig. 10 denote streamlines. Streamlines originating from the mid-plane ($y = 0$) does not penetrate the wake of prisms. It should be noted that flows studied here are three-dimensional and streamlines from other planes will fill the wake region. Flow characteristics near two tandem prisms are strongly influenced by the spacing between prisms, as also reported by Sohankar [12]. When two prisms are apart by $5D$, the downstream prism, P2, is shielded by the shear layers induced by the upstream prism, P1. The intensity of vorticity field downstream and upstream of P2 is similar, hence it is expected that the pressure difference between the upstream and

388 downstream face of P2 is smaller. This translates to radical drop in drag coefficient of this prism.
389 For $G/D = 7$, shear layers formed from P1 reattach onto P2, upstream and downstream of P2
390 displays noticeable difference in flow patterns. As a result the drag coefficient of P2 rebounds, as
391 illustrated in Fig. 9. For $G/D = 10$, both prisms start generating the Karman vortex streets
392 individually. Vortices generated from P1 still impinge on P2 periodically, as shown in Fig. 10(e)
393 and Fig. 10(f). The intensity of vortices impinging on P2 is decreased significantly for $G/D = 20$.
394 The drag coefficient of both P1 and P2 becomes nearly the same for $G/D = 20$ (see Fig. 9).

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396 Fig. 10. (a,c,e,g) Isosurface of vorticity at $Q = 0.01$ and (b,d,f,h) contours of instantaneous
 397 vorticity in flows past two tandem prisms for different values of G/D . Prisms are separated by
 398 $5D$ (case 4), $7D$ (case 5), $10D$ (case 6), and $20D$ (case 7), as illustrated in the figure.

5.3. *Flows past an yawed prism*

Flows past a single prism for different values of the yaw angle are simulated. Drag coefficient for the yaw angle of 45° (case 1), 60° (case 2), and 90° (case 3), is calculated to be 1.08, 1.42, and 1.50, respectively. Isosurface and contours of instantaneous vorticity for these three cases are depicted in Fig. 11. Contours are acquired at the $y = 0$ plane. In cases 1 and 2, there is a stationary tip vortex attached to the leading edge of the prism, as shown in Fig. 11(a)-Fig. 11(d) and Fig. 11(b)-Fig. 11(e). Flow patterns in the wake become less turbulent. Absence of large and small eddies are obvious from images. As the yaw angle is decreased, the turbulent intensity decreases and the region where turbulent eddies present shrinks. This well-documented streamlining effect will result in decrease in the drag coefficient, as predicted in the present study.

Fig. 11. (a,b,c) Isosurface of vorticity at $Q = 0.01$ and (d,e,f) contours of instantaneous vorticity. Images are acquired for the yaw angle of (a,d) 45° (case 1), (b,e) 60° (case 2), and (c,f) 90° (case 3).

5.4. Flows past yawed prisms in staggered arrays

Flows past two yawed prisms in a staggered array are studied for a yaw angle of 45° (case 8) and 60° (case 9). Simulations for both cases are conducted for G/D of 10. Drag coefficient of prism A1 and A2 for case 8 is calculated to be 1.06 and 1.13, respectively. For case 9, drag coefficient of prism A1 and A2 is 1.41 and 1.46. For each case, drag coefficient of downstream prism is slightly higher than that of upstream prism. This slight increase in drag coefficient of downstream prism can be ascribed to the observation that the fluid accelerating passing prism A1 is transported toward prism A2. Flow pattern for case 8 and case 9 are depicted in Fig. 12. Stationary tip vortex is present in both prisms. Vortices shed from each prism roll up toward the center, and are convected downstream, as shown in Fig. 12. The region where vortices are

concentrated becomes smaller as the yaw angle is decreased. It is also noticed that as the yaw angle is increased the shear layer near the centerline is formed. The wake of each prism is separated by the shear layer.

Fig. 12. (a,b) Isosurface of vorticity at $Q = 0.01$ and (c,d) contours of instantaneous vorticity for two prisms in a staggered arrangement. Prisms have the yaw angle of (a,c) 45° (case 8) and (b,d) 60° (case 9).

Simulations of three-dimensional flows past four yawed prisms are conducted with the yaw angle of 45° (case 10) and 60° (case 12) in a staggered arrangement for G/D of 10. Drag coefficient of each prism in cases 10 and 12 is calculated. Drag coefficient of A1, A2, A3 and A4 is 1.06, 1.13, 1.09, and 1.09, respectively, for case 10. Drag coefficient of prisms for case 12 is

436 1.38, 1.46, 1.32, and 1.34, respectively, for A1, A2, A3 and A4. It is noted that drag coefficient
437 for A2 again is slightly larger than that of others for both cases. Drag coefficient of A3 and A4
438 are similar to that of A1; indicating that the turbulent wake flow patterns induced by upstream
439 prisms are hardly influencing the force exerted on these prisms. Flow images depict that vorticity
440 shed from upstream prisms still impinges on part of the downstream prisms, as shown in Fig. 13.
441 For yaw angle of 45° and smaller, vortices impinge on a small region near trailing edge of
442 upstream prisms. Stationary tip vortex is attached to the leading edge of each prism in both cases,
443 as depicted in Fig. 13. As the number of prisms in staggered arrays is increased the drag
444 coefficient of downstream prisms remains the same. This implies that wake flow patterns are
445 repeated behind each prism. Drag coefficient of six yawed prisms in a staggered array (case 14)
446 are 1.06, 1.12, 1.10, 1.10, 1.12, and 1.12, respectively, for A1 through A6. Prisms with spacing
447 G/D of 10 are situated at 45° yaw angle against the oncoming flow. Flow patterns near prisms for
448 case 14 are depicted in Fig. 14. Downstream prisms are not experiencing highly turbulent wake
449 flows as oncoming flow toward them. That could be an important design consideration for
450 offshore and marine structures. Highly turbulent flows impinging on structures increase the risk
451 of failures in these systems.

Fig. 13. (a,b) Isosurfaces of vorticity at $Q = 0.01$ and (c,d) contours of instantaneous vorticity for four staggered prisms. Images are acquired for the yaw angle of (a,c) 45° (case 10) and (b,d) 60° (case 12).

456

(b)

457 Fig. 14. (a) Isosurface of vorticity at $Q = 0.01$ and (b) contours of instantaneous vorticity for six
 458 staggered prisms with the yaw angle 45° (case 14).

459 Fig. 15 illustrates flow patterns near four yawed prisms with yaw angle of 45° and
 460 spacing of $G/D = 2.5$ and 5. Even though prisms are tightly spaced drag coefficient of
 461 downstream prism is still 66% of the drag coefficient of upstream prisms for the case of $G/D =$
 462 2.5. Drag coefficient is calculated to be 1.04, 1.08, 0.66, and 0.66. It is clear that when prisms are
 463 closely spaced, staggered arrays of yawed prisms will perform much better, as listed in Table 5.

Drag coefficient of downstream and upstream prisms in a tandem and a staggered array are compared in Table 5. For staggered yawed prisms drag coefficient of downstream prism is slightly smaller for tight spacing. For perpendicular prisms in a tandem arrangement with spacing of $G/D = 5$ - not as tight as in case 11 - drag coefficient drops nearly 78% for the downstream prism, as listed in Table 5 for case 4. Tightly packed arrangement is desirable to be used in energy harvesting devices for marine current applications. Staggered arrangements of yawed prisms could be an integral part of design and optimization processes in these systems.

Fig. 15. (a,b) Isosurfaces of vorticity at $Q = 0.01$ and (c,d) contours of instantaneous vorticity for four staggered prisms. Prisms are placed with the yaw angle of 45° and the spacing of (a,c) $2.5D$ (case 11) and (b,d) $5D$ (case 13) .

Table 5. Average value of drag coefficient of upstream and downstream prisms in a tandem arrangement of perpendicular prisms and a staggered arrangement of yawed prisms.

G/D	Tandem (case 4-7)		Staggered (case 11, 13, 10)	
	$\bar{C}_{D,P1}$	$\bar{C}_{D,P2}$	$\bar{C}_{D,A2}$	$\bar{C}_{D,A4}$
2.5	-	-	1.08	0.66
5	1.47	0.32	1.11	0.92
7	1.47	1.11	-	-
10	1.48	1.38	1.13	1.09
20	1.49	1.46	-	-

6. Conclusions

LES simulations are carried out to study three-dimensional flows past arrays of finite yawed prisms at $Re = 50,000$ by FLUENT and OpenFOAM solver. Wake flow patterns are characterized for different values of spacing G/D and yaw angle in tandem and staggered arrays. Finite prisms of $L/D = 10$ with two free ends are used in simulations. The average value of drag coefficient for each prism in tandem and staggered arrays are calculated. Results predicted by FLUENT and OpenFOAM solvers agree well. Mathematical models and the numerical method employed in the current study are validated by comparing simulated results against experimental results reported by Liu and Chen [10].

As expected, drag coefficient of finite prism situated perpendicular to the oncoming flow is lower compared to that of infinitely long prism. This is attributed to the three-dimensional wake flows induced by the presence of tip vortices at free ends of finite prisms. The effect of spacing in tandem arrays predicted by three-dimensional simulations is vastly different compared to that predicted by two-dimensional modeling. For $G/D = 5$, three-dimensional flow modeling predicts that average drag coefficient of downstream prism is about 22% percent of drag coefficient of upstream prism while simulations in two-dimensional geometry predict average drag of upstream prism is nearly zero. Three-dimensional flow model also predicts that drag

coefficient of downstream prism rebound quickly as G/D is increased to 7 and recover almost fully as G/D is increased to 10. On the other hand, drag coefficient predicted by simulations in two-dimensional geometry rebound very slowly. Drag coefficient of downstream prism for G/D of 20 is still about 30% of the drag coefficient of upstream prism predicted by two-dimensional modeling. Flow transitions observed in wake flows are responsible for the influence of spacing on drag coefficients. For $G/D \leq 7$, the shear layers induced by P1 wraps around or reattaches to P2 and influences oncoming flows of downstream prism. For $G/D = 10$ and higher, both upstream and downstream prisms generate Karman vortex street almost independently.

Due to simply streamlining effects, drag coefficient of yawed prisms is less than that of prisms oriented perpendicular to oncoming flows. Nevertheless, when yawed prisms placed in staggered arrays outcome could be very favorable in terms of stability and power generation for marine current energy harvesting applications. Yawed prisms situated downstream part of the array could be outside of the turbulent wake flows induced by upstream prisms. As presented by the present study even for very tightly spaced yawed prisms ($G/D = 2.5$) drag coefficient of downstream prisms can still be 63% of the drag coefficient of upstream prisms. Tighter spacing of prisms aids in compact design of energy harvesting devices for the same power generation. This study shows that designing and optimizing offshore structures requires accurate spatial and temporal characteristics of the flows near structures. Orientation, arrangement and spacing of these systems are critical variables in design and optimization of energy harvesting modules.

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References

1. X. Tian, M. C. Ong, J. M. Yang, and D. Myrhaug, *Ocean Eng.* **58**, 208 (2013). doi:10.1016/j.oceaneng.2012.10.013.
2. C. W. Knisely, *J. Fluids Struct* **4**, 371 (1990). doi:10.1016/0889-9746(90)90137-T.
3. A. Fage and F. C. Johansen, *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences* **116**, 170 (1927). doi:10.1098/rspa.1927.0130.
4. F. M. Najjar and S. P. Vanka, *Int. J. Numer. Methods Fluids* **21**, 525 (1995). doi:10.1002/fld.1650210702.
5. X. L. Tian, M. C. Ong, J. M. Yang, and D. Myrhaug, *J. Fluids Struct* **49**, 149 (2014). doi:10.1016/j.jfluidstructs.2014.04.008.
6. H. Sakamoto, H. Hainu, and Y. Obata, *J. Wind Eng. Ind. Aerod.* **26**, 85 (1987). doi:10.1016/0167-6105(87)90037-7.
7. I.-H. Liu, J. Riglin, W. C. Schleicher, and A. Oztekin, *Ocean Eng.* **111**, 323 (2016). doi:10.1016/j.oceaneng.2015.11.009.
8. T. P. Lloyd and M. James, *Appl. Ocean Res.* **59**, 676 (2016). doi:10.1016/j.apor.2015.11.009.
9. S. M. Yeon, J. M. Yang, and F. Stern, *Appl. Ocean Res.* **59**, 663 (2016). doi:10.1016/j.apor.2015.11.013.
10. C. H. Liu and J. M. Chen, *J. Wind Eng. Ind. Aerod.* **90**, 1019 (2002). doi:10.1016/S0167-6105(02)00234-9.
11. S. C. Yen, K. C. San, and T. H. Chuang, *Exp. Therm. Fluid Sci.* **32**, 927 (2008). doi:10.1016/j.expthermflusci.2007.07.001.
12. A. Sohankar, *Theor. Comp. Fluid Dyn.* **28**, 531 (2014). doi:10.1007/s00162-014-0329-2.
13. D. Sumner, *J. Fluids Struct* **26**, 849 (2010). doi:10.1016/j.jfluidstructs.2010.07.001.
14. M. Zhao, L. Cheng, B. Teng, and D. F. Liang, *Appl. Ocean Res.* **27**, 39 (2005). doi:10.1016/j.apor.2004.10.002.
15. F. Auteri, M. Belan, C. Cassinelli, and G. Gibertini, *J. Visualization* **12**, 307 (2009). doi:10.1007/bf03181875.

- 547 16. M. Shirakashi, A. Hasegawa, and S. Wakiya, *Bulletin of JSME* **29**, 1124 (1986).
548 doi:10.1299/jsme1958.29.1124.
- 549 17. D. Yeo and N. P. Jones, *J. Wind Eng. Ind. Aerod.* **96**, 1947 (2008).
550 doi:10.1016/j.jweia.2008.02.040.
- 551 18. M. Zhao, L. Cheng, and T. M. Zhou, *J. Fluids Struct* **25**, 831 (2009).
552 doi:10.1016/j.jfluidstructs.2009.02.004.
- 553 19. S. A. Jordan, *Int. J. Heat Fluid Flow* **31**, 489 (2010).
554 doi:10.1016/j.ijheatfluidflow.2010.04.009.
- 555 20. H. F. Wang, Y. Zhou, C. K. Chan, and K. S. Lam, *Phys. Fluids* **18**, 065106 (2006).
556 doi:10.1063/1.2212329.
- 557 21. J. F. McClean and D. Sumner, *J. Fluids Eng.* **136**, 081206 (2014). doi:10.1115/1.4027138.
- 558 22. D. Sumner, *J. Fluids Struct* **43**, 41 (2013). doi:10.1016/j.jfluidstructs.2013.08.007.
- 559 23. T. Kawamura, M. Hiwada, T. Hibino, I. Mabuchi, and M. Kumada, *B Jsmc* **27**, 2142
560 (1984). doi:10.1299/jsme1958.27.2142.
- 561 24. N. Rostamy, J. McClean, D. Sumner, D. Bergstrom, and J. Bugg, *Proceedings of the 7th*
562 *International Colloquium on Bluff Body Aerodynamics and Applications*
563 *(BBAA7)Shanghai, China, 2012.*
- 564 25. M. M. Zdravkovich, V. P. Brand, G. Mathew, and A. Weston, *J. Fluid Mech.* **203**, 557
565 (1989). doi:10.1017/S002211208900159X.
- 566 26. O. Inoue and A. Sakuragi, *Phys. Fluids* **20**, 033601 (2008). doi:10.1063/1.2844875.
- 567 27. H. F. Wang and Y. Zhou, *J. Fluid Mech.* **638**, 453 (2009).
568 doi:10.1017/s0022112009990693.
- 569 28. G. Hu, K. T. Tse, K. C. S. Kwok, and Y. Zhang, *J. Wind Eng. Ind. Aerod.* **146**, 172
570 (2015). doi:10.1016/j.jweia.2015.08.008.
- 571 29. F. Nicoud and F. Ducros, *Flow Turbul Combust* **62**, 183 (1999).
572 doi:10.1023/A:1009995426001.
- 573 30. ANSYS FLUENT Theory Guide 14.5 (2012).
- 574 31. ANSYS FLUENT User Guide 14.5 (2012).
- 575 32. C. J. Greenshields, *The Open Source CFD Toolbox, Programmer's Guide, Version 3.0.1,*
576 *2015.*
- 577 33. F. M. Najjar and S. Balachandar, *J. Fluid Mech.* **370**, 101 (1998).
578 doi:10.1017/S0022112098002110.
- 579 34. J. C. R. Hunt, A. A. Wray, and P. Moin, *1988 Summer program of the center for turbulent*
580 *researchNASA Ames/ Stanford University, 1988, pp. 193.*