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A Bilevel Model Formulation for the Distributed Wood Supply Planning Problem

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Abstract: The classic wood supply optimisation model maximises even-flow harvest levels, and implicitly assumes infinite fibre demand. In many jurisdictions, this modelling assumption is a poor fit for actual fibre consumption, which is typically a subset of total fibre allocation. Failure of the model to anticipate this bias in industrial wood fibre consumption has been linked to increased risk of wood supply failure. In particular, we examine the *distributed wood supply planning problem* where the roles of forest owner and fibre consumer are played by independent agents. We use game theory to frame interactions between public forest land managers and industrial fibre consumers. We show that the distributed wood supply planning problem can be modelled more accurately using a bilevel formulation, and present an extension of the classic wood supply optimisation model which explicitly anticipates industrial fibre consumption behaviour. We present a solution methodology that can solve a convex special case of the problem to global optimality, and compare output and solution times of classic and bilevel model formulations using a computational experiment on a realistic dataset. Experimental results show that the bilevel formulation can mitigate risk of wood supply failure.

Key words: forest management, fibre procurement, annual allowable cut, industrial consumption, bilevel model, game theory.

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Résumé : La formulation classique du modèle d'optimisation de la possibilité forestière maximise le rendement soutenu, et pose l'hypothèse implicite que la demande de fibre est infinie. Dans plusieurs juridictions, cette hypothèse représente mal la réelle consommation de fibre, qui est typiquement un sous-ensemble de l'attribution totale. L'incapacité du modèle classique de prévoir l'effet de ce biais entre la possibilité forestière et la consommation réelle est liée à une hausse du risque de ruptures de stock. En particulier, nous examinons le *problème de planification distribuée de l'approvisionnement en bois* où des agents indépendants jouent les rôles de propriétaire de la ressource forestière et de consommateur de la fibre. Nous utilisons la théorie du jeu pour encadrer les interactions entre le gestionnaire de la ressource et le consommateur industriel. Nous démontrons que le problème de planification distribuée de l'approvisionnement en bois peut être modélisé plus justement avec une formulation biniveau, et nous présentons une extension du modèle d'optimisation classique pouvant anticiper le comportement du consommateur de fibre industriel. Nous présentons une méthodologie pour résoudre un cas spécial convexe à l'optimalité globale, et nous comparons les performances expérimentales des formulations classiques et biniveaux à l'aide d'un jeu de données synthétique réaliste. Les résultats expérimentaux montrent que le modèle biniveau peut atténuer le risque de ruptures de stock.

Mots clés : planification forestière, approvisionnement en fibre, possibilité forestière, consommation industrielle, modèle biniveau, théorie du jeu.

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1. Introduction

In Canada, provincial government authorities allocate timber licences (TL) to industrial fibre consumers. These TLs, which grant rights to harvest fibre on public forest land, set species-wise upper bounds on periodic harvest volume, however there is typically no policy requirement to set matching lower bounds. Timber licences are typically valid for a pre-determined period (e.g. 5 years), after which point licences may be renewed, subject to re-evaluation of available wood supply. Maximal sustainable TL volume is commonly referred to as *annual allowable cut*, or AAC.

The process by which AAC is determined is referred to as the *wood supply planning problem* (WSPP). We focus on a particular problem variant, which we call the *distributed wood supply planning problem* (DWSPP), where the roles of forest land owner and industrial fibre consumer are played by independent agents. The DWSPP is common in Canada and other jurisdictions, where government planners are responsible for determining AAC and industrial fibre consumers are responsible for planning the actual harvesting activities.

In practice, AAC is typically determined by solving a linear programming (LP) optimisation model, which finds the maximum sustainable harvest level for a given forest (Davis et al., 2001; Gunn, 2007). We will refer to this LP optimisation model formulation as the *classic* wood supply model.

The concept of sustainable forest management (SFM) has long been a central theme of public forest policy in Canada (Canadian Council of Forest Ministers, 2008). According to the *Criteria and Indicators of Sustainable Forest Management in Canada* (CCFM C&I) (Canadian Council of Forest Ministers, 2003), harvesting is generally deemed to be sustainable if it is below AAC. This notion, that any harvest level below AAC is sustainable, is also implicitly used at the forest management unit (FMU) scale when government uses species-wise AAC volumes as contractual upper-bounds in TLs.

The classic model simulates a finite alternating sequence of harvesting and growth, and implicitly assumes that all available fibre will be consumed in every planning period (regardless of operability, quantity, quality, cost, or value creation potential). In practice this assumption is rarely respected. Figure 1 shows the species-wise proportion of Canadian AAC consumed from 1990 to 2012. On average, 80% of softwood AAC and 45% of hardwood AAC were consumed, indicating a clear industrial preference for softwood during this period. The classic model always simulates harvesting of the entire AAC in every planning period, ignoring local constraints on processing capacity of certain types of

30 fibre, fibre procurement cost, primary breakdown cost, and market price for primary harvested wood
31 products. Thus, the classic model implicitly assumes infinite fibre demand. This assumption is unre-
32 alistic in many cases, which explains why historical harvest levels are systematically lower than AAC.
33 This bias is related to the infinite-demand assumption implicitly embedded in the classic wood supply
34 optimisation model and can, to a certain extent, be attributed to a poor alignment between industrial
35 fibre demand and wood supply planning.

36 Local industrial processing capacity may be insufficient to consume some parts of AAC. Other parts
37 of AAC may be economically unattractive (Mathey et al., 2009) or be operationally inaccessible. Thus,
38 the optimal solution of the classic model will likely never be executed, and the long-term state of the
39 forest will be systematically different from that predicted by the wood supply model. If this systematic
40 difference between predicted and actual outcomes is not entirely corrected by regular rolling-horizon
41 AAC re-planning, then the current policy may not be achieving the best possible compromise between
42 realizing short-term value-creation opportunities and maintaining long-term fibre supply stability and
43 forest productivity.

44 Paradis et al. (2013) simulate repeated wood supply planning cycles, modelling interaction be-
45 tween government wood supply planners and industrial fibre consumers. They show that the classic
46 wood supply model may fail due to the aforementioned species-skewed negative fibre consumption
47 bias, and conclude that the wood supply planning process currently in place on public land in Canada
48 may be either overly constraining short-term value-creation opportunities, or not providing credible
49 assurance of the long-term sustainability of the wood supply. Given the pervasiveness of this bias in
50 practice, the classic wood supply optimisation model formulation does not constitute a rational basis
51 for the implementation of sustainable forest management. We endeavor to eliminate this bias from the
52 optimisation model used to determine AAC, thereby improving credibility of the wood supply planning
53 process.

54 In many jurisdictions, sustainable forest management is implemented using a hierarchical planning
55 framework (Van Bueren and Blom, 1997; Tittler et al., 2001). It has been common practice for several
56 decades that planning tools and forestry decision support systems be designed around the hierarchical
57 planning paradigm (Weintraub and Cholak, 1991; Weintraub et al., 2007; Davis et al., 2001). The no-
58 tion of hierarchical planning was first formally developed in a manufacturing context, under the desig-
59 nation of *hierarchical production planning* (HPP) (Bitran and Hax, 1977). In its original manufacturing

context, HPP was typically implemented within a centralized decision-making environment. Effective implementation of HPP includes both coherent linkages to lower levels and effective feedback loops to higher levels, to ensure coherent disaggregation of information from upper through to lower planning levels. In a centrally-managed organisation, goals should be well aligned across planning levels, and lower planning levels are typically subordinate to upper planning levels. In this context, there is clear incentive to implement proper linkages and feedback loops in the HPP process.

In a *distributed planning problem* DPP (Schneeweiß, 2003) environment, where independent agents with imperfectly aligned goals are responsible for different levels of the planning hierarchy, the incentive to implement proper linkages and feedback loops may be weakened or absent, as is the case for hierarchical forest management planning on public land. Despite the widespread and long-standing use of hierarchical planning as a conceptual framework for forest management planning (Barros and Weintraub, 1982; Gunn and Rai, 1987; Weintraub and Davis, 1996), the linkages between long- and short-term forest management planning remain weak, and adaptive feedback loops are essentially non-existent. These deficiencies, which are linked to failure to recognize the distributed nature of the forest management problem, may compromise the effectiveness of the planning process in achieving its stated goals. We can gain further insight into these issues by analyzing the relationships that bind these government and industry actors from a game-theoretic perspective.

Beaudoin et al. (2010) successfully used an agent-based modelling approach to anticipate interaction between participants in a distributed fibre procurement process at the tactical-operational planning level. They leverage this anticipation to better coordinate the planning process between horizontally-linked agents (i.e. firms sourcing fibre from the same procurement area in the public forest), resulting in local and global profitability gains relative to the *ad hoc* base case procurement planning process. We conjecture that a sequential leader-follower anticipation approach could help coordinate the vertical integration between government and industry planners, at a strategic-tactical planning level, in an effort to close the gap between planned and executed harvest volume in the context of the DWSP. From a game-theoretic perspective, this type of sequential leader-follower anticipation corresponds to a *Stackelberg game* (Stackelberg, 2010).

A Stackelberg game is a non-cooperative game with two players who move sequentially. One player (the *leader*) moves first, and the other player (the *follower*) moves second. The follower can observe the leader's move before selecting his move, and the leader knows *ex ante* that this will be the case.

We can describe the DWSPP, from a game-theoretic perspective, in terms of a Stackelberg leader-follower game. The role of *leader* is played by government planners, and the role of *follower* is played by industrial fibre consumers. The leader has the responsibility to ensure long-term integrity of the ecosystem and the wood supply, and aims to maximise AAC. The follower aims to maximise short-term profit by transforming an optimal subset of the wood supply into forest valuable products.

From an operations research perspective, a Stackelberg game can be formulated as a *bilevel planning problem* (BLPP) (Colson et al., 2007). Hence, the DWSPP leader-follower problem can be formulated as a bilevel optimisation problem. We developed a bilevel model formulation for the DWSPP and a novel methodology to solve a special case of the bilevel problem to global optimality.

We hypothesized that the bilevel model formulation would improve stability of the wood supply. To verify this hypothesis, we designed a computational experiment comparing output from classic and bilevel model formulations after 30 sequential rolling-horizon replanning cycles.

The remainder of this paper is organized as follows. Mathematical problem formulations, solution methodology, and experimental methods are presented in §2. Results from the computational experiment are presented in §3, followed by discussion in §4 and concluding remarks in §5.

2. Methods

A BLPP can be described in terms of upper- and lower-level problems (Dempe et al., 2015). We present mathematical formulations for upper- and lower-level problems of the DWSPP in §2.1 and §2.2, which we combine into a bilevel formulation in §2.3. We describe a solution methodology to solve the bilevel model in §2.4, and a computational experiment comparing the performance of classic and bilevel model formulations in §2.5.

2.1. Upper-Level Problem Formulation

The following formulation of the upper-level problem of the DWSPP is an adaptation of the formulation presented in Paradis et al. (2013). This is essentially a general form of the *de facto* standard wood supply optimization model that is used extensively in Canada and elsewhere to determine species-wise maximum even-flow annual allowable cut (AAC) levels. Model input includes a description of the initial forest area (stratified by stand type and age class), growth and yield curves (linked to stand types), a set of treatments (with complex eligibility criteria), state transition matrix (keyed on stand state and

118 treatment), and some aggregate outputs used to define the objective function and constraints.

$$\begin{aligned}
 [1] \quad & \max \sum_{i \in Z} \sum_{j \in J_i} c_{ij} x_{ij} \\
 [2] \quad & \text{s.t.} \sum_{j \in J_i} x_{ij} = 1, \quad \forall i \in Z \\
 [3] \quad & y_{p1} \leq \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijpt} x_{ij} \leq (1 - \varepsilon_p) y_{p1}, \quad \forall p \in O', t \in T \\
 [4] \quad & v_{ot}^- \leq \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijot} x_{ij} \leq v_{ot}^+, \quad \forall o \in O, t \in T \\
 [5] \quad & 0 \leq x_{ij} \leq 1, \quad \forall i \in Z, j \in J_i
 \end{aligned}$$

119 where

Z := set of spatial zones

J_i := set of available prescriptions for zone $i \in Z$

O := set of forest outputs

$O' \subseteq O$:= set of targeted forest outputs

T := set of time periods in the planning horizon

ε_p := admissible level of variation on yield of targeted output $p \in O'$

μ_{ijot} := quantity of output $o \in O$ produced in period $t \in T$ by prescription $j \in J_i$ in zone $i \in Z$

v_{ot}^- := lower bound on yield of output $o \in O$ in period $t \in T$

v_{ot}^+ := upper bound on yield of output $o \in O$ in period $t \in T$

c_{ij} := objective function contribution of prescription $j \in J_i$ in zone $i \in Z$

x_{ij} := proportion of zone $i \in Z$ on which prescription $j \in J_i$ is applied

$y_{p1} := \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijp1} x_{ij}$, i.e. first-period harvest volume for targeted output $p \in O'$

120 The territory is divided into a set Z of spatial zones. For every zone $i \in Z$ we have a set of
 121 prescriptions J_i , where each prescription is a sequence of forest operations to be applied to the stands
 122 in the zone for the entire long-term planning horizon.

123 The objective function [1] maximises harvest volume over the planning horizon. Variables x_{ik} are

linear, with domain $\{x_{ik} \in \mathbb{R} | 0 \leq x_{ik} \leq 1\}$ specified in [5]. These variables are aggregated into first-period targeted-output-wise harvest volume y_{p1} . The set $O' \subseteq O$ represents targeted forest outputs (i.e. species groups for which we determine AAC and enforce even-flow constraints). Constraint [2] requires prescriptions applied to a zone to cover the entire territory. Note that doing nothing for the entire planning horizon is considered a prescription that could generate some outputs (e.g. carbon flows, wildlife habitat, old-growth forest area, etc.). Constraint [3] enforces an *even-flow* policy on targeted outputs. We have expressed the even-flow constraint in terms of y_{p1} , which represents first-period harvest volume for output $p \in O'$. Constraint [4] sets upper and lower bounds on periodic yield for forest outputs. If we set $c_{ij} = \sum_{p \in O'} \sum_{t \in T} \mu_{ijpt}$, the upper-level objective function maximises targeted output harvest volume. The symbol y_{p1} corresponds to the maximum wood supply, for a given species group, that the upper level might offer the lower level in a given planning cycle—thus, y_{p1} constitutes the primary linkage interface between upper- and lower-level models.

Periodic harvest volume is stabilized using *even-flow* constraints, which is common in practice. The objective of the optimisation problem is to maximise species-wise AAC over the planning horizon. This corresponds to the *status quo* strategic planning model in many jurisdictions.

2.2. Lower-Level Problem Formulation

The following formulation of the lower-level problem of the DWSPP is an adaptation of a multi-period problem formulation presented in Jerbi et al. (2012). The lower-level problem essentially emulates industrial fibre-consumption behaviour using a linear network flow optimization model formulation. The model formulation is very general, and as such can accommodate a wide range of network configurations. The network is made up of a number of business units (processing nodes) that consume one or more products and produce one or more products. Input to each processing node can originate from the forest (i.e. raw logs) or from other processing nodes (e.g. chips or sawdust from a sawmill going to a pulpmill). Processes consume set proportions of input products (several inbound arcs can supply the same product), and output set mixes of output products (several outbound arcs can ship the same product). Processes also consume capacitated resources, which can limit flow through a node if a resource becomes saturated. The network features special (forest) source nodes and (market) sink nodes that essentially interface the network with upstream and downstream activities.

$$\begin{aligned}
[6] \quad & \max \sum_{u \in U} \left(\sum_{p \in P} \rho_{up} D_{up} - \sum_{i \in Z} \sum_{p \in O'} \pi_{iup} \beta_{iup} - \sum_{w \in W_u} c_w Y_{uw} \right) - \sum_{e \in E} \sum_{p \in P} \sigma_{ep} F_{ep} \\
[7] \quad & \text{s.t. } \beta_{iup} + \sum_{w \in W_u} (\gamma_{pw} - \alpha_{pw}) Y_{uw} + \sum_{e \in \delta_u^+} F_{ep} - \sum_{e \in \delta_u^-} F_{ep} - D_{up} = 0, \quad \forall i \in Z, u \in U, p \in P \\
[8] \quad & \sum_{w \in W_u} \lambda_{kuw} Y_{uw} \leq q_{ku}, \quad \forall u \in U, k \in K \\
[9] \quad & D_{up} \leq d_{up}, \quad \forall u \in U, p \in P \\
[10] \quad & \sum_{p \in P} F_{ep} \leq f_e^+, \quad \forall e \in E \\
[11] \quad & f_{ep}^- \leq F_{ep} \leq f_{ep}^+, \quad \forall e \in E, p \in P \\
[12] \quad & \sum_{i \in Z} \sum_{u \in U} \beta_{iup} \leq y_{p1}, \quad \forall p \in O' \\
[13] \quad & \beta_{iup} = 0, \quad \forall i \in Z, u \in U, p \in P \setminus O' \\
[14] \quad & D_{up} \geq 0, \quad \forall u \in U, p \in P \\
[15] \quad & \beta_{iup} \geq 0, \quad \forall i \in Z, u \in U, p \in P \\
[16] \quad & Y_{uw} \geq 0, \quad \forall u \in U, w \in W \\
[17] \quad & F_{ep} \geq 0, \quad \forall e \in E, p \in P
\end{aligned}$$

152 where

U := set of business units

K := set of resource capacity types (machine capacities, stock limits)

W := set of processes (machines, inventories)

$W_u \subseteq W$:= set of processes available at business unit $u \in U$

O := set of outputs (from upper-level model)

P := set of products

$O' \subseteq P$:= set of sustainable forest outputs, from the upper-level model

E := set of links between business units

$\delta_u^+ \subseteq E$:= set of inbound links for business unit $u \in U$

$\delta_u^- \subseteq E :=$ set of outbound links for business unit $u \in U$

$q_{ku} :=$ capacity of type $k \in K$ at business unit $u \in U$

$f_{ep}^- :=$ lower bound on flow of product $p \in P$ through link $e \in E$

$f_{ep}^+ :=$ upper bound on flow of product $p \in P$ through link $e \in E$

$f_e^+ :=$ upper bound on flow of all products through link $e \in E$

$\pi_{iup} :=$ unit cost of procuring product $p \in O'$ from forest zone $i \in Z$ to business unit $u \in U$

$c_w :=$ unit cost of process $w \in W$

$\sigma_{ep} :=$ unit cost of transporting product $p \in P$ on link $e \in E$

$\alpha_{pw} :=$ quantity of product $p \in P$ required for one unit of process $w \in W$

$\gamma_{pw} :=$ quantity of product $p \in P$ produced for one unit of process $w \in W$

$\lambda_{kuw} :=$ capacity of type $k \in K$ utilized by one unit of process $w \in W$ at business unit $u \in U$

$d_{up} :=$ demand for product $p \in P$ at business unit $u \in U$

$\rho_{up} :=$ price of product $p \in P$ at business unit $u \in U$

$y_{p1} :=$ maximum external supply of sustainable forest output $p \in O'$ (from upper-level model)

and

$D_{up} :=$ quantity of product $p \in P$ sold by business unit $u \in U$

$\beta_{iup} :=$ external supply of product $p \in P$ from forest zone $i \in Z$ to business unit $u \in U$

$Y_{uw} :=$ quantity of process $w \in W$ performed at business unit $u \in U$

$F_{ep} :=$ flow of product $p \in P$ on link $e \in E$

153 The variables in the lower-level problem are given by D_{up} , β_{iup} , Y_{uw} , and F_{ep} .

154 Note that storage of unsold products and end-product consumption by external clients are mod-
155 elled as special cases of processes. Fibre procurement from the forest is also modelled as a process,
156 although we present it here using a separate symbol β_{iup} , as this facilitates description of the linkage
157 with the upper-level model, via symbol y_{p1} . For simplicity, we have chosen to model industrial fibre
158 consumption in the lower level for the first planning period only, although we could extend anticipation
159 of industrial fibre consumption behaviour to an arbitrary number of periods without loss of generality.

160 The objective function [6] maximises total network profit. Constraint [7] enforces flow conservation
161 in the network. Constraint [8] models process capacity constraints. Constraint [9] ensures that sale

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of products to end-customers does not exceed demand. Constraints [10] and [11] enforce total and product-wise link flow constraints. Constraint [12] ensures flow of fibre from the forest does not exceed species-wise upper bounds from the upper-level model. Constraint [13] ensures null external supply for any product $p \in P$ that is not in set O' . Constraints [14] through [17] are non-negativity constraints on decision variables.

2.3. Bilevel Problem Formulation

Having formulated the optimisation problems of the upper and lower levels, we can combine both levels to form a BLPP. Recall that the motivation for modelling the DWSPP as a BLPP is to anticipate industrial fibre consumption in the wood supply model, thereby eliminating the species-skewed gap between AAC and the volume of fibre actually consumed by industry.

$$\begin{aligned}
 [18] \quad & \max \quad \sum_{i \in Z} \sum_{j \in J_i} c_{ij} x_{ij} \\
 [19] \quad & \text{s.t.} \quad \sum_{j \in J_i} x_{ij} = 1, \quad \forall i \in Z \\
 [20] \quad & y_{p1} \leq \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijpt} x_{ij} \leq (1 - \varepsilon_p) y_{p1}, \quad \forall p \in O', t \in T \\
 [21] \quad & v_{ot}^- \leq \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijot} x_{ij} \leq v_{ot}^+, \quad \forall o \in O, t \in T \\
 [22] \quad & y_{p1} \leq \sum_{i \in Z} \sum_{u \in U} \beta_{iup}, \quad \forall p \in O' \\
 [23] \quad & 0 \leq x_{ij} \leq 1, \quad \forall i \in Z, j \in J_i \\
 [24] \quad & \beta_{\bar{p}} \in \operatorname{argmax}_{\beta_{\bar{p}}} \sum_{u \in U} \left(\sum_{p \in P} \rho_{up} D_{up} - \sum_{i \in Z} \pi_{i\bar{u}\bar{p}} \beta_{i\bar{u}\bar{p}} - \sum_{w \in W_u} c_w Y_{uw} \right) - \sum_{e \in E} \sum_{p \in P} \sigma_{ep} F_{ep}, \quad \forall \bar{p} \in O' \\
 [25] \quad & \text{s.t.} \quad \beta_{i\bar{u}\bar{p}} + \sum_{w \in W_u} (\gamma_{pw} - \alpha_{pw}) Y_{uw} + \sum_{e \in \delta_u^+} F_{ep} - \sum_{e \in \delta_u^-} F_{ep} - D_{up} = 0, \quad \forall i \in Z, u \in U, p \in P \\
 [26] \quad & \sum_{w \in W_u} \lambda_{kuw} Y_{uw} \leq q_{ku}, \quad \forall u \in U, k \in K \\
 [27] \quad & D_{up} \leq d_{up}, \quad \forall u \in U, p \in P \\
 [28] \quad & \sum_{p \in P} F_{ep} \leq f_e^+, \quad \forall e \in E \\
 [29] \quad & f_{ep}^- \leq F_{ep} \leq f_{ep}^+, \quad \forall e \in E, p \in P
 \end{aligned}$$

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$$\begin{aligned}
[30] \quad & \sum_{i \in Z} \sum_{u \in U} \beta_{iu\bar{p}} \leq y_{\bar{p}1} \\
[31] \quad & \sum_{j \in J_i} \mu_{ij\bar{p}1} \hat{x}_{ij} = \sum_{u \in U} \beta_{iu\bar{p}}, \quad \forall i \in Z \\
[32] \quad & \sum_{u \in U} \left(\sum_{p \in P} \rho_{up} D_{up} - \sum_{i \in Z} \pi_{iu\bar{p}} \beta_{iu\bar{p}} - \sum_{w \in W_u} c_w Y_{uw} \right) - \sum_{e \in E} \sum_{p \in P} \sigma_{ep} F_{ep} \geq 0 \\
[33] \quad & \sum_{j \in J_i} \hat{x}_{ij} = 1, \quad \forall i \in Z \\
[34] \quad & v_{o1}^- \leq \sum_{i \in Z} \sum_{j \in J_i} \mu_{ijo1} \hat{x}_{ij} \leq v_{o1}^+, \quad \forall o \in O \\
[35] \quad & D_{up} \geq 0, \quad \forall u \in U, p \in P \\
[36] \quad & \beta_{iu\bar{p}} \geq 0, \quad \forall i \in Z, u \in U, p \in O' \\
[37] \quad & Y_{uw} \geq 0, \quad \forall u \in U, w \in W \\
[38] \quad & F_{ep} \geq 0, \quad \forall e \in E, p \in P \\
[39] \quad & 0 \leq \hat{x}_{ij} \leq 1, \quad \forall i \in Z, j \in J_i
\end{aligned}$$

where $\beta_{\bar{p}}$ represents the vector of fibre flows for targeted output $\bar{p} \in O'$ from all forest zones $i \in Z$ to all units $u \in U$, and $\hat{x}_{ij}$ represents the lower-level (i.e. as-implemented) harvest solution.

The upper-level objective function [18] maximises harvest volume over the planning horizon. Upper-level forest treatment variables x_{ik} are linear, with domain $\{x_{ik} \in \mathbb{R} | 0 \leq x_{ik} \leq 1\}$ specified in [23]. These variables are aggregated into first-period targeted-output-wise harvest volume y_{p1} . The set $O' \subseteq O$ represents targeted outputs (i.e. species groups for which we determine AAC and enforce even-flow constraints). Upper-level constraint [19] requires prescriptions applied to a zone by the upper level to cover the entire territory. Upper-level constraint [20] enforces an *even-flow* policy on targeted outputs. We have expressed the even-flow constraint in terms of y_{p1} . Upper-level constraint [21] bounds minimal and maximal periodic yield on the general outputs. If we set $c_{ij} = \sum_{p \in O'} \sum_{t \in T} \mu_{ijpt}$, the upper-level objective function maximises AAC. Upper-level constraint [22] ensures that first-period harvest volume can and will be entirely consumed by the profit-maximizing lower level. This constraint was not present in the upper-level model formulation presented in §2.1.

The lower-level objective function [24] maximises total network profit, for subproblems considering flows from each targeted output $\bar{p} \in O'$ individually. Lower-level constraint [25] enforces flow conservation in the network. Lower-level constraint [26] models process capacity constraints. Lower-level

constraint [27] ensures that sale of products to end-customers does not exceed demand. Lower-level constraints [28] and [29] enforce total and product-wise link flow bounds. Lower-level constraint [30] ensures flow of fibre from the forest does not exceed species-wise upper bounds from the upper-level model. Lower-level constraint [31] ensures that all fibre harvested in the lower-level harvest vector $\hat{x}$ is consumed by the network. Lower-level constraint [32] ensures that profit is non-negative for the subproblem. Lower-level harvest variables $\hat{x}_{ik}$ are linear, with domain $\{\hat{x}_{ik} \in \mathbb{R} | 0 \leq \hat{x}_{ik} \leq 1\}$ specified in [39]. Lower-level constraints [35] through [38] are non-negativity constraints on lower-level flow variables. Lower-level constraint [33] requires prescriptions applied to a zone by the lower level to cover the entire territory. Lower-level constraint [34] bounds minimal and maximal first-period yield on the general outputs.

Note that symbols y_{p1} and β_{iup} appear in both upper- and lower-level parts of the bilevel model. y_{p1} is controlled by the upper level, whereas β_{iup} is controlled by the lower level. The interpretation of these symbols (i.e. as variables or parameters) depends on where they appear in the model. $\sum_{i \in Z} \sum_{u \in U} \beta_{iup}$ represents an upper bound on y_{p1} in the upper-level problem, whereas y_{p1} represents an upper bound on $\sum_{i \in Z} \sum_{u \in U} \beta_{iup}$ in the lower-level model.

Strictly speaking, this is not a well-posed optimisation problem, as there may be multiple feasible β flow vectors and $\hat{x}$ harvest vectors that induce the same optimal objective function value in the lower-level problem, for an given $\bar{p} \in O'$. If we assume that the leader can select a single follower-optimal $\hat{x}$ harvest vector that maximises his own objective function (i.e. the *optimistic bilevel planning problem*, or OBLPP, see Dempe et al. (2015) and Paradis (2016)), then the problem is well-formed and can potentially be solved to global optimality. The $\hat{x}$ harvest vector corresponds to the first-period lower-level harvest plan, which may (in the most general interpretation of the bilevel DWSPP) differ from the first-period harvest plan selected by the upper level. If we set $\hat{x} = x$, this corresponds to the optimistic assumption which induces the OBLPP. Scenario 2, presented later in this paper, simulates the OBLPP (i.e., simulates the long-term outcome of the system if the optimistic assumption holds true).

The bilevel feasible region is the subset of the upper-level feasible region for which the lower-level model consumes the entire wood supply, while ensuring non-negative profit for each lower-level subproblem. In other words, a bilevel-feasible wood supply solution must be upper level feasible and be entirely consumed by the lower-level model, while ensuring non-negative profit for all species-wise lower-level subproblems.

218 The classic definition of AAC fails to account for the species-skewed negative fibre consumption
 219 bias (i.e. fails to account for realistic industrial fibre consumption behaviour). We propose an extended
 220 definition of AAC, which is *the maximum sustainable harvest level that will be entirely consumed by a*
 221 *profit-maximizing network of industrial consumers*. Given this extended definition of AAC, even-flow
 222 wood supply solutions that will not be entirely consumed by the follower are no longer feasible. If de-
 223 terministic assumptions hold true—the assumption of determinism is implicit to linear programming,
 224 which we use to model both upper and lower levels of the problem—this extended definition will com-
 225 pletely eliminate the problematic species-skewed negative fibre consumption bias. Although defining
 226 bilevel AAC this way is relatively straightforward, the resulting bilevel solution space is non-convex,
 227 which makes it difficult to solve the bilevel wood supply problem to optimality.

228 Early model development efforts focused on what we will hereafter refer to as the *general bilevel*
 229 *case*, which we hoped to solve to global optimality using an iterative solution methodology, based on
 230 the pioneering work of Fortuny-Amat and McCarl (1981) and Bard and Falk (1982). After develop-
 231 ing the mathematical formulation for the general bilevel case, we recognised the potential for non-
 232 convexity of the bilevel solution space. This limited the worst-case performance of our initial solution
 233 methodology approach to local optimal solutions. Given the strategic importance of wood supply plan-
 234 ning, we considered local optimal solutions to be of limited interest in the absence of reliable bounds
 235 on the optimality gap. Thus, we decided to focus subsequent efforts on identifying special cases of the
 236 bilevel problem that we could solve to global optimality.

237 Non-convex cases can occur when the lower-level model features one or more convergent processes
 238 with saturated capacity constraints. A convergent process accepts input derived from two or more sus-
 239 tainable forest outputs $\bar{p} \in O'$ (e.g. a mix of softwood and hardwood). For example, this could occur
 240 if both hardwood and softwood chips in a network must flow through a single pulpmill (i.e. conver-
 241 gent process), with a single digester that runs non-stop (i.e. shared resource with saturated capacity
 242 constraints).

243 By restricting the problem domain to non-problematic (i.e. convex) instances, we can define a
 244 special case of the bilevel problem that can be solved more easily. The special case, which is defined
 245 by excluding problematic instances from the general problem domain, occurs in two types of instances.
 246 Special case 1 corresponds to strictly divergent networks (i.e. no convergence of product flows at any
 247 process), and can be easily identified by analyzing network topology of the lower-level model. Special

case 2 corresponds to partially convergent networks with sufficient capacity at convergent processes so these will not be saturated at the bilevel optimal solution. Special case 2 may only be observable through empirical analysis of the lower level model, and may not be detectable *a priori*, as checking for potential capacity constraint saturation requires us to first solve the lower level model.

Both types of species cases are separable in O' (i.e. the species groups used to segment AAC in the upper level model). Thus, we can develop a solution methodology for the special case that decomposes the lower level problem into convex species-wise subproblems, which can be solved to determine optimal species-wise upper bounds on harvest levels in the upper level problem. These species-wise upper bounds represent valid cuts for the upper level solution space. These cuts form the basis for our solution methodology to solve the special cases. We cannot generate similar valid cuts for the general case, using this methodology, due to potential non-convexity of the general bilevel DWSPP solution space.

The species-wise upper-bound volume for output $\bar{p} \in O'$ (i.e. solution to lower level subproblem for output $\bar{p}$) is equal to $\text{argmax}_{\beta_{\bar{p}}} f(\beta_{\bar{p}})$, where $f(\beta_{\bar{p}})$ is a concave piece-wise linear function describing sub-problem profit as a function of the volume of product $\bar{p}$ consumed by the lower-level model. The piecewise profit function is concave for each separable output $\bar{p} \in O'$, for any non-negative consumption volume $\sum_{i \in Z} \sum_{u \in U} \beta_{iu\bar{p}}$ (see Figure 2). At the lower extremity of the piecewise linear function domain, the follower utilises his limited resources to produce his most profitable product mix until one of his resource constraints is saturated. The follower then switches production to the next most profitable product mix. The slope of each line segment in the piecewise function represents unit profit from a given product mix. By definition, the slopes of these line segments will be monotonically decreasing, hence the concave shape of the piece-wise linear profit function.

2.4. Solution Methodology

We developed a solution methodology to solve the bilevel problem presented in §2.3. By limiting the problem domain to the special cases, we eliminate the possibility of interaction between outputs $\bar{p} \in O'$ (i.e. the bilevel problem becomes separable in O'). This property allows us to compute output-wise upper bounds on lower-level fibre consumption behaviour, which can be used to optimally constrain the upper-level problem to produce the global optimal bilevel solution to the DWSPP. This is the basis of our solution methodology. Algorithm 1 describes this solution methodology using pseudo-code.

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Algorithm 1 solves the bilevel model presented in §2.3. The lower-level subproblems are equivalent to equations [24] to [39]. The optimal solution to each lower-level subproblem yields the maximum volume $\sum_{i \in Z} \sum_{u \in U} \beta_{iu\bar{p}}$ of output $\bar{p}$ that we can expect the follower will consume. Conceptually, this corresponds to $\operatorname{argmax}_{\beta_{\bar{p}}} f(\beta_{\bar{p}})$ (see Figure 2).

For the special case, each upper bound volume $\operatorname{argmax}_{\beta_{\bar{p}}} f(\beta_{\bar{p}})$ constitutes a valid cut for the bilevel solution space. Applying similar cuts to the general case would possibly exclude the global optimal solution, due to non-convexity of the solution space.

Lower-level datasets corresponding to one or the other of the special cases can be solved to optimality using Algorithm 1. Convergent lower-level datasets that have saturated shared capacity constraints can always be transformed to special case 2 by relaxing the saturated shared capacity constraint. The test dataset used in our computational experiment, which we describe in the next section, is an example of special case 2.

2.5. Computational Experiment Methodology

This section describes the computational experiment we conducted to compare the performance of classic and bilevel wood supply models.

2.5.1. Test Dataset

We tested our bilevel solution methodology on a realistic synthetic dataset from Quebec, Canada. The upper-level dataset represents FMU 031–53, located in the boreal forest region. The lower-level dataset represents a hypothetical network of 5 mills.

Our upper-level test dataset is adapted from a government wood supply model for FMU 031–53 in Quebec, Canada. We use the same dataset as that used in Paradis et al. (2013). Although the upper- and lower-level problem formulations described earlier allow the forest dataset to be subdivided into

```
Output: Global optimal wood supply solution  $\mathbf{x}^*$ 
foreach  $\bar{p} \in O'$  do
    Solve lower-level subproblem to determine upper bound  $\sum_{i \in Z} \sum_{u \in U} \beta_{iu\bar{p}}$  on consumption of  $\bar{p}$ .
    Set upper bound constraint  $y_{\bar{p}1} \leq \sum_{i \in Z} \sum_{u \in U} \beta_{iu\bar{p}}$  in the upper-level problem.
end
Solve constrained upper-level model.
```

Algorithm 1: Bilevel model solution algorithm (special case)

any number of zones (for example, to account for differences in transportation cost, stumpage fees, or eligibility for harvest treatments), our test dataset has only a single zone. This simplifies interpretation of our simulation results for the purposes of comparing the output of our bilevel formulation with the classic model. Initial forest inventory, silviculture treatment eligibility and operability, yield curves, and state transition matrix were all compiled by government wood supply analysts for the 2013–2018 AAC planning period.

The original dataset has a total area of 116 538 hectares, 104 413 hectares of which are included in the wood supply analysis (10.4% area netdown). The productive area is made of up 24 373 stands. Figure 3 shows initial stand size distribution for our test dataset.

The test area is in the boreal forest type. The majority (88%) of initial growing stock is softwood (mostly black spruce [*Picea mariana*] and balsam fir [*Abies balsamea*]), with presence (12%) of hardwood species (mostly white birch [*Betula papyrifera*] and trembling aspen [*Populus tremuloides*]). Figure 4 shows the initial growing stock for FMU 031–53.

Some pure softwood stands occur naturally, and plantations are generally pure spruce. A significant proportion of the forest cover is made up of mixed-wood stands containing different proportions of hardwood mixed in with the softwood. Figures 5 and 6 show initial cover type and density class distributions of this management unit. Figure 7 shows the initial age class distribution for our test dataset.

The most recently published official AAC (determined by government planners) is 101 000 m³ for softwood and 10 000 m³ for hardwood.

The original government wood supply model features an array of silviculture treatment options. Our simplified test dataset implements one commercial treatment (clearcut) and two non-commercial treatments (planting, pre-commercial thinning). For our test model, no species-wise selective cutting is allowed (i.e. hardwood must be harvested if present) in mixed-wood stands.

For the lower-level planning model, we compiled an illustrative case study using realistic parameters (production capacities, unit cost of raw logs, transformation process input/output maps, unit prices of finished products). The case study dataset does not represent an actual network of industrial fibre consumers, however these parameters were synthesized from realistic data that was compiled for previous research projects realized by the FORAC Research Consortium. Parameter values used were originally collected from industry partners in the course of previous research projects.

The lower level data model allows for an arbitrary number of external fibre suppliers, with each supplier potentially having distinct input parameters. Our test dataset features a single fibre procurement zone, thus all input units of fibre in the lower-level model have the same unit procurement cost. In reality, procurement cost may vary due to a number of factors (e.g. distance from mill, accessibility, terrain, choice of harvesting system and silviculture prescription, species mix, etc.). However, modelling variable procurement cost was not necessary given the objectives of our computational experiments, and would only have served to obfuscate the results. We model the forest (i.e. external supply of fibre for the network) as a business unit, which encapsulates processes that convert raw fibre to assortments of logs. The forest business units produce three types of softwood logs (small, medium, large) and one type of hardwood log.

The three types of softwood logs can flow to any of the three softwood sawmills in the network. The softwood sawmills produce four types of softwood lumber (2×3 , 2×4 , 2×6 , 2×8) and softwood chips. Softwood lumber can be sold to a single softwood lumber external customer.

The hardwood logs are processed by the single hardwood sawmill, which produces a single type of hardwood lumber and hardwood chips. Hardwood lumber can be sold to a single hardwood lumber external customer.

Both hardwood and softwood chips flow from the sawmills to the paper mill, where they can be converted to paper. The paper can be sold to one of two paper external customers. There is no external customer for chips.

Output from the upper-level model is aggregated into two outputs: *softwood* and *hardwood*. The lower-level model has limited capacity for transforming hardwood (approximately one third of maximum biophysical AAC).

Our test dataset corresponds to an instance of special case 2. Although chip flows from both hardwood and softwood sawmills converge at the pulp mill, we have determined empirically that the shared resource capacity at the pulpmill is sufficient to avoid saturation problems.

2.5.2. Iterative Simulation Framework

We use the two-stage rolling-horizon simulation framework presented in Paradis et al. (2013) to compare the performance of classic and bilevel wood supply model formulations. At each simulated planning cycle, the leader and the follower make their moves sequentially, in a two-stage game. For our

computational experiments, we chose to simulate 30 (5-year) replanning cycles, as this corresponds to the length of our wood supply model planning horizon. The framework simulates forest growth between each 5-year rolling-horizon replanning cycle. The leader has the advantage of the first move, which means he can set AAC to any level of his choosing. Algorithm 2 describes the iterative simulation algorithm using pseudo-code.

foreach $t \in T$ **do**

First stage: leader determines AAC (species-wise bounds on follower consumption).

Second stage: follower consumes a subset of AAC.

    Roll planning horizon forward one period.

end

Algorithm 2: Two-stage rolling-horizon simulation framework algorithm

The simulation algorithm can be summarized as follows:

1. *First stage: the leader determines his wood supply offer.* We simulate the wood supply planning process by solving a wood supply optimisation model. Which model we solve at this stage—either the classic (single-level) model or the extended (bilevel) model—depends on the scenario. The wood supply offer is communicated to the follower in terms of species-wise upper bounds on volume that can be harvested in the second stage.
2. *Second stage: the follower consumes a subset of the wood supply.* We simulate fibre consumption by solving a network flow model, to determine the profit-maximizing subset of wood supply that the follower would willingly consume. Species-wise upper bounds from the first stage (i.e. AAC) are applied at this stage.
3. Simulate rolling horizon forward one period (simulate evolution of forest state using growth and yield curves from long-term wood supply model).

2.5.3. Experimental Methodology

We present five scenarios. Within each scenario, simulation parameters for the industrial fibre consumption network are held constant for all 30 planning cycles. Table 1 summarises simulation parameters for each scenario.

Scenario 1 simulates *status quo* behaviour for both leader and follower, and acts as a control scenario. At each planning cycle, the leader maximises even-flow AAC (30-period horizon) using the

classic wood supply model, then the follower maximises first-period profits (1-period horizon) by consuming an optimal subset of the wood supply offered by the leader. The leader does not consider the follower’s fibre consumption capacity when determining AAC.

Scenario 2 presents a perfect-implementation bilevel scenario—rather than being allowed to replan harvesting on a one-period horizon (as is the case for all other scenarios), we force the follower to exactly implement the first period of the leader’s wood supply solution. This scenario shows the best-case performance of the bilevel model solution, and corresponds to the OBLPP.

Scenario 3 is the basic bilevel scenario. The leader uses the bilevel model to determine AAC, and the follower is allowed to replan harvesting on a one-period horizon, choosing any profit-maximizing subset of available wood supply. Due to the optimal formulation of the bilevel model and perfect anticipation of volume consumption, the follower always chooses to harvest the entire wood supply volume. However, during the second stage of each simulation iteration, the follower may select to harvest this volume from a different combination of forest types than that which was prescribed in the first period of the leader’s optimal solution. Conceptually, the contrast between scenarios 2 and 3 is an estimate of sensitivity of long-term wood supply to deviations from the leader’s optimal bilevel wood supply plan (i.e. sensitivity to the assumption that this problem can be formulated as an OBLPP).

Scenarios 4 and 5 are variants of scenario 3, simulating reduced upper bounds on softwood supply volume. This indirectly creates a buffer stock to protect against the effects of the follower replanning harvest decision vector $\hat{x}$ in the second stage of each simulation iteration (i.e. compensation for the leader’s incomplete control of the DWSP).

3. Results

We present experimental results in two stages. Figure 8 and Table 2 compare output from the first planning cycle for scenarios 1 and 3. Figure 9 shows results of simulating 30 sequential rolling-horizon planning cycles for scenarios 1 through 5.

For scenario 1, the control scenario, potential hardwood fibre supply is 64 583 m³, whereas actual consumption is only 20 800 m³ (see Figure 8). The difference between planned and executed hardwood fibre consumption volumes is due to the limited processing capacity at the (single) hardwood sawmill in the lower-level dataset. The entire softwood fibre supply is consumed by the follower, as we would expect, as both end-product demand and processing capacity for the softwood line are high enough to

accommodate more softwood fibre than the forest can supply. This phenomenon can be observed to varying extents in practice, as processing capacity and market demand are often misaligned with the proposed wood supply. To consume the full softwood supply, while only harvesting one third of the hardwood supply, the follower must harvest stands that have a higher proportion of softwood and lower proportion of hardwood in the second stage of each simulation iteration.

For scenario 3 (i.e. the basic bilevel scenario), the anticipation mechanism completely eliminates over-estimation of hardwood fibre consumption. Fibre consumption is exactly equal to wood supply volumes ($20\,800\text{ m}^3$ for hardwood, and $323\,759\text{ m}^3$ for softwood, see Figure 8). This is the desired outcome from the bilevel model, and represents a global optimal solution for this instance. Note that the follower plans his own harvesting in the second stage of the simulation (using his single-period profit-maximizing model), so harvest areas may not match the upper-level solution, although harvest volume is exactly equal to AAC.

Table 2 presents intermediate results from each step of the bilevel solution method for scenario 3. We present this data as an example of how we derive the optimal upper bounds to the wood supply problem from solutions to output-wise sub-problems. Hardwood consumption capacity ($20\,800\text{ m}^3$) is the binding constraint in this case. The softwood line could have consumed up to $584\,861\text{ m}^3$ in the first planning period. As expected, fibre volume consumed by the follower in the second phase of the simulation is exactly equal to the wood supply. The bilevel model eliminates the gap between planned and executed fibre consumption levels, thereby fulfilling its intended purpose.

We solve the bilevel model in less than twice the time required to solve the classic model. The classic model can be solved in a single step, which corresponds to approximately 13 seconds of CPU time for our test setup. The bilevel model requires $|O'| + 1$ steps to solve, which corresponds to approximately $(4+6)+10$ seconds of CPU time using our test setup. Note that each rolling-horizon-replanning simulation scenario requires solution of the model for each period (30 times, for our test case). The solution times reported for classic and bilevel models are averages over the 30 solutions constituting each of scenarios 1 and 3, respectively. We ran our tests on an Intel® Xeon® E5-2670 processor (20 MB cache, 2.60 GHz).

Figure 9 shows sequential replanning simulation results for the five scenarios described in §2.5.3. For each scenario, we plot both species-wise AAC and fibre consumption for each of the 30 rolling-horizon replanning simulation iterations.

4. Discussion

Scenario 1 shows the relative instability of the classic wood supply model. This is attributable to the species-skewed gap between AAC and fibre consumption volumes. Despite the harvest levels being systematically lower than AAC, the follower’s preference for harvesting high-softwood-content stands gradually shifts the composition of the residual forest cover towards a higher hardwood content. This undesirable shift in forest composition is not predicted by the leader when using the classic wood supply model. Paradis et al. (2013) use the term *systematic drift effect* to describe this phenomenon.

The leader uses the bilevel model to determine AAC for scenarios 2 through 5. By virtue of its formulation, the bilevel model completely eliminates the volume gap between AAC and fibre consumption in scenarios 2 and 3. Note that we simulate perfect anticipation of industrial fibre consumption *volume*. For scenarios 3 through 5, we allow the follower to plan his own harvesting in the second phase of each planning cycle simulation—this explains the residual instability (or *drift*) in long-term wood supply, relative to initial AAC.

Scenario 2 forces the follower to harvest the exact forest units that form the basis of the first period of the leader’s optimal bilevel solution. This scenario simulates the long-term outcome if the optimistic assumption in the OBLPP holds true, and represents best-case outcome of the bilevel planning model. The purpose of this scenario is to show that wood supply tracks almost perfectly along the initial bilevel AAC solution (even after 30 rolling-horizon replanning cycles) under these (optimistic) best-case conditions (i.e. when the leader controls wood procurement planning and execution all the way to the mill gate). In practice, the decoupling point between leader and follower is not typically located this far downstream.

Scenarios 3 through 6 simulate the residual drift induced by a more conventional decoupling point, where the follower is allowed some control over first-period harvest planning. Haley and Nelson (2007) discuss alternative tenure systems for managing public forest in Canada, which can impact the decoupling point between government planners and industrial fibre consumers. Better coordination between wood supply planning and industrial fibre consumption planning could potentially reduce fibre-production-related pressure on the forest ecosystem, thereby providing some measure of freedom in the solution space for accommodation of non-timber values in the forest management process. Note that in Quebec, the wood supply planning process has recently been decentralized, and regional plan-

ning offices have a certain level of autonomy in customizing the formulation of wood supply planning models to accommodate local needs and conditions.

Scenario 3 shows improved softwood supply stability, relative to the control scenario. Note the gradual downward trend of the softwood fibre supply for scenario 3, which contrasts with the more even supply profile simulated in scenario 2. The contrast between scenarios 2 and 3 shows that the bilevel model is sensitive to deviations from the optimal bilevel wood supply model solution. Sensitivity to deviations from the optimal solution is typical of deterministic optimisation models, as optimal solutions are invariably located along the boundary of the feasible region—even the slightest deviations from the optimal solution (or error in constraint right-hand-side values) can induce problem infeasibility.

Scenarios 4 and 5 show the effect of reducing the proportion of softwood AAC that is allocated to the follower, in an attempt to compensate for the residual drift seen in scenario 3. Reducing allocation is an indirect way for the leader to induce a buffer stock in the standing timber inventory. This tends to move the executed (second-stage) solution away from the feasible boundary of the planned (first-stage) solution space, thereby improving the robustness of the distributed wood supply planning process. Scenario 4 shows a marked reduction in drift compared with scenario 3. Residual drift is virtually eliminated in scenario 5. Intuitively, withholding 40% of AAC seems like a high penalty to eliminate the residual drift in the bilevel model, and the leader would likely meet with political opposition if he chose to implement such a policy. We conjecture that, using a more direct management approach to maintaining a buffer stock in standing timber inventory, as described in Raulier et al. (2014), it may be possible to achieve higher stable bilevel AAC levels. This represents a promising direction for further research.

By stabilising the long-term wood supply, scenario 5 succeeds in restoring credibility to the wood supply planning process, albeit at a relatively high cost in terms of withheld AAC. Furthermore, scenario 5 makes less optimistic assumptions regarding follower behaviour than scenarios 2 (which simulates a perfectly compliant follower). Scenario 5 respects the even-flow pattern prescribed by the classic wood supply model constraints. Assuming that the even-flow constraints are valid and necessary (although not sufficient) conditions for sustainability of the forest management plan, and that the leader's responsibility to ensure sustainability must absolutely supersede any desire to maximise short-term TL volume allocations, scenario 5 represents the only example of a sustainable policy in this case study.

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499 Note that if the leader could bundle economically unattractive portions of potential wood supply
500 with more interesting portions, thereby forcing the follower to consume the unattractive portions as
501 a condition on gaining access to the interesting portions, then the antagonism between leader and
502 follower would disappear. This bundling policy option, if it were available to the leader, would change
503 the nature of the problem that we address here, and the bilevel model formulation may no longer be
504 optimal. However, this bundling option would require a more highly-constrained contract binding the
505 follower to the leader. If this bundling policy option is not available to the leader in practice, he has no
506 leverage with which to force the follower to consume economically unattractive fibre. In this case, his
507 only rational choice is to lower the wood supply offer until the follower willingly consumes it all.

508 Recall that our decomposition methodology converges to a globally optimal solution for special
509 cases (with no saturated capacity constraints on convergent nodes in the lower-level network), and
510 that any problematic general instance can be transformed to an instance of special case 2 by relax-
511 ing the saturated capacity constraint just enough to avoid saturation. Alternatively, saturation of the
512 convergent node could be avoided by reducing flow from upstream nodes (i.e. by tightening upstream
513 resource capacity constraints). Finally, our methodology could be used to solve problematic general
514 instances directly (i.e. without any modification of resource capacity constraints), at the expense of
515 guaranteed convergence on a globally optimal solution. For many complex forest management opti-
516 mization problems (e.g. most spatially-explicit multi-period harvest scheduling problems), no solution
517 method guarantees convergence on global optimality (hence the extensive literature on metaheuristic
518 methods for these types of problems). We are assuming that the substantial benefits of anticipating in-
519 dustrial fibre consumption outweigh any potential distortions in the consumption anticipation function
520 induced by relaxing the shared capacity constraint (to convert a problematic general case to special case
521 2)—we have not, however, conducted an elaborate sensitivity analysis on this particular assumption,
522 as this falls outside the scope of this particular study.

523 Conceptually, our bilevel formulation would be applicable to a case where the lower-level model
524 includes a stand-level spatially-explicit harvest scheduling component (harvest scheduling is currently
525 aspatial, notwithstanding the possibility of augmenting the spatially-referenced nature of a given in-
526 stance by partitioning the forest dataset into a large number of zones, even down to stand level). How-
527 ever, spatially-explicit harvest scheduling optimization models (Gunn and Richards, 2005; Lockwood
528 and Moore, 1993; Martins et al., 2005; Liu et al., 2006; Baskent and Keles, 2005) tend to be NP-hard

combinatorial problems, which would tend to considerably extend solution times relative to results reported here (possibly beyond the point of practical tractability, although this has not been tested yet). Ideally, the lower-level model in our bilevel framework would provide a perfect anticipation of the fibre-consumption process, including the exact location and nature of harvest treatments. Extending the method presented here to include a more realistic spatially-explicit harvest schedule optimization component would likely further improve the model's ability to mitigate risk of future wood supply shortages, insofar as this refinement of the model more closely reflects the fibre that will actually be harvested in the first period. However, we cannot say, without further testing, the extent to which this additional level of detail would improve the solution (given other sources of uncertainty in the model, which may dominate the solution).

We tested our proposed methodology on a single realistic dataset. The upper-level dataset is compiled from a real (albeit simplified) government wood supply model used to derive AAC. The lower-level dataset is synthetic, to a certain extent, but was compiled from real mill data collected in the course of previous studies undertaken by the FORAC Research Consortium. Although we made every effort to compile a realistic case study, the result presented here are nonetheless derived from a single dataset, which required considerable effort to compile and validate. We recommend further testing on additional datasets prior to deploying the method in a production setting.

Solutions times for both classic and bilevel models were on the order of seconds. Both upper- and lower-level models are linear, so we expect solution times to scale up or down in as an approximately linear function of the complexity of input datasets for upper- and lower-level problems. Given the large spatial and temporal scale of these wood supply models, and the fact that they need to be re-solved at periodic (5 or 10 year) intervals, the difference between classic and bilevel model solution times is trivial. Furthermore, we consider accuracy (i.e. absence of bias) in the long-term projections to be much more important than model runtime in this context.

We simulated the DWSPP as a two-stage sequential game, where the leader proposes his wood supply in the first stage and the follower consumes a profit-maximizing subset of the wood supply in the second stage, albeit at a high cost in terms of short-term fibre allocation. At this point, we can conjecture that stable increases in AAC, relative to the bilevel solutions presented here, may be achievable if the leader and the follower were allowed to iteratively adjust their respective supply and demand offers within a given planning cycle. This represents a promising direction for further wood supply

559 policy research. From a game-theoretic perspective, extending the two-stage game simulated in this
560 study to include an iterative negotiation dimension corresponds to a *repeated game* or *supergame* in
561 game theory. Under certain conditions *supergames* are known to converge on *socially optimum* equilib-
562 rium solutions (i.e. collaborative solutions) that are globally superior to the (optimal) selfish behaviour
563 in the context of non-repeated (i.e. one-shot) games (Fudenberg and Tirole, 1991). The concept of su-
564 pergames could also be used on a larger scale, to model leader and follower anticipation of upcoming
565 planning cycles (and, potentially, memory of past planning cycles). Ultimately, both scales could be
566 nested (i.e. iterative negotiation within each planning cycle, combined with anticipation of upcoming
567 planning cycles). These hypothetical nested-supergame models might be harnessed for practical ap-
568 plication using a *metagaming* approach (Howard, 1971), potentially providing a wealth of valuable
569 insight to guide high-level government policy-makers.

570 5. Conclusion

571 Paradis et al. (2013) conjectured that extending the classic wood supply model formulation to
572 anticipate industrial fibre consumption would improve wood supply stability (i.e. mitigate risk of wood
573 supply failure). We framed the DWSPP as a Stackelberg leader-follower game, and proposed a bilevel
574 formulation that optimises the outcome from the perspective of the leader. We isolated two special
575 cases of the bilevel problem, which can be decomposed into output-wise convex subproblems, and
576 presented an algorithm that solves the special case of the bilevel problem to global optimality.

577 We tested our solution methodology on a synthetic dataset of realistic size and complexity, and
578 compared results to output from the classic (single-level) wood supply optimisation model. Using a
579 series of five scenarios, we showed that the bilevel model improves long-term wood supply stability,
580 although instability is not completely eliminated by the bilevel model when the follower is allowed to
581 replan first-period harvesting in the second stage of each planning cycle. We showed that the leader can
582 compensate for this residual instability by withholding (i.e. not attributing) a fraction of bilevel AAC.
583 We conjecture that a similar stabilising effect could be achieved more efficiently (i.e. at a lower cost in
584 terms of withheld bilevel AAC) using a more direct buffer stock modelling approach.

585 The bilevel model has the same output data format as the classic model and can be solved us-
586 ing comparable computational effort. As such, the bilevel model formulation constitutes a technically
587 compatible and conceptually superior alternative to the classic model.

The current study examines the performance of a bilevel model formulation in the context of a two-stage Stackelberg game. We recommend that research effort on bilevel wood supply model formulations be extended to *supergame* contexts, both in terms of intra-cycle leader-follower negotiations, and inter-cycle anticipation of future wood supply planning games. To cope with the complexity of using these hypothetical nested-supergame models in a practical government-policy-setting environment, we suggest the adoption of a metagaming approach to wood supply planning as an appropriate starting point for further study.

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Table 1. Summary of scenario parametres

Scenario	Upper-Level Model	Lower-Level Model	Description
1	Classic	Basic	Control scenario (<i>status quo</i>)
2	Bilevel	Slave	Best-case bilevel
3	Bilevel	Basic	Base bilevel
4	Bilevel (80% attribution [†])	Basic	Reduced attribution bilevel
5	Bilevel (60% attribution [†])	Basic	Reduced attribution bilevel (stable)

[†]Proportion of bilevel softwood AAC volume determined in scenario 3.

Table 2. Bilevel solution method (intermediate results)

Stage	Description	Volume (m ³)	
		Hardwood	Softwood
1 (leader)	Upper bound on <i>hardwood</i> consumption	20 800	–
1 (leader)	Upper bound on <i>softwood</i> consumption	–	584 861
1 (leader)	Maximum even-flow wood supply levels	20 800	323 759
2 (follower)	Fibre consumption	20 800	323 759

Fig. 1. Proportion of species-wise AAC from public forests consumed in Canada for period 1990 to 2012 [source: National Forestry Database (2014)]

Fig. 2. Illustration of concave profit function of a hypothetical separable output $\bar{p}$

Fig. 3. Initial stand size distribution for forest management unit 031–53

Fig. 4. Initial growing stock, by species group, for forest management unit 031–53. The two species groups correspond to *S*: softwood and *H*: hardwood.

Fig. 5. Initial cover type distribution for forest management unit 031–53. The three cover types correspond to *S*: softwood, *M*: mixedwood, and *H*: hardwood.

Fig. 6. Initial density class distribution for forest management unit 031–53. The four density classes, expressed as a proportion of complete crown closure, correspond to *A*: (0.25, 0.40], *B*: (0.40, 0.60], *C*: (0.60, 0.80], and *D*: (0.80, 1.00].

Fig. 7. Initial age class distribution for forest management unit 031–53

Fig. 8. Comparison of planned and executed volumes, using classic and bilevel wood supply model formulations

Fig. 9. Species-wise bilevel AAC and fibre consumption for scenarios 1 to 5. The top row of subfigures shows hardwood volume, whereas the bottom row shows softwood volume. Dotted lines represent initial AAC, black circles show re-planned AAC, and grey bars show volume that was actually consumed by the lower-level model in the second stage of each simulation cycle.