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Numerical studies for a better understanding of static ice loads on dams

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20

21 Abstract

22 It is important to anticipate potential maximum ice loads to ensure the structural
23 stability of dams in cold climates. Finite element modeling (FEM) can provide some
24 insights into process mechanisms. Four important ice-loading events on dams are
25 presented and simulated. The measured loads were caused by the thermal expansion
26 of ice together with intermediate water level fluctuations. Only the thermal expansion
27 is modeled by the FEM but the impact of water level fluctuations can increase lateral
28 confinement that increases the predicted load by 36% to 106%, particularly when the
29 cover contains mostly columnar ice. It is demonstrated that the presence of snow ice
30 in the cover can decrease the predicted load by 35% to 53%. The study also
31 demonstrates how initial stresses in the ice can affect the ultimate load and show that
32 the very-difficult-to-manage delayed-elastic strain term need **not** be included in the
33 material model where preliminary results suffice.

34
35 Keywords: ice loads, finite element model (FEM), lateral confinement, delayed
36 elasticity, choice of a material model.

37

38 **1. Introduction**

39 Ice loading is an important factor in the design of safe dams in cold climates. In
40 pioneer research, the prevailing wisdom was that it was only changes in reservoir ice
41 cover temperature that generated ice loads at dams (e.g. Drouin and Michel 1971).
42 Based on a nine-year field program at nine sites, Comfort et al. (2003) found that
43 thermal ice loads can be amplified by intermediate water level fluctuations (of about
44 an amount equal to the ice thickness – h . Here and further, definitions of symbols and
45 abbreviations are given in the Table 6). Different theories are proposed to explain this
46 finding (e.g. tidal jacking; Stander 2006) but increases due to the changes in material
47 properties of columnar ice under lateral confinement may be another and this effect is
48 explored in this paper.

49 Although, dams have been built and operated for many years, a considerable
50 scatter in design ice line-load values (kN/m) exists (between $90 \cdot h$ and $300 \cdot h$ (m)
51 according to Westberg 2007). To harmonize design ice loads, several field studies
52 were conducted, suggesting that design loads could be substantially higher than the
53 recommended values (Carter et al. 1998; Comfort et al. 2003; Taras et al. 2011).
54 Clearly, the harmonization of design criteria has not yet been achieved as we are still
55 grappling with loading mechanism drivers and limitations. One approach for design
56 load assessment is to determine the ultimate load of an ice cover before its instability
57 (Carter et al. 1998). Another avenue is through numerical simulation. Parametric
58 studies of historically-measured events can help better understand field processes and
59 help us move forward the objective of harmonization of ice load design criteria.

60 In numerical simulations of reservoir ice loads on dams, the ice cover is often
61 assumed to be isotropic and to consist of one type of ice, although several ice types
62 can be present (often, S2 transversely isotropic columnar ice superimposed by T1

isotropic snow ice, Michel and Ramseier's classification 1971). Unfortunately, there are no constitutive equations for ductile behavior of ice deduced from medium-scale/large-scale experiments at low strain rates. And material (rheological) models based on uniaxial small-scale laboratory experiments have to be used for simulations (e.g. Côté et al. 2016). Sometimes the resulting uniaxial load is increased by 15% - 20% to account for plane strain conditions (e.g. Lupien et al. 2013). In the case of T1 ice, it seems to be sufficient to represent ductile behavior of ice cover under lateral confinement. But for S2 ice, it is not. Frederking (1977) shows that, under lateral confinement, due to its transversely isotropic structure, S2 ice becomes significantly stronger (4 times or more) at the strain rates $<10^{-7} \text{ s}^{-1}$. Therefore, when loads are amplified as a result of confinement, particularly during increased water levels, it is essential to take into account the correct material behavior of S2 ice.

Full ice rheological models contain elastic, delayed-elastic and viscous terms. Calculation of delayed-elastic term is computationally complex for researchers because many commercially-available FEMs (such as ANSYS used here) do not easily track the required ice load history. As a result some investigators use elasto-viscous-only (EVO) models for simulations (Petrich et al. 2015) and a question arises: "Is it absolutely required to include the delayed elastic term in a model for assessing ice loads on dams?" No comparative study of full and EVO models for static ice loads has been reported in the literature yet. However, Azarnejad and Hruday (1996) conducted preliminary analysis for ice stress measured in a field test. They concluded that their full model did not give much improvement in comparison to their EVO model.

This paper addresses the problem of reservoir ice loads on dams under thermal expansion of ice with particular emphasis on the role of lateral confinement. While

there are a few laboratory studies on confined freshwater ice samples at strain rates $< 3 \cdot 10^{-7} \text{ s}^{-1}$, the considerable increase in strength of columnar ice under confinement at those strain rates has never been adequately recognized in design ice load calculations for dams. In this study, ice material model is generalized into 3D using equivalent-stress/equivalent-strain method with von Mises and Zhan (1993) equivalent stress formulations for snow and columnar ice, respectively. The model is calibrated on confined and unconfined experiments on both snow and columnar ice (Frederking 1977). Through the simulation of four field events, it is demonstrated that transversely isotropic behavior of S2 ice cannot be neglected while assessing loads of confined ice on dams. The paper also compares result loads calculated with EVO and full models. Previously reported studies on the influence of ice type and initial stresses on ice load (Kharik et al. 2015; 2016) are broadened and supplemented by additional simulations. Ice loads are discussed from a numerical/deterministic point of view (Table 1) and the reader is encouraged to interpret results presented herein in light of other analytical methods that can also describe ice loading behavior, including empirical (e.g., Comfort et al. 2003), stability (Carter et al. 1998) and statistical/chaotic analyses (Sanderson 2010; Taras et al. 2011).

2. Methodology

This section is divided into two parts. First, the historical (reference) field events are presented. Second, the numerical model is outlined.

2.1. Major historical Canadian ice load events

2.1.1. Multi-day event (Seven Sisters, MB)

One of the largest Canadian multi-day ice load events (≈ 137 hours) was registered at Seven Sisters dam, MB, mid-January 1999. The ice was 0.51 m thick and consisted of S2 ice only (Comfort and Gong 1999). Pressures in ice cover were measured with

113 two BP sensor groups (Duckworth and Westermann 1989) at about 24 m from the
114 dam face (considered to be “far field” – FF). Comfort et al. (2003) reported that the
115 loads resulted from superposition of: (i) large ice temperature changes produced by
116 very cold winter; and (ii) intermediate water level fluctuations (an especially
117 important contributor). The thermal strain rate at the time of peak load was $\sim 10^{-9} \text{ s}^{-1}$.
118 Figure 1 presents a summary of the event (sign convention: compression is positive,
119 tension is negative). There were no instruments deployed to measure confinement or
120 the loads generated at the dam face.

121 **2.1.2. Diurnal events (Barrett Chute, ON)**

122 The events recorded at Barrett Chute dam, ON, in February 2011 (Taras et al.
123 2011) are among the largest Canadian diurnal load events ever measured against a
124 dam face. Two BP sensor groups were deployed at 27 m from the wall (FF). Pressures
125 in ice cover were also measured with 11 Carter panels (Carter et al. 1998) against the
126 dam wall (near field – NF) separated by 2.5 m from each other.

127 Three significant events were measured on February 22, 23 and 24. The
128 maximum FF line loads were respectively 50, 76 and 81 kN/m (Figure 2). Given that
129 the contact of the ice cover with dam and reservoir shoreline is quite erratic (due to
130 the presence of cracks in the ice sheet parallel to the boundaries) and given that the ice
131 sheet can contain many material imperfections and some lateral instability, the line
132 load in the FF has less time-dependent variability and less amplitude than maximum
133 loads recorded at (and near) the dam face.

134 The line load recorded at the dam face depends on the width of measurements
135 (Taras et al. 2011) because the degree of contact between the ice sheet and the dam
136 face depends on highly variable contacts through the ice cracks. When observed
137 values are averaged over the 11 panels (spanning 25 m), the maximum near field line

loads for the events were 117, 145 and 128 kN/m, respectively. These average NF values correspond to about 1.9 times the FF values reported above. The local maximum NF values were 208, 295 and 199 kN/m, respectively, and are, on average, 3.4 times maximum FF values.

Such high loads were recorded during a normal winter with 0.45 m thick reservoir ice (0.225 m of S2 ice covered by 0.225 m of T1 ice). They resulted from: (i) a thin or no snow cover; (ii) a cold spell followed by a warm period with rain; and (iii) intermediate water level fluctuations. The thermal strain rates at the time of peak loads were $\sim 10^{-8} \text{ s}^{-1}$.

Four rows of biaxial gauges deployed in the ice near the dam (Morse et al. 2011) revealed that stresses parallel to the dam face were, on average 0.56, 0.60 and 0.68 times stresses perpendicular to the dam face respectively for peak loads of February 22, 23, and 24. These stresses parallel to the dam face demonstrate lateral confinement and possible stresses generated in this direction.

2.2. Numerical model

A finite element model was used to simulate field events presented in the Section 2.1. In the FEM, the total strain $[\varepsilon]$ was a sum of thermal and mechanical strains:

$$[\varepsilon] = [\varepsilon_{th}] + [\varepsilon_m] \quad (1)$$

The coefficient of thermal dilatation of the ice (α) was $(0.22T - 5.06) \cdot 10^{-6} \text{ K}^{-1}$ (Drouin and Michel 1971). Although, the temperature dependent coefficient of linear thermal dilatation is used in all following calculations, it gives 5% lower peak load than constant coefficient ($60 \cdot 10^{-6} \text{ K}^{-1}$) - supplementary Figures S1 and S2.

Sinha's 1D model (1979) was chosen as a primary material model for ice (Table 2), because it had been based on good quality experiments that captured elastic, delayed elastic and viscous behavior of ice. The model is developed for S2 ice, but

163 Sinha (1979) had also shown that it was able to describe T1 ice uniaxial behavior. The
164 numerical inclusion of the full model in ANSYS was made possible by the user-
165 programmable subroutine USERMAT. Sinha's equation was generalized into 3D
166 using Hooke's law and the method of equivalent-stress/equivalent-strain (Appendix
167 A).

168 In the section 3.1, this model is populated with sets of parameters ($\dot{\epsilon}_v$, n , $b \approx 1/n$)
169 that were assigned to the model after its calibration on Frederking's (1977)
170 experiments with and without lateral confinement. (Frederking's experiments (1977)
171 show that neither type of ice have common creep distribution properties, as maximum
172 axial stresses from both uniaxial and plane strain experiments have different slopes
173 from one another. The reported FEM with the coefficients from Frederking's uniaxial
174 experiments do not provide correct values for his plane strain experiments. The right
175 coefficients for equivalent creep could have been deduced from experiments had the
176 stresses been measured in all directions. To overcome this problem, the FEM was
177 fitted to the reported axial stresses - supplementary Figures S3 and S4).

178 The material models with different parameters that were used in the FEM are
179 designated by "the author of experiments (S or F after Sinha or Frederking)" and "the
180 presence of confinement (UC - UnConfined, or FC - Fully Confined)": S-UC, F-UC,
181 F-FC (Table 3). Ice thickness and ice type were selected according to each problem.

182 In simulations of the cases confined in one x -direction ("-u"), the ice cover was
183 represented by a 3D solid element with dimensions equal in each direction ($h \times h \times h$).
184 In simulations of the cases confined in x - and z -directions ("-b"), the ice cover was
185 represented by a 2D plane strain solid element. The front view for the 3D solid (or
186 structural scheme for 2D solid) is given on the Figure 3 (z -axis is directed towards the
187 observer). Such representation of ice does not directly include the effect of reservoir

width and instability of ice field. Actually, the aspect ratio (dam width / ice thickness) for the studied problems is $\gg 1$.

The ice solids were subjected to the ice temperature change with depth and time (Figure 1a for Seven Sisters, Figure 2a for 3 events at Barrett Chute). For Barrett Chute, three events were modeled separately (02/22 07:15 – 02/22 23:35; 02/23 07:15 – 02/24 01:00; 02/24 06:00 – 02/25 06:00). Water level fluctuations were not directly included in the FEM.

The 3D and 2D-plane-strain FEMs used a $10 \times 10 \times 10$ and a 10×10 meshes. A time step of 150 s was used for all calculations. The choice of the mesh and time steps was validated based on the results of previous parametric studies (supplementary Figures S5 and S6). Initial stresses in ice were assumed to be zero except for the study on initial conditions presented in section 3.4, where the initial internal stress values arose from rapid temperature jumps in ice to its initial temperature distribution prior the event.

3. Results and discussion

3.1. The importance of the selected material constants and confinement

To probe the importance of the material model constants and the role of confinement on calculated line loads, five series of calculations are done: S-UC-u, S-UC-b, F-UC-u, F-UC-b and F-FC-b. The results of the simulations are presented in Figures 4 and 5 and are summarized in Table 4.

For Seven Sisters event, the maximum far field line load is 356 kN/m. Based on the difference between far field and maximum near field loads at Barrett Chute (Section 2.1.2), the local maximum near field load would reached 1210 kN/m (3.4 times far field load). The simulated line loads are generated from the reservoir's 100% columnar ice sheet. The S-UC-u and F-UC-u models (without lateral confinement) are

0.09 times the maximum near field load. The results are close to each other because S-UC and F-UC models have similar constants (Table 3). When lateral confinement is intruded in the FEM on UC models (UC-b), the simulated loads increase by 2.0 to 2.1 times, but still are not high enough (0.18 and 0.19 times the estimated maximum near field load). The F-FC-b simulated load is much more satisfactory as it corresponds to 0.77 of the estimated maximum near field load.

For Barrett Chute events, the simulated line loads are generated from the reservoir's ice sheet that had T1 ice in its upper half and S2 ice in the lower half. Here the F-FC-b model is 0.97 – 1.34 times the maximum near field load. The difference between the peak loads of F-UC-b and F-FC-b models is 13% - 19%, that is consistent with Frederking's laboratory results. The confinement increases the peak loads by 36% - 38% and by 45% - 48% for F-UC-u/b models and S-UC-u/b, respectively. The difference between S- and F-models is explained by the difference in their respective material constants for T1 ice.

The F-FC-b model seems to be the most appropriate model for ice load calculations as (i) it captures the behavior of modeled events, and (ii) it is the closest model to the local maximum near field loads (measured or estimated). Indeed, it is expected that the FEM results match the local maximum measured line load as the model is simulating loads under ideal conditions because (i) the material model parameters come from ideal laboratory conditions, (ii) there is no ice field instability included and (iii) the boundary conditions imposed in the FEM are ones of perfect contact and perfect confinement (when the confinement is applied). However, the perfect contact or the perfect confinement are rarely achieved in the field because of the presence of circumferential cracks. Tidal cracks represent one of the limiting mechanisms that are not included in the FEM simulations and may be responsible for

238 a significant source of discrepancy between the predicted and measured far field loads
239 (Table 1). However, the calculation of idealized thermal ice load from the FEM
240 allows the researcher to isolate and explore the importance of different assumptions
241 such as lateral confinement and ice type. To explore this further, the results of
242 parametric tests are reported in the following section.

243 **3.2. The importance of ice type**

244 The cover on a hydro-electric reservoir has a certain structure that is influenced
245 by the sum of meteorological conditions present during the winter. If winters in a
246 certain region are mild and wet, there will be significant snow ice in the cover. For
247 dams in dry cold climates (e.g., Manitoba), the covers will be primarily composed of
248 columnar ice. Depending on location and weather, the amount of snow ice in the ice
249 cover may vary from $\approx 0\%$ (Comfort and Gong 1999) to 90% (Leppäranta 2015). In
250 addition to snow influencing ice type, it can play a very significant limiting effect on
251 ice temperature dilation rates and therefore on strain rates and therefore on maximum
252 ice loads.

253 The Seven Sisters event is simulated using F-FC-b material model for covers
254 containing only 50% or 10% of S2 ice compared to the previously simulated 100% S2
255 cover. Figure 6 shows that the resulting reduction in peak load is massive (35% and
256 53%, respectively) under these assumptions.

257 For space-saving considerations, the simulations for Barrett Chute are presented
258 in the supplementary Figure S7. Had the cover there been columnar ice only, the
259 maximum simulated load on 02/22/2011 would have been 47% higher.

260 **3.3. Necessity of including the delayed elastic term in the material model for** 261 **assessing ice loads on dams**

262 The delayed elastic term of ice material model is calculated on the basis of the
263 whole previous history of ice loading (Sinha 1983) and that complicates its inclusion
264 in FEMs. To document the importance of the presence of this term for assessing ice
265 loads on dams, the application of F-FC-b model is compared with a simplified version
266 F-FC-b-EVO. The numerical studies have been conducted on the multi-day event at
267 Seven Sisters (maximum strain rate = $3 \cdot 10^{-7} \text{ s}^{-1}$) and on one diurnal event at Barrett
268 Chute ($3 \cdot 10^{-8} \text{ s}^{-1}$).

269 Numerical calculations at Barrett (Figure 7) show that EVO model over predicts
270 the peak reference load by 10%. (For more pronounced delayed-elastic creep ($b =$
271 0.34), the difference between EVO and full models is 18% - supplementary Figure
272 S8). To avoid redundancy, the multi-day event is shown in the supplementary Figure
273 S9. The EVO model predicts a peak load 6% higher than the full material model (10%
274 higher for more pronounced delayed-elastic creep).

275 Calculations with the EVO models only are 5% to 22% faster. Given the advances
276 in computer power, these gains are not very significant. The real gain is in avoiding
277 the very awkward programming required to include the delayed elastic term and the
278 very limiting data storage capabilities of some commercial models such as ANSYS
279 (remark A1 of Appendix A).

280 **3.4. The influence of initial ice stress condition on peak ice load**

281 An ice load event could be influenced by the initial stress conditions within the
282 ice sheet (Comfort et al. 2003). After a typical cold period, one could expect the
283 internal stress at the top of the ice sheet to be at -200 kPa and the bottom to be at +200
284 kPa. For a very severe cold snap, the tension at the ice surface could reach -600 kPa
285 (without generating micro-fissures) causing the bottom to be in compression at about

286 +600 kPa. Although the stress distributions within an ice cover due to such scenarios
287 may be quite non-linear, linear initial stress distributions are assumed in this study.

288 The impact of two initial conditions on the ultimate maximum load is shown in
289 the Figure 8a for one diurnal event. The simulated peak loads for the first (-/+200
290 kPa) and second (-/+600 kPa) scenarios are 8% and 25% higher than the maximum
291 load assuming no initial stresses. The reason the simulated peak loads are higher after
292 a cold snap is because the preloading of the ice cover in the bottom half of the ice
293 sheet eventually contributes to the peak load when the top half of the ice has warmed
294 up. Figure 8b shows the stress profile evolution from initial stress distribution (dashed
295 lines) to the stress distribution at peak load (dotted lines) for both the reference event
296 and the -/+600 kPa event. At the time of peak load, whereas the bottom 0.2 m of the
297 ice contributes virtually no compressive stress for the reference event, there is a
298 considerable contribution for the -/+600 kPa event.

299 Figure 8c shows more clearly how the stresses evolve at both the bottom of the
300 ice sheet and the top of the ice sheet. At the bottom, the +600 kPa reduces
301 continuously to +537 kPa through stress relaxation due to creep. At the top of the ice
302 sheet, the effect of the -600 kPa gradually (and completely) disappears over a period
303 of 4.5 hours (7:15 to 11:45). In summary, the effect of pre-compression in the bottom
304 portion of the ice is present at the time of peak load whereas the effect of pre-tension
305 at the ice surface has essentially vanished.

306 The results of numerical simulations of the multi-day event predict an increase of
307 1% and 4% in the peak load for the first and the second scenarios, respectively
308 (supplementary Figure S10). The effect is much smaller here because the whole depth
309 of initially stress-free ice cover contributes to the load during the multi-day event.

310 While for the diurnal event, only upper 0.25 m of initially stress-free ice cover
311 contributes to the load.

312 **4. Conclusions**

313 The present paper is one of a series by the authors and fellow contributors that
314 explores ice loads on dams. The paper uses a finite element model that simulates the
315 stresses in x -, y - and z - directions within an ice sheet as a function of imposed thermal
316 strain rates using different sets of model constants that rely on the experiments with
317 and without lateral confinement. Peak line loads from each of the models are
318 presented in the paper for four important ice load events measured in Canada.
319 Calculated and observed peak line loads are summarized in Tables 4 and 5.

320 The results show that the choice of the material model (Sinha/Frederking from
321 Unconfined/Fully-Confined experiments), confinement conditions and the amount of
322 T1 snow ice in the top part of the cover all significantly affect the simulated peak
323 local line load of the ice on the dam face. For both theoretical and practical reasons,
324 the authors prefer the use of the coefficients based on Frederking's-FullyConfined
325 experiments because they are well suited for ice types and confinement conditions.
326 Simulations clearly show that ice type must be correctly represented and biaxial data
327 clearly show significant stress confinement (about 68% as measured by stress gauges)
328 at the time of peak loads.

329 Furthermore, given the significant computational difficulty of including the
330 delayed-elastic term, the authors would use the F-FC-b-EVO material model for many
331 applications except those requiring very detailed considerations. FEM simulations
332 also show that the distribution of initial stress conditions (sum = zero) from a
333 warming period after a cold snap can increase the line load particularly for diurnal
334 events (8% to 25%) but is of limited importance for multi-day events (1% to 4%).

335 The main conclusion is that laterally-confined columnar ice, possibly accentuated
336 from moderate water level increases, is capable of generating larger loads than those
337 generated in a snow-ice/columnar ice sheet. Engineers should carefully think of what
338 type of ice cover there could be on a reservoir and what type of water level
339 fluctuations could occur before proposing design line load values.

340 **5. Further work**

341 The results presented in this study do not account for all factors that influence and
342 limit ice loads on dams (Table 1). In interpreting the results and conclusions presented
343 here, we encourage engineers to carefully examine considerations that have been
344 documented by other researchers particularly using other methodologies. We will
345 attempt to address some of these issues in upcoming papers. We also encourage the
346 deduction of constitutive laws for ductile behavior of T1 and S2 ice types from
347 uniaxial and biaxial experiments on medium and large ice samples at strain rates
348 $< 3 \cdot 10^{-7} \text{ s}^{-1}$ and the development of a non-linear 3D representation of ice stress-strain
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444 Figure 1. One of the most important multi-day ice load events measured in Canada
445 (Seven Sisters 1999): a) ice temperature, b) ice temperature profile area, c) air
446 temperature; d) average measured far field load, e) water level rise or drop.

447 Figure 2. Some of the most important diurnal ice load events measured in Canada
448 (Barrett Chute 2011): a) ice temperature (white boundaries show the periods of
449 temperature changes chosen for the modeling of each of three events), b) ice
450 temperature profile area, c) air temperature; d) average measured far field and near
451 field line loads and near field line load variation, e) water level rise or drop.

452 Figure 3. The front view for a 3D solid (or the structural scheme for a 2D solid), z -
453 axis is directed towards the observer (plane strain for 2D solid in z -axis).

454 Figure 4. Observed and calculated line loads (Seven Sisters multi-day event).

455 Figure 5. Observed and calculated line loads (Barrett Chute diurnal events).

456 Figure 6. Observed average and calculated loads for Seven Sisters multi-day event
457 (different ratios between T1 and S2 ice types: 0%/100%, 50%/50%, 90%/10% of the
458 ice cover).

459 Figure 7. Observed and calculated loads (F-FC-b and F-FC-b-EVO models for 02/22
460 Barrett Chute diurnal event).

461 Figure 8. Diurnal event: a) observed and calculated loads with different initial
462 stresses; b) calculated stress profiles within the ice cover (initial conditions and time
463 of maximum calculated load); c) calculated stresses and temperatures (ice surface and
464 ice bottom).

Figure 1. One of the most important multi-day ice load events measured in Canada (Seven Sisters 1999): a) ice temperature, b) ice temperature profile area, c) air temperature; d) average measured far field load, e) water level rise or drop.

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Table 1. Overall objectives of the study in comparison to other analytical methods.

Objectives	<ul style="list-style-type: none"> - provide recommendations for finite element modeling of ice loads on dams, - study the effect of lateral confinement on ice loads, - study the effect of ice type on ice loads, - study the effect of delayed elastic creep on calculated ice loads, - study the effect of initial stresses on ice loads.
Mechanisms	<p><i>included:</i></p> <ul style="list-style-type: none"> - thermal expansion of ice - ice material behavior with creep (elasticity, delayed elastic creep, viscous creep) dependent on ice type - lateral confinement <p><i>not included (mechanisms that can increase the ice load)</i></p> <ul style="list-style-type: none"> - water level fluctuations <p><i>not included (mechanisms that can limit the ice load):</i></p> <ul style="list-style-type: none"> - water level fluctuations - bending / buckling / ice field instability - ice damage history / fatigue - nature, shape and size of tidal cracks - material properties of field ice covers (e.g. imperfections, lower strength and better creep than in case of small samples)

Table 2. Base material model parameters for S-UC, F-UC and F-FC models (* parameter value at the reference temperature of 263°K: for others, refer to Sinha (1979)).

Base model	
$\varepsilon_m = \varepsilon_e + \varepsilon_d + \varepsilon_v$	
$\varepsilon_m = \frac{\sigma}{E} + c_1 \left(\frac{d_1}{d} \right) \left(\frac{\sigma}{E} \right) \left\{ 1 - \exp \left[- (a_T t)^b \right] \right\} + \dot{\varepsilon}_{v_1} \left(\frac{\sigma}{\sigma_1} \right)^n t$	
Parameters shared by S-UC, F-UC and F-FC models	
Symbol (units)	Value (if applicable)
σ (kPa)	
E (E_{TI} or E_p) (kPa)	9500000
c_1 (-)	9
d_1 (mm)	1
d (mm)	5
a_T (s ⁻¹)	(2.5·10 ⁻⁴)*
t (s)	
σ_I (kPa)	1000

Table 3. Parameters for S-UC, F-UC and F-FC models (* parameter value at the reference temperature of 263°K; refer to Sinha (1979)).

Model	Ice type	$\dot{\epsilon}_{v_1}$ (s ⁻¹)	n (-)	b (-)
S-UC	T1	$(1.76 \cdot 10^{-7})^*$	3.00	0.340
	S2	$(1.76 \cdot 10^{-7})^*$	3.00	0.340
F-UC	T1	$(9.15 \cdot 10^{-9})^*$	4.90	0.204
	S2	$(2.37 \cdot 10^{-7})^*$	3.34	0.299
F-FC	T1	$(4.89 \cdot 10^{-9})^*$	6.70	0.149
	S2	$(1.55 \cdot 10^{-9})^*$	5.15	0.194

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Table 4. Observed (Obs.) and Calculated (Calc.) peak line loads and their respective ratios. (*Estimated maximum near field line load is 3.4 times measured maximum far field line load. All other observed line loads in table are observed.)

Event	Obs.	(kN/m)	Calc. (kN/m)	Calc./ Max NF
Seven Sisters	Far Field: Max Near*: 1210	356	S-UC-u:	105 0.09
			S-UC-b:	212 0.18
			F-UC-u:	109 0.09
			F-UC-b:	224 0.19
			F-FC-b:	933 0.77
Barrett Chute Event 1	Far Field: Ave. Near: Max Near: 208	50	S-UC-u:	66 0.32
			S-UC-b:	96 0.46
			F-UC-u:	152 0.73
			F-UC-b:	209 1.00
			F-FC-b:	237 1.14
Barrett Chute Event 2	Far Field: Ave. Near: Max Near: 295	76	S-UC-u:	81 0.27
			S-UC-b:	120 0.41
			F-UC-u:	185 0.63
			F-UC-b:	252 0.85
			F-FC-b:	287 0.97
Barrett Chute Event 3	Far Field: Ave. Near: Max Near: 199	81	S-UC-u:	70 0.35
			S-UC-b:	103 0.52
			F-UC-u:	164 0.82
			F-UC-b:	224 1.13
			F-FC-b:	266 1.34

Table 5. Observed and simulated peak line loads during the reported event at Seven Sisters (01/15/1999) and during the reported Event 1 at Barrett Chute (02/22/2011).

Characteristic	Seven Sisters	Barrett Chute
Measurements and characteristics for base case		
Time to peak (hours)	61	7
Total ice thickness (m)	0.51	0.45
Snow ice thickness (m)	0	0.225
Initial stress (kPa)	–	–
Observed peak far field line load (kN/m)	356	50
Observed max peak (NF, kN/m)	–	208
Observed average peak (NF, kN/m)	–	117
Simulated loads (base case, 0 initial stress)		
S-UC-u (kN/m)	105	66
S-UC-b (kN/m)	212	96
F-UC-u (kN/m)	109	152
F-UC-b (kN/m)	224	209
F-FC-b (kN/m)	933	237
F-FC-b-EVO (kN/m)	992	260
Simulated line loads		
F-FC-b (0% of snow ice, kN/m)	base case	348
F-FC-b (50% of snow ice, kN/m)	602	base case
F-FC-b (90% of snow ice, kN/m)	443	239
F-FC-b (-/+ 200 initial stress, kN/m)	945	256
F-FC-b (-/+ 600 initial stress, kN/m)	971	297

Table 6. List of symbols and abbreviations.

Symbol	Definition (Units)
α	Coefficient of thermal dilatation (degrees K ⁻¹)
ε	Total strain (-)
ε_d	Delayed-elastic strain (-)
ε_e	Elastic strain(-)
ε_m	Mechanical strain (-)
ε_{th}	Thermal strain (-)
ε_v	Viscous strain (-)
$\dot{\varepsilon}_{v_1}$	Viscous strain rate corresponding to σ_I (s ⁻¹)
σ	Stress (kPa)
σ_I	Nominal unit stress (kPa)
a_T	Inverse relaxation time at temperature T (s ⁻¹)
b	Time exponent (-)
-b	biaxial confinement of the FEM
c_I	A constant (-)
d	Grain size (mm)
d_I	Nominal unit grain size (mm)
E	Young's modulus (kPa)
E_{T1}	Young's modulus of T1 ice (kPa)
E_p	Young's modulus of S2 ice (plane of isotropy xz) (kPa)
EVO	Elastic-Viscous-Only model
F	Frederking
FC	FullyConfined

FEM	Finite Element Model
FF load	Far Field load (kN/m)
h	Ice thickness (m)
n	Stress exponent (-)
NF load	Near Field load (kN/m)
S	Sinha
S2	Columnar ice
T	Temperature (degrees K)
t	Time (s)
T1	Snow ice
-u	uniaxial confinement of the FEM
UC	UnConfined

Appendix A. Material model implementation into ANSYS via USERMAT.

Elastic strain is calculated from Hooke's law in engineering notation:

$$[\varepsilon_e] = [S][\sigma] \quad (A1)$$

(Ugural and Fenster 2003 for isotropic material; Gere and Timoshenko 1984 for transversely isotropic material). The elastic constants for ice are given in the Table A1. Here and further, definitions of symbols are given in the Table A2.

Delayed elastic strain and viscous strain are calculated using the method of equivalent-stress/equivalent-strain (ANSYS 2013). The method consists of three following stages.

1) *Equivalent stress calculation:*

$$\sigma_{eq} = \frac{1}{\sqrt{m_1 + m_2}} \sqrt{m_2 (\sigma_{xx} - \sigma_{yy})^2 + m_2 (\sigma_{yy} - \sigma_{zz})^2 + m_1 (\sigma_{zz} - \sigma_{xx})^2 + 6(m_5 \sigma_{xy}^2 + m_5 \sigma_{yz}^2 + m_4 \sigma_{xz}^2)} \quad (A2)$$

where m_i -coefficients are given in the Table A1.

2) *Equivalent creep strain calculation:* The creep part of Sinha's 1D equation in the Table 2 is used for the equivalent creep strain calculation at any given time step (Remark A1):

$$\left\{ \begin{array}{l} (\Delta \varepsilon_{cr})^{k+1} = (\Delta \varepsilon_d)^{k+1} + (\Delta \varepsilon_v)^{k+1} \\ (\Delta \varepsilon_d)^{k+1} = \frac{c_1}{E} \frac{d_1}{d} b (a_T \Delta t)^b \sigma_{eq}^{k+1} \exp[-(a_T \Delta t)^b], \quad k=0 \\ (\Delta \varepsilon_d)^{k+1} = \Delta t \frac{c_1}{E} \frac{d_1}{d} \sum_{i=1}^{k+1} \left\{ (\Delta \sigma_{eq})_i (a_T [k+1-i] \Delta t)^b \frac{b}{[k+1-i] \Delta t} \exp[-(a_T [k+1-i] \Delta t)^b] \right\}, \quad k>0 \\ (\Delta \varepsilon_v)^{k+1} = \dot{\varepsilon}_v \left(\frac{\sigma_{eq}^{k+1}}{\sigma_1} \right)^n \Delta t \end{array} \right. \quad (A3)$$

3) *Generalization of the equivalent creep strain in 3D* for normal and shear components of creep strain in engineering notation, respectively (Naumenko 2006):

$$\begin{cases} (\varepsilon_{cr})_{jj}^{k+1} = (\varepsilon_{cr})_{jj}^k + \frac{3}{2} \frac{(\Delta\varepsilon_{cr})^{k+1}}{\sigma_{eq}^{k+1}} (\sigma_{jj}^{dev})^{k+1} \\ 2(\varepsilon_{cr})_{jl}^{k+1} = 2(\varepsilon_{cr})_{jl}^k + 3 \frac{(\Delta\varepsilon_{cr})^{k+1}}{\sigma_{eq}^{k+1}} (\sigma_{jl}^{dev})^{k+1} \end{cases} \quad (A4)$$

The model is validated using analytical solutions and Sinha's (1982) experiments (Kharik et al. 2015). However, for calculation of S2 ice loads in the direction of its columns or triaxial ice loads, detailed literature review on these topics and additional model improvement are recommended.

Remark A1: $\Delta\sigma_{eq}$ should be saved for previous time steps. USERMAT has a limited number of state variables (2093) for data storage. Once the maximum number of state variables is reached, the first $\Delta\sigma_{eq}$ and delayed elastic strain due to it are cumulated in two separate values, that allow liberate one by one position to store new $\Delta\sigma_{eq}$ at 2093+ time steps. Such approach imposes a restriction on a time step depending on an investigated temperature range. Indeed, full ε_d (Table 2) for given stress can be reached only at $t=+\infty$. Therefore, the storage of $\Delta\sigma_{eq}$ prior to $0.9 \cdot \varepsilon_d$ is a good approximation, as last 10% of ε_d accumulates slowly. The dependence of time to get $0.9 \cdot \varepsilon_d$ is exponential: ≈ 0.2 day at 0°C (273 K), ≈ 0.6 day at -10°C (263 K), ≈ 1.9 days at -20°C and ≈ 3.2 days at -24°C . Thus, a time step should be chosen such that all $\Delta\sigma_{eq}$ prior to $0.9 \cdot \varepsilon_d$ are stored. For diurnal events, all stress history is stored as maximum 576 time steps are required. For the multi-day event, $\Delta\sigma_{eq}$ are stored for 3.6 days. It is enough to get $>0.9 \cdot \varepsilon_d$ at -24°C temperature and higher. Moreover, peak load occurs at 1662 time step before the limit of positions for storing $\Delta\sigma_{eq}$ is reached.

Thus, peak loads for both diurnal and multi-day events reported in the paper are not affected by the approach of delayed elastic strain calculation.

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Table A1. Elastic constants (Sinha 1989; Sanderson 2010) and coefficients for equivalent stress formulation (Zhan 1993; Naumenko 2006 – von Mises) used in the finite element model.

T1 ice	
$E = E_{Tl}$ (kPa)	9500000
ν_{Tl} (-)	0.33
m_1 (-)	1.00
m_2 (-)	1.00
m_4 (-)	1.00
m_5 (-)	1.00
S2 ice	
$E = E_p$ (kPa)	9500000
E_y (kPa)	9610000
ν_p (-)	0.33
ν_{yp} (-)	0.05
G_{yp} (kPa)	3170000
m_1 (-)	6.20
m_2 (-)	0.70
m_4 (-)	26.30
m_5 (-)	25.40

Table A2. List of symbols for Appendix A.

Symbol	Definition (Units)
Δ	Increment of a value
ε_{cr}	Creep strain ($\varepsilon_d + \varepsilon_v$) (-)
ε_d	Delayed-elastic strain (-)
ε_e	Elastic strain (-)
ε_v	Viscous strain (-)
$\dot{\varepsilon}_{v_1}$	Viscous strain rate corresponding to σ_I (s^{-1})
ν_p	Poisson ratio of S2 ice at plane of isotropy xz (-)
ν_{T1}	Poisson ratio of T1 ice (-)
ν_{yp}	Poisson ratio in the direction of columns of S2 ice (-)
σ	Stress (kPa)
σ_I	Unit stress (kPa)
σ^{dev}	Stress deviator (kPa)
σ_{eq}	Equivalent stress (kPa)
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{yz}, \sigma_{xz}$	Normal and shear stress components of creep strain in engineering notation (kPa)
a_T	Inverse relaxation time at temperature T (s^{-1})
b	Time exponent (-)
c_I	A constant (-)
d	Grain size (mm)
d_I	Unit of grain size (mm)
E	Young's modulus (kPa)
E_{T1}	Young's modulus of T1 ice (kPa)

E_p	Young's modulus of S2 ice (plane of isotropy xz) (kPa)
E_y	Young's modulus in the direction of columns of S2 ice (kPa)
G_{yp}	Shear modulus in y -direction for S2 ice (kPa)
jj	xx , yy , or zz (-)
jl	xy , yz , or zx (-)
k	Index number of a time step (-)
m_1, m_2	Coefficients for equivalent stress calculation (-)
m_4, m_5	Coefficients for equivalent stress calculation (-)
n	Stress exponent (-)
$[S]$	Compliance matrix for Hooke's law
S2	Columnar ice
t	Time (s)
T1	Snow ice