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Experimental and Numerical Investigation of Lateral Torsional Buckling of Wood I-Joists

Rémi St-Amour¹, Ghasan Doudak²

Abstract This paper investigates the elastic lateral torsional buckling capacity of wood I-joists. A sensitivity analysis determined that the orthotropic material properties that affect the critical buckling load of wood I-joists are the longitudinal modulus of elasticity, the transverse shear modulus of the flanges and the elastic modulus of the web. A 3D finite element model was developed using experimentally determined material properties and initial imperfections. The study found that FE linear predictions provided reasonable agreement with the experimental buckling loads. The FE geometric nonlinear analysis was able to replicate the experimental nonlinear behavior that was observed during the test. Comparison with contemporary North American design standards showed significant conservatism in the design approach.

Keywords Wood I-joist, Engineered wood product, lateral instability, lateral torsional buckling, finite element analysis, non-linear analysis, initial imperfection, full-scale testing.

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17 **Introduction**

18 The slender nature of wood I-joists may be desirable for flexural resistance but comes at
19 the price of lateral instability as they become susceptible to lateral torsional buckling
20 (LTB) failure. An I-shaped beam develops a tendency to undergo lateral torsional
21 buckling when it is bent about its major axis, if it is not restrained against lateral and
22 angular displacements. Several examples of construction details exist where wood I-joists
23 may not be adequately braced, such as continuous beams, cantilevers and design cases
24 where wind uplift on roofs governs the design.

25 North American timber design standards consider lateral torsional buckling as a potential
26 failure mode for long and slender members. The development of present provisions on
27 the lateral torsional buckling of rectangular wood beams is based on the work by Hooley
28 & Madsen (1964). Research by Bell and Eggen (2001), Suryoatmono and Tjondro
29 (2008), Hindman (2005a), and Xiao (2014) validated the lateral buckling provisions
30 through experimental and numerical investigations. The stability of more complex
31 systems such as beam and deck assemblies were also assessed through numerical studies
32 including those by Zahn (1973) and Du et al. (2016).

33 A closed form solution for the lateral torsional buckling of an I-beam was originally
34 derived for a simply supported configuration and a uniform moment based on the
35 principle of total potential energy (Timoshenko & Gere 1961). The critical moment M_{cr}
36 for a simply supported I-beam of unsupported length L , weak-axis bending stiffness EI_y ,

37 torsional rigidity GJ and a warping stiffness EC_w subjected to a uniform moment M is
 38 given as:

$$39 \quad M_{cr} = \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (1)$$

40 Several factors influence the lateral torsional buckling resistance of an I-section,
 41 including the section geometric properties, moment distribution along the span, boundary
 42 conditions and the position of the applied load relative to the shear center. Closed form
 43 solutions for boundary conditions, moment distribution and load eccentricity effect other
 44 than those assumed for Equation 1 are often unattainable using closed form solutions.
 45 Finite element analysis can be used to find the critical moment of such configurations,
 46 and factors can be derived for various moment gradient (C_m) and load positions relative
 47 to the shear center (C_e).

48 An underlying assumption of Eq. 1 is that it is based on a homogenous material, whereas
 49 wood I-joists consists of a composite assembly of wooden flanges, usually made of
 50 machine stress-rated lumber or laminated veneer lumber, and a web panel made of
 51 oriented strand board (OSB). Transformed section properties, including St-Venant
 52 constant, J_{tr} , warping constant, $C_{w\ tr}$, and moment of inertia about the weak axis, $I_{y\ tr}$,
 53 for composite I-sections with different materials for the flanges and the web can be
 54 derived from the theorem of internal strain energy and Vlasov theory (Du 2016). An
 55 analytical solution is derived (Eq. 2) to predict the critical buckling load of the wood I-

joists by including the moment gradient and load eccentricity factors ($C_m C_e$) as well as the transformed section properties ($E I_{y\ tr}$, $G J_{tr}$ and $C_{w\ tr}$):

$$M_{cr} = C_m C_e \frac{\pi}{L} \sqrt{E I_{y\ tr} G J_{tr} + \left(\frac{\pi E}{L}\right)^2 I_{y\ tr} C_{w\ tr}} \quad (2)$$

In the derivation of Eq. 2, it was assumed that the section is made of isotropic material. However, wood is an anisotropic material but can be simplified and modelled as a rectangular orthotropic material (Isopescu et al. 2012). Also, it has been demonstrated that only 2 of the 9 orthotropic properties of wood influence the elastic lateral torsional buckling response of solid sawn wooden beams (Xiao et al. 2014), namely the longitudinal modulus of elasticity and the transverse shear modulus. Material properties of engineered wood products has been investigated by Hindman (2005a) including the modulus of elasticity and the shear modulus of machine stress-rated lumber, laminated-veneer lumber, parallel strand lumber, and laminated strand lumber.

Although experimental data on the modulus of elasticity and the shear modulus of OSB panel has been provided in the literature (e.g. Hindmann 2003, FPL 2010, Thomas 2011), little is known about the torsional shear modulus and the Poisson's ratio. Grandmont (2011) studied the effect that the material properties of the web has on the deflection of wood I-joist using a finite element model. It was demonstrated that the density variations along the OSB panels affect the shear resistance of wood I-joist and that the properties determined from small-scale specimens differed significantly from those of large scale

specimens. No consistency is found in the literature on how to experimentally determine the material properties of wood I-joists for model input.

Hindman (2005b) conducted an experimental study on the lateral buckling of cantilevered wood I-joist subjected to a concentrated force at the free end. The author showed that the elastic buckling beam theory (Timoshenko & Gere 1961) yielded reasonably accurate results, although there was a significant difference in the range of 12 to 43.4 %. Differences were attributed to considering wood I-joist as a homogenous isotropic material. Zhu et al. (2005) studied the buckling behavior of wood I-joists by comparing experimental tests to finite element simulations in ABAQUS. An empirical formula was derived for the buckling capacity of the web. The authors concluded that the global buckling of wood I-joist was driven by the span, flange size and the location of lateral restraints. Finite element model including geometrical non-linearity provided predictions that agreed well with experimental data. The findings by Zhu et al. (2005) support what was stated previously by Kirby and Nethercot (1979) that initial geometric imperfections reduce the load-carrying capacity of thin-walled members and that effects on the nonlinear response becomes larger with increase in initial imperfections.

Burow et al. (2006) investigated the behavior of wood I-joists in lateral buckling through cantilever and simply supported configurations. Three mathematical models were developed and they included various loading and lateral bracing configurations. Bending stiffness and torsional rigidity of the wood I-joists were determined experimentally. Aguilar (2012) investigated the lateral buckling behavior of wood I-joists under human

walking loads. The author concluded that the critical buckling loads were influenced by the end support condition and the location of the braces. Hindman et al. (2014) studied the effect of bracing stiffness, and human participants' weight on the lateral torsional buckling behavior of wood I-joist. The study showed that the brace stiffness inhibited the angular and lateral displacements of the wood I-joists, while the brace configuration did not influence the lateral acceleration.

The current research study aims at expanding the knowledge base on the lateral instability of engineered wood I-joists through full-scale testing and numerical analysis of deeper and longer specimens than was found in the literature. The objective of the study is to develop a finite element model with experimentally determined material properties and initial imperfections to predict the behavior of wood I-joist tested in full-scale. The paper also evaluates the adequacy of current timber code provisions on lateral stability of engineered wood I-joists.

Sensitivity analysis

Prior to the commencement of the experimental program, a sensitivity analysis was conducted to determine the parameters that significantly influence the lateral torsional buckling behavior of wood I-joists. S4R shell elements were used for modeling the I-joist web. This general-purpose element considers transverse shear deformation for thick shell, however, it becomes negligible when the shell thickness is small. It has a total of 24 degrees of freedom per element (3 displacement and 3 rotation components per node).

Solid C3D8R elements were considered for the I-joist flanges, featuring a solid brick that consists of 24 degrees of freedom (3 translational degrees of freedom per node) with reduced integration and hourglass control. These elements have previously been used in the literature to model solid wood beams and wood I-joists (e.g. Zhu 2005 and Xiao 2014). The wood I-joists were composed of solid-sawn lumber and oriented strand board web, which are both modelled as rectangular orthotropic elastic materials. Table 1 presents the material properties of the reference case. The flange properties are based on the species Sitka Spruce, while the properties of the web are based on mean values found in the literature (FPL 2010 and Thomas 2010). In Table 1, E_i, G_i and μ_i denotes the modulus of elasticity, shear modulus and Poisson’s ratio in the i^{th} direction as illustrated in Figure 1.

Table 1: FE Material properties for the reference case in the sensitivity analysis

Material property	Flanges	Web
E_1 (MPa)	11200	2500
E_2 (MPa)	678	5000
E_3 (MPa)	678	500
μ_{12} (MPa)	0.372	0.195
μ_{13} (MPa)	0.467	0.195
μ_{23} (MPa)	0.435	0.195
G_{12} (MPa)	700	500

G_{13} (MPa)	700	500
G_{23} (MPa)	33.6	100

128 Figure 1 illustrates the inactive translational degrees of freedom (DOFs) at the member
 129 ends that provide a pinned-roller support in the transverse (axis 2) and lateral (axis 3)
 130 directions as well as rotation restraint about the longitudinal axis (axis 1). The web-to-
 131 flange connectivity nodes share the same DOFs using the “SHELL TO SOLID
 132 COUPLING” feature in ABAQUS. This feature is used to couple the displacement and
 133 rotation DOFs of the shell nodes to the solid surface at the web-to-flange interface, which
 134 helps document the effect of distortion of the web on the buckling capacity of the
 135 I-joist.

Figure 1 Inactive translational DOFs (RED arrows) at each end of a simply supported wood I-joist

The load application consisted of a uniform moment about the strong axis. This was done using concentrated forces that were applied at the boundary nodes of the flanges resulting in compression in the top flange and tension in the bottom flange. An eigenvalue buckling analysis was performed to evaluate the critical load. Mesh sensitivity indicated that the critical load converges when using 232 elements along the length, eight elements for the flange width, five elements across the flange thickness and 16 elements for the web height. Further mesh refinements did not result in significant improvement of the solution convergence.

146 The buckling capacity obtained from the finite element (FE) model was analyzed for each
 147 parameter by varying the reference value (from Table 1) from 50% to 150%, while
 148 keeping all other material parameters constant, as shown in Table 2. The effects of
 149 varying these parameters on the buckling capacity are shown as a ratio of the critical
 150 moment obtained for each case to that of the reference case.

151 Table 2 FE model buckling capacity response to material properties fluctuations

Model #	Material property fluctuation									M_{cr} / M_{cr-ref}		
	$\frac{E_1}{E_{1ref}}$	$\frac{E_2}{E_{2ref}}$	$\frac{E_3}{E_{3ref}}$	$\frac{\mu_{12}}{\mu_{12ref}}$	$\frac{\mu_{13}}{\mu_{13ref}}$	$\frac{\mu_{23}}{\mu_{23ref}}$	$\frac{G_{12}}{G_{12ref}}$	$\frac{G_{13}}{G_{13ref}}$	$\frac{G_{23}}{G_{23ref}}$	Compression flange	Tension flange	Web
Ref.	1	1	1	1	1	1	1	1	1	1.00	1.00	1.00
1	1.5	1	1	1	1	1	1	1	1	1.24	1.04	1.00
2	0.5	1	1	1	1	1	1	1	1	0.75	0.94	1.00
3	1	1.5	1	1	1	1	1	1	1	1.00	1.00	1.02
4	1	0.5	1	1	1	1	1	1	1	1.00	1.00	0.97
5	1	1	1.5	1	1	1	1	1	1	1.00	1.00	1.00
6	1	1	0.5	1	1	1	1	1	1	1.00	1.00	1.00
7	1	1	1	1.5	1	1	1	1	1	1.00	1.00	1.00
8	1	1	1	0.5	1	1	1	1	1	1.00	1.00	0.99
9	1	1	1	1	1.5	1	1	1	1	1.00	1.00	1.00
10	1	1	1	1	0.5	1	1	1	1	1.00	1.00	1.00
11	1	1	1	1	1	1.5	1	1	1	1.00	1.00	1.00
12	1	1	1	1	1	0.5	1	1	1	1.00	1.00	1.00
13	1	1	1	1	1	1	1.5	1	1	1.02	1.02	1.01
14	1	1	1	1	1	1	0.5	1	1	0.96	0.96	0.99
15	1	1	1	1	1	1	1	1.5	1	1.05	1.05	1.00
16	1	1	1	1	1	1	1	0.5	1	0.92	0.92	1.00
17	1	1	1	1	1	1	1	1	1.5	1.00	1.00	1.00

18	1	1	1	1	1	1	1	1	0.5	0.99	0.99	0.99
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From Table 2, it can be concluded that the most sensitive parameter is the modulus of elasticity of the compression flange in the longitudinal direction (E_1). The shear modulus of the flanges (G_{12} and G_{13}), also play a significant role in the lateral buckling response. Such results are consistent with the classical theory. Additionally, the FE sensitivity analysis indicates a moderate effect of web distortion on the buckling capacity. The modulus of elasticity (MOE) and shear modulus in the remaining planes as well as the Poisson’s ratios had no effect on the buckling capacity. The above formed the basis for the material parameters that were determined experimentally.

Experimental program

Component tests and determining model inputs

Two approaches were considered to investigate the longitudinal elastic modulus and shear modulus of the I-joists’ flanges, namely those involving the whole I-joist or considering flange components alone. Figure 2 illustrates the dimensions for the two I-joist cross sections investigated.

Figure 2 : I-joist cross section dimensions (mm)

The determination of the flanges' longitudinal modulus of elasticity involved subjecting both the whole I-joist as well as the flange components to bending tests in accordance with the ASTM standard (D198-14). Specimens were loaded to 25 % of the estimated elastic limit at a prescribed rate of 20 mm/minute. The flanges were supported at each end with bearing plates to allow unrestricted longitudinal deformation and rotation during the loading phase. The MOE resulting from the lateral flexural tests on whole I-joist were compared to the one obtained from their respective flanges. The results yielded a difference in the range of 3 to 11%. This difference is likely due to the fact that the component testing approach used only half of the flange length, while the lateral flexural test used the whole flange length. Both methods showed similar variability in the resulting MOE values. Given the similarities in the results it can be assumed that both test methods could be used to derive the model inputs for MOE. The MOE results from the entire I-joist was used as input in the current study since it involves the whole wood I-joist specimen length, and thus accounts for the strength variations along the flanges.

Furthermore, the bending deformations takes place at the same critical section as for the lateral torsional buckling tests.

The determination of the flanges' transverse shear modulus also involved subjecting the whole I-joist or the flange components to torsion tests. The torsion test procedure followed the ASTM standard (D198-14), as shown in Figure 3. Inclinometers were used to measure the angle of twist. The loading was applied until the specimen reached approximately 8° angle of twist to avoid any inelastic deformation.

Figure 3 I-joist torsional rigidity test configuration for whole I-joist (a) and the flange only (b)

The transverse modulus of elasticity of the web was determined based on the procedure described in the American Standard Test Methods for panel in flexure (D3043). A point load was applied at the mid-span of a simply supported web test specimen. The web

specimen was cut from selected I-joists after the lateral torsional buckling test was conducted. These specimens were taken from regions near the support to avoid any damage that may have occurred from the previous test.

Full scale lateral torsional buckling capacity

The lateral torsional buckling capacity of 42 I-joists specimens of various dimensions (Table 3) was evaluated using a simply supported beam configuration with a concentrated load applied at mid-span on the specimen top flange (Figure 4

(a)). In Table 3, IB stands for bending tests on whole I-joists, CT is for component testing which involves bending tests on the flanges, torsion tests on the flanges and bending tests on the webs. LTB represents full-scale lateral torsional buckling test and IT indicates torsion test on whole I-joists.

Table 3: Wood I-joist specimen breakdown and moisture content

Sample group #	Specimen length m [feet]	I-joist section depth, m	Number of specimens	Average moisture content (%)	Moisture content COV (%)	Test conducted
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1	1.83 [6]	0.4	10	12.7	4.67	IB, CT
2	2.44 [8]	0.4	10	13.0	7.03	IB,
3	3.05 [10]	0.4	10	12.6	5.51	IB,
4	3.66 [12]	0.4	10	12.8	6.48	IB,
5	4.27 [14]	0.4	10	12.6	2.73	IB, LTB, CT
6	4.88 [16]	0.4	5	12.8	4.57	IB, IT, LTB
7	5.49 [18]	0.4	5	12.5	2.13	IB, IT, LTB
8	6.10 [20]	0.4	26	12.6	3.52	IB(11), IT, LTB
9	1.83 [6]	0.6	10	15.8	4.61	IB,
10	2.44 [8]	0.6	10	15.9	6.69	IB,
11	3.05 [10]	0.6	10	14.8	6.23	IB,
12	3.66 [12]	0.6	10	16.0	6.99	IB,
13	4.27 [14]	0.6	10	15.3	2.96	IB,
14	4.88 [16]	0.6	5	15.3	7.34	IB,
15	5.49 [18]	0.6	5	15.5	4.14	IB,
16	6.10 [20]	0.6	11	15.4	7.15	IB, IT, LTB, CT

217

218 Figure 4: Schematic drawing of the full-scale lateral torsional buckling test setup (a) and
219 load transfer device (b)

220 The initial out-of-straightness of the top flange was documented prior to testing.
221 Measurements taken during the test consisted of mid-height lateral deflection at mid-
222 span, vertical displacement at mid-span, angular displacement at mid-span and lateral
223 deflection of the top flange at quarter-span. The test specimen was loaded manually by
224 adding weights on a loading platform attached to a load transfer device mounted on the
225 top flange. This allowed the loading apparatus to follow the I-joist through its buckling
226 motion without providing any lateral restraint. The end supports were designed to restrain
227 the vertical displacement, lateral displacement, and rotation about the I-joist longitudinal
228 axis. To that purpose, the specimen was confined in an HSS frame preventing vertical
229 and lateral displacement as well as rotation about the longitudinal axis (Figure 5). The
230 frame was placed on a thrust bearing to allow the specimen to rotate freely about the
231 vertical axis. A steel shaft, welded to the frame base plate, was inserted through the thrust
232 bearing into a steel base that had a fitted hole to prevent the HSS frame from translating
233 in the lateral-longitudinal plane (Figure 5). The load transfer device was designed to
234 transmit the load from the loading platform to the top flange as a concentrated force. A
235 steel hinged support allowed the load to remain vertical during the buckling motion of the
236 test specimen, which is consistent with the underlying assumptions of the reference case
237 for lateral torsional buckling. To achieve this configuration, the hinged support bottom

was screwed on the specimen top flange while the upper part was welded to the load transfer device.

Figure 5: Schematic drawing of end supports design and its restrained DOFs (a) and actual testing end support (b)

Figure 6 shows the buckling motion of the I-joist specimen during the loading phase. Most of the test specimen started deflecting laterally as soon the loading was applied. Very few specimens remained straight during the loading phase until they reached their lateral torsional buckling capacity.

Figure 6 Lateral torsional buckling test deformations. (a) buckling shape, (b) ultimate material failure reached.

Figure 7 displays the buckling shape of eight specimens which are representative of the observed force-displacement curves in the current study. The results reveal two lateral torsional buckling deformation behaviors, one where the I-joist behaves as a perfectly straight beam undergoing lateral torsion buckling when reaching its critical buckling load, and another where the behavior is nonlinear due majorly to initial imperfections. It

can be observed that the nonlinear buckling shapes will generally fail in the vicinity of the critical buckling load of those that exhibited a linear behavior.

Figure 7 : Experimental lateral buckling behavior of wood I-joist specimen from sample group #16

Flexural capacity

The flexural capacity was also evaluated in accordance with the ASTM standard (D198-14) to provide a reference material strength for the test specimens used in this study. Steel hinge supports at the load-bearing location were installed as load distributors to avoid local failure. Web stiffeners were installed at support and force application locations to avoid local web buckling failure. The ultimate flexural capacity test was conducted on 15

I-joist specimens. It was observed that rupture tended to take place in regions with inclined grain orientation and defects such as knots and splits. The average ultimate flexural capacity was found to be 29.9 kNm with a coefficient of variation (COV) of 0.19.

Numerical analysis

The FE models used here is similar to that described in the sensitivity analysis section. The model accounts for the composite nature of the wood I-joist by using the mechanical properties determined from the experimental program. An average flange St-Venant shear modulus of 966 MPa, and a web modulus of elasticity of 3392 MPa were used based on the experimental component test results. The web shear modulus is assumed to be 100 MPa, based on average values in the literature (Grandmont et al. 2008). A concentrated load was symmetrically positioned at mid-span.

A nonlinear analysis was also conducted to describe the behaviour of the joists prior to the ultimate buckling failure. The analysis is characterized by a nonlinear response of a given system where the structure's stiffness varies with respect to the displacement. When the structure's stiffness depends on the displacement, the initial stiffness can no longer be multiplied by the applied load to determine the displacement and can be expressed as:

$$[K(v)]\{v\} = \{P\} \quad (3)$$

286 where $[K(v)]$ denotes the stiffness matrix that is dependent on the nodal displacement
287 $\{v\}$ under loading $\{P\}$.

288 In ABAQUS, a nonlinear problem is solved incrementally with the Newton-Raphson
289 method (Simulia; 2012). This method solves for the single equilibrium path of a
290 nonlinear problem with an iterative process. The nonlinear relationship observed from the
291 experimental investigation of the wood I-joist critical buckling load can be primarily
292 attributed to geometric non-linearity. An eigenvalue buckling analysis were carried out to
293 provide an estimate of the critical buckling load, followed by a nonlinear analysis to
294 reproduce the lateral instability behavior of the wood I-joists observed during the
295 experiments. Modified Riks method was used to conduct a geometric nonlinear analysis,
296 which incorporated the measured initial imperfections on the wood I-joist, using the
297 preceding Eigen mode shape scaled to match the out-of-straightness of the top flange of
298 each test specimens.

299 **Discussion**

300 **Effect of length-to depth aspect ratio on component parameters**

301 Currently, no standardized procedure exists to specify the aspect ratio of span-to-depth
302 needed for testing wood I-joists. As part of the current study the effect of the span-to-
303 depth ratio on the torsional stiffness (GJ) was therefore investigated. As seen in Figure 8,
304 the results show that the span-to-depth ratio has a significant effect on the resulting
305 torsional stiffness (GJ). This effect is likely due to warping deformations which increase

as the test span decreases. It can be observed from Figure 8 that, for the 0.4m I-joist, the torsional stiffness value stabilizes as the test configuration reaches a span-to-depth ratio around 11.5. A similar level of span-depth ratio was not feasible for the 0.6m I-joist since the length of the specimens did not allow for a span-to-depth ratio larger than 9.5. The trend observed in Figure 8 supports the deduction that a similar aspect ratio could also be used for the 0.6m joists. This assumption needs to be substantiated by future studies before a general statement about suitable span-to-depth ratios can be made.

Figure 8 : Length study on the torsional rigidity of entire wood I-joist specimens

Predicting the buckling behaviour of wood I-joists

Figure 9 shows the comparison between the experimental results and those obtained from the linear FE model and the analytical solution (Eq. 2). Both analysis methods yielded critical buckling loads that were lower than the experimental capacity. As expected, the

classical analytical model yielded higher predictions than the FE model. This can in part be attributed to the fact that the analytical solution does not account for the distortion of the web which, as mentioned in the sensitivity analysis section, can reduce the LTB capacity. In general, both the FE analysis and the analytical solution seem to provide appropriate and conservative estimates of the buckling loads.

Figure 9: Experimental CBLs vs. model predictions for the wood I-joist specimens

Figure 10 shows comparison between the non-linear buckling analysis and the experimental results. It can be observed that both the experimental angular displacement and the experimental lateral displacement have similar stiffness relationships. It can also be observed that the nonlinear FE model provides a reasonable estimate of the joist

buckling behaviour when the effect of initial imperfections is included in the model. The I-joist specimens' initial imperfection was measured as the out-of-straightness of the top flange. The value was included in the non-linear analysis in such a way that the pre-buckled shape of the joist in the model is the same as that measured in the test specimen. The deviation between the FE analysis and the experimental results observed could be attributed to the variation in material properties and accuracy in the estimate of the initial imperfection. The sensitivity analysis showed that the initial imperfections appear to affect the lower stiffness branch of the overall nonlinear behavior. Also, the measured imperfections on the wood I-joist test specimens did not account for the initial twist at mid-span, nor the out-of-straightness of the bottom flange, which could have led to more discrepancy between test and modelling results.

a)

b)

Figure 10 Buckling displacements of specimen 5-5. a) Angular displacement at mid-span and b) mid-height lateral displacement at mid-span

Comparison with design standard

Current provisions in the timber standards (CSA 2014; NDS 2015) only address the design of wood I-joist members in a cursory manner. The American design standard and the Canadian design commentary state that the material and geometric properties of only the compression flange should be taken into account when performing lateral stability calculations (NDS 2015; CWC 2014). This statement is partially supported by the FE sensitivity analysis which showed that the most influential material parameter in lateral-torsional buckling is found to be the longitudinal modulus of elasticity in the compression flange.

Moment gradient and eccentricity factors were derived using the FE model developed in the current project and compared with those obtained using the AFPA approach (AFPA; 2003) in Figure 11. The AFPA factors were derived with the same values for the modulus of elasticity, the shear modulus and the geometric properties used in the FE model which considered a simply supported wood I-joist. Although the AFPA (2003) procedure for the derivation of the eccentricity factors (C_e) is based on a rectangular section and neglects warping, it provides eccentricity factors that are similar to those derived with the FE model. A small deviation is observed as the span-to-depth ratio diminishes, suggesting that the warping effects are more pronounced as the model reaches smaller spans. The derived moment gradient (w_2) was almost constant with values ranging from 1.24 to 1.27 for a span to height ratio range of 8.5 to 14.5.

371
372 Figure 11: Comparison between FE and proposed moment gradient and load eccentricity
373 factors

374 The ability of the NDS (2015) design method to predict the critical buckling load was
375 also investigated. Table 4 shows the comparison between the calculated buckling load
376 and those obtained experimentally.

377 Table 4: comparison between experimental buckling capacity and code predictions

Sample group	Specimen #	MOE (MPa)	CBL (kN)		NDS / Exp.
			Experimental	NDS	
5	1	12773	5.03	2.42	0.481
	2	13545	5.53	2.56	0.463
	3	13417	5.39	2.54	0.471
	4	12233	5.30	2.32	0.438
	5	13351	5.44	2.53	0.465
	6	13372	5.06	2.53	0.500
	7	13490	5.48	2.55	0.465
	8	13756	5.66	2.60	0.459
	9	12504	5.30	2.37	0.447
	10	13505	5.44	2.55	0.469

6	1	12205	4.33	1.50	0.346
	2	13301	4.24	1.63	0.384
	3	11950	4.41	1.47	0.333
	4	12394	4.06	1.52	0.374
	5	13095	4.15	1.61	0.388
7	1	11335	3.58	0.88	0.246
	2	13377	3.49	1.03	0.295
	3	12813	3.40	0.99	0.291
	4	13845	3.28	1.07	0.326
	5	11161	2.95	0.86	0.292
8	1	13706	2.57	0.74	0.288
	2	14310	2.66	0.77	0.289
	3	14223	2.75	0.76	0.276
	4	14068	2.48	0.75	0.302
	5	13696	2.75	0.74	0.269
	6	19282	2.75	1.03	0.375
	7	14537	2.66	0.78	0.293
	8	13742	2.57	0.74	0.288
	9	14066	2.57	0.75	0.292
	10	14028	2.66	0.75	0.282
	11	13370	2.48	0.72	0.290
16	1	13490	5.67	3.73	0.658
	2	11393	5.35	3.16	0.591
	3	12160	5.53	3.37	0.609
	4	11735	5.76	3.25	0.564
	5	11357	5.58	3.15	0.565
	6	11846	5.50	3.28	0.596
	7	11597	5.67	3.22	0.568
	8	12286	5.58	3.40	0.609
	9	12753	5.85	3.53	0.603
	10	12593	5.50	3.49	0.635
	11	13204	5.85	3.65	0.624

378

379 The design standard provided predictions of the critical buckling loads that were in the
380 range of 34 to 75% smaller than the experimental capacity. These results expose the high
381 conservatism found in the design standard. The approach used in the design standard
382 seems too simplistic as it substitutes the lateral buckling of the wood I-joist by simple

column buckling of compression flange restrained in the plane of the web. The design standard clearly neglects the restoring action effect of the tension flange on the critical buckling load and the torsional resistance provided by the warping of the I-joist cross-section.

A methodology to determine the lateral buckling capacity of wood I-joists using the analytical solution found in Eq. 2, and modified for moment gradient and load eccentricity factors (AFPA 2003) is presented next. The analytical solution can take the following form;

$$M_{cr} = C_b C_e \frac{\pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \quad (3)$$

Where C_b is a moment gradient factor and C_e is the load eccentricity factors.

In order to compare the numerical example presented here with results obtained experimentally, a test specimen consisting of 38 x 64 mm flanges, 9.53 mm thick web section height of 406 mm and a span (l_u) of 3970 mm is considered. A concentrated load is assumed to be applied at mid-span. The average values of the elastic and shear modulus determined from component testing for sample group #5 are 12,800 MPa and 966 MPa, respectively. The moment gradient factor, C_b , is computed as (AFPA 2003):

$$C_b = \frac{1,25M_{max}}{2,5M_{max} + 4M_A + 3M_B + 3M_C} \quad (4)$$

Where M_A is the absolute value of the moment at $L/4$, M_B is the absolute value of the moment at $L/2$ and M_C is the absolute value of the moment at $3/4 L$. Having $M_A = M_C = 2M_B = M_{max}$, C_b is calculated as 1.32.

The AFPA expression for the load eccentricity factor is as follows:

$$C_e = \sqrt{\left(\frac{kd}{2l_u} \sqrt{\frac{EI_y}{GJ}}\right)^2 + 1} - \frac{kd}{2l_u} \sqrt{\frac{EI_y}{GJ}} \quad (5)$$

Where k is given as 1.72 (AFPA; 2003).

The resulting C_e is 0.82. The web contribution in determining the geometrical properties of the section (I_y, J and C_w) is found to be negligible. The computed moment of inertia about the weak axis calculated using only the properties of flanges was found to be $1.62 \times 10^6 \text{ mm}^4$. Neglecting the web contribution, the warping constant (C_w) of an I-section (Gjelsvik 1981) with thick flanges is $C_w = \{I_y d'^2 / 4\} + \{2t^3 b^3 / 144\}$, where b is the flange width, d' is the web height taken between flange centroids, t is the flange thickness and I_y the moment of inertia about the weak axis. An approximate form for the St-Venant torsional constant $J_e = \sum_{i=1,2,3...} \left(\frac{b_i h_i^3}{\phi}\right)$ was proposed by Kirby and Nethercot (1979), where b is the width, h the height and ϕ is 4.85 for a flange cross-section of $38 \times 63.5 \text{ mm}$. The resulting warping constant and St-Venant torsional constant of the flanges are computed to be $5.52 \times 10^{10} \text{ mm}^6$ and $1.44 \times 10^6 \text{ mm}^4$, respectively.

417 Using these inputs, the resulting critical load applied at mid-span can be determined to be
418 5.29 kN. This value is considered reasonably close to the experimental lateral buckling
419 load of specimen group #5, i.e. 5.36 kN.

420 The above example illustrate that the proposed procedure can be used to estimate the
421 lateral torsional buckling capacity of wood I-joists with reasonable accuracy. It should be
422 noted that the example is based on average values in order to compare with the test
423 results. The approach used to determine the design level safety factor for the buckling
424 failure is different for different standards and therefore no attempt was made here to
425 suggest what the design level capacity should be. Instead the validity of the approach is
426 demonstrated and it is left to the appropriate code and standard committees to debate the
427 level of safety needed.

428 **Conclusions**

429 The conclusions that can be drawn from the current study are summarized as follows:

- 430 1) A sensitivity analysis was conducted on the mechanical properties of wood I-
431 joists and it revealed that the most influential mechanical properties are the
432 longitudinal modulus of elasticity and the torsional shear modulus of the
433 compression flange. To a lesser extent, these same properties in the tension flange
434 and the transverse modulus of elasticity of the web also impact the buckling
435 response.

- 2) The lateral flexural stiffness test of the entire I-joist and the torsion test on the flange alone were found to be appropriate and provide adequate input parameters for the analytical models.
- 3) The FE linear predictions were found to be in reasonable agreement with the experimental buckling loads. The FE geometric nonlinear analysis incorporated the initial imperfection into the models and was able to replicate the experimental nonlinear behavior that was observed during the test.
- 4) The Analytical solution for lateral torsional buckling of I-beam can provide a conservative estimate of the critical buckling load of simply supported wood I-joist providing that proper moment gradient and load eccentricity factors are used.
- 5) The NDS model provided predictions that were 34 to 75% smaller than the experimental lateral torsional buckling failure load, which shows significant conservatism in the approach. An approach based on the classical solution modified for moment gradient and load eccentricity factors was presented to demonstrate a possible methodology to design I-joists for lateral torsional buckling failure.

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