Canadian Geotechnical Journal
Revue canadienne de géotechnique

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Journal:	<i>Canadian Geotechnical Journal</i>
Manuscript ID	cgj-2017-0006.R1
Manuscript Type:	Article
Date Submitted by the Author:	09-May-2017
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Keyword:	subgrade reaction, tunnel, mobilized strength, undrained clay

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2 **on Mobilized Strength of Undrained Clay**

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Nonlinear Subgrade Reaction Solution for Circular Tunnel Lining Design Based on Mobilized Strength of Undrained Clay

Abstract:

This paper presents a nonlinear solution of radial subgrade reaction–displacement (p_k – u_r) curve for circular tunnel lining design in undrained clay. With the concept of soil shear strength nonlinearly mobilized with shear strain, an analytical solution of p_k is obtained using the mobilized strength design (MSD) method. Two typical deformation modes are considered, namely oval and uniform modes. A total of 197 orthogonally designed cases are used to calibrate the proposed nonlinear solution of p_k using the finite element method (FEM) with the Hardening soil (HS) model. The calibration results are summarized using a correction factor η , which is defined as the ratio of p_{k_FEM} over p_{k_MSD} . It is shown that η is correlated to some input parameters. If this correlation is removed by a regression equation f , the modified solution $f \times p_{k_MSD}$ agrees very well with p_{k_FEM} . Although the mobilized soil strength varies with principal stress direction in reality, it is found that a simple average of plane strain compression and extension results is sufficient to produce the above agreement. The proposed nonlinear p_k – u_r curve is applied to an actual tunnel lining design example. The predicted tunnel deformations agree very well with the measured data. In contrast, a linear p_k model would produce an underestimation of tunnel convergence and internal forces by 2–4 times due to the overestimation of p_k at large strain level.

Keywords:

Radial subgrade reaction, Tunnel, Mobilized strength design (MSD), Nonlinear, Undrained clay.

45 Introduction

46 The embedded beam spring model is the most frequently used design model among all the
47 models recommended by design codes for the structural design of tunnel linings (ITA 2004;
48 JSCE 2007; GB50157 2013). Some key aspects in lining design, e.g., soil load, lining
49 structure and soil-structural interactions, are considered explicitly in this model.
50 Specifically, the tunnel lining is represented by beams supported on soil springs around
51 tunnel perimeter to simulate the soil-structural interaction caused by initial earth pressure.
52 However, the soil spring constant, also named as the radial subgrade modulus k_r , is difficult
53 to evaluate and it is often selected from empirical correlations with some engineering
54 judgment (Mair 2008). For example, the standard penetration test (SPT) blow count N has
55 been related to a range of k_r in some local design regulations (JSCE 2007; DGJ08-10 2010).
56 It is a matter of judgment to pick a specific value of k_r from this broad guideline, which
57 would translate to significant uncertainty in the calculated lining thrust and moment (Lee et al.
58 2001; Gong et al. 2015). Hence, some analytical solutions of k_r have been proposed as a
59 more rational method to estimate the magnitude of this parameter for design (Arnaud and
60 Molins 2011).

61 These analytical solutions of k_r are usually derived with a basic assumption of elastic soil
62 in infinite or semi-infinite space. The value of k_r is defined as the ratio of calculated
63 subgrade reaction p_k over soil radial displacement u_r around the tunnel perimeter given a
64 prescribed deformation mode of soil-lining interaction. Wood (1975) proposed a solution of
65 k_r under the assumption of oval deformation mode shape in infinite space. Sagaseta (1987)
66 presented a solution of k_r under a uniform deformation mode shape in semi-infinite space.
67 Later on, more general ground deformation modes are developed by combining these two
68 basic mode shapes (Verruijt and Booker 1996; Park 2005; Pinto and Whittle 2014). Based
69 on this observation, Zhang et al. (2014) proposed a general elastic solution of k_r that can be

70 decomposed into a series of basic k_r , each produced by a simple soil deformation mode
71 shape.

72 However, it has been observed from centrifuge test results that tunneling may produce
73 non-linear soil behavior around the tunnel (Mair 1979; Taylor 1998). In addition, in-situ
74 measurements of tunnels in operation have shown that large deformations far beyond the
75 elastic range can happen (Shen et al. 2014). Huang and Zhang (2016) presented a detailed
76 field case where the tunnel deformation caused by extreme surface surcharge is almost seven
77 times larger than the design value. The large tunnel deformation inevitably will drive the
78 soil around the tunnel to an elasto-plastic state (Osman et al. 2006b; Klar et al. 2007). Thus,
79 the elastic solution of k_r might not be sufficient to describe the full range of behavior of this
80 key input parameter in a tunnel lining design. Hence, there is a practical motivation to
81 develop an elasto-plastic solution of k_r to model tunnel lining behavior in non-linear soils.

82 The task at hand is to obtain a non-linear solution of k_r , which is a non-linear curve of
83 subgrade reaction p_k versus the radial displacement u_r . This is conceptually similar to the
84 well-known non-linear p - y curve for laterally loaded piles (Nogami et al. 1992; Bransby 1999;
85 McGann et al. 2011). However, unlike the p - y curve for laterally loaded pile design, limited
86 solutions have been proposed for the non-linear p_k - u_r curve for tunnel lining design. A
87 non-linear curve with the assumption of a hyperbolic function for subgrade reaction has been
88 proposed and utilized for the design of rock tunnel supports (Oreste 2007; Do et al. 2013; Do
89 et al. 2014). The rationality of using the hyperbolic shape function is not well described and
90 its relation to rock properties is unclear.

91 The mobilized strength design (MSD) method can provide a more rational method to
92 derive the non-linear p_k - u_r curve and can provide an explicit linkage to relevant soil
93 properties. The MSD method is built around an assumed plastic deformation mechanism of
94 the soil movements due to soil-lining interaction and the concept of “mobilized soil strength”

that is founded on measured soil behavior (Bolton and Powrie 1988; Bolton et al. 1990). For the problem with tunneling induced ground movement, Osman et al. (2006a) has developed a non-linear curve of initial support pressure p varying with the ground surface settlement s_m by using the MSD method. The empirical Gaussian function (Peck 1969; Osman et al. 2006b) is adopted to describe the ground movement in the model. However, the soil-lining interactions that cause the common oval deformation mode (Klar et al. 2007; Pinto et al. 2014) are ignored in their model. The MSD method is useful, because it provides an analytical solution to a fairly complex soil-structure interaction problem. Although the finite element method (FEM) can solve the same problem, it is arguably less physically insightful than the MSD method and it may require a more detailed characterization of soil behavior (in terms of parameters for constitutive model) beyond what is produced by a routine site investigation. The cost of achieving analytical simplicity in the MSD method is that it is potentially less accurate than the FEM (assuming all information required in the FEM method is fully available). Nonetheless, Zhang et al. (2015) showed that the MSD method and the FEM could produce comparable solutions if the former is corrected by a simple factor. A key contribution of this paper is the characterization of this correction factor for the tunnel lining problem.

The objective of this paper is to develop a non-linear curve of subgrade reaction p_k varying with radial displacement u_r for a circular lined tunnel in elasto-plastic undrained soil using the MSD method. The non-linear solution of radial subgrade modulus k_r is derived as the derivative of this p_k - u_r curve. The mobilized soil strain ε_{mob} is expressed as a function of radial displacement u_r using a typical uniform or oval deformation mode for soil-lining interactions in semi-infinite space (Verruijt and Booker 1996). The subgrade reaction p_k is then calculated with the concept of mobilized soil strength $s_{u,\text{mob}}$ under the minimum energy principle. The finite element method (FEM) with a widely used nonlinear constitutive

model for undrained soil, namely the Hardening Soil (HS) model, is adopted to validate the applicability of the proposed nonlinear solution of p_k . The MSD solution (p_{k_MSD}) can be corrected to agree with the FEM solution (p_{k_FEM}) using a simple approach proposed in Zhang et al. (2015). Finally, the performance of the corrected MSD method is demonstrated using a tunnel lining design example based on a field case reported by Huang and Zhang (2016).

Deformation modes subjected to subgrade reaction

Mode shape for tunnel lining

A tunnel lining interacts with the surrounding ground after it is installed. It deforms under the initial earth pressure coming from the ground. The ground responds to the lining deformation by a subgrade reaction. The lining deformation would adjust to the subgrade reaction and subsequently induce a new but small subgrade reaction. This iteration would finally converge and the lining would form a specific distribution of deformation that is consistent to the distribution of subgrade reaction. Although this distribution is complicated and different from case to case, it could be reasonably characterized by basic mode shapes such as uniform and oval shapes or more general shapes that can be viewed as combinations of these basic mode shapes (Pinto and Whittle 2014; Pinto et al. 2014; Zhang et al. 2014). It should be noted that all deformation mechanisms discussed in this paper take place under the plane strain condition since a tunnel is a long linear structure.

In the case of a uniform mode shape (see in Fig. 1a), the lining deforms uniformly around the tunnel perimeter at a constant value, saying Δ (Sagasetta 1987):

$$u_{r1} = \Delta \quad (1a)$$

where u_{r1} means the uniform radial convergence at the perimeter of the tunnel lining. In the case of an oval mode shape (see in Fig. 1b), the lining deforms following a cosine function of the sectional angle θ with vertical axis, as shown below (Wood 1975):

$$u_{r2} = \delta \cos 2\theta \quad (1b)$$

145 where u_{r2} means the oval radial convergence at tunnel perimeter. The parameter δ is the
146 value of deformation at the tunnel crown (i.e., $\theta = 0^\circ$). More general deformation shapes
147 can be formed by combining these two basic mode shapes. It is clear that the parameters Δ
148 and δ are the maximum magnitude of the lining convergence around the tunnel perimeter. It
149 is more convenient to use these parameters as key performance indicators of the tunnel lining
150 under earth pressure including subgrade reaction, rather than to consider the entire shape.

151 It has been reported that these deformation indicators (i.e., Δ and δ) are closely related to
152 the internal forces of lining and structural defects, e.g., crack, leakage, concrete spalling or
153 joint opening for segmental lining of shield tunnel (Yuan et al. 2012; Huang and Zhang 2016).
154 Large internal forces or severe structural defects could affect the serviceability and safety of
155 tunnels. Hence, it is not surprising these deformation indicators appear in the definition of
156 ultimate and serviceability limit states in tunneling codes and standards. For example, the
157 British Tunnel Standard (BTS 2004) has set an ultimate limit for the maximum convergence
158 normalized by tunnel radius (Δ/R or δ/R , R is the tunnel radius) at 2%. As for serviceability,
159 the Chinese code (GB50157 2013) has set a serviceability limit of Δ/R (or δ/R) at 0.3%-0.4%,
160 while the Shanghai local regulation (DGJ08-10 2010) relaxes the serviceability limit to 0.5%.

161 ***Deformation modes for soils around the tunnel***

162 Assuming that soil and tunnel lining deforms in a compatible way, one expects the soil
163 around the deformed tunnel to follow the two main deformation modes, namely uniform and
164 oval modes. Equations characterizing these two modes in infinite space and semi-infinite
165 space are available (Timoshenko and Goodier 1970; Wood 1975; Pinto and Whittle 2014).
166 Although these equations are derived analytically with a deformation field based on elasticity,
167 measured data reported in the literature have justified this assumption, namely the observed
168 deformation field for mobilized soils in a nonlinear state is proportional to those elastic
169 deformation fields (Mair 1979; Klar et al. 2007; Pinto et al. 2014). Since the mode in

semi-infinite space is suitable for both the shallow and deep tunnels, it is applied to describe the elasto-plastic deformation field of mobilized soils in this paper. Based on the solutions proposed by Verruijt and Booker (1996), the uniform and oval deformation modes for undrained soil with a Poisson's ratio ν of 0.5 in semi-infinite space are briefly introduced below.

a). Uniform mode

A typical uniform deformation field is plotted in Fig. 2a. The tunnel with a radius of R is buried in a depth of h below the ground surface, as shown in Fig. 2a. The horizontal ground movement u_x could be represented by Eq. 2a, while the vertical ground movement u_z could be represented by Eq. 2b, as shown below:

$$u_x = -\Delta \cdot R \left\{ \frac{x}{x^2 + (z-h)^2} + \frac{x}{x^2 + (z+h)^2} - \frac{4xz(z+h)}{[x^2 + (z+h)^2]^2} \right\} \quad (2a)$$

$$u_z = -\Delta \cdot R \left\{ \frac{z-h}{x^2 + (z-h)^2} + \frac{z+h}{x^2 + (z+h)^2} - \frac{2(z+h)}{x^2 + (z+h)^2} + \frac{2z[x^2 - (z+h)^2]}{[x^2 + (z+h)^2]^2} \right\} \quad (2b)$$

where x is the horizontal distance of calculated soil element from tunnel axis, and z is the depth of the element. The parameter Δ is the radial convergence of the deformed tunnel lining in a uniform shape mentioned previously. Although the tunnel deforms uniformly, it is clear from Fig. 2a that the surrounding soils do not deform uniformly due to the effect of the image part of singularity solution in a semi-infinite space (Sagaseta 1987).

In a semi-infinite space, the boundary of the deformation field is located at the ground surface above the tunnel. There is no boundary at the two sides of the tunnel and below the tunnel. But it should be noted from Eq. 2 that, as the radial distance r between soil and the center of tunnel increases, the soil displacement vanishes quickly following a decay function of $1/r$. That is to say, when the distance r is about 100 times of radius R , the corresponding soil element would have a displacement less than 1% of Δ , which exert a negligible effect on

the behaviors of soil around tunnel lining (Mair et al. 1993).

In this deformation mode, the calculation of soil strain by a first derivation of Eq. 2 have revealed that the direction of principal strain is horizontal for soils locating above tunnel crown and along its vertical axis, while the direction of principal strain is vertical for soils locating at the two sides of the springline. It has been validated by ground movement in 1g model test (Seneviratne 1979; Osman et al. 2006a). Hence, the soil behaviors along the vertical axis are more likely to be modelled under the condition of plane strain extension (PSE), while the soils at two sides of springline are to be modelled under the condition of plane strain compression (PSC), as shown in Fig. 2b.

b). Oval mode

The typical oval deformation field is plotted in Fig. 3a. Similar to the uniform mode, the tunnel has a radius of R and a cover depth of h . The horizontal ground movement u_x is expressed by Eq. 3a, and the vertical ground movement u_z is represented by Eq. 3b, as shown below:

$$u_x = -\delta \cdot R \left\{ \frac{x(x^2 - z^2)}{(x^2 + z^2)^2} + \frac{x[x^2 - (2h - z)^2]}{[x^2 + (2h - z)^2]^2} - 4xh \frac{(h - z)[x^2 - 3(2h - z)^2]}{[x^2 + (2h - z)^2]^3} \right\} \quad (3a)$$

$$u_z = \delta \cdot R \left\{ \frac{-z(x^2 - z^2)}{(x^2 + z^2)^2} + \frac{(2h - z)[x^2 - (2h - z)^2]}{[x^2 + (2h - z)^2]^2} - 2h \frac{[x^2 - (2h - z)^2]}{[x^2 + (2h - z)^2]^2} - 4h \frac{(h - z)(2h - z)[3x^2 - (2h - z)^2]}{[x^2 + (2h - z)^2]^3} \right\} \quad (3b)$$

where x is the horizontal distance between the calculated soil element and the tunnel axis, and z is the depth of the element. The parameter δ is the magnitude of lining displacement at crown as the tunnel deforms into an oval shape mentioned previously. Similar to the case of uniform mode, the boundary of the oval mode is located at the ground surface above tunnel, and the soil displacement will decrease with the increase of radial distance r following a

214 decay function of $1/r$.

215 In the oval deformation mode, similar to the case of uniform mode, the calculated
216 principal strain of soil along the vertical axis from Eq. 3 is in the horizontal direction. The
217 soil at both sides of the springline experiences principal strains in the horizontal direction due
218 to the extension of soil at the springline location as part of an oval shape. This extension
219 ground movement mode at the springline location can be seen from measured data in field
220 cases (Hashimoto et al. 1996; Standing and Selemetas 2013). Hence, the plane strain
221 extension (PSE) condition is more suitable for modeling mobilized soil behavior in an oval
222 mode, as shown in Fig. 3b.

223 Energy Conservation in Mobilized Soils

224 Following the MSD concept that the soil is in continuous yielding corresponding to a
225 mobilized strength, the potential energy loss caused by deformed soil induced by soil-lining
226 interaction is equal to the work done by corresponding subgrade reaction load p_k around the
227 tunnel perimeter. This energy conservation formulation is the same as those general
228 formulation used for elastic solutions of subgrade reaction (Verruijt and Booker 1996; Zhang
229 et al. 2014). It should be emphasized that this energy conservation formulation only
230 accounts for soil-lining interaction at a distance behind the tunnel face. It does not include
231 the work done by the face support pressure during tunnel excavation. The MSD framework
232 for this tunneling procedure has been presented elsewhere by Osman et al. (2006a) and Klar
233 and Klein (2014). With the above consideration in mind, the formulation of energy
234 conservation for subgrade reaction can be expressed as below:

$$235 \quad \int_{\pi D} p_k u_r ds = \int_{Area} s_{u,mob} \epsilon_s dA \quad (4)$$

236 where the u_r is soil radial displacement around the tunnel perimeter calculated by using the u_x
237 and u_z mentioned previously ($u_r = u_z \cos \theta - u_x \sin \theta$). The one-dimensional integral length for
238 left side of Eq. 4 is essentially along the tunnel perimeter i.e., πD in Eq. 4, while the

two-dimensional integral area for the right side, i.e., *Area* in Eq. 4, is the semi-infinite space described in previous section (see Fig. 1). The parameters $s_{u,mob}$ and ε_s are the mobilized soil undrained shear strength and the corresponding mobilized engineering shear strain, respectively. Adopting the rule-of-thumb that the engineering shear strain ε_s is 1.5 times of the deviatoric strain ε_q leads to:

$$\varepsilon_s = 1.5\varepsilon_q = \sqrt{\frac{3}{2}} \varepsilon_{ij} \varepsilon_{ij}, \quad (i, j \text{ principal directions}) \quad (5)$$

where ε_{ij} are the shear strain in the principal directions. All these strains could be calculated by Eqs. 2 and 3 in closed form. The mobilized undrained shear strength $s_{u,mob}$ is obtained by a suitable nonlinear interpolation of the stress-strain curve given a calculated mobilized engineering shear strain ε_s for each soil element within the semi-infinite integral area. The soil elements around tunnel are in a PSC, PSE, or an intermediate state, depending on the rotation of the principal strain. Hence, soils within the integral area in the right side of Eq. 4 theoretically should be discretized into separate soil element that follows a stress-strain curve compatible to the rotation of the principal strain calculated by Eqs. 2 and 3. In other words, for this tunnel problem, the stress-strain curve varies with spatial location, because the rotation of the principle strain varies with spatial location.

Practical Approximation of Energy Conservation

It is evident that the exact solution of Eq. 4 can be tedious if each soil element is assigned a stress strain curve as a function of the principal strain rotation at each location. A practical solution to Eq. 4 is to examine the possibility of replacing $s_{u,mob}$ which varies according to principal strain rotation by an equivalent $s_{u,mob}^*$ that does not depend on principal strain rotation. Since both PSC and PSE states are present in the soil elements around the deformed tunnel (as see in Fig. 2b), a simple way to approximate the exact mobilized shear strength $s_{u,mob}$ is to average the results of $s_{u,mob}$ from plane strain compression tests and the results of $s_{u,mob}$ from plane strain extension tests, as recommended by Koutsoftas and Ladd

(1985):

$$s_{u,mob}^* = 0.5(s_{u,mob|PSE} + s_{u,mob|PSC}) \quad (6)$$

Note that both the $s_{u,mob|PSE}$ from plane strain extension test and $s_{u,mob|PSC}$ from plane strain compression tests are evaluated at the depth of the tunnel axis following the assumption adopted by Osman and Bolton (2006a) for a tunneling-induced ground settlement problem. The mobilized shear strength $s_{u,mob}$ in Eq. 4 could be replaced by the above approximate $s_{u,mob}^*$ in Eq. 6. It is worth pointing out here that the fundamental value of MSD is related to its ability to produce an answer sufficiently accurate for design at a significantly lower cost than FEM. If MSD is only marginally less costly than FEM, there is no reason to adopt MSD. Eq. 6 is adopted with this pragmatic consideration in mind. The focus of this study is to clarify if the error associated with this approximation as benchmarked against the finite element solution is acceptable for design purposes.

In addition, when Eq. 4 is integrated numerically, the semi-infinite space is approximated by a large circular domain with a radius about 100 times the tunnel radius R , which is reasonably large enough to minimize the effect of ignoring soil displacements outside this domain on the integral results (Klar et al. 2007). Hence, equation 4 is re-written based on the above simplification as follows:

$$\int_{\pi D} p_k u_r ds = \int_{Area} s_{u,mob}^* \varepsilon_s dA \quad (Area \in \{z \geq 0, r \leq 100R\}) \quad (7)$$

As the tunnel lining deforms into a prescribed mode shape (uniform or oval), the distribution of subgrade reaction would match a similar shape to the deformation shape following the basic Winkler assumption. Hence, two distribution shapes for subgrade reaction p_k is assumed based on the previously assumed deformation shapes, namely the uniform and oval shapes. Equation 7 is further developed based on each specific deformation mode below.

In the case of the uniform mode, subgrade reaction p_k is distributed with a constant value

289 p_{k0} at any position along lining as below:

$$290 \quad p_k = p_{k0} \quad (8)$$

291 By substituting Eq. 8 and Eq. 1a into Eq. 7, the p_{k0} can be derived as following equation:

$$292 \quad p_{k0} = \frac{\int_{Area} s_{u,mob}^* \times \varepsilon_s(\Delta) dA}{\Delta \int_{\pi D} ds} \quad (9)$$

293 where Δ is the radial displacement of lining under uniform shape (see in Eq. 1a). When
 294 given a specific value of Δ , the engineering shear strain ε_s could be derived by Eq. 5. The
 295 approximate mobilized shear strength $s_{u,mob}^*$ can then be read off a representative shear
 296 stress-strain curve from Eq. 6 corresponding to the calculated ε_s . It is worth pointing out
 297 that the stress-strain curve is an input obtained from an actual laboratory test. Subsequently,
 298 subgrade reaction is derived by using Eq. 9. Hence, subgrade reaction p_k is a nonlinear
 299 function of the parameter Δ given a specific soil stress-strain curve from the average of the
 300 PSC test and PSE test. The subgrade modulus k_r is just the first-order derivative of Eq. 9
 301 with parameter Δ .

302 In the case of oval shape, subgrade reaction p_k is distributed in an oval shape represented
 303 by following equation:

$$304 \quad p_k = p_{k0} \cos 2\theta \quad (10)$$

305 where p_{k0} is the subgrade reaction at tunnel crown and the angle θ is equal to zero. By
 306 substituting Eq. 1b and Eq. 10 into Eq. 7, the p_{k0} is further derived as below:

$$307 \quad p_{k0} = \frac{\int_{Area} s_{u,mob}^* \cdot \varepsilon_s(\delta) dA}{\delta \int_{\pi D} \cos^2 2\theta ds} \quad (11)$$

308 where δ is the tunnel radial displacement at tunnel crown (see in Eq. 1b). Similar to the
 309 uniform case, when the parameter δ is obtained, the subgrade reaction p_{k0} is derived as a
 310 function of radial displacement δ given a specific soil stress-strain curve from the average of
 311 the PSE and PSC tests. Subgrade modulus k_r under oval mode is also obtained from the

312 first-order derivative of Eq. 11.

313 **Validation of Proposed Nonlinear Solution of p_k**

314 The proposed nonlinear subgrade reaction p_k is derived analytically using the mobilized
 315 strength concept and an approximate mobilized shear strength $s_{u,mob}^*$ that is independent of
 316 principal strain rotation. In this section, this nonlinear subgrade reaction p_k is validated by
 317 comparing with the numerical result of p_k from a two-dimensional finite element method
 318 (FEM) analysis. A homogeneous soil condition is assumed in FEM and MSD to simplify
 319 the validation. The FEM analysis requires a soil constitutive model, such as the Hardening
 320 soil (HS) model. The MSD does not require a soil constitutive model. In practice, it uses
 321 the measured laboratory test data from the average of PSE and PSC test results. In this
 322 section, the PSE and PSC stress-strain curves are calculated numerically using a commercial
 323 FEM code PLAXIS2D (Brinkgreve et al. 2006) on an element extracted at the depth of tunnel
 324 axis with an element length, width and height equal to $1\text{m} \times 1\text{m} \times 1\text{m}$. The same Hardening
 325 soil (HS) model is used in this numerical element soil test. Hence, given a prescribed
 326 displacement u_r , the analytical p_{k0} is calculated by using Eqs. 9 and 11 with a calculated
 327 stress-strain curve from the average of numerical PSC and PSE test results.

328 The PLAXIS2D code is also used for FEM analysis. Figure 4 is a representative finite
 329 element mesh used in this paper. A tunnel with radius R of 3m is buried 22m below the
 330 ground surface. The cover depth h is thus equal to 25m. Due to the symmetrical feature of
 331 this problem, only half of the model is established as shown in Fig. 4a. The half-width and
 332 depth of the mesh is equal to 200m and 120m, respectively, which are sufficiently large to
 333 minimize boundary effects. The horizontal displacement along the vertical boundaries at
 334 two sides of the mesh are constrained, while the vertical displacement along the horizontal
 335 boundary at the bottom of mesh is constrained. An example of stress boundary conditions
 336 following Eqs. 8 and 10 is shown in Fig. 4b and 4c.

337 The input stress boundary around the tunnel perimeter is the subgrade reaction p_k . The
 338 response obtained from PLAXIS2D with a specific constitutive soil model (Hardening soil
 339 model) is the soil *radial* displacement u_r at the *tunnel crown*. Note that the radial subgrade
 340 modulus k_r varies around the tunnel perimeter – it is smaller at the tunnel crown than at the
 341 invert (Verruijt 1997). In other words, a uniform subgrade reaction around the tunnel
 342 perimeter produces a larger soil radial displacement u_r and consequently a smaller subgrade
 343 modulus k_r (i.e., $k_r = p_k / u_r$) at the tunnel crown compared to the results at other positions
 344 around the tunnel perimeter. Hence, a design based on the smallest subgrade modulus k_r at
 345 the tunnel crown would be most conservative, i.e. the calculated internal lining forces are the
 346 highest. This study focuses on the nonlinear p_k versus u_r curve at the tunnel crown for this
 347 reason.

348 The Hardening soil (HS) model that is widely used in tunnel designs is selected for this
 349 validation study. The HS model is essentially a hyperbolic nonlinear model that considers
 350 both shear hardening and compression hardening. Details of this soil model are given
 351 elsewhere (Möller and Vermeer 2008). Typical stress strain curves for HS model are
 352 illustrated in Fig. 5, i.e., solid line for the plane strain compression condition and dash line for
 353 the plane strain extension condition. The average mobilized strength obtained by averaging
 354 the results from both PSC and PSE tests is also plotted against shear strain in Fig. 5 as the
 355 dot-dash line. A set of input soil parameters required for HS model are provided in Table 1
 356 for typical undrained Shanghai soft clay following the suggestion by Zhang et al. (2015).
 357 Note that the soil unit weight input for FEM analysis is set to zero because the work done by
 358 soil weight is not included in the energy conservation equation (see in Eq. 4). Undrained
 359 analysis with undrained strength parameter (s_u) (method B in Plaxis) is carried out in this
 360 validation study. For method B in PLAXIS, note that the stiffness modulus is no longer
 361 stress level dependent, because the effective friction angle is equal to zero. Hence, the HS

model adopted in this paper exhibits no compression hardening. As a result, the soil used in FEM model exhibits homogeneous soil stiffness. But the model retains its unloading-reloading modulus and shear hardening characteristics. The nonlinear shear hardening in the HS model contributes to the nonlinear mobilization of the soil undrained shear strength.

Comparison with FEM Analysis

Figure 6 compares the calculated nonlinear curve of $p_{k0}-u_r$ between MSD and FEM under the *uniform* deformed shape condition. The horizontal axis refers to the ratio of the tunnel convergence Δ over the tunnel radius R , and the vertical axis refers to the representative subgrade reaction p_{k0} at tunnel crown. The solid square dots in Fig. 6 are the FEM results p_{k0_FEM} from PLAXIS, and the solid line denotes the MSD results p_{k0_MSD} with $s_{u,mob}^*$ determined from the average of $s_{u,mob}$ from PSC and PSE test condition. It is clear from Fig. 6 that the MSD results p_{k0_MSD} using an average stress strain curve is almost equal to the numerical results of p_{k0_FEM} . Thus, the approximation $s_{u,mob}^*$ composed by the average of PSC and PSE test results (Eq. 6) appears to be reasonable for this example. Similar findings have also be presented for the tunneling-induced ground movement calculation where the same approximation of mobilized strength is adopted (Osman et al. 2006a).

In addition, the linear elastic solution p_{k0_linear} presented by Verruijt and Booker (1996) and Zhang et al. (2013) is also plotted against the convergence ratio (Δ/R) in Fig. 6 (denoted by dotted lines). When the Δ/R is relatively small, say within a value of 0.1-0.2%, the nonlinear p_{k0_MSD} is almost equal to the linear solution, which shows that the nonlinear solution reduces correctly to the linear solution at small strain level. However, the nonlinearity of p_{k0_MSD} could become significant as the tunnel convergence increases. The shape of nonlinear variation of p_k with Δ is similar to the nonlinear shape of stress strain curve. Hence, these figures indicate that linear p_{k0_linear} is only appropriate for the design of

tunnel lining where the convergence ratio (Δ/R) is smaller than 0.2%. The convergence ratios recommended in various codes are indicated by arrows in Fig. 6, where 0.2% is still below the lower bound of the ratio for serviceability limit state (SLS). Otherwise, the tunnel structural design, i.e., deformation and internal forces, could be significantly underestimated since the subgrade reaction p_k is overestimated by the linear solution. This design impact will be further discussed by a design example presented later in this paper.

Figure 7 compares the calculated p_{k0} between MSD and FEM for the *oval* deformed shape condition. The solid line in Fig. 7 is the MSD results p_{k0_MSD} with the average stress-strain curve from PSE and PSC condition, while the solid dots denote the p_{k0_FEM} calculated from FEM analysis. The variation of MSD results (p_{k0_MSD}) with convergence ratio (δ/R) matches the same trend from FEM results reasonably well. The linear solution of p_k varying with the convergence ratio is also illustrated in Fig. 7 (denoted by dotted line). The nonlinear solution of p_{k0_MSD} at small strain range also agree well with the linear solution presented in Fig. 7. It should be specifically noted that the magnitude of p_{k0} under the oval mode is almost 1.5-2 times of the magnitude of p_{k0} under the uniform mode. It is consistent with results from the elastic solution presented by Zhang et al. (2013). One possible explanation is that an oval deformation shape absorbs more soil energy at a given strain level compared to the uniform deformation shape. Overall, the proposed nonlinear solution of p_k produced by the MSD method agrees reasonably well with the FEM solution for both uniform and oval shapes for one example. More extensive validation is discussed in the following section.

Correction factor η for proposed nonlinear solution of p_k

One baseline example with input parameters given in Table 1 has been studied above. From an application perspective, engineers would be interested to know the detailed difference between this analytical MSD solution and the actual field measurement over a wider range of conditions encountered in practice. This difference is typically characterized

by a model factor (ratio of measured p_k over the calculated value) in geotechnical reliability based design (Phoon and Kulhawy 2005). Strictly speaking, the numerical subgrade reaction p_k from FEM analysis is not equal to the measured p_k , but comparisons between FEM solutions and field data for a number of geotechnical structures indicate that this difference (i.e., model factor ε_{FEM}) is small. Several calibrations of finite element analysis including finite element limit analysis (FELA) with field cases or physical model tests show that mean of ε_{FEM} is between 0.95 and 1.01 and COV is between 0.06 and 0.18, as shown in Table 2 (Phoon and Tang 2015a, 2015b; Tang and Phoon 2016a, 2016b, 2017). It is reasonable to say that mean and COV of ε_{FEM} should be around 1 and around 0.1, respectively. Nonetheless, to maintain a strict distinction between FEM solutions and field measurements, Zhang et al. (2015) defined a ratio of FEM solution over calculated MSD value as a *correction* factor. The model factor can be constituted as the product of the correction factor and the model factor for FEM (typically unbiased, i.e. mean close to 1 and precise, i.e. coefficient of variation around 10%). The purpose of this section is to characterize the correction factor for the nonlinear analytical solution of p_k using the method proposed in Zhang et al. (2015). The characterization of the correction factor for the oval mode shape is presented in detail below.

It is clear from Eqs. 2, 3, 9 and 11 that the calculated nonlinear solution p_k depends on the input parameters such as cover depth h , tunnel diameter D (or radius R), soil undrained shear strength s_u and convergence deformation δ . Besides, the mobilized stress-strain curve could also contribute to the evaluation of p_k in the calculation. Because the Hardening Soil model is applied, the soil unload-reloading modulus E_{ur} is also included as an input parameter. In summary, a total of four dimensionless parameters from the above five parameters are considered in the orthogonal design of numerical cases required for characterization of the correction factor, i.e., cover depth ratio h/D , undrained strength ratio s_u/σ_v' , soil

437 unload-reloading modulus ratio E_{ur}/s_u , and the tunnel convergence ratio δ/R .

438 Table 3 shows the typical ranges of these four parameters for tunnels in soft undrained
 439 clay condition. The 197 numerical cases are designed in two stages. In the first stage, only
 440 the parameters h/D , s_u/σ_v' , E_{ur}/s_u are included to generate 25 orthogonal FEM models. Each
 441 parameter has five different levels. In the second stage, for each one of the generated 25
 442 models, different levels of convergence ratio (δ/D) are generated within the range of 0.001%
 443 – 3%, i.e., typical range of tunnel deformation on site (Huang et al. 2016). Similar to the
 444 procedure carried out for the baseline example, the p_k is prescribed as the stress boundary
 445 condition for FEM model in PLAXIS2D, while the calculated displacement δ from
 446 PLAXIS2D is applied as an input parameter for MSD calculation to obtain an analytical
 447 solution of p_k . In total, there are 197 cases generated using these two-stage orthogonal
 448 design principle.

449 Figure 8 plots the calculated p_k from MSD (denoted as p_{k_MSD} hereon) against the
 450 prescribed p_k in FEM model (denoted as p_{k_FEM} hereon) for the 197 cases in a logarithm form
 451 of the axes, denoted by grey square dots. It is observed from Fig. 8 that the p_{k_MSD}
 452 distributed relatively closely around the 45 degree line (i.e., the equality line $p_{k_MSD} = p_{k_FEM}$).
 453 However, it is evident from Fig. 8 that the discrepancy between grey dots and 45 degree line
 454 becomes larger as the absolute value of p_k increases. To characterize this discrepancy
 455 statistically, the correction factor η is defined as the ratio of the magnitude of p_{k_FEM} over the
 456 magnitude of p_{k_MSD} .

$$457 \quad \eta = \frac{p_{k_FEM}}{p_{k_MSD}} \quad (12)$$

458 The model factor for p_{k_MSD} is related the correction factor η as follows:

$$459 \quad \varepsilon_{MSD} = \frac{p_{k_m}}{p_{k_MSD}} = \frac{p_{k_FEM}}{p_{k_MSD}} \times \frac{p_{k_m}}{p_{k_FEM}} = \eta \times \varepsilon_{FEM} \quad (13)$$

460 where p_{k_m} is the measured subgrade reaction. Table 2 shows that the mean and coefficient

of variation (*COV*) of ε_{FEM} is close to 1 (unbiased) and relatively small (less than 20%) for a number of different geotechnical systems studied thus far. The mean of η for the 197 cases is equal to 1.02, which means the calculated p_{k_MSD} is almost equal to the p_{k_FEM} on the average. The minimum and maximum value of η are 0.66 and 1.28, respectively, as shown by the grey dashed lines in Fig. 8. The *COV* of the correction factor η is 0.15, which is quite small compared to the *MSD* for other geotechnical structures, e.g., 0.47 for cantilever deflection as shown in Table 2 (Zhang et al. 2015). The mean and *COV* of the correction factor η for a variety of geotechnical problems such as foundation systems in different type of soils (Table 2) appear to be around 1 and 0.10, respectively.

It is worthwhile to examine if the correction factor η is dependent on input parameters, because it is clearly dependent on the magnitude of p_k as shown in Fig. 8. The customary practice for engineers to correct for model bias using the average model factor (deterministic method) or as a random variable (reliability method) is only correct when the model factor is random. To clarify this critical dependency issue, the calculated correction factor η is plotted against the four input parameters h/D , s_u/σ_v' , E_{ur}/s_u and δ/R as shown in Fig. 9a, 9b, 9c and 9d, respectively. Note that the vertical axis in Fig. 9a – 9c is represented by averaged correction factor $\eta_{ave,i}$ for 25 orthogonally designed FEM models in the first stage, which is calculated as below:

$$\eta_{ave,i} = \frac{\sum_j^m \eta_{i,j}}{m} \quad (14)$$

where $\eta_{i,j}$ is the calculated correction factor for the total of 197 designed FEM cases, i is equal to 1, 2, ..., 25 which stands for the number of 25 orthogonal designed case in the first stage, and j is equal to 1, 2, ..., m which stands for the number of m cases in the second stage given the parameter set of h/D , s_u/σ_v' , E_{ur}/s_u have been selected in the first stage. For example, the averaged $\eta_{ave,1}$ is obtained by taking the arithmetic average of $\eta_{1,j}$ values from 6

485 simulations (i.e., $m = 6$) with the following input parameters:

486 1. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 0.025\%$, $\eta_{1,1} = 0.91$;

487 2. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 0.061\%$, $\eta_{1,2} = 0.94$;

488 3. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 0.122\%$, $\eta_{1,3} = 0.97$;

489 4. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 0.242\%$, $\eta_{1,4} = 1.02$;

490 5. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 0.605\%$, $\eta_{1,5} = 1.15$;

491 6. $h/D = 2$, $s_u/\sigma_v' = 0.5$, $E_{ur}/s_u = 200$, $\delta/R = 1.419\%$, $\eta_{1,6} = 1.19$;

492 Thus, the calculated averaged $\eta_{ave,1}$ is equal to 1.04, i.e., $(0.91+0.94+0.97+1.02+1.15+1.19)/6$.

493 By doing so, the averaged correction factor η_{ave} is plotted against the input parameters in Fig.

494 9a to 9c. It is clear that η_{ave} is strongly dependent on the cover depth ratio h/D , but is

495 relatively independent of strength ratio s_u/σ_v' and modulus ratio E_{ur}/s_u . The dependency of

496 η_{ave} on the parameter h/D seems to follow a sigmoid function. In Fig. 9d, the calculated

497 correction factor η for all the 197 cases is plotted directly against δ/D on a logarithm scale.

498 A linear correlation between η and $\ln(\delta/R)$ is obtained as shown in Fig. 9d. By conducting

499 Spearman correlation tests for all these four parameters, the p -value associated with a null

500 hypothesis of zero-rank (Spearman) correlation is much less than the strict 1% level of

501 significance for parameter h/D and δ/R . Hence, the null hypothesis that the correction factor

502 η is independent of the input parameter h/D and δ/R can be rejected at a level of significance

503 of 1%.

504 Because the correction factor is correlated to two input parameters, it cannot be modeled

505 as a random variable (Phoon and Kulhawy 2005). Figure 9 indicates that the correction

506 factor η varies with the input parameter h/D as a sigmoid function and varies with the input

507 parameter $\ln(\delta/R)$ as a linear function. This systematic variation should be removed by

508 regression. Based on the above observed trends, the proposed regression equation is:

$$f = \exp \left[a_0 + a_1 \left(\frac{h}{D} \right)^{-1} \right] \times \left(a_2 \ln \frac{\delta}{R} + a_3 \right) \quad (15)$$

where a_i is the coefficients in Eq. 15. A multiple regression analysis is carried out to determine the regression coefficients a_i . The values of the regression coefficients a_i are shown in Table 4. The coefficient of determination (R^2) of Eq. 15 associated with the coefficients given in Table 4 is about 0.9. The R^2 is relative high, which means Eq. 15 describes the systematic part of correction factor η very well.

The cover depth ratio h/D affects the displacement field (Eq. 3) and the tunnel convergence ratio δ/R is considered by the energy conservation equation (Eq. 11). Equation 15 describes the systematic part of the ratio between the subgrade reaction computed by FEM versus MSD. The existence of this regression equation as a function of h/D and δ/R implies that these input parameters produce secondary effects beyond the primary effects already covered by the displacement field and the energy conservation equation. Figure 10 compares the tunnel deflection at the springline s produced by FEM and MSD against the parameters h/D and δ/R shown in Eq. 15. The tunnel deflection s (an output) is distinct from the imposed displacement at the tunnel boundary δ (a “loading” input). The data points refer to the results produced by the 197 orthogonal scenarios involving a wide range of input parameters described previously. Figure 10a shows that the discrepancy between FEM and MSD becomes significant as the tunnel cover depth decreases (say h/D less than 1 or 2). The average ratio of $s_{\text{FEM}}/s_{\text{MSD}}$ converges to 0.94 for deep tunnels. For shallow tunnels, the average ratio can be as low as 0.64 for $h/D = 1$. It is possible that the elastic displacement field assumed in MSD model departs from the elasto-plastic field used in FEM for shallow tunnels. Figure 10b shows the ratio of $s_{\text{FEM}}/s_{\text{MSD}}$ versus the convergence ratio δ/R . With the exception of results from $h/D = 1$ (closed markers), the ratio generally increases with δ/R . One expects the FEM displacement field to be close to elastic (assumed in MSD) and

therefore $s_{FEM}/s_{MSD} \approx 1$ for small “loading” δ/R . However, this is not the trend shown in Fig. 10b. This may imply that there are other reasons besides the elastic displacement field assumption in MSD, such as the lack of a soil unload-reload modulus in the stress-strain curve for MSD and the averaging strength assumption in Eq. 6. The physical reasons underlying Eq. 15 were not systematically studied in this paper.

The residual of the regression equation f , denoted as η^* , should be the random part of the correction factor η .

$$\eta = f \times \eta^* \quad (16)$$

The residual η^* is also plotted against the four parameters in Fig. 9 (denoted by hollow dots in each figure). It is clear that the residual η^* is not correlated to any input parameter, particularly to h/D and δ/R , and it is appropriate to model it as a random variable. The histogram of calculated residual η^* is plotted in Fig. 11 (grey bars). The residual η^* is observed to be lognormally distributed. The mean of residual η^* is equal to 1.00 and the COV is about 0.05, which suggest the precision of the proposed nonlinear solution of p_{k_MSD} could be further improved if Eq. 16 is adopted. In summary, the analytical MSD solution of p_k under the oval deformation mode can be as good as the corresponding FEM solution when it is corrected as follows:

$$p_{k0} = \frac{\int_{Area} s_{u,mob}^* \cdot \varepsilon_s(\delta) dA}{\delta \int_{\pi D} \cos^2 2\theta ds} \times f \times \eta^* \quad (17)$$

The only marginal cost is the model uncertainty η^* , the associated COV of 0.05 is negligible for all practical purposes within the geotechnical context. The proposal in this paper is to modify the original MSD subgrade reaction (p_{k_MSD}) by the regression function f . Figure 8 also plots the modified MSD subgrade reaction ($f \times p_{k_MSD}$) against the p_{k_FEM} , denoted by the solid circular dots. It is clear from Fig. 8 that the discrepancy between the p_{k_MSD} and the p_{k_FEM} is reduced in particular at a large magnitude of p_k . The range between upper and

lower bound limits for η^* has narrowed down to a maximum of 1.11 and a minimum of 0.85.

The same characterization procedure is carried out for the uniform deformation mode.

The regression equation f is derived by using 146 orthogonally designed numerical cases.

The analytical MSD solution of p_k for the uniform deformation mode can be as good as the corresponding FEM solution when it is corrected as follows:

$$p_{k0} = \frac{\int_{A_{red}} s_{u,mob}^* \cdot \varepsilon_s(\Delta) dA}{\Delta \int_{\pi D} ds} \cdot \exp \left[b_0 + b_1 \frac{h}{D} + b_2 \left(\frac{\Delta}{R} \right)^{-1} \right] \cdot \eta^* \quad (18)$$

The coefficients b_i in the correlation function f are also given in Table 4. The original correction factor η has a mean of 1.08 with a COV at 0.15. However, by removing the dependency on input parameters using the regression equation f , the residual η^* can be modeled as a lognormally distributed random variable with a mean of 1.00 and a COV of 0.07.

Application Example

The proposed nonlinear solution of subgrade reaction p_k for tunnel lining design is applied to a field case reported by [Huang and Zhang \(2016\)](#). The site of this case located in southeast part of Shanghai in China. A running metro tunnel with a diameter of 6.2m and a wall thickness of 0.35m was buried 16.4m below the initial ground surface. A sectional profile is shown in Fig. 12. Unfortunately, 360 lining rings of this tunnel has been subjected to an unexpected ground surcharge due to soil dumping during daily metro operation. The dumped soil has a height H ranging from 1.7m to 7m (shaded area in Fig. 12) causing extreme surcharge loads from 30kPa to 120kPa. The design level for surcharge is 20kPa, which is much smaller than this accidental surcharge. Hence, performance of the segmental lining experienced a severe disruption as many of structural defects could be observed on site ([Huang and Zhang 2016](#)). In this circumstance, the structural loads in the segmental lining can be re-calculated to better understand the structural behavior and performance robustness

581 of this tunnel subjected to such an unexpected surcharge.

582 Since the tunnel lining ring of this case is composed of six concrete segments jointed by
 583 steel bolts, the embedded beam model incorporating discontinuous joint are adopted in the
 584 design. This structural model is proposed by Hashimoto et al. (1994). A typical layout of
 585 the model is shown in Fig. 13a. Segmental joint is simulated by a three dimensional joint
 586 spring representing shear, axial and rotational discontinuous between segments. Thus,
 587 spring constants in these three dimensions are the input parameters for modeling of the joint.
 588 These parameters are extracted from the literature (Ding et al. 2004). The loads acting on
 589 the lining, e.g., vertical total pressure (p_1 , p_2), lateral total pressure (q_1 , q_2) and dead load (p_g),
 590 are determined by following the ITA guideline (ITA 2004). Note that the extreme surcharge
 591 is assumed to be fully added on the calculation of vertical load p_1 ignoring the effect of load
 592 spreading along depth. It is acceptable due to the relative shallow depth of the tunnel in
 593 Shanghai soft clay (DGJ08-10 2010). The properties of soil and segmental linings used for
 594 the calculation are provided in Table 5. The mobilized stress-strain curve is built based on
 595 the HS model from the element soil tests in PLAXIS2D using the soil properties given in
 596 Table 5. The measured sectional radial displacements of this case indicate a significant oval
 597 deformation mode (Huang et al. 2016). Hence, the nonlinear solution of p_k under oval mode
 598 is applied in this example. The nonlinear p_k - u_r curve is first calculated by using the
 599 proposed MSD method both for the corrected solution (Eq. 17) and the uncorrected solution
 600 (Eq. 11). The calculated nonlinear p_k - u_r curves are plotted in Fig. 13b (p_k without correction
 601 represented by solid dot line and p_k with correction represented by hollow dot line). For
 602 comparison, the embedded beam model with linear soil spring is also included in the design.
 603 The linear p_k - u_r curve with a slope (k_r) identically equal to 6870kN/m^3 is also plotted in Fig.
 604 13b (represented by dash line). The discrepancy between linear and nonlinear p_k - u_r curve is
 605 quite significant especially when convergence is large.

The structural responses of tunnel to the extreme surcharge are also analyzed by finite element method using the PLAXIS2D code. The finite element mesh corresponding to this case is illustrated in Fig. 14. Only half of the model is simulated assuming that the surcharge is also distributed symmetrically with the vertical axis of metro tunnel. This assumption generally agrees well with the field condition, as shown in Fig. 12. The mesh has a width of 200m and a depth of 120m. The HS model is used to simulate the soil behavior, and the elastic model is used to simulate the lining behavior. Input parameters for these two models are shown in Table. 5. The joint between two segment linings is simulated by a continuous rotation spring, as shown in the insert in Fig. 14, with a rotation stiffness equal to that of the embedded beam spring model given in Table 5. Compared to the embedded beam spring model, it should be noted that the shear and axial discontinuity have not been simulated in this PLAXIS2D model. The stress reduction method, also named as β -method, is adopted to simulate the installation of tunnel lining in numerical analysis (Möller and Vermeer 2008). A reduction factor β equal to 0.25 is selected to match the typical volume loss at 0.5% commonly encountered in tunneling practice in Shanghai (Ding et al. 2004). The extreme surcharge above the ground surface is simulated by line load with a same distribution range as that in the field. The load levels for surcharge are set equivalent to the height of dumped soils from 2m to 8m. Hence, for this design example, the tunnel convergence, bending moment and axial forces are calculated by four design models, namely embedded beam with the corrected nonlinear soil spring model (nonlinear corrected in short), embedded beam with the uncorrected nonlinear soil spring model (nonlinear uncorrected in short), the embedded beam with linear soil spring model (linear model) and the FEM model.

Figure 15 plots the calculated tunnel convergence δ at crown by using four design models mentioned previously. The in-situ tunnel convergence at the crown for 360 rings also has

631 been measured and is plotted in Fig. 15 (solid circular dots). To minimize the effect of
632 initial cover depth on the measured convergence, the horizontal axis only covers the relative
633 surcharge H over initial cover depth to tunnel crown C . It is clear from Fig. 15 that both the
634 predicted and measured convergence increase as the surcharge level increases. Among the
635 results from the four types of design models, the convergence predicted by nonlinear
636 corrected model and FEM model could capture both the trend and magnitude of measured
637 convergence more accurately. The prediction by using linear model is approximately half of
638 the magnitude of measured convergence. The prediction by using nonlinear uncorrected
639 model is slightly larger than the measured data. Corresponding to the measured
640 convergence ratio δ/R ranging from 1.5% to 3.5%, the magnitude of subgrade reaction p_k for
641 linear spring model could be 2-4 times of the value for nonlinear spring model (see the
642 shaded area in Fig. 13b). The overestimation of p_k by linear spring model could be the main
643 reason that produces an unsafe prediction of tunnel convergence in Fig. 15.

644 The prediction of internal forces from the above four models are plotted in Fig. 16 (Fig.
645 16a for bending moments and Fig. 16b for axial forces). The predictions both from
646 nonlinear corrected and nonlinear uncorrected models are quite comparable to the results
647 from FEM model, but they are much larger than the results from linear model. It is not
648 surprising to obtain these results due to the overestimation of p_k in the linear model. An
649 overestimated subgrade reaction would over-confine the lining deformation and thus result in
650 a small bending moment. Similar results can be observed in Fig. 16b for axial forces. The
651 axial force from linear spring model is much smaller than those from nonlinear corrected
652 model, nonlinear uncorrected model and FEM model. But it is also interesting to find that
653 the axial forces from FEM model is larger than those from nonlinear models. The
654 discrepancy of results will increase as the surcharge increases. This observation could be
655 explained by the fact that the discontinuity of segmental lining are not simulated in FEM

model as mentioned previously. A joint model with shear and axial discontinuity would absorb some of the shear and axial forces. Hence, the axial forces in the segmental lining would be smaller compared to those results produced by a joint model with rotational discontinuity only. Similar numerical results could also be observed in the paper elsewhere (Ding et al. 2004). By comparing to the measured axial forces, the discontinued model for segmental lining could predict the results much better than the conventional continuous FEM model does. To sum up from Fig. 16, it is clear that a nonlinear subgrade reaction can be important when the tunnel convergence is large.

Conclusion

In the practical application of embedded beam model for structural design of tunnels, the soil spring is always regarded to be linear elastic. This could hardly match the reality of significant nonlinear properties of soils on site. To solve this problem, this paper has presented an analytical nonlinear solutions of subgrade reaction-displacement (p_k - u_r) curve by using the mobilized strength design (MSD) method. Two typical deformation modes for soil-lining interaction, i.e., uniform and oval shape functions, are used to calculate the energy loss of ground movement induced by subgrade reaction. With the concept of nonlinearly mobilized shear strength with engineering shear strain by averaging the curves both from compression test and extension test, the subgrade reaction p_k is thus obtained via the energy conservation in the mobilized undrained clays.

The proposed nonlinear solution of p_k varying with tunnel convergence (Δ or δ) is successfully validated by a rigorous 2D FEM analysis with the Hardening soil (HS) model. From the statistical point of view, the subgrade reaction p_k from FEM analysis could be predicted by the p_k from MSD analysis multiplying a correction factor η of 1.02. However, the η is correlated with the input parameters. If the systematic correlation characterized by a regressed function f is removed in the correction factor ($\eta^* = \eta/f$), the bias of proposed

681 nonlinear model of p_{k_MSD} compared to FEM result p_{k_FEM} could be further reduced to a
682 residual η^* . The residual η^* could be a lognormally distributed random variable with a mean
683 of 1.00 and a COV of 0.05 for oval mode and with a mean of 1.00 and a COV of 0.07 for
684 uniform mode, which are negligible for all practical purposes within the geotechnical context.

685 As shown in the application example at the end of this paper, the merit of nonlinear
686 solution of p_k should be greatly appreciated when tunnel has a large deformation. The
687 prediction of convergence from nonlinear solution of p_k has a very well agreements with
688 measured data and FEM model in this example. However, the magnitude of p_k from linear
689 solution could be overestimated by 2-4 times the value from nonlinear solution, resulting in a
690 significant underestimation of the predicted tunnel convergence and inner forces by 2-4 times.
691 Hence, the proposed nonlinear solution of p_k should be helpful in the structural design of
692 tunnel lining especially for the case that soil nonlinear behavior is mobilized significantly.

693 However, it should be noted that the proposed nonlinear solution of p_k is largely based on
694 either the uniform mode or the oval mode. It also has been found in this paper that the value
695 of p_k for oval shape could be 1.5-2 times the value for uniform shape. In other words, a
696 pre-judgement might be made before the application of proposed method in that the dominant
697 deformation mode should be selected. Based on the application example, the oval
698 deformation mode appears to agree with the measured data. This is the cost of analytical
699 solution by using MSD compared to the purely numerical analysis. Furthermore, soil
700 shearing may occur before the installation of the tunnel lining, resulting in a possible
701 reduction in soil strength among others. This aspect and other construction aspects that could
702 affect the determination of internal forces in the lining are not considered in this paper.

703 **Acknowledgement:**

704 This study is funded by the Natural Science Foundation Committee Program (51538009,
705 51608380), the Shanghai Science and Technology Committee Research Funding Program

(15220721600), and the Ministry of Transport Construction Technology Project (2013318J11300). Their support is gratefully acknowledged.

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Captions for tables and figures

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894

Table 1 Input parameters for HS model used in the validation (modified after data from [Zhang et al. \(2015\)](#))

Model	Parameter (Unit)	Value
General	Reference vertical earth pressure p^{ref} (kPa)	200
	Soil unit weight γ (kN/m ³)	18
	Soil initial void ratio e_0	1
HS	Soil secant stiffness in standard triaxial test E_{50} (kPa)	9800
	Soil tangent stiffness for primary oedometer loading E_{oed} (kPa)	7840
	Soil unloading-reloading stiffness E_{ur} (kPa)	29400
	Soil undrained shear strength s_u (kPa)	80

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Table 2 Summary of statistics for correction factors and FEM (or FELA) model factor for different geotechnical systems

Problem	$\eta^{(a)}$		$\eta^{*(b)}$		$\varepsilon_{FEM} \text{ or } \varepsilon_{FELA}^{(c)}$		$N^{(d)}$
	mean	COV	mean	COV	mean	COV	
Lateral deflection at top of cantilever wall (MSD method) (Zhang et al., 2015)	1.01	0.47	1.01	0.18	1.01	0.18	59
Strip footings on sand under positive combined loading (Phoon and Tang 2015a)	1.65	0.22	1.03	0.09	1.01	0.06	120
Strip footings on sand under negative combined loading (Phoon and Tang 2015b)	2.00	0.24	1.06	0.09	1.01	0.06	72
Strip footings on sand under general combined loading (Phoon and Tang 2015b)	2.00	0.25	1.04	0.09	1.01	0.09	192
Helical anchors in clay under tension loading (Tang and Phoon 2017)	1.37	0.26	0.98	0.14	0.95	0.08	78
Circular footings on dense sand (Tang & Phoon 2016a)	1.18	0.32	1.02	0.11	1.00	0.10	26
Bearing capacity of dense sand overlying clay (punching shear method) (Tang and Phoon 2016b)	2.63	0.31	1.01	0.12	1.01	0.11	62
Bearing capacity of dense sand overlying clay (load spread method $\tan\alpha_p=1/3$) (Tang and Phoon 2016b)	1.83	0.30	1.01	0.11	1.01	0.11	62
Bearing capacity of dense sand overlying clay (load spread method $\tan\alpha_p=1/5$) (Tang and Phoon 2016b)	2.32	0.33	1.02	0.11	1.01	0.11	62

Note:

a) η = correction factor

b) η^* = residual part of the correction factor η , namely, $\eta^* = \eta/f$.

c) ε_{FEM} (or ε_{FELA}) = model factor of finite element method analysis or finite element limit analysis, defined as $\varepsilon_{FEM} = Q_m/Q_{C_FEM}$ (or $\varepsilon_{FELA} = Q_m/Q_{C_FELA}$), where Q_m = measured data.

d) N = number of measured data for field cases and model tests.

Table 3 Parameters and Their Ranges of Values used in the Numerical Simulations

Parameters	Ranges of Values
h/D	1.00 – 5.00
s_u/σ_v'	0.30 – 0.70
E_{ur}/s_u	200.00 – 900.00
δ/R (Δ/R) (%)	0.001 – 3.000

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Table 4 Best regressed coefficient a_i for systematic part f in correction factor η

Oval mode		Uniform mode	
Coefficient	Value	Coefficient	Value
a_0	0.172	b_0	-0.109
a_1	-0.361	b_1	0.0404
a_2	0.0591	b_2	0.00520
a_3	1.03		
R^2	0.90	R^2	0.91

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Table 5 Input parameters for the design example (modified after data from Ding et al. 2004; Huang and Zhang 2016).

Design objective	Design input parameter	Value
Tunnel Lining	Outer-diameter D (m)	6.2
	Wall thickness t (m)	0.35
	Elastic modulus E (kPa)	34.5×10^6
	Joint compressive stiffness k_{n_joint} (kN/m)	1×10^4
	Joint shear stiffness k_{s_joint} (kN/m)	8×10^4
	Joint rotate stiffness k_{r_joint} (kN/rad)	3.5×10^4
Soil	Cover depth to tunnel crown C (m)	16.4
	Soil unit weight γ (kN/m ³)	16.7
	Coefficient of earth pressure at-rest K_0	0.54
	Surcharge at ground surface H (m)	1.7 – 7
	Soil undrained shear strength s_u (MPa)	46
	Soil unloading-reloading modulus E_{ur} (MPa)	21.3

Figure 1 Deformation modes for tunnel lining: a) uniform; and b) oval.

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(a)

(b)

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(a)

(b)

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