Unparticle effects on Bhabha scattering

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Complete List of Authors:	Anh, Sa Thi Lan; Faculty of Physics, Ha Noi University of Viet Nam, Faculty of Physics Trang, Trieu Quynh; Faculty of Physics, Ha Noi University of Science, Faculty of Physics Huong, Nguyen Thu; Vietnam National University, Ha Noi University of Science Bang, Ha Huy; Faculty of Physics, Ha Noi University of Sciences
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UNPARTICLE EFFECTS ON BHABHA SCATTERING

*Sa Thi Lan Anh*¹, *Trieu Quynh Trang*², *Nguyen Thu Huong*³ and *Ha Huy Bang*⁴* Faculty of Physics, Ha Noi University of Sciences, Vietnam National University
334 Nguyen Trai Street, Hanoi, VietnamEmail: ¹lananhkhtn@gmail.com; ²mecuathugiang@gmail.com;
³thnguyen@gmail.com; ⁴hahuybang@hus.edu.vn**Abstract**

In this paper, we investigate the possible unparticle signatures through Bhabha scattering. The numerical results show that the angular distribution with unparticle effects can be up to 42% of the one that is confirmed by Born QED and Born + 1 - loop QED calculation and the total cross section with unparticle effects are about 0.04 – 4.24 pico barn. This could have important implications for unparticle searches and for the measurement of the cross section of the Bhabha scattering.

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Keywords: unparticle, Bhabha scattering.

I. Introduction

Bhabha scattering is among the most important and carefully studied processes in particle physics. It provides a very efficient tool for luminosity at electron - positron colliders and thus mediates the process of extracting physical information from the raw experimental data [1, 2]. Since the theoretical accuracy directly affects the luminosity determination, remarkable efforts have been devoted to the study of the radiative corrections. The one – loop corrections have been known in the full electroweak theory for several decades [3]. Recently, the calculation of the two-loop QED corrections to Bhabha scattering has been completed [4-6].

A decade ago, Georgi [7] made an interesting observation that a nontrivial scale invariance sector of scale dimension d_u might manifest itself at low energy as a non integral number d_u of invisible massless particle, dubbed unparticle. If unparticles exist, their phenomenological implications should be discussed. In the literature, there have been many discussions which investigate various features of unparticle physics [8-24]. In the some of these researches, the authors focused on Hawking radiation of unparticles [10], astrophysical constraints on unparticle - inspired models of gravity [11] and the effects of the peculiar CP conserving phases in unparticle propagator. In particular, it has been illustrated in Ref. [13] that

the annihilation cross section of dark matter pair is sensitive to the dark matter mass and the scaling dimension of scalar unparticle. Recently, the effects of unparticle modified static potentials on the energy levels of the hydrogen atom have been studied in Ref. [14].

It is interesting that the virtual effects of vector unparticle in the Möller scattering have been investigated and the analytic expression for scattering amplitudes with unpolarized beams has been derived [15]. Next, the effects of scalar and tensor unparticles on the differential and total cross section for different polarization configurations of gamma gamma scattering have been shown in Refs. [16-18]. In addition, the authors have shown that the cross section via unparticle exchange can be substantially larger than the standard model contribution [18]. The impact of unparticle physics on the annihilation of relic neutrinos with the neutrinos identified as primary source of ultra high energy cosmic ray events, producing a cascade of photons and charged particles has been investigated in Ref. [19]. Çakir and Ozansoy [20] have studied the possible unparticle signatures through tau pair production $e^+e^- \rightarrow \tau^+\tau^-$ process at the low energy particle factories. Noteworthy, Chen and He [21] have discussed different features of producing unparticle at e^+e^- collider and other phenomenologies.

Very recently, by considering not only the self-interactions of unparticle but also all the other

possible contributions, which are dominant, a detailed study of the processes, within a scalar unparticle scenario $pp \rightarrow 4\gamma$, $pp \rightarrow 2\gamma 2g$, $pp \rightarrow 2\gamma 2l$ and $pp \rightarrow 2e 2\mu$ at $\sqrt{S}=14\text{TeV}$ at the LHC has been carried out in Ref. [22]. A search for dark matter and unparticle production at the LHC has been performed in Ref. [23]. Searching for the new physics effects, the e^+e^- linear colliders have an exceptional advantageous for its appealing clean background, and the possibility for the options of $e\gamma$, and $\gamma\gamma$ colliders based on it.

Recently, we have considered the photon – photon scattering via radion or unparticle exchange [24, 25]. In 2015, we experienced the 80th anniversary of Bhabha scattering that is an important process for e^+e^- colliders. In this paper, we investigate virtual unparticle effects through Bhabha scattering.

II. Cross sections for Bhabha scattering

For our study, we consider the following set of effective interactions for the unparticle operators which have a coupling to leptons

$$\frac{C_V^{ll}}{\Lambda_u^{d_u-1}} \bar{l} \gamma_\mu l O_u^\mu + \frac{C_A^{ll}}{\Lambda_u^{d_u-1}} \bar{l} \gamma_\mu \gamma_5 l O_u^\mu. \quad (1)$$

The scalar operator with ordinary SM fermions and in principle can contribute to the problem under consideration. However, their contributions are proportional to the fermion mass and are suppressed, as we have electrons (positrons) in the final state. For this reason, we shall consider only the vector (axial - vector) unparticle operator in this paper.

Figure 1: Feynman diagram for $e^+e^- \rightarrow e^+e^-$ in Standard Model.

Figure 2: Feynman diagram for $e^+e^- \rightarrow e^+e^-$ in unparticle physics.

The Bhabha scattering is described by the Feynman diagrams presented in the figure 1 and 2. As such, in obtaining the matrix element for the Bhabha scattering, we need to know the propagator of the vector unparticles. This propagator has the explicit form [8, 26]

$$D_{\mu\nu}(x) = \int d^4x e^{ipx} \langle 0 | T | O_\mu^\mu(x) O_\nu^\mu(0) | 0 \rangle \\ = \frac{iA_{d_u}}{2\sin(d_u\pi)} \left(-g_{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \right) (-P^2 - ie)^{d_u-2}, \quad (2)$$

where A_{d_u} is given by

$$A_{d_u} = \frac{16\pi^{5/2}}{(2\pi)^{2d_u}} \frac{\Gamma(d_u+1/2)}{\Gamma(d_u-1)\Gamma(2d_u)}. \quad (3)$$

After making these deductions, we can write the amplitude for the $e^+e^- \rightarrow e^+e^-$ scattering as

$$M = M_1 + M_2, \quad (4)$$

with

$$iM_1 = \bar{u}(p_3) \left[\frac{C_V^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu + \frac{C_A^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu \gamma_5 \right] v(p_4) \\ \times \frac{iA_{d_u}}{2\sin(d_u\pi)} \left(-g_{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \right) \times [-(p_1+p_2)^2 - i\epsilon]^{d_u-2} \\ \times \bar{v}(p_2) \left[\frac{C_V^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu + \frac{C_A^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu \gamma_5 \right] u(p_1), \quad (5)$$

$$iM_2 = \bar{u}(p_3) \left[\frac{C_V^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu + \frac{C_A^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu \gamma_5 \right] u(p_1) \\ \times \frac{iA_{d_u}}{2\sin(d_u\pi)} \left(-g_{\mu\nu} + \frac{P^\mu P^\nu}{P^2} \right) \times [-(p_3-p_1)^2 - i\epsilon]^{d_u-2} \\ \times \bar{v}(p_2) \left[\frac{C_V^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu + \frac{C_A^{ll}}{\Lambda_u^{d_u-1}} \gamma_\mu \gamma_5 \right] v(p_4). \quad (6)$$

The second term in the vector unparticle propagator will give a contribution which is proportional to the lepton mass and thus will be neglected in our calculation. Since there are only three independent four - vectors in this kind of scattering, there is no interference between the

vector and the axial - vector couplings. Thus, we can consider these two different contributions independently. For simplicity, we consider the vector coupling by taking $C_A^{ee} = 0$. The square of the amplitude averaged over initial polarizations is given by

$$\begin{aligned} \sum |M|^2 = & 8 \frac{A_u^2 |C_V^{ee}|^4}{\Lambda_u^{4d_u-4}} \frac{1}{\sin^2(d_u \pi)} \\ & \times \left\{ S^{2d_u-2} (1 + \cos^2 \theta) + S^2 |t|^{2d_u-4} \left[\frac{1}{16} (1 + \cos \theta)^2 + \frac{1}{4} \right] \right. \\ & \times \left. \frac{1}{8} S^{d_u} |t|^{2d_u-2} (1 + \cos \theta)^2 \cos(d_u \pi) \right\}, \quad (7) \end{aligned}$$

where θ is the scattering angle (the angle between the 3-momentum of electron: $\mathbf{p}_1$ and $\mathbf{p}_3$).

Note that we have written the Mandelstam variables as $S, |t|, |u|$ where $|t| = \frac{S(1 - \cos \theta)}{2}$.

The differential cross section is given by

$$\begin{aligned} \frac{d\sigma}{d\cos\theta} = & \frac{A_u^2 (C_V^{ee})^4}{256\pi \Lambda_u^{4d_u-4} \sin^2(d_u \pi)} S^{2d_u-3} \\ & \times \left\{ 2(1 + \cos^2 \theta) + \left(\frac{1 - \cos \theta}{2} \right)^{2d_u-4} \left[(1 + \cos \theta)^2 + 4 \right] \right. \\ & \times \left. + 2 \left(\frac{1 - \cos \theta}{2} \right)^{2d_u-2} (1 + \cos \theta)^2 \cos(d_u \pi) \right\}. \quad (8) \end{aligned}$$

The total cross section can be obtained as follows

$$\begin{aligned} \sigma = & \frac{A_u^2 (C_V^{ee})^4}{16\pi \Lambda_u^{4d_u-4} \sin^2(d_u \pi)} S^{2d_u-3} \\ & \times \left[\frac{1}{3} + \frac{1}{2(2d_u-1)(2d_u-3)(d_u-1)} \right. \\ & \times \left. \frac{2\cos(d_u \pi)}{d_u(d_u+1)(d_u-1)} \right]. \quad (9) \end{aligned}$$

III. Numerical results and discussions

Let us now turn to the numerical analysis. We take $C_V^{ee} = 1$, $\Lambda_u = 1 \text{ TeV}$, $\sqrt{S} = 300; 500; 1000$ GeV as input parameters.

In the figure 3a, 3b, 3c, we plot angular distributions for unparticle contribution for different values of the scaling parameter d_u .

Figure 3a: The angular distributions for unparticle contribution at $\sqrt{S} = 300$ GeV. Here we assumed $\Lambda_u = 1 \text{ TeV}$.

Figure 3b: The angular distributions for unparticle contribution at $\sqrt{S} = 500$ GeV. Here we assumed $\Lambda_u = 1 \text{ TeV}$.

Figure 3c: The angular distributions for unparticle contribution at $\sqrt{S} = 1000$ GeV. Here we assumed $\Lambda_u = 1$ TeV.

As we can observe from Fig. 3a, 3b and 3c, the $\frac{d\sigma}{d\cos\theta}$ increases with $\cos\theta$. Furthermore, the unparticle effects quickly go up as $\cos\theta$ becomes larger. These results are different from ones in the processes $e^+e^- \rightarrow \tau^+\tau^-$ [20] and $\gamma\gamma \rightarrow \gamma\gamma$ [17], where the angular distributions for unparticle contribution are almost flat. In addition, the angular distributions via unparticle exchange can be substantially much larger than the standard model contribution and unparticle contributions [20, 17].

In the following, we give the numerical values of the angular distribution with unparticle effects in Table 1.

$\cos\theta$	$\frac{d\sigma_u}{d\cos\theta}(pb)$			
	du=1.6	du=1.7	du=1.8	du=1.9
-0.9	1.6242×10^{-1}	0.7829×10^{-1}	0.4808×10^{-1}	0.5305×10^{-1}
-0.5	1.6565×10^{-1}	0.7809×10^{-1}	0.4681×10^{-1}	0.5026×10^{-1}
0.0	2.2841×10^{-1}	1.0104×10^{-1}	0.5681×10^{-1}	0.5719×10^{-1}
0.5	4.5184×10^{-1}	1.7377×10^{-1}	0.8572×10^{-1}	0.7658×10^{-1}
0.9	18.165×10^{-1}	5.0002×10^{-1}	1.7974×10^{-1}	1.2078×10^{-1}

0.9999	4.5951×10^2	0.3074×10^2	2.6392	0.4276
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Table 1: The angular distributions with unparticle effects at $\sqrt{S} = 500$ GeV for $C_V^{ee} = 1, \Lambda_u = 1$ TeV.

In Ref. [27], the authors have determined the angular distributions for Bhabha scattering without unparticle effects at $\sqrt{S} = 500$ GeV.

$\cos\theta$	$(\frac{d\sigma_{\text{Born QED}}}{d\cos\theta})(pb)$	$(\frac{d\sigma_{\text{Born+1-loop QED}}}{d\cos\theta})(pb)$
-0.9	0.523873	0.387798
-0.5	0.611600	0.471451
0.0	1.17253	0.916946
0.5	5.50440	4.35535
0.9	0.189118×10^3	0.152861×10^3
0.9999	0.20843×10^9	0.180178×10^9

Table 2: The angular distributions for Bhabha scattering without unparticle effects at $\sqrt{S} = 500$ GeV in Born QED and Born +1-loop QED.

Finally, we have obtained the ratio of the angular distributions with unparticle effects to one without unparticle effects in Born QED and Born +1-loop QED [27] at $\sqrt{S} = 500$ GeV in Table 3 and Table 4.

$\cos\theta$	$\frac{d\sigma_u / d\cos\theta}{d\sigma_{\text{Born-QED}} / d\cos\theta}$			
	du=1.6	du=1.7	du=1.8	du=1.9
-0.9	0.3100	0.1494	0.0918	0.1012
-0.5	0.2708	0.1277	0.0765	0.0822
0.0	0.1948	0.0862	0.0484	0.0488
0.5	0.0821	0.0316	0.0156	0.0139
0.9	0.0096	0.0026	0.9504×10^{-3}	0.6386×10^{-3}
0.9999	2.2046×10^{-6}	0.17475×10^{-6}	0.01266×10^{-6}	2.0516×10^{-9}

Table 3: The ratio of the angular distributions with unparticle effects to one without unparticle effects at $\sqrt{S} = 500$ GeV in Born QED.

$\cos\theta$	$\frac{d\sigma_u / d\cos\theta}{d\sigma_{\text{Born}+1\text{-loop QED}} / d\cos\theta}$			
	$d_u=1.6$	$d_u=1.7$	$d_u=1.8$	$d_u=1.9$
-0.9	0.4188	0.2019	0.1239	0.1368
-0.5	0.3514	0.1656	0.0991	0.1066
0.0	0.2490	0.1102	0.0619	0.0624
0.5	0.1037	0.0398	0.0197	0.0176
0.9	0.1109	0.0033	1.1758×10^{-3}	0.7901×10^{-3}
0.9999	2.5505×10^{-6}	0.1706×10^{-6}	0.0146×10^{-6}	2.3732×10^{-9}

Table 4: The ratio of the angular distributions with unparticle effects to one without unparticle effects at $\sqrt{S} = 500$ GeV in Born + 1-loop QED.

So, direct computations have showed that the ratio in Table 3 varies from 31% to 0% for $\cos\theta = -0.9$ to 0.9999 at $d_u = 1.6$. In case of $d_u = 1.7$, $d_u = 1.8$ and $d_u = 1.9$, the ratio varies from 14% to 0%, 9% to 0%, and 10% to 0% for $\cos\theta = -0.9$ to 0.9999 , respectively.

Similarly, in Table 4 the ratio varies from 42% to 0% for $\cos\theta = -0.9$ to 0.9999 at $d_u = 1.6$. For $d_u = 1.7$, $d_u = 1.8$ and $d_u = 1.9$, the ratio varies from 20% to 0%, 12% to 0%, and 13% to 0% for $\cos\theta = -0.9$ to 0.9999 , respectively.

For the next step, Fig.4 shows the behaviour of the total cross section as function of the center of mass energy of the e^+e^- system for $d_u = 1.7; 1.8; 1.9$.

Figure 4: Total cross section due to unparticle contribution depending on the center of mass energy. Here we assume $C_V^{ee} = 1$, $\Lambda_u = 1$ TeV.

These total cross sections strongly increase with increasing $\sqrt{S}$. Furthermore, the cross sections can surpass the one without unparticle at a high enough energy.

Let us present the variation of σ as a function of scaling dimension d_u (Fig. 5).

Figure 5: Dependence of the unparticle contributed total cross section on d_u . Here we take

$$C_V^{ee} = 1, \Lambda_u = 1 \text{ TeV}.$$

As we can observe from Fig. 5, the σ decreases with increasing d_u .

Next, to compare the contributions we give the numerical values of the total cross section with unparticle effects in Table 5.

$\sqrt{s}$ (GeV)	σ_u (pb)			
	$d_u = 1.6$	$d_u = 1.7$	$d_u = 1.8$	$d_u = 1.9$
300	6.7184×10^{-1}	1.5468×10^{-1}	6.1219×10^{-2}	4.7039×10^{-2}
500	8.2415×10^{-1}	2.3277×10^{-1}	11.301×10^{-2}	10.651×10^{-2}
800	9.9461×10^{-1}	3.3902×10^{-1}	19.863×10^{-2}	22.584×10^{-2}
1000	10.874×10^{-1}	4.0528×10^{-1}	25.962×10^{-2}	32.289×10^{-2}
1500	12.789×10^{-1}	5.6057×10^{-1}	42.232×10^{-2}	61.774×10^{-2}
3000	16.875×10^{-1}	9.7601×10^{-1}	97.025×10^{-2}	187.26×10^{-2}
5000	20.701×10^{-1}	14.687×10^{-1}	179.10×10^{-2}	424.08×10^{-2}

Table 5: The cross section for the unparticle contributions for $C_\nu^{ee} = 1$, $\Lambda_u = 1 \text{ TeV}$.

We can see from this table that the total cross sections in Bhabha scattering are about $0.04 - 4.24$ pico barn, and are larger than the cross sections in photon photon scattering [16, 17] by one order of magnitude, and those in the processes $pp \rightarrow 4\gamma$ and $pp \rightarrow 2\gamma 2l$ [22] by from two to five order of magnitude.

IV. Conclusions

In this work, we have considered the unparticle physics at Bhabha scattering. We have analyzed the cross sections, and have found that the effects of the unparticle can be strong. These could have important implications for unparticle searches. We hope that unparticle physics effects can be observed at LHC or ILC. Our work can be extended for other scatterings, for example dark matter pair production inside supernovae from e^+e^- annihilation [28] or $e^+e^- \rightarrow \gamma a$ process, here a is axion - like particle. Works along these line are in progress.

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