RESOURCE MANAGEMENT AND INTERFERENCE CONTROL IN DISTRIBUTED MULTI-TIER AND D2D SYSTEMS

by

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Abstract

Resource Management and Interference Control in Distributed Multi-Tier and D2D Systems

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In order to improve the capacity and spectrum efficiency of next generation wireless networks, multi-tier wireless networking and device-to-device (D2D) communication are widely considered as strong candidates for 5G. In this thesis, I have developed new theories and design guidelines to improve the performance of large-scale multi-tier and D2D networks by studying their resource optimization and interference management.

In the first part of this thesis, we study optimal power allocation for distributed relays in a multi-channel system with multiple source-destination pairs and an individual power budget for each relay. We focus on designing the optimal relay beamformers, aiming at minimizing per-relay power usage while meeting minimum signal-to-noise guarantees. Showing that strong Lagrange duality holds even for this non-convex problem, we solve it in the dual domain. Further, we investigate the effect of imperfect channel information by quantifying the performance loss due to either quantization error with limited feedback or channel estimation error.

In the second part of this thesis, we study optimal inter-cell interference control for distributed relays in a multi-channel system. We design optimal relay beamforming to minimize the maximum interference caused at the neighboring cells, while satisfying minimum signal-to-noise requirements and per-relay power constraints. Even though the problem is non-convex, we propose an iterative algorithm that provides a semi-closed-form solution. We extend this algorithm to the problem of maximizing the minimum signal-to-noise subject to some pre-determined maximum interference constraints at neighboring cells. In order to gain insight into designing this system in practice, we further study the received worst-case signal-to-interference-and-noise ratio versus the maximum interference target.

In the third part of this thesis, we consider D2D communication underlaid in a cellular system for uplink resource sharing. Under optimal cellular user (CU) receive beamforming, we jointly optimize the powers of CUs and D2D pairs for their sum rate maximization, while satisfying minimum quality-of-service (QoS) requirements and worst-case inter-cell interference limit in multiple neighboring cells. The formulated joint optimization problem is non-convex. We propose an approximate power control algorithm to maximize the sum rate and provide an upper bound on the performance loss by the proposed
algorithm and conditions for its optimality.

We further extended the results of the third part in the fourth part of this thesis, where we jointly optimize the beam vector and the transmit powers of the CU and D2D transmitter under practical system settings. We consider a multi-cell scenario, where perfect channel information is available only for the direct channels from the CU and D2D to the base station. For other channels, only partial channel information is available. The uncertain channel information, the non-convex expected sum rate, and the various power, interference, and QoS constraints, lead to a challenging optimization problem. We propose an efficient robust power control algorithm based on a ratio-of-expectation approximation to maximize the expected sum rate, which is shown to give near-optimal performance by comparing it with an upper bound of the sum rate.
To my family
I am honored to have you.
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Chapter 1

Introduction

It is projected that the demand for high date rate and spectrum efficiency will grow significantly in the future to handle a plethora of various devices and services due to the emergence of smartphones, tablets, and data hungry applications. Only in 2015, the global mobile traffic was increased by 74% [2]. It is expected that there will be 50 billion connected devices by 2020, i.e., approximately 7 times larger than the human population [3]. Further, the emergence of new applications such as remote surgery, autonomous driving, and industrial robots, which require ultra-low latency and ultra-high reliability, characterizes Tactile Internet. Some requirements for future generation wireless systems compared with those of the current generation are shown in Table 1.1.

New technologies and design guidelines are required for the current generation of Long Term Evolution (LTE) and LTE-Advanced wireless systems to meet the stringent demands of future wireless connectivity. In this respect, the following general approaches are widely considered as strong candidates for 5G: 1) multi-tier wireless networking, where smaller cells (e.g., pico or femto cell within buildings) and relays are to be deployed concurrently with the conventional macro cells [4–59], and 2) device-to-device (D2D) communication, to offload traffic from the cellular macro cells and to support direct machine-to-machine connectivity [60–82].

Cooperative relaying is one of the key techniques to improve quality-of-service (QoS) and efficient resource usage in our wireless systems. The relays could be viewed as smaller base stations (BSs), relay stations, or user equipments (UEs). Installation of relays within the macro cells is a promising strategy, since their deployment time and cost will be less than those of the macro BSs. It has been adopted in the current and future multi-channel based broadband access systems, such as the 4th generation (4G) orthogonal frequency division multiple access (OFDMA) systems with LTE and LTE-Advanced standards [83, 84]. It is also the underlying technique for many potential features for 5G evolution [85]. In such a network, there are typically multiple communicating pairs as well as available relays. Efficient physical layer design of cooperative relaying to support such simultaneous transmissions is crucial.

In order to improve the capacity and spectrum efficiency of next generation wireless networks, D2D communication has been developed as a strong candidate for 5G standardization, where nearby users can establish a direct communication link to transmit data to each other without going through the backhaul network [86–88]. D2D communication can improve the overall network utilization due to resource reuse by both the cellular users (CUs) and the D2D pairs. It leads to low delay and power consumption due to the proximity of users. Offloading cellular traffic reduces congestion in the backhaul network, which
Table 1.1: Requirements for 5G wireless systems compared with those of 4G [1]

<table>
<thead>
<tr>
<th>Measure</th>
<th>5G</th>
<th>Compared with 4G</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak data rate</td>
<td>10 Gb/s</td>
<td>100 times higher</td>
</tr>
<tr>
<td>Guaranteed data rate</td>
<td>50 Mb/s</td>
<td>-</td>
</tr>
<tr>
<td>Number of devices</td>
<td>1 M/km²</td>
<td>1000 times higher</td>
</tr>
<tr>
<td>Total number of IoT terminals</td>
<td>≥ 1 trillion</td>
<td>-</td>
</tr>
<tr>
<td>Energy consumption</td>
<td>-</td>
<td>90% less</td>
</tr>
</tbody>
</table>

benefits CUs. Furthermore, D2D communication is a strong candidate for 5G to support the stringent demands of future Internet of Things (IoT) connectivity.

1.1 New Challenges in Designing and Optimizing Future Communication Systems

Multi-tier wireless networking improves QoS. However, there are many challenges in the design of those networks under stringent QoS requirements of various applications. Interference is the other main impediment to achievable coverage and capacity of the networks [41]. For example, small cells and D2D pairs may cause interference to users communicating with a nearby macro BS. In particular, inter-cell interference (ICI) from the nodes in adjacent cells to the desired cell is a challenging issue. Although interference mitigation techniques are well studied for users within a cell, the ICI management is a key barrier of modern cellular networks. Because of diverse interference patterns, the design of transmission strategies for relays and D2D pairs with heterogeneous service requirements becomes very challenging.

1.1.1 Per-Relay Power Minimization

Available resources in the relay networks are strictly limited. Efficient algorithms are necessary to share limited available power among the relays. Typically, each relay has its own power budget. This is a more practical scenario, especially for distributed relay systems. Furthermore, minimizing per-relay power usage in a cell can lead to ICI reduction in the neighboring cells. We are interested in designing multi-relay systems to minimize the power budget, such that no single relay uses a much higher power than the rest. Since any relay may be the one using the highest power at any given time due to randomness in the channel condition, in real-life deployment, we will have to build the hardware of every relay to be able to offer the maximum required power. Another alternative is to minimize the total power among all relays. One important disadvantage of sum-power minimization is that it does not consider the power consumption of individual relays, i.e., the optimal solution may require some relays to transmit at a much higher power than the others. This leads to more stringent requirements in the hardware design of all relays, since we need to provision for the higher power output level possibly at any relay. From this point of view, sum-power minimization will result in an inefficient design for practical implementation.

1.1.2 Interference Management

Next-generation wireless networks will be characterized by heterogeneous infrastructure consisting of BS and relays in cooperative communication. Furthermore, the new IoT paradigm will foster a large popula-
Chapter 1. Introduction

In the age of the Internet of Things (IoT), the diversity of users (UE) and the complexity of communication patterns pose significant challenges for future-generation wireless networks. Traditional BS-UE transmission and UE-UE transmission, e.g., in a D2D mode, require efficient management of interference (ICI). In these systems, radio interference is a crucial yet challenging issue, due to the many randomly located transmitters and receivers. The management of ICI remains a critical issue in the design of future-generation wireless networks.

1.1.3 Design of D2D Systems

For D2D communication underlaid in a cellular system to reuse spectrum resource assigned to CUs, uplink resource sharing is in general preferred for several reasons [64, 65]. Downlink spectrum reuse for D2D communication would require a D2D pair to have a new transmit chain, which is more costly than uplink spectrum reuse, where only a new receive chain is required at the D2D receiver (such as in LTE systems). In addition, uplink traffic is often lighter than downlink traffic, with uplink resources more likely being available for D2D communication. Furthermore, it is easier to manage the interference incurred at the BS [88]. Using LTE as the baseline for designing a D2D system, peer discovery methods, physical layer procedures, and radio resource management algorithms have been studied in [87]. When a D2D pair reuses the channel resource of a CU, they generate intra-cell interference to each other. Furthermore, both D2D and CU transmissions cause ICI to neighboring cells. Thus, the use of D2D communication and corresponding resource allocation need to ensure satisfactory QoS for both D2D pairs and CUs, as well as to maintain a satisfactory ICI limit to neighboring cells.

1.1.4 Robust Design of D2D Systems

A vast majority of the existing literature on D2D communication is focused on resource allocation when perfect channel state information (CSI) is available at the scheduling BS. However, this assumption imposes substantial signaling overhead due to the requirement of CSI feedback for channels that are away from the BS. In practical scenarios, the BS may not have perfect CSI knowledge. It is essential to design and optimize D2D systems assuming only partial CSI is available at the scheduling BS.

1.2 Thesis Outline and Main Contributions

The goal of this thesis is to develop new theories and design guidelines to improve the performance of distributed multi-tier and D2D systems by studying their resource optimization and interference management. The main contributions are summarized as follows.

1.2.1 Per-Relay Power Minimization with Cooperative Relay Beamforming

In Chapter 3, we study the optimal relay beamforming problem for multi-user peer-to-peer communication in a multi-channel system. Assuming perfect CSI, we formulate the multi-channel relay beamforming problem as a min-max per-relay power minimization problem with minimum signal-to-noise ratio (SNR) guarantees. Showing that strong Lagrange duality holds for this non-convex problem, we solve it in the
dual domain. Through transformations, we express the dual problem as a semi-definite programming (SDP) problem to determine the optimal dual variables, which has a much smaller problem size than that of the original problem and can be solved efficiently.

We identify that the optimal relay beamforming solution of the original problem can be obtained in three cases depending on the values of the optimal dual variables. These cases reflect, at optimality, whether the minimum SNR requirement at each source-destination (S-D) pair is met with equality, and whether the power consumption at a relay is the maximum among relays. Among these three cases, the first one corresponds to the feasibility of the original problem. For the second and third cases, we obtain a semi-closed-form solution structure of relay beam vectors, and design an iterative approach to determine the relay beam vector for each S-D pair.

We further study the reverse problem of max-min SNR subject to per-relay power constraints. We show the inverse relation of the two problems and propose an iterative bisection algorithm to solve the max-min SNR problem.

Through simulation, we analyze the effect of the number of relays, as well as the number of S-D pairs on the power and SNR performance under the optimal relay beam vector solution. Furthermore, we investigate the effect of imperfect CSI of the second hop. We quantify the performance loss due to either quantization error with limited feedback or channel estimation error. It is found that the loss due to imperfect CSI is mild. Furthermore, the loss due to quantization is less sensitive to the number of relays than that due to channel estimation error.

1.2.2 Interference Minimization with Cooperative Relay Beamforming

In Chapter 4, we first formulate the relay beamforming problem in order to minimize the maximum interference in multiple neighboring cells under minimum SNR requirements and per-relay power constraints. Although the problem is non-convex, we show that it has zero duality gap and hence can be solved in the Lagrange dual domain. We then transform the dual problem into an SDP problem with a much fewer number of variables and constraints compared to the original optimization problem and as such can be solved efficiently using interior-point methods.

Depending on the values of the optimal dual variables, we identify three cases to obtain the optimal beam vectors accordingly. These cases represent whether the minimum SNR requirement and per-relay power constraint are met with equality, and whether at optimality the interference at a destination in a neighboring cell is the maximum among destinations in all neighboring cells. The first case corresponds to the infeasibility of the min-max interference problem. For the other two cases, we propose an iterative algorithm to obtain optimal relay beam vectors with a semi-closed-form structure.

We also consider the problem of maximizing the minimum received SNR subject to maximum interference at each neighboring cell and per-relay power constraints. We show that the max-min SNR is the inverse problem of minimizing the maximum interference subject to a pre-determined SNR constraint. We propose an algorithm to solve the max-min SNR problem iteratively using the solution to the problem of maximum interference minimization and bisection search. Furthermore, by limiting the interference from each neighboring cell, we propose a solution to the problem of maximizing the worst-case received signal-to-interference-and-noise ratio (SINR). To this end, we solve the max-min SNR problem under an appropriate maximum interference target.

In order to gain insight into designing this system in practice, we study the received worst-case SINR versus the maximum interference target numerically. Interestingly, a maximum worst-case SINR is iden-
tified for different system setups. Using the obtained optimal relay beamforming solution, we investigate the effect of the number of relays, S-D pairs, and neighboring cells on the maximum interference and worst-case SINR. We further study the performance of the proposed algorithm when the knowledge of interference CSI is imperfect due to either limited feedback or channel estimation error.

1.2.3 Power Optimization for D2D Communication with Interference Control

In Chapter 5, we consider D2D communication underlaid in a cellular system for uplink resource sharing. We assume all users are equipped with a single antenna, while the BS is equipped with multiple antennas. First, we focus on one D2D pair sharing the uplink resource assigned to one CU. We aim at jointly optimizing the power control at the CU and D2D transmitters, under optimal BS receive beamforming, to maximize the sum rate of the D2D and CU while satisfying minimum SINR requirements and obeying worst-case ICI to multiple neighboring cells. The formulated joint optimization problem is non-convex. We propose a two-step approach to find a solution:

- We determine the admissibility of the D2D pair under the power, SINR, and ICI constraints, through a feasibility test. The optimal beam vector for the CU is provided, as well as the necessary and sufficient condition for the D2D admissibility.

- Assuming the D2D pair is admissible, we propose an approximate power control algorithm to maximize the sum rate. We obtain the power solution of the CU and D2D in closed form, by analyzing the feasible solution region of the problem and the characteristics of the solution in the feasible region. We show that, depending on the severity of the ICI that D2D and CU each may cause to the neighboring cell, the shape of the feasible solution region can be categorized into five cases, each of which may further include several scenarios depending on the minimum SINR requirements. With a total of sixteen unique scenarios, we derive the joint power solution in closed form for each scenario. The proposed algorithm is optimal when ICI to a single neighboring cell is considered. For ICI to multiple neighboring cells, we provide an upper bound on the performance loss by the proposed algorithm and conditions for its optimality.

Next, we extend our consideration to the scenario of multiple CUs and D2D pairs, and formulate the joint power control and CU-D2D matching problem. The joint optimization problem is a mixed integer programming problem that is difficult to solve. Instead, as a suboptimal approach, we show how our provided solution for one CU and one D2D pair can be utilized to find a solution by breaking down the joint optimization problem into a joint power optimization problem and a CU-D2D matching problem.

Simulation shows that substantial performance gain is achieved by the proposed power control algorithm over two alternative approaches for a single CU and D2D pair. Furthermore, for multiple CUs and D2D pairs, simulation shows that our proposed approach provides close to optimal performance.

1.2.4 Robust Power Optimization for D2D Communication with Interference Control

In Chapter 6, we consider a multi-cell uplink scenario, where both the CUs and D2D pairs may generate significant ICI at multiple neighboring BSs. The CU and D2D users are each equipped with a single antenna, and the BS is equipped with multiple antennas. First, we focus on one D2D pair sharing a CU’s
assigned uplink channel resource. To capture the tradeoff between performance and signaling overhead due to the requirement of CSI feedback, we consider two scenarios. In Scenario 1, we assume perfect knowledge of instantaneous CSI for the communication channels and intra-cell interfering channels. We jointly optimize power control at the CU and D2D transmitter, under optimal receive beamforming for the CU, to maximize the sum rate of the D2D pair and CU while satisfying the minimum SINR requirements and limiting the worst-case ICI caused to multiple neighboring cells. In Scenario 2, we assume the perfect knowledge of instantaneous CSI only for the direct channels from the CU and D2D transmitter to the BS. For other channels, only partial CSI in terms of average channel quality is available. We jointly optimize receive beamforming for the CU and the transmit powers of the CU and D2D transmitter. Our objective is to maximize the expected uplink sum rate of the CU and D2D pair under minimum CU and D2D SINR requirements, as well as per-node maximum power and ICI constraints in multiple neighboring cells. The resulting joint optimization problem is challenging because of the uncertainty of instantaneous CSI, the non-convex expected sum rate objective, and the various power, interference, and SINR constraints imposed.

- In Scenario 1, the formulated joint optimization problem is non-convex. We propose a two-step approach to find the solution. First, through a feasibility test, the admissibility of the D2D pair is determined. If the D2D pair is admissible, we propose an optimal power control algorithm to maximize the sum rate by generalizing our algorithm in Chapter 5, which was proposed for a single ICI constraint, to a scenario with multiple ICI constraints. We obtain the optimal power solution of the CU and D2D in closed form, by analyzing the feasible solution region of the problem and the characteristics of the solution in the feasible region. We systematically find all candidates of an optimal solution.

- In Scenario 2, we first study D2D admissibility and obtain a simple feasibility test under minimum SINR requirements, maximum power, and ICI constraints. Assuming the D2D pair is admissible, we propose an efficient robust power control algorithm based on a ratio-of-expectation (ROE) approximation to maximize the expected sum rate. For performance benchmarking, we also develop an upper bound on the maximum expected sum rate.

- Then, we extend our consideration to the scenario of multiple D2D pairs and CUs. We formulate the joint power control and CU-D2D matching problem. The joint optimization problem is a difficult mixed integer program. We show how our solution for one D2D pair and one CU can be leveraged to find a solution by decomposing the joint optimization problem into a joint power optimization problem and a CU-D2D matching problem.

- Simulation shows that the proposed ROE algorithm gives performance that is close to the upper bound, and it substantially outperforms two baseline algorithms.
Chapter 2

Related Works

In this chapter, we summarize the prior research in design and optimization of multi-tier and D2D networks, and discuss the relation between this thesis and the prior works.

2.1 Relay Power Minimization

The vast majority of the existing literature on cooperative relay beamforming design is focused on a single S-D pair, considering perfect or imperfect CSI [4–7], multi-antenna relay processing matrix design [8–11], and relay beamforming design for two-way relaying [12–15]. For multi-user peer-to-peer relay networks, relay beamforming design has been considered for single-carrier systems [16–25]. For multi-user transmission in a single-carrier system, each S-D pair suffers from the interference from other pairs, causing significant performance degradation and is the main challenge in relay beamforming design. Due to the complexity involved in such a problem, an optimal solution is difficult to obtain. Typically, approximate solutions through numerical approaches are proposed or suboptimal problem structures are considered for analytical tractability.

In contrast, cooperative relay beamforming in a multi-channel system can avoid multi-user interference through subchannel orthogonalization. However, it adds a new design challenge of creating additional dimensions of power sharing. For each relay, its power is shared among subchannels for relaying signals of all S-D pairs. For each S-D pair, all relays participate in beamforming the transmitted signal, affecting the power usage of all relays. Thus, the optimal design of relay beamformers for per-relay power minimization remains a challenging problem.

The problem of optimal relay beamforming design for a single S-D pair has been extensively studied under total and per-relay power constraints [4–15]. For the multi-user downlink broadcast channel, multiple-input-multiple-output (MIMO) relay beamforming has been considered in [26, 27]. For transmission of multiple S-D pairs, the design of relay beam vectors has been studied under different metrics, including sum rate, sum mean square error (MSE), relay power, and total source and relay power, for single-carrier systems [16–25] and for multi-channel systems [28, 29]. Most of these works consider only the total power across relays either as the constraint or objective of the optimization problem, which renders the optimization problems analytically more tractable [16–24, 28].

There has been much study on MIMO relay beamforming for multiple S-D pairs. For example, in [16], a robust design of MIMO relay processing matrix to minimize the worst-case relay power has
been proposed for multiple S-D pairs, where the relays have only CSI estimates. With multiple MIMO relays, the MIMO relay processing design has been considered to minimize the total relay power subject to SINR guarantees in [21] for the perfect CSI case and in [18] when only second-order statistics of CSI are known at the relays. In [19], a robust MIMO relay processing design with CSI estimates is considered for sum MSE minimization and MSE balancing under a total relay power constraint. For a network with multiple MIMO S-D pairs, the total source and relay power minimization problem subject to minimum received SINR is considered in [23] and an iterative algorithm is proposed to jointly optimize the source, relay, and receive beam vectors and the source transmission power.

For single-antenna cooperative relay beamforming, the problem of total relay power minimization subject to minimal SINR guarantees has been considered for multiple S-D pairs in [17], where an approximate solution is proposed based on the semi-definite relaxation approach. Joint optimization of the source power and distributed relay beamforming is considered for the total power minimization in [22]. For a single-carrier relay beamforming system with multiple S-D pairs, the relay sum power minimization problem is studied in [24] using an interference zero-forcing approach. In contrast, we consider a multi-channel system and we solve the per-relay power with optimal beamforming, which is technically far more challenging.

To the best of our knowledge, the per-relay power minimization problem in multi-channel multi-relay systems has been studied only in [29]. However, the solution provided there is incomplete. In Chapter 3, we propose an algorithm to provide a complete solution in several possible cases. It can be shown that the solution in [29] is one special case of our solution (i.e., Case 3 in Section 3.2.2). Our algorithm transforms the dual problem into an efficient SDP problem and uses an iterative approach to find the solution. In [29], however, the dual problem is directly solved using a subgradient method. Moreover, we have investigated the effect of imperfect CSI due to quantization error or channel estimation error, while only the perfect CSI is assumed in [29].

2.2 Interference Management

The design of relay beamforming in order to minimize ICI is challenging. Most ICI mitigation techniques for relay networks in the literature focus on the scheduling problem, i.e., resource block allocation [30–34]. These techniques could not precisely control the amount of interference at the neighboring cells. To the best of our knowledge, the problem of minimizing the maximum interference at neighboring cells by relay beamforming has not been studied in the literature. Furthermore, most of the existing results in beamforming consider only a total power constraint across the antennas, which increases analytical tractability. However, in practical scenarios, we often need to consider an individual power limit for each relay [11, 90].

For single-channel systems, joint encoding and decoding across the BSs has been proposed to mitigate the ICI [35, 36]. In [37], joint optimization of source power allocation and relay beamforming to maximize the minimum SINR has been studied for a single-carrier FDMA system. Further BS cooperation or coordination, in the form of “virtual” or “network” MIMO systems, have been extensively studied in the literature [38–40]. These BS coordination techniques demand a huge amount of back-haul communication to share the data streams among the cells. In Chapter 4, we do not consider data sharing between the BSs or relays.

For multi-channel systems, such as those based on OFDMA, ICI coordination techniques have been
studied in [41–51]. The proposed approaches in the literature include power control, network MIMO, opportunistic spectrum access, adaptive frequency reuse factor, sphere decoding, and dirty paper decoding. The problem formulation in Chapter 4 is different from all of those available in the literature. In order to mitigate ICI, we consider relay beamforming, which leads to a uniquely complicated optimization problem. Furthermore, relay cooperative communication in interference limited environments has been considered under various criteria such as capacity, throughput, area spectral efficiency, and received SINR [52–56]. However, the objectives of these works do not include ICI reduction.

Most similar to Chapter 4, ICI mitigation techniques for relay networks in multi-channel systems have been studied in [30–34], which focus on scheduling and resource management. The authors of [30] have proposed a radio resource management strategy for relay-user association, resource allocation, and power control, along with four scheduling methods for power allocation in the ICI environment. In [31], the performance of different relay strategies, one-way, two-way, and shared relays, has been studied in interference-limited cellular systems. Assuming Gaussian signaling, the achievable rate for each strategy is derived. In [32], a joint subcarrier allocation, scheduling, and power control scheme has been proposed for ICI-limited networks. For relay-aided cellular OFDMA-based systems, the authors of [33] have proposed an interference coordination heuristic scheme consisting of two phases, each performing a resource allocation algorithm. In [34], a game theoretic framework called interference coordination game has been developed to mitigate interference in OFDMA-based relay networks, and a low complexity algorithm is proposed to reach its equilibrium in a distributed way.

However, none of the above works consider relay beamforming, which leads to a complex optimization problem as shown in Chapter 4. Furthermore, none of these works aims to directly minimize ICI, which could be significant if the interference channel is strong. Finally, it is important to find the maximum worst-case received SINR in a multi-channel system with multiple S-D pairs, especially for delay-sensitive applications requiring guaranteed worst bit-rate. Chapter 4 is the first to address the min-max interference and max-min SINR problems with relay beamforming.

In Chapter 3, we study the problem of relay beamforming to minimize per-relay power usage in a multi-user peer-to-peer network. Different from Chapter 3, in Chapter 4 we consider interference to multiple neighboring cells in a cellular system under relay power constraints. The new formulation and constraints add more difficulty to solving the problem. Although both problems use the dual method and involve dual variable case discussions, the case discussions involved in finding the optimal solutions in Chapter 4 are much more complicated and challenging than those in Chapter 3. In addition, as shown through simulation in Chapter 4, the min-max interference approach significantly outperforms the per-relay power approach for the maximum interference caused to neighboring cells.

2.3 Design of D2D Systems

For a D2D underlaid cellular network, interference management to D2Ds and CUs in the same cell has been investigated in various aspects in the literature [60–76]. The works in [60–69] focus on interference management to meet minimum QoS requirements for both D2Ds and CUs. Maximizing the sum rate of D2Ds and CUs while meeting the minimum QoS requirements through power control, resource allocation, or association techniques among users is considered in [70–73]. The problem of joint D2Ds and CUs association and power control to maximize the sum rate has been considered in [74, 75]. Despite the above results, the ICI due to D2D communication has not been investigated in the existing literature.
For a practical system, the ICI caused by both D2Ds and CUs in a neighboring cell should be carefully controlled to not exceed a certain level. In addition, due to the challenges involved in the problem, existing power allocation schemes for interference mitigation proposed in the literature are typically heuristics whose performance gap from the optimal cannot be guaranteed.

To limit the intra-cell interference due to resource sharing by CUs and D2Ds, different approaches have been proposed to meet minimum QoS requirements. As one of the earliest works, [60] has proposed a simple power control scheme for the D2D pair to constrain the SINR degradation of the CU, with limited interference coordination available between D2D and CU. In [67], the interference link condition between D2D and CU is obtained through the D2D pairs’ received power measurement during uplink transmission, and an interference-aware resource allocation scheme for D2D pairs has been proposed. To limit the interference at the D2D receiver, [65] has proposed an interference limited cell area, where a D2D pair and multiple CUs cannot coexist for channel reuse. For uplink resource sharing, [68] has proposed to scale the power of a D2D transmitter according to the pathloss between the D2D transmitter and the BS to satisfy the CU SINR requirement.

Without ICI consideration, the sum rate maximization of D2D and CU under their respective minimum QoS requirements has been studied in the literature. In [70], optimal time-frequency resource allocation and power control for sum rate maximization of a D2D pair and a CU has been studied, under rate limitation due to modulation and coding, and the CU’s minimum QoS requirement. For a single-antenna system, optimal power allocation for sum rate maximization of a D2D pair and a CU has been obtained in [71]. In contrast, in Chapter 5, we consider a multi-antenna BS and ICI constraints when optimizing the CU and D2D powers, which is more general and technically more challenging. With only the statistics of the interfering link from a CU to a D2D pair being available at the BS, [72] have proposed a probabilistic access control for the D2D pair to maximize the expected sum rate for uplink resource sharing. A low complexity D2D-CU association scheme has been proposed in [73] to maximize the sum rate of D2D pairs and CUs under power and QoS constraints.

The gaming approach has also been considered for D2D resource sharing [76, 91]. The problem of joint association and power control for D2D pairs has been studied in [76] using a pricing-based game theoretical approach to satisfy the SINR requirements of D2D pairs and CUs. A nontransferable coalition formation game has been considered in [91] to solve the energy-efficient resource sharing problem for mobile D2D multimedia communication.

To the best of our knowledge, neither ICI to neighboring cells nor BS receive beamforming has been considered in the literature studying the joint power optimization of CU and D2D for sum rate maximization under their respective minimum SINR requirements.

### 2.4 Robust Design of D2D Systems

To meet the SINR requirements for CUs and D2D pairs, various schemes have been proposed in the literature to limit the intra-cell interference due to resource reuse by CUs and D2D pairs [60, 61, 64, 65, 67–69]. None of these works directly concern sum rate maximization. Furthermore, existing power control schemes for interference management proposed in D2D communication are typically heuristics due to the complications involved in the problem. Hence, there is no performance guarantee for those schemes.

Some studies in the literature aim to directly maximize the sum rate of a D2D pair and a CU under
Chapter 2. Related Works

minimum SINR requirements. In [70], time-frequency resource allocation and power control has been studied for this problem in a cellular network with one CU and one D2D pair under rate constraints and a minimum SINR requirement for the CU. For a single-antenna system, optimal power control of a D2D pair and a CU has been obtained in [71] under minimum CU and D2D SINR requirements. Furthermore, low-complexity CU-D2D matching algorithms have been proposed in [73–75] to maximize the sum rate of CUs and D2D pairs. None of these works consider the ICI. In contrast, in Chapter 6, we consider a multi-antenna BS, ICI constraints, and partial CSI when optimizing the CU and D2D powers, which is more general and technically more challenging. Finally, in Chapter 5, we have jointly optimized the power of a CU and a D2D pair for their sum rate maximization, while satisfying minimum SINR requirements and a worst-case ICI limit in a neighboring cell. In 6, we generalize those results to a scenario with multiple ICI constraints from one or multiple neighboring cells.

Furthermore, a vast majority of the existing literature on D2D communication is focused on resource allocation when perfect CSI is available at the scheduling BS [60, 61, 64, 65, 67–71, 73–75, 80, 81]. In practical scenarios, the BS may not have perfect CSI knowledge. With only partial CSI available at the BS, the works in [77–79] study the problem of maximizing the expected sum rate of D2D pairs and CUs in a single-cell scenario. However, the effect of ICI due to D2D communication has not been considered in these works.

Under an assumption of partial CSI, the expected sum rate maximization of a D2D pair and CU under minimum SINR requirements has been studied in the literature. With only partial CSI knowledge of the interfering link from a CU to a D2D pair, probabilistic access control has been proposed in [77] for the D2D pair to maximize the expected sum rate for uplink resource sharing. A channel assignment algorithm based on dynamic programming is proposed in [78] to maximize the network utility with partial CSI. In [79], a low-complexity CU-D2D matching algorithm is proposed to maximize the ergodic sum rate under maximum power and outage constraints. Despite the results in [77–79], the effect of ICI due to D2D communication has not been investigated in the existing literature with partial CSI.

2.5 Notation

We use $A \triangleq B$ to denote that $A$ by definition is equivalent to $B$. We use $\| \cdot \|$ to denote the Euclidean norm of a vector. ⊙ stands for the element wise multiplication. We use $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^\dagger$ to denote transpose, Hermitian, and matrix pseudo-inverse, respectively. The conjugate is represented by $(\cdot)^*$. Notation diag($a$) denotes a diagonal matrix with diagonal elements being the elements of vector $a$, and diag($A$) denotes a diagonal matrix consisting of the diagonal elements of matrix $A$. We use $\mathbb{E}[\cdot]$ to denote the expectation and $\text{tr}(B)$ to represent the trace of $B$. $I$ denotes an $N \times N$ identity matrix. We use $Y \succeq Z$ to indicate that $Y - Z$ is a positive semi-definite matrix.

2.6 A Brief Review on Convex Optimization

In this section, we briefly review some preliminaries for convex optimization [92], which are employed in the analysis in Chapters 3 and 4.

Let $x = [x_1, \cdots, x_n]^T \in \mathbb{R}^n$ denote the vector of optimization variable. The mathematical programming is the problem of minimizing an objective function subject to some constraints, i.e.,
\[ \min_{x} f_0(x) \]
subject to \( f_i(x) \leq 0, \ i = 1, \cdots, m \)

where \( f_0 : \mathbb{R}^n \rightarrow \mathbb{R} \) and \( f_i : \mathbb{R}^n \rightarrow \mathbb{R} \) denote the objective function and constraint \( i \), respectively.

### 2.6.1 Convex Set

A set \( C \subset \mathbb{R}^n \) is convex if \( \forall x_1, x_2 \in C \) and \( \forall \theta \) such that \( 0 \leq \theta \leq 1 \), we have \( \theta x_1 + (1 - \theta)x_2 \in C \).

### 2.6.2 Convex Function

A function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \) is convex if the domain of \( f \) is a convex set and \( \forall x, y \in \text{dom} \ f, \ 0 \leq \theta \leq 1 \), we have \( f(\theta x + (1 - \theta)y) \leq \theta f(x) + (1 - \theta)f(y) \).

### 2.6.3 Cone

A set \( C \) is a cone if \( \forall x \in C \) and \( \forall \theta \geq 0 \), we have \( \theta x \in C \).

For a vector \( x \in \mathbb{R}^N \), a conic constraint is written in the form of \((Ax + b, c^T x + d) \in K\) where \( K \) is a cone.

### 2.6.4 Convex Problem

An optimization problem \( P \) is convex if it is expresses as

\[
P: \min_{x} f_0(x) \\
\text{subject to } f_i(x) \leq 0, \ i = 1, \cdots, m \\
a_i^T x - b_i = 0, \ i = 1, \cdots, p
\]

where \( f_0, f_1, \cdots, f_m \) are convex functions.

**Second-order Cone Programming (SOCP)**

\[
\min_{x} f^T x \\
\text{subject to } \|A_i x + b_i\| \leq c_i^T x + d_i, \ i = 1, \cdots, m \\
Fx = g.
\]

**Semi-definite Programming (SDP)**

\[
\min_{x} f^T x \\
\text{subject to } F_0 + x_1 F_1 + \cdots + x_n F_n \preceq 0 \\
F_i = F_i^T, \ i = 0, \cdots, n.
\]
2.6.5 Duality

Lagrangian

Consider an optimization problem $\mathbf{P}$ in the standard form:

$$\begin{align*}
\mathbf{P}: & \quad \min_{\mathbf{x}} f_0(\mathbf{x}) \\
& \text{subject to } f_i(\mathbf{x}) \leq 0, \ i = 1, \ldots, m \\
& \quad h_i(\mathbf{x}) = 0, \ i = 1, \ldots, p.
\end{align*}$$

The Lagrangian of $\mathbf{P}$ is defined as

$$L(\mathbf{x}, \lambda, \mu) \triangleq f_0(\mathbf{x}) + \sum_{i=1}^{m} \lambda_i f_i(\mathbf{x}) + \sum_{i=1}^{p} \mu_i h_i(\mathbf{x}).$$

Dual Problem

The dual problem of $\mathbf{P}$ is given by

$$\mathbf{D}: \quad \max_{\lambda, \mu} g(\lambda, \mu)$$

$$\text{subject to } \lambda \succeq 0$$

where $g(\lambda, \mu) \triangleq \min_{\mathbf{x}} L(\mathbf{x}, \lambda, \mu)$.

Let us denote the optimal value of the primal problem $\mathbf{P}$ and the dual problem $\mathbf{D}$ by $p^*$ and $d^*$, respectively. It can be shown that $p^* \geq d^*$ regardless of convexity of $\mathbf{P}$. Note that $p^* - d^*$ is known as the duality gap.

Duality Theorem

For convex problems satisfying a condition known as “constrained qualification”, we have $p^* = d^*$, i.e., the duality gap is 0.

2.7 Publications Related to this Thesis

A version of Chapter 3 is published in [57]; a version of Chapter 4 is published in [58, 59]; a version of Chapter 5 is published in [80, 81]; and a version of Chapter 6 is published in [82].
Chapter 3

Power Management for Distributed Relays

In this chapter, we investigate the optimal relay beamforming problem for multi-user peer-to-peer communication with amplify-and-forward relaying in a multichannel system. Assuming each S-D pair is assigned an orthogonal channel, we formulate the problem as a min-max per-relay power minimization problem with minimum SNR guarantees. After showing that strong Lagrange duality holds for this non-convex problem, we transform its Lagrange dual problem to an SDP problem and obtain the optimal relay beamforming vectors. We identify that the optimal solution can be obtained in three cases, depending on the values of the optimal dual variables. These cases correspond to whether the minimum SNR requirement at each S-D pair is met with equality, and whether the power consumption at a relay is the maximum among relays at optimality. We obtain a semi-closed-form solution structure of relay beam vectors, and propose an iterative approach to determine relay beam vector for each S-D pair. We further show that the reverse problem of maximizing the minimum SNR with per-relay power budgets can be solved using our proposed algorithm with an iterative bisection search. Through simulation, we analyze the effect of various system parameters on the performance of the optimal solution. Furthermore, we investigated the effect of imperfect channel side information of the second hop on the performance and quantify the performance loss due to either channel estimation error or limited feedback.

3.1 System Model and Problem Formulation

3.1.1 System Model

We consider a two-hop wireless AF relaying system where $M$ S-D pairs transmit data through $N$ relays in a multi-channel communication system. All the nodes in the network are equipped with a single antenna. We assume that a direct link is not available between each S-D pair (e.g., due to long distances). The multi-channel system is assumed to contain at least $M$ frequency subchannels. Each S-D pair is pre-assigned a subchannel for its data transmission which is orthogonal to all other S-D pairs. Each relay can transmit received signals from all sources over their assigned respective subchannels. The system model is illustrated in Fig.3.1.

Since each S-D pair is pre-assigned a subchannel, without loss of generality, we assume S-D pair $m$
Figure 3.1: The system model for multi-pair multi-channel relay communications.

 communicates through $N$ relays over subchannel $m$. The S-D transmission is established in two phases. In phase one, each source transmits its signal to all the relays. The received signal at relay $i$ over subchannel $m$ is given by

$$y_{m,i} = \sqrt{P_0} h_{m,i} s_m + n_{r,m,i}$$  \hspace{1cm} (3.1)$$

where $h_{m,i}$ is the channel coefficient on subchannel $m$ between source $m$ and relay $i$, $s_m$ is the transmitted symbol from source $m$ with unit power, i.e., $E[|s_m|^2] = 1$, $P_0$ is the transmission power\(^1\), and $n_{r,m,i}$ is the additive white Gaussian noise (AWGN) at relay $i$ on subchannel $m$ with zero mean and variance $\sigma_r^2$, which is i.i.d. across subchannels and relays. The received signal vector at all relays over subchannel $m$ is given by

$$y_m = \sqrt{P_0} h_m s_m + n_{r,m}$$  \hspace{1cm} (3.2)$$

where $h_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$ and $n_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the first-hop channel vector and the relay noise vector for S-D pair $m$, respectively.

In phase two, each relay $i$ multiplies its received signal over subchannel $m$ with a beamweight $w_{m,i}$ and forwards it to destination $m$. The received signal at destination $m$ from all relays over subchannel $m$ is given by

$$r_m = \mathbf{g}_m^T \mathbf{W}_m y_m + n_{d,m}$$

$$= \sqrt{P_0} \mathbf{g}_m^T \mathbf{W}_m h_m s_m + \mathbf{g}_m^T \mathbf{W}_m \mathbf{n}_{r,m} + n_{d,m}$$  \hspace{1cm} (3.3)$$

where $\mathbf{g}_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$ is the second-hop channel vector for S-D pair $m$, with $g_{m,i}$ being the channel coefficient on subchannel $m$ from relay $i$ to destination $m$, $\mathbf{W}_m \triangleq \text{diag}(\mathbf{w}_m)$, with $\mathbf{w}_m \triangleq [w_{m,1}, \cdots, w_{m,N}]^T$ being the relay beam vector for S-D pair $m$, and $n_{d,m}$ is the AWGN at destination $m$ with zero mean and variance $\sigma_d^2$, respectively.

\(^1\)Note that for simplicity, we assume the transmit power $P_0$ is the same for all sources. It is straightforward to extend our results to the scenario with different transmit power at different sources.
We assume that the data transmissions for the sources in phase one and for the relays in phase two are perfectly synchronized. In order to achieve perfect synchronization, multiple carrier frequency offsets, multiple timing offsets, and multiple channel gains corresponding to different communication links should be estimated and forwarded to a central controller. This central controller is assumed to be present in each cell, and may be co-located with the network node, i.e., the BS. Each network node at the transmitting end of a communication link sends pilot or reference symbols periodically or on-demand. Each network node at the receiving end of a communication link estimates the carrier frequency offset, timing offset, and the channel gain of this link by observing the pilot symbols sent by the network node at the transmitting end. The receiving network node sends a quantized version of the CSI to the beamforming controller in its own cell via a separate control channel. Achieving synchronization is beyond the scope of this thesis. In [93], estimation algorithms have been proposed to obtain the above parameters in a multi-relay network with one S-D pair.

The power usage of relay $i$ is given by

$$P_{r,i} = \sum_{m=1}^{M} E[|w_{m,i}y_{m,i}|^2] = \sum_{m=1}^{M} w_{m}^H R_{m} D_{i} w_{m}$$

(3.4)

where $R_{m} \triangleq \text{diag}([R_{y,m}]_{1,1}, \cdots, [R_{y,m}]_{N,N})$, with $R_{y,m} \triangleq P_{0} h_{m} h_{m}^H + \sigma_r^2 I$, for $m = 1, \cdots, M$, and $D_{i}$ denotes the $N \times N$ diagonal matrix with 1 in the $i$-th diagonal entry and 0 otherwise.

Define $f_{m} \triangleq g_{m} \odot h_{m} = [h_{m,1}g_{m,1}, \cdots, h_{m,N}g_{m,N}]^T$. The received signal power at destination $m$ is obtained by

$$P_{S,m} = P_{0}[g_{m}^T W_{m} h_{m} h_{m}^H W_{m}^H g_{m}^*] = P_{0} w_{m}^H F_{m} w_{m}$$

(3.5)

where $F_{m} \triangleq (f_{m} f_{m}^H)^*$. The total noise power at destination $m$ including both the receiver noise and the relay amplified noise is given by

$$P_{N,m} = \mathbb{E}[n_{r,m}^H W_{m}^H g_{m}^* g_{m}^T W_{m} n_{r,m}] + \sigma_d^2$$

$$= w_{m}^H G_{m} w_{m} + \sigma_d^2$$

(3.6)

where $G_{m} \triangleq \sigma_r^2 \text{diag} ((g_{m} h_{m}^H)^*)$. Thus, the SNR at destination $m$ is given by

$$\text{SNR}_m = \frac{P_{0} w_{m}^H F_{m} w_{m}}{w_{m}^H G_{m} w_{m} + \sigma_d^2}$$

(3.7)

We use SNR as the QoS metric. Many other QoS metrics, such as BER and data rate, are monotonic functions of SNR. We assume perfect knowledge of CSI, i.e., $\{h_{m}, g_{m}\}_{m=1}^{M}$, in designing the relay beam vectors.

### 3.1.2 Problem Formulation

We focus on a power efficient design of relay beamforming for multi-pair communications. Our goal is to minimize the maximum per-relay power usage by optimizing the relay beam vectors, while meeting the received SNR requirement at each destination. This min-max relay power optimization problem is
Chapter 3. Power Management for Distributed Relays

Given by

\[
\min_{\{w_m\}} \max_{1 \leq i \leq N} P_{r,i} \quad (3.8)
\]

subject to \(P_0 w_m^H f_m w_m \geq \gamma_m, m = 1, \cdots, M.\) \( (3.9)\)

Denoting \(P_{r,\text{max}} = \max_i P_{r,i}\), the min-max optimization problem (3.8) is equivalent to the following problem

\[
\min_{\{w_m\}, P_{r,\text{max}}} P_{r,\text{max}} \quad (3.10)
\]

subject to \(\sum_{m=1}^{M} w_m^H R_m D_i w_m \leq P_{r,\text{max}}, i = 1, \cdots, N,\) \( (3.11)\)

and (3.9).

### 3.2 Minimizing Maximum Per-Relay Power Usage

The per-relay power minimization problem (3.10) is non-convex due to the SNR constraint (3.9). To solve it, we first examine the feasibility of the problem. Then we show that the solution can be obtained in the dual domain. The dual problem is further converted into an SDP with polynomial worst-case complexity. We obtain a semi-closed-form structure of the beam vectors \(\{w_m\}\) and propose our algorithm to obtain the optimal dual variables in determining \(\{w_m\}\).

We first give the necessary condition for which the optimization problem (3.10) is feasible, which can be used to stop execution of the proposed algorithm if there exists \(m \in \mathcal{M}\) such that SNR constraint (3.9) cannot be satisfied.

**Proposition 1.** A necessary condition for the feasibility of the relay power minimization problem (3.10) is

\[
\min_{1 \leq m \leq M} \frac{P_0}{\gamma_m} f_m^H G_m f_m > 1. \quad (3.12)
\]

**Proof.** See Appendix A.

Note that the condition in (3.12) directly reflects the feasibility of the SNR constraint in (3.9), as shown in Appendix A. In other words, if the condition in (3.12) is not satisfied, the SNR constraint (3.9) cannot be satisfied for all \(m\) no matter what \(\{w_m\}\) is used.

#### 3.2.1 The Dual Approach

Although the optimization problem (3.10) is non-convex, we show that the strong duality holds and hence the problem (3.10) can be solved in the Lagrange dual domain. The result is given below.

**Proposition 2.** The per-relay power minimization problem (3.10) has zero duality gap.

**Proof.** See Appendix B.
By Proposition 2, since the zero duality gap holds for the problem (3.10), the optimal beam vectors \( \{ w_m^o \}_{m=1}^M \) can be obtained through the Lagrange dual domain. Let \( \lambda \triangleq [\lambda_1, \cdots, \lambda_N]^T \) and \( \alpha \triangleq [\alpha_1, \cdots, \alpha_M]^T \) denote the Lagrange multipliers associated with the per-relay power constraint (3.11) and SNR constraint (3.9), respectively. The dual problem of the problem (3.10) is given by

\[
\begin{align*}
\max_{\lambda, \alpha} \quad & \min_{\{w_m\}, P_{r,\text{max}}} L(\{w_m\}, P_{r,\text{max}}, \lambda, \alpha) \\
\text{subject to} \quad & \lambda \succeq 0, \alpha \succeq 0.
\end{align*}
\]  

(3.13)

(3.14)

The Lagrangian \( L(\{w_m\}, P_{r,\text{max}}, \lambda, \alpha) \) in (3.13) is given by

\[
L(\{w_m\}, P_{r,\text{max}}, \lambda, \alpha) = \sum_{m=1}^M \alpha_m \sigma_d^2 + P_{r,\text{max}}(1 - \sum_{i=1}^N \lambda_i) + \sum_{m=1}^M w_m^H (K_m - \frac{\alpha_m P_0}{\gamma_m} f_m f_m^H) w_m
\]

(3.15)

where

\[
K_m \triangleq R_m D_{\lambda} + \alpha_m G_m
\]

(3.16)

and \( D_{\lambda} \triangleq \text{diag}(\lambda_1, \cdots, \lambda_N) \).

The dual problem (3.13) can be shown to be equivalent to the following problem:

\[
\begin{align*}
\max_{\lambda, \alpha} \quad & \sum_{m=1}^M \alpha_m \sigma_d^2 \\
\text{subject to} \quad & K_m \succeq \frac{\alpha_m P_0}{\gamma_m} f_m f_m^H, \ m = 1, \cdots, M, \\
& \sum_{i=1}^N \lambda_i \leq 1, \\
& \text{and (3.14)}.
\end{align*}
\]  

(3.17)

(3.18)

(3.19)

To see the equivalence, note that if either (3.18) or (3.19) is not satisfied, there exists some \( \{w_m, P_{r,\text{max}}\} \) resulting in \( L(\{w_m\}, P_{r,\text{max}}, \lambda, \alpha) = -\infty \), which cannot be an optimal solution of the dual problem (3.13). Therefore, the constraints (3.18) and (3.19) are met at the optimality of the problem (3.13).

After the inner minimization with respect to (w.r.t.) \( \{w_m\} \) and \( P_{r,\text{max}} \), the objective of the dual problem (3.13) is equivalent to that in (3.17).

To solve the problem (3.17) for the optimal dual variables \( \{\lambda^o, \alpha^o\} \), we now show that it can be reformulated into an SDP given below to obtain the solution.
\[
\begin{align*}
\min_{y} & \quad a^T y \\
\text{subject to} & \quad b^T y \leq 1, \quad y \succeq 0 \\
& \quad \sum_{j=1}^{M+N} y_j \Psi_{m,j} \leq 0, \quad m = 1, \cdots, M
\end{align*}
\]

where \( y \triangleq [\alpha^T, \lambda^T]^T \), \( a \triangleq [-\sigma_d^2 f_{M \times 1}^T, 0_{N \times 1}^T]^T \), \( b \triangleq [0_{M \times 1}^T, 1_{N \times 1}^T]^T \), \( \Psi_{m,m} \triangleq \frac{P_0}{\gamma_m} f_m f_m^H - G_m \), \( \Psi_{m,M+i} \triangleq -R_m D_i \) for \( m = 1, \cdots, M \), \( i = 1, \cdots, N \), and all other \( \Psi_{m,j} \) are zeros.

The above SDP can be solved efficiently using a standard SDP solver [92]. Obtaining the optimal beam vector solution \( \{w^o_m\}_{m=1}^M \) of the problem (3.13) depends on the values of the optimal dual variables \( \{\lambda^o, \alpha^o\} \). In the following, we partition the values of \( \{\lambda^o, \alpha^o\} \) into three cases and derive \( \{w^o_m\}_{m=1}^M \) in each case. We first present the following lemma showing a certain condition on the value of \( \alpha^o \).

**Lemma 1.** If \( \lambda^o > 0 \), then \( \alpha^o > 0 \).

**Proof.** See Appendix C. \( \square \)

Note that \( \lambda^o \) and \( \alpha^o \) are the optimal dual variables associated with the per-relay power constraint (3.11) and SNR constraint (3.9), respectively. The Karush-Kuhn-Tucker (KKT) conditions require complementary slackness. Thus, Lemma 1 indicates that if the per-relay power constraint is active (i.e., attained with equality) at optimality, then the SNR constraint is also active at optimality. However, note that \( \alpha^o_m \) could be zero for some \( m \), if \( \lambda^o_i \) is zero for some \( i \).

### 3.2.2 The Optimal Beam Vector \( \{w^o_m\} \)

Using Lemma 1, in the following, we partition the values of \( \{\lambda^o, \alpha^o\} \) into three cases to derive \( \{w^o_m\}_{m=1}^M \).

**Case 1**

\( \lambda^o = 0 \). In this case, \( K_m \) in (3.16) reduces to \( \alpha_m G_m \). For the constraint (3.18) to hold, we have \( \alpha^o = 0 \) (also see Appendix C). As a result, the objective in (3.17) becomes zero. If the SNR constraint (3.9) could be satisfied for all \( m \), i.e., the original problem (3.10) is feasible, the optimal objective has to be strictly greater than zero which is a contradiction. This implies the per-relay power minimization problem (3.10) is infeasible. In other words, if the optimization problem (3.10) is feasible, there should be at least one \( i \) such that (3.11) is active at optimality, i.e., \( \lambda^o_i > 0 \).

**Case 2**

\( \lambda^o \neq 0 \) and \( \alpha^o \neq 0 \). In this case, we have \( \lambda^o_i = 0 \) for some \( i \)'s and \( \alpha^o_m = 0 \) for some \( m \)'s. In the following, we first consider the case in which at optimality, only one entry in \( \lambda^o \) and \( \alpha^o \) is strictly positive. In other words, only one S-D pair and one relay meet the SNR constraint and power constraint with equality, respectively. Then, we explain how to extend our solution to the case in which \( \lambda^o_i > 0 \), \( \alpha^o_m > 0 \) for arbitrary \( i \)'s and \( m \)'s. Denote \( \tilde{m} \) and \( \tilde{i} \) such that \( \alpha^o_{\tilde{m}} > 0 \) and \( \lambda^o_{\tilde{i}} > 0 \), respectively, and \( \alpha^o_m = 0 \) for \( m \neq \tilde{m} \) and \( \lambda^o_i = 0 \) for \( i \neq \tilde{i} \). In this case, we have \( \lambda^o_i = 1 \) from the maximization problem (3.17), since its optimal objective is increasing w.r.t. \( \lambda^o_i \).
In the following, we first obtain the optimal beam vector $w_{\tilde{m}}^{o}$. For $m \neq \tilde{m}$, the optimal beam vector $w_{m}^{o}$ cannot be derived in a similar way as that for $w_{\tilde{m}}^{o}$. Instead, we formulate a new optimization problem to obtain $w_{m}^{o}$.

**Proposition 3.** Assume $\alpha_{\tilde{m}}^{o} > 0$. The optimal beam vector $w_{\tilde{m}}^{o}$ for the per-relay power minimization problem (3.10) is given by

$$w_{\tilde{m}}^{o} = \zeta_{\tilde{m}} K_{\tilde{m}}^{o} \dagger f_{\tilde{m}}$$

(3.21)

where

$$\zeta_{\tilde{m}} = \sigma_{d} \left[ \frac{P_{0}}{\gamma_{\tilde{m}}} |f_{\tilde{m}}^{H} K_{\tilde{m}}^{o} \dagger f_{\tilde{m}}|^{2} - f_{\tilde{m}}^{H} K_{\tilde{m}}^{o} \dagger G_{\tilde{m}} K_{\tilde{m}}^{o} \dagger f_{\tilde{m}} \right]^{-\frac{1}{2}}$$

(3.22)

with $K_{\tilde{m}}^{o}$ obtained by substituting the optimal dual variables $\alpha_{\tilde{m}}^{o}$ and $\lambda_{\tilde{m}}^{o}$ into (3.16).

**Proof.** See Appendix D.

Define $\mathcal{M} = \{1, \ldots, M\} \setminus \{\tilde{m}\}$, and define $P_{i,\tilde{m}}^{o} = w_{\tilde{m}}^{o} R_{\tilde{m}} D_{i} w_{\tilde{m}}^{o}$ as the power used at relay $i$ for S-D pair $\tilde{m}$. The beamforming vectors $\{w_{m}, m \in \mathcal{M}\}$ are determined through solving the following feasibility problem

$$\text{find } \{w_{m}, m \in \mathcal{M}\}$$

subject to

$$\max_{1 \leq i \leq N} P_{i,\tilde{m}}^{o} + \sum_{m \in \mathcal{M}} w_{m}^{H} R_{m} D_{i} w_{m} = P_{r,\max}^{o},$$

$$\frac{P_{0} w_{m}^{H} F_{m} w_{m}}{w_{m}^{H} G_{m} w_{m} + \sigma_{d}^{2}} \geq \gamma_{m}, \ m \in \mathcal{M}. \tag{3.24}$$

There is no unique solution for the feasibility problem (3.23). However, we can always scale $w_{m}$ such that (3.24) meets with equality for $m \in \mathcal{M}$. Since we assume $\alpha_{\tilde{m}}^{o} = 0$ for $m \neq \tilde{m}$, the optimal objective of the original problem (3.10) is $P_{r,\max}^{o} = \alpha_{\tilde{m}}^{o} \sigma_{d}^{2}$. By Proposition 2, this means, at optimality, the Lagrangian in (3.15) is $\alpha_{\tilde{m}}^{o} \sigma_{d}^{2}$. It follows that, under the assumed $\alpha^{o}$, $\lambda^{o}$, we have $\sum_{m \in \mathcal{M}} w_{m}^{H} R_{m} D_{i} w_{m} = 0$. Since $\lambda_{i} > 0$, the power constraint (3.11) for $i$ is met with equality, and we have $P_{i,\tilde{m}}^{o} = P_{r,\max}^{o}$.

As analyzed above, at optimality, except S-D pair $\tilde{m}$, relay $\tilde{i}$ does not forward signal from any other source $m \in \mathcal{M}$. Thus, to obtain $w_{m}^{o}$, for $m \in \mathcal{M}$, we now propose the following relay power minimization problem by excluding the consideration of S-D pair $\tilde{m}$ and restricting the power usage on relay $\tilde{i}$

$$\min_{\{w_{m}, m \in \mathcal{M}\}, \tilde{P}_{r}} \tilde{P}_{r}$$

subject to

$$\sum_{m \in \mathcal{M}} w_{m}^{H} R_{m} D_{i} w_{m} \leq 0,$$

$$\sum_{m \in \mathcal{M}} w_{m}^{H} R_{m} D_{i} w_{m} \leq \tilde{P}_{r}, \forall i \neq \tilde{i}, \tag{3.26}$$

and (3.24).

Following similar argument as Proposition 2, we can show that zero duality gap holds for the problem (3.25). This problem can be reformulated in the dual domain into an SDP, given by
Proof. The first equality in (3.28) is due to the zero duality gap by Proposition 2. As shown in Appendix D for Case 3, we have \( \frac{\alpha_m^o}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m \mathbf{f}_m \geq 1 \) at optimality. Substituting \( \alpha_m^o \) into the objective of (3.17), we arrive at the expression at the right-hand side of (3.28).

Corollary 1. The maximum per-relay power for the original problem (3.10) is given by

\[
P_{r,\text{max}}^o = \sum_{m=1}^{M} \alpha_m^o \sigma_d^2 = \sigma_d^2 \sum_{m=1}^{M} \frac{\gamma_m}{P_0^f \mathbf{K}_m^H \mathbf{f}_m^H \mathbf{f}_m}. 
\]

Proof. The first equality in (3.28) is due to the zero duality gap by Proposition 2. As shown in Appendix D for Case 3, we have \( \frac{\alpha_m^o}{\gamma_m} \mathbf{f}_m^H \mathbf{K}_m \mathbf{f}_m \geq 1 \) at optimality. Substituting \( \alpha_m^o \) into the objective of (3.17), we arrive at the expression at the right-hand side of (3.28).
Algorithm 1 Solving the per-relay power minimization problem (3.10)

1: Check the feasibility condition (3.12).
2: Solve the SDP problem (3.20) to obtain the optimal dual variables \( \{\alpha^o, \lambda^o\} \).
3: Obtain \( \mathcal{I}_\alpha = \{m \mid \alpha^o_m > 0\} \) and \( \mathcal{I}_\lambda = \{i \mid \lambda^o_i > 0\} \).
4: Set \( \Pi_\alpha = \mathcal{I}_\alpha \).
5: while \( \mathcal{I}_\alpha \neq \{1, \cdots, M\} \) do
6: Compute \( \mathbf{K}^o_m \) and find \( \mathbf{w}^o_m \) in (3.21) for all \( m \in \Pi_\alpha \).
7: Update \( \mathbf{c} \) and \( \mathbf{d} \) as defined below the problem (3.27).
8: Solve the SDP problem (3.27).
9: Find \( \Pi_\alpha = \mathcal{I}_\alpha \cup \Pi_\alpha \) and \( \Pi_\lambda = \mathcal{I}_\lambda \cup \Pi_\lambda \).
10: Update \( \mathcal{I}_\alpha = \mathcal{I}_\alpha \cup \Pi_\alpha \) and \( \mathcal{I}_\lambda = \mathcal{I}_\lambda \cup \Pi_\lambda \).
11: end while
12: Compute \( \mathbf{K}^o_m \) and find \( \mathbf{w}^o_m \) in (3.21) for all \( m \in \Pi_\alpha \).

Combining Cases 2 and 3, we summarize our algorithm for solving per-relay power minimization problem (3.10) in Algorithm 1.

Note that for both Cases 2 and 3, the beam vector solution has the semi-closed-form structure given in (3.21). Hence, we can provide the necessary and sufficient condition for the feasibility of (3.10). Note that for \( \zeta_m \) in (3.22) to be real, the term in the bracket at the right-hand side of (3.22) should be positive. Therefore, the problem (3.10) is feasible if and only if there exists \( \alpha \succeq 0, \lambda \succeq 0, \) with \( \sum_{i=1}^N \lambda_i \leq 1 \) such that

\[
\min_{1 \leq m \leq M} \frac{P_0}{\gamma_m} |f^H_m \mathbf{K}^\dagger_m f_m|^2 - f^H_m \mathbf{K}^\dagger_m \mathbf{G}_m \mathbf{K}^\dagger_m f_m > 0.
\]  

(3.29)

3.2.3 Complexity Analysis

Now we analyze the complexity of Algorithm 1. Note that the optimization problem (3.10) has been converted to an SDP problem in (3.20) with \( M+N \) variables and \( M \) linear matrix inequality constraints. The SDP can be solved efficiently using interior-point methods with standard SDP solvers such as SeDuMi [94, 95]. In the following, we analyze the complexity based on the standard SDP form in [94]. Based on the complexity analysis of the standard SDP form, for the SDP with \( M+N \) variables, and \( M \) linear matrix inequality constraints of the size given, the computation complexity per iteration to solve (3.20) is \( O((M+N)^2MN^2) \). The number of iterations to solve SDP is typically between 5 to 50 regardless of problem size [94]. Thus, the complexity to solve the SDP is \( O((M+N)^2MN^2) \).

Note that the overall computation complexity to solve the optimization problem (3.10) depends on the values of the optimal dual variables. As shown in Section 3.2.2, if Case 3 happens, only one SDP problem (3.20) is solved, i.e., the complexity is given by \( O((M+N)^2MN^2) \). If Case 2 happens, at most \( M \) SDP problems formulated as (3.27) are solved, i.e., the worst-case complexity is given by \( O((M+N)^2M^2N^2) \). In both cases, the algorithm has a polynomial worst-case complexity w.r.t. the number of relays and S-D pairs. Note that the above analysis is based on worst-case complexity estimates. In practice, the complexity is much lower than the worst-case estimate [94].
3.3 Maximizing Minimum SNR

The ultimate end-to-end performance measures of the network such as the data rate or bit-error-rate (BER) are direct functions of the received SNR. It is often desirable to maximize the worst received SNR at the destinations under power constraints. In this section, we formulate the max-min SNR problem subject to per-relay power constraints, and show that it is the inverse problem of the min-max per-relay power subject to SNR constraints. Thus, we propose an iterative algorithm through bisection search to solve the max-min SNR problem.

In a typical system, the relays have the same front-end amplifiers and the destinations have the same minimum SNR requirements. In the following, we assume identical per-relay power budgets and minimum SNR requirements for the relays and destinations, respectively. Extension to the non-uniform power and/or SNR requirement scenarios can follow a similar approach, and is omitted for simplicity.

The problem of maximizing the minimum received SNR under a maximum per-relay power budget can be formulated as

\[
\max_{\{w_m\}, \gamma} \gamma
\]

subject to

\[
\sum_{m=1}^{M} w_m^H R_m D_i w_m \leq P_{r,0}, \quad i = 1, \cdots, N,
\]

\[
\text{SNR}_m \geq \gamma, \quad m = 1, \cdots, M
\]

where \(P_{r,0}\) denotes the relay power budget. The min-max relay power optimization problem (3.10) with a common SNR target \(\gamma_0\) is given by

\[
\min_{\{w_m\}, P_r} P_r
\]

subject to

\[
\text{SNR}_m \geq \gamma_0, \quad m = 1, \cdots, M,
\]

\[
\sum_{m=1}^{M} w_m^H R_m D_i w_m \leq P_r, \quad i = 1, \cdots, N.
\]

We use the notations \(\gamma^o(P_{r,0})\) and \(P_r^o(\gamma_0)\) to denote the optimal objectives in problems (3.30) and (3.31), to emphasize their dependency on \(P_{r,0}\) and \(\gamma_0\), respectively. The following proposition shows the property of \(\gamma^o(P_{r,0})\) as a function of \(P_{r,0}\).

**Proposition 4.** The optimal max received SNR \(\gamma^o(P_{r,0})\) is a continuous and strictly monotonically increasing function of \(P_{r,0}\), and any \(\gamma < \gamma^o(P_{r,0})\) is achievable.

**Proof.** See Appendix E. \(\square\)

Following Proposition 4, the min-max per-relay power \(P_{r,0}\) is achieved when \(\gamma^o(P_{r,0}) = \gamma_0\), for any \(\gamma_0\), i.e., \(P_{r}^o(\gamma^o(P_{r,0})) = P_{r,0}\). Hence the optimization problem (3.30) is the inverse problem of (3.31), i.e.,

\[
P_{r}^o(\gamma^o(P_{r,0})) = P_{r,0}, \quad \gamma^o(P_r^o(\gamma_0)) = \gamma_0.
\]

As a result, the SNR maximization problem (3.30) can be solved iteratively by solving the per-relay power minimization problem (3.31) with bisection search on the maximum per-relay power target \(P_r\) such that
Algorithm 2 Solving the min SNR maximization problem (3.30)

1: Set $\gamma_{0,\min}$ such that $P_r(\gamma_{0,\min}) < P_{r,0}$ and $\gamma_{0,\max}$ such that $P_r(\gamma_{0,\max}) > P_{r,0}$. Set $\varepsilon$.
2: Set $\gamma_0 = \frac{\gamma_{0,\min} + \gamma_{0,\max}}{2}$.
3: Solve the optimization problem (3.31) under $\gamma_0$.
4: if $P_r(\gamma_0) > P_{r,0}$ then
5: Set $\gamma_{0,\max} = \gamma_0$ and $P_r = 0$ (or $P_r < P_{r,0} - \varepsilon$).
6: else
7: Set $\gamma_{0,\min} = \gamma_0$ and $P_r = P_r(\gamma_0)$.
8: end if
9: if $P_r < P_{r,0} - \varepsilon$ then
10: Repeat (3)–(9); otherwise, return $\gamma_0$.
11: end if

Figure 3.2: CDF of maximum normalized relay power with $M = 2$.

$P_r \to P_{r,0}$. The steps to solve the max-min SNR problem (3.30) using bisection search are summarized in Algorithm 2. It is shown in [94] that SDP problems have nearly linear convergence regardless of the problem size. Furthermore, it is well-known that the bisection algorithm used in Algorithm 2 converges in $\log(\gamma_{0,\max} - \gamma_{0,\min}) - \log \varepsilon$ iterations.

### 3.4 Numerical Study

In this section, we provide numerical results to evaluate the performance of the proposed min-max relay power algorithm. In simulation, the noise powers at the relay and destination are set to $\sigma_r^2 = \sigma_d^2 = 1$. The first and second hop channels $h_m$ and $g_m$ are assumed i.i.d. zero-mean Gaussian with variance 1. The normalized source transmit power (against destination noise power) is set to $P_0/\sigma_d^2 = 10$ dB. A total of 1000 feasible realizations are used. Unless otherwise specified, the default minimum SNR guarantees are set to $\gamma_m = \gamma_0 = 5$ dB for $m = 1, \cdots, M$.

#### 3.4.1 Effect of the Number of Relays

In order to study the effect of the number of relays, $N$, on the maximum relay power, we plot the CDF of $P_r/\sigma_d^2$ obtained in problem (3.10) under different channel realizations, as shown in Fig. 3.2. We set $M = 2$. The number of relays are chosen as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. It can be noticed that as
N increases, the CDF is shifted to the left, and it also becomes more concentrated. In addition, the CDF curves do not converge as \( N \) becomes very large. In fact, those curves are uniformly shifted to the left. The uniform shift is because of the power gain achieved by relay beamforming. The tightening of CDF curves reflects the “hardening” of the effective channel due to beamforming, in the sense that the distribution of the effective channel becomes tighter.

The CDFs of the average received signal in (3.5) and noise power in (3.6), each normalized against \( \sigma_d^2 \), with \( N = 2^i \) for \( i \in \{0, \cdots, 5\} \) and \( M = 2 \) are shown in Fig. 3.3 and Fig. 3.4, respectively. In both figures, we observe that, as \( N \) increases, the CDF is shifted to the left. Furthermore, the amount of shift decreases, and the CDF shape becomes tighter. In Fig. 3.4, as \( N \) increases, the amplified noise is reduced to zero, and the overall noise converges to the receiver noise, which is 0 dB. This happens because the beam vector norm \( \|w_m\| \) decreases as \( N \) increases. For Fig. 3.3, as \( N \) increases, the normalized received signal power converges to 5 dB which is the minimum SNR requirement.

To demonstrate the result of the max-min SNR problem (3.30), in Fig. 3.5, the average minimum received SNR, i.e., \( \min_m SNR_m \) versus average \( P_{r,\text{max}}/\sigma_d^2 \) is plotted with \( M = 4 \), and \( N = 2^i \) for \( i \in \{1, \cdots, 5\} \). To generate each curve, we set the minimum SNR requirement \( \gamma_0 \) from -10 dB to 10 dB. For each \( \gamma_0 \) value, 1000 realizations are generated and the average \( P_{r,\text{max}}/\sigma_d^2 \) and \( \min_m SNR_m \) are computed for each realization. We see from Fig. 3.5 that, \( \min_m SNR_m \) is a monotonically increasing
function of $P_{r,\text{max}}/\sigma_d^2$. Also, for fixed $P_{r,\text{max}}/\sigma_d^2$, the minimum received SNR $\min_m \text{SNR}_m$ increases by more than 5 dB as $N$ doubles.

### 3.4.2 Effect of the Number of S-D Pairs

For fixed $N = 4$, the CDF of maximum relay power $P_{r,\text{max}}$ from the problem (3.10), normalized against $\sigma_d^2$, under various channel realizations is shown in Fig. 3.6, with $M = 2^i$ for $i \in \{1, \cdots, 4\}$. As expected, as $M$ increases, more relay power is needed, i.e., the CDF is shifted to the right.

In Fig. 3.7, the average minimum received SNR ($\min_m \text{SNR}_m$) versus average $P_{r,\text{max}}/\sigma_d^2$ is presented with $N = 4$, and $M = 2^i$ for $i \in \{1, \cdots, 5\}$. We see that, as expected, the average $\min_m \text{SNR}_m$ increases with average $P_{r,\text{max}}/\sigma_d^2$, while it decreases as $M$ increases because the number of SNR constraints increases. Consequently, the relays increase transmission power in order to satisfy the SNR requirement $\gamma_0$ for all destinations.

### 3.4.3 Effect of Imperfect CSI

So far, perfect CSI is assumed. To observe the robustness of the proposed algorithm w.r.t. the limited number of CSI feedback bits and channel estimation error, we consider the following two scenarios when
second-hop perfect CSI is not available.

In Scenario 1, there is no error in estimating the second-hop CSI. However, there is a limited number of feedback bits in order to send data to the relays. We consider equiprobable quantization of channel coefficients [96]. Let $B$ denote the number of available feedback bits. In the equiprobable quantization, every real and imaginary part of the channel coefficient on a subchannel is quantized with equal probability according to the CSI distribution, which is complex Gaussian.

In Scenario 2, the second-hop channels are estimated with estimation error; however, no feedback limit is imposed. Specifically, let us define $\hat{h} = h + \alpha \tilde{h}$, where $h$ is the true subchannel, $\hat{h}$ is the estimated subchannel used in the optimization problem. The estimation error $\tilde{h}$ is assumed Gaussian, i.e., $\tilde{h} \sim \mathcal{CN}(0, 1)$. The weight $\alpha$ is set to adjust the variance of error w.r.t. the variance of perfect CSI.

In Fig. 3.8, the CDF of $P_{r,\text{max}}/\sigma_d^2$ under perfect CSI is compared with that under imperfect CSI Scenario 1 with $2B$ bits ($B$ bits for each real and imaginary parts), where $B = 2$ and 3. Note that the performance under limited feedback is close to the case of perfect CSI. The degradation is similar for all $N$ values.

Finally, Fig. 3.9 shows the CDF of $P_{r,\text{max}}/\sigma_d^2$ of perfect CSI as compared with that under imperfect CSI Scenario 2 with the channel estimation error being $\alpha = 0.1$ and 0.3. Again, we observe that the performance gap from the perfect CSI case is relatively small. Furthermore, we observe that, unlike
Scenario 1, the performance is sensitive to $N$. In particular, the performance degradation increases as $N$ increases.

### 3.4.4 Comparison with Relay Selection

We may consider relay selection as an alternate approach to control the maximum per-relay power usage. In this scheme, we assign a relay to each subchannel, where the relay power usage is obtained such that the SNR requirement at the destination, which pre-assigned the subchannel, is met. To solve the relay selection problem, we enumerate among all feasible assignments to find a solution that minimizes the maximum per-relay power.

In Fig. 3.10, the CDF of $P_{r,max}/\sigma_d^2$ under Algorithm 1 is compared with that under relay selection. We set $M = 2$ and $N = 2$. Note that the min-max per-relay power approach significantly outperforms the relay selection approach. There is no known efficient method to find the optimal relay selection among all feasible assignments except an exhaustive search.\(^3\) If we consider a random relay selection, the performance will substantially degrade. Furthermore, in the relay selection approach, the number of

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\(^3\)To maximize the minimum rate among S-D pairs, some relay selection schemes have been proposed with quadratic complexity in the number of relays and S-D pairs \([97, 98]\).
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relays should be greater than or equal to the number of S-D pairs. However, this is not required in the
min-max per-relay power approach. Hence, our approach is better than the relay selection in terms of
performance, complexity, and practicality.

3.5 Summary

In this chapter, we have investigated the problem of relay beamforming design in a multi-user peer-
to-peer relay network in a multi-channel system. Assuming perfect CSI, the problem of minimizing
the maximum per-relay power usage subject to minimum received SNR guarantees is formulated. It
is shown that the non-convex problem satisfies strong duality. We have expressed its dual problem as
an SDP with polynomial worst-case complexity. Based on the values of the optimal dual variables,
we have studied the optimal relay beamforming vectors of the original problem in three cases. These
cases have reflected at optimality whether the minimum SNR requirement at each S-D pair is met with
equality, and whether the power consumption at a relay is the maximum among relays. Furthermore,
we have shown that maximizing the minimum received SNR subject to a fixed maximum relay power
constraint is the inverse problem of min-max relay power subject to a minimum SNR constraint. The
max-min SNR problem is solved iteratively using a bisection search. We have numerically evaluated the
proposed algorithm, and analyzed the effect of various system parameters on the performance of the
optimal solution. Furthermore, we have investigated the effect of imperfect CSI over the second hop,
and quantified the performance loss due to limited feedback or channel estimation error.

Appendix A

Proof of Proposition 1

Proof. Please note that $G_m \succeq 0$. If $w_m$ is in the null space $G_m$, denoted by null{$G_m$}, then $\text{SNR}_m$ becomes zero. Hence, a feasible solution $w_m$ will be not be in null{$G_m$}, i.e., we only consider $w_m \notin \text{null}{G_m}$.

The upper-bound of $\text{SNR}_m$ is given by (3.7) by ignoring the receiver noise $\sigma_d^2$ in the denominator, i.e.,

$$
\text{SNR}_m \triangleq \frac{P_0 |f_m^H w_m|^2}{w_m^H G_m w_m}.
$$

(A.1)

The upper-bound (A.1) is invariable w.r.t. the scale of $w$. For a fixed SNR upper-bound, the per-relay
power constraint (3.11) can be satisfied by scaling $\{w\}$. Hence, a necessary feasibility condition of (3.10)
is given by

$$
\max_{w_m \notin \text{null}(G_m)} \frac{P_0 |f_m^H w_m|^2}{w_m^H G_m w_m} > \gamma_m, \ m = 1, \cdots, M.
$$

(A.2)

Using the solution of the generalized eigenvalue problem, the left-hand side of (A.2) is maximized by
substituting $w_m = G_m^\dagger f_m$ into (A.1). Noting that the maximum value of (A.1) is $P_0 f_m^H G_m^\dagger f$, (3.12) is
obtained and the proof is complete.

□
Appendix B

Proof of Proposition 2

Proof. In order to prove the strong duality property, (3.10) is rewritten as an SOCP problem in conic form. The SOCP in conic form is convex and therefore has zero duality gap [92]. We need to show that the dual of (3.10) is equivalent to the dual of the SOCP.

The per-relay power constraint (3.11) is convex w.r.t. \( w \mapsto [w_1^T, \cdots, w_M^T]^T \). However, the minimum received SNR constraint (3.9) is non-convex. Reformulating the SNR constraint (3.9) in a conic form, we have

\[
\sqrt{P_0} |w_m^H f_m| \geq \sqrt{\gamma_m} \left\| \begin{bmatrix} G_{m}^{1/2} w_m \\ \sigma_d \end{bmatrix} \right\|, \quad m = 1, \cdots, M. \tag{B.1}
\]

Note that \( w_m \) can have any arbitrary phase, i.e., it is obtained uniquely up to a phase shift. The phase could be adjusted such that \( w_m^H f_m \) becomes real-valued for \( m = 1, \cdots, M \). Hence, the optimization problem (3.10) can be recast as

\[
\begin{align*}
\min_{\{w_m\}, P_{r,\text{max}}} & \quad P_{r,\text{max}} \\
\text{subject to} & \quad \sqrt{P_0} \frac{w_m^H f_m}{\gamma_m} \geq \sqrt{\gamma_m} \left\| \begin{bmatrix} G_{m}^{1/2} w_m \\ \sigma_d \end{bmatrix} \right\|, \quad m = 1, \cdots, M, \tag{B.3}
\end{align*}
\]

which is an SOCP. The problem (B.2) is non-convex since the constraint (B.3) is not in conic form. It is known that strong duality holds for SOCP in the conic form, but it may not hold in general forms [92]. However, the primal-dual optimality conditions for the problems with constraints in the form of (B.3) are provided in [99, Proposition 3]. Following a similar proof, it can be shown that (B.2) has zero duality gap. In the following, we show that the Lagrangian of (3.10) is the same as the Lagrangian of (B.2) using a similar proof as in [90, Proposition 1].

The Lagrangian of (3.10) is given by

\[
L_1 = P_{r,\text{max}} + \sum_{i=1}^{N} \lambda_i \left( \sum_{m=1}^{M} w_m^H R_m D_i w_m - P_{r,\text{max}} \right) \\
+ \sum_{m=1}^{M} \alpha_m \left( \sigma_d^2 + w_m^H G_m w_m - \frac{P_0}{\gamma_m} |w_m^H f_m|^2 \right). \tag{B.4}
\]

The Lagrangian of (B.2) is obtained by

\[
L_2 = P_{r,\text{max}} + \sum_{i=1}^{N} \tilde{\lambda}_i \left( \sum_{m=1}^{M} w_m^H R_m D_i w_m - P_{r,\text{max}} \right) \\
+ \sum_{m=1}^{M} \tilde{\alpha}_m \left( \left\| \begin{bmatrix} G_{m}^{1/2} w_m \\ \sigma_d \end{bmatrix} \right\| - \sqrt{\frac{P_0}{\gamma_m}} |w_m^H f_m| \right). \tag{B.5}
\]
Denoting \( \varphi_m \triangleq \left\| \left[ G_m^{1/2} w_m \right] \right\| + \sqrt{\frac{P_0}{\gamma_m}} |w_m^H f_m| \geq \sigma_d \) and converting the last term of the Lagrangian (B.5), it is equivalent to

\[
L_2 = P_{r, \text{max}} + \sum_{i=1}^{N} \lambda_i \left( \sum_{m=1}^{M} w_m^H R_m w_m - P_{r, \text{max}} \right) + \sum_{m=1}^{M} \alpha_m \left( \sigma_d^2 + w_m^H G_m w_m - \frac{P_0}{\gamma_m} |w_m^H f_m|^2 \right).
\]

Since \( \varphi_m \geq \sigma_d \), by changing the variables \( \alpha_m = \frac{\alpha_m}{\varphi_m} \), there exists \( \alpha_m \geq 0 \) for any \( \tilde{\alpha}_m \geq 0 \) and \( m = 1, \cdots, M \) such that (B.4) and (B.5) become exactly the same. As a result, strong Lagrange duality holds for the non-convex problem (3.10).

\begin{appendices}
\section*{Appendix C}
\section*{Proof of Lemma 1}

\textit{Proof.} Substituting (3.16) into (3.18), the constraint (3.18) is equivalent to

\[
R_m D_\lambda + \alpha_m \left( G_m - \frac{P_0}{\gamma_m} f_m f_m^H \right) \succeq 0.
\]

Using contradiction, we show that \( G_m - \frac{P_0}{\gamma_m} f_m f_m^H \) is an indefinite matrix. Suppose that \( G_m \succeq \frac{P_0}{\gamma_m} f_m f_m^H \). Since \( G_m \) is a positive-definite matrix, we have \( P_0 f_m H G_m^{-1} f_m \leq \gamma_m \). (\cite[Lemma 1]{90}). This contradicts the necessary condition for the feasibility of (3.10) as shown in Proposition 1. If \( \lambda^o \succ 0 \), there exists \( \alpha_m^o > 0 \) such that constraint (3.18) is satisfied. Note that the objective of the dual problem increases as \( \alpha_m \) increases. If there exists \( \lambda_i^o = 0 \) for some \( i \), then \( \alpha_m^o \) can be zero for some \( m \).

\section*{Appendix D}
\section*{Proof of Theorem 3}

\textit{Proof.} Suppose that \( \lambda^o \) satisfies the necessary condition in Lemma 1, \textit{i.e.}, the optimal dual variables are in the set defined by Lemma 1. The constraint (3.18) can be rewritten as an equivalent inequality using \cite[Lemma 1]{90} as follows. The dual problem (3.17) is equivalent to

\[
\begin{align*}
\max_{\lambda} \max_{\alpha} & \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} & \frac{\alpha_m P_0}{\gamma_m} f_m \gamma_m^{-1} f_m \leq 1, \ m = 1, \cdots, M,
\end{align*}
\]

(3.19), and (3.14).
In the following, we show the duality between (D.1) and SIMO beamforming problem similarly to [90]. Comparing (D.1) with the optimization problem
\[
\max_{\lambda} \min_{\alpha} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} \quad \frac{\alpha_m P_0}{\gamma_m} f_m^H K_m^{-1} f_m \geq 1, \quad m = 1, \cdots, M,
\]
we see that the inner maximization in (D.1) becomes minimization in (D.3) and the SNR inequality is reversed. Substituting (3.16) into the left-hand side of (D.2), we define
\[
\Phi_m(\alpha_m) = \frac{\alpha_m P_0}{\gamma_m} f_m^H K_m^{-1} f_m.
\]
which is a monotonically increasing function of \(\alpha_m > 0\) for \(\lambda^o\). Therefore, the constraints (D.2) and (D.4) are met with equality at optimality. The two problems (D.1) and (D.3) have the same optimal value \(\alpha_m^o\) satisfying \(\Phi_m(\alpha_m^o) = 1\) for \(m = 1, \cdots, M\), i.e., the optimization problems (D.1) and (D.3) are equivalent. The SIMO beamforming problem (D.3) is given by substituting \(\tilde{w}_m = \sum_{m=1}^{M} \alpha_m \sigma_d^2\) into
\[
\max_{\lambda} \min_{\alpha, w_m} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} \quad \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma_d^2} K_m f_m \geq 1, \quad m = 1, \cdots, M,
\]
we see that the inner maximization in (D.1) becomes minimization in (D.3) and the SNR inequality is reversed. Substituting (3.16) into the left-hand side of (D.2), we define
\[
\Phi_m(\alpha_m) = \frac{\alpha_m P_0}{\gamma_m} f_m^H K_m^{-1} f_m,
\]
which is a monotonically increasing function of \(\alpha_m > 0\) for \(\lambda^o\). Therefore, the constraints (D.2) and (D.4) are met with equality at optimality. The two problems (D.1) and (D.3) have the same optimal value \(\alpha_m^o\) satisfying \(\Phi_m(\alpha_m^o) = 1\) for \(m = 1, \cdots, M\), i.e., the optimization problems (D.1) and (D.3) are equivalent. The SIMO beamforming problem (D.3) is given by substituting \(\tilde{w}_m = \sum_{m=1}^{M} \alpha_m \sigma_d^2\) into
\[
\max_{\lambda} \min_{\alpha, w_m} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} \quad \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma_d^2} K_m f_m \geq 1, \quad m = 1, \cdots, M,
\]
we see that the inner maximization in (D.1) becomes minimization in (D.3) and the SNR inequality is reversed. Substituting (3.16) into the left-hand side of (D.2), we define
\[
\Phi_m(\alpha_m) = \frac{\alpha_m P_0}{\gamma_m} f_m^H K_m^{-1} f_m,
\]
which is a monotonically increasing function of \(\alpha_m > 0\) for \(\lambda^o\). Therefore, the constraints (D.2) and (D.4) are met with equality at optimality. The two problems (D.1) and (D.3) have the same optimal value \(\alpha_m^o\) satisfying \(\Phi_m(\alpha_m^o) = 1\) for \(m = 1, \cdots, M\), i.e., the optimization problems (D.1) and (D.3) are equivalent. The SIMO beamforming problem (D.3) is given by substituting \(\tilde{w}_m = \sum_{m=1}^{M} \alpha_m \sigma_d^2\) into
\[
\max_{\lambda} \min_{\alpha, w_m} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} \quad \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma_d^2} K_m f_m \geq 1, \quad m = 1, \cdots, M,
\]
we see that the inner maximization in (D.1) becomes minimization in (D.3) and the SNR inequality is reversed. Substituting (3.16) into the left-hand side of (D.2), we define
\[
\Phi_m(\alpha_m) = \frac{\alpha_m P_0}{\gamma_m} f_m^H K_m^{-1} f_m,
\]
which is a monotonically increasing function of \(\alpha_m > 0\) for \(\lambda^o\). Therefore, the constraints (D.2) and (D.4) are met with equality at optimality. The two problems (D.1) and (D.3) have the same optimal value \(\alpha_m^o\) satisfying \(\Phi_m(\alpha_m^o) = 1\) for \(m = 1, \cdots, M\), i.e., the optimization problems (D.1) and (D.3) are equivalent. The SIMO beamforming problem (D.3) is given by substituting \(\tilde{w}_m = \sum_{m=1}^{M} \alpha_m \sigma_d^2\) into
\[
\max_{\lambda} \min_{\alpha, w_m} \sum_{m=1}^{M} \alpha_m \sigma_d^2 \\
\text{subject to} \quad \frac{\alpha_m P_0}{\sum_{m=1}^{M} \alpha_m \sigma_d^2} K_m f_m \geq 1, \quad m = 1, \cdots, M,
\]
Appendix E

Proof of Proposition 4

Proof. Using contradiction, it can be shown that the optimal $\gamma^o(P_{r,0})$ is strictly monotonically increasing function of $P_{r,0}$. Suppose that $\{w_m\}_{m=1}^M$ is the optimal beam vector of the max-min problem (3.30) achieving $\gamma^o(P_{r,0})$. Let us assume $P_{r,1} > P_{r,0}$ and $\gamma^o(P_{r,1}) \leq \gamma^o(P_{r,0})$ for some $P_{r,1}$ and $P_{r,0}$. The beam vectors $\{w_m\}_{m=1}^M$ can be scaled by a real-valued $0 < \chi < 1$ such that, under $\{\chi w_m\}_{m=1}^M$, the SNR becomes $\gamma^o(P_{r,1})$ with the resulting maximum per-relay power usage $\chi^2 P_{r,0} < P_{r,1}$. This contradicts with the assumption that $P_{r,1}$ is optimal for $\gamma = \gamma^o(P_{r,1})$. It is not difficult to show that $\gamma^o(P_{r,0})$ is continuous w.r.t. $P_{r,0}$. In order to show that any $\gamma < \gamma^o(P_{r,0})$ is achievable, let us denote $\nu \triangleq \arg \min_{m=1,\cdots,M} \text{SNR}_m$ and

$$\eta \triangleq \frac{\sigma_d}{\left( \frac{P_0}{\gamma} w^H_{\nu} F_{\nu} w_{\nu} - w^H_{\nu} G_{\nu} w_{\nu} \right)^{\frac{1}{2}}} > 0. \quad (E.1)$$

Note that the denominator of $\eta$ is positive since $\gamma < \gamma^o(P_{r,0})$. After some manipulation, it can be shown that $\{\eta w_m\}_{m=1}^M$ achieves any arbitrary $\gamma < \gamma^o(P_{r,0})$. \qed
Chapter 4

Interference Management for Distributed Relays

In this chapter, we consider a cellular network where each cell contains multiple source-destination pairs communicating through multiple amplify-and-forward relays using orthogonal channels. We propose an optimal relay beamforming design that minimizes the maximum interference at the neighboring cells subject to per-relay power limits and minimum received SNR requirements. Even though the problem is non-convex, we show that it has zero Lagrange duality gap, and we convert its dual problem to an SDP problem. Depending on the values of the optimal dual variables, we study three cases to obtain the optimal beam vectors accordingly. This results in an iterative algorithm that provides a semi-closed-form optimal solution. We extend our algorithm to the problem of maximizing the minimum SNR subject to some pre-determined maximum interference constraints at neighboring cells, by the solution to the min-max interference problem along with a bisection search. The solution to this max-min SNR problem gives insight into the worst-case SINR given some maximum interference target. The performance of the proposed algorithm is studied numerically, both for when the knowledge of interference channel is perfect and for when it is imperfect due to either limited feedback or channel estimation error. It is demonstrated that this min-max interference approach substantially outperforms the alternative where we simply minimize the maximum relay transmission power.

4.1 System Model and Problem Formulation

4.1.1 System Model

As illustrated in Fig. 4.1, we consider a cellular system where each cell contains $M$ S-D pairs, $N$ relays, and $b$ neighboring cells, and all nodes are equipped with a single antenna. A multichannel communication system (e.g., OFDMA) consisting of $M$ orthogonal subchannels is used in each cell. Each source transmits data to its destination through the relays using a specific subchannel, and each subchannel is assigned to one S-D pair, so that the S-D pairs within the same cell do not interfere with each other. In this chapter, we study the interference caused by the relays in one cell (desired cell) to the destinations in its neighboring cells.

We consider the half-duplex AF protocol for relaying, where the direct path is ignored. Assume that
Figure 4.1: The system model for multiple communication pairs. The solid and dashed lines show the desired and interference channels, respectively.

S-D pair $m$ communicates through $N$ relays over subchannel $m$. The S-D communication is established in two phases. In phase one, each source transmits its signal to all the relays. In the desired cell, the received signal at relay $i$ over subchannel $m$ is given by

$$z_{m,i} = \sqrt{P_m h_{m,i}} s_m + n_{r,m,i}$$  \hspace{1cm} (4.1)$$

where $s_m$ is the transmitted symbol with unit power, i.e., $\mathbb{E}[|s_m|^2] = 1$, $P_m$ is the transmission power, and $n_{r,m,i}$ denotes the AWGN at relay $i$ on subchannel $m$ with zero mean and variance $\sigma^2_r$, which is i.i.d. across subchannels and relays. The vector of received signals at all relays over subchannel $m$ is given by

$$z_m = \sqrt{P_m} h_m s_m + n_{r,m}$$  \hspace{1cm} (4.2)$$

where $h_m \triangleq [h_{m,1}, \cdots, h_{m,N}]^T$ and $n_{r,m} \triangleq [n_{r,m,1}, \cdots, n_{r,m,N}]^T$ are the first-hop channel vector and the relay noise vector for S-D pair $m$, respectively.

In phase two, each relay $i$ multiplies the received signal over subchannel $m$ by a complex coefficient $w_{m,i}$ and forwards it to destination $m$, for $1 \leq m \leq M$. The received signal at destination $m$ from all relays over subchannel $m$ is given by

$$r_m = g_{m}^T W_m z_m + n_{d,m}$$  \hspace{1cm} (4.3)$$

$$= \sqrt{P_m} g_{m}^T W_m h_m s_m + g_{m}^T W_m n_{r,m} + n_{d,m}$$  \hspace{1cm} (4.4)$$

where $g_m \triangleq [g_{m,1}, \cdots, g_{m,N}]^T$ is the second-hop channel vector for S-D pair $m$, with $g_{m,i}$ denoting the channel coefficient over subchannel $m$ from relay $i$ to destination $m$, $W_m \triangleq \text{diag}(w_m)$, with $w_m \triangleq$ 

\footnote{Note that multiple transmissions in a cooperative relay system may not be synchronized at a destination. An asynchronous transmission scheme is proposed in [100,101].}
denoting the relay beam vector for S-D pair $m$, and $n_{d,m}$ is the AWGN at destination $m$ with zero mean and variance $\sigma_d^2$.

The power usage of relay $i$ is expressed as

$$P_i = \sum_{m=1}^{M} \mathbb{E}[|w_{m,i}\hat{z}_{m,i}|^2] = \sum_{m=1}^{M} w_m^H R_m D_i w_m$$

where $R_m \triangleq \text{diag}([R_{y,m}],1,\cdots,[R_{y,m}])$, with $R_{y,m} \triangleq P_m h_m h_m^H + \sigma_r^2 I$, for $m = 1, \cdots, M$, and $D_i$ denotes the $N \times N$ diagonal matrix with 1 in the $i$-th diagonal and zero otherwise. We assume that the total power available at each relay, $P_r$, can be allocated across different subchannels.

The received signal power at destination $m$ is given by

$$P_{S,m} = P_m |g_m^T w_m h_m h_m^H w_m^T| = P_m w_m^H F_m w_m$$

where $F_m \triangleq (f_m f_m^H)^*$, with $f_m = g_m \otimes h_m \triangleq [h_{m,1} g_{m,1}, \cdots, h_{m,N} g_{m,N}]^T$. The total noise power at destination $m$, including both the receiver noise and the relay amplified noise, is obtained as

$$P_{N,m} = \mathbb{E}[n_m^H w_m^H \hat{G}_m w_m] + \sigma_d^2$$

$$= w_m^H G_m w_m + \sigma_d^2$$

where $G_m \triangleq \sigma_r^2 \text{diag}([g_m g_m^H]^*)$. Hence, the SNR at destination $m$ is given by

$$\text{SNR}_m = \frac{P_m w_m^H F_m w_m}{w_m^H G_m w_m + \sigma_d^2}.$$

Each relay causes interference to the $M$ destinations in each of neighboring cells. Let $\hat{g}_{m,j}$ denote the interference channel vector over subchannel $m$ from the $N$ relays of the desired cell to destination $m$ in neighboring cell $j$. The received interference at destination $m$ in neighboring cell $j$ is given by

$$\hat{r}_{m,j} = \hat{g}_{m,j}^T W_m (\sqrt{P_m h_m s_m} + n_{r,m}).$$

The received interference power at destination $m$ in neighboring cell $j$, including both the forwarded signal and the relay amplified noise, is given by

$$\mathcal{I}_{m,j} = P_m w_m^H \hat{F}_{m,j} w_m + w_m^H \hat{G}_{m,j} w_m$$

where $\hat{F}_{m,j} \triangleq (\hat{f}_{m,j} \hat{f}_{m,j}^H)^*$, $\hat{f}_{m,j} \triangleq \hat{g}_{m,j} \otimes h_m$, and $\hat{G}_{m,j} \triangleq \sigma_r^2 \text{diag}([\hat{g}_{m,j} \hat{g}_{m,j}^H]^*)$ for $j = 1, \cdots, b$.

We assume the perfect knowledge of CSI, i.e., $\{h_m, g_m, \hat{g}_{m,j}\}_{m=1}^{M}$, in designing the relay beam vectors, where a central controller in each cell may collect all intra- and inter-cell CSI for computing relay beam weights. In Section 4.4.4, we further study the case where the interference CSI is imperfect through simulation.

### 4.1.2 Problem Formulation

Our focus is on designing the relay beam weights of the desired cell to minimize the maximum interference at the neighboring cells under per-relay power constraint and the received SNR requirement at each
destination. This is expressed as the following optimization problem:

\[ \text{P0: } \min_{\{w_m\}} \max_{m \in \mathcal{M}, j \in \mathcal{B}} \mathcal{I}_{m,j} \]

subject to

\[ \sum_{m=1}^{M} w_m^H \mathbf{R}_m \mathbf{D}_i w_m \leq P_r, \quad i \in \mathcal{N}, \quad (4.11a) \]

\[ \frac{P_m w_m^H \mathbf{F}_m w_m}{w_m^H \mathbf{G}_m w_m + \sigma_d^2} \geq \gamma_m, \quad m \in \mathcal{M} \]

\[ (4.11b) \]

where \( I_{m,j} \) is as defined in (4.10), \( \mathcal{M} \triangleq \{1, \cdots, M\} \), \( \mathcal{N} \triangleq \{1, \cdots, N\} \), and \( \mathcal{B} \triangleq \{1, \cdots, b\} \). To remove the inner maximization in P0, we note that the min-max optimization problem P0 is equivalent to the following:

\[ \text{P1: } \min_{\{w_m\}, \mathcal{I}_{\text{max}}} \mathcal{I}_{\text{max}} \]

subject to

\[ w_m^H \tilde{\mathbf{B}}_{m,j} w_m \leq \mathcal{I}_{\text{max}}, \quad m \in \mathcal{M}, \ j \in \mathcal{B}, \]

\[ (4.12a) \]

\[ (4.11a), \text{ and } (4.11b) \]

where \( \mathcal{I}_{\text{max}} = \max_{m \in \mathcal{M}, j \in \mathcal{B}} \mathcal{I}_{m,j} \) and \( \tilde{\mathbf{B}}_{m,j} \triangleq P_m \tilde{\mathbf{F}}_{m,j} + \tilde{\mathbf{G}}_{m,j} \).

Note that, in this chapter, we consider the interference minimization problem under per-relay total power constraint. For a fixed source transmit power, we can show that considering the direct ICI link from sources to destinations in neighboring cells does not change our analysis.\(^2\) To see this, note that the total received interference at destination \( m \) in neighboring cell \( j \) contains interference from both relays and sources, given by

\[ \bar{I}_{m,j} = I_{m,j} + P_m |\tilde{h}_{m,j}|^2 \]

where \( \tilde{h}_{m,j} \) denotes the direct channel from source \( m \) to destination \( m \) in neighboring cell \( j \). Assume \( |\tilde{h}_{m,j}|^2 \) is known in the desired cell. Given the fact that \( P_m |\tilde{h}_{m,j}|^2 \) does not depend on \( w_m \), we can treat it as a constant term when designing the beam vectors. To include the inference coming from the sources, we can replace the left-hand side of constraint (4.12a) with \( w_m^H \tilde{\mathbf{B}}_{m,j} w_m + P_m |\tilde{h}_{m,j}|^2 \). Then, a similar procedure as in our proposed algorithm can be followed to obtain the optimal beam vectors.

### 4.2 Minimizing Maximum Interference

The solution of P1 is provided in this section. Since the SNR constraint (4.11b) is not convex w.r.t. \( w_m \), P1 is non-convex. In order to solve this problem, we first provide a necessary condition for its feasibility. Then we show that P1 can be reformulated as a second-order-conic programming (SOCP) problem, and more importantly, the SOCP’s conic dual and Lagrange dual are equivalent, so that P1 has zero Lagrange duality gap. In order to obtain the optimal dual variables, an SDP-based algorithm is proposed with polynomial worst-case complexity. We then propose an iterative algorithm to obtain the optimal beam vectors \( \{w_m\} \) with a semi-closed-form structure. Through complexity analysis, we show that our proposed algorithm is computationally more efficient in finding an optimal solution than

\[ \text{If the source transmission power } p \triangleq |P_1, \cdots, P_M |^T \text{ is also an optimization variable, we have a joint optimization problem with } \{p, w\} \text{ as variables. This joint optimization problem becomes much more difficult to solve, as it is jointly non-convex. Whether it can be solved needs to be carefully investigated and is an open problem left for future research.} \]
4.2.1 Necessary Condition for Feasibility

We first introduce necessary condition for feasibility following the similar arguments in Chapter 3, which can be used to stop execution of the proposed algorithm if there exists \( m \in \mathcal{M} \) such that SNR constraint (4.11b) cannot be satisfied.

A necessary condition for the feasibility of the min-max interference problem \( \text{P1} \) is

\[
\min_{m \in \mathcal{M}} P_m \frac{\gamma_m}{f_m} G_m^H f_m > 1. \tag{4.13}
\]

Note that not satisfying (4.13) means that regardless of the values of \( \{ w_m \} \) there always exists \( m \in \mathcal{M} \) such that SNR constraint (13) cannot be satisfied, and thus \( \text{P1} \) is infeasible. On the other hand, even if (4.13) holds, that does not guarantee that \( \text{P1} \) is feasible. In that case, we will see later that Case 1 in Section 4.2.3 will identify the infeasibility of \( \text{P1} \).

4.2.2 The Lagrange Dual Approach

In the following, we show that, despite \( \text{P1} \) being non-convex, it has zero duality gap and can be solved in the Lagrange dual domain.

**Proposition 5.** Strong duality holds for the min-max interference problem \( \text{P1} \).

**Proof.** We first show that \( \text{P1} \) can be reformulated as an SOCP problem. It is known that the SOCP has zero conic duality gap [92]. Then we show that the Lagrange dual of \( \text{P1} \) and conic dual of the SOCP are equivalent. For further details, see Appendix A. \( \square \)

Using Proposition 5, we can obtain the optimum solution of \( \text{P1} \) through the Lagrange dual approach. Let \( \mu \triangleq [\mu_1, \cdots, \mu_M]^T \) with \( \mu_m \triangleq [\mu_{m,1}, \cdots, \mu_{m,b}]^T \), \( \lambda \triangleq [\lambda_1, \cdots, \lambda_N]^T \), and \( \alpha \triangleq [\alpha_1, \cdots, \alpha_M]^T \) denote the Lagrange multipliers associated with the interference constraint (4.12a), per relay power constraint (4.11a), and SNR constraint (4.11b), respectively. The Lagrangian of \( \text{P1} \) is given by

\[
L(\{w_m\}, I_{\text{max}}, \lambda, \mu, \alpha) = \sum_{m=1}^{M} w_m^H (K_m - \frac{\alpha_m P_m}{\gamma_m} f_m f_m^H) w_m + \sum_{m=1}^{M} \alpha_m \sigma_d^2 + I_{\text{max}}(1 - \sum_{m=1}^{M} \sum_{j=1}^{b} \mu_{m,j}) - P_r(\sum_{i=1}^{N} \lambda_i) \tag{4.14}
\]

where

\[
K_m \triangleq R_m D_{\lambda} + \sum_{j=1}^{b} \mu_{m,j} \tilde{B}_{m,j} + \alpha_m G_m \tag{4.15}
\]

and \( D_{\lambda} \triangleq \text{diag}(\lambda_1, \cdots, \lambda_N) \).

\(^3\)We can show that solving SOCP directly increases complexity as compared with the proposed algorithm for the typical scenario of large number of relays and S-D pairs. In addition to complexity reduction, one can gain insights on the optimal solution structure using our proposed algorithm. However, the SOCP-based method does not provide any insight on the structure of the solution for \( w^* \).
The dual problem of P1 is obtained by

\[
\begin{align*}
&\text{D0: } \max_{\lambda, \mu, \alpha} \min_{\{w_m\}, \mathcal{I}_{\text{max}}} \mathcal{L}(\{w_m\}, \mathcal{I}_{\text{max}}, \lambda, \mu, \alpha) \\
&\text{subject to } \lambda \succeq 0, \mu \succeq 0, \alpha \succeq 0.
\end{align*}
\] (4.16a)

Furthermore, the dual problem D0 can be reformulated as the following problem:

\[
\begin{align*}
&\text{D1: } \max_{\lambda, \mu, \alpha} \sum_{m=1}^{M} \alpha_m \sigma_d^2 - P_r \left( \sum_{i=1}^{N} \lambda_i \right) \\
&\text{subject to } K_m \succeq \alpha_m P_m f_m H_m, m \in \mathcal{M} \quad (4.17a) \\
&\sum_{m=1}^{M} b \sum_{j=1}^{N} \mu_{m,j} \leq 1, \quad (4.17b)
\end{align*}
\]

and (4.16a).

The equivalence of D0 and D1 can be shown by showing that constraints (4.17a) and (4.17b) are satisfied at optimality of D0. Suppose one of the constraints (4.17a) or (4.17b) is not satisfied. Then there is some \(\{w_m, \mathcal{I}_{\text{max}}\}\) such that the inner minimization of D0 leads to \(L(\{w_m\}, \mathcal{I}_{\text{max}}, \lambda, \mu, \alpha) = -\infty\), but clearly this cannot be the optimal objective of the dual problem. Hence, the optimal solution of D0 satisfies constraints (4.17a) and (4.17b). In this case, after the inner minimization of the Lagrangian in D0, we have the objective of D1. Thus, both D0 and D1 lead to the same optimal \(\{\lambda^o, \mu^o, \alpha^o\}\).

To solve the dual problem D1 we show that it can be reformulated as an SDP problem to determine the optimal \(\{\alpha^o, \lambda^o, \mu^o\}\).

**Proposition 6.** The dual problem D1 can be expressed as

\[
\begin{align*}
&\text{D2: } \min_{x} a^T x \\
&\text{subject to } \sum_{i=1}^{M(b+1)+N} x_i \Psi_{m,i} \preceq 0, \ m \in \mathcal{M}, \\
&\quad x \succeq 0, \ b^T x \leq 1
\end{align*}
\] (4.18a)

where \(x_i\) is the \(i\)-th entry of the vector \(x \triangleq [\alpha^T, \lambda^T, \mu^T]^T\), \(a \triangleq [-\sigma_d^2 1_{M \times 1}, P_r 1_{N \times 1}, 0_{Mb \times 1}]^T\), \(b \triangleq [0^T_{(M+N) \times 1}, 1_{Mb \times 1}]^T\), \(\Psi_{m,m} = \frac{\alpha_m P_m f_m^H f_m}{\gamma_m} - G_m\), \(\Psi_{m,M+i} = -R_m D_i\) for \(i \in \mathcal{N}\), \(\Psi_{m,M+N+(m-1)b+j} = -B_{m,j}\) for \(m \in \mathcal{M}\), \(j \in \mathcal{B}\), and all other \(\Psi\) are zeros.

**Proof.** It is not difficult to show that D1 is equivalent to D2, and (4.16a) and (4.17b) are equivalent to (4.18b). Then, substituting (4.15) into (4.17a) and after some manipulation, (4.18a) is obtained.

Note that standard interior point-based solvers, e.g., CVX, could be used to solve D2 efficiently [92]. Then, depending on the values of the optimal dual variables \(\{\alpha^o, \lambda^o, \mu^o\}\), we identify three cases to obtain the optimal beam vectors \(\{w_m^o\}\). We first investigate a useful property of the constraint (4.17a) in the following lemma.

**Lemma 2.** If either \(\mu_{m,j}^o > 0\) for some \(\{m,j\}\) or \(\lambda^o > 0\), then \(\alpha_{m}^o > 0\), i.e., the Lagrange dual variable associated with the SNR requirement at destination \(m\) is strictly positive.
Proof. See Appendix B.

Recall that $\mu_{m,j}$, $\lambda^o$, and $\alpha^o_m$ are the optimal dual variables corresponding to the interference constraint (4.12a), per-relay power constraint (4.11a), and SNR constraint (4.11b), respectively. Due to Proposition 5, the KKT conditions for $P_1$ are satisfied. Hence, the complementary slackness condition holds. According to Lemma 2, if the interference constraint over subchannel $m$ is active at optimality, i.e., attained with equality, or the per-relay power constraint is active for each relay, then the SNR constraint for S-D pair $m$ is also active at optimality.

4.2.3 The Optimal Beam Vector $\{w_m^o\}$

Using Lemma 2, we classify the optimal dual variables $\{\lambda^o, \mu^o_m, \alpha^o_m\}$ into three cases to obtain $\{w_m\}$.

Case 1

$\mu^o = 0$. In this case, we show that the min-max interference problem $P_1$ is infeasible. Suppose the per-relay power constraint (4.11a) and minimum SNR requirement (4.11b) could be satisfied for every S-D pair $m$ and relay $i$, i.e., the original problem $P_1$ is feasible. Since the objective of $P_1$ clearly is sensitive to changes in the RHS of (4.12a), $\mu^o = 0$ implies that (4.12a) is inactive for all $m$ and $j$. Then the optimal objective $T_{\text{max}}^o$ is strictly greater than $\hat{T} \triangleq \max_{m \in M, j \in B} w_m^o H B_{m,j} w_m^o$ at optimality. However, $T_{\text{max}}^o$ can be replaced by $\hat{T}$ resulting in a smaller objective while satisfying all the constraints which is a contradiction. In this case, the only possible conclusion is that the min-max interference problem $P_1$ is infeasible. Hence, if $P_1$ is feasible, there should be at least one $\{m, j\}$ such that (4.12a) is active at optimality, i.e., $\mu^o_{m,j} > 0$.

Case 2

$\mu^o_m \neq 0$ for all $m$ or $\lambda^o > 0$. According to Lemma 2, we have $\alpha^o > 0$ in $D_1$, i.e., if $K_m^o - \alpha^o_m G_m > 0$, then $\alpha^o_m > 0$ for all $m \in M$, and the solution is given by the following proposition.

Proposition 7. Suppose $\alpha^o > 0$. The optimum beam vector $w_m^o$ of the min-max interference problem $P_1$ for $m \in M$ is given by

$$w_m^o = \zeta_m K_m^o f_m$$

(4.19)

where

$$\zeta_m \triangleq \sigma_d \left[ \frac{p_m}{\gamma_m} |f_m^H K_m^o f_m|^2 - |f_m^H K_m^o G_m K_m^o f_m|^2 \right]^{-\frac{1}{2}}$$

(4.20)

and $K_m^o$ is obtained by substituting the optimum dual variables $\{\lambda^o, \mu^o_m, \alpha^o_m\}$ into (4.15).

Proof. See Appendix C.

The following corollary provides the structure for the optimal value of $P_1$ as a function of the optimal dual variables.

\footnote{Note that for Cases 2 and 3, it is implicitly assumed $\mu^o \neq 0$.}
Corollary 2. The maximum received interference of P1 is given by

\[ T_{\text{max}} = \sum_{m=1}^{M} \alpha_m^o \sigma_d^2 - P_r(\sum_{i=1}^{N} \lambda_i^o) \]

\[ = \sigma_d^2 \sum_{m=1}^{M} \frac{\gamma_m}{P_m f_m^o K_m^o - f_m} - P_r(\sum_{i=1}^{N} \lambda_i^o). \]  

(4.21)

Proof. The first equality follows from Proposition 5 due to the zero duality gap. According to the proof in Appendix C, \( \alpha \) is equivalent to solving the following feasibility problem: 

\[ \{w_m, m \in \mathcal{M}\} \] 

subject to \( w_m^H B_m, j w_m \leq T_{\text{max}}, m \in \mathcal{M}, j \in \mathcal{B}, \) \n
(4.22a)

\[ P_{\tilde{m}, i} + \sum_{M_{\tilde{m}}} w_m^H R_m D_i w_m \leq P_r, \ i \in \mathcal{N}, \]  

(4.22b)

\[ \frac{P_m w_m^H F_m w_m}{w_m^H G_m w_m + \sigma_d^2} \geq \gamma_m, m \in \mathcal{M}_{\tilde{m}}. \]  

(4.22c)

Case 3

\( \mu^o \neq 0, \mu_m^o = 0 \) for some \( m \), and \( \lambda^o \neq 0 \). Using Lemma 2, we have \( \alpha_m^o = 0 \) for some \( m \). In the following, we first consider the case where only one entry in \( \alpha^o \) is strictly positive. In other words, only one S-D pair meets the SNR requirement with equality. Later, we will generalize the solution to the case where multiple entries in \( \alpha^o \) are positive. Denote \( \tilde{m} \) such that \( \alpha_m^o > 0 \) and \( \alpha_m^o = 0 \) for \( m \neq \tilde{m} \).

Following the proof in Appendix C, we can show that \( \alpha_m^o, \mu_m^o f_m^o K_m^o - f_m = 1 \) in Case 2 for \( m \in \mathcal{M} \). Substituting \( \alpha_m^o \) into the objective of D1, the second equality in (4.21) is derived.

\( \Box \)

Denote \( \mathcal{M}_{\tilde{m}} \triangleq \mathcal{M} \setminus \{\tilde{m}\} \). Using the fact that \( \alpha_m^o = 0 \) for \( m \in \mathcal{M}_{\tilde{m}} \) and Proposition 5, we see that the optimal objective of P1 is \( T_{\text{max}} = \alpha_m^o \sigma_d^2 - P_r(\sum_{N} \lambda_i^o) \). Further define \( P_{\tilde{m}, i} \triangleq w_m^H R_m D_i w_m \) as the power usage at relay \( i \) over subchannel \( \tilde{m} \). Then, obtaining the optimal beam vectors \( \{w_m, m \in \mathcal{M}_{\tilde{m}}\} \) is equivalent to solving the following feasibility problem:

\[ \text{P2: find } \{w_m, m \in \mathcal{M}_{\tilde{m}}\} \]

subject to \( w_m^H B_m, j w_m \leq T_{\text{max}}, m \in \mathcal{M}, j \in \mathcal{B}, \) \n
(4.22a)

\[ P_{\tilde{m}, i} + \sum_{M_{\tilde{m}}} w_m^H R_m D_i w_m \leq P_r, \ i \in \mathcal{N}, \]  

(4.22b)

\[ \frac{P_m w_m^H F_m w_m}{w_m^H G_m w_m + \sigma_d^2} \geq \gamma_m, m \in \mathcal{M}_{\tilde{m}}. \]  

(4.22c)

Note that the solution to P2 is not unique. This is because the SNR constraint (4.22c) may not be active at optimality for \( m \in \mathcal{M}_{\tilde{m}} \) since \( \alpha_m^o = 0 \). However, we can always scale \( w_m \) such that (4.22c) meets with equality for \( m \in \mathcal{M}_{\tilde{m}} \) while satisfying the max interference constraint (4.22a) and per-relay power constraint (4.22b). In what follows, we provide the details of using this approach to find a solution to P2.

Since the optimal beam vector \( w_{m_{\tilde{m}}} \) is already obtained, we can reduce \( P_{\tilde{m}, i} \) from the maximum per-relay power target \( P_r \) to find the maximum available power that can be used over other subchannels. This motivates the following interference minimization problem by excluding S-D pair \( \tilde{m} \) from consideration.
and limiting the power usage on each relay based on the new maximum power target, \( i.e., \)

\[
P_3: \min_{\{w_m, m \in M_{\tilde{m}}\}} \sum_{m \in M_{\tilde{m}}} w_m \tilde{I}, \quad m \in M_{\tilde{m}}, \quad j \in B, \quad \tilde{I}
\]

subject to

\[
\sum_{m \in M_{\tilde{m}}} w_m^H B_{m,j} w_m \leq \tilde{I}, \quad m \in M_{\tilde{m}}, \quad j \in B, \quad (4.23a)
\]

\[
\sum_{m \in M_{\tilde{m}}} w_m^H R_m D_i w_m \leq P_r - P_{\tilde{m},i}, \quad i \in N, \quad (4.23b)
\]

and (4.22c).

Similar to Proposition 5, we can show that zero duality gap holds for \( P_3 \). We can reformulate the Lagrange dual problem of \( P_3 \) into an SDP as follows:

\[
D_3: \min_{x} c^T x
\]

subject to

\[
\sum_{i=1}^{M(b+1)+N} x_i \Psi_{m,i} \preceq 0, \quad m \in M_{\tilde{m}}, \quad (4.24a)
\]

\[
x \succeq 0, \quad d^T x \leq 1 \quad (4.24b)
\]

where \( x \) is as defined in \( D_2 \); \( c \) is defined similarly to \( a \) in \( D_2 \) except that the entries \( a_{(M+1):M+N} \), and \( a_{(M+N+(\tilde{m}-1)b+1):(M+N+\tilde{m}b)} \) are zero, \( [P_r - P_{\tilde{m},1}, \ldots, P_r - P_{\tilde{m},N}]^T \), and zero, respectively; and \( d \) is defined similarly to \( b \) in \( D_2 \) except that the entries \( b_{(M+N+(\tilde{m}-1)b+1):(M+N+\tilde{m}b)} \) are zero. Thus, the essence of the proposed algorithm is to eliminate the terms associated with S-D pair \( \tilde{m} \) in both the objective and the constraints of \( D_3 \) such that the per-relay maximum power targets are updated.

In order to obtain \( \{w_m, m \in M_{\tilde{m}}\} \), the above procedure is repeated to update the values of \( \{\alpha_{m}^o, m \in M_{\tilde{m}}\} \) through solving \( D_3 \). If \( \alpha_{m}^o > 0 \) for all \( m \in M_{\tilde{m}} \), then we can obtain \( \{w_m, m \in M_{\tilde{m}}\} \) similar to Case 2. Otherwise, the steps to find the solution in Case 3 are repeated. As an example, after solving SDP problem \( D_3 \), suppose Case 3 happens, \( i.e., \alpha_{m'} > 0 \) for some \( m' \in M_{\tilde{m}} \) (the SNR constraint (4.22c) is active for S-D pair \( m' \)). Following the proof in Appendix C, we can find \( w_m \) for a similar structure as in (4.19) through substituting the optimal dual variables given by \( D_3 \) into (4.15). As long as \( P_1 \) is feasible, this procedure can be repeated until \( w_m \) for all \( m \) are found.

So far, we have assumed only one entry in \( \alpha^o \) is strictly positive in the solution to the dual problem \( D_1 \). We can extend our algorithm to the general case where the number of positive entries in \( \alpha^o \) is arbitrary. Define \( P_\alpha \triangleq \{m \mid \alpha_{m}^o > 0\} \). Using Proposition 7, we can obtain the optimal beam vector \( w_m^o \) for \( m \in P_\alpha \) with a similar expression as in (4.19). To obtain \( w_m^o \) for \( m \in M \setminus P_\alpha \), we can solve a feasibility problem similar to \( P_2 \). We can show zero duality gap holds and formulate the dual problem into an SDP similar to \( D_3 \) through updating \( c, d, \) and \( \Psi_{m,i} \) according to \( P_\alpha \).

### 4.2.4 Summary of Algorithm

The steps proposed to solve the min-max interference problem \( P_1 \) are summarized in Algorithm 3.\footnote{Algorithm 3 requires at most \( M - 1 \) iterations to complete.} We can further obtain a necessary and sufficient condition for the feasibility of \( P_1 \) since both Cases 2 and 3 lead to a solution with the semi-closed-form structure in (4.19). Note that for \( \zeta_m \) in (4.20) to be real, the expression in RHS of (4.20) should be strictly positive. Furthermore, substituting (4.19) into
Algorithm 3 Minimizing the maximum interference

1: Check the feasibility condition (4.13).
2: Solve the SDP problem D2 finding the optimal dual variables \( \{\alpha^o, \mu^o, \lambda^o\} \).
3: Obtain \( \mathcal{P}_\alpha = \{ m \mid \alpha^o_m > 0 \} \).
4: Set \( \Pi = \mathcal{P}_\alpha \).
5: while \( \mathcal{P}_\alpha \neq \mathcal{M} \) do
6: Compute \( \mathbf{K}^o_m \) (4.15) and find \( \mathbf{w}^o_m \) (4.19) for all \( m \in \Pi \).
7: Update available power at each relay, \( \mathbf{c} \) and \( \mathbf{d} \).
8: Solve D3 finding \( \Pi = \{ l \in \mathcal{M} \setminus \mathcal{P}_\alpha \mid \alpha^o_l > 0 \} \).
9: Update \( \mathcal{P}_\alpha = \mathcal{P}_\alpha \cup \Pi \).
10: end while
11: Compute \( \mathbf{K}^o_m \) (4.15) and find \( \mathbf{w}^o_m \) (4.19) for all \( m \in \Pi \).

(4.11a), the per-relay power usage should not exceed the maximum target \( P_r \). As a result, the necessary and sufficient conditions for feasibility of P1 is as follows.

**Corollary 3.** P1 is feasible if and only if there exists \( \alpha \succeq 0, \lambda \succeq 0, \mu \succeq 0 \) with \( \sum_{m \in \mathcal{M}} \sum_{j \in \mathcal{B}} \mu_{m,j} \leq 1 \) such that

\[
\min_{m \in \mathcal{M}} \frac{P_m}{\gamma_m} |f_m^H \mathbf{K}^\dagger_m f_m|^2 - f_m^H \mathbf{K}^\dagger_m \mathbf{G}_m \mathbf{K}^\dagger_m f_m > 0, \quad (4.25)
\]

\[
\max_{i \in \mathcal{N}} \sum_{m \in \mathcal{M}} \zeta_i^2 f_m^H \mathbf{K}^\dagger_m \mathbf{R}_m \mathbf{D}_i \mathbf{K}^\dagger_m f_m \leq P_r. \quad (4.26)
\]

4.2.5 Complexity Analysis

To determine the complexity of our proposed algorithm, note that P0 has been converted to an SDP D2 with \( M(b+1) + N \) variables and \( M \) linear matrix inequality constraints of size \( N \). Typically there are only a few neighboring cells with dominant interference, so \( b \) is a small number. The SDP can be solved efficiently using interior-point methods. Based on the complexity analysis for the standard SDP form in [94, Section 5], the computation complexity per iteration to solve the SDP D2 is \( \mathcal{O}((M+N)^2 M N^2) \). The number of iterations to solve an SDP is typically between 5 to 50 regardless of problem size [94, Section 5]. Thus, the complexity to solve the SDP is \( \mathcal{O}((M+N)^2 M N^2) \).

The overall computation complexity to solve P0 depends on the values of the optimal dual variables. As shown in Section 4.2.3, if Case 2 happens, only one SDP problem D2 is solved, i.e., the complexity is given by \( \mathcal{O}((M+N)^2 M N^2) \). If Case 3 happens, at most \( M \) SDP problems formulated as D3 are solved, i.e., the worst-case complexity is given by \( \mathcal{O}((M+N)^2 M^2 N^2) \). In both cases, the algorithm has a polynomial worst-case complexity w.r.t. the number of relays and S-D pairs. Note that the above analysis is based on worst-case complexity estimates. In practice, the complexity is much lower than the worst-case estimate [94, Section 5].

As shown in Appendix B, we can also reformulate P1 into an SOCP problem (A.2) given in Appendix B. It has \( MN+1 \) variables and \( M(b+1)+N \) constraints. This SOCP can be directly solved using interior-point methods with the complexity per iteration of \( \mathcal{O}((M+N)^3 M^2 N) \). The number of iterations to solve an SOCP does not depend on the problem size [102]. Thus, the complexity of the SOCP compared with the worst-case complexity of our proposed algorithm (i.e., the maximum number of iterations in Case 3 of Section 4.2.3) is increased by a factor of \( \mathcal{O}(MN/(M+N)) \). We note that \( MN \) is typically much larger than \( M + N \). Therefore, our proposed algorithm to obtain the optimal solution offers much
lower complexity than the SOCP method.

### 4.3 SNR and SINR as Objectives

We can use the method presented in Section 4.2 to maximize the minimum received SNR or SINR subject to per-relay power constraint and a maximum interference limit at the neighboring cells.

#### 4.3.1 Maximizing the Minimum SNR

In the following, we first formulate the max-min SNR problem and show that this problem and P1 are inverse problems. Then using the bisection search, an iterative algorithm is proposed to solve the max-min SNR problem.

Typically the relays have the same front-end amplifiers and the destinations have the same minimum SNR requirements and received interference threshold. In the following, we assume identical per-relay power budget, minimum SNR requirements for destinations in the desired cell, and maximum interference limit for destinations in the neighboring cells. Extension to the case of non-uniform power, SNR, and interference requirement can follow a similar approach.

The problem of maximizing the minimum SNR under a pre-determined maximum interference threshold, $I_0$, is formally stated as follows:

\[
\text{P4: } \max \{\gamma\} \quad \text{subject to} \quad w_m^H \tilde{B}_{m,j} w_m \leq I_0, \ m \in \mathcal{M}, \ j \in \mathcal{B}, \quad (4.27a) \\
\text{SNR}_m \geq \gamma, \ m \in \mathcal{M}, \quad (4.27b) \\
\text{and (4.11a)}
\]

where $I_0$ denotes a pre-determined maximum interference threshold.

The min-max interference problem P1 with a common SNR target $\gamma_0$ is given by

\[
\text{P5: } \min \{I\} \quad \text{subject to} \quad w_m^H \tilde{B}_{m,j} w_m \leq I, \ m \in \mathcal{M}, \ j \in \mathcal{B}, \quad (4.28a) \\
\text{SNR}_m \geq \gamma_0, \ m \in \mathcal{M}, \quad (4.28b) \\
\text{and (4.11a)}.
\]

We denote the optimal objective of P4 and P5 as $\gamma^o(I_0)$ and $I^o(\gamma_0)$ to focus on their dependencies on $I_0$ and $\gamma_0$, respectively. We first study the optimal maximum SNR $\gamma^o(I_0)$ as a function of $I_0$ following the similar arguments in Chapter 3.

The optimal objective $\gamma^o(I_0)$ is continuous and strictly monotonically increasing function of $I_0$; for given $I_0$ any $\gamma < \gamma^o(I_0)$ is achievable.

Hence, for any $\gamma_0$, the minimum interference $I_0$ is obtained when $\gamma^o(I_0) = \gamma_0$, i.e., $I^o(\gamma^o(I_0)) = I_0$. This indicates that P4 and P5 are inverse problems, i.e.,

\[
I^o(\gamma^o(I_0)) = I_0, \ \gamma^o(I^o(\gamma_0)) = \gamma_0.
\]
Algorithm 4 Maximizing the minimum SNR

1: Set convergence threshold $\delta > 0$.
2: Find $\gamma_{0,\min}$ such that $I_o(\gamma_{0,\min}) < I_0$.
3: Find $\gamma_{0,\max}$ such that $I_o(\gamma_{0,\max}) > I_0$.
4: Set $\gamma_0 = \frac{\gamma_{0,\min} + \gamma_{0,\max}}{2}$.
5: Solve P5 under $\gamma_0$.
6: if $I_o(\gamma_0) > I_0$ then
7: Set $\gamma_{0,\max} = \gamma_0$ and $I = I_o(\gamma_0)$.
8: else
9: Set $\gamma_{0,\min} = \gamma_0$ and $I = I_o(\gamma_0)$.
10: end if
11: while $I < I_0 - \delta$ do
12: $\text{(4)}$–$\text{(10)}$.
13: end while
14: Return $\gamma_0$.

The solution for P4 can be obtained by iteratively solving the min-max interference problem P5 with bisection search on the max interference threshold $I$. The stopping criterion is when $I \rightarrow I_0$.

In Algorithm 4, we provide the steps to solve P4 using P5 and bisection.\(^6\)

### 4.3.2 Maximizing the Worst-Case Received SINR

Since the performance measures for cellular networks, e.g., data rate or BER, are direct functions of the received SINR, our ultimate goal is to maximize the worst-case SINR at the destinations. For many practical scenarios, we make local decisions in each cell. Without jointly optimizing all cells, each cell optimizes its own resources in a distributed way based on some messages passed between cells over the backhaul.

The received SINR at destination $m$ in the desired cell is given by $\text{SINR}_m = \frac{P_{S,m} I_m}{I_m + P_{N,m}} = \frac{\text{SNR}_m}{I_m + P_{N,m} + 1}$, where $I_m$ denotes the total interference at destination $m$. It can be determined by solving the max-min SNR problem P4, if we constrain the interference from each neighboring cell to be below a given value $I_0$ such that $I_m \leq b I_0$, for all $m \in \mathcal{M}$. Thus, we propose solving P4 under different values of $I_0$. Then, an optimal $I_0$ value can be chosen to maximize the worst-case SINR among all destinations in a cell, given by $\min_m \text{SINR}_m$. Intuitively, when $I_0$ is too low, SINR at the destination is noise dominant. Also, low $I_0$ limits the relay power, and the received signal power at the intended user is low as well, resulting in low SINR. As $I_0$ increases, SINR increases due to power (and beamforming) gain. As $I_0$ continues increasing, the interference becomes dominant over the gain received by relay beamforming, and SINR decreases. The $I_0$ values that maximize the worst-case SINR are illustrated in Section 4.4.

### 4.4 Numerical Study

In this section, we applied the proposed min-max interference algorithm in various simulation settings. We are mainly interested in how the maximum interference and the worst-case SINR behave under different system parameter values, i.e., different number of relays, S-D pairs, and neighboring cells. We set $\sigma_r^2 = \sigma_d^2 = 1$, $P_m = P_0$ for $m \in \mathcal{M}$ with $P_0/\sigma_r^2 = 10$ dB, and $P_r/\sigma_d^2 = 20$ dB. The minimum SNR\(^6\)

---

\(^6\)The values of $\gamma_{0,\min}$ and $\gamma_{0,\max}$ can be set based on the typical range of SNRs in a particular application. For simplicity, we can always set $\gamma_{0,\min} = 0$ and $\gamma_{0,\max} = \max_m P_m G_m^H f_m$. 

\[^{6}\text{The values of } \gamma_{0,\min} \text{ and } \gamma_{0,\max} \text{ can be set based on the typical range of SNRs in a particular application. For simplicity, we can always set } \gamma_{0,\min} = 0 \text{ and } \gamma_{0,\max} = \max_m P_m G_m^H f_m.\]
target is set to $\gamma_m = \gamma_0 = 5$ dB for $m \in M$. The first and second hop channels $h_m$ and $\{g_m, \tilde{g}_{m,j}\}$ are assumed i.i.d. zero-mean Gaussian with variance 1. This model essentially captures the worst-case interference scenario, where the distance from relays to cell-edge users at the neighboring cells is similar to that between the relays and destinations, causing strong interference. We further study the effect of imperfect interference CSI in Section 4.4.4, and consider the scenario of random user locations in Section 4.4.5.

### 4.4.1 Effect of the Number of Relays

To study the behavior of maximum interference as the number of relays $N$ increases, we plot the CDF of $I_{\text{max}}$ in the objective of P1, normalized against noise variance $\sigma_d^2$, with $M = 2$ and $b = 1$ in Fig. 4.2. Also shown in Fig. 4.2 is the maximum interference under an alternate optimization problem where the objective is to minimize the maximum transmission power over all relays while meeting the minimum SNR requirements as in Chapter 3. This min-max relay power problem may be viewed as a simpler alternative to reduce interference, which is created by the relays. The number of relays are chosen as $N = 2^i$ for $i \in \{0, \cdots, 5\}$. We see that as $N$ increases, the interference CDF curves are shifted to the left for both optimization approaches. Note that the min-max interference approach significantly
outperforms the per-relay power approach for each $N$, and the performance gap increases as $N$ increases.

The average received noise power (4.7) versus average normalized $\mathcal{I}_\text{max}$ with $N = 2, 4, 8, M = 2$, and $b = 4$ is shown in Fig. 4.3. It can be seen that noise power increases as $N$ increases. Note that received noise is the total amplified noise and AWGN at the destination. The amplified noise decreases to zero as $N$ increases, and the overall noise converges to the destination noise, i.e., 0 dB. This happens as the beam vector norm $\|w_m\|$ decreases as $N$ increases due to the power (and beamforming) gain achieved by relay beamforming.

To evaluate the performance of the max-min SNR problem P4, in Fig. 4.4, the average received worst-case SINR, i.e., $\min_m \text{SINR}_m$ versus average normalized $\mathcal{I}_\text{max}$ is represented with $b = 2$, $M = 8$, and $N = 2^i$ for $i \in \{1, \cdots, 4\}$. To plot each curve, the minimum SNR requirement $\gamma_0$ is set to $-10 \text{ dB}$ to $24 \text{ dB}$. For each $\gamma_0$, 500 realizations are generated, and then $\mathcal{I}_\text{max}$ and $\min_m \text{SINR}_m$ are computed for each realization. As discussed at the end of Section 4.3.2, we see that $\min_m \text{SINR}_m$ first increases and then decreases as a function of $\mathcal{I}_\text{max}$. Hence, we can numerically identify the maximum $\min_m \text{SINR}_m$ for each $N$.

In Fig. 4.5, the true $\min_m \text{SINR}_m$ is compared with the SINR lower bound with $N = 2, 4, 8$ and
4.4.2 Effect of the Number of S-D pairs

In Fig. 4.6, the average min\(_m\) SINR\(_m\) versus average normalized I\(_{\text{max}}\) is shown for N = 4, and M = 2\(^i\) for i ∈ \{1, · · · , 4\}, and b = 2. For each curve, a maximum min\(_m\) SINR\(_m\) is observed. We see that min\(_m\) SINR\(_m\) decreases as M increases because the number of SNR constraints in each cell increases. Hence, the relays increase transmission power in each cell and generate more interference at the neighboring cells.

4.4.3 Effect of the Number of Neighboring Cells

In Fig. 4.7, the average min\(_m\) SINR\(_m\) versus average normalized I\(_{\text{max}}\) is shown for N = 4, M = 8, and b ∈ \{1, 2, 4, 6\}. A maximum min\(_m\) SINR\(_m\) for each curve is identified. For fixed average I\(_{\text{max}}\), increasing
b leads to degradation on min$_m$ SINR$_m$. As b increases, the number of interference sources corresponding to each subchannel increases. The total received interference increases and hence, min$_m$ SINR$_m$ decreases as b increases.

4.4.4 Effect of Imperfect CSI

So far, true interference CSI is assumed to be known perfectly at the relays. In practice, obtaining such interference CSI may not be possible. In order to observe how robust the proposed algorithm is w.r.t. imperfect CSI, we consider the following scenarios with two types of imperfect CSI: limited number of CSI feedback bits and channel estimation error.

In Scenario 1, the receiver knows the interference CSI perfectly. However, the feedback bits to the relays are limited. We consider equiprobable quantization of channel values. Let B denote the number of available feedback bits. Every real and imaginary part of a channel is quantized with equal probability according to the CSI distribution, which is complex Gaussian.

In Scenario 2, the channels are estimated at the receiver with error and the estimated channel is fed back to the relays. In order to model the channel estimation error, let us define $\hat{h} = h + \alpha \tilde{h}$, where $h$ is the true channel, $\hat{h}$ is the estimated channel used for optimization, $\tilde{h} \sim \mathcal{CN}(0, 1)$, and the weight $\alpha$ is set to adjust the variance of error w.r.t. the variance of perfect CSI.
Figure 4.10: Average min\_m SINR\_m under limited feedback (Scenario 1) with $M = 2$ and $b = 4$.

Figure 4.11: CDF of normalized $I_{max}$ when $M = 2$ and $b = 1$.

In Fig. 4.8, the CDF of normalized $I_{max}$ under true interference CSI is compared with that of the imperfect CSI in Scenario 1 with 6 feedback bits (3 bits for each real and imaginary parts). It can be seen the interference in this limited feedback scenario is very close to the perfect CSI case even when the number of relays is large (e.g., $N = 8$). As expected, the performance gap between the limited feedback scenario and perfect CSI case increases as $N$ increases. In addition, the min-max interference approach under limited feedback still substantially outperforms the min-max per-relay power approach.

Fig. 4.9 shows the CDF of normalized $I_{max}$ under imperfect CSI in Scenario 2 with the channel estimation error being $\alpha = 0.01$. The interference in this case is close to that of perfect CSI. In addition, we see that the performance degradation increases as $N$ increases. Again, the min-max interference approach under imperfect CSI in Scenario 2 outperforms the min-max per-relay approach.

The average min\_m SINR\_m versus average normalized $I_{max}$ compared with that of the imperfect CSI in Scenario 1 with 4 feedback bits is shown in Fig. 4.10 for $N = \{2, 4, 8\}$, $M = 2$, and $b = 4$. The performance degradation due to limited feedback increases as $N$ increases. Note that $I_{max}$ corresponding to the maximum min\_m SINR\_m decreases as $N$ increases, reflecting higher diversity gain attained with more relays through achieving smaller $I_{max}$.
Figure 4.12: Average min\textsubscript{m} SINR\textsubscript{m} versus average normalized $I_{\text{max}}$ for $M = 8$ and $b = 2$.

4.4.5 Performance under Random Relay and User Locations

Previous simulation setup has captured the worse-case interference scenario, by assuming i.i.d. channel distribution for all relays and users. If some relays and destinations are far away from the cell edge, or users in neighboring cells are away from the cell edge, the ICI caused to these neighboring users is low and less critical.

We now study the pattern of the maximum interference in a scenario with random user and relay locations. We set the distance between each source and relay, relay and destination in the desired cell, and relay and destination in the neighboring cells by $\kappa R$, where $R$ is the cell radius and $\kappa$ is a random variable with uniform distribution in the range $[0, 0.5]$, $[0.5, 1]$, and $[1, 1.5]$, respectively. The channel over each link is generated as zero-mean Gaussian with variance using the distance-based pathloss. We assume the path loss exponent is 3.

Similar to Fig. 4.2, we plot the CDF of normalized $I_{\text{max}}$ with $M = 2$, $b = 1$, and increasing $N$ in Fig. 4.11, where we compared $I_{\text{max}}$ under our solution for P1 with that under the per-relay power objective. As expected, based on the above discussion, comparing Fig. 4.11 with Fig. 4.2, we observe that the interference CDF is shifted to the left, indicating a smaller $I_{\text{max}}$ as the the users are randomly located. However, the general trend remains the same.

Similar to Fig. 4.4, we also plot the average min\textsubscript{m} SINR\textsubscript{m} versus average normalized $I_{\text{max}}$ with $b = 2$, $M = 8$, and increasing $N$ in Fig. 4.12, where we evaluate the performance of the max-min SNR problem P4. As expected, we see that min\textsubscript{m} SINR\textsubscript{m} first increases and then decreases as a function of $I_{\text{max}}$. Comparing Fig. 4.12 with Fig. 4.4, we observe that the average received worst-case SINR increases. It verifies that the $I_{\text{max}}$ decreases for random user locations, but again the general trend remains the same.

4.5 Summary

In this chapter, we have considered a multi-relay cellular network, where each cell has multiple S-D pairs communicating in orthogonal channels with assistance from the relays. In order to manage ICI, we have formulated the min-max interference problem under per-relay power constraints and minimum SNR requirements. We have shown that the strong duality property holds for this non-convex problem. Solving the Lagrange dual problem, three cases have been identified based on the optimal dual variables.
We then propose an iterative algorithm to obtain the optimal beam vectors in semi-closed-form expressions. Numerical results have shown 10 dB reduction in the maximum interference with 4 relays for the min-max interference approach over the per-relay power approach, while the performance degradation when only 6 CSI feedback bits are used is within 3 dB. We have also solved the max-min SNR problem, under maximum interference and per-relay power constraints, using bisection search. Under different problem setups, we have evaluated the maximum interference and the corresponding worst-case received SINR. A maximum worst-case SINR has been observed as we vary the maximum interference target, which provides insight into designing relay beamforming in a multi-cell network.

Appendix A

Proof of Proposition 5

Proof. The interference constraint (4.12a) and per-relay power constraint (4.11a) are convex w.r.t. \( w \in [w_1^T, \cdots, w_M^T]^T \). However, the minimum received SNR constraint (4.11b) is non-convex. Reformulating the SNR constraint (4.11b) in a conic form, we have

\[
\sqrt{P_m |w_m^H f_m|} \geq \sqrt{\gamma_m \left\| \begin{bmatrix} G_m^{1/2} w_m \\ \sigma_d \end{bmatrix} \right\|}, \quad m \in \mathcal{M}.
\] (A.1)

Note that \( w_m \) can have any arbitrary phase, i.e., it is obtained uniquely up to a phase shift. The phase could be adjusted such that \( w_m^H f_m \) becomes real-valued for \( m \in \mathcal{M} \). The min-max interference problem P1 can be recast as

\[
\min_{w_1, \cdots, w_M, I_{max}} I_{max}
\] (A.2)

subject to (A.1), (4.11a), and (4.12a).

The primal-dual optimality conditions for the problems with constraints in the form of (A.1) are provided in [99, Proposition 3]. Following a similar proof, it can be shown that (A.2) has zero duality gap with its Lagrangian dual. To prove Proposition 5, we are left to show that the Lagrangian of P1 is the same as the Lagrangian of (A.2) by using a similar proof as in [90, Proposition 1].

The Lagrangian of P1 is given by

\[
L = I_{\text{max}} + \sum_{m=1}^{M} \sum_{j=1}^{b} \mu_{m,j} \left( w_{m,j}^H \tilde{B}_{m,j} w_m - I_{\text{max}} \right) + \sum_{i=1}^{N} \lambda_i \left( \sum_{m=1}^{M} w_{m,i}^H R_m D_i w_m - P_r \right) + \sum_{m=1}^{M} \alpha_m \left( \sigma_d^2 + w_m^H G_m w_m - \frac{P_m}{\gamma_m} |w_m^H f_m|^2 \right).
\] (A.3)
The Lagrangian of (A.2) is obtained by

\[
\hat{L} = I_{\text{max}} + \sum_{m=1}^{M} \sum_{j=1}^{b} \hat{\mu}_{m,j} (w_m^H \tilde{B}_{m,j} w_m - I_{\text{max}}) + \sum_{i=1}^{N} \hat{\lambda}_i (\sum_{m=1}^{M} w_m^H R_m D_i w_m - P_r) + \sum_{m=1}^{M} \hat{\alpha}_m \left( \left\| \frac{G_m^{1/2}}{\sigma_d} w_m \right\| - \sqrt{\frac{P_m}{\gamma_m} |w_m^H f_m|} \right). \tag{A.4}
\]

Denoting \( u_m \triangleq \left\| \frac{G_m^{1/2}}{\sigma_d} w_m \right\| + \sqrt{\frac{P_m}{\gamma_m} |w_m^H f_m|} \geq \sigma_d \) and converting the last term of the Lagrangian (A.4), it is equivalent to

\[
\hat{L} = I_{\text{max}} + \sum_{m=1}^{M} \sum_{j=1}^{b} \hat{\mu}_{m,j} (w_m^H \tilde{B}_{m,j} w_m - I_{\text{max}}) + N \sum_{i=1}^{\hat{\lambda}_i} (\sum_{m=1}^{M} w_m^H R_m D_i w_m - P_r) + M \sum_{m=1}^{\hat{\alpha}_m} u_m \left( \sigma_d^2 + w_m^H G_m w_m - \frac{P_m}{\gamma_m} |w_m^H f_m|^2 \right). \tag{A.5}
\]

Since \( u_m \geq \sigma_d \), by changing the variables \( \alpha_m = \frac{\hat{\alpha}_m}{\sigma_d} \), there exists \( \alpha_m \geq 0 \) for any \( \hat{\alpha}_m \geq 0 \) and \( m \in \mathcal{M} \) such that (A.3) and (A.4) become exactly the same. As a result, the strong Lagrange duality holds for the non-convex problem P1.

**Appendix B**

**Proof of Lemma 2**

*Proof.* Substituting (4.15) into (4.17a), the constraint (4.17a) is equivalent to

\[
R_m D_A + \sum_{j=1}^{b} \mu_{m,j} \tilde{B}_{m,j} + \alpha_m (G_m - \frac{P_m}{\gamma_m} f_m^H f_m) \succeq 0. \tag{B.1}
\]

Using contradiction, we show that \( G_m - \frac{P_m}{\gamma_m} f_m^H f_m \) is an indefinite matrix. Suppose that \( G_m \geq \frac{P_m}{\gamma_m} f_m^H f_m \). Since \( G_m \) is a positive-definite matrix, we have \( P_m f_m^H G_m^{-1} f_m \leq \gamma_m \) ( [90, Lemma 1]). This contradicts the necessary condition for the feasibility of P1 as shown in Section 4.2.1. If either \( \mu_{m,j}^{\alpha} > 0 \) for some \( \{m, j\} \) or \( \lambda^0 \succ 0 \), there exists \( \alpha_m^{\alpha} > 0 \) such that (4.17a) is satisfied. Note that the objective of the dual problem increases as \( \alpha_m \) increases. If \( \mu_{m,j}^{\alpha} = 0 \) and there exists \( \lambda^0_i = 0 \) for some \( i \), then \( \alpha_m^{\alpha} \) can be zero.
Appendix C

Proof of Proposition 7

Proof. Suppose the necessary condition in Lemma 2 is satisfied for all $m \in M$, i.e., $\alpha^o > 0$. Then we have $K_m^o > 0$ for all $m \in M$. Using [90, Lemma 1] and rewriting the expression of the matrix inequality (4.17a), the dual problem D1 can be expressed as

$$\max_{\lambda, \mu} \max_{\alpha} \sum_{m=1}^{M} \alpha_m^2 \sigma_d^2 - Pr \left( \sum_{i=1}^{N} \lambda_i \right)$$

subject to \( \frac{\alpha_m P_m}{\gamma_m} f_m^H K_m^{-1} f_m \leq 1, \ m \in M \) \hspace{1cm} (C.1)

\( \frac{\alpha_m P_m}{\gamma_m} f_m^H K_m^{-1} f_m \geq 1, \ m \in M \) \hspace{1cm} (4.17b), and (4.16a).

Since the optimal beam vector solution of the SIMO beamforming problem is known, in the following, we establish the duality between (C.1) and SIMO beamforming problem similar to [90]. Considering the dual problem (C.1) and the optimization problem

$$\max_{\lambda, \mu} \min_{\alpha} \sum_{m=1}^{M} \alpha_m^2 \sigma_d^2 - Pr \left( \sum_{i=1}^{N} \lambda_i \right)$$

subject to \( \frac{\alpha_m P_m}{\gamma_m} f_m^H K_m^{-1} f_m \geq 1, \ m \in M \) \hspace{1cm} (C.3)

\( \frac{\alpha_m P_m}{\gamma_m} f_m^H K_m^{-1} f_m \leq 1, \ m \in M \) \hspace{1cm} (4.17b), and (4.16a),

we have the inner maximization in (C.1) becomes minimization in (C.3) and the SNR inequality is reversed. Substituting (4.15) into LHS of (C.2), we define

$$\Phi_m(\alpha_m) = \frac{P_m}{\gamma_m} f_m^H \left( \frac{1}{\alpha_m} (R_m D + \sum_{j \in B} \mu_{m,j} \tilde{B}_{m,j}) + G_m \right) f_m$$

which is a monotonically increasing function of $\alpha_m$. Hence, both (C.2) and (C.4) are met with equality at optimality for a given $\{\lambda^o, \mu^o, \alpha^o\}$ satisfying Lemma 2. Furthermore, problems (C.1) and (C.3) have the same solution $\alpha^o$ satisfying $\Phi_m(\alpha_m) = 1$ for $m \in M$. This implies that the optimization problems (C.1) and (C.3) are equivalent.

Consider the following optimization problem

$$\max_{\lambda, \mu} \min_{w_m, \alpha} \sum_{m=1}^{M} \alpha_m^2 \sigma_d^2 - Pr \left( \sum_{i=1}^{N} \lambda_i \right)$$

subject to \( \alpha_m P_m |w_m f_m|^2 \geq \gamma_m, \ m \in M \) \hspace{1cm} (C.6)

\( \frac{1}{\|K_m^{1/2} w_m\|^2} \geq \gamma_m, \ m \in M \) \hspace{1cm} (4.17b), and (4.16a).

The inner minimization of problem (C.6) is the receive SIMO beamforming for power minimization problem where $M$ receivers each are equipped with $N$ antennas. The transmit power and noise covariance
matrix for receiver \( m \) are \( \sum_{m=1}^{M} \alpha_m \sigma_d^2 - P_r(\sum_{i=1}^{N} \lambda_i) \) and \( \bar{K}_m = \frac{\sum_{m=1}^{M} \alpha_m \sigma_d^2 - P_r(\sum_{i=1}^{N} \lambda_i)}{\alpha_m P_m} \), respectively.

The solution of the SIMO beamforming problem, \( i.e. \), the inner minimization of problem (C.6), is given by \( \bar{w}_m = \bar{K}_m^{-1} f_m \). Substituting \( \bar{w}_m = \frac{\bar{K}_m^{-1} f_m}{P_m} \) into problem (C.6), we have problem (C.3). Note that the optimal \( \bar{w}_m \) can be scaled by any non-zero coefficient \( \xi \) such that \( \xi \bar{w}_m \) is also an optimal solution. Hence, the dual problem D1 is equivalent to the SIMO beamforming problem (C.6), and we can use the solution of (C.6) to obtain \( \bar{w}_m \) in the min-max interference problem P1.

Since P1 has zero duality gap as shown in Proposition 5 and \( \bar{w}_m \) is unique up to a scale factor, the optimal beam vector \( w_m \) is given by \( w_m = \frac{1}{\bar{K}_m} \bar{w}_m \). In order to obtain \( \zeta_m \), note that the SNR constraint (4.11b) is met with equality based on the slackness condition. Substituting \( w_m \) into the equation \( \frac{P_m w_m^H \bar{F}_m w_m}{w_m^H \bar{G}_m w_m + \sigma_d^2} = \gamma_m \) and after some manipulations, (4.20) is obtained and the proof is complete.
Chapter 5

Power Optimization for D2D Systems

For D2D communication underlaid in a cellular network with uplink resource sharing, both cellular and D2D pairs may cause significant ICI at a neighboring BS. In this chapter, under optimal BS receive beamforming, we jointly optimize the power of a CU and a D2D pair for their sum rate maximization, while satisfying minimum SINR requirements and worst-case ICI limit in multiple neighboring cells. We solve this non-convex joint optimization problem in two steps. First, the necessary and sufficient condition for the D2D admissibility under given constraints is obtained. Next, we consider joint power control of the CU and D2D transmitters. We propose a power control algorithm to maximize the sum rate. Depending on the severity of ICI that D2D and CU may cause, we categorize the feasible solution region into five cases, each of which may further include several scenarios based on minimum SINR requirements. The proposed algorithm is optimal when ICI to a single neighboring cell is considered. For multiple neighboring cells, we provide an upper bound on the performance loss by the proposed algorithm and conditions for its optimality. We further extend our consideration to the scenario of multiple CUs and D2D pairs, and formulate the joint power control and CU-D2D matching problem. We show how our proposed solution for one CU and one D2D pair can be utilized to solve this general joint optimization problem. Simulation demonstrates the effectiveness of our power control algorithm and the nearly optimal performance of the proposed approach in the setting of multiple CUs and D2D pairs.

5.1 System Model and Problem Formulation

5.1.1 System Model

We study the underlaying D2D communication in a cellular system, where D2D devices reuse the spectrum resource already assigned to the CUs for their uplink communication. We assume orthogonal spectrum resource allocation among the CUs within a cell. Thus, these CUs do not interfere with each other. When a D2D pair communicates using the channel assigned to a CU, the D2D pair and the CU cause intra-cell interference to each other. We first focus on a single CU and a single D2D pair attempting to reuse the CU’s channel as shown in Fig. 5.1. We assume that all users are equipped with a single
antenna and the BS is equipped with $N$ antennas. The BS centrally coordinates the transmission from the CU and the D2D pair. In Section 5.4, we extend our consideration to the scenario of multiple CUs and D2D pairs.

Let $P_D$ and $P_C$ denote the transmit power of the D2D pair and the CU, respectively. The SINR at the D2D receiver, denoted by $\gamma_D$, is given by

$$\gamma_D = \frac{P_D |h_D|^2}{\sigma_D^2 + P_C |g_C|^2} \quad (5.1)$$

where $h_D \in \mathbb{C}$ is the channel between the D2D pair, $g_C \in \mathbb{C}$ is the interference channel between the CU and the D2D receiver, and $\sigma_D^2$ is the noise variance at the D2D receiver. The uplink received SINR at the BS from the CU is given by

$$\gamma_C = \frac{P_C |w^H h_C|^2}{\sigma^2 + P_D |w^H g_D|^2} \quad (5.2)$$

where $h_C \in \mathbb{C}^{N \times 1}$ is the channel between the CU and the BS, $g_D \in \mathbb{C}^{N \times 1}$ is the interference channel between the D2D transmitter and the BS, $w$ is the receive beam vector at the BS with unit norm, i.e., $\|w\|^2 = 1$, and $\sigma^2$ is the noise variance at the BS.\(^1\)

Both D2D and CU transmissions cause ICI in a neighboring cell. In this chapter, we focus on ICI for uplink transmission at neighboring BSs. However, our approach can also be applied to considering ICI in the downlink transmission scenario. Let $b$ denote the number of neighboring cells. Let $f_{C,i} \in \mathbb{C}^{N \times 1}$ and $f_{D,i} \in \mathbb{C}^{N \times 1}$ denote the ICI channels from the CU and the D2D transmitter to neighboring BS $i$, respectively, for $i = 1, \ldots, b$. Since the beam vector at neighboring BS $i$ is typically unknown to the CU and D2D pair, we consider the worst-case ICI, denoted by $P_{I,i}$, given by

$$P_{I,i} = P_C \|f_{C,i}\|^2 + P_D \|f_{D,i}\|^2. \quad (5.3)$$

Note that $P_I$ is an upper bound of the actual ICI, because for a neighboring BS with the beam vector

\(^1\)The noise term in SINR expressions, i.e., $\sigma^2$ and $\sigma_D^2$, can be treated as the receiver noise plus inter-cell interference power.
\( \tilde{w} \), the received signal is \( |\tilde{w}^H f| \leq \|f\| \).

We assume perfect knowledge of the communication channels and intra-cell interfering channels, i.e., \( h_C, h_D, g_c, \) and \( g_D \). For the ICI channels, note that only channel power gains are needed to obtain \( P_{L,i} \). They can be measured at neighboring BSs and shared with the BS of the desired cell through the backhaul.

### 5.1.2 Problem Formulation

Let \( P_{C}^{\text{max}} \) and \( P_{D}^{\text{max}} \) denote the maximum transmit power at the CU and D2D transmitters, respectively. Our goal is to maximize the sum rate of the D2D pair and the CU by optimizing the transmit powers \( \{P_D, P_C\} \) and the receive beam vector \( \tilde{w} \), while satisfying the worst-case ICI and minimum SINR requirements under per-node power constraints. The problem is formulated as follows:

\[
P_1: \max_{P_D, P_C, \tilde{w}} \log(1 + \gamma_C) + \log(1 + \gamma_D) \tag{5.4}
\]

subject to

\[
\gamma_C \geq \tilde{\gamma}_C, \tag{5.5}
\]

\[
\gamma_D \geq \tilde{\gamma}_D, \tag{5.5}
\]

\[
P_C \leq P_C^{\text{max}}, \quad P_D \leq P_D^{\text{max}}, \tag{5.6}
\]

\[
P_{L,i} \leq \tilde{I}, \quad i = 1, \ldots, b \tag{5.7}
\]

where \( \tilde{\gamma}_C \) and \( \tilde{\gamma}_D \) are the minimum SINR requirements of the CU and D2D pair, respectively, and \( \tilde{I} \) is the worst-case ICI threshold in neighboring cells.

The optimization problem \( P_1 \) is non-convex, due to the non-convex objective function. Solving \( P_1 \) requires two steps. First, we need to determine whether the D2D pair can be admitted to reuse the CU’s channel. Second, if the D2D pair can be admitted, we optimize the powers and beam vector to maximize the sum rate in \( P_1 \). The first problem can be cast as a feasibility test as shown in the next section. For the second problem, we will derive the optimal power solution \( \{P_D^o, P_C^o\} \) when \( b = 1 \). For \( b > 1 \), we will propose an approximate power control algorithm by applying the results obtained for \( b = 1 \).

### 5.2 Admissibility of D2D

Given the power constraints, SINR requirements, and ICI threshold, the admissibility of the D2D pair can be determined by evaluating the feasibility of the problem in \( P_1 \) given by

\[
\text{find } \{P_D, P_C, \tilde{w}\} \tag{5.8}
\]

subject to \((5.4), (5.5), (5.6), (5.7)\).

A necessary condition for \( P_1 \) being feasible is that CU SINR constraint (5.4) under the optimal \( \tilde{w} \) should be met for some \( \{P_D, P_C\} \). For any given \( \{P_C, P_D\} \), we obtain the optimal beam vector \( \tilde{w}^o \) that maximizes \( \gamma_C \). This is a receive beamforming problem given by

\[
\max_{\tilde{w}: \|\tilde{w}\|^2 = 1} \frac{P_C \tilde{w}^H H_C \tilde{w}}{\tilde{w}^H A_D \tilde{w}} \tag{5.9}
\]

\(^2\)Note that the additional signaling overhead due to D2D communication is mainly on the feedback of channels \( \{h_D, g_C\} \) to the BS. These two channels are typically necessary for establishing D2D communication.
where \( \mathbf{H}_C \triangleq \mathbf{h}_C \mathbf{h}_C^H \) and \( \mathbf{A}_D \triangleq \sigma^2 \mathbf{I} + \mathbf{P}_D \mathbf{g}_D \mathbf{g}_D^H \). The maximization problem (5.9) is a generalized eigenvalue problem, and the optimal beam vector is given by

\[
\mathbf{w}^o = \frac{\mathbf{A}_D^{-1} \mathbf{h}_C}{\| \mathbf{A}_D^{-1} \mathbf{h}_C \|}.
\]

(5.10)

The SINR of the CU in (5.2) under the optimal \( \mathbf{w}^o \) is given by

\[
\max_{\mathbf{w}} \gamma_C = P_C \mathbf{h}_C^H \mathbf{A}_D^{-1} \mathbf{h}_C
\]

which is a function of \( \{P_D, P_C\} \).

From the definition of \( \mathbf{A}_D \), and applying the matrix inversion lemma [103], we derive \( \mathbf{A}_D^{-1} \) as

\[
\mathbf{A}_D^{-1} = \frac{1}{\sigma} \left( \mathbf{I} - \frac{\mathbf{P}_D \mathbf{g}_D \mathbf{g}_D^H}{\sigma^2 + \mathbf{P}_D \| \mathbf{g}_D \|^2} \right). \]

Substituting the expression of \( \mathbf{A}_D^{-1} \) into (5.11) and after some algebraic manipulation, the SINR constraint (5.4) under the optimal beamforming can be re-expressed as

\[
\frac{P_C \| \mathbf{h}_C \|^2}{\sigma^2} \left( 1 - \frac{\rho^2}{1 + \frac{\sigma^2}{\mathbf{P}_D \| \mathbf{g}_D \|^2}} \right) \geq \gamma_C
\]

(5.12)

where \( \rho \triangleq \frac{\| \mathbf{h}_C \| \| \mathbf{g}_D \|}{\| \mathbf{h}_C \| \| \mathbf{g}_D \|^2} \) is the correlation coefficient of the channels \( \mathbf{h}_C \) and \( \mathbf{g}_D \), and \( |\rho| \leq 1 \).

For \( \{x, y\} \) to be feasible, there should exist at least one power solution pair \( \{P_D, P_C\} \) such that constraints (5.5)-(5.7) and (5.12) hold. For notation simplicity, in the following, we denote \( x \triangleq P_D \) and \( y \triangleq P_C \). We now study the SINR constraints (5.5) and (5.12) and state the following lemma.

**Lemma 3.** For \( \gamma_D = \gamma_D \) and \( \gamma_C = \gamma_C \), the power solution \( \{x_I, y_I\} \) is unique and is given by

\[
\begin{align*}
x_I &= \frac{\xi}{2(1 - K_1)}, \quad y_I = \frac{\xi}{2(1 - K_1) \beta K_3} - \frac{\sigma_D^2}{K_3} \quad (5.13)
\end{align*}
\]

where \( \alpha \triangleq \frac{\sigma_D^2 \gamma_D}{\| \mathbf{h}_C \|^2}, \beta \triangleq \frac{\gamma_D}{\| \mathbf{h}_D \|^2}, K_1 \triangleq \rho^2, K_2 \triangleq \frac{\sigma_D^2}{\| \mathbf{g}_D \|^2}, K_3 \triangleq \| \mathbf{g}_C \|^2, K_4 \triangleq \beta (\alpha K_3 + \sigma_D^2 (1 - K_1)) - K_2, K_5 \triangleq 4(1 - K_1) \beta K_2 (\alpha K_3 + \sigma_D^2), \) and

\[
\xi = \beta (\alpha K_3 + \sigma_D^2 (1 - K_1)) - K_2 + \sqrt{K_4^2 + K_5}.
\]

**Proof.** See Appendix A. \( \square \)

By Lemma 3 and combining constraints (5.6) and (5.7), the necessary and sufficient condition for the D2D pair to be admissible is given as follows.

**Necessary and sufficient condition:** The D2D pair is admissible if \( \{x_I, y_I\} \) in (5.13) satisfies

\[
\begin{align*}
0 &< x_I \leq P_D^{\text{max}}, \quad (5.14) \\
0 &< y_I \leq P_C^{\text{max}}, \quad (5.15) \\
c_{1,i} x_I + c_{2,i} y_I &\leq 1, \quad i = 1, \ldots, b 
\end{align*}
\]

(5.16)

where \( c_{1,i} \triangleq \frac{\| \mathbf{h}_C,i \|^2}{I} \), and \( c_{2,i} \triangleq \frac{\| \mathbf{g}_D,i \|^2}{I} \).

Note that \( \{x_I, y_I\} \) is the minimum power level required to satisfy the minimum SINR requirements. Thus, for any feasible \( \{x, y\} \), we have \( x \geq x_I \) and \( y \geq y_I \). Constraints (5.14) and (5.15) ensure the
maximum power at the D2D and CU are enough to meet their respective SINR requirements. Constraint (5.16) ensures the ICI constraints can be satisfied.

## 5.3 Power Control for D2D and CU

Assuming the D2D pair is admissible, we now solve the sum rate maximization problem P1. With the optimal \( w^o \) given in (5.10), we need to solve P1 with respect to \( \{P_D, P_C\} \). Due to the non-convex objective, finding an optimal solution is challenging. Instead, we propose the following approximation to obtain the power solution.

Note that the ICI constraints in (5.7) can be equivalently written as

\[
c_{1,i}x + c_{2,i}y \leq 1, \quad i = 1, \cdots, b.
\]

(5.17)

We replace these ICI constraints with a single ICI constraint and ensure that it satisfies the ICI requirements in all neighboring cells. We denote this constraint by \( c_{1,y} + c_{2,x} \leq 1 \), where \( c_1 \) and \( c_2 \) are determined as follows.

Define \( c_{1,max} \triangleq \max_i c_{1,i}, \quad c_{2,max} \triangleq \max_i c_{2,i} \); \( \hat{x} \triangleq \min_i \frac{1-c_{1,i}P_{C}^{max}}{c_{2,i}}, \quad \hat{\gamma} \triangleq \min_i \frac{1-c_{2,i}P_{D}^{max}}{c_{1,i}} \). Note that if there exists \( i \) such that \( c_{1,i}P_C^{max} \geq 1 \), then from (5.17), \( y \leq \frac{1}{\max_i c_{1,i}} \), and we have \( c_1 = c_{1,max} \). Otherwise, if \( c_{1,i}P_C^{max} < 1 \) for all \( i \), then the intersection of \( c_{1,y} + c_{2,x} = 1 \) and \( y = P_C^{max} \) is given by \( (\hat{x}, P_C^{max}) \). Similarly, if there exists \( j \) such that \( c_{2,j}P_D^{max} \geq 1 \), then \( c_2 = c_{2,max} \). Otherwise, the intersection of \( c_{1,y} + c_{2,x} = 1 \) and \( x = P_D^{max} \) is given by \( (P_D^{max}, \hat{y}) \). Based on this, we determine \( c_1 \) and \( c_2 \) in four possible cases:\(^3\)

1. If \( \exists i, j \), such that \( c_{1,i}P_C^{max} \geq 1 \) and \( c_{2,j}P_D^{max} \geq 1 \): \( c_1 = c_{1,max} \) and \( c_2 = c_{2,max} \).
2. If \( \exists i \), such that \( c_{1,i}P_C^{max} \geq 1 \), and \( \forall j \), \( c_{2,j}P_D^{max} < 1 \): \( c_1 = c_{1,max} \) and \( c_2 = \frac{1-c_{1,max}\hat{y}}{P_D^{max}} \).
3. If \( \forall i, c_{1,i}P_C^{max} < 1 \), and \( \exists j \), such that \( c_{2,j}P_D^{max} \geq 1 \): \( c_1 = \frac{1-c_{2,max}\hat{x}}{P_C^{max}} \) and \( c_2 = c_{2,max} \).
4. If \( \forall i, j \), \( c_{1,i}P_C^{max} < 1 \) and \( c_{2,j}P_D^{max} < 1 \): \( c_1 = \frac{P_D^{max}\hat{x}}{P_D^{max}P_C^{max}-\hat{y}} \) and \( c_2 = \frac{P_C^{max}\hat{y}}{P_D^{max}P_C^{max}-\hat{y}} \).

Note that when \( b = 1 \), the approximation becomes accurate to represent the ICI constraint. Substituting the expression of \( \gamma_C \) under the optimal \( w^o \) at the left hand side of (5.12) in P1, and approximating multiple ICI constraints in (5.17) with a single ICI constraint, we modify P1 into the following problem

\[
P2: \quad \max_{(x, y)} \log R(x, y)
\]

subject to

\[
y \left(1 - \frac{K_1 x}{K_2 + x}\right) \geq \tilde{\gamma}_C,
\]

\[
\frac{a x}{\sigma_D^2 + K_4 y} \geq \tilde{\gamma}_D,
\]

\[
y \leq \frac{P_D^{max}}{x} \leq P_D^{max},
\]

\[
c_{1,y} + c_{2,x} \leq 1
\]

\^3\text{Note that other values of } c_1 \text{ and } c_2 \text{ may also be chosen; however, our method strives to create a new feasible region that is close to the original one, so that the loss due to approximation is small.}
where
\[ R(x, y) \triangleq \left( 1 + \frac{ax}{\sigma_D^2 + K_3y} \right) \left( 1 + y\left(1 - \frac{K_1x}{K_2 + x}\right)l \right), \tag{5.23} \]
with \( a \triangleq |h_D|^2 \) and \( l \triangleq \|h_C\|^2/\sigma^2 \).

Let \( A_{xy} \) denote the feasible solution region of the problem P2. Note that by modifying P1 to P2, we shrink the original feasible region of P1 to \( A_{xy} \). This is done by replacing the feasible region boundaries formed by the intersections of the multiple ICI constraints in (5.17) with a single boundary described by (5.22). The following lemma gives the locations of the optimal power pair in \( A_{xy} \).

**Lemma 4.** The optimal power solution pair \((x^o, y^o)\) is at the vertical, horizontal, or tilted boundary of \( A_{xy} \), given by \( x = P_{D}^{\text{max}}, \ y = P_{C}^{\text{max}}, \) or \( c_1y + c_2x = 1 \), respectively.

**Proof.** See Appendix B.

Note that, depending on the system parameter setting, the shape of the feasible region \( A_{xy} \) varies. The boundaries of \( A_{xy} \) may or may not include the tilted boundary segment \( c_1y + c_2x = 1 \). In the following, we consider both cases and obtain the optimal solution to P2 for each case.

If the boundaries of \( A_{xy} \) do not include \( c_1y + c_2x = 1 \), the optimal solution satisfies (5.22) with strict inequality. By Lemma 4, it follows that at least one of \( x^o \) and \( y^o \) equals its maximum value (\( P_{D}^{\text{max}} \) or \( P_{C}^{\text{max}} \)), and the optimal \((x^o, y^o)\) is at either the vertical or horizontal boundary line of \( A_{xy} \). In this case, the optimal power pair is given in the following proposition.

**Proposition 8.** If the boundaries of the feasible region \( A_{xy} \) do not include \( c_1y + c_2x = 1 \), then the optimal power pair \((x^o, y^o)\) for P2 is at one end point of the vertical or horizontal boundary line segment of \( A_{xy} \).

**Proof.** See Appendix C.

If the boundaries of \( A_{xy} \) include \( c_1y + c_2x = 1 \), then at optimality, \((x^o, y^o)\) is on the horizontal, vertical, or tilted boundary line of \( A_{xy} \). The following proposition provides the solution to P2 in this case.

**Proposition 9.** If the boundaries of the feasible region \( A_{xy} \) include \( c_1y + c_2x = 1 \), then the optimal power pair \((x^o, y^o)\) is given in one of the two cases: 1) An end point of the horizontal, vertical, or tilted boundary line segment of \( A_{xy} \); or 2) an interior point of tilted boundary line segment of \( A_{xy} \), whose \( x \)-coordinate is one of the roots of the following quartic equation
\[ e_4x^4 + e_3x^3 + e_2x^2 + e_1x + e_0 = 0 \tag{5.24} \]
where \( \{e_i\}_{i=0}^{4} \) are given as in (D.1)–(D.5) in Appendix D. The optimal CU power is \( y^o = (1 - c_2x^o)/c_1 \).

**Proof.** See Appendix D.

Note that the roots of a quartic equation have closed-form expressions. Furthermore, we do not need to compute all the roots of (5.24), since not all of them are in \( A_{xy} \). In the following, we classify different scenarios leading to different types of the boundaries for \( A_{xy} \). We obtain simple inequalities to check the conditions under which each scenario applies. For each scenario, we discuss the corresponding optimal power solution \((x^o, y^o)\).
5.3.1 Moderate ICI from CU and D2D

We first consider the case where the boundary line $c_1 y + c_2 x = 1$ intersects both the horizontal and vertical boundary lines. This means that $c_1 P_{\text{max}}^C + c_2 P_{\text{max}}^D \geq 1$, and the $x$-intercept and $y$-intercept of $c_1 y + c_2 x = 1$ are greater than $P_{\text{max}}^D$ and $P_{\text{max}}^C$, respectively. These result in the following conditions

$$\frac{1 - c_2 P_{\text{max}}^D}{c_1} \leq P_{\text{max}}^C \leq \frac{1}{c_1}, \quad (5.25)$$

$$\frac{1 - c_1 P_{\text{max}}^C}{c_2} \leq P_{\text{max}}^D \leq \frac{1}{c_2}. \quad (5.26)$$

Note that given the definitions of $c_1$ and $c_2$, conditions (5.25) and (5.26) mean that the ICI caused by the CU or D2D transmitter alone, at each maximum power, is less than the ICI threshold, while combined the ICI caused by both of them is greater than the ICI threshold. In other words, both the CU and the D2D transmitter cause relatively moderate ICI to the neighboring cell.

In this case, depending on SINR requirements (5.19) and (5.20), there are several different shapes of
Chapter 5. Power Optimization for D2D Systems

I
F
G
y
x
T
Q

(a) Scenario 13
(b) Scenario 14
(c) Scenario 15
(d) Scenario 16

Figure 5.4: (a) Strong ICI from CU and D2D; (b)-(d) Weak ICI.

the feasible region $A_{xy}$, as shown in Figs. 5.2a-5.2f, where $A_{xy}$ is the shaded area. The curve and line passing through point I correspond to constraints (5.19) and (5.20) with equality. The feasible region $A_{xy}$’s in Figs. 5.2a-5.2f correspond to six scenarios, depending on whether the line and curve passing through point I intersect the vertical, horizontal, or tilted boundary. In the following, we derive the optimal power control solution in each of these six scenarios.

Scenario 1

The feasible solution region $A_{xy}$ is depicted in Fig. 5.2a as the shaded area. Points A, B, C, E, and I are the intersections of two lines (or a line and curve). Both curve I-E and line I-A intersect with the horizontal boundary line $y = P_{C}^{\text{max}}$. In this scenario, we have $x_E \leq x_B$. It occurs under the following condition:

$$K_2 \left( \frac{K_1}{\frac{\gamma C}{P_{C}^{\text{max}}} - 1} \right)^{-1} \leq \frac{1 - c_1 P_{C}^{\text{max}}}{c_2}.$$  (5.27)

By Lemma 4 and Proposition 8, the optimal power pair $(x^o, y^o)$ can be one of points A and E. Therefore, the set of candidate power pairs is given by

$$P^{(A,1)} = \left\{ \left( \beta \left( \sigma_D^2 + K_3 P_{C}^{\text{max}} \right), P_{C}^{\text{max}} \right), \left( K_2 \left( \frac{K_1}{\frac{\gamma C}{P_{C}^{\text{max}}} - 1} \right)^{-1}, P_{C}^{\text{max}} \right) \right\}.  \quad (5.28)$$

Scenario 2

As shown in Fig. 5.2b, in this scenario, the curve I-F and line I-A intersect the tilted boundary line and horizontal boundary line, respectively. Note that $x_F$ is the $x$-coordinate of point F, which is the intersection of the curve I-F and the tilted boundary line B-F. We can find $x_F$ by setting constraints (5.19) and (5.22) with equality. This results in a quadratic equation given by

$$c_2(1 - K_1)x^2 - \theta x + K_2(\alpha c_1 - 1) = 0 \quad (5.29)$$

where $\theta \triangleq 1 - K_1 - c_2 K_2 - \alpha c_1$. The uniqueness of the feasible solution of (5.29) is stated in the following lemma.
Lemma 5. For $A_{xy}$ as shown in Fig. 5.2b, the feasible solution of the quadratic equation (5.29) is unique and is given by

$$x_F = \frac{\theta + \sqrt{\theta^2 - 4c_2(1 - K_1)K_2(\alpha c_1 - 1)}}{2c_2(1 - K_1)}.$$  \hspace{1cm} (5.30)

Proof. See Appendix E. \hfill \square

Note that in Scenario 2, we have $x_A \leq x_B \leq x_F \leq P_D^{\text{max}}$. The conditions for this scenario to happen are as follows:

$$\beta(\sigma_D^2 + K_3P_C^{\text{max}}) \leq \frac{1 - c_1P_C^{\text{max}}}{c_2},$$  \hspace{1cm} (5.31)

$$\frac{1 - c_1P_C^{\text{max}}}{c_2} \leq x_F \leq P_D^{\text{max}}.$$  \hspace{1cm} (5.32)

By Proposition 9, the candidate pairs for $(x^o, y^o)$ are points A, B, F, and any interior point of line B-F. For the last case to happen, $x^o$ should be within the range $\frac{1 - c_1P_D^{\text{max}}}{c_2} < x^o < x_F$, which means the roots of (5.24) should satisfy this range constraint. Let $S_2$ be the set of roots that meet the above range constraint. The corresponding set of points on the interior of line B-F is given by

$$A_2 \triangleq \{(x, (1 - c_2x)/c_1) : x \in S_2\}.$$  \hspace{1cm} (5.33)

Now, we have the set of candidate pairs for $(x^o, y^o)$ as

$$P(A.2) = \left\{\left(\beta(\sigma_D^2 + K_3P_C^{\text{max}}), P_C^{\text{max}}\right), \left(x_F, (1 - c_2x_F)/c_1\right), \left((1 - c_1P_C^{\text{max}})/c_2, P_C^{\text{max}}\right), A_2\right\}.$$  \hspace{1cm} (5.33)

where the first three pairs are the coordinates for points A, F, and B, respectively.

Scenario 3

As illustrated in Fig. 5.2c, in this scenario, the curve I-D and line I-A intersect the horizontal and vertical boundary lines, respectively. The entire tilted boundary B-C is in the feasible region. In this scenario, we have $x_A \leq x_B$ and $y_D \leq y_C$. The conditions for this scenario to occur are given by

$$\alpha \left(1 - \frac{K_1}{1 + K_2/P_D^{\text{max}}}\right)^{-1} \leq \frac{1 - c_2P_D^{\text{max}}}{c_1},$$  \hspace{1cm} (5.34)

and (5.31).

By Proposition 9, $(x^o, y^o)$ could be either points A, B, C, D, or if $(x^o, y^o)$ lies on the interior of line B-C, $x^o$ should be within the range $x_B < x^o < x_C$, i.e., $\frac{1 - c_1P_D^{\text{max}}}{c_2} < x^o < P_D^{\text{max}}$.

Let $S_3$ denote the set of roots (5.24) satisfying the above range constraint. By Proposition 9, the set of candidate pairs on the interior of line B-C is given by $A_3 \triangleq \{(x^o, (1 - c_2x)/c_1) : x \in S_3\}.$
Thus, in Scenario 3, the set of candidate pairs is given by
\[
\mathcal{P}^{(A,3)} = \left\{ \left( \beta \left( \sigma_D^2 + K_3 P^\text{max}_C \right), P^\text{max}_C \right), \left( \left(1 - c_1 P^\text{max}_C \right)/c_2, P^\text{max}_C \right), \left( P^\text{max}_D, (1 - c_2 P^\text{max}_D)/c_1 \right), \left( P^\text{max}_D, a \left(1 - \frac{K_1}{1 + K_2 P^\text{max}_D} \right)^{-1} \right), A_3 \right\}
\] (5.35)

where the first four pairs are the coordinates of points A, B, C, and D, respectively.

Scenario 4

As shown in Fig. 5.2d, in this scenario, both curve I-F and line I-G intersect the tilted boundary line B-C. The condition for this scenario to happen is as follows:
\[
1 - c_1 P^\text{max}_C/c_2 \leq x_G \leq x_F \leq P^\text{max}_D
\] (5.36)

where \(x_G\) can be obtained by setting constraints (5.20) and (5.22) with equality, given by
\[
x_G = \frac{\sigma_D^2 \beta + \beta K_3 / c_1}{1 + \beta K_3 / c_1}.
\] (5.37)

By Proposition 9, to find the optimal power pair, we need to consider points G and F. The set of candidate power pairs on the interior of line G-F is given by \(A_4 \triangleq \left\{ (x^o, (1 - c_2 x^o)/c_1) : x \in S_4 \right\} \) where \(S_4\) denotes the set of roots (5.24) which meet \(x_G < x^o < x_F\).

Thus, the set of candidate pairs is given as follows:
\[
\mathcal{P}^{(A,4)} = \left\{ (x_G, (1 - c_2 x_G)/c_1), (x_F, (1 - c_2 x_F)/c_1), A_4 \right\}.
\] (5.38)

Scenario 5

As shown in Fig. 5.2e, in this scenario, line I-G intersects tilted boundary line G-C, while curve I-D intersects the vertical boundary line \(x = P^\text{max}_D\). In this scenario, we have \(x_B \leq x_G \leq P^\text{max}_D\) and \(y_D \leq y_C\).

The conditions under which this scenario happens are given by
\[
1 - c_1 P^\text{max}_C/c_2 \leq x_G \leq P^\text{max}_D,
\] (5.39)
and (5.34).

Based on Proposition 9, \((x^o, y^o)\) could be either points G, C, D, or if \((x^o, y^o)\) is on the interior of line G-C, \(x^o\) should be within the range \(x_G < x^o < P^\text{max}_D\).

Let \(S_5\) denote the set of roots of (5.24) within the above range of \(x^o\). The set of candidate points on the interior of line G-C is \(A_5 \triangleq \left\{ (x^o, (1 - c_2 x^o)/c_1) : x \in S_5 \right\} \).
Table 5.1: Moderate ICI from CU and D2D (under conditions (5.25) and (5.26))

<table>
<thead>
<tr>
<th>Condition</th>
<th>Set of candidates for the optimal powers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.27)</td>
<td>$P^o = P^{A.1}$ in (5.28)</td>
</tr>
<tr>
<td>(5.31)</td>
<td>$P^o = P^{A.2}$ in (5.33)</td>
</tr>
<tr>
<td>(5.34)</td>
<td>$P^o = P^{A.3}$ in (5.35)</td>
</tr>
<tr>
<td>(5.36)</td>
<td>$P^o = P^{A.4}$ in (5.38)</td>
</tr>
<tr>
<td>(5.39)</td>
<td>$P^o = P^{A.5}$ in (5.40)</td>
</tr>
<tr>
<td>(5.41)</td>
<td>$P^o = P^{A.6}$ in (5.42)</td>
</tr>
</tbody>
</table>

Hence, the set of candidate pairs for $(x^o, y^o)$ in this scenario is given by

$$P^{(A.5)} = \left\{ \left( x_G, \frac{1 - c_2 x_G}{c_1}, \left( \frac{P_{D_{\max}}}{c_1}, \left(1 - c_2 P_{D_{\max}}\right)/c_1 \right), \left( P_{D_{\max}}, \alpha \left(1 - K_1 1 + K_2/P_{D_{\max}} \right)^{-1} \right), A_5 \right\}. \quad (5.40)$$

**Scenario 6**

As depicted in Fig. 5.2f, this scenario happens when both curve I-D and line I-H intersect the vertical boundary line $x = P_{D_{\max}}$. In this scenario, we have $y_H \leq y_C$. The condition for this scenario to happen is given by

$$P_{D_{\max}} - \beta \sigma^2_D \leq \frac{1 - c_2 P_{D_{\max}}}{c_1}. \quad (5.41)$$

In this scenario, the boundaries of $A_{xy}$ do not include the tilted boundary line. By Proposition 8, it is sufficient to consider only points H and D to find the optimal power solution. The set of candidates for the optimal powers is thus given by

$$P^{(A.6)} = \left\{ \left( P_{D_{\max}}, \alpha \left(1 - K_1 1 + K_2/P_{D_{\max}} \right)^{-1} \right), \left( P_{D_{\max}}, P_{D_{\max}} - \beta \sigma^2_D \right) \right\}. \quad (5.42)$$

We summarize in Table 5.1 the candidate solution sets for the above six scenarios for the case where conditions (5.25) and (5.26) hold.

### 5.3.2 Strong ICI from CU and Moderate ICI from D2D

Consider the case in which the tilted line $c_1 y + c_2 x = 1$ only intersects the vertical line $x = P_{D_{\max}}$, but does not intersect the horizontal line $y = P_{C_{\max}}$. This means that the $x$-intercept and $y$-intercept of $c_1 y + c_2 x = 1$ are greater than $P_{D_{\max}}$ and less than $P_{C_{\max}}$, respectively. The resulting conditions are given by

$$\frac{1}{c_1} \leq P_{C_{\max}}, \quad (5.43)$$

and (5.26).

Note that the condition (5.43) means the ICI caused by the CU under its maximum power is greater
than the ICI threshold. Along with (5.26), these conditions correspond to the case where the ICI caused by the CU is strong, while the ICI caused by the D2D transmitter is moderate.

In this case, by SINR requirements (5.19) and (5.20), the necessary and sufficient condition to set up D2D communication is given by (5.14) and (5.16). The feasible region $A_{xy}$ can have three different shapes as shown in Figs. 5.2g–5.3a, depending on whether the line and curve passing through point I intersect the vertical or tilted boundary line. Accordingly, we derive the optimal power control solution in these three scenarios as follows.

**Scenario 7**

As shown in Fig. 5.2g, in this scenario, both curve I-F and line I-G intersect the tilted boundary line. The condition under which this scenario happens is given by

$$0 \leq x_G \leq x_F \leq P_{D\text{max}}$$

(5.44)

where $x_F$ and $x_G$ are given in (5.30) and (5.37), respectively.

Let $P^{(B.1)}$ denote the set of candidate pairs in this scenario. Note that the feasible region $A_{xy}$ has the same shape as that in Scenario 4 in Fig. 5.2d. Thus, these two scenarios have the same set of candidate pairs, i.e., $P^{(B.1)} = P^{(A.4)}$.

**Scenario 8**

As depicted in Fig. 5.2h, in this scenario, line I-G and curve I-D intersect the tilted boundary line G-C and vertical boundary line, respectively. This scenario occurs when $x_G \leq x_C$ and $y_D \leq y_C$, which results in the following conditions:

$$0 \leq x_G \leq P_{D\text{max}},$$

and (5.34).

From Fig. 5.2h, we see that the shape of $A_{xy}$ is the same as that of Scenario 5 in Fig. 5.2e. Thus, these two scenarios have the same set of candidate pairs. Let $P^{(B.2)}$ denote the set of candidate power pairs in Scenario 8. We have $P^{(B.2)} = P^{(A.5)}$.

**Scenario 9**

As illustrated in Fig. 5.3a, this scenario happens when both curve I-D and line I-H intersect the vertical boundary line. By similar discussion as the above, the condition for this scenario to happen is the same as that in Scenario 6 in Fig. 5.2f, which is given by (5.41). As a result, the set of candidate pairs, denoted by $P^{(B.3)}$, is the same as that in Scenario 6, i.e., $P^{(B.3)} = P^{(A.6)}$.

The candidate solution sets for these three scenarios for the case with conditions (5.26) and (5.43) are summarized in Table 5.2.

### 5.3.3 Moderate ICI from CU and Strong ICI from D2D

Now consider the case in which line $c_1 y + c_2 x = 1$ only intersects the horizontal line $y = P_{C\text{max}}$, but does not intersect the vertical line $x = P_{D\text{max}}$. Similar to the discussion in Section 5.3.2, this means that the
Table 5.2: Strong ICI from CU and Moderate ICI from D2D (under conditions (5.26) and (5.43))

<table>
<thead>
<tr>
<th>Condition</th>
<th>Set of candidates for the optimal powers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.44)</td>
<td>$P^o = P^{(B.1)} = P^{(A.4)}$</td>
</tr>
<tr>
<td>(5.34) and (5.45)</td>
<td>$P^o = P^{(B.2)} = P^{(A.5)}$</td>
</tr>
<tr>
<td>(5.41)</td>
<td>$P^o = P^{(B.3)} = P^{(A.6)}$</td>
</tr>
</tbody>
</table>

$x$-intercept and $y$-intercept of $c_1 y + c_2 x = 1$ are greater than $P^\text{max}_C$ and less than $P^\text{max}_D$, respectively, which are equivalent to the following conditions:

$$\frac{1}{c_2} \leq P^\text{max}_D,$$

and (5.25).

These conditions correspond to the case where the ICI caused by the CU is moderate, while the ICI caused by the D2D transmitter is strong. Similar to the previous cases, depending on the SINR requirements (5.19) and (5.20), the feasible region of $A_{xy}$ can have three different shapes as shown in Figs. 5.3b–5.3d. The optimal power pair for each of these three scenarios are discussed below.

**Scenario 10**

The feasible region is shown in Fig. 5.3b. In this scenario, both curve I-E and line I-A intersect the horizontal boundary line. The shape of the feasible region $A_{xy}$ is the same as that of Scenario 1, with the condition to occur given by (5.27). Thus, the set of candidate pairs, denoted by $P^{(C.1)}$, is given by $P^{(C.1)} = P^{(A.1)}$.

**Scenario 11**

As shown in Fig. 5.3c, this scenario happens when curve I-F and line I-A intersect the tilted boundary line B-F and horizontal boundary line, respectively. In this scenario, we have $x_A \leq x_B \leq x_F \leq x_T$, which results in the following conditions:

$$\frac{1 - c_1 P^\text{max}_C}{c_2} \leq x_F \leq \frac{1}{c_1},$$

and (5.31).

Similarly, the shape of $A_{xy}$ is the same as that of Scenario 2 in Fig. 5.2b. As a result, the set of candidate pairs, denoted by $P^{(C.1)}$, is given by $P^{(C.1)} = P^{(A.2)}$.

**Scenario 12**

As depicted in Fig. 5.3d, in this scenario, both curve I-F and line I-G intersect the tilted boundary line. With a similar approach, we can derive the condition for this scenario to happen as follows:

$$\frac{1 - c_1 P^\text{max}_C}{c_2} \leq x_G \leq \frac{1}{c_2},$$

and (5.48).

Again, $A_{xy}$ in this scenario has the same shape as that in Scenario 4, and thus, the set of candidate pairs, denoted by $P^{(C.3)}$, is given by $P^{(C.3)} = P^{(A.4)}$. 

Table 5.3: Moderate ICI from CU and Strong ICI from D2D (under conditions (5.25) and (5.46))

<table>
<thead>
<tr>
<th>Condition</th>
<th>Set of candidates for the optimal powers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.27)</td>
<td>$P^o = P^{(C,1)} = P^{(A,1)}$</td>
</tr>
<tr>
<td>(5.31) and (5.47)</td>
<td>$P^o = P^{(C,2)} = P^{(A,2)}$</td>
</tr>
<tr>
<td>(5.48)</td>
<td>$P^o = P^{(C,3)} = P^{(A,4)}$</td>
</tr>
</tbody>
</table>

The candidate solution sets for these three scenarios for the case with conditions (5.25) and (5.46) are summarized in Table 5.3.

5.3.4 Strong ICI from CU and D2D

When the tilted line $c_1 y + c_2 x = 1$ does not intersect either the vertical or horizontal line, we have the feasible region $A_{xy}$ as shown in Fig. 5.4a. In this case the $x$-intercept and $y$-intercept of $c_1 y + c_2 x = 1$ are less than $P^\text{max}_D$ and $P^\text{max}_C$, respectively, which are equivalent to conditions (5.43) and (5.46). In this scenario, the necessary and sufficient condition to have an admissible D2D pair is reduced to (5.16).

Denote this case by Scenario 13. In this scenario, we have

$$0 \leq x_G \leq x_F \leq \frac{1}{c_2}.$$  \hspace{1cm} (5.49)

Since $A_{xy}$ in both this scenario and Scenario 4 have the same shape, the set of candidate pairs, denoted by $P^{(D,1)}$, is given by $P^{(D,1)} = P^{(A,4)}$.

5.3.5 Weak ICI

In all above cases, the line $c_1 y + c_2 x = 1$ intersects at least one of the horizontal and vertical boundary lines (i.e., $y = P^\text{max}_D$ and $x = P^\text{max}_C$).

Now we consider the case where there is no intersection between $c_1 y + c_2 x = 1$ and either of the horizontal and vertical boundary lines. This case happens when

$$c_1 P^\text{max}_C + c_2 P^\text{max}_D < 1.$$ \hspace{1cm} (5.50)

Note that condition (5.50) occurs when both the CU and the D2D transmitter cause weak ICI to the neighboring cell, e.g., they both are located near the center of the cell.

Shown in Figs. 5.4b–5.4d, there are three possible shapes for the feasible region $A_{xy}$ in this case, depending the SINR requirements (5.19) and (5.20). The optimal power pairs for these three scenarios are discussed as follows.

Scenario 14

As shown in Fig. 5.4b, this scenario happens when curve I-E and line I-A intersect the horizontal boundary line. The condition for this scenario to happen is $x_E \leq P^\text{max}_D$, i.e.,

$$K_2 \left( \frac{K_1}{1 - \frac{P^\text{max}_C}{P^\text{max}_D}} - 1 \right)^{-1} \leq P^\text{max}_D.$$ \hspace{1cm} (5.51)
Table 5.4: Weak ICI (under condition (5.50))

<table>
<thead>
<tr>
<th>Condition</th>
<th>Set of candidates for the optimal powers</th>
</tr>
</thead>
<tbody>
<tr>
<td>(5.51)</td>
<td>( \mathcal{P}^o = \mathcal{P}^{(E.1)} = \mathcal{P}^{(A.1)} )</td>
</tr>
<tr>
<td>(5.52) and (5.53)</td>
<td>( \mathcal{P}^o = \mathcal{P}^{(E.2)} ) in (5.54)</td>
</tr>
<tr>
<td>(5.55)</td>
<td>( \mathcal{P}^o = \mathcal{P}^{(E.3)} = \mathcal{P}^{(A.6)} )</td>
</tr>
</tbody>
</table>

Since the shape of \( \mathcal{A}_{xy} \) is the same as that in Scenario 1, the set of candidates for the optimal powers, denoted by \( \mathcal{P}^{(E.1)} \), is given by \( \mathcal{P}^{(E.1)} = \mathcal{P}^{(A.1)} \).

**Scenario 15**

As illustrated in Fig. 5.4c, in this scenario, line I-A and curve I-D intersect the horizontal and vertical boundary lines, respectively. In this scenario, we have \( x_A \leq P^\text{max}_D \) and \( y_D \leq P^\text{max}_C \). Solving for \( x_A \) and \( y_D \), we have the following conditions:

\[
\beta (\sigma_D^2 + K_3 P^\text{max}_C) \leq P^\text{max}_D, \quad (5.52)
\]

\[
\alpha \left(1 - \frac{K_1}{1 + K_2/P^\text{max}_D}\right)^{-1} \leq P^\text{max}_C. \quad (5.53)
\]

By Proposition 8, \((x^o, y^o)\) could be at either of points A, D, or M, and the set of candidate pairs is given by

\[
\mathcal{P}^{(E.2)} = \left\{ (\beta (\sigma_D^2 + K_3 P^\text{max}_C), P^\text{max}_C), \left( P^\text{max}_D, P^\text{max}_C \right), \left( P^\text{max}_D, \alpha \left(1 - \frac{K_1}{1 + K_2/P^\text{max}_D}\right)^{-1} \right) \right\}. \quad (5.54)
\]

**Scenario 16**

The feasible region is shown in Fig. 5.4d. This scenario happens when both line I-H and curve I-D intersect the vertical boundary line. The condition for this scenario to occur is \( y_H \leq P^\text{max}_C \), i.e.,

\[
\frac{P^\text{max}_D - \beta \sigma_D^2}{\beta K_3} \leq P^\text{max}_C. \quad (5.55)
\]

Since the shape of \( \mathcal{A}_{xy} \) is the same as that in Scenario 6, the set of candidate pairs, denoted by \( \mathcal{P}^{(E.3)} \), is given by \( \mathcal{P}^{(E.3)} = \mathcal{P}^{(A.6)} \).

The candidate solution sets for these three scenarios for the case with condition (5.50) are represented in Table 5.4.

Finally, we summarize the steps to solve the optimization problem P1 in Algorithm 5. We note that the optimal solution in any scenario can be obtained in closed form.

### 5.3.6 Performance Bound Analysis

Note that Algorithm 1 is very efficient in obtaining the power solution pair \( \{P^o_D, P^o_C\} \), as the candidate pairs are all given in closed-form. However, it only provides the optimal solution for \( b = 1 \). For \( b > 1 \), since \( \mathcal{A}_{xy} \) is a subset of the original feasible region of P1, Algorithm 1 is suboptimal. In the following, we
Algorithm 5 Approximate power control algorithm

Input: $\alpha, \beta, a, l, K_1, K_2, K_3, \{c_{1,i}, c_{2,i}\}_{i=1}^l, \gamma_C, \gamma_D, P_C^{\max}, P_D^{\max}$
Output: $P_C^c, P_D^c$, and $w^o$
1: Check the feasibility condition (5.14)–(5.16).
2: Determine $c_1, c_2, a_1, a_2, b_1, b_2, x_F$, and $x_G$.
3: if (5.25) and (5.26) hold then
4: Compute candidate solution set in Table 5.1.
5: else if (5.26) and (5.43) hold then
6: Compute candidate solution set in Table 5.2.
7: else if (5.25) and (5.46) hold then
8: Compute candidate solution set in Table 5.3.
9: else if (5.43) and (5.46) hold then
10: Compute candidate solution set $P^o = P^{(D, 1)}$ in Section 5.3.4.
11: else if (5.50) holds then
12: Compute candidate solution set in Table 5.4.
13: end if
14: Enumerate among candidate solution set $P^o$ to find the optimum solution.
15: Obtain the optimum beam vector (5.10) using $P_C^c$ and $P_D^c$.

provide an upper bound on the performance loss of the proposed algorithm and provide the conditions for its optimality.

Let $A_{xy}^o$ denote the original feasible region of P1, and we have $A_{xy} \subseteq A_{xy}^o$. An example for $b = 2$ is shown in Fig. 5.5, where $A_{xy}$ and $A_{xy}^o$ are given by $A_1$ and $A_1 \cup A_2$, respectively. Let points $U_1$ and $U_2$ denote the intersections of the boundary of $A_{xy}$ with the line and curve corresponding to minimum SINR requirements (5.20) and (5.19), respectively. It follows that $x_{U_1} \leq x_{U_2}$ and $y_{U_1} \leq y_{U_2}$. Similarly, let points $L_1$ and $L_2$ denote the intersections of the boundary of $A_{xy}$ with the same line and curve, and we have $x_{L_1} \leq x_{L_2}$ and $y_{L_1} \leq y_{L_2}$. We denote the set of all corner points of $A_{xy}$ by $L$, e.g., $L = \{L_1, L_2, L_3\}$ in Fig. 5.5. Note that $|L| \leq 4$. We define a new point $U_3 = (\max(x_{U_1}, x_{U_2}), \max(y_{U_1}, y_{U_2}))$ (as shown in Fig. 5.5 for $b = 2$). For simplicity, we use $R_Z$ to denote $\log R(x_Z, y_Z)$ in (5.18) for point $Z$ in $A_{xy}$. Also, let $R^{A_1}$ and $R^{opt}$ denote the sum rate achieved by Algorithm 5 and the maximum sum rate under an optimal solution of P1, respectively. We have the following results on the performance of Algorithm 5.

**Proposition 10.** For $b = 1$, $R^{opt} = R^{A_1}$. For $b > 1$, the performance loss of Algorithm 5 is bounded by

$$R^{opt} - R^{A_1} \leq \max\{R_{U_1}, R_{U_2}, R_{U_3}\} - \max_{l \in L}\{R_l\}. \tag{5.56}$$

Furthermore, $R^{opt} = R^{A_1}$ if one of the following conditions holds:

1. ICI constraints in (5.17) results in a single tilted boundary line for $A_{xy}^o$.
2. $x_{L_1} = x_{L_2}$ or equivalently $x_{U_1} = x_{U_2}$.
3. $y_{L_1} = y_{L_2}$ or equivalently $y_{U_1} = y_{U_2}$.

**Proof.** See Appendix F.

Note that in Proposition 10, Condition 1) means $A_{xy} = A_{xy}^o$. For Conditions 2) or 3), the line and curve associated with (5.20) and (5.19) both intersect either the vertical or horizontal boundary of $A_{xy}^o$. 

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5.4 Extension to Multiple CUs and D2D Pairs

So far, we have provided a power control solution for one CU and one D2D pair. We now extend our consideration to the scenario of multiple CUs and D2D pairs. Consider a multichannel communication system (e.g., OFDMA) with $N_C$ orthogonal subchannels in each cell. We assume a fully loaded network with $N_C$ CUs, and there are $N_D$ D2D pairs with $N_D \leq N_C$. Without loss of generality, we assume CU $j$ uses subchannel $j$ for $j \in \mathcal{C} \triangleq \{1, \cdots, N_C\}$. Each D2D pair reuses at most one subchannel, and the subchannel of each CU can be reused by at most one D2D pair. Define indicator $x_{k,j} \in \{0, 1\}$ such that $x_{k,j} = 1$ if D2D pair $k$ reuses CU $j$'s subchannel and $x_{k,j} = 0$ otherwise. Let $\mathbf{P} \triangleq [P_{D,1}, \cdots, P_{D,N_D}, P_{C,1}, \cdots, P_{C,N_C}]^T$, $\mathbf{x} \triangleq [x_{1,1}, \cdots, x_{1,N_C}, \cdots, x_{N_D,1}, N_C]^T$, and $\mathbf{w} \triangleq [\mathbf{w}_1^T, \cdots, \mathbf{w}_{N_C}^T]^T$.

The objective is to maximize the overall sum rate of all D2D pairs and CUs by optimizing the transmit power vector $\mathbf{P}$, the indicator vector $\mathbf{x}$, and the receive beam vector $\mathbf{w}$, while satisfying the worst-case ICI and minimum SINR requirements under per-node power constraints. The formulated
problem is given by:

\[
\text{P3:} \quad \max_{P, w, x} \sum_{k \in D} \sum_{j \in C} \log(1 + \gamma_{C,j}) + x_{k,j} \log(1 + \gamma_{D,k})
\]

subject to

\[
\frac{P_{C,j}|w_j^H h_{C,j}|^2}{\sigma^2 + x_{k,j}P_{D,k}|w_j^H g_{D,k}|^2} \geq \tilde{\gamma}_C, \quad \forall j \in C
\]

\[
\frac{P_{D,k}|h_{D,k}|^2}{\sigma_{D,k}^2 + x_{k,j}P_{C,j}|g_{j,k}|^2} \geq \tilde{\gamma}_D, \quad \forall k \in D
\]

\[
P_{C,j} \leq P_{C,max}, \quad P_{D,k} \leq P_{D,max}, \quad \forall j \in C, k \in D
\]

\[
P_{I,i,j} \leq \tilde{I}, \quad \forall j \in C, i = 1, \ldots, b
\]

\[
\sum_{k \in D} x_{k,j} \leq 1, \quad \sum_{j \in C} x_{k,j} \leq 1, \quad \forall j \in C, k \in D
\]

\[
x_{k,j} \in \{0, 1\}, \quad \forall j \in C, k \in D
\]

where \(D\) denotes the set of admissible D2D pairs. D2D pair \(k\) is admissible if it can reuse at least one subchannel from \(C\).

Note that problem P3 is a mixed integer programming problem and is challenging to solve. Instead, we consider a suboptimal approach by utilizing our proposed Algorithm 1 as follows:

1. Determine the admissibility of any D2D pair \(k\) to reuse CU \(j\)'s subchannel, for \(k = 1, \ldots, N_D, j = 1, \ldots, N_C\).

2. For all \(k\) and \(j\), if D2D pair \(k\) is admissible to use CU \(j\)'s subchannel, we jointly optimize their transmit powers to maximize their sum rate, which is given by problem P1 with the solution provided by Algorithm 1.

3. We solve the CU-D2D matching problem to optimally assign each admissible D2D pair to a CU. In particular, we define a bipartite graph between CUs and D2D pairs. Each edge between a D2D pair and a CU indicates that the pairing of the D2D pair and the CU is feasible. The weight of the edge is given by the sum rate or rate gain of the D2D pair and the CU, under the approximate power control solution provided by Algorithm 5. This CU-D2D matching problem is an assignment problem, whose optimal solution can be achieved by using the well-known Hungarian algorithm [105].

Remark 1. Note that the suboptimality of the above approach lies only in the approximation of multiple ICI constraints by a single ICI constraint. It follows that this approach is optimal for P3 if one of the conditions in Proposition 10 is satisfied.

We can further reduce the complexity of the CU-D2D assignment problem in Step 3 above by proposing two suboptimal CU-D2D matching schemes. Instead of the reward (rate or gain), we define the cost on an edge between D2D pair \(k\) and CU \(j\) in the bipartite graph as follows:

\[\text{Remark 1. Note that the suboptimality of the above approach lies only in the approximation of multiple ICI constraints by a single ICI constraint. It follows that this approach is optimal for P3 if one of the conditions in Proposition 10 is satisfied.}\]
The distances between the CU and the BS, the CU and the D2D receiver, and the D2D transmitter and receiver are located at \((0, 0), (0, -0.75d_0 - d_D/2),\) and \((0, -0.75d_0 + d_D/2),\) respectively. Unless otherwise mentioned, we consider one neighboring cell with its BS located at \((2d_0, 0).\) Let \(d_C, d_{DC},\) and \(d_D\) denote the distances between the CU and the BS, the CU and the D2D receiver, and the D2D transmitter and receiver, respectively.

### 5.5 Numerical Study

In our simulation, we consider that the cell of interest contains one CU and one D2D pair. Assume that the BS is located at coordinates \((0, 0),\) and that the CU, the D2D transmitter, and the D2D receiver are located at \((0, 0.5d_0), (0, -0.75d_0 - d_D/2),\) and \((0, -0.75d_0 + d_D/2),\) respectively. Unless otherwise mentioned, we consider one neighboring cell with its BS located at \((2d_0, 0).\) Let \(d_C, d_{DC},\) and \(d_D\) denote the distances between the CU and the BS, the CU and the D2D receiver, and the D2D transmitter and receiver, respectively.
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the BS, respectively, and let $d_{IC}$ and $d_{ID}$ denote the distance between the CU and the neighboring BS, and the D2D transmitter and the neighboring BS, respectively. We set $d_C = 0.5d_0$, $d_{gC} = 1.25d_0 - d_D/2$, $d_{gD} = 0.75d_0 + d_D/2$, $d_{IC} = 2.0616d_0$, and $d_{ID} = \sqrt{2^2 + (0.75 + 0.5d_D/d_0)^2}d_0$. For all the links, we assume the path loss exponent is set to 4. The channel coefficients are assumed to be Gaussian with zero-mean and variance $(d/d_0)^{-4}$. We set $\sigma^2 = \sigma_D^2 = 1$, $\tilde{\gamma}_C = \tilde{\gamma}_D = 3$ dB, $P_C^{\max} = P_D^{\max} = P^{\max}$, and $\tilde{I} = N\tilde{I}_0$ where $\tilde{I}_0$ is the ICI threshold reference, such that $\tilde{I}$ is a function of the number of antennas at the BS. We set $\tilde{I}_0/\sigma^2 = 3$ dB. We use 5000 channel realizations to evaluate the average performance.

We evaluate the rate gain obtained by adding D2D communication. It is the difference of the maximum sum rate of P1 provided by Algorithm 5 and that when there is no D2D pair in the cell. Furthermore, for performance comparison, we consider two baseline algorithms: 1) boost-and-limit (BL) heuristic, where the unique power solution $(x_I, y_I)$ in (5.13) is boosted proportionally with a common factor $\zeta_{\text{max}}$ such that either the maximum power constraint (5.21) or the ICI limit (5.22) is met with equality, i.e., further boosting the powers would violate at least one constraint. This BL algorithm is also promising in a practical point of view. Note that the scheduling BS can easily compute $(x_I, y_I)$

Figure 5.8: The sum rate and rate gain versus $d/d_0$ ($N = 4, P^{\max}/\sigma^2 = 10$ dB).

Figure 5.9: The sum rate and rate gain versus $I_0/\sigma^2$ ($d_D/d_0 = 0.2, P^{\max}/\sigma^2 = 20$ dB).
and then boost the power of the CU and D2D pair until either the maximum power is achieved or a neighboring cell alerts regarding the ICI level. This can be achieved by simple binary feedback through the backhaul. Hence, there is no need for any ICI channel exchange among the cells, which reduces signaling overhead substantially. 2) **CU-priority heuristic**, aiming at maximizing SINR $\gamma_C$ of the CU. It selects the maximum feasible CU power with the minimum feasible D2D power, satisfying constraints (5.19)–(5.22). Note that in this CU-priority heuristic, we prioritize the CU in terms of choosing a specific end point of the feasible region.

### 5.5.1 Single Neighboring Cell

We first evaluate how the performance changes with the maximum transmit power. The sum rate and rate gain versus the normalized maximum power, $P_{\text{max}}/\sigma^2$, under Algorithm 5, the BL heuristic, and the CU-priority heuristic, are shown in Figs. 5.6a and 5.6b, respectively, for $N = 2, 4, 8$. In Fig. 5.6a, for the increment of the sum rate and rate gain over $P_{\text{max}}/\sigma^2$ under Algorithm 5, we observe two regimes: i) Regime 1, where the sum rate is an increasing function of $P_{\text{max}}$. In this regime, the ICI is relatively weak, which is similar to the case in Section 5.3.5. In this case, as shown in Scenarios 14–16, the feasible region is not affected by the ICI constraint, and the candidates for the optimal power pair in Table 5.4 are directly functions of $P_{\text{max}}$. As a result, the sum rate increases linearly with $P_{\text{max}}$ in this regime. ii) Regime 2, where the sum rate converges. In this regime, the ICI is relatively strong, which is similar to the case in Section 5.3.4. In this case, as shown in Scenario 13, the feasible region is not changed by $P_{\text{max}}$, and the candidates for the optimal power pair are functions of $\tilde{I}$. Hence, the sum rate is controlled by the fixed ICI threshold. We observe in Fig. 5.6b that the rate gain is increasing in Regime 1 and decreasing in Regime 2. To see why this happens, notice that, the ICI in the D2D mode is caused by both the CU and the D2D transmitter, while in the non-D2D mode, it is caused by the CU only. Hence, in non-D2D mode, the CU can use a higher power for transmission, and the corresponding rate gain by the D2D mode is reduced. Furthermore, comparing these three algorithms, we see that the optimal solution by the proposed algorithm provides significant sum rate improvement over the BL and CU-priority heuristics in both regimes for all values of $N$. Note that the BL heuristic outperforms the
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In order to study the effect of the D2D distance on the performance, the sum rate and rate gain versus $P_{\text{max}}/\sigma^2$ for $d_D/d_0 = 0.1$ and 0.2 are shown in Figs. 5.7a and 5.7b, respectively. We set $N = 4$. Note that for the proposed algorithm, the sum rate and rate gain improve significantly as $d_D/d_0$ decreases. However, the performance of the CU-priority heuristic is not sensitive to $d_D/d_0$. This is due to the objective of the sum rate of both the D2D and CU in Algorithm 5, resulting in significant rate improvement when the D2D channel is very strong, i.e., the D2D distance is small.

Let $\tilde{d}$ denote the D2D-BS distance, which is defined as the distance between the middle point of the D2D pair and the BS. The D2D transmitter and receiver are placed at $(0, -\tilde{d} - d_D/2)$ and $(0, -\tilde{d} + d_D/2)$, respectively. Then we have $d_{gC} = 0.5d_0 + \tilde{d} - d_D/2$, $d_{gD} = \tilde{d} + d_D/2$ and $d_{fD} = \sqrt{2\tilde{d}^2 + (d_{gD}/d_0)^2d_0}$. Figs. 5.8a and 5.8b show the sum rate and rate gain versus $\tilde{d}/d_0$, respectively. We set $N = 4$, $P_{\text{max}}/\sigma^2 = 10$ dB, and $d_D/d_0 = 0.1$ and 0.2. Increasing $\tilde{d}$ increases the distance between the CU and the D2D pair, resulting in reduced intra-cell interference at the BS and at the D2D receiver. Furthermore, the distance between the D2D transmitter and the neighboring BS increases, which leads to ICI reduction. As expected, both the sum rate and rate gain improve as $\tilde{d}/d_0$ increases. Furthermore, we observe from Fig. 5.8b that the gap of the rate gain between the optimal solution by Algorithm 5 and the CU-priority heuristic is significant; and it increases as the D2D-BS distance increases.

We now study the effect of the ICI threshold reference $I_0$ on the performance. The sum rate and the rate gain versus $I_0/\sigma^2$ are shown in Figs. 5.9a and 5.9b, respectively, for $d_D/d_0 = 0.2$ and $P_{\text{max}}/\sigma^2 = 20$ dB. We observe that both the sum rate and rate gain improve when $I_0$ increases. For small $I_0$ values, the sum rate is an increasing function of $I_0/\sigma^2$ since the ICI is relatively strong (the case in Section 5.3.4), and the candidates for the optimal power pair are functions of $I_0/\sigma^2$. As $I_0$ increases, the ICI constraint becomes inactive as the case in Section 5.3.5 and the sum rate converges due to the fixed $P_{\text{max}}$.

### 5.5.2 Multiple Neighboring Cells and Multiple Users

We now consider multiple neighboring cells and study the performance of Algorithm 5. Besides multiple neighboring cells, we further consider multiple CUs and D2D pairs in the cell of interest as discussed in Section 5.4. We set the number of neighboring cells as $b = 6$. We consider 3 CUs and 3 D2D pairs that are randomly dropped in the cell of interest. We compare the following schemes: 1) optimal power control and optimal CU-D2D matching, where the optimal power control is obtained by exhaustive search; 2) proposed approach in Section 5.4, i.e., the approximate power control (Algorithm 5) and optimal CU-D2D matching; 3) BL heuristic power control and optimal CU-D2D matching; 4) CU-priority heuristic power control and optimal CU-D2D matching; 5) approximate power control and suboptimal CU-D2D matching A given in Section 5.4; 6) approximate power control and suboptimal CU-D2D matching B given in Section 5.4.

In Figs. 5.10a and 5.10b, the sum rate and rate gain versus $P_{\text{max}}/\sigma^2$ under different power control methods and matching schemes are shown. We set $N = 4$ and $d_D/d_0 = 0.1$. We observe that the performance of our proposed approach is close to that of the optimal power control with optimal CU-D2D matching. In particular, in the region where the sum rate is an increasing function of $P_{\text{max}}$, the performance by both power control schemes overlap. This demonstrates the merit of our proposed approximate power control algorithm to provide a simple closed-form solution. In addition, it can be seen that the approximate power control algorithm with any of the three CU-D2D matching schemes
outperforms the BL and CU-priority heuristics with optimal CU-D2D matching. Furthermore, for the approximate power control algorithm, the gap between optimal CU-D2D matching and suboptimal CU-
D2D matching A is small. This is because when CU-D2D matching A is used to define the bipartite graph, the intra-cell interference a CU causes to the matched D2D receiver is small. This results in a high D2D rate.

Note that unlike the optimal CU-D2D matching solution, where the optimal powers for pairing each D2D pair and a CU are needed to determine the best matching, for the suboptimal CU-D2D matching A scheme, only the channel power of the intra-cell interference channel needs to be known at the BS. As a result, the computational complexity of suboptimum CU-D2D matching A is drastically reduced.

### 5.6 Summary

In this chapter, we have studied power control to maximize the sum rate of a CU and a D2D pair, subject to minimum SINR requirements, power constraints, and worst-case ICI constraints to neighboring cells. With optimal BS receive beamforming for uplink transmission, we have proposed an efficient approximate power control algorithm to obtain the powers of the CU and D2D transmitters in closed form. Depending on the ICI conditions from the CU and the D2D pair, we have divided the problem into five cases, each including several different scenarios due to minimum SINR requirements. The proposed algorithm is optimal when the ICI to a single neighboring cell is considered. For multiple neighboring cells, we have given a performance bound on our proposed algorithm, and further provided conditions for which our approximation becomes optimal.

We have further considered the general scenario of multiple CUs and D2D pairs, and have shown how our previously proposed solution can be utilized to find a solution to the joint power control and CU-D2D matching problem. Simulation demonstrates that substantial performance gain can be achieved by our proposed power control algorithm over two alternative approaches. It also shows our proposed approach provides close to optimal performance for the scenario of multiple CUs and D2D pairs, despite its low complexity.

### Appendix A

**Proof of Lemma 3**

*Proof.* Considering (5.12) with equality, we rewrite it as

\[
y = \eta(x) = \alpha \left( 1 - \frac{K_1}{1 + K_2} \right)^{-1}
\]  

(A.1)

where \( \alpha \triangleq \frac{\sigma^2 \tilde{\gamma}}{\|h_C\|^2}, \ K_1 \triangleq \rho^2, \) and \( K_2 \triangleq \frac{\sigma^2}{\|g_D\|^2}. \) Taking the first and second derivatives, we have

\[
\frac{d\eta(x)}{dx} = \frac{\alpha K_1 K_2}{(x + K_2)^2} \left(1 - \frac{K_1}{1 + K_2/x}\right)^{-2} > 0,
\]

\[
\frac{d^2\eta(x)}{dx^2} = \frac{2\alpha K_1 K_2 (K_1 - 1)}{(x + K_2)^3} \left(1 - \frac{K_1}{1 + K_2/x}\right)^{-3} \leq 0,
\]
since $K_1 \leq 1$ and $K_2 > 0$, i.e., $\eta(x)$ is a concave strictly increasing function. Note that constraint (5.5) is characterized by a line on the power plane, i.e., $\frac{x}{\sigma_D^2 + K_3y} = \beta$.

Solving the intersection of this line and the curve (A.1), we obtain (5.13).

\[ \text{Appendix B} \]

**Proof of Lemma 4**

Proof. Given any power pair $(x, y)$ in the interior of $\mathcal{A}_{xy}$, there exists $\zeta > 1$, such that $(\zeta x, \zeta y) \in \mathcal{A}_{xy}$. In the following, we show that $\mathcal{R}(\zeta x, \zeta y) > \mathcal{R}(x, y)$. Substituting $(\zeta x, \zeta y)$ into (5.23), we have

\[ \mathcal{R}(\zeta x, \zeta y) = (1 + \frac{ax}{\sigma_D^2/\zeta + K_3y})(1 + \Phi(\zeta)) \quad (B.1) \]

where $\Phi(\zeta) = \zeta y\left(1 - \frac{K_2}{K_2/\zeta + x}\right)l$.

It is straightforward to show that $1 + \frac{ax}{\sigma_D^2/\zeta + K_3y} > 1 + \frac{ax}{\sigma_D^2 + K_3y}$ for $\zeta > 1$. In order to complete the proof, it is sufficient to show that $\Phi(\zeta)$ is an increasing function of $\zeta$ for a given $(x, y)$. Taking the first derivative, we have

\[ \frac{d\Phi(\zeta)}{d\zeta} = ly\frac{xK_2(1-K_1) + \varphi}{\zeta(x + K_2/\zeta)^2} > 0 \quad (B.2) \]

where $\varphi = \zeta x^2(1-K_1) + K_2/\zeta(K_2/\zeta + x(1-K_1))$ since $K_1 \leq 1$ and $\varphi > 0$. Therefore, we have $\mathcal{R}(\zeta x, \zeta y) > \mathcal{R}(x, y)$, i.e., the optimal solution pair $(x^o, y^o)$ is not in the interior of $\mathcal{A}_{xy}$.

\[ \text{Appendix C} \]

**Proof of Proposition 8**

Proof. Substituting $y = P_C^{max}$ and $x = P_D^{max}$ into (5.23), we define $h(x) \triangleq \mathcal{R}(x, P_C^{max})$ and $g(y) \triangleq \mathcal{R}(P_D^{max}, y)$. To show that the maximum of $g(y)$ for $\bar{a} \leq y \leq \bar{b}$ is obtained when $y^o$ is at either $\bar{a}$ or $\bar{b}$, it is sufficient to show that $g(y)$ is a strictly monotonic function, or $g(y)$ is a strictly convex function. In the following, we show that $g(y)$ is such a function. In both cases, since $P2$ is a maximization problem, $y^o$ is an end point of the domain determined by $\mathcal{A}_{xy}$. A similar proof is also provided for $h(x)$.

The function $g(y)$ can be written as $g(y) = (1 + \frac{\alpha_1}{\alpha_2 + y})(1 + \alpha_3y)$ where $\alpha_1 \triangleq aP_D^{max}/K_3$, $\alpha_2 \triangleq \sigma_D^2/K_4$, and $\alpha_3 \triangleq l\left(1 - \frac{K_1P_D^{max}}{K_2 + P_D^{max}}\right)$.

Taking the first derivative of $g(y)$, we have

\[ \frac{dg(y)}{dy} = \alpha_3y^2 + 2\alpha_2\alpha_3y + \mu \quad (C.1) \]

where $\mu \triangleq \alpha_3\alpha_2^2 + \alpha_1(\alpha_2\alpha_3 - 1)$. Since $\alpha_2 > 0$, $\alpha_3 > 0$, and $y \geq 0$, either $\frac{dg(y)}{dy} > 0$, i.e., $g(y)$ is a strictly increasing function or $\frac{dg(y)}{dy} = 0$ may have a valid solution only if $\mu < 0$. Supposing $\mu < 0$ and taking the second derivative, we have $\frac{d^2g(y)}{dy^2} = 2\alpha_1(1 - \alpha_2\alpha_3)\alpha_3y^2 > 0$, since $\mu < 0$ implies $\alpha_1(1 - \alpha_2\alpha_3) > 0$. In other words, $g(y)$ is a convex function.
Similarly, \( h(x) \) can be written as \( h(x) = (1 + \beta_1 x) \left( 1 + \beta_2 \left( 1 - \frac{K_1}{K_2 + x} \right) \right) \) where \( \beta_1 \triangleq \frac{b}{\sigma_D^2 + K_3 P_C^\text{max}} \) and \( \beta_2 \triangleq P_C^\text{max} \). Taking the first derivative of \( h(x) \), we have \( \frac{d h(x)}{dx} = \frac{\dot{h}(x) + \omega}{(x + K_2)^2} \) where \( \dot{h}(x) \triangleq \beta_1 \left( 1 + \beta_2 (1 - K_1) \right) \).

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Appendix D

Proof of Proposition 9

Proof. By Proposition 8, if \( x^o = P_D^\text{max} \) or \( y^o = P_C^\text{max} \), then \((x^o, y^o)\) is an end point of the vertical or horizontal boundary line segment of \( \mathcal{A}_{xy} \), respectively. If ICI constraint (5.22) is active at optimality, the optimal power is the solution of the following optimization problem:

\[
\max_{(x,y)} \left( 1 + \frac{ax}{\sigma_D^2 + K_3 y} \right) \left( 1 + y \left( 1 - \frac{K_1 x}{K_2 + x} \right) l \right)
\]

subject to \( c_1 y + c_2 x = 1 \).

Substituting \( y = (1 - c_2 x)/c_1 \) into the objective function above, we have \( \max_x \tilde{R}(x) \), where \( \tilde{R}(x) \triangleq (1 + \frac{ax}{a_1 - K_4 x}) \left( 1 + (b_1 - b_2 x) (1 - \frac{K_1 x}{K_2 + x}) \right) \). Since \( \tilde{R}(x) \) is continuous and has a first-order derivative, the optimum \( x^o \) is either the \( x \)-coordinate of an end point of the tilted line segment of \( \mathcal{A}_{xy} \) or obtained by solving \( \frac{d \tilde{R}(x)}{dx} = 0 \), which results in the quartic equation \( e_4 x^4 + e_3 x^3 + e_2 x^2 + e_1 x + e_0 = 0 \) where

\[
\begin{align*}
e_0 & \triangleq a_1 K_2^2 (b_1 + 1) - a_1^2 b_1 K_1 K_2 - a_1^2 b_2 K_2^2 \quad (D.1) \\
e_1 & \triangleq -2a_1 b_2 K_2^2 + a_1 K_2 (b_1 + 1) - 2a_1 K_1 K_2 b_1 \\
 & \quad + a_1 b_1 K_2 + 2a_1^2 b_2 K_2 (K_1 - 1) + 2a_1 a_2 b_2 K_2^2 \\
 & \quad + a_1 K_2 + 2a_1 a_2 b_1 K_1 K_2 \quad (D.2) \\
e_2 & \triangleq a_1 b_2 K_2 (3K_1 - 4) + a_1 (1 + b_1 (1 - K_1)) \\
 & \quad + a_2 b_1 K_2 (a - a_2) - 4a_1 a_2 b_2 K_2 (K_1 - 1) \\
 & \quad + a_2 b_2 K_2^2 (a - a_2) - a_1^2 b_2 (1 - K_1) \quad (D.3) \\
e_3 & \triangleq -2a_1 b_2 (1 - K_1) + 2a_1 a_2 b_2 (1 - K_1) \\
 & \quad - 2a_2 b_2 K_2 (K_1 - 1) (a - a_2) \quad (D.4) \\
e_4 & \triangleq a_2 (a - a_2) b_2 (1 - K_1), \quad (D.5)
\end{align*}
\]

with \( a_1 \triangleq \sigma_D^2 + K_3/c_1, a_2 \triangleq K_3 c_2/c_1, b_1 \triangleq l/c_1, \) and \( b_2 \triangleq l c_2/c_1 \). □
Appendix E
Proof of Lemma 5

Proof. We show that the alternative solution of the quadratic equation is not feasible, i.e.,
\[
\hat{x}_F = \frac{\theta - \sqrt{\theta^2 - 4c_2(1 - K_1)K_2(\alpha c_1 - 1)}}{2c_2(1 - K_1)} < 0. \tag{E.1}
\]
In order to reject \(\hat{x}_F\), it is sufficient to show that \(\alpha c_1 < 1\). Substituting \(P_C^{\text{max}}\) into (5.12) and considering \(\rho^2 \leq 1\), we have
\[
\frac{P_C^{\text{max}} \| h_C \|^2}{\sigma^2} > \tilde{\gamma}_C. \tag{E.2}
\]
Rearranging the inequality (E.2), we have \(\alpha c_1 < c_1 P_C^{\text{max}} < 1\), where the first and second inequality hold due to the definition of \(\alpha\) and condition of moderate ICI from CU (5.25), respectively. \(\square\)

Appendix F
Proof of Proposition 10

Proof. Defining \(R_L \overset{\Delta}{=} \max_{l \in L} \{R_l\}\), we have
\[
R^{A_1} \geq R_L. \tag{F.1}
\]
A new feasible region \(\mathcal{A}_{xy} = \mathcal{A}_{xy}^o \cup \mathcal{A}_3\) is formed by replacing the corner of \(\mathcal{A}_{xy}^o\) with a rectangular one including \(U_3\). Since \(\mathcal{A}_{xy}^o \subseteq \mathcal{A}_{xy}\), we have
\[
R^{\text{opt}} \leq R_U = \max \{R_{U_1}, R_{U_2}, R_{U_3}\} \tag{F.2}
\]
where the right-hand side of (F.2) is the optimal sum rate within \(\mathcal{A}_{xy}\) by Proposition 8. Combining (F.1) and (F.2), we have the upper bound on the sum-rate loss of of Algorithm 5 given by (5.56).

Based on Condition 1), we have \(\mathcal{A}_{xy} = \mathcal{A}_{xy}^o\), and thus the solution of Algorithm 5 is optimal. For Conditions 2), it means the line and curve associated with (5.20) and (5.19), respectively, both intersect the vertical boundary of \(\mathcal{A}_{xy}^o\), i.e., \(L_1 = U_1\) and \(L_2 = U_2\). In this case, by definition of \(U_3\), we have \(U_3 = U_1\). By (F.1) and (F.2), we have \(\max \{R_{U_1}, R_{U_2}\} \leq R^{A_1} \leq R^{\text{opt}} \leq \max \{R_{U_1}, R_{U_2}\}\). Thus, \(R^{A_1} = R^{\text{opt}}\). For Condition 3), the line and curve both intersect the horizontal boundary of \(\mathcal{A}_{xy}^o\) with \(L_1 = U_1\) and \(L_2 = U_2\). Following a similar argument, we have \(U_3 = U_2\), and \(R^{A_1} = R^{\text{opt}}\). \(\square\)
Chapter 6

Robust Power Optimization for D2D Systems

For D2D underlaid cellular networks with uplink resource sharing, D2D pairs cause ICI to multiple neighboring cells which should be carefully considered in the design of resource sharing. In this chapter, we jointly optimize the CU receive beamforming and power control of CUs and D2D pairs under ICI constraints to neighboring cells in a multi-cell network. We first consider maximizing the sum rate of a CU and a D2D pair under perfect CSI, while satisfying minimum SINR requirements, power budget, and worst-case ICI limits at neighboring cells, and present an analytical method to obtain a semi-closed-form solution to the joint optimization problem. Then, assuming only partial CSI is available, we study the robust power control problem by maximizing the expected sum rate for the CU and D2D pair under similar constraints with robust consideration. We solve this non-convex joint optimization problem in two steps. First, we consider the D2D admissibility problem to determine the suitability for reusing the channel resource of the CU. Then, we propose a robust power control algorithm using a ratio-of-expectation (ROE) approximation to maximize the expected sum rate. For benchmarking, we provide an upper bound on the maximum expected sum rate, and simulation results show that the proposed ROE algorithm gives performance close to the upper bound and hence is nearly optimal. Finally, we extend our consideration to the scenario of multiple CUs and D2D pairs, and formulate the joint power control and CU-D2D matching problem. We show how our proposed solution for one CU and one D2D pair can be leveraged to solve this more general joint optimization problem.

6.1 System Model and Problem Formulation

6.1.1 Cellular System with D2D Pairs

Consider a cellular system where the D2D pairs reuse the spectrum resource already assigned to the CUs for uplink communication. We assume that all users are equipped with a single antenna, and the BS is equipped with $N$ antennas. The BS coordinates the transmission of the CU and D2D pair. We follow the conventional assumption of orthogonal spectrum resource allocation among CUs in a cell. Thus, these CUs do not interfere with each other. When a D2D pair communicates using the channel of a CU, they cause intra-cell interference to each other. Due to orthogonal channelization within each cell,
we may initially focus on one CU and one D2D pair as shown in Fig 6.1. In Section 6.4, we extend our consideration to the scenario of multiple CUs and D2D pairs.

### 6.1.2 SINR and ICI Expressions

Let $P_D$ and $P_C$ denote the transmit power of the D2D pair and the CU, respectively. The uplink received SINR at the BS from the CU is given by

$$\gamma_C = \frac{P_C |w^H h_C|^2}{\sigma^2 + P_D |w^H g_D|^2}$$

(6.1)

where $h_C \in \mathbb{C}^{N \times 1}$ is the channel between the CU and the BS, $g_D \in \mathbb{C}^{N \times 1}$ is the interference channel between the D2D transmitter and the BS, $w$ is the receive beam vector at the BS with unit norm, i.e., $\|w\|^2 = 1$, and $\sigma^2$ is the noise variance at the BS. The SINR at the D2D receiver is given by

$$\gamma_D = \frac{P_D |h_D|^2}{\sigma_D^2 + P_C |g_C|^2}$$

(6.2)

where $h_D \in \mathbb{C}$ is the channel between the D2D pair, $g_C \in \mathbb{C}$ is the interference channel between the CU and the D2D receiver, and $\sigma_D^2$ is the noise variance at the D2D receiver.\(^1\)

In a multi-cell network, both D2D and CU transmissions cause ICI in neighboring cells. In this chapter, we consider ICI for uplink transmission at $b$ neighboring BSs. However, our approach can be applied also to ICI in a downlink scenario. Let $f_{C,j} \in \mathbb{C}^{N \times 1}$ and $f_{D,j} \in \mathbb{C}^{N \times 1}$ denote the ICI channels from the CU and the D2D transmitter to neighboring BS $j$, respectively, for $j = 1, \cdots, b$. Since the beam vector at neighboring BS $j$ is typically unknown to the CU and D2D pair, we consider the worst-case

\(^1\)The noise term in SINR expressions, i.e., $\sigma^2$ and $\sigma_D^2$ can be treated as the receiver noise plus inter-cell interference power.
ICI given by\(^2\)

\[ P_{I,j} = P_C \| f_{C,j} \|^2 + P_D \| f_{D,j} \|^2. \]  

(6.3)

### 6.1.3 CSI Availability

To capture the tradeoff between performance and signaling overhead due to the requirement of CSI feedback, we consider the following two scenarios.

In **Scenario 1**, we assume instantaneous CSI is available for the communication channels and intra-cell interfering channels, *i.e.*, \( h_C \), \( h_D \), \( g_c \), and \( g_D \). For the ICI channels, only channel power gains are known at the BS scheduler. These parameters can be estimated in neighboring BSs and shared with the BS in the desired cell through the wired backhaul.

In **Scenario 2**, we assume instantaneous CSI is available only for \( h_C \) and \( g_D \), *i.e.*, the direct channels from the CU and D2D to the BS in Fig. 6.1. However, only partial CSI is available for \( h_D \), \( g_C \), \( \{f_{D,j}\}_{j=1}^b \), and \( \{f_{C,j}\}_{j=1}^b \). In particular, only distance-based statistical knowledge is available at the BS scheduler. We assume \( |h_D|^2 \sim \exp(\eta_1) \) and \( |g_C|^2 \sim \exp(\eta_2) \), which corresponds to the common Rayleigh fading model. Instead of instantaneous CSI, we assume \( \eta_1 \) and \( \eta_2 \) are known at the BS. Note that measuring and transmitting these statistical parameters is much easier than the instantaneous CSI [56]. This substantially reduces the signaling overhead due to D2D communication. For the ICI channels, we assume \( \mathbb{E}[|f_{D,j}|^2] = \lambda_{D,j} \) and \( \mathbb{E}[|f_{C,j}|^2] = \lambda_{C,j} \) for \( j = 1, \cdots, b \), where only \( \{\lambda_{D,j}\}_{j=1}^b \) and \( \{\lambda_{D,j}\}_{j=1}^b \) are known at the BS scheduler.

The sum rate and expected sum rate maximization problems for Scenario 1 and 2 are formulated in the following section.

### 6.1.4 Problem Formulation

#### Scenario 1 (Perfect CSI)

Let \( P_{C}^{\text{max}} \) and \( P_{D}^{\text{max}} \) denote the maximum transmit power at the CU and D2D transmitters, respectively. Our goal is to maximize the sum rate of the D2D pair and the CU uplink transmission by optimizing the transmit powers \( \{P_D, P_C\} \) and the beam vector \( \mathbf{w} \), under per-node power and ICI constraints, as well as SINR requirements for both the CU and the D2D pair. The problem is formulated as follows:

\[
P_1: \max_{(P_D, P_C, \mathbf{w})} \left( \log_2(1 + \gamma_C) + \log_2(1 + \gamma_D) \right)
\]

subject to\(^2\)

\[ \gamma_C \geq \gamma_C^{\text{min}}, \]  

(6.4)

\[ \gamma_D \geq \gamma_D^{\text{min}}, \]  

(6.5)

\[ P_C \leq P_C^{\text{max}}, \quad P_D \leq P_D^{\text{max}}, \]  

(6.6)

\[ P_{I,j} \leq I_{\text{max}}, \quad j = 1, \cdots, b \]  

(6.7)

where \( \gamma_C^{\text{min}} \) and \( \gamma_D^{\text{min}} \) are the minimum SINR requirements for the CU and D2D pair, respectively, and \( I_{\text{max}} \) is the worst-case ICI threshold in neighboring cells.

\(^2\)Note that \( P_{I,j} \) in (6.3) is an an upper bound of the actual ICI. Let \( \tilde{\mathbf{w}}_j \) denote the beam vector at neighboring BS \( j \). If \( \tilde{\mathbf{w}}_j \) is known, then we can consider the actual ICI through replacing \( \|f_j\| \) by \( \|\tilde{\mathbf{w}}_j f_j\| \) in (6.3).
Scenario 2 (Partial CSI)

Our goal is to maximize the expected sum rate by optimizing \( \{P_D, P_C, w\} \), under SINR requirements for both the CU and the D2D pair, per-node maximum power, and ICI constraints. Due to the partial CSI assumptions explained in Section 6.1.3, the D2D SINR and ICI at each neighboring BS are random variables. For the D2D pair’s SINR requirement, we consider a probabilistic constraint limiting the outage probability of the D2D pair. We also limit the expected worst-case ICI. Thus, the expected sum rate maximization problem is given by

\[
Q_1 : \max_{\{P_D, P_C, w\}} \left( \log_2(1 + \gamma_C) + E[\log_2(1 + \gamma_D)] \right)
\]

\[
\Pr{\{\gamma_D \leq \gamma_{D,\text{min}}\}} \leq \epsilon, \quad (6.8)
\]

\[
E[P_{I,j}] \leq I_{\text{max}}, \quad j = 1, \ldots, b, \quad (6.9)
\]

(6.4) and (6.6).

where \( \epsilon \) is the maximum probability of the D2D SINR dropping below \( \gamma_{D,\text{min}} \).

In the following, we solve optimization problems \( \mathcal{P}_1 \) and \( \mathcal{Q}_1 \). These problems are non-convex, since the objective functions are non-convex. We solve each of \( \mathcal{P}_1 \) and \( \mathcal{Q}_1 \) in two steps. First, we need to ensure whether the D2D pair can be admitted to reuse the CU’s assigned channel. Then, if the D2D pair is admissible, we attempt to optimize the powers and beam vector to maximize the (expected) sum rate. We recast the first problem as a feasibility test. Then we obtain the power solution.

6.2 Admissibility Test and Power Control for \( \mathcal{P}_1 \)

6.2.1 Admissibility Condition

Given the power constraints, SINR requirements, and ICI constraints, the admissibility of the D2D pair in Scenario 1 can be determined by solving the feasibility test given by

\[
\text{find } \{P_D, P_C, w\} \quad (6.10)
\]

subject to (6.4), (6.5), (6.6), and (6.7).

Following a similar argument in Chapter 5, we first obtain the optimal beam vector \( w \) in terms of \( \{P_C, P_D\} \) that maximizes \( \gamma_C \) at the left-hand side of constraint (6.4). For a given set of \( \{P_C, P_D\} \), the optimal beam vector is given by

\[
w^* = \frac{\Lambda_D^{-1}h_C}{\|\Lambda_D^{-1}h_C\|} \quad (6.11)
\]

where \( \Lambda_D \triangleq \sigma^2 I + P_D g_D g_D^H \).

For notation simplicity, in the following, we denote \( x \triangleq P_D \) and \( y \triangleq P_C \). The D2D admissibility
condition in Scenario 1 is given by
\begin{align}
0 < x_T & \leq P_D^{\text{max}}, \\
0 < y_T & \leq P_C^{\text{max}}, \\
c_{1,j}y_T + c_{2,j}x_T & \leq 1, \ j = 1, \ldots, b
\end{align}
(6.12, 6.13, 6.14)

where \(c_{1,j} \triangleq ||f_{C,j}||^2/T_C^{\text{max}}\) and \(c_{2,j} \triangleq ||f_{D,j}||^2/T_D^{\text{max}}\) and \(s_T \triangleq [x_T, y_T]^T\) is uniquely given by
\begin{align}
x_T &= \frac{\xi}{2(1 - K_1)}, \quad y_T = \frac{\xi}{2(1 - K_1)}\beta K_3 - \frac{\sigma_D^2}{K_3}
\end{align}
(6.15)

where \(\alpha \triangleq \frac{\sigma_C^2 \gamma_C^{\min} \xi}{||h_C||^2}, \beta \triangleq \frac{\gamma_D^{\min} \xi}{||h_D||^2}, K_1 \triangleq \frac{||h_D||^2 ||s_D||^2}{||h_C||^2 ||s_D||^2}, K_2 \triangleq \frac{\sigma_D^2}{||s_D||^2}, K_3 \triangleq \frac{||g_C||^2}{2}, K_4 \triangleq \beta(\alpha K_3 + \sigma_D^2(1 - K_1)) - K_2, K_5 \triangleq 4(1 - K_1)\beta K_2(\alpha K_3 + \sigma_D^2), \) and
\[\xi = \beta(\alpha K_3 + \sigma_D^2(1 - K_1)) - K_2 + \sqrt{K_4^2 + K_5}.
\]

We note that constraints (6.12) and (6.13) ensure the maximum powers at the D2D and CU are enough to satisfy both SINR requirements. Constraint (6.14) guarantees the ICI limits can be met.

### 6.2.2 Overview of Power Control Solution

Assuming the D2D pair is admissible, we solve \(P_1\) as an optimal power allocation problem. After substituting \(w^o\) into (6.1), and defining \(x\) and \(y\) as in the previous subsection, we transform \(P_1\) into the following:
\begin{align}
P_2 : \quad & \max_{(x,y)} R(x,y) \\
\text{subject to} \quad & y \left(1 - \frac{K_1 x}{K_2 + x}\right) \geq \gamma_C^{\min}, \\
& \frac{ax}{\sigma_D^2 + K_3 y} \geq \gamma_D^{\min}, \\
& y \leq P_C^{\text{max}}, \quad x \leq P_D^{\text{max}}, \\
& c_{1,j}y + c_{2,j}x \leq 1, \ j = 1, \ldots, b
\end{align}
(6.16, 6.17, 6.18, 6.19)

where \(R(x,y) = \log_2 \left(1 + \frac{y(1 - K_1 x)}{K_2 + x}(1 + \frac{ax}{\sigma_D^2 + K_3 y})\right), l = ||h_C||^2/\sigma^2, \) and \(a \triangleq ||h_D||^2/\sigma^2.\)

Let \(A_{xy}\) denote the feasible solution region of problem \(P_2.\) By definition, \(A_{xy}\) is non-empty as long as the D2D pair is admissible. An example of \(A_{xy}\) for a specific scenario is shown in Fig. 6.2.

We can now solve \(P_2\) by generalizing our algorithm in Chapter 5, which was proposed for a single ICI constraint, to a scenario with multiple ICI constraints. Following similar arguments in Chapter 5, we first summarize the properties of the optimal power pair \((x^o, y^o):\) The optimal power solution pair is at the vertical, horizontal, or a tilted boundary of \(A_{xy}.\) If the boundaries of the feasible region \(A_{xy}\) do not include any tilted boundary line segment, then the optimal power pair is at one corner point of the vertical or horizontal boundary. If the boundaries of the feasible region \(A_{xy}\) include \(c_{1,j}y + c_{2,j}x = 1\) for some \(j,\) then the optimal power pair is given in one of two cases: 1) A corner point of the horizontal, vertical, or tilted boundary line segment(s) of \(A_{xy};\) or 2) an interior point of the tilted boundary line...
segment(s) of $A_{xy}$, whose $x$-coordinate is one of the roots of the following quartic equation

$$e_4x^4 + e_3x^3 + e_2x^2 + e_1x + e_0 = 0$$

(6.20)

where $\{e_i\}_{i=0}^{4}$ are given in Chapter 5.

### 6.2.3 Algorithm Details

In the proposed algorithm, we first iteratively compute the feasible region when a new tilted line is considered one at a time (i.e., when a new ICI constraint is considered in a neighboring cell). We present a means to efficiently check whether the feasible region is non-empty, thereby testing the admissibility of the D2D pair. Then, the candidates for optimal power solution are identified and computed through root finding in equations of the form of (6.20). The detailed steps of the algorithm are as follows.

#### Initialization

We define two matrices:

$$C \triangleq \begin{bmatrix} 0 & 0 & P_{D}^{\text{max}} & P_{D}^{\text{max}} & 0 \\ 0 & P_{C}^{\text{max}} & P_{C}^{\text{max}} & 0 & 0 \end{bmatrix},$$

(6.21)

which includes the corner points of the feasible region by considering constraints (6.18) and (6.19), i.e., ignoring the SINR requirements for the CU and D2D pair; and

$$A \triangleq \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & P_{C}^{\text{max}} & P_{D}^{\text{max}} & 0 \end{bmatrix},$$

(6.22)
which specifies the line segment connecting any two consecutive corner points in \( \mathbf{C} \). Note that this initial \( \mathbf{C} \) is constructed using only constraint (6.18). The first and last columns of the initial \( \mathbf{C} \) are \([0 0]^T\), i.e., the origin coordinates. The other corner points are in the columns of the initial \( \mathbf{C} \) in a clock-wise order. For matrix \( \mathbf{A} \), the column \( \mathbf{A}_{:,i} \) is \([A_{i1} A_{i2} A_{i3}]^T\) when the line segment between \( \mathbf{C}_{:,i} \) and \( \mathbf{C}_{:,i+1} \) is \( A_{i1}x + A_{i2}y = A_{i3} \). The intersection of this line segment and \( c_{1,j}y + c_{2,j}x = 1 \) is \( s_{i,j} = \begin{bmatrix} s_{x,i,j} \\ s_{y,i,j} \end{bmatrix} \) where

\[
\begin{align*}
\frac{A_{i2} - c_{1,j}A_{i3}}{c_{2,j}A_{i2} - c_{1,j}A_{i1}} & \quad \text{and} \quad \frac{c_{2,j}A_{i3} - A_{i1}}{c_{2,j}A_{i2} - c_{1,j}A_{i1}}.
\end{align*}
\]

We further define

\[
\Delta \triangleq \begin{bmatrix} -1 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \delta \triangleq \begin{bmatrix} 0 \\ 0 \\ P_{\text{max}}^D \\ P_{\text{max}}^C \end{bmatrix}.
\]

Admissibility Test

Let \( \mathbf{A}_{xy,j} \) denote the updated feasible region after considering ICI constraint \( j \). Then we add a new tilted line \( l \) due to ICI constraint \( j + 1 \) as shown in Fig (6.3). Note that \( l \) intersects \( \mathbf{A}_{xy,j} \) in exactly two points if there is any intersection at all. We denote \( \{P_{i1}, P_{i1+1}, \ldots, P_{i2}, P_{i2+1}\} \subset \mathbf{A}_{xy,j} \) a set of corner points in \( \mathbf{A}_{xy,j} \) such that the intersections of \( l \) with \( \mathbf{A}_{xy,j} \) are on the lines specified by \( \mathbf{A}_{:,i1} \) and \( \mathbf{A}_{:,i2} \). Since \( P_{i1} \) and \( P_{i2} \) are the corner points of the new feasible region \( \mathbf{A}_{xy,j+1} \), we update \( \mathbf{C} \) by keeping the corner points \( \{P_{i1}, P_{i2+1}\} \) and removing \( \{P_{i1+1}, \ldots, P_{i2}\} \), i.e., all the middle points. The new feasible
region $A_{xy,j+1}$ includes $\{P_i, \hat{P}_i, \tilde{P}_i, P_{i+1}\}$. Accordingly, we update the matrices $C$ and $A$.

In order to test the admissibility of the D2D pair, we consider the intersection of $A_{xy,b}$ with the curve and line associated with minimum SINR requirements (6.16) and (6.17). A necessary and sufficient condition for the D2D pair to be admissible is that the solution $s_I$ in (6.15) satisfies $\Delta \cdot s_I \preceq \delta$ where $\Delta$ and $\delta$ are obtained iteratively through the algorithm. The feasible region is specified as the shaded area in Fig. 6.2. The feasible region may include some tilted, horizontal, or vertical boundary line segments.

**Finding Corner Points**

In order to solve problem $P_2$, we need to find all candidates to be an optimal solution. Let $P$ and $Q$ denote the points where line $I - P$ and curve $I - Q$ intersect $A_{xy,b}$ as shown in Fig. 6.2. As discussed earlier, the optimal power pair $(x^o, y^o)$ can be one of points $\{P, P_1, \cdots, P_j, Q\}$ in this figure. The intersections of line $I - P$ and curve $I - Q$ with the horizontal boundary line segment $y = P_{max}^C$ are given by

$$P_H = \left[ \beta (\sigma_D^2 + K_3 P_{max}^C) P_{max}^C \right]^T,$$

(6.25)

$$Q_H = \left[ K_2 \left( \frac{K_1}{1 - \alpha / P_{max}^C} - 1 \right) \right]^{-1} P_{max}^C^T.$$

(6.26)

The intersections of line $I - P$ and curve $I - Q$ with the vertical boundary line segment $x = P_{max}^D$ are given by

$$P_V = \left[ P_{max}^D \frac{P_{max}^D - \beta \sigma_D^2}{\beta K_3} \right]^T,$$

(6.27)

$$Q_V = \left[ P_{max}^D, K_1 \left( \frac{1}{1 + K_2 / P_{max}^D} \right) \right]^{-1} P_{max}^C^T.$$

(6.28)

The intersections of line $I - P$ and curve $I - Q$ with a tilted boundary line segment $c_1, j y + c_2, j x = 1$ are given by

$$P_{T,j} = \left[ \psi_{1,j} \frac{1 - c_2, j \psi_{2,j}}{c_1, j} \right]^T,$$

(6.29)

$$Q_{T,j} = \left[ \psi_{1,j} \frac{1 - c_2, j \psi_{1,j}}{c_1, j} \right]^T$$

(6.30)

where

$$\psi_{1,j} \equiv \theta_j + \sqrt{\theta_j^2 - 4 c_2, j (1 - K_1) K_2 (\alpha c_1, j - 1) \over 2 c_2, j (1 - K_1)}$$

(6.31)

with $\theta_j \equiv 1 - K_1 - c_2, j K_2 - \alpha c_1, j$ and

$$\psi_{2,j} \equiv \frac{\sigma_D^2 \beta + \beta K_3 / c_1, j}{1 + \beta K_3 c_2, j / c_1, j}$$

(6.32)

for $j = 1, \cdots, b$. 

Finding Roots

Let $\mathbf{T}$ denote the set of all feasible corner points. Then we find all roots of (6.20), whose $x$-coordinates are within the range of two consecutive corner points in $\mathbf{T}$. Let $\mathbf{S}_j$ denote the set of roots that meet the range constraint for $\tilde{c}_{1,j} y + \tilde{c}_{2,j} x = 1$. Then the set of candidate points on the interior of line segment $c_{1,j} y + c_{2,j} x = 1$ is given by

$$
\mathbf{Z}_j = \left\{ \left[ x_r, (1 - c_{2,j} x_r)/c_{1,j} \right]^T : x_r \in \mathbf{S}_j \right\}.
$$

(6.33)

for $j = 1, \cdots, b$. Thus, the set of candidate pairs for $(x^o, y^o)$ is given by $\mathbf{P}^o = \mathbf{T} \cup_{j=1}^b \mathbf{Z}_j$.

The steps to solve $\mathcal{P}_2$ are summarized in Algorithm 6.

6.3 Admissibility Test and Power Control for $\mathcal{Q}_1$

6.3.1 Admissibility Condition

Given the power constraints, SINR requirements, and ICI constraints, the admissibility of the D2D pair in Scenario 2 can be determined by solving the feasibility test given by

$$
\text{find } \{P_D, P_C, w\} \quad \text{subject to } (6.4), (6.6), (6.8), \text{ and } (6.9).
$$

(6.34)

Following a similar argument as Section 6.2.1, for a given set of $\{P_C, P_D\}$, the optimal beam vector is given by (6.11). Substituting (6.11) into (6.1), the SINR constraint (6.4) is given by

$$
\frac{P_C \| \mathbf{h}_C \|^2}{\sigma^2} \left( 1 - \frac{K_1 P_D \| \mathbf{g}_D \|^2}{P_D \| \mathbf{g}_D \|^2 + \sigma^2} \right) \geq \gamma_{C \min}.
$$

(6.35)

Then, the necessary and sufficient condition for the D2D pair to be admissible in Scenario 2 is given in the following proposition.

Proposition 11. The necessary and sufficient condition for the D2D pair to be admissible in Scenario 2 is given by

$$
\tilde{c}_{1,j} y_I + \tilde{c}_{2,j} x_I \leq 1, \ j = 1, \cdots, b,
$$

(6.36)

(6.12) and (6.13)

where $\tilde{c}_{1,j} \triangleq \lambda_{C,j}/I_{\max}^C$, $\tilde{c}_{2,j} \triangleq \lambda_{D,j}/I_{\max}^D$, and $\tilde{s}_I \triangleq [\tilde{x}_I \tilde{y}_I]^T$ is the unique power solution of the following system of equations

$$
\begin{align*}
y &= \alpha \left( 1 - \frac{K_1}{1 + K_2/x} \right)^{-1} \\
y &= l_1 x \left( \frac{\exp(-l_2/x)}{1 - \epsilon} - 1 \right)
\end{align*}
$$

(6.37)

(6.38)

where $l_1 = \frac{\eta_2}{\eta_1 \gamma_D \min}$ and $l_2 = \eta_1 \sigma_D^2 \gamma_D \min$.
6.3.2 Overview of Power Control Solution

Assuming the D2D pair is admissible, we solve $Q_1$ as an optimal power allocation problem. After substituting $w^o$ into (6.1), we transform $Q_1$ into the following:

$$Q_2 : \max_{(x,y)} \bar{\mathcal{R}}(x,y)$$

$$y \leq l_1 x \left( \frac{\exp(-l_2/x)}{1 - \epsilon} - 1 \right),$$

(6.39)

$$\tilde{c}_{1,j} y + \tilde{c}_{2,j} x \leq 1, \ j = 1, \cdots, b, \hfill (6.40)$$

(6.16) and (6.18)

where $\bar{\mathcal{R}}(x,y) = \log_2 \left( \frac{1 + y(1 - \frac{\tilde{k}_{1,x}}{\tilde{k}_{2,x} + x})l) \right) + \mathbb{E}[\log_2(1 + \gamma_D)]$.

Let $\tilde{A}_{xy}$ denote the feasible solution region of problem $Q_2$, i.e., $\tilde{A}_{xy}$ is non-empty as long as the D2D pair is admissible. An example of $\tilde{A}_{xy}$ for a specific scenario is shown in Fig. 6.4.

Two properties of the objective function in $Q_2$ are provided in the following lemmas.

**Lemma 6.** The optimal power solution pair $(x^o, y^o)$ is at the vertical, horizontal, or a tilted boundary of $\tilde{A}_{xy}$, given respectively by $x = P_D^{\text{max}}$, $y = P_C^{\text{max}}$, or $\tilde{c}_{1,j} y + \tilde{c}_{2,j} x = 1$ for some $j$.

**Proof.** See Appendix B.

**Lemma 7.** The expected D2D rate is given by

$$\mathcal{R}_D = \frac{\eta_2 x \log_2(e)}{\eta_1 y - \eta_2 x} \left( E'(\eta_2 \sigma_D^2/y) - E'(\eta_1 \sigma_D^2/x) \right)$$

(6.41)
where \( E'(x) = \exp(x)E_1(x) \) and
\[
E_1(x) = \int_0^\infty \frac{\exp(-t)}{t} \, dt. \tag{6.42}
\]

Proof. See Appendix C. \( \square \)

In order to simplify the notations, we change the variables \( \hat{x} = \frac{n_2 \sigma_D^2}{x} \) and \( \hat{y} = \frac{n_2 \sigma_D^2}{y} \) in the following proposition. The vertical boundary line \( x = P_D^{\text{max}} \) and horizontal boundary line \( y = P_C^{\text{max}} \) can be represented by \( \hat{x} = v_1 \) and \( \hat{y} = v_2 \), respectively with \( v_1 = \frac{\eta_1 \sigma_D^2}{P_D^{\text{max}}} \) and \( v_2 = \frac{\eta_2 \sigma_D^2}{P_C^{\text{max}}} \). Similarly, the tilted boundary line \( \hat{c}_{1,j}/\hat{y} + \hat{c}_{2,j}/\hat{x} = 1 \) can be written as \( \hat{c}_{1,j}/\hat{y} + \hat{c}_{2,j}/\hat{x} = 1 \) with \( \hat{c}_{1,j} = \hat{c}_{1,j} \eta_2 \sigma_D^2 \) and \( \hat{c}_{2,j} = \hat{c}_{2,j} \eta_2 \sigma_D^2 \) for \( j = 1, \ldots, b \).

**Proposition 12.** The optimal power pair \((\hat{x}^*, \hat{y}^*)\) to maximize \( Q_2 \) is given in one of two cases: 1) A corner point of the horizontal, vertical, or tilted boundary line segment(s) of \( \bar{A}_{xy} \); or 2) an interior point of the horizontal, vertical, or tilted boundary line segment(s) of \( \bar{A}_{xy} \), where \( \hat{x} \) and \( \hat{y} \) are obtained in any of the following cases

1. Vertical boundary line: \( \hat{x} = v_1 \) and \( \hat{y} \) is a simple root of the equation
\[
-\hat{\alpha}_3(\hat{y} - v_1)^2 \frac{\hat{v}_1 E'(\hat{v}_1)}{\hat{f}_V(\hat{y})} - \frac{\hat{v}_1 v_1}{\hat{f}_V(\hat{y})} + \hat{y} - v_1 - E'(\hat{y}) = 0 \tag{6.43}
\]
where \( \hat{\alpha}_3 = \eta_2 \sigma_D^2 (1 - K_1 P_D^{\text{max}}/(K_2 + P_C^{\text{max}})) \) and \( \hat{f}_V(\hat{y}) = \hat{y}^2 - \hat{y} v_1 - v_1 \).

2. Horizontal boundary line: \( \hat{y} = v_2 \) and \( \hat{x} \) is a simple root of the equation
\[
-\hat{\beta} K_1 (v_2 - \hat{x})^2 \frac{\hat{v}_2 E'(\hat{v}_2)}{\hat{f}_H(\hat{x})} + \frac{\hat{v}_2 (v_2 - \hat{x})}{\hat{x} \hat{f}_H(\hat{x})} - E'(\hat{x}) = 0 \tag{6.44}
\]
where \( \hat{\beta} = \eta_2 \sigma_D^2 K_1, \hat{K}_1 = K_1 \eta_2 \sigma_D^2 /K_2, \hat{K}_2 = \eta_1 \sigma_D^2 /K_2, \hat{f}_H(\hat{x}) = v_2^2 + v_2 - v_2 \hat{x} \), and \( \hat{f}_H(\hat{x}) = (K_2 + \hat{x})(\hat{\beta} + 1)(\hat{K}_2 + \hat{x}) - \hat{K}_1 \hat{\beta} \hat{f}_H(\hat{x}) \).

3. Tilted boundary line: \( \hat{c}_{1,j}/\hat{y} + \hat{c}_{2,j}/\hat{x} = 1 \) where \( \hat{x} \) is a simple root of the equation
\[
f_{1,T}(\hat{x}) + f_{2,T} E'(\hat{x}) - f_{3,T} E'(g_T(\hat{x})) = 0 \tag{6.45}
\]
where \( \hat{c}_{1,j} = \eta_2 \sigma_D^2 \hat{c}_{1,j}, \hat{c}_{2,j} = \eta_1 \sigma_D^2 \hat{c}_{2,j}, \hat{b}_{1,j} = l/\hat{c}_{1,j}, \hat{b}_{2,j} = l \hat{c}_{2,j} /\hat{c}_{2,j}, \hat{f}_{2,T}(\hat{x}) = \frac{\hat{c}_{1,j}^2 \hat{c}_{2,j} - \hat{c}_{1,j} \hat{c}_{2,j} (\hat{c}_{1,j} + \hat{c}_{2,j} - x)}{(\hat{c}_{1,j} + \hat{c}_{2,j} - x)^2 + \hat{c}_{1,j} \hat{c}_{2,j}}, \hat{f}_{1,T}(\hat{x}) = -\frac{\hat{c}_{1,j}^2 \hat{c}_{2,j} - \hat{c}_{1,j} \hat{c}_{2,j}}{\hat{c}_{1,j} \hat{c}_{2,j} (\hat{c}_{1,j} + \hat{c}_{2,j} - x)^2}, \hat{f}_{1,T}(\hat{x}) = \hat{x} (\hat{c}_{1,j} + \hat{c}_{2,j} - x) \), \( f_{1,T}(\hat{x}) = -\frac{\hat{c}_{1,j}^2 \hat{c}_{2,j} - \hat{c}_{1,j} \hat{c}_{2,j}}{\hat{c}_{1,j} \hat{c}_{2,j} (\hat{c}_{1,j} + \hat{c}_{2,j} - x)^2 + \hat{c}_{1,j} \hat{c}_{2,j}}, \hat{f}_{1,T}(\hat{x}) = -\frac{\hat{c}_{1,j}^2 \hat{c}_{2,j} - \hat{c}_{1,j} \hat{c}_{2,j}}{\hat{c}_{1,j} \hat{c}_{2,j} (\hat{c}_{1,j} + \hat{c}_{2,j} - x)^2 + \hat{c}_{1,j} \hat{c}_{2,j}}, \hat{g}_T(\hat{x}) = \frac{\hat{c}_{1,j} \hat{x}}{\hat{x} \hat{c}_{2,j}}. \)

Proof. See Appendix D. \( \square \)

In order to solve \( Q_2 \) optimally, we need to solve a numerical equation to find the set of candidate power pairs over each boundary line. Unfortunately, there is no closed-form solution or efficient algorithm to solve those equations.\(^3\) Hence, to avoid such high computational complexity we propose to obtain the powers through approximating the objective function as follows.

---

\(^3\)Our extensive simulation results suggest that (6.43) and (6.44) do not have a valid solution in the specific intervals determined by \( \bar{A}_{xy} \). However, we still need to solve (6.45) numerically.
We replace $E[\log_2(1+\gamma_D)]$ in the objective of $Q_2$ by the ratio of expectation, i.e., $\log_2 \left( 1 + \frac{x E[|h_D|^2]}{\sigma_D^2 + y E[|g_C|^2]} \right)$. In other words, we propose to solve the following problem:

$$Q_3 : \max_{(x,y)} \tilde{R}(x,y)$$

subject to (6.16), (6.18), (6.39), and (6.40)

where $\tilde{R}(x,y) = \log_2 \left( (1 + y(1 - \frac{K_1}{K_2 x + x})l) \left( 1 + \frac{x/m_1}{\sigma_D^2 + y/m_2} \right) \right)$. Note that the objective of $Q_3$ has a form similar to that of $P_3$. Therefore, it can be solved by a similar method as in Algorithm 6. We term this the ROE approximation algorithm. We observe through simulation and comparison with an upper bound presented in Section 6.3.4 that the ROE algorithm incurs little performance degradation for a wide range of parameter settings.

6.3.3 Algorithm Details

In the proposed algorithm, we first iteratively compute the feasible region and present an efficient method to test the admissibility of the D2D pair. Then, the candidates for an optimal power solution of $Q_3$ are obtained. The detailed steps of the algorithm are as follows.

Initialization

We initialize $\tilde{C}$, $\tilde{A}$, and $\{\tilde{\delta}, \tilde{\Delta}\}$, as in the right-hand side of (6.21), (6.22), and (6.24), respectively. For matrix $\tilde{A}$, the column $\tilde{A}_{i,:}$ is $[\tilde{A}_{i1} \tilde{A}_{i2} \tilde{A}_{i3}]^T$ when the line segment between $\tilde{C}_{i,:}$ and $\tilde{C}_{i+1,:}$ is $\tilde{A}_{i1} x + \tilde{A}_{i2} y = \tilde{A}_{i3}$. The intersection of this line segment and $c_{1,j} y + c_{2,j} x = 1$ is $\tilde{s}_{i,j} = [\tilde{s}_{x,i,j} \tilde{s}_{y,i,j}]^T$ where

$$\tilde{s}_{x,i,j} = \frac{\tilde{A}_{i2} - \tilde{c}_{1,j} \tilde{A}_{i3}}{\tilde{c}_{2,j} \tilde{A}_{i2} - \tilde{c}_{1,j} \tilde{A}_{i1}} \quad \text{and} \quad \tilde{s}_{y,i,j} = \frac{\tilde{c}_{2,j} \tilde{A}_{i3} - \tilde{A}_{i1}}{\tilde{c}_{2,j} \tilde{A}_{i2} - \tilde{c}_{1,j} \tilde{A}_{i1}}. \quad (6.46)$$

Admissibility Test

Let $\tilde{A}_{xy,b}$ denote the updated feasible region after considering ICI constraint $j$. Following similar argument as Section 6.2.3, we update the matrices $\tilde{C}$ and $\tilde{A}$ by adding a new tilted line due to ICI constraint $j + 1$. In order to test the admissibility of the D2D pair, we consider the intersection of $\tilde{A}_{xy,b}$ with the curves associated with minimum SINR requirements (6.16) and (6.39). A necessary and sufficient condition for the D2D pair to be admissible is that the solution $\tilde{s}_j$ in Proposition 11 satisfies $\tilde{\Delta} \cdot \tilde{s}_j \preceq \tilde{\delta}$ where $\Delta$ and $\delta$ are obtained iteratively through the algorithm. The feasible region is specified as the shaded area in Fig. 6.4.

Finding Corner Points

In order to solve problem $Q_3$, we need to find all candidates for an optimal solution. Let $\hat{P}$ and $\hat{Q}$ denote the points where the curves $I - \hat{P}$ and $I - \hat{Q}$ intersect $\tilde{A}_{xy,b}$ as shown in Fig. 6.4. As discussed earlier, the optimal power pair $(x^o, y^o)$ can be one of points $\{\hat{P}, \hat{P}_1, \cdots, \hat{P}_j, \hat{Q}\}$ in this figure. The intersection
of curve $I - \tilde{P}$ with the horizontal boundary line segment $y = P_C^{\text{max}}$ is given by

$$\tilde{P}_H = \begin{bmatrix} x_H & P_C^{\text{max}} \end{bmatrix}^T,$$  \quad (6.47)

where $x_H$ is given by solving $l_1 x_H \left( \frac{\exp(-l_2/x_H)}{1-\epsilon} - 1 \right) = P_C^{\text{max}}$ using bisection. The intersection of curve $I - \tilde{P}$ with the vertical boundary line segment $x = P_D^{\text{max}}$ is given by

$$\tilde{P}_V = \begin{bmatrix} P_D^{\text{max}} & l_1 P_D^{\text{max}} \left( \frac{\exp(-l_2/P_D^{\text{max}})}{1-\epsilon} - 1 \right) \end{bmatrix}^T.$$

(6.48)

The intersections of curves $I - \tilde{P}$ and $I - \tilde{Q}$ with a tilted boundary line segment $c_{1,j} y + c_{2,j} x = 1$ are given by

$$\tilde{P}_{T,j} = \begin{bmatrix} \tilde{\psi}_{2,j} 1 - \tilde{c}_{2,j} \tilde{\psi}_{2,j} \end{bmatrix}^T,$$

(6.49)

$$\tilde{Q}_{T,j} = \begin{bmatrix} \tilde{\psi}_{1,j} 1 - \tilde{c}_{2,j} \tilde{\psi}_{1,j} \end{bmatrix}^T,$$

(6.50)

where $\tilde{\psi}_{1,j}$ is given by the right-hand side of (6.31) except with $\{c_{1,j}, c_{2,j}\}$ substituted by $\{\tilde{c}_{1,j}, \tilde{c}_{2,j}\}$, and $\tilde{\psi}_{2,j}$ is given by solving

$$l_1 \tilde{\psi}_{2,j} \left( \frac{\exp(-l_2/\tilde{\psi}_{2,j})}{1-\epsilon} - 1 \right) = \frac{1 - \tilde{c}_{2,j} \tilde{\psi}_{2,j}}{\tilde{c}_{1,j}}$$

(6.51)

using bisection for $j = 1, \cdots, b$.

### Finding Roots

Let $\tilde{T}$ and $\tilde{S}_j$ denote the sets of all feasible corner points and roots of (6.20) that meet the range constraint for $\tilde{c}_{1,j} y + \tilde{c}_{2,j} x = 1$, respectively. Then the set of candidate points on the interior of line segment $\tilde{c}_{1,j} y + \tilde{c}_{2,j} x = 1$ is given by

$$\tilde{Z}_j \triangleq \left\{ [x_r (1 - \tilde{c}_{2,j} x_r)/\tilde{c}_{1,j}]^T : x_r \in \tilde{S}_j \right\}.$$  \quad (6.52)

Thus, the set of candidate pairs for $(x^o, y^o)$ is given by $\tilde{P}^o = \tilde{T} \bigcup_{j=1}^b \tilde{Z}_j$.

The steps to solve Problem $Q_3$ are summarized in Algorithm 7.

### 6.3.4 An Upper Bound to the Maximum Objective of $Q_2$

In the previous section, we have presented an algorithm to solve the ROE approximation problem $Q_3$ exactly. Let $(x^*, y^*)$ denote the optimal solution of $Q_3$. Substituting $(x^*, y^*)$ into the objective of $Q_2$, we have $\mathcal{R}(x^*, y^*) \leq \mathcal{R}^o$ where $\mathcal{R}^o$ denotes the optimal value of the objective in $Q_2$, as well as the original problem $Q_1$. Since it is difficult to compute $\mathcal{R}^*$, to evaluate the gap between $\mathcal{R}(x^*, y^*)$ and $\mathcal{R}^*$, we next propose an upper bound on $\mathcal{R}^o$. 
Proposition 13. An upper bound on the optimal objective of $Q_2$ can be obtained by solving the problem

$$Q_4 : \max_{(x,y)} \hat{R}(x,y)$$

subject to (6.16), (6.18), (6.39), and (6.40)

where $\hat{R}(x,y) = \log_2 \left( \left( 1 + y(1 - \frac{K_1 x}{K_2 + x}) \right) \left( 1 + G \frac{x/y}{\sigma^2_D + y/\eta_2} \right) \right)$ and $G = (1 + \sigma^2_D \eta_2 / P_{\text{max}} C) E' \left( \frac{\sigma^2_D \eta_2 / P_{\text{max}} C}{P_{\text{max}}} \right) > 1$.

Proof. See Appendix E.

We note that, since the objectives of problems $Q_3$ and $Q_4$ have similar structures, we can simply modify Algorithm 7 to solve $Q_4$.

Note that $G \to 1$ as $P_{\text{max}} \to 0$, which can be shown using the following inequalities [106]:

$$\frac{1}{2} \ln \left( 1 + \frac{2}{7} \right) < E'(t) < \ln \left( 1 + \frac{1}{7} \right) \text{ for all } t > 0. \quad (6.53)$$

This suggests that the solution of the ROE approximation algorithm is optimal when $P_{\text{max}}$ is small enough.

6.4 Extension to Multiple CUs and D2D Pairs

So far, we have provided the power control solution for one CU and one D2D pair. We now extend our consideration to the scenario of multiple CUs and D2D pairs. For CSI availability, we consider Scenario 2 in Section 6.1.3. Extension to Scenario 1 can follow a similar approach, and is omitted to avoid redundancy.

Consider a multichannel communication system (e.g., OFDMA) with $N_C$ orthogonal subchannels in each cell. We assume a fully loaded network with $N_C$ CUs and $N_D$ D2D pairs. Without loss of generality, we assume CU $j$ uses subchannel $j$ for $j \in \mathcal{C} \triangleq \{1, \cdots, N_C\}$. Each D2D pair reuses at most one subchannel, and the subchannel of each CU can be reused by at most one D2D pair. Let $x_{k,j} \in \{0,1\}$ indicate D2D pair $k$ reuses CU $j$’s subchannel, i.e., $x_{k,j} = 1$ if D2D pair $k$ reuses CU $j$’s subchannel; otherwise, $x_{k,j} = 0$. Let $p \triangleq [P_{D,1}, \cdots, P_{D,N_D}, P_{C,1}, \cdots, P_{C,N_C}]^T$, $x \triangleq [x_{1,1}, \cdots, x_{1,N_C}, \cdots, x_{N_D,N_C}]^T$, and $w \triangleq [w_1^T, \cdots, w_{N_C}^T]^T$.

The objective is to maximize the overall expected sum rate of all D2D pairs and CUs by optimizing the transmit power vector $p$, the indicator vector $x$, and the receive beam vector $w$, while satisfying the worst-case ICI and SINR requirements under the per-node power constraints. The formulated problem
is given by

$$
\mathcal{R}_1 : \max_{\mathbf{p}, \mathbf{w}, \mathbf{x}} \sum_{k \in D} \sum_{j \in C} \log(1 + \gamma_{C,j}) + x_{k,j} E[\log(1 + \gamma_{D,k})]
$$

subject to

$$
\frac{P_{C,j}|w_{C,j}^H h_{C,j}|^2}{\sigma^2 + x_{k,j} P_{D,k}|w_{D,k}^H g_{D,k}|^2} \geq \gamma_{C,j}^{\min}, \forall j \in C
$$

$$
Pr\{\gamma_{D,k} \leq \gamma_{D,k}^{\min}\} \leq \epsilon, \forall k \in D
$$

$$
P_{C,j} \leq T_{C,j}^{\max}, P_{D,k} \leq T_{D,k}^{\max}, \forall j \in C, k \in D
$$

$$
E[P_{l,i,j}] \leq T_{l,i,j}^{\max}, \forall j \in C, i = 1, \cdots, b
$$

$$
\sum_{k \in D} x_{k,j} \leq 1, \sum_{j \in C} x_{k,j} \leq 1, \forall j \in C, k \in D
$$

$$
x_{k,j} \in \{0, 1\}, \forall j \in C, k \in D
$$

where $D$ denotes the set of admissible D2D pairs. Here, we say that D2D pair $k$ is admissible if it can reuse at least one subchannel from $C$.

Note that problem $\mathcal{R}_1$ is a mixed integer programming problem and is challenging to solve. Instead, we consider a suboptimal solution by utilizing our proposed ROE approximation algorithm as follows.

1. Determine the admissibility of any D2D pair $k$ to reuse CU $j$’s subchannel, for $k = 1, \cdots, N_D$, $j = 1, \cdots, N_C$.

2. For $\forall k$ and $j$, if D2D pair $k$ is admissible to use CU $j$’s subchannel, we jointly optimize their transmit powers to maximize their expected sum rate, which is given by problem $\mathcal{Q}_3$ with the solution provided by Algorithm 7.

3. We solve the CU-D2D matching problem to optimally assign each admissible D2D pair to a CU. In particular, we define a bipartite graph between CUs and D2D pairs. Each edge between a D2D pair and a CU indicates that the pairing of the D2D pair and the CU is feasible. The weight of the edge is given by the sum rate or rate gain of the D2D pair and the CU, under the ROE approximation solution provided by Algorithm 7. This CU-D2D matching problem, to maximize the expected sum rate, can be solved by using the well-known Hungarian algorithm in polynomial time [105].

The optimal CU-D2D matching approach in Step 3 above requires computing a power control solution for each admissible CU-D2D pair. We can further reduce the computational complexity of this step by using the following suboptimal CU-D2D matching schemes. Instead of the sum rate or rate gain, we define the cost on an edge between D2D pair $k$ and CU $j$ in the bipartite graph as one of the two choices below:

- **Suboptimal CU-D2D matching A**: the intra-cell interference channel gain between CU $j$ and D2D receiver $k$, i.e., $|g_{j,k}|$;
- **Suboptimal CU-D2D matching B**: the weight of CU transmit power in the ICI constraint.\(^4\)

We will show through simulation that these two approximate matching schemes often perform close to the jointly optimal solution for $\mathcal{R}_1$.

\(^4\)When all ICI constraints are replaced with a single ICI constraint as in Chapter 5.
6.5 Numerical Study

We provide numerical results to illustrate the performance of Algorithms 6 and 7, with respect to the original problems $P_1$ and $Q_1$, respectively, along with the CU-D2D matching methods presented in Section 6.4. We consider 5 CUs and one D2D pair that are randomly dropped in a cell of interest, while the number of neighboring cells is $b = 6$. The BS coordinates D2D communication by associating the D2D pair with a CU to achieve the maximum sum rate. The cell radius is $d_0 = 0.5$ km and the D2D distance is denoted by $d_D$. We assume Rayleigh fading for each channel with path loss $128.1 + 37.6 \log_{10}(d)$. We set $\sigma^2 = \sigma^2_D = -103$ dBm, $\gamma_{C_{\min}}^\text{min} = \gamma_{D_{\min}}^\text{min} = 3$ dB, $P_{C_{\max}}^\text{max} = P_{D_{\max}}^\text{max} = P_{\max}$, and $I_{0_{\max}}^\text{max} = NI_0$ where $I_0$ is the ICI threshold reference and $I_0/\sigma^2 = 5$ dB. We use 5000 channel realizations to evaluate the average performance.

For performance comparison, we use the upper bound developed in Section 6.3.4. Furthermore, we consider two baseline algorithms: 1) boost-and-limit (BaL) heuristic, where the unique power solution $[x_I \ y_I]^T$ in Scenario 1 or $[\tilde{x}_I \ \tilde{y}_I]^T$ in Scenario 2 is boosted proportionally with a common factor $\zeta_{\text{max}}$ such that either the maximum power constraint (6.18) or the ICI limit (i.e., (6.19) in Scenario 1 and (6.40) in Scenario 2) is met with equality, i.e., further boosting the powers would violate at least one
constraint. Note that the scheduling BS can easily compute this unique power solution and then boost the power of the CU and D2D pair until either the maximum power is achieved or a neighboring cell alerts regarding the ICI level. 2) **CU-priority heuristic**, where a corner point of the feasible region is selected to maximize the SINR of the CU. It selects the maximum feasible CU power with the minimum feasible D2D power.

### 6.5.1 Scenario 1

We compare the following schemes: 1) the proposed optimal power control (Algorithm 6) and optimal CU-D2D matching; 2) approximate power control (obtained by replacing all ICI constraints with a single ICI constraint and then using the algorithm proposed in Chapter 5) and optimal CU-D2D matching; 3) BaL heuristic power control and optimal CU-D2D matching; 4) CU-priority heuristic power control and optimal CU-D2D matching; 5) approximate power control and suboptimal CU-D2D matching A given in Section 6.4; 6) approximate power control and suboptimal CU-D2D matching B given in Section 6.4.
We evaluate both the sum rate and the rate gain obtained by adding D2D communication, which is the difference of the maximum sum rate of $P_1$ using the optimal solution provided by Algorithm 6 and the maximum sum rate when there is no D2D pair in the cell.

In Figs. 6.5 and 6.6, the sum rate and rate gain versus $P_{\text{max}}$ under different power control methods and matching schemes are shown. We set $N = 4$ and $d_D/d_0 = 0.1$. We observe that the proposed approach with optimal CU selection outperforms all other schemes. However, the performance of the approximate power control scheme can be close to that of the optimal power control. In addition, it can be seen that approximate power control with any of the three CU-D2D matching schemes outperforms the BaL and CU-priority heuristics with optimal CU-D2D matching. Furthermore, for approximate power control, the gap between optimal CU-D2D matching and suboptimal CU-D2D matching $A$ is small. This is because when CU-D2D matching $A$ is used to define the bipartite graph, the intra-cell interference a CU causes to the matched D2D receiver is small. This results in a high D2D rate.
Table 6.1: The expected sum rate (bit/channel use) versus $P_{\text{max}}$ (dBm).

<table>
<thead>
<tr>
<th>$P_{\text{max}}$</th>
<th>20</th>
<th>22</th>
<th>24</th>
<th>26</th>
<th>28</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opt.</td>
<td>45.6879</td>
<td>49.1297</td>
<td>51.9008</td>
<td>54.1057</td>
<td>55.2725</td>
<td>55.8749</td>
</tr>
<tr>
<td>ROE</td>
<td>45.6879</td>
<td>49.1297</td>
<td>51.9001</td>
<td>54.1031</td>
<td>55.2639</td>
<td>55.8623</td>
</tr>
</tbody>
</table>

6.5.2 Scenario 2

We first evaluate how the expected sum rate changes with the maximum power $P_{\text{max}}$, under Algorithm 7 and both baseline algorithms in Fig. 6.7 for $N = 2$ and 8. We observe two regimes. When the expected sum rate is an increasing function of $P_{\text{max}}$ (Regime 1), the ICI is relatively weak, and the feasible region is not affected by the ICI constraint. As a result, the expected sum rate increases linearly with $P_{\text{max}}$. When the expected sum rate converges (Regime 2), the ICI is relatively strong, and the feasible region is not changed by $P_{\text{max}}$. Hence, the expected sum rate is controlled by the fixed ICI threshold. It can be seen that the proposed ROE algorithm significantly outperforms the BaL and CU-priority heuristic algorithms for all values of $N$. Furthermore, the gap between the ROE algorithm and the upper bound is small, at less than 5% of the optimal expected sum rate.

To study the effect of the D2D distance on the performance, the expected sum rate versus $P_{\text{max}}$ for $d_D/d_0 = 0.02$ and 0.1 is shown in Fig. 6.8. We set $N = 4$. For the ROE algorithm, the expected sum rate improves significantly as $d_D/d_0$ decreases. However, the performance of the CU-priority heuristic is not sensitive to $d_D/d_0$. This is because the proposed ROE algorithm results in a significant rate improvement when the D2D channel is very strong, i.e., the D2D distance is small. The expected sum rate versus the normalized D2D distance $d_D/d_0$ for $N = 2$ and 8 is shown in Fig. 6.9. We observe that, when the D2D channel is strong, i.e., the D2D distance is small, significant expected sum rate is achievable even while knowing only partial CSI.

We next investigate the effect of the ICI threshold reference $I_0$ on performance. The expected sum rate versus $I_0$ is demonstrated in Figs. 6.10 for $d_D/d_0 = 0.1$, $P_{\text{max}} = 24$ dBm, and $N = 2$ and 8. We observe that the expected sum rate improves when $I_0$ increases. For small $I_0$ values, the expected sum rate is an increasing function of $I_0$ since the ICI is relatively strong (Regime 2). As $I_0$ increases, the ICI constraint becomes inactive (Regime 1) and the expected sum rate converges due to the fixed $P_{\text{max}}$.

In order to more precisely quantify the performance loss due to the ROE approximation, the expected sum rate versus $P_{\text{max}}$ under Algorithm 7 and the optimal power control algorithm obtained by exhaustive search, for $d_D/d_0 = 0.1$ and $N = 4$, is shown in Table 6.1. We observe that the performance of the ROE approximation algorithm is close to that of optimal power control. In particular, in Regime 1 the performance by both power control schemes overlap. Hence, the ROE approximation algorithm offers nearly optimal performance with drastically reduced computational complexity.

6.6 Summary

We have studied the sum rate maximization of CUs and D2D pairs sharing the uplink cellular spectrum, under minimum SINR requirements, per-node maximum power, and ICI constraints in multiple neighboring cells. We accommodate receive beamforming at the BS, and consider both the perfect CSI and partial CSI cases. We obtain a simple feasibility test to determine whether a D2D pair can reuse the channel resource of a CU. An efficient robust power control algorithm based on ROE approximation has
been developed to obtain the transmit powers of the CU and D2D transmitter when some CSI is available only in terms of statistics, along with an algorithm to compute an upper bound of the maximum expected sum rate. We have further shown how the proposed power control solutions can be integrated with different CU-D2D matching schemes, comparing their performance. Numerical results demonstrate that the proposed algorithms are close to optimal in both the perfect CSI and partial CSI cases.

Appendix A

Proof of Proposition 11

\textit{Proof.} We first obtain the cumulative distribution function for random variable \( Z = \frac{X}{\sigma_D^2 + Y} \)
where \( X \sim \exp(\eta_1/x) \) and \( Y \sim \exp(\eta_2/y) \).

\[ F_Z(z) = \Pr \left\{ \frac{X}{\sigma_D^2 + Y} \leq z \right\} = \int_0^\infty f_Y(t)F_X(z(\sigma_D^2 + t)) \, dt \]
\[ = \int_0^\infty \frac{\eta_2 \exp(-\eta_2 t/y)}{y} (1 - \exp(-\eta_1 z(\sigma_D^2 + t)/x)) \, dt \]
\[ = 1 - \frac{\eta_2/y}{\eta_1 z/x + \eta_2/y} \exp(-\eta_1 z\sigma_D^2/x). \quad (A.1) \]

The constraint (6.8) can be written as \( F_Z(\gamma_D^{\text{min}}) \leq \epsilon, \) i.e.,
\[ y \leq l_1 x \left( \frac{\exp(-l_2/x)}{1 - \epsilon} - 1 \right). \quad (A.2) \]

It is not difficult to show that \( g(x) = l_1 x \left( \frac{\exp(-l_2/x)}{1 - \epsilon} - 1 \right) \) is a convex and increasing function of \( x \). Furthermore, the D2D SINR requirement (A.2) can be satisfied only if
\[ x \geq \tilde{x}_{\text{min}} = -\eta_1 \sigma_D^2 \gamma_D^{\text{min}} \ln(1 - \epsilon). \quad (A.3) \]

Considering both (6.35) and (A.2) with equality, the unique power solution is given by \( \{ x_T, y_T \} \).

Note that \( x_T \) is the solution of
\[ \alpha \left( 1 - \frac{K_1}{1 + K_2/x} \right)^{-1} = l_1 x \left( \frac{\exp(-l_2/x)}{1 - \epsilon} - 1 \right), \quad (A.4) \]
which can be obtained efficiently using a bisection search algorithm within the range \( \tilde{x}_{\text{min}} \leq x \leq P_D^{\text{max}} \).

Appendix B

Proof of Lemma 6

\textit{Proof.} Given any \((x, y)\) in the interior of \( \hat{A}_{xy} \), there exists \( \zeta > 1 \), such that \((\zeta x, \zeta y) \in \hat{A}_{xy} \). We show that \( \mathcal{R}(\zeta x, \zeta y) > \mathcal{R}(x, y) \) for any \( \zeta > 1 \). Note that the objective function in \( Q_2 \) can be written as
\[
\mathcal{R}(x, y) = \mathbb{E}[\log_2(\Omega_{x,y})]
\]
where
\[
\Omega_{x,y} \triangleq \left( 1 + y(1 - \frac{K_1 x}{K_2 + x})t \right) \left( 1 + \frac{x|h_D|^2}{\sigma_D^2 + y|g_C|^2} \right).
\] (B.1)

For any realization of random channels \(\{h_D, g_C\}\), we have \(\Omega_{\xi_x, \xi_y} > \Omega_{x,y}\) following the similar arguments in Lemma 4. Then we have \(\mathcal{R}(\xi_x, \xi_y) > \mathcal{R}(x, y)\) since \(\log_2(\cdot)\) is a monotonically increasing function. As a result, the optimal power pair \((x^o, y^o)\) cannot be in the interior of \(\mathcal{A}_{xy}\).

**Appendix C**

**Proof of Lemma 7**

*Proof.* Since \(|h_D|^2 \sim \exp(\eta_1)\) and \(|g_C|^2 \sim \exp(\eta_2)\), we can obtain \(\mathbb{E}[\log_2(1 + \gamma_D)]\) by taking a double integral as follows:

\[
\mathbb{E}[\log(1 + \gamma_D)] = \int_0^\infty \int_0^\infty \log \left( 1 + \frac{x u}{\sigma_D^2 + y v} \right) f(u, v) \, d u \, d v
\]
\[
= \int_0^\infty \int_0^\infty \frac{\eta_2 x \exp(-\eta_1 u - \eta_2 v)}{\sigma_D^2 + y v + x u} \, d u \, d v
\]
\[
= \int_0^\infty \int_{\eta_1 y}^{\infty} \frac{\eta_2 x}{\eta_1 y} \, d t \exp \left( - \frac{\eta_2 x}{\eta_1 y} + \frac{\eta_2 \sigma_D^2}{y} \right)
\]
\[
= \left[ \frac{\eta_2 x}{\eta_1 y - \eta_2 x} \left( E'\left( \frac{\eta_2 \sigma_D^2}{y} \right) - E'\left( \frac{\eta_1 \sigma_D^2}{x} \right) \right) \right]_{y=0}^{y=\infty}
\] (C.1)

where \(f(u, v) \triangleq \eta_1 \exp(-\eta_1 u)\eta_2 \exp(-\eta_2 v)\).

**Appendix D**

**Proof of Proposition 12**

*Proof.* By Lemma 6, the optimal power solution pair \((\hat{x}^o, \hat{y}^o)\) to maximize \(Q_2\) is given by a corner point or an interior point of the horizontal, vertical, or tilted boundary line segment(s) of \(\mathcal{A}_{xy}\). If \((\hat{x}^o, \hat{y}^o)\) is an interior point, we prove only for the case of tilted boundary line. The other cases can be proved similarly. If ICI constraint \(j\) in (6.40) is active at optimality, the optimal power is the solution of the following optimization problem

\[
\max_{(\hat{x}, \hat{y})} \ln \left( 1 + \frac{\hat{b}}{\hat{y}} \left( 1 - \frac{K_1}{K_2 + \hat{x}} \right) \right) + \frac{\hat{y}}{\hat{y} - \hat{x}} \left( E'(\hat{x}) - E'(\hat{y}) \right)
\]

subject to \(\hat{c}_{1,j} + \hat{c}_{2,j} = 1\)

where \(\hat{b} \triangleq \eta_2 \sigma_D^2\). Substituting \(\hat{y} = \frac{\hat{c}_{1,j}}{\hat{x} - \hat{c}_{2,j}}\) into the objective function above, we have \(\max_{\hat{x}} \hat{R}(\hat{x})\), where

\[
\hat{R}(\hat{x}) \triangleq \ln \left( 1 + \left( \hat{b}_{1,j} - \hat{b}_{2,j} / \hat{x} \right) \left( 1 - \frac{K_1}{K_2 + \hat{x}} \right) \right) + \frac{\hat{c}_{1,j}}{\hat{c}_{1,j} + \hat{c}_{2,j} - \hat{x}} \left( \exp(\hat{x})E_1(\hat{x}) - \exp(g_T(\hat{x}))E_1(g_T(\hat{x})) \right).
\] Since
\( \hat{R}(\hat{x}) \) is continuous and has a first-order derivative, the optimum \( \hat{x}^o \) is either an end point of the interval defined by \( \tilde{A}_{xy} \) or obtained by solving \( d\hat{R}(\hat{x})/d\hat{x} = 0 \), which results in the equation in (6.45). \( \square \)

Appendix E
Proof of Proposition 13

Proof. We consider two random variables \( Z_1 \triangleq xE[|h_D|^2] \) and \( Z_2 \triangleq \sigma_D^2 + yE[|g_C|^2] \) for notation simplicity. First, we show the following inequality holds:

\[
\frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]} \leq \mathbb{E}\left[ \frac{Z_1}{Z_2} \right] = \frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]} \left( 1 + \frac{\sigma^2 \eta^2}{y} \right) \left( \frac{\sigma^2 \eta^2}{y} \right).
\]

(E.1)

Note that \( Z_1 \) and \( Z_2 \) are independent random variables. Hence, we have

\[
\mathbb{E}\left[ \frac{Z_1}{Z_2} \right] = \mathbb{E}[Z_1]\mathbb{E}\left[ \frac{1}{Z_2} \right] \geq \frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]} \quad (E.2)
\]

by Jensen’s inequality, since \( f(y) = 1/y \) is a convex function.

Now, we show that \( \varphi(t) = (1 + t)E'(t) \) is a strictly decreasing function of \( t \). The continued fraction expansion of \( E_1(t) \) is given by \[106\]

\[E'(t) = \frac{1}{t + \frac{1}{t + \frac{1}{t + \cdots}}}.\]

(E.3)

Ignoring high order terms in (E.3), we have

\[
E'(t) < \frac{t + 1}{t(t + 2)} \quad \text{for all } t.
\]

(E.4)

Using the inequality (E.4) and taking the first order derivative of \( \varphi(t) \), we have

\[
\frac{d\varphi(t)}{dt} = (2 + t)E'(t) - 1 - \frac{1}{t} < 0.
\]

(E.5)

Since \( \varphi(t) \) is a strictly decreasing function, the right-hand side of (E.1) is maximized by substituting \( y = P_C^{\text{max}} \), i.e.,

\[
\mathbb{E}\left[ \frac{Z_1}{Z_2} \right] < G \frac{\mathbb{E}[Z_1]}{\mathbb{E}[Z_2]} \quad \text{for all } (x, y).
\]

(E.6)

Finally, we note that \( \mathbb{E}[\log_2(1 + Z_1/Z_2)] \leq \log_2(1 + \mathbb{E}[Z_1/Z_2]) \) for any given \((x, y)\) due to Jensen’s inequality. Hence, the optimal objective of \( Q_4 \) is always an upper bound on the objective of \( Q_2 \) under the optimal solution. \( \square \)
Algorithm 6 Maximizing the objective of problem $P_2$

**Input:** $\alpha, \beta, a, l, K_1, K_2, K_3, \{c_{1,j}\}_{j=1}^b, \{c_{2,j}\}_{j=1}^b, P_C^{\text{max}}, P_B^{\text{max}}$

**Output:** $x^o, y^o, \text{ and } w^o$

1. **Step 1) Initialization:**
   
   1. Set $k = 0$, $C$ as in (6.21), $A$ as in (6.22), $\delta$ and $\Delta$ as in (6.24).

2. **Step 2) Admissibility Test:**
   
   1. for $j = 1 : b$
      
   2. if $\Delta \cdot s_{i,j} \leq \delta$ and $k == 0$ then
      
   3. Set $i_1 = i$, $k = 1$, and $s_1 = s_{i,j}$.
      
   4. else if $\Delta \cdot s_{i,j} \leq \delta$ and $k == 1$ then
      
   5. Set $i_2 = i$ and $s_2 = s_{i,j}$.
      
   end if
   
5. end for

6. if $k > 0$ then
   
5. Set $C_1 = C_{i=1}, C_2 = C_{i=2}^{n_C}, A_1 = A_{i=1},$
   
5. and $A_2 = A_{i=2}^{n_A}$.

7. Update $C = [C_1 = s_1 s_2 C_2]$, $A = [A_1 \cdot C A_2]$ where $C = [c_{2,j} c_{1,j} 1]^T$.

8. Update $\Delta = [\Delta^T s_{1,j}]^T$ and $\delta = [\delta^T 1]^T$

9. end if

10. end for

11. Check $\Delta \cdot s_{1,j} \leq \delta$ where $s_{1,j}$ is given in (6.15).

12. **Step 3) Finding Corner Points:**
   
5. Set $i_s = 2$ and $i_f = n_A - 1$.

13. if $A_{1:2,i_s} == [0 1]^T$ then
   
5. Set $i_s = 3$, $P_{s,1}$ as in (6.25), and $Q_{s,1}$ as in (6.26).

14. else if $A_{1:2,i_f} == [1 0]^T$ then
   
5. Set $i_f = n_A - 2$, $P_{s,1} = n_A - 2$ as in (6.27), and $Q_{s,1}$ as in (6.28).

15. end if

16. for $j = i_s : i_f$
   
5. Set $P_{j,1}$ as in (6.29) and $Q_{j,1}$ as in (6.30).

17. end for

18. Find $j_1$ and $j_2$ such that $\Delta \cdot P_{j,1} \not\preceq \delta$ and $\Delta \cdot Q_{j,2} \not\preceq \delta$.

19. Define $T = \{P_{j,1}, C_{j=1+2:j+1}, Q_{j,2}\}$ and set $P^o = T$.

20. **Step 4) Finding Roots:**
   
5. for $k = 1 : n_T - 1$
   
5. if $A_{1:2,k+1} == [1 0]^T$ or $A_{1:2,k+1} == [0 1]^T$ then return
   
5. else
   
5. Compute $Z$ in (6.33) with $T_{1,k} \leq x_r \leq T_{1,k+1}$.
   
5. Update $P^o = P^o \cup \{z\}$
   
5. end if
   
5. end for

21. Enumerate among candidate solution set $P^o$ to find the optimal solution.

22. Obtain the optimal beam vector.
Algorithm 7 Maximizing the objective of problem $Q_3$ (the ROE approximation algorithm)

Input: $\alpha, K_1, K_2, l, l_1, l_2, x_H, \{\hat{c}_{1,j}\}_{j=1}^b, \{\hat{c}_{2,j}\}_{j=1}^b, P_C^{\max}, P_D^{\max}$

Output: $x^o, y^o, \text{ and } w^o$

Step 1) Initialization and Step 2) Admissibility Test:
1: Line 1-16 in Algorithm 6 except replacing $s_{i,j}$ and $c$ with $\tilde{s}_{i,j}$ and $\tilde{c}$, respectively.
2: Check $\Delta \cdot \tilde{s}_I \preceq \tilde{\delta}$ where $\tilde{s}_I$ is given in Proposition 11.

Step 3) Finding Corner Points:
3: Set $i_s = 2$ and $i_f = n_A - 1$.
4: if $\tilde{A}_{1:2,i_s} == [0 1]^T$ then
5: Set $i_s = 3, \tilde{P}_{1:1}$ as in (6.47), and $\tilde{Q}_{1:1}$ as in the right-hand side of (6.26).
6: else if $\tilde{A}_{1:2,i_f} == [1 0]^T$ then
7: Set $i_f = n_A - 2, \tilde{P}_{1:n_A-2}$ as in (6.48), and $\tilde{Q}_{1:n_A-2}$ as in the right-hand side of (6.28).
8: end if
9: for $j = i_s : i_f$ do
10: Set $\tilde{P}_{1:j-1}$ as in (6.49) and $\tilde{Q}_{1:j-1}$ as in (6.50).
11: end for
12: Find $j_1$ and $j_2$ such that $\tilde{\Delta} \cdot \tilde{P}_{1:j_1} \preceq \tilde{\delta}$ and $\tilde{\Delta} \cdot \tilde{Q}_{1:j_2} \preceq \tilde{\delta}$.
13: Define $\tilde{T} = \{\tilde{P}_{1:j_1}, \tilde{C}_{1:j_1+2:j_2+1}, \tilde{Q}_{1:j_2}\}$ and set $\tilde{P}^o = \tilde{T}$.

Step 4) Finding Roots:
14: Line 29-37 in Algorithm 6 except replacing $Z$ with $\tilde{Z}$. 
Chapter 7

Conclusions

In this thesis, we have developed new theories and design guidelines to improve the performance of distributed multi-tier and D2D systems by investigating their resource optimization and interference management.

In Chapter 3, we study power allocation for a multi-relay cellular network with multiple source-destination pairs and an individual power budget for each relay. We focus on designing optimal relay beam vectors to minimize the maximum per-relay power usage subject to minimum SNR guarantees. It is shown that strong Lagrange duality holds for this non-convex problem. We investigate the optimal relay beamformers of the min-max relay power problem in three cases based on the values of the optimal dual variables. Further, we show that max-min SNR subject to a fixed maximum relay power constraint is the inverse problem of min-max relay power subject to a minimum SNR constraint. Finally, we analyze the effect of imperfect CSI by evaluating the performance loss due to either quantization error with limited feedback or channel estimation error.

In Chapter 4, we study optimal ICI control in a multi-relay cellular network, where each cell has multiple S-D pairs. We focus on designing optimal relay beam vectors to minimize the maximum interference caused at the neighboring cells under per-relay power constraints and minimum SNR requirements. It is shown that the strong duality property holds for this non-convex problem. We propose an iterative algorithm to provide a semi-closed-form solution. Using bisection search, we extend this algorithm to solve the max-min SNR problem, under some pre-determined maximum interference constraints at neighboring cells and per-relay power constraints. We further study the received worst-case SINR versus the maximum interference. A maximum worst-case SINR is identified for different system setups, which provides insight into designing relay beamforming in a multi-cell network.

In Chapter 5, we consider D2D communication underlaid in a cellular system. We study power control to maximize the sum rate of a CU and a D2D pair, while satisfying minimum SINR requirements and worst-case ICI limit in multiple neighboring cells. Under optimal BS receive beamforming, we propose an efficient approximate power control algorithm to obtain the powers of the CU and D2D transmitters in closed form. It is shown that the proposed algorithm is optimal when the ICI to a single neighboring cell is considered. For multiple neighboring cells, we provide an upper bound on the performance loss by the proposed algorithm and conditions for its optimality. We further consider the scenario of multiple CUs and D2D pairs, and show how our provided solution for one CU and one D2D pair can be utilized to solve the joint power control and CU-D2D matching problem. Simulation demonstrates that our
Chapter 7. Conclusions

The proposed algorithm provides close to optimal performance for the scenario of multiple CUs and D2D
pairs.

In Chapter 6, we further extended the results of Chapter 5, where we jointly optimize the beam
vector and the transmit powers of the CU and D2D transmitters under practical system settings. We
study a multi-cell scenario, and consider both the perfect CSI and partial CSI cases. For the partial CSI
case, we assume perfect channel information is available only for the direct channels from the CU and
D2D to the BS. For other channels, only partial channel information is available. An efficient robust
power control algorithm based on ROE approximation is proposed to maximize the expected sum rate
when only partial CSI is available, along with an algorithm to compute an upper bound of the maximum
expected sum rate. We further extend our consideration to the scenario of multiple CUs and D2D pairs,
and solve the joint power control and CU-D2D matching problem. Numerical results demonstrate that
the proposed algorithms are nearly optimal in both the perfect CSI and partial CSI cases.

As a future work, we might study designing relay beamforming with multiple-antenna relays and/or
S-D pairs, aiming at minimizing per-relay power or minimizing the maximum interference.


[83] 3GPP TS 36.211 V8.2.0, Rel-8 Evolved universal terrestrial radio access (E-UTRA); physical channels and modulation, Mar. 2008.

[84] 3GPP TR 36.814, Rel-9 Evolved universal terrestrial radio access (E-UTRA); further advancements for E-UTRA physical layer aspects, Mar. 2010.


