Economics of Parking: Short, Medium, and Long-Term Planning

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy

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Abstract

Worldwide surveys have shown that cruising for parking substantially increases the travel time of individuals and leads to higher traffic congestion as well. Motivated by the need to mitigate adverse effects of parking, this thesis investigates the impact of emerging short, medium, and long-term parking policies. Hourly parking pricing is a short-term policy whereby vehicles pay based on their dwell time. We show that hourly parking pricing can actually increase congestion if imposed imprudently, but if devised optimally, it can lead to better traffic management. Parking enforcement is a medium-term policy comprised of a citation fine and a level-of-enforcement (e.g., number of inspection cameras in streets). We quantify the effect of enforcement policies and we investigate the impact of enforcement technology (i.e., on-foot inspection or camera-based inspection) on illegal parking behaviour. For long-term planning, we model the impact of autonomous vehicles (AV) on parking. With the anticipated growth in AVs, passengers no longer need to park near their final destination. Instead, passengers are dropped off by AVs at their final destination and the AVs travel to the nearest and most affordable available parking lot. We investigate the parking patterns of AVs and show that they substantially decrease parking search time.
Acknowledgments

I like to thank my supervisor, Prof. Matthew Roorda. Matt, thank you for your guidance throughout the last five years. I learned so much from you as a supervisor, yet, I feel that you have been so much more: a confidant, a mentor, and a friend. Thank you for taking a chance on me as a Masters student and keeping me around as PhD. I will truly miss you when I leave.

I like to thank my other mentors who have helped me shape my research including Professors Joseph Chow, Eric Miller, and Khandker Habib. Joe, I cannot thank you enough for your ever-continuing guidance. It is always a pleasure working with you and frankly, it is a lot of fun too. Eric, you are a true inspiration. I hope to one day live up to the example you have set as a world-renowned scientist. Habib, I appreciate all the advice you kindly gave me on everything whenever we got a chance to chat. I hope to one day be half as dedicated as you in my career.

I also like to thank the members of my defence committee, Professors Eric Miller, Marianne Hatzopoulou, and Baher Abdulhai for their constructive comments on my thesis. I have greatly enjoyed sharing my research with you. I like to thank Prof. Hani Mahmassani for agreeing to be my external reviewer despite his incredibly high workload.
Publications

The chapters of this thesis either are under review or published in selected journals. Chapters 2 and 3 are under review as:


Chapter 4 is published as


Chapter 5 is published as

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Chapter 1
Introduction

1.1 Motivation

Parking is a cumbersome part of auto travel. A worldwide survey of 20 cities, conducted by IBM, shows that drivers spend an average of 20 minutes in search of parking (Gallivan, 2011). As drivers search for parking, they create additional traffic which can be detrimental in streets that are already highly congested. For example, analysis of parking in 11 major cities showed that about 30% of vehicles are on average in search of parking where each vehicle cruises for an average of 8.1 minutes. In another study, Shoup (2011) claims that even in cities where it is easy to find parking, the environmental consequences of cruising even for short time can be severe. To validate this claim, Shoup (2011) studied parking patterns in 15 blocks of the Upper West Side of Manhattan in 2008 and found that although the average search time was 3.1 min, the vehicles generated 325 tons of CO₂ (per year) while searching for parking. This is equivalent to 366,000 excess vehicle miles of travel which is about 14 trips around the earth (Shoup 2011). The environmental consequences of parking are even more severe at the life-cycle level when impacts of maintenance and construction are taken into account. At the life-cycle level, parking is found to create more SO₂ than driving (which leads to respiratory harm) and equal levels of PM₁₀ compared to driving (which leads to cardiovascular harm).

In addition to environmental pollution, the sheer number of parking spaces in North America comes at a high social cost. In the U.S., there are close to billion parking spaces throughout the country, most of which are located in the suburban regions where land is less expensive. Comparing this to the 253 million passenger and light-duty freight vehicles, there are four times more parking spaces than vehicles (Chester et al., 2011). The total parking space in the U.S. is roughly 6,500 square miles making it bigger than the state of Connecticut (Chester et al., 2011). Hence, it is argued that parking supply is not optimally distributed in major cities.

Another major problem with parking is the illegal parking behaviour of passengers and commercial vehicles. In the U.S., it is estimated that commercial vehicles cause 47 million vehicle-hours of delay every year due to their recurrent pattern of illegal parking (Han et al,
This impact is so severe that illegal commercial vehicle parking is the third leading cause of delay in the U.S. right behind construction and crashes (Han et al., 2005).

It is critical that cities design and implement policies that minimize GHG emissions (by lowering the parking search time and road congestion), deter illegal parking behaviour, and promote the use of emerging technologies to make alleviate the burden of parking. Parking policies are implemented either at the supply or at the demand level. Supply-side policies focus on reshaping existing parking facilities to make them more convenient or to mitigate the negative impacts of parking, while demand policies are designed to lower parking demand to the point where social cost is minimized. Parking policies, whether at the supply or the demand level, are designed for short, medium, and long-term planning.

We investigate the impact of emerging short, medium, and long-term parking policies. Short-term policies are implemented within a short time window of less than six months and they do not require construction of new infrastructure. Examples of short-term policies include parking pricing (Cats et al., 2016), on street parking allocation (Nourinejad et al., 2014), parking space sharing (Xu et al., 2016), and parking permits (Rosenfield et al., 2016). Among these, parking pricing is a well-established policy that was implemented in many cities as early as the 1950s. Given the importance and popularity of pricing, this thesis, in part, investigates the impact of pricing as a tool for optimal management of parking.

**Figure 1-1: Examples of short, medium, and long-term parking policies.**
Medium-term policies require some form of new infrastructure or recruitment of staff. Parking enforcement (Nourinejad and Roorda, 2016) is a medium-term policy where city planners choose an appropriate enforcement technology that is both effective and affordable. Parking enforcement, in essence, is vital for successful implementation of any parking policy to ensure that travellers adhere to a set of rules. This thesis investigates the impact of parking enforcement, as a medium-term policy, on commercial and passenger vehicles.

Long-term policies are developed in anticipation of emerging technologies in the next two decades. With autonomous vehicles expected to change our driving patterns in the near future, it is inevitable that our parking patterns would change as well. When in an autonomous vehicle, we no longer need to find a parking spot. Instead, we can be dropped off at our final destination and the autonomous vehicle heads off to find a spot that is affordable and geographically close to the drop-off zone. In light of this, autonomous vehicles will become our personal valets. This thesis investigates the impact of autonomous vehicles on parking patterns, parking land use, and road traffic.

1.2 Thesis organization

This thesis consists of six chapters. The chapters are sequentially dedicated to long, medium, and short term parking policies. Given the importance of autonomous vehicles, we begin the next chapter (i.e., Chapter 2) with a discussion and a model of autonomous vehicle parking. In Chapters 3 and 4, we study the impact of parking enforcement as a medium-term parking policy. Chapter 3 is dedicated to enforcement policies for passenger vehicles and Chapter 4 is dedicated to enforcement policies for commercial vehicles. In Chapter 5, we consider short-term policies and discuss the impact of hourly parking pricing on travel demand. The chapters can be read independently. A summary of the chapters is presented below.

1.3 Summary of chapters

1.3.1 Chapter 2: Long-term planning

Parking is an inconvenient part of any trip because travellers have to search for a spot and walk from that spot to their final destination. This conventional method of parking will change with the arrival of autonomous vehicles (AV). In the near future, users of AVs will be dropped off at their final destination and the occupant-free AVs will search for the nearest and most convenient
and affordable parking spot. Hence, individuals no longer bear the inconvenience of cruising for parking while sitting in their vehicle. This chapter quantifies the impact of AVs on parking occupancy and traffic flow on a corridor that connects a home zone to a downtown zone. The model considers a heterogeneous group of AVs and conventional vehicles (CV) and captures their parking behaviour as they try to minimize their generalized travel costs. Insights are obtained from applying the model on two case studies with uniform and linear parking supply. We show that (i) CVs park closer to the downtown zone in order to minimize their walking distance whereas AVs park farther away from the downtown zone to minimize their parking search time, (ii) AVs experience a lower search time than CVs, and (iii) higher AV penetration rates reduce the parking search cost for both AVs and CVs. Our analysis of traffic shows that AVs reduce the peak traffic only when they take less than half the road space of a CV.

1.3.2 Chapter 3: Medium-term planning

A parking enforcement policy, in its simplest form, is comprised of a citation fine and a level-of-enforcement. The citation fine is the penalty paid by illegally parked vehicles that get a parking ticket and the level-of-enforcement is the number of enforcement units (e.g., cameras or on-foot officers) deployed in a region to find illegally parked vehicles. In this chapter, we investigate how to optimally devise a parking enforcement policy to maximize social welfare and profit. Given that it takes time to find and cite each illegally parked vehicle, there is friction present in the searching process. To quantify the friction, we use the bilateral-search-and-meet function and we characterize key factors of illegal parking behaviour such as parking duration, probability that a vehicle parks illegally, citation probability, and rate of citations. Using these factors, we present an equilibrium model of illegal parking where each driver first decides to park legally or illegally and next chooses the parking duration. The model yields several insights: (i) the citation probability increases with the illegal dwell time because vehicles that are parked for a long time are more susceptible to getting a citation, (ii) the citation probability decreases with the number of illegally parked vehicles, (iii) vehicles are more likely to park illegally when their dwell time is short, and (iv) the citation fine and the level-of-enforcement are lowered as the enforcement technology becomes more efficient.
1.3.3 Chapter 4: Medium-term planning II

Central business districts (CBDs) are major destinations for goods pickup and delivery in Canada’s urban centres. “Last mile” delays in CBDs are one of the most expensive components of urban freight (O’Laughin et al., 2007). In this “last mile”, truckers must navigate congested urban streets and search for appropriate parking. When parking is unavailable or inappropriately located, delivery vehicles frequently park illegally, often considering the parking tickets as a cost of doing business. This cost is increasing over time. From 2006 to 2009 parking fines in Toronto increased 70%, and there is little evidence that illegal parking problems are being reduced. In Toronto, FedEx, UPS and Purolator paid an estimated $2.5 M in parking fines in 2009 (Haider, 2009).

The problem is significant and growing. The Toronto CBD, for example, receives a daily average of 81,000 packages from express delivery alone (Haider, 2009). Parking and loading spaces are limited in the CBD because of competing demands for curbside space and because many buildings were constructed before the invention of the automobile. Increasing land values have resulted in the conversion of surface parking lots to high-rise buildings, which in turn are increasing the demands for goods delivery.

Freight parking issues are common in other North American cities as well. The U.S. Department of Transportation together with the Federal Highway Administration and the Office of Freight Management and Operations prepared a series of case studies documenting best practices for urban goods movement. Reports were prepared for Washington DC, Orlando, New York City, and Los Angeles. The purpose of these studies is to investigate initiatives aimed at mitigating congestion and improving efficiency of commercial vehicle operations, including parking.

Commercial vehicles are of particular interest in parking enforcement because of their heavy presence in central business districts and their recurrent behaviour of illegal parking. To deter illegal commercial vehicle parking, enforcement policies are defined by the citation fine and level of enforcement. This chapter investigates how rational carriers react to a policy under steady state equilibrium conditions. To model the equilibrium, the chapter uses the theory of bilateral searching and meeting where enforcement units meet illegally parked commercial vehicles at a rate which depends on the size of the two sets of agents (illegally parked commercial vehicles and enforcement units). In assessing policy effectiveness, two objectives are
defined: profit maximization and social cost minimization. With the two objectives, the chapter presents three market regimes and studies the equilibrium of each market. The proposed model covers several gaps in the parking literature by introducing illegal parking behaviour elasticity with respect to parking dwell time, level of enforcement, citation fine, and citation probability. The model is applied on a case study of the City of Toronto and the results show that the citation probability increases with dwell time and the level of enforcement. Increasing either the citation fine or level of enforcement will hinder illegal parking but the obtained profit remains approximately constant. Sensitivity analysis on the meeting rate elasticity shows that profits are low when both elasticities are either high or low.

1.3.4 Chapter 5: Short-term planning

Efficient parking management strategies are vital in central business districts of cities where space is limited and congestion is intense. Hourly parking pricing is a common parking management strategy where vehicles pay based on their parking duration (dwell time). In this chapter, we derive comparative static effects for a small network to show that road pricing and hourly parking pricing are structurally different in how they influence the traffic equilibrium with elastic demand. Whereas road pricing strictly reduces demand, hourly parking pricing can reduce or induce demand depending on the parking dwell time elasticity (to the hourly parking price). When dwell time is elastic, travel demand always increases with parking price. However, when dwell time is inelastic, demand may increase or decrease with the parking price. Hence, hourly parking pricing can actually cause higher congestion and decay social welfare if imposed imprudently. For larger networks, we present a Variational Inequality model that characterizes the emergent equilibrium. Numerical experiments on a large network validate our analytical findings from a smaller and stylized case study. Our results also show a lower standard deviation in the parking search time (i.e., time to find a parking spot) when dwell time is highly elastic to the hourly parking price.

1.4 References


Chapter 2 Long-Term Planning

2. The End of Cruising for Parking in the Era of Autonomous Vehicles

2.1 Introduction

There are close to one billion parking spaces in the U.S., which is roughly four times more than the existing number of passenger cars and light-duty trucks in the country (Chester et al., 2011). This abundance in parking does not imply that parking is ample and easy to find everywhere. Lack of available parking in business districts of major cities makes drivers search for a long time to find a coveted spot that is close enough to their final destination. A worldwide survey conducted by IBM on parking in 20 cities shows that drivers spend an average of 20 minutes to find a spot (Gallivan, 2011). Cruising for parking adversely affects traffic as well. Shoup (2006) shows that vehicles in search of parking make up between 8% to 74% of traffic.

Resolving the parking dilemma has been a challenge for city planners for decades and the solution is not yet clear. On one hand, cities cannot afford to increase parking supply because parking facilities take up valuable land that can otherwise be used as road capacity (in the case of on-street parking) or real-estate (in the case of off-street parking). On the other hand, decreasing parking supply further exacerbates cruising for parking and leads to longer search times. As a result, cities have resorted to managing parking demand instead of supply using pricing strategies (Qian and Rajagopal, 2014; Cats et al., 2016), parking permits (Liu et al., 2014a,b; Rosenfield et al., 2016), and parking time restrictions (Simićević et al., 2013; Arnott and Rowse, 2013). Parking demand management strategies, however, have drawbacks as well. Parking pricing, for instance, causes inequality between a heterogeneous group of travellers and parking permits require proper enforcement measures which can be costly.

 Autonomous vehicles (AV) have great potential to resolve many of the current parking problems. With the proliferation of AVs, users (i.e., passengers of AVs) will no longer need to park close to their final destination. Instead, users alight the AVs at their final destination, and the occupant-free AVs drive to park at the nearest or most affordable parking spot. Fig. 2-1 illustrates the round-trip home-to-work journey (i.e., travel tour) of a conventional vehicle (CV) driver and an AV user. As illustrated, the CV driver first finds a parking spot and then walks from that spot to
her work place, whereas the AV user is dropped off right at her workplace without having to walk or search for parking while in the vehicle.

Motivated by the impact of AV parking, cities are partnering with car manufacturers to rethink parking in major urban areas. Audi, for instance, is planning to implement a parking pilot in Somerville, Boston, for AVs. The pilot is estimated to save up to 62% in parking space (or equivalently $100 million USD in real-estate value) in the district of Assembly Row where the heart of the project lies (DesignBoom, 2015). With AVs taking two square meters less parking space than CVs, city planners are able to pack more AVs into each parking facility. It is anticipated that AV parking will influence traffic as well. Audi estimates that the transformation of on-street parking spots into lanes of traffic will reduce congestion by 20-50% because of the higher road capacity, reduction of spillbacks at intersections, and fewer vehicles searching for parking (DesignBoom, 2015).

The emerging pattern of AV parking (as shown in Fig. 2-1) has several advantages for urban planners, AV users, and CV drivers. Urban planners no longer need to allocate as much space to parking in areas where renting costs are high and land is valuable. Thus, space can be used more efficiently with improved aesthetics since large parking lots are not a pleasant sight. AV users benefit because (i) they no longer have to find a parking spot and (ii) they do not have to walk from their parking spot to their final destination. Lastly, CVs benefit from AV parking because of lower competition for spots that are closer to major activity centers such as central business districts. That is, as AVs park farther away, CVs have the advantage of finding vacant spots at a lower search time and closer to their final destination.

Although AVs promise a modern way of parking, many questions remain to be answered about new parking patterns, impact on traffic, cruising time, land-use, and the role of AV penetration rates. While it is hypothesized that AVs park farther from their final destination, it is not yet clear how far away they are willing to park. Parking too close to the destination may still create some competition with CVs and parking too far increases travel costs. The second question is whether AVs increase traffic congestion by introducing additional legs to the tour of each AV or if they reduce traffic congestion because of their potentially more efficient use of road capacity. Third, as AVs park farther away, parking land use may change so that more parking facilities are developed farther away from downtown cores at a lower cost. Motivated to answer the above
questions, we present an equilibrium model to quantify the impact of AVs on parking facilities and roads.

The model is generic and is sensitive to several key parameters including parking supply, AV road occupancy, AV parking occupancy, and the value-of-time of drivers. By varying these parameters, we are able to find analytical insights from a case study with a uniform parking capacity along a corridor. For other complex parking supply structures, we present a discretization method that divides the parking supply into segments for easier analysis.

The remainder of this chapter is organized as follows. We present a literature review of existing studies on models of parking behaviour in Section 2.2. We present the model is Section 2.3. An extension of the model is presented in Section 2.4 to find the optimal parking land use in a city. We provide analytical results on a case study with uniform parking supply distribution in Section 2.5. Additional insights are provided in Section 2.6 from numerical experiments. We present the conclusions of this study in Section 2.7 along with directions for future research.

![Figure 2-1: Parking pattern of autonomous and conventional vehicle drivers.](image)


2.2 Background

In this section, we review the existing studies related to the impact of AVs on urban parking patterns. We advocate the importance of AV parking in Section 2.2.1 and we present a review of relevant parking studies in Section 2.2.2.

2.2.1 Autonomous vehicles

Autonomous vehicles are now in the testing phase for many car manufacturers (including Audi, Ford, GM, Toyota, Nissan, Volvo, Volkswagen, BMW, and Cadillac) and technology investors. Google, among the key investors in the automated driving technology, has tested AVs over more than 2 million miles in four cities: Mountain View (since 2009), Austin (since 2015), Metro Phoenix (since 2016), and Kirkland (since 2016) (Waymo, 2016). Successfully completing the testing phase, AVs are anticipated to be available to the public on a mass scale by 2025 (Fagnant and Kockelman, 2015). As AVs are an inevitable reality, a number of studies have investigated their impact in terms of fuel economy (Mersky and Samaras, 2016), induced traffic (Harper et al., 2016), willingness-to-pay for AVs (Bansal and Kockelman, 2016), traffic flow (Levin and Boyles, 2016; de Almeida Correia and van Arem, 2016; Talebpour and Mahmassani, 2016; Mahmassani, 2016; de Oliveira, 2017), safety (Katrakazas et al., 2015; Kalra and Paddock, 2016), intersection control (Le Vine et al., 2015; Yang and Monterola, 2016), and the use of AVs as shared fleet between a group of users (Fagnant and Kockelman, 2014; Chen et al., 2016a,b; Krueger et al., 2016). In a recent survey, Fagnant and Kockelman (2015) argue that AVs will change parking in two ways. First, parking patterns may change as AVs are able to self-park in less-expensive facilities, and second, parking facilities will be relocated from central business districts to areas with less-expensive renting costs. Motivated by the impact of AV parking, we provide a synopsis of the relevant literature on parking and we discuss the application of existing parking models on AVs.

2.2.2 Parking models

There are many studies on practical aspects of parking (see Inci (2015) for a thorough review) including parking pricing (Qian and Rajagopal, 2014; Mackowski et al., 2015; He et al., 2015), cruising for parking (Arnott and Inci, 2006; Liu and Geroliminis, 2016), enforcement (Nourinejad and Roorda, 2016), parking competition (Arnott, 2006; Inci and Lindsey, 2015), optimal parking control strategies (Qian and Rajagopal, 2014; Zheng and Geroliminis, 2016),
and parking for commercial vehicles (Nourinejad et al., 2014; Wenneman et al., 2015; Marcucci et al., 2015; Amer and Chow, 2016). The existing literature is not explicitly applicable to AV parking because AVs do not have the same parking pattern as CVs. Nevertheless, the common ground between AVs and CVs is that they both need to cruise to find appropriate parking. Although there is strong evidence that CVs have to search for parking, we argue that AVs also have to search when dispatched by vehicle-owners to a parking facility. In such cases, the search time is interpreted as the waiting time of an AV until one spot becomes empty at a full parking facility.

The literature on cruising-for-parking is abundant. Arnott and Inci (2006) analyze the economic impacts of cruising-for-parking in a “bathtub” model that considers the influence of cruising-for-parking vehicles on traffic congestion. They show that it is always efficient to increase the parking price to the point where cruising-for-parking is eliminated while onstreet parking is unsaturated. Arnott and Inci (2010) investigate the stability of equilibrium solutions in a parking model with cruising-for-parking. Arnott and Rowse (2009) extend the model of Arnott and Inci (2006) with the assumption that demand is completely inelastic to simplify the computational complexity of the model. In an effort to relax the parking space contiguity assumption in Arnott and Inci (2006), Levy et al. (2013) compare the results of an analytical parking model PARKANALYST with a geosimulation model PARKAGENT and show that parking space heterogeneity (i.e., distinction between on-street, off-street, paid, and free parking) becomes critically important when the occupancy rate is above 92%.

Cruising-for-parking is analyzed at a network scale as well. The existing models consider cruising cost to be the Lagrange multiplier associated with the parking capacity constraint at each parking node. The Lagrange multiplier is non-zero when the parking constraint is binding which indicates the next vehicle that enters the parking facility has to incur a cruising cost (Li et al., 2007, 2008). We use a similar strategy by imposing a constraint that restricts parking occupancy to be lower than (or equal to) parking supply at each facility. With this assumption, we estimate the parking cruising cost at each parking facility.
2.3 Equilibrium model

2.3.1 Problem setting

Consider a two-zone city where the two zones are connected with a corridor of length $D$ km as shown in Fig. 2-2(a). A total of $V$ vehicles per hour leave the “home zone” and head for the “downtown zone”. Out of the $V$ vehicles (per hour), a ratio of $r$ are autonomous and a ratio of $1 - r$ are conventional vehicles such that the AV demand is $Vr$ vehicle per hour and the CV demand is $V(1 - r)$ vehicles per hour. The vehicles can park anywhere along the corridor as long as there is available parking. The vehicles drive in both directions.

![Figure 2-2: (a) Two zone city, and (b) Two zone city with the corridor divided into e elements.](image)

Consider point $X$ on the corridor located $x$ km away from the downtown zone as shown in Fig. 2-2(a). A CV that parks at point $X$ has to drive $D - x$ km to reach point $X$ and has to walk (or use any other access mode) $x$ km to reach the downtown zone. Hence, the travel cost of a CV that parks at $X$ is

$$C_{cv}(x) = 2(D - x)t + 2wx$$

where $t$ is the driving cost per km, $w$ is the walk (access) cost per km, and the factor 2 accounts for the return trip. Naturally, $w > t$ because CV owners would otherwise walk from the home zone to the downtown zone instead of driving.

AVs, as explained earlier, have a different parking pattern. They drive $D$ km to drop off their occupants downtown, then drive back (right-ward in Fig. 2-2) $x$ km to park at point X. On the return trip, AVs drive $x$ km to the downtown zone to pickup their occupants, and then drive $D$ km back to the home zone. Hence, the travel cost of an AV that parks at X is
where $\bar{t}$ is the travel cost per km of an occupant-free AV. Naturally, $t > \bar{t}$ because the time-value of occupants is included in $t$ but not in $\bar{t}$. As an example, at an average fuel consumption of $10L/100km$ in urban areas and average fuel cost of $1/L$, we have $\bar{t} = $ $0.1/km$ (excluding maintenance costs). To find $t$, at an average speed of $40km/hour$, and a time value of $20/hour$ for one driver, we have $t = \bar{t} + (20/40) = $ $0.6/km$. Hence, $t$ is six times larger than $\bar{t}$ in this example.

Parking capacity at point $X$ is denoted by $k(x)$ and measured in vehicles per hour per km. CVs take one unit of parking capacity but AVs take $\alpha$ units of capacity where $\alpha < 1$ because of (i) their higher maneuverability in parking facilities (ii) AVs can be packed in parking facilities, and (iii) AVs do no need the gap required to open passenger doors when they are occupant-free.

CVs take one unit of road capacity and AVs take $\beta$ units of road capacity where either $\beta \leq 1$ due efficient road-space usage of of AVs or $\beta > 1$ due to the higher inter-vehicle gap for safety assurance.

We make the following assumption throughout the rest of this chapter to ensure that the results of the model are practical when the following inequality is imposed

$$w - t > \bar{t}.$$  \hspace{1cm} (3)

Eq. 3 is realistic because $w$ (i.e., walking cost per km) is generally large. As an example, $w = $ $4/km$ (as a lower-bound on the true value of $w$) when the average walking speed is $5km/hour$ and the value of time is $20/hour$. Hence, Eq. 3 holds with $w = $ $4/km$, $\bar{t} = $ $0.1/km$, and $t = $ $0.6/km$. Eq. 3 implies that $C_{av}(x) < C_{cv}(x)$ which means that an AV that parks at X always experiences a lower travel cost than a CV that parks at X for all $x \in [0,D]$. Moreover, with vehicles becoming electrified on a mass scale in the near future, it is reasonable to assume that $\bar{t}$ is small.

2.3.2 User equilibrium conditions

We assume that an equilibrium exists in the parking choice of travellers. Each vehicle that parks at X incurs a parking search cost (the cruising cost until a spot becomes available), denoted by
s(x), and a travel time cost which is \( C_{cv}(x) \) for CVs and \( C_{av}(x) \) for AVs. The generalized cost of a CV that parks at \( X \) is

\[
g_{cv}(x) = C_{cv}(x) + s(x)
\]  

(4)

and the generalized cost of an AV that parks at \( X \) is

\[
g_{av}(x) = C_{av}(x) + s(x).
\]  

(5)

Under user equilibrium conditions, all CVs have the same generalized cost denoted by \( u_{cv} \) such that

\[
g_{cv}(x) = u_{cv}
\]  

for all \( x \in [0, B] \) as long as at least one CV parks at \( x \). Similarly, all AVs have the same generalized travel cost denoted by \( u_{av} \) such that

\[
g_{av}(x) = u_{av}
\]  

for all \( x \in [0, B] \) as long as at least one AV parks at \( x \).

2.3.3 Mathematical model

Let \( v_{cv}(x) \) and \( v_{av}(x) \) be the number of CVs and AVs per km that park at \( X \). That is, \( v_{cv}(x) \) and \( v_{av}(x) \) are the probability-density-functions of a continuous distribution that represents parking occupancy on the corridor for CVs and AVs, respectively, when \( Vr \) and \( V (1 - r) \) are normalized to one.

We now proceed to formulate the traffic flow on the corridor. Let \( f(x) \) be the (inbound) flow of vehicles at \( X \) (i.e., the leftward flow in Fig. 2-2(a)):

\[
f(x) = \int_{0}^{x} v_{cv}(y) \, dy + \beta \int_{0}^{x} v_{av}(y) \, dy + 2\beta \int_{x}^{D} v_{av}(y) \, dy \quad \forall x \in [0, B]
\]  

(8)

where the first term is the flow of CVs that park downstream of \( X \), the second term is the flow of AVs that park downstream of \( X \) discounted by \( \beta \) to account for AV impact on road capacity, and the third term is the flow of AVs that park upstream of \( X \) discounted by factor \( \beta \) and multiplied
by 2 because these AVs pass point $X$ twice: once to drop off their occupants and a second time to pick them up. An alternative and equivalent way of defining flow at $X$ is

$$f(x) = \int_0^x v_{cv}(y) \, dy + \beta Vr + \beta \int_x^D v_{av}(y) \, dy \quad \forall x \in [0,B]$$

(9)

where the first term is the flow of CVs that park downstream of $X$, the second term indicates that all AVs pass point $X$ because they need to drop off their occupants at the downtown zone, and the third term indicates that only the AVs that park upstream of $X$ have to pass $X$ for a second time to pick up their occupants from the downtown zone.

We now present the following lemma which is applied in the mathematical model that follows.

**Lemma 1.** CVs and AVs do not share any parking facility such that $v_{av}(x)$ and $v_{cv}(x)$ cannot both be positive at any $x \in [0,D]$.

Proof. The prerequisites to the proof are the following. Consider two points on the corridor located at $x$ and $x + \varepsilon$ where $\varepsilon > 0$ is an infinitesimal distance. Assume that CVs and AVs are present on both points (i.e., CVs and AVs share parking facilities) such that $v_{av}(x), v_{av}(x + \varepsilon), v_{cv}(x), v_{cv}(x + \varepsilon) > 0$. Let $\gamma > 0$ be an arbitrary value representing the drop in the parking search cost as travellers park $\varepsilon$ km farther away from the downtown zone. Finally, assume $\alpha = 1$, although this assumption is not restrictive.

We now show that a CV (located at $x$) and an AV (located at $x + \varepsilon$) would both benefit if they change parking spots. By showing this for one CV and one AV, we establish that all vehicles on at $x$ and $x + \varepsilon$ would change spots to the point where CVs and AVs do not share any parking facility.

A CV that moves from $x$ to $x + \varepsilon$ incurs an additional cost of

$$\gamma \varepsilon - 2\varepsilon(w - t) \quad (I)$$

and an AV that moves from $x + \varepsilon$ to $x$ incurs an additional cost of

$$2\varepsilon t - \gamma \varepsilon \quad (II).$$
It is evident from (I) and (II) that the additional cost of the AV and the CV is negative (i.e., they both incur a lower cost) as long as $\bar{t} < \gamma < (w - t)$ which holds true because $\bar{t} < (w - t)$ as was previously assumed in Eq. 3. ■

The following mathematical model stipulates the user-equilibrium conditions in the parking choice of CVs and AVs:

Minimize $\int_0^D v_{av}(x)C_{av}(x)dx + \int_0^D v_{cv}(x)C_{cv}(x)dx$  \quad (10)

subject to

$\int_0^D v_{av}(x)dx = Vr$  \quad (11)

$\int_0^D v_{cv}(x)dx = V(1 - r)$  \quad (12)

$v_{cv}(x) + \alpha v_{av}(x) \leq k(x)$  \quad $\forall x \in [0, D]$  \quad (13)

$f(x), v_{cv}(x), v_{av}(x) \geq 0$  \quad $\forall x \in [0, D]$  \quad (14)

where Eq. 11 and Eq. 12 ensure that a total of $Vr$ AVs and $V (1 - r)$ CVs are assigned to all the parking facilities. Constraints 13 ensure that occupancy in all parking facilities is within the available capacity where $k(x)$ is the number of parking spaces per km. Constraints 14 ensure non-negativity where $f(x)$ is obtained from Eq. 9. The mathematical model has a feasible solution if there is enough parking supply on the corridor to station all the vehicles. Precisely, the following should be satisfied for feasibility

$\int_0^D k(x) \geq V r\alpha + V (1 - r)$

where the left-term is the total parking supply and the right-term is the total parking demand.

We now proceed to show that the mathematical model (Eq. 10-14) is equivalent to the user equilibrium conditions of Section 2.3.2. Let $\lambda_{av}$, $\lambda_{cv}$, and $\lambda_p(x)$ be the Lagrange multipliers of Constraints 11 to 13, respectively. The Lagrange multipliers have the following interpretation. $\lambda_{av}$ and $\lambda_{cv}$ are the generalized cost of one additional AV and one additional CV, respectively, and $\lambda_p(x)$ is the parking search cost of a vehicle that parks at $X$. 

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According to Lemma 1, CVs and AVs do not share any parking facility. Hence, the parking search cost of a CV is \( s(x) = \lambda_p(x) \) and the parking search cost of an AV is \( s(x) = \alpha \lambda_p(x) \).

If Lemma 1 did not hold (i.e., if CVs and AVs shared parking facilities), then Constraint 13 would be incorrect because AVs would experience a lower search cost (of \( \alpha \lambda_p(x) \)) compared to CVs (who experience \( \lambda_p(x) \)) for parking at the same location which would be unrealistic. With Lemma 1, however, the mathematical model is valid.

We now present the Karush-Kuhn-Tucker (KKT) conditions of the mathematical model as

\[
\begin{align*}
[C_{cv}(x) + \lambda_p(x) - \lambda_{cv}] v_{cv}(x) &= 0 & \forall x \in [0, B] \\
C_{cv}(x) + \lambda_p(x) - \lambda_{cv} &\geq 0 & \forall x \in [0, B] \\
[C_{av}(x) + \alpha \lambda_p(x) - \lambda_{av}] v_{av}(x) &= 0 & \forall x \in [0, B] \\
C_{av}(x) + \alpha \lambda_p(x) - \lambda_{av} &\geq 0 & \forall x \in [0, B] \\
[k(x) - v_{cv}(x) - \alpha v_{av}(x)] \lambda_p(x) &= 0 & \forall x \in [0, B]
\end{align*}
\]

(11) - (14)

where Eq. 15 and 18 are equivalent the user-equilibrium conditions in Eq. 4 and 5, respectively. For CVs, \( g_{cv}(x) = C_{cv}(x) + \lambda_p(x) \) and \( u_{cv} = \lambda_{cv} \). Eq. 15 and 16 indicate that \( g_{cv}(x) = u_{cv} \) when \( v_{cv}(x) > 0 \) and \( g_{cv}(x) > u_{cv} \) when \( v_{cv}(x) = 0 \). Hence, all CVs experience the same generalized cost of \( u_{cv} = \lambda_{cv} \). Similarly, for AVs, \( g_{av}(x) = C_{av}(x) + \alpha \lambda_p(x) \) and \( u_{av} = \lambda_{av} \). Eq. 17 and 18 indicate that \( g_{av}(x) = u_{av} \) when \( v_{av}(x) > 0 \) and \( g_{av}(x) > u_{av} \) when \( v_{av}(x) = 0 \). Hence, all AVs experience the same generalized cost of \( u_{av} = \lambda_{av} \).

2.3.4 Solution algorithm

We present a solution algorithm for complex cases that cannot be solved analytically. The mechanism behind the algorithm is to segment the corridor into finite elements whereby each element represents the location of one parking facility with a given capacity. This strategy allows us to transform the mathematical problem 10-14 into a linear program.
The algorithm has the following steps. We divide the corridor into \( e \) elements of length \( \Delta \) such that the elements collectively make up the length of the corridor: \( D = e\Delta \). Let \( E = \{1, \ldots, i, \ldots, e\} \) be the set of elements such that element \( i = 1 \) touches the downtown zone and element \( i = e \) touches the home zone as shown in Fig. 2-1(b).

The parking capacity of element \( i \) is denoted by \( k^i \) and defined as:

\[
k^i = \int_{\Delta(i+1)}^{\Delta(i)} k(x) \, dx,
\]

(21)

and travel cost of CVs and AVs that park at element \( i \) is denoted as \( C^i_{cv} \) and \( C^i_{av} \), respectively, and defined as

\[
C^i_{cv} = 2(D - i\Delta)t + 2wi\Delta
\]

(22)

\[
C^i_{av} = 2Dt + 2i\Delta t
\]

(23)

where the two above equations are derived by replacing \( x \) with \( i\Delta \) in Eq. 1 and 2. Let \( v^i_{av} \) and \( v^i_{cv} \) be the flow of CVs and AVs at element \( i \), and let \( f^i \) be the traffic flow at element \( i \)

\[
f^i = \sum_{j=1}^{j=i} v^i_{cv} + \beta rV + \beta \sum_{j=i+1}^{j=e} v^i_{av} \]

(24)

where the first term is the flow of CVs, the second term is the flow of all AVs that drop off their passengers downtown, and the third term is the flow of AVs that drive downtown for a second time to pick up their passengers.

The following mathematical model is the linear transformation of the original model (Eq. 10-14):

Minimize \( \sum_{i=1}^{i=e}[v^i_{av} C^i_{av} + v^i_{cv} C^i_{cv}] \)

subject to

\[
\sum_{i=1}^{i=e} v^i_{av} = Vr
\]

(26)

\[
\sum_{i=1}^{i=e} v^i_{cv} = V(1 - r)
\]

(27)

\[
\sum_{i=1}^{i=e} v^i_{cv} + \alpha v^i_{av} \leq k(i) \quad \forall i \in E
\]

(28)
\[ f^i, v_{cv}^i, v_{av}^i \geq 0 \quad \forall i \in E \quad (29) \]

which can be solved using the Simplex method.

### 2.4 Optimal parking land use

It is expected that the large-scale adoption of AVs would transform the existing parking land use in major cities. In the near future, urban planners will have higher flexibility in choosing the location of parking facilities because AVs do not need to park near their destination. This flexibility grants cities the chance to build parking in areas that have a lower rental cost. In this section, we present a mathematical model to investigate changes in parking land use.

Let \( R(x) \) be the rent cost per parking space per km and let \( k(x) \) be the number of spaces per km at \( X \). Here, we consider \( k(x) \) to be decision of a central agent that plans to invest \( B \) dollars to build parking. The rent cost \( R(x) \) is a non-increasing function of \( x \) which indicates that, according to bid rent theory, it is more expensive to place parking near the downtown zone. We later discuss other rent structures in the conclusions section. The mathematical model is

\[
\text{Minimize } \sum_{i=1}^{l=x} [v_{av}^i C_{av}^i + v_{cv}^i C_{cv}^i] \quad (30)
\]

subject to

\[
\int_0^D k(x) R(x) \leq B \quad (31)
\]

\[
k(x) \geq 0 \quad \forall x \in [0, D] \quad (32)
\]

\[
(11) - (14) \quad (33)
\]

where Constraint 31 ensures that the total expenditure on parking is lower than the available budget, Constraint 32 ensures non-negativity, and Constraint 33 implies that constraints 11-14 are applied as well.

We can apply the same method in Section 2.3.4 to find the optimal parking land use. This is done by adding the following two equations, which are discretized versions of Eq. 31 and 32, to the mathematical program (Eq. 25-29) of the solution algorithm:
$$\sum_{i=1}^{i=q} k^i R^i \leq B \quad (34)$$

$$k^i \geq 0 \quad \forall i \in E \quad (35)$$

where \(k(x)\) is the number of parking spaces per km and \(R^i\) is the expected rent cost of a parking facility at element \(i\) on the corridor: \(R^i = \int_{i \Delta}^{(i+1) \Delta} R(x)/\Delta\).

### 2.5 Analytical insights from a city with a uniform parking distribution

We consider first the case with a mixed fleet of CVs and AVs, and second a case where the entire fleet is either exclusively made of CVs or AVs.

#### 2.5.1 Mixed fleet of CVs and AVs

As a simple case, we assume that parking is uniformly distributed throughout the corridor at a density of \(\bar{k}\) parking spaces per km such that \(k(x) = \bar{k}, \ \forall x \in [0,D]\). We also established previously that (i) for CVs, the walking (access) cost per km, \(w\), is larger than the driving cost per km, \(t\), such that \(w > t\), (ii) for AVs, \(t > \bar{t}\) such that the driving cost per km with occupants, \(t\), is larger than the driving cost per km without occupants, \(\bar{t}\), and (iii) AVs that park at \(X\) experience a lower total cost than CVs, i.e., \(C_{av}(x) < C_{cv}(x)\), because \(w > t - \bar{t}\) as discussed in Eq. 3.

The parking pattern that emerges from the above assumptions has two properties. First, CVs always park closer to the downtown zone than AVs because this pattern leads to a lower objective function (10) since \(C_{av}(x) < C_{cv}(x)\). In other words, the objective function (10) would always be lower if we move CVs closer to the downtown zone. Second, CVs and AVs do not share any of the parking spaces as discussed in Lemma 1.

Parking occupancy, \(v_{cv}(x)\) and \(v_{av}(x)\), is illustrated in Fig. 2-3. It is evident that CVs park on a stretch of the corridor that is \(V (1 - r)/\bar{k}\) km long and AVs park on a stretch that is \(V \alpha r/\bar{k}\) km long. As \(\alpha\) gets smaller (i.e, each AV take a smaller parking space), the AVs take a shorter stretch of the corridor and the height of \(v_{av}(x)\) increases in Fig. 2-3 because more AVs can be packed into each parking facility.
The parking search cost, \( s(x) \), and the parking capacity Lagrange multiplier, \( \lambda_p(x) \), are depicted in Fig. 2-4 and their parameters are derived in Appendix 2-A. The relationship between the two is that the search time is discounted by a factor of \( \alpha \) in the stretch of the corridor where AVs park. Fig. 2-4 shows that vehicles experience a higher search cost as they park closer to the downtown zone because they have better accessibility. CVs that are closer to the downtown zone have to search for a longer time but they walk a shorter distance and AVs that park closer to the downtown zone experience a longer search time but they travel a shorter distance when picking up and dropping off their occupants. Fig. 2-4(b) shows that the maximum search cost of AVs is smaller than the minimum search cost of CVs by a factor of \( \alpha \) (see point \( x = V (1 - r)/\bar{k} \) in Fig. 2-4(b)). Hence, AVs experience a lower search cost than CVs as \( \alpha \) increases.

Figure 2-4: (a) Parking capacity Lagrange multiplier, and (b) Parking search cost.
Traffic flow along the corridor is depicted in Fig. 2-5. Fig. 2-5(a) shows that CVs approach the downtown zone at \( V (1 - r) \) vehicles per hour. As they get closer to downtown, some vehicles park on the corridor and the overall flow decays to the point where there is no CV flow at the downtown zone. AVs, on the other hand, show a completely different traffic flow pattern. AVs approach the downtown zone at a flow of \( V r \beta \) vehicles per hour. Every AV visits downtown twice: once to drop off a passenger and once to pick up a passenger. Hence, the AV flow at the downtown zone is \( 2V r \beta \) vehicles per hour. The total flow of vehicles (i.e., sum of AV and CV traffic flow) is depicted in Fig. 2-5(c) where it is evident that the peak flow occurs at \( x = \frac{V (1 - r)}{\bar{k}} \) which is the point where AV and CV parking is separated. From Fig. 2-5(c) it is evident that increasing \( r \) (i.e, the ratio of AVs) pushes the peak flow location on the corridor closer to the downtown zone which may be problematic if one’s objective is to reduce downtown traffic.

![Figure 2-5: (a) Conventional vehicle flow, (b) Autonomous vehicle flow, and (c) Total vehicle flow.](image)

We now investigate the changes in the height of the peak traffic flow which is \( V (1 - r + 2r \beta) \) as shown in Fig. 2-5(c). The peak flow has an expansion factor of \( \zeta = 1 - r + 2r \beta \) compared to the no AV case with \( r = 0 \) where the peak flow is \( V \). We illustrate the peak flow expansion factor \( \zeta = 1 - r + 2r \beta \) in Fig. 2-6 for several values of \( r \) and \( \beta \). When \( \beta > 0.5 \) (with each AV taking more than half the road space allocated to each CV), \( \zeta \) (the expansion factor) increases with \( r \) (AV ratio) because each AV traverses the peak location twice while taking more than half the space of a CV. Hence, increasing the AV ratio leads to higher peak traffic flow on the corridor. Moreover, when \( \beta > 0.5 \), then \( \zeta > 1 \) which indicates that the introduction of AVs increases traffic.

Consider now the case where \( \beta < 0.5 \) with AVs each taking less than half the road space allocated to each CV. The results here are the opposite of the previous case. When \( \beta < 0.5 \),
then $\zeta$ (the expansion factor) decrease with $r$ which shows that AVs can reduce traffic. Moreover, when $\beta < 0.5$, then $\zeta < 1$ which shows that the peak flow is lowered with the introduction of AVs.

**Figure 2-6: Peak traffic flow expansion factor: $\zeta = 1 - r + 2r\beta$.**

The generalized travel cost of CVs and AVs is depicted in Fig. 2-7. CVs experience the lowest generalized cost in the domain $[0, V(1 - \beta)/\bar{k}]$ where they park and the AVs experience their lowest cost in the domain $[V(1 - r)/\bar{k}, V(1 - r + \alpha r)/\bar{k}]$ where they park.
2.5.2 Fleet of only CVs or only AVs

We now consider a special case of the previous section where the entire fleet is either made of AVs (i.e., \( r = 1 \)) or CVs (i.e., \( r = 0 \)). The parking search cost is presented in Fig. 2-8(a) and Fig. 2-8(b) for general \( \alpha \) (AV parking capacity) and for \( \alpha = 1 \), respectively. As is evident in Fig. 2-8(a), the maximum parking search cost of AVs is lower than CVs by a factor of \( (\alpha \bar{t})/(w - t) \) which shows that AVs experience a lower search cost when \( \alpha \) is small so that more AVs are packed into each parking facility, or when \( \bar{t} \) is small so that AVs can cruise around at a low cost of occupant-free driving per km. At \( \alpha = 1 \) (where CVs and AVs take the same amount of parking space), Fig 2-8(b) shows that AVs take the same stretch of the corridor to park but they still experience a lower search cost compared to CVs.

The traffic flow is presented in Fig. 2-9 for four separate ranges of \( \beta \) (AV road occupancy). When \( \beta > 1 \), as shown in Fig. 2-9(a), the AV traffic flow is higher than the CV traffic flow all along the corridor. When \( \beta = 1 \), as shown in Fig. 2-9(b), AVs and CVs have the same traffic flow at the home zone but AVs have a higher traffic flow at the downtown zone which is larger than \( V \). When \( 0.5 \leq \beta < 1 \), as shown in Fig. 2-9(c), the AVs have a smaller traffic flow than CVs at the home zone but a larger flow at the downtown zone which is also larger than \( V \).

Finally, when \( \beta < 0.5 \), as shown in Fig. 2-9(d), the AV traffic flow is smaller than \( V \) all along the corridor. Hence, the maximum traffic flow of AVs is smaller than CVs only in the fourth scenario where \( 0.5 < \beta \). This was also shown in the form of an expansion factor in Fig. 2-6.

![Figure 2-8: (a) Parking search cost with a general \( \alpha \), and (b) Parking search cost with \( \alpha = 1 \).](image)
2.6 Insights from numerical experiments

The analytical results of the previous section were derived for a case where parking was uniformly distributed throughout the corridor. In this section, we consider an alternative linear parking distribution in Section 2.6.1. We then find the optimal parking supply distribution in Section 2.6.2 for a given budget.

2.6.1 Linear parking distribution

Consider a linear parking supply of \( k(x) = 90 + 20.5 x \) where parking supply is 90 \( \text{spaces/km} \) at the downtown zone and 500 \( \text{spaces/km} \) at the home zone. Let \( \alpha = 0.8, r = 0.7, W = 0.09, t = 0.25, \) and \( \bar{t} = 0.001 \). The results of the linear parking supply case are presented in Fig. 2-10. As is evident from Fig. 2-10(a) and Fig. 2-10(b), the CVs park closer to the downtown zone compared to the AVs. The generalized cost of CVs and AVs is presented in Fig. 2-10(c) and Fig. 2-10(d), respectively, where it is shown that each vehicle type incurs its lowest generalized cost in the stretch of the corridor where it parks. The parking search cost is presented in Fig. 2-10(e) where it is shown that vehicles incur a larger search cost closer to the downtown zone and the traffic flow along the corridor is presented in Fig. 2-10(f) where it is shown that flow increases and decreases non-linearly compared to the case in the previous section with a uniform parking supply.
Figure 2-9: Traffic flow at (a) $\beta > 1$, (b) $\beta = 1$, (c) $0.5 < \beta < 1$, and (d) $0 < \beta \leq 0.5$. 
Figure 2-10: (a) User equilibrium for conventional vehicles, and (b) User equilibrium for autonomous vehicles.

We present the user equilibrium costs $u_{cv}$ and $u_{av}$ with respect to $r$ (AV ratio) and $\alpha$ (parking space per AV) in Fig. 2-11. Fig. 2-11(a) shows that $u_{av}$ decreases with $r$ because the AVs are allowed to park close the downtown zones as $r$ increases whereas $u_{av}$ increases with $\alpha$ because each AV takes more parking space and AV have to search for a longer time to find an empty spot. The contours of Fig. 2-11 show that AVs experience the same $u_{av}$ for a range of $r$ and $\alpha$. That is, the negative impact of a large $\alpha$ is mitigated if $r$ is large and there are enough AVs in the city.

For CVs, $u_{cv}$ decreases with $r$ because each CV competes with fewer CVs as $r$ increases. On the other hand, $u_{cv}$ is not sensitive to $\alpha$ because CVs do not compete with AVs for parking. That is,
an increase in $\alpha$ benefits all AVs because they share the parking facilities together but it does not influence the CVs. We conclude here that increasing $r$ (AV ratio) generally helps both CVs and AVs, and decreasing $\alpha$ helps AVs because they compete with each other. However, decreasing $\alpha$ does not impact CVs because CVs do not compete with AVs for parking spaces.

Figure 2-11: (a) Equilibrium cost of autonomous vehicles, and (b) Equilibrium cost of conventional vehicles.

2.6.2 Optimal parking supply

We now find the optimal parking supply distribution along the corridor. Let $R(x) = 5.2 \times 103 - 258.6x$ be the rent cost (and maintenance cost) of one parking space per km at x, and let the budget be $B = $3 $\times 107$. The rest of the parameters are the same as the previous section. The optimal parking supply is presented in Fig. 2-12. The CV parking occupancy distribution $v_{cv}(x)$ is depicted in Fig. 2-12(a) where it is shown that (i) CV drivers only have to walk a maximum of 2.7 km, and (ii) there are more CVs parking closer to the downtown zone to minimize the walking distance. The AV parking occupancy distribution is depicted in Fig. 2-12(b) where it is shown that AVs drive all the way back to the home zone to park. Comparing Fig. 2-12(a) and (b) shows that the priority is to develop parking near the downtown zone for CVs and have AVs drive a longer distance back to park at a facility with the lowest rent cost. The total parking supply on the corridor is the presented in Fig. 2-12(c). The parking search cost, $s(x)$ is presented in Fig. 2-12(d) where it is shown that the search cost strictly decreases with $x$. The search cost of AVs is zero because parking supply is larger than parking demand at $x = D$. 

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2.7 Conclusions

City planners have tried many solutions to fix the half-century long parking problem. Some have resorted to optimize the supply of parking and others have applied demand management strategies. Nevertheless, the problem still persists as parking search time is at an all-time high in many urban areas. It is anticipated that autonomous vehicles can provide a viable solution to the parking problem. In the next decade, AV drivers can use their vehicle as a personal valet where the AVs drop off their passengers and then head off occupant-free to find a parking space.

This chapter quantifies the impact of AVs on parking occupancy and traffic flow on a corridor that connects a home zone to a downtown zone. The model considers a heterogeneous group of AVs and CVs and captures their parking behaviour as they try to minimize their generalized travel costs. The model is solved using a finite element method that transforms the problem into
a linear program that can be solved using the Simplex method. Insights are obtained from applying the model on two case studies with uniform and linear parking supply. The results of the model show that CVs park closer to the downtown zone because CV drivers cannot walk a long distance from where they park to their final destination. AVs, on the other hand, park far from the downtown zone and choose parking facilities where the search time is low. In light of this, we show that AVs experience a lower search time than CVs: The maximum AV search time is lower than the minimum CV search time. We investigate the impact of AV parking on traffic flow and show that AVs increase traffic flow because they make an additional trip to the downtown zone compared to CVs. In some instances, however, AVs can reduce the maximum traffic flow when they are highly connected and take less than half the road capacity compared to a CV.

2.8 Appendix 2-A

In this Appendix, we derive the search costs of Fig. 2-4. Let $x_1 = V(1 - r)/\bar{k}$ and $x_2 = V\alpha r/\bar{k}$ be the length of the corridor where CVs and AVs park, respectively. The AV that parks at the point $x_1 + x_2$ does not incur any search cost, i.e., $s(x_1 + x_2) = 0$, because this AV can always move an infinitesimal distance to the right where parking occupancy and subsequently search time are both zero. Hence, the generalized AV travel cost at $x_1 + x_2$ is $g_{av}(x_1 + x_2) = 2Dt + 2(x_1 + x_2)\bar{t}$.

Consider now the AV that parks at point $x_1$ on the corridor. This AV experiences the same generalized cost, $g_{av}(x_1) = s(x_1) + 2Dt + 2x_1\bar{t}$, as the AV that parks at $x_1 + x_2$. By setting $g_{av}(x_1) = g_{av}(x_1 + x_2)$, which holds under user equilibrium conditions, the search cost of AVs at point $x_1$ is

$$s(x_1 + \epsilon) = 2V\alpha r\bar{t}/\bar{k}$$ (36)

where $\epsilon$ is an infinitesimal distance.

We now move on finding the search cost of CVs at points $x = x_1$ and $x = 0$. We start with the CV search cost $s(x_1)$. Recall that, according to Lemma 1, $s(x) = \lambda_p(x)$ for CVs and $s(x) = \alpha \lambda_p(x)$ for AVs. The following equality holds according to Lemma 1:
\[ \lambda_p(x_1 + \varepsilon) = s(x_1 + \varepsilon)/\alpha \]  \hspace{1cm} \text{(37)}

because point \( x_1 + \varepsilon \) is occupied by AVs. To obtain \( s(x_1) \), we need to use the continuity condition of \( \lambda_p(x) \) which states that

\[ \lambda_p(x) = \lambda_p(x + \varepsilon) \]  \hspace{1cm} \text{(38)}

for small \( \varepsilon \). Using Eq. 36, 37, and 38, the search cost of CVs at point \( x_1 \) is

\[ s(x_1) = 2V \frac{r\bar{\varepsilon}}{k} \]  \hspace{1cm} \text{(39)}

We now move on to derive the search cost of CVs at point \( x = 0 \). We do this using the user equilibrium condition which states that \( g_{cv}(0) = g_{cv}(x_1) \). Given that \( g_{cv}(0) = s(0) + 2t, g_{cv}(x_1) = s(x_1) + 2(D - x_1), g_{cv}(0) = g_{cv}(x_1) \), we have

\[ s(0) = 2V[(w - t)(1 - r) + r\bar{\varepsilon}]/\bar{k} \]  \hspace{1cm} \text{(40)}

2.9 References


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Chapter 3 Medium-Term Planning I

Economics of Parking Enforcement

3.1 Introduction

It is estimated that illegal parking causes 47 million vehicle-hours of delay each year in the United States, which makes illegal parking the third leading cause of delay behind construction and crashes (Han et al., 2005). In response to the adverse social impacts of illegal parking, enforcement policies are implemented in many major cities. A parking enforcement policy, in its simplest form, is comprised of the citation fine and the level-of-enforcement. The citation fine is the penalty paid for parking illegally and the level-of-enforcement is the number of enforcement units (e.g. on-foot officers or cameras) deployed in a region to find illegal vehicles. In this chapter, we investigate how to optimally develop an enforcement policy (i.e., the citation fine and the level-of-enforcement) that helps cities better manage illegal parking behaviour.

The impact of an enforcement policy on illegal parking behaviour is conceptually illustrated in Fig. 3-1. Illegal parking behaviour is influenced by the citation probability and the citation fine; drivers are less likely to park illegally when the citation probability is high and the citation fine is large. While the fine is directly stipulated by parking authorities, derivation and analysis of the citation probability is more intricate. When a vehicle is parked illegally for a long time, it has a higher citation probability because of its longer exposure to getting caught by an enforcement unit. Although it is acknowledged that the citation probability is sensitive to parking duration (i.e., dwell time), most studies assume the citation probability to be a fixed parameter (Elliot and Wright, 1982; Cullinane, 1993; Thomson and Richardson, 1998). The drawback of this assumption is that the proposed models are limited in representing dwell time which is a critical component of illegal parking behaviour. In addition to dwell time, the citation probability is also sensitive to the level-of-enforcement; the citation probability is higher when more enforcement units are deployed to find illegal vehicles. The outcome of the enforcement policy is the number of vehicles that park illegally which can be controlled by parking authorities when the optimal policy is implemented.

Parking enforcement policies are commonly developed to achieve two objectives. The first objective is to improve social welfare by mitigating the negative effects of illegal parking
(Cullinane and Polak, 1992) and the second objective is to raise profits generated from legal and/or illegal parking. The citation profits are substantial in many cities. In 2013, New York City, Los Angeles, and Chicago each generated 534, 250, and 176 million dollars, respectively. In some instances, target profits are defined annually and policies are devised to reach them. We investigate how parking authorities can optimize each of these objectives while taking into account the reactive illegal parking behaviour of drivers.

This chapter is organized as follows. A review of research on illegal parking is presented in Section 3.2 and the gaps in the literature are highlighted. A choice model of parking legally or illegally is presented in Section 3.3. The equilibrium that arises from this choice model is presented in Section 3.4. Properties of the equilibrium model are investigated in Section 3.5. An optimal parking enforcement policy is derived for social welfare and profit maximization in Section 3.6. A numerical experiment is conducted and analyzed in Section 3.7. Conclusions are presented in Section 3.8.

![Figure 3-1: Impact of parking enforcement policies on illegal parking behaviour.](image)

### 3.2 Background

#### 3.2.1 Parking enforcement and illegal parking behaviour

Many studies have investigated the causes of illegal parking behaviour by focusing on aspects such as illegal parking behaviour in central business districts (Brown, 1983), impact of the parking citation fine on public transportation ridership (Auchincloss et al., 2014), and illegal parking behaviour of commercial vehicles (Nourinejad et al., 2014; Wang and Gogineni, 2015;
Wenneman et al., 2015). Most of these studies, however, are empirical investigations of the factors that influence illegal parking behaviour. While identifying these influential factors brings us a step closer to understanding illegal parking behaviour, there is still a need for models that can quantitatively assess the impact of enforcement policies. In a recent review of the literature on parking, Inci (2014) emphasizes the immediate need for theoretical and analytical models of parking enforcement that take into account illegal parking behaviour.

There are currently only a handful of studies that develop analytical models of illegal parking and parking enforcement. Petiot (2004) presents a parking model where each driver makes a binary choice of parking legally or illegally based on the utility obtained from each choice. Petiot’s (2004) model, which is an extension of the model of Arnott and Rowse (1999), assumes that the citation probability is an exogenous fixed parameter in the model that negatively impacts the expected utility of illegal parking. This assumption, although vital in simplifying the derived model, is limiting because it does not capture the relationship between the parking dwell time and the citation probability. To account for this relationship, Nourinejad and Roorda (2016) develop a model of illegal commercial vehicle parking and show that commercial vehicles only park illegally if their dwell time is below some threshold. The model of Nourinejad and Roorda (2016), however, does not capture the reverse impact of citation probability on the dwell time. In this chapter, we hypothesize and prove that the citation probability influences the dwell time such that vehicles park illegally for a shorter dwell time when the citation probability high.

Accounting for parking dwell time and level-of-enforcement is non-trivial as these two factors strongly influence the citation probability. A larger citation probability reduces the utility of illegal parking which in turn influences a driver’s choice of parking legally or illegally. Lack of a representation of the citation probability, illegal dwell time, and level-of-enforcement is evident in other theoretical studies of illegal parking such as Elliot and Wright (1982), Cullinane (1993), and Thomson and Richardson (1998).

In this chapter, we focus on developing an analytical model of illegal parking with an explicit representation of these important factors and we investigate the changes in illegal parking behaviour with respect to the implemented enforcement policy. To proceed, we first review in the next section the topic of inspection games as it is widely used to model the impact of enforcement on any type of illegal behaviour.
3.2.2 The inspection game

The inspection game is a classical methodology for modelling environments where enforcement units, known as inspectors, seek potential parking violators called inspectees (Ferguson and Mlolidakis, 1998; Avenhaus and Canty, 2005; Avenhaus and Canty, 2012). Examples of the inspection game include ticket-inspection by barrier-free transit providers (Sasaki, 2014; Barabino et al., 2014; Barabino et al., 2015), arms control agreements (Avenhaus and Kilgour, 2004), and doping in sports (Kirstein, 2014). Each player in this game-theoretic formation has two strategies. The inspector’s strategy is to check (or not check) if the inspectee has adhered to a set of rules and the inspectee’s strategy is to either break the rules or to comply with them. In the inspection game with a mixed strategy equilibrium, the driver (the inspectee) parks illegally with probability \( \beta \) and the enforcement unit (the inspector) inspects the driver with a probability \( \alpha \). There are four possible outcomes in this game and the two players obtain a payoff from each outcome. The payoff matrix is presented in Table 3-1 where the first term in each entry is the driver’s payoff and the second term is the enforcement unit’s payoff. The payoffs are comprised of the following terms\(^1\): The driver pays \( p_0 \) dollars for parking legally and \( f \) dollars for parking illegally and getting cited. The enforcement unit pays \( c_0 \) dollars per vehicle inspection. These costs are ideally set up such that \( c_0 < p_0 < f \): The condition \( c_0 < p_0 \) ensures that the enforcement unit has monetary incentive for inspecting the driver and the condition \( p_0 < f \) ensures that drivers have incentive to park legally as well illegally. The presented mixed-strategy inspection game has a unique Nash equilibrium with \( \beta = c_0 / f \) and \( \alpha = p_0 / f \).

Table 3-1: Parking enforcement as an inspection game. The two components in each entry are the driver and the enforcement unit payoffs, respectively.

<table>
<thead>
<tr>
<th>Driver / Enforcement unit</th>
<th>Inspect</th>
<th>Do not inspect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Park Illegally</td>
<td>((-f, f - c_0))</td>
<td>((0,0))</td>
</tr>
<tr>
<td>Park Legally</td>
<td>((-p_0, p_0 - c_0))</td>
<td>((-p_0, p_0))</td>
</tr>
</tbody>
</table>

\(^1\) The enforcement unit payoffs in Table 3-1 can be revised to represent social welfare instead of profit.
There are three major drawbacks in the presented classical game-theoretic approach that hinder its applicability to model parking enforcement. First, the inspection game is not sensitive to the number of enforcement units that are deployed and hence cannot be used to find the optimal level-of-enforcement. Second, the inspection game does not capture how the citation probability is related to the dwell time of illegal vehicles and hence cannot be used to assess illegal parking behaviour at a desirable level of detail. Third, the inspection game is limited in replicating realistically how the rate of citations is non-linearly related to the number of illegally parked vehicles and the number of enforcement units in a region (Wright, 1983). To accommodate these features in the inspection game, we use the concept of bilateral searching and matching (or bilateral meeting) which models the search friction between illegal vehicles and enforcement units as the latter searches for the former.

3.2.3 Bilateral meeting

Bilateral meeting models explicitly quantify the friction between two sets of agents as they seek each other out in an aggregated market. In a frictionless market, the meetings between the two sets of agents would occur instantaneously which is not the case in parking enforcement. Examples of bilateral meeting in economics include taxi-passenger meeting (Yang et al., 2010; Yang and Yang, 2011; Yang et al., 2014), buyer-seller meeting (Burdett et al., 2001), and employer-employee meeting in the labor market (Andolfatto, 1996; Berman, 1997; Barnichon and Figura, 2015). Mathematically, the bilateral meeting process is formulated so that the meeting rate between the two sets of agents (say vacant taxis and passengers) is a function of the size of each set of agents in an aggregated market. As an example, the rate that vacant taxis meet passengers is a function of (i) the number of passengers waiting for a taxi and (ii) the number of vacant taxis cruising to find passengers. This quantification of the meeting rate has time and again proven to be very useful in various topics of economics because it enables the modelling of frictions in otherwise conventional models, with a minimum of added complexity (Petrongolo and Pissarides, 2001). In this chapter, we use the bilateral meeting function to model the friction between enforcement units and illegal vehicles. The simplicity of the meeting function allows us to develop an analytical model of parking enforcement which provides insights about the interplay between key factors that influence illegal parking behaviour.
The proposed model of parking enforcement is timely for the following reasons. First, the literature on parking is extensive and covers many diverse topics such as pricing of parking facilities (Qian et al., 2012; He et al., 2015; Zheng and Geroliminis, 2016), parking reservation (Yang et al., 2013; Liu et al., 2014), and parking equilibrium (Boyles et al., 2015). However, only a handful of studies are dedicated to parking enforcement despite its practical importance and widespread application in many cities. Second, many studies of transportation systems that are subject to enforcement either disregard the impact of enforcement or assume perfect enforcement (Yang et al., 2012; He et al., 2013). Although these assumptions are valid in many contexts, they can also be limiting in situations where enforcement costs are non-trivial. Third, illegal parking is a unique type of violation because it is a time-based. That is, it does not only matter that drivers are parking illegally but how long they engage in the illegal activity (i.e., the dwell time). Hence, some of our findings can be applied to other similar violations such as speed limit violations in freeways where the duration of the violation is an important factor.

3.3 The choice to park legally or illegally

3.3.1 Arrival rate, dwell time, and utility of legal and illegal vehicles

Vehicles enter a region at the rate of $T$ [vehicles per hour] and they all need to park. Each vehicle chooses to park legally or illegally. The arrival rate of illegal vehicles is $T^v$ [vehicles per hour] (superscript “v” represents a “violator”) and the arrival rate of legal vehicles is $T^n$ [vehicle per hour] (superscript “n” represents a “non-violator”). The total flow of vehicles is the sum of legal and illegal vehicle flows such that the following equation holds

$$T = T^n + T^v$$ (1).

The probability that a randomly arriving vehicle parks illegally is $\beta$ so that $T^v = T\beta$ and $T^n = T(1 - \beta)$. The probability $\beta$ increases with the illegal parking utility. Let $U^n$ and $U^v$ denote the utility of parking legally and illegally, respectively. By assuming that the choice of parking follows a logistic function (i.e., assuming a random Gumbel-distributed error term is added to the systematic utilities), the illegal parking probability $\beta$ is defined as

$$\beta = \frac{\exp(\theta U^v)}{\exp(\theta U^v) + \exp(\theta U^n)}$$ (2).
where \( \theta \) is a non-negative dispersion parameter that can be estimated empirically. The probability \( \beta \) is also referred to as the non-compliance ratio in Cullinane and Polak (1992). Note that the logistic function is an assumption that admittedly needs to be justified using real data in future research. Nevertheless, we perform sensitivity analysis on the dispersion parameter \( \theta \) to test a range of choice structures including a deterministic choice where \( \theta \to \infty \) and a random choice where \( \theta = 0 \).

The utilities of parking \((U^n \text{ and } U^n)\) are the benefit minus the cost of parking. The benefit is a direct result of preforming an activity. Each driver receives a marginal benefit of \( s(l) \) at the \( l^{th} \) hour of parking (i.e., parking to perform an activity). The function \( s(l) \) is non-negative, strictly decreasing, convex, and asymptotic to zero, thus implying that travellers always obtain a higher marginal utility from the earlier hours of parking. This corresponds to the law of diminishing marginal benefit.

The cost of parking, on the other hand, depends on whether the vehicle is parked legally or illegally. Legal vehicles park for \( l^n \) [hours] and pay a variable hourly-based price of \( p \) [dollars per hour] so that they incur a total cost of \( pl^n \) dollars. Illegal vehicles, on the other hand, park for a period of \( l^v \) hours but only have to pay a fine if they are cited by an enforcement unit. By denoting \( \alpha \) as the probability that an illegal vehicle is cited (also known as the citation probability), and denoting \( f \) [dollars] as the citation fine, the expected cost of parking illegally is \( \alpha f \).

With the defined costs and benefits, the systematic utility of parking legally is

\[
U^n(l^n) = \int_0^{l^n} s(l) \cdot dl - p l^n
\]  

(3)

and the systematic utility of parking illegally is

\[
U^v(l^v) = \int_0^{l^v} s(l) \cdot dl - \alpha f
\]

(4).
3.3.2 The meeting rate and the citation probability

We derive the citation probability and identify the factors that influence it. Let $N^v$ be the expected number of illegal vehicles in a region under steady state conditions. According to Little’s Law, $N^v$ is

$$N^v = T^v l^v$$  \hspace{1cm} (5).

The $N^v$ illegal vehicles are sought out by $k$ enforcement units. The event where an enforcement unit finds an illegal vehicle is called a “meeting”; a meeting is synonymous to citing an illegal vehicle. The meeting rate is denoted by $m$ [citations per hour] and is a function of the number of illegal vehicles $N^v$ and number of enforcement units $k$ as follows

$$m = M(N^v, k)$$  \hspace{1cm} (6)

where $\partial m / \partial N^v > 0$ and $\partial m / \partial k > 0$ in their domains $N^v \geq 0$ and $k \geq 0$, indicating that the meeting rate increases with the number of illegal vehicles or enforcement units. Moreover, $m \to 0$ as either $N^v \to 0$ or $k \to 0$, indicating that no meetings occur if there are no illegal vehicles or no enforcement units present.

To express the rate of change in the meeting rate, we define two elasticities $\gamma_1$ and $\gamma_2$:

$$\gamma_1 = \frac{\partial M}{\partial N^v} \frac{N^v}{M}$$  \hspace{1cm} (7)

and

$$\gamma_2 = \frac{\partial M}{\partial k} \frac{k}{M}$$  \hspace{1cm} (8)

which, within their domains $0 < \gamma_1, \gamma_2 \leq 1$, represent the enforcement technology that is implemented in the region. For instance, a human-based inspection technology is distinguished from a camera-based inspection technology based on the two elasticities.
We now use the meeting function $M$ to express the citation probability, $\alpha$, as the ratio of the meeting rate, $m$, over the arrival rate of illegal vehicles $T^v$:

\[
\alpha = \frac{m}{T^v} = \frac{M(N^v, k)}{T^v}
\]  

(9).

Differentiating Eq. (9) with respect to $l^v$ (Lemma A1, Appendix 3-A), the following two properties are derived:

\[
\frac{\partial \alpha}{\partial T^v} = \frac{\alpha(\gamma_1-1)}{T^v} 
\]  

(10)

and

\[
\frac{\partial \alpha}{\partial l^v} = \frac{\alpha \gamma_1}{l^v} 
\]  

(11).

which yield the following insights. First, a higher arrival rate of illegal vehicles $T^v$ eventually lowers the citation probability $\alpha$ because the enforcement units have to search within a larger pool of illegal vehicles. That is, each illegal vehicle has a lower chance of getting cited because there are too many vehicles parked illegally. This observation is mathematically confirmed because $\partial \alpha / \partial T^v < 0$ in Eq. (10). Second, illegal vehicles are more susceptible to getting cited if they are parked for a long time. That is, by parking illegally for a long time, the drivers increase their chances of getting a citation. This observation is mathematically confirmed because $\partial \alpha / \partial l^v > 0$ in Eq. (11). A graphical representation of the two equations (Eq. 10 and 11) is presented in Fig. 3-2 which illustrates the meeting rate and the citation probability for a given range of illegal arrival rates and dwell times. As illustrated, the meeting rate increases with $T^v$ and $l^v$ whereas the citation probability increases with $l^v$ but decreases with $T^v$. Third, as $\gamma_1$ increases from 0 to 1, the citation probability $\alpha$ becomes less sensitive to $T^v$ and more sensitive to $l^v$ (as shown in Eq. 10 and 11). For instance, at $\gamma_1 = 1$, the citation probability is only and highly sensitive to $l^v$ but not sensitive to $T^v$ at all, whereas at $\gamma_1 = 0$, the citation probability, $\alpha$, 

---

2 The citation probability is analogous the probability of finding a job in the job-market literature (Mortensen and Pissarides, 1994).

3 The meeting rate in Fig. 3-2 is a Cobb-Douglas function of the form $M = A_0(N^v)^{\delta_1}(k)^{\delta_2}$ where $A_0 = 0.6, k = 10, \delta_1 = 0.6, \delta_2 = 0.3$. 

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depends only on illegal arrival rate $T^v$. Having defined the citation probability, $\alpha$, we now explain the equilibrium in the choice of parking.

Figure 3-2: Meeting rate and citation probability as functions of the illegal arrival rate and dwell time.

3.4 An equilibrium model of illegal parking

When looking for parking, each driver first chooses the type of parking behaviour (i.e., legal or illegal), and next, the parking duration. This choice structure yields an equilibrium where the equilibrium durations ($l^n*$ and $l^v*$) and the equilibrium arrival rates ($T^n*$ and $T^v*$) satisfy the following two conditions:

*Condition 1 (lower-level):* $l^n*$ and $l^v*$ are chosen to maximize the utilities $U^n$ and $U^v$.

*Condition 2 (upper-level):* $T^n*$ and $T^v*$ are chosen based on the utilities $U^n$ and $U^v$ obtained from the lower-level according to Eq. (2).

This setup represents a two-stage game. We now elaborate the properties of the above two conditions using backward induction whereby the lower-level condition is used in the upper-level condition, and we show in the next section that the above two conditions stipulate an equilibrium.
3.4.1 Condition 1 (lower-level): $l^n$ and $l^v$ are chosen to maximize $U^n$ and $U^v$

At the lower-level of the equilibrium, the dwell times $l^n$ and $l^v$ are chosen to maximize the utilities $U^n$ and $U^v$. For legal vehicles, $U^n$ is maximized at $l^n^*$ based on the first order optimality condition ($dU^n/dl^n = 0$):

$$s(l^n^*) = p \iff l^n^* = s^{-1}(p). \quad (12)$$

which shows that $l^n^*$, and consequently $U^n$, only depends on the parking price $p$. Hence, if the parking price $p$ is fixed, $U^n$ and $l^n^*$ are both fixed as well. Hereafter, we assume that $p$, $l^n^*$, and $U^n$ are all fixed.\(^4\)

For illegal vehicles, $U^v$ is maximized at $l^v^*$ based on the first order optimality condition ($dU^v/dl^v = 0$) which leads to

$$s(l^v^*) = f \frac{d\alpha}{dl^v} \quad (13).$$

To interpret (13), we first need to compute the full derivative $d\alpha/dl^v$. Given that the citation probability $\alpha$ is a function of $l^v$ and $T^v$ (based on Eq. 10 and 11), the full derivative of $\alpha(l^v, T^v)$ with respect to $l^v$ is:

$$\frac{d\alpha}{dl^v} = \frac{\partial \alpha}{\partial l^v} + \frac{\partial \alpha}{\partial T^v} \frac{dT^v}{dl^v} \quad (14).$$

which is simplified by showing that the second term on the right-side of (14) is zero at $l^v = l^v^*$. This can be shown by expanding $dT^v/dl^v$ to

$$\frac{dT^v}{dl^v} = \frac{\partial T^v}{\partial l^v} \frac{dU^v}{dl^v} \quad (15)$$

and setting $dU^v/dl^v = 0$ at $l^v = l^v^*$ based on the first order of optimality. Hence, we have $dT^v/dl^v = 0$ in (15) which simplifies (14) to

\(^4\) The second order optimality condition can also be easily checked to show that $U^n$ is strictly concave and $l^n^*$ is unique. The legal utility $U^n$ is concave because the benefit function $\int s(l) \cdot dl$ is concave and the cost of legal parking is convex as it increases linearly with the parking dwell time $l^n$.\(^4\)
\[
\frac{d\alpha}{dl^v} = \frac{\partial \alpha}{\partial l^v} = \frac{\alpha \gamma_1}{l^v}
\]  
(16).

Eq. (16) shows that the partial and full derivatives of \(\alpha\) with respect to \(l^v\) are equal at \(l^v^*\). Hence, the citation probability is not sensitive to \(T^v\) at \(l^v^*\). Using the result of (16), the first order optimality condition in (13) can now be rewritten as

\[s(l^v^*)l^v^* = f\alpha \gamma_1 \]  
(17)

which is an important expression used throughout the rest of the chapter. The first condition of equilibrium is satisfied as long as (17) is applied. Solving (17), however, is not particularly easy in its current form because \(\alpha(l^v, T^v)\) is a function of both \(l^v\) and \(T^v\). To solve (17), let us define \(l^v^* = L(T^v)\) as a function that finds the optimal illegal dwell time \(l^v^*\) for a given arrival rate \(T^v\). We show in Lemma B2 (Appendix 3-B) that \(l^v^*\) is unique for each \(T^v\). Given the uniqueness of \(l^v^*\), the challenge now is to find the equilibrium illegal arrival rate \(T^v^*\) that satisfies \(l^v^* = L(T^v^*)\). We explore this condition as the second condition of equilibrium in the next section.

The following two remarks can be inferred from (17). Remark 1 states that in an environment where the optimal dwell times are equal, i.e., \(l^v^* = l^n^*\), the drivers always receive a higher utility from parking legally. An indirect conclusion of this observation is that vehicles are more inclined to park illegally if their dwell time is short and they are more inclined to park legally if their dwell time is long. This remark is intuitive because a vehicle that parks illegally for a long time has a high chance of getting cited. Hence, it is commonly logical to park illegally only when the parking dwell time is short.

**Remark 1:** For the same dwell time \(l^v^* = l^n^*\), legal vehicles receive a higher utility than illegal vehicles such that \(U^v < U^n\).

**Proof:** We need to show that \(U^v < U^n\) whenever \(l^v^* = l^n^* = \hat{l}\). We proceed by investigating the benefit and cost components of the two utilities \(U^v\) and \(U^n\). It is easy to establish that the benefit components (first terms in (3) and (4)) are both equal to each other as they are both \(\int_0^{\hat{l}} s(l) \cdot dl\).

Hence, it suffices to show that the cost of illegal parking \(\alpha f\) is higher than legal parking \(p\hat{l}\) such that \(U^v < U^n\) holds. Our task now is to prove the following

\[p\hat{l} < \alpha f \]  
(18)
To show the cost inequality \( p \hat{l} < \alpha f \) holds, we use (12) to rewrite (17) as \( p \hat{l} = f \alpha \gamma_1 \). Using this result, we have \( \gamma_1 f \alpha < \alpha f \) which is always true when \( \gamma_1 < 1 \).

In Remark 2 we show how \( \gamma_1 \) influences the expected cost of illegal parking \( \alpha f \) at the two boundaries \( \gamma_1 \to 0 \) and \( \gamma_1 = 1 \). To have \( \gamma_1 \to 0 \) is analogous to having a fixed parking price and to have \( \gamma_1 = 1 \) is analogous to having a cost that increases linearly with the dwell time such as in the case of legal parking. The insight from Remark 2 is that the cost of illegal parking is very generic and can take different forms according to the elasticity \( \gamma_1 \).

**Remark 2:** To have \( \gamma_1 \to 0 \) is analogous to having a fixed legal parking price and to have \( \gamma_1 = 1 \) is analogous to having a cost that increases linearly with the dwell time.

**Proof:** Consider first the case where \( \gamma_1 \to 0 \). As \( \gamma_1 \to 0 \), we have, according to (17), \( s(l^*v)l^* = 0 \), which happens when \( l^*v = 0 \) or when \( l^*v \to \infty \). By showing later that \( l^*v = 0 \) is an unstable solution, we can only have \( l^*v \to \infty \). The statement \( l^*v \to \infty \) is analogous to having a fixed parking price so that vehicles pay the fixed price and park as long as they want.

Consider now the case where \( \gamma_1 = 1 \). When \( \gamma_1 = 1 \), the citation probability \( \alpha \) increases linearly and exclusively with the dwell time \( l^*v \) which means that the expected illegal cost \( \alpha f \) is also only linearly dependent on the dwell time \( l^*v \). We first show that \( \alpha \) is exclusively dependent on \( l^*v \) and second show that the dependence is linear as well. Exclusivity is verified as \( \alpha \) no longer depends on \( T^v \) because \( \partial \alpha / \partial T^v = \alpha (\gamma_1 - 1) / T^v = 0 \) at \( \gamma_1 = 1 \) according to (10). Linearity is verified as elasticity of \( \alpha \) with respect to \( l^*v \) is strictly equal to 1 because \( (\partial \alpha / \partial l^*v)(l^*v / \alpha) = \gamma_1 = 1 \) according to (11).

3.4.2 Condition 2 (upper level): \( T^n \) and \( T^v \) are dependent on the utilities

At the upper level of the equilibrium, the arrival rates \( T^n \) and \( T^v \) are materialized according to the obtained utilities \( U^n \) and \( U^v \) from the lower level of the equilibrium. From the two utilities, the legal utility \( U^n \) is assumed to be known and fixed because the legal parking price \( p \) is fixed. The illegal utility \( U^v \), however, is itself a function of \( T^v \) as any change in \( T^v \) influences the citation probability \( \alpha \) (Eq. 10) which in turn impacts \( U^v \) (Eq. 4). As an example, when \( T^v \) increases, the citation probability declines because there are more illegal vehicles to be cited. This decline in the citation probability lowers the expected cost of illegal vehicles \( f \alpha \) which in
turn compels more drivers to park illegally and consequently increases \( T^v \). Hence, \( T^v \) depends on \( U^v \) and \( U^v \) depends on \( T^v \).

To present this relationship mathematically, let \( \Gamma: T^v \to T^v \) be a continuous function mapping a set \( T^v \) of all illegal arrival rates to itself. The function \( \Gamma \) is defined based on (2) as

\[
\Gamma(T^v) = \frac{T}{1 + \exp(\theta(U^a - U^v(T^v, l^v)))} \quad \forall T^v \in T^v
\]  

(19)

where \( l^v = L(T^v) \). As shown in (19), we can compute the illegal utility \( U^v \) for a given \( T^v \) (right-hand-side of (19)) and we can use the computed \( U^v \) to recalculate the illegal arrival rate \( T^v \). At equilibrium, the illegal arrival rate \( T^v^* \) must be chosen such that the upper-level equilibrium condition \( T^v^* = \Gamma(T^v^*) \) and the lower-level equilibrium condition \( l^v^* = L(T^v^*) \) are simultaneously applied. The upper level condition ensures that at \( T^v^* \), no legally parked vehicle likes to change to parking illegally and no illegally parked vehicle likes to change to parking legally. We now explore the properties of this equilibrium and prove that the solution \((T^v^*, l^v^*)\) exists and is unique.

### 3.5 Properties of the equilibrium

We now analyze the presented equilibrium by proving its existence and uniqueness. We then investigate the properties of the equilibrium and present an algorithm to find the equilibrium solution.

#### 3.5.1 Existence of an equilibrium

We prove the existence of a steady-state equilibrium that meets the two conditions of an equilibrium using Brouwer’s fixed-point theorem (Fuente, 2000). Brouwer’s fixed point theorem states that if \( \Gamma: T^v \to T^v \) is a continuous function mapping a compact and convex set \( T^v \) to itself, then there exists a \( T^v \in T^v \) such that \( \Gamma(T^v) = T^v \). To prove the existence of an equilibrium, first we show that the set \( T^v \) is a compact and convex set, and second we establish that continuity condition is satisfied. Using the two proofs, we readily prove the existence of an equilibrium. The proofs are presented in Appendix 3-B.
3.5.2 Interpretation of the equilibrium

Consider the two-dimensional space illustrated in Fig. 3-3 with $l^v$ on the horizontal axis and $T^v$ on the vertical axis. In this space, two $N^v (≡ T^v l^v)$ contours are plotted (in red) for $N_1^v$ and $N_2^v$ such that $N_1^v < N_2^v$. The two curves $L$ and $\Gamma$ are defined as follows with slight abuse of notation of the function $L$. The first curve $L(N^v)$ represents the dwell time of a newly arriving illegal vehicle when $N^v$ vehicles are already parked illegally. This curve is obtained by simply dividing the two sides of (17) (first equilibrium condition) by $l^v*$ which leads to

$$s(l^v*) = mf \gamma_1 / N^v$$

(20)

We precisely define $L(N^v)$ as the solution of (20) which is strictly a function $N^v$ (proof is not shown for brevity).

The second curve $\hat{T}^v(N^v)$ is defined as follows. When the illegal dwell time is fixed at $L(N^v)$, the illegal utility is so large that it attracts an illegal arrival rate of $\hat{T}^v$. In other words, the arrival rate $\hat{T}^v$ also depends on the number of illegally parked vehicles $N^v$. Let us further explain this concept graphically. Consider the contour $N_1^v$ in Fig. 3-3 where $l_1^v = L(N_1^v)$ is the optimal dwell time occurring at the arrival rate $T_1^v = N_1^v / l_1^v$. For the given dwell time $l_1^v$ and arrival rate $T_1^v$, the arrival rate should ideally rise from $T_1^v$ to $\hat{T}_1^v$ as more vehicles are inclined to park illegally because of the high utility obtained from the parking dwell time $l_1^v$. Loosely speaking, $\hat{T}_1^v$ is the induced illegal arrival rate when the dwell time is fixed is $l_1^v$. It is clear that the equilibrium occurs at the point where the two curves cross. This crossing point $(l^v*, T^v*)$ occurs when the induced demand $\hat{T}^v$ is equal to the materialized demand $T^v*$ (see proof of existence) as is presented in Fig. 3-3. At this equilibrium solution, the illegal dwell time $l^v* = L(N^v*)$ is such that no vehicle is inclined to switch between legal and illegal parking.
3.5.3 Algorithm

We present an algorithm to find the equilibrium \( (l^\nu, T^\nu) \). Before explaining the steps of the algorithm, we use the example in Fig. 3-4 to illustrate graphically how the algorithm searches the solution space to find the equilibrium. We start off with \( N_0^\nu \) as an initial number of illegal vehicles which is chosen randomly. Given \( N_0^\nu \), we find the optimal dwell time \( l_0^\nu = L(N_0^\nu) \) and the corresponding arrival rate \( T_0^\nu = N_0^\nu / l_0^\nu \). The point \( (T_0^\nu, l_0^\nu) \) is the starting point of the algorithm. Next, we find the induced demand \( T_1^\nu \) which is larger than \( T_0^\nu \) thus showing that illegal parking is still profitable for drivers. As the illegal arrival rate increases, the number of illegal vehicles also increases from \( N_0^\nu \) to \( N_1^\nu \). Now, we repeat the procedure by finding \( l_1^\nu = L(N_1^\nu) \) and continuing the process until we reach the equilibrium. The steps of the algorithm are presented as follows.

Step 1 - Initialization

Set the counter \( a = 0 \). Set the number of illegal vehicles \( N_a^\nu \) to a randomly selected value.
Step 2- Illegal dwell Time $L(N_a^v)$

Use the Newton-Raphson algorithm to solve Eq. (17) and find the optimal illegal dwell time $(N_a^v)$.

Step 3- Find the materialized demand $T_a^v = N_a^v / L(N_a^v)$.

Step 4- Find the induced demand $\hat{T}_a^v = \Gamma(\hat{T}_a^v, L(N_a^v))$ using the following sub-steps.

Step 4.1- Set the counter $b := 0$. Choose a random illegal arrival rate $\hat{T}_b^v$.

Step 4.2- Find the illegal utility $U^v$ as a function of $\hat{T}_b^v$ and $L(N_a^v)$ using Eq. 4.

Step 4.3- Set $b := b + 1$ and find $\hat{T}_b^v = \Gamma(\hat{T}_{b-1}^v, L(N_a^v))$ using Eq. 19.

Step 4.4- Inner convergence check: Go to Step 5 if the following convergence condition is satisfied. Otherwise, go to step 4.2. The convergence condition is the following

$$|\hat{T}_b^v - \hat{T}_{b-1}^v| \leq \varepsilon$$

(21)

Step 5- Update the optimal illegal arrival rate and number of illegal vehicles.

Set $a := a + 1$. Let $T_a^v = \hat{T}_b^v$ and let $N_a^v = T_a^v \cdot L(N_{a-1}^v)$.

Step 6- Outer convergence check

Terminate the algorithm if the following convergence condition is satisfied. Otherwise, go to Step 2. The convergence condition is the following

$$|T_a^v - T_{a-1}^v| \leq \varepsilon$$

(22)
3.5.4 Elasticities at the equilibrium solution

We now present the elasticity of key variables with respect to $f$ and $k$ at the equilibrium solution $(l^v, T^v)$. Defining these elasticities improves substantially the process of finding the optimal enforcement policy which involves choosing the appropriate values of $f$ and $k$. To denote any elasticity, we use the notation $\mu_x^y$ to show that a one percent increase of $x = \{f, k\}$ will change $y = \{m, T^v, l^v\}$ by $\mu_x^y$ percent. Although some of the derived elasticities are not easily interpretable in their current form, we show in the next section that they can be simplified under special (deterministic and random) equilibrium conditions where they provide meaningful insight. Moreover, the defined elasticities are helpful in finding the optimal enforcement policies in the next section. The derivation of all elasticities is presented in Appendix 3-C.

We start off with the elasticity of the meeting rate $m$ with respect to the citation fine $f$ (Eq. 23) and level-of-enforcement $k$ (Eq. 24). As shown in (23), $\mu_f^m$ depends on the elasticities $\gamma_1^v$ and $\mu_f^{N^v}$; the first elasticity $\gamma_1^v$ shows that the impact of the citation fine $f$ on the meeting rate $m$ is contingent on what kind of inspection technology is implemented and the second elasticity, $\mu_f^{N^v}$,
shows that the number of illegal vehicles $N^v$ is also influential as well. Eq. (24) shows that the elasticity $\mu^m_k$ is linearly dependent on $\mu^m_f$.

$$\mu^m_f = \gamma_1 \mu^v_f$$  \hspace{1cm} (23)

$$\mu^m_k = \gamma_2 \mu^m_f$$  \hspace{1cm} (24)

We now proceed to define the elasticity of the illegal arrival rate $T^v$ and dwell time $l^v$ with respect to $f$ and $k$. These elasticities are derived in Appendix 3-C and presented in Eq. (25)-Eq. (28). Although these elasticities do not have any direct interpretation, they are later explored using numerical experimentation to better understand the forces that influence illegal parking behaviour.

$$\mu^v_f = \frac{-(1-\beta) \theta f a}{1+(1-\beta)(1-\gamma_1) \theta f a}$$  \hspace{1cm} (25)

$$\mu^v_l = \frac{s(l^v)/[s'(l^v)l^v+s(l^v)(1-\gamma_1)]}{1+(1-\beta)(1-\gamma_1) \theta f a}$$  \hspace{1cm} (26)

$$\mu^v_k = \gamma_2 \mu^v_f$$  \hspace{1cm} (27)

$$\mu^v_l = \gamma_2 \mu^v_l$$  \hspace{1cm} (28)

3.6 Optimal parking enforcement policies

The two commonly pursued objectives of parking enforcement are profit and social welfare maximization. To reach either of the two objectives, parking authorities implement strategic policies by choosing the citation fine $f$ and the level-of-enforcement $k$ due to their explicit influence on parking behaviour. The effect of a policy, comprised of the pair $(k, f)$, on each objective is investigated in this section.

3.6.1 Maximizing the profit of parking enforcement

The profit of parking enforcement is the revenue generated from the tickets that are issued minus the cost of employing enforcement units. The expected revenue is defined as $f m$ [dollars per hour] which is the product of the citation fine and the number of citations in one hour. To define the cost of enforcing parking, let $c$ be fixed the cost of acquiring one enforcement unit for one
hour so that the total cost of enforcement is \( ck \) [dollars per hour]. Then, the expected profit is denoted by \( \pi \) and calculated as\(^5\)

\[
\pi = f m - ck
\]  
(29).

To maximize the profit \( \pi \), we take the derivative of Eq. (29) as

\[
\frac{d\pi}{df} = f \frac{dm}{df} + m \equiv m(1 + \mu_f^m)
\]  
(30).

Expected profit is maximized by setting \( d\pi/df = 0 \) or equivalently

\[
\mu_f^m = -1
\]  
(31)

which implies that at the optimal citation fine \( f^* \), one percent increase in the fine must decrease the meeting rate by one percent. This decline in the meeting rate occurs with either a decrease the arrival rate \( T^v \) or the dwell time \( l^v \) or both. Hence, at \( f^* \) the number of illegal vehicles \( N^v \) (\( \equiv T^v l^v \)) is negatively impacted with an increase in the fine. We later explore, in the next section, how \( f^* \) can be calculated under special deterministic and random equilibrium conditions.

The second influencing factor in the profit \( \pi \) is the number of enforcement units \( k \). To find the optimal \( k^* \), we take the derivative of Eq. (29) as

\[
\frac{d\pi}{dk} = f \frac{md}{k} (\gamma_2 + \mu_k^m) - c
\]  
(32)

By setting \( d\pi/dk = 0 \), we have

\[
\mu_k^m = \frac{ck}{mf} - \gamma_2
\]  
(33)

which shows that the optimal elasticity \( \mu_k^m \) is equal to the cost-benefit ratio \( ck/mf \) offset by the second technology parameter \( \gamma_2 \). Eq. (33) yields the following two insights. When \( \frac{ck}{mf} < \gamma_2 \), we have \( \mu_k^m < 0 \) which indicates that the inspection technology is efficient enough (because of a

\(^5\) The profit in Eq. (29) can be revised to include the revenue generated from legal parking as well. This can be done by adding the term \( pT^n \) to Eq. (29).

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Let us now consider the case of optimizing simultaneously the pair \((k^*, f^*)\) to maximize the profit \(\pi\). Given that \(\mu_k^m = \gamma_2 \mu_f^m\) (Eq. 24), we can rewrite Eq. (32) as
\[
\frac{d\pi}{dk} = \left( \frac{f \gamma_2}{k} \cdot \frac{d\pi}{df} \right) - c
\]
which shows that the profit \(\pi\) cannot be simultaneously maximized with respect to \(f\) and \(k\); that is, the two terms \(d\pi/f\) and \(d\pi/dk\) cannot be equal to zero together. By setting \(d\pi/df = 0\), we have \(d\pi/dk = -c\) which implies that lowering the level-of-enforcement at \(f^*\) can further raise the generated profit. This indicates that, ideally, it is most profitable to have a very small level-of-enforcement and a high citation fine so that a few illegal vehicles are caught with little inspection but they pay a large sum of money. Practically, however, this type of policy cannot be applied as there are social restrictions that limit the upper bound of the citation fine. Hence, in real-life scenarios, the optimal citation fine must be set to the largest socially acceptable price. When the fine is set, the level-of-enforcement can be obtained using Eq. (32).

### 3.6.2 Maximizing social welfare

In defining social welfare for parking, both legal and illegal drivers must be taken into account. Illegal vehicles cause a negative externality because they increase congestion by creating bottlenecks in the traffic flow as a result of parking in appropriate places. Legal vehicles, on the hand, are also important as they have not violated the parking laws and therefore should be rewarded for their behaviour. Mathematically, social welfare in the context of parking enforcement is defined as follows. Let \(W(f, k)\) denote the social welfare as a function of the citation fine \(f\) and the level-of-enforcement \(k\) as
\[
W(f, k) = T^n \int_0^{l^n} s(l) \cdot dl - Q(N^v) - cN^v \tag{35}
\]
where the first term is the net benefit obtained by all legal vehicles, the second term is a function \(Q\) which defines the negative externality caused by all illegal vehicles \(N^v\), and the third term is...
the total cost of enforcement. Arguably, one could also consider the total benefit of the illegal vehicles \( T^v \int_0^\nu s(w)dw \). This consideration, however, is quite controversial as enforcement policies are implemented to reduce illegal parking when social welfare is of interest.

By taking the derivative of \( W \) with respect to \( f \) and \( k \), we have the following two equations

\[
\frac{dw}{df} = -\frac{\tau^v}{f} \left[ \mu_f \int_0^l s(w)dw - l^v g'(N^v) \left[ \mu_f^v + \mu_f^{lv} \right] \right]
\]

(36)

\[
\frac{dw}{dk} = -\frac{\tau^v \gamma_2}{k} \left[ \mu_f^v \int_0^l s(w)dw - l^v g'(N^v) \left[ \mu_f^v + \mu_f^{lv} \right] \right] - c
\]

(37)

Eq. (36) and (37) cannot be easily investigated analytically. Hence, we proceed to obtain managerial insights using numerical experimentation in the next section.

3.7 Numerical experiments

In this section, we perform numerical experiments to illustrate our findings. The three main inputs in the analyses are (i) the marginal benefit function \( s \), (ii) the meeting function \( M \), and (iii) the logit choice model dispersion parameter \( \theta \). These inputs are defined as follows. The marginal benefit function has the following form

\[
s(l) = B_0 \cdot (B_1)^l
\]

(38)

where \( B_0 = 30 \) and \( B_1 = 0.2 \) are fixed parameters. The function \( s(l) \) in Eq. (38) has all the required features of the marginal benefit function because it is strictly decreasing, convex, and asymptotic to zero. The second input is the meeting function which is defined using the Cobb-Douglas relationship as the following

\[
M(N^v, k) = A_0 (N^v)^{\gamma_1} (k)^{\gamma_2}
\]

(39)

where \( A_0 = 1 \) is a parameter and \( \gamma_1, \gamma_2 \) are the elasticities. We perform sensitivity analysis on the elasticities to analyze numerically their impact on the equilibrium and the optimal enforcement policy. The third input is the dispersion parameter which is set to \( \theta = 0.1 \) unless stated otherwise. Sensitivity analysis is performed on \( \theta \) as well. Finally, the cost of legal parking is set to \( p = 3 \) [dollars per hour] which leads to a legal parking duration of \( l^{n^*} = 0.68 \) [hours].
3.7.1 Analysis of the optimal policy

We first assess the impact of each policy, defined by the pair \((k, f)\), on the equilibrium. The results are presented in Fig. 3-5 where the two-dimensional space with \(k\) on the horizontal axis and \(f\) on the vertical axis is used to illustrate changes in the illegal arrival rate \(T^v_*\), dwell time \(l^v_*\), meeting rate \(m\), and citation probability \(\alpha\). The following insights are observed in Fig. 3-5.

First, the illegal arrival rate and dwell time are shown to be highest when \(f\) and \(k\) are both low because (i) illegal vehicles have a low chance of getting cited and (ii) even if they get cited, they pay only a small penalty; increasing either \(f\) or \(k\), however, lowers both \(T^v_*\) and \(l^v_*\). Second, the meeting rate \(m\) and the citation probability \(\alpha\) are shown to increase with \(k\) and decrease with \(f\) for the following reasons. The increase with \(k\) occurs because more enforcement units can catch more illegal vehicles. The decrease with \(f\), on the other hand, occurs because fewer vehicles park illegally when \(f\) is large to avoid a large penalty which leads to a lower meeting rate. Third, the white region in Fig. 3-5 illustrates policies where \(f\) and \(k\) are so large that no vehicle parks illegally. We call this the “no illegal parking” region.

Next, we assess the impact of each policy, i.e. pair \((k, f)\), on the profit \(\pi\) and social welfare \(W\). The results are presented in Fig. 3-6 and the following insights are observed. First, the maximum profit occurs at a low \(k\) and a large \(f\) which shows that it is best to have a few enforcement units that cite illegal vehicles for a large penalty. Second, the profit in the “no illegal parking” region (white area in Fig. 3-5) is negative because the city does not make any citation money is this zone regardless of the level-of-enforcement. Third, the optimal social welfare occurs in the “no illegal parking” region as no negative externality, associated with illegal parking, is imposed. Moreover, the optimal social welfare occurs at a \(k\) to avoid the cost deploying the enforcement units. Instead, vehicles are deterred to park illegally due to the high cost of the fine \(f\).
Figure 3-5: Impact of each enforcement policy on the illegal arrival rate, dwell time, meeting rate, and citation probability.

Figure 3-6: Profit and social welfare at different enforcement policies.
3.7.2 Analysis of meeting technology

We investigate the impact of the meeting technology parameters $\gamma_1$ and $\gamma_2$ on profit and social welfare. We present the impact of simultaneously changing $\gamma_1$ and $f$ on the profit. The results are presented in Fig. 3-7 and the following are observed. First, the lowest profit is obtained at highest $\gamma_1$ because drivers avoid parking illegally due to the efficient inspection technology. We show in the next section that the impact of $\gamma_1$ on profit is substantially dependent on the dispersion parameter $\theta$. Second, the citation probability $\alpha$, illegal arrival rate $T^v*$, and dwell time $l^v*$ all decrease with the fine $f$. Their rate of change, however, depends on the elasticity $\gamma_1$; a larger $\gamma_1$ increases the rate of change in $\alpha$, $T^v*$, and $l^v*$. This observation shows the parking behaviour is more sensitive to the enforcement policy when the inspection technology is associated with a large $\gamma_1$. Third, the two elasticities $\mu^v$ and $\mu^T$ are both negative for all values of citation fine, thus confirming that both $T^v*$ and $l^v*$ decrease with $f$. 

Next, we present the impact of changing $\gamma_2$ and $k$ on social welfare. The results are presented in Fig. 3-8 and the following are observed. First, social welfare is negative when $k$ is too low because too many vehicles park illegally. Similarly, social welfare is also negative when $k$ is too large because of the high cost of hiring the enforcement units. Second, citation probability $\alpha$ is shown to increase with $k$ because more enforcement units are searching for illegal vehicles. Third, both illegal arrival rate $T^*_v$ and dwell time $l^*_v$ are shown to decrease with $k$ because of the higher citation probability $\alpha$. Fourth, the two elasticities $\mu^*_f$ and $\mu^*_v$ are both negative for all values of $k$, thus confirming that both $T^*_v$ and $l^*_v$ decrease with $k$. 

Figure 3-7: Impact of the meeting elasticity $\gamma_1$ and citation fine $f$ on profit $\pi$. 

Legend

<table>
<thead>
<tr>
<th>$\gamma_1$</th>
<th>$\gamma_1$</th>
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</thead>
<tbody>
<tr>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>0.6</td>
<td>0.8</td>
</tr>
</tbody>
</table>
3.7.3 Analysis of the dispersion parameter

We investigate the impact of the dispersion parameter on the optimal profit and social welfare. The results are illustrated in Fig. 3-9 and the following insights are observed. First, a higher dispersion parameter \( \theta \) (i.e. a higher level of information in the system) is shown to decrease the optimal profit but increase the social welfare because fewer vehicles park illegally and get cited as they become aware of the higher cost of illegal parking. The conclusion from this observation
is that cities should provide details of their parking enforcement plans if they wish to mitigate the negative externalities of illegal parking and improve social welfare. Second, increasing the elasticity $\gamma_1$ also improves social welfare at all values of $\theta$. For profit, however, a different trend is observed: When $\theta$ is large, it is more profitable to have a large $\gamma_1$ and when $\theta$ is small, it is more profitable to have a $\gamma_1$ that is neither too large nor too low.

![Figure 3-9: Impact of the dispersion parameter $\theta$ on profit and social welfare.](image)

### 3.8 Conclusions

We present an equilibrium model of illegal parking and use the model to devise optimal parking enforcement policies. By investigating the properties of the equilibrium model, we determine the interrelationship between the factors that affect illegal parking behaviour and we illustrate these effects in Fig. 3-10. Our findings on the properties of the equilibrium model are summarized as the following. First, the citation probability increases with the illegal dwell time because vehicles that are parked illegally for a long time are more susceptible to get cited. In contrast, the citation probability decreases with the illegal arrival rate because the enforcement units have more vehicles to inspect and cite. Second, increasing the illegal dwell time raises both the benefit and the cost of illegal parking. The raise in benefit occurs because the vehicle enjoys a longer duration for completing a given activity. The raise in the cost, on the other hand, occurs because of the respective increase in the citation probability. Third, the meeting rate increases with the level-of-enforcement, illegal arrival rate, and illegal dwell time. Fourth, the illegal dwell time
decreases with the citation probability, thus showing that vehicles park illegally for a shorter time when there is a high chance of getting cited. Finally, for a fixed dwell time, each vehicle is better off parking legally because of the larger resulting utility.

![Diagram](image)

**Figure 3-10: Factors that affect illegal parking behaviour.**

Our findings on devising the optimal parking enforcement policy are summarized as follows: First, when maximizing the profit, it is best to set the citation fine so large that some vehicles still park illegally but not large enough to deter illegal parking completely. Second, when maximizing social welfare, it is best to set the fine so large such that no vehicle parks illegally and to have a small level-of-enforcement. This policy ensures that no vehicle parks illegally. Finally, profit cannot be simultaneously optimized with respect to the citation fine and the level-of-enforcement.

**Appendix 3-A: Impact of the illegal dwell time and arrival rate on the citation probability**

In this appendix, we describe the relationship between the citation probability with the illegal arrival rate and dwell time.
**Lemma A1.** The partial derivatives of the citation probability $\alpha$ with respect to illegal arrival rate $T^v$ and dwell time $l^v$ are defined respectively as $\partial \alpha / \partial T^v = \alpha (\gamma_1 - 1) / T^v$ and $\partial \alpha / \partial l^v = \alpha \gamma_1 / l^v$.

**Proof:** The proof lies in differentiating $\alpha = M(N^v, k)/T^v$ with respect to $l^v$ which leads to the following

$$\frac{d\alpha}{dl^v} = \left[ \frac{dM}{dl^v} T^v - \frac{dT^v}{dl^v} M \right] / (T^v)^2$$

(40)

To simplify Eq. (40), we need to find $\frac{dM}{dl^v} = \frac{\partial M}{\partial N^v} \frac{dN^v}{dl^v}$ which is derived as

$$\frac{dM}{dl^v} = \frac{\alpha \gamma_1}{l^v} \left( \frac{dT^v}{dl^v} l^v + T^v \right)$$

(41)

Using Eq. (41) in Eq. (40), we can rewrite Eq. (40) as

$$d\alpha = \frac{\alpha (\gamma_1 - 1) dT^v}{T^v} + \frac{\alpha \gamma_1 dl^v}{l^v}$$

(42)

Lemma A1 is readily proven from this result.

**Appendix 3-B: Proof of the existence of an equilibrium**

In this appendix, we prove the existence of an equilibrium solution $(l^v^*, T^v^*)$. The proof is involves showing that $T^v$ is compact and convex (Lemma B1) and continuous (Lemma B3).

**Lemma B1.** The feasible set $T^v$ is compact and convex when $U^v \geq U^n$.

**Proof:** We first prove that $T^v$ is compact and second prove that it is convex. To prove compactness, we need to show that $T^v$ is closed and bounded. We do this by proving that the illegal utility $U^v$ is closed and bounded as well. Let us define the lower and upper boundaries of the illegal utility $U^v$ as $U^v$ and $U^\overline{v}$, respectively. The lower boundary can be defined at the very extreme case where an illegal vehicle receives no benefit of parking because of a short dwell time $l^v = 0$ but pays a fine of $f$ as a result of getting cited right away. Hence, the lower boundary is defined as $U^v = -f$. The upper boundary occurs at the other extreme end of the spectrum where an illegal vehicle parks for a very long time $l^v \to \infty$ but never gets cited which leads to a zero expected cost for illegal parking. This condition leads to a utility comprised strictly a benefit...
that is equal to $U^v = \int_0^\infty s(l) \cdot dl$. Hence we have $U^v \leq U^p \leq U^{\overline{v}}$ which shows that $T^v$, a one-to-one logit function of $U^v$, is bounded and closed and hence compact. Next, we prove that that $T^v$ is convex. This is a straightforward result when $U^v \geq U^n$ which indicates that only the convex part of the logit choice function is considered. ■

To show that $T^v$ is continuous, we first present Lemma B2 as a prerequisite.

Lemma B2. The optimal dwell time $l^v^* = L(T^v)$ is unique for each $T^v \in [\underline{T^v}, T]$ where $\underline{T^v}$ is a lower bound on the illegal arrival rate.

Proof: To show that $l^v^* = L(T^v)$ is unique, we first show that $L(T^v)$ exists for $T^v \in [\underline{T^v}, T]$. As indicated in the first condition of equilibrium (Eq. 17), the dwell time $L(T^v)$ is obtained as the solution of $s(l^v^*) = \alpha l^v f_1 l^v^*$; the two sides of this equation are plotted in Fig. 3-11. It is easy to show that the right-hand-side of this equation (i.e. $\alpha f_1 l^v$) tends infinity when $l^v \to 0$, and it tends to zero as $l^v \to \infty$. As is illustrated, when $T^v > \underline{T^v}$, the two curves cross each other at two points, and when $T^v < \underline{T^v}$, the two curves do not cross each other. Hence, it is clear that $l^v^* = L(T^v)$ has a solution as long as the two curves cross each other when the condition $T^v \in [\underline{T^v}, T]$ is satisfied.

We now show that $L(T^v)$ is unique. Given that $L(T^v)$ is obtained from the first-order optimality condition on $U^v$, to prove uniqueness it is sufficient to show that $U^v$ is concave based on the second-order optimality condition. $U^v$, however, is not concave; the first term of $U^v$ (the benefit of parking) is concave but the second term (the negative of the cost) is convex. Despite non-concavity of $U^v$ with respect to $l^v$, we can still show that the solution is unique. Given that $U^v$ has two components (benefit and cost) and given that each component is either strictly convex or strictly concave, there are at most two solutions that satisfy the first-order optimality condition. These two solutions are the points where the two curves cross each other in Fig. 3-11 and only the second solution (with a larger $l^v$) the is global maximum of $U^v$. Hence, only one of the two solutions is valid and $l^v^* = L(T^v)$ is unique. ■

Lemma B3. The feasible set $T^v$ is continuous when $T^v \in [\underline{T^v}, T]$.
Proof: Continuity of $\mathbf{T}^v$ can be easily established based on the uniqueness proof presented in Lemma B2. ■

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure3_11.png}
\caption{Influential factors that affect illegal parking behaviour.}
\end{figure}

Appendix 3-C: Comparative static effects and elasticities

In this appendix, we present the comparative static effects of regulatory variables $k$ and $f$ on transitional variables and the interrelationship between the transitional variables.

We start with the citation fine $f$ and investigate its impact on $\alpha, l^v, T^v, U^v, M$ by taking the following derivatives.

\begin{align*}
\frac{dM}{df} &= \frac{\partial M}{\partial N^v} \frac{dN^v}{df} = \frac{\alpha y_1}{N^v} \left( \frac{dl^v}{df} T^v + \frac{dT^v}{df} l^v \right) & (43) \\
\frac{d\alpha}{df} &= \frac{\partial \alpha}{\partial l^v} \frac{dl^v}{df} + \frac{\partial \alpha}{\partial T^v} \frac{dT^v}{df} & (44) \\
\frac{dl^v}{df} &= \frac{y_1 (\alpha + \frac{d\alpha}{df} f) / [s'(l^v)l^v + s(l^v)]}{df} & (45)
\end{align*}
\[
\frac{dT^v}{df} = \frac{\partial T^v}{\partial U^v} \frac{dU^v}{df} = T^v \beta (1 - \beta) \frac{dU^v}{df}
\]

(46)

\[
\frac{dU^v}{df} = s(l^v) \frac{dU^v}{df} - f \frac{da}{df} - \alpha
\]

(47)

As is evident from the above conditions, we have five equations and five unknowns. Eq. (43) and (44) are straightforward. Eq. (45) is obtained from taking the implicit differentiation of Eq. (17), the first condition of equilibrium, with respect to \( l^v \), Eq. (46) has the term \( \partial T^v / \partial U^v \) which is obtained by taking the derivative of the logit choice model, and Eq. (47) is obtained by taking implicit differentiation of Eq. (4) with respect to \( f \). Solving this system of equations yields the following two equations

\[
\frac{dT^v}{df} = \frac{-(1-\beta)\theta_f \alpha}{1+f \alpha \theta (1-\beta)(\gamma_1-1)}
\]

(48)

\[
\frac{dl^v}{df} = \frac{s(l^v)[s'(l^v)l^v + s(l^v)(1-\gamma_1)]}{1+f \alpha \theta (1-\beta)(\gamma_1-1)}
\]

(49)

which are used to derive the following two elasticities

\[
\mu_T^v = \frac{-(1-\beta)\theta_f \alpha}{1+(1-\beta)(1-\gamma_1)\theta_f \alpha}
\]

(50)

\[
\mu_l^v = \frac{s(l^v)[s'(l^v)l^v + s(l^v)(1-\gamma_1)]}{1+(1-\beta)(1-\gamma_1)\theta_f \alpha}
\]

(51)

A very similar approach can be followed to obtain all other equations in Section 3.5.4.

3.9 References


Optimal Parking Enforcement Policies for Commercial Vehicles

4.1 Introduction

Illegal parking leads to adverse societal impacts such as reduced traffic speeds, loss of revenue from legal parking, and more accidents caused by safety violations. In response to these detrimental consequences, policies are imposed to alleviate illegal parking. Parking enforcement, the most prevalent policy, has been implemented in major cities for many years. Clearly, an effective enforcement plan requires an in-depth understanding of the causes and patterns of illegal parking. Commercial vehicles (CV) are of particular interest in parking enforcement because of their heavy presence in central business districts and their recurrent behaviour of illegal parking. In 2014 alone, a total of 691,240 tickets were issued to CVs in Toronto, Canada, almost a quarter of the total number of parking tickets (City of Toronto, 2015). The CV tickets generated $30,516,000 for the city which the carriers are willing to pay as part of the high cost of the “last mile” in the supply chain. To exacerbate the situation, illegally parked CVs create other problems such as increased traffic delay and unsafe conditions. Estimates show that illegal CV parking results in approximately 47 million vehicle-hours of delay each year in the United States, making illegal CV parking the third leading cause of delay behind construction and crashes (Han et al., 2005). Moreover, CVs commonly park on bike lanes in order to reduce their egress time to the delivery destinations. In New York City, an average of 14% of CV on-street parking results in a conflict with a cyclist (Conway et al., 2013). With all these complications, parking enforcement policies must be designed with consideration of illegal CV parking.

The three fundamental components of any parking enforcement policy are detection technology, level of enforcement, and the citation fine. Detection technology is the method of finding illegally parked CVs and the two prominent methods are human\(^6\) surveillance and video detection (Mithun et al., 2012); level of enforcement is the density of the enforcement units (e.g.

\(^6\) Human surveillance includes on-foot, cycling, and driving officers.)
cameras or on-foot officers) in the region; and citation fine is the imposed penalty for illegal parking. While the choice of the detection technology is a long term decision, policy-makers generally have more power over choosing the level of enforcement and the citation fine. The City of Toronto, for instance, has practiced human surveillance since the initiation of its enforcement policy, but has changed the citation fine many times, such as in 2015 when parking fines during peak periods were raised from $60 to $150 and the number of parking enforcement officers were increased as well (Powell and Clarke, 2015). Hereafter, we use the term “policy” to refer to a chosen level of enforcement and the citation fine.

Enforcement policies can influence the parking behaviour of CVs. A large fine can deter CVs from parking illegally whereas a small fine may be considered by carriers as “the cost of doing business” (Nourinejad et al., 2014). Similarly, a high level of enforcement increases the probability of receiving a citation thus discouraging illegal CV parking. For the city, choosing the right policy depends on what objective is pursued. Two common objectives are profit maximization and social cost minimization. The citation profits are substantial in many cities. In 2013, New York City, Los Angeles, and Chicago each generated 534, 250, and 176 million dollars, respectively. In some instances, target profits are defined annually and policies are devised to reach them. Profit maximization must consider the reactive behaviour of CVs as well. For instance, increasing the fine does not always lead to a higher profit because some CVs might start to park legally in order to avoid the penalty. As the second objective, social cost is seldom quantified but equally important. The extra traffic delay that CVs generate is cost borne by society. The two objectives are not naturally obtained from one policy. The policy that maximizes profits may compromise social welfare. In this chapter, we formulate the two objectives, model the reactive behaviour of CVs, and present the tradeoff between the two objectives.

This chapter is organized as follows. A review of research on illegal parking is presented in Section 4.2 and the gaps in the literature are highlighted. A model of parking enforcement with special treatment of CVs is presented in Section 4.3. Two policy objectives are formulated and three market regimes are discussed in Section 4.4. Numerical experiments are performed in Section 4.5 using the City of Toronto as a case study. Conclusions are presented in Section 4.6.
4.2 Background

Despite the abundance of research on parking enforcement, most studies provide only a descriptive analysis of illegal parking such as on-street parking meter behaviour (Adiv and Wang, 1987), illegal parking behaviour in central business districts (Bradley and Layzell, 1986; Brown, 1983), impact of illegal parking on local businesses (May, 1985), impact of parking fines on public transportation ridership (Auchincloss et al., 2014), CV illegal parking behaviour (Wang and Gogineni, 2015; Wenneman et al., 2015), and non-CV illegal parking in loading bays (Aiura and Taniguchi, 2005; Alho and Silva, 2014). A review of descriptive models is presented in one (if not the only) literature review of parking enforcement by Cullinane and Polak (1992) that focuses on the relationship between illegal parking and parking controls and the factors that influence the choice of illegal parking.

Descriptive models, although useful for identifying the factors that influence illegal parking behaviour, do not provide a tool for finding the optimal enforcement policy. The need for prescriptive parking enforcement models is advocated in a recent literature review by Inci (2015) where the need for theoretical modelling techniques is stressed. In theory, the parking enforcement problem is an inspection game where the enforcement units are the inspectors and the CVs are the inspectees (Ferguson and Mlolidakis, 1998; Avenhaus and Canty, 2005; Sasaki, 2014). In the classical inspection game, there are two players called the inspector and the inspectee. The inspector’s strategy space is to audit the inspectee or not and the inspectee’s strategy space is to violate the rules or not. The conditional probability that a violating inspectee is caught (i.e. the citation probability) is equal to the audit probability of the inspector. In the illegal parking problem, however, the citation probability (i.e. probability of catching a violating inspectee) is a function of how long the illegal CV parks (i.e. the dwell time) and the number of enforcement units. An increase in either the level of enforcement or the dwell time of the illegally parked CV increases the citation probability. This feature of the illegal parking problem merits an appropriate modelling approach.

To accommodate this feature in the inspection game, we use the concept of bilateral searching and matching (or bilateral meeting) which models the meeting friction between two sets of agents. Examples of bilateral meeting in economics include taxi-passenger meeting (Yang et al., 2010; Yang and Yang, 2010), buyer-seller meeting (Burdett et al., 2001), and employer-
employee meeting in the labor market (Andolfatto, 1996; Berman, 1997). The meeting function is formulated so that the meeting rate of the two agents is a function of the number of agents. For instance, the rate of taxis meeting passengers depends on how many vacant taxis and passengers are available (Yang and Yang, 2010). A review of the bilateral meeting function is conducted by Petrongolo and Pissarides (2001). In this chapter, we use the bilateral meeting function to model the searching friction present in the inspector-inspectee (i.e. enforcement unit - illegally parked CV) agents. The presented analytical formulation has two advantages. First, compared to other descriptive black box models, it clarifies the interplay between the factors that influence illegal parking behaviour, citation probability, and the meeting rate. Second, it provides a quantitative tool for assessing optimality of policies.

Although the meeting function has never been used to model the parking enforcement problem as an inspection game, some studies have used other approaches to develop prescriptive solutions. Elliot and Wright (1982) study the relationship between parking compliance and enforcement and show that the relationship can be potentially unstable due to presence of hysteresis. In their proposed hysteresis theory, the inspection rate decreases with decreasing compliance simply because each enforcement unit spends more time citing the illegally parked CVs and has less time for inspection. Petiot (2004) extends the original parking model of Arnott and Rowse (1999) to model parking violations where each driver makes a binary choice of parking legally or illegally. If parked illegally, there is a fixed probability of receiving a citation. The binary choice is made in order to maximize the obtained utility of parking. The model captures the impact of the citation fine on illegal parking behaviour but is not sensitive to parking duration or the level of enforcement. Kladeftiras and Antoniou (2013) present a microsimulation model to analyze the effects of illegal parking on traffic congestion and show that average traffic speeds can be increased by 10-15% if double parking is limited and that it can be increased by up to 44% if completely eliminated. The simulation model, although detailed in capturing parking dynamics, is developed only for modelling double-parking and does not capture the impact of enforcement on illegal parking behaviour. Moving away from policy, Summerfields et al. (2015) develop a model for parking enforcement by formulating a Chinese Postman Problem to optimize the routing of the enforcement officers to maximize the total number of citations and minimize the total distance travelled by the officers. The model of Summerfields et al. (2015) is useful as a tool for operational decision-making but cannot be used for policy-making.
As it is evident from these studies, no research has yet been dedicated to modelling the relationship between parking dwell time, citation probability, and illegal parking behaviour. Moreover, no research yet investigates the quantitative influence of parking enforcement policies (defined by a citation fine and the level of enforcement) on illegal parking behaviour and the role of policies in helping cities reach their objectives of profit maximization and social cost minimization. Finally, there are few analytical models on parking enforcement that consider explicitly the parking patterns and delivery features of CVs (Alho, 2015; Alho et al., 2016). The model of this chapter is developed to address these gaps in the literature.

4.3 The model

4.3.1 Problem setting

Notation

\[ I = \{1, \ldots, i, \ldots, n\} \] set of \( n \) carriers

\[ T_i \] shipment frequency of carrier \( i \) [deliveries per hour]

\[ F_i \] fleet size of carrier \( i \)

\[ C_i \] capacity of each vehicle in the fleet of carrier \( i \) [shipping units]

\[ X_i \] vector of dwell time distribution parameters for carrier \( i \)

\[ d_i \] dwell time distribution for carrier \( i \)

\[ g_i(d_i; X_i) \] probability density function of the carrier \( i \)'s dwell time

\[ w_i \] average walking cost of carrier \( i \) if parked legally [Dollars]

\( f \) fine of parking illegally [Dollars]

\[ \alpha(d) \] probability that an illegally parked vehicle is cited with dwell time \( d \)

\( p \) cost of parking legally

\[ G_i^l \] expected cost of all legal parking for carrier \( i \)
\( G_i^v \) expected cost of all illegal parking for carrier \( i \)

\( m \) rate of parking enforcement units finding illegally parked vehicles

\( N^v \) total number of illegally parked vehicles

\( N^e \) total number of enforcement units

\( \gamma_1 \) elasticity of the meeting rate with respect to the enforcement units

\( \gamma_2 \) elasticity of the meeting rate with respect to the illegally parked vehicles

A total of \( n \) carriers defined by the set \( I = \{1, ..., i, ..., n\} \) deliver daily shipments to a service area. The shipment frequency of each carrier \( i \in I \) is denoted by \( T_i \) deliveries per hour. Each carrier \( i \in I \) owns a fleet of \( F_i \) CVs, each with a capacity of \( C_i \) shipping units. Hence, for a given operating time (shift time) of \( S \) hours, we have:

\[
F_i = ST_i / C_i \quad \forall i \in I
\]  

(1)

where \( ST_i \) is the total number of deliveries for carrier \( i \in I \). Eq. 1, as it stands, assumes that CVs are filled to capacity. This assumption is not restrictive because \( C_i \), which is physical capacity of the CVs, can easily be replaced with the effective capacity \( \tilde{C}_i \) (where \( \tilde{C}_i \leq C_i \)) which is the actual used capacity of each CV. Since there is no logical interaction between parking enforcement policies and CV capacity, we assume \( \tilde{C}_i = C_i = C, \forall i \in I \), without any loss of generality.

Each delivery of each CV has a dwell time. The parking dwell time of carrier \( i \)'s deliveries follows a continuous distribution \( d_i \sim D(X_i) \) where \( X_i \) is the vector of distribution parameters and \( g_i(d_i; X_i) \) is the pdf of the distribution. Each CV of each carrier \( i \in I \), when close to its delivery destination, chooses to park legally or illegally. If parked legally, the CV pays a fixed parking fee of \( p \) dollars (to park as long as required to load or unload) and an average walking cost of \( w \) dollars (product of walking distance and the value of time)\(^7\). The walking cost can also

\(^7\) The term \( w \) can also include the parking search time associated with legal parking.
be carrier specific so that \( w_i \) is the average walking cost of carrier \( i \in I \). If parked illegally, the CV does not pay a parking fee but will have to pay a penalty of \( f \) dollars if cited by parking enforcement. It is assumed that illegally parked CVs (hereafter referred to as illegal CVs) park so close to their destination that their walking distance is equal to 0. That is, all legal parking is far-sided and all illegal parking is near-sided. In cases where there is an average walking distance with illegal parking as well, then \( w_i \) can be interpreted as the difference in the walking cost of parking legally and illegally.

The probability that an illegal CV is cited is denoted by \( \alpha(d) \), which is a function of the CV’s parking duration \( d \). The citation probability \( \alpha(d) \) is strictly increasing; the longer the parking duration, the higher the probability of getting a citation. Hence, for a given dwell time \( d \), the expected cost of parking illegally is \( f \alpha(d) \) and the cost of parking legally is \( p + w \). The two costs are illustrated in Fig. 4-1. As depicted, the cost of legal parking is fixed regardless of the parking duration\(^8\) whereas the expected cost of illegal parking increases with dwell time and it converges to \( f \) when \( d \rightarrow \infty \). This indicates that an illegal CV is bound to get a citation if parked for a very long time. Proposition 1 shows that \( p \) and \( f \) should be selected so that \( p + w \leq f \), otherwise the legal parking supply will be underutilized.

**Proposition 1:** The parking fee \( p \) and citation fine \( f \) should be chosen so that the inequality \( p + w \leq f \) holds.

**Proof:**

As mentioned earlier, the expected cost of legal and illegal parking are \( p + w \) and \( f \alpha(d) \), respectively. Hence, the maximum cost of illegal parking is \( f \) when \( \alpha(d) \rightarrow 1 \). Now, assume the opposite of Proposition 1 is true so that \( p + w > f \). In this case, every CV would park illegally because even under the toughest enforcement conditions with \( \alpha(d) = 1 \), the CV still pays less for illegal parking. The inequality \( p + w > f \) indicates an underutilization of the legal parking supply which is not logically sound. Hence, \( p + w \leq f \).

\(^{8}\) The assumption of a fixed price for legal parking does not compromise the generality of the model. This cost could also be a function of dwell time which makes the equations more complex without adding any insight.
The following Lemma is now imposed.

Lemma 1: At every delivery, each carrier $i \in I$ makes a choice of parking legally or illegally based on the duration of that stop. When the dwell time $d_i$ is shorter than some threshold $\bar{d}_i$ the carrier parks illegally. Alternatively, the carrier parks legally when the dwell time $d$ is longer than $\bar{d}_i$.

Proof:

Under steady state equilibrium conditions, $\bar{d}_i$ is chosen so that $f(\bar{d}_i) = p + w$ as illustrated in Fig. 4-1. Given that the cost of illegal parking $f(\bar{d}_i)$ is strictly increasing, for $d > \bar{d}_i$ we have $f(\bar{d}_i) > p + w$. Hence, a CV with $d > \bar{d}_i$ will park legally due to its lower cost. However, when $d \leq \bar{d}_i$ we have $f(\bar{d}_i) \leq p + w$. Hence, a CV with $d \leq \bar{d}_i$ will park illegally due to its lower cost. The rationale for Lemma 1 is that a CV is less likely to be cited for illegal parking when the parking duration is short.

\[ G_i^l = \int_{\bar{d}_i}^{\infty} (p + w) T_i g_i(v) \, dv \quad \forall i \in I \]
where the bounds of the integral represent the domain of parking duration for legal parking. In Eq. 2, the term $T_i g_i(v)$ is the parking frequency of carrier $i \in I$ with dwell time $v$, $dv$ is the infinitesimal dwell time, and the term $p + w$ is the cost per delivery. Hereafter we set $p = 0$. This assumption does not compromise the generality of the equations because Eq. 2 is still dependent on $w$. In other words, eliminating $p$ from Eq. 2 is equivalent to choosing a larger $w$ that incorporates the cost $p$ as well.

According to Lemma 1, every CV of carrier $i \in I$ with dwell time $d \leq \bar{d}_i$ will park illegally. Hence, $G^p_i$ is calculated as:

$$G^p_i = \int_0^{\bar{d}_i} f(\alpha(v)) T_i g_i(v) \, dv \quad \forall i \in I$$

(3)

where $f(\alpha(v))$ is the cost of an illegal CV for duration of $v$ and $T_i g_i(v)$ is the parking frequency of carrier $i$ with dwell time $v$. By the law of total expectation, the expected cost of each carrier $i \in I$ is denoted by $G_i$ which is the sum of legal and illegal parking costs:

$$G_i = G^l_i + G^p_i \quad \forall i \in I$$

(4)

The objective of each carrier $i \in I$ is to minimize $G_i$ via the proper selection of $\bar{d}_i$. It is clear that the optimal $\bar{d}_i$ occurs at the point where $f(\alpha(\bar{d}_i)) = p + w$ as indicated by Lemma 1.

4.3.2 Citation probability function

The following assumptions are imposed. An inspection can lead to a citation if the CV is illegally parked and has not already been cited. If a CV is caught multiple times by enforcement units during its dwell time, it receives only one citation. Each vehicle is prone to receiving a citation for illegal parking regardless of how many citations it received on its former deliveries.

Let the term “meet” define the event where an enforcement unit inspects an illegally parked CV. The meeting rate is measured in CVs per hour and denoted by $m$. Given that the meetings are independent from each other, it can be assumed that the citation process follows a Poisson distribution and the inter-arrival time between each two meetings, denoted by the continuous variable $t$, follows an Exponential distribution. Other distributions can be used as well. Using the law of random incidence (Larson and Odoni, 1981), the probability that an illegal CV with dwell
time $d$ is cited is equal to the probability that the citation inter-arrival time is smaller than or equal to $d$. Hence, the citation probability $\alpha(d, m)$ is calculated as:

$$\alpha(d, m) = \Pr(t \leq d) = 1 - e^{-md} \quad (5)$$

Eq. 5 shows that the citation probability depends on both the dwell time and the meeting rate $m$. Note that Eq. 5 has the strictly increasing feature that was considered in Lemma 1. Eq. 5 also indicates a citation probability of 1 when the dwell time is very long, i.e. $\lim_{d \to \infty} \alpha(d) = 1$, which is rationally correct since a CV with a very long dwell time is bound to get a citation.

The meeting rate $m$ depends on the following three factors: (i) the level of enforcement, (ii) the number of illegal CVs, and (iii) the enforcement technology. The enforcement technology is assumed to be fixed and defined. Let $N^e$ denote the number of enforcement units (where the superscript ‘$e$’ refers to enforcement) homogenously scattered in the service area and let $N^v$ denote the total number of illegal CVs in the service area. The citation rate can formally be defined as:

$$m = M(N^e, N^v) \quad (6)$$

where the function $M$ is defined so that $\partial M / \partial N^e > 0$ and $\partial M / \partial N^v > 0$ which indicates that increasing the enforcement $N^e$ or illegal CVs $N^v$ causes the meeting rate to rise. Furthermore, $\lim_{N^v \to 0} m = 0$ and $\lim_{N^e \to 0} m = 0$, indicating that no meetings will occur when there are no illegal CVs in the service area to be cited or when there is no enforcement available. Moreover, let

$$\gamma_1 = \frac{\partial m}{\partial N^e} \frac{N^e}{m} \quad (7)$$

$$\gamma_2 = \frac{\partial m}{\partial N^v} \frac{N^v}{m} \quad (8)$$

represent the elasticity of the meeting rate with respect to $N^e$ and $N^v$, respectively. We have $\gamma_1, \gamma_2 > 0$ indicating that increasing either $N^v$ or $N^e$ leads to a higher meeting rate. The elasticities are later used in the sensitivity analysis of Section 4.5.

Calculation of $N^v$ and $N^e$ is elaborated as follows. The enforcement level $N^e$ is a policy decision made by the enforcement authority (the city) and the number of illegal CVs $N^v$ is dependent on
the parking behaviour of all the carriers. To obtain \( N^v \), the number of each carrier’s illegal CVs at the steady state equilibrium conditions must be calculated in the following way. There are \( T_i g_i(v) \) deliveries per hour belonging to carrier \( i \) whose dwell times are between \( v \) and \( v + dv \). The product of the delivery rate \( T_i g_i(v) \) and the dwell time \( v \) (which is equal to \( v T_i g_i(v) \)) gives carrier \( i \)'s number of deliveries at steady state equilibrium with dwell times that are between \( v \) and \( v + dv \). Given a dwell time domain of \([0, \bar{d}_i]\) for illegal CVs of carrier \( i \), the expected number of illegally parked carrier \( i \) CVs, denoted by \( N^v_i \), is calculated as

\[
N^v_i = \int_0^{\bar{d}_i} v T_i g_i(v) \frac{C_i}{d_i} dv \quad \forall i \in I \tag{9}
\]

Note that \( C_i \) (capacity of carrier \( i \) CVs) in the denominator of Eq. 9 converts the number of deliveries into the number of CVs. Since \( C_i \) and \( T_i \) are both constants, we drop \( C_i \) for brevity from hereafter and reformulate Eq. 9 as\(^9\):

\[
N^v_i = \int_0^{\bar{d}_i} v T_i g_i(v) \quad \forall i \in I \tag{10}
\]

The total number of all illegal CVs, the summation of \( N^v_i \) across all carriers, is calculated as:

\[
N^v = \sum_{i \in I} N^v_i = \sum_{i \in I} \int_0^{\bar{d}_i} v T_i g_i(v) dv \tag{11}
\]

### 4.3.3 Equilibrium conditions

So far we have shown that each carrier \( i \) minimizes its cost \( G_i \) by choosing \( \bar{d}_i \) at which the cost of legal parking \( w_i \) is equal to the expected cost of illegal parking \( f\alpha(\bar{d}_i, m) \). Thus, for each carrier \( i \), we have:

\[
w_i = f\alpha(\bar{d}_i, m) \quad \forall i \in I \tag{12}
\]

where \( \alpha(\bar{d}_i, m) \) is obtained from Eq. 5. With a bit of simplification and using Eq. 5, Eq. 12 can be rewritten as:

\(^9\) Dropping the capacity \( C_i \) from Eq. 9 is equivalent to defining \( T_i \) as the number of deliveries per vehicle per hour.
\[ \tilde{d}_i = \frac{-\ln(1 - \frac{w_i}{f})}{m} \quad \forall i \in I \] (13)

which shows that \( \tilde{d}_i \forall i \in I \) can be obtained if the meeting rate \( m \) is known. For a fixed level of enforcement \( N^e \), the meeting function itself only depends on \( N^v \) which according to Eq. 10 is a function of the vector of dwell time thresholds of all carriers. In mathematical form, with a fixed \( N^e \), we have

\[ m = M(\tilde{d}_i, \forall i \in I) \] (14)

The equilibrium occurs at the vector \( \tilde{d}_i \forall i \in I \) where Eq. 13 and Eq. 14 are simultaneously satisfied. The existence conditions of the equilibrium are presented in Proposition 2.

**Proposition 2:** There exists an equilibrium solution if the meeting rate \( m \) is always bounded from below and above such that \( \bar{m} \leq m \leq \hat{m} \).

**Proof:**

Let \( \bar{d} = (\tilde{d}_i, \forall i \in I) \) be the vector of carrier dwell time thresholds and let \( \Omega \) represent the feasible set of \( \bar{d} \). Brouwer's fixed-point theory states the following: If \( \Gamma : \Omega \rightarrow \Omega \) is a continuous function mapping a compact and convex set \( \Omega \) into itself, then there is some vector \( \bar{d} \) in \( \Omega \) such that \( \bar{d} = \Gamma(\bar{d}) \). Hence, in order to prove the existence of a solution, we need to show that \( \Gamma \) is compact and convex.

To show that \( \Gamma \) is compact, we have to show that it is closed and bounded. According to Eq. 13 and given the condition \( \bar{m} \leq m \leq \hat{m} \), it is easy to infer that the following condition holds:

\[ \frac{-\ln(1 - \frac{w_i}{f})}{\bar{m}} \leq \tilde{d}_i \leq \frac{-\ln(1 - \frac{w_i}{f})}{\hat{m}} \] (15)

where the upper-bound of Eq. 15 occurs at \( m = \bar{m} \) and the lower-bound occurs at \( m = \hat{m} \). Given Eq. 15, it is clear that \( \Gamma \) is bounded and closed and hence compact.

We now show that \( \Gamma \) is convex by showing that its second derivative is positive. With some simplification, the second derivative of Eq. 13 is positive when the following condition is justified
\[ m \leq 2 \frac{dm}{dd_i} \frac{d^2m}{dd_i^2} \quad \forall i \in I \] (16)

By choosing the upper-bound of \( m \) such that \( \hat{m} = \min \{ \frac{2}{d_i} \frac{d^2m}{dd_i^2} \} \), the second derivative of Eq. 13 will always be positive and \( \Gamma \) will always be convex.

In finding the equilibrium of the highly non-linear system, we use Brouwer’s fixed-point theory which is explained in Algorithm I (Fuente, 2000).

**Algorithm I:** Brouwer’s fixed-point algorithm

Step 0 – Initialization: Set the iteration counter \( k \) to 0. Set the dwell time threshold of each carrier to randomly chosen value \( \bar{d}_i, \forall i \in I \), so that \( \bar{d}_i^k = \bar{d}_i, \forall i \in I \).

Step 1 – Update the meeting rate: given \( \bar{d}_i^k \), use Eq. 14 to find the meeting rate \( m \).

Step 2 – Update the iteration counter: Set \( k \leftarrow k + 1 \).

Step 3 – Update the dwell time thresholds: given \( m \) from Step 1, use Eq. 13 to obtain the new vector of dwell time thresholds \( \bar{d}_i^k, \forall i \in I \).

Step 4 – Convergence criterion: terminate the algorithm if the convergence criterion is satisfied. Otherwise go to Step 1. The convergence criterion is the following condition

\[ \sum_{i \in I} (\bar{d}_i^k - \bar{d}_i^{k-1})^2 \leq \varepsilon \] (17)

where \( \varepsilon \) is a predetermined convergence tolerance.

To ensure convergence, we use the following modified algorithm with the addition of the method of successive averages (MSA) to solve for the equilibrium. The modified algorithm uses MSA in Step 3 of Algorithm I. The revised Step 3 is presented below in Algorithm II. All other steps of Algorithm II are similar to those of Algorithm I. Algorithm II converges in all the empirical numerical experimentations.

**Algorithm II:** Method of successive averages algorithm
Step 3 – Update the dwell time thresholds: given $m$ from Step 1, use Eq. 13 to obtain the new vector of dwell time thresholds $\bar{d}_i^{new}, \forall i \in I$. Use the method of successive averages to find the dwell time thresholds of the current iteration as

$$\bar{d}_i^k = \frac{k-1}{k} \bar{d}_i^{k-1} + \frac{1}{k} \bar{d}_i^{new}$$

(18)

### 4.3.4 Comparative static effects of regulatory variables

This section presents the comparative static effects of the regulatory variables $N^e$ and $f$ on transitional variables $\bar{d}_i$, $m$, and $N^v$ and the intrarelationship between the transitional variables. The results of the static effects are presented in Eq. 19 to Eq. 30 and their proofs are given in Appendix 4-A. To avoid unnecessary detail, we have omitted the proofs that are too obvious.

The static effect of the regulatory variables on the transitional variables is presented through Eq. 19 to Eq. 24.

$$\frac{dm}{dN^e} > 0$$

(19)

$$\frac{dN^v}{dN^e} < 0$$

(20)

$$\frac{d\bar{d}_i}{dN^e} < 0$$

(21)

$$\frac{dN^v}{df} < 0$$

(22)

$$\frac{dm}{df} < 0$$

(23)

$$\frac{d\bar{d}_i}{df} < 0$$

(24)

Eq. 19 shows that increasing enforcement raises the meeting rate $m$ as more enforcement units would be searching for illegal CVs (Lemma 6). The increase in the meeting rate as a consequence of Eq. 19 will deter CVs from illegal parking which in turn leads to a lower $N^v$ as is shown in Eq. 20 (Lemma 5). The decrease in $N^v$ as a consequence of Eq. 20 is equivalent to carriers reducing their dwell time threshold $\bar{d}_i$ as shown in Eq. 21. Eq. 22 shows that increasing the citation fine lowers $N^v$ as CVs would avoid illegal parking due to the higher penalty of
getting a citation (Lemma 8). As $N^v$ decreases, the meeting rate is lowered as well because there are less illegal CVs to be found as indicated in Eq. 23 (Lemma 7). Finally, the decrease in $N^v$ as a consequence of Eq. 22 is equivalent to carriers reducing their dwell time threshold $\bar{d}_l$ as shown in Eq. 24.

The static relationship between the transitional variables is presented through Eq. 25 to Eq. 30.

\[
\frac{d \bar{d}_l}{dm} < 0 \quad (25)
\]

\[
\frac{d N^v}{dm} < 0 \quad (26)
\]

\[
\frac{\partial m}{\partial N^v} > 0 \quad (27)
\]

\[
\frac{d \bar{d}_l}{d N^v} < 0 \quad (28)
\]

\[
\frac{\partial N^v}{\partial \bar{d}_l} > 0 \quad (29)
\]

\[
\frac{\partial m}{\partial \bar{d}_l} > 0 \quad (30)
\]

The impact of the meeting rate $m$ on $N^v$ and $\bar{d}_l$ is shown in Eq. 25 and 26. Increasing the meeting rate is equivalent to a higher citation probability (See Eq. 5) which reduces $\bar{d}_l$ (Lemma 3) and consequently $N^v$ (Lemma 4) as CVs would be less inclined to park illegally. The impact of $N^v$ on $\bar{d}_l$ and $m$ is shown in Eq. 27 and Eq. 28. Increasing $N^v$ raises the pool of illegal CVs leading to a higher rate $m$ as shown in Eq. 27. As a consequence of Eq. 27 a higher meeting rate reduces $\bar{d}_l$ (according to Eq. 25) thus justifying Eq. 28. The impact of $\bar{d}_l$ on $m$ and $N^e$ is shown in Eq. 29 and Eq. 30. Eq. 29 is obvious (Lemma 2). As a consequence of Eq. 29, when $N^v$ increases, the meeting rate increases as well due to more illegal CVs as shown in Eq. 30.

### 4.4 Profit, social cost, and markets

The two objective functions of the enforcement authority are (i) profit maximization denoted by $PR$ and (ii) social cost minimization denoted by $SC$. Profit is equal to the generated revenue from collecting citation fines minus the cost of enforcement (i.e. price of inspection units). Parking
enforcement revenue itself is composed of the fines paid by each of the carriers that were cited. Let $R_i$ denote the expected hourly revenue obtained from carrier $i \in I$ which is equal to

$$R_i = \int_0^{\bar{a}_i} f \alpha(v) T_i g_i(v) \, dv \quad \forall i \in I$$ \hspace{1cm} (31)$$

The total generated revenue, denoted by $R$, is equal to the sum of the revenue obtained from all carriers. Hence, revenue is computed as $R = \sum_{i \in I} R_i$ which is measured in dollars per hour. By letting $\beta$ denote the cost of each enforcement unit per hour, the total cost of enforcement per hour is $\beta N^e$. The total profit from parking enforcement is the difference between the revenue and cost of enforcement which is given by

$$PR = R - \beta N^e$$ \hspace{1cm} (32)$$

The social cost of each enforcement policy is composed of three negative externalities. The first component is the cost of enforcement which is equal to $\beta N^e$. The second component is the walking cost of the carriers associated with each legal parking delivery. The third component is the cost imposed by each illegal CV on through traffic. Let us assume that the negative externality of the extra travel time cost from each illegal CV on through traffic is $\delta$ dollars per illegal CV. Then social cost is computed as:

$$SC = \beta N^e + \sum_{i \in I} \int_{\bar{a}_i}^{\infty} w_i T_i g_i(v) \, dv + \delta N^v$$ \hspace{1cm} (33)$$

where the second term on the RHS is the cost of walking for all carriers.

Assume that the city grants a single firm the monopoly rights of managing parking enforcement through policy-making that involves choosing a citation fine and the level of enforcement. Under this monopoly system, the single expected profit-maximizing firm would choose a policy that maximizes the generated profit. Hence, we have

**Monopoly:** Maximize $PR$

Conventionally, the first-best social optimum policy is one at which the marginal social welfare benefit from adding one additional enforcement unit is equal to the marginal cost per enforcement unit. Given that the demand for parking is assumed to be fixed in this chapter, we define the first-best social optimum as the policy at which social cost is minimized.
First-best: Minimize $SC$

The first-best social optimum policy may lead to a negative profit. Hence, the second-best social optimum regime is set up with the addition of the constraint that the generated revenue must cover the cost the enforcement (i.e. profits are nil). Despite the relevance of other second-best policies with $PR > 0$, we focus only of cases where $PR = 0$. The second-best market is defined as

Second-best: Minimize $SC$

Subject to: $PR = 0$

4.5 Commercial vehicle parking in the City of Toronto

The central business district of the City of Toronto is chosen as a case study. The central business district, as shown in Fig. 4-2, is approximately 1 km by 1 km containing the highest employment density of Toronto with 8 of the 60 most heavily ticketed locations in Toronto reported by the Canadian Courier Logistics Association (Nourinejad et al., 2014). These locations are depicted by black squares in Fig. 4-2. A survey was conducted in the region in August 2010 to obtain information about carrier deliveries in Toronto’s central business district (Kwok, 2010). The surveyor recorded details of carrier deliveries on individual road segments on weekdays between the hours of 9:00 AM and 3:00 PM. The surveyor collected the arrival time, departure time, parking location, type of location, and the company that owned the CV. A total of 1940 observations were recorded. Table 4-1 presents the percentage of the 17 company types that performed the deliveries. Among the 17 company types, the courier, food, private, office products, rental, and shredding companies have the largest presence. We focus only on first five prominent company types and ignore shredding companies mainly because shredding companies do not have a choice of parking legally as they are obliged by law to be on site (close to their destination) when shredding documents. The relative dwell time frequency of the five company types is illustrated in Fig. 4-3 and the mean and variance of the dwell times are presented in
Table 4-2. The dwell times are assumed to follow an exponential distribution\textsuperscript{10}. All of the following analysis is performed in Matlab R2014.

The following costs are chosen for the case study. The cost of inspection $\beta$ is set to 15 dollars\textsuperscript{11} per enforcement unit per hour and the marginal social cost per each illegal CV (i.e. $\delta$) is set to 250 dollars per illegal CV per hour. To obtain the walking cost $w$, we assume an average walking distance of 200 meters, a walking speed of 5 km per hour, and a cost of 30 dollars per each hour of delay for each carrier.

![Figure 4-2: Toronto’s central business district.](image)

\textsuperscript{10} A Chi-squared goodness-of-fit test is conducted with a 90\% confidence interval. All dwell times fit the exponential distribution without violating the null hypothesis of the test.

\textsuperscript{11} All monetary units are in Canadian Dollars.
<table>
<thead>
<tr>
<th>Company Type</th>
<th>Delivery Percentage</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Courier</td>
<td>30%</td>
<td>Includes both corporate and smaller companies that offer package or mail deliveries</td>
</tr>
<tr>
<td>Food</td>
<td>24%</td>
<td>Any catering company or wholesale food supplier or company specializing in utensils and tools related to food</td>
</tr>
<tr>
<td>Private</td>
<td>8%</td>
<td>No visible label on the vehicle or dashboard of the associated company</td>
</tr>
<tr>
<td>Shredding</td>
<td>8%</td>
<td>On-site shredding and data processing companies</td>
</tr>
<tr>
<td>Office Products</td>
<td>6%</td>
<td>Office supplies such as paper, toner cartridges necessary for daily operations</td>
</tr>
<tr>
<td>Rental</td>
<td>4%</td>
<td>Any vehicle labeled with a freight rental company name</td>
</tr>
<tr>
<td>Secure Courier</td>
<td>3%</td>
<td>High security couriers employing armored trucks</td>
</tr>
<tr>
<td>Building Material</td>
<td>2%</td>
<td>Deliver or provide construction materials such as steel</td>
</tr>
<tr>
<td>Cartage</td>
<td>2%</td>
<td>Operators of large transport trucks specializing in LTL or TL deliveries</td>
</tr>
<tr>
<td>Cleaner</td>
<td>2%</td>
<td>Laundromat companies that deliver clean or pick up dirty office clothes</td>
</tr>
<tr>
<td>Electronics</td>
<td>2%</td>
<td>Any company that deliver electronic products</td>
</tr>
<tr>
<td>Furniture</td>
<td>2%</td>
<td>Office furniture such as chairs and desks</td>
</tr>
<tr>
<td>Other</td>
<td>2%</td>
<td>Other company types</td>
</tr>
<tr>
<td>Plants</td>
<td>2%</td>
<td>Decorative plants and flowers</td>
</tr>
<tr>
<td>Sanitation</td>
<td>1%</td>
<td>Sanitation products such as hand cleanser and toilet paper</td>
</tr>
<tr>
<td>Waste and Recycling</td>
<td>1%</td>
<td>Waste collection and disposal or recycling companies</td>
</tr>
<tr>
<td>Mover</td>
<td>1%</td>
<td>Office moving or relocating companies</td>
</tr>
</tbody>
</table>
Figure 4-3: Relative dwell time frequency of the main company types.

Table 4-2: Mean dwell time of company types.

<table>
<thead>
<tr>
<th>Company Type</th>
<th>Dwell Time Mean (minutes)</th>
<th>Dwell Time Variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Courier</td>
<td>9.86</td>
<td>84.33</td>
</tr>
<tr>
<td>Food</td>
<td>15.91</td>
<td>99.21</td>
</tr>
<tr>
<td>Office Products</td>
<td>12.75</td>
<td>129.12</td>
</tr>
<tr>
<td>Private</td>
<td>9.34</td>
<td>51.45</td>
</tr>
<tr>
<td>Rental</td>
<td>9.65</td>
<td>39.05</td>
</tr>
</tbody>
</table>

Let us assume that meeting rate is obtained from a Cobb-Douglas type meeting function as is customary in the bilateral meeting literature (Varian, 1992, Yang et al., 2010, Yang and Yang, 2011). This function has the following form

$$M(N^v, N^e) = A(N^e)^{\gamma_1}(N^v)^{\gamma_2}$$

(34)

where $A$ is a positive function parameter that depends on the spatial characteristics of the market and it can be negatively related to the size of the searching and meeting areas. As already mentioned, $\gamma_1$ and $\gamma_2$ are the elasticities of the meeting function with respect to $N^e$ and $N^v$, respectively, and we have $0 < \gamma_1, \gamma_2 \leq 1$. 
Consider the following three scenarios. In Scenarios I, II, and II, we set $\gamma_1, \gamma_2 = 0.3, \gamma_1, \gamma_2 = 0.5$, and $\gamma_1, \gamma_2 = 0.7$, respectively. Increasing $\gamma_1$ and $\gamma_2$ (when moving from Scenario I to Scenario III) is indicative of improved inspection technology. For each of the three scenarios, we have computed the city’s profit ($PR$), social cost ($SC$), the meeting rate ($m$), and the number of illegal CVs ($N^w$). Fig. 4-4 illustrates the contours of the city’s profit for a number of policies at each of the three scenarios. Each policy is defined by a given citation fine $f$ and level of enforcement $N^e$. The horizontal axis on each panel of Fig. 4-4 represents the citation fine measured in dollars and the vertical axis represents the level of enforcement. The optimal policy for maximizing $PR$ is shown by an “X” (which occurs at the peak of the contours) in each of the three scenarios. Fig. 4-4 shows that moving from Scenario I to Scenario III reduces the optimal citation fine $f$ and enforcement $N^e$ as a result of improved inspection and the higher meeting rate elasticities. However, the maximum profit remains approximately constant at 800 dollars per hour indicating that despite the improvement in the inspection technology, the reactive behaviour of the CVs at each scenario defuses the potential of increasing the profit.
Figure 4-4: Profit (Dollars) as a function of the citation fine and enforcement at three scenarios.

The social cost contours associated with each scenario are illustrated in Fig. 4-5. The optimal policy for minimizing $SC$ is shown by an “X” sign (which occurs at the peak of the contours) in each of the three scenarios. In Scenario I, the city has to increase the citation fine substantially up to 500 dollars to compensate for the inefficient inspection as a result of a low $\gamma_1$ and $\gamma_2$. The increased citation fine discourages CVs from parking illegally without having to increase enforcement too much. Scenarios II and III, however, require a lower citation fine of 250 and 160 dollars per hour, respectively, due to improved inspection technology.
The number of illegal CVs at each scenario is illustrated in Fig. 4-6 where it is shown that an increase in the citation fine or enforcement lowers $N^p$ as a result of the increased expected cost of parking illegally. Fig. 4-6 also shows that Scenario III, compared to the other two scenarios, has a lower $N^p$ under all policies as a result of more efficient inspection. The meeting rate of the policies under each scenario is illustrated in Fig. 4-7 where it is shown that Scenario III has the highest meeting rate due to the larger meeting function elasticities. It is also evident from Fig. 4-7 that the meeting rate is positively related to enforcement and negatively related to the citation fine, thus validating the results of the comparative static effects of Section 4.3.4.

Figure 4-5: Social cost (Dollars) as a function of the citation fine and enforcement at three scenarios.
We now perform sensitivity analysis on the meeting function elasticities $\gamma_1$ and $\gamma_2$. In light of this, we assume a fixed and given citation fine of $250 with a total of 20 enforcement units. The $\gamma_1$ and $\gamma_2$ values are both increased from 0 to 1. Profit, social cost, the meeting rate, and the number of illegal CVs are computed for each pair $(\gamma_1, \gamma_2)$ and the results are illustrated in Fig. 4-8.

Figure 4-6: Number of illegal CVs as a function of the citation fine and enforcement at three scenarios.
We partition the elasticity space into three collectively exhaustive and mutually exclusive sets where $\gamma_1$ and $\gamma_2$ are both small, both large, and neither small nor large. Let us first analyze the part of the elasticity space where the $\gamma_1$ and $\gamma_2$ are both small. In this region, the meeting rate is small (Fig. 4-8A), due to the low values of $\gamma_1$ and $\gamma_2$, even though there are a lot of illegal CVs (Fig. 4-8B). Given the city’s inability to cite the illegal CVs due to the small meeting rate, very little revenue is generated from illegal parking and profit becomes small (Fig. 4-8C). For the same reason, the high $N^e$ leads to a large social cost due the induced congestion on through traffic (Fig. 4-8D). Let us now analyze the part of the elasticity space where the $\gamma_1$ and $\gamma_2$ are both large. When $\gamma_1$ and $\gamma_2$ are both large, the meeting rate is high due to its high sensitivity to $N^e$ and $N^v$ (Fig. 4-8A). The high meeting rate discourages illegal parking which leads to a low $N^v$ (Fig. 4-8B) and a low profit as there are not enough illegal parked CVs to be cited (Fig. 4-8C). Analysis of the final segment of the elasticity space where $\gamma_1$ and $\gamma_2$ are neither low nor high shows that the highest profit is generated in this region since there are enough illegal CVs to be cited and the inspection technology is efficient enough to find and cite them.
Figure 4-7: Meeting rate (vehicles per hour) as a function of the citation fine and enforcement at three scenarios.
Figure 4-8: Profit (Dollars), social cost (Dollars), illegal commercial vehicles (vehicles), and the meeting rate (vehicles per hour) as functions of meeting elasticity.

Sensitivity analysis on the cost of an enforcement unit is presented in Fig. 4-9. Social cost is shown to first decrease and then increase with the number of enforcement units. The initial decrease occurs because the enforcement units deter CVs to park illegally, thus lowering the delay cost on through traffic and consequently the social cost. The latter increase occurs because the cost of acquiring enforcement units no longer justifies the benefits of having fewer illegally parked CVs. Fig. 4-9 also shows that the optimal number of enforcement units (which occurs at the minimum social cost) decreases with $\beta$, thus indicating that more enforcement units should be acquired when they are inexpensive.
Figure 4-9: Impact of the enforcement cost on social cost.

We now analyze the three defined markets of Section 4.4. Fig. 4-10 illustrates the iso-profit and iso-social cost contours for a number of policies with the assumption that \( \gamma_1 = \gamma_2 = 0.4 \). The equilibrium location of each market is identified in Fig. 4-10. The monopoly solution occurs at the policy with the maximum \( PR \) and the first best solution occurs at the policy with the minimum SC. Fig. 4-10 also identifies all policies at which the profit is nil (i.e. \( PR = 0 \)). As mentioned earlier, the second best solution is a point on the \( PR = 0 \) line at which the social cost is minimal (Fig. 4-10). The second-best solution for this example is dominated by the first-best solution which leads simultaneously to a higher profit and social welfare. In addition to the three markets, the Pareto front of the two objective functions is illustrated in Fig. 4-10 as well.
4.6 Conclusions

This chapter investigates how rational carriers react to parking enforcement policies under steady state equilibrium conditions. In modelling the equilibrium, this chapter uses the concept of bilateral searching and meeting to capture the inherent friction in how parking enforcement units find illegally parked commercial vehicles. The use of the meeting function is helpful in capturing CV illegal parking sensitivity with respect to parking dwell time, level of enforcement, citation fine, and citation probability. The chapter also introduces two objective functions for policy-making along with three parking enforcement market regimes. Sensitivity analysis is performed on a case study of the City of Toronto and the three market regimes are analyzed as well. Results show that the citation probability increases with dwell time and the level of enforcement. Increasing either of the citation fine or enforcement will hinder illegal parking but the obtained profit remains approximately constant. Sensitivity analysis on the meeting rate elasticity shows that profits are low when both elasticities are either high or low.

The City of Toronto has made major changes in its parking enforcement policy in 2015 by changing its previous revenue-based enforcement regime to one that is targeted at minimizing congestion especially during rush hours (CBC, 2015). Under the new policy, the citation fine has increased from $60 to $150 and intensive towing is implemented at the cost of the drivers –
commercial vehicles, for instance, are charged $1000 for retrieval. This new enforcement blitz makes it substantially more difficult for courier and other commercial vehicles, which are under strict time windows, to complete their deliveries. The higher cost of delivery is in many cases considered the “cost of doing business” which eventually turns into higher user service costs and potentially lower social welfare. The prevalent tradeoffs that are present in any parking enforcement policy necessitate the need for models that quantify the impacts of parking enforcement policies. This chapter presented a first attempt of quantifying the influential factors in parking enforcement to find the optimal policy.

Appendix 4-A

The following lemmas are prepared as proofs of the relationships presented in Section 4.3.4.

**Lemma 2:** For all \( \tilde{d}_i \), we have \( \frac{\partial N^v}{\partial \tilde{d}_i} > 0 \).

**Proof:**

By taking the derivative of Eq. 10, we have \( \frac{\partial N^v}{\partial \tilde{d}_i} = \tilde{d}_i T_i g_i(\tilde{d}_i) \) which is a positive value since \( \tilde{d}_i, T_i, g_i(\tilde{d}_i) > 0 \). This completes the proof.

**Lemma 3:** For all \( m \), we have \( \frac{\partial \tilde{d}_i}{\partial m} < 0 \).

**Proof:**

\[ \frac{\partial \tilde{d}_i}{\partial m} = \ln \left( 1 - \frac{w_i}{f} \right) / m^2 \] (derivative of Eq. 13) is a negative value. This completes the proof.

**Lemma 4:** For all \( m \), we have \( \frac{d N^v}{m} < 0 \).

**Proof:**

\( \frac{d N^v}{m} \) can be rewritten as

\[ \frac{\partial N^v}{\partial m} = \sum_i \frac{\partial N^v}{\partial \tilde{d}_i} \cdot \frac{\partial \tilde{d}_i}{\partial m} \] (35)
where \( \frac{\partial N^v}{\partial d_i} = \ddot{d}_i T_i g_i(\ddot{d}_i) \) is a positive value according to Lemma 2 and \( \frac{\partial d_i}{\partial m} = \ln \left(1 - \frac{w_i}{f}\right)/m^2 \) is a negative value according to Lemma 3, thus making \( \frac{\partial N^v}{\partial m} \) negative. This completes the proof.

**Lemma 5:** For all \( N^e \), we have \( \frac{dN^v}{dN^e} < 0 \).

**Proof:**

Let \( \frac{dN^v}{dN^e} \) be presented as:

\[
\frac{dN^v}{dN^e} = \frac{dN^v}{dm} \frac{dm}{dN^e}
\]  

(36)

where \( \frac{dm}{dN^e} \) (last term on the RHS of Eq. 36) is

\[
\frac{dm}{dN^e} = \frac{\partial m}{\partial N^e} + \frac{\partial m}{\partial N^v} \cdot \frac{dN^v}{dN^e}
\]  

(37)

Substituting Eq. 36 into Eq. 37 gives:

\[
\frac{dN^v}{dN^e} = \frac{\partial N^v}{\partial m} \frac{dm}{dm} / \left(1 - \frac{\partial N^v}{\partial m} \frac{dm}{dN^e}\right)
\]  

(38)

Note that \( \frac{\partial m}{\partial N^e} = \frac{m y_1}{N^e} \) and \( \frac{\partial m}{\partial N^v} = \frac{m y_2}{N^v} \) are both positive in Eq. 38. Hence by showing that \( \frac{\partial N^v}{\partial m} < 0 \), which is proved in Lemma 4, it is clear that \( \frac{dN^v}{dN^e} < 0 \) for all \( N^e \). This completes the proof.

**Lemma 6:** For all \( N^e \), we have \( \frac{dm}{dN^e} > 0 \).

**Proof:**

With some simplifications, \( \frac{dN^v}{dN^e} \) can be calculated as:

\[
\frac{dN^v}{dN^e} = \frac{-k y_1}{(1 + k y_2 / N^e) N^e}
\]  

(39)

where \( k = \sum_i T_i g_i(\ddot{d}_i)\ddot{d}_i^2 > 0 \). Eq. 39 is consistent with Lemma 5 since it is strictly negative.

We now show through Lemma 6 the impact of enforcement on the meeting rate \( m \).
By substituting Eq. 39 into Eq. 37, we have:

\[ \frac{dm}{dN^e} = \frac{my_1}{(1 + \frac{k}{N^e})N^e} \]  

(40)

Given that \( k > 0 \), it is obvious from Eq. 40 that \( \frac{dm}{dN^e} > 0 \). This completes the proof.

**Lemma 7:** For all \( f \), we have \( \frac{dm}{df} < 0 \).

**Proof:**

Let \( \frac{dm}{df} \) be presented as

\[ \frac{dm}{df} = \frac{\partial m}{\partial N^v} \frac{dN^v}{df} \]  

(41)

where \( \frac{dN^v}{df} \) (the second term on the RHS of Eq. 41) may be written as

\[ \frac{dN^v}{df} = \sum_i \frac{dN^v}{d\bar{d}_i} \frac{d\bar{d}_i}{df} \]  

(42)

Substituting Eq. 41 into Eq. 42 gives

\[ \frac{dm}{df} = \frac{\partial m}{\partial N^v} \sum_i \frac{dN^v}{d\bar{d}_i} \frac{d\bar{d}_i}{df} \]  

(43)

where \( \frac{d\bar{d}_i}{df} \) (the second term on the RHS of Eq. 43) is

\[ \frac{d\bar{d}_i}{df} = - \left[ \frac{dm}{df} \bar{d}_i + \frac{a(\bar{d}_i)}{f(1 - \alpha(\bar{d}_i))} \right] / m \]  

(44)

Substituting Eq. 44 into Eq. 43 gives:

\[ \frac{dm}{df} = \frac{\frac{\partial m}{\partial N^v} \sum_i \frac{dN^v}{d\bar{d}_i} \frac{a(\bar{d}_i)}{d\bar{d}_i f(1 - \alpha(\bar{d}_i))}} {m + \frac{\partial m}{\partial N^v} \sum_i \frac{dN^v}{d\bar{d}_i} \frac{d\bar{d}_i}{df}} \]  

(45)

which is a negative number for all \( f \). This completes the proof.
Lemma 8: For all $f$, we have $\frac{dN^p}{df} < 0$.

Proof:

Let $\frac{dN^p}{df}$ be presented as

$$\frac{dN^p}{df} = \frac{dN^p}{dm} \frac{dm}{df} \quad (46)$$

According to Lemma 4, we have $\frac{dN^p}{m} < 0$ and according to Lemma 7 we have $\frac{dm}{df} < 0$. Hence, it is clear that $\frac{dN^p}{df} < 0$. This completes the proof.

4.7 References


Chapter 5
Short-Term Planning

Impact of Hourly Parking Pricing on Travel Demand

5.1 Introduction

5.1.1 Motivation

Efficient parking management strategies are vital in Central Business Districts (CBDs) of cities where space is restricted and congestion is intense. A great deal of parking demand in these regions is generated by travellers who visit their destination for some specified period (called dwell time) before returning to their origin location (Anderson and de Palma, 2007). To find parking for these activities (e.g., shopping activities), travellers incur a cost comprised of traveling to a chosen parking area, searching for a spot, paying the parking price, and walking to the final destination.

In day-to-day equilibrium conditions or in the presence of information systems such as mobile apps, travellers adjust their travel patterns to minimize their experienced costs. This adjustment includes choosing an affordable parking area in the vicinity of the final destination. Parking areas are underground or multi-floor parking garages, surface lots, or a collection of on-street parking spots. They can be public or private and generally require a parking fee with a fixed price (e.g. $3 for entrance) and an hourly price (e.g. $0.5 per hour). The hourly price plays a key role in parking management. Its impact on parking demand is twofold. First, increasing the hourly price of a parking facility increases user costs and explicitly reduces demand. Second, the same increase in the hourly price motivates travellers to shorten their dwell time which leads to lower parking occupancy and searching time (to find a spot), but higher demand. Hence, the hourly parking price influences travel demand in two counterbalancing ways. This chapter investigates the impact of hourly parking pricing on traffic equilibrium conditions and parking search time. We show that, despite intuition, hourly parking pricing can actually increase demand if imposed imprudently.

Parking pricing has long been advocated and deployed as a policy to reduce congestion. Whereas a wealth of research is dedicated to parking pricing in areas such as mall parking services
(Change et al., 2014), commercial vehicle parking (Marcucci et al., 2014; Nourinejad et al., 2014; Nourinejad and Roorda, 2016; Amer and Chow, 2016), curbside parking (Millard-Ball et al., 2014), private firm parking (Tsai and Chu, 2006; Nourinejad and Roorda, 2014), morning commute parking (Qian et al., 2011; Liu et al., 2014; Liu and Geroliminis, 2016), dynamic parking pricing (Zheng and Geroliminis, 2016; Zakharenko, 2016), and parking permits (Rosenfield et al., 2016), fewer studies have investigated the role of hourly pricing as a travel demand management (TDM) policy. Among the few, Glazer and Niskanen (1992) showed that if roads are sub-optimally priced or not priced at all, then fixed parking pricing can increase welfare but hourly pricing may not. Glazer and Niskanen (1992) conclude that, despite intuition, an increase in the hourly parking price will induce demand because more parking spaces become available as drivers shorten their dwell times. In this chapter, we show that this is not always the case by proving that the changes in demand (with respect to the hourly price) are sensitive to the elasticity of parking dwell time (with respect to the hourly price). Thereafter, we define three market regimes, investigate the equilibrium conditions, and present a network-based model to capture the spatial effects of hourly parking pricing.

5.1.2 Background

Parking studies are broadly categorized based on modelling framework, search mechanism, and turnover. The two main modelling frameworks are simulation and analytic formulations. Simulations capture complex dynamics of parking but require detailed data for calibration. Often, lack of sufficient data is accommodated by applying behavioural assumptions which are mostly inconsistent among different studies (Benenson et al., 2008; Gallo et al., 2011; Nourinejad et al., 2014). In Benenson et al. (2008), for instance, vehicles relinquish their on-street parking search after some time threshold (10 minutes) and head for off-street parking instead. In Nourinejad et al. (2014), on the other hand, vehicles start to search for parking when within 500 meters of their final destination. In comparison, analytical models, with a few exceptions, are less data-hungry and more insightful but are generally aggregate and not amenable to detailed results (Arnott and Inci, 2006; Arnott and Rowse, 1999; Anderson and de Palma, 2004). In Arnott and Inci (2006), for instance, a parking model is developed for downtown areas with equal-sized blocks and a constant demand over the region. Although aggregate, the model provides very useful insights such as showing that it is efficient to raise the on-street parking fee to the point where cruising for parking is eliminated without parking
becoming unsaturated. More recently, there is growing advocacy for network-based analytical and traffic assignment models that allow for a finer level of policy support. Boyles et al. (2014) and Qian and Rajagopal (2014) formulate equilibrium models to assign vehicles to spatially disaggregate parking areas.

Searching mechanisms are either zone-based or link-based. In zone-based searching, vehicles only start searching for a spot when they reach a zone and each zone is associated with a search time which is assumed to be a function of the zone’s occupancy (i.e., number of occupied parking spots) (Qian and Rajagopal, 2014). Applications of zone-based searching are not limited to parking. In taxi equilibrium models, taxi drivers search for passengers in different zones and incur a searching cost which is generally assumed to be a function of the total number of searching taxis and passengers in that zone. Taxi searching time is usually lower when there are more passengers and fewer taxis in the zone (Yang and Wong, 1998; Yang et al., 2002; Yang et al., 2010a; Yang et al., 2010b). In link-based searching, vehicles search for a spot in any of the links that are on their route to a final destination zone. One of the interesting implications of a link-based search model, as is shown in Boyles et al. (2014), is the smooth transition of vehicles from “driving” to “searching for parking” which is inherent in the equilibrium structure of the model.

Parking studies are classified into zero and non-zero turnover rate models. Turnover refers to the rate at which vehicles leave a parking area. Hence, zero turnover parking indicates that vehicles only enter parking areas without leaving. This type of parking is common in the morning commute context where the major concern is the dynamic arrival pattern of vehicles at the parking zones. These studies are usually defined for stylized settings such as a single bottleneck linear city (Zhang et al., 2008; Qian et al., 2012) or a parallel bottleneck city with several corridors (Zhang et al., 2011). A more general network-based zero turnover model is developed by Qian and Rajagopal (2014). Non-zero turnover models, on the other hand, are suitable for short duration activities such as shopping. These models capture both the arrival and departure rate of vehicles from each parking area. Under steady-state conditions, the arrival rate is equal to the departure rate from each parking area (Arnott, 2006; Arnott and Rowse, 2009; Arnott an Inci,

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12 By zone, we refer to either an off-street parking lot or a collection of on-street parking spaces.
2010; Arnott and Rowse, 2013; Arnott, 2014; Arnott et al., 2015). In non-zero turnover simulation models such as Guo et al. (2013) and Nourinejad et al. (2014), the sum of vehicles entering and leaving each parking area are assumed to be equal.

The policy implications of parking have also been the subject of many studies (Inci, 2014). Among the more innovative ones are parking permit schemes that involve distributing a fixed number of permits among travellers and restricting vehicles to spend the permits for parking (Zhang et al., 2011; Liu et al., 2014). He et al. (2015) study the optimal assignment of vehicles to parking spots while considering the competition game between the vehicles. They show the existence of multiple equilibria and propose a robust pricing scheme. Qian and Rajagopal (2014) study parking pricing strategies using real-time sensors to manage parking demand. Using parking pricing and information provision systems, Qian and Rajagopal (2014) propose a dynamic stabilized controller to minimize the total travel time in the system. Parking prices are then adjusted in real-time according to occupancy information collected from parking sensors. Finally, Xu et al. (2016) consider a policy where private parking slots can be shared between a pool of drivers.

5.1.3 Contributions and organization

In this chapter, we present a non-zero turnover, zone-based search, analytical model for parking. Given the non-zero turnover rate, we consider both arrival and departure rates of vehicles to parking areas which are assumed to be equal under steady-state conditions. Our model is therefore distinguished from Qian and Rajagopal (2014) which is a zero turnover model. Contrary to Boyle et al. (2014), we use the zone-based search mechanism which, due to its simplicity, helps derivation of the analytical results and improves policy evaluation. The presented model is also distinguished from the analytical models of Arnott and Inci (2006), Arnott (2014), and Arnott et al. (2015) since it investigates parking patterns at a network level. Although our analytical findings are only derived for a simple case study, we show through numerical experimentation that are results are generalizable to larger networks as well.

We particularly focus on unassigned parking where drivers have to cruise to find a spot. These trips have shorter dwell times and tend to belong to frequent drivers. We present cases where parking supply can be varied such as in Arnott and Inci (2006) and cases where parking supply is fixed. Each parking zone has a specified capacity and can either be an off-street parking facility
or a group of on-street parking spots. The modelled network is a CBD where travellers reside far away. This assumption is previously imposed by Anderson and de Palma (2004) as well.

The remainder of this chapter is organized as follows. The model is presented in Section 5.2. Comparative statics effects of regulatory variables are investigated in Section 5.3. Equilibrium conditions are discussed in Section 5.4. Three market regimes are presented in Section 5.5. Numerical experiments are provided in Section 5.6. Conclusions are presented in Section 5.7.

5.2 The model

Nomenclature

Sets

\[ G(N, A) \] Graph with node set \( N \) and arc set \( A \)

\( N \) Set of nodes

\( R \) Set of external nodes

\( I \) Set of parking nodes

\( S \) Set of internal zones

\( A \) Set of arcs

\( A_d \) Set of driving arcs

\( A_w \) Set of walking arcs

\( V \) Set of O-D pairs

\( \Omega(r, s) \) Parking choice set of O-D pair \( (r, s) \in V \) travellers

\( \psi(r, s, i) \) Set of routes for O-D pair \( (r, s) \in V \) travellers who choose parking \( i \in \Omega(r, s) \)

Constants

\( w_b \) Walking time on walking link \( b \in A_w \)
\( k_i \) Capacity of parking \( i \in I \)

\( \Delta_{a,b} \) Link-path incidence matrix

\( \sigma_i \) Maintenance cost of one spot at parking zone \( i \in I \)

\( l_i \) Average searching time at parking \( i \in I \)

\( \mu_i \) Constant representing how drivers adopt occupancy information at parking \( i \in I \)

\( \alpha \) Marginal cost of each hour of driving time

\( \beta \) Marginal cost of each hour of parking search time

\( \gamma \) Marginal cost of each hour of walking time

\( \theta \) Dispersion parameter in the parking choice model

Decision variables

\( x_b \) Flow on link \( b \in A_d \)

\( \tau_b(x_b) \) Travel time on driving link \( b \in A_d \)

\( d^i_{rs} \) Flow of O-D pair \( (r, s) \in V \) travellers who choose parking \( i \in \Omega(r, s) \)

\( d_{rs} \) Flow of O-D pair \( (r, s) \in V \) travellers

\( d^i_{rs,a} \) Flow of O-D pair \( (r, s) \in V \) who choose parking \( i \in \Omega(r, s) \) via route \( a \in \psi(r, s, i) \)

\( d^i \) Flow of travellers into parking \( i \in I \)

\( q_i \) Occupancy of parking \( i \in I \)

\( h^i_{rs} \) Dwell time of O-D pair \( (r, s) \in V \) travellers who choose parking \( i \in \Omega(r, s) \)

\( \pi^i_{rs} \) Probability that an O-D pair \( (r, s) \) traveller chooses parking \( i \in \Omega(r, s) \)

\( \eta_{rs} \) Expected perceived travel cost of O-D pair \( (r, s) \in V \) travellers
\( D_{rs}(\eta_{rs}) \) Demand function of O-D pair \((r,s) \in V\) travellers

\( C^i_{rs} \) Observed cost of O-D pair \((r,s)\) travellers who choose parking \(i \in \Omega(r,s)\)

\( \varepsilon^i_{rs} \) Unobserved cost of O-D pair \((r,s)\) travellers who choose parking \(i \in \Omega(r,s)\)

\( p_i \) Hourly price of parking at zone \(i \in I\) measured in dollars per hour

\( \Gamma \) Feasible region of the Variational Inequality program

\( u^i_{rs} \) Lagrange multiplier associated with conservation of flow for O-D pair \((r,s)\) travellers who choose parking \(i \in \Omega(r,s)\)

\( \lambda_{rs} \) Lagrange multiplier associated with conservation of flow for O-D pair \((r,s)\)

\( \delta_i \) Lagrange multiplier associated with conservation of flow at each parking zone \(i \in I\)

\( \varphi^i_{rs,a} \) Lagrange multiplier associated with conservation of flow for O-D pair \((r,s)\) travellers who choose parking \(i \in \Omega(r,s)\) via route \(a \in \psi(r,s,i)\)

Functions

\( H_{rs}(p_i) \) Dwell time function for O-D pair \((r,s)\) at parking zone \(i \in \Omega(r,s)\)

\( F_i(q_i) \) Searching time at parking \(i \in I\)

\( PM \) Profit maximization function

\( SS \) Social surplus function

5.2.1 The network

Consider a transportation network \(G(N,A)\) with node and arc sets \(N\) and \(A\), respectively. To model parking, we further partition the node set \(N\) into external nodes denoted by \(R\), parking zones denoted by \(I\), and internal zones denoted by \(S\) so that \(N = R \cup I \cup S\). Let \(R = \{1,\ldots,r,\ldots,|R|\}, I = \{1,\ldots,i,\ldots,|I|\}\), and \(S = \{1,\ldots,s,\ldots,|S|\}\). The reason for this terminology is that external zones are located at the boundary of a region whereas internal zones are within the
study region as shown in Fig. 5-1. External zones are gateways that provide accessibility to a region and internal zones are attraction locations (e.g. a shopping center) that vehicles want to visit.

Each vehicle completes two types of trips: inbound and outbound. In the inbound trip, vehicles leave an external zone, \( r \), and drive to a parking zone, \( i \). After parking, the inbound traveller walks from the parking zone, \( i \), to an internal zone, \( s \), as is shown in Fig. 5-1a. Hence, the path of every inbound traveller includes the sequence \( r \to i \to s \). Outbound trips are the reverse direction of inbound trips. The path of every outbound vehicle includes the sequence \( s \to i \to r \).

Fig. 5-1a depicts the general inbound and outbound trip trajectories and Fig. 5-1b illustrates an example of internal and external zones where the internal zones are attraction locations in the Toronto CBD and the external zones represent the gateways to the CBD. Let us also partition the link Set \( A \) into \( A_d \) and \( A_w \) representing the driving and walking links, respectively, as is also shown in Fig. 5-1a.

![Diagram](image)

**Figure 5-1:** (a) Inbound and outbound trip trajectories; (b) Example of zone types.
The following sets are now defined. Let $V = R \times S$ be the set of external-internal zone pairs. For each pair $(r, s)$, let $\mathcal{Q}(r, s)$ be the set of parking zones that are within the parking zone choice-set of these travellers. The Set $\mathcal{Q}(r, s)$ can be defined according to features such as walking distance from parking $i$ to destination $s$ and the cost of parking. Clearly, parking zones that are too far from the internal destinations zones are less likely to be included in the choice set. For every pair $(r, s) \in V$ and parking zone $i \in \mathcal{Q}(r, s)$, let $\psi(r, s, i)$ be the set of routes for the segments of the tour that include driving links. Each route is comprised of a set of driving links connecting zone $r$ to $i$ and zone $i$ to $s$. For instance, in Fig. 5-1a, there is only one route that includes the sequences of zones $r \rightarrow i \rightarrow s$.

Let $d_{rs,a}^i$ denote the flow of vehicles belonging to pair $(r, s) \in V$ that choose parking area $i \in \mathcal{Q}(r, s)$ via route $a \in \psi(r, s, i)$. Let $x_b$ be the flow and $\tau_b(x_b)$ the travel time of driving link $b \in A_d$, and let $w_b$ be the walking time on link $b \in A_w$. It is commonly assumed that the travel time on driving link $b \in A_d$ is a continuous and monotonically increasing function of link flow $x_b$ and the travel time on walking link $b \in A_w$ is independent of the flow. Let $\Delta$ represent the path-link incidence matrix where $\Delta_{a,b} = 1$ if link $b \in A_d$ is included in route $a \in \psi(r, s, i)$ and $\Delta_{a,b} = 0$, otherwise. Hence we have $x_b = \sum_{(r,s)\in V} \sum_{i\in \mathcal{Q}(r,s)} \sum_{a\in \psi(r,s,i)} d_{rs,a}^i \Delta_{a,b}$.

### 5.2.2 The non-zero turnover parking process

The parking search process is explained in this section. First, the following assumption is imposed:

**Assumption 1:** Under equilibrium conditions travellers park at a zone with the lowest generalized cost.

Assumption 1 is justified under at least two conditions. First, if the trips are recurrently performed, travellers become familiar with the process and choose to park at a zone with the lowest generalized cost. Second, when parking information such as parking occupancy is provided to users via apparatus such as mobile apps, travellers are better informed about which zone to choose for parking. Assumption 1 implies that travellers will not hop between parking zones (i.e. they will not drive from one parking area to another) and will instead choose the one with the lowest generalized cost. That is, traveller cruise for parking at only one parking area. The cost of parking is comprised of the cost of traveling from the external zone to a parking
zone, the cost of searching for parking, the parking fee which can include both a fixed and an hourly component, the cost of walking from the parking area to the internal zone, the cost of walking from the internal zone to the parking area, and the cost of driving from the parking area to the external zone.

Using Assumption 1, we can now analyze the parking pattern of travellers. Let $d_{rs}^i, \forall (r, s) \in V, i \in \Omega (r, s)$, be the flow of vehicles that originate at zone $r$, terminate at zone $s$, and park at zone $i$, and let $d_{rs} = \sum_{i \in \Omega (r, s)} d_{rs}^i$ be the total flow from $r$ to $s$. All travellers of pair $(r, s)$ that park at $i$, remain there for $h_{rs}^i$ [hours] called the dwell time. This assumption is justified as travellers belonging to the same origin-destination pair are likely to be homogenous (Yang and Huang, 2005).

Let $q_i$ be the total occupancy of parking $i \in I$ under equilibrium and let $k_i$ be the capacity of parking $i$ measured in vehicles. Note that $k_i$ is a given whereas $q_i$ is obtained from the equilibrium:

$$q_i = \sum_{(r, s)} d_{rs}^i h_{rs}^i \quad \forall i \in I \quad (1)$$

Parking search time is typically assumed to be a convex function of parking occupancy $q_i$ and capacity $k_i$ (Axhausen et al., 1994; Anderson and de Palma, 2004; Levy et al., 2012; Qian and Rajagopal, 2014). The general form of this function $F_i(q_i)$, as explained in Axhausen et al. (1994), is:

$$F_i(q_i) = \frac{l_i \mu_i}{1 - \frac{q_i}{k_i}} \quad \forall i \in I \quad (2)$$

where $l_i$ is the average searching time in parking area $i$ when occupancy is low or medium and $\mu_i$ is a constant representing how drivers react to occupancy information. When $\mu_i = 0$, drivers are unaware of the searching time and when $\mu_i = 1$ drivers are completely aware of searching time. Axhausen et al. (1994) estimated the search time function with a coefficient of determination $R^2 = 0.91$ for Frankfurt, Germany. The searching time function $F_i(q_i)$ asymptotically goes to infinity as $q_i$ approaches $k_i$, i.e., $\lim_{q_i \to k_i} F_i(q_i) = \infty$. This implies that a driver entering a full occupancy parking facility will never find a spot.
5.2.3 Generalized travel costs

The hourly price of parking at $i \in I$ is $p_i$ [dollars per hour]. Hence, for the pair $(r, s)$ a traveller who chooses parking $i$ pays $p_i h^i_{rs}$ dollars for parking. We can now derive the generalized travel costs. Let $C^i_{rs,a}$ be the generalized travel cost for travellers of pair $(r, s)$ who choose parking $i \epsilon \Omega(r, s)$ via route $a \epsilon \psi(r, s, i)$. This cost is composed of the following six terms: (i) traveling from external zone $r$ to parking $i$ via route $a$ with a travel time $t^i_{ri,a}$, (ii) searching for parking for a period of $F_i(q_i)$, (iii) a parking cost of $p_i h^i_{rs}$ dollars, (iv) walking from parking $i$ to zone $s$, (v) walking from zone $s$ to parking $i$, and (vi) traveling from parking $i$ to external zone $r$ via route $a$:

$$C^i_{rs,a} = \alpha t^i_{ri,a} + \beta F_i(q_i) + p_i h^i_{rs} + \gamma w_{is} + \gamma w_{si} + \alpha t^i_{ir,a}$$

$\forall (r, s) \in W, \forall i \in \Omega (r, s), \forall a \in \psi (r, s, i)$ (3)

In Eq. 3, $\alpha$, $\beta$, and $\gamma$ are the marginal cost of travel time, parking search time, and walking time, respectively. For the first term on the right-side of Eq. 3, we have $t^i_{ri,a} = \sum_b \tau^i_{b} \Delta_{a,b}$.

The minimum cost of the shortest route for a traveller of pair $(r, s) \in V$ that parks at zone $i \in \Omega (r, s)$ is $C^i_{rs} = \min_{a \in \psi (r, s, i)} C^i_{rs,a}$. However, $C^i_{rs}$ only represents the observed cost of travel. Let us also assume an additional unobserved cost of $\varepsilon^i_{rs}$ which is independently and identically Gumbel distributed for all parking zones $i \in I$ that can be chosen by travellers of pair $(r, s)$. With this assumption, the probability that a pair $(r, s) \in V$ traveller chooses parking $i \in \Omega (r, s)$ is denoted by $\pi^i_{rs}$ which is obtained from the following logit-based probability function:

$$\pi^i_{rs} = \frac{\exp(-\theta C^i_{rs})}{\sum_{j \epsilon \Omega (r, s)} \exp(-\theta C^j_{rs})}$$

$\forall i \in \Omega (r, s)$ (4)

where $\theta$ is a dispersion parameter representing the variation in the cost perception of travellers.

Eq. 4 relies on the following assumption:

**Assumption 2:** Travellers are stochastic in choosing a parking area but deterministic in choosing routes. This assumption is justified due to the availability and accuracy of route-guidance advanced traveller information systems.
We also assume that travel demand is a continuous and decreasing function of the generalized travel cost. The demand function is denoted by \( D_{rs}(\cdot) \) and the expected, generalized travel cost is denoted by \( \eta_{rs} \) for each \((r, s) \in V\). Hence, we have:

\[
d_{rs} = D_{rs}(\eta_{rs}) \quad \forall (r, s) \in W \tag{5}
\]

Given the logit-based parking choice model in Eq. 4, the expected minimum cost for each \((r, s) \in V\) is:

\[
\eta_{rs} = E\left(\min_{i \in \Omega(r,s)} \{C^i_{rs}\}\right) = -\frac{1}{\theta} \ln\left(\sum_{i \in \Omega(r,s)} \exp(-\theta C^i_{rs})\right) \quad \forall (r, s) \in W \tag{6}
\]

### 5.2.4 Parking dwell time

Recall that parking dwell time \( h^i_{rs} \) is the time spent by \((r, s)\) travellers at parking zone \( i \in \Omega(r, s)\). The following assumption is now imposed:

**Assumption 3:** The dwell time of pair \((r, s)\) travellers at parking zone \( i \in \Omega(r, s)\) is assumed to be a function of the hourly parking cost \( p_i \). As \( p_i \) increases, dwell time decreases.

Let \( H_{rs}(p_i) \) denote this function which is assumed to be convex and monotonically decreasing with \( p_i \). It is also sound to assume that dwell time approaches zero as \( p_i \) tends to infinity, i.e.

\[
\lim_{p_i \to \infty} H_{rs}(p_i) = 0.
\]

The dwell time function is:

\[
h^i_{rs} = H_{rs}(p_i) \quad \forall (r, s) \in W, \forall i \in \Omega(r, s) \tag{7}
\]

We now investigate the impact of the hourly price, \( p_i \), on the out-of-pocket cost of parking, \( p_i h^i_{rs} \). The hourly parking price, \( p_i \), may or may not increase \( p_i h^i_{rs} \). If \( h^i_{rs} \) decreases slowly with \( p_i \), then the term \( p_i h^i_{rs} \) increases with \( p_i \), thus showing that the travellers pay more when the hourly parking price is increased. On the other hand, if \( h^i_{rs} \) decreases rapidly with \( p_i \), then travellers pay less when \( p_i \) is increased.

### 5.3 Comparative static effects of parking pricing

We use comparative static effects (Fuente, 2000) to show that road pricing and parking fares are structurally different in how they influence the traffic equilibrium. Whereas road pricing reduces
demand, hourly parking pricing may reduce or induce demand. Mathematically, we have $\frac{dD}{d\hat{p}} < 0$ where $\hat{p}$ is the road toll, whereas $\frac{dD}{dp} > 0$ or $\frac{dD}{dp} < 0$ where $p$ is the hourly parking price and $D$ is the demand function. Consider the network of Fig. 5-1a with one origin $r$, one destination $s$, and one parking area $i$. A road toll $\hat{p}$ is imposed on the driving link $(r, i)$ and an hourly parking price $p$ is imposed on parking area $i$. The demand function is defined such that demand decreases with generalized cost, i.e., $\frac{dD(\eta)}{d\eta} < 0$. For the remainder of this section, we drop the subscripts $r, i,$ and $s$ for brevity.

The following two lemmas demonstrate the changes in demand with respect to the road toll, $\hat{p}$, and the hourly parking price, $p$.

Lemma 1: Demand strictly decreases with the road toll, i.e., $\frac{dD}{d\hat{p}} < 0$.

Proof:

Let us rewrite $\frac{dD}{d\hat{p}}$ as

$$\frac{dD}{d\hat{p}} = \frac{dD}{d\eta} \frac{d\eta}{d\hat{p}}. \tag{8}$$

It is already assumed that $\frac{dD}{d\eta} < 0$ as demand decreases with the generalized cost. It is also evident that $\frac{d\eta}{d\hat{p}} > 0$ because $\hat{p}$ is the out-of-pocket money paid by travellers to traverse road $(r, i)$. Hence, the product of the two terms on the RHS of Eq. (8) is negative and $\frac{dD}{d\hat{p}} < 0$. □

Lemma 2: Changing the hourly parking price, $p$, may induce or reduce demand, i.e., $\frac{dD}{dp} < > 0$.

Proof:

Let us rewrite $\frac{dD}{dp}$ as

$$\frac{dD}{dp} = \frac{dD}{d\eta} \frac{d\eta}{dp}. \tag{9}$$
It is already assumed that \( \frac{dD}{d\eta} < 0 \) as demand decreases with the generalized cost. Hence, we focus on the second term on the RHS of Eq. (9). By taking the derivative, \( \frac{d\eta}{dp} \), we have

\[
\frac{d\eta}{dp} = \frac{d(Hp)}{dp} + \frac{dF}{dp}.
\] (10)

By taking the derivative, \( \frac{dF}{dp} \), the second term on the RHS of Eq. (10) can be rewritten as

\[
\frac{dF}{dp} = \frac{\mu\left[(dH/dp)D+(dD/dp)H\right]}{k(1-Hd/k)^2}.
\] (11)

By inputting Eq. (11) into Eq. (10), inputting Eq. (10) into Eq. (9), and simplifying the terms, we have

\[
\frac{dD}{dp} = \frac{dD}{d\eta} \left[ \frac{(d(Hp)/dp)+\omega(dH/dp)D}{1-\omega(Hd/d\eta)} \right]
\] (12)

where \( \omega = \mu/\left[k(1-HD/k)^2\right] > 0 \). Analysis of Eq. (12) concludes the following:

\[
\frac{dD}{dp} > 0 \quad if \quad D > D^* \quad (13a)
\]

\[
\frac{dD}{dp} < 0 \quad if \quad D < D^* \quad (13b)
\]

where \( D^* = \frac{-(dHp/dp)}{\omega (dH/dp)} \). Eq. (13) shows that marginal change of demand with respect to the hourly parking price, \( p \), depends on the value of the materialized demand, \( D \). If \( D > D^* \), then travel demand, \( D \), increases with the hourly parking price, \( p \), and if \( D < D^* \), then travel demand, \( D \), decreases with the hourly parking price, \( p \). ■

Lemma 1 and 2 show that although road pricing reduces demand, variable parking pricing can reduce or induce demand. This implies that parking pricing is not always an appropriate policy for lowering congestion. Lemma 2 also has the following remark:

Remark 1: The hourly parking price has the same effect as the road toll when travellers’ dwell time is insensitive to the hourly parking price.

Proof:
When traveller dwell time time is insensitive to the hourly parking cost (i.e. \( dH/dp \to 0 \)), we have \( D^* = \frac{-H}{\omega (dH/dp)} \to \infty \) which, according to Eq. (13b) indicates, that demand, \( D \), strictly decreases with hourly parking price, \( p \). In other words, when \( dH/dp \to 0 \), the hourly parking price has a similar impact on demand as a road toll. ■

Let \( e_p^h = \frac{\partial H}{\partial p} p \) be the elasticity of dwell time with respect to hourly parking price. By definition, dwell time is elastic when \( e_p^h \leq -1 \) and inelastic when \( -1 < e_p^h \leq 0 \). We now summarize the findings of this section in the following proposition:

Proposition 1: Demand increases with hourly parking price (i.e., \( \frac{dD}{dp} > 0 \)) when dwell time is elastic (i.e., \( e_p^h \leq -1 \)). However, when dwell time is inelastic (i.e., \( -1 < e_p^h \leq 0 \)), demand may increase or decrease with hourly parking price (i.e., \( \frac{dD}{dp} <> 0 \)).

Proof:

Given that \( e_p^h = \frac{\partial H}{\partial p} p \) and \( \frac{\partial (pH)}{\partial p} = (1 + e_p^h)H \), we can rewrite \( D^* \) in Eq. (13) as \( D^* = -p(1 + e_p^h)/\omega e_p^h \). When dwell time is inelastic (i.e., \( -1 < e_p^h \leq 0 \)) we have \( D^* > 0 \) indicating that the demand both increases and decreases with the hourly parking price, \( p \). However, when dwell time is elastic (i.e., \( e_p^h \leq -1 \)), we have \( D^* \leq 0 \) which, according to Eq. (13), indicates that demand strictly increases with hourly parking price, \( p \). ■

5.4 Equilibrium conditions

5.4.1 A variational inequality formulation

In this section, we formulate the equilibrium problem using Variational Inequality (VI). The idea behind VI is to find an equilibrium point (say vector \( d^* \)) within a feasible (closed and compact) solution space such that for all other points (say \( d \)) in the solution space, we have \( Z(d^*)^T (d - d^*) \geq 0 \), where \( Z \) is a continuous function. Informally, \( d^* \) is a point in the feasible region that does not bear any force from the function \( Z \). For novel applications of VI, see Wong et al. (2008) which presents a taxi equilibrium model and Nourinejad et al. (2016) which uses VI to model activity patterns in the presence of vehicle-to-grid technology.
Let us define the feasible region $\Gamma$ route flows of the equilibrium model as the following set of equations where $d^i$ is the flow of vehicles to parking zone $i \in I$ and the variables in brackets are the dual variables.

$$\sum_{a \in \psi(r,s,i)} d^i_{rs,a} = d^i_{rs} \quad [u^i_{rs}] \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \quad (14a)$$

$$\sum_{i \in \Omega(r,s)} d^i_{rs} = d_{rs} \quad [\lambda_{rs}] \quad \forall (r,s) \in V \quad (14b)$$

$$d^i = \sum_{(r,s) \in V} d^i_{rs} \quad [\delta_i] \quad \forall i \in I \quad (14c)$$

$$d^i_{rs,a} \geq 0 \quad [\varphi^i_{rs,a}] \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \quad (14d)$$

Constraints (14a) and (14b) ensure conservation of flow, constraints (14c) represent occupancy of parking $i$, and constraints (14d) ensure non-negativity of path flows.

For clarity, let us now partition the cost $C^i_{rs,a}$ (as shown in Eq. 3) into the following terms:

$$C^i_{rs,a} = \zeta^i_{rs,a} + \beta F_i(q_i) \quad \forall (r,s) \in V, \forall i \in \Omega(r,s), \forall a \in \psi(r,s,i) \quad (15)$$

where $\zeta^i_{rs,a} = \alpha t_{ri,a} + \gamma w_{is} + \gamma w_{si} + \alpha t_{ir,a} + (g_i + p_i h^i_{rs})$ represents the total observed travel cost including the cost of driving from $r$ to $i$, walking from $i$ to $s$, walking from $s$ to $i$, parking at parking area $i$, and driving from $i$ to $r$. The VI model is now presented as follows. Let $d = \{d^i_{rs,a} \in \Gamma \}$ be a feasible solution. We plan to find the equilibrium solution $d^* = \{d^i_{rs,a} \in \Gamma \}$ by showing that it always satisfies the following inequality:

$$\sum_{(r,s) \in V} \left( \sum_{i \in \Omega(r,s)} \left( \sum_{a \in \psi(r,s,i)} \zeta^i_{rs,a}(d^*) \left(d^i_{rs,a} - d^i_{rs,a}^*\right) + \frac{1}{\theta} \ln d^i_{rs} \left(d^i_{rs} - d^i_{rs}^*\right) - \frac{1}{\theta} \ln d^i_{rs} (d^i_{rs} - d^i_{rs}^*) \right) - D^{-1}_{rs}(d^i_{rs} - d^i_{rs}) \right) + \beta \sum_i F_i(q^*_i)(d^i - d^i) \geq 0 \quad \forall d \in \Gamma \quad (16)$$

The Karush-Kuhn-Tucker (KKT) conditions of the VI program in Eq. 16 are derived as

$$d^i_{rs,a} : \quad \zeta^i_{rs,a}(d^*) - u^i_{rs} - \varphi^i_{rs,a} = 0 \quad \forall (r,s) \in V, \forall i \in \Omega(r,s) \quad (17)$$
\[ d_{rs}^i : u_{rs}^i + \delta_i - \lambda_{rs} + \frac{1}{\theta} \ln d_{rs}^i = 0 \quad \forall (r, s) \in V, \forall i \in \Omega (r, s) \tag{18} \]

\[ d_{rs} : \lambda_{rs} - D_{rs}^{-1}(d_{rs}) - \frac{1}{\theta} \ln d_{rs} = 0 \quad \forall (r, s) \in V \tag{19} \]

\[ d^i : \beta F_i(q_i) - \delta_i = 0 \quad \forall i \in I \tag{20} \]

The complementarity conditions include constraints (14a) to (14d) and the following two:

\[ d_{rs,a}^i \varphi_{rs,a}^i = 0 \quad \forall (r, s) \in V, \forall i \in \Omega (r, s), \forall a \in \psi(r, s, i) \tag{21} \]

\[ \varphi_{rs,a}^i \geq 0 \quad \forall (r, s) \in V, \forall i \in \Omega (r, s), \forall a \in \psi(r, s, i) \tag{22} \]

At equilibrium, \( \delta_i \) is interpreted as the cost of searching at parking area \( i \) as per Eq. (29) and \( u_{rs}^i \) is interpreted as the minimum generalized travel cost (both driving and walking) of pair \( (r, s) \in W \) travellers parking at zone \( i \) as per (Eq. 17). We now show that the presented VI is equivalent to the equilibrium conditions of Section 5.2.

First, assume that demand is always non-negative \( d_{rs,a}^i > 0 \), so that \( \varphi_{rs,a}^i = 0 \). Given that \( \varphi_{rs,a}^i = 0 \), taking the exponential function of both sides of Eq. (18) and simplifying the terms, we have

\[ d_{rs}^i = \exp(-\theta (u_{rs}^i + \delta_i - \lambda_{rs})) \quad \forall (r, s) \in V, \forall i \in \Omega (r, s) \tag{23} \]

Using Eq. (14b), (Eq. 23) can be rewritten as:

\[ \sum_i d_{rs}^i = \exp(\theta \lambda_{rs}) \sum_i \exp(-\theta (u_{rs}^i + \delta_i)) = d_{rs} \quad \forall (r, s) \in V \tag{24} \]

Thus,

\[ \exp(\theta \lambda_{rs}) = \frac{d_{rs}}{\sum_i \exp(-\theta (u_{rs}^i + \delta_i))} \quad \forall (r, s) \in V \tag{25} \]

Substituting Eq. (25) into Eq. (23) gives

\[ d_{rs}^i = \frac{\exp(-\theta (u_{rs}^i + \delta_i))}{\sum_j \exp(-\theta (u_{rs}^j + \delta_j))} d_{rs} \quad \forall (r, s) \in V, \forall i \in \Omega (r, s) \tag{26} \]
where the term $\delta_i$ can be related to the cost of searching at parking area $i$. This makes Eq. (26) equivalent to the logit-based choice probability indicating that $d_{rs}^i = \pi_{rs}^i d_{rs}$.

Eq. (19) can also be reorganized as

$$\lambda_{rs} = \frac{1}{\theta} \ln d_{rs} - \frac{1}{\theta} \ln \sum_i \exp(-\theta(u_{rs}^i + \delta_i)) \quad \forall (r, s) \in V$$

(27)

Substituting Eq. (27) into Eq. (25) gives:

$$D_{rs}^{-1}(d_{rs}) = -\frac{1}{\theta} \ln \sum_i \exp(-\theta(u_{rs}^i + \delta_i)) \quad \forall (r, s) \in V$$

(28)

which is equivalent to Eq. (6) as the demand function.

We have shown that the solution of the VI program satisfies all the functional relationships that are required by the parking model as defined in Section 5.2. The VI program has at least one solution when its feasible region, $\Gamma$, is a compact and convex set. Given that the feasible region $\Gamma$ as is a set of linear constraints, and given that the VI function in Eq. (16) is continuous within the feasible region, we conclude that the VI has at least one solution (Florian, 2002).

5.4.2 Solving for equilibrium

An extensive review of solution algorithms for finding the traffic equilibrium is presented by Patriksson (2004). To solve the VI, traffic flows are assigned to parking areas $(d_i^l, \forall i)$ to find parking search times. Calculating parking search times can lead to infeasible solutions when the parking occupancy is larger than the parking capacity, i.e. $q_i \geq k_i$, because the search time function (Eq. 2) is discontinuous with a vertical asymptote. To rectify this issue, the parking search time function is replaced with the following BPR-type equation:

$$F_i(q_i) = l_i \mu_i \left[ 1 + \left( \frac{q_i}{k_i} \right)^\theta \right]$$

(29)

where $l_i$ is the is the average searching time in parking area $i$, $\mu_i$ is a constant representing how drivers adopt occupancy information, and $\theta$ is a calibration parameter. The parking search times are then used to find generalized costs and the origin-destination demands. The algorithm terminates upon convergence. The steps of the algorithm are the following:
Step 1. Initialization

Set the iteration number \( v = 0 \). Select an initial feasible demand solution \( d^v \). The feasible solution can be obtained by setting all travel times equal to free-flow travel times and setting the parking search time equal to zero for all parking areas.

Step 2. Computation of generalized costs

First, using \( d^v \), find the flow of vehicles into each parking area. The product of vehicle flows into each parking area and the parking dwell times (obtained for a given hourly price) gives parking occupancy which is input to Eq. (29) to find the search time of each parking area.

Second, using \( d^v \), find the travel times and the generalized costs as per Eq. (3).

Step 3. Direction finding

Perform a stochastic network loading procedure on the current set of link travel times. This yields an auxiliary link flow vector \( \tilde{d} \).

Step 4. Method of successive averages

Using the demand obtained from Step 3, find the new flow pattern by setting

\[
d^{v+1} = \frac{v-1}{v} d^v + \frac{1}{v} \tilde{d}
\]  

(30)

Step 5. Convergence test

Terminate if the following condition is satisfied with \( \kappa \) being a small number. Otherwise, set \( v \to v + 1 \) and go to Step 2.

\[
\frac{\sqrt{\sum (d^{v+1} - d^v)^2}}{\sum d^v} \leq \kappa
\]  

(31)

5.5 Market regimes

Let us first assume that a single operator is in charge of managing all the parking facilities. This operator can be either a public or a private entity. In such cases, the two objectives of interest are profit maximization (denoted by \( PM \)) and social surplus maximization (denoted by \( SS \)). The
former can be associated to the private and the latter to public authorities. The parking profit, $PM$, is:

$$PM = \sum_{(r,s) \in V} \sum_{i \in \Omega_{(r,s)}} \left[ (p_i h_{rs}^i + g_i) d_{rs}^i \right] - \sum_{i \in I} k_i \sigma_i \quad (32)$$

where the first term is the revenue from parking and the second term is the maintenance cost of all parking spots with $\sigma_i$ denoting the maintenance cost of one parking spot at parking zone $i \in I$. The maintenance cost is not necessarily the cost of physical rehabilitation and can include other supervisory costs such as the cost of parking enforcement for on-street parking. The second objective function is social surplus which is:

$$SS = \sum_{(r,s) \in V} \int_{0}^{d_{rs}} D_{rs}^{-1}(z)dz - \sum_{i \in I} k_i \sigma_i \quad (33)$$

where $D_{rs}^{-1}(z)$ is the inverse of the demand function. With the two objective functions, we can now define the following three markets: (i) monopoly, (ii) first best, and (iii) second best. Let us assume for now that the parking operator has monopoly rights and can simultaneously decide on the capacity and the fee structure of all parking zones. Under this market, the objective is to maximize the total profit as shown in Eq. 32. Alternatively, in the first-best market, the objective is to maximize social surplus and in the second-best market, the objective is to maximize social welfare while ensuring a positive profit. Hence, under the second-best market we have:

$$\text{maximize} \quad \sum_{(r,s) \in V} \int_{0}^{d_{rs}} D_{rs}^{-1}(z)dz - \sum_{i \in I} k_i \sigma_i$$

subject to

$$\sum_{(r,s) \in V} \sum_{i \in \Omega_{(r,s)}} \left[ (p_i h_{rs}^i + g_i) d_{rs}^i \right] \geq \sum_{i \in I} k_i \sigma_i \quad (34)$$

5.6 Numerical experiments

The first set of numerical experiments are performed on a simple case study with variable parking capacity. Thereafter, we present a network with fixed parking capacity and show that the analytical remarks are generalizable to larger networks.
5.6.1 A simple example with one origin, one destination, and one parking area

We present a simple example to visually present the three defined market regimes of Section 5.5. Consider the network in Fig. 5-1a with one O-D pair \((r, s)\) and one parking zone \(i \in \Omega(r, s)\).

Let \(\alpha = \beta = 10\) dollars per hour, \(w_{sl} = w_{ls} = 0\) hours, \(\gamma = 0\) dollars per hour, \(\theta = 1\), \(\mu_i = 1\), and \(l_i = 3\) minutes. The functions are defined as follows. Let \(t_{ri} = t_{ir} = 0.5 + \frac{x^2}{1000}\) measured in hours where \(x\) is the total demand obtained from the demand function \(x = D_{rs}(\eta_{rs}) = 20 - \eta_{rs}\).

The dwell time function is \(H_{rs}(p_i) = 3p_i^{-0.4}\).

The profit and social surplus contours are depicted in Fig. 5-2 for the simple example. As illustrated, the monopoly equilibrium occurs at the optimum of the profit objective function and the first-best equilibrium occurs at the optimum of the social surplus contours. The second-best equilibrium has to lie on the zero profit line where social surplus is maximized.

![Figure 5-2: Profit and social surplus contours for the simple example.](image)

We further investigate the generated profits from the following three dwell time function scenarios:

\[
H_{rs}(p_i) = 3p_i^{-1} : \text{Unit elastic, } e_p^h = -1
\]

\[
H_{rs}(p_i) = 3p_i^{-0.4} : \text{Inelastic, } e_p^h = -0.4
\]
\( H_{rs}(p_i) = 3p_i^{-1.4} \): Elastic, \( e_p^h = -1.4 \)

The demand and profit for Scenarios I, II, and III are illustrated in Fig. 5-3, Fig. 5-4, and Fig. 5-5, respectively. For each scenario, demand and profit are plotted for five parking capacities.

Before discussing the scenarios, let us redefine \( C_{rs}^l \) by substituting Eq. (1) and Eq. (2) into Eq. (3):

\[
C_{rs}^l = \alpha t_{rl} + \beta \frac{l_{rl}k_{rl}}{\sum_k l_{rl}k_{rl}} + (g_l + p_l h_{rs}^l) + \gamma w_{ls} + \gamma w_{sl} + \alpha t_{lr} \quad \forall (r,s) \in V, \forall i \in \Omega \quad (r,s)(35)
\]

As is now shown in Eq. (35), \( h_{rs}^l \) generally influences \( C_{rs}^l \) in two separate terms (second and third terms of Eq. (35)). However, under Scenario I, given that \( p_l h_{rs}^l = p_l 3p_i^{-1} = 3 \) is a constant, \( h_{rs}^l \) influences \( C_{rs}^l \) only via the second term. Hence, as \( p_i \) increases, \( h_{rs}^l \) decreases causing \( C_{rs}^l \) and consequently \( d_{rs} \) to approach their asymptotic values as is shown in Fig. 5-3.

The profit of this scenario also reaches its asymptotic value for the same reason. In Scenario II, demand initially increases with price and then decreases as is shown in Fig. 5-4. The initial increase occurs because increasing \( p_i \) leads to a lower dwell time and lower generalized cost, which in turn increases demand as elaborated in Section 5.3. The latter decrease in demand occurs because \( p_i \) directly contributes to the generalized cost which reduces demand. The demand in Scenario III somewhat follows the same pattern as Scenario I (as shown in Remark 2 of Section 5.3) but the profit patterns are different as shown in Fig. 5-5. In Scenario III, the profit reaches a peak value due to the higher influence of price on reducing dwell time. In all three scenarios, cases with higher parking capacities have higher demand, profit, and occupancy due to the lower cost of searching for parking (second term of Eq. (35)). Moreover, for all parking capacities in all three scenarios, demand, profit, and occupancy converge. The reason of convergence is that at high \( p_i \) values, dwell time and parking occupancy become so low that the parking capacity no longer imposes any restriction.
Figure 5-3: Demand, profit, and occupancy for Scenario I with unit elasticity dwell time.

Figure 5-4: Demand, profit, and occupancy for Scenario II with inelastic dwell time.

Figure 5-5: Demand, profit, and occupancy for Scenario III with elastic dwell time.
5.6.2 A network example

The second network is a grid network with 32 origins (external) nodes, 49 destination (internal) nodes, and 64 parking areas as shown in Fig. 5-6. The network includes a total of 144 bidirectional traffic links and a total of 196 bidirectional walk paths that connect the parking areas to the final destination zones. Travel time on each walking link is fixed and equal to 5 minutes but the travel time of each traffic link is obtained from the BPR function $t = f[1 + (x/cap)^4]$ where $f = 5$ minutes is the free-flow travel time and $cap=1000$ vehicles per hour is the capacity of each link. The parking search time at each parking area is obtained from the BPR-type function $F(q) = \mu l[1 + (q/k)^2]$ where $\mu = 0.5$ minutes $l = 1$ and $k = 30$ vehicles is the capacity of each parking area. The dispersion parameter in the stochastic equilibrium model is set to $\theta = 0.9$. The demand function is $D_{rs}(\eta_{rs}, z) = 9(1 - z/20) \exp(-0.07\eta_{rs})$ where $z$ is the distance from node $r$ to the geometrical center of the network. The demand function leads to higher demand in the center of the network, thus replicating a CBD. The travellers of all OD pairs are assumed to homogenous. Parking dwell times are obtained from $H_{rs}(p_i) = 1.5 p^e_p$ where $e^h_p$ is the dwell time elasticity to hourly parking price.

The parking search time and demand are depicted in Fig. 5-7 when dwell time is inelastic (i.e., $-1 < e^h_p \leq 0$). As illustrated, increasing the hourly parking price, $p$, from $1$ to $3$, reduces parking search time because dwell time is shorter at $p =3$ and more spots are available. This reduction in parking search time increases demand because of the lower generalized cost of travel. From $p = 3$ to $p = 5$, on the other hand, parking demand decreases because the low parking search time is offset by the high hourly price of $5$ dollars per hour. To sum up, when dwell time is inelastic, the hourly parking price may increase or decrease demand but the parking search time always decreases. This remark is consistent with Proposition 1 which was proved for a simple case with only one origin, one destination, and one parking area. The parking search time and demand for the elastic case (i.e., $e^h_p < -1$) are presented in Fig. 5-8. As illustrated, increasing $p$ strictly decreases search time and increases demand. This remark is also consistent with Proposition 1.

To investigate the impact of hourly parking price on parking behaviour, Fig. 5-9 illustrates the average parking search time and demand (for the 64 parking areas) for a range of dwell time elasticities and hourly parking prices. As shown, when dwell time elasticity, $e^h_p$, is very low and
close to zero, parking demand strictly decreases with $p$. This shows that at $e_p^h \approx 0$, the hourly parking price has the same impact as a road toll on demand as they both decay demand. However, when $-1 < e_p^h < 0$, parking demand first increases then decreases with $p$, and when $e_p^h < -1$, parking demand strictly increases with $p$. Fig. 5-9 also shows that search time always decreases when $p$ regardless of the elasticity.

Fig. 5-10 shows the scatter of parking demand and search time for inelastic dwell time (Fig. 5-10a and 5-10b) and elastic dwell time (Fig. 5-10c and 5-10d). It is evident from Fig. 5-10b and Fig. 5-10d, that search time has a lower standard deviation when dwell time is elastic (compared to the inelastic case) because drivers are more sensitive to the hourly parking price. Precisely, the high sensitivity to $p$ leads to a faster reaction (of drivers) to hourly parking prices. For the same reason, the mean of search time (as marked in the box plots) reaches zero at a faster rate when dwell time is elastic. Parking demand is shown to be fairly constant in the inelastic case shown in Fig. 5-10a (although it slightly increases and then decrease with $p$) but it increases at a fast rate in the elastic case. Finally, Fig. 5-11 shows the convergence of the algorithm by depicting demand at each parking area at 8 iterations of the algorithm for $e_p^h = -0.5$.

Figure 5-6: Example network.
<table>
<thead>
<tr>
<th>Parking Demand [veh per hour]</th>
<th>p = $1 dollar/hour</th>
<th>p = $3 dollar/hour</th>
<th>p = $5 dollar/hour</th>
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<tr>
<th>Parking Search Time [minutes]</th>
<th>p = $1 dollar/hour</th>
<th>p = $3 dollar/hour</th>
<th>p = $5 dollar/hour</th>
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**Figure 5-7:** Parking demand and search time for inelastic dwell time with $e_p^h = -0.3$.

<table>
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<tr>
<th>Parking Demand [veh per hour]</th>
<th>p = $1 dollar/hour</th>
<th>p = $3 dollar/hour</th>
<th>p = $5 dollar/hour</th>
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**Figure 5-8:** Parking demand and search time for elastic dwell time with $e_p^h = -1.3$.  

135
Figure 5-9: (a) Average parking search time [hours], and (b) Average parking demand [vehicles per hour].

Figure 5-10: (a) Parking demand, and (b) Search time for inelastic dwell time \( e_p^h = -0.5 \).
(c) Parking demand, and (d) Search time for elastic dwell time \( e_p^h = -1.5 \).
Figure 5-11: Demand at each parking area at eight iterations of the algorithm.

5.7 Conclusions

This chapter investigates the impact of hourly parking pricing on travel demand. Parking pricing, if imposed wisely, has the potential to complement or even substitute road pricing. When imposed imprudently, however, it can increase demand and create more congestion. This chapter shows that road pricing and hourly parking pricing are structurally different in how they influence the traffic equilibrium. While road pricing reduces demand, parking pricing can reduce or induce demand depending on the elasticity of parking dwell time to the hourly parking price which can be estimated from parking surveys. Hence, neglecting the dwell time elasticity can lead to suboptimal pricing and reduced social-welfare. To capture the impact of the hourly price on demand, a simple case is presented with one origin, one destination, and one parking area. For this simple case, we prove that increasing the hourly parking price always induces demand when dwell time is highly elastic to the parking price. On the other hand, when dwell time is inelastic to parking price, an increase in the hourly parking price may increase or decrease demand. Hence, dwell time elasticity requires special attention in design of parking policy.
For more realistic networks, we present a Variational Inequality model that captures the parking (and route) choice equilibrium. To gain managerial insight, we perform sensitivity analysis on a network with 64 parking areas. Numerical results show that the dwell time elasticity is still a key factor in travel demand. When dwell time elasticity is equal to zero, the hourly parking price has the same impact as a road toll on travel demand where increasing the parking price decreases demand. The numerical experiments also show a lower standard deviation in the parking search time (i.e., time to find a parking spot) of the 64 parking areas when dwell time is highly elastic to the hourly parking price. Moreover, when dwell time is inelastic, demand is fairly constant, whereas when dwell time is elastic, demand is asymptotic.

5.8 References


Parking is an inconvenience for travellers as well as city planners, operators, and regulators. Travellers find parking cumbersome because they have to search for a spot that is convenient and affordable. In some cities, travellers spend as much as 40% of their total travel time in search of parking (Gallivan, 2011). To mitigate these negative impacts, parking policies are implemented for short, medium, and long-term planning. As a long-term policy, we investigate the impact of autonomous vehicles on parking land-use. We show that autonomous vehicles park farther away from downtown to avoid a long parking search-time. As a medium-term policy, we propose models to optimize parking enforcement for passenger and commercial vehicles. We quantify the impact of enforcement technology on social welfare and profit. Finally, as a short-term policy, we study the impact of hourly parking pricing on travel demand and show that, despite intuition, demand is induced when the hourly parking price is raised. We summarize our findings in detail below and we follow it by future research directions.

6.1 Autonomous vehicle parking

With the arrival of autonomous vehicles (AVs), travellers no longer need to search for parking. Instead, they get dropped off at their final destination and the occupant-free AVs search for the nearest and most convenient parking spot. Having investigated this emerging parking pattern, we conclude that

- Conventional vehicles always park closer to the downtown zone to avoid walking a long distance. AVs, on the other hand, park farther away from the downtown zone to avoid a long parking-search time.
- AVs experience a much lower search time compared to conventional vehicles. The longest parking search of AVs is shorter than the shortest parking search time of conventional vehicles.
- Increasing the AV penetration rate leads to a lower search time for conventional vehicles and other AVs as well.
- The parking pattern of AVs leads to higher traffic. AVs can only reduce traffic if they take less than half the road space occupied by a conventional vehicle.
While this thesis is the first to investigate the impact of AV parking, there are several remaining questions that need to be addressed in future research. The developed model in Chapter 2 can be extended in the following ways. First, to keep the model in tractable form, we did not account for the congestion effects on the corridor and assumed a fixed cost per km travelled. Relaxing this assumption improves the accuracy of the congestion estimates of the model. Second, the model assumes that travellers originate from and terminate at one zone. A reasonable extension is to assume that vehicles originate from one zone but their destination location is randomly distributed along the corridor as is done in Inci and Lindsey (2015). With random destination locations, new parking occupancy patterns may emerge. Third, for practical application, it is imperative to capture the network-scale impact of AV parking patterns. An initial step in this direction is to use a Beckmann-type model or a Variational Inequality model with additional parking constraints. Fourth, to find the optimal parking land use, we assumed a fixed rent cost (per space) along the corridor. This cost structure in real-life is comprised of a fixed construction cost and an amortized renting cost. We hypothesize that inclusion of the fixed parking cost leads to larger vertical economies of scale (Arnott, 2006) for AVs than CVs so that AVs would be packed in a few high-rise parking facilities that are far from downtown. We believe that addressing these questions would advance our understanding of the impact of AV parking on the future of cities.

In addition to the above, AVs will change the future of parking in several other ways. It is anticipated that AVs will promote collaborative consumption (with a fleet of AVs shared between several families). When a fleet is shared between multiple families, it is important to decide where to park each vehicle in the fleet so that all families have sufficient accessibility. Choosing the optimal parking location requires knowledge of the activity schedules of each family. With this information, vehicles can be relocated between parking lots so that each family can complete its daily activities without conflict.

AVs will also change the layout of the existing parking facilities in the future. Currently, parking facilities are designed with a lot of redundant space so that vehicles can easily maneuver while parking or coming out of a parking spot as shown in Fig. 6.1. With autonomous vehicles, however, the geometric design of parking facilities will change for two reasons. First, AVs can be parked closer to each other and less space will be taken in total. Second, AVs can be relocated automatically within the parking lot. Hence, by optimally relocating (re-handling) the AVs, more
vehicles can be packed into each parking facility. Questions that remain to be answered are (i) what is the optimal geometric design for AV parking facilities, (ii) how to relocate the vehicles within the facility to maximize the expected vehicle capacity, and (iii) how to stack the vehicles in the facility to minimize the expected number of relocations.

Figure 6-1: Parking lot design for conventional and autonomous vehicles.

6.2 Parking enforcement

Illegal parking leads to adverse societal impacts such as reduced traffic speeds, loss of revenue from legal parking, and more accidents caused by safety violations. It is estimated that illegal parking causes 47 million vehicle-hours of delay each year in the United States, which makes illegal parking the third leading cause of delay behind construction and crashes (Han et al., 2005). In response to these detrimental consequences, parking enforcement policies are implemented to hinder illegal parking. A parking enforcement policy, in its simplest form, is comprised of choosing a citation fine and the level-of-enforcement. The citation fine is the penalty paid by illegally parked vehicles that get caught by an enforcement unit (e.g. on-foot officers or cameras) and the level-of-enforcement is the number of enforcement units that search for illegally parked vehicles. Given that it takes time to find and cite each illegally parked vehicle, there is friction present in the searching process. To quantify the friction, we use the
bilateral-search-and-meet function and we characterize key factors of illegal parking behaviour such as parking dwell time, probability of parking illegally, citation probability, and rate of citations. Using these factors, we present an equilibrium model of illegal parking where each driver first decides to park legally or illegally and next chooses the parking duration. The model yields several insights: (i) the citation probability increases with the illegal dwell time because vehicles that are parked for a long time are more susceptible to getting a citation, (ii) the citation probability decreases with the number of illegally parked vehicles, (iii) vehicles are more likely to park illegally when their dwell time is short, and (iv) the citation fine and the level-of-enforcement are lowered as the enforcement technology becomes more efficient.

Despite the widespread application and ubiquity of parking enforcement policies in many cities, research in this area is still scarce and there is a need for studies that address the following extensions to the presented model. First, our model explicitly defines a policy as a given citation fine and level-of-enforcement. In reality, however, parking enforcement policies are multifaceted and include sub-policies that regulate parking through towing, issuing parking permits, or wheel clamping. A question that remains to be answered is when is it beneficial to introduce any of these sub-policies to parking enforcement and what is the effect of these sub-policies on social welfare and profit? As an example, a city that prioritizes social welfare is better off with towing illegally parked vehicles to take them off the street. Second, there is a need for optimizing the details of each of such sub-policies. As an example, if towing is a viable option, then what is the required number of towing trucks, enforcement units that find illegal vehicles, and the towing penalty paid by drivers? Similarly, if parking permits are the favorable sub-policy, then how many permits should be issued? Third, there are a number of assumptions made in this chapter. Relaxing each of these assumptions, such as accounting for the parking search time of legal and illegal vehicles, is an avenue of future research. Finally, there is still a need for addressing other sources of uncertainty (such as travel time, number of deliveries per day, and the choice of parking illegally), investigating the role of other policies, and validating the proposed models using empirical data. Addressing the sources of uncertainty, coupled with rigorous surveys of influential factors in the choice of parking, can lead to better calibrated models that are essential for policy-making.
6.3 Hourly parking pricing

Efficient parking management strategies are vital in central business districts of mega-cities where space is restricted and congestion is intense. Assessing the impact of hourly parking pricing as a short-term policy, we conclude that

- Hourly parking pricing can reduce or induce demand depending on the parking dwell time elasticity (to the hourly parking price). Hence, it is imperative that hourly pricing is implemented after conducting surveys to infer parking dwell time elasticity with respect to price.

- When dwell time is elastic, demand always increases with parking price. This happens because the hourly price motivates travellers to park for a shorter time which in turn reduces parking occupancy and search time. Consequently, there is induced demand when parking search time is low.

- When dwell time is inelastic, demand may increase or decrease with the parking price. In such cases, we have to consider the sign and value of the elasticity.

- Analysis of parking pricing on a large network shows a lower standard deviation in the parking search time when dwell time is highly elastic to the hourly parking price. Hence, policy makers have higher certainty in their decision making when the dwell time is highly elastic.

The main finding of Chapter 5 is that parking pricing policies should be devised with sufficient knowledge of dwell time elasticities. While this chapter emphasizes the role of dwell time in parking policy, there are several aspects that are worthy of future research. First, user heterogeneity should be addressed by segmenting travellers based on their value of time and dwell-time elasticity. In a heterogeneous setting, demand of each traveller segment is sensitive to that segment’s dwell time elasticity as well as the dwell time elasticity of all other segments. Second, with the growing interest in dynamic parking pricing in many major cities, the model can be modified with hourly prices changing within a day. This implies that parking occupancies are dynamic as well. Third, non-linear pricing can further improve social welfare where the hourly parking price increases with time. Finally, the impact of
hourly parking pricing is amplified in the presence of multiple public and private parking management authorities who are generally in competition with each other (Arnott and Rowse, 2009; Arnott and Rowse, 2013). This leads to competitive price-setting environment which requires further research when prices are hourly-based.

6.4 Further research

While this thesis evaluates the impact of three major policies, there remain unanswered questions that require additional research. Future research is needed to (1) explore the impact of emerging technologies in the parking industry and (2) to unify all parking policies in a standard test-bed. Emerging technologies, in addition to vehicle automation, will substantially change the way we move people and goods. In the freight industry, for instance, the use of drones to perform deliveries can change the parking patterns of commercial vehicles; Instead of visiting all delivery points, commercial vehicles can park at one (optimal) location from where drones are dispatched to complete the deliveries. Other emerging technologies include the variety of data collection sources that can be used to accurately estimate the state of parking occupancy.

The second area the requires additional research is the unification of parking policies in a standard test-bed. Currently, research on parking is dispersed and the available studies focus on one particular policy without considering its relationship with other policies. As an example, there are many models that try to find the optimal parking price and there are other models that investigate optimal enforcement measures. Nevertheless, the connection between pricing and enforcement is not explored. Hence, there is a need for a test-bed platform where all existing and future policies can be evaluated simultaneously. An ideal methodology for the platform is a simulation model where multiple policies can be tested at once. The simulation model would also allow for model calibration for accurate emulation of real-life situations.

6.5 References


