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One-dimensional large strain thaw consolidation using nonlinear effective stress–void ratio–hydraulic conductivity relationships

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Abstract

A one-dimensional model for the consolidation of thawing soils is formulated in terms of large strain consolidation and heat transfer equations. The model integrates heat transfer due to conduction, phase change and advection. The hydro-mechanical behaviour is modelled by large strain consolidation theory. The equations are coupled in a moving boundary scheme developed in Lagrangian coordinates. Finite strains are allowed and nonlinear effective stress–void ratio–hydraulic conductivity relationships are proposed to characterize the thawing soil properties. Initial conditions and boundary conditions are presented with special consideration for the moving boundary condition at the thaw front developed in terms of large strain consolidation. The proposed model is applied and compared with small strain thaw consolidation theory in a theoretical working example of a thawing fine-grained soil sample. The modelling results are presented in terms of temperature, thaw penetration, settlements, void ratio and excess pore water pressures.

Key words: Thaw consolidation, permafrost, thaw settlement, thawing soil

Résumé

Un modèle unidimensionnel de consolidation au dégel est formulé en combinant les théories de consolidation à grandes déformations et de transfert de chaleur. Le modèle inclut les transferts de chaleur dus à la conduction, au changement de phase et à l'advection. Les équations sont couplées dans un système à frontières mobiles développé en coordonnées Lagrangiennes. Les déformations finies sont permises et l'évolution des propriétés du sol est caractérisée par des relations contrainte effective-indice des vides-conductivité hydraulique non linéaires. Les conditions initiales et les conditions limites sont présentées avec une attention particulière portée au développement des conditions limites mobiles au front de dégel en fonction de la consolidation à grandes déformations. Le modèle proposé est appliqué et comparé à la théorie de consolidation au dégel à petites déformations dans un exemple théorique du dégel d'un échantillon de sol à grains fins. Les résultats de modélisation sont présentés en fonction de la température, de la pénétration du dégel, du tassement, de l'indice des vides et des pressions interstitielles excédentaires.

Introduction

The assessment of instabilities related to thawing soils is paramount for the sustainable design of infrastructure built on permafrost. There is thus a substantial incentive for developing a comprehensive engineering theory to predict the magnitude and rate of settlement as well as the magnitude of the pore water pressures generated upon thawing of a frozen soil.

There is general agreement that modelling this complex thermo-hydro-mechanical process involves coupling heat transfer with consolidation theory. The first complete thaw consolidation theory was proposed by Morgenstern and Nixon (1971); it was formulated in terms of conventional small strain consolidation theory which may not be applicable to ice-rich soils undergoing large thaw strains due to the imposed limitation of small strains and the use of linear soil properties. A solution for thaw consolidation with a nonlinear void ratio–effective stress relationship was presented by Nixon and Morgenstern (1973a) indicating that the pore water pressure at the thaw front increases with increasing degree of nonlinearity. This solution was nevertheless derived in a small strain configuration. Subsequently, Foriero and Ladanyi (1995) modelled thaw consolidation using large strain consolidation theory which allows for finite strains and accounts for the variation of the hydraulic conductivity and compressibility during consolidation.

Nevertheless, the aforementioned studies reduced thaw consolidation to an uncoupled moving boundary consolidation problem. The rate of thaw penetration was evaluated by a close-form solution to heat conduction for simple boundary conditions prior to any assessment of consolidation. The displacement of the surface thermal boundary condition as settlement proceeds was ignored which led to an underestimation of the rate of thaw penetration (Sykes et al. 1974). For complex thermal boundary conditions, the rate of thaw penetration can be determined more rigorously from the temperature profile calculated using the heat transfer equation including conduction, phase change and advection implemented in a moving boundary modelling scheme (Sykes et al. 1974).

To comply with the intricate coupled nature of the problem at hand, this paper presents the development and application of a theory of generalized coupled thaw consolidation for saturated soils. The formulation retains the simplicity of the problem statement of one-dimensional thaw consolidation while gaining the generality of large strain consolidation and heat transfer theories. Nonlinear effective stress–void ratio–

hydraulic conductivity relationships are used for the characterization of the hydro-mechanical behaviour of the soil upon thawing. The theory is numerically implemented in a moving boundaries finite-element model formulated in Lagrangian coordinates. Finally, the model is applied to the theoretical case of thawing fine-grained soil in a working example. Model results are compared with the analytical solution to the problem of small strain thaw consolidation derived by Morgenstern and Nixon (1971) to highlight the main implications inherent to the large strain configuration adopted in this study.

Problem statement

Changes in the thermal conditions at the surface of a semi-infinite mass of frozen soil may initiate thawing. Thaw penetration then proceeds at a rate controlled mainly by the surface thermal conditions and by the thermal properties of the soil. Water liberated at the thaw front from the melting of the ice contained in the ground is subjected to a pore water pressure gradient generated from the combination of thawed soil self-weight and applied load. Consequently, water in excess of the amount that can be absorbed by the soil skeleton in the existing stress conditions flows towards the surface. The length of the drainage path is initially infinitely small and consolidation of the upper thawed layers of the soil is almost instantaneous. This leads to the creation of a thawed soil layer with reduced permeability impeding the drainage of the water subsequently generated at the thaw front and excess pore water pressures may be generated if drainage is insufficient. In addition to the deformations due to phase change from ice to water, thaw settlement proceeds by seepage of the excess melt water. In thaw consolidation theory, the frozen soil can be considered impermeable and incompressible as water flow and deformations in the thawed layer are significantly larger than in the frozen region.

It should also be noted that the rate of thaw penetration is affected by the consolidation process. As settlement proceeds, surface thermal boundary conditions move towards the thaw front accelerating the rate of thaw penetration. Also, the soil thermal properties which are largely controlled by the volumetric fractions of ice, water and soil particles are affected by the void ratio changes upon consolidation. Furthermore, seepage of water towards the surface upon consolidation generates heat transfer by advection (Nixon 1975; Sykes et al. 1974).

Consequently, the interaction between consolidation and heat transfer needs to be considered in the assessment of thaw consolidation. Accordingly, the coupled large strain thaw consolidation model proposed hereafter circumvents the shortcomings of previously proposed theories by considering consolidation and heat transfer concurrently within a unified modelling domain. This is facilitated by using the void ratio as a state variable as it governs most physical processes involved in thaw consolidation. The modelling domain, also defined in terms of void ratio, is thus capable of efficiently handling soil deformations and multiple moving boundaries such as surface settlement and advancing thaw front.

Modelling domain

A schematic representation of one-dimensional large-strain thaw consolidation is presented in Figure 1 in Lagrangian coordinates (a). The modelling domain is divided into two regions: the thawed region and the frozen region. Heat transfer is effectively modelled over the full height of the soil column while consolidation modelling is limited to the thawed region as the frozen region is assumed to be impermeable and incompressible.

The heat transfer domain is defined in Lagrangian coordinates between an upper boundary at the surface of the ground $a=0$ and a lower boundary set at an arbitrary depth $a=H_i$. At the initial state $t=0$, the soil temperature is below the freezing point of the soil T_f . Thawing is initiated by changing thermal boundary conditions at the surface which is commonly modelled as a step increase in surface temperature. The lower thermal boundary conditions at $a=H_i$ can be defined by a geothermal flux. During thaw consolidation at any given time $t>0$, the freeze-thaw interface is located at thaw depth $a=Z(t)$. Heat transfer continuity is ensured between the thawed and frozen regions at $a=Z(t)$.

The consolidation domain is defined between a free-draining upper boundary at the surface of the ground $a=0$, where a uniformly distributed load P_0 is applied, and a moving impervious lower boundary at $a=Z(t)$. Settlement $s(a,t)$ is defined as a function of depth and time. During thaw consolidation, the position of the upper boundary at $a=0$ for consolidation and heat transfer is thus given by the surface settlement denoted $s(0,t)$.

Coordinate systems

Before proceeding to the mathematical development of the thaw consolidation model, it is convenient to properly differentiate the two coordinate systems used in the current study. The following description is adapted from Gibson et al. (1981) for the case of the consolidation of thawing soils. One-dimensional consolidation within the thawed soil layer is presented in Figure 2 in terms of Lagrangian and Convective coordinates. The configuration presented in Lagrangian coordinates in Figure 2a is analogous to the state before consolidation of the thawed soil occurs. The bottom boundary taken as the datum plane is at the thaw front $a=Z(t)$ and the upper boundary is at the soil surface $a=0$. An element of soil $A_0B_0C_0D_0$ is located at the Lagrangian coordinate position a and has a thickness ∂a . At any given time $t>0$ during consolidation, the state of the thawed soil layer can be represented by the Convective coordinates configuration shown in Figure 2b. The element of soil, now bounded by ABCD, is located at the Convective coordinate position ξ and has a thickness $\partial \xi$. The datum plane remains at the thaw front which is expressed in Convective coordinates as $\xi=Z(t)$. In the current study, thaw depth $Z(t)$ is always expressed with respect to the initial position of the ground surface; the height of the thawed layer at any given time $t>0$ is thus given by:

$$(1) \quad H(t) = Z(t) - s(0, t).$$

The upper boundary moved with surface settlements $s(0, t)$. The Convective coordinate position of the upper boundary located at the surface of the ground $a=0$ is thus always given by:

$$(2) \quad \xi(0, t) = s(0, t).$$

This definition can be extended for any depth such that Convective coordinates (ξ) is related to Lagrangian coordinates (a) and to settlement ($s(a, t)$) by:

$$(3) \quad \xi = a + s(a, t).$$

The schematic representation of the change in void ratio during thaw consolidation presented in Figure 3 illustrates that the conversion between Lagrangian coordinates and Convective coordinates is given by:

$$(4) \quad \frac{\partial \xi}{\partial a} = \frac{1+e}{1+e_f}$$

where $e=e(a,t)$ is the void ratio profile at time t and $e_f=e(a,0)$ is the initial void ratio profile of the frozen soil. Accordingly, Lagrangian coordinates can always be referenced back to the initial state characterized by the frozen void ratio.

Convective coordinates (ζ) move along with the material deformations. Consequently, they would be physically convenient for the description of large strain consolidation as they constitute a lifelike portrayal of the problem (Gibson et al. 1981); however, the Convective system is time dependent and is therefore numerically impractical. Indeed, the location of the thaw consolidation upper boundary is defined as a function of surface settlement $s(0,t)$ which is unknown prior to the assessment of consolidation. Convective coordinates will only be used in this study for the graphical representation of the modelling results.

The main governing equations for thaw consolidation are derived hereafter in Lagrangian coordinates (a) which relate all events to an initial configuration as a function of the void ratio (Gibson et al. 1981). As a result, boundaries can always be identified and boundary conditions can seamlessly be implemented without knowing the exact location of the boundaries prior to the analysis. For example, the surface is always located at a Lagrangian position $a=0$ as shown in Figure 2b while surface settlement $s(0,t)$ can still be fully accounted for numerically. Although Lagrangian coordinates and Eulerian coordinates commonly used in soil mechanics might appear to be the same, they differ in that Eulerian coordinates are fixed in space for all time.

Large strain consolidation

The large strain consolidation theory proposed by Gibson et al. (1967) and Gibson et al. (1981) is assumed to be valid within the thawed region of compressible saturated soil. Gibson's theory of consolidation is effectively a one-dimensional model of mass transfer based on the vertical equilibrium of soil mass and pore water. The simplest and most convenient form of the theory is derived in terms of void ratio. It is thus primarily oriented towards the prediction of deformations from which other geotechnical quantities of interest such as excess pore water pressures can be calculated.

General equations for large strain consolidation

Under the assumption that soil particles and pore water are incompressible, large strain consolidation of thawing saturated soils can be described by the following set of equations formulated in Lagrangian coordinates (Gibson et al. 1967; Gibson et al. 1981; Xie and Leo 2004).

The vertical equilibrium of the soil mass is given by:

$$(5) \quad \frac{\partial \sigma_v}{\partial a} = \frac{(G_s + e)\gamma_w}{(1 + e_f)}$$

where σ_v is the total vertical stress, G_s the specific gravity of the solid particles and γ_w the unit weight of water.

Also, the equilibrium of pore water requires that:

$$(6) \quad \frac{\partial u}{\partial a} = \frac{\partial u_e}{\partial a} + \frac{(1 + e)\gamma_w}{(1 + e_f)}$$

where u is the total pore water pressures and u_e the excess pore water pressures.

Darcy's law relates the relative velocity of the soil skeleton and pore water to the excess pore water pressure gradient such that:

$$(7) \quad v_w - v_s = -\frac{k_v(e)(1 + e_f)}{\gamma_w e} \frac{\partial u_e}{\partial a}$$

where v_w and v_s are the velocities of the pore water and soil particles relative to the datum plane and $k_v(e)$ the vertical hydraulic conductivity as a function of the void ratio.

Additionally, the continuity of pore water flow requires that:

$$(8) \quad \frac{\partial}{\partial a} \left[\frac{e(v_w - v_s)}{1 + e} \right] = \frac{1}{1 + e_f} \frac{\partial e}{\partial t}$$

Equations 7 and 8 can be combined to yield the following equation governing large strain consolidation:

$$(9) \quad \frac{1}{\gamma_w} \frac{\partial}{\partial a} \left[\frac{k_v(e)(1 + e_f)}{1 + e} \frac{\partial u_e}{\partial a} \right] = \frac{1}{1 + e_f} \frac{\partial e}{\partial t}$$

Equation 9 is mathematically inconvenient as both excess pore water pressures and void ratio are used as dependent variables. It can be reduced to an equation expressed only in terms of void ratio by using the principle of effective stress given by:

$$(10) \quad \sigma_v = \sigma'_v + u$$

where σ'_v is the vertical effective stress.

The derivative of Equation 10 with respect to depth is:

$$(11) \quad \frac{\partial \sigma_v}{\partial a} = \frac{\partial \sigma'_v}{\partial a} + \frac{\partial u}{\partial a}$$

Equations 5 and 6 can be substituted into Equation 11 such that:

$$(12) \quad \frac{(G_s+e)\gamma_w}{(1+e_f)} = \frac{\partial \sigma'_v}{\partial a} + \frac{\partial u_e}{\partial a} + \frac{(1+e)\gamma_w}{(1+e_f)}$$

Accordingly, the excess pore water pressure gradient is given by:

$$(13) \quad \frac{\partial u_e}{\partial a} = \frac{(G_s-1)\gamma_w}{(1+e_f)} - \frac{\partial \sigma'_v}{\partial a}$$

Equations 9 and 13 can be combined to yield the following equation governing void ratio:

$$(14) \quad \frac{d}{de} \left[\frac{k_v(e)(G_s-1)}{1+e} \right] \frac{\partial e}{\partial a} + \frac{1}{\gamma_w} \frac{\partial}{\partial a} \left[- \frac{k_v(e)(1+e_f)}{1+e} \frac{d\sigma'_v(e)}{de} \frac{\partial e}{\partial a} \right] = \frac{1}{1+e_f} \frac{\partial e}{\partial t}$$

where the dependent variable e is spatially dependent such that $e=e(a,t)$ and $e_f=e(a,0)$, and $\sigma'_v(e)$ is the relationship between void ratio and effective stress.

Equation 14 is the main governing equation used in the model to describe the hydro-mechanical processes within the thawed region. It is developed under the assumption that the effective stress and hydraulic conductivity are related to the void ratio alone.

Evaluation of geotechnical parameters

By formulating the equation for consolidation in terms of void ratio, soil deformations are obtained directly from the modelling results. However, additional relations need to be developed for the evaluation of other geotechnical quantities of interest.

The settlement as a function of depth and time is given by:

$$(15) \quad s(a, t) = \int_a^{Z(t)} \frac{e_f - e}{1 + e_f} da.$$

The settlement at the thaw front $a=Z(t)$ is obviously null since all points deeper than $Z(t)$ are in the frozen region. The settlement of the ground surface $a=0$ is:

$$(16) \quad s(0, t) = \int_0^{Z(t)} \frac{e_f - e}{1 + e_f} da.$$

The degree of consolidation as a function of depth and time is given by:

$$(17) \quad U(a, t) = \int_a^{Z(t)} \frac{e_f - e(a, t)}{e_f - e(a, \infty)} da.$$

The degree of consolidation for the whole thawed layer is calculated with $a=0$.

Thaw strain for the thawed layer is defined as a function of the thaw depth in Lagrangian coordinates as:

$$(18) \quad \delta(0, t) = \frac{s(0, t)}{Z(t)}.$$

In addition to the deformations, assessment of thaw consolidation related instabilities requires the evaluation of the stress conditions in the ground.

The total vertical stress can be calculated by integrating Equation 5 such that:

$$(19) \quad \sigma_v = \sigma_v(a, t) = \sigma_v(0, t) + \int_0^a \frac{(G_s + e)\gamma_w}{1 + e_f} da$$

where $\sigma_v(0, t)$ is total stress at the surface given by:

$$(20) \quad \sigma_v(0, t) = P_0(t) + \gamma_w H_w(0, t)$$

where $H_w(0,t)$ is the height of water accumulating at the surface from consolidation. If the water seeping out of the soil skeleton during consolidation is drained away from the ground surface, $H_w(0,t)$ is equal to zero. If water accumulation is allowed, $H_w(0,t)$ is equal to surface settlement $s(0,t)$.

The total pore water pressure is defined as:

$$(21) \quad u(a, t) = u_h(a, t) + u_e(a, t)$$

where u_h is the hydrostatic pore water pressure given by:

$$(22) \quad u_h(a, t) = \gamma_w(a + s(a, t) - s(0, t) + H_w(0, t)).$$

Under the assumption that the effective stress is a function of the void ratio alone, the vertical effective stress profile is given by:

$$(23) \quad \sigma'_v(a, t) = \sigma'_v(e(a, t)).$$

Based on the principle of effective stress presented in Equation 10, the excess pore water pressure can be calculated from the combination of Equations 19 to 23:

$$(24) \quad u_e(a, t) = P_0(t) + \int_0^a \frac{(G_s + e)\gamma_w}{1 + e_f} da - \sigma'_v(e(a, t)) - \gamma_w(a + s(a, t) - s(0, t)).$$

Nonlinear effective stress–void ratio–hydraulic conductivity relationships

Determination and characterization of the consolidation properties of thawing soils have scarcely been discussed in previous thaw consolidation studies despite their paramount role to the development of a comprehensive theory of practical engineering relevance (Morgenstern and Nixon 1971; Sykes et al. 1974; Foriero and Ladanyi 1995). As shown in Equation 14, large-strain consolidation is primarily governed by void ratio dependent functions for the effective stress and the hydraulic conductivity. While those relationships have been studied thoroughly for unfrozen soils, the properties of thawing soils are drastically different which warrants the ensuing analysis.

Figure 4 presents the effective stress–void ratio–hydraulic conductivity relationships for thawing soils in a unified semi-logarithmic workspace. To provide a physical interpretation of thaw consolidation with

regards to the soil properties, the theoretical case of a soil sample at a frozen void ratio e_f is now considered. If the sample is thawed in undrained conditions, the change in void ratio is only due to the 9% volume contraction of ice to water. The frozen void ratio e_f is thus related to the initial thawed void ratio e_0 such that:

$$(25) \quad e_0 = \frac{e_f}{1.09}.$$

If a constant external load P_0 is applied on the sample during undrained thawing, pore water pressures can develop. The effective stress in the sample upon thawing is thus equal to the difference between the applied load and the pore water pressure. The effective stress that is sustained by the soil skeleton in undrained conditions is called the residual stress σ'_0 (Nixon and Morgenstern 1973b). If the applied load is equal to the residual stress, no excess pore water pressures develop upon thawing. For applied loads larger than the residual stress, excess pore water pressures develop and consolidation occurs if drainage is allowed. Void ratio changes during consolidation are then governed by the effective stress–void ratio relationship of the thawed soil.

It is generally accepted that the change in void ratio with effective stress for thawing fine-grained soils is nonlinear over the range of stresses of practical interest (Konrad 2010; Konrad and Samson 2000; Nixon and Morgenstern 1973a, 1973b). Indeed, some small limited loading is required to induce substantial thaw settlement for soils with high frozen void ratio. However, expulsion of additional pore water leading to further settlement is only achieved at significantly higher stresses. For thawed fine-grained soils, experimental evidence (Konrad 2010; Konrad and Samson 2000) demonstrates that this behaviour can be defined by a semi-logarithmic relationship given by:

$$(26) \quad (e - e_0) = -C_{cT} \log \left(\frac{\sigma'_v(e)}{\sigma'_0} \right)$$

where C_{cT} is the compression index of the thawed soil.

In addition to the effective stress–void ratio relationship, consolidation of thawing soils is regulated by the hydraulic conductivity of the soil. Studies have demonstrated that the variation of hydraulic conductivity with void ratio can be described by a semi-logarithmic relationship characterized by a slope parameter C_{kT}

called the hydraulic conductivity change index of the thawed soil (Konrad 2010; Konrad and Samson 2000). The void ratio–hydraulic conductivity relationship for thawed fine-grained soils is given by:

$$(27) \quad (e - e_0) = C_{kT} \log \left(\frac{k_v(e)}{k_{v0}} \right).$$

The initial state of the thawed soil is fully defined by the combination of the residual stress σ'_0 , the initial thawed void ratio e_0 and the initial hydraulic conductivity k_{v0} . From the initial state of the thawed soil, the hydro-mechanical behaviour during thaw consolidation is governed by the stress increment which is the difference between the applied effective stress P_0 and the residual stress σ'_0 . Morgenstern and Nixon (1971) indicated that the residual stress σ'_0 of soils with high initial void ratio could be assumed to be equal to zero for first order estimates. However, this approach overlooks the nonlinearity of the effective stress–void ratio relationship and the interdependence between the effective stress–void ratio–hydraulic conductivity relationships over the particular stress path followed by the thawing soil.

Initial conditions and boundary conditions for consolidation

The initial conditions for consolidation are simply described by the initial void ratio profile of the frozen soil:

$$(28) \quad e(a, 0) = e_f(a).$$

Although this definition may appear trivial, calculation of settlements and excess pore water pressures is contingent on a precise description of the frozen void ratio profile as it is used extensively in the formulation of large strain consolidation equations and in the characterization of the soil properties upon thawing.

Modelling consolidation of thawing soils typically involves two boundary conditions: a free-draining boundary at the ground surface and a moving impervious boundary at the thaw front. Although handling of these boundary conditions is relatively simple in classical consolidation theory developed in terms of pore water pressures, their implementation into a large strain consolidation scheme is more complex because of the formulation in terms of void ratio and of the movement of the lower boundary at the thaw front.

At the free-draining boundary, no excess pore water pressure is sustained. From Equation 26, the condition at the ground surface $a=0$ in terms of excess pore water pressure is thus given by:

$$(29) \quad u_e(0, t) = P_0(t) - \sigma'_v(e(0, t)) = 0.$$

Taking advantage of the effective stress–void ratio relationship defined in Equation 24, Equation 29 can be rearranged such that the boundary condition at $a=0$ in terms of void ratio is given by:

$$(30) \quad e(0, t) = e_0 - C_{cT} \log \left(\frac{P_0(t)}{\sigma'_0} \right).$$

The conditions at the thaw front were first formulated by Morgenstern and Nixon (1971). The derivation of the lower boundary conditions is reformulated here in terms of large strain consolidation. Also, volume changes due to phase change from ice to water, which were never explicitly integrated in any thaw consolidation theory, are directly included into the following development at no additional computation costs due to the formulation in terms of void ratio.

As described in the problem statement, water liberated at the thaw front flows towards the surface due to the pore water pressure gradient it is subjected to. According to Darcy's law, the volumetric discharge of water at the thaw front is given by:

$$(31) \quad \Delta V = - \frac{k_v(e)(1+e_f)}{\gamma_w e} \frac{\partial u_e}{\partial a} A \Delta t$$

where A is the cross-section area of the soil element and Δt the time increment.

Consequently, the volumetric strain due to water discharge of a layer of thickness ΔZ is:

$$(32) \quad \frac{\Delta V}{V} = \frac{\Delta V}{A \Delta Z} = - \frac{k_v(e)(1+e_f)}{\gamma_w e} \frac{\partial u_e}{\partial a} \frac{1}{\frac{\Delta Z}{\Delta t}}.$$

The additional volumetric strain due to phase change of ice to water is given by:

$$(33) \quad \frac{\Delta V}{V} = - \frac{e_0 - e_f}{1 + e_f}.$$

Per the principle of compressibility, the total volume change is related to a corresponding change in effective stress such that:

$$(34) \quad \frac{\Delta V}{V} = -m_v \Delta \sigma'_v = \frac{\Delta e}{\Delta \sigma'_v} \frac{\Delta \sigma'_v}{1+e_f}$$

where m_v is the coefficient of volume change.

If volumetric strain from water discharge and phase change are accommodated by a change in effective stress, the combination of Equations 32 to 34 yields:

$$(35) \quad -\frac{k_v(e)(1+e_f)}{\gamma_w e} \frac{\partial u_e}{\partial a} \frac{1}{\frac{\Delta Z}{\Delta t}} - \frac{e_0 - e_f}{1+e_f} = \frac{\Delta e}{\Delta \sigma'_v} \frac{\Delta \sigma'_v}{1+e_f}$$

By considering volume changes due to phase change and consolidation separately, Equation 35 becomes:

$$(36) \quad \frac{\Delta e}{\Delta \sigma'_v} \frac{\Delta \sigma'_v}{1+e_f} = \frac{e_0 - e_f}{\sigma'_0 - \sigma'_v(e_f)} \frac{\sigma'_0 - \sigma'_v(e_f)}{1+e_f} + \frac{e - e_0}{\sigma'_v(e) - \sigma'_0} \frac{\sigma'_v(e) - \sigma'_0}{1+e_f}$$

For infinitely small time step, the contribution of phase change can be removed from both sides of Equation 36 such that:

$$(37) \quad -\frac{k_v(e)(1+e_f)}{\gamma_w e} \frac{\partial u_e}{\partial a} \frac{1}{\frac{dZ(t)}{dt}} = \frac{1}{\frac{d\sigma'_v(e)}{de}} \frac{(\sigma'_v(e) - \sigma'_0)}{1+e_f}$$

By replacing the excess pore water pressure gradient in Equation 37 by Equation 13, the conditions at the thaw front in terms of void ratio are given by:

$$(38) \quad \frac{d\sigma'_v(e)}{de} \frac{\partial e}{\partial a} = \frac{(G_s - 1)\gamma_w}{1+e_f} + \frac{dZ(t)}{dt} \frac{\gamma_w e}{k_v(e)(1+e_f)} \frac{1}{\frac{d\sigma'_v(e)}{de}} \frac{\sigma'_v(e) - \sigma'_0}{1+e_f}$$

Heat transfer

As shown in the previous section, the consolidation of thawing soils is geometrically controlled by the thaw depth. However, reducing the assessment of heat transfer to the calculation of the rate of thaw penetration offers very little coupling possibilities between the consolidation and heat transfer components. For more

complex analysis, the heat transfer component of thaw consolidation needs to be implemented within the moving boundary scheme previously defined by large-strain consolidation theory. Initiation and progression of thaw penetration should thus be calculated using the equation of heat transfer which includes phase change, conduction and advection.

General equations for heat transfer

The one-dimensional convection-conduction heat transfer equation in Lagrangian coordinates is:

$$(39) \quad \left(C - L\rho_i \frac{\partial \theta_i}{\partial T} \right) \frac{\partial T}{\partial t} - \frac{\partial}{\partial a} \left[\lambda \frac{(1+e_f)^2}{(1+e)^2} \frac{\partial T}{\partial a} \right] + c_w \rho_w v_a \frac{1+e_f}{1+e} \frac{\partial T}{\partial a} = 0$$

where C is the volumetric heat capacity of the soil, L the latent heat of fusion of water per unit mass, ρ_i and ρ_w the density of ice and water, θ_i the volumetric ice fraction, T the temperature in Kelvin, λ the thermal conductivity of the soil, c_w the heat capacity of water by mass, and v_a the water flow velocity in the direction of a .

The relative water flow velocity in the direction of a calculated by combining Equations 7 and 13 is given by:

$$(40) \quad v_a = - \frac{k_v(e)(1+e_f)}{\gamma_w e} \left(\frac{(G_s-1)\gamma_w}{(1+e_f)} - \frac{d\sigma'_v(e)}{de} \frac{\partial e}{\partial a} \right).$$

Soil thermal properties

The geothermal properties introduced in Eq. 39 are mainly dependent on the relative contribution of the soil components which are readily obtained from the void ratio.

The volumetric fraction of the soil particles is given by:

$$(41) \quad \theta_s = \frac{1}{1+e}.$$

while, the volumetric fraction of the voids is given by:

$$(42) \quad \theta_v = \frac{e}{1+e}.$$

In a saturated soil, the voids are entirely filled with either ice, water or a mix of both such that:

$$(43) \quad \theta_v = \theta_w + \theta_i = \frac{e}{1+e}$$

where θ_w and θ_i are the volumetric fractions of liquid water and ice.

The volumetric fraction of liquid water is given by:

$$(44) \quad \theta_w = \begin{cases} \frac{e}{1+e}, & T > T_f \\ \theta_u, & T \leq T_f \end{cases}$$

where θ_u is the unfrozen water content and T_f the freezing point.

The relationship between unfrozen water content and temperature, also called the phase composition curve, is often represented using a power function such that:

$$(45) \quad w_u = \alpha(T_f - T)^\beta$$

where w_u is the unfrozen water content by mass, α and β coefficients characteristic of the soil. Typical values for α and β can be found in the literature (Nixon 1991) or determined experimentally. The volumetric fraction of unfrozen water is:

$$(46) \quad \theta_u = \alpha(T_f - T)^\beta \frac{G_s}{1+e}.$$

From the volumetric contribution of each of the soil components, the volumetric heat capacity of the soil is given by:

$$(47) \quad C = \rho_w c_w \theta_w + \rho_s c_s \theta_s + \rho_i c_i \theta_i$$

where ρ is the density and c the heat capacity by mass with subscripts w , s and i denoting liquid water, solids and ice respectively.

Finally, the generalized model proposed by Côté and Konrad (2005) states that the thermal conductivity of a saturated soil is given by:

$$(48) \quad \lambda = \lambda_s^{\theta_s} \times 2.24^{\theta_i} \times 0.6^{\theta_w}$$

where λ_s is the thermal conductivity of solid particles.

Initial conditions and boundary conditions for heat transfer

The initial conditions for heat transfer are typically given by the temperature profile of the frozen soil:

$$(49) \quad T_0 = T(a, 0) \leq T_f.$$

For the simplest case of thaw consolidation, thawing is initiated by a step increase in temperature at the surface given by boundary conditions:

$$(50) \quad T_s = T(0, t) > T_f.$$

The surface temperature T_s may vary with time given that it remains strictly above the freezing point T_f because the proposed model does not integrate the effect of freeze-back.

For simple analysis, the lower boundary conditions at $a=H_i$ may be given as a function of temperature:

$$(51) \quad T_{H_i} = T(H_i, t),$$

or as a function of heat flux:

$$(52) \quad q_{H_i} = q(H_i, t).$$

To reproduce field conditions, the heat flux (q_{H_i}) may be defined by the geothermal heat flux such that:

$$(53) \quad q_g(H_i) = -k_g G$$

where k_g is the thermal conductivity of the soil below H_i and G the geothermal gradient defined in permafrost by the temperature gradient with depth below the depth of zero annual amplitude.

A major benefit of the current model is its ability to handle any type of thermal boundary conditions that may be required for more complex analysis. For example, a combination of boundary conditions can be used to simulate the energy balance at the surface of a natural ground. However, it should be noted that the model is limited to monotonic top-down thawing. Accordingly, boundary conditions at the top should not initiate freeze-back and lower boundary conditions should not initiate thawing of the soil from the bottom.

Coupling

A thaw consolidation model is established by coupling the consolidation and heat transfer components. Most aspects of the interaction between the two components have already been explicitly or implicitly introduced in previous sections. Still, the coupling scheme is best defined by describing sequentially the contributions of each component to the overall model. It should be noted that all contributions are occurring simultaneously as each time step is solved in an iterative process over the unified modelling domain.

The consolidation component is used to define the coordinates system of the modelling the domain and the position of the boundaries from the calculation of the void ratio at all time steps. The modelling domain is kept physically coherent as the Lagrangian coordinates are normalized at each time step as a function of the initial void ratio and the current void ratio profiles. Accordingly, the boundary conditions at the surface, namely the heat source causing thawing of the frozen soil and the free-draining boundary for consolidation, are always physically coherent with surface settlements. Additionally, the void ratio is used in the calculation of the geothermal properties of the soil. The consolidation component is also analogous to a mass transfer model which provides the pore water flow velocity for calculation of the advective heat transfer.

The heat transfer component contributes to the model by defining the position of the interface between the thawed and frozen regions where the lower boundary for consolidation is located. The depth where the temperature is equal to the freezing point T_f is calculated at each time step and the lower boundary for consolidation is moved accordingly. The rate of thaw penetration is equal to the rate of displacement of the thaw depth during each time step. The movement of the boundary is mathematically accommodated by the formulation of the impervious boundary condition at the thaw front in Equation 38.

Numerical implementation

The proposed model can be implemented using any numerical calculation tool capable of solving nonlinear partial differential equations systems for multiple coupled components and moving boundaries. While several commercial and free software solutions are capable of such feats, COMSOL Multiphysics® version 5.2a was used for the current study. The model is implemented using a combination of two *Coefficient Form PDE Mathematics Interfaces* with a *Fully Coupled Direct MUMPS Time-Dependent*

Solver. The one-dimensional geometry is formed of two *Intervals* for the thawed and frozen regions respectively. The consolidation component is first activated in the initially infinitely small thawed region to allow the top boundary conditions for consolidation to stabilize. The heat transfer component is then initiated by the step increase in temperature at the surface which triggers the movement of the interface between the thawed and frozen regions. The moving boundary is handled by a *Deformed Mesh* component of the type *Deformed Geometry (dg)*. Both the thawed and frozen intervals are composed of very fine mesh elements to facilitate numerical stability of the deforming geometry. A *Strict BDF* time stepping is adopted. It is important to mention that none of the pre-programmed physics modules offered in COMSOL was used to set up the model and that all the properties and equations were user-defined.

Working example

The model developed herein is applied to a typical thaw consolidation laboratory problem. Athabasca Clay is used in this example since all soil properties pertaining to the assessment of thaw consolidation were reported by Smith (1972). This example considers a 50 mm thick sample of Athabasca Clay at an initial frozen void ratio e_f of 2.83 which corresponds to the initial thawed void ratio e_0 of 2.60 for which the effective stress–void ratio–hydraulic conductivity relationships were determined by Smith (1972). The sample is at an initial uniform temperature T_0 of $-5\text{ }^{\circ}\text{C}$ and thawing is initiated by increasing the temperature at the top of the sample T_s to $5\text{ }^{\circ}\text{C}$. The thermal boundary conditions at the bottom of the sample simulate a semi-infinite mass of frozen soil to reproduce the laboratory conditions used by Smith (1972). A constant load P_0 of 15 kPa is applied at the top of the sample. The thermal and loading conditions were specifically selected to yield a combination of thaw penetration and consolidation rates in the range of practical interest with regards to the properties of Athabasca Clay.

Soil properties

Athabasca Clay is a clay of low plasticity with a clay content of 45 %, a silt content of 54 % and a sand content of 1 % (Smith 1972). The properties of Athabasca Clay are similar to those of the lean silty clays commonly found in permafrost regions of Northern Canada which are characterized by high compressibility and low hydraulic conductivity (Morgenstern and Smith 1973). Athabasca Clay has a specific gravity G_s of 2.65, a thermal conductivity of the solid particles λ_s of $2.1\text{ W/m}^{\circ}\text{C}$ and a volumetric

heat capacity of the solid particles C_s of $712 \text{ MJ/m}^3\text{°C}$ (Smith 1972). Parameters α and β for the unfrozen water content are set equal to 9.0 and -0.45 respectively in concordance with literature data for soils with a similar grain size distribution curve (Andersland and Ladanyi 2004). The freezing point T_f is set at 0°C (Smith 1972).

The effective stress–void ratio–hydraulic conductivity relationships of thawed Athabasca Clay are shown in Figure 5. The relationships were experimentally determined by oedometric consolidation tests performed on thawed samples with applied loading ranging from 1.3 to 230 kPa (Smith 1972). The experimental curves are extrapolated to the initial thawed void ratio as indicated by the empty dots in Figure 5 to fully characterize both relationships over the full range of void ratio changes expected upon thawing. The effective stress–void ratio relationship of thawed Athabasca Clay is given by:

$$(54) \quad (e - 2.60) = -0.421 \log \left(\frac{\sigma'_v(e)}{0.0028} \right),$$

while the void ratio–hydraulic conductivity relationship of thawed Athabasca Clay is given by:

$$(55) \quad (e - 2.60) = 0.305 \log \left(\frac{k_v(e)}{8.1 \times 10^{-6} \text{ m/s}} \right).$$

Numerical modelling

The previously described strategy is used for the numerical implementation of the model in COMSOL Multiphysics® version 5.2a. The initial height of the thawed *Interval* is a hundred times smaller than the sample height. The thawed *Interval* is divided in 500 mesh elements and the frozen *Interval* in 100 mesh elements. Several *Probes* and *Coefficient Form PDE Mathematics Interfaces* are set up to facilitate post-processing of the simulation data by automating the calculation of settlements and stress state variables. Time stepping is automatically adapted by COMSOL during computing of the model. The conversion from Lagrangian coordinates to Convective coordinates is computed during the simulation to allow for a physically coherent representation of the modelling results as a function of the actual geometry.

Thaw consolidation testing is typically designed to reproduced field conditions (Smith 1972). The boundary conditions at the bottom of the sample for the heat transfer component should thus simulate thawing of a semi-infinite column of soil. This is achieved by increasing the modelling domain for heat

transfer to 5 times the height of the frozen soil sample which effectively creates a heat sink at the bottom of the modelled sample. Thaw penetration is automatically stopped when the maximum thaw depth which is equal to the frozen height of the sample is reached. The simulation then enters the post-thaw consolidation phase and an impervious boundary condition is automatically imposed at the bottom of the sample.

Typical results

It is convenient to first examine the thermal component of thaw consolidation because of its importance in defining the rate of thaw penetration which regulates the rate at which pore water pressures are generated at the thaw front. Figure 6 shows the model predicted temperature profiles in the sample at times 0, 5, 25, 95 and 348 min. The y-axis represents depth in Convective coordinates below the initial position of the top of the frozen sample. The constant initial temperature set equal to -5°C is shown by the $t=0$ min isochrone while it can be seen that the temperature at the top of the sample is set equal to 5°C at $t>0$. The temperature isochrones are characterized by a break at the 0°C isotherm which is typical of thawing soils due to the difference between the thermal properties of the frozen and thawed soil. The downward progression of thaw depth $Z(t)$ is defined by the intersection between the temperature isochrones and the 0°C isotherm. At $t=348$ min, the sample is fully thawed as thaw depth reaches the bottom of the sample and the post-thaw consolidation phase is initiated.

A simulation performed without the inclusion of the advection heat transfer term indicated that heat transfer by advection cause a reduction of the thaw depth of less than 1% for the working example presented herein. This result confirms that heat transfer by advection due to seepage of water caused by consolidation is not significant in thaw consolidation (Nixon 1975). However, it should be mentioned that the advection term is added at no additional computational costs due to the formulation of the model in terms of void ratio. Also, the proposed formulation allows for the addition of an external hydraulic gradient which might have a more significant impact on thaw depth.

The position of the top of the sample corresponding to the progression of surface settlement $s(0,t)$ is indicated by dashed lines in Figure 6. Accordingly, the boundary conditions applied at the top of the sample in the proposed model moves along with the soil deformations in concordance with the physical reality of the problem. In comparison, a small strain thaw consolidation theory implemented in Eulerian coordinates

(Morgenstern and Nixon 1971) always considers that the upper boundary conditions are applied at the initial position of the top of the sample at $\xi=0$. In the example presented herein for ice-rich Athabasca Clay undergoing large thaw strain, the model results indicate that the top of the sample when thaw depth reaches the bottom of the sample is 21.5 mm lower than its initial position. Small strain thaw consolidation theory would thus underestimate the rate of thaw penetration by not considering the movement of the 5 °C temperature boundary condition towards the thaw front.

Figure 7 shows model predicted void ratio profiles in the thawed layer during the thaw consolidation and post-thaw consolidation phases. The alternate x-axis at the bottom represents the corresponding hydraulic conductivity of the soil. The constant initial void ratio of the frozen sample equal to 2.83 is not shown in Figure 7. At the free-draining boundary at the top of the sample, the void ratio which is given by Equation 54 for an applied load P_0 of 15 kPa is equal to 1.03. For all time steps, the void ratio profile is plotted between the top of the sample as a function of surface settlement and the bottom of the thawed layer which is always equal to the thaw depth $Z(t)$.

The surface settlement is indicated for each time step in Figure 7. The model results indicate that most consolidation occurred during the thaw consolidation phase for the ice-rich Athabasca Clay sample subjected to the specified thermal conditions. In the current example, the void ratio at the thaw front decreases as thaw consolidation proceeds which indicates that the excess water is drained out of the soil skeleton faster than it is liberated at the thaw front. The hydro-mechanical behaviour during thaw consolidation is regulated by three main factors: the hydraulic conductivity of the thawed layer, the velocity of the thaw front and the length of the drainage path which is controlled by the compressibility of the soil. As anticipated, the hydraulic conductivity in the thawed layer decreases with time as shown in Figure 7. The rate of drainage of the excess water liberated at the thaw front is thus expected to decrease with time. However, the velocity of the thaw front $dZ(t)/dt$, which is a key parameter in Equation 38 in regulating the boundary conditions at the thaw front, also decreases with time which means that the incremental thaw depth $dZ(t)$ for a constant time step dt is getting smaller. Consequently, the volume of water liberated at the thaw front decreases with time which offset the decreasing hydraulic conductivity of the thawed layer.

Moreover, the length of the drainage path does not increase as much as it might be anticipated due to surface settlement which also contributes to facilitating the drainage of the excess water.

Comparison with small strain thaw consolidation theory

This section presents a comparison between the proposed large strain model and the small strain thaw consolidation approach as proposed by Morgenstern and Nixon (1971). The large strain and small strain configurations for thaw consolidation are compared in Figure 8. In the large strain configuration at time $t > 0$, thaw depth is equal to $Z(t)$ and the position of the surface is equal to $s(0, t)$ which is below the datum given by the initial position of the soil surface $\xi = 0$. After incremental time Δt , thaw depth and surface settlement both increases such that

$$(56) \quad Z(t + \Delta t) > Z(t)$$

and

$$(57) \quad s(0, t + \Delta t) > s(0, t).$$

The increase in thaw depth over time step Δt in small strain configuration is given by

$$(58) \quad \Delta Z = Z(t + \Delta t) - Z(t).$$

In the small strain configuration, the surface is always located at the datum given by the initial position of the soil surface $\xi = 0$ and the small strain thaw depth is always given by the height of the thawed layer as defined in Eq. 1 such that

$$(59) \quad Z_z(t) = Z(t) - s(0, t)$$

where Z_z is the small strain thaw depth.

After incremental time Δt , the small strain thaw depth is thus equal to

$$(60) \quad Z_z(t + \Delta t) = Z(t + \Delta t) - s(0, t + \Delta t).$$

The increase in small strain thaw depth over time step Δt is thus given by

$$(61) \quad \Delta Z_z = Z_z(t + \Delta t) - Z_z(t) = (Z(t + \Delta t) - s(0, t + \Delta t)) - (Z(t) - s(0, t)),$$

which can be rearranged as

$$(62) \quad \Delta Z_z = Z_z(t + \Delta t) - Z_z(t) = (Z(t + \Delta t) - Z(t)) - (s(0, t + \Delta t) - s(0, t)).$$

By comparing Eq. 58 and 62, it can be seen that the incremental thaw depth is always smaller when calculated in the small strain configuration given that the condition formulated in Eq. 57 is satisfied. This effectively means that the small strain approach underestimates the volume of water liberated at the thaw front which may ultimately lead to an underestimation of the excess pore water pressure at the thaw front. It should be mentioned that the length of the drainage path given by the height of the thawed layer is the same in both large and small strain configurations.

The difference between the large and small strain approaches can be efficiently analyzed using the modelling results from the previously discussed working example. Figure 9a presents the thaw depth and the surface settlement as a function of the square root of time. The small strain thaw depth is always defined relative to the position of the surface. It is obtained by subtracting the surface settlement curve from the large strain thaw depth curve. As the soil sample of the working example is subjected to very large thaw strain, the difference between the large and small strain thaw depths is most noticeable. The rates of thaw penetration are thus considerably different. Nevertheless, the length of the drainage path is the same for both configurations.

Figure 9b presents the model computed excess pore water pressure isochrones in the thawed layer at times 5, 95 and 348 min. Also plotted on this figure are the values obtained by the use of the analytical solution based on the thaw consolidation ratio proposed by Morgenstern and Nixon (1971). At each time step, the thaw depth is indicated by empty dots on the excess pore water pressure isochrones and on the thaw penetration curves of Figure 9a. The position of the surface is indicated by full dots on the excess pore water pressure isochrones and on the settlement curve of Figure 9a. For the small strain configuration, the surface is always at $\xi=0$ as seen in Figure 9a.

The thaw consolidation ratio which parametrizes the theoretical balance between the generation and the dissipation of pore water pressure in Morgenstern and Nixon (1971) theory is defined as:

$$(63) \quad R = \frac{\alpha r}{2\sqrt{c_v}}$$

where α_T is the rate of thaw penetration and c_v is the coefficient of consolidation.

If thaw penetration is assumed to be proportional to the square root of time (Morgenstern and Smith 1972), the rate of thaw penetration can be calculated as:

$$(64) \quad \alpha_T = \frac{Z(t_{thaw}) - s(0, t_{thaw})}{\sqrt{t_{thaw}}}$$

where $Z(t_{thaw}) - s(0, t_{thaw})$ represents the thaw depth relative to the position of the surface in the small strain configuration and t_{thaw} is the time when the thaw depth reaches the bottom of the sample.

From Figure 9a, $Z(t_{thaw}) - s(0, t_{thaw}) = 0.05 - 0.0215$ m and $t_{thaw} = 348$ min which yields a rate of thaw penetration α_T in the sample equal to $0.00153 \text{ m/min}^{1/2}$.

By assuming that the consolidation of thin thawing soil samples is controlled by the conditions at the surface, the coefficient of consolidation can be calculated by:

$$(65) \quad c_v = \frac{k(e(P_0))}{\frac{C_c T}{P_0 \ln(10)} \frac{1}{1+e_f} \gamma_w}.$$

Using the effective stress–void ratio–hydraulic conductivity relationships of thawed Athabasca Clay defined by Equations 54 and 55, the coefficient of consolidation for $P_0 = 15$ kPa is thus equal to $1.24 \times 10^{-8} \text{ m}^2/\text{sec}$.

The thaw consolidation ratio for this example is thus equal to 0.89. The analytical solution derived by Morgenstern and Nixon (1971) for $R = 0.89$ gives a normalized maximum excess pore water pressure at the thaw front of 0.75 which is equal to a value of 11.2 kPa for an applied load of 15 kPa. The excess pore water pressure profile in the thawed layer plotted in Figure 9b is obtained from the graphical solution developed by Morgenstern and Nixon (1971).

Figure 9b efficiently illustrates the fundamental difference between the large strain and the small strain configurations. The magnitude and the shape of the excess pore pressure isochrones predicted by both approaches are similar. However, without a direct assessment of soil deformations, excess pore water pressures are predicted outside of the soil sample by the small strain thaw consolidation theory. This is also

typically observed when comparing small and large strain consolidation theories (Gibson et al. 1981). Furthermore, thaw consolidation theory presents an added level of complexity inherent to the moving thaw depth.

Figure 10 presents the excess pore water pressures isochrones in the thawed layer in large strain configuration. The small strain results were adapted to the large strain configuration by plotting the excess pore water pressure isochrones from the top of the sample as a function of the settlement computed by the proposed model to provide a visually improved comparison between the proposed model and the small strain approach.

Figure 10 shows that the excess pore water pressures at the thaw front computed by the proposed model are slightly larger than the ones predicted by small strain theory. This is mainly due to the large strain configuration which allows for a physically coherent assessment of the thaw penetration rate. In a small strain configuration, the thaw depth is defined relative to the position of the surface. As the displacement of the surface is not accounted for in small strain consolidation, the small strain thaw penetration rate is thus always smaller than the actual thaw penetration rate. This divergence is expected to increase with increasing thaw strain. It is acknowledged that the large strain thaw penetration rate can be used in Morgenstern and Nixon (1971) theory. However, this is only possible if surface settlements are known prior to the analysis as Morgenstern and Nixon (1971) theory does not provide a direct assessment of deformations. For example, the large strain thaw penetration rate was used in the analysis of the experimental Inuvik warm-oil pipeline (Morgenstern and Nixon 1975).

The model computed excess pore water pressure isochrones are slightly skewed compared to the small strain theory isochrones. This is due to the non-uniform hydraulic conductivity profile in the thawed layer. Indeed, the excess pore water pressure gradient is larger near the top of the sample where the hydraulic conductivity is smaller. As a result, the relative difference between the excess pore water pressures predicted by small strain theory and the ones computed by the proposed model is larger in the upper part of the sample and smaller near the thaw front. These results which are typical of small strain consolidation theory indicate that Morgenstern and Nixon (1971) theory may underestimate the excess pore water

pressures which may lead to an overestimation of the effective stress not only at the thaw front but in the whole thawed layer.

Finally, the excess pore water pressure profile predicted by Morgenstern and Nixon (1971) theory is constant with time. This is due to the thaw consolidation ratio approach which considers a constant rate of thaw penetration α_T and a constant and uniform coefficient of consolidation c_v . However, the conditions at the thaw front are controlled by the velocity of the thaw front $dZ(t)/dt$ which decreases with time whereas the rate of thaw penetration α_T is constant. Also, the hydraulic conductivity of the thawed layer varies with time as consolidation proceeds notwithstanding its spatial variation. The thaw consolidation ratio should change during thaw consolidation.

Discussion

In addition to the advantages provided by the large strain configuration discussed in the previous section, the model presented herein proposes two main improvements for the assessment of thaw consolidation: the implementation into a large strain configuration of both the consolidation and the heat transfer components and the characterization of the thawing soil properties by nonlinear effective stress–void ratio–hydraulic conductivity relationships.

The application of large strain consolidation theory to thaw consolidation was first proposed by Foriero and Ladanyi (1995). However, the innovation of the proposed model lies in the direct integration of heat transfer into the coupled large strain modelling scheme. The model is thus capable of considering the impact of surface settlements on the movement of the upper boundary conditions for consolidation and heat transfer concurrently. The working example demonstrated that for ice-rich soils undergoing large thaw strain this would affect the evaluation of thaw penetration and of the length of the drainage path even for simple thaw consolidation problems.

The model's most important contribution is, however, the comprehensive description of the thawed soil properties. The use of physically meaningful relationships relieves the ambiguity inherent of previous thaw consolidation theories which sometimes required subjective determination of the soil properties.

Nixon (1973) proposed that the coefficient of consolidation determined during the post-thaw consolidation phase could be used to evaluate the consolidation behaviour during the thaw consolidation phase. However, this assumption is based on Terzaghi's small strain consolidation theory and on the evaluation of the stress state when thaw depth reaches the bottom of a frozen soil. The post-thaw coefficient of consolidation is thus dependent on the specific loading condition and thaw penetration rate combination under which it is experimentally determined and it is unclear how it can be generalized to various field conditions. In contrast, the void ratio dependent soil properties used in the proposed framework can effectively model both the thaw consolidation and post-thaw consolidation phases and their determination is independent of the experimental thermal conditions (Smith 1972). In practice, the added benefit is that the effective stress–void ratio–hydraulic conductivity relationships determined for a thawed soil can be used with any thaw penetration rate and applied loading combination anticipated in the field.

Furthermore, it is convenient to mention that the effective stress–void ratio–hydraulic conductivity relationships can be derived from limited testing. The properties of thawed Athabasca Clay presented in Figure 5 were determined from only three samples (Smith 1972). Only one sample would have been sufficient as the same thawed sample can be tested under multiple loads (Smith 1972). Limiting the number of samples is particularly beneficial in the case of permafrost soils since retrieving intact permafrost frozen core samples constitutes a significant challenge and a costly endeavour especially in remote areas. Likewise, the relationships could be established from remoulded permafrost samples (McRoberts and Morgenstern 1974). On the other hand, the evaluation of the coefficient of consolidation under different conditions requires either multiple intact frozen samples (Nixon and Morgenstern 1974) or refreezing the thawed sample (Morgenstern and Smith 1973) which could affect the properties of the soil.

Finally, Morgenstern and Nixon (1971) assessment of thaw consolidation is primarily based on the assumption that the coefficient of consolidation is constant and homogeneous in the thawed soil layer. Yet, determination of the proper coefficient of consolidation value applicable to thaw consolidation analysis is highly subjective due to the absence of a generalized characterization method. Morgenstern and Nixon (1975) defined the coefficient of consolidation from the ultimate average effective stress for the analysis of a pipeline built on permafrost. However, the ultimate effective stress state which is established only at the

very end of the post-thaw consolidation phase may not be representative of the whole thaw consolidation process. Indeed, the void ratio isochrones presented in Figure 7 indicate that the properties of the soil vary both as a function of depth and time. The spatial variation is expected to be even more significant in the field where the thickness of the thawed layer is much larger than in the laboratory. The proposed framework is supported by void ratio dependent constitutive relationships which ensure a continuous description of the soil properties. Therefore, no subjective assessment needs to be made prior to the analysis of thaw consolidation.

The nonlinear effective stress–void ratio–hydraulic conductivity relationships for thawed fine-grained soils are defined in the proposed framework as a function of the initial state of the thawed soil and of the slope parameters C_{cT} and C_{kT} . The characteristics of these parameters will be analyzed in detail in a subsequent paper. However, early evidences indicate that the residual stress and hydraulic conductivity values are intrinsic properties of the soil that otherwise depend only on the frozen void ratio (Smith 1972). They also appear to be independent of thermal and stress history (Nixon and Morgenstern 1974). Furthermore, the slope parameters C_{cT} and C_{kT} which changes with initial thawed void ratio can be predicted from the index properties of thawed soils (Dumais and Konrad 2016).

Conclusion

A framework for the evaluation of the consolidation of saturated thawing soils has been proposed based on a one-dimensional model formulated by coupling large strain consolidation and heat transfer into a Lagrangian moving boundary scheme. The model considers nonlinear effective stress–void ratio–hydraulic conductivity relationships. The mathematical formulation of the critical impervious boundary conditions at the thaw front has been revised to integrate the nonlinearity of the soil properties and the deformations due to phase change.

The model has been applied to the experimental case of a thawing ice-rich fine-grained soil sample which showed that:

- (1) The movement of the heat source and the reduction of the length of the drainage path due to surface settlement is duly accounted for in the proposed large-strain configuration.

- (2) The hydro-mechanical behaviour is effectively modelled from nonlinear effective stress–void ratio–hydraulic conductivity relationships characterized from the initial thawed state of the soil by the residual stress and its hydraulic conductivity.
- (3) Both the thaw consolidation and post-thaw consolidation phases can be modelled successively using a consistent model configuration from the same set of effective stress–void ratio–hydraulic conductivity relationships.

A comparison with conventional small strain thaw consolidation theory showed that:

- (1) Small strain thaw consolidation theory has the potential to underestimate excess pore water pressures in the thawed layer mainly due to the underestimation of the thaw penetration rate inherent to the small strain configuration.
- (2) The large strain configuration provides a more physically coherent representation of thaw consolidation.
- (3) The evolution and the interdependence of the soil parameters as thaw consolidation proceeds is handled seamlessly by the proposed model.

The practical implication of the proposed model is that the assessment of thaw consolidation is facilitated by the comprehensive and objective characterization of the properties of thawing soils. The effective stress–void ratio–hydraulic conductivity relationships can be determined experimentally and applied to a wide range of field conditions for the calculation of the thaw strain and excess pore water pressures.

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Figure Captions

- Figure 1: One-dimensional large strain thaw consolidation in Lagrangian coordinates
Figure 2: Coordinate systems for consolidation of thawing soil (adapted from Gibson et al. (1981))
Figure 3: Change in void ratio during thaw consolidation (adapted from Gibson et al. (1981))
Figure 4: Nonlinear effective stress–void ratio–hydraulic conductivity relationships for thawing soils
Figure 5: Properties of thawed Athabasca Clay (adapted from Smith (1972))
Figure 6: Temperature isochrones during the thaw consolidation phase
Figure 7: Void ratio and hydraulic conductivity isochrones in thawed layer
Figure 8: Thaw consolidation in (a) large strain and (b) small strain configurations
Figure 9: (a) Thaw depth and surface settlement progression and (b) excess pore water pressure isochrones in thawed layer
Figure 10: Excess pore water pressures in thawed layer in large strain configuration

Draft

Figure 1.

69x60mm (200 x 200 DPI)

Figure 2.

74x53mm (200 x 200 DPI)

Figure 3.

81x38mm (200 x 200 DPI)

Figure 4.

85x68mm (200 x 200 DPI)

Figure 5.
85x73mm (200 x 200 DPI)

Figure 6.

85x90mm (200 x 200 DPI)

Figure 7.

85x100mm (200 x 200 DPI)

Figure 8.

86x70mm (198 x 198 DPI)

Figure 9.

181x100mm (200 x 200 DPI)

Figure 10.

85x100mm (200 x 200 DPI)