Comparison of methods for spatial interpolation of fire weather in Alberta, Canada

<table>
<thead>
<tr>
<th>Journal:</th>
<th>Canadian Journal of Forest Research</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript ID</td>
<td>cjfr-2017-0101.R2</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>03-Oct-2017</td>
</tr>
<tr>
<td>Complete List of Authors:</td>
<td>Jain, Piyush; University of Alberta, Renewable Resources Flannigan, Mike; University of Alberta, Renewable Resources</td>
</tr>
<tr>
<td>Keyword:</td>
<td>interpolation, Kriging, Fire Weather Index, Inverse distance weighting, Splines</td>
</tr>
<tr>
<td>Is the invited manuscript for consideration in a Special Issue? :</td>
<td>N/A</td>
</tr>
</tbody>
</table>

https://mc06.manuscriptcentral.com/cjfr-pubs
Comparison of methods for spatial interpolation of fire weather
in Alberta, Canada

P. Jain^A and M.D. Flannigan^A

^ADepartment of Renewable Resources, University of Alberta, Edmonton, Alberta, Canada
Abstract

Spatial interpolation of fire weather variables from station data allow fire danger indices to be mapped continuously across the landscape. This information is crucial to fire management agencies, particularly in areas where weather data are sparse. We compare the performance of several standard interpolation methods (inverse distance weighting, spline and geostatistical interpolation methods) for estimating output from the Canadian Fire Weather Index (FWI) System at unmonitored locations. We find geostatistical methods (Kriging) generally outperform the other methods, particularly when elevation is used as a covariate. We also find that interpolation of the input meteorological variables as well as the previous day’s moisture codes to unmonitored locations followed by calculation of the FWI output variables is preferable to first calculating the FWI output variables and then interpolating, in contrast to previous studies. Alternatively, when the previous day’s moisture codes are estimated from interpolated weather, rather than directly interpolated, errors can accumulate and become large. This effect is particularly evident for the duff moisture code and drought moisture code, due to their significant autocorrelation.

Keywords: Interpolation, Kriging, Fire Weather Index, Inverse distance weighting, Splines
Introduction

Wildfire is not only a natural disturbance on ecosystems but is also a major component of forest management, particularly in areas where fire coexists with human activity and values. In addition, anthropogenic climate change is expected to increase both fire frequency and severity in the coming century (Wang et al. 2015). On a daily timescale wildfire potential has a strong dependence on antecedent and current weather conditions. For example, in Canada, 97% of the area burned by wildland fire is the result of just 3% of fire ignitions (Stocks et al. 2002). These large fires occur on days with extreme fire weather (Flannigan and Harrington 1988); that is, days with a combination of meteorological conditions conducive to a highly elevated wildfire risk. Various fire weather indices are used by many countries to inform fire management agencies and the public about potential fire danger due to meteorological conditions. This information is used to employ preventative measures (eg. prescribed burning), for the allocation of resources for fire control and suppression, or for emergency response (eg. evacuations). These fire management goals are facilitated by the accurate spatial mapping of fire danger on the landscape.

Widely used fire danger indices include the US National Fire Danger Ratings System (NFDRS) (Deeming et al. 1977), the Australian McArthur Forest Fire Danger Index (McArthur 1967), the Haines Index (Haines, 1988), and the Fire Weather Index (FWI) System, which is part of the Canadian Forest Fire Danger Rating System (CFFDRS) (Van Wagner 1987). The FWI system is used extensively by provinces and territories in Canada to monitor wildfire risk due to weather conditions. The input to the FWI system consists of daily noon observations of temperature, relative humidity, wind speed and 24 hour accumulated precipitation. The system works by calculating three intermediate moisture codes which represent the moisture content of three ground layers. These are the Fine Fuel Moisture Code (FFMC), the Duff Moisture Code...
(DMC) and the Drought Code (DC). Subsequently two indices, the Build-up index (BUI) and Initial Spread Index (ISI) are calculated, which are combined to give an overall Fire Weather Index (FWI) or its transformed equivalent the Daily Severity Rating (DSR). The variable dependence of the FWI system is illustrated in Figure 1. An important feature of the system is that the FWI System outputs for the current day depend not only on the current day’s weather but the previous day’s moisture codes. For this reason, the system is essentially a bookkeeping system for fuel moisture.

Most provincial fire management agencies in Canada deploy their own weather station networks specifically to monitor fire weather during the fire season. The number and location of weather stations is constrained by cost, location of values and convenience. The mapping of fire danger on the landscape therefore usually requires spatial interpolation of meteorological variables or fire weather indices onto a grid. Spatial interpolation refers to any method that estimates a value of a variable (or variables) at an unobserved location (predictand) using data at observed locations (predictors). The accuracy of interpolated weather in general depends on the following: i) the spatio-temporal characteristics of meteorological variables ii) measurement error, iii) station network density, iv) station network distribution, iv) topography or other local climate factors; and vi) chosen interpolation method. Based on an early study using data from New Brunswick, FWI values are expected to be representative of conditions within a 40 km radius surrounding a weather station, becoming unreliable beyond a radius of 160 km (Williams 1963). These values have subsequently been used as a guide for Canadian fire management agencies (Lawson and Armitage 2008). In the 1990s the spatial Fire Management System (sFMS) was developed, adding GIS capabilities to the CFFDRS and providing a computerized forest fire management system to be used operationally (Lee et al. 2002). Part of these added capabilities included the ability to map fire danger using spatial interpolation, with the inverse
distance weighting scheme being used by default (Englefield et al. 2000). Currently the sFMS
interpolates fire weather by interpolating the meteorological variables to grid points and then
calculating the FWI components. The previous day’s moisture codes are taken from the previous
day’s calculated codes on the grid, unless no value is available (as occurs early in the fire
season), in which case the codes are interpolated from available weather stations (P. Englefield
(personal communication, 2017)).

There are a few studies specifically examining interpolation of fire danger rating indices.
Flannigan and Wotton (1989) previously compared interpolation methods for the Canadian FWI
system in the North Central Region of Ontario. They tested a second order polynomial fit, cubic
spline (thin plate spline) and weighted interpolation (inverse distance weighting) for
interpolation of the FWI. Their results suggest the smoothed cubic spline gives the most
consistent results. Flannigan et al. (1998) investigated whether interpolation of FWI could be
improved by using precipitation estimates from radar. Tait and Zheng (2005) also compared a
number of interpolation schemes for the FWI over New Zealand and found the thin plate
smoothed spline performed best. Significantly both of these studies concluded that direct
interpolation of the FWI was preferable to interpolation of the underlying meteorological
variables followed by calculation of the FWI. Sanabria et al. (2013) examined the spatial
interpolation of the Australian fire danger rating system, the McArthur Forest Fire Danger Index
(FFDI). They achieved the best results with an algorithm that combined random forest and
inverse distance weighting interpolation. This study focused specifically on return intervals of
extreme values of the FFDI. More generally, the interpolation of meteorological variables has
been treated extensively in the literature and we refer the interested reader to Li and Heap (2014)
who review a number of commonly applied interpolation methods.
More recently, a number of alternatives to spatial interpolation have been suggested for real-time monitoring or forecasting of fire danger. Satellite remote sensing has been investigated for real-time monitoring of live fuel moisture (Leblon 2001). Although satellite remote sensing of land surface temperature (Li et al. 2013) and precipitation (Stephens and Kummerow 2007) is possible, the technology has not yet advanced sufficiently for determination of land surface wind speed and relative humidity, which are also required meteorological inputs of the FWI System. The use of gridded products such as the Canadian Precipitation Analysis (CaPA) (Mahfouf et al. 2007), which blends gauge data, numerical weather forecasts and radar estimates, has also been investigated for improving precipitation and fuel moisture estimates using the FWI System (Hanes et al., in press). On the other hand, there are (early warning) fire danger forecasting systems based on numerical weather models at both the continental (San-Miguel-Ayanz et al. 2002) and global (de Groot et al. 2006, Di Giuseppe et al. 2016) scale. These forecasting systems are expected to improve as the performance of numerical weather prediction systems improve (Bauer et al. 2015).

In general, the presence of systemic bias in modelled weather data remains an obstacle to their adoption for fire weather monitoring where it is desirable that fire danger indices are unbiased – that is, they are exact at spatial locations with observations. Therefore, while remote sensing technologies and numerical weather models continue to improve, it is not yet clear that their performance surpasses that of a dedicated ground station network combined with spatial interpolation procedures. For example, Hanes et al. (in press) found that using precipitation estimates from CaPA improved fire danger estimates only in regions with radar coverage.

The spatial interpolation of FWI component variables is challenging for several reasons. First, due to resource and geographical constraints it is not always feasible to achieve a sufficiently dense station density (ie. corresponding to the 40 km representation radius). Second,
interpolating precipitation and wind speed is highly problematic, owing to their high variability, particularly over complex terrain. It has previously been noted that summer precipitation is difficult to interpolate due to the occurrence of convective (i.e. short range) rain events (Flannigan and Wotton 1989). Last, because each of the fuel moisture codes (FFMC, DMC, DC) track fuel moisture using the previous days’ values, there is a significant temporal autocorrelation in their values. This is reflected in the time lag constants, which represent the time required for each of the corresponding layers to lose approximately 63% (i.e. $1-1/e$) of their moisture due to evapotranspiration (assuming dry equilibrium conditions). For FFMC, DMC and DC these are respectively $2/3$, 15 and 53 days (Lawson and Armitage 2008). This means that any errors in estimation of moisture codes can propagate to estimations made for successive days.

In this paper, we revisit the issue of spatial interpolation of fire danger studied previously (Flannigan and Wotton 1989; Flannigan et al. 1998; Tait and Zheng 2005; Sanabria et al. 2013). In particular, we compare several common interpolation schemes for spatial mapping of fire danger using the FWI System. Our study region is the Canadian province of Alberta, of which 57% is covered by the Boreal forest, an area that requires substantial fire management efforts. The interpolation methods we consider include inverse distance weighting, two spline methods and Kriging, a geostatistical method that offers several advantages over the other methods. We also compare interpolation of the FWI input variables followed by calculation of FWI components to directly interpolating the FWI components.

Data

Fire weather data were obtained for Alberta from the weather section of the Wildfire Management Branch of Alberta Agriculture and Forestry. Specifically, daily values of primary meteorological variables at 12:00 LST included 2m temperature, relative humidity, 10m wind
speed and 24 hour accumulated precipitation as required by the FWI system. Secondary variables included the calculated fuel moisture codes FFMC, DMC, and DC and the fire weather indices FWI, BUI, ISI and DSR. The data were subset to include 20 years of fire weather data from 1995-2014. This relatively long data record should prevent some of the issues associated with large inter-annual variability of weather patterns, particularly with regard to precipitation as noted in previous studies that used shorter data records (Flannigan et al. 1998, Hanes et al., in press).

Provincial fire weather stations are typically only operational during the fire season (March to October in Alberta) with the majority of stations operational after May. Therefore, the data were further subset to only include data for each year between the dates June 1st and August 31st (92 days) and to only include stations that were operational for the entire duration of this period. Although the shorter time period restricts our study to summer fire weather, this compromise allowed the inclusion of the greatest number of fire weather stations for each year of analysis. Over the 20 year period the number of validation stations (total number of stations) varied between a minimum of 68 (106) in 1995 to a maximum of 102 (149) in 2012.

Interpolation error is bounded only for points inside the convex hull of the interpolation points (Powell 1994). Outside of the convex hull, estimations are considered extrapolation, and are subject to greater uncertainty since their values may not be bounded. Therefore, in order to reliably conduct the leave-one-out cross-validation procedure (see methods below) it was necessary to interpolate fire weather to station locations within the convex hull formed by the station network (see Figure 2). One of the interpolation procedures we tested, Akima spline interpolation, often failed for points close to (but inside) the convex hull boundary so for this method it was further necessary to only choose validation stations no closer than 75km to the Alberta border (see Figure 3).
Interpolation Variables

In this study, we compared the accuracy of four different interpolation techniques (and their variations) for estimating fire weather variables. Specifically, we tested interpolation of the daily meteorological variables that are inputs to the FWI System: Temperature (T), Relative Humidity (RH), 24 hourly accumulated Precipitation (P) and Wind speed (WS); as well as FWI System components: Fine Fuel Moisture Code (FFMC), Duff Moisture Code (DMC), Drought Code (DC), Build-Up Index (BUI), Initial Spread Index (ISI), Fire Weather Index (FWI). Daily Severity Rating (DSR), also an output of the FWI system, was not investigated because it is a transformed value of the FWI output, and so does not represent any additional information output of the FWI system. Because the inputs to the FWI System consist of the four meteorological variables and the three previous days’ moisture codes there are three different procedures that can be used for estimating the FWI System components at new locations.

1. “Calculate first, interpolate later” (CI): Calculate FWI components at observed locations and interpolate FWI components to unobserved locations.

2. “Interpolate first, calculate later” (IC): Interpolate meteorological variables and previous day’s moisture codes (denoted FFMC-PD, DMC-PD, DC-PD) to unobserved locations, then calculate FWI components.

3. “Interpolate weather only first, calculate later” (IC2): Interpolate meteorological variables to unobserved locations, calculate moisture codes at unobserved locations using previous day’s values (and interpolated meteorological variables) at those locations, and then calculate FWI components. If previous day’s moisture codes are not yet calculated at unobserved locations, use interpolated values.
In this paper, we test the performance of each of these procedures using a cross-validation procedure. Previous works have concluded that the CI procedure provides the best results with respect to root mean squared error (Flannigan et al. 1998, Tait and Zheng 2005); we revisit this conclusion and show that the temporal autocorrelation in the moisture codes has a significant influence on estimation error and bias for each procedure.

**Spatial interpolation Methods**

Currently the Alberta Wildfire Management Branch estimates fire weather between stations using either the method of Inverse Distance Weighting (IDW) with weights given by the power parameters $p = 2$ or using the thin plate spline with smoothing (TPSS) (personal communication). Therefore, these methods were included as baselines for comparison with the other methods. The interpolation methods compared in this study were chosen because they are commonly used methods, and they can be easily implemented in a number of open source and propriety packages. Moreover, they are computationally fast making them suitable for daily operational use. We outline the different methods used below.

**Thin Plate Spline (TPS)**

The Thin Plate Spline is a variational method that is a two dimensional generalization of cubic splines (Duchon 1977; Wahba and Wendelberger 1980). In two dimensions the interpolating surface is estimated by the function $f(r) = \sum_{i=1}^{K} \omega_i \varphi(||r - r_i||)$ for $K$ control points and where

$$
\varphi(r) = r^2 \log(r).
$$

The function is found by minimizing

$$
\sum_{i=1}^{n} ||z(r_i) - f(r_i)||^2 + \lambda \int \int \left[ \left( \frac{\partial^2 f}{\partial x^2} \right)^2 + 2 \left( \frac{\partial^2 f}{\partial x \partial y} \right)^2 + \left( \frac{\partial^2 f}{\partial y^2} \right)^2 \right] dx \, dy
$$
where \( z(\tau_i) \) is the observed value at \( \tau_i \) and the integral represents the deformation energy of the function. The parameter \( \lambda \) determines the level of smoothing. \( \lambda = 0 \) corresponds to no smoothing so that the spline function passes through the data points (an unbiased estimator), whereas \( \lambda \neq 0 \) can further minimize the deformation energy subject to a trade-off with goodness-of-fit. We respectively denote these methods as TPSNS (thin plate spline with no smoothing) or TPSS (thin plate spline with smoothing). Thin plate smoothing was implemented using the *fields* package available for R (Nychka et al. 2015). By default, the smoothing parameter is chosen automatically by a generalized cross validation procedure. The number and location of knots is taken as the default, which corresponds to all unique locations.

**Akima Spline (AS)**

Akima spline (AS) (Akima 1978) interpolation is based on a triangular tessellation of all the data points. A continuously differentiable piecewise function is then fit with each piece corresponding to a cubic polynomial that passes through the vertices of each triangle. Specifically, within each triangle the function is estimated by

\[
\hat{z}(\tau_i) = \sum_{j=0}^{3} \sum_{k=0}^{3-j} q_{jk} x_i^j y_i^k
\]

where the 10 coefficients \( q_{jk} \) are found by solving the set of linear equations formed using the set of points given by \( \tau_i = (x_i, y_i) \) and its 9 closest neighbors. As this is a local approximator, it can lead to a surface robust to outliers. This feature of the Akima spline motivates the inclusion of this method in our study as it may lead to improved estimates for precipitation, a quantity that can exhibit large spatial variability. Moreover, the Akima spline does not use additional smoothing, and since it passes through all data points, is therefore an unbiased estimator. To implement the Akima spline we used the *akima* package available for R (Akima et al. 2015).
Inverse Distance Weighting (IDW)

Inverse distance weighting (IDW) is a method that approximates a value at an unknown location using a linear weighted average of the data at known points using the formula (Shepard 1968)

\[
\hat{z}(r_i) = \frac{\sum_{j=1}^{n} z(r_j) \, d_{ij}^{-p}}{\sum_{j=1}^{n} d_{ij}^{-p}}
\]

where \(d_{ij}\) is the distance between the \(i\)th and \(j\)th points. Here we take \(p=2\), which is the value used by the Alberta wildfire management branch, which therefore provides a practical baseline with which to compare the other methods. However, it should be noted that in general, \(p\) can be optimized by choosing the value which minimizes the absolute error. To implement IDW we used the \(gstat\) package available for R (Pebesma 2004).

Kriging

Kriging is a geostatistical interpolation method (Cressie 2015; Wackernagel 2013) based on the spatial covariance of the data. The method requires fitting a statistical model to the so-called semivariogram, given by

\[
\gamma(h) = \frac{1}{2N_h} \sum_{i=1}^{N_h} [z(r_i) - z(r_i + h)]^2
\]

where \(N_h\) is the number of pairs of observation points separated by a distance \(h\). Using the fitted model, the ordinary kriging (OK) estimator is given by the linear weighted average of known values at neighboring points

\[
\hat{z}(r_i) = \sum_{j=1}^{n} \omega_j z(r_j)
\]

where the weights \(\omega_j\) are determined by minimizing the error variance \(\text{var}[\hat{z}(r_i) - z(r_i)]\) with the constraint that the expected value of the error is zero. OK is the most commonly used form of
kriging, and is equivalent to spatial regression around a local mean estimated from the data. Under the assumption that the data has a multivariate Gaussian distribution, OK provides the best linear unbiased estimator (BLUE). In this paper, we employ OK as well as two variants: Regression Kriging (RK) and Universal Kriging (UK). RK allows incorporation of auxiliary variables by first fitting a regression model to the auxiliary variables and performing (ordinary) kriging with the regression residuals. An obvious covariate for both meteorological and FWI variables was elevation, available for each weather station in the Alberta fire weather database. UK is a special case of RK where the spatial coordinates (longitude and latitude) are used as the covariates. In this paper, we implemented OK, RK using elevation as a covariate (RKE) and UK. Based on testing with several semivariogram models, we fit the semivariogram using the spherical model for all variables considered here, which provided the best fit for the majority of test cases. We also fit the semivariogram model separately for each day; we found fitting a single semivariogram model for the entire season resulted in a poor fit due to the large temporal variability in the weather data. To implement Kriging, we used the gstat package available for R (Pebesma 2004).

Transformations

Data transformations can improve interpolation estimates for some variables, particularly those that exhibit skewness in their distributions. The procedure is to transform the data using a transformation function, perform the interpolation on the transformed data, and then back-transform the interpolated values using the inverse of the transformation function. It should be noted that this procedure can lead to a biased estimator for nonlinear transformations, for which a bias correction term may be necessary (Cressie 2015). Both precipitation and wind speed are non-negative quantities that exhibit skewness in their distributions (ie. non-normality), and their interpolation may therefore be improved by transformation. It has been shown that the square...
root transform can reduce interpolation error for precipitation when using the thin plate spline (Hutchinson 1998). In the current implementation of the Canadian Precipitation Analysis (CaPA), that combines surface observations, numerical weather model predictions and radar observations, the cube root transform has also been found to reduce estimation error (Fortin 2007). In the case of Kriging, if the data (or residuals for regression Kriging) are normally distributed, the best unbiased predictor and best linear unbiased predictor are the same, so that making the data approximately normal can similarly improve interpolation estimates. In this paper, we considered three transformations (and corresponding back transformations) for precipitation and wind speed, in addition to the default case of no transformation: (i) the square root transform (SRT); (ii) the cube root transform (CBT); and (iii) a transformation based on the natural logarithm (ln) of the form \( \ln(x+a) \) for a positive constant \( a \), taken to be \( a=0.1 \). However, we found the log transform consistently underperformed relative to the square root and cube root transforms and therefore omit those results from the analysis. For interpolating relative humidity, we tested a transformation to vapor pressure based on the Clausius-Clapeyron equation (Satoh 2013); we found this method did not lead to any improvement over interpolating relative humidity directly, so we omit these results also. For the kriging estimates, when the square root or cube root transform was applied, we also tested the bias-corrected back-transformed values, given respectively by (Gregoire et al. 2008)

\[
\hat{z}(r) = \hat{y}(r)^2 + \sigma_y^2 \\
\hat{z}(r) = \hat{y}(r)^3 + 3 \hat{y}(r) \sigma_y^2
\]

where \( \hat{y}(r) \) is the kriging estimate for the transformed variable and \( \sigma_y^2 \) is its variance. Note, it was not possible to apply the bias correction procedure for either IDW, AS, TPSS or TPSNS since none of the specific implementations used here provided the variance.
Cross-Validation

In order to assess the performance of each interpolation method, a Leave-one-out cross-validation (LOOCV) procedure was employed. For each year of analysis, the validation stations were determined by the stations interior to the complex hull (Fig. 2). LOOCV proceeds by looping through each validation station, removing observations of fire weather variables corresponding to that station from the data, and interpolating the target variables to the location of the removed station. The removed station data were then replaced and the procedure repeated until all validation stations had been tested. The overall procedure was performed for each day June 2nd to August 31st. June 1st was removed from the validation period as data from this date was used to determine the values of the moisture codes, required as inputs to the FWI calculation for June 2nd. Note that the interpolation procedure used the full set of available weather stations comprising the validation stations as well as those on the complex hull boundary. In the special case of Akima spline interpolation, the procedure was the same as above except that the validation stations consisted only of those interior to a 75km buffered region of the province (Fig. 3).

To assess the performance of the various interpolation methods three continuous metrics were considered: the mean absolute error (MAE), the mean rank absolute error (MRAE) and the mean bias. They are defined respectively as follows:

\[
MAE = \frac{1}{n} \sum_{i=1}^{n} | z_{m,i} - z_{o,i} |
\]

\[
MRAE = \frac{1}{n} \sum_{i=1}^{n} \text{rank}(| z_{m*,i} - z_{o,i} | \mid m \leq M)
\]
where \( z_{o,i} \) and \( z_{m,i} \) are the observed and predicted values at location \( r_i \), \( n \) is the total number of observation-prediction value pairs and \( m \) is a label that indexes the interpolation method. For the MRAE, the rank is calculated between the set of methods \( m \ast \) with \( M \) being the total number of methods being compared. In our case \( n = n_t \times n_s \) where \( n_t \) is the number of days in the validation period (June 2 – August 31\(^{st} \), ie. 91 days) and \( n_s \) is the number of validation stations, which varies by year. The MAE and mean bias represent the mean error across pairs of observations and predicted values. MAE has the advantage over the root-mean-square error (RMSE) metric of giving less weight to outliers. The MRAE is a non-parametric rank based measure.

To account for both spatial and temporal correlations in the data, as well as any non-normality in the data, we employed a non-parametric vector block bootstrap method to determine the 95% confidence intervals for each of the three metrics. This procedure involves repeated resampling of the data (with replacement) using the stationary block bootstrap (SBB) (Politis and Romano 1994) where each data element in the time series consisted of the vector of observations (or predictions) at all validation stations corresponding to each day. Since each time point in the resampled data includes observations at all spatial correlations, the spatial as well as temporal correlation structure is preserved in the resampled data. The block length for each variable (see Table 1) was determined by calculating the 95% confidence intervals for a range of block lengths, and selecting the smallest block length above which the confidence intervals converged, ie. were approximately constant. This ensured the minimum valid block length was chosen, particularly important for DMC, DC and BUI, which exhibit the longest autocorrelation times.

**Results**
Estimating FWI input variables

We first compared the seven interpolation methods (IDW, TPSS, TPSWS, AS, OK, UK, RKE) for the meteorological variables that serve as inputs to the FWI System: ie. 24 hour precipitation, wind speed, temperature and relative humidity. For both precipitation and wind speed we additionally combined each method with either the square root transform (SRT) or cube root transform (CRT). In the cases where kriging (OK, UK, RKE) was used with a transformation we also tested the corrections to bias transformation as given in the methods section (denoted SRTBC and CRTBC for SRT and CRT respectively). This resulted in a total of 27 tested methods for these variables. The results are shown in Fig. 4. In the case of precipitation, we find OK with the CRT transform give the lowest MAE, AS with CRT gives the lowest MRAE and OK with SRT (bias corrected) gives the lowest mean bias. It should be noted that for precipitation MAE values for each method did not differ appreciably and had relatively large 95% confidence intervals for all tested methods. In contrast, the MRAE showed greater differentiation between the methods compared with the confidence intervals. For wind speed the RKE (no transformation) gives the lowest MAE and MRAE, whereas TPSS (no transformation) gives the lowest mean bias. For both temperature and relative humidity, RKE (no transformation) gives the best results with respect to all three metrics. Moreover, for temperature it can be seen that the use of elevation as a covariate gives a significant improvement in the interpolated estimates.

Estimating FWI output variables

To estimate the FWI System variables at new locations we test each of the three procedures outlined in the methods section: The CI procedure corresponds to calculating the FWI components at station locations and then interpolating the FWI output variables to new
locations; The IC procedure corresponds to interpolating weather and the previous day’s moisture codes to unobserved locations and calculating FWI output variables; and the IC2 procedure proceeds similarly to IC except that the previous day’s moisture codes are calculated from the previous day’s moisture codes at the unobserved locations. For IC we selected as inputs the interpolated meteorological variables with the lowest MAE in each case. These were OK with CRT for precipitation and RKE for wind speed, temperature and relative humidity. We also tested two other combinations of interpolated weather variables, corresponding to either reduced bias in precipitation or wind speed, to examine their influence on the FWI output variables.

Table 2 summarizes how each FWI input variable was selected for procedures IC and IC2.

Results for estimation of the moisture codes (FFMC, DMC, DC) at validation locations using the three procedures is given in Fig. 5. We find for IC that calc-3 gives the lowest MAE for FFMC, whereas calc-2 gives the lowest MAE for DMC and DC. IC2 (calc-4) performs relatively poorly with respect to MAE and has the largest positive bias amongst all the estimation methods for all moisture codes. In particular, for the DC, IC2 gives the largest MAE, MRAE and mean bias. To further investigate the source of bias in the moisture code estimations we plot in Fig. 6 the daily mean bias (averaged over all validation stations) for procedures IC and IC2 for a typical year, 2012 in this case. Here, calc-3 is omitted for clarity since it’s results are identical to calc-1 for DMC and DC, which do not depend on wind speed, and calc-1 and calc-3 show almost identical results for FFMC. For FFMC, calc-2 leads to the lowest bias of the calculated methods in the first half of the study period, although there are some large negative deviations of bias in the second half of the study period. For DMC and DC, calc-4 has a large and positive bias compared with calc-1 and calc-2, which increases during the evaluation period. For DMC, calc-1 and calc-2 have negative biases that decrease during the evaluation period. For DC, calc-1 and calc-2 do not change significantly during the evaluation period.
Results for estimation of the FWI output indices (BUI, ISI and FWI) at validation locations using the three methods is given in Fig. 7. We find IC with interpolation of precipitation using OK with SRT (bias corrected) (calc-2) leads to the lowest MAE for BUI, ISI and FWI estimation. Calc-1, Calc-2 and Calc-3 perform similarly well with respect to MRAE for ISI, BUI and FWI; whereas calc-4 (IC2) has smallest negative bias for ISI, AS the smallest positive bias for ISI, calc-1/2 (IC) the smallest (negative) bias for BUI and TPSNS the smallest (negative) bias for FWI. Similar to Fig. 6 we also plot the daily mean bias for the FWI output indices (BUI, ISI and FWI) for 2012 in Fig. 8. For ISI there is no apparent trend in the mean bias for any of the tested methods over the evaluation period. In contrast for both BUI and FWI, calc-4 exhibits a positive trend in mean bias, whereas calc-1 and calc-2 exhibit a negative trend in mean bias.

**Autocorrelation of each variable**

To characterize the temporal autocorrelation of each variable we fit an exponential function to the autocorrelation function (ACF) of each variable and defined the autocorrelation time (ACT) as the time corresponding to a decay of the fitted ACF to a value of 0.1 (see Table 1). These values were then averaged over the 20 years of data used in the study. It is worth noting that wind speed and precipitation show the shortest ACT values, whereas DMC, DC and BUI show the longest ACT values. The long ACT times for DMC and DC are consistent with the long (drying) lag times of those variables. Overall, in contrast to earlier studies (Flannigan et al. 1998; Tait and Zheng 2005) we achieved the best performance for estimating FWI variables with long autocorrelation times (DMC, DC, BUI and FWI) when meteorological variables and previous day’s moisture codes were interpolated to unobserved locations and FWI was then calculated at the unobserved locations using the interpolated values (ie. the IC procedure).
Last, we examine the individual influence of interpolating each FWI input variable on the estimation of each FWI output variable. The procedure is as follows. For each validation station in the LOOCV procedure and each FWI input variable (T, RH, WS, P, FFMC-PD, DMC-PD, DC-PD), we form the set of FWI input variables by taking the value of the selected variable from the interpolated method which minimizes the MAE (given by RKE for T, RH, WS, FFMC-PD, DMC-PD, DC-PD and OK-CRT for P) with the values of the remaining variables given by observations. The set of input variables (interpolated and observed) is then used to calculate the FWI output variables (FFMC, DMC, DC, ISI, BUI, FWI) and the corresponding MAE and mean bias are calculated. The results are shown for the moisture codes (FFMC, DMC, DC) in Fig. 9 and for the fire behavior indexes (ISI, BUI, FWI) in Fig. 10.

For FFMC, DMC and DC the input variables that give the largest MAE are respectively precipitation, DMC-PD and DC-PD. For FFMC, precipitation gives the largest (positive) bias; for DMC, DMC-PD gives the largest (negative) bias; and for DC, DC-PD gives the largest (positive) bias. For ISI, BUI and FWI the input variables that respectively give the largest MAE are wind speed, DMC-PD and wind speed. For ISI, wind speed, relative humidity, precipitation and FFMC-PD all contribute significantly to the overall bias; for BUI, DMC-PD gives the largest (negative) bias; and for FWI, FFMC-PD and DMC-PD are the largest sources of (negative) bias.

Discussion

Improving the performance of spatial interpolation methods for the FWI System outputs is crucial for more accurate spatial mapping of fire danger on the landscape, which is of obvious benefit to fire management agencies. In this study, we compared several common interpolation schemes for estimating FWI System outputs at unknown locations, and readdressed the question
of whether it is best to first interpolate meteorological variables to unobserved locations and then
calculate the FWI System outputs or directly interpolate the FWI System outputs to unobserved
locations.

For the input meteorological variables, we found the geostatistical method Kriging performed best with respect to mean absolute error. Specifically, temperature, relative humidity
and wind speed were interpolated with the lowest MAE using regression Kriging with elevation
as a covariate, whereas precipitation was interpolated with the lowest MAE using ordinary
Kriging with observations transformed using a cube root transform. It is worth noting that for
precipitation, although it is a local approximator, Akima spline interpolation did not perform
better than the Thin plate spline, indicating this feature may not have been exploited for the
spatial density of the data. Apart from reduced error, a benefit of using Kriging is that it is an
exact interpolator, meaning interpolated values are equal to observed values at sampled
locations; therefore, since FWI inputs are interpolated exactly at sampled locations, the
calculated FWI outputs are also exact at those locations.

Local orography (eg. elevation, slope aspect) can have a significant influence on climate
variables. There is, for example, a well-known inverse linear relationship between temperature
and elevation in the troposphere (ie. positive lapse rate). This relationship has previously been
exploited to improve interpolation of temperature (eg. Hudson and Wackernagel 1993, Daly et
al. 2002). Likewise, relative humidity, which varies with temperature and atmospheric pressure,
also has an implicit dependence on elevation. Although there is relatively little work on
interpolation of relative humidity, one study did find using cokriging with elevation as a
covariate led to the lowest estimation errors amongst several methods (Apaydin et al. 2004).
Elevation has also been previously used to improve interpolated values of wind speed (Luo et al.
2008). The positive correlation between precipitation and elevation in mountainous areas (Basist
et al. 1994) has previously been exploited to improve interpolation estimates of precipitation (eg. Goovaerts 2000, Daly et al. 2002). Including elevation as a covariate in our study led to the most significant improvements for interpolation of temperature, with more modest improvements for relative humidity and wind speed. Conversely, the three best performing methods – with similar performance – for interpolating precipitation were ordinary kriging, universal kriging and regression kriging with elevation (all using the cube root transform). The fact that regression kriging with elevation did not outperform either ordinary nor universal kriging indicates may be due the high spatial variability of precipitation combined and the supposition that the linear relationship between precipitation and elevation is not as good as it is for the other variables (ie. temperature, relative humidity and wind speed).

The performance of spatial interpolation of the moisture codes (FFMC, DMC, DC) and fire behavior indexes (ISI, BUI, FWI) was significantly influenced by the procedure employed. For FFMC we found both IC (calc-1, calc-2, calc-2) and IC2 (calc-4) perform well with respect to MAE but that calc-2 gives a lower bias. For DMC and DC, IC (calc-1, calc-2, calc-3) and CI (with RKE) perform well but IC2 (calc-4) performs relatively poorly. The lower bias achieved with calc-2 for FFMC reflects the fact that the interpolated precipitation input has a lower bias. For all moisture codes, calc-4 lead gave a large positive mean bias compared to the other methods tested. This can be attributed to the propagation of estimation error from the previous day’s codes to the current day’s codes, an effect which is larger for DMC and DC with large autocorrelation times (Table 1).

To understand the performance of the estimation methods on the fire behavior indices (ISI, BUI, and FWI) it is necessary to consider the variable dependence of the FWI System (Fig. 1). Overall, the IC and IC2 procedures outperformed CI for all three indexes. In particular, ISI, which depends only on FFMC and wind speed, was best estimated by IC (calc-1, calc-2, calc-3)
and IC2 (calc-4). Interestingly, calc-4 gives the lowest (negative) bias of the four methods for BUI, but the largest (positive) bias for FFMC; whereas calc-2 gives the largest (negative) bias of the four methods for BUI, but the smallest (positive) bias for FFMC. In contrast, BUI is best estimated by IC (calc-1, calc-2, calc-3) or CI (RKE), but is poorly estimated by IC2 (calc-4); this result reflects the performance of estimation methods for DMC and DC, which are inputs to BUI. FWI, which combines ISI and BUI, is best estimated by IC (calc-1, calc-2, calc-3). The middling performance of IC2 (calc-4) for FWI is likely due to a compromise between its good performance for ISI and poor performance for BUI. In general, IC2 (calc-4) performed poorly for any of the FWI System outputs with relatively large correlation times (Table 1); that is, for DMC, DC, and BUI.

In a spatial modeling context, we have investigated the differences between a “calculate first, interpolate later” (CI) approach and a “interpolate first, calculate later” (IC) approach (Stein et al. 1991). In general, unless the model is linear, these approaches may lead to different results (Addiscott and Tuck 1996). Due to model nonlinearities, CI may not reproduce spatial heterogeneities of the actual (unknown) model, whereas IC might lead to significant uncertainty in the model inputs, depending on the number of input parameters to be interpolated and their spatio-temporal characteristics. Thus, whether CI or IC yields better performance is likely to be a trade-off between uncertainty in the model inputs (ie. interpolation error) and model nonlinearity. For example, in a study of drought index estimation (Rhee and Carbone 2011) IC was found to perform better than CI, whereas a study of areal interpolation of soil moisture (Stein et al. 1991) found conversely that CI performed better than IC. In yet another study looking at the estimation of reference evapotranspiration (Mardikis et al. 2005) found IC and CI procedures gave very similar results.
As the FWI System is a nonlinear empirical model, the IC and CI procedures will in general yield different results, depending on the spatio-temporal characteristics of the FWI input parameters. In the current study, we found that in most cases IC outperformed CI with respect to both MAE and mean bias. In contrast, an earlier study on interpolation of the FWI System indexes in north-western Ontario by Flannigan et al. (1998) found the CI procedure (ie. interpolating the FWI indexes directly) performed better than the IC procedure. They attributed this to the short-range nature of convective precipitation, which leads to relatively poor estimation of this variable by interpolation. Tait and Zheng (2005) similarly suggested the CI procedure should also be used for interpolation of fire weather in New Zealand. There are a number of differences between the present study and that of Flannigan et al. (1998) that may account for the contrasting results: (i) Flannigan et al. (1998) considered north-western Ontario, whereas the present study considers Alberta. These regions have different climatologies and the station networks also differ by location and distribution. In particular, the station density in north-western Ontario was approximately one third of that considered in the present study; (ii) Flannigan et al. (1998) did not test geostatistical methods, which in the present study have further reduced interpolation error by inclusion of elevation or location as a covariate; (iii) Flannigan et al. (1998) used 2 years of data with 6 validation stations, whereas the present study uses a larger data set with 20 years of (summer) fire weather data with between 68 and 102 validation stations.

Since we found the IC procedure performed best overall, we also examined how interpolation of each FWI input variable independently contributes to the output error and bias of the FWI output variables (see Figs. 9 and 10). These results suggest which FWI input variable interpolation estimates should be improved to give the largest reduction in estimation error (or bias) for the FWI System outputs, thus providing guidance for possible future work. For
example, improving estimates of precipitation, which leads to the largest MAE and mean bias for
FFMC, should lead to the largest improvement in FFMC. Likewise, improving interpolation of
DMC-PD and DC-PD should give the largest improvement in DMC and DC respectively. For
the fire behavior indexes, ISI and FWI may be best improved (with respect to MAE) by
improving interpolation of wind speed, and BUI should be best improved by improving
interpolation of DMC-PD.

Conclusions

In summary, the results of this study lead to two main recommendations for improving
interpolation of fire danger ratings based on the FWI System: (1) One should first interpolate the
FWI System input variables (temperature, relative humidity, wind speed, precipitation and
previous days’ moisture codes) to unobserved locations and then calculate the FWI System
outputs; (2) regression kriging with elevation as a covariate should be used for estimating all
FWI System input variables, except for precipitation which may be better estimated using
ordinary kriging and the cube root transform.

There are two main directions any future work could take. First, an obvious strategy for
improving estimation of fire weather danger ratings is to improve interpolation of the FWI input
variables (particularly, those identified above) by either improving the methods used in this study
or by considering additional interpolation methods. In this study, we found the use of elevation
as a covariate improved interpolation for most of the considered variables. Incorporating
additional information may further improve interpolation estimates. For example, Wagner et al.
(2012) used wind and satellite remote sensing data as covariates for improved spatial
interpolation of precipitation; Jarvis and Stuart (2001) used topographic and land use variables to
improve interpolation of daily maximum and minimum temperatures; and Di Luzio et al (2008)
considered additional topographic features for improving temperature and precipitation estimates. Moreover, in addition to the interpolation methods considered in this study (ie. IDW, splines and kriging) there are a number of other methods that are appropriate for environmental variables including trend surface analysis (Tabios and Salas 1985), multivariate regression (Stahl et al. 2006), conditional interpolation (Hewitson et al. 2005), objective analysis (Barnes 1964), and machine learning algorithms (Li et al. 2011). Second, as an alternative to spatial interpolation, gridded products for near-real-time monitoring of weather may be used. Such systems are becoming more prevalent, including those based on satellite remote sensing (see Thies and Bendex 2011 for a review), precipitation estimates that combine radar, observations and numerical weather models (Mahfouf et al. 2007), and physics based wind models (Forthofer et al. 2009). As these technologies continue to improve in the future, they are expected to be incorporated into fire weather monitoring systems.

In Canada, the FWI System is an integral part of fire management operations at both a daily and seasonal time scale. The use of the system is particularly important in Alberta where total annual fire management expenditures have exceeded $300 million (CAD) in recent years (Stocks and Martell 2016). Fire suppression costs can be associated with area burned, with 97% of the total area burned in Canada occurring for only 3% of the fires, corresponding to days with extreme fire weather (Stocks et al. 2002). Accurate spatial mapping of fire weather is therefore critical for fire management agencies to assess potential fire danger in regions where weather stations are not located and to accordingly allocate resources for fire prevention or suppression, or for emergency response.
Acknowledgements

The authors would like to thank Alberta Environment and Parks for providing the fire weather data. We also thank Peter Englefield for useful comments.
References


Wahba, G., and Wendelberger, J. 1980. Some new mathematical methods for variational...


Table 1: Autocorrelation times (ACT) and Stationary block bootstrap block lengths used for FWI input and output variables.

<table>
<thead>
<tr>
<th></th>
<th>T (days)</th>
<th>RH</th>
<th>WS</th>
<th>P</th>
<th>FFMC</th>
<th>DMC</th>
<th>DC</th>
<th>ISI</th>
<th>BUI</th>
<th>FWI</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACT</td>
<td>5.6</td>
<td>3.4</td>
<td>2.2</td>
<td>2.2</td>
<td>4.4</td>
<td>13.1</td>
<td>24.4</td>
<td>4.6</td>
<td>13.8</td>
<td>6.1</td>
</tr>
<tr>
<td>SBB block length (days)</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>3</td>
<td>5</td>
<td>10</td>
<td>14</td>
<td>5</td>
<td>10</td>
<td>7</td>
</tr>
</tbody>
</table>
Table 2: Estimation method used for each of the FWI System input variables for procedures IC and IC2.

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>RH</th>
<th>WS</th>
<th>P</th>
<th>FFMC(_{t-1})</th>
<th>DMC(_{t-1})</th>
<th>DC(_{t-1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calc-1 (IC)</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
<td>OK-CRT</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
</tr>
<tr>
<td>Calc-2 (IC)</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
<td>OK-SRTBC</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
</tr>
<tr>
<td>Calc-3 (IC)</td>
<td>RKE</td>
<td>RKE</td>
<td>TPSS</td>
<td>OK-CRT</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
</tr>
<tr>
<td>Calc-4 (IC2)</td>
<td>RKE</td>
<td>RKE</td>
<td>RKE</td>
<td>OK-CRT</td>
<td>FFMC(_{t-2})</td>
<td>DMC(_{t-2})</td>
<td>DC(_{t-2})</td>
</tr>
</tbody>
</table>
Figure 1: Variable dependence in the FWI system.
Figure 2: Operational fire weather stations for Alberta, June 1 - August 31 2014. Solid blue circles indicate stations interior to the convex hull of all stations, whereas red hollow circles indicate stations comprising the convex hull.
Figure 3: Operational fire weather stations for Alberta, June 1 - August 31 2014. Blue solid circles indicate stations at least 75km from the provincial border, whereas red hollow circles indicate stations located within 75km of the provincial border.
Figure 4: MAE, MRAE and mean bias for tested interpolation methods for FWI input meteorological variables.

Horizontal lines represent the 95% percentile confidence intervals calculated using the bootstrap method outlined in the main text.
Figure 5: MAE, MRAE and mean bias for FWI System moisture codes estimated using different methods (see Table 2 for definitions of calc-1, calc-2, calc-3 and calc-4 methods). Horizontal lines represent the 95% percentile confidence intervals calculated using the bootstrap method outlined in the main text.
Figure 6: Mean daily bias in moisture codes for different estimation methods. See main text for further details.
Figure 7: MAE, MRAE and mean bias for FWI System output variables estimated using different methods.

Horizontal lines represent the 95% percentile confidence intervals calculated using the bootstrap method outlined in the main text.
Figure 8: Mean daily bias in FWI output indices for different estimation methods. See main text for further details.
Figure 9: Variable importance for FWI moisture codes. Hollow circles indicate the moisture code has no dependence on the variable.
Figure 10: Variable importance for FWI fire behavior indexes. Hollow circles indicate the moisture code has no dependence on the variable.