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Transient electroosmotic flow of generalized second grade fluids under slip boundary conditions

Xiaoping Wang, Haitao Qi, and Huanying Xu

Abstract: This work investigates the transient slip flow of viscoelastic fluids in a slit microchannel under the combined influences of electroosmotic and pressure gradient forcings. We adopt the generalized second grade fluid model with fractional derivative as the constitutive equation and the Navier linear slip model as the boundary conditions. The analytical solution for velocity distribution of the electroosmotic flow is determined by employing the Debye-Hückel approximation and the integral transform methods. The corresponding expressions of classical Newtonian and second grade fluids are obtained as the limiting cases of our general results. These solutions are presented as sum of steady-state part and the transient one. The combined effects of slip boundary conditions, fluid rheology, electroosmotic and pressure gradient forcings on the fluid velocity distribution are also discussed in terms of the pertinent dimensionless parameters graphically. By comparison with the two cases corresponding to the Newtonian fluid and the classical second grade fluid, it is found that the fractional derivative parameter β has a significance effect on the fluid velocity distribution and the time when the fluid flow reaches the steady state. Additionally, the slip velocity at the wall increases in a noticeable manner the flow rate in an electroosmotic flow.

Key words: Generalized second grade fluid, Electroosmotic flow, Fractional calculus, Slip velocity, Analytical solutions.

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1. Introduction

Today, fractional calculus has obtained more and more attention due to its demonstrated applications in many diverse fields of science and engineering. For an extensive bibliography on fractional calculus, see the books written by Podlubny [1], Hilfer [2], Mainardi [3], Uchaikin [4] and West [5]. As a kind of intermediate process and critical phenomenon, the constitutive relation of viscoelastic materials is one of the most successful applications of the theory of fractional calculus [6]. The simplest fractional model of a viscoelastic material, the fractional element model, was first introduced by Scott Blair [7]: $\sigma \sim d^\alpha \varepsilon / dt^\alpha$, where α are material-dependent constants, σ and ε are the stress and strain, respectively. This model can be realized physically through hierarchical arrangements of springs and dashpot, such as ladders, trees or fractal structures [2, 3]. Through the rheological experiments, the fractional constitutive relations have been found to be quite flexible in describing viscoelastic properties of viscoelastic materials [2, 8, 9].

In recent years, interest in the flows of viscoelastic fluids describing with fractional derivative models has increased considerably [4, 6]. For the unsteady flows of a generalized second grade fluid, a lot of research has been done. For example, Huang et al. [10] studied the flow of a generalized second grade fluid in double cylinder rheometer. Xu and Tan [11] investigated the velocity field, stress field and vortex sheet of a generalized second grade fluid with fractional anomalous diffusion. Furthermore, a series of work was carried out to study laminar flows

of a generalized second grade fluid [12–17]. The flow of a generalized second grade fluid past a porous plate was analyzed with the homotopy analysis method by Hayat and Khan [18]. Shen et al. [19] studied the Rayleigh-Stokes problem for a generalized second grade fluid subject to a flow on a heated flat plate and within a heated edge. Tripathi [20] studied the peristaltic flow behavior of a fractional second grade fluid through the cylindrical tube. More recently, the corresponding numerical algorithms and parameter estimation were also developed to solve the Rayleigh-Stokes problem of a heated generalized second grade fluid [21–25].

On the other hand, with the rapid development of microfluidic technology, the fluid flow through small channels has become a popular research topic [26–28]. Most studies of electrokinetics in literature usually assume Newtonian fluid [29, 30]. But in reality, microfluidic devices are usually involved in analyzing and/or processing biofluids, polymeric solutions and colloidal suspensions. These fluids can not be treated as Newtonian fluids. Therefore, many researchers have recently focused on non-Newtonian fluid behavior of viscoelastic fluids in electrokinetically driven microflows. A comprehensive review of electrokinetics pertaining to non-Newtonian fluids was presented in the review by Zhao and Yang [31]. More recently, electroosmotic flows of viscoelastic fluids in microchannels were received increasing attention, where the second grade model, the Maxwell model, the Oldroyd-B model and the simplified Phan-Thien-Tanner model were used in describing the fluid rheological behavior, respectively [32–41]. Up to date, various researchers have introduced the fractional calculus approach into the studies of electroosmotic flow of viscoelastic fluids [42–45], where the generalized second grade fluid, Maxwell fluid and Oldroyd-B fluid with fractional derivatives are used as the constitutive relationships. It is worth noting that blood as one of typical biofluid is essentially a shear-thinning

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Fig. 1. Sketch of the electroosmotic flow in a slit microchannel.

fluid and the modified second grade fluid is the simplest constitutive model that can describe shear-thinning (or shear-thickening) and normal stress differences [46]. It is for this reason that Wang et al. [43] studied the electroosmotic flow of biofluid firstly using the generalized second grade fluid with fractional derivative. But to our best knowledge, exact solutions for the electroosmotic flow of viscoelastic fluids are rarely reported yet. In present study, the generalized second grade constitutive model with fractional derivative is introduced to describe the rheological behavior of viscoelastic fluids. The electroosmotic slip flow of generalized second grade fluid in a slit microchannel under the combined influences of electroosmotic and pressure forcings is investigated. The analytical solution for the velocity distribution of this electroosmotic slip flow is obtained by using the Laplace transform and finite Fourier cosine transform. Then we also discuss the combined effects of parameters on the fluid velocity distribution.

2. Governing equation

The governing equations for the electroosmotic slip flow studied in this work are the continuity equation

$$\nabla \cdot \mathbf{u} = 0, \quad [1]$$

and the general Cauchy momentum equation

$$\rho \frac{D\mathbf{u}}{Dt} = -\nabla p + \nabla \cdot \boldsymbol{\tau} + \rho_e \mathbf{E}, \quad [2]$$

where $\mathbf{u}$ is the velocity vector, ρ is the fluid density, $\boldsymbol{\tau}$ is the shear stress tensor. The term $\rho_e \mathbf{E}$ represents a body force term, where $\mathbf{E}$ is the applied external electric field and ρ_e is the net electric charge density associated with the spontaneously formed electric double layers (EDL), which are assumed here not to be affected by the imposed electric field [38].

We consider an unsteady electroosmotic flow of viscoelastic fluid in a slit microchannel. An illustration of the problem

and the coordinate system is sketched in Fig. 1. The channel is filled with a liquid solution of dielectric constant, ϵ . It is assumed that the channel wall is uniformly charged with a zeta potential, ψ_w , and the constitutive equation of the liquid solution is the well-known generalized second grade fluid model with fractional derivative. The fluid is considered to be initially at rest, and when an external electric field $\mathbf{E} = (E_x, 0, 0)$ is imposed along the x -axis direction, the fluid in the microchannel sets in motion due to electroosmosis. Because of geometric symmetry, half of the slit channel ($0 \leq y \leq H$) is considered.

For this unidirectional flow, we assume the velocity distribution of the form

$$\mathbf{u} = (u(y, t), 0, 0), \quad 0 \leq y \leq H, \quad t > 0, \quad [3]$$

and the continuity equation, Eq. (1), is satisfied automatically.

According to the theory of electrostatics, the net electric charge density ρ_e in the diffuse layer of the wall EDL can be expressed by means of a Poisson equation

$$\frac{d^2\psi}{dy^2} = -\frac{\rho_e}{\epsilon}, \quad [4]$$

where ψ denotes the EDL potential. For small values of electrical potential ψ of the EDL, the Debye-Hückel approximation can be applied. The electrical potential profile in the EDL is governed by the following linear Poisson-Boltzmann equation expressed by

$$\frac{d^2\psi}{dy^2} = \kappa^2\psi, \quad [5]$$

which is subject to the following boundary conditions

$$\left. \frac{d\psi}{dy} \right|_{y=0} = 0, \quad \psi(H) = \psi_w. \quad [6]$$

Here, $\kappa^2 = 2n_0e^2z^2/(\epsilon k_B T)$ is the Debye-Hückel parameter, where n_0 is the bulk concentration of the ions, e is the elementary electronic charge, z is the valence of the ions, ϵ is the

dielectric constant of the fluid, k_B is the Boltzmann constant, T is the absolute temperature [28]. From Eqs. (4)–(6), the net charge density ρ_e can be expressed as a function of EDL potential

$$\rho_e(y) = -\varepsilon\kappa^2\psi(y), \quad \psi(y) = \psi_w \frac{\cosh(\kappa y)}{\cosh(\kappa H)}. \quad [7]$$

For the particular flow conditions considered, the general Cauchy momentum equation (2) can be simplified to

$$\rho \frac{\partial u}{\partial t} = -p_{,x} + \frac{\partial \tau_{xy}}{\partial y} + \rho_e(y)E_x, \quad [8]$$

where $p_{,x} = \partial p / \partial x$. And the constitutive equation for a generalized second grade fluid with fractional derivative can be expressed in terms of [10–12]

$$\tau_{xy} = \left(\mu + \alpha \frac{\partial^\beta}{\partial t^\beta} \right) \frac{\partial u}{\partial y}, \quad [9]$$

where τ_{xy} is the shear stress, μ is the dynamic viscosity coefficient, α is the fractional viscoelastic constant, $\partial^\beta / \partial t^\beta$ is the Caputo fractional derivative operator and may be defined as [1]

$$\frac{\partial^\beta}{\partial t^\beta} u(y, t) = \frac{1}{\Gamma(1-\beta)} \int_0^t \frac{u'(y, t')}{(t-t')^\beta} dt', \quad 0 \leq \beta \leq 1. \quad [10]$$

By combining Eqs. (8) and (9), the following fractional momentum equation is obtained

$$\rho \frac{\partial u}{\partial t} = -p_{,x} + \left(\mu + \alpha \frac{\partial^\beta}{\partial t^\beta} \right) \frac{\partial^2 u}{\partial y^2} + \rho_e(y)E_x, \quad [11]$$

which is subject to the initial and boundary conditions

$$u(y, t) = 0, \quad \text{at } t \leq 0 \text{ for } 0 \leq y \leq H. \quad [12]$$

$$\frac{\partial u}{\partial y} \Big|_{y=0} = 0, \quad \text{and} \quad \left(u(y, t) + d \frac{\partial u}{\partial y} \right) \Big|_{y=H} = 0, \quad t > 0. \quad [13]$$

Here, we apply the symmetry boundary condition at the centerline of the channel and the Navier linear slip law at the channel walls, where d is the slip length provided at the channel wall. The reason that we introduce the slip boundary condition is that experiments and molecular dynamic simulations have demonstrated that the fluid can slip at solid surfaces. Furthermore, Matías et al. [47] has pointed out that the slip boundary condition is very important at level of micro scales in their study on the electroosmotic flow of a viscoelastic fluid.

3. Analytical solution

For convenience in the subsequent analysis, we introduce the following dimensionless quantities

$$\begin{aligned} \psi^* &= \frac{\psi}{\psi_w}, \quad u^* = \frac{u}{U_{HS}}, \quad y^* = \frac{y}{H}, \quad t^* = \frac{t}{\rho H^2 / \mu}, \\ \alpha^* &= \frac{\alpha / \mu}{(\rho H^2 / \mu)^\beta}, \quad \kappa^* = \kappa H, \quad d^* = \frac{d}{H}, \end{aligned} \quad [14]$$

where $U_{HS} = -\varepsilon\psi_w E_x / \mu$ is the Helmholtz-Smoluchowski electroosmotic velocity. After inserting dimensionless parameters (14) into Eq. (11), the dimensionless momentum equation and relevant initial boundary conditions have the following form (for simplicity, the superscript “*” is omitted hereafter)

$$\frac{\partial u}{\partial t} = -\Gamma + (1 + \alpha D_t^\beta) \frac{\partial^2 u}{\partial y^2} + \kappa^2 \frac{\cosh(\kappa y)}{\cosh \kappa}, \quad [15]$$

$$u(y, t) = 0, \quad t \leq 0, \quad 0 \leq y \leq 1, \quad [16]$$

$$\frac{\partial u}{\partial y} \Big|_{y=0} = 0, \quad \left(u(y, t) + d \frac{\partial u}{\partial y} \right) \Big|_{y=1} = 0, \quad t > 0, \quad [17]$$

where $\Gamma = -H^2 p_{,x} / (\varepsilon\psi_w E_x)$ represents the ratio of pressure to electroosmotic driving force. The cases $\Gamma > 0$ and $\Gamma < 0$ correspond to mixed Poiseuille/electroosmotic flows with adverse and favorable pressure gradients, respectively.

In order to obtain the analytical solution, we use the joint Laplace transform with respect to time t and the finite Fourier cosine transform with respect to y defined by

$$\bar{u}(\lambda_n, s) = \int_0^\infty e^{-st} dt \int_0^1 u(y, t) \cos(\lambda_n y) dy, \quad [18]$$

and the inverse finite Fourier cosine transform [48]

$$u(y, t) = 2 \sum_{n=1}^\infty \frac{(1 + d^2 \lambda_n^2) \bar{u}(\lambda_n, t) \cos(\lambda_n y)}{1 + d + d^2 \lambda_n^2}, \quad [19]$$

where $\bar{u}(\lambda_n, t)$ is the finite Fourier cosine transform of $u(y, t)$, λ_n are the positive roots of $\cos \lambda_n - d \lambda_n \sin \lambda_n = 0$. And we introduce the following two functions

$$v_1(y) = \frac{y^2 - 1 - 2d}{2}, \quad [20a]$$

$$v_2(y) = 1 + d\kappa \tanh \kappa - \frac{\cosh(\kappa y)}{\cosh \kappa}. \quad [20b]$$

Their corresponding finite Fourier cosine transforms are

$$\bar{v}_1(\lambda_n) = -\frac{\sin \lambda_n}{\lambda_n^3}, \quad [21a]$$

$$\bar{v}_2(\lambda_n) = \frac{\kappa^2 (1 + d\kappa \tanh \kappa) \cos \lambda_n}{d\lambda_n^2 (\kappa^2 + \lambda_n^2)}. \quad [21b]$$

respectively. It should be noted that $v_1(y)\Gamma + v_2(y)$ is the steady solution of Eq.(15) satisfying the boundary condition (17).

Application of the double transform (18) to Eqs. (15)–(17) gives the solution for $\bar{u}(\lambda_n, s)$ as

$$\bar{u} = \left(\frac{1}{s} - \frac{1 + \alpha \lambda_n^2 s^{\beta-1}}{s + \alpha \lambda_n^2 s^\beta + \lambda_n^2} \right) (\bar{v}_1(\lambda_n)\Gamma + \bar{v}_2(\lambda_n)). \quad [22]$$

Here the Laplace transform formula for the Caputo fractional derivative (see Eq. (1.27) in Ref. [3]) is used.

The inverse Laplace transform gives

$$\bar{u}(\lambda_n, t) = (1 - A(\lambda_n, t))(\bar{v}_1(\lambda_n)\Gamma + \bar{v}_2(\lambda_n)). \quad [23]$$

Fig. 2. Dimensionless centerline velocity profiles for various values of β ($\beta = 0.2, 0.4, 0.6, 0.8, 1.0$) under pure electroosmotic slip flow ($d = 0.002$ and $\Gamma = 0$). The arrow indicates plots with increasing β .

In Eq. (23), $A(\lambda_n, t)$ can be given by

$$A(\lambda_n, t) = \mathcal{L}^{-1} \left\{ \frac{1 + \alpha \lambda_n^2 s^{\beta-1}}{s + \alpha \lambda_n^2 s^\beta + \lambda_n^2} \right\} \\ = \sum_{m=0}^{\infty} \frac{(-1)^m \lambda_n^{2m} t^{(2-\beta)m}}{m!} \times E_{1-\beta, 1+(2-\beta)m}(-\alpha \lambda_n^2 t^{1-\beta}), \quad [24]$$

using the following formula

$$\mathcal{L} \left[t^{\beta-1} E_{\alpha, \beta}^{\gamma}(\lambda t^\alpha) \right] (s) = \frac{s^{\alpha\gamma-\beta}}{(s^\alpha - \lambda)^\gamma}, \quad [25]$$

where $E_{\alpha, \beta}^{\rho}(\cdot)$ is the three-parametric Mittag-Leffler function [49], $Q(t) = \int_{-1}^1 u(y, t) dy = 2 \int_0^1 u(y, t) dy$. [28]

defined by

$$E_{\alpha, \beta}^{\gamma}(z) = \sum_{n=0}^{\infty} \frac{(\gamma)_n z^n}{n! \Gamma(\alpha n + \beta)}, \\ (\gamma)_0 = 1, (\gamma)_n = \gamma(\gamma+1) \cdots (\gamma+n-1). \quad [26]$$

Thus, the inverse finite Fourier cosine transform (19) yields the analytical solution for velocity distribution

$$u(y, t) = v_1(y)\Gamma + v_2(y) - 2 \sum_{n=1}^{\infty} A(\lambda_n, t) \\ \times (\Gamma \bar{v}_1(\lambda_n) + \bar{v}_2(\lambda_n)) \frac{(1 + d^2 \lambda_n^2) \cos(\lambda_n y)}{1 + d + d^2 \lambda_n^2}. \quad [27]$$

And, the dimensionless flow rate $Q(t)$ can also be obtained by the following integration, i.e.

Fig. 3. Dimensionless velocity profiles for both $\kappa = 100$ ($-1 < y < 0$) and $\kappa = 20$ ($0 < y < 1$) cases when $\alpha = 1$, $\beta = 0.6$ and $d = 0.002$ for different values of Γ and t (solid lines : $t = 0.01$; dashed lines : $t = 0.1$; dash-dot lines : $t = 1$; dot lines : $t = \infty$).

The solution for the velocity distribution given by Eq. (27) will now be analyzed with some special cases.

1. When $t \rightarrow \infty$, the solution for the electroosmotic flow in the steady state is obtained as

$$u(y, t) = v_1(y)\Gamma + v_2(y), \quad [29a]$$

$$Q(t) = -\frac{2}{3}(1 + 3d)\Gamma + 2 + 2\left(d\kappa - \frac{1}{\kappa}\right)\tanh\kappa. \quad [29b]$$

Eqs. (29) predict negative centraline velocities (or negative flow rate) when

$$\Gamma > \frac{1 + d\kappa \tanh\kappa - \operatorname{sech}\kappa}{1/2 + d}, \quad [30]$$

or

$$\Gamma > \frac{1 + (d\kappa - \kappa^{-1})\tanh\kappa}{1/3 + d}. \quad [31]$$

2. When $\alpha \neq 0$ and $\beta = 1$ in (27), we get the solution for the electroosmotic flow of the classical second grade fluid, which can be expressed as

$$u(y, t) = v_1(y)\Gamma + v_2(y) - 2 \sum_{n=1}^{\infty} \exp\left(-\frac{\lambda_n^2 t}{1 + \alpha\lambda_n^2}\right) \times (\Gamma\bar{v}_1(\lambda_n) + \bar{v}_2(\lambda_n)) \frac{(1 + d^2\lambda_n^2)\cos(\lambda_n y)}{1 + d + d^2\lambda_n^2}. \quad [32]$$

3. The Newtonian fluid can be viewed as a special case of generalized second grade fluid with $\alpha = 0$. From Eq. (27), the analytical solutions take the following forms

$$u(y, t) = v_1(y)\Gamma + v_2(y) - 2 \sum_{n=1}^{\infty} \exp(-\lambda_n^2 t) \times (\Gamma\bar{v}_1(\lambda_n) + \bar{v}_2(\lambda_n)) \frac{(1 + d^2\lambda_n^2)\cos(\lambda_n y)}{1 + d + d^2\lambda_n^2}. \quad [33]$$

4. It should be noted that for $d = 0$, Eq. (17) reduces to the conventional no-slip boundary condition. The solution for this case can be obtained by taking the limit $d \rightarrow 0$ in Eq. (27)

$$u(y, t) = \frac{y^2 - 1}{2}\Gamma + 1 - \frac{\cosh(\kappa y)}{\cosh\kappa} - 2 \sum_{n=1}^{\infty} A(\lambda_n, t) \times \left(\frac{(-1)^n \Gamma}{\lambda_n^3} + \frac{(-1)^{n-1} \kappa^2}{\lambda_n(\kappa^2 + \lambda_n^2)} \right) \cos(\lambda_n y), \quad [34]$$

where $\lambda_n = n\pi - \frac{\pi}{2}$.

4. Results and Discussions

Now, in order to understand the influence of pertinent parameters, α , β , d , κ and Γ , on the fluid flow, we present the

Fig. 4. Dimensionless transient velocity profiles for both Newtonian fluid ($-1 < y < 0$) and generalized second grade fluid ($0 < y < 1$) cases when $\kappa = 20$, $\beta = 0.6$ and $d = 0.002$ for different values of Γ and t .

velocity profiles of the electroosmotic slip flow of a generalized second grade fluid. We all know that the index of the generalized second grade fluid model β plays an important and fundamental role in this fractional model [11]. Thus, we first discuss the effect of β on the centerline velocity. Fig. 2 demonstrates the centerline velocity profiles for various values of β with different α and κ . It can be seen that the flow for the fractional second grade fluid with smaller β is easier to start up in the initial time. However, it seems their influence on centerline velocity changes inversely at a critical point. The reason is a small β means the fluid elasticity prevails in the transient state and then the electroosmotic flow of generalized second grade fluid shows the competition between viscous, elastic and electrical effects. For the fractional viscoelastic constant α , the flow needs more time to reach the steady flow for greater values of α . And we can also see that κ contribute to an increase in the centerline velocity.

Fig. 3 shows the dimensionless velocity profiles for two relative microchannel ratios $\kappa = 20$ and $\kappa = 100$ for various ratios of pressure to electroosmotic driving forces Γ with slip at the wall. The velocity profiles corresponding to a pluglike flow when $\Gamma = 0$. After comparing these figures, we can conclude that it requires larger adverse pressure gradients to obtain negative velocities for large values of κ . This can also be found from Eq. (30) for the steady cases, as found in Refs. [36, 38]. As we all know that the influence of κ on the velocity profile is restricted to the effective EDL thickness. We can also see clearly that the velocity profiles for large values of κ exhibit thinner EDL layers and larger velocity gradients because a lar-

ger κ means a stronger EDL electrical potential gradient when the wall zeta potential is prescribed. The transient velocity variations of Newtonian fluid, second grade fluid and generalized second grade fluid under different values of Γ are compared in Figs. 4 and 5, respectively. It is found that the velocity of Newtonian fluid changes faster than the generalized second grade fluid and the velocity of generalized second grade fluid changes faster than that of classical second grade fluid. This phenomenon can also be found in Fig. 6 and Fig. 7, which present the volumetric flow rates of Newtonian fluid, generalized second grade fluid and classical second grade fluid with different values of slip coefficient d . It can be observed that the necessary time to reach the steady flow rate for the Newtonian fluid and the classical second grade fluid is shorter than the fractional second grade fluid. Finally, Fig. 8 illustrates the variation of the dimensionless flow rate $Q(t)$ with favorable and adverse pressure gradients for different values of the slip coefficient d . We can see that the presence of slip velocity affects the velocity and lead to a significant increase in $Q(t)$. Further, the flow rate $Q(t)$ is linear dependent on the slip coefficient d in the steady state flow as shown in Fig. 8 and Eq. (29).

5. Conclusions

In this work, the unsteady-state electrokinetic slip flow of generalized second grade fluid in a slit microchannel under the combined influence of electroosmotic and pressure forces was studied. We derived exact solutions for the velocity distribution

Fig. 5. Dimensionless transient velocity profiles for both second grade fluid ($-1 < y < 0$) and generalized second grade fluid ($0 < y < 1$) cases when $\kappa = 20$, $\alpha = 0.6$ and $d = 0.002$ for different values of Γ and t .

under the Debye-Hückel approximation. This solution is obtained by solving the linearized Poisson-Boltzmann equation, the Cauchy momentum equation and the constitutive equation of a generalized second grade fluid with fractional derivative. With the help of integral transforms, the distributions of velocity and flow rate in the microchannel are obtained analytically and several special cases are also discussed. The results demonstrate that the fractional derivative parameter β strongly influences the distribution of fluid velocity and the time when the fluid flow reaches the steady state. We also found that the slip velocity at the wall increases in a noticeable manner the flow rate in an electroosmotic flow.

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Fig. 6. Effect of the slip coefficients d on the flow rates $Q(t)$ for the Newtonian fluid ($\alpha = 0$) and the generalized second grade fluid ($\alpha = 1, \beta = 0.6$) ($d = 0$ solid lines, $d = 1/128$ dashed lines, $d = 1/64$ dash-dot lines)

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Fig. 7. Effect of the slip coefficients d on the flow rates $Q(t)$ for the second grade fluid ($\alpha = 1$, $\beta = 1$) and the fractional second grade fluid ($\alpha = 1$, $\beta = 0.6$) ($d = 0$ solid lines, $d = 1/128$ dashed lines, $d = 1/64$ dash-dot lines)

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Fig. 8. Variation of the dimensionless flow rate $Q(t)$ with $\Gamma = 1$ and $\Gamma = -1$ for different values of the slip coefficient d .

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