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Copula Parameter Estimation Using Bayesian Inference for Pipe Data Analysis

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Abstract: Water main systems are aging and becoming a growing concern for maintenance. The structural deterioration of water mains is affected by different factors, such as pipe age, pipe material, soil condition, and pipe size, among others. Various methods of modeling have been used to predict the failure of water mains. Since pipe networks are underground and obtaining data on pipe conditions is very costly, statistical modeling has been widely used for pipe condition assessment. An emerging statistical method known as copula modeling is used here for pipe data analysis. The copula method is very useful in cases where marginals belong to different families of distributions. It is also useful for generating a large number of data points when it is difficult to obtain a data set, as is the case for pipe condition assessment, and where data sets have random variables belonging to non-Gaussian family distributions. Different copula families are applied here to model the dependency between the pipe age and repair age of pipes. The paper uses a Bayesian framework to estimate the parameter values in the copula model. This approach offers an additional option for estimating copula parameters for pipe data.

Keywords: pipe data, copula modelling, Bayesian statistics

Introduction

The underground water infrastructure system in the United States was installed mainly in the periods of the 1800s, 1900-1945, and post-1945. These pipelines are aging and will start to fail in the coming decades due to age, corrosion, and improper installation. In 2013, The American Society of Civil Engineers (ASCE) issued an infrastructure report card. Drinking water and wastewater infrastructure received D grades (ASCE 2013).

Nearly 170,000 public drinking water systems are located across the United States, of which 54,000 are community water systems. Failures in drinking water infrastructure can result

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in water supply disruptions, impediments to emergency responses, and damage to other types of infrastructure. Contamination of water due to broken pipes is also a health concern. Broken water pipes can damage roadways and structures. They can also cause disruption in fire control measures. Unscheduled pipe repair work can cause transportation and commerce disruption (ASCE 2013).

Infrastructure asset management is an approach which can help maintain utility at a desired level of service at the lowest life-cycle cost. Asset management practices applied to underground infrastructure can help utility companies understand the timing and cost associated with rehabilitating, repairing, and replacing pipelines. Knowledge gained from these efforts also helps to develop pipe material selection criteria as part of the replacement strategy (Folkman 2012). Knowing the condition of the pipelines and making accurate predictions about the repair and maintenance work is vital for conducting proper asset management for pipe infrastructure.

Many factors affect the condition of the pipes. The structural deterioration of water mains and their eventual failure are caused by static (pipe material, size, age, soil type, loads on pipe, installation quality, groundwater table, etc.) and dynamic (climate, pressure zone change, etc.) factors. The physical mechanism that leads to pipe breakage is a complex process. Pipes are buried underground, and as such there is not enough data to understand the deterioration mechanism. Information on the current condition of the water main along with an understanding of the failure mode can help utility companies better manage their assets in a cost-effective manner. Data required for physical models are costly to obtain. However, high costs can be justified for major transmission mains. For small diameter pipe networks, other methods such as statistical analysis models are more cost effective.

One of the emerging methods of statistical modeling, being used in financial sectors and

in hydrological engineering is Copula. Copulas are useful for generating joint distributions by combining given marginal distributions according to a specified form of a copula function. Copulas are appealing because they capture dependencies more broadly than the standard multivariate normal framework. This paper uses copulas to model the bivariate distribution by drawing samples from the joint distributions of dependent variables. The paper then uses the Bayesian statistical method using Markov Chain Monte Carlo (MCMC) sampling to obtain the copula parameter.

Background

There are three modeling procedures used to predict pipe failures (Opila and Atttoh-Okine 2011): descriptive analysis models, physical/mechanical models, and statistical models. Descriptive analysis models summarize pipe failure data, which can be used to determine trends in pipe failures. These types of models can provide information on the breakage rates for groups of pipes but not information about individual pipes.

Physical models use the physical mechanism that leads to pipe failure to predict future pipe failures. The physical mechanisms that affect pipe failure are influenced by the pipe structural properties, internal and external loads and material deterioration. The data required for physical models are hard to obtain. Some physical model data include continuous freezing index, soil backfill properties such as porosity, segregation potential, unfrozen water content and thermal gradient at the freezing, elastic soil reaction modulus, the pipe-soil reaction modulus etc (Kleiner and Rajani 2001b). The high cost of obtaining data makes these models appropriate for large lines but very costly for smaller pipe networks. The third type, the statistical models, has been widely used for pipe failure modeling. Kleiner and Rajani (2001a), have summarized and provided list of the statistical models that have been

proposed for water mains condition assessment. The statistical methods for predicting water main breaks use historical data on past failures to predict a future pipe breakage pattern. The statistical models are classified into two groups: deterministic and probabilistic models. The deterministic models predict breakage rates using two or three parameters (Opila and Attoh-Okine 2011). Many other factors affect the breakage of pipes and to incorporate other factors, pipes are classified into groups that are uniform and homogeneous with respect to factors not included in the models. These models use the grouping criteria as covariates in the analysis while retaining a simple mathematical framework. Probabilistic single-variate models use probabilistic processes on grouped data to derive probabilities of pipe life expectancy, probabilities of breakage, and the probability of break clustering phenomenon. The probabilistic multivariate models can consider most of the covariates in the analysis. They do not require the pre-partitioning of the data. Table 1 shows some of the statistical failure models.

Table 1. Sample statistical failure models (adapted from Opila and Attoh-Okine 2011)

Deterministic models	Probabilistic single variate models	Probabilistic multivariate models
Counting process	Cohort-survival models	Accelerated lifetime models
Time-exponential models	Semi-Markov process	Time-dependent Poisson models
Time linear models	Break clustering	Neural networks
Generalized linear model	Bayesian diagnostic	Proportional hazard models (PHM)
		-Bathtub effect PHM
		-Poisson PHM
		-Weibull PHM

Copula modeling is an emerging method of statistical modeling. Even though the use of copulas has become dominant in actuarial science, the financial sector, and biology, there has been limited use of copula modeling in the civil engineering research field. There have been a few research papers on copula modeling in hydrology (Genest and Favre 2007), infrastructure

dependence modeling (Attoh-Okine 2013), water pipe dependence modeling (Atique and Okine 2016), and modelling of vehicle axle-weight (Srinivas et al. 2006).

In this paper, copula dependence modeling is introduced to pipe condition data. The study of multivariate distributions has been dominated by the normal distribution (Frees and Valdes 1998). The choice of the multivariate normal distribution is appealing as the marginal distributions are also normal and the association between two random variables can be fully described by knowing a) their marginal distribution and b) the correlation coefficient. However, in engineering applications, non-normality can occur in the following ways: a) the marginal distribution of some of the variables may not be normal or b) in some cases, even though all the marginal distributions are normal, jointly these variables may not be multivariate normal (Yan 2006). Copulas are useful for generating joint distributions by combining given marginal distributions according to a specified form of a copula function. Copulas are appealing because they capture dependence more broadly than the standard multivariate normal framework.

Linear correlation is used as a measure of dependence between variables following a multivariate normal or elliptical distribution. The dependence measure most frequently used is Pearson's correlation coefficient, which is related with linear dependence and the normal distribution (Accioly and Chiyoshi 2004). However, individual infrastructure conditions may have fat tails, skewness, and non-normal characteristics (Attoh-Okine 2013). This makes inference of the dependency based on correlation inaccurate for non-Gaussian data. The copula approach uses Kendall's rank correlation and Spearman's rank correlation, which measure dependence across the entire distribution, as opposed to Pearson's mean correlation, and are able to model tail dependencies in distributions. The advantages of copula functions have been summarized as follows (Srinivas et al. 2006):

- 1) This process is useful when multivariate simulation is to be carried out from dependent random variables from different classes of marginal distributions
- 2) The copula approach provides a way to separate marginals from the dependence structure, hence reducing the study of a multivariate distribution to a study of a multivariate dependence structure.
- 3) The limitation of using the linear correlation coefficient is avoided.
- 4) The invariance property of the copula dependence structure under transformations and its independence from the marginal distribution can provide efficient simulations.

Berg (2008) has listed the following features of copula modeling:

- 1) The copula contains all the information about the dependence between random variables.
- 2) Copulas provide an alternative and more useful representation of multivariate distribution functions compared to traditional approaches such as multivariate normality.
- 3) Most traditional representations of dependence are based on the linear correlation coefficient, which is restricted to multivariate elliptical distributions. Copula representations of dependence are free of such limitations.
- 4) Copulas enable marginal distributions and the dependence structure to be modeled separately.
- 5) Copulas provide greater modeling flexibility; given a copula, we can obtain many multivariate distributions by selecting different margins.
- 6) Any multivariate distribution can serve as a copula.
- 7) A copula is invariant under strictly increasing transformations.

8) Most traditional measures of dependence are measures of pairwise dependence. Copulas measure the dependence between all d random variables.

Kelly (2007) used copulas to model dependence in simulation risk assessment. The paper used the copula as a tool for risk assessment, stating that common-cause failure models have limitations and substantial subjectivity is associated with parameter estimation for those models. The paper applied the Marshall-Olkin copula and the Frank copula for analysis and stated that marginal distributions in a joint distribution will not be Gaussian; typically, they will be exponential, Weibull, gamma, or lognormal, which complicates the process of drawing Monte Carlo samples from the joint distribution. Romeo and Tanaka (2006) proposed a Bayesian modeling for bivariate survival data and used it for a diabetic retinopathy study. Their method of analysis included slice sampling and the Metropolis Hastings algorithm to sample the distribution. Silva and Lopes (2008) used a Bayesian approach to estimate the parameters of copula models using a two-step method by fitting empirical distribution functions to marginals and maximum likelihood to the copula model.

Copulas

Copulas were introduced in 1959 by Sklar. An n -dimensional copula is a multivariate distribution function defined on the unit cube $[0,1]^n$ with uniformly distributed marginals. The idea of dependence modeling with copula functions is based on Sklar's theorem (Srinivas et al. 2006). It states that if H is an n -dimensional cumulative distribution function with continuous marginal cumulative distributions $F_1, F_2, \dots, F_n$, then there exists an n -dimensional copula C such that for all the real values of $s = [s_1 s_2 \dots s_n]^T$,

$$H(s) = C(F_1(s_1), F_2(s_2), \dots, F_n(s_n)) \quad (1)$$

Where C is unique for marginal distribution functions which are continuous.

Sklar's theorem also shows that for an n -dimensional joint distribution function, the n marginal distributions and the dependence structure can be separated. The copula function can completely describe the dependence between the n variables. By using the converse of Sklar's theorem, the multivariate distribution can be defined by linking the n univariate continuous marginal distributions of any type with a copula function. From equation (1), the multivariate probability density function h can be written as (Srinivas et al. 2006)

$$h(s) = \frac{\partial^n [C(F_1(s_1), \dots, F_n(s_n))]}{\partial F_1(s_1), \dots, \partial F_n(s_n)} \prod_{i=1}^n f_i(s_i) \quad (2)$$

$$h(s) = c(F_1(s_1), \dots, F_n(s_n)) \prod_{i=1}^n f_i(s_i) \quad (3)$$

where $c(F_1(s_1), \dots, F_n(s_n))$ = copula density associated with the copula function $C(F_1(s_1), \dots, F_n(s_n))$ and f_i = marginal probability density function corresponding to F_i (Srinivas et al. 2006).

There are many functions which satisfy the mathematical condition for being a copula function. The functions are known as families of copula functions, and there are two major types of copula: 1) elliptical copulas and 2) Archimedean copulas.

Elliptical copulas are related to elliptical distributions. They have properties that are similar to multivariate normal distributions. The class of elliptical distributions provides a good source of multivariate distributions, which share many of the tractable properties of the multivariate normal distributions, and enables modeling of multivariate extremes and other forms of non-normal dependences (Embrechts et al. 2001).

Archimedean Copulas

Archimedean copulas are a very popular copula family and have a wide range of applications because of the following reasons (Bacigal 2006):

- a. They can be constructed easily.

b. They can be applied whether the correlation between variables is positive or negative.

The Archimedean bivariate copula is expressed as (Nelsen 1999)

$$C(u, v) = \varphi^{-1}(\varphi(u) + \varphi(v)), \quad 0 < u, v < 1 \tag{4}$$

where φ is a convex decreasing function from $[0,1]$ to $[0,\infty]$ such that $\varphi(1) = 0$. By having different generators, there are derivations of several important copulas. Some of the widely known Archimedean copulas are summarized in Table 2 (Attoh-Okine 2013).

Table 2. Archimedean copulas (adapted from Attoh-Okine 2013)

Family of copula	Generator ϕ_t	Parameter θ	Bivariate copula $C_\varphi(u, v)$
Independence	$-\ln t$		uv
Gumbel	$(-\ln t)^\theta$	$\theta \geq 1$	$e^{-[(-\ln u)^\theta + (-\ln v)^\theta]^{-1/\theta}}$
Clayton	$t^{-\theta} - 1$	$\theta > 0$	$(u^{-\theta} + v^{-\theta} - 1)^{-1/\theta}$
Frank	$-\ln\left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right)$	$\theta \in \mathbb{R}$	$\frac{1}{\theta} \ln\left(1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1}\right)$

The Clayton copula tends to work well where there is strong left tail dependence; the Gumbel copula is good for positive tail dependence rather than negative tail dependence. Meanwhile, the Frank copula is a symmetric Archimedean copula where tail dependence is weak. The comparisons between elliptical and Archimedean copulas are shown in Table 3.

Table 3. The comparison between elliptical copulas and Archimedean copulas (adapted from Naimy 2012)

	Elliptical copula	Archimedean copula
Definition	Elliptical copulas are of elliptically contoured distributions. Widely known elliptical distributions are: normal (Gaussian) and student's t copula.	Archimedean copulas are easily constructed and have attractive properties. Commonly used distributions are: Clayton, Frank, and Gumbel.
Advantages	The correlation between the marginals can be easily determined.	They are easily deduced.
Disadvantages	Absence of closed form expressions and impossible to have radial symmetry	The definition does not extend to a multivariate data set of n variables as there will be multiple values of tau.

Since Archimedean copulas are easily constructed and deduced, for analysis of copula parameter using Bayesian approach Archimedean copulas are chosen for analysis of pipe data in this research.

Measure of Dependence

Common measures that are used for analyzing dependence are the 1) Pearson, b) Spearman, and 3) Kendall coefficients. For copula dependence measurements, Spearman's rho and Kendall's tau are used. The Kendall's tau (τ) of pair (x, y) , distributed according to H, can be expressed as the difference between probabilities of concordance and discordance for two independent pairs (x_1, y_1) and (x_2, y_2) , each with H distribution (Attoh-Okine 2013):

$$\tau(x, y) = \Pr((x_1 - x_2)(y_1 - y_2) > 0) - \Pr((x_1 - x_2)(y_1 - y_2) < 0) \quad (5)$$

where $(x_1 - x_2)(y_1 - y_2) > 0$ is concordant and $(x_1 - x_2)(y_1 - y_2) < 0$ is discordant.

204 Kendall's tau can be expressed as

$$\tau(x, y) = \frac{\#Concordant\ pairs - \#Discordant\ pairs}{\#pairs} \quad (6)$$

205

206 *Parameter Estimation*

207 The dependence structure captured by copulas does not have anything to do with the
 208 individual behavior of the variables, so any inference about the parameter indexing a family of
 209 copulas should rely on the rank of the observations (Genest and Favre 2007). Methods normally
 210 used for estimation are estimates based on Kendall's tau, Spearman's rho, and maximum
 211 pseudolikelihood.

212

213 Here the maximum pseudolikelihood estimator is discussed in detail. The method requires that
 214 C_θ be continuous with density c_θ and involves maximizing a rank-based, log-likelihood of the
 215 form (Genest and Favre 2007)

216

$$l(\theta) = \sum_{i=1}^n \log \left\{ c_\theta \left(\frac{R_i}{n+1}, \frac{S_i}{n+1} \right) \right\} \quad (7)$$

217

218 where R_i stands for the rank of X_i among $X_1, \dots, X_n$ and S_i stands for the rank of Y_i among
 219 $Y_1, \dots, Y_n$. Letting $\dot{c}_\theta(u, v) = \partial c_\theta(u, v) / \partial \theta$, Genest et al. (1995) showed that

220

$$l(\dot{\theta}) = \frac{\partial}{\partial \theta} l(\theta) = \sum_{i=1}^n \frac{c_\theta \left(\frac{R_i}{n+1}, \frac{S_i}{n+1} \right)}{\dot{c}_\theta \left(\frac{R_i}{n+1}, \frac{S_i}{n+1} \right)} = 0 \quad (8)$$

221

222 is unique.

223 *Basics of Bayesian Inference*

224 Bayes' theorem is based on the conditional probability as denoted by

$$p(y|x) = p(y, x)/p(x) \quad (9)$$

225 In words, the definition implies that the probability of y given x is the probability that they
 226 happen together relative to the probability that x happens at all. Multiplying both sides of Eq. 9
 227 by $p(x)$ results in $p(y|x)p(x) = p(y, x)$. Similarly, starting with $p(x|y) = p(y, x)/p(y)$ results
 228 in $p(x|y)p(y) = p(y, x)$. With two similar expressions for $p(y, x)$, one finds $p(y|x)p(x) =$
 229 $p(x|y)p(y)$. Dividing both sides by $p(x)$ results in

$$p(y|x) = \frac{p(x|y)p(y)}{p(x)} \quad (10)$$

231
 232 $p(x)$ can be expanded as $p(x) = \sum_y p(x, y) = \sum_y p(x|y)p(y)$; substituting this into Eq. 10
 233 yields

$$p(y|x) = \frac{p(x|y)p(y)}{\sum_y p(x|y)p(y)} \quad (11)$$

235 Eq. 10 and Eq. 11 are called Bayes' rule, and this simple relationship lies at the core of Bayesian
 236 inference. For a continuous variable, Eq. 11 becomes

237

238

$$p(y|x) = \frac{p(x|y)p(y)}{\int dy p(x|y)p(y)} \tag{12}$$

239 Bayes’ rule is applied to models and data. It can be used to know how strongly we should
240 believe the model, given the data:

241

$$p(\text{parameters values and model structure} \mid \text{data values})$$

242 For a model with data value D and parameter value θ , Bayes’ rule can be expressed as

243

$$p(\theta|D) = p(D|\theta) \times p(\theta) / p(D) \tag{13}$$

244 Posterior likelihood prior evidence

Where

245

$$p(D) = \int d\theta p(D|\theta)p(\theta) \tag{14}$$

246 The prior $p(\theta)$ is the strength of the belief in θ without the data D. The posterior $p(\theta|D)$ is
247 the strength of belief in θ when the data D have been taken into account. The likelihood $p(D|\theta)$
248 is the probability that the data could be generated with parameter value θ .

249 When there is observed data, Bayes’ rule can be used to determine beliefs across competing
250 parameter values in a model and to determine beliefs across competing models. The evidence
251 $p(D)$ is the probability of the data according to the model, determined by summing across all
252 possible parameter values weighted by the strength of belief in those parameter values.

For copula parameter estimation, maximum likelihood estimation is often straightforward.

However, there are cases where a Bayesian analysis can be preferable. Smith (2011) noted that:

- For more complex marginal models and/or copula functions, the likelihood can be hard to maximize directly. One of the solutions mentioned in the paper is to use a two-stage estimator where the marginal model parameters θ_j are estimated first and then $\emptyset$ estimated conditionally on these. Another Bayesian alternative in this circumstance is to construct an inference from the joint posterior $f(\theta, \emptyset|y)$, evaluated in a Monte Carlo manner, with θ and $\emptyset$ generated separately in a Gibbs sampling scheme.
- Bayesian hierarchical modeling has been proven successful for the modeling of multivariate data.
- When estimating a copula model, the objective is often to construct inferences on the measures of dependence, quantiles, and/or functionals of random variable vector Y or parameters $(\theta, \emptyset)$. The evaluation of the posterior distribution of these quantities is often easy using Markov Chain Monte Carlo (MCMC) methods.

Pipe Data Analysis Using Copula Modeling

Figure 1 shows a general flow chart for pipe data analysis using copula method. In the beginning pipe inspection data obtained from methods described in Section 2.2 are combined with pipe inventory data such as pipe length, pipe age etc. If GIS data are available, that can also be used for the pipe data analysis. In the data analysis phase, copula modeling can be used to determine the dependence between variables. In pipe deterioration process there is not only a large number of processes that can lead to the degradation of pipes, but also a correspondingly

large number of exploratory variables (Scheidegger et al. 2011). Pipe construction has evolved through many different construction processes and use of different construction materials. In addition, the site conditions, i.e. soil conditions, water table location, water quality etc. can all affect the rate of deterioration. It is important to find, determine and chose exploratory variables for useful prediction models. For cases where the dependence is non linear and skewed, Spearman's rho and Kendall's tau can be better methods for dependence analysis. Dependence analysis can help to determine the relevancy of pipe deterioration variables. Dependence of water pipe data is covered in paper Atique and Okine (2016). Generated large data sets can be used for different objectives. Based on bivariate copula dependence modeling, generated data points are used for regression analysis. Multivariate simulation for copula modeling can also be used to determine the effectiveness and comparison of the existing pipe deterioration models.

A set of water pipe data was obtained from a utility company in the desert west region. The data set included pipes ranging from 4" diameter to 66" diameter. The oldest pipe was installed in 1943, and the newest one was installed in 2009. The data notation used for the variables is shown in Table 4. For analysis, 978 observations were used. These pipes had repairs done to them and were in active status. From Material variable M indicates material type indicated by code. Distribution class varieties include distribution, fire hydrant lateral, service lateral, and transmission. The scatter plots of the variables are shown in Figure 2. For an example, the second column and top row gives the dependency figure for pipe material and pipe age. The x-axis shows the material code from 10 to 60 whereas the pipe age varies from 1 to 40,000. Since the age of the pipe can be a maximum of 24,000 days based on installation day, 40,000 value was an error. The data point is a PVC pipe that had an arbitrary installation date of 1/1/1900. The scatter plot provides an initial revelation that the variables do not provide joint normal distribution. Marginals of some of the variables are also found to be non-normal in further analysis. This provides a good opportunity to study copula modeling for this set of variables.

* In case available

Figure 1. Flow chart of pipe asset management with copula modeling

Table 4. Notation used for data analysis

Variable	Meaning
D	Diameter of pipe
M	Pipe material (code number indicated different material)
L	Length of pipe
PA	Pipe age (in days)
RA	Repair age (in days)

Figure 2. Scatter plots of different variables of pipe data

Pipe age and repair age were the two variables chosen for analysis using the copula method and application of Bayesian inference for parameter analysis. Pipe age is the age recorded at the time the inspection was conducted. Repair age is based on the time the pipe repair took place. The pipe age and repair age were converted to years instead of days for simplicity. The refined data set gave a correlation value for Spearman’s rho of 0.936 for pipe age and repair

age. Figure 3 shows the histogram for pipe age and demonstrates that the general extreme value
type distribution fits the data well.

347

348 **Figure 3.** Pipe age data distribution

Figure 4. Repair age data distribution

Corresponding to the Spearman’s rho value of 0.936, a Gumbel copula parameter of 4.5 was obtained. Using the general extreme value marginal and the Gumbel copula 4.5, 1000 data points were generated. The points were plotted to the original data set as shown in Fig. 5. The figure shows that the generated points match the original points well.

Similarly, using the normal marginal, 1000 data points were generated. Both of these data sets were used for regression analysis, and the results are shown in Table 5. It is apparent that using the GEV as a marginal gives a better result for regression analysis with an R squared value of 0.88 and a standard error of 4.89.

Figure 5. Original data points in hollow circles and copula-generated data in solid circles. PA denotes pipe age, and RA denotes repair age.

Table 5. Comparison of regression analysis using copula modeling and original data

Regression statistics			
	Original data	Gumbel copula GEV marginal generated data points	Gumbel copula normal marginal generated data points
R Squared	0.88	0.88	0.87
Standard error	4.92	4.89	5.06
Observations	902	1000	1000

Applications of the Bayesian inference for the pipe data using the Gumbel copula and the Frank copula are discussed in the following sections.

Gumbel Copula Parameter Estimation Applying Bayesian Inference

368 From Eq. 13, the likelihood function for Bayesian inference for the copula parameter can also be
 369 written as (Kelly 2007)

$$f(t_1, t_2) = f_1(t_1)f_2(t_2)c[F_1(t_1), F_2(t_2)] \quad (15)$$

370 where c is the copula density, and for a marginal of normal distribution,

$$f_i(t_i) = \sqrt{\frac{\tau}{2\pi}} e^{-\frac{\tau}{2}(t_i - \mu_i)^2} \quad (16)$$

371 where μ denotes the mean of the distribution and τ is known as precision, which equals $1/\sigma^2$.

372

373 The “zero tricks” as described in WinBUGS manual is used to encode the likelihood function
 374 (Kelly 2007). In zero tricks, the Poisson (ϕ) observation of zero has likelihood $\exp(-\phi)$, so if
 375 observed data is a set of 0's and $\phi[i]$ is set to $-\log(L[i])$, the correct likelihood is obtained.

376 Bivariate Gumbel copula is given by the following formula:

$$C(u, v; \theta) = e^{-[(-\ln u)^\theta + (-\ln v)^\theta]^{-1/\theta}} \quad (17)$$

377

378 The bivariate copula density

$$c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v} \quad (18)$$

379

$$c(u, v; \theta) = \frac{C(u, v; \theta)(\log(u) \cdot \log(v))^{\theta-1}}{uv((-\log(u))^\theta + (-\log(v))^\theta)^{2-\frac{1}{\theta}}} x((-\log(u))^\theta + (-\log(v))^\theta)^{\frac{1}{\theta}} + \theta - 1 \quad (19)$$

where $u = F_1(t_1)$ and $v = F_2(t_2)$ are in Eq. 19.

For the Gumbel copula, the density function in Eq. 19 was used as a likelihood function in Bayesian analysis. The theta prior of uniform value of 1 to 100 was also used.

Using the Spearman's rho value of 0.93, the Gumbel copula 4.5 was chosen to generate 100 random data points. The parameter estimated by the maximized pseudolikelihood method yielded a value of 4.95 with a standard error of 0.69. For the same data set of repair age and pipe age, using a normal marginal resulted in parameter estimation of 4.81 with a standard error of 0.67.

The generated data using a normal marginal was used to obtain the parameter by Bayesian inference. The mean value of the parameter was 4.106. The range for the 2.5th percentile was 3.40 and for the 97.5th percentile was 4.912, as shown in Table 6. Comparing this theta value with those obtained from the maximized pseudolikelihood method shows that the value obtained from Bayesian inference is of a narrower range than the 95% confidence interval for the theta value obtained from the maximized pseudolikelihood method as shown in Fig. 6.

Table 6. Results for Gumbel copula parameter estimation

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
mu1	31.56	0.8399	0.04827	29.81	31.58	33.15	4001	6000
mu2	26.77	0.7877	0.04559	25.2	26.75	28.29	4001	6000
tau1	0.004936	4.293E-4	1.128E-5	0.004128	0.004922	0.005831	4001	6000
tau2	0.005282	4.651E-4	1.26E-5	0.004428	0.00526	0.006252	4001	6000

theta	4.106	0.3857	0.007917	3.402	4.085	4.912	4001	6000
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Figure 6 Comparison of parameter estimation based on maximized pseudolikelihood method and Bayesian inference method

The results from WinBUGS are shown in Fig. 7, 8, and 9, where the results of quantile, kernel density, and autocorrelation values for theta are shown, respectively.

Figure 7. Quantile figure from WinBUGS analysis for Gumbel copula parameter estimation

Figure 8. Density figure from WinBUGS analysis for Gumbel copula parameter estimation

Figure 9. Convergence of simulation - autocorrelation plot

The trace plots for some of the parameters used in modeling are shown in Fig. 7. The trace plots, which show the iterations versus generated values, are all within a parallel zone without strong periodicities; thus, convergence can be said to have been reached. The autocorrelation plot for the theta parameter is shown in Fig. 9. For the autocorrelation plot, there is no obvious pattern with increasing lag, justifying the conclusion that convergence has been reached. An important parameter used as a check on the simulation is the Monte Carlo (MC) error. It measures the variability of the estimate due to the simulation. A low MC error is required to calculate the parameter of interest with increased precision (Mills and Atttoh-Okine 2014). From Table 6, for all the parameter calculations, the MC error value is significantly low.

Frank Copula Parameter Estimation

The bivariate Frank copula is expressed as

$$C(u, v; \theta) = \frac{1}{\theta} \ln \left(+ \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{e^{-\theta} - 1} \right) \quad (20)$$

Since the Frank copula is continuous, the bivariate copula density can be written as

$$c(u, v) = \frac{\partial^2 C(u, v)}{\partial u \partial v} \quad (21)$$

$$= \frac{\theta(1 - e^{-\theta})e^{-\theta(u+v)}}{[1 - e^{-\theta} - (1 - e^{-\theta u})(1 - e^{-\theta v})]^2}$$

For the Frank copula, a value of 50 for θ corresponds to a Spearman's rho of 0.99, which is almost equal to 1. Therefore, a uniform distribution between 0 and 50 was used as the prior for theta. The normal distribution was chosen for the mean parameter, and the gamma distribution was chosen for the precision parameter tau.

Using two normal marginal distributions, the Frank copula was chosen as 14, corresponding to a Spearman's rho dependency of 0.936, to obtain the joint distribution. Fifty random samples were generated. This sample data was then used to obtain parameter θ by the maximized pseudolikelihood method for the Frank copula, which was 12.07 with a standard error of 2.086. The 95% confidence level interval for the copula parameter yielded the lower limit of 7.98 and the upper limit of 16.16. Bayesian analysis was conducted to obtain parameter θ . The result yielded a mean value of theta 11.25 with the 2.5th and 97.5th percentile values as 8.129 and 14.7, respectively. The Bayesian analysis result fit with a narrower variation within the 95% confidence level than the likelihood method. The summary for the result is shown in Table 7. μ_1 and τ_1 correspond to a value of mean and $\tau_1 = 1/\text{std.dev}^2$ of the pipe age marginal.

Table 7. Summary of results for Frank copula parameter

node	mean	sd	MC error	2.5%	median	97.5%	start	sample
mu1	34.07	1.011	0.0289	32.08	34.08	36.07	4001	17000
mu2	27.66	0.9524	0.02786	25.77	27.64	29.56	4001	17000
tau1	0.007146	9.604E-4	8.536E-6	0.0054	0.007097	0.009143	4001	17000
tau2	0.008054	0.001074	9.752E-6	0.00609	0.007995	0.01029	4001	17000
theta	11.25	1.679	0.01584	8.129	11.19	14.7	4001	17000

From the above analysis, it is shown that Bayesian analysis gives a parameter close to the original parameter of the chosen Frank copula. The values of the mean and standard deviation are also close to original values of the marginals. This shows that Bayesian inference can be used as an alternative method of parameter estimation for copula modeling. The copula parameter estimation is a step of the modeling of non-gaussian pipe data shown in Figure 1, which in return can generate large data set for water pipes.

Remarks

Infrastructure asset management is an approach that can help maintain utility at a desired level of service at the lowest life-cycle cost. Asset management practices applied to underground infrastructure can help utility companies understand the timing and cost associated with rehabilitating, repairing, and replacing pipelines. Knowledge gained from these efforts also helps to develop pipe material selection criteria. Pipe failure is a complex process involving the work of many factors. It is important to have an understanding of the different factors that cause pipe

leakage and to understand the common mechanisms of failure. Different pipe deterioration models are applied to predict the condition of the water pipes.

In this research paper, copula modeling was applied to pipeline engineering. Copula modeling is an emerging method of modeling and is useful in the case where the marginals belong to different families of distributions. Copula modeling is also useful for generating large numbers of data points when it is difficult to obtain data sets. This is the case for pipe condition assessment, where underground pipe data information is expensive to obtain and where data sets have random variables belonging to non-Gaussian family distributions. The large data sets generated can then be used for evaluating the current pipe condition models and the appropriateness of those models for determining the remaining life of a pipe or its condition.

In the pipe data set analysis presented here, it was shown that the general extreme value distribution is a good fit for the marginals, and data simulated with these marginals provide more accurate values for regression analysis for future predictions. The parameter estimation of a copula can be achieved through different methods, such as maximized likelihood, inversion of Spearman's rho, inversion of Kendall's tau, and maximized pseudolikelihood, among which the maximized pseudolikelihood method is based on ranked data. These parameter estimation methods are newly developed methods. Bayesian inference is an alternative method that can be used to obtain the parameters of the copulas. The results indicated that Bayesian inference can provide a good estimate of copula parameters. Two types of copula, the Frank and Gumbel copulas, were tested with a normal marginal, and both gave good estimations. In comparison to the pseudolikelihood method of parameter estimation, the Bayesian result provided a narrower range for the parameter value.

For more complex marginal models and/or copula functions, the likelihood can be hard to maximize directly. In such cases, Bayesian inference using MCMC is a better alternative. For further research, the analysis can be extended to include methods applying Bayesian inference to choose a model of different copulas to fit the pipe condition data based on the Akaike information criterion (AIC).

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