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Complete List of Authors:	Guo, Peijun; McMaster University, Stolle, Dieter; McMaster University,
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2. Authors:

- a) Peijun Guo
- b) Dieter F. E. Stolle

3. Affiliation and address:

- a) Peijun Guo (corresponding author)
Professor
Department of Civil Engineering, McMaster University
1280 Main Street West
Hamilton, ON
Canada L8S 4L7
Email: guop@mcmaster.ca
Tel: 905-525-9140 ext. 27903
Fax: 905-529-9688

- b) Dieter F. E. Stolle
Professor
Department of Civil Engineering, McMaster University
1280 Main Street West
Hamilton, ON
Canada L8S 4L7
Email: stolle@mcmaster.ca
Tel: 905-525-9140 ext. 28600
Fax: 905-529-9688

A physically meaningful homogenization approach to determine equivalent elastic properties of layered soil

Peijun Guo, Dieter F. E. Stolle

Department of Civil Engineering, McMaster University

Abstract

A simple, yet physically meaningful, homogenization approach to determine the “equivalent” elastic properties for a layered medium is proposed. The proposed approach makes use of the Reuss and Voigt approximations without employing auxiliary stresses and strains related to the different elastic properties of the material layers. When assuming the constituent materials are isotropic, some special features of the equivalent homogeneous medium are discussed for special cases in which all layers have either the same Poisson's ratio, or elastic modulus, or shear modulus. A methodology to determine the equivalent anisotropic elastic properties of multilayered soils is proposed, in which all required quantities are physically meaningful and can be determined directly. The applicability of the proposed methodology is demonstrated through tests on two types of soils, one of which has varved structure while the other is isotropic.

Keywords: Layered soil, equivalent elastic properties, anisotropy, homogenization.

Introduction

Varved soils and stratified sedimentary rocks usually consist of a series of parallel layers, with the thickness and properties of the layers varying with depth in either a random or systematic alternative pattern. Each of these layers may have isotropic or orthorhombic properties with the thickness of layers varying between several millimeters to several metres or more. When the thicknesses of the individual layers are much smaller than the characteristic length of interest, a

25 layered media can be represented by an equivalent homogeneous medium using various
26 homogenization methodologies. To obtain meaningful representation of the equivalent material,
27 both its failure characteristics (e.g., strength) at large deformation and the elastic properties at
28 small strain levels must be obtained.

29 Both theoretical and experimental work has been carried out to investigate the properties of
30 media which are equivalent in behaviour to layered media. Among the research work regarding
31 the strength and failure of stratified rocks or soils, Arthur and Philips (1975) investigate the
32 failure of layered sand in triaxial compression and conclude that the strength of samples
33 comprising layers of loose and dense sand is mainly dominated by the proportion of loose and
34 dense sand in a sample. Yielding tends to be confined to the loose material layer, with a line of
35 zero extension locally in the loose material being approximately parallel to the layer and
36 differing appreciably from that of the plane of maximum stress obliquity. Lydzba *et al.* (2003)
37 propose a framework to develop a criterion for the macroscopic failure of stratified rocks, in
38 which the shear strength parameters of the stratified rock are uniquely expressed as a function of
39 the volume fraction and the strength parameters of each individual constituent layer. Lourenço
40 (1996) and Niemunis *et al.* (2000) explicitly take into account the interaction between material
41 layers and the discontinuity of velocity gradients across material interfaces when determining the
42 failure criterion of layered materials. Guo and Stolle (2009) derive the lower limit and upper
43 limit for the strength of layered soils, which correspond to failure in one constituent and all
44 constituents, respectively. It should be noted that the homogenization approach (e.g., Kaw
45 2006) used to determine the ultimate strength of laminate materials may also be used to estimate
46 the strength of layered media, however with caution.

47 Regarding the research on the failure characteristics, considerable work has been
48 undertaken to determine the elastic properties of layered media and composite materials, using

various homogenization methodologies; see, for example by Hornung (1997), Cioranescu and Saint Jean Paulin (1999), Nemat-Nasser and Hori (1999), Milton (2004), Kachanov and Sevostianov (2013) and Skrzypek and Ganczarski (2015). Among this group of researches, the original studies of Voigt (1907) and Reuss (1929) are still of considerable interest, particularly for layered media. The Voigt and Reuss models are however over-simplified, even though they have clear physical meaning. Nevertheless, their models tend to provide upper and lower bounds of the elastic moduli (Kachanov and Sevostianov 2013). While various advanced homogenization methodologies have been developed for composite materials, when dealing with stratified rock mass or layered medium, many investigators define an equivalent homogeneous and continuous medium within the framework of conventional volume-averaging techniques, by following the work of Postma (1955) and Salamon (1968). In their approaches, the interaction between layers is taken into account together with deformation compatibility. More details are provided by Postma (1955), Backus (1962) and Salamon (1968), as well as more recent work by Gerrard (1982), Thomsen (1986), Lai et al. (1997), and Niemunis et al. (2000). In all these works, a multilayered medium is represented by an equivalent homogeneous material. Expressions for the elastic constants of this equivalent material are given in terms of the elastic properties and volume fraction of the constituent layers. When each constituent layer is considered as isotropic elastic, the equivalent homogeneous material is shown to resemble a homogeneous transversely isotropic material that has five independent constants.

This paper first revisits the methods of Postma (1955) and Salamon (1968) to determine the effective elastic constants of a transversely isotropic material that is equivalent to a layered medium. Then a simple, yet physically meaningful, homogenization approach is proposed to determine effective elastic properties of layered material. This approach does not require the determination of the auxiliary stresses or strains in individual constituent layers, but rather only

the well-known Voigt and Reuss approximations. A scheme to determine the effective elastic moduli of stratified soils is proposed, in which soil properties with clear physical meaning are determined from laboratory tests. This approach is demonstrated for two select soils. In addition, some special features of the homogenized stratified medium are discussed.

Basic principles for homogenization of stratified elastic materials

Anisotropic elasticity

The stress-strain relation of a general anisotropic linear elastic material can be expressed as $\varepsilon_{ij} = C_{ijkl} \sigma_{kl}$, in which the elastic compliance tensor C_{ijkl} has at most 21 independent elastic constants.

For transversely isotropic (or cross-anisotropic) linear elastic behaviour, when the axis of symmetry is selected as the z -axis, the stress-strain relationship can be expressed in the matrix form as (e.g., Skrzypek and Ganczarski 2015):

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{bmatrix} 1/E_h & -\nu_{hh}/E_h & -\nu_{vh}/E_v & & & \\ -\nu_{hh}/E_h & 1/E_h & -\nu_{vh}/E_v & & & \\ -\nu_{hv}/E_h & -\nu_{hv}/E_h & 1/E_v & & & \\ & & & 1/G_{hh} & & \\ & & & & 1/G_{hv} & \\ & & & & & 1/G_{vh} \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} \quad (1)$$

where ν_{hv} = principal Poisson's ratio of strain in vertical direction caused by strain in the horizontal direction; ν_{vh} = principal Poisson's ratio of strain in horizontal direction caused by strain in the vertical direction; ν_{hh} = Poisson's ratio of strain in horizontal direction caused by strain in the horizontal direction normal to the former; G_{hh} , G_{hv} and G_{vh} are shear modulus appropriate to the respective planes. The symmetry in the elastic compliance matrix requires

$$\frac{\nu_{vh}}{E_v} = \frac{\nu_{hv}}{E_h} \quad (2)$$

The shear modulus G_{hh} is not independent since it can be related to E_h and ν_{hh} by $G_{hh} = E_h / 2(1 + \nu_{hh})$. As a result, the five independent elastic constants are $E_v, E_h, \nu_{hh}, \nu_{hv}$ and G_{hv} . The elastic compliance matrix in the principal stress space is then

$$C_{ijkl} = \begin{bmatrix} 1/E_h & -\nu_{hh}/E_h & -\nu_{hv}/E_h \\ -\nu_{hh}/E_h & 1/E_h & -\nu_{hv}/E_h \\ -\nu_{hv}/E_h & -\nu_{hv}/E_h & 1/E_v \end{bmatrix} \quad (3)$$

The requirement for positive strain energy limits the value of ν_{vh} via

$$-\sqrt{\frac{1-\nu_{hh}}{2n}} \leq \nu_{vh} \leq \sqrt{\frac{1-\nu_{hh}}{2n}}$$

where $n = \nu_{hv} / \nu_{vh} = E_h / E_v$. In addition, the value of ν_{hv} must satisfy the constraint

$$-\sqrt{\frac{n}{2}(1-\nu_{hh})} \leq \nu_{hv} \leq \sqrt{\frac{n}{2}(1-\nu_{hh})}.$$

Considering the stress-strain relation in terms of stiffness, $\sigma_{ij} = D_{ijkl} \epsilon_{kl}$, the elasticity matrix in the principal stress space can be simplified as (Skrzypek and Ganczarski 2015)

$$D_{ijkl} = \begin{bmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{bmatrix} \quad (4)$$

with

$$D_{11} = D_{22} = \frac{E_h (E_v - E_h \nu_{vh}^2)}{(1 + \nu_{hh}) [E_v (1 - \nu_{hh}) - 2E_h \nu_{vh}^2]} \quad (5a)$$

$$D_{12} = D_{21} = \frac{E_h (\nu_{hh} E_v + E_h \nu_{vh}^2)}{(1 + \nu_{hh}) [E_v (1 - \nu_{hh}) - 2E_h \nu_{vh}^2]} \quad (5b)$$

$$D_{13} = D_{31} = D_{23} = D_{32} = \frac{E_h E_v \nu_{hh}}{E_v (1 - \nu_{hh}) - 2E_h \nu_{vh}^2} \quad (5c)$$

$$D_{33} = \frac{E_v^2 (1 - \nu_{hh})}{E_v (1 - \nu_{hh}) - 2E_h \nu_{vh}^2} \quad (5d)$$

It should be noted that the expressions for components of C_{ijkl} and D_{ijkl} in Eqs. (3) and (5) are applicable to the coordinate frame when the axis of symmetry is selected as the z -axis. For an arbitrary coordinate frame, C_{ijkl} and D_{ijkl} must be transformed by use of appropriate transformation. More details can be found, for example, in Milton (2004) and Skrzypek and Ganczarski (2015).

Effective elastic modulus of homogenized stratified materials

Salamon (1968) considers an elastic system composed of parallel layers, each of which is homogeneous and cross-anisotropic. The layers, which are of random sequence, are aligned so that the axes of rotational symmetry of their material properties are all perpendicular to the interfaces between layers, as illustrated in Figure 1. The thickness of each layer is small when compared to the scale of the system so that a homogenization operation can be used to seek an equivalent homogeneous system. If the deformation remains elastic and if there is no slipping between layer boundaries in the entire domain, Salamon shows that the equivalent material also has cross-anisotropic symmetry, with the axis of symmetry coinciding with the direction perpendicular to the layer interfaces. The stress-strain relations of the equivalent medium are derived from the condition that the strain energy stored in the cube cut from the constituent layers is equal to that in the cube of the equivalent medium. It should be noted that the auxiliary stresses and strains that develop in the different material layers due to each having different mechanical properties are taken into account when determining the elastic energy of the real

system. Moreover, the principal directions of stresses and strains are not necessarily aligned with the principal axes of the anisotropy. The five independent elastic constants of the equivalent elastic material are expressed in terms of the normalized thicknesses (or volume fractions φ_i) and elastic properties of each constituent layer as follows:

$$E_h = (1 - \nu_{hh}^2) \sum \frac{\varphi_i E_{hi}}{1 - \nu_{hhi}^2}, \quad \frac{1}{E_v} = \sum \frac{\varphi_i}{E_{hi}} \left(\frac{E_{hi}}{E_{vi}} - \frac{2\nu_{vhi}^2}{1 - \nu_{hhi}} \right) + \frac{2\nu_{hv}^2}{(1 - \nu_{hh})E_h}$$

$$\nu_{hh} = \frac{\sum \frac{\varphi_i \nu_{hhi} E_{hi}}{1 - \nu_{hhi}^2}}{\sum \frac{\varphi_i E_{hi}}{1 - \nu_{hhi}^2}}, \quad \nu_{hv} = (1 - \nu_{hh}) \sum \frac{\varphi_i \nu_{hvi}}{1 - \nu_{hhi}}, \quad \frac{1}{G_{hv}} = \sum \frac{\varphi_i}{G_{hvi}} \quad (6)$$

Details for the derivation of expressions in Eq. (6) can be found in Salamon (1968). The modulus G_{hh} of the equivalent material, which is related to E_h and ν_{hh} by $G_{hh} = E_h / 2(1 + \nu_{hh})$, can be alternatively expressed as

$$G_{hh} = \frac{E_h}{2(1 + \nu_{hh})} = \sum \varphi_i G_{hhi} \quad (7)$$

In a later study by Gerrard (1982), special cases for elastic modulus, Poisson's ratio and shear modulus in the constituent layers are considered, including isotropic constituent layers and isotropic layers with identical elastic modulus or the Poisson's ratio.

If each individual layer is isotropic with $E_{hi} = E_{vi} = E_i$, $\nu_{hhi} = \nu_{hvi} = \nu_i$ and $G_{hvi} = G_{vhi} = E_i / 2(1 + \nu_i)$, the expressions of elastic properties of the equivalent material given in Eq. (6) simplify as

$$E_h = (1 - \nu_{hh}^2) \sum \frac{\varphi_i E_i}{1 - \nu_i^2}, \quad \frac{1}{E_v} = \sum \frac{\varphi_i}{E_i} \left(1 - \frac{2\nu_i^2}{1 - \nu_i} \right) + \frac{2\nu_{hv}^2}{(1 - \nu_{hh})E_h}$$

$$\nu_{hh} = \frac{\sum \frac{\varphi_i \nu_i E_i}{1 - \nu_i^2}}{\sum \frac{\varphi_i E_i}{1 - \nu_i^2}}, \quad \nu_{hv} = (1 - \nu_{hh}) \sum \frac{\varphi_i \nu_i}{1 - \nu_i}, \quad \frac{1}{G_{hv}} = \sum \frac{\varphi_i}{G_i} \quad (8)$$

Using the equivalent cross-anisotropic elastic constants, the average stresses or strains of the stratified system can be determined by considering the system as an equivalent homogeneous cross-anisotropic medium. The auxiliary stresses and strains resulting from interaction between different constituent layers are required to determine local stresses and strains. More details can be found in Gerrard (1982) and Salamon (1968).

A physically-based approach for homogenization of multi-layered system

It is known that the Reuss and the Voigt models give accurate equivalent stiffnesses of springs arranged in parallel and in series, respectively. For a layered system, when there is no interaction between adjacent layers, these models can be used to approximate the equivalent elastic moduli within the plane of transverse isotropy and in the direction perpendicular to it. In reality, owing to interaction between layers of different elastic properties, auxiliary stresses and strains must develop, requiring rigorous analyses as presented by Salamon (1968) and Gerrard (1982). On the other hand, if a layered system deforms under specific constraints that do not allow auxiliary stresses to develop due to incompatible deformation between adjacent layers, then the Reuss and the Voigt models can still be applied. This section presents a simple, yet physically-based, homogenization approach to determine the equivalent elastic properties of a layered medium composed of isotropic constituent layers. The details of the mathematical derivation for the elastic properties of equivalent homogeneous materials are presented by Salamon (1968) and Gerrard (1982). We provide a physically-based interpretation of the results based on simple deformation mechanisms. The Voigt model and the Reuss model are adopted in this analysis.

163 **Constrained compression with $\varepsilon_{xx} = \varepsilon_{yy} = 0$**

164 Let us first examine a layered system subject to compression in the vertical direction
 165 without lateral deformation by applying vertical stress $\sigma_{zz} = \sigma_v$, as shown in Figure 2. Each layer
 166 is isotropic with elastic modulus E_i and the Poisson's ratio ν_i . All layers have the same vertical
 167 stress $\sigma_{zzi} = \sigma_v$, but different lateral stresses ($\sigma_{xxi} = \sigma_{yyi} = \sigma_{hi}$) owing to the zero lateral strain
 168 constraint ($\varepsilon_{xxi} = \varepsilon_{yyi} = 0$) and the effects of the individual layer Poisson's ratios. The lateral
 169 stress in each layer can be easily determined as

$$170 \quad \sigma_{hi} = \frac{\nu_i}{1 - \nu_i} \sigma_v \quad (9)$$

171 For the equivalent homogeneous, cross-anisotropic material, the average lateral stress can be
 172 determined from Eq. (1) as

$$173 \quad \sigma_h = \frac{\nu_{hv}}{1 - \nu_{hh}} \sigma_v \quad (10)$$

174 The equilibrium condition in the lateral directions requires $\sigma_h = \sum \sigma_{hi}$, which yields a relation
 175 between ν_{hv} and ν_{hh} as

$$176 \quad \frac{\nu_{hv}}{1 - \nu_{hh}} = \sum \frac{\varphi_i \nu_i}{1 - \nu_i} \quad (11)$$

177 This relation turns out to be identical to that in Eq. (8). It can be used to determine the average
 178 at-rest earth pressure coefficient K_0 of stratified soil. Referring to Eq. (1), the K_0 -value of the
 179 equivalent cross-anisotropic material is

$$180 \quad K_0 = \left. \frac{\sigma_{xx}}{\sigma_{zz}} \right|_{\varepsilon_{xx} = \varepsilon_{yy} = 0} = \frac{\nu_{hv}}{1 - \nu_{hh}} \quad (12)$$

181 By combining Eq. (11) and (12), we arrive at $K_0 = \sum \varphi_i K_{0i}$ with K_{0i} being the at-rest earth
 182 pressure coefficient of each constituent soil layer.

183 With regard to the modulus in the vertical direction, the Reuss model can be applied to
 184 determine the modulus of the homogenized equivalent material since there is no interaction
 185 between layers in the lateral direction. Owing to the lateral constraint, the constrained modulus
 186 must be used for this case. Recalling the definition of the constrained modulus
 187 $M_v = d\sigma_{zz} / d\varepsilon_{zz} \big|_{\varepsilon_{xx}=\varepsilon_{yy}=0}$, for a cross-anisotropic material, the constrained modulus M_v is derived
 188 from Eq. (1) as

$$189 \quad \frac{1}{M_v} = \frac{1}{E_v} - \frac{2\nu_{hv}^2}{(1-\nu_{hh})E_h} \quad (13)$$

190 On the other hand, the constrained modulus of the isotropic material in each material layer is
 191 expressed as

$$192 \quad \frac{1}{M_{vi}} = \frac{1}{E_i} - \frac{2\nu_i^2}{E_i(1-\nu_i)} = \frac{(1+\nu_i)(1-2\nu_i)}{E_i(1-\nu_i)} \quad (14)$$

193 yielding via the Reuss assumption

$$194 \quad \frac{1}{M_v} = \sum \frac{\varphi_i}{M_{vi}} \quad (15)$$

195 It follows that

$$196 \quad \frac{1}{E_v} - \frac{2\nu_{hv}^2}{(1-\nu_{hh})E_h} = \sum \varphi_i \left(\frac{1}{E_i} - \frac{2\nu_i^2}{E_i(1-\nu_i)} \right) \quad (16)$$

197 It may be observed that this equation is the same as the expression for E_v in Eq. (8).

198 **Plane strain compression with $\varepsilon_{yy} = 0$ and $\sigma_{zz} = 0$**

199 The second case to examine is a layered system subject to plane strain compression with
 200 $\varepsilon_{yyi} = \varepsilon_{yy} = 0$, $\sigma_{zz} (= \sigma_v) = 0$ and $\varepsilon_{xxi} = \varepsilon_{xx}$, as illustrated in Figure 3. For the equivalent
 201 homogeneous material, when applying $\varepsilon_{yyi} = \varepsilon_{yy} = 0$ and $\sigma_v = 0$ to Eq. (1), one has

$$\begin{aligned} \varepsilon_{xx} &= \frac{\sigma_{xx}}{E_h} - \nu_{hh} \frac{\sigma_{yy}}{E_h} \\ \varepsilon_{yy} &= -\frac{\nu_{hh}}{E_h} \sigma_{xx} + \frac{\sigma_{yy}}{E_h} = 0 \end{aligned} \quad (17)$$

203 where σ_{xx} and σ_{yy} are the average stresses in the equivalent material. Given Eq. (16), both σ_{xx}
 204 and σ_{yy} can be related to ε_{xx} by

$$\sigma_{xx} = \frac{E_h}{1 - \nu_{hh}^2} \varepsilon_{xx}, \quad \sigma_{yy} = \nu_{hh} \sigma_{xx} \quad (18)$$

206 Owing to the constraint in the lateral directions and the different material properties, each
 207 constituent layer has different lateral stresses, which are expressed as

$$\sigma_{xxi} = \frac{E_i}{1 - \nu_i^2} \varepsilon_{xx}, \quad \sigma_{yyi} = \nu_i \sigma_{xxi} \quad (19)$$

209 By applying $\sigma_{xx} = \sum \varphi_i \sigma_{xxi}$ and $\sigma_{yy} = \sum \varphi_i \sigma_{yyi}$, the following relations are obtained from Eqs.
 210 (18) and (19):

$$\frac{E_h}{1 - \nu_{hh}^2} = \sum \frac{\varphi_i E_i}{1 - \nu_i^2}, \quad \frac{E_h \nu_{hh}}{1 - \nu_{hh}^2} = \sum \frac{\varphi_i E_i \nu_i}{1 - \nu_i^2} \quad (20)$$

212 which yields the expressions for E_h and ν_{hh} as

$$E_h = (1 - \nu_{hh}^2) \sum \frac{\varphi_i E_i}{1 - \nu_i^2}, \quad \nu_{hh} = \left(\sum \frac{\varphi_i \nu_i E_i}{1 - \nu_i^2} \right) \left(\sum \frac{\varphi_i E_i}{1 - \nu_i^2} \right)^{-1}$$

214 These relations are also identical to those in Eq. (8).

Eq. (20) can be alternatively obtained by applying the Voigt model for the modulus in the x -direction. According to Eqs. (18) and (19), for this specific plane strain condition with $\varepsilon_{yyi} = \varepsilon_{yy} = 0$, $\sigma_v = 0$, the elastic moduli in the x -direction for the equivalent homogeneous material and each individual layer are

$$\bar{E}_x = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{E_h}{1 - \nu_{hh}^2}, \quad \bar{E}_{xi} = \frac{\sigma_{xxi}}{\varepsilon_{xxi}} = \frac{E_i}{1 - \nu_i^2}$$

Equation (20) is readily recovered after applying $\bar{E}_x = \sum \varphi_i \bar{E}_{xi}$.

Shear perpendicular to the axis of symmetry

The last case to examine is simple shear of a layered system, in which the shear stresses in all layers are equal as shown in Figure 4. When there is no sliding along the interface of any two adjacent layers, the average shear strain can be determined via homogenization as $\gamma = \sum \varphi_i \gamma_i$.

Thus, we have

$$\frac{\tau}{G_{hv}} = \sum \varphi_i \frac{\tau}{G_i}$$

This relation implies that the Reuss average is appropriate for G_{hv} :

$$\frac{1}{G_{hv}} = \sum \frac{\varphi_i}{G_i} \quad (21)$$

which is identical to the last expression in Eq. (8).

When the layered system is subjected to shear parallel to the layers (i.e., in the x - y plane) without relative sliding between layers, the condition for the Voigt model is satisfied, which results in $G_{hh} = \sum \varphi_i G_{hhi}$.

The above analyses clearly show that a simple, yet physically based, approach using the Voigt and the Reuss models can reproduce the five independent elastic properties of

homogeneous, transversely isotropic medium that is equivalent to a layered material system. Even though the principal directions of stresses and strains are aligned with the principal axes of the anisotropy in both the constrained compression test and the plane strain compression test, the results are applicable to general stress states as the effect of shear is considered in the case of shear perpendicular to the axis of symmetry. The following section discusses some special features of this homogenized transversely isotropic material.

Some special features of a homogenized layered material

The special features of the equivalent homogenized medium are explored by examining three special cases in which each individual layer is isotropic and one of their elastic properties (Poisson's ratio, elastic modulus and shear modulus) is the same for all layers. For each case, the expressions for the cross-anisotropic properties of the equivalent medium are significantly simplified. Moreover, the five elastic properties are not all independent, which is different from a regular cross-anisotropic medium.

Special case A: Equal Poisson's ratio in all layers

Let us examine special case A when the Poisson's ratios of all layers are equal and assuming that each layer is isotropic. In this case, the elastic constants of the layers are: $E_{hi} = E_{vi} = E_i$, $\nu_{hhi} = \nu_{hvi} = \nu$ and $G_{hvi} = G_{hhi} = E_i / 2(1 + \nu)$. The equivalent elastic constants ($E_v, E_h, \nu_{hh}, \nu_{hv}$ and G_{hv}) of the layered materials given in Eq. (8) simplify as

$$\begin{aligned} \nu_{hh} = \nu_{hv} = \nu, \quad E_h = \sum \varphi_i E_i; \quad G_{hh} = \frac{E_h}{2(1 + \nu)} \\ E_v = \frac{(1 - \nu)E_h}{(1 + \nu)(1 - 2\nu)E_h \sum \frac{\varphi_i}{E_i} + 2\nu^2}, \quad G_{hv} = \frac{1}{2(1 + \nu) \sum \frac{\varphi_i}{E_i}} \end{aligned} \quad (22)$$

The elastic constants satisfy the following relation

$$E_v = \frac{(1-\nu)E_h}{(1-2\nu)E_h / (2G_{hv}) + 2\nu^2} \quad (23)$$

In other words, the equivalent cross-anisotropic medium only has three independent elastic constants.

If all constituent layers are incompressible with $\nu_i = 1/2$, Eq. (23) becomes $E_v = E_h$. However, the equivalent medium is not isotropic since $G_{hv} \neq G_{hh}$ and still has three independent elastic constants.

Special case B: Equal elastic modulus in all layers

Once again assuming isotropy for each layer, the elastic constants of the layers for this case are:

$E_{hi} = E_{vi} = E$, $\nu_{hhi} = \nu_{hvi} = \nu_i$. Substitution of these constants into Eq. (8) yields

$$\begin{aligned} E_h &= E(1-\nu_{hh}^2) \sum \frac{\varphi_i}{1-\nu_i^2}, \quad \frac{1}{E_v} = \sum \left(\frac{\varphi_i}{E} \frac{(1+\nu_i)(1-2\nu_i)}{1-\nu_i} \right) + \frac{2\nu_{hv}^2}{(1-\nu_{hh})E_h} \\ \nu_{hh} &= \frac{\sum \varphi_i \nu_i / (1-\nu_i^2)}{\sum \varphi_i / (1-\nu_i^2)}, \quad \nu_{hv} = (1-\nu_{hh}) \sum \frac{\varphi_i \nu_i}{1-\nu_i}, \quad G_{hv} = \frac{E}{\sum 2(1+\nu_i)\varphi_i} \end{aligned} \quad (24)$$

After some algebraic manipulations, the relation among the five elastic properties is

$$\frac{1}{E_v} = \frac{1}{G_{hv}} - \frac{(1-\nu_{hh})^2 + 3\nu_{hv}(1-\nu_{hh})}{(1-\nu_{hh})E_h} = \frac{1}{G_{hv}} - \frac{1-\nu_{hh} + 3\nu_{hv}}{E_h} \quad (25)$$

For this particular case, the equivalent cross-anisotropic medium has four independent elastic constants.

Special case C: Equal shear modulus in all layers

When the shear modulus of each layer has the same value, i.e., $G_i = E_i / 2(1+\nu_i) = G$, the expressions of the elastic properties in Eq. (8) are significantly simplified to

$$E_h = E_v = G(1+\bar{\nu}), \quad \nu_{hh} = \nu_{hv} = \bar{\nu} \quad (26)$$

in which $\bar{u}$ is determined by the following relation

$$\frac{1}{1-\bar{v}} = \sum \frac{\varphi_i}{1-\nu_i} \quad (27)$$

The equivalent homogeneous medium is isotropic, even though the constituent layers have different elastic properties.

Special case D: Multilayered medium with two constituent materials

The analyses in Section 3 clearly show that the Reuss average yields exact results for M_v and G_{hv} , while the Voigt average is exact for G_{hh} . However, neither the Voigt average nor the Reuss average yields the exact expression for E_v, E_h, ν_{hh} and ν_{hv} . On the other hand, in engineering practice, the Voigt average and the Reuss average are considered appropriate to represent the upper and the lower bounds for different effective properties of a mixture composed of different constituents (Mavko et al. 2009). Now we examine the applicability of these averages as the bounds for E_v, E_h, ν_{hh} and ν_{hv} of a layered material. To simplify the exposition, a multilayered medium with two constituent materials that are both isotropic is examined.

Referring to Eq. (8), the equivalent elastic properties of a layered medium containing two constituent materials are expressed as

$$\nu_{hh} = \frac{\frac{E_1}{E_2} \varphi_1 \nu_1 (1-\nu_2^2) + \varphi_2 \nu_2 (1-\nu_1^2)}{\frac{E_1}{E_2} \varphi_1 (1-\nu_2^2) + \varphi_2 (1-\nu_1^2)}, \quad \nu_{hv} = (1-\nu_{hh}) \left(\frac{\varphi_1 \nu_1}{1-\nu_1} + \frac{\varphi_2 \nu_2}{1-\nu_2} \right)$$

$$E_h = (1-\nu_{hh}^2) \left(\frac{E_1 \varphi_1}{1-\nu_1^2} + \frac{E_2 \varphi_2}{1-\nu_2^2} \right), \quad \frac{1}{E_v} = \frac{\varphi_1}{E_1} \left(1 - \frac{2\nu_1^2}{1-\nu_1} \right) + \frac{\varphi_2}{E_2} \left(1 - \frac{2\nu_2^2}{1-\nu_2} \right) + \frac{2\nu_{hv}^2}{(1-\nu_{hh})E_h}$$

$$\frac{1}{G_{hv}} = \frac{1}{2} \left(\frac{\varphi_1}{E_1} (1+\nu_1) + \frac{\varphi_2}{E_2} (1+\nu_2) \right)$$

For such a layered medium, the values of the effective Poisson's ratio only depend on the ratio of the elastic moduli of the constituent layers rather than their absolute values. When a medium has thin layers of very soft material (Material 2) with large Poisson's ratio, i.e., $E_1 \gg E_2$ and $\varphi_1 \gg \varphi_2$, the variation of ν_{hv} with φ_1 can be approximated as

$$\nu_{hv} \approx \nu_1 + (1 - \varphi_1) \left(\nu_2 \frac{1 - \nu_1}{1 - \nu_2} - \nu_1 \right)$$

and ν_{hh} is very close to the Poisson's ratio of the stiffer material (Material 1), or $\nu_{hh} \approx \nu_1$. On the other hand, when the elastic moduli of the layers are close, ν_{hh} can be generally estimated by the Voigt average for a large range of the Poisson's ratio, regardless the relative layer thicknesses φ_1 and φ_2 .

Regarding the effective moduli of the equivalent medium, the theoretical values of E_h and the Voigt average E_{Voigt} are in close agreement; as shown in Figure 5, in which the variation of E_{Voigt} / E_h with the relative thickness of the stiffer material for different conditions is illustrated. On the other hand, the Reuss average E_{Reuss} tends to underestimate E_v considerably, depending on the properties of the constituent layers, as shown in Figure 6. The difference between E_v and E_{Reuss} decreases when E_1 approaches E_2 (Figure 6a) or when the constituent layers have low Poisson's ratio (Figure 6b). In addition, the value of E_v is sensitive to the difference between the Poisson's ratio of the constituent layers (Figure 6a).

For a large range of the Poisson's ratio, E_h is generally higher than E_v . However, as the difference between the Poisson's ratio of the constituent layers increases, the value of E_v increases significantly and E_v may become equal to or higher than E_h . It should be noted

$E_v = E_h$ when the constituent materials are all incompressible (i.e., $\nu_1 = \nu_2 = 1/2$) regardless of the values of E_1 and E_2 . Nevertheless, the homogenized medium is still anisotropic, as indicated previously.

Laboratory determination of elastic constants via tests along select strain paths

Various laboratory testing methods have been used to determine the elastic constants of cross-anisotropic materials (e.g., Tatsuoka et al. 1994; Tatsuoka et al. 1997; Chaudhary et al. 2004; Clayton 201 and Liu 2011). In this study, triaxial tests along the following select strain and stress paths are carried out to determine E_v, E_h, ν_{hh} and ν_{hv}

(1) K_0 -compression test in which $\varepsilon_{xx} = \varepsilon_{yy} = 0$;

(2) Plane strain compression test with $\varepsilon_{zz} = 0, \sigma_{xx} = \sigma_{yy}$; and

(3) Compression under spherical pressure $\sigma_{xx} = \sigma_{yy} = \sigma_{zz}$.

The above strain and stress paths are selected because quantities with clear physical meaning can be obtained from these tests. More specifically, the results of a K_0 -compression test can be used to determine both the earth pressure coefficient at rest and the constrained modulus (or compressibility) in the vertical direction. Horizontal compression with $\varepsilon_{zz} = 0$ and $\sigma_{xx} = \sigma_{yy}$ is a plane strain test, which provides a direct measure of the in-plane compressibility of layered soils. The deformation of material in different directions under a hydrostatic stress state provides visible evidence for material anisotropy.

Test A: K_0 -compression test in which $\varepsilon_{xx} = \varepsilon_{yy} = 0$

By applying the constraint $\varepsilon_{xx} = \varepsilon_{yy} = 0$ together with $\sigma_{xx} = \sigma_{yy}$ to the stress-strain relationship in Eq. (1), the earth pressure coefficient at rest is determined as

$$K_0 = \frac{\sigma_{xx}}{\sigma_{zz}} = \frac{E_h}{E_v} \frac{\nu_{vh}}{1 - \nu_{hh}} = \frac{\nu_{hv}}{1 - \nu_{hh}} \quad (28)$$

Both the K_0 -value and the constraint modulus that is defined as $M_{vz} = \sigma_{zz} / \varepsilon_{zz} \big|_{\varepsilon_{xx} = \varepsilon_{yy} = 0}$ can be readily obtained from the measured stresses and strains. M_{vz} can be related to other elastic properties via

$$\frac{1}{M_{vz}} = \frac{1}{E_v} - \frac{2\nu_{hv}}{E_h} = \frac{1}{E_v} - 2K_0 \frac{\nu_{hv}}{E_h} = \frac{1}{E_h} \left(\frac{E_h}{E_v} - \frac{2\nu_{hv}^2}{1 - \nu_{hh}} \right) \quad (29)$$

Test B: Plane strain compression with $\varepsilon_{zz} = 0$ and $\sigma_x = \sigma_y$

In this case, the plane strain constraint requires

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E_v} - 2 \frac{\nu_{hv}}{E_h} \sigma_{xx} = 0$$

The relation between the lateral strain ε_{xx} and the applied stress σ_{xx} is obtained from Eq. (1) as

$$\varepsilon_{xx} = \frac{1}{E_h} \left[(1 - \nu_{hh}) \sigma_{xx} - \nu_{hv} \sigma_{zz} \right] = \frac{\sigma_{xx}}{E_h} \left[(1 - \nu_{hh}) - 2 \left(\frac{E_v \nu_{hv}}{E_h} \right)^2 \right]$$

It follows that

$$K_{v0} = \frac{\sigma_{zz}}{\sigma_{xx}} \bigg|_{\varepsilon_{zz}=0} = 2\nu_{hv} \frac{E_v}{E_h} \quad (30)$$

$$\frac{1}{M_{vx}} = \frac{\varepsilon_{xx}}{\sigma_{xx}} = \frac{1}{E_h} \left[(1 - \nu_{hh}) - 2 \left(\frac{E_v \nu_{hv}}{E_h} \right)^2 \right] \quad (31)$$

Both K_{v0} and M_{vx} can be directly determined from the measured data in the plane strain compression test using a triaxial apparatus by increasing the cell pressure while maintaining zero deformation of the specimen in the axial direction.

348 **Test C: Compression under hydrostatic pressure** $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma$

349 According to Eq. (1), the stress-strain relations in a hydrostatic compression test are explicitly
350 expressed as

$$351 \quad \varepsilon_{xx} = \frac{1}{E_h}(1 - \nu_{hh} - \nu_{hv})\sigma, \quad \varepsilon_{zz} = \left(\frac{1}{E_v} - \frac{2\nu_{hv}}{E_h} \right) \sigma \quad (32)$$

352 in which σ is the applied hydrostatic pressure. The volumetric strain ε_{vol} varies with σ by
353 following

$$354 \quad \varepsilon_{vol} = 2\varepsilon_{xx} + \varepsilon_{zz} = \frac{1}{E_h} \left[2(1 - \nu_{hh} - 2\nu_{hv}) + \frac{E_h}{E_v} \right] \sigma \quad (33)$$

355 Equations (32) and (33) yield the volumetric to axial strain ratio as

$$356 \quad \frac{\varepsilon_{vol}}{\varepsilon_{zz}} = \frac{2(1 - \nu_{hh} - 2\nu_{hv}) + E_h / E_v}{E_h / E_v - 2\nu_{hv}} \quad (34)$$

357 In a hydrostatic compression test using triaxial apparatus, both the volume change and axial
358 deformation are measured. For an isotropic material, $\varepsilon_{vol} / \varepsilon_{zz} = 3$ is recovered from Eq. (34).
359 However, $\varepsilon_{vol} / \varepsilon_{zz} = 3$ or $\varepsilon_{xx} = \varepsilon_{yy} = \varepsilon_{zz}$ under hydrostatic pressure is not a sufficient condition
360 for an isotropic material. It can be shown that weakly anisotropic materials with
361 $E_h / E_v = 1 - \nu_{hh} + \nu_{hv}$ also have $\varepsilon_{vol} / \varepsilon_{zz} = 3$ when subject to hydrostatic pressure.

362 After K_0 , K_{v0} , M_{vz} , M_{vx} and $\varepsilon_{vol} / \varepsilon_{zz}$ are obtained from the three tests, the elastic
363 constants E_v, E_h, ν_{hh} and ν_{hv} can be determined. Since there are five equations for four quantities,
364 an optimization method can be used. One may alternatively find the best estimates for ν_{hh}, ν_{hv}
365 and the E_h / E_v ratio from Eqs. (28), (30) and (34), then use Eqs. (29) and (31) to determine the
366 values of E_v and E_h .

Regarding the shear modulus, G_{hh} can be obtained from Eq. (7) after E_h and u_{hh} are known. However, an additional test is required to determine G_{hv} .

Example experimental study

The objective of the experimental work is to demonstrate how to carry out the three types of tests discussed in the previous section rather than to verify expressions for equivalent cross-anisotropic elastic constants given in Eq. (6). To achieve this goal, a series of tests were carried out using a triaxial apparatus with a “rigid” cell having a specially designed loading head and frictionless seal. The design and configuration of the “rigid” cell is similar to that described in Campanella and Vaid (1972). The cylinder of the “rigid” triaxial cell is machined from heavy wall stainless steel pipe to keep the compliance of the cell-water system to minimum. The loading ram is guided in its vertical motion by a pair of high precision stainless steel linear Thompson Ball Bushings and is sealed to the head by means of a Bellofram having an effective area equal to the area of the sample. An essentially frictionless vertical movement of the ram is obtained as the flexible Bellofram simply rolls on or off the inner periphery of the head. The friction in the guide bushings is negligible unless lateral loading becomes excessive. A pressure-volume controller is attached to the “rigid” cell so that either the lateral pressure or the lateral deformation of the specimen can be controlled. The axial stress applied to the specimen can be load control or displacement control.

Cylindrical specimens 61 mm diameter and 50 mm high were trimmed from soil cores obtained from different depths. A varved silt (specimen 6011 at the depth of 26.5m) and a clayey sand (specimen 4002 at the depth of 4.8 m) were tested. As shown in Figure 7, Specimen 6011 has varved structure with the layering thickness varying approximately from 2mm to 15mm. All

layers are nearly horizontal. Specimen 4002 is a homogeneous clayey sand without visible layering or other type of heterogeneity.

In all tests, the specimens were first saturated by gradually increasing the backing pressure to 400 kPa with the effective stress being 10 kPa. The backpressure was maintained overnight until the pore pressure coefficient B reached 0.96. Then the specimen was consolidated at the effective stress of 20 kPa, followed by Test A, B and C, respectively. To determine the elastic properties of soil reasonably, both K_0 and K_{v0} were determined using measured data corresponding an axial strain $\varepsilon_{zz} = 0.1\%$ and the $\Delta\varepsilon_{vol} / \Delta\varepsilon_{zz}$ ratio was determined in the range of $\varepsilon_{zz} = 0.08\% - 0.1\%$. The test results and the best estimates for ν_{hh}, ν_{hv} and the E_h / E_v ratio from Eqs. (28), (30) and (34) as well as the values of E_h and E_v using Eq. (29) are summarized in Table 1

(insert Table 1 here)

As expected, the varved silt (Specimen 6011) has noticeable anisotropy, with the elastic modulus E_h in the direction parallel to the layer being 40% higher than E_v in the vertical direction. The clayey sand at shallower depth (4.8m) is practically isotropic. The values of earth pressure coefficient at rest of these two soils can be considered as the same, even though the soil properties are quite different.

Final remarks

The equivalent, homogenized, elastic properties of the layered medium were derived using the Reuss and Voigt models taking into account the physics associated with homogenization. The equivalent medium was shown to be generally cross-anisotropic. However, when assuming that

the constituent materials are isotropic, the cross-anisotropic properties of the equivalent material are more restrictive than those for a 'general' cross-anisotropic material. In particular, it was found that the equivalent material has only three elastic constants when the Poisson's ratios of all layers are equal, while the general cross-anisotropic material has five. On the other hand, if all material layers have the same elastic modulus, the equivalent material has four independent elastic properties. As a very special case, when all layers have the same shear modulus, the equivalent medium becomes isotropic, even though the other elastic properties of the material layers are different.

The equivalent anisotropic elastic properties of a multilayered soil can be obtained by performing laboratory tests, to estimate E_v, E_h, ν_{hh} and ν_{hv} . These tests include stress (or strain paths) under K_0 -compression, plane strain compression and compression under hydrostatic pressure. It is demonstrated that these tests are sufficient to determine E_v, E_h, ν_{hh} and ν_{hv} of cross-anisotropic soil.

It should be emphasized that the technique in this paper and the resulting elastic constants of the equivalent material are applicable only when all constituent materials are linear, or at most piece-wise linear, elastic materials.

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Figure captions

Figure 1: System of parallel homogeneous cross-anisotropic layers

Figure 2: Constrained 1D compression of a layered material

Figure 3: Plane-strain compression of a layered material

Figure 4: Constrained shear of a layered material

Figure 5: Comparison of $\bar{\sigma}$ with the Voigt averages for layered medium with two constituent materials

Figure 6: Comparison of $\bar{\sigma}$ with the Reuse averages for layered medium with two constituent materials

Figure 7: Varved and homogeneous soil specimens

Table Caption

Table 1: Summary of test results and estimated equivalent material properties

Figure 1: System of parallel homogeneous cross-anisotropic layers

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75x48mm (300 x 300 DPI)

Figure 2: Constrained 1D compression of a layered material

Figure 2: Constrained 1D compression of a layered material

81x45mm (300 x 300 DPI)

Figure 3: Plane-strain compression of a layered material

Figure 3: Plane-strain compression of a layered material

87x51mm (300 x 300 DPI)

Figure 4: Constrained shear of a layered material

Figure 4: Constrained shear of a layered material

86x56mm (300 x 300 DPI)

Figure 5: Comparison of E_h with the Voigt averages E_{Voigt} for layered medium with two constituent materials

109x77mm (300 x 300 DPI)

Figure 6: Comparison of E_V with the Reuse averages E_{Reuss} for layered medium with two constituent materials

91x53mm (300 x 300 DPI)

(a) Specimen 6011 (depth 26.5 m)

(b) Specimen 4002 (depth 4.8 m)

Figure 7: Varved and homogeneous soil specimens

Figure 7: Varved and homogeneous soil specimens

88x62mm (300 x 300 DPI)

Table 1: Summary of test results and estimated equivalent material properties

Specimen	K_0	K_{v0}	$\frac{\Delta \varepsilon_{vol}}{\Delta \varepsilon_{zz}}$	M_{vz} (MPa)	ν_{hh}	ν_{hv}	E_h / E_v	E_h (MPa)	E_v (MPa)
#6011	0.38	0.56	2.54	31.90	0.10	0.35	1.40	35.98	25.70
#4002	0.39	0.55	2.93	38.58	0.28	0.28	1.01	30.56	30.26
#2004 (sand@60')	0.313	0.5	3.117	46.19	0.245	0.235	0.96	56.38	58.73

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