DESIGN AND DEVELOPMENT OF GLASS-FIBRE-REINFORCED POLYMER AND ULTRA-HIGH-PERFORMANCE FIBRE-REINFORCED CONCRETE GUIDEWAY GIRDERS

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
Graduate Department of Civil Engineering
University of Toronto

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Abstract

Design and Development of Glass-Fibre-Reinforced Polymer and Ultra-High-Performance Fibre-Reinforced Concrete Guideway Girders

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An innovative design for prefabricated rail guideway girders is developed and validated. The proposed concept uses compositely acting glass-fibre-reinforced polymer (GFRP) and ultra-high-performance fibre-reinforced concrete (UHPFRC), achieving a girder that is 30% lighter than state-of-the-art prestressed concrete designs. Questions regarding the structural behaviour of compositely acting GFRP–UHPFRC are addressed through prototyping and experimental studies. A robust structural connector for GFRP–UHPFRC, which can be fabricated during the same vacuum infusion process that produces the GFRP shell, is developed and tested. The tensile fatigue behaviour of UHPFRC is characterized through a series of direct tension cyclic tests, showing that for peak cyclic tensile stress below 7.5 MPa, there is no progressive damage to the UHPFRC. The intrinsic damping of UHPFRC is investigated through a series of free-vibration tests, demonstrating that a loss factor of $\eta = 0.04$ is typical for UHPFRC under tensile cyclic loading, with higher values possible under reversed cyclic and short-duration loading. The unprecedented lightness of the proposed guideway, together with the high strength of GFRP, means that serviceability and fatigue limit states govern the dimensioning of the girders. This calls for a rational means of assessing the vibration response of the guideway system, from the points of view of passenger comfort and girder fatigue life. A frequency-domain method for the analysis of train-girder interaction dynamics is developed and presented. Relative to existing analysis methods, this approach eliminates the need for Monte Carlo simulation of random track irregularities, and allows for rapid assessment of the effects of long-term deformations.
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Nomenclature

\( \alpha \) Mass-proportional Rayleigh damping constant

\( \beta \) Newmark method acceleration variation constant

\( \beta \) Stiffness-proportional Rayleigh damping constant

\( \ddot{u}_g \) Harmonic girder input vector

\( \ddot{u}_t \) Harmonic train input vector

\( \ddot{a}_{\text{rms}} \) Harmonic vector representation of one-sided root mean square acceleration spectrum of train response

\( \Delta t \) Spatial discretization increment

\( \delta \) Logarithmic decrement

\( \epsilon_a \) Amplitude of tensile fatigue strain

\( \epsilon_{cr} \) UHPFRC cracking strain

\( \epsilon_c \) Fixed point strain for fatigue creep-softening model

\( \epsilon'_c \) UHPFRC compressive elastic limit strain

\( \epsilon_{\text{min}} \) Minimum tensile fatigue strain

\( \epsilon_m \) Mean tensile fatigue strain

\( \epsilon_p \) Peak tensile fatigue strain

\( \epsilon_{tu} \) UHPFRC ultimate tensile strain

\( \eta \) Loss factor

\( \gamma \) Material density

\( \gamma \) Newmark method numerical damping constant

\( \gamma_s \) Composite action factor for mechanically jointed beam

\( \hat{r}_k \) Fourier coefficients of vertical track irregularity profile

\( \Omega \) Frequency domain differentiation multiplier matrix
\( \Omega_a \)  Frequency domain differentiation multiplier matrix for axle degrees of freedom

\( \Omega_g \)  Frequency domain differentiation multiplier matrix for girder degrees of freedom

\( \Omega_t \)  Frequency domain differentiation multiplier matrix for train degrees of freedom

\( \Phi_{agn} \)  Axle-location mode shape matrix for all girders in time-domain analysis

\( \Phi_{ag} \)  Matrix of magnitudes of girder mode shapes at axle locations

\( \Theta_a \)  Harmonic comb matrix for axles

\( \Theta_g \)  Harmonic comb matrix for girders

\( A_{gg} \)  Girder submatrix of train-girder system matrix

\( A_{gt} \)  Girder-train submatrix of train-girder system matrix

\( A_{tg} \)  Train-girder submatrix of train-girder system matrix

\( A_{tt} \)  Train submatrix of train-girder system matrix

\( C_{at} \)  Axle-train coupling damping matrix

\( C_a \)  Axle damping matrix

\( C_{gn} \)  Damping matrix for all girders in time-domain analysis

\( C_g \)  Girder damping matrix

\( C_{ta} \)  Train-axle coupling damping matrix

\( C_t \)  Train damping matrix

\( G \)  Harmonic transfer function

\( G_{gg} \)  Girder submatrix of harmonic transfer matrix

\( G_{gt} \)  Girder-train submatrix of harmonic transfer matrix

\( G_{tf} \)  Harmonic transfer matrix for static force input, girder response

\( G_{tt} \)  Harmonic transfer matrix for static force input, train response

\( G_{tg} \)  Train-girder submatrix of harmonic transfer matrix

\( G_{tr} \)  Harmonic transfer matrix for track irregularity input, girder response

\( G_{tt} \)  Harmonic transfer matrix for track irregularity input, train response

\( G_{tw} \)  Train submatrix of harmonic transfer matrix

\( G_{tw} \)  Harmonic transfer matrix for girder shape input, girder response

\( G_{tw} \)  Harmonic transfer matrix for girder shape input, train response

\( I \)  Identity matrix
\( \mathbf{K}_{\text{at}} \) Axle-train coupling stiffness matrix

\( \mathbf{K}_a \) Axle stiffness matrix

\( \mathbf{K}_{\text{gn}} \) Stiffness matrix for all girders in time-domain analysis

\( \mathbf{K}_g \) Girder stiffness matrix

\( \mathbf{K}_{\text{ta}} \) Train-axle coupling stiffness matrix

\( \mathbf{K}_t \) Train stiffness matrix

\( \mathbf{M}_a \) Axle mass matrix

\( \mathbf{M}_{\text{gn}} \) Mass matrix for all girders in time-domain analysis

\( \mathbf{M}_g \) Girder mass matrix

\( \mathbf{M}_t \) Train mass matrix

\( \Omega \) Spatial frequency

\( \omega \) Radial frequency

\( \omega_f \) Fundamental frequency

\( \omega_n \) Undamped natural frequency

\( \phi \) Phase angle

\( \phi_{\text{GFRP}} \) GFRP material reduction factor

\( \sigma' \) Stored stress

\( \sigma'' \) Coercive stress

\( \sigma_a \) Amplitude of tensile fatigue stress

\( \sigma_{\text{bc}} \) Critical compressive stress for plate buckling

\( \sigma_{\text{bf}} \) Factored compressive bending stress in webs

\( \sigma_{\text{cr}} \) UHPFRC cracking stress

\( \sigma_{\text{cu}} \) Ultimate compressive strength

\( \sigma_c \) Fixed point stress for fatigue creep-softening model

\( \sigma'_{\text{c}} \) UHPFRC compressive strength

\( \sigma_{\text{min}} \) Minimum applied tensile fatigue stress

\( \sigma_m \) Mean tensile fatigue stress

\( \sigma_{\text{pre}} \) Peak stress of initial static loading for fatigue tests

\( \sigma_p \) Peak tensile fatigue stress
\( \sigma_{u} \)  UHPFRC ultimate tensile stress 
\( \sigma_{u,post} \)  Ultimate tensile stress at failure, post-fatigue static tensile test

\( \tau_{sc} \)  Critical shear stress causing plate buckling

\( \tau_{sf} \)  Factored shear stress

\( \hat{C}_{gn} \)  Inertia-modified damping matrix for all girder degrees of freedom in time-domain analysis

\( \hat{K}_{gn} \)  Inertia-modified stiffness matrix for all girder degrees of freedom in time-domain analysis

\( \hat{M}_{gn} \)  Inertia-modified mass matrix for all girder degrees of freedom in time-domain analysis

\( \hat{p}_{gn} \)  Inertia-modified force vector for all girder degrees of freedom in time-domain analysis

\( f_{a} \)  Static axle load

\( r_{a} \)  Vector of magnitudes of vertical track irregularity profile of girder at axle locations

\( w_{a} \)  Vector of magnitudes of static displaced shape of girder at axle locations

\( y_{a} \)  Vector of axle vertical displacements

\( y_{gn} \)  Displacement vector for all girder degrees of freedom in time-domain analysis

\( y_{t} \)  Vector of internal train degree of freedom vertical displacements

\( \xi \)  Frequency, in Hz

\( \zeta \)  Damping ratio

\( a \)  Larger dimension of rectangular plate, used in buckling checks

\( A_{UHPFRC} \)  Cross-sectional area of UHPFRC

\( A_{box} \)  Enclosed area of box girder

\( a_{w} \)  Frequency-weighted acceleration

\( b \)  Smaller dimension of rectangular plate, used in buckling checks

\( b_{n} \)  Width of bottom UHPFRC slab

\( b_{sc} \)  Width of shear-connected interface

\( c \)  Damping coefficient

\( c_{a} \)  Train car primary suspension damping coefficient

\( c_{b} \)  Train car secondary suspension damping coefficient

\( D \)  Energy dissipated per cycle

\( D \)  Isotropic flexural stiffness

\( d \)  Structural depth of girder
$D_{11}$ Longitudinal flexural stiffness of an anisotropic plate
$D_{22}$ Transverse flexural stiffness of an anisotropic plate
$E'$ Storage modulus
$E''$ Loss modulus
$E_f$ Final elastic modulus, after 5 million cycle run-out or just prior to failure
$E_{pre}$ Initial elastic modulus
$E_{UHPFRC}$ UHPFRC elastic modulus
$f(t)$ Forcing function
$F_{a1}$ Force in actuator 1
$F_{a2}$ Force in actuator 2
$F_a$ Cyclic force amplitude
$F_{contr}$ Actuator control force
$G(\omega)$ Transfer function
$h$ Free vibration specimen height
$h_w$ Web depth
$I_c$ Train car pitching inertia
$I_t$ Transformed second moment of area
$k$ Stiffness constant
$k'$ Real component of complex stiffness
$k''$ Imaginary component of complex stiffness
$k^*$ Complex stiffness
$k_a$ Train car primary suspension stiffness coefficient
$k_b$ Train car secondary suspension stiffness coefficient
$K_{sc}$ Stiffness of shear connector
$k_s$ Buckling factor for shear plate buckling
$L$ Span length
$l_a$ Half-length between train car axles
$l_b$ Half-length between train car bogies
$l_c$ Train car length
$l_g$  Girder analysis length

$l_t$  Length between trains

$m$  Mass per unit length

$m$  Mass

$m_c$  Train car mass

$M_{L/2}$  Midspan moment

$n$  Number of cycles per train passage

$N_a$  Number of axle degrees of freedom per train car

$N_c$  Number of train cars per train

$N_{fat}$  Number of fatigue stress cycles over service life

$n_f$  Number of spatial increments in fundamental length

$N_g$  Number of girder degrees of freedom

$n_h$  Number of harmonics kept in analysis

$n_{tc}$  Number of degrees of freedom per train car

$N_t$  Number of internal degrees of freedom per train car

$N_t$  Number of train traversals over service life

$n_t$  Number of degrees of freedom per train

$P_v$  Power spectral density for vertical track irregularity

$q$  Shear flow

$Q_b$  First moment of area of bottom UHPFRC slab

$R$  Horizontal radius of curvature

$R^2$  Coefficient of determination

$t$  Time

$T_f$  Fundamental period of train-girder system

$T_g$  Time for train to traverse girder

$T_t$  Time between trains

$t_w$  Web thickness

$U$  Strain energy amplitude

$u$  Displacement
\( u_{a1} \)  Displacement of actuator 1
\( u_{a2} \)  Displacement of actuator 2
\( u_a \)  Displacement amplitude
\( v \)  Train speed
\( V_f \)  Factored shear force
\( x_{\text{max}} \)  Free vibration test initial lateral displacement
\( x_s \)  Longitudinal spacing of transverse web stiffeners
\( y \)  Vertical displacement
\( y_{a1} \)  Front train car axle vertical displacement
\( y_{a2} \)  Second train car axle vertical displacement
\( y_{a3} \)  Third train car axle vertical displacement
\( y_{a4} \)  Fourth train car axle vertical displacement
\( y_{t1} \)  Front of train car vertical displacement
\( y_{t2} \)  Back of train car vertical displacement
\( y_{t3} \)  Front of front bogie vertical displacement
\( y_{t4} \)  Back of front bogie vertical displacement
\( y_{t5} \)  Front of back bogie vertical displacement
\( y_{t6} \)  Back of back bogie vertical displacement
Chapter 1

Introduction

This thesis presents the conceptualization, development, and validation of an innovative structural system using emerging construction materials. The structural system developed is a prefabricated hybrid composite rail guideway girder, and the materials are ultra-high-performance fibre-reinforced concrete (UHPFRC) and glass-fibre-reinforced polymer (GFRP), acting compositely as GFRP–UHPFRC. In addition to the guideway girder design development, the following contributions are made:

1. A strong, reliable, and easily-fabricated bond mechanism for GFRP–UHPFRC is invented and tested.
2. The fatigue performance of UHPFRC in direct tension is characterized.
3. The potential for cracked UHPFRC to provide significant damping for service loads is investigated, and ultimately ruled out.
4. A dynamic analysis method for preliminary analysis of the passenger comfort performance of a guideway structure is developed and presented.

1.1 Motivation: Potential for improved guideway design

Innovation in civil engineering progresses at a slow pace by twenty-first century standards. The reasons for this slow progress are diverse [68, 94], but perhaps the most important factor is that for a large infrastructure project there is a strong preference, from a financial management point of view, to minimize risk. Therefore, significant innovations are unlikely to arise in direct response to industry demand, and are more often sought as a creator of competitive advantage. Indeed, a study by Nam and Tatum [69] of ten construction innovations implemented in the US between 1985 and 1989 found that, as they state in their concluding remarks:
Chapter 1. Introduction

It turns out that, in many innovative projects, solutions often precede problems. Owners’ demands do not necessarily come first; the designers’ or contractors’ advanced technology often shapes the owners’ demands. [69]

Thus, academic research can play an important role both in responding to demands for innovation from industry and in increasing the supply of innovative technology from which industry can draw. This thesis falls into the latter category, pursuing the design and development of an innovative system for guideway structures.

Elevated transit systems can be an attractive public transit solution in many situations, as they offer the benefit of grade separation from automobile traffic, with less risk, cost, and disruption than the tunnelling work required for subway construction. In order to fully realize these benefits, the elevated guideway structure that supports the transit line must be efficient and economical to build. It must also have an aesthetic impact that is least acceptable to the population living and working around it. Recent trends in guideway structure construction are toward speed and simplicity of construction, these benefits evidently outweighing considerations of material efficiency or aesthetics. Precast concrete, in either segmental or girder form, has become the most common structural system.

These considerations suggest an opportunity for significant competitive advantage to be captured by a structural system for guideway construction that is fast and simple to build, and that offers material use and aesthetic benefits over current precast solutions.

1.2 Background: Opportunities offered by GFRP and UHPFRC

GFRP and UHPFRC are both relatively new materials for the construction industry. Their potential value is reflected by an expanding body of academic research into the properties and applications of the materials, and by the ongoing efforts in Canada, and around the world, to integrate recommendations and requirements for both materials into design standards.

The work presented in this thesis demonstrates that there are a number of advantages to combining UHPFRC with GFRP into a hybrid composite material, which will be referred to as GFRP–UHPFRC.

This section provides a brief overview of the properties of UHPFRC, GFRP, and GFRP–UHPFRC, and a review of state-of-the-art applications of these materials to bridge construction.
1.2.1 Glass-fibre-reinforced polymer

GFRP is a composite material consisting of glass fibres embedded in a polymeric matrix. Mechanical properties are dependent on the specific type of glass and polymer, the fibre architecture, and the fabrication method.

Reinforcing materials

E-glass fibres are the cheapest and most common type of glass reinforcement. Other types of glass reinforcement include S-glass (having higher strength), and AR-glass (having a higher alkali resistance). Glass-fibre-reinforced concrete, where fibres are in direct contact with the alkaline cement matrix, requires type AR glass. A study in Canada [67] found that alkali attack on GFRP is negligible, as the polymeric matrix effectively protects the fibres.

Reinforcing materials other than glass are also possible. In civil engineering applications, carbon fibre is sometimes used for its higher stiffness and strength, and its improved fatigue performance. The primary application of CFRP in civil engineering is in retrofit applications, where the thinness of the laminate is desired to minimize the aesthetic impact to the existing structure. For application in new construction, the increased cost of CFRP makes it is generally more economical to use an equivalent, thicker, GFRP laminate.

All design and experimental work in this thesis uses E-glass fibre reinforcement.

Polymer matrix materials

The polymeric matrix can, in general, be thermoset or thermoplastic. Thermosets generally provide the superior durability and thermal stability. All design and experimental work in this thesis uses a thermoset epoxy matrix.

Durability concerns

Epoxy resins are prone to discoloration under prolonged exposure to ultraviolet sunlight. This is typically only a cosmetic concern [41], but severe discoloration could be problematic from an aesthetic point of view. UV-resistant coatings are available, which can mitigate the effects of sunlight exposure.

Moisture exposure can also lead to degradation of composite materials [8]. Again, protective coatings are available.

E-glass/epoxy laminates have been used extensively in the manufacture of wind-turbine blades, which in offshore wind farms are exposed to far harsher conditions than would be seen by most guideway
structures.

Fire resistance can also be a concern with polymer composites. This consideration is addressed with reference to the proposed design in Chapter 3.

**Fabrication**

In civil engineering structures, pultrusion is a common means of fabricating reinforcing bars and structural members with GFRP. The advantage of pultrusion is the high glass fibre volume fraction that can be achieved, which improves stiffness and strength, and the reliability and consistency of a fully mechanized manufacturing process. However, pultrusion has a number of limitations: (1) the standard process is limited to straight members; (2) the force required to pull the part through the die increases with size, making large pultrusions difficult; and (3) the surfaces of the part, which slide through the die as the resin cures, are smooth in the longitudinal direction, precluding the possibility of transverse ridges or texture.

A different manufacturing process, vacuum infusion (also referred to as vacuum-assisted resin transfer moulding), can overcome some of these limitations. In this process, the dry glass fibre fabric is arranged in the mold, before being sealed under a flexible bag. This “lay-up” process allows the fibre architecture to be tailored to the mechanical requirements of the part. Evacuating the air from inside the sealed mold creates a pressure differential which is used to both compress the lay-up, and to draw the resin into the mold and through the fabric. The flexibility of this method allows the manufacture of large and irregularly shaped members. As discussed below, vacuum infusion has been used with success in the fabrication of full-scale GFRP bridge girders.

**Specific materials used for experimental studies**

In this study, GFRP consisting of continuous E-glass fibres in an epoxy matrix is used. GFRP plates were fabricated using vacuum infusion. The lay-up consisted of 8 layers of non-crimp Vectorply E-LM 3610 (1210 g/m² in the 0° direction plus 305 g/m² as a chopped-strand mat backing), with one layer of Vectorply E-BX 1200 (213 g/m² in each of the ±45° directions) on each outer face. Fibre volume fraction is approximately 40 to 45%. The longitudinal stiffness for this laminate was measured as 20 GPa, and the ultimate tensile strength as 410 MPa. Further details of the laminate are given in Appendix A.

The polymer matrix was Gurit Prime 20LV, a low-viscosity epoxy, with long working time to allow for the infusion process. This epoxy system has four hardener options: fast, slow, extra slow and high-Tg. The fast, slow and extra slow hardeners can be mixed to achieve the optimal working time for a given infusion. The infusions undertaken for this thesis involved approximately 5 L of epoxy resin per
part, and it was found that given the thickness of the part, the slower hardener was preferable to keep the temperature generated by the exothermic reaction of resin and hardener low. A summary of the properties of this epoxy are given in Table 1.1, below.

<table>
<thead>
<tr>
<th>Table 1.1: Properties of Gurit Prime 20LV epoxy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial mixed viscosity, 20°C</td>
</tr>
<tr>
<td>Pot life (extra slow hardener, 500 g in air, 20°C)</td>
</tr>
<tr>
<td>Cure schedule</td>
</tr>
<tr>
<td>Linear shrinkage</td>
</tr>
<tr>
<td>Tg, dynamic mechanical analysis (DMA)</td>
</tr>
<tr>
<td>Tg, differential scanning calorimetry (DSC)</td>
</tr>
<tr>
<td>Tensile strength</td>
</tr>
<tr>
<td>Tensile modulus</td>
</tr>
<tr>
<td>Strain to failure</td>
</tr>
</tbody>
</table>

An epoxy with a higher glass transition temperature (Tg) would be required for application in girders. The Canadian Specification for fibre-reinforced polymers, CSA S807-10 [21], specifies a minimum glass transition temperature for high-durability applications of 110 °C by DMA or 100 °C by DSC.

**Mechanical behaviour**

The mechanical stress-strain behaviour of GFRP is dependent on the particular quantity and alignment of fibres in the laminate. Properties of the laminates considered in this thesis are given in Appendix A. Theoretical uniaxial stress-strain curves can be developed for a given laminate by calculating a maximum failure strain for each lamina by micromechanics theory (see, for example, Christensen [13]), and progressively increasing the strain applied to the laminate (uniform through the thickness), eliminating the stiffness of each lamina after its failure strain is exceeded [5]. Stress-strain curves for the E-glass–epoxy laminates considered in this thesis are shown in Figure 1.1. In reality, laminae will fail gradually and sequentially, which will smooth over the discontinuities in the theoretical stress-strain curves shown.

A particular concern for GFRP is creep rupture, a phenomenon whereby long-term softening of the polymer matrix redistributes stress to the fibres, which can then cause fibre rupture and ultimate failure of the entire laminate. To avoid this danger, the service load tensile stress in GFRP should be kept below 25% of the ultimate tensile stress [20].

In general, GFRP is strong and soft. As such, the dimensioning of GFRP structures is often dictated by serviceability considerations such as deflection and vibration control, rather than ultimate strength requirements.
Figure 1.1: Stress-strain curves for the predominantly unidirectional (UD) and quadraxial (QX) GFRP laminates detailed in Appendix A.

**Bridge design precedents using GFRP**

For a thorough overview of bridges built with GFRP acting compositely with concrete, see Cheng and Karbhari [11]. Two recent projects are of particular relevance, and are briefly described below.

Acciona, a Spanish contractor, built a footbridge in the Canary Islands in 2010, using a vacuum infused GFRP box shell (fabricated in Madrid and shipped to Lanzarote by boat). This project used a hybrid laminate of glass and carbon reinforcement. Though details of this project are scarce in the literature, the size and geometrical form of the girder appears to be very similar to the guideway concept proposed in this thesis. A brief description of the project is given by Hurtado et al. [44].

Siwowski et al. [85] describe a 22 m simply-supported road bridge, recently completed in Poland, consisting of four parallel GFRP U-girders supporting a compositely acting concrete deck reinforced with GFRP bars. The girders for this project were also vacuum infused, in an 8-hour sequential infusion process. Laminate thickness in the flanges is 20 to 28 mm; the girder webs are sandwich panels consisting of 4 mm-thick laminates each side of a 15 mm-thick PVC foam core.

These projects demonstrate that the experience and capacity needed to fabricate large GFRP bridge girders through vacuum infusion already exists in industry.
1.2.2 Ultra-High-Performance Fibre-Reinforced Concrete (UHPFRC)

UHPFRC is a cementitious composite reinforced with short, straight steel fibres. The UHPFRC mix used in this study was developed by Shao and Gauvreau [81, 82] at the University of Toronto (U of T). The mix is designed to use materials that are locally available in Ontario, and it uses type GUL cement, which has a higher limestone content than general use cement, reducing its ecological impact [43]. The composition of the U of T UHPFRC mix is given in Table 1.2. The minimum compressive strength, measured by a standard 100 mm cube test, is 130 MPa. First cracking in tension is observed at approximately 7 MPa and the peak direct tensile strength is approximately 10 MPa.

Table 1.2: Composition of UHPFRC

<table>
<thead>
<tr>
<th>Component</th>
<th>kg/m$^3$</th>
<th>% weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>GUL cement (10% to 12% limestone powder)</td>
<td>724</td>
<td>31</td>
</tr>
<tr>
<td>Ground granulated blast-furnace slag</td>
<td>362</td>
<td>16</td>
</tr>
<tr>
<td>Densified silica fume</td>
<td>121</td>
<td>5</td>
</tr>
<tr>
<td>Silica sand</td>
<td>669</td>
<td>29</td>
</tr>
<tr>
<td>Polycarboxylate superplasticizer</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>Steel fibre (13 mm × 0.2 mm; 2.5 %vol)</td>
<td>196</td>
<td>8</td>
</tr>
<tr>
<td>Water</td>
<td>241</td>
<td>10</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2330</strong></td>
<td><strong>100</strong></td>
</tr>
</tbody>
</table>

Mixing & casting

The components must be mixed in a high-energy mixer to ensure a complete and uniform dispersion of fibres. The wet mix has a low viscosity, leading to a self-levelling behaviour, which is advantageous for casting flat slabs, but presents challenges for fabricating complex, voided members.

The method of casting is very important to the mechanical properties. Maya Duque and Graybeal [63] report the tensile strength in the direction of flow to be approximately twice the tensile strength in the perpendicular direction.

Mechanical behaviour

The approximate stress-strain curve for UHPFRC in the direction of flow, simplified for design purposes, is shown in Figure 1.2.
Bridge design precedents using UHPFRC

An overview of applications of UHPFRC in bridge construction is given by Wang [91]. Precast I-girders, and double-tee "π-girders" have been used in highway bridge construction, but have not yet found widespread acceptance. Box sections are less common, because of the aforementioned fabrication difficulties. The PS34 bridge on autoroute A51, in France [77], is an externally post-tensioned segmental UHPFRC box girder. Segments were cast vertically to eliminate the need for an interior box form, and to improve the fibre distribution. Segmental construction with UHPFRC is, however, not able to take full advantage of the tensile strength of the UHPFRC, as joints themselves typically are assumed to have no tensile strength.

A number of precast UHPFRC bridges have also been successfully built in Malaysia by Dura Technology [89, 90]. Segmental U-girders using a combination of internal and external post-tensioning have been used to support a cast-in-place or precast deck slab. The Dura UBG1250 girder [24], slightly modified to bring the webs into alignment with standard gauge rails, could be applied to guideway structures. This girder is studied in more detail in Chapter 3.

1.3 The GFRP–UHPFRC guideway girder concept

The cross section of the proposed guideway girders is shown in Figure 1.3, along with a typical light rail vehicle, for scale.

The numbered elements in the figure are explained as follows:
To form each girder, a GFRP shell is fabricated by vacuum infusion with the required geometry, including in-plan and elevation curvature, twist to accommodate track superelevation, and camber. To simplify the infusion and subsequent casting of UHPFRC, the shell is fabricated in two pieces: one comprising the bottom slab, webs and wings, and the other being a flat plate closing the top of the box. This shell provides 100% of the ultimate strength of the section.

Within this shell, two slabs of UHPFRC are cast. The GFRP shell is unshored when the UHPFRC is cast, such that the UHPFRC does not participate in carrying the self weight of the girder.

The webs of the box are stiffened for shear with pultruded GFRP channels glued with epoxy to the inner surface of the webs.

Inserts for attaching the continuous-welded rail to the girders are cast into the UHPFRC top slab. Placement of these inserts is precisely controlled to ensure that the rails can be easily installed, within the required tolerance, on site. The unshored casting of the UHPFRC is helpful in this regard, as the deflection under self-weight does not need to be calculated in order to set the locations of the rail attachments.

Inserts for attaching the barriers are also cast into the UHPFRC top slab. Barriers do not participate in the longitudinal stiffness or strength of the girder. Aside from providing the required impact load capacity, their design is primarily driven by aesthetics. They are shown here as built.
up from pultruded GFRP sections.

6. A maintenance and emergency walkway formed of GFRP planks is supported between the two girders.

Diaphragms would be required, especially in curved girders, to enhance the torsional stiffness. These could be provided by pultruded bracing elements, or by a diaphragm wall. End diaphragms would be formed by filling the box, over a short length, with UHPFRC.

Further details of the proposed girder are presented in Chapter 3.

1.3.1 Combining UHPFRC and GFRP

The benefit of GFRP is evident from the comparison of materials in Table 1.3: it has, by far, the highest strength-to-weight ratio of any construction material, and it also has a good stiffness-to-weight ratio—approximately 75% of the value for steel. This suggests that GFRP could be used to design extremely light guideway girders. However, GFRP has a number of drawbacks:

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\gamma$ kN/m$^3$</th>
<th>Ultimate tensile strength $\sigma_{tu}$ MPa</th>
<th>Ultimate compressive strength $\sigma_{cu}$ GPa</th>
<th>Elastic Modulus $E$ $\times 10^3$ m</th>
<th>Tensile strength to weight ratio $\sigma_{tu}/\gamma$ $\times 10^6$ m</th>
<th>Stiffness to weight ratio $E/\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>77</td>
<td>400</td>
<td>400</td>
<td>200</td>
<td>5.2</td>
<td>2.6</td>
</tr>
<tr>
<td>Concrete</td>
<td>23.5</td>
<td>4</td>
<td>80</td>
<td>35</td>
<td>0.17</td>
<td>1.5</td>
</tr>
<tr>
<td>UHPFRC</td>
<td>25.5</td>
<td>10</td>
<td>140</td>
<td>45</td>
<td>0.4</td>
<td>1.8</td>
</tr>
<tr>
<td>GFRP</td>
<td>20.5</td>
<td>450</td>
<td>450</td>
<td>42</td>
<td>22</td>
<td>2.0</td>
</tr>
</tbody>
</table>

1. The useable tensile stress is significantly lower than the ultimate tensile stress. The Canadian specifications for FRP materials in construction [21], specify a maximum service load stress of 25% of ultimate, and a fatigue stress of less than 35% of ultimate.

2. The material cost of GFRP is relatively high.

3. Fabrication can be difficult for thick GFRP parts, especially those with complex geometrical form.

In the proposed guideway girder design, these drawbacks are overcome, to some extent, by combination with UHPFRC. In the bottom slab, the thickness of GFRP can be kept manageable by adding a compositely acting slab of UHPFRC. The ductility of the UHPFRC in tension adds some measure of softening (i.e. reduction in flexural stiffness) to the flexural response at ultimate limit states. Under service
loads, the presence of the UHPFRC adds significantly to the stiffness to the section, keeping the stresses in the GFRP low, while the added softening prior to failure improves the ultimate capacity utilization. In the top slab, UHPFRC can make up most of the slab thickness, without any loss of flexural capacity due to the high compressive strength of UHPFRC, and the fact that the neutral axis shifts up as the bottom slab cracks. The top slab UHPFRC also provides a practical means of connecting the rails and barriers to the structure. The reduction in quantity of GFRP relative to an all-GFRP design significantly reduces the cost, and despite the added weight of the UHPFRC, the girder weight is still very light.

The higher strength-to-weight and stiffness-to-weight ratios of UHPFRC relative to ordinary concrete also suggest the possibility for a light weight guideway design using steel-prestressed UHPFRC instead of conventional steel-prestressed concrete. The recent work described by Voo et al. [90] follows this path. However, the reduction in weight offered by UHPFRC is significantly less than is possible with GFRP, especially considering the following challenges to designing with UHPFRC:

1. As mentioned above, the casting method of UHPFRC significantly impacts the tensile strength, making it difficult to cast complex shapes in such a way as to achieve the full potential tensile capacity of UHPFRC.

2. The random steel fibre reinforcement of UHPFRC cannot be relied upon as part of a primary load path. As a result, prestressing is required. The additional material needed for anchorages and deviators of external post-tensioning, and cover requirements for internal post-tensioning adds weight.

3. Relatively thick webs, especially if shear keys must be accommodated, also add weight.

4. If webs and slabs are made as slender as they could be by capacity demand alone, they would often be so slender that concerns could be raised regarding the resilience of the girder to accidental impacts during transportation and erection.

Each of these concerns are, to some extent, addressed by combination of UHPFRC with GFRP in the proposed guideway girder design. The GFRP can act as a stay-in-place form to simplify fabrication, and the need to cast only two flat slabs of UHPFRC means that the optimal fibre orientation can be achieved. The continuous tensile reinforcement provided by the GFRP means that prestressing is not required, which greatly simplifies the detailing of the girder and the process of construction. The webs are stiffened GFRP plate, which can carry shear forces in an efficient and reliable manner. Finally, the high strength of the GFRP provides robust protection against accidental impacts.
While many other researchers have explored structural components using GFRP acting compositely with conventional concrete [11], only a few research projects to date have explored the potential of GFRP–UHPFRC. Keller et al. [50] tested GFRP–UHPFRC sandwich panels with a lightweight concrete core. El-Hacha and Chen [25] have tested hybrid composite beams consisting of pultruded GFRP sections acting compositely with UHPFRC and Carbon-Fibre Reinforced Polymer (CFRP). These works will be discussed in more detail in Chapter 4.

1.4 Chapter overview

The structure of this thesis is as follows: Chapter 2 presents the proposed girder design in further detail, tracing its emergence from a conceptual design process to its validation through design checks using experimentally-determined material properties. Of vital importance to the success of the proposed design is a reliable means of connecting UHPFRC and GFRP. This is addressed in Chapter 3, where the invention, fabrication, and testing of a connection mechanism enabling reliable composite action between UHPFRC and GFRP is described in detail. Chapters 4 and 5 present the results of experimental investigations to determine material properties for UHPFRC and GFRP–UHPFRC that are needed for design validation: Chapter 4 describes an experimental investigation to assess the potential of UHPFRC to provide beneficial levels of damping, while Chapter 5 presents the results of an experimental exploration of the fatigue life of UHPFRC and GFRP–UHPFRC. Chapter 6 describes a dynamic analysis method suitable for lightweight guideway girders where the moving inertia of the train cannot be neglected. The method described is a frequency-domain approach that has not been previously applied in civil engineering. Finally, Chapter 7 summarizes the conclusions and contributions of the work.
Chapter 2

Guideway Structures: Design
Considerations and State-of-the-Art
Review

2.1 Design considerations for guideway structures

The design of guideway structures is constrained and influenced by a set of challenges and functional requirements particular to this type of structure. These considerations are outlined in the following subsections.

2.1.1 Functional requirements

An important characteristic of guideway structures is the fact that the transverse location of service live loads is fixed: it is always on the rails. This means that the transverse bending demand in the top slab of a guideway girder is much lower than what is typically required for a highway bridge. However, susceptibility to fatigue is increased, since each passing train produces the same stress fluctuations throughout the girder.

The need to control fatigue stresses increases the importance of dynamic assessment of guideway girders. The dynamic behaviour is also important from the point of view of passenger comfort: vibrations felt by passengers are intensified by poor dynamic response of the guideway. The requirement for maintaining passenger comfort is effectively the only serviceability requirement for guideway struc-
tures, as the stiffness demanded by this requirement will generally exceed the stiffness required to avoid unsightly deflections or cracking.

Related to vibration control considerations is the need to install the rails within a tight tolerance. As will be shown in Chapter 7, track irregularity is a significant factor in the deterioration of passenger comfort. Misalignment of rails also leads to increased noise, and faster wear on the rails. The method by which the rails are attached to the structure is an important design decision that must be made early on.

Most transit lines consist of two parallel tracks, one for each direction of travel. An important design choice is whether to support both tracks on a single girder, or to have parallel girders each supporting a single track.

Finally, guideway structures typically carry long transit lines which include a variety of tangent and curved track alignments, as well as changes in elevation. In curved sections of track, superelevation (transverse inclination) of the track is required to reduce centrifugal forces on the train as it follows the curve. The need to accommodate these varying track geometries is an important factor driving the choice of structural system.

2.1.2 Economics

Guideway structures are typically part of a revenue-generating system. This is in contrast to most bridges, which are often provided by governments for free public use. Therefore, in the construction of transit systems, there is a large incentive to minimize the construction time in order to start generating fare revenue as soon as possible. This creates a preference for prefabrication, shifting as much work as possible off-site.

The selection of elevated transit over subway is often driven by economic considerations. However, even though guideway structure is significantly lower in cost than tunneling, the guideway is typically the largest cost in an elevated transit line construction project.

2.1.3 Aesthetics

Many bridges are highly visible elements of the public environment, but because public transit is obviously needed most in populated areas where many people live and work, guideway structures are constant visual fixtures in the daily lives of many people. This is in contrast to highway overpasses, for instance, which are also seen by many on a daily basis, but only in a fleeting way. The unavoidable visual impact of a guideway structure on its neighbourhood means that elevated transit construction
projects are often met with fierce opposition from local residents who fear a permanent negative visual change to their environment. Intensifying the scrutiny to which the aesthetics of a proposed guideway structure is subjected is the fact that elevated transit construction is very often considered alongside the option of subterranean transit, which of course has no visual impact at all. Whereas for many bridges the baseline for aesthetics can be set by a cheap and utilitarian design option, for guideway structures the aesthetic baseline is no bridge at all.

Good aesthetic design can therefore be valuable as a means of achieving buy-in from members of the public who are anxious about changes to their neighbourhood. However, given that the main reason for considering elevated transit in the first place is to keep costs down, guideway structures are pushed toward an aesthetic that is derived from structural efficiency and thoughtful detailing, rather than ornamentation or exhibitionism.

2.2 Structural systems for guideway

A variety of structural systems have been used for guideway structures over the years. Historically, steel and masonry were the most common material choices; more recently, precast concrete has seen widespread use. Since masonry construction is generally not economical for new construction, the focus here will be restricted to steel and concrete structural systems.

The trade-offs between different structural systems are best discussed with reference to actual structures. To this end, a selection of cross sections of steel, cast-in-place, and precast concrete guideway structures built around the world are shown in Tables 2.1, 2.2, 2.3 and 2.4. Typical span lengths, span-to-depth ratios, and weights per metre of track are given, where such information was available, as comparators.

2.2.1 Steel

Many early elevated transit systems, as were constructed in New York, Chicago, Paris, and Berlin, used riveted steel plate or truss girders. These structures were sometimes very utilitarian and minimalist, such as the New York Ninth Ave cross section shown in Table 2.1. In other cases, however, they typified an aesthetic, as discussed above, of careful detailing and efficient design, resulting in structures which were and are points of civic pride. A good example of this is Line 2 of the Berlin Hochbahn, pictured in Figure 2.1, and shown in cross section in Table 2.1.

Modern steel construction is typified by the Docklands Light Rail and Busan-Gimhae Transit structures, also shown in Table 2.1. For the Busan-Gimhae structure, poor geotechnical conditions were
present throughout the corridor of the transit line. In such a case, longer spans and a lightweight structure are both advantageous. On the Docklands Light Rail project, speed of construction and a preference for a slender aesthetic [78] are cited as having dictated the choice of steel.

The above examples show that steel structural systems can be well suited for particular conditions, and can achieve a worthy aesthetic quality. However, concerns regarding durability and noise (due to the low damping of steel structures) present challenges for modern steel construction, which have led to a preference for concrete guideway systems.
Table 2.1: Steel structural systems.

<table>
<thead>
<tr>
<th>Location</th>
<th>Year</th>
<th>Longest Span (m)</th>
<th>Spans</th>
<th>L/d</th>
</tr>
</thead>
<tbody>
<tr>
<td>New York City</td>
<td>1875</td>
<td>10</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td>Ninth Avenue Metro</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Berlin Hochbahn Metro</td>
<td>1920</td>
<td>9.5</td>
<td></td>
<td>10</td>
</tr>
<tr>
<td>Docklands Light Rail</td>
<td>1987</td>
<td>12.5 to 26</td>
<td></td>
<td>17</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Busan-Gimhae Light Rail</td>
<td>2011</td>
<td>50</td>
<td></td>
<td>20</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.2.2 Cast-in-place concrete

Since guideway structures are often built in populated urban or suburban neighbourhoods, the cast-in-place on falsework method of construction is not typically suitable. The extension of the Rotterdam Metro Line B to Nesselande, completed in 2005, is a rare example of a cast-in-place system. From photographs it appears that the elevated portion of this line follows a relatively undeveloped canal bank, a unique condition that perhaps allowed a cast-in-place solution to be viable. The cross section is shown in Table 2.2. The lack of prestress dictates a very heavy structure, as measured by a weight per track (the average weight per metre length of superstructure divided by the number of tracks supported, in this case 2) of 78 to 115 kN/m.

Table 2.2: Cast-in-place concrete structural systems.

<table>
<thead>
<tr>
<th>Rotterdam Line B Metro 2005 [52]</th>
<th>- No prestressing</th>
</tr>
</thead>
<tbody>
<tr>
<td>- 20 m spans</td>
<td>- 3-span continuous</td>
</tr>
<tr>
<td>- $L/d = 9$ to $11$</td>
<td>- Weight per track = 78 to 115 kN/m</td>
</tr>
</tbody>
</table>

2.2.3 Precast concrete

In guideway construction, precast concrete is far more common than cast-in-place concrete, because of the aforementioned incentives to minimize on-site work and neighbourhood disruption, and to speed up the erection process as much as possible. Precast systems fall into two broad categories, described in further detail in the following sections: those using short precast segments from which the superstructure is assembled on-site (a process known as precast segmental construction), and those using full-length precast girders. In recent years, precast segmental has become the dominant technology for guideway construction, for a number of compelling reasons. However, the system is not without its drawbacks, and building with full-length precast girders offers its own advantages.

Precast segmental construction

In precast segmental construction, short segments are cast at a precast plant and are post-tensioned together on-site. The casting typically follows the “match-casting” method, whereby one cross sectional face of each segment is formed by the matching face of the preceding segment of the span. Match-casting ensures a perfect fit between adjacent segments, and allows the forms for each segment to be precisely set, based on a survey of the previous segment, in order to ensure the proper geometry for the final
assembled girder. The length of segments is limited by the maximum cargo width for road transport, which is typically 3 m.

Erection can follow the balanced cantilever method, where segments are added to growing cantilevers on each side of the piers before casting small closure pours at the midspans; or the span-by-span method, in which an overhead gantry temporarily supports all the segments of one span before they are post-tensioned together and set into place. The span-by-span approach is the most common for guideway construction, as it is a faster and more straightforward method.

Table 2.3: Precast segmental concrete structural systems.

<table>
<thead>
<tr>
<th>Location</th>
<th>Remarks</th>
<th>Spans</th>
<th>L/d</th>
<th>Weight per track</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bangkok BTS Light Rail</td>
<td>1999 [6]</td>
<td>30 to 35 m</td>
<td>15 to 18</td>
<td>≈ 48 kN/m</td>
</tr>
<tr>
<td>Lille Line 2 Metro</td>
<td>1999</td>
<td>33 m</td>
<td>21</td>
<td>≈ 49 kN/m</td>
</tr>
<tr>
<td>Singapore Bukit Panjang</td>
<td>1999 [55]</td>
<td>15 to 34 m</td>
<td>7 to 16</td>
<td>≈ 45 kN/m</td>
</tr>
<tr>
<td>Vancouver Millennium Line</td>
<td>2000</td>
<td>37 m</td>
<td>18</td>
<td>≈ 37 kN/m</td>
</tr>
<tr>
<td>Hong Kong West Commuter</td>
<td>2002 [15]</td>
<td>35 m</td>
<td>14</td>
<td>≈ 80 kN/m</td>
</tr>
<tr>
<td>London Air Link Light Rail</td>
<td>2005 [3]</td>
<td>37 m</td>
<td>15</td>
<td>≈ 56 kN/m</td>
</tr>
<tr>
<td>Dubai Metro</td>
<td>2009 [86]</td>
<td>28 to 36 m</td>
<td>14 to 18</td>
<td>≈ 70 kN/m</td>
</tr>
</tbody>
</table>
The main advantage of segmental construction is that a wide range of curved and/or twisting girder geometries can be accommodated without any significant change to the construction process. Curvature and twist are created simply by varying the relative position of the preceding segment to the forms in the match-casting process, resulting in geometrically similar segments that can be transported and erected in a standardized way. Segmental construction has seen widespread application in highway bridge construction, and, as such, in most parts of the world there are contractors who possess the necessary experience and equipment to undertake the work.

Table 2.3 shows seven representative precast segmental guideway cross sections. Four of the seven, the Bangkok, Singapore, Vancouver, and London cross sections, are single-cell boxes. This standard cross section has been used in many other guideway structures in addition to these four, as well as in countless highway bridges around the world. The Lille and Dubai guideways were clearly designed with aesthetics foremost in the design process, leading to less common design choices such as continuous spans in Lille, and a very heavy, shallow cross section in Dubai. The Hong Kong viaduct differs in that it supports a commuter rail rather than a rapid transit line. This means it is likely subject to higher live loads, and stricter vibration limits due to faster, heavier trains, requiring a heavier girder.

Some of the limitations of segmental construction are evident from inspection of Table 2.3. For example, of the seven structures illustrated, six are dual-track (meaning a single structure carries the tracks for both directions of travel). A dual-track design is nearly ubiquitous because it essentially cuts the erection time in half, since only a single girder must be assembled for each span. A downside to dual-track girders, however, is that they dictate that stations be designed with separated platforms for each direction of travel, thus losing the flexibility to choose central-platform stations in situations where they may be preferred.

Another trend amongst the segmental designs in Table 2.3 is that six out of seven are simply-supported. Simple spans are preferred because they greatly simplify the span-by-span erection process; to create continuity across spans requires closure pours on each side of the pier segment, and a more complex post-tensioning sequence involving multiple spans. However, simple spans are inherently less efficient than continuous spans, and require deeper cross sections.

Segmentally constructed bridges come with another source of inefficiency in that they offer no possibility of continuous passive steel reinforcement. This means that there can be no tolerance of tensile stress in the concrete. The result of this stricter limit for the prestress design is that span-to-depth ratios for segmental bridges, which range from 14 to 18 in Table 2.3, are typically lower than for comparable precast girders or cast-in-place prestressed concrete designs.
Precast girder construction

Full-span precast girders can overcome many of the aforementioned limitations of segmental construction, though working with precast girders presents challenges of its own. Table 2.4 shows three guideway designs using full-span precast girders. The Vancouver and Delhi structures are fully precast; the Toronto structure consists of a cast-in-place concrete deck acting compositely with precast concrete I-girders.

Table 2.4: Precast concrete girder structural systems.

<table>
<thead>
<tr>
<th>Structure</th>
<th>Description</th>
<th>Details</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vancouver Expo Line</td>
<td>Light Rail</td>
<td>Fully precast, 33 m spans, 2 or 3-span continuous, L/d = 24, Weight per track ≈ 25 kN/m</td>
</tr>
<tr>
<td>Delhi Metro</td>
<td>2010 [84]</td>
<td>Fully precast, 25 m spans, Simply supported, L/d = 13, Weight per track ≈ 52 kN/m</td>
</tr>
<tr>
<td>Toronto UP Express</td>
<td>Commuter Rail</td>
<td>Cast-in-place deck, 30 to 38 m spans, 2-span continuous, L/d = 11 to 15, Weight per track ≈ 64 kN/m</td>
</tr>
</tbody>
</table>

In general, a fully precast girder design offers the following benefits over segmental construction:

- Faster erection, without the need for a specialised gantry: girders are simply picked up with a crane and set into place.

- Girders can be pretensioned instead of post-tensioned, which simplifies the on-site work, and eliminates the need for bulky anchorages.

- Stations can be designed with central or separated platforms.

The primary challenge with precast girders is to keep the size and weight of the girders manageable and to accommodate curvature and twist in rational and aesthetically acceptable ways.

The two fully precast design examples shown in Table 2.4 are both single-track, in order to keep the size and weight of the girders manageable. The same road transport size limitation that dictated a 3 m length for segments dictates a maximum width for girders. The Vancouver girders are 3 m wide; while the Delhi girders are wider than 3 m (the benefits of this additional width, discussed later, evidently outweighed the additional cost of transporting oversized loads), a dual-track girder would need to be at least 7.5 m wide, and would certainly be too large to transport by road.
The combined system of precast girders with cast-in-place deck used for the Toronto Union-Pearson (UP) Express is a nearly ubiquitous solution for short to medium span highway bridges in Ontario. This is evidence of the attractiveness of full-span precast girder construction. Despite the popularity of this structural system, it suffers from a number of problems:

- Standard precast I-girders are available only as straight members, which makes accommodating curves awkward. The usual solution is to cast a curved deck on straight girders that kink at the piers. This solution inevitably introduces transverse bending into the deck due to misalignment between the girder centre lines and the rails, and yields an aesthetically inferior structure.

- Although the cast-in-place deck provides a means of accommodating curves, the large quantity of cast-in-place concrete in this system diminishes the potential construction acceleration offered by the use of precast concrete. Furthermore, it requires the involvement of additional subcontractors in the project, adding complexity and risk to the project management.

- Typically, the deck is not prestressed; if continuity is required across spans, this lack of prestress creates an onerous reinforcement requirement to control cracking in the negative moment regions of the deck.

The Delhi Metro design (which has been replicated for a number of other projects around the world [65]), uses U-shaped girders which integrate the barriers as part of the load-bearing structure. This seems like an efficient way to achieve a more slender girder, but without a balanced pair of top and bottom flanges with which to develop moment, girders of this form struggle to provide sufficient stiffness. This same problem is evident in the cross section of the Dubai Metro, shown in Table 2.3, which is one of the heaviest segmental girders in that table. For the Delhi Metro, stiffness was achieved by shortening the spans and widening the girders. These two design choices simultaneously provide a solution to the problem of curves, as shorter spans allow for tighter curvatures to be accommodated with straight segments, and the additional width provides room for curved track alignment to deviate from the girder centreline. A downside to supporting curved rail on straight girders is that thermal expansion/contraction of the rails and girders will produce larger lateral forces on the girder than if both track and girder have the same curvature.

The design of the Vancouver Expo Line used a box girder cross section. The girders were precast in a segmented formwork which could be arranged for casting with the required curve and/or twist. Straight girders have a combination of pretensioned and post-tensioned prestressing, while curved girders are entirely post-tensioned. The box section, being more efficient than a U- or I-girder section, allows an even higher span-to-depth ratio than the steel girder designs.
2.2.4 Aesthetic comparison

As discussed in the introduction, the aesthetics of guideway structures matters. Of particular interest here is the relative aesthetic merit of precast segmental vs precast girder guideway. The guideway structures of the Vancouver SkyTrain offer an interesting case study for this comparison. This section presents a brief background of the SkyTrain construction history, followed by a visual comparison and critique of the SkyTrain guideways.

The first elevated line in Vancouver, called the Expo Line, was completed in 1986, to coincide with the World Exposition on Transportation and Communication, which was held in Vancouver in that year. Since then, three subsequent expansions have been completed: in 2002, the Millennium Line was opened; in 2009 the Canada Line added service to Vancouver airport; and in 2016 the Evergreen Line, an extension of the Millennium Line, was completed. As discussed in the previous section, the Expo Line used precast concrete box girders, while all of the more recent expansions have used precast segmental construction.

Figures 2.2 and 2.3 show comparable views of the Expo, Millennium and Canada Line structures, from common viewpoints. In Figure 2.2, where the structures are viewed from the side, similar to a standard elevation view, the relative slenderness of the structures is striking. As shown in Table 2.3, the Millennium Line span-to-depth ratio is approximately 18 (typical of precast segmental construction), while the Expo girder span-to-depth ratio is approximately 24. The superior detailing of the Expo guideway is also evident in these views. Compare the distinct and unbroken soffit line created by the dapped girder ends at expansion piers of the Expo Line with the visually prominent bearings of the segmental designs. The local widening of the box at the ends of the Canada Line girders is a further aesthetic deterioration in this regard.

As shown in Figure 2.3, the structural slenderness is less evident in a view from beneath or close beside the structure. From this visual perspective, the girder width and overall height and slenderness of the piers are more significant than the girder slenderness, and the detailing of the girder/pier interface is perhaps even more noticeable.

2.2.5 Opportunities for innovation

The preceding discussion has shown that for guideway structures, two design considerations are of the utmost importance:

1. Curved track alignment must be accommodated by the girders in a rational way.
Figure 2.2: Top: Expo Line (photo by Paul Gauvreau); Middle: Millennium Line (Google Street View, June 2016); Bottom: Canada Line (Google Street View, June 2016)
Figure 2.3: Top: Expo Line; Middle: Millenium Line; Bottom: Canada Line. (Google Street View, June 2016)
2. The erection process must be accelerated as much as possible, while minimizing the disruption to
neighbourhoods during construction.

To satisfy both criteria, fully precast girders would be ideal, if only they could be more easily fab-
ricated, transported, and handled. The current dominance of precast segmental systems seems to stem
from a compromise between meeting the above two requirements and standardizing the fabrication, trans-
portation, and erection procedures. An innovative precast girder, with reduced weight and a streamlined
process for curved fabrication, would offer many marketable advantages over current state-of-the-art seg-
mental construction methods, especially considering the aesthetic and station-design limitations of that
technology.

UHPFRC and GFRP provide the higher material performance demanded by such a girder, and through
their combination, a feasible fabrication process can be designed. The next chapter presents the details
of the innovative precast GFRP–UHPFRC girder design, which is the focus of this thesis.
Chapter 3

GFRP–UHPFRC Guideway Girders

3.1 Introduction

A detailed cross section of a GFRP–UHPFRC girder suitable for straight 30 m simply-supported spans is shown in Figure 3.1, expanding on the elements of the cross section which are described in Section 1.3.

The closure plate can be longitudinally discontinuous, to simplify fabrication, without adversely affecting the girder performance. Shear connection between the UHPFRC and GFRP, a key detail in the design, is provided by a novel stud-and-texture shear connection developed as part of this work, and discussed in detail in Chapter 4.

The girder shown has a self-weight of 1860 kg/m, so a full 30 m girder would weigh 55.7 tonnes. Compare this with the Expo girders, which weighed 2600 kg/m, corresponding to 78 tonnes for a 30 m girder, or the Delhi Metro U-girders which weighed approximately 120 tonnes per 25 m girder. This 30% reduction in weight relative to the Expo girders will have benefits for transportation, handling, and erection, as well as for pier and foundation design.

3.2 Fabrication

The fabrication process for the GFRP–UHPFRC girder is split into two distinct stages: fabrication of the GFRP shell and casting of the UHPFRC are completely independent activities that could be carried out in different factories by different fabricators. This is important, since a fabrication company with the industrial expertise and equipment needed for working with both GFRP and UHPFRC is currently nonexistent. The fabrication process is as follows:

1. The GFRP shell is vacuum infused, using a segmented reusable mould that can be set to the
Figure 3.1: GFRP–UHPFRC guideway girder cross section.
Figure 3.2: Longitudinal section at pier.
required geometry of the girder. Closure plates are all identical, matching in length to the straight segments of the shell mould. The closure plates are infused on a separate flat plate mould. The stud-and-texture shear connectors are formed simultaneously with the infusion of the shell and closure plates, as described in Chapter 4.

2. Pultruded GFRP web stiffeners and diaphragms are glued to the inside of the shell.

3. GFRP shells and closure plates are shipped to a precast plant for addition of the UHPFRC.

4. At the precast plant, the GFRP shell is supported at its bearing points and the bottom UHPFRC slab is cast into the unshored shell. Because this is a simple flat slab, a directional casting method can be used to bias the fibre orientation in the longitudinal direction.

5. Closure plates are epoxied into place to close the box.

6. Inserts for barriers, running rails, and linear induction motor (LIM) rails are positioned and held in place from above by a support frame.

7. With the girder remaining in the unshored condition, the top slab of UHPFRC is cast, along with the pier diaphragms, again with a directional casting method producing a longitudinal bias in fibre orientation.

8. The completed girders are now ready to be shipped to site and erected.

3.3 Conceptual development

The GFRP–UHPFRC girder design presented above arose out of an extensive conceptual design process beginning with the conviction that the high-performance properties of UHPFRC could enable a competitive precast girder solution for guideway structures. This section provides a brief overview of the conceptual design path from this starting point to the proposed GFRP–UHPFRC design.

The first girder cross section considered is shown in the upper left of Figure 3.3. This is a prestressed UHPFRC girder, where the tendons have been moved out of the webs to the interior of the box to allow the web thickness to be minimized. As discussed in more detail in the next section, the flexural behaviour of this cross section is promising. However, it suffers from a number of problems:

1. As has been mentioned, fabrication of a UHPFRC box section is difficult due to the low viscosity of wet UHPFRC.
2. The slender webs and slabs of this cross section, reinforced only with randomly distributed short fibres, would be susceptible to damage due to accidental impacts, especially during construction handling.

3. The light weight of the girder means that the live load to dead load ratio is high. An efficient use of prestressing is difficult to achieve in this situation, as the prestress force becomes restricted to a more narrow zone near the centroid of the section.

To overcome these problems, it was determined that UHPFRC would be better used in combination with a second material. Steel was eliminated from consideration, as it would introduce a maintenance burden by requiring painting, and, in general, would compromise the outstanding durability of the UHPFRC. Combination with cross-laminated timber (CLT) was considered, as shown in the top right of Figure 3.3. Although the CLT can add additional reinforcement to the UHPFRC, it adds a substantial amount of weight, and the cross section shown would be even more difficult to build than the plain UHPFRC box. The goal of realizing the potential for a very light weight girder offered by UHPFRC led to the consideration of GFRP, which possesses a very high strength-to-weight ratio. The initial concepts used pultruded GFRP (bottom left of Figure 3.3), or filament-wound GFRP (bottom right of Figure 3.3). However, neither of these fabrication methods offered a rational way to customize the geometry for curved girders, and an effective means of connecting the GFRP to the UHPFRC, to achieve composite action, was not readily evident. The vacuum-infused GFRP design presented above offered a way of overcoming these difficulties.
3.4 Comparative study of flexural behaviour

The advantages of the proposed girder design are evident through a comparison with three other designs, shown in Figure 3.4. The three other cross sections are described below.

**Vancouver Expo girders.** This is not a direct comparison, as the Expo girders are two or three-span continuous. As such, the flexural stiffness and strength of the cross-section can be lower. However, as these are existing girders in an operational transit system, they serve as a valuable point of reference. The girder weight of 25.5 kN/m was shown in Chapter 2 to be low by contemporary standards.

**Dura Technology UBG1250.** This is a precast UHPFRC girder that is currently commercially available in Malaysia. The cross-section has been slightly modified to bring the webs into alignment with the rails of a standard gauge track. This is achieved by reducing the slope of the webs, and has very little effect on the vertical bending behaviour of the section. The girder is a precast U; a 150 mm thick UHPFRC slab has been added, matching the top slab thickness of Dura’s BBG4000 precast box girder [24]. The girder weight is effectively the same as the Expo girders. The main benefit of the UHPFRC in this design is therefore not to achieve lightness, but to allow for simple spans of nearly the same slenderness as the Expo girder continuous girders, thereby simplifying the erection process.

**An all-GFRP cross section.** This cross section would provide the maximum possible structural lightness. However, it requires a 120 mm thick bottom slab of GFRP in order to achieve the required stiffness. This large amount of GFRP would lead to a very expensive girder. The large variation in GFRP thickness would also present significant challenges for fabrication.

The moment-curvature diagrams for each cross sections in vertical bending are shown in Appendix B.

The proposed GFRP–UHPFRC girder achieves a 30% reduction in weight relative to the Expo girders and current state-of-the-art UHPFRC girders, while keeping the quantity and thickness of GFRP manageable. The initial uncracked stiffness is higher than necessary for vibration control, but long-term degradation of the UHPFRC under fatigue stress decreases the stiffness. The quantity of UHPFRC in the bottom slab is thus dictated by fatigue considerations, as described in section 3.5.6.
Chapter 3. GFRP–UHPFRC Guideway Girders

(a) Vancouver Expo Line
   Prestressed concrete full-span girders
   \( w_{sw} = 25.5 \text{ kN/m} \)
   \( EI = 6020 \text{ MNm}^2 \)
   \( L = 33 \text{ m (2 span continuous)} \)
   \( L/d = 24 \)
   Total girder weight = 85 700 kg

(b) Dura Technology UBG1250
   Prestressed segmental UHPFRC
   \( w_{sw} = 26.1 \text{ kN/m} \)
   \( EI = 13 \ 500 \text{ MNm}^2 \)
   \( L = 30 \text{ m (simply supported)} \)
   \( L/d = 21.4 \)
   Total girder weight = 79 800 kg

(c) GFRP Girder with Composite
    UHPFRC Deck
    \( w_{sw} = 12.7 \text{ kN/m} \)
    \( EI = 12 \ 100 \text{ MNm}^2 \)
    \( L = 30 \text{ m (simply supported)} \)
    \( L/d = 17.1 \)
    Total girder weight = 38 800 kg

(e) GFRP with Composite UHPFRC
    \( w_{sw} = 18.2 \text{ kN/m} \)
    \( EI_l = 18 \ 450 \text{ MNm}^2 \)
    \( EI_f = 12 \ 000 \text{ MNm}^2 \)
    \( L = 30 \text{ m (simply supported)} \)
    \( L/d = 17.1 \)
    Total girder weight = 55 500 kg

Figure 3.4: Comparison of possible guideway girder cross sections with Expo Line girders.
3.5 Design validation

3.5.1 Overview

Table 3.1, below, identifies the main aspects of structural assessment required before the proposed girder could be close to application in public infrastructure. Appendices A, B, C, and D present details of the assumed material and cross-sectional properties and the specified loading, using the Vancouver Millennium Line design specification [14] as a guide. The following sections provide simple calculations and discussion to demonstrate that the capacity of the proposed design is sufficient to meet these load demands. Situations in which more detailed analysis is required, and where modifications to the design could be necessary are highlighted. Aspects of the design that are outside the scope of this thesis are flagged for future work.

Table 3.1: Overview of design considerations

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Approximate consideration of horizontally curved spans

Design of curved girders is outside the scope of this thesis, as the design for curved spans would need to be treated on a site-specific basis. However, it is important that the proposed girders be able to accommodate curved spans without requiring significant changes to the design. Therefore, an approximate preliminary check of the torsional behaviour is warranted.

The torsion arising from the longitudinal bending of horizontally curved girders can be approximately accounted for by approximating the longitudinal moment as a force couple in the top and bottom flanges of the box, and calculating the horizontal forces required to deviate these forces through the curve (this is sometimes referred to as the “M/R” method). If it is also assumed that there is torsional fixity at both ends of the curved girders, and the torsional force is resisted entirely by a St. Venant shear flow around the closed box section, then the torsional shear flow arising from the curvature is given by

\[ q = \frac{M_{L/2}L}{6RA_{\text{box}}} \]  

where \( R \) is the radius of horizontal curvature, and \( A_{\text{box}} \) is the area enclosed by the box. For the proposed cross-section, \( A_{\text{box}} \approx 1.9 \text{ m}^2 \). Taking the minimum radius of curvature to be 35 m [64], and using the maximum factored midspan moment \( M_{L/2} = 8170 \text{ kNm} \) gives \( q = 614 \text{ kN/m} \), equivalent to a vertical shear force in each web of \( V = 850 \text{ kN} \).

3.5.2 Sectional flexure and shear ultimate capacity

The moment-curvature response of the girder is shown in Appendix B. The unfactored ultimate capacity of the section is 39 600 kNm, nearly five times the factored moment demand of 8170 kNm. Thus even after applying the most conservative reduction factors for GFRP, the ultimate capacity for vertical bending is more than sufficient.

Assuming the entire shear force is carried by the webs, the maximum factored shear of 1256 kN corresponds to a shear stress in the webs of approximately 25 MPa, which is again well below the 70 MPa shear strength of the GFRP laminate. In maximally curved spans, given the assumptions outlined above, the factored shear force is 1480 kN per web, which assuming a reduced shear strength of 50 MPa requires a minimum thickening of the webs to approximately 22 mm.
3.5.3 Web buckling

Shear buckling

Using the derivation of the critical buckling stress for an isotropic plate in shear given by Timoshenko and Gere [87], and a reduction factor for GFRP of \( \phi_{\text{GFRP}} = 0.75 \), the critical shear stress in a web causing shear buckling is:

\[
\phi_{\text{GFRP}} \tau_{sc} = \phi_{\text{GFRP}} \frac{k_s \pi^2 D}{b^2 t_w}. \tag{3.2}
\]

where \( b \) is the smaller dimension of the plate, \( t_w \) is the plate thickness, \( D \) is the isotropic flexural stiffness, and the factor \( k_s \) is given by the expression

\[
k_s = 5.35 + 4 \left( \frac{b}{a} \right)^2, \tag{3.3}
\]

where \( b \) is again the smaller dimension of the plate, and \( a \) is the larger dimension. The rectangular web panel is bounded horizontally by the top and bottom slabs and vertically by stiffeners. For relatively tall, slender, webs, the spacing between stiffeners, \( x_s \), will be smaller than the web depth, so \( a = h_w \), and \( b = x_s \).

For the specified quadraxial fabric layup (see Appendix A), the longitudinal flexural stiffness is \( D_{11} = 16.4 \text{kNm} \), and the transverse flexural stiffness is \( D_{22} = 9.1 \text{kNm} \). Conservatively, the weaker of these stiffnesses, \( D = D_{22} \), is used. This partially offsets the unconservative assumption of fully rigid restraint provided by the stiffeners.

Assuming that the entire shear force is carried in the webs, the factored shear stress is \( 1256 \text{kN/(}2 \cdot 1.37 \text{m} \cdot 0.018 \text{m}) = 25.5 \text{MPa} \). Setting \( \tau_{sc} \) equal to this value, and combining equations 3.2 and 3.3 leads to a quadratic equation for stiffener spacing. The minimum spacing at the ends of tangent girders calculated in this way is 1.1 m.

A more detailed analysis taking account of the anisotropy of the GFRP laminate is described by Nemeth [70]. The more detailed analysis should allow even fewer stiffeners.

Horizontally curved spans

For curved spans, assuming the web is thickened to 22 mm, the factored shear stress in the outer web is \( \tau_{sf} = 49 \text{MPa} \). This dictates that the maximum stiffener spacing at the ends of tightly curved girders would again be approximately 1.1 m.
Flexural compression buckling

From Timoshenko and Gere [87], the critical buckling stress for a thin plate subject to pure bending is given by

$$\sigma_{bc} = \frac{25.6\pi^2 D}{h_w^2 t_w}.$$  \hspace{1cm} (3.4)

Conservatively using $D = D_{22} = 9.1 \text{kNm}$, with an unsupported web height of $h_w = 1.37 \text{m}$, and a web thickness of $t_w = 0.018 \text{m}$ gives $\sigma_{bc} = 64.5 \text{MPa}$, which exceeds the factored mid-span bending stress, even accounting for the additional stress transferred to the GFRP by fatigue softening of the UHPFRC.

At the quarter points, the webs acting under combined bending and shear must satisfy the relationship

$$\left(\frac{\sigma_{sf}}{\phi_{GFRP}\tau_{sc}}\right)^2 + \left(\frac{\sigma_{bf}}{\phi_{GFRP}\sigma_{bc}}\right)^2 \leq 1.0,$$ \hspace{1cm} (3.5)

which is taken from the CHBDC provisions for thin-walled aluminum structures [20]. The factored quarter point bending stress is $\sigma_{bf} = 17 \text{MPa}$; the factored quarter point shear stress is $\sigma_{sf} = 15 \text{MPa}$. Assuming a stiffener spacing of $x \geq 1.37 \text{m}$, the combined utilization according to equation 3.5, is 0.84. Therefore, stiffeners are only needed within the end regions of the girder.

3.5.4 Interface shear

Interfacial shear strength

Based on the factored shear demand of 1256 kN, the maximum interfacial shear stress demand between UHPFRC and GFRP is

$$\tau_{GFRP-UHPFRC} = \frac{V_f Q_b}{I_b} = \frac{1256 \text{kN} \cdot 0.337 \text{m}^3}{0.615 \text{m}^4 \cdot 1.2 \text{m}} = 0.57 \text{MPa}.$$  \hspace{1cm} (3.6)

In Chapter 4 it is shown that the proposed epoxy texture alone provides a capacity of 4 MPa. Studs should still be provided to prevent separation at the interface. A typical stud spacing of, $100 \times 100 \text{mm}$ on centre is recommended. The resulting capacity will include a generous reserve to accommodate interfacial shear stresses arising from differential thermal expansion or contraction, UHPFRC shrinkage, and rail-structure interaction. The broken rail force, for example, exerts 21 kN/m along the length of a rail; spread over a width of approximately 0.5 m, this creates only 0.042 MPa of interfacial stress. The non-brittle failure response of the studs will allow for redistribution of locally high shear stresses in the event of an extreme load.
Interfacial shear stiffness

The $\gamma$ method [27] can be used to estimate the degree to which plane sections remain plane across two different components in a composite cross section. This method defines a factor, $\gamma_s$, as

$$
\gamma_s = \frac{1}{1 + \frac{\pi^2 EA}{b_{sc} K_{sc} L^2}}, \tag{3.7}
$$

where $L$ is the span length, $EA$ is the axial stiffness of one of the components, $b_{sc}$ is the width of the shear connection, and $K_{sc}$ is the distributed shear connector stiffness per unit of connected area. If $K_{sc} = \infty$, $\gamma_s = 1$, indicating full composite action (plane sections remain perfectly plane). On the other hand, if $K_{sc} = 0$, $\gamma_s = 0$, indicating that the two components each bend about their own centroid, with unrestrained shear slip at the interface.

Defining effectively full composite action as $\gamma_s > 0.9$, and assuming the measured interfacial connector stiffness of 20 MPa/mm, equation 3.7 can be rearranged to calculate the minimum span length for which the plane sections assumption is reasonable:

$$
L > \sqrt{\frac{\pi^2 E_{\text{UHPFRC}} A_{\text{UHPFRC}}}{(\gamma_s - 1) b_{sc} K_s}} = \sqrt{\frac{\pi^2 \cdot 45000 \text{MPa} \cdot 0.3 \text{m}^2}{0.11 \cdot 3000 \text{mm} \cdot 20 \text{MPa/mm}}} = 4.5 \text{ m}. \tag{3.8}
$$

Therefore the assumption of full composite behaviour is reasonable over the range of expected spans.

3.5.5 Vibration

Dynamic analysis of the proposed girder is presented in Chapter 7. Using the existing Expo Line structure as a baseline for comparison, it is shown that the passenger comfort performance of the girders is acceptable.

Final assessment of an actual guideway structure would require a more detailed analysis, taking into account the knowledge and experience of the train manufacturer, and the specific span arrangements of the guideway structure.

3.5.6 Fatigue

The required fatigue life is $N_{\text{fat}} = n N_t$ cycles, where $n$ is the number of fatigue cycles per train traversal, and $N_t$ is the number of train traversals expected during the service life of the girder. The Vancouver Millennium Line Design Manual specifies an expected $N_t = 10.8$ million train trips per track over a 100 year period. Referring to Figure 7.5, a conservative estimate for $n$ would be three cycles per train
passage for a three-car train. Typical trains could have as many as 5 cars, so \( n = 5 \) and \( N_t = 50 \times 10^6 \) is a reasonable, though conservative, estimate of fatigue life demand. However, even 10 million is beyond the “run-out” length of most experimental fatigue studies, including the experimental work presented in Chapter 5. Given the large number of fatigue cycles expected, and the difficulty of acquiring high-cycle fatigue test data, the fatigue requirement must be set to keep stresses low enough that an indefinite fatigue life can be expected.

**Flexural fatigue in UHPFRC**

For unreinforced UHPFRC, the tests described in this thesis have shown that a reliable fatigue length can be achieved if the peak tensile stress is kept below the cracking stress of the UHPFRC. For the U of T UHPFRC this limit was identified as approximately 7.5 MPa. This is a relatively low stress, and for un prestressed cross sections, maintaining the requirement of no cracking in the UHPFRC is an onerous constraint that leads to a much higher material use than would be required by all other design considerations.

Though it is only one test, the GFRP–UHPFRC specimen fatigue test discussed in section 5.6 showed that a stress level of 7.9 MPa was sustainable over 5 million cycles, with degradation of the UHPFRC arrested by stress transfer to the GFRP. Similar results have been reported by Makita and Bruhwiler [59, 60] regarding UHPFRC reinforced with steel reinforcing bars, which they refer to as R-UHPFRC. In their studies, a reduction of the elastic modulus of the UHPFRC of approximately 30% is reported, for specimens subjected to force-controlled fatigue cycling with an initial peak strain between 1%\( e \) and 1.5%\( e \).

The results of Makita and Bruhwiler [60] can be generalized to other cross sections and different reinforcement stiffnesses in the following way. First, it is assumed that the degradation of the UHPFRC proceeds as observed in the tests performed for this thesis and described in section 5.5.3, with the secant stiffness of all hysteresis loops passing through a fixed point. Based on the results in section 5.5.3, this fixed point is taken to lie on the uncracked compressive stress-strain curve, at a stress equal to 1.5 times the cracking stress of the UHPFRC. In the presence of reinforcement, degradation of the UHPFRC leads to redistribution of stresses into the reinforcement, and a drop in stress in the UHPFRC. Based on the previous assumptions, when the UHPFRC stiffness is reduced by 30%, the peak stress in the UHPFRC will have dropped to approximately 75% of the cracking stress. Although further fatigue tested is needed for validation, it is a reasonable hypothesis that regardless of the amount or type of reinforcement, the elastic modulus will stabilize once redistribution reduces the peak stress in the UHPFRC below a certain threshold. While the results of Makita and Bruhwiler [60] are consistent with a threshold of 0.75\( \sigma_{cr} \), for
now a conservative value of $0.5\sigma_{cr}$ will be used.

Applying this criterion to the moment-curvature response of the proposed cross section allows the prediction of the long-term flexural stiffness if the UHPFRC is allowed to crack under service loads. Degradation of the UHPFRC causes a rotation of the uncracked stiffness about the fixed point in the negative bending side of the diagram. Given the prebending in the section due to the unshored construction method, this fixed point is beyond the negative cracking moment, but as it is a fictitious analytical point, this does not matter. This rotation proceeds until the peak fatigue moment equals the moment causing a peak stress of $0.5\sigma_{cr}$ in the UHPFRC. The fatigue design requirement is that this long-term stiffness be satisfactory from the point of view of all other serviceability limit states, most importantly the passenger comfort requirement. For the proposed cross-section, given the peak fatigue moment indicated, the flexural stiffness is reduced to 65% of its uncracked value. The dynamic analysis of the proposed girder described in Chapter 7 uses this reduced stiffness value, and the passenger comfort performance is shown to be adequate.

![Figure 3.5: Moment-curvature diagram showing fatigue degradation.](image)

**Flexural fatigue in GFRP**

A study of fatigue of unidirectional E-glass fabric laminates is reported by Kensche [51], showing that a peak strain of 4% can reliably sustain 100 million cycles. Both wet and dry specimens were tested, including specimens cut from plates that had previously sustained hailstone impact damage. Conservatively using the crush-loaded vehicle as the fatigue loading, and using section properties based on the reduced stiffness of the UHPFRC, the peak fatigue stress in the GFRP is $5680 \times 1.2/0.4 = 17$ MPa, which would produce a peak strain of only 0.6%, well below the limit observed by Kensche of 4%.
Shear fatigue in GFRP

For tangent girders, the maximum fatigue shear force can be conservatively taken as 900 kN, which corresponds to a shear stress resultant in each web of 320 kN/m. For the laminate described in Appendix A.2.1, the shear compliance is 6.28 mm/MN. Therefore the maximum shear strain in the webs is 2.0%. Neglecting the effect of any combined axial strain, this corresponds to a principal strain of 1.0%. Tests on angle ply E-glass/epoxy laminates with fibre volume ratio of 0.65 performed by Rotem and Hashin [79] show that this level of strain is below the fatigue threshold for the off-ply laminae of the laminate.

An assessment of the fatigue performance of the webs of curved girders is outside the present scope of work.

GFRP–UHPFRC interfacial shear stress fatigue

Because the shear capacity of the GFRP–UHPFRC bond is much higher than the demand, fatigue degradation of the shear connection between UHPFRC and GFRP is not expected to be a problem. However, fatigue testing of the shear connection should be undertaken to allow a quantitative assessment of this limit state.

3.5.7 Top slab under barrier impact

Barrier design is outside the scope of this thesis. However, the collision loads applied to the barrier must be transferred through the top slab of the girder, which must have sufficient ultimate capacity to withstand the resulting combined action of axial tension and negative bending moment. The magnitude of these effects depends on the ability of the barrier to spread the impact load longitudinally.

For the proposed cross-section, the top slab consists of a 100 mm thick UHPFRC slab on a 18 mm thick GFRP plate. At the ultimate capacity of this plate, approximately the full depth of the UHPFRC is carrying a reduced stress of 7.5 MPa in tension, and the moment arm to the compression force in the GFRP is approximately 55 mm, giving a capacity of approximately 40 kNm/m. Compare this to the 100 mm thick reinforced concrete slab provided in the Vancouver Expo girders. Here, the tensile force in the top reinforcement is at least 30 mm below the top of the slab, and is coupled with a concrete compressive block that is at most 30 mm deep, giving a moment arm of approximately 45 mm. For 50 MPa concrete, the average reduced stress in the compression block is about 30 MPa, so the resulting capacity is again approximately 40 kNm/m. Thus, a barrier with similar longitudinal load distribution capability to the concrete barrier provided on the Expo Line girders will not unduly stress the slab.

At the ends of the girder, where the barrier can spread load in one direction only, there may be a
need for some local strengthening of the slab. This could be easily achieved by the addition of a few steel or GFRP reinforcing bars.

3.5.8 Fire

Because the polymer matrix of GFRP is combustible, concerns relating to fire safety are often raised in relation to applications of GFRP in construction. However, whereas most building codes contain extensive fire safety provisions, bridge design codes do not, in general, include provisions for fire. The Canadian Highway Bridge Design Code [20], which forms the basis for most guideway structure specifications in Canada, is typical in this manner: the only mention of fire is in regard to operator’s houses on moveable bridges. Despite this lack of regulation, the heightened level of concern related to fire with GFRP justifies a careful consideration of the level of fire risk associated with a proposed design. Building codes can provide some guidance: in building design the purpose of fire safety requirements is the protection of life—not the protection of the structure. Aiming at the same goal in the fire safety assessment of a guideway structure gives the requirement that girders must maintain their load-bearing capacity in the presence of fire long enough for all people to get away from danger without loss of life.

The time it takes for a structural member to lose its functionality is referred to as its fire resistance. The review paper by Correia et al. [16] summarizes a number of tests of cellular or tubular beams and slabs subjected to fire. The wall thickness ranged from 7 to 15 mm. For fire exposure on only one side of the member, which would be the case in a guideway structure exposed to a vehicular fire from below, the fire resistance was in all tests longer than a half hour, and was closer to one hour for the thicker laminates. Ignition time and degradation rate varies with the type of polymer matrix: phenolics provide the best fire resistance, followed by epoxies and then polyester and vinylester [16].

With these considerations in mind, the following scenarios can be envisioned for guideway structures:

1. **Fire in a train or other equipment on top of a girder.** In this case, the top UHPFRC slab will provide insulation to the GFRP, and a fire rating of greater than one hour can reasonably be expected. This would allow ample time for the safe evacuation of the train.

2. **Fire below a girder.** A half-hour fire resistance gives ample time for a moving trains to evacuate a dangerous span; once a fire alarm was raised, operations would cease so that oncoming trains would not approach the danger zone. A reasonable operational restriction could forbid trains from stopping over road crossings, in the highly unlikely event that a stopped train had to be evacuated from a fire zone, a half an hour would still provide ample time, given that trains are typically no more than 5 cars long. The area under the girder is not enclosed by typical guideway spans, so
evacuation of the danger zone below the structure would take only minutes.

Often it is convenient and economical to use the same track support structure in the stations as is used for the bulk of the elevated structure. Inside stations, longer fire resistance times may be required by the applicable building code. For interior applications, fire resistant intumescent coatings and insulating boards are available, which can prolong the fire resistance [19].
Chapter 4

GFRP–UHPFRC Bond

4.1 Introduction

The key to enabling symbiotic composite behaviour between GFRP and UHPFRC is the bond mechanism joining the two materials. This connection must meet a number of requirements: (1) it must provide sufficient shear strength and stiffness; (2) it must be easily fabricated at a large scale; (3) it must not be susceptible to degradation due to differential volume change or moisture ingress; and (4) it must not create large stress concentrations that would unduly compromise fatigue performance. This chapter presents the design of a bond mechanism to meet these criteria.

Following a review of the current state-of-the-art methods for connecting GFRP to ordinary reinforced concrete (RC) and to UHPFRC, the conceptual design process leading to the ultimate proposed connector is described. Shear capacity testing through simple push-off tests demonstrates that the proposed connector provides ample strength and stiffness relative to the demands of the proposed girder described in Chapter 2. The fabrication method for the proposed connector is simple, reliable, and scalable to large industrial projects. The commencement of an experimental investigation into the fatigue performance of GFRP–UHPFRC connected by the bond mechanism described here is presented in Chapter 5.

4.2 Review of GFRP bond mechanisms

The growing academic and industrial interest in GFRP–concrete and GFRP–UHPFRC hybrid composites is clear from the expanding body of literature exploring these material combinations. Table 4.1 presents an overview of some past and ongoing projects combining GFRP with concrete and with UHPFRC. The proposals and results of these projects, and their implications for the design of an improved GFRP–
4.2.1 GFRP and ordinary concrete

Composite systems combining GFRP and ordinary concrete fall into three categories: (1) GFRP bars as a replacement for conventional steel bars in reinforced concrete; (2) external or near-surface bonded GFRP strips or wraps installed as a repair or strengthening measure on existing concrete structures; and (3) systems combining GFRP structural members (e.g. I-sections or tubes) with compositely acting concrete. This third category is of most interest for the present design development. However, the substantial research and development efforts centred around GFRP bar and externally-bonded GFRP technology have provided an important impetus for the acceptance of GFRP by the construction industry.

**GFRP bar reinforcement**

GFRP bars are typically pultruded and use a combination of E-glass fibre with epoxy or vinyl ester resin. Various bond mechanisms have been developed, including epoxy ridges and rough sand coatings added following pultrusion. An extensive body of literature exists characterizing the bond behaviour of such bars—see, for example, Johnson [49]. The \(360^\circ\) confinement of the bar within the surrounding concrete is helpful in creating bond. The surface textures that have been successfully used as the bond mechanism for GFRP bars rely on this complete encapsulation. A planar bond surface between flat GFRP plate and a concrete or UHPFRC slab will require an additional mechanism, such as headed studs or adhesive, to prevent separation and loss of bond due to differential volumetric change or freeze-thaw cycles.

**Externally-bonded GFRP**

Externally bonded reinforcement relies predominantly on adhesive connection, and a large body of literature has developed characterizing the properties of adhesive bonds [83]. For retrofit applications involving existing structures, adhesive bonding is often the only option, and the technology has been applied with success on many projects. For new construction, however, it is preferable to avoid reliance on an adhesive bond. A technical report by the International Federation for Structural Concrete (FIB) on externally bonded reinforcement for reinforced concrete structures [29] states: “Field experience has shown that the bond between FRP and concrete cannot always be assured. Bond quality is influenced by the condition of the existing concrete surface preparation of the concrete substrate, quality of the FRP application, quality of the FRP itself, and durability of the resin.” While ongoing research will no doubt overcome many of these challenges, the superiority of a mechanical bond over an adhesive one, when the
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<td>Pultruded GFRP, RC</td>
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</tr>
<tr>
<td>Fam and Honickman [28]</td>
<td>Girder</td>
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<td>Girder</td>
<td>Prepreg CFRP, RC</td>
<td>Transverse GFRP I-sections</td>
</tr>
<tr>
<td>Siwowski et al. [85]</td>
<td>Girder</td>
<td>VI GFRP, RC</td>
<td>Steel bolts</td>
</tr>
<tr>
<td>Hall and Mottram [37]</td>
<td>Girder/Slab</td>
<td>Pultruded GFRP, Concrete</td>
<td>T upstands on GFRP panel &amp; Epoxy</td>
</tr>
<tr>
<td>Zou et al. [97]</td>
<td>Girder</td>
<td>Pultruded GFRP, RC</td>
<td>Perforated GFRP web with intersecting rebar</td>
</tr>
<tr>
<td>Al-Ramahee et al. [2]</td>
<td>Bridge deck</td>
<td>GFRP infused against hardened UHPFRC</td>
<td>Grid of grooves cut into UHPFRC surface</td>
</tr>
<tr>
<td>Keller et al. [50]</td>
<td>Bridge deck</td>
<td>Pultruded GFRP panel, lightweight concrete and UHPFRC</td>
<td>T upstands on GFRP panel</td>
</tr>
<tr>
<td>El-Hacha and Chen [25]</td>
<td>Girder</td>
<td>Pultruded GFRP box section, UHPFRC</td>
<td>Steel or GFRP studs &amp; epoxy bonded sand</td>
</tr>
<tr>
<td>Nguyen et al. [72]</td>
<td>Girder</td>
<td>Pultruded GFRP/CFRP</td>
<td>Steel bolts</td>
</tr>
<tr>
<td></td>
<td></td>
<td>I-girder, precast UHPFRC</td>
<td></td>
</tr>
<tr>
<td>Gonilha et al. [33]</td>
<td>Girder</td>
<td>Pultruded GFRP, precast FRC</td>
<td>Stainless steel bolts &amp; epoxy</td>
</tr>
</tbody>
</table>
situation allows it, seems clear.

**GFRP structural members**

In composite systems which combine GFRP structural sections with reinforced concrete, the most common means of shear connection, as illustrated in Table 4.1, has been to install steel studs into holes drilled into the GFRP, around which concrete is cast. Other connections that have been studied include: (1) pultruded GFRP I-section “upstands” glued to the top flange of the GFRP girder; (2) gravel chips distributed and glued over the top flange surface; (3) epoxy adhesive which is uncured when the concrete is cast; and (4) perforated GFRP webs which support the steel reinforcement and are cast into the concrete.

Since GFRP is essentially linearly elastic to failure, some designers purposely choose to have a ductile shear connection fail first at ultimate limit states as a way to introduce some softening of the system prior to ultimate failure. This behaviour is sometimes referred to as “pseudo-ductility”.

### 4.2.2 GFRP and UHPFRC

Ideally, the size of the texture, studs, or other connecting structure will be of the same scale as the constituents of the materials being connected. If the connector is too small, it will not fully engage with the adjacent material. On the other hand, if the connector is too large, the stress transfer across the connection will be discontinuous, leading to stress concentrations.

In contrast to ordinary reinforced concrete with 20 mm aggregate, and a first reinforcement layer approximately 50 mm from the interface, the largest particle in UHPFRC is sand, with diameter less than 1 mm, and the fibre reinforcement is uniformly distributed throughout. Therefore, the standard shear stud connections for ordinary concrete, which are large enough to extend past the reinforcement and are of roughly the same diameter as the aggregate, are oversized for UHPFRC. Despite this fact, as shown by Table 4.1, the current state-of-the-art for connecting GFRP and UHPFRC is to use stud-type shear connectors similar to what has been used for connecting GFRP and ordinary reinforced concrete. The promising compatibility of GFRP and UHPFRC, described earlier, makes a compelling case for the development of a shear connection that is specifically tailored to the properties of UHPFRC.

The relative ductility of UHPFRC in comparison to ordinary unreinforced concrete provides a means of achieving pre-failure softening through micro-cracking and softening of the UHPFRC. Therefore the need to introduce pseudo-ductility through plastic failure of the shear connection is lessened relative to GFRP-concrete systems.

The importance of the shear connection is a common theme amongst the conclusions of the research
cited in Table 4.1. Al-Ramahee et al. [2], Deskovic [22], El-Hacha and Chen [25], Keller et al. [50] and Honickman and Fam [42] all report premature or undesirable failure of the interface in the course of their experimental tests. The need for a well-designed, rational, reliable, shear connection mechanism is clear.

4.3 Development of a GFRP–UHPFRC shear connection

4.3.1 Development process

Placing a high priority on the requirement for easy, scalable fabrication dictates the abandonment of any shear connection concept based on individual studs installed into drilled holes, which would require excessive labour—especially if the size of the studs were decreased to be more compatible in scale with UHPFRC.

A connection based on an adhesive layer that must be placed just prior to casting the UHPFRC was also eliminated from consideration, as such a process is also labour intensive and heavily reliant on the cleanliness of the surfaces, and the timing of the cast. For example, Deskovic [22] attempted a shear connection based on adhesive bond, and ultimately concluded that for the connection to be reliable, “the surface should be cleaned repeatedly with a clean tissue, and then rasped until some fibers become visible. Finally, the whole surface should be blasted by compressed air.” Such a process is not economically feasible at an industrial scale.

The goal was set to design a connector which can be formed as part of the same fabrication effort as the underlying GFRP part. This has two advantages: first, it streamlines the fabrication process by minimizing the need for “post-processing” the part by machining or assembly; second, the system of connectors and/or texture which creates the bond with UHPFRC is monolithic with the underlying GFRP and is created simultaneously in the sealed, pressurized, and controlled environment as the underlying bulk of the GFRP structure.

This type of “one-shot” fabrication of a bond mechanism is possible when the GFRP part is made by vacuum infusion. Several potential concepts are pictured in Figure 4.1. These are briefly described, below, outlining the reasons that ultimately lead to the proposed bond mechanism.

The challenge with a “one-shot” vacuum infusion method is to find a way to create an elevated structure or texture while still allowing the vacuum bag to compress the lay-up. There are numerous considerations which must be kept in mind in order to successfully overcome this challenge:

1. A perfect seal is required for a successful infusion, and any puncture or tear in the bag could lead
Figure 4.1: Preliminary concepts for bond mechanisms

to a failed infusion.

2. If the vacuum bag must compress against an uneven surface, channels can form (called “race-tracks”) along which resin can preferentially flow, limiting the impregnation of the lay-up itself, leading to a failed infusion.

3. Flow medium can be used to help disperse resin, but this adds complexity to the lay-up and adds to cost, as it is a single-use, disposable material. Other consumables such as peel ply and release film can also be used to facilitate the infusion process.

4. Even pressure applied by the vacuum bag is necessary to achieve a uniform thickness and fibre volume density in the laminate.

5. All mould surfaces and supplemental elements which come in contact with epoxy must be coated with a release agent, a product which ensures these elements can be separated from the part after it is fully cured.

A simple concept, building on existing approaches described in the literature, is illustrated in (a). Strips of a pultruded “upstand” section are used, similar to Hall and Mottram [37] and Keller et al. [50]. The pultrusions would be placed on top of the primary glass-fibre lay-up, followed by strips of additional fabric arranged over the lower flanges, ensuring a well-reinforced connection with the underlying GFRP. This entire arrangement would be covered by the vacuum bag. This concept is also similar to the approach used by Gutiérrez et al. [35], except here the adhesive bond with the pultrusions is created monolithically with the bulk GFRP matrix, as opposed to being glued on in a later post-production stage.
In concepts (b), (c), and (d), steel connectors are inserted into the dry fibre lay-up and infused into the GFRP part. These steel connectors could be individual nails or bent wire hooks, as in (b) and (c), or a continuous length of bent wire as in (d). Steel connectors have the advantage of ductility, which allows for a more even sharing of the ULS shear stress demand.

While these initial concepts are feasible, they all present serious practical challenges for the vacuum bagging process, related to the considerations listed above. For instance, the bag would require many folds to accommodate each line of upstanding connectors, and the compression of the bag against the upstand-covered surface of the fibre lay-up would have to be carried out with great care to ensure that even and thorough compression of the part is achieved without voids which could act as resin race-tracks. Furthermore, the possible presence of sharp ends, edges or burs on the inserts would greatly add to the risk of puncturing the vacuum bag, causing loss of pressure and potentially jeopardizing the infusion process. When imagining an industrial scale operation, these concerns eliminate concepts (a) through (d) from consideration.

Concept (e) was developed by the author during a training visit to the FibreLab at the Danish Technical University (DTU). The goal was to create a shear connection entirely from vacuum-infused GFRP. The fabrication method and final prototype infusion are pictured in Figure 4.2. The angled tabs, 70 mm long by 30 mm tall, are produced by cutting slits following one of the 45° orientations of two layers of 450 g/m² bi-axial stitched fabric, and folding the resulting strips up around a release-treated aluminum bar. A fairly complex arrangement of flow medium (visible between the bars) was required, both below and above the lay-up, to ensure thorough wetting of the thick fabric lay-up and the tabs. Two plates were fabricated in this way, and their bond capacity with UHPFRC was tested at the University of Toronto as described in a following section.

While a successful proof-of-concept was achieved for concept (e), again, several practical challenges were evident. The hope had been that the aluminum bars could be extracted sideways from under the tabs. However, they were sufficiently embedded in the plate that they had to be hammered out longitudinally. This would certainly be impossible at a large scale. The bagging operation was manageable, though a fair amount of excess bag was required, as can be seen in Figure 4.2, and care was required to ensure that the bag settled tightly around the aluminum bars as the vacuum was applied.

Concept (f) sought to overcome the practical challenges of the previous concepts. Slightly tapered holes are drilled into a thick but flexible sheet of rubber or silicone, which is placed on top of the fabric lay-up. During infusion, the holes fill with epoxy, creating an array of studs. This allows the fabrication of surface structure while maintaining a flat surface for the vacuum bag to compress against evenly. This concept was ultimately refined and combined with the successful aspects of concept (e) to arrive at the
proposed shear connector, which is described in detail in the following section.

Figure 4.2: Concept (e): GFRP plate and shear connector prototype manufactured at DTU. Counterclockwise from top left: dry fibre lay-up; infusion in progress; finished prototype.

4.3.2 Physical description of stud-and-texture shear connector

A detail of the GFRP–UHPFRC shear connector, showing surface texture and fibre-reinforced studs, is illustrated in Figure 4.3, and hereafter referred to as the “stud-and-texture” shear connector. A rough texture is formed on the interfacial surface of the GFRP. This texture has a depth of 3 mm, and the individual peaks and valleys in the texture repeat over a length of 4 to 7 mm. The scale of the texture is compatible with the scale of the steel fibres in the UHPFRC, which are 13 mm long by 0.2 mm in diameter. Isometric views of the texture are shown in Figure 4.4.

In addition to the texture, an array of 17 mm tall headed studs are provided. These studs are reinforced by a loop of glass fibre that is continuous through a line of multiple studs. Spacing of studs
can be varied to provide the required capacity; prototypes created for the current study used studs on a $50 \times 50\text{mm}$ or $50 \times 100\text{mm}$ grid. Details of the fabrication method for the stud-and-texture shear connector are given in the following section.

![Figure 4.3: Schematic elevation of shear connection elements.](image)

![Figure 4.4: Isometric view of texture. Left: GFRP surface with representative steel fibres; Right: matching mould surface.](image)

### 4.3.3 Fabrication

The method of forming and infusing the texture and studs for experimental test specimens is outlined below. The same procedure could be scaled up for production of full-size girders without requiring significant changes.

1. The lay-up for the GFRP member is completed on top of a release-treated glass plate, with flow medium, peel ply, and release film arranged as necessary, as shown in Figure 4.5.
2. Ribbons of unidirectional fabric are hooked into holes punched through two sheets of rubber, which will form the texture and studs. This “stitching” process is shown in Figure 4.6. The bottom, texture-forming, sheet is a 6 mm thick high-friction conveyor belt, and the upper, stud-forming, sheet is untextured 6 mm thick neoprene. Each hole in the upper sheet is countersunk to form the stud head. Both rubber sheets are treated with mould sealer and release coat, and can be cleaned and reused after demoulding the part. The underside of the fully stitched texture and stud forming assembly is shown in Figure 4.7. In a large-scale operation, it is not hard to imagine this stitching process being mechanized.

3. The fully stitched texture and stud forming assembly is placed on top of the fibre lay-up, as shown
in Figure 4.8. In final preparation for infusion, a sheet of peel ply is placed over the full assembly, followed by the vacuum bag. Peel ply is a synthetic woven fabric that does not bond to epoxy, and has a high resistance to resin flow. Its inclusion on top of the stud-forming assembly ensures that the vacuum pressure is not cut off from the studs as the resin flows through their stems. After sealing the vacuum bag and installing the inlet and outlet resin lines, the part is ready to infuse.

Figure 4.8: Complete fibre lay-up including texture and stud forming assembly.

4. The infusion is shown, in progress, in Figure 4.9. The resin flow path can be set up in a number of ways, depending on the overall geometry of the part. The textured rubber sheet creates a flow layer, thus eliminating the need for a specific flow medium in the lay-up.

Figure 4.9: Infusion in progress.

5. After the resin is fully cured and the vacuum film and peel ply layers are removed, the rubber sheets can be removed one at a time. Short radial cuts at the quarter points of the perforations allow the sheets to be pulled over the stud heads without damaging the studs. This is shown in Figure 4.10.
Figure 4.10: Removing the stud-forming layer of the rubber sheet assembly.

6. The finished part, shown in Figure 4.11, is demoulded from the glass plate. A GFRP–UHPFRC composite plate can now be formed by casting UHPFRC against this part.

Figure 4.11: The final, demoulded, part.

4.4 Experimental program to measure shear capacity

4.4.1 Introduction

To assess the shear capacity of the GFRP–UHPFRC texture and stud connection, a series of push-off tests was performed. The objective of these tests was solely to determine the ultimate capacity of the shear connection; in the interests of keeping the setup simple and economical, the specimens were kept at a small scale, and the instrumentation was minimal. Despite the simplicity of the tests, the results provide
an important proof-of-concept for this innovative and previously untested structural connection.

Tabulated results of all tests are provided in Appendix F.

4.4.2 Specimens

Other than the preliminary DTU prototypes, all bond test specimens are 100 mm wide by 100 or 200 mm tall, cut from a 910 mm × 320 mm GFRP–UHPFRC plate. The GFRP thickness is 12 mm, and the UHPFRC is 50 mm thick. Plan and elevation drawings for each specimen series are shown in Figure 4.12

The fibre lay-up used for the GFRP plates is described in Appendix A.

UHPFRC was cast using a directional pour method, to bias the orientation of fibres in the direction of loading. Specimens were wet-cured for 7 days and allowed to age for at least 28 days prior to testing. The compressive strength at 28 days was 135 MPa. Further details of the casts are given in Appendix E.

![Figure 4.12: Plan and elevation views of bond specimens tested.](image)

4.4.3 Test set-up & procedure

The bond test set-up is pictured in Figure 4.13. Two GFRP–UHPFRC blocks are positioned back-to-back (GFRP sides together). The blocks are supported from below by metal plates and shims which bear on the UHPFRC alone, while the loading head at the top bears only on the GFRP. The spreading of load necessitates a transverse tie across the bottom of the specimen, which is provided by a hand-tightened
C-clamp. Under load, the two halves of the specimen visibly separate at the bottom, confirming the absence of any confining effect due to the C-clamp. After the peak load is reached and the GFRP begins to slip along the failure interface, the C-clamp may be imposing a confining effect, which should be borne in mind when interpreting the test results.

![Figure 4.13: Push-out test set-up](image)

4.5 Experimental results

4.5.1 Preliminary tests

Three preliminary specimen types were created in an effort to gain familiarity with the vacuum infusion process, and to test the basic texture and stud forming techniques. These initial tests are briefly described in the following sections.

**DTU panels**

Two 25 mm thick panels were infused, with tabs formed by folding strips of biaxial fabric around release-coated aluminum bars. The lay-up, infusion, and the finished panel are shown in Figure 4.2.

The panels were trimmed, and UHPFRC was cast against each panel, embedding the tabs. The hybrid
panels were subjected to push-off tests. The load-displacement response is shown in Figure 4.14. Peak load was 635 kN, which corresponds to 26.5 kN per tab, or dividing by the total interfacial area, 2.7 MPa shear stress. The initial stiffness of the connection (measured as the secant stiffness between 200 kN and 400 kN) was 993 kN/mm, or 4.2 MPa/mm.

![Figure 4.14: DTU panel test: load-displacement curve](image)

The specimen was still carrying 130 kN upon termination of the test, and although there was nearly 4 mm of relative displacement between the GFRP and UHPFRC, the two parts of the hybrid slab were still held together. To investigate the failure surface, it was necessary to pry apart the two parts. The failed interface that was revealed is shown in Figure 4.15. It can be seen that the failure was entirely in the GFRP, with the UHPFRC remaining intact. The GFRP failure was by delamination and rupture of the biaxial fabric plies that formed the tabs.

![Figure 4.15: DTU panel: failed interface. Left: UHPFRC side. Right: GFRP side.](image)

While the strength of the DTU panels was sufficient, the stiffness was too low, and as discussed in Section 4.3.1, the fabrication method was too cumbersome for use at the scale of a full bridge girder, so
further study of the concept was abandoned.

**Texture and unreinforced studs (series BU)**

In pursuit of maximum simplicity of fabrication, it was desired to see if a pure epoxy connection mechanism, in the vein of concept (f) of Figure 4.1, could provide sufficient performance. To this end, 100 mm tall GFRP–UHPFRC specimens were created with texture and unreinforced headed studs as shear connection. Unfortunately, the unreinforced studs proved to be very fragile, with many studs snapping off in the process of demoulding the part. Only four specimens remained intact, allowing for two push-off tests. The stress-displacement curves are shown in Figure 4.16. Failure is brittle, with abrupt drops in capacity as individual studs rupture, before the UHPFRC side of the surface texture fails.

![Figure 4.16: Series BU: shear stress-displacement curves. Ultimate shear strengths indicated by points on stress axis.](image)

**Texture and reinforced studs (series BR)**

To overcome the fragility and brittleness of the unreinforced studs, glass-fibre reinforcement was added to the studs in the form of a glass fibre rope running longitudinally along the surface of the part and hooked up into each stud-forming hole in the rubber sheets. As the focus is on overcoming fabrication challenges and rapidly producing a low-cost prototype, a glass-fibre woven rope, available at hardware stores for thermal gasket applications, was used for the stud reinforcement.

The reinforced studs proved to be durable enough to reliably survive the demoulding process, and contributed to a 50% increase in ultimate shear strength over the unreinforced connection. The stress-displacement curves for these bond tests are shown in Figure 4.17.
4.5.2 Tests to characterize the stud-and-texture connector

Given the promising results from the preliminary tests, an expanded set of experiments was undertaken. The first objective was to isolate the contributions from the texture and the studs. Following this, the scale of the tests was increased to produce a purer state of shear loading.

For all of these tests, studs were reinforced with ribbons cut from Vectorply E-LM 3610 unidirectional fabric. The presence of the chopped mat layer in this fabric was not ideal from either a handling or a mechanical performance perspective, but the quantity required did not justify the purchase of a different fabric, and it was preferable to use the Vectorply material, which has fully documented properties, over the repurposed thermal gasket. The width of the ribbon was between 1.0 cm and 1.4 cm, corresponding to a unidirectional weight of 13 to 17 g/m, and a chopped mat weight of 3 to 4 g/m. The stud base area is 28 mm², so assuming a density of E-glass of 2540 kg/m³, the fibre volume ratio in the stud base is approximately 51%.

Tests to isolate stud capacity (series BS)

Four tests were carried out on 100 × 100 mm GFRP–UHPFRC specimens with only studs as shear connection. The results are shown in Figure 4.18. The average ultimate load is 45 kN, or 5.6 kN per stud. Ultimate failure is by rupture of the studs in the plane of the interface.

The capacity of the studs can be understood with reference to the simple truss model shown in Figure 4.19. The failure load of 5.6 kN per stud, applied at the mid-height of the stud, is resisted by a tensile force in the stem of the stud and a conical compression strut in the UHPFRC below the stud head. The angle of the compression stud would be determined by the frictional capacity of the GFRP–UHPFRC interface: higher friction would allow a larger shear component of this force to be developed, which would enhance the capacity of the stud. For a smooth interface as was the case in these tests, the line of action
Figure 4.18: Series BS: shear force-displacement curves. Ultimate shear strengths indicated by points on stress axis.

Figure 4.19: Truss model of stud capacity

of this strut is nearly perpendicular to the interfacial plane. The tensile force is ultimately carried at the root of the stud by those fibres that extend upward from the stud in Figure 4.19; these fibres occupy half of the cross-sectional area of the stem. In the system shown, the ultimate tensile stress in the GFRP (7000/14 = 500 MPa) is in equilibrium with the observed ultimate load per stud of 5.6 kN.

Tests to isolate texture capacity (series BT)

Four tests were carried out on specimens with no studs. The results are shown in Figure 4.20. The average ultimate load is 82 kN, or 4.1 MPa shear stress.

This capacity can be understood by assuming a failure plane tangent to the upper extent of the GFRP texture, as is illustrated in Figure 4.22. The UHPFRC area intersected by this plane is about 70% of the total interfacial area. Thus, the shear capacity of 4.1 MPa corresponds to a shear failure in the UHPFRC at approximately 5.9 MPa. For a pure shear stress this corresponds to 5.9 MPa tensile strength, which is reasonable given that first cracking stress of UHPFRC is typically observed around 7 MPa, and the interface zone can be expected to have a lower fibre content, a greater prevalence of small voids and stress concentrations, and possibly a distribution of pre-existing shrinkage-induced cracks caused by the
restraint afforded by the GFRP.

There is one outlier in terms of stiffness among the four stress-displacement curves in Figure 4.20. The most likely explanation for the high apparent stiffness of this specimen is that the loading block was slightly overlapping the interface, leading to a direct compressive load path through the UHPFRC. The stress concentration caused by a misalignment like this would cause local crushing of the concrete at a relatively low load level, leaving the ultimate failure load unaffected.

Larger scale tests: studs at 50 × 100 mm centres (series BLA)

Increasing the height of the specimen from 100 mm to 200 mm creates a more pure state of shear stress, as the line of action between the top load point on the GFRP and the bottom reaction below the UHPFRC is more vertical. Therefore, given the promising results of the small-scale tests, a set of seven larger-scale, 200 mm tall by 100 mm wide test specimens were prepared. These specimens had studs at 50 × 100 mm centres.

Due to practical considerations regarding the width of the high-friction conveyor belt from which the texture-forming sheets are made, the orientation of the texture was rotated 90° for these larger tests. The results are consistent with the isolated texture bond strength obtained with the other orientation, indicating that the orientation of the texture does not have a significant impact on the shear strength or stiffness of the connection.

The stress-displacement curves for these seven tests are illustrated in Figure 4.21. The average ultimate capacity is 5.03 MPa, and the initial stiffness is approximately 20 MPa/mm. Both surfaces of the failed interface of specimen BLA-4 are shown in Figure 4.22.

The results of the previous texture and stud capacity tests are consistent with these results. Smearing the 5 kN capacity of each stud over its tributary area of 5000 mm² gives a 1 MPa expected capacity from
Figure 4.21: Series BLA: shear stress-displacement curves. Ultimate shear strengths indicated by points on stress axis.

Figure 4.22: Test BLA-4: failed interface. Left: UHPFRC side. Right: GFRP side.
the studs plus the 4 MPa capacity of the texture for an expected ultimate capacity of 5 MPa, a nearly perfect agreement with the observed 5.03 MPa average capacity. The fact that the contributions of the texture and studs appear to be additive in this way suggests that the studs, by maintaining contact between the UHPFRC and GFRP after the textured interface cracks, are able to prevent the complete shedding of load from the texture to the studs.

Test BLA-2, which appears as an outlier in terms of stiffness among the set, failed in part by delamination of the ribbon from the underlying GFRP plate, as shown in Figure 4.23. This test also saw a relatively large rotation during the initial compaction of the shims (see Appendix F). Delamination of the short length of ribbon extending above the top studs to the top edge of the plate was also observed on a number of other specimens (including BLA-4, as pictured in Figure 4.22). A potential improvement to the design which could prevent premature failure by delamination would be to have the stud reinforcing ribbon run underneath a biaxial fabric layer, with the stud loops pulled through this fabric and into the rubber sheets.

Figure 4.23: Test BLA-2: failed interface. Left: UHPFRC side. Right: GFRP side.

For design, a simple bilinear model of the stress-slip behaviour of the interface is useful. For studs at 50 \( \times \) 100 mm, a conservative bilinear model is shown in Figure 4.24. Because the tests have two shear interfaces, there are two peaks to each test; the behaviour of a single interface is better reflected by
ignoring the second peak.

Figure 4.24: Series BLA: simplified model for design use.

**Larger scale tests: studs at 50 \times 50\text{ mm centres (series BLB)}**

Figure 4.24 shows that the eventual plateau stress level is below the initial texture failure stress. This is undesirable, as it means that under force-controlled conditions, the brittle failure of the textured interface would immediately cause the total failure of the interface. It was speculated that a preferable behaviour could be achieved if there were more studs. To test this hypothesis, a set of tests with studs at 50 \times 50\text{ mm centres was undertaken.} The expected capacity is 4 MPa from texture plus 2 MPa from studs for a total of 6 MPa. The average ultimate capacity from tests was 7.3 MPa. As can be seen from Figure 4.25, formation of a crack at the texture interface, at around 4 MPa, does not cause an immediate drop in stress-carrying capacity, and the eventual plateau that is reached, as the studs fail, is close to this same 4 MPa level. This allows the assumption of a bilinear stress-slip model without an abrupt drop in capacity, as shown by the dashed line in Figure 4.26. Given that providing studs on a denser grid may not add significant cost in terms of materials or labour, this approach may be advisable.

Figure 4.25: Series BLB: shear stress-displacement curves. Ultimate shear strengths indicated by points on stress axis.
4.6 Conclusions

Push-off tests of the connection between UHPFRC and GFRP have shown that a bond mechanism based on epoxy texture and fibre reinforced studs, infused together with the GFRP plate provides sufficient stiffness and strength for many applications. The lower bound shear stiffness of the bond is 20 MPa/mm. The ultimate strength comprises contributions of approximately 4 MPa from the texture plus 5 kN per stud. Providing studs in a 50 mm square grid provides good ductility, allowing the assumption of a bilinear stress-displacement relationship for design. The connection can be fabricated at a large scale with relative ease and good reliability, in comparison with bond mechanisms based on steel studs or epoxy adhesives.
Chapter 5

Fatigue Behaviour of UHPFRC & GFRP–UHPFRC

5.1 Introduction

This chapter presents the results of a series of axial tensile fatigue tests on UHPFRC and GFRP–UHPFRC. Given the high live load to dead load ratio for the proposed guideway structure, and the absence of any prestressing, knowledge of the fatigue capacity of UHPFRC in tension is of particular importance to the development of the concept. Few studies of the fatigue performance of UHPFRC have been undertaken to date, and no previous data exists for the mix developed at the University of Toronto. The test results presented here do not constitute a full characterization of UHPFRC in fatigue, but they provide a valuable reference for validating the proposed girder design concept.

5.2 Literature review

5.2.1 UHPFRC

Recommendations in preliminary codes and standards for UHPFRC

The Japan Society of Civil Engineers (JSCE), in their 2008 recommendations on design with UHPFRC [48], specify a bilinear S-N curve, and state that flexural stress less than half of the static flexural strength will withstand more than 2 million cycles. A material safety factor of 1.3 is recommended.

The Association Française de Génie Civil (AFGC), in their 2013 recommendations for UHPFRC [4],
only specify that the peak tensile stress must be below the tensile elastic limit stress. On the basis of flexural tests, it is suggested that for peak stress below the elastic limit, no irreversible damage occurs.

**Academic studies**

Makita and Brühwiler [58] have performed direct tension fatigue tests on prisms of UHPFRC and UHPFRC reinforced with steel reinforcing bars. For fatigue of UHPFRC, they propose a bilinear damage model, based on their observation that the rate of stiffness loss, \( \frac{dE}{dN} \), is constant after the first 20% of the fatigue life.

Fitik et al. [31, 32] report a series of axial reversed cyclic fatigue tests on UHPFRC dog-bone shaped specimens. They varied the peak compressive stress between 0 and 50% of the compressive strength, and the peak tensile stress between 20% and 80% of the ultimate tensile strength. There is large scatter in the reported results.

Parant et al. [74] performed bending fatigue tests using the CEMTEC multiscale UHPFRC mix. They report a linear relationship between the peak strain reached in an initial static test and the number of subsequent cycles to failure, when the initial strain exceeds a threshold between 1.27%/perthousandzero and 1.44%/perthousandzero. This threshold corresponds to a flexural stress of approximately 65% of the ultimate flexural stress, which they report to be 61.5 MPa.

Lappa et al. [53] report preliminary results of bending fatigue tests, with relatively short fatigue lives between 30 000 and 170 000 cycles observed for stress ranging from 15% to 75% of the ultimate flexural strength. The UHPFRC used was CERACEM, with a mean cube compressive strength reported as 217 MPa.

This set of studies shows that there is little consensus amongst the results of the few reported studies of UHPFRC in fatigue. This is not surprising given the lack of standardization in both mix design and test methods. At this early stage in the development of UHPFRC, test data for the specific mix and loading conditions envisioned for a design are therefore a prerequisite for confidence regarding fatigue performance.

### 5.2.2 GFRP fatigue

The fatigue performance of GFRP is dependent on many factors, including the type of matrix; the fibre type and architecture; the ambient temperature and humidity; the frequency of loading; and the mean, peak, and fluctuating stress magnitudes [40]. Therefore, despite the more developed body of research regarding fatigue in GFRP, design-specific testing is necessary if an accurate assessment of fatigue life is required.
This is often not required, however, because the high strength-to-weight ratio of GFRP leads to the situation that stiffness requirements govern the dimensioning of structural elements, and the stress range is small in comparison to the ultimate strength. In this case, the limit on the peak strain below which no damage occurs can be used without governing the design. This was shown in section 3.5.6 to be the case in the proposed girder design.

5.3 Experimental setup

5.3.1 Test frame

The system used for fatigue testing is illustrated in isometric view in Figure 5.1, in elevation in Figure 5.2, and is pictured in Figure 5.3. Two actuators react against two wide-flange spreader beams which transfer the load through bolted grips to the specimen. The actuators, spreader beams, and specimen form a self-equilibrating system which is hung vertically by bolting the top spreader beam between two columns. Sliding surfaces on the sides of the bottom spreader beam provide out-of-plane restraint.

5.3.2 Design for continuous load redistribution

Because the damage progression in GFRP and UHPFRC is expected to occur at much different rates, it was important that the experimental setup be able to redistribute the applied load throughout the test in order to maintain a uniform axial state of strain in the specimen. This is reflective of the situation in the GFRP–UHPFRC guideway girder cross section, where the large eccentricity of each of the top and bottom slabs, relative to their thickness, produces an approximately uniform strain, regardless of any degradation of the UHPFRC. The capacity for this continuous applied load redistribution was achieved by the setup described above, with the hydraulic actuators on either side of the specimen applying potentially unequal loads. One of the actuators, the “master”, is controlled in force control, and the other, the “slave”, is controlled in displacement control to match the master actuator’s displacement. If during the test, for example, the specimen centroid shifts toward the master actuator, the force applied by the slave will decrease, shifting the centroid of the applied load toward the master actuator to keep it aligned with the specimen centroid.

5.3.3 Control system

The control system for slaving one actuator in displacement control to a master actuator in force control is an important element of the test apparatus. This control system is illustrated in Figure 5.4. The
Figure 5.1: Isometric view of direct tension load frame.
Figure 5.2: Elevation view of direct tension load frame.
control signal, $F_{\text{contr}}$, is input to the master actuator, and is compared with the feedback from the load cell, $F_{a1}$. The output from the master actuator’s internal LVDT, $u_{a1}$, is used as the control signal for the second actuator, using the second actuator’s internal LVDT signal, $u_{a2}$, as feedback. Implementation of the control system was completed using an MTS 793 FlexTest 40 controller [66].

5.3.4 Hydraulic power supply

In order to allow continuous, 24-hour operation of the test machine for fatigue tests lasting up to a week in duration, a dedicated hydraulic power unit was required, with the proper safety mechanisms in place to allow unattended operation. The hydraulic power unit features a variable displacement piston pump with max flow rate of 14.9 GPM (56 L/min), which allows a maximum displacement rate for the two actuators of 70 mm/s (equivalent to a maximum displacement range of 2 mm at 10 Hz). The advantage of variable displacement is that the pump can switch to a lower displacement if the flow demand is low, for example if a test ends outside of working hours, reducing heat buildup in the oil, and wear on the pump. The pump draws from a 32 L reservoir of hydraulic oil, and the return line oil is passed through a water-cooled heat exchanger. A temperature sensor and level gauge can cut power to the pump if the oil in the reservoir overheats or drops in volume.
Figure 5.4: Control system for direct tension load frame.
5.3.5 Data acquisition

For cyclic testing at 10 Hz, a sampling rate of 1024 Hz gives approximately 100 samples per cycle, which is sufficient for preserving the shape of the response and plotting hysteresis loops. However, for tests that last up to a week, continuous sampling at this rate would produce an unmanageable quantity of data. Therefore, a data acquisition schedule was required that would yield the salient data without producing an overwhelming output. The schedule that was used is as follows:

**Peak/valley data:** A real-time peak/valley detector records the coincident data from all outputs when a peak or valley is detected in the master actuator force.

**Millennial cycle data:** Every thousand cycles, two cycles’ worth of data are recorded from all outputs.

**Degradation-triggered data:** A degradation detector monitors one of the two auxiliary LVDTs mounted to the specimen, triggering the recording of 0.4 seconds of data from all channels whenever the peak displacement increases by 0.02 mm relative to the previous trigger level.

5.4 Tests & results: plain UHPFRC

5.4.1 Specimens

Plain UHPFRC specimens were designed in keeping with established practice for static direct tension testing of UHPFRC. Specimens were prismatic, with no notching and no expansion in cross section toward the grips. Graybeal and Baby [34] and Shao [81] have used tapered aluminum plates epoxied to the sides of the specimen to provide an interfacial layer at the grip that is able to plastically deform to any irregularities in the concrete surface, thereby facilitating a smooth introduction of stress into the UHPFRC. For the test set-up described in section 5.3, the grip plates were themselves tapered, and they were designed for the longest possible grip length. This eliminated the need for aluminum plates as an interface, simplifying the preparation of each specimen, and reducing costs.

Specimens were cut from a UHPFRC plate with dimensions $910 \times 320 \times 50$ mm. The $910$ mm length of the specimens was dictated by the length capacity of the available diamond saw. The saw-cut faces of the specimen provided a flat, straight grip surface. Cutting specimens from a large plate also ensured that the flow conditions of the UHPFRC cast were similar to the reality of a girder-scale cast.

The specimen thickness of 50 mm was selected as large enough to avoid influencing the fibre distribution, but small enough to keep the specimen weight and volume of casts manageable. Width was 100 mm for specimens UC1 through UC5, and was reduced to 50 mm for the remainder of the specimens,
to allow a greater number of tests using the available material. The cross sections are shown in Figure 5.5.

![Specimen cross sections](image)

Figure 5.5: Specimen cross sections.

### 5.4.2 Tests

Fifteen tests on plain UHPFRC were completed. An overview of these tests is provided by Table 5.1 and Figure 5.7. Plots of stress versus strain and strain range versus number of cycles are provided in Appendix H.

For each test, the specimen was loaded quasi-statically up to the stress $\sigma_{\text{pre}}$ given in Table 5.1. The initial elastic modulus, as displayed during this static loading, is recorded in the table as $E_0$. The specimen was then subjected to sinusoidal cyclic loading at 10 Hz, with stress varying between a minimum stress of $\sigma_{\text{min}}$ and a peak stress of $\sigma_p$. In most tests, this cyclic loading continued for 5 million cycles, unless failure occurred sooner. In tests UC10, 11, and 12, the stress range was incrementally increased until failure under cyclic loading occurred. In all other tests that survived the cyclic loading, a final quasi-static test was carried out, taking the specimen to its ultimate tensile failure. Specimen UC9 is shown in Figure 5.6 after the ultimate load has been reached.

Figure 5.7 shows the stress range of each test in the form of a constant life diagram (also called a Goodman diagram). The horizontal axis is the mean tensile stress, and the vertical axis the magnitude of the alternating stress. Points below the solid $45^\circ$ line correspond to tension-only cycling; points above this line correspond to reversed-cyclic loading. These stress ranges were chosen to avoid encountering the play in the actuator clevises, the connecting elements which transfer load from the actuators to the spreader beams through spherical bearings. Each actuator has two clevises, one at the top and one at the bottom. For each clevis there is a load level at which the spherical bearing in the clevis is disengaged. For the bottom clevises, this is the load corresponding to the full weight of the bottom steel spreader beam...
Table 5.1: Summary of UHPFRC fatigue tests

<table>
<thead>
<tr>
<th>Test</th>
<th>$E_{\text{pre}}$ (GPa)</th>
<th>$\sigma_{\text{pre}}$ (MPa)</th>
<th>$\sigma_{\text{min}}$ (MPa)</th>
<th>$\sigma_{\text{p}}$ (MPa)</th>
<th>Cycles</th>
<th>$E_f$ (GPa)</th>
<th>$\sigma_{u,\text{post}}$ (MPa)</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC1</td>
<td>33.6</td>
<td>7.1</td>
<td>2.0</td>
<td>6.6</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>UC2</td>
<td>40.7</td>
<td>8.7</td>
<td>2.0</td>
<td>8.0</td>
<td>$7.5 \times 10^6$</td>
<td>—</td>
<td>7.7</td>
<td>13.6</td>
</tr>
<tr>
<td>UC3</td>
<td>46.3</td>
<td>9.3</td>
<td>2.0</td>
<td>8.0</td>
<td>$2.3 \times 10^6$</td>
<td>—</td>
<td>—</td>
<td>g</td>
</tr>
<tr>
<td>UC4</td>
<td>49.2</td>
<td>9.2</td>
<td>2.0</td>
<td>8.0</td>
<td>$5.0 \times 10^6$</td>
<td>14.4</td>
<td>11.5</td>
<td>l</td>
</tr>
<tr>
<td>UC5</td>
<td>47.0</td>
<td>10.9</td>
<td>2.5</td>
<td>8.5</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>UC6</td>
<td>30.6</td>
<td>8.2</td>
<td>4.25</td>
<td>8.25</td>
<td>$5.0 \times 10^6$</td>
<td>19.3</td>
<td>10.6</td>
<td>g,u</td>
</tr>
<tr>
<td>UC7</td>
<td>27.8</td>
<td>6.8</td>
<td>4.0</td>
<td>6.35</td>
<td>$5.0 \times 10^6$</td>
<td>30.9</td>
<td>10.8</td>
<td>u</td>
</tr>
<tr>
<td>UC8</td>
<td>32.1</td>
<td>5.8</td>
<td>3.5</td>
<td>5.75</td>
<td>$5.0 \times 10^6$</td>
<td>28.4</td>
<td>8.1</td>
<td>u</td>
</tr>
<tr>
<td>UC9</td>
<td>46.4</td>
<td>6.1</td>
<td>3.75</td>
<td>6.0</td>
<td>$5.0 \times 10^6$</td>
<td>29.6</td>
<td>8.9</td>
<td>u</td>
</tr>
<tr>
<td>UC10</td>
<td>39</td>
<td>8.0</td>
<td>3.6</td>
<td>7.9</td>
<td>$1.5 \times 10^6$</td>
<td>3.6</td>
<td>8.7</td>
<td>0.17 $\times 10^6$</td>
</tr>
<tr>
<td>UC11</td>
<td>39.6</td>
<td>8.3</td>
<td>3.9</td>
<td>8.4</td>
<td>$3.4 \times 10^6$</td>
<td>3.9</td>
<td>8.8</td>
<td>1.84 $\times 10^6$</td>
</tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3.9</td>
<td>9.2</td>
<td>1.58 $\times 10^6$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>3.9</td>
<td>9.6</td>
<td>0.32 $\times 10^6$</td>
</tr>
<tr>
<td>UC12</td>
<td>41.4</td>
<td>8.5</td>
<td>3.9</td>
<td>8.5</td>
<td>1000</td>
<td>3.9</td>
<td>9.3</td>
<td>35300</td>
</tr>
<tr>
<td>UC13</td>
<td>38.1</td>
<td>8.7</td>
<td>3.8</td>
<td>8.5</td>
<td>$5.0 \times 10^6$</td>
<td>3.9</td>
<td>9.3</td>
<td>35300</td>
</tr>
<tr>
<td>UC14</td>
<td>36.1</td>
<td>7.0</td>
<td>3.7</td>
<td>7.0</td>
<td>$5.0 \times 10^6$</td>
<td>31.7</td>
<td>12.9</td>
<td>u</td>
</tr>
<tr>
<td>UC15</td>
<td>37.1</td>
<td>2.0</td>
<td>-0.5</td>
<td>1.8</td>
<td>$5.0 \times 10^6$</td>
<td>42.4</td>
<td>8.5</td>
<td>g,r</td>
</tr>
</tbody>
</table>

8 Ultimate failure occurred in gripped length.
1 Ultimate failure occurred at or just outside of LVDT mount.
\textsuperscript{u} Initial stiffness, $E_o$, calculated from first unloading to avoid bedding-in effects. See Appendix H.
\textsuperscript{r} Final static test was reversed cyclic.
and clamp plate assembly; for the top clevis it is this load plus the weight of the actuators themselves. Despite the fact that the clevises can be tightened to reduce the play in the spherical bearings, it remains the fact that if either of these load levels is encountered under cyclic loading at 10 Hz, the impact forces arising from the spherical bearing engaging and disengaging creates unacceptable noise in the observed hysteresis curves. The wear on the bearings over a total of more than 75 million cycles over all the tests would also be unacceptable.

5.5 Results

5.5.1 Fatigue limit

Figure 5.8 shows the peak tensile stress for each test, plotted against the number of cycles sustained at that stress level. For tests UC10,11, and 12, which were tested under incrementally increasing peak stress, the unfilled circles are plotted for the cumulative number of cycles under at least the given peak stress. The final failure point is plotted corresponding to the total number of cycles (under at most the final peak stress). For each test, the unfilled circles define a lower-bound for the constant amplitude fatigue life, and the solid circle an upper bound. Note that this interpretation assumes that the cycling
Figure 5.7: Overview of fatigue tests.

itself does not affect the fatigue life, which may not be strictly true, but any effect is likely to be small.

In total, eight constant-amplitude tests were run-out to at least 5 million cycles, with the only failure occurring in test UC3, which failed in the gripped length after 2.3 million cycles. UC1 and UC5 (not shown) had sustained significant plastic strain during the initial static loading, and so, in a sense, were failed even before the fatigue test began (stress-strain records from each test are given in Appendix H). Setting aside UC1, 3, and 5, these results suggest a “mega-cycle” endurance limit as high as 8.5 MPa for the U of T UHPFRC mix.

The usefulness of this representation of the fatigue performance is questionable, however, given that a number of the specimens which did not fail nevertheless showed a significant loss of stiffness over the course of the test. This is discussed in the following section.

5.5.2 Damage progression

Makita and Bruhwiler [60] have proposed a damage progression model as follows: 50% of initial stiffness is lost in the first 20% of the fatigue life; an additional 25% of stiffness is lost at a constant rate over the remaining 80% of the fatigue life, and the loss of the remaining 25% of stiffness occurs very rapidly at failure.

The results obtained in this study do not fit such a consistent pattern. The degradation of the secant elastic modulus (stress range divided by strain range) over the length of the tests is shown in Figure 5.9, and is repeated in Figure 5.10, with the number of cycles on a logarithmic scale. Two distinct behaviours are exhibited:
1. For tests with a peak stress below 7.5 MPa, the elastic modulus is stable over the entirety of the test.

2. For peak stress above 7.5 MPa, the elastic modulus decreases in an erratic manner over the course of the test.
Figure 5.9: Modulus of elasticity vs number of cycles (linear scale).
Figure 5.10: Modulus of elasticity vs number of cycles (log scale).
Figure 5.11 shows the final values of elastic modulus (at run-out or just prior to failure) for each test, plotted against the peak stress. Also shown are \((E, \sigma_p)\) points from a quasi-static cyclic test, with incrementally increasing peak stress. This test is described in more detail in Chapter 6. For peak stress below 7.5 MPa (the zone shaded darker grey in the figure), the elastic modulus is stable. The stable stiffness after cycling is in all cases lower than the initial static stiffness of 45 GPa, but as shown in Table 5.1, in many cases the damage in the softening occurred in the static loading, with little change in stiffness over the course of cycling. When the peak stress is above 7.5 MPa (the lighter grey zone in the figure), the reduction in elastic modulus is more severe and more scattered, in some cases leading to failure. These results indicate that the peak fatigue stress in UHPFRC must remain below the cracking stress.

Figure 5.11: Long-term modulus of elasticity vs peak stress.
5.5.3 Creep–softening relationship

As fatigue damage progresses under constant stress range, both the mean strain and the strain amplitude increase, as there is both a softening and a plastic elongation of the specimen. Figure 5.12 shows the relationship between the mean strain and the strain amplitude over the course of each test. As can be seen, the two strains increase in nearly perfect linear relation to each other.

![Figure 5.12: Strain amplitude vs. mean strain.](image)

The linear relationship between $\epsilon_m$ and $\epsilon_a$ implies that damage of UHPFRC can be modelled as a rotation of the hysteresis stiffness line, joining maximum and minimum stress and strain points, about a fixed compressive stress-strain point. This point could be conceptualized as the point at which cracks would close under reversed, compressive stress. This is illustrated in Figure 5.13, below. Selected hysteresis loops (from test UC10) are shown, along with the extrapolated secant lines joining the maximum and minimum stress-strain points for each hysteresis loop. As can be seen, these lines converge near a point on the uncracked compressive stress-strain line. Similar damage behaviour has been observed in concrete in compression [75], and fibre reinforced concrete in compression [47]. For each test that displayed fatigue damage, the best-fit intersection point as shown in Figure 5.13 was determined by an ordinary least squares fit. The coordinates of the points determined in this way, $(\epsilon_c, \sigma_c)$, are summarized in Table 5.2, and plotted in Figure 5.14, along with the compressive stress-strain line for $E = 45$ GPa. For tests where damage progressed in a halting manner, the intersection point was found for each duration of the test over which damage occurred. The coefficients of determination, $R^2$, are in all cases very close to one, indicating that the relationship between plastic strain and softening is well-captured by the
Figure 5.13: Determination of fixed point for fatigue degradation of UHPFRC in tension.
fixed point model. For all tests, the fixed points lie close to the compressive stress-strain line, assuming $E = 45 \text{ GPa}$ in compression.

Table 5.2: Fixed point of damage progression

<table>
<thead>
<tr>
<th>Test</th>
<th>$\epsilon_c$</th>
<th>$\sigma_c$</th>
<th>Cycle range</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>UC2</td>
<td>-0.131</td>
<td>-8.67</td>
<td>All</td>
<td>0.945</td>
</tr>
<tr>
<td>UC3</td>
<td>-0.118</td>
<td>-8.43</td>
<td>1 - 350</td>
<td>0.992</td>
</tr>
<tr>
<td>UC4</td>
<td>-0.120</td>
<td>-8.27</td>
<td>All</td>
<td>0.988</td>
</tr>
<tr>
<td>UC6</td>
<td>-0.186</td>
<td>-9.02</td>
<td>2 - 200</td>
<td>0.988</td>
</tr>
<tr>
<td>UC10a</td>
<td>-0.246</td>
<td>-11.03</td>
<td>1 - 500</td>
<td>0.986</td>
</tr>
<tr>
<td>UC10b</td>
<td>-0.180</td>
<td>-8.92</td>
<td>1515 - 1678</td>
<td>0.981</td>
</tr>
<tr>
<td>UC11a</td>
<td>-0.347</td>
<td>-16.19</td>
<td>75 - 3400</td>
<td>0.985</td>
</tr>
<tr>
<td>UC11b</td>
<td>-0.189</td>
<td>-13.71</td>
<td>3410 - 5240</td>
<td>0.908</td>
</tr>
<tr>
<td>UC11c</td>
<td>-0.426</td>
<td>-17.08</td>
<td>5260 - 5385</td>
<td>0.989</td>
</tr>
<tr>
<td>UC11d</td>
<td>-0.285</td>
<td>-15.00</td>
<td>6825 - 7125</td>
<td>0.986</td>
</tr>
<tr>
<td>UC12</td>
<td>-0.273</td>
<td>-19.91</td>
<td>2 - 34</td>
<td>0.946</td>
</tr>
<tr>
<td>UC13</td>
<td>-0.365</td>
<td>-16.30</td>
<td>1 - 15</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Figure 5.14: Fixed point for fatigue damage.

5.5.4 Influence of load redistribution

Relative to typical tensile test setups, the test setup used here has the ability, in one direction, to maintain alignment of the applied load with the centroid of the cracked section (as described in section 5.3. This could be an explanation for why the damage progression observed in these tests does not generally occur at a constant rate. If the applied load cannot shift, the formation of a crack inevitably leads to bending strains that will further contribute to the propagation of the crack. If however, the applied load is able to shift in order to maintain an axial state of strain, then crack propagation will not
be encouraged by bending effects.

The top and bottom slabs of a box girder are effectively in a state of axial strain within the service load range. Therefore, for assessment of the fatigue performance of the proposed girder design, the results yielded by the test setup used here are more applicable. However, in other applications of UHPFRC, the stress-strain conditions may be more accurately reflected by a test in which the resultant of the applied load does not move.

5.6 Tests & results: GFRP–UHPFRC

5.6.1 Specimens

GFRP–UHPFRC specimens were fabricated from 12 mm thick GFRP plates, infused as described in Section 4.3.3, and having the laminate composition detailed in Appendix A. Studs were spaced at 100 mm along the length of the plate, and at 50 mm across the width. The $0^\circ$ direction of the unidirectional fabric reinforcement is aligned with the length of the specimen.

For an initial set of specimens, a 50 mm thick slab of concrete was cast against the GFRP slab, and once hardened, the hybrid slab was saw-cut into strips. Unfortunately, these specimens proved insufficient to yield meaningful results, due to difficulties encountered in gripping the specimens. Since the coefficient of friction between steel and UHPFRC is higher than for steel and GFRP, the applied force is introduced into the specimen primarily through the UHPFRC. The stud and texture shear connection between GFRP and UHPFRC is therefore relied upon to transfer the full applied load once the UHPFRC cracks and softens. Not surprisingly, premature failure occurred at this interface well below the full tensile capacity of the GFRP.

Adding to the difficulty of gripping was the fact that even though the saw cuts generally produced a uniform, flat grip surface, inspection of the grip length after testing indicated that the grip pressure was often confined to less than the full area of the specimen’s grip length. Once the UHPFRC was damaged, and the specimen centroid had shifted toward the centroid of the GFRP, there was an eccentricity between the line of action of the force in the gauge length and the centroid of the grip interfacial shear stress. In the absence of a full gripped length, the specimen rotated within the grips as a result. This caused bending in the specimen, and prying apart of the GFRP and UHPFRC, leading to a failure mode that was not reflective of the actual tensile behaviour of the GFRP–UHPFRC composite.

These problems were ultimately overcome by (1) reinforcing the GFRP–UHPFRC interface, within the grip length, with perforated steel sheets; (2) casting UHPFRC all around the GFRP; and (3) tapering the
specimen within the gauge length, to create a “dog-bone” type specimen, as shown in Figures 5.15 and 5.16. This specimen design allowed the full tensile capacity to be reached in the GFRP in tests GC2 and HC3. For specimen GC2, UHPFRC was not cast against the GFRP within the gauge length, in order to assess the properties of the bare GFRP. HC3 had a gauge length width of 45 mm.

Figure 5.15: Tapered reinforced GFRP–UHPFRC specimens - isometric view.

Figure 5.16: Tapered reinforced GFRP–UHPFRC specimens.

5.6.2 Instrumentation

Specimens were instrumented with two LVDTs, as shown in Figures 5.17 and 5.18. In order to keep the instrumentation simple, and allow the use of the same LVDTs and mounts that were used for the
UHPFRC tests, LVDTs were mounted to the UHPFRC above and below the gauge length, as shown. This arrangement is not recommended for future tests. Although the strength of the GFRP–UHPFRC interface is adequately increased by the perforated steel reinforcement, there is still significant cracking and deformation along this interface. The vertical crack visible near the top grip in Figure 5.18 is evidence that some delamination and shear slip has occurred. As a result of this slip, a larger gauge length would need to be used to calculate the strain in the GFRP from the measured LVDT displacements, and this larger effective gauge length is unknown. In future tests, it is recommended that an adhesively mounted strain gauge be used in place of the LVDT on the GFRP side of the specimen, to provide a more direct strain measurement.

Figure 5.17: Specimen GC2.

5.6.3 Results

Because of the inadequacy of the instrumentation, only limited conclusions can be drawn from these tests. However, these limited results show promising behaviour, which warrants a further, more refined investigation of the static and fatigue behaviour of GFRP–UHPFRC building upon the experience presented here.
Chapter 5. Fatigue Behaviour of UHPFRC & GFRP–UHPFRC

Static composite behaviour

It can be reasonably assumed that the strain at ultimate failure of tests GC2 and HC3 was the same. With this assumption, the measured static tests can be compared on the basis of a normalized strain, as shown in Figure 5.19. Theoretically, the difference between the stress resultant versus strain curves for HC3 and GC2 will represent the contribution of the UHPFRC to the composite behaviour of test HC3. This is shown by the dotted line in Figure 5.19. The peak stress in the UHPFRC, which corresponds to the onset of strain softening behaviour, is reached at approximately 1.2% of the ultimate strain. Assuming the theoretical ultimate strain for the GFRP laminate of 20% is accurate, this would correspond to an onset of strain softening in the UHPFRC at 2.4%, which is in good agreement with the results of the other static tests shown in Appendix H.

The magnitude of the ultimate stress in the UHPFRC is, according to Figure 5.19, 24 MPa. However, given the instrumentation and gripping challenges discussed above, this should be considered a spurious result.

As shown in Figure 5.18 two distinct, large, cracks formed in specimen HC3. Although these cracks are at the extremes of the narrowed gauge length, which indicates that stress concentrations caused by the specimen geometry are affecting the result, the test nevertheless shows that GFRP is able to maintain bond with the UHPFRC at high strains, allowing the formation of multiple large cracks, and greatly
extending the ductility of the UHPFRC. Further testing is warranted to more thoroughly explore this behaviour.

**Damage progression**

Before being tested to failure, specimen HC3 was subjected to 5 million cycles at a peak load of 21 kN. Assuming an uncracked stiffness for the UHPFRC of 45 GPa, and a cracking stress of 7.5 MPa, gives a cracking load of 19.5 kN. Thus the peak cyclic stress of 21 kN is 8% beyond cracking, into the strain hardening domain of the UHPFRC. Despite this, after the initial cycles, the strain range in the specimen was constant throughout the entire test, as shown in Appendix section H. Cycling UHPFRC on its own at 8% beyond the cracking stress was shown above to lead to significant loss of stiffness. The lack of any progressive damage observed in test HC3 indicates that the GFRP is able to absorb redistributed stresses and arrest fatigue damage in the UHPFRC.

Further testing is needed to draw firm conclusions regarding fatigue damage progression in GFRP–UHPFRC.
5.7 Conclusions

Fifteen direct tension cyclic tests on UHPFRC, one on bare GFRP and one on composite GFRP–UHPFRC
connected by the stud-and-texture shear connector described in Chapter 4 have been carried out. Al-
though more tests are needed to fully characterize the behaviour of UHPFRC and GFRP–UHPFRC under
tensile fatigue loading, the following conclusions can be drawn based on the work presented in this
chapter.

• UHPFRC exhibits stable elastic modulus and limited plastic deformation for peak fatigue stress
  below approximately 7.5 MPa.

• Damage progression in UHPFRC for peak stress above this limit progresses in an erratic manner,
  with no consistent rate of damage evident.

• When damage does occur, the elastic modulus decreases and the plastic strain increases, with the
  proportionality between these two changes dictated by the fact that the secant stiffness lines of
  the hysteresis loops pass through a common fixed point in the compression side of the material
  stress-strain curve.

• Compositely acting GFRP–UHPFRC exhibits the ability to redistribute load from UHPFRC to GFRP
  as damage progresses in the UHPFRC.

• GFRP–UHPFRC possesses the ability to form multiple large cracks in the UHPFRC, thereby signifi-
cantly increasing the ductility of the UHPFRC.
Chapter 6

Damping of UHPFRC

6.1 Introduction

The light weight and high strength of the proposed guideway girders means that vibration and fatigue are important considerations, as was demonstrated in Chapter 2. Evaluation of dynamic structural response requires knowledge of dynamic material properties.

While the damping characteristics of GFRP composites have been extensively studied due to their applications in mechanical and aerospace engineering [10], studies of the intrinsic damping of UHPFRC are nearly non-existent in the current literature. Gonilha et al. [33] performed dynamic tests on a GFRP–UHPFRC composite footbridge, and report a fundamental mode damping ratio of $\zeta = 1.7\%$. In this structure, the UHPFRC formed the top slab of a simply supported bridge, and as such it was presumably uncracked.

It was speculated that if the UHPFRC was allowed to be cracked under service loads, potentially much higher damping values could be attained, which could improve the dynamic performance. This chapter presents the assessment of the damping capacity of cracked UHPFRC through both free and forced vibration tests. While higher damping is observed in cracked UHPFRC in free vibration tests, the forced vibration tests show that under repeated load cycles, the damping drops back to a baseline value of approximately $\zeta = 2\%$.

Although the tests presented here did not yield the benefit for design that was hoped for, the characterization of the damping capacity of UHPFRC remains a useful result. Following a brief review of analytical representations of material and structure damping, the setup and results of the experimental investigations are presented.
6.2 Review of damping theory

Damping is commonly quantified by the loss factor, $\eta$, defined as the ratio between the energy dissipated per radian of cyclic loading ($D/2\pi$, where $D$ is the energy dissipated per cycle), and a quantification of the strain energy amplitude, $U$:

$$\eta = \frac{D}{2\pi U}. \quad (6.1)$$

The energy dissipated per cycle is equal to the area contained within the hysteresis loop corresponding to cyclic loading; the strain energy amplitude can be defined in different ways, depending on the type of damping under consideration, as described in the following sections.

Damping can be characterized by how it varies with strain rate and stress/strain amplitude. If the loss factor is constant with varying strain rate, the damping is said to be rate-independent; damping that varies with strain rate is rate-dependent. Damping is said to be linear if the loss factor is constant over varying stress amplitude, and nonlinear otherwise.

Another important distinction in the character of damping is between viscous and non-viscous damping. Viscous damping is a specific type of linear damping where the dissipated energy is the area inside an elliptical hysteresis loop. This property greatly simplifies the dynamic analysis of damped structural systems, so most structures damped by nonlinear mechanisms are analysed based on an equivalent viscous damping. Given the uncertain nature of damping in real structures, the accuracy given by such an approach is usually sufficient.

Two broad categories of damping will be described in more detail in the following sections: linear viscous damping, and nonlinear, rate-independent damping. The small energy dissipation of uncracked or unyielded materials can be modelled as viscous. Larger energy dissipation in construction materials typically arises from yielding in steel or cracking and debonding in reinforced concrete, and is generally nonlinear and rate-independent.

6.2.1 Viscous damping

Viscous damping is described by the model shown in Figure 6.1. This model, consisting of a spring, with stiffness constant $k$, and dashpot with damping coefficient $c$, in parallel, is known as a Voigt model.

An alternative to the Voigt model is the Maxwell model, in which the spring and dashpot are arranged in series. The Maxwell model exhibits creep under a static sustained load, but does not display an increased stiffness under dynamic loads. An accurate material model will typically consist of a
combination of Maxwell and Voigt elements, but for the purposes of this discussion, the Voigt model will suffice.

A viscous damper produces a force that is proportional to the velocity of its deformation. This type of damping is analytically convenient, as it gives rise to an elliptical hysteresis loop, and can be simply described using a complex-valued stiffness. Because of these properties of viscous damping, other types of damping are often idealised as viscous. This approach will be taken for the quantification of damping in UHPFRC.

**Complex stiffness**

Consider the single degree of freedom system of the axial fatigue setup described in section 5.3. Figure 6.2 shows the free body diagram for the lower spreader beam and clamp assembly. Using the Voigt model defined above to describe the material behaviour of the specimen, the total force \( F = F_a \cos(\omega t) \) is resisted by the inertia of the assembly, \( m\ddot{u} \), the elastic stiffness of the specimen, \( ku \), and a viscous
damping force from the specimen, $c \dot{u}$. The equation of motion for the system is

$$m \ddot{u} + c \dot{u} + k u = F_a \cos (\omega t) .$$

(6.2)

The particular solution to the above equation can be found by assuming the solution to be the real part of

$$u = u_a e^{-i(\omega t - \phi)} .$$

(6.3)

Substituting into the differential equation gives

$$(k - i\omega - m\omega^2) u_a = F_a e^{-i\phi} .$$

(6.4)

The stiffness and damping of the specimen can be expressed by a complex stiffness, defined as

$$k^* = k' + ik'' = k - i\omega c ,$$

(6.5)

such that equation 6.4 can be written as

$$k^* u_a = F_a e^{-i\phi} + m\omega^2 u_a .$$

(6.6)

The real component of the complex stiffness, $k'$, is taken as equal to the static stiffness $k$ (this is appropriate for viscous damped materials, but other definitions may be used when dealing with materials having nonlinear damping). The imaginary component is the product of the damping coefficient and the frequency:

$$k'' = c \omega .$$

(6.7)

If the damping coefficient, $c$, is taken to be constant, then $k''$ is a function of $\omega$, and the damping is rate-dependent. Conversely, if $k''$ is taken as constant, implying that $c$ is inversely proportional to $\omega$, the damping is rate-independent.
Solving for $u_a$ and $\phi$ gives

$$u_a = \frac{F_a}{\sqrt{(k - m\omega^2)^2 + k''^2}}, \quad (6.8)$$

$$\tan \phi = \frac{k''}{k \left(1 - \left(\frac{\omega}{\omega_n}\right)^2\right)}, \quad (6.9)$$

where $\omega_n$ is the undamped natural frequency of the system,

$$\omega_n = \sqrt{\frac{k}{m}}. \quad (6.10)$$

If the specimen in Figure 6.2 is homogeneous, then the complex stiffness can be related to the material level complex modulus by the familiar equation

$$E^* = E' + iE'' = \frac{k^*L}{A}. \quad (6.11)$$

The imaginary component of the complex modulus, $E''$, is called the loss modulus, and the real component, $E'$, is called the storage modulus.

**Hysteresis**

Under sinusoidal cyclic loading, a viscously damped material displays an elliptical hysteresis curve, as in Figure 6.3. For reversed cyclic stress, the strain energy amplitude is typically taken as the triangular cross-hatched area, as shown in the figure.
The expressions for $D$ and $U$ shown in Figure 6.3 lead to the following convenient simplification of the definition of the loss factor:

$$\eta = \frac{D}{2\pi U} = \frac{\pi \sigma'' \epsilon_a}{2\pi \frac{\sigma''}{\sigma''} \epsilon_a} = \frac{\sigma'' \epsilon_a}{\sigma''} = \frac{E''}{E'}.$$

Equation 6.12 shows that for viscous damping, the loss factor is the ratio of the loss component of the stress amplitude, $\sigma''_a$ (sometimes called the coercive stress), to the storage component of the amplitude, $\sigma'_a$, or equivalently, the ratio between the loss modulus, $E''$, and the storage modulus, $E'$.  

If the loading is not reversed, then the definition of strain energy amplitude is less intuitive. In order for the loss factor to be unaffected by the superposition of a non-zero constant stress, the definition of strain energy must be maintained. Thus, the strain energy, $U$, under non-reversed cyclic loading is still defined as the triangular area shown in Figure 6.4. This is not the total change in strain energy between the mean and peak strain, which would also include the rectangular area under the triangle.

![Hysteresis curve and energy definitions for viscous damping of non-reversed cyclic loading.](image)

**Figure 6.4**: Hysteresis curve and energy definitions for viscous damping of non-reversed cyclic loading.

### 6.2.2 Nonlinear, rate-independent damping

The mechanisms of nonlinear rate-independent damping can be understood through simple models analogous to the Voigt and Maxwell models, but with the dashpot replaced by a Coulomb friction element which is rigid up to a threshold force, at which point slip occurs and no additional force can be carried. These models can be used to capture the behaviour of plastic yielding materials, and interfacial slip in composite materials, or friction at bearing surfaces and joints in a structure. The simplest such models are illustrated below. A spring and friction element in series describes bilinear elasto-plastic yield behaviour as in Figure 6.5; a spring and friction element in parallel describe a rigid stick – elastic slip...
behaviour, as in Figure 6.6.

For nonlinear hysteresis curves, the strain energy amplitude is typically taken as the triangular area under the secant joining the extreme stress-strain points, as shown in the figures. For materials with highly nonlinear behaviour other definitions may be more appropriate.

\[ k_f = \begin{cases} \infty, & |F| \leq F_{\text{slip}} \\ 0, & |F| > F_{\text{slip}} \end{cases} \]

\[ F = \left( \frac{1}{k} + \frac{1}{k_f} \right)^{-1} x \]

Figure 6.5: Elasto-plastic model and corresponding hysteresis.

Accurate modelling of the nonlinear hysteretic behaviour of real structures typically requires a more complex combination of springs and Coulomb friction elements to achieve more smoothly curved loading and unloading paths between the characteristically pointed load reversal points. To avoid the computational complexity required by such a model, it is advantageous to linearize the observed damping by fitting an ellipse to the hysteresis curve, having the same area, and the same maximum and minimum stress and strain, as shown in Figure 6.7. It is important to remember that the resulting equivalent viscous damping is valid only for that particular stress/strain range.

6.2.3 Other quantifications of damping

In addition to the loss factor, \( \eta \), defined by equation 6.1, a number of other quantifications of damping are used. The tangent of the phase lag, \( \phi \), between force and load, and the logarithmic decrement, \( \delta \), are more directly measurable, and the damping ratio, \( \zeta \), is more directly applicable to structural analysis. Relationships between these quantities and the loss factor are derived in the sections below.
\[ k_l = \begin{cases} \infty, & |F| \leq F_{\text{slip}} \\ 0, & |F| > F_{\text{slip}} \end{cases} \]

\[ F = (k + k_l) x \]

Figure 6.6: Stick-slip model and corresponding hysteresis.

Figure 6.7: Hysteresis curves for plastic strain damping, and equivalent viscous damping.
**Tangent of phase lag, \( \tan \phi \)**

The tangent of the phase lag between the force and the response, given by equation 6.9, is sometimes directly used as a measure of damping. The phase lag is related to the loss factor through the equation

\[
\tan \phi = \frac{\eta}{1 - \left(\frac{\omega}{\omega_n}\right)^2}.
\]

Equation 6.13 gives a means of experimentally determining the loss factor, \( \eta \), if the natural frequency of the system is known or can be estimated from the measured stiffness and mass. If the inertia of the system is negligible, \( \left(\frac{\omega}{\omega_n}\right)^2 \approx 0 \), then

\[
\tan \phi = \eta.
\]

**Damping ratio, \( \zeta \)**

The damping ratio, \( \zeta \), for a single degree of freedom system is defined as

\[
\zeta = \frac{c}{2m\omega_n},
\]

where \( c \) is the damping coefficient, \( m \) is the mass, and \( \omega_n \) is the natural frequency.

The relationship between damping ratio, \( \zeta \), and loss factor, \( \eta \), can be derived by substituting \( c = \frac{k''}{\omega} \), where \( \omega \) is the damped free vibration frequency \( \omega = \omega_n \sqrt{1 - \zeta^2} \):

\[
\zeta = \frac{k''}{2m\omega_n^2 \sqrt{1 - \zeta^2}} = \frac{\eta}{2\sqrt{1 - \zeta^2}}.
\]

For typical damping values, \( \zeta < 0.1 \), the square root term is very close to one, so

\[
\zeta = \frac{\eta}{2}.
\]

**Logarithmic decrement, \( \delta \)**

The logarithmic decrement, \( \delta \), of a free-vibration response is the natural logarithm of the ratio of amplitudes one period apart:

\[
\delta = \ln \left( \frac{u_a(t)}{u_a(t + T)} \right) = \ln \left( \frac{e^{-\zeta \omega_n t}}{e^{-\zeta \omega_n t}e^{-\zeta \omega_n T}} \right) = \zeta \omega_n T.
\]
Chapter 6. Damping of UHPFRC

The period of vibration is $T = \frac{2\pi}{\omega_n \sqrt{1 - \zeta^2}}$, so the logarithmic decrement can be related to the loss factor by

$$\delta = \frac{2\pi \zeta}{\sqrt{1 - \zeta^2}}. \quad (6.19)$$

For typical damping values, $\zeta < 0.1$, the denominator is very close to one, giving

$$\delta = 2\pi \zeta = \pi \eta \quad (6.20)$$

6.2.4 Material vs. member damping

In the same way that the Young’s modulus, the cross sectional properties, and the global geometry influence the global elastic stiffness of a structure, the global damping capacity is also influenced by material, cross sectional, and global properties.

If the storage and loss moduli are homogeneous throughout the structure, then the loss factor is clearly the same at the material, cross sectional and structure scales. However, for materials with nonlinear storage or loss behaviour, the material damping can vary widely throughout the volume of the structure, and the peak material damping will be larger than the global damping of the whole structure. Calculation of this conversion between material and structure-scale loss factor is illustrated in Figure 6.8.

The storage behaviour at the material level is described by a nonlinear stress-strain curve, which for UHPFRC can be approximated as bilinear, as shown in the top left. This is a modified stress-strain curve, neglecting the non-recoverable strain. After stressing the material up to a given stress-strain point, the reverse cyclic stress strain behaviour is assumed to be linear elastic with the secant modulus corresponding to that stress-strain point. This is a substantial simplification of reality, but it is sufficiently accurate to illustrate the relationship between material and member scale damping.

From the stress strain curve and the cross section geometry, the corresponding moment-curvature diagram can be constructed, for example by dividing the cross section into layers and finding states of strain that lead to equilibrating layer forces. The top middle diagram shows the moment-curvature diagram for a rectangular UHPFRC cross section. From the moment-curvature diagram and the overall geometry of the structure, a push-over analysis will yield a load-displacement diagram, such as is shown in the top right for a UHPFRC cantilever with constant rectangular cross section.

A relationship between coercive stress, $\sigma''$, and strain is shown in the middle left diagram. This diagram describes a material which has low damping up to the yield strain, and high damping thereafter. Using this material level curve, a “coercive moment”-curvature diagram can be constructed based on
Figure 6.8: Relationship between loss factor at material, cross section and structure scale.
the same self-equilibrating states of strain as give rise to the moment-curvature diagram. Similarly, a “coercive load”-displacement diagram can be constructed by integrating over the same curvature diagrams as give rise to the load-displacement diagram.

The loss factor is the ratio of the coercive stress, moment, or load to the stored stress, moment or load, as shown in the bottom row of Figure 6.8. The three loss factor diagrams are drawn at the same vertical scale, showing that for lightly cracked cantilevers, the ratio between the peak material loss factor and the global structure loss factor can be quite large.

A detailed theoretical discussion of the relationship between material and member damping is given by Lazan [54].

6.3 Experimental program to measure damping

Damping of UHPFRC was measured experimentally in three different ways. Quasi-static cyclic tests under incrementally increasing axial load show the rate-independent energy dissipation for low numbers of cycles. Forced vibration tests under 10 Hz cyclic axial load, run up to 5 000 000 cycles show how the energy dissipation changes with higher strain rate and higher number of cycles. Finally, free vibration tests provide a simple and clean measurement, with few parasitic sources of damping. A detailed characterization by any of these methods is beyond the scope of this thesis. However, even the relatively limited results described here provide valuable insight into this little-studied property of UHPFRC.

The following sections describe the setup, analysis and results for each type of damping measurement.

6.3.1 Quasi-static cyclic tests

Two quasi-static cyclic tests were performed using the axial load frame described in section 5.3, one in tension-only cyclic loading, the other under reversed cyclic loading.

Tension-only cyclic load

In the first test, the stress in the specimen ranged from a minimum of approximately 4.5 MPa to an incrementally increasing maximum up to the ultimate tensile strength of 10 MPa. The minimum stress was chosen to avoid the play in the actuator clevises, which occurs at a stress of approximately 4 MPa (corresponding to the weight of the lower steel assembly hanging on the specimen, with no load applied by the actuators). Under quasi-static loading, the effect of this play is negligible, however under dynamic loading the change in system stiffness distorts the observed hysteretic response. The stress-strain record from the test is shown in Figure 6.9.
The energy dissipated per cycle corresponds to a loss factor ranging between $\eta = 0.11$ and $\eta = 0.16$. The loss factor versus peak stress is plotted in Figure 6.14, along with the results of the forced vibration tests described in the next section.

**Reversed cyclic load**

The second test subjected the specimen to reversed cyclic loading, starting with a range of $\pm 4$ MPa and incrementally increasing to $\pm 8$ MPa. Post-peak behaviour was not captured in this test, as formation of a macro-crack occurred outside the gauge length. The stress-strain record is shown in Figure 6.10.

The initial unloading stiffness in the reversed test is the same as in the tension-only tests, and the initial reloading stiffness is close to the original uncracked stiffness, with a characteristically pointed hysteresis curve resulting. The plastic strains incurred mean that for a constant stress amplitude, the strains eventually become tensile throughout the entire stress range.

The loss factor ranges between 0.01 and 0.03 up to the tensile elastic limit of 7.5 MPa, beyond which the damping increases sharply to $\eta = 0.17$ for a stress amplitude of 8.5 MPa. The loss factor versus peak stress is plotted in Figure 6.14, along with the results of the forced vibration tests described in the next section.
Hysteresis

Figure 6.11 shows three representative normalized hysteresis loops for tension-only cycling at three different peak stress levels. It can be seen that the loops could be well approximated with an ellipse, and the energy dissipated is similar for all three stress levels.

In comparison, normalized hysteresis loops for reversed cyclic loading are shown in Figure 6.12. At stress ranges below the tensile elastic limit of UHPFRC, the dissipated energy is very low. Once plastic strains occur, the opening and partial closing of micro-cracks produces a much larger, pointed hysteresis loop.

The comparatively low loss factor under reversed vs tension-only cyclic loading at low peak stress levels is due in part to the rate-independence of the energy dissipation. If the dissipated energy increases linearly with strain amplitude (i.e. if the coercive stress, $\sigma''_a$, remains constant), while the strain energy amplitude increases with the square of the strain amplitude, the loss factor will decrease linearly with strain amplitude.
normalized strain = \frac{(\epsilon - \epsilon_{\text{min}})}{\epsilon_{\text{max}} - \epsilon_{\text{min}}} \\
normalized stress = \frac{(\sigma - \sigma_{\text{min}})}{\sigma_{\text{max}} - \sigma_{\text{min}}} \\
\sigma_p = 6 \text{ MPa} \\
\sigma_p = 8 \text{ MPa} \\
\sigma_p = 9.7 \text{ MPa} \\

Figure 6.11: Normalized hysteresis loops: tension-only quasi-static cyclic load \\

normalized strain = \frac{(\epsilon - \epsilon_{\text{min}})}{\epsilon_{\text{max}} - \epsilon_{\text{min}}} \\
normalized stress = \frac{(\sigma - \sigma_{\text{min}})}{\sigma_{\text{max}} - \sigma_{\text{min}}} \\
\sigma_p = 4 \text{ MPa} \\
\sigma_p = 7 \text{ MPa} \\
\sigma_p = 8.4 \text{ MPa} \\

Figure 6.12: Normalized hysteresis loops: reversed quasi-static cyclic load
6.3.2 Forced vibration tests

Setup & analysis

The axial tension fatigue tests described in section 5.3 also yield information regarding the material damping, through analysis of the observed hysteresis.

Damping is calculated on the basis of the stress-strain hysteresis loops where strain is calculated using the average displacement measured by the two auxiliary LVDTs mounted to the specimen. These LVDTs are part of an independent data acquisition system, and as such there is a systematic time delay between their displacement readings and the synchronous outputs from the load cells. In order to see the true hysteresis it is necessary to time-shift the auxiliary LVDT data to eliminate this delay. This synchronization was completed by mounting the auxiliary LVDTs directly to the actuators, such that they were subjected to the identical displacement as the LVDTs in the actuators themselves. Cycling at 10 Hz, the phase shift between the two actuators was measured as $0.43\,\text{rad}$ (if due to damping, this would correspond to $\eta = 0.45$), or $0.007\,\text{s}$.

Results

Figure 6.13 shows the loss factor, calculated according to equation 6.1, over the length of each fatigue test, up to 5 000 000 cycles.

The results can be better understood by plotting the long-term loss factor (at run-out or just prior to failure) versus the peak stress, as shown in Figure 6.14. As was shown in Figure 5.10, two shaded zones are identified: below 7.5 MPa, where the UHPFRC remains stable, with $E \approx 35\,\text{GPa}$; and from 7.5 to 10 MPa, which under static loading sees strain hardening, and under fatigue loading sees a continual loss of stiffness down to as low as 8 GPa prior to fatigue failure. For comparison, the static tension-only and reversed cyclic results are shown as well.

It can be seen that the highest damping occurs in the $\sigma_p = 3.75$ to 7.5 MPa range. The mechanical explanation for this may be as follows: at low stress levels, the UHPFRC matrix remains intact, and the damping is due only to microscopic internal friction effects. At higher stresses, the matrix sustains some micro-cracking. The incremental propagation of these cracks, and the stress concentrations they produce increases the damping, but the cracks are disparate, isolated, and are stabilized by fibre bridging, so there is not an unbounded degradation of the material. At still higher stress levels, these micro-cracks propagate and combine more rapidly, approaching a fully crack-saturated condition where, in the absence of stress reversals to close the cracks, the energy dissipation is even lower than in the uncracked UHPFRC.

The long-term damping under 10 Hz cyclic loading is uniformly lower than the damping corresponding
Figure 6.13: Loss factor remains constant throughout fatigue life.

Figure 6.14: Loss factor vs. peak stress.
to quasi-static cyclic loading, indicating that damping is highest over the first few cycles at a given stress range. This observation also supports the conclusion that damping in UHPFRC can be considered rate-independent.

Figure 6.15 shows representative normalized hysteresis loops from four different tests, at approximately 500,000 cycles. Though the resolution is obscured by noise in the data, it can be said that linearization of the damping behaviour, corresponding to an elliptical idealisation of the hysteresis loops, is an appropriate simplification.

\[
\text{normalized strain} = \frac{(\epsilon - \epsilon_{\text{min}})}{(\epsilon_{\text{max}} - \epsilon_{\text{min}})}
\]

\[
\text{normalized stress} = \frac{(\sigma - \sigma_{\text{min}})}{(\sigma_{\text{max}} - \sigma_{\text{min}})}
\]

Figure 6.15: Normalized hysteresis loops

6.3.3 Free vibration tests

Setup

A series of free vibration tests was carried out using the setup illustrated in Figure 6.16 and pictured in Figure 6.17. The steel base plate is fixed to the strong floor of the lab with four high-strength post-tensioned anchor bolts. A 300 mm tall buttressed vertical steel plate is welded to the base plate. The specimen is clamped between this plate and a second steel plate with six high-strength threaded bars. The nuts on the threaded bars are tightened to greater than 500 ft-lb of torque.

A frame designed to carry the initial lateral load is also welded to the base plate. The specimen is
connected to the load frame by a turnbuckle, a loop of high-strength wire, and an eye screw cast into the UHPFRC. Tightening the turnbuckle imposes a lateral displacement at the top of the cantilevered specimen. The test is started by cutting the wire, abruptly removing the displacement constraint. The ensuing free vibrations are measured by an LVDT that is mounted on a separate column, and therefore isolated from the vibrations of the specimen and the loading frame.

Weights are added to the top of the specimen to vary the frequency of vibrations. Total weight varies from 0 kg to 150 kg, producing a range of natural frequencies from 2 Hz to 26 Hz.

**Specimens & test summary**

The specimens tested are summarized in Table 6.1. The specimens were cast prior to completion of the mix development work by Shao and Gauvreau [82], and as such, they used a slightly different UHPFRC mix, described by Wang [91]. All specimens were 70 mm × 100 mm, and were oriented as shown in Figure 6.16, bending in the weak direction.

![Free vibration test setup](image)

*Figure 6.16: Free vibration test setup.*

Three tests were performed at each weight level.
Figure 6.17: Free vibration test setup.

Table 6.1: Overview of free vibration tests

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</table>
Results & analysis

A representative vibration decay record is shown in Figure 6.18. The raw displacement record is processed in the following way to yield the loss factor:

1. Any non-zero average displacement caused by residual deformation after cracking of the cantilever is subtracted from the data, and the absolute value is taken.

2. Peak points of the resulting data are extracted.

3. The natural logarithm of the peaks is taken and plotted versus number of cycles.

4. A linear fit over a moving window of five peaks is used to determine the slope of the logarithmic decay, which is the logarithmic decrement, $\delta$.

5. The loss factor is calculated according to equation 6.20: $\eta = \frac{\delta}{\pi}$.

The extent of cracking can be measured by the change in the natural frequency of the cantilever, which for a weighted cantilever is given by the equation

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{\hat{m}}},$$

(6.21)

where $\hat{k}$ and $\hat{m}$ are the modal stiffness and modal mass of the weighted cantilever. Neglecting the change in the mode shape due to cracking, the ratio of the frequency to the initial uncracked natural frequency, $f_0$, is equal to the square root of the ratio between the stiffness and the initial uncracked stiffness:

$$\frac{f}{f_0} = \sqrt{\frac{k}{k_0}},$$

(6.22)
Figure 6.19 shows the loss factor versus frequency ratio points obtained in this way from all free vibration tests.

![Diagram](image)

Free vibration tests

Theoretical

Figure 6.19: Relationship between cracked modulus of elasticity and damping ratio

The theoretical curve shown in Figure 6.19 is obtained following the procedure outlined in section 6.2.4, assuming a material loss factor of 0.007 up to a cracking strain of 0.175%, and approximately 0.1 beyond, up to an ultimate strain of 0.95%, at which point strain localization and softening is assumed to occur. Beyond this point, the assumptions of linear elasticity and damping are no longer justifiable, and a more sophisticated analysis would be required. Given the simplifying assumptions involved in this calculation, the general agreement in trend and magnitude with the observed free vibration damping shows that the assumed material behaviour is consistent with the experimental results.

The loss factor measured by the forced vibration tests covers the same range as is shown in Figure 6.19, as shown in Figure 6.14. However, the forced vibration tests show that the higher damping observed in cracked UHPFRC, which the above discussion shows is approximately $\eta = 0.1$ ($\zeta = 5\%$), and the quasi-static cyclic tests showed to be as high as $\eta = 0.16$ ($\zeta = 8\%$), is not, in general, sustainable for long-term non-reversing cyclic loading.
6.4 Conclusions

The results of quasi-static cyclic, forced vibration and free vibration tests on UHPFRC allow the following conclusions regarding the damping capacity of UHPFRC:

- UHPFRC damping is predominantly rate-independent.

- Baseline damping for UHPFRC is approximately \( \eta = 0.04 \), or \( \zeta = 2\% \).

- Under reversed cyclic loading extending into strain hardening, the damping can increase to approximately \( \eta = 0.16 \) or \( \zeta = 8\% \), and potentially higher if the peak compressive stress is higher.

- Under tension-only cyclic loading, damping as high as \( \eta = 0.12 \) or \( \zeta = 6\% \) is possible, but this is only sustainable over a high number of cycles for peak stress near 6 MPa.

For design of UHPFRC structures under dynamic service loads, a damping value of \( \zeta = 2\% \) is advised, regardless of stress range. Potentially much higher values could be expected under high-stress, low-cycle seismic loading.
Chapter 7

Dynamic Analysis of Guideway Structures

7.1 Introduction

The primary serviceability requirement for transit rail guideways is that the dynamic response of the girders to the passage of a train must not cause train car vibrations that would be uncomfortable to passengers. Dynamic analysis more often focuses on the response of the girders, with the train simplified to a set of moving loads. This, of course, yields no information about the effect that the girder response has on the train. In order to analyse this interaction between girder and train, a more sophisticated analysis is required, which includes a mass-spring-damper representation of the train. Most commonly, the dynamic model of the train is then included in a finite element analysis of the structure, and a time-stepping numerical integration scheme is used to calculate the response time-history [12, 95].

The finite element analysis approach has the benefit that the mechanics of the calculation are generally familiar to most engineers. However, the implementation of such a model is often complex and time-consuming. Given the importance of the passenger comfort requirement in dictating the required stiffness of the structure, it is preferable to have a means of analysis which can take the basic girder and train dynamic properties as inputs and yield a reasonable estimate of the magnitude of vibrations that would be experienced by passengers. Such a tool, which could be more readily used earlier in the design process than a specialised finite element model, would allow the train-girder system to be designed as a whole, starting from the conceptual stage.

For detailed design, a finite element model will, in most situations, play an important role in the
analysis. However, confidence in finite element results is often challenging to achieve, given that there is much scope for misjudgement or error in setting the many parameters of a complex finite element analysis. An independent analysis, following a completely different method, is always helpful in this regard. In addition to being useful at the preliminary design stage, the method presented here can serve a useful purpose in this way.

The analysis method presented here is based on a frequency domain representation of the train-girder system. Frequency domain methods are commonly used in the analysis of linear time-invariant systems, such as a building subjected to seismic ground motions, or a girder subject to a set of moving loads. When the girder is treated not as a set of loads, but as a mass-spring-damper model, the inertia of the passing train effectively modifies the girder’s mass, stiffness and damping, which makes a frequency domain analysis of the train-girder interaction more complex, as the system is no longer time-invariant.

An advantage of a frequency domain approach is that the track irregularity, which is a stochastic process represented by a power spectral density, can be included directly in the analysis without the need for a Monte Carlo simulation. Similarly, the effects of pier settlement and long-term deflections can be investigated quickly once the initial analysis is complete.

Following a brief literature review of studies of train-girder dynamic interaction, the frequency domain analysis approach for periodically-varying dynamic systems will be presented. The statistical representation of track irregularity and the evaluation of passenger comfort will then be outlined, and the method will be applied to the evaluation of the Vancouver Expo girders and the proposed GFRP–UHPFRC girder concept.

### 7.2 Review of literature on train-girder dynamic interaction

The textbook by Yang et al. [96] provides a thorough overview of the fundamentals of moving load analysis and of finite element methods for train-girder interaction, with an emphasis on the design of high-speed rail bridges. Elmaraghy [26] presents a ride quality analysis for a train on stochastically modelled rough track, though with no consideration of the influence of flexible girder support. The analytical formulation of the train/girder system follows, in some ways, the analytical formulation of Cheung et al. [12], though these authors recommend solution of the system via the Wilson-θ time-stepping method. The study by Wang et al. [92] includes consideration of camber and pier settlement effects, again computing the response via a time-stepping analysis of a finite element model.
7.3 Frequency domain methods for linear periodically-varying systems

For linear time-invariant systems, such as a girder subject to a set of moving loads, dynamic behaviour is characterized in the frequency domain by the system’s transfer function. The transfer function, \( G(\omega) \), is a complex-valued function of frequency that multiplies the frequency-domain representation of the loading, \( \hat{f}(\omega) \), to yield the frequency-domain representation of the response, \( \hat{y}(\omega) \):

\[
\hat{y}(\omega) = G(\omega) \hat{f}(\omega) .
\] (7.1)

(The \( \hat{\cdot} \) accent is used to indicate the frequency-domain representation of a given function of time.)

If the loading is periodic, then its frequency domain representation is discrete, and the continuous functions in the above equation can be replaced by a system of equations, one for each harmonic of the fundamental frequency of the loading \( \omega_f \):

\[
\hat{y}(k\omega_f) = G(k\omega_f) \hat{f}(k\omega_f) , \quad k = 0, 1, \ldots, \infty .
\] (7.2)

This system of equations can be represented in “harmonic vector” form, where the vector elements are the amplitudes of each harmonic of \( \omega_f \):

\[
\hat{\mathbf{y}} = G \hat{\mathbf{f}} .
\] (7.3)

(The \( \hat{\cdot} \) accent is used to indicate the harmonic vector representation of a variable.)

\( G \) will be called the harmonic transfer matrix. For time-invariant systems, \( G \) is diagonal, since the system of equations 7.2 is uncoupled.

If the system is not time-invariant, but rather has mass, damping and stiffness that vary periodically in time with the same fundamental frequency as the loading, the system can still be represented in the form of equation 7.3, but the harmonic transfer matrix \( G \) will no longer be diagonal. In other words, there will be some coupling between the harmonics of the system.

The harmonic transfer matrix is infinite-dimensional. In physical systems, however, the participation of high-frequency harmonics is usually negligible, which allows the harmonic transfer matrix to be truncated, keeping only a finite number of salient harmonics.

For multiple-degree of freedom systems, each element in the harmonic vectors \( \hat{\mathbf{y}} \) and \( \hat{\mathbf{f}} \) is replaced with a vector spanning the degrees of freedom of the system. In this case, \( G \) will be a block matrix with each block relating a specific loading harmonic to a specific response harmonic.
For linear time-invariant systems, the diagonal entries of the harmonic transfer matrix are samples of the transfer function, which can be calculated by taking the Fourier transform of the equation of motion and making use of the fact that differentiation in the time domain becomes multiplication by $i\omega$ in the frequency domain.

For periodically varying systems, the equation of motion contains products of time varying mass, damping and stiffness with the displacement response and its derivatives. Upon taking the Fourier transform of the equation of motion, these time-domain products become convolution integrals in the frequency domain. In matrix form, each convolution integral becomes a matrix product involving a circulant matrix, or a block-circulant matrix for multiple-degree of freedom systems.

This strategy of solving periodically time-varying systems is sometimes referred to as the “harmonic balance” approach. Theoretical development and discussion of the method is given by Sandberg et al. [80], and Werely [93]. It has been developed primarily for applications in mechanical and electrical engineering.

### 7.4 Application to train–girder interaction dynamics

#### 7.4.1 Overview of train–girder system

A two-dimensional train–girder system is illustrated in Figure 7.1. A three-dimensional analysis could be completed following a similar method to what will be described here in two dimensions. However, the method is likely to be of most use in allowing relatively fast evaluations at the preliminary design stage, when a two dimensional analysis will often suffice. The system consists of an infinite series of identical spans, of length $l_g$, being traversed by an infinite number of trains, with spacing of length $l_t$ between them, travelling at constant speed, $v$. The distance between trains must be long enough that the girder response has almost fully decayed before the arrival of the next train, as this will typically be the case for transit operations. The infinite extent of the system simplifies the analysis, as it allows a steady-state solution to be sought.

![Figure 7.1: Train-girder system](image-url)
The system will accept three inputs: the static component of the moving axle loads, a random track irregularity profile, and a static displaced shape for the girders, being the camber plus the deflection due to permanent loads.

### 7.4.2 Train model

A multi-body system modelling a single car of the train is illustrated in Figure 7.2.

![Figure 7.2: Dynamic model of single train car](image)

There are $N_t = 6$ internal degrees of freedom per car: translation and rotation (pitching) of each of the three masses in a single car (the main car body and the two bogies). There are also $N_a = 4$ axle degrees of freedom per car (rotational inertia of the axles is negligible). Dynamic equilibrium of each
car degree of freedom yields the following six equations:

\[
\begin{align*}
\frac{m_c}{2} \ddot{y}_{k1} + \ddot{y}_{k2} &= -k_b(y_{k1} - \frac{1}{2}y_{k3} - \frac{1}{2}y_{k4}) - c_b(\dot{y}_{k1} - \frac{1}{2}\dot{y}_{k3} - \frac{1}{2}\dot{y}_{k4}) - k_b(y_{k2} - \frac{1}{2}y_{k5} - \frac{1}{2}y_{k6}) - c_b(\dot{y}_{k2} - \frac{1}{2}\dot{y}_{k5} - \frac{1}{2}\dot{y}_{k6}) \quad (7.4) \\
I_c \frac{\ddot{y}_{k2} - \ddot{y}_{k1}}{2l_b} &= k_b(y_{k1} - \frac{1}{2}y_{k3} - \frac{1}{2}y_{k4})I_c + c_b(\dot{y}_{k1} - \frac{1}{2}\dot{y}_{k3} - \frac{1}{2}\dot{y}_{k4})I_c - k_b(y_{k2} - \frac{1}{2}y_{k5} - \frac{1}{2}y_{k6})I_c - c_b(\dot{y}_{k2} - \frac{1}{2}\dot{y}_{k5} - \frac{1}{2}\dot{y}_{k6})I_c \quad (7.5) \\
\frac{m_b}{2} \ddot{y}_{k4} + \ddot{y}_{k3} &= k_b(y_{k1} - \frac{1}{2}y_{k3} - \frac{1}{2}y_{k4}) + c_b(\dot{y}_{k1} - \frac{1}{2}\dot{y}_{k3} - \frac{1}{2}\dot{y}_{k4}) - k_a(y_{k3} - y_{a1}) - c_a(\dot{y}_{k3} - \dot{y}_{a1}) - k_a(y_{k4} - y_{a2}) - c_a(\dot{y}_{k4} - \dot{y}_{a2}) \quad (7.6) \\
I_b \frac{\ddot{y}_{k4} - \ddot{y}_{k3}}{2l_b} &= k_a(y_{k3} - y_{a1})I_b + c_a(\dot{y}_{k3} - \dot{y}_{a1})I_b - k_a(y_{k4} - y_{a2})I_b - c_a(\dot{y}_{k4} - \dot{y}_{a2})I_b \quad (7.7) \\
\frac{m_b}{2} \ddot{y}_{k5} + \ddot{y}_{k6} &= k_b(y_{k2} - \frac{1}{2}y_{k5} - \frac{1}{2}y_{k6}) + c_b(\dot{y}_{k2} - \frac{1}{2}\dot{y}_{k5} - \frac{1}{2}\dot{y}_{k6}) - k_a(y_{k5} - y_{a3}) - c_a(\dot{y}_{k5} - \dot{y}_{a3}) - k_b(y_{k6} - y_{a4}) - c_a(\dot{y}_{k6} - \dot{y}_{a4}) \quad (7.8) \\
I_b \frac{\ddot{y}_{k6} - \ddot{y}_{k5}}{2l_b} &= k_a(y_{k5} - y_{a3})I_b - c_a(\dot{y}_{k5} - \dot{y}_{a3})I_b - k_a(y_{k6} - y_{a4})I_b - c_a(\dot{y}_{k6} - \dot{y}_{a4})I_b \quad (7.9)
\end{align*}
\]

For a train with \( N_c \) cars, these equations can be written in matrix form as follows:

\[
M_t \ddot{y}_t + C_t \dot{y}_t + K_t y_t = C_{ta} \dot{y}_a + K_{ta} y_a,
\]

(7.10)

where the vector representation of the train degrees of freedom, \( y_t \), and the vector representation of the axle degrees of freedom, \( y_a \), both contain the degrees of freedom for each car, in sequence:

\[
y_t = \left[ \begin{array}{cccccc} y_{t1} & y_{t2} & y_{t3} & y_{t4} & y_{t5} & y_{t6} \end{array} \right]^\top, \quad y_a = \left[ \begin{array}{cccc} y_{a1} & y_{a2} & y_{a3} & y_{a4} \end{array} \right]^\top,
\]

(7.11)

and the mass, damping and stiffness matrices are given by Kronecker products with the \( N_c \times N_c \) identity
matrix, $I_{N_c}$ (having the effect of repeating the individual car matrices along a block diagonal):

$$M_t = I_{N_c} \otimes \begin{bmatrix}
\frac{m_c}{2} & \frac{m_c}{2} & 0 & 0 & 0 & 0 \\
-\frac{l_c}{2l_c} & \frac{l_c}{2l_c} & 0 & 0 & 0 & 0 \\
0 & 0 & \frac{m_b}{2} & \frac{m_b}{2} & 0 & 0 \\
0 & 0 & -\frac{l_b}{2l_b} & \frac{l_b}{2l_b} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{m_b}{2} & \frac{m_b}{2} \\
0 & 0 & 0 & 0 & -\frac{l_b}{2l_b} & \frac{l_b}{2l_b}
\end{bmatrix},$$  \hspace{1cm} (7.12)

$$C_t = I_{N_c} \otimes \begin{bmatrix}
c_b & c_b & -\frac{1}{2}c_b & -\frac{1}{2}c_b & -\frac{1}{2}c_b & -\frac{1}{2}c_b \\
-c_b l_b & c_b l_b & \frac{1}{2}c_b l_b & \frac{1}{2}c_b l_b & -\frac{1}{2}c_b l_b & -\frac{1}{2}c_b l_b \\
-c_b & 0 & \frac{1}{2}c_b + c_a & \frac{1}{2}c_b + c_a & 0 & 0 \\
0 & 0 & -c_a l_a & c_a l_a & 0 & 0 \\
0 & -c_b & 0 & 0 & \frac{1}{2}c_b + c_a & \frac{1}{2}c_b + c_a \\
0 & 0 & 0 & 0 & -c_a l_a & c_a l_a
\end{bmatrix},$$ \hspace{1cm} (7.13)

$$K_t = I_{N_c} \otimes \begin{bmatrix}
k_b & k_b & -\frac{1}{2}k_b & -\frac{1}{2}k_b & -\frac{1}{2}k_b & -\frac{1}{2}k_b \\
-k_b l_b & k_b l_b & \frac{1}{2}k_b l_b & \frac{1}{2}k_b l_b & -\frac{1}{2}k_b l_b & -\frac{1}{2}k_b l_b \\
-k_b & 0 & \frac{1}{2}k_b + k_a & \frac{1}{2}k_b + k_a & 0 & 0 \\
0 & 0 & -k_a l_a & k_a l_a & 0 & 0 \\
0 & -k_b & 0 & 0 & \frac{1}{2}k_b + k_a & \frac{1}{2}k_b + k_a \\
0 & 0 & 0 & 0 & -k_a l_a & k_a l_a
\end{bmatrix},$$ \hspace{1cm} (7.14)

$$C_{ta} = I_{N_c} \otimes \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
c_a & c_a & 0 & 0 \\
-c_a l_a & c_a l_a & 0 & 0 \\
0 & 0 & c_a & c_a \\
0 & 0 & -c_a l_a & c_a l_a
\end{bmatrix}, \quad K_{ta} = I_{N_c} \otimes \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
k_a & k_a & 0 & 0 \\
-k_a l_a & k_a l_a & 0 & 0 \\
0 & 0 & k_a & k_a \\
0 & 0 & -k_a l_a & k_a l_a
\end{bmatrix},$$ \hspace{1cm} (7.15)

By this formulation, the mass, damping and stiffness matrices are not symmetrical, since the equations of motion were written for the translation and rotation of each mass, while the degrees of freedom were defined by the end displacements of the spring-damper connections between masses. Traditional modal analysis of multi-body systems requires that these matrices be diagonal; this is not a requirement for
the present analysis.

7.4.3 Girder model

The girder is represented in modal coordinates. The number of girder modes included in the analysis, \( N_g \), can be relatively small: it has been found that 3 modes are usually sufficient to capture the response. The equation of motion for the girder in terms of the modal displacements \( y_g \) is

\[
M_g \ddot{y}_g(t) + C_g \dot{y}_g(t) + K_g y_g(t) = -\sum_{k=-\infty}^{\infty} \Phi_{ag}^\top(t-kT_t) \left[ f_a + M_a \ddot{y}_a(t-kT_t) + C_a \dot{y}_a(t-kT_t) + K_a y_a(t-kT_t) \right] - C_{at} \dot{y}_t(t-kT_t) - K_{at} y_t(t-kT_t)], \quad (7.16)
\]

where \( M_g, C_g, \) and \( K_g \) are the modal mass, damping and stiffness matrices for the girder. The right side of equation 7.16 is the modal load on the girder: i.e. the summation, over the infinite number of trains, of the modal displacement at each axle location multiplied by the force in each axle. The \( k^{\text{th}} \) train produces the identical effect to the \( i^{\text{th}} \) train, but shifted in time by \( kT_t \). The force in a given axle has a static component, corresponding to the weight of the car, and a dynamic component arising from the forces in the spring and damper of the axle suspension.

The modal displacements at the axle locations for a single train are given by \( \Phi_{ag}(t) \), which is a \( n_a \times n_g \) matrix function of time: the \( (i,j) \)th element of \( \Phi_{ag}(t) \) gives the displacement of the \( j^{\text{th}} \) mode of the girder at the location occupied by the \( i^{\text{th}} \) axle at time \( t \). For positions outside the extent of the girder, the modal displacement is zero. Therefore, as long as the distance between trains, \( vT_t \), is greater than the span length (which will be the case during normal operation), there is no overlap between terms in the summation on the right side of equation 7.16, and the infinite series converges to a periodic function of time.

The static axle loads are represented by the vector \( f_a \); the dynamic axle loads are related to the axle displacement vector, \( y_a \), and train displacement vector, \( y_t \), through the matrices \( M_a, C_a, K_a, C_{at} \), and
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\( K_{at} \), defined as follows:

\[
M_a = I_{N_c} \otimes \begin{bmatrix}
    m_a & 0 & 0 & 0 \\
    0 & m_a & 0 & 0 \\
    0 & 0 & m_a & 0 \\
    0 & 0 & 0 & m_a
\end{bmatrix}, \quad C_a = I_{N_c} \otimes \begin{bmatrix}
    c_a & 0 & 0 & 0 \\
    0 & c_a & 0 & 0 \\
    0 & 0 & c_a & 0 \\
    0 & 0 & 0 & c_a
\end{bmatrix}, \quad (7.17)
\]

\[
K_a = I_{N_c} \otimes \begin{bmatrix}
    k_a & 0 & 0 & 0 \\
    0 & k_a & 0 & 0 \\
    0 & 0 & k_a & 0 \\
    0 & 0 & 0 & k_a
\end{bmatrix}, \quad (7.18)
\]

\[
C_{at} = I_{N_c} \otimes \begin{bmatrix}
    0 & 0 & c_a & 0 & 0 & 0 \\
    0 & 0 & 0 & c_a & 0 & 0 \\
    0 & 0 & 0 & 0 & c_a & 0 \\
    0 & 0 & 0 & 0 & 0 & c_a
\end{bmatrix}, \quad K_{at} = I_{N_c} \otimes \begin{bmatrix}
    0 & 0 & k_a & 0 & 0 & 0 \\
    0 & 0 & 0 & k_a & 0 & 0 \\
    0 & 0 & 0 & 0 & k_a & 0 \\
    0 & 0 & 0 & 0 & 0 & k_a
\end{bmatrix}. \quad (7.19)
\]

7.4.4 Train–girder coupling

The coupling between train and girder is created by the assumption that the wheels are in constant contact with the rails (which are assumed to be continuously connected to the girder). This constraint is expressed by

\[
y_a = \sum_{j=-\infty}^{\infty} \Phi_{ag}(t-jT_g)y_g(t-jT_g) + w_a(t-jT_g) + r_a(t). \quad (7.20)
\]

where the vectors \( w_a(t) \) and \( r_a(t) \) give the values of the static displaced shape of the girder, and the track irregularity profile, respectively, at the axle locations at time \( t \). The summation in the above equation is over each of the infinite number of girders.

7.4.5 Transformation to a consistent fundamental period

Because the infinite sums on the right-hand sides of equations 7.16 and 7.20 are periodic sums with finite non-zero extent, substitution of \( y_a \) into equations 7.10 and 7.16 yields a system of two differential equations, each with periodic forcing. The forcing period for the train is the time it takes to traverse a girder, \( T_g \); the forcing period for the girder is the time between trains, \( T_t \). Though these periods are generally different, the forcing on both train and girder can both be expressed in terms of harmonics of
a fundamental period, $T_f$, equal to the least common multiple of $T_g$ and $T_t$:

$$T_f = \text{lcm} \{T_t, T_g\} = n_t T_t = n_g T_g .$$

(7.21)

To reflect this change of fundamental period, the periodic sums are regrouped, yielding:

$$M_g \ddot{y}_g(t) + C_g \dot{y}_g(t) + K_g y_g(t)$$

$$= - \sum_{j=0}^{n_t-1} \sum_{k=-\infty}^{\infty} \Phi_{ag}(t - kT_t - jT_t) \left[ f_a + M_a \ddot{y}_a(t - kT_t - jT_t) + C_a \dot{y}_a(t - kT_t - jT_t) + K_a y_a(t - kT_t - jT_t) \right] ,$$

(7.22)

$$y_a = \sum_{j=0}^{n_g-1} \sum_{k=-\infty}^{\infty} \left[ \Phi_{ag}(t - kT_t - jT_t) y_g(t - kT_t - jT_g) + w_a(t - kT_t - jT_t) \right] + r_a(t)$$

(7.23)

### 7.4.6 Transformation to the frequency domain

Equations 7.10 and 7.16 can be transformed to the frequency domain via the Fourier transform. Because the forcing functions for both the train and girder are periodic functions with period $T_f$, the response will also have period $T_f$. The Fourier transform of a periodic function is an infinite discrete Fourier series. The elements of the series are coefficients multiplying sinusoidal basis functions with frequencies that are multiples of the fundamental frequency $1/T_f$.

In the implementation of this method, time will be discretized to allow a numerical computation using the discrete Fourier transform. The details of this discretization are relatively straightforward with a numerical computation programming language, and will not be discussed here.

In this derivation, the transform to ordinary frequency $\xi$ (in Hz), is used:

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(t) e^{-i2\pi \xi t} dt .$$

(7.24)

The radial frequency, $\omega = 2\pi \xi$, will also be used at times.

After substitution of the expression for $y_a$ given by equation 7.20, the right sides of equations 7.10 and 7.16 both contain periodic sums of products of two functions of time, each with two time shifts. Taking the Fourier transform of such an expression is possible by application of the shift property of the Fourier transform, the Poisson summation formula, and the convolution theorem. For two general
functions $f$ and $g$, shifted by $T_1$ and $T_2 = T_1/n$,

\[
\sum_{j=0}^{n-1} \sum_{k=-\infty}^{\infty} f(t-kT_1-jT_2)g(t-kT_1-jT_2) = \sum_{k=-\infty}^{\infty} \left( \sum_{j=0}^{n-1} e^{-i2\pi kjT_2/T_1} \right) \frac{1}{T_1} \{\hat{f} \ast \hat{g}\}(k/T_1)e^{i2\pi kt/T_1}, \quad (7.25)
\]

where $\ast$ is the convolution operator, defined as

\[
\{f \ast g\}(t) \overset{\text{def}}{=} \int_{-\infty}^{\infty} f(\tau) g(t-\tau) d\tau . \quad (7.26)
\]

The summation in brackets on the right side of equation 7.25 acts as a harmonic “comb”: it is $n$ when $k$ is a multiple of $n$, and zero otherwise. The remaining expression before the exponential term is the $k^{th}$ Fourier coefficient of the Fourier series of the periodic summation of $f \cdot g$, which will be denoted in this way:

\[
\frac{1}{T_1} \{\hat{f} \ast \hat{g}\}(k/T_1) = \{\hat{f} \ast \hat{g}\}_k . \quad (7.27)
\]

In practice, the Fourier coefficients of the periodic summation are calculated directly via the discrete Fourier transform, rather than via the continuous Fourier transform as derived above.

In other words, the transformation from equations 7.16 and 7.20 to equations 7.22 and 7.23 adds zero terms to the summations of the Fourier transform samples given by the Poisson summation formula, such that both sums cover the harmonics of the same fundamental frequency.

Also present in equations 7.22 and 7.23 are the time derivatives of $y_a$, $y_g$, and $y_t$. These derivatives become multiplication by $\omega$ in the frequency domain.

Applying all of these relations, taking the Fourier transforms of equations 7.10, 7.22, and 7.23, and discretizing the frequency domain to harmonics, $\omega_k = k\omega_f$, of the fundamental frequency $\omega_f = 2\pi/T_1$, gives

\[
\left( -\omega_k^2 M_t + i\omega_k C_t + K_t \right) \hat{y}_{tk} = (i\omega_k C_{ta} + K_{ta}) \hat{y}_{ak} , \quad k = -\infty \ldots \infty \quad (7.28)
\]

\[
\left( -\omega_k^2 M_g + i\omega_k C_g + K_g \right) \hat{y}_{gk} = \sum_{j=0}^{n_t-1} e^{-i2\pi kjT_t/T_1} \left( \hat{\Phi}^T_{ag} f_a - \{\hat{\Phi}^T_{ag} \ast \omega^2 M_a \hat{y}_a \}_k + \{\hat{\Phi}^T_{ag} \ast i\omega C_a \hat{y}_a \}_k \right.
\]

\[
\left. + \{\hat{\Phi}^T_{ag} \ast K_a \hat{y}_a \}_k - \{\hat{\Phi}^T_{ag} \ast i\omega C_{at} \hat{y}_t \}_k - \{\hat{\Phi}^T_{ag} \ast K_{at} \hat{y}_t \}_k \right), \quad k = -\infty \ldots \infty \quad (7.29)
\]
\[
\hat{y}_{ak} = \sum_{j=0}^{n_k-1} e^{-i2\pi kjT_f/T_i} \left( \{\hat{\Phi}_{ag} * \hat{y}_g\}_k + \hat{w}_{ak}\right) + \hat{r}_{ak}, \quad k = -\infty \ldots \infty
\] (7.30)

### 7.4.7 Transformation into harmonic matrix form

The discrete samples of the convolutions in the above equations can be calculated by matrix multiplication. Arranging the harmonics of \(\{\hat{\Phi}_{ag} * \hat{y}_g\}\) into a block vector, the infinite sum above can be cast as a matrix multiplication as follows:

\[
\begin{bmatrix}
  \vdots \\
  \{\hat{\Phi}_{ag} * \hat{y}_g\}_{-2} \\
  \{\hat{\Phi}_{ag} * \hat{y}_g\}_{-1} \\
  \{\hat{\Phi}_{ag} * \hat{y}_g\}_0 \\
  \{\hat{\Phi}_{ag} * \hat{y}_g\}_1 \\
  \{\hat{\Phi}_{ag} * \hat{y}_g\}_2 \\
  \vdots 
\end{bmatrix} =
\begin{bmatrix}
  \vdots \\
  \ldots \hat{\Phi}_{ag0} \hat{\Phi}_{ag-1} \hat{\Phi}_{ag-2} \hat{\Phi}_{ag-3} \hat{\Phi}_{ag-4} \ldots \\
  \ldots \hat{\Phi}_{ag1} \hat{\Phi}_{ag0} \hat{\Phi}_{ag-1} \hat{\Phi}_{ag-2} \hat{\Phi}_{ag-3} \ldots \\
  \ldots \hat{\Phi}_{ag2} \hat{\Phi}_{ag1} \hat{\Phi}_{ag0} \hat{\Phi}_{ag-1} \hat{\Phi}_{ag-2} \ldots \\
  \ldots \hat{\Phi}_{ag3} \hat{\Phi}_{ag2} \hat{\Phi}_{ag1} \hat{\Phi}_{ag0} \hat{\Phi}_{ag-1} \ldots \\
  \ldots \hat{\Phi}_{ag4} \hat{\Phi}_{ag3} \hat{\Phi}_{ag2} \hat{\Phi}_{ag1} \hat{\Phi}_{ag0} \ldots \\
  \vdots 
\end{bmatrix}
\begin{bmatrix}
  \vdots \\
  \hat{y}_{g-2} \\
  \hat{y}_{g-1} \\
  \hat{y}_0 \\
  \hat{y}_1 \\
  \hat{y}_2 \\
  \vdots 
\end{bmatrix}
\] (7.31)

\[
= \tilde{\Phi}_{ag} \hat{y}_g
\] (7.32)

To allow the complete transformation of equations 7.10, 7.22, and 7.23 into harmonic form, a few final definitions are required: The “comb” term in equation 7.25, described above, is given a matrix representation in both its manifestation in the girder forcing function, \(\Theta_g\), and in the axle displacement equation, \(\Theta_a\). These matrices are defined as follows:

\[
\Theta_a = \text{diag} \left\{ \sum_{j=0}^{n_k-1} e^{-i2\pi kjT_f/T_i} \right\} \otimes I_{N_cN_a}
\] (7.33)

\[
\Theta_g = \text{diag} \left\{ \sum_{j=0}^{n_k-1} e^{-i2\pi kjT_f/T_i} \right\} \otimes I_{N_g}
\] (7.34)

where \(\text{diag} \{f(k)\}\) denotes an infinite diagonal matrix with the \(k^{th}\) diagonal element equal to \(f(k)\).

A differentiation multiplier matrix \(\Omega\): a diagonal matrix with diagonal entries

\[
\Omega = \text{diag} \{i\omega_k\}
\] (7.35)
which is expanded across the train, girder, and axle degrees of freedom as

\[ \Omega_t = \Omega \otimes I_{N_tN_t} \]  
\[ \Omega_g = \Omega \otimes I_{N_g} \]  
\[ \Omega_a = \Omega \otimes I_{N_aN_a} . \]

Finally, harmonic expansions of the constant mass, damping and stiffness matrices for the train, girder, and axles are each given by a Kronecker product with an infinite-dimensional identity matrix:

\[ \tilde{X} = I_\infty \otimes X . \]  

Given all of the above, equations 7.28 and 7.29 can be written in harmonic matrix form as

\[
\begin{bmatrix}
\tilde{M}_t \Omega_t^2 + \tilde{C}_t \Omega_t + \tilde{K}_t \\
\tilde{M}_g \Omega_g^2 + \tilde{C}_g \Omega_g + \tilde{K}_g + \Theta_g \tilde{\Phi}_{ag}^T \left( \tilde{M}_a \Omega_a^2 + \tilde{C}_a \Omega_a + \tilde{K}_a \right) \\
\tilde{M}_a \Omega_a^2 + \tilde{C}_a \Omega_a + \tilde{K}_a
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\
\tilde{y}_g \\
\tilde{y}_a
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{u}_t \\
\tilde{u}_g \\
\tilde{r}_a
\end{bmatrix},
\]

or

\[
\begin{bmatrix}
A_{tt} & A_{tg} \\
A_{gt} & A_{gg}
\end{bmatrix}
\begin{bmatrix}
\tilde{y}_t \\
\tilde{y}_g
\end{bmatrix}
= 
\begin{bmatrix}
\tilde{u}_t \\
\tilde{u}_g
\end{bmatrix},
\]
where

\[ A_{tt} = \hat{M}_t \Omega_t^2 + \hat{C}_t \Omega_t + \hat{K}_t \]  

(7.43)

\[ A_{gg} = \hat{M}_g \Omega_g^2 + \hat{C}_g \Omega_g + \hat{K}_g + \theta_g \bar{\phi}_{ag} \left( \hat{M}_a \theta_a \hat{\phi}_{ag} \Omega_g^2 \right. \]  

\[ + \left( 2\hat{M}_a \Omega_a + \hat{C}_a \right) \theta_a \bar{\phi}_{ag} \Omega_g + \left( \hat{M}_a \Omega_a^2 + \hat{C}_a \Omega_a + \hat{K}_a \right) \theta_a \bar{\phi}_{ag} \right) \]  

(7.44)

\[ A_{tg} = -\hat{C}_{ta} \theta_a \bar{\phi}_{ag} \Omega_g - \left( \hat{C}_{ta} \Omega_a + \hat{K}_{ta} \right) \theta_a \bar{\phi}_{ag} \]  

(7.45)

\[ A_{gt} = -\theta_g \bar{\phi}_{ag} \left( \hat{C}_{at} \Omega_t + \hat{K}_{at} \right) \]  

(7.46)

\[ \ddot{u}_t = \left( \hat{C}_{ta} \Omega_a + \hat{K}_{ta} \right) \theta_a \ddot{w}_a + \left( \hat{C}_{ta} \Omega_a + \hat{K}_{ta} \right) \ddot{r}_a \]  

(7.47)

\[ \ddot{u}_g = -\theta_g \bar{\phi}_{ag} \left[ \theta_a \ddot{f}_a + \left( \hat{M}_a \Omega_a^2 + \hat{C}_a \Omega_a + \hat{K}_a \right) \theta_a \bar{w}_a \right. \]  

\[ + \left. \left( \hat{M}_a \Omega_a^2 + \hat{C}_a \Omega_a + \hat{K}_a \right) \theta_a \bar{r}_a \right] . \]  

(7.48)

### 7.4.8 Calculation of the harmonic transfer matrix

One form of the harmonic transfer matrix can be found by inverting the matrix of \( A \)'s in equation 7.42 to give \( \ddot{y} = G \ddot{u} \) where \( G = A^{-1} \). The inversion can be completed using block-wise inversion formulae:

\[ G_{gg} = (A_{gg} - A_{gt} A_{tt}^{-1} A_{tg})^{-1} \]  

(7.51)

\[ G_{tt} = A_{tt}^{-1} + A_{tt}^{-1} A_{tg} G_{gg} A_{gt} A_{tt}^{-1} \]  

(7.52)

\[ G_{tg} = -A_{tt}^{-1} A_{tg} G_{gg} \]  

(7.53)

\[ G_{gt} = -G_{gg} A_{gt} A_{tt}^{-1} \]  

(7.54)

However, it is more convenient to have harmonic transfer matrices that directly multiply the harmonic vector representations of the static axle load, the static shape and the track irregularity. Therefore, three harmonic transfer functions for the train response and three for the girder response will be calculated, such that the response of the system is given by the two equations:

\[ \ddot{y}_t = G_{tt} \ddot{f}_a + G_{tw} \ddot{w}_a + G_{tr} \ddot{r}_a , \]  

(7.55)

\[ \ddot{y}_g = G_{gt} \ddot{f}_a + G_{gw} \ddot{w}_a + G_{gr} \ddot{r}_a . \]  

(7.56)
where

\[ G_{tf} = -G_{tg} \Theta_{g} \Phi_{ag}^* \Theta_{a} \]  
\[ G_{gf} = -G_{gg} \Theta_{g} \Phi_{ag}^* \Theta_{a} \]  
\[ G_{tr} = \left[ G_{tt} \left( \tilde{C}_{ta} \Omega_{a} + \tilde{K}_{ta} \right) - G_{tg} \Theta_{g} \Phi_{ag}^* \left( \tilde{M}_{a} \Omega_{a}^2 + \tilde{C}_{a} \Omega_{a} + \tilde{K}_{a} \right) \right] \]  
\[ G_{gr} = \left[ G_{gt} \left( \tilde{C}_{ta} \Omega_{a} + \tilde{K}_{ta} \right) - G_{gg} \Theta_{g} \Phi_{ag}^* \left( \tilde{M}_{a} \Omega_{a}^2 + \tilde{C}_{a} \Omega_{a} + \tilde{K}_{a} \right) \right] \]  
\[ G_{tw} = G_{tr} \Theta_{a} \]  
\[ G_{gw} = G_{gr} \Theta_{a} . \]

If a time-domain response history is desired, it can be calculated for the train from \( \hat{y}_t \) (if a specific irregularity simulation is used for \( \hat{r}_a \)) by rearranging the block harmonic vector into a \( n_t \times n_H \) matrix and taking the inverse discrete Fourier transform of each row. The time-domain response history for the girder can be calculated in the same way.

### 7.5 Time-domain solution

As noted above, the more common alternative to this frequency domain approach is to solve the system in the time domain using a time-stepping algorithm. In this approach, the system cannot be infinite, but should consist of a single train and a finite number of spans. The number of spans must be large enough to allow the analysis to reach steady state. For a system with \( n_g \) spans, the mathematical system consists of \( n_g \) equations in the form of equation 7.16, but without the infinite sum as there is only one train; equation 7.10; and equation 7.20 with the infinite sum replaced with a finite sum over the \( n_g \) spans. Neglecting, for simplicity, the camber term \( w_a \), this system of equations can be arranged into the form

\[
\begin{bmatrix}
M_t & 0 \\
0 & M_{gn}
\end{bmatrix}
\begin{bmatrix}
\dot{y}_t \\
\dot{y}_{gn}
\end{bmatrix}
+ \begin{bmatrix}
C_t & -C_{ta} \Phi_{agn} \\
-\Phi_{agn}^T C_{at} & \tilde{C}_{gn}
\end{bmatrix}
\begin{bmatrix}
\dot{y}_t \\
\dot{y}_{gn}
\end{bmatrix}

+ \begin{bmatrix}
K_t & -v C_{ta} \Phi_{agn} - K_{ta} \Phi_{agn} \\
-\Phi_{agn}^T K_{at} & \tilde{K}_{gn}
\end{bmatrix}
\begin{bmatrix}
y_t \\
y_{gn}
\end{bmatrix}
= \begin{bmatrix}
C_{ta} \dot{v} r'_a + K_{ta} r_a \\
\tilde{p}_{gn}
\end{bmatrix},
\]  
(7.63)
where

\[
\begin{align*}
\tilde{M}_{gn} &= M_{gn} + \Phi_{agn}^\top M_a \Phi_{agn} \\
\tilde{C}_{gn} &= C_{gn} + \Phi_{agn}^\top C_a \Phi_{agn} + 2v \Phi_{agn}^\top M_a \Phi' \\
\tilde{K}_{gn} &= K_{gn} + \Phi_{agn}^\top K_a \Phi_{agn} + v \Phi_{agn}^\top C_a \Phi' + v^2 \Phi_{agn}^\top M_a \Phi'' \\
\tilde{p}_{gn} &= -\Phi_{agn}^\top f_a - \Phi_{agn}^\top K_a r_a - v \Phi_{agn}^\top C_a r'_a - v^2 \Phi_{agn}^\top M_a r''_a
\end{align*}
\]

(7.64)

(7.65)

(7.66)

(7.67)

The above expressions are the modal mass, damping, stiffness, and loading of the girders, modified to account for the inertial effect of the unsprung axle masses.

Defining the combined mass, damping and stiffness matrices in equation 7.63, evaluated at discretized time value \( t_i \), as \( M_i \), \( C_i \), and \( K_i \), and the loading vector in equation 7.63 at time \( t_i \) as \( p_i \), equation 7.63 can be solved by the Newmark \( \beta \) method as follows:

\[
\ddot{y}_{i+1} = \left[ M_{i+1} + \gamma \Delta t C_{i+1} + \beta (\Delta t)^2 K_{i+1} \right]^{-1} \begin{bmatrix} p_{i+1} - C_{i+1} \dot{u}_i - (1 - \gamma) \Delta t C_{i+1} \ddot{u}_i - K_{i+1} u_i - \Delta t K_{i+1} \dot{u}_i - (0.5 - \beta)(\Delta t)^2 K_{i+1} \ddot{u}_i \end{bmatrix}
\]

(7.68)

where the constants \( \gamma \) and \( \beta \) are taken as \( \gamma = 1/2 \), \( \beta = 1/4 \) (see Yang et al. [96] for further explanation the Newmark method).

Care must be taken to ensure that discontinuities in the analytical representation of the mode shapes (or at rigid supports of a finite element model) do not induce artificial impact loads to the system. The mode shape can be smoothed or the loads artificially ramped at the beginning and the end of each span to overcome this numerical problem [7]. In the frequency domain analysis, this smoothing is achieved as a byproduct of truncating the number of harmonics in the Fourier transform of the girder mode shapes.

### 7.6 Modelling track irregularity

Tolerances in the fabrication and placement of track lead to random deviations from the desired elevation, gauge, and cross-level. To allow simulation of track irregularity that is consistent with the irregularities observed on in-service track, power spectral densities are recommended by the U.S. Federal Railroad Administration (FRA) [38]. These power spectral densities give the mean-squared deviation from level per unit of spatial frequency as a function of spatial frequency. They are one-sided power spectral densities, meaning that they are defined over the positive frequencies only.

The FRA defines six classes of track, with Class 1 being the roughest, and Class 6 the smoothest.
The PSD for elevation irregularity in Class 6 track is

\[
P_v(\Omega) = 0.60 \text{mm}^2 \text{rad m}^{-1} \times \frac{(0.82 \text{rad m}^{-1})^2(\Omega^2 + (0.15 \text{rad m}^{-1})^2)}{\Omega^4(\Omega^2 + (0.82 \text{rad m}^{-1})^2)}.
\] (7.69)

The above power spectral density is valid over spatial wavelengths from 3 m to 300 m. Short wavelength irregularity is caused by the manufacturing process, and by increased local wear around welds and material defects. Longer wavelength components correspond to irregularities caused by differential compaction of ballast, survey error, frost heave, etc., most of which are not applicable to direct-fixation continuously-welded track supported on guideway structure. Therefore, in the context of this chapter, only the wavelength range from 3 m to 30 m will be considered in modelling track irregularities. The power spectral density over this range is plotted in Figure 7.3.

From this power spectral density, the Fourier coefficients of a representative irregularity profile can be found by “lumping” the power spectral density at the harmonics of the fundamental frequency, and giving each frequency component a uniformly distributed random phase angle, \( \alpha_k \):

\[
\hat{r}_k = \begin{cases} 
0 & \text{for } k = 1 \\
\frac{e^{i2\pi\alpha_k}}{\sqrt{2}} P_v(\Omega_k) \frac{2\pi}{l_f} & \text{for } k = 2, \ldots, n_h \\
\frac{e^{-i2\pi\alpha_{2n_h-k+1}}}{\sqrt{2}} P_v(\Omega_{2n_h-k+1}) \frac{2\pi}{l_f} & \text{for } k = n_h + 1, \ldots, n_H
\end{cases}
\] (7.70)

where

\[
\Omega_k = \frac{2\pi k}{l_f}.
\] (7.71)

The root mean square amplitudes of each harmonic are the absolute values of the above coefficients,
obtained by leaving out the phase angle, and discarding frequencies above the Nyquist frequency.

### 7.7 Passenger comfort evaluation according to ISO 2631

The one-sided root mean square acceleration spectrum of the train vibrations can be calculated as

\[
\dot{a}_{\text{rms}} = \sqrt{2 |\Omega t^2 G_{\text{tt}} \ddot{f} + \Omega t^2 G_{\text{tw}} \ddot{w}|^2 + 2 |\Omega t^2 G_{\text{tr}} \ddot{r}_a|^2}
\]  

(7.72)

In the above equation, the square root and squares operate element-by-element, and the harmonics above the Nyquist frequency are to be discarded. The terms relating to the static train load, \( \ddot{f} \), and the static girder deflected shape, \( \ddot{w} \), are grouped together, as they are both deterministic. The track irregularity, being a random process, is combined with these deterministic responses in a root mean square manner.

The level of passenger discomfort caused by this vibration can be evaluated following the procedure outlined in ISO 2631: Mechanical vibration and shock – Evaluation of human exposure to whole-body vibration \[45, 46\]. The response spectrum is divided into third-octave bands, and the root mean square accelerations falling within each band are summed. Then, a total frequency-weighted RMS acceleration is calculated by

\[
a_w = \sqrt{\sum_i (W_i \dot{a}_{\text{rms}i})^2},
\]  

(7.73)

with the weighting values \( W_i \) as specified by ISO 2631-4 \[46\] and plotted in Figure 7.4.

![Figure 7.4: ISO 2631 frequency weighting functions \[46\]](image)

The frequency-weighted acceleration can be interpreted qualitatively according to Table 7.1. An intuitive sense of these acceleration values can be gained by converting them to a percentage of gravitational acceleration, which can for convenience be taken as 10 m/s\(^2\). Thus, the table indicates that accelerations
below 3% of gravity are not uncomfortable, with increasing discomfort up to 20% of gravity, which is extremely uncomfortable.

Table 7.1: Comfort reactions to public transport vibration levels [45]

<table>
<thead>
<tr>
<th>Frequency-weighted acceleration, $a_w$ (m/s²)</th>
<th>Comfort level</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 0.315</td>
<td>not uncomfortable</td>
</tr>
<tr>
<td>0.315 − 0.63</td>
<td>a little uncomfortable</td>
</tr>
<tr>
<td>0.5 − 1</td>
<td>fairly uncomfortable</td>
</tr>
<tr>
<td>0.8 − 1.6</td>
<td>uncomfortable</td>
</tr>
<tr>
<td>1.25 − 2.5</td>
<td>very uncomfortable</td>
</tr>
<tr>
<td>&gt; 2</td>
<td>extremely uncomfortable</td>
</tr>
</tbody>
</table>

7.8 Optimal camber for live load

The train response can be minimized by determining $\vec{w}$ in order to minimize the quantity $|G_{tt}f_a + G_{tw}\vec{w}_a|$ in equation 7.72. This could be solved as a linear, least-squares, optimization problem, which would yield a specific static shape, different for each axle, that would minimize the response of a given train degree of freedom. However, the static shape must, of course, be the same for all axles, and furthermore, the camber can only practically be specified as a smooth, symmetrical profile. Therefore, the most straightforward way to investigate the effect of cambering for live load is to take the cambered shape to be the same as the fundamental mode shape of the girder, and run the analysis for a range of maximum midspan camber values. Results of such an analysis are presented and discussed in the applications described in later sections of this chapter.

It must be borne in mind, however, that long-term deflection of the girder will decrease any initial camber. Creep and shrinkage are more limited to early age in UHPFRC than in ordinary concrete, but if the UHPFRC is cracked under fatigue loads, as is proposed for the GFRP–UHPFRC girder design presented in this thesis, there will be some permanent loss of camber due to fatigue damage to the UHPFRC. Based on the assumptions outlined in Section 3.5.6, a downward deflection of 14 mm is expected long-term.

7.9 Comparison with solution by numerical integration

Figure 7.5 shows a comparison of response time-histories calculated by the method described here, and by a Newmark method numerical integration. The same system, consisting of a three-car MkII train traversing a simply-supported 30 m span at 22 m/s is analysed by both methods. The total length of the train is 51 m, so there are axles on the girder for $(51 + 30)/22 = 3.6$ s, as reflected by the duration of
large deflection in the girder, which is plotted in the bottom of Figure 7.5. During this time, the front axle of the train traverses \((3.6 \times 22)/30 = 2.7\) spans, as shown in the top of Figure 7.5. The solid lines are the response calculated by the frequency-domain analysis described above. The dotted lines show the response calculated by numerical integration of the coupled differential equations 7.10 and 7.16 using Newmark’s method. The grey line shows the quasi-static displacement, as a base line. It can be seen that the agreement is good.

Figure 7.5: Response time-history for example system
7.10 Application

7.10.1 Introduction

The efficacy of the analysis method presented here will be demonstrated through analysis of two guideway structures. First, the Expo Line girders will be assessed, carrying the Bombardier MkI ART vehicles for which they were designed. As this is a fully serviceable existing structure, this analysis will provide a general validation of the analysis method, and a baseline for comparison. Then the proposed GFRP–UHPFRC girder will be analysed, using the more modern MkII ART vehicle.

7.10.2 General assumptions

Rayleigh damping

The girder damping is mass and stiffness proportional:

\[ c_{gk} = \alpha m_{gk} + \beta k_{gk} \]  \hspace{2cm} (7.74)

Substituting the relations \( c_{gk} = 2\zeta_g \sqrt{k_{gk} m_{gk}} \) and \( \omega_{gk} = \sqrt{k_{gk} / m_{gk}} \) gives

\[ \zeta_{gk} = \frac{1}{2} \left( \frac{\alpha}{\omega_{gk}} + \beta \omega_{gk} \right) \]  \hspace{2cm} (7.75)

Values of \( \alpha = 0.663 \), \( \beta = 0.000604 \) will be used in all analyses. These constants give damping ratios between 2\% and 3\% for natural frequencies between 2 and 13 Hz.

Track irregularity

A sample track irregularity profile is generated by taking the inverse discrete Fourier transform of the Fourier coefficients given by equation 7.70. The random phase angle in the Fourier coefficients produces randomness in the track profile. Several such profiles are plotted in Figure 7.6.
Figure 7.6: Track irregularity profile samples.
7.10.3 Vancouver Expo Line

The train, girder, and system parameters for the Vancouver Expo Line are summarized in Table 7.2, below.

Table 7.2: Vancouver Expo Line system parameters

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bombardier MkI Train</strong></td>
<td></td>
</tr>
<tr>
<td>Number of cars</td>
<td>( N_c )</td>
</tr>
<tr>
<td>Degrees of freedom per car</td>
<td>( n_{tc} )</td>
</tr>
<tr>
<td>Degrees of freedom per train</td>
<td>( n_t )</td>
</tr>
<tr>
<td>Axle degrees of freedom per car</td>
<td>( n_a )</td>
</tr>
<tr>
<td>Car length</td>
<td>( l_c )</td>
</tr>
<tr>
<td>Length between bogies</td>
<td>( 2l_b )</td>
</tr>
<tr>
<td>Length between bogie axles</td>
<td>( 2l_a )</td>
</tr>
<tr>
<td>Mass of car body</td>
<td>( m_c )</td>
</tr>
<tr>
<td>Mass of bogie</td>
<td>( m_b )</td>
</tr>
<tr>
<td>Mass of axle and wheels</td>
<td>( m_a )</td>
</tr>
<tr>
<td>Pitching inertia of car body</td>
<td>( I_c )</td>
</tr>
<tr>
<td>Pitching inertia of bogie</td>
<td>( I_b )</td>
</tr>
<tr>
<td>Primary suspension damping</td>
<td>( c_a )</td>
</tr>
<tr>
<td>Secondary suspension damping</td>
<td>( c_b )</td>
</tr>
<tr>
<td>Primary suspension stiffness</td>
<td>( k_a )</td>
</tr>
<tr>
<td>Secondary suspension stiffness</td>
<td>( k_b )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Expo Line Girders</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Spatial discretization increment</td>
<td>( \Delta x )</td>
</tr>
<tr>
<td>Number of continuous spans</td>
<td>( N_s )</td>
</tr>
<tr>
<td>Span length</td>
<td>( L )</td>
</tr>
<tr>
<td>Elastic modulus</td>
<td>( E )</td>
</tr>
<tr>
<td>Area moment of inertia</td>
<td>( I )</td>
</tr>
<tr>
<td>Mass per unit length</td>
<td>( m )</td>
</tr>
<tr>
<td>Rayleigh damping mass proportionality</td>
<td>( \alpha )</td>
</tr>
<tr>
<td>Rayleigh damping stiffness proportionality</td>
<td>( \beta )</td>
</tr>
<tr>
<td>Natural frequencies</td>
<td>( f_g )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>System</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Train speed</td>
<td>( v )</td>
</tr>
<tr>
<td>Distance between trains</td>
<td>( l_t )</td>
</tr>
<tr>
<td>Girder analysis length</td>
<td>( l_g )</td>
</tr>
<tr>
<td>Fundamental analysis length</td>
<td>( l_f = l_t, l_g )</td>
</tr>
<tr>
<td>Number of spatial increments in analysis</td>
<td>( n_f )</td>
</tr>
<tr>
<td>Number of harmonics included in analysis</td>
<td>( n_h )</td>
</tr>
</tbody>
</table>

Following the analysis described above and converting the train and girder response back to the time domain gives the vertical displacement time histories shown in Figures 7.7 and 7.8, below.

The average frequency-weighted acceleration value for the car body degrees of freedom is 0.51 m/s²
Figure 7.7: Vertical displacement time history of the front of the first train car traversing the Expo Line girder. $v = 80 \text{ km/h}$ (solid) and $v \approx 0$ (dotted).

Figure 7.8: Vertical displacement time history of the Expo Line girder modes. $v = 80 \text{ km/h}$: mode 1 (solid black), modes 2 and 3 (grey). $v \approx 0$: mode 1 (dotted).
for a speed of 80 km/h (22.2 m/s), and 0.31 for 40 km/h (11.1 m/s). Thus at the maximum operating speed, the ride quality might be described as a little uncomfortable, while for slower speeds the vertical accelerations approach the "not uncomfortable" range. This is especially true if it is considered that the above numbers are for the accelerations directly above the bogies. Passengers in the middle of the cars would experience lower levels of acceleration, due to the averaging of the effectively independent random excitations at the front and back of each car.

### 7.10.4 GFRP–UHPFRC Girders

Table 7.3, below, gives the parameters used for dynamic analysis of the proposed GFRP–UHPFRC guideway girder.

The vertical displacement responses of the first, representative train degree of freedom, and the girder modes are illustrated in Figures 7.9 and 7.10, along with the quasi-static displacements, for comparison. The reference point for displacements is the undisplaced level of the girder; the track irregularity means that even in the quasi-static case, the train displacements do not return to zero at every support location.

![Figure 7.9](image)

Figure 7.9: Vertical displacement time history of the front of the first train car traversing a GFRP–UHPFRC girder. \( v = 80 \text{ km/h} \) (solid) and \( v \approx 0 \) (dotted).

The frequency-weighted accelerations for the GFRP–UHPFRC girder system are summarized in Table 7.4, below. At the initial uncracked stiffness, with a slight upward camber, the response is only slightly worse than the Expo girders, and falls at all speeds within the top two comfort levels of Table 7.1. Long-term, as the UHPFRC softens, the vibration response degrades, to the point that at top speed, the ride quality falls into the "fairly uncomfortable" range. However, given the number of assumptions that have gone into both the calculation of the long-term stiffness degradation, and the response itself, this result alone does not indicate a concerning deficiency in the design. At this preliminary stage, of design, the point is that even considering a reduced stiffness accounting for long-term fatigue effects, the
Table 7.3: GFRP–UHPFRC guideway system parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bombardier MkII Train</strong></td>
<td></td>
</tr>
<tr>
<td>Number of cars</td>
<td>$N_c$</td>
</tr>
<tr>
<td>Degrees of freedom per car</td>
<td>$n_{tc}$</td>
</tr>
<tr>
<td>Degrees of freedom per train</td>
<td>$n_t$</td>
</tr>
<tr>
<td>Axle degrees of freedom per car</td>
<td>$n_a$</td>
</tr>
<tr>
<td>Car length</td>
<td>$l_c$</td>
</tr>
<tr>
<td>Length between bogies</td>
<td>$2l_b$</td>
</tr>
<tr>
<td>Length between bogie axles</td>
<td>$2l_a$</td>
</tr>
<tr>
<td>Mass of car body (crush loaded)</td>
<td>$m_c$</td>
</tr>
<tr>
<td>Mass of bogie</td>
<td>$m_b$</td>
</tr>
<tr>
<td>Mass of axle and wheels</td>
<td>$m_a$</td>
</tr>
<tr>
<td>Pitching inertia of car body</td>
<td>$I_c$</td>
</tr>
<tr>
<td>Pitching inertia of bogie</td>
<td>$I_b$</td>
</tr>
<tr>
<td>Primary suspension damping (nominal value ≈ 0)</td>
<td>$c_a$</td>
</tr>
<tr>
<td>Secondary suspension damping</td>
<td>$c_b$</td>
</tr>
<tr>
<td>Primary suspension stiffness</td>
<td>$k_a$</td>
</tr>
<tr>
<td>Secondary suspension stiffness</td>
<td>$k_b$</td>
</tr>
<tr>
<td><strong>GFRP–UHPFRC Girders</strong></td>
<td></td>
</tr>
<tr>
<td>Spatial discretization increment</td>
<td>$\Delta x$</td>
</tr>
<tr>
<td>Number of continuous spans</td>
<td>$N_s$</td>
</tr>
<tr>
<td>Span length</td>
<td>$L$</td>
</tr>
<tr>
<td>Base elastic modulus</td>
<td>$E$</td>
</tr>
<tr>
<td>Transformed area moment of inertia, initial</td>
<td>$I_i$</td>
</tr>
<tr>
<td>Transformed area moment of inertia, long-term</td>
<td>$I_f$</td>
</tr>
<tr>
<td>Mass per unit length</td>
<td>$m$</td>
</tr>
<tr>
<td>Rayleigh damping mass proportionality</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Rayleigh damping stiffness proportionality</td>
<td>$\beta$</td>
</tr>
<tr>
<td>Natural frequencies, initial</td>
<td>$f_g$</td>
</tr>
<tr>
<td>Natural frequencies, long-term</td>
<td>$f_g$</td>
</tr>
<tr>
<td><strong>System</strong></td>
<td></td>
</tr>
<tr>
<td>Train speed</td>
<td>$v$</td>
</tr>
<tr>
<td>Distance between trains</td>
<td>$l_k$</td>
</tr>
<tr>
<td>Girder analysis length</td>
<td>$l_g$</td>
</tr>
<tr>
<td>Fundamental analysis length</td>
<td>$l_f = \text{lcm} {l_k, l_g}$</td>
</tr>
<tr>
<td>Number of spatial increments in analysis</td>
<td>$n_f = l_f/\Delta x$</td>
</tr>
<tr>
<td>Number of harmonics included in analysis</td>
<td>$n_h$</td>
</tr>
</tbody>
</table>


girder performance is at close to acceptable. A more refined analysis, in three dimensions, would include rail-structure interaction of the specific rail fastener system to be used, the specific geometry of the structure, and the expected peak operating speed given the distance between stations. If at this detailed level of design the system was still exhibiting deficiency with regard to passenger comfort, the most practical solution would be local stiffening of specific girders in problematic areas, either by thickening of cross-section elements, or shortening the span lengths.

Table 7.4: Frequency-weighted accelerations for Expo Line girders and proposed GFRP–UHPFRC girders

<table>
<thead>
<tr>
<th></th>
<th>LL camber</th>
<th>40 km/h</th>
<th>80 km/h</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expo Line</td>
<td>0</td>
<td>0.31</td>
<td>0.51</td>
</tr>
<tr>
<td>GFRP–UHPFRC, uncracked stiffness</td>
<td>14 mm</td>
<td>0.36</td>
<td>0.58</td>
</tr>
<tr>
<td>GFRP–UHPFRC, long-term stiffness</td>
<td>0</td>
<td>0.41</td>
<td>0.71</td>
</tr>
</tbody>
</table>

7.11 Conclusions

A frequency domain approach for determining the steady-state behaviour of a train-girder system has been presented. While the approach is complex in its derivation, once implemented it allows for rapid evaluation of the impact of various fundamental parameters on the train and girder response.

The method is applied to the analysis of the Vancouver Expo Line girders, to provide a baseline for comparison, before assessing the dynamic behaviour of the proposed GFRP–UHPFRC girder, demonstrating that the girder can provide passenger comfort within a reasonable range. Fine-tuning of the dynamic behaviour would be necessary once an actual track alignment and span arrangement was in hand.
Chapter 8

Conclusions

8.1 Summary of Design & Research Contributions

An innovative precast girder design for rail guideway applications has been presented.

By conceptualising and developing a design that combines UHPFRC and GFRP in an innovative way, this thesis adds to the body of design knowledge allowing industry to take advantage of the currently untapped potential offered by these high-performance materials.

GFRP is a particularly suitable composite match for UHPFRC, and vice-versa: GFRP–UHPFRC can offer structural performance that is greater than the sum of its parts. Through the course of the work presented here, the ability to vacuum infuse GFRP plate has been introduced to the University of Toronto structures lab, opening the possibility of studying this intriguing material combination.

The conceptual design of the proposed guideway girder was informed by an extensive study of the functional requirements for guideway structures, and of the historical development of structural systems for guideway. A critical review of fifteen existing guideway structures provides a valuable context for understanding the important considerations for guideway design, and points to the opportunity for an innovative precast girder system to provide a competitive advantage over current state-of-the-art guideway construction methods.

Out of a number of considered concepts, a girder consisting of a GFRP box shell which provides webs and forms for a top and bottom flange of composite GFRP–UHPFRC was identified as a promising design. A study of the flexural behaviour of this girder in comparison with other conceptual and conventional designs supports the conviction that the GFRP–UHPFRC design offers significant potential value.

Four important questions were identified regarding the proposed GFRP–UHPFRC guideway girder
1. Can GFRP and UHPFRC be connected by a strong, reliable bond, to create full composite action, and can this bond mechanism be fabricated in an economically feasible way?

2. Given that serviceability limit states govern the design, can a dynamic analysis method for the evaluation of passenger comfort performance be developed?

3. Can UHPFRC be relied upon to carry tensile stresses under fatigue loading?

4. Is there potential for microcracked UHPFRC to add damping to the system, improving both the dynamic and fatigue performance?

To answer these questions, a suite of experimental and analytical studies was undertaken, resulting in the following conclusions:

1. A novel means of efficiently creating a reliable shear connection between GFRP and UHPFRC has been invented, as presented in Chapter 4. This stud-and-texture connector is capable of providing the stiffness and strength required by the proposed girder design, and can be fabricated in an efficient way that is scalable to full-size girder production.

2. To better understand the dynamic performance of light-weight guideway girders, a frequency domain analysis method was developed, as presented in Chapter 7, based on the theory of linear, periodically time-varying, systems. In combination with the passenger comfort assessment guidelines set by the relevant ISO standards, and a spectral density representation of the random track irregularities, the presented method can be used to assess the dynamic performance of a proposed girder without the need for numerical time-history integration, or Monte Carlo simulation. Studies of the vertical vibration response indicate that the dynamic performance of the proposed girder is within a reasonable range.

3. Fatigue testing of UHPFRC in direct tension, as presented in Chapter 5, shows that there is an endurance limit for peak stress, below which no progressive damage of the UHPFRC occurs. For the U of T UHPFRC mix, this limit is approximately 7.5 MPa, which is the approximate stress level coinciding with the onset of micro-cracking. Beyond this limit, damage progression was erratic, leading to the recommendation that tensile fatigue stresses in UHPFRC be kept below 7.5 MPa.

4. Damping in UHPFRC was measured by both free and forced vibration tests, presented in Chapter 6. It is concluded that damping in UHPFRC is primarily rate-independent, with damping ratio $\zeta = 2\%$.
(or, equivalently, loss factor $\eta = 0.04$). These values appear to hold true for any stable level of damage in the UHPFRC, increasing temporarily with the onset of further damage. Under reversed cyclic loading much higher damping is observed. Based on these observations, it is concluded that UHPFRC does not offer much possibility for damping of repeated dynamic service loads, but may offer benefits for seismic resilience.

Simple design checks indicate that the proposed guideway girder design could be taken to a more detailed level with the confidence that no unsurmountable challenges would arise. Complete detailed structural design is impractical from an academic position, isolated from a specific client and site. However, the results contained here will hopefully provide a valuable resource for the advancement of high-quality guideway structure design, and of GFRP–UHPFRC structural systems.

8.2 Recommendations for future work

A number of directions for further study are evident at the conclusion of the present work:

1. Fabrication of a full girder prototype would provide a valuable proof-of-concept of the fabrication method and the overall girder behaviour.

2. Since each fatigue test to 5 million cycles lasts just under one week, the number of tests that could be undertaken as part of this thesis was limited. Further testing of fatigue under different stress ranges, and replication of the reported results are required before full confidence can be attained regarding the fatigue performance of UHPFRC.

3. Due to the difficulty encountered in gripping the GFRP–UHPFRC specimens, very limited results were obtained regarding the tensile static and fatigue behaviour of GFRP–UHPFRC. The two tests described in Chapter 5 show some intriguing results, which warrant a more extensive program of tests.
Bibliography


Appendix A

Laminate Properties
A.1 Experimental laminate

The following laminate was used for all experimental work. Based on the measured modulus of elasticity, the fibre volume fraction is approximately 40%. Eight layers of primarily unidirectional fabric make up the bulk of the plate, with layers of biaxial fabric on each face to help spread the load from shear connector studs throughout the plate. The chopped strand mat layers provide a degree of isotropy, and promote resin flow during infusion, but cause a significant reduction in stiffness. In future work, it is recommended that the chopped strand mat laminae be eliminated, if possible.

![Figure A.1: GFRP lay-up used for experimental work](image)

Table A.1: Experimental laminate properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal elastic modulus (E_x)</td>
<td>30 GPa</td>
</tr>
<tr>
<td>Transverse elastic modulus (E_y)</td>
<td>13 GPa</td>
</tr>
<tr>
<td>Shear modulus (G_{xy})</td>
<td>5 GPa</td>
</tr>
<tr>
<td>Longitudinal coefficient of thermal expansion (\alpha_x)</td>
<td>0.0085 %/°C</td>
</tr>
<tr>
<td>Transverse coefficient of thermal expansion (\alpha_y)</td>
<td>0.022 %/°C</td>
</tr>
<tr>
<td>Density (\gamma_{GFRP})</td>
<td>1920 kg m(^{-3})</td>
</tr>
</tbody>
</table>

![Stress-strain graph](image)
A.2 Laminates assumed for girder design

The fibre volume fraction is assumed to be 60%. This fibre volume fraction is higher than was achieved in our lab, but is a reasonable value for vacuum infusion under good conditions [9].

A.2.1 Webs, cantilever wings, and top slab closure plate

The following 18 mm thick quadraxial laminate is used for the webs and top slab of the proposed GFRP–UHPFRC girder.

![GFRP lay-up used for girder top slab and webs.](image)

Figure A.2: GFRP lay-up used for girder top slab and webs.

Table A.2: Girder laminate properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal elastic modulus $E_x$</td>
<td>30 GPa</td>
</tr>
<tr>
<td>Transverse elastic modulus $E_y$</td>
<td>18 GPa</td>
</tr>
<tr>
<td>Shear modulus $G_{xy}$</td>
<td>9 GPa</td>
</tr>
<tr>
<td>Longitudinal coefficient of thermal expansion $\alpha_x$</td>
<td>0.008 °C⁻¹</td>
</tr>
<tr>
<td>Transverse coefficient of thermal expansion $\alpha_y$</td>
<td>0.015 °C⁻¹</td>
</tr>
<tr>
<td>Density $\gamma_{GFRP}$</td>
<td>2080 kg m⁻³</td>
</tr>
</tbody>
</table>

![Stress-strain curve](image)
Appendix A. Laminate Properties

Shear stress, $\tau$ (MPa) vs. shear strain, $\gamma$ (%)

$\cdot 10^{-2}$
A.2.2 Bottom plate

The following 30 mm thick predominantly unidirectional laminate is used for the bottom slab plate of the proposed GFRP–UHPFRC girder.

![GFRP lay-up used for girder bottom slab.](image)

Figure A.3: GFRP lay-up used for girder bottom slab.

Table A.3: Girder laminate properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal elastic modulus $E_x$</td>
<td>42 GPa</td>
</tr>
<tr>
<td>Transverse elastic modulus $E_y$</td>
<td>16 GPa</td>
</tr>
<tr>
<td>Shear modulus $G_{xy}$</td>
<td>6 GPa</td>
</tr>
<tr>
<td>Longitudinal coefficient of thermal expansion $\alpha_x$</td>
<td>0.007 %/perthousandzero°C</td>
</tr>
<tr>
<td>Transverse coefficient of thermal expansion $\alpha_y$</td>
<td>0.020 %/perthousandzero°C</td>
</tr>
<tr>
<td>Density $\gamma_{GFRP}$</td>
<td>2080 kg m$^{-3}$</td>
</tr>
</tbody>
</table>

![Stress-strain curve](image)

strain, $\epsilon$ (%): $\cdot 10^{-2}$
Appendix B

Cross Sections

The following sections present moment-curvature diagrams of the cross sections that are compared in Chapter 3.
B.1 All-GFRP

Moment-curvature response

\[
\begin{align*}
\text{moment, } M (\text{kN}\cdot\text{m}) & \quad -20,000 \quad 0 \quad 20,000 \quad 40,000 \\
\text{curvature, } \kappa (\%e/\text{m}) & \quad -2 \quad -1.5 \quad -1 \quad -0.5 \quad 0 \quad 0.5 \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4 \quad 4.5 \quad 5
\end{align*}
\]

Crushing of UHPPRC
Creep rupture limit for GFRP
UHPPRC cracking
Limit of UHPPRC strain hardening
B.2 GFRP–UHPFRC

Moment-curvature response

![Diagram of cross sections and moment-curvature response graph]

- Creep rupture limit for GFRP
- Limit of UHPFRC strain hardening
- UHPFRC cracking
- Limit of UHPFRC strain hardening

Curvature, $\kappa$ (%/m) vs. Moment, $M$ (kNm)
B.3 Dura UBG1250

Moment-curvature response

![Diagram of Dura UBG1250 cross section with moment-curvature graph.]
B.4 Expo Line

\[ \kappa \text{ (\%/per thousand)} \quad M \text{ (kNm)} \]

Moment-curvature response

\begin{figure}
\centering
\includegraphics[width=\textwidth]{moment_curvature_response.png}
\end{figure}
Appendix C

Design Load Cases

The train loading and design load combinations used in validating the proposed GFRP–UHPFRC guideway girder design are outlined in this section. Most of the specifications are taken from the Vancouver Millennium Line Design Manual [14]. The maximum sectional forces arising in a 30 m span having the GFRP–UHPFRC cross section described in Appendix B are given for each load case. Factored combinations of these load cases to be used in limit states design checks are described in Appendix D.

C.1 Load Case Definitions

The following definitions of load cases used for design are reproduced from the Vancouver Millennium Line Design Manual [14].

<table>
<thead>
<tr>
<th>Table C.1: Load Cases</th>
</tr>
</thead>
<tbody>
<tr>
<td>D Dead load</td>
</tr>
<tr>
<td>L Live load</td>
</tr>
<tr>
<td>I Dynamic load allowance</td>
</tr>
<tr>
<td>LC Rolling/lurching load</td>
</tr>
<tr>
<td>CF1 Normal centrifugal force</td>
</tr>
<tr>
<td>CF2 Extreme centrifugal force</td>
</tr>
<tr>
<td>HF Hunting forces</td>
</tr>
<tr>
<td>LN Braking load - normal operation</td>
</tr>
<tr>
<td>LE Braking load - emergency</td>
</tr>
<tr>
<td>COLH Horizontal collision load</td>
</tr>
<tr>
<td>COLV Vertical collision load</td>
</tr>
<tr>
<td>W1 Wind load on girder and train</td>
</tr>
<tr>
<td>W2 Wind load on girder only</td>
</tr>
<tr>
<td>EQ Earthquake load</td>
</tr>
<tr>
<td>T Uniform temperature change</td>
</tr>
<tr>
<td>TG Temperature gradient</td>
</tr>
<tr>
<td>SIV Snow/ice on train</td>
</tr>
<tr>
<td>SIG Snow/ice on girder</td>
</tr>
<tr>
<td>SE Settlement</td>
</tr>
<tr>
<td>LR1 Longitudinal restraint of rails - SLS</td>
</tr>
<tr>
<td>LR2 Longitudinal restraint of rails - ULS</td>
</tr>
<tr>
<td>RT1 Radial restraint of curved rails - SLS</td>
</tr>
<tr>
<td>RT2 Radial restraint of curved rails - ULS</td>
</tr>
<tr>
<td>BR Broken rail force</td>
</tr>
<tr>
<td>S Concrete shrinkage</td>
</tr>
<tr>
<td>ST Street traffic collision</td>
</tr>
</tbody>
</table>
C.2 Summary of sectional forces for 30 m simply-supported tangent spans

Moments and shear forces corresponding to each load case considered for preliminary design validation are summarized in the table below.

Table C.2: Moments and shear forces for preliminary design validation

<table>
<thead>
<tr>
<th>Moment, M</th>
<th>0</th>
<th>L/4</th>
<th>L/2</th>
<th>0</th>
<th>L/4</th>
<th>L/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>0</td>
<td>2110</td>
<td>2810</td>
<td>D</td>
<td>375</td>
<td>188</td>
</tr>
<tr>
<td>L</td>
<td>0</td>
<td>2590</td>
<td>2590</td>
<td>L</td>
<td>444</td>
<td>280</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td>277</td>
<td>277</td>
<td>I</td>
<td>48</td>
<td>31</td>
</tr>
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<td>W1</td>
<td>0</td>
<td>115</td>
<td>154</td>
<td>W1</td>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>W2</td>
<td>0</td>
<td>194</td>
<td>258</td>
<td>W2</td>
<td>34</td>
<td>17</td>
</tr>
<tr>
<td>SIV</td>
<td>0</td>
<td>295</td>
<td>394</td>
<td>SIV</td>
<td>32</td>
<td>21</td>
</tr>
<tr>
<td>SIG</td>
<td>0</td>
<td>295</td>
<td>394</td>
<td>SIG</td>
<td>53</td>
<td>26</td>
</tr>
<tr>
<td>BR</td>
<td>365</td>
<td>274</td>
<td>183</td>
<td>BR</td>
<td>12</td>
<td>12</td>
</tr>
</tbody>
</table>

C.3 Dead Load

Self-weight and superimposed dead load (D)

A value of 20 kN/m will be used for girder self-weight. This includes an allowance for intermediate diaphragms, web stiffeners, and a contingency allowing for some thickening of the cross-section if necessary.

Superimposed dead loads are summarized in the Table C.3, below.

Table C.3: Superimposed dead loads

<table>
<thead>
<tr>
<th></th>
<th>kN/m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power Rails</td>
<td>0.5</td>
</tr>
<tr>
<td>Conduits &amp; Cables</td>
<td>0.5</td>
</tr>
<tr>
<td>LIM &amp; Attachments</td>
<td>1.0</td>
</tr>
<tr>
<td>Running Rails &amp; Attachments</td>
<td>1.5</td>
</tr>
<tr>
<td>Walkway</td>
<td>1.0</td>
</tr>
<tr>
<td>Barriers</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>5.0</td>
</tr>
</tbody>
</table>
Snow and ice on girder (SIG)

The Vancouver Millennium Line Design Manual specifies a snow and ice load on the girder of 1.15 kPa. Applying this over the 3 m width of the deck gives 3.5 kN/m.

C.4 Vehicle loading

Axle spacing for Bombardier ART vehicle

The Bombardier Mark-II ART (Advanced Rapid Transit) vehicle axle spacing is shown in Figure C.1, below.

![Axle spacing diagram](image)

Figure C.1: Bombardier ART Mk II axle spacing (m)

Normalized live load envelope for ART vehicle

Moment and shear diagrams for a straight, 30 m, simply-supported girder, subject to a moving train of 1 kN loads, spaced as shown in Figure C.1, are shown in Figure C.2, below. The gray lines are the envelopes resulting from moving the train across the span in 1 m increments. The dashed black lines represent linearized simplifications of the envelopes used to simplify preliminary design checks.

Vertical axle loads (L, SIV)

Vehicle axle loads are summarized in Table C.4. For preliminary design, the crush load axle loads will be used in all design checks. The load due to snow and ice accumulation on the vehicle (SIV) is specified as an axle load.

<table>
<thead>
<tr>
<th></th>
<th>kN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Empty vehicle</td>
<td>58.3</td>
</tr>
<tr>
<td>199 passengers per car (crush load)</td>
<td>34.1</td>
</tr>
<tr>
<td>Snow and ice on vehicle</td>
<td>6.6</td>
</tr>
</tbody>
</table>
Appendix C. Design Load Cases

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Midspan L/4 3L/4

Moment Shear

4.8 kN 1.3 kN 28 kNm

Figure C.2: Moment and shear force envelopes per 1 kN of axle load

Dynamic load allowance (I)

The Vancouver Millennium Line Design Manual specifies a dynamic load allowance of 10% of the live load (potentially including the snow and ice on the vehicle) on typical track, and 30% at locations of switches and rail expansion joints. For preliminary design, a value of 0.1(L+SIV) will be used.

Lurching load (LC)

The lurching load reflects the possibility of an unequal distribution of the vehicle weight among its axles, which creates a torque on the girder, and increases the local bearing forces. An imbalance of ±10% of the wheel load is specified. It is only applied to a single car at a time, and is thus only of significance for local effects such as bearing on the webs.

Centrifugal force (CF)

Trains travelling around curves at speed create centrifugal forces which apply lateral and torsional loads to the girder. Superelevation of the track reduces the centrifugal force experienced by the girders, since more of the centrifugal force is transferred as vertical and lateral bending. The load transferred to piers is unaffected by superelevation.
Following the specifications for the Vancouver Millennium Line, the centrifugal force on a curved girder is given by:

\[ CF = \left( \frac{v^2}{Rg} - e \right) \cdot L \]  

where \( v \) is the maximum expected speed of the train, \( R \) is the radius of curvature, \( g \) is gravitational acceleration, \( e \) is the tangent of the superelevation angle, and \( L \) is the vehicle live load. The force is applied horizontally, in the radial direction of the curve, at the centre of gravity of the vehicle.

Application of the above equation requires knowledge of the specific track alignment, designed with the operational parameters in mind. As the dimensioning of horizontally curved girders is beyond the scope of this thesis, centrifugal forces will be neglected.

**Hunting force (HF)**

Trains travelling on horizontally straight (tangent) sections of track can oscillate back and forth within the play between wheel flanges and rails, a phenomenon called hunting or nosing. The probability of more than two axles oscillating in phase is low, so this is predominantly a local effect. Hunting does not occur on curved track, since the centrifugal forces keep the wheel flanges in contact with one side of the rails.

For preliminary design, hunting forces will be neglected.

**Braking load (LN, LE)**

Deceleration of a train applies longitudinal braking forces to the guideway. For normal operations, this braking force is specified as 15\% of the vehicle live load corresponding to a five-car train. This is the load used for SLS checks (LN). Under extreme conditions, considered at ULS, the braking load is 50\% of vehicle live load (LE).

Distributing the axle loads uniformly along the train length gives a distributed load of 24 kN/m. Therefore, the longitudinal braking load is 3.6 kN/m, applied at the centre of gravity of the train, which is 1.5 m above the top of the rails.

Braking loads are of primary concern for design of piers, and will be neglected for preliminary design.

**Maintenance and other vehicles**

Other vehicles such as rail grinders and general maintenance service vehicles are often specified. In the case of the Vancouver Millennium project, the axle loads of these vehicles are less than the maximum
axle load of the transit train. Smaller axle spacing may create higher local effects from these load cases, but for preliminary design only the standard transit train described above is considered.

### C.5 Wind (W)

The wind loads specified for the Vancouver Millennium Line will be used here. The 100 year hourly mean wind pressure for Vancouver is specified in the CHBDC as 530 Pa, which is fairly average for Canada’s major cities (Toronto is 580 Pa; Montreal is 440 Pa; Calgary is 540 Pa). Two wind pressures are specified: W1 is the maximum wind speed under which operations would continue, and is applied to the frontal area of girders plus train; W2 is the maximum design wind condition, and is applied to the girder only. The values are given in Table C.5, below, reproduced from [14].

<table>
<thead>
<tr>
<th>Condition</th>
<th>Vehicle Lateral</th>
<th>Structure Vertical</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>1.16</td>
<td>1.56</td>
</tr>
<tr>
<td>W2</td>
<td>—</td>
<td>2.62</td>
</tr>
</tbody>
</table>

### C.6 Earthquake (EQ)

The earthquake loading condition is of greatest importance for the design of the piers. The lightness of the girders relative to typical concrete girders will be a significant benefit in reducing the inertial forces caused by ground movement. Detailed design for earthquake effects is beyond the scope of this thesis.

### C.7 Temperature effects and rail–structure interaction

#### Material coefficients of thermal expansion

The coefficients of thermal expansion for steel, UHPFRC and GFRP are assumed as follows:

<table>
<thead>
<tr>
<th>Material</th>
<th>Coefficient (%/°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>(a_{\text{steel}} = 0.012)</td>
</tr>
<tr>
<td>UHPFRC</td>
<td>(a_{\text{UHPFRC}} = 0.011)</td>
</tr>
<tr>
<td>Top slab &amp; web GFRP</td>
<td>(a_{\text{GFRP},0} = 0.008)</td>
</tr>
<tr>
<td>Top slab &amp; web GFRP</td>
<td>(a_{\text{GFRP},90} = 0.015)</td>
</tr>
<tr>
<td>Bottom slab GFRP</td>
<td>(a_{\text{GFRPb},0} = 0.007)</td>
</tr>
<tr>
<td>Bottom slab GFRP</td>
<td>(a_{\text{GFRPb},90} = 0.020)</td>
</tr>
</tbody>
</table>
Rail fastener longitudinal stiffness and slip force

The coefficient of friction for the rail clips can be taken as $\mu_{\text{clip}} = 0.5$. The clamping force of the clip can vary depending on the fastener, but 16 kN is a reasonable assumption [1, 73]. Given two clips per rail fastener, the longitudinal capacity of each fastener is 16 kN.

Arrangement of longitudinal fixity at piers

The preferred arrangement of longitudinal fixity in guideway structures is as shown in Figure C.3, where alternating piers provide fixity for both supported girders. This is contrast to the usual approach for highway viaducts, where it is more common to have longitudinal fixity for one span at every pier, as this reduces the loads on the piers. The arrangement shown is preferable since, for consistent span lengths, it aligns the neutral points for thermal shrinkage or elongation of the girders with the fixed piers. This means that the rail-structure interaction does not apply longitudinal forces to the piers, and more importantly, it means that there is no accumulation of axial force in the rails along the length of the guideway. It is also possible to provide partial longitudinal restraint at all piers, such that the neutral points coincide with girder midspans, and therefore also cancel out at the piers, however this is a less common arrangement [1].

Uniform temperature change (T, LR1, LR2)

Due to the difference between the thermal expansion coefficients of UHPFRC and GFRP, and the additional restraint of the continuously welded rail, temperature changes will induce stresses and bending moments in the girders. The installation, maximum and minimum rail temperatures for design are specified for the Vancouver Millennium Line as For the girder itself, the larger thermal mass and lower conductivity of GFRP and UHPFRC relative to steel justify a lower maximum temperature. Assuming a maximum mean daily temperature of 32°C, and following the CHBDC specification of a maximum effective design temperature 10°C above this value (for concrete superstructures) gives a maximum temperature of $T_{S,\text{max}} = 42°C$. The same minimum temperature will be used for the structure ($−20°C$ is on the warm side for Canada, but is conservative for much of the rest of the developed world).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$</td>
<td>20°C</td>
</tr>
<tr>
<td>$T_{\text{R,min}}$</td>
<td>$−20°C$</td>
</tr>
<tr>
<td>$T_{\text{R,max}}$</td>
<td>55°C</td>
</tr>
</tbody>
</table>
The longitudinal rail force arising from extreme cold is

\[ P_{\text{rail}} = EA\alpha \Delta T \]  \hspace{1cm} (C.2)

where \( \alpha \) is the coefficient of thermal expansion for steel, taken as 0.012%/%\(^\circ\)C, and \( \Delta T \) is the difference between the stress-free temperature (which can be taken as 20 \(^\circ\)C) and the lowest expected temperature at the site, specified above as \(-20 \(^\circ\)C\), giving \( \Delta T = -40 \(^\circ\)C \). Given \( E = 200 \text{ GPa} \) and \( A = 7250 \text{ mm}^2 \), the force in the rail is

\[ P_{\text{rail}} = 200 \text{ GPa} \cdot 7250 \text{ mm}^2 \cdot 0.012 \% /{^\circ}\text{C} \cdot 40 \(^\circ\)C \]

\[ = 696 \text{ kN} \]  \hspace{1cm} (C.3)

**Temperature gradient (TG)**

Temperature gradients are caused by radiant heating of the top slab during the day, and by different rates of temperature change due to different conductivity and thermal mass of the steel rails, GFRP shell, and UHPFRC slabs. These gradients induce vertical bending in the girders.

The CHBDC specifies a 10 \(^\circ\)C gradient for summer conditions, and a ±5 \(^\circ\)C gradient for winter conditions, for steel or concrete superstructures greater than 1 m deep. Positive gradient means warmer at the top of the cross section. These gradients will be used, with the entire rail cross section taken to be the same temperature as the top surface of the girder.

Temperature gradient loads are of most concern in multi-span continuous girders, where the thermally-induced vertical curvature creates secondary reactions at the piers and bending moments. In statically determinate structures, the curvature induced by thermal gradients is unrestrained, so the only stresses arising are those due to strain compatibility in the cross section. In the SLS load combinations described in Appendix D, the temperature gradient is combined with 80% of the uniform temperature effect. Given the uniform temperature range of approximately ±40 \(^\circ\)C, the combination of 0.8T+TG is not likely to
govern, and will be neglected at this stage of design.

**Radial restraint of curved rails (RT1, RT2)**

In horizontally curved spans, the tension developed in the continuously welded rail must be deviated around the horizontal curve by radial restraint forces at the fastener locations. If the rails are fully restrained (i.e. the nearest expansion joint is remote from the curved span under consideration), the axial force developed was calculated above as $P_{\text{rail}} = 696 \text{ kN}$. For the minimum curvature of $R = 35 \text{ m}$, this would require a distributed horizontal deviation force of $P_{\text{rail}}/R = 21 \text{ kN/m per rail}$, for a total force of $42 \text{ kN/m on a girder}$. This is nearly twice the magnitude of the live load, and is therefore likely to be a governing load case for tightly-curved spans. Expansion joints and fasteners with low longitudinal stiffness can be used to relieve such forces, but the need to limit the differential movement at an expansion joint can lead to large forces on the structure as well. The design of an economical solution for this condition would depend on the specifics of the site conditions, and is therefore beyond the scope of this thesis.

**Broken rail force (BR)**

In the event of a rail breaking in extreme cold, the thermal shrinkage of the rail is resisted by friction forces in the rail fasteners, resulting in longitudinal forces applied to the structure. The Transit Cooperative Research Program (TCRP) Report on Direct-Fixation Track Design and Example Specifications [88] provides guidance on the calculation of these effects, follows.

After a rail break, the thermal force in the rail of $P_{\text{rail}} = 696 \text{ kN}$ must be developed by the longitudinal slip capacity of the rail clips, which is assumed, above, as $16 \text{ kN per fastener}$. Therefore, $696/16 = 44$ fasteners are required to develop the rail break force.

Fastener spacing is typically $750 \text{ mm}$, so 44 fasteners cover a length of $33 \text{ m}$, or approximately one span. For the girder bearing arrangement shown in Figure C.3, the maximum thermal force in the rail will be over the expansion piers, so it is most likely that a rail break will occur close to the end of a girder. Although a break in one rail adds stress to the other rail increasing the likelihood of a second break, it is considered too conservative to design for two rails breaking simultaneously. Therefore, the rail break force corresponds to a longitudinal force of $16/0.75 = 21.3 \text{ kN/m}$ applied over the entire length of a span. The eccentricity of this force above the girder centroid is $570 \text{ mm}$, creating a uniform applied moment of $21.3 \cdot 0.57 = 12.2 \text{ kNm/m}$. At the fixed pier end of the girder, these moments will accumulate to a total of $12.2 \cdot 30 = 365 \text{ kNm}$.
Broken rail forces are only considered as an ultimate limit state. The high moment capacity of the GFRP shell means that these forces will not be a governing factor in the design.

C.8 Settlement (SE)

Differential settlement is not a concern for simply supported spans, as imposed displacements cause no stresses in statically determinate structures. The continuity of the rails across spans means there will be some stress development, but this is negligible at the preliminary design stage.

C.9 Concrete shrinkage

Tests by Shao [81] show that the ultimate total drying and autogenous shrinkage for the UofT UHPFRC (with water/binder ratio of 0.18) is

\[ \epsilon_{cs0} = 0.4\% \text{.} \]  

(C.4)

Approximately 75\% of this strain is autogenous shrinkage, which occurs in the first 10 days after casting. During this time, the concrete elastic modulus is still developing, and its ability to creep is relatively high. As such, in a restrained condition, a portion of this strain can be accommodated by the higher compliance of the young UHPFRC. Tests by Habel et al. [36] on fully restrained UHPFRC specimens showed a 60\% reduction in shrinkage due to creep.

For preliminary design, a value of \( \epsilon_{cs0} = 0.2\% \) will be used, which assumes a 50\% of shrinkage due to creep. The shrinkage effect is included in the strain compatibility analysis used to construct the cross section moment-curvature diagram—in other words, it is integrated into the analysis on the capacity side rather than the demand side.

Shrinkage-reducing and expansive admixtures are available, and have been shown to provide an approximately 40\% reduction in shrinkage. Of course, the effects of a change in the UHPFRC mix on mechanical properties would require study before admixtures could be proposed for use.

C.10 Derailment loads (COLV)

The vertical collision load (COLV) corresponds to the weight of the ends of two cars being off the track centreline, as would happen if the train “buckled”. For the Bombardier MkII train, this produces a concentrated torque of 86 kNm.
C.11 Barrier collision loads (COLV)

The horizontal collision load (COLH) consists of two 155.7 kN concentrated loads, 4760 mm apart, applied to the barrier at a height 230 mm above the top of the rails.
Appendix D

Load Combinations and Resistance Factors

D.1 Load Combinations

Load combinations are taken from the Vancouver Millennium Line Design Manual [14]. The ULS and SLS combinations are summarized in Tables D.1 and D.2.

### Table D.1: ULS Load Combinations

<table>
<thead>
<tr>
<th>Case</th>
<th>Constituent loads</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>U1</td>
<td>Train</td>
<td>$1.25(D+L+I)+(1.6(L+I+(HF or CF)))$</td>
</tr>
<tr>
<td>U2</td>
<td>Train + Wind + Snow/Ice</td>
<td>$1.25(D+L+I+(HF or CF)+W_1)+T+(LR_2 or RT_2 or BR)\cdot S$</td>
</tr>
<tr>
<td>U3</td>
<td>Train + Wind + Temperature + Rail Interaction + Shrinkage</td>
<td>$1.25(D+L+I+L+T+T_s)+(LR_2 or RT_2 or BR)\cdot S$</td>
</tr>
<tr>
<td>U4</td>
<td>Wind + Temperature + Rail Interaction + Shrinkage</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
<tr>
<td>U5</td>
<td>Train + Wind + Collision</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
<tr>
<td>U6</td>
<td>Train + Earthquake + Emergency Braking</td>
<td>$1.0(D+L+I+L+T+T_s)$</td>
</tr>
<tr>
<td>U7</td>
<td>Train + Street Traffic Collision</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
<tr>
<td>U8</td>
<td>Train + Extreme Centrifugal Force</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
<tr>
<td>U9</td>
<td>Wind</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
<tr>
<td>U10</td>
<td>Train + Wind</td>
<td>$1.25(D+L+I+T+T_s)$</td>
</tr>
</tbody>
</table>

### Table D.2: SLS Load Combinations

<table>
<thead>
<tr>
<th>Case</th>
<th>Constituent loads</th>
<th>Combination</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>Train</td>
<td>$D+L+I+(HF or CF)+L+LR_1$</td>
</tr>
<tr>
<td>S2</td>
<td>Train + Wind + Snow/Ice</td>
<td>$D+L+I+(HF or CF)+LC+LN+(LR_1 or RT_1)+W_1\cdot SIG+SIG+SIV$</td>
</tr>
<tr>
<td>S3</td>
<td>Train + Wind + Temperature</td>
<td>$D+L+I+(HF or CF)+LC+LN+(LR_1 or RT_1)+W_1+T+S$</td>
</tr>
<tr>
<td>S4</td>
<td>Wind + Temperature</td>
<td>$D+L+I+(LR_1 or RT_1)+T+S$</td>
</tr>
<tr>
<td>S5</td>
<td>Train + Temperature Gradient</td>
<td>$D+L+I+(LR_1 or RT_1)+0.8+T+TG+S$</td>
</tr>
<tr>
<td>S6</td>
<td>Temperature Gradient</td>
<td>$D+(LR_1 or RT_1)+0.8+T+TG+S$</td>
</tr>
</tbody>
</table>
Relevant combinations for girder design

Of the ULS combinations specified in Table D.1, several can be eliminated from consideration at the preliminary girder design stage. The earthquake load combination, U6, and the street traffic collision load combination, U7, are primarily of concern for the pier design, which is outside the present scope of work. Combination U4, which is the same as U3 but with no live load, is of importance in checking uplift conditions in multi-span continuous structures and so can be neglected. U8 is only applicable to curved spans, which are not treated in detail here. For simple spans, combination U9 will govern for uplift. Combination U10 is the same as combination U3, but with no temperature or rail interaction effects, and will also not govern design of simply supported girders. Therefore, the salient combinations for the present study are U1, U2, and U3, which cover longitudinal bending, and shear demand; and U5, which covers transverse bending demand in the slab due to barrier collision.

At SLS, combinations S2 and S3 have the potential to govern for positive bending, torsion, and shear. S5 is unlikely to govern for simply supported spans. S1 is the combination used for fatigue checks. S4 and S6 are the relevant combinations for checking for potential cracking in the top slab of UHPFRC.

Because shrinkage in UHPFRC occurs more rapidly than in ordinary concrete, the shrinkage effect is included in the capacity for all load cases, rather than just S3 to S6 as specified for Vancouver Millennium Line, where conventional concrete construction was expected.

D.1.1 Summary of factored loads for 30 m tangent spans

Table D.3: Moments and shear forces for preliminary design validation

<table>
<thead>
<tr>
<th>Factored Moment, $M_f$</th>
<th>0</th>
<th>L/4</th>
<th>L/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULS1</td>
<td>0</td>
<td>7220</td>
<td>8100</td>
</tr>
<tr>
<td>ULS2</td>
<td>0</td>
<td>7120</td>
<td>8170</td>
</tr>
<tr>
<td>ULS3</td>
<td>365</td>
<td>6640</td>
<td>7470</td>
</tr>
<tr>
<td>SLS2</td>
<td>0</td>
<td>5570</td>
<td>6410</td>
</tr>
<tr>
<td>SLS3</td>
<td>0</td>
<td>4820</td>
<td>5550</td>
</tr>
<tr>
<td>FLS</td>
<td>0</td>
<td>4980</td>
<td>5680</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Factored Shear, $V_f$</th>
<th>0</th>
<th>L/4</th>
<th>L/2</th>
</tr>
</thead>
<tbody>
<tr>
<td>ULS1</td>
<td>1256</td>
<td>733</td>
<td>213</td>
</tr>
<tr>
<td>ULS2</td>
<td>1241</td>
<td>712</td>
<td>185</td>
</tr>
<tr>
<td>ULS3</td>
<td>1121</td>
<td>648</td>
<td>178</td>
</tr>
<tr>
<td>SLS2</td>
<td>972</td>
<td>499</td>
<td>133</td>
</tr>
<tr>
<td>SLS3</td>
<td>839</td>
<td>478</td>
<td>120</td>
</tr>
<tr>
<td>FLS</td>
<td>867</td>
<td>499</td>
<td>133</td>
</tr>
</tbody>
</table>

D.2 Material reduction factors

Material reduction factors (partial safety factors) for GFRP and UHPFRC are given in this section.
D.2.1 GFRP

Material reduction factors for GFRP, taken from the Eurocomp design handbook, are as follows:

<table>
<thead>
<tr>
<th>Partial factor</th>
<th>Source of error</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{g,1}$</td>
<td>Measurement of material properties</td>
<td>0.87</td>
</tr>
<tr>
<td>$\phi_{g,2}$</td>
<td>Fabrication process (vacuum infusion)</td>
<td>0.91</td>
</tr>
<tr>
<td>$\phi_{g,3}$</td>
<td>Load duration</td>
<td>1.0 for transient loads</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4 for sustained loads</td>
</tr>
</tbody>
</table>

D.2.2 UHPFRC

For UHPFRC in tension, the AFGC recommends a partial safety factor of $\phi_{cf} = 0.77$ for normal loading, and $\phi_{cf} = 0.95$ for “accidental situations”. For UHPFRC in compression, the same reduction factor for normal concrete is recommended. The safety factor for ordinary concrete specified in the CHBDC is $\phi_c = 0.75$, so this value seems justified for use with UHPFRC in both compression and tension.
Appendix E

UHPFRC Casts

Table E.1, below, shows the specifics of each cast from which test specimens were cut:

1. $f_c$, the compressive strength, as measured by a 100 mm cube test

2. $f_f$, the flexural strength, as measured by a $100 \times 100 \times 450$ mm modulus of rupture test

3. $f_t$, the tensile strength, as measured by a direct tension test on a $50 \times 50 \times 585$ mm specimen

4. The casting method used: Work by Shao [81] showed the substantial benefit of a directional casting method, which was used for all casts starting in summer 2015.

<table>
<thead>
<tr>
<th>Cast</th>
<th>Mix</th>
<th>Date</th>
<th>$f_c$ MPa</th>
<th>$f_f$ MPa</th>
<th>$f_t$ MPa</th>
<th>Method</th>
<th>Use</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Wang [91]</td>
<td>2013-06-13</td>
<td>141</td>
<td>—</td>
<td>—</td>
<td>Static</td>
<td>Damping</td>
</tr>
<tr>
<td>2</td>
<td>Shao [81]</td>
<td>2014-12-15</td>
<td>—</td>
<td>—</td>
<td>—</td>
<td>Static</td>
<td>Preliminary Bond</td>
</tr>
<tr>
<td>3</td>
<td>Shao</td>
<td>2015-03-17</td>
<td>136</td>
<td>12.4</td>
<td>6</td>
<td>Static</td>
<td>Bond, Fatigue</td>
</tr>
<tr>
<td>4</td>
<td>Shao</td>
<td>2015-07-23</td>
<td>135</td>
<td>30</td>
<td>—</td>
<td>Directional</td>
<td>Fatigue</td>
</tr>
<tr>
<td>5</td>
<td>Shao</td>
<td>2015-08-17</td>
<td>135</td>
<td>—</td>
<td>10</td>
<td>Directional</td>
<td>Bond, Fatigue</td>
</tr>
<tr>
<td>6</td>
<td>Shao</td>
<td>2016-09-26</td>
<td>147</td>
<td>—</td>
<td>10</td>
<td>Directional</td>
<td>Not Used</td>
</tr>
<tr>
<td>7</td>
<td>Shao</td>
<td>2016-10-27</td>
<td>135$^a$</td>
<td>—</td>
<td>—</td>
<td>Directional</td>
<td>Bond, Fatigue</td>
</tr>
<tr>
<td>8</td>
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$^a$ Projected from 14-day cube strength of 122 MPa.

$^b$ Projected from 21-day cube strength of 126 MPa.
Appendix F

Bond Test Results

The following tables give the peak load, interfacial area, and corresponding ultimate shear stress for each GFRP–UHPFRC bond test described in Chapter 4.

Table F.1: Results of 100 × 100 mm specimen tests

<table>
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<th>Specimen</th>
<th>Peak Load (kN)</th>
<th>Area (cm²)</th>
<th>Ultimate Shear Stress (MPa)</th>
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Appendix G

Drawings of axial forced vibration setup
NOTES:
1. Adjust dimension A if necessary to ensure minimum clearance of 2" between bottom beam and floor when actuator is in neutral position. Maintain minimum distance of 1 5/8" from centre of hole to edge of plate.
Fatigue Test Setup
Clamp & Spacer Plates
David Hubbell
April 23, 2015

23 Steps
3/16 long x 1/16 deep
1-1/16 Dia. Hole
1" Threaded Hole

CLAMP PLATE

SPACER PLATE
Appendix H

Fatigue test results
Fatigue test UC1

Cast: 3; test start: 2016-07-20
Fatigue test UC2

Cast 3; test start 2016-07-21.
Fatigue test UC3

Cast 4; test start 2016-08-08.
Fatigue test UC4

Cast 4; test start 2016-08-18.
Fatigue test UC5

Cast 4; test start 2016-08-04.
Fatigue test UC6

Cast 4; test start 2017-01-11.
Fatigue test UC7

Cast 4; test start 2017-01-19.
Appendix H. Fatigue test results

Fatigue test UC8

Cast 4; test start 2017-01-31.
Fatigue test UC9

Cast 4; test start 2017-02-13.
Appendix H. Fatigue test results

Fatigue test UC10

Cast 4; test start 2017-05-17.

[Graph showing stress-strain cycles for fatigue test UC10]
Appendix H. Fatigue test results

Fatigue test UC11

Cast 7; test start 2017-05-27.
Fatigue test UC12

Cast 7; test start 2017-06-06.
Fatigue test UC13

Cast 3; test start 2017-06-08.
Appendix H. Fatigue test results

Fatigue test UC14

Cast 7; test start 2017-06-20.
Fatigue test UC15

Cast 7; test start 2017-06-26.
Fatigue test GC2

Cast 9; test start 2017-06-29.
Appendix I

Free vibration test data

The following tables present the results of each free vibration test. Each table gives the following information for each individual test:

1. The adjusted mass, $M_{adj}$, which is adjusted to account for the height of the stack of weights.
   Weights on the top of the stack add more to the modal mass than weights on the bottom.

2. The initial displacement, $x_0$.

3. The frequency of vibration, $f$.

4. The loss factor, $\eta$. 
### Table I.1: Free vibration test

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## Appendix I. Free vibration test data

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A20C2

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### Table I.9: Free vibration test
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