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Strength behavior of plain cement concrete subjected to true-triaxial Compression

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Abstract

True-triaxial compression test was conducted on 100 mm x 100 mm x 100 mm cubical sample of plain cement concrete using the true-triaxial testing machine. A total of 30 tests were conducted on two mix of concrete having characteristic strengths of 24.39 MPa and 34.67 MPa. The minor and intermediate principal stresses were varied from 2.5 MPa to 10 MPa and 2.5 MPa to 30 MPa respectively. The samples were loaded till failure under the stress rate of 1 MPa/min. The failure mode and direction of fracture planes were observed carefully. It was observed that the ratio of intermediate and minor principal stress had an incremental effect on the strength of the concrete. The results obtained from our experimental study as well as the true-triaxial compression test data published in literature has been used to conduct a quantitative evaluation of three commonly used failure criteria presented for the concrete. The three failure criteria considered are Drucker-Prager (DP) criterion, Bresler-Pester (BP) criterion and Hsieh-Ting-Chen (HTC) criterion. It was observed that for the normal strength concrete, the HTC criterion predicts the strength with least error while for the high strength concrete, the DP criterion have a better prediction. Overall the DP and BP criterion underestimates the strength while the HTC criterion overestimates.

Keyword: True-triaxial compression test, Plain cement concrete, Intermediate principal stress, Hsieh-Ting-Chen criterion, Drucker-Prager criterion

1.0 Introduction

The constitutive behavior and relationship of the plain cement concrete (PCC) must be understood before modeling the structural response of any concrete structure. The behavior of concrete varies with the state of confinement and hence the constitutive relation also changes. Concrete structure such as the shear wall, spiral columns, nodes of the building, pressure container of the nuclear reactors are some of the cases, where concrete in general remains in state of confinements (He and Song 2010). The confinement states in mentioned structures are generally uniaxial ($\sigma_1 > \sigma_2 = \sigma_3 = 0$), biaxial ($\sigma_1 \geq \sigma_2 > \sigma_3 = 0$) and multiaxial state of stress which include both triaxial compression ($\sigma_1 > \sigma_2 = \sigma_3 > 0$) and true-triaxial compression ($\sigma_1 > \sigma_2 > \sigma_3 > 0$) conditions. The indices σ_1 , σ_2 , and σ_3 represent the major, intermediate and minor principal stresses respectively. The mechanical response of the concrete depends upon the state of stress and hence intermediate principal stress influences the constitutive response. Experimental testing of concrete under the various condition of confinement helps in establishing fundamental characteristic of concrete. Analytical models are developed to describe these fundamental characteristics which were further refined and calibrated using experimental data.

The behavior of concrete under the multiaxial state of stresses has been experimentally studied for last 7 decades. The pioneering work are those of Balmar (1949), Akroyd (1961), Chin and Zimmerman (1965), Kupfer et al. (1969), Mills and Zimmerman (1970), Lounay and Gachon (1972), Kotsovos and Newman (1978), Wastiels (1979), Gerstle et al. (1980), Kosaka et al. (1984), Van Mier (1986), Michelis (1985), Wang et al. (1987), Smith et al. (1989), Imran and Pantazopoulou (1996), Li and Ansari (1999), Sfer et al. (2002), Ahmad and Ajarloo (2005), Petkovski et al. (2006), Hampel et al. (2009), Gabet et al. (2008), Vu et al. (2009 a and b), Liu and Song (2010), Malecot et al. (2010), Vu et al. (2011), Petkovski (2013), Shang and Li (2014), Zhou et al. (2014) and Vu et al. (2015). However, most of the

researchers have conducted testing under the triaxial conditions only on cylindrical or cubical samples. Most of these studies are also conducted at very moderate confining pressure. Gabet et al. (2008), Vu et al. (2009 a and b), Vu et al. (2011) have conducted triaxial compression test on concrete using GIGA press facility in Grenoble which has cell pressure capacity of upto 1 GPa. Testing under true-triaxial conditions is very rare. Very few researchers attempted true-triaxial testing using their self-designed machines which can apply all the three principal stresses independently. True-triaxial test on concrete was attempted by some researchers (Mills and Zimmerman 1970; Van Mier 1986; Wang et al. 1987; Hampel et al. 2009; He and Song 2010; Liu and Song 2010; Shang and Li 2014; Zhou et al. 2014). Petkovski et al. (2006) in university of Sheffield have developed an apparatus for testing concrete under true-triaxial compression at elevated temperature. He along with his colleagues studied the effect of different heat-load regimes on the strains under transient hygro-thermal states, stress-strain behaviour and damage evolution in concrete subjected to true-triaxial compression (Petkovski and Crouch 2008; Petkovski 2010 and Petkovski 2013). He and Song (2010) also studied the behaviour of high strength and high-performance concrete under true-triaxial compression before and after exposure to high temperatures.

Several failure criteria have been presented by various researchers to model the behavior of concrete under the multiaxial state of stress. Most of these criteria had been developed with the basics of the failure theories developed for the metals. These theories have been modified for the concrete using biaxial and multiaxial experimental data as well as considering the various shape of failure surface in the deviatoric plane. These failure criteria have been categorized based on the number of parameters included in the same. **Table 1** presents a list of commonly used concrete failure criteria under the multiaxial state of stress.

Based on the literature review, it has been felt that the laboratory studies on the true-triaxial behavior of concrete are very few. To have better and more realistic constitutive

relationships, more studies are the need of the hour. A quantitative evaluation of strength criteria is also needed. The present article discusses in detail, an experimental study wherein concrete of two different characteristic strengths have been tested under the true-triaxial state of compression. The strength behavior of plain cement concrete has been studied. The result of present experimental study along with the results published in literature has been used to statistically evaluate the three classical multiaxial strength criteria developed for the concrete.

2.0 Experimental program

2.1 Materials

Ordinary Portland cement (OPC) conforming to IS: 8112-1989 (2005) was used as the binder. **Table 2** presents the physical properties of OPC. Locally available river sand having fineness modulus in the range of 2.6 to 2.9 conforming to IS: 383-1970 (2002) was used as the fine aggregate. Aggregate in its natural form (uncrushed river gravel) having the maximum size of 12.5 mm was used as the coarse aggregate. **Table 3** presents the physical properties of the coarse and fine aggregates. The Impact value and aggregate crushing value as presented in Table 3 were obtained as per procedure given in IS: 2386 (part IV)-1963 (1999). Potable water was used for mixing of constituents. Two types of concrete having 28-day target strength of 20 MPa and 30 MPa respectively were designed. These mixes are identified as M20 and M30 concrete respectively. The concretes were designed using the absolute volume method as per IS 10262-2009 (2009). The mix proportions of the M20 and M30 Concrete have been presented in **Table 4**. The constituent materials were mixed thoroughly. The concrete cubes were cast in steel molds which were vibrated well to ensure uniformity. The samples were cured for 28 days.

Cubical samples having the nominal dimension of 100 mm x 100 mm x 100 mm were prepared and used for the true-triaxial tests and the uniaxial tests. The samples were carefully grounded and lapped on the entire surface and their orthogonality was checked.

2.2 Testing apparatus

The testing was conducted in the true-triaxial testing machine developed by the Wille-Geotechnik, Germany. The machine consists of four different units as shown in **Fig. 1**. The units are the sample holder, the power-pack, pressure intensifiers and the workstation. The sample holder has three pairs of piston system attached with it from X, Y and Z directions. The stress on the concrete is applied with the help of pressure or loading platen having the dimension of 94 mm x 94 mm. The loading platens are attached with the sample holder's piston system. **Fig. 2** shows the closer view of sample holder, the pressure platen and the piston system. The power-pack generates the initial pressure which gets intensified in the intensifiers. The intensifier maintains the pressure in the sample holder's piston system. The maximum stress that can be applied by the machine on the concrete samples from any direction is 140 MPa. The LVDT associated with the loading platen measures the displacement in all the six directions. The displacement measured has been used to calculate the strain in the samples. The complete system is fully automated and controlled by the GEOsys software. The testing is controlled and coordinated with the help of the workstation.

2.3 Test program

The tests have been conducted for compression only. Five uniaxial compression strength tests have been conducted on the cubical samples of the same size for each concrete mix. The average value of uniaxial compressive strength has been designated as characteristics strength (f_c) of concrete mix.

A total of thirty true-triaxial tests have been conducted on cubical sample plain cement concrete samples. Twenty tests were conducted on M20 mix while ten tests were conducted

on M30 mix. For M20 mix, tests have been conducted at four different minor principal stresses (σ_3) of 2.5 MPa, 5 MPa, 7.5 MPa, and 10 MPa. The intermediate principal stress (σ_2) had been kept such that the ratio between intermediate and minor principal stress was 1, 1.5, 2, 2.5 and 3. For M30 mix, σ_3 was kept at 5, 7.5 and 10 MPa while σ_2 was kept such that the ratio of σ_2 and σ_3 was 1, 2, and 3. One test was conducted at σ_3 of 10 MPa and σ_2 of 15 MPa respectively. The results of experimental study on M20 concrete can be also found in Sajwan et al. (2016) where in the concrete has been considered as artificial rocks.

2.4 Testing procedure

The friction between the concrete surface and the loading platens affect the characteristic strength of concrete (Kupfer et al. 1969). A number of friction reducing measures have been discussed by Launay and Gachon (1972). For the present study, two layers of 0.25 mm thick Teflon sheet sandwiching a uniform layer of grease have been used as a friction reducing agent. The concrete sample wrapped with friction reducing agent was placed in the sample holder and the loading platens were touched to it. An initial seating stress of 1 MPa was applied in all the three directions to make sure that the loading platens were in proper contact with the sample. The stress in all the three directions was then hydrostatically increased at a constant rate of 1 MPa/min till the all-around stress equal to σ_3 was reached. Now the stress in the X direction was maintained at the σ_3 value while stress from the Y and Z direction were further increased at the same rate till it reaches σ_2 value. Now the stress from Y direction was maintained at σ_2 value and the stress from Z direction was increased at the same rate till the failure.

Since the testing was conducted at stress controlled conditions, the post-peak behavior of concrete was not obtained. A sudden shooting of deformation in major principal stress direction was observed which indicated the failure of the sample. Once the failure of the

sample was observed, the testing was stopped and the sample was carefully taken out from the sample holder and the details of the failure mode were noted.

3.0 Experimental results

3.1 Uniaxial compression test results

Five uniaxial compression tests were conducted for both M20 and M30 mix. The samples were of same size i.e. 100 mm x 100 mm x 100 mm. The tests were conducted at the same stress rate of 1 MPa/min. The average compressive strength observed for M20 mix was 24.39 MPa, and for M30 mix it was 34.67 MPa. The average compressive strength of the mixes has been designated as the characteristic strength (f_c) of concrete mix.

3.2 True-triaxial compression test results

Fig. 3 shows the typical failed sample and failure pattern for concrete under the true-triaxial compression testing. It was observed that many cracks have developed throughout the sample. The cracks propagated through the aggregate-mortar interface or through the mortar alone. The fracture plane was observed to have strike almost aligned with the plane of the intermediate principal stress (σ_2). The dip of the fracture plane was oriented toward the direction of the minor principal stress (σ_3). Similar failure pattern was also observed by previous researchers (He and Song 2010; Shang and Li 2014; Zhou et al. 2014).

In this study, we have assumed a positive sign convention for compression and negative sign convention for tension. The results of the true-triaxial tests in terms of the principal stresses (σ_1 , σ_2 , σ_3) at failure have been presented in **Table 5**. It was observed that, the strength of concrete also depends upon the intermediate principal stress. For the range of σ_2 and σ_3 considered, the strength (σ_1) increases with the increase in σ_2 . **Fig. 5 a and b** show the failure stress (or major principal stress) as a function of the ratio of the intermediate to the minor

193 principal stress, for both M20 and M30 mix. The increasing trend of strength is very clear.
 194 Few typical stress-strain plots for the true-triaxial testing are presented in **Fig. 4**.

195 **4.0 True-triaxial strength database for plain cement concrete**

196 An attempt has been made to collect all the available experimental strength data of the
 197 plain cement concrete subjected to true-triaxial states of stress. The authors have been able to
 198 identify fourteen such datasets from literature. **Table 6 a and b** shows the detail of the true-
 199 triaxial tests database conducted on plain concrete. The database consists of number of test
 200 data, characteristic strength (f_c), range of minor principal stress, intermediate principal stress
 201 and major principal stress for all data set. The concrete classified into two categories namely
 202 the Normal Strength Concrete (NSC) having the characteristic strength less the 60 MPa and
 203 the High Strength Concrete (HSC) having the characteristic strength greater than 60 MPa and
 204 accordingly mentioned in **Table 6 a and b**. The complete detail of the strength database is
 205 presented in Appendix-I.

206 **5.0 Multiaxial strength criteria for concrete**

207 Strength criteria or failure models are developed to predict the strength of concrete under
 208 various state of stress. A number of strength criteria were developed in the past. A list of
 209 major multiaxial strength criteria is presented in the introduction section of this article. Most
 210 of these criteria were developed considering the shape of the failure surface in the deviatoric
 211 plane. The parameters associated with these criteria are calculated on the basis of some
 212 standard laboratory test conducted on the same concrete mix. The standard laboratory tests
 213 conducted for the calculation of criterion parameters are uniaxial tension tests, uniaxial
 214 compression tests, equal biaxial compression tests and triaxial compression tests in tensile and
 215 compression meridians. However, for the most of the projects in India, only the uniaxial
 216 compression tests are conducted on the cubical sample of concrete. The standard correlation

between strength obtained from other standard laboratory test mentioned earlier and uniaxial compression test was used to calculate the parameters associated with the strength criterion. A strength criterion can be called robust if it can predict the strength under true-triaxial conditions using the parameters obtained from the conventional technique with higher accuracy.

Three failure criteria were selected and their predictive capabilities with respect to the database generated have been evaluated. The three strength criteria considered for the present study are the Drucker-Prager (DP) criterion (Drucker-Prager 1952), the Bresler-Pester (BP) criterion (Bresler-Pester 1958) and the Hsieh-Ting-Chen (HTC) criterion (Hsieh et al. 1982). These criteria are classified as two parameter, three parameter and four parameter criteria respectively. These criteria are chosen as they are relatively simple and most commonly used for structural analysis. The brief detail and the calculation of the value of parameters associated with these criteria are given hereunder.

5.1 Drucker-Prager (DP) criterion

Drucker and Prager (1952) presented a modification to the classical von-Mises criterion which can be said to be a smooth approximation of the Mohr-Coulomb hexagonal surface.

The criterion can be presented as:

$$f(I_1, J_2) = 0 \quad (1)$$

$$\text{or, } \alpha I_1 + \sqrt{J_2} - k = 0 \quad (2)$$

where, α and k are material constants or criterion parameters.

I_1 and J_2 are stress invariant defined as:

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (3)$$

$$J_2 = \frac{1}{6} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2] \quad (4)$$

where, σ_1 , σ_2 and σ_3 are major, intermediate and minor principal stress.

The material constant or the criterion parameters associated with the strength criteria are generally obtained by considering two failure states namely:

$$\text{Uniaxial compressive strength: } \sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (5)$$

$$\text{Equal biaxial strength: } \sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (6)$$

Considering the standard correlation between equal biaxial strength and uniaxial compressive strength as:

$$f_{bc} = 1.15 \cdot f_c \quad (7)$$

The value for the strength parameters are obtained as:

$$\alpha = -0.0666 \quad (8)$$

$$\frac{k}{f_c} = 0.5107 \quad (9)$$

5.2 Bresler-Pester (BP) criterion

Bresler and Pester (1958) presented a new strength criterion of the form:

$$f(\tau_{oct}, \sigma_{oct}, f_c) = 0 \quad (10)$$

$$\text{or, } \frac{\tau_{oct}}{f_c} = a - b \cdot \frac{\sigma_{oct}}{f_c} + c \cdot \left(\frac{\sigma_{oct}}{f_c} \right)^2 \quad (11)$$

where, a , b and c are material constants or criterion parameters

σ_{oct} and τ_{oct} are octahedral normal and shear stress defined as:

$$\sigma_{oct} = \frac{I_1}{3} \quad (12)$$

$$\text{and, } \tau_{oct} = \sqrt{\frac{2}{3} J_2} \quad (13)$$

The material constant or the criterion parameters associated with the strength criteria are generally obtained by considering three failure states namely:

$$\text{Uniaxial tensile strength: } \sigma_1 = \sigma_2 = 0; \sigma_3 = -f_t \quad (14)$$

$$\text{Uniaxial compressive strength: } \sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (15)$$

$$\text{Equal Biaxial strength: } \sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (16)$$

Considering the standard correlation between equal biaxial strength, uniaxial tensile strength and uniaxial compressive strength as:

$$f_{bc} = 1.15 \cdot f_c \quad (17)$$

$$\text{and, } f_t = 0.10 \cdot f_c \quad (18)$$

The value for the strength parameters are obtained as:

$$a = 0.099 \quad (19)$$

$$b = -1.532 \quad (20)$$

$$c = -1.245 \quad (21)$$

5.3 Hsieh-Ting-Chen (HTC) criterion

Hsieh et al. (1982) presented a four-parameter criterion for concrete in the terms of stress invariant as:

$$f(I_1, J_2, \sigma_1) = 0 \quad (22)$$

$$\text{or, } a \frac{J_2}{f_c^2} + b \frac{\sqrt{J_2}}{f_c} + c \frac{\sigma_1}{f_c} + d \frac{I_1}{f_c} = 1 \quad (23)$$

where, a , b , c and d are material constant or criterion parameters.

The material constant or the criterion parameters associated with the criterion are generally obtained by considering the three failure states namely:

$$\text{Uniaxial tensile strength: } \sigma_1 = \sigma_2 = 0; \sigma_3 = -f_t \quad (24)$$

$$\text{Uniaxial compressive strength: } \sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (25)$$

$$\text{Equal Biaxial strength: } \sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (26)$$

$$\text{Confined triaxial compression strength: } \sigma_2 = \sigma_3 = f_{pc}; \sigma_1 = f_{cc} \quad (27)$$

Considering the standard correlation between equal biaxial strength, uniaxial tensile strength, confined compression strength and uniaxial compressive strength as:

$$f_{bc} = 1.15 \cdot f_c \quad (28)$$

$$f_t = 0.10 \cdot f_c \quad (29)$$

$$f_{pc} = 0.8 \cdot f_c \quad (30)$$

$$f_{cc} = 4.2 \cdot f_c \quad (31)$$

The value for the strength parameters can be obtained as:

$$a = 2.002 \quad (32)$$

$$b = 16.806 \quad (33)$$

$$c = -9.128 \quad (34)$$

$$d = -0.236 \quad (36)$$

The calculation for criterion parameter has been presented in appendix-II. Similar method for calculation of parameters associated with strength criterion was discussed by some previous researchers (Chen 2007; Labbane et al. 1993). The values of the parameters associated with criteria presented above will be mentioned as the model value of the parameters hereafter for further reference in this article.

5.4 Analysis method

Two different types of procedure for analysis have been considered in the present study. The first procedure compares the value of parameters associated with these criteria. The value of parameters obtained from the true-triaxial experimental study has been compared to the model values as obtained in the previous section. This analysis has been conducted for DP and BP criteria only as these can be represented easily in two dimensional space. The experimental data are plotted on the respective two dimensional spaces to obtain the experimental value of criterion parameters. A best fit line in the $\sqrt{J_2} - I_1$ space for the DP criterion and $\frac{\tau_{oct}}{f_c} - \frac{\sigma_{oct}}{f_c}$ space for the BP criterion can easily give the criterion parameters.

The second procedure calculates the value of major principal stress, σ_I considering the model value of parameters. The error between the calculated and the experimental value of σ_I has been used to determine the predictive capability of the strength criteria. A MATLAB program has been developed to calculate the value of σ_I . The predictive capability of the

criteria was determined using three indices namely root mean square error (RMSE), coefficient of accordance (COA) and coefficient of determination (R^2).

Root mean square error (RMSE) was calculated as:

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^n (\sigma_{1,i}^{cal} - \sigma_{1,i}^{exp})^2} \quad (37)$$

where $\sigma_{1,i}^{cal}$ and $\sigma_{1,i}^{exp}$ are the calculated and the experimental value of the major principal stress for i^{th} data respectively and 'n' is the number of data points.

Coefficient of accordance (COA) was calculated as

$$\psi^2(COA) = \frac{\sum (\sigma_{1,i}^{exp} - \sigma_{1,i}^{cal})^2}{\sum (\sigma_{1,i}^{exp} - \sigma_1^{avg})^2} \quad (38)$$

where σ_1^{avg} is average of experimental value of the major principal stress. RMSE and ψ^2 are the measure of goodness of prediction. The lower value of the RMSE and ψ^2 represents a better prediction. The coefficient of determination (R^2) was used to assess the scatter between the experimental and calculated value of σ_1 relative to the best fit line.

6.0 Analysis results

Figure 6 (a) to (d) show the plot for experimental results obtained for M20 and M30 mix

in $\sqrt{J_2} - I_1$ space for the DP criterion and $\frac{\tau_{oct}}{f_c} - \frac{\sigma_{oct}}{f_c}$ space for BP criterion. The equation for

the best fitting straight line for DP criterion and second order curve for BP criterion has been also shown. The experimental values of the parameters have been obtained from best fitting equation for the complete database and presented as **Table 7 a and b**. The absolute error and RMSE in prediction of the parameters is presented in the **Table 8 a and b**. For normal strength concrete, the RMSE in parameters associated with the DP criterion are 0.141 for α and 0.157 for k/f_c . For high strength concrete, the RMSE in parameters associated with DP criterion are 0.093 for α and 0.036 for k/f_c . **Fig. 7 a and b** shows the plot of the variation of

the DP criterion parameter with respect to characteristic strength. A significant scatter can be observed clearly. Similarly, for normal strength concrete, the RMSE in parameters associated with the BP criterion are 0.193 for a, 0.798 for b and 1.126 for c. For high strength concrete, the RMSE in parameters associated with the BP criterion are 0.194 for a, 0.809 for b and 1.084 for c. **Fig. 8 a, b and c** show the plot of the variation of the parameters with respect to the characteristic strength of concrete for the BP criterion.

Table 9 a and b shows the value of the various predictability indices obtained for all the sixteen data set considered in the present study using the second approach. **Fig. 9** shows the plot of experimental to calculated value of the major principal stress obtained from the three criteria for both High Strength Concrete (HSC) and Normal Strength Concrete (NSC) concrete. It can be clearly seen from **Table 9 a**, that for NSC, overall HTC criterion shows the least error having RMSE of 19.345 and COA of 0.289 followed by DP criterion having RMSE of 33.837 and COA of 0.884. For HSC, The DP criterion shows the least error with RMSE of 59.283 and COA of 1.142 followed by BP criterion having RMSE of 62.314 and COA of 1.262. **Fig. 9** shows that the DP and BP criteria underestimate the major principal stress while the HTC criterion overestimates.

Concluding remarks

The paper discusses in details, the aspects related to the true-triaxial strength of plain cement concrete. A comprehensive true-triaxial testing program was conducted on two different mixes of concrete. Thirty tests were conducted and the result of experimental study along with that of published articles in literature have been used to quantitatively evaluate the three most commonly used strength criteria for concrete. Two different approaches were used to evaluate the criteria.

From the study, it was observed that the model value of the parameters obtained using standard correlations and failure states shows significant error compared to that obtained by

fitting best fit line in DP and BP criteria space. This again significantly affects the overall predictive capability of the criteria. For the database considered in present study, it was also observed that for NSC, HTC criterion with model value of parameters shows the least error while for HSC, DP criterion shows the least error. Overall, BP and DP criterions underestimate while the HTC criterion overestimates.

Finally, it can be concluded from this study that very high error is associated with these criteria when used with the model values of the parameters. The experimental value of the parameters must be supplied to these criteria for better accuracy when used for numerical simulations. Thus, outlining that engineers must not rely only on uniaxial compressive strength/characteristic strength of concrete for designing and analyzing any concrete structure.

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Figure 1. True-triaxial Testing Machine at IIT Roorkee

Figure 2. Sample Holder

Figure 3. Typical Failed Specimen and failure mode in concrete for the polyaxial test

Figure 4. Typical stress-strain plot for the true-triaxial test (Values in the box are in order of : $\sigma_1, \sigma_2, \sigma_3$)

(a) For M20 Mix

(b) For M30 Mix

Figure 5. Plot for Major Principal stress (σ_1) vs. ratio of intermediate to minor principal stress (σ_2/σ_3)

(a) For M20 Mix (BP)

(b) For M30 Mix (BP)

(c) For M20 Mix (DP)

(d) For M30 Mix (DP)

Figure 6. Plot for M20 and M30 Mix data in the BP and DP criterion space

(a)

(b)

Figure 7. Plot for experimental value and model value of DP criterion parameter

Figure 8. Plot for experimental value and model value of BP criterion parameter

(a) Bresler-Pester Criterion for NSC

(b) Bresler-Pester Criterion for HSC

(c) Drucker-Prager Criterion for NSC

(d) Drucker-Prager Criterion for HSC

(e) Hsieh-Ting-Chen criterion for NSC

(f) Hsieh-Ting-Chen criterion for HSC

Figure 9. Plot for calculated value and experimental values of major principal stress for all criteria

Table 1: List of commonly used failure criterion for concrete

Sl. No.	Classification of criteria	Name of criteria
1	Single Parameter criteria	Rankine
		Tresca
		Von Mises
2	Double Parameter criteria	Mohr-Coulomb
		Drucker-Prager
3	Three Parameter criteria	Bresler-Pester
		William Warnke
4	Four Parameter Criteria	Ottosen
		Reimann
		Hsieh-Ting-Chen
5	Five Parameter Criteria	Extended Version of William Warnke

Table 2: Physical Property of Portland cement

Physical Property	IS 8112: 1989, 2005 requirement	Test results	Units
Blaine’s fineness	≥ 2250	2900	Cm ² /g
Relative Density	-	3.12	-
Soundness (Le Chatelier’s method)	≤ 10	3.14	mm
Autoclave expansion	≤ 0.80	0.12	%
Setting time			Min
Initial	≥ 30	54	
Final	≤ 600	215	
Compressive strength			MPa
3-days	≥ 23	24.87	
7-days	≥ 33	36.92	
28-days	≥ 43	45.76	

Table 3: Physical Property of course aggregate and Fine aggregate

Physical Property	Course Aggregate	Fine Aggregate
Maximum Size (mm)	12.5	4.75
Relative Density	2.64	2.67
Water Absorption (%)	1.10	1.2
Fineness Modulus	6.35	2.65
Impact value (%)	15.9	-
Crushing Value (%)	21.2	-

Draft

Table 4: Mix proportion for concrete

Mix ID	Ingredient (kg/m ³)			
	Cement	Course Aggregate	Fine Aggregate	Water
M20	336	890	946	221
M30	380	935	894	207

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Table 5: Results of true triaxial testing

Mix ID	σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	I_1 (MPa)	$\sqrt{J_2}$ (MPa)
M30	57.95	5	5	67.95	30.57
	68.31	10	5	83.31	35.20
	71.42	15	5	91.42	35.81
	65.56	7.5	7.5	80.56	33.52
	82.79	15	7.5	105.29	41.47
	97.80	22.5	7.5	127.8	48.39
	78.96	10	10	98.96	39.81
	86.82	15	10	111.82	42.98
	111.20	20	10	141.20	55.77
	119.67	30	10	159.67	58.41
M20	35.74	2.5	2.5	40.74	19.19
	37.62	3.75	2.5	43.87	19.93
	38.65	5.0	2.5	46.15	20.19
	39.77	6.25	2.5	48.52	20.52
	43.00	7.5	2.5	53	22.08
	47.92	5	5	57.92	24.78
	50.66	7.5	5	63.16	25.67
	54.28	10	5	69.28	27.12
	56.16	12.5	5	73.66	27.63
	48.43	15	5	76.48	27.30
	59.33	7.5	7.5	74.33	29.92
	65.08	11.25	7.5	83.83	32.22
	56.67	15	7.5	83.17	28.78
	60.67	18.75	7.5	89.07	29.24
	70.11	22.5	7.5	100.11	32.69
	73.82	10	10	93.82	36.85
	74.06	15	10	100.06	36.20
	72.33	20	10	104.33	34.62
	73.93	25	10	114.93	36.82
	89.91	30	10	129.91	41.58

Table 6 a: Available database of true-triaxial compression test conducted on plain concrete (Normal Strength Concrete, NSC)

Sl. No.	Reference	No. of test data	f_c (MPa)	Type	Minor Principal Stress (MPa)		Intermediate principal stress (MPa)		Major Principal stress (MPa)	
					Min	max	Min	max	Min	max
1	Liu and Song (2010)	11	20.00	NSC	0	47.00	0	176.17	20.00	188.01
2	Shang and Ji (2014)	5	34.20	NSC	0	14.48	0	112.12	34.20	141.82
3	Zhou et al. (2014) – I	16	44.13	NSC	0	14.96	0	132.42	44.13	149.45
4	Hampel et al. (2009) – I	10	46.09	NSC	0	34.57	0	176.99	46.09	231.83
5	Wang et al. (1987) – I	13	9.37	NSC	0	29.21	0	29.45	9.37	96.11
6	Wang et al. (1987) – II	15	10.74	NSC	0	18.47	0	45.11	10.74	91.30
7	Mills and Zimmerman (1970) – I	37	23.03	NSC	2.90	8.69	2.90	72.41	41.03	82.07
8	Mills and Zimmerman (1970) – II	16	26.93	NSC	0.62	7.24	1.45	62.76	41.52	71.24
9	Mills and Zimmerman (1970) - III	16	36.10	NSC	0.62	4.83	1.45	67.59	48.28	73.10
10	M20 (Present Study)	21	24.23	NSC	0	10	0	30	24.23	89.91
11	M30 (Present study)	11	34.67	NSC	0	10	0	30	34.67	119.67

Table 6 b: Available database of true-triaxial compression test conducted on plain concrete (High Strength Concrete, HSC)

Sl. No.	Reference	No. of test data	f_c (MPa)	Type	Minor Principal Stress (MPa)		Intermediate principal stress (MPa)		Major Principal stress (MPa)	
					Min	max	Min	max	Min	max
1	He and Song (2010)	7	60.16	HSC	0	15.77	0	131.64	60.16	157.74
2	Zhou et al. (2014) – II	16	66.43	HSC	0	20.99	0	190.26	66.43	209.73
3	Zhou et al. (2014) – III	16	90.64	HSC	0	27.10	0	238.11	90.64	270.92
4	Hampel et al. (2009) – II	10	72.73	HSC	0	40.00	0	226.92	72.73	268.37
5	Hampel et al. (2009) - III	9	83.44	HSC	0	44.22	0	268.68	83.44	292.87

**Table 7 a: Calculation of criterion parameters for experimental data present in literature
(Normal Strength concrete, NSC)**

Sl. No.	Reference	f_c (MPa)	Material constant				
			Bresler-Pester criterion			Drucker Prager criterion	
			a	b	c	a	k/f_c
1	Liu and Song (2010)	20.00	0.108	-0.82	-0.052	-0.185	0.654
2	Shang and Ji (2014)	34.20	-0.092	-1.864	-0.525	-0.203	0.469
3	Zhou et al. (2014)	44.13	0.353	-0.440	0.001	-0.170	0.527
4	Hampel et al. (2009) -I	46.09	0.231	-0.745	-0.084	-0.186	0.525
5	Wang et al. (1987) - I	9.37	0.251	-0.842	-0.049	-0.218	0.773
6	Wang et al. (1987) - II	10.74	0.326	-0.545	0.002	-0.226	0.385
7	Mills and Zimmerman (1970) - I	23.03	0.387	-0.635	-0.098	-0.146	0.688
8	Mills and Zimmerman (1970) – II	26.93	0.169	-1.177	-0.420	-0.144	0.642
9	Mills and Zimmerman (1970) – III	36.10	0.408	-0.564	-0.199	-0.095	0.651
10	M20 (Present Study)	24.23	0.208	-0.820	-0.092	-0.256	0.364
11	M30 (Present study)	34.67	0.243	-0.669	0.056	-0.317	0.243

**Table 7 b: Calculation of criterion parameters for experimental data present in literature
(High Strength concrete, HSC)**

Sl. No.	Reference	f_c (MPa)	Material constant				
			Bresler-Pester criterion			Drucker Prager criterion	
			a	b	c	a	k/f_c
1	He and Song (2010)	60.16	0.100	-1.295	-0.483	-0.145	0.562
2	Zhou et al. (2014) – II	66.43	0.375	-0.505	-0.042	-0.165	0.522
3	Zhou et al. (2014) – III	90.64	0.378	-0.498	-0.053	-0.155	0.535
4	Hampel et al. (2009) – II	72.73	0.207	-0.861	-0.170	-0.164	0.562
5	Hampel et al. (2009) - III	83.44	0.250	-0.733	-0.120	-0.167	0.529

**Table 8 a: Error in prediction of criterion parameters for experimental data present in literature
(Normal Strength concrete, NSC)**

Sl. No.	Reference	f_c (MPa)	Absolute error in calculation of material constant				
			Bresler-Pester criterion			Drucker Prager criterion	
			a	b	c	a	k/f_c
1	Liu and Song (2010)	20.00	0.009	0.712	1.193	0.119	0.143
2	Shang and Ji (2014)	34.20	0.191	0.332	0.721	0.137	0.042
3	Zhou et al. (2014)	44.13	0.254	1.092	1.245	0.103	0.0162
4	Hampel et al. (2009) -I	46.09	0.132	0.787	1.161	0.120	0.014
5	Wang et al. (1987) - I	9.37	0.152	0.690	1.196	0.151	0.263
6	Wang et al. (1987) - II	10.74	0.227	0.987	1.247	0.160	0.126
7	Mills and Zimmerman (1970) - I	23.03	0.288	0.897	1.147	0.080	0.178
8	Mills and Zimmerman (1970) – II	26.93	0.07	0.355	0.825	0.077	0.131
9	Mills and Zimmerman (1970) – III	36.10	0.309	0.968	1.046	0.028	0.141
10	M20 (Present Study)	24.23	0.109	0.712	1.153	0.190	0.147
11	M30 (Present study)	34.67	0.144	0.863	1.301	0.250	0.268
Overall RMSE			0.193	0.798	1.126	0.141	0.157

**Table 8 b: Error in prediction of criterion parameters obtained from experimental data
(High Strength concrete, HSC)**

Sl. No.	Reference	f_c (MPa)	Absolute error in calculation of criterion parameters				
			Bresler-Pester criterion			Drucker Prager criterion	
			α	b	c	α	k/f_c
1	He and Song (2010)	60.16	0.001	0.237	0.762	0.078	0.052
2	Zhou et al. (2014) – II	66.43	0.276	1.027	1.204	0.099	0.011
3	Zhou et al. (2014) – III	90.64	0.279	1.034	1.192	0.088	0.024
4	Hampel et al. (2009) – II	72.73	0.108	0.672	1.075	0.097	0.051
5	Hampel et al. (2009) - III	83.44	0.151	0.800	1.125	0.101	0.0184
Overall RMSE			0.194	0.809	1.084	0.093	0.036

Table 9 a: Predictability indices for various criteria for Normal Strength Criterion (NSC)

Reference	Predictability indices								
	Bresler-Pester criterion			Drucker Prager criterion			Hsieh-Ting-Chen criterion		
	RMSE	ψ^2	R^2	RMSE	ψ^2	R^2	RMSE	ψ^2	R^2
Liu and Song (2010)	63.603	1.186	0.424	62.838	1.157	0.421	12.517	0.046	0.959
Shang and Ji (2014)	51.439	1.729	0.244	47.331	1.464	0.342	20.570	0.277	0.906
Zhou et al. (2014)	35.583	1.564	0.256	35.263	1.536	0.427	37.515	1.739	0.649
Hampel et al. (2009) -I	63.231	1.379	0.388	58.034	1.162	0.518	39.805	0.547	0.805
Wang et al. (1987) - I	41.318	3.017	0.547	31.610	1.766	0.933	6.756	0.081	0.970
Wang et al. (1987) - II	36.620	2.083	0.223	36.026	1.958	0.237	5.431	0.046	0.958
Mills and Zimmerman (1970) - I	26.006	3.912	0.174	24.06	3.349	0.321	17.282	1.726	0.699
Mills and Zimmerman (1970) – II	19.069	2.668	0.075	20.01	2.937	0.074	18.046	2.389	0.377
Mills and Zimmerman (1970) – III	12.691	1.715	0.358	14.90	2.365	0.247	13.211	1.859	0.667
M20 (Present Study)	26.854	2.671	0.880	23.835	2.104	0.942	8.583	0.273	0.938
M30 (Present study)	36.002	2.426	0.601	35.982	2.423	0.589	6.590	0.081	0.947
overall	35.904	0.996	0.532	33.837	0.884	0.614	<u>19.345</u>	<u>0.289</u>	<u>0.865</u>

Table 9 b: Predictability indices for various criteria for High Strength Criterion (HSC)

Reference	Predictability indices								
	Bresler-Pester criterion			Drucker Prager criterion			Hsieh-Ting-Chen criterion		
	RMSE	ψ^2	R^2	RMSE	ψ^2	R^2	RMSE	ψ^2	R^2
He and Song (2010)	47.922	2.306	0.239	41.305	1.713	0.476	66.341	4.418	0.772
Zhou et al. (2014) – II	49.382	1.560	0.358	48.673	1.515	0.438	60.327	2.327	0.644
Zhou et al. (2014) – III	60.556	1.631	0.339	59.586	1.580	0.434	89.260	3.544	0.595
Hampel et al. (2009) – II	76.207	1.719	0.315	71.9	1.53	0.418	79.169	1.855	0.742
Hampel et al. (2009) - III	77.0123	1.531	0.361	71.09	1.304	0.468	77.991	1.570	0.475
overall	62.314	1.262	0.45	<u>59.283</u>	<u>1.142</u>	<u>0.543</u>	75.878	1.870	0.67

Appendix – A

True-triaxial strength Database for plain cement concrete

Table A1. He and Song (2010)

f_c	σ_3	σ_2	σ_1
60.16	0	0	60.16
60.16	12.03	12.03	120.28
60.16	14.36	38.78	143.63
60.16	15.34	64.43	153.4
60.16	15.77	82.03	157.74
60.16	15.25	117.45	152.53
60.16	13.16	131.64	131.64

Table A4a. Hampel et al. (2009)

f_c	σ_3	σ_2	σ_1
46.09	0	0	46.09
46.09	5.07	20.28	102.78
46.09	5.07	63.60	106.47
46.09	4.61	88.49	93.10
46.09	14.29	29.04	144.72
46.09	16.59	100.02	166.85
46.09	13.37	129.51	136.43
46.09	30.42	41.02	204.64
46.09	34.57	139.19	231.83
46.09	27.65	176.99	186.20

Table A2. Liu and Song (2010)

f_c	σ_3	σ_2	σ_1
20	0.00	0.00	20.00
20	5.24	5.24	52.43
20	7.36	18.40	73.60
20	7.18	21.53	71.77
20	7.02	35.10	70.20
20	6.21	46.60	62.13
20	6.17	61.71	61.71
20	37.34	37.34	149.35
20	47.00	56.40	188.01
20	46.82	93.64	187.28
20	44.04	176.17	176.17

Table A4b. Hampel et al. (2009)

f_c	σ_3	σ_2	σ_1
72.73	0	0	72.73
72.73	7.27	28.36	144.01
72.73	8.00	100.37	167.28
72.73	7.27	141.82	149.10
72.73	20.36	40.73	203.64
72.73	22.55	134.55	224.74
72.73	20.36	192.73	202.92
72.73	40.00	53.09	266.19
72.73	40.00	161.46	268.37
72.73	35.64	226.92	238.55

Table A3. Shang and Ji (2014)

f_c	σ_3	σ_2	σ_1
34.2	0.00	0.00	34.20
34.2	13.47	33.67	134.66
34.2	14.18	70.91	141.82
34.2	12.68	95.09	126.78
34.2	11.21	112.12	112.12

Table A4c. Hampel et al. (2009)

f_c	σ_3	σ_2	σ_1
83.44	0	0	83.44
83.44	8.34	33.38	167.71
83.44	8.34	102.63	171.89
83.44	7.51	148.52	156.87
83.44	22.53	45.89	228.63
83.44	23.36	141.01	235.30
83.44	20.86	201.09	211.94
83.44	44.22	58.41	292.87
83.44	42.55	268.68	282.86

Table A5a. Zhou et al. (2014)

f_c	σ_3	σ_2	σ_1
44.13	0.00	0.00	44.13
44.13	1.85	7.45	74.48
44.13	1.94	19.61	78.45
44.13	2.07	41.23	82.47
44.13	2.03	60.63	80.84
44.13	1.90	75.76	75.76
44.13	4.24	8.43	84.32
44.13	4.68	23.40	93.59
44.13	5.03	50.30	100.61
44.13	4.85	72.51	96.68
44.13	4.63	92.22	92.22
44.13	10.99	10.98	109.78
44.13	14.12	35.27	141.07
44.13	14.96	74.73	149.45
44.13	14.30	107.22	142.97
44.13	13.24	132.42	132.42
44.13	0.00	0.00	44.13
44.13	1.85	7.45	74.48
44.13	1.94	19.61	78.45
44.13	2.07	41.23	82.47

66.43	20.26	151.92	202.55
66.43	19.00	190.26	190.26
66.43	0.00	0.00	66.43
66.43	2.66	10.72	107.16
66.43	2.86	28.65	114.60
66.43	2.99	59.99	119.98

Table A5c. Zhou et al. (2014)

f_c	σ_3	σ_2	σ_1
90.64	0.00	0.00	90.64
90.64	3.63	14.52	145.21
90.64	3.90	38.59	154.36
90.64	3.99	79.08	158.17
90.64	3.81	113.80	151.73
90.64	3.63	146.29	146.29
90.64	8.16	16.37	163.70
90.64	8.88	44.62	178.47
90.64	9.70	96.94	193.88
90.64	9.34	140.38	187.17
90.64	8.70	174.48	174.48
90.64	20.76	20.76	207.57
90.64	25.47	63.61	254.43
90.64	27.10	135.46	270.92
90.64	25.56	191.77	255.70
90.64	23.84	238.11	238.11
90.64	0.00	0.00	90.64
90.64	3.63	14.52	145.21
90.64	3.90	38.59	154.36
90.64	3.99	79.08	158.17

Table A5b. Zhou et al. (2014)

f_c	σ_3	σ_2	σ_1
66.43	0.00	0.00	66.43
66.43	2.66	10.72	107.16
66.43	2.86	28.65	114.60
66.43	2.99	59.99	119.98
66.43	2.92	87.84	117.12
66.43	2.72	108.95	108.95
66.43	6.18	12.32	123.17
66.43	6.78	34.03	136.12
66.43	7.44	74.44	148.88
66.43	7.24	108.62	144.82
66.43	6.78	135.26	135.26
66.43	15.41	15.39	153.93
66.43	20.46	51.20	204.81
66.43	20.99	104.86	209.73

Table A6a. Wang et al. (1987)

f_c	σ_3	σ_2	σ_1
9.37	0.00	0.00	9.37
9.37	3.90	3.87	37.00
9.37	2.98	3.87	29.79
9.37	4.39	8.51	45.29
9.37	5.70	14.26	46.61
9.37	4.55	18.82	46.13
9.37	4.23	17.14	43.25
9.37	5.05	26.57	52.86
9.37	15.69	15.22	74.97
9.37	12.42	11.54	58.15
9.37	14.79	14.10	67.28
9.37	29.03	28.65	96.11
9.37	29.45	29.21	96.11

Table A6b. Wang et al. (1987)

f_c	σ_3	σ_2	σ_1
10.74	0.00	0.00	10.74
10.74	5.05	9.46	45.17
10.74	4.88	14.26	49.02
10.74	4.72	23.85	47.57
10.74	4.72	36.00	48.05
10.74	4.23	32.64	43.25
10.74	3.90	34.08	38.44
10.74	3.74	37.12	38.44
10.74	3.74	38.40	38.44
10.74	18.47	27.37	91.30
10.74	17.49	25.93	86.02
10.74	17.98	34.88	86.50
10.74	18.31	36.48	91.30
10.74	17.82	43.03	84.58
10.74	18.39	45.11	91.30

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Table A7a. Mills and Zimmerman

f_c	σ_3	σ_2	σ_1
23.03	0.00	0.00	23.03
23.03	5.79	5.79	53.10
23.03	5.79	5.79	50.69
23.03	5.79	11.59	57.93
23.03	5.79	11.59	57.93
23.03	5.79	17.38	53.10
23.03	5.79	17.38	62.76
23.03	5.79	23.17	65.17
23.03	5.79	28.97	67.59
23.03	5.79	34.76	67.59
23.03	5.79	40.55	60.34
23.03	5.79	47.86	56.83
23.03	2.90	50.69	50.69
23.03	8.69	67.59	77.24
23.03	8.69	8.69	62.76
23.03	8.69	14.48	72.41
23.03	8.69	20.28	79.66
23.03	8.69	26.07	79.66
23.03	8.69	31.86	77.24
23.03	8.69	37.66	82.07
23.03	8.69	43.45	82.07
23.03	8.69	49.24	82.07
23.03	8.69	55.72	81.52
23.03	8.69	60.83	82.07
23.03	8.69	72.41	72.41
23.03	2.90	2.90	41.03
23.03	2.90	8.69	53.10
23.03	2.90	14.90	53.10
23.03	2.90	24.14	57.93
23.03	2.90	20.28	55.52
23.03	2.90	31.86	57.93
23.03	2.90	37.24	55.52
23.03	2.90	43.45	55.52
23.03	2.90	49.31	57.93
23.03	2.90	53.10	53.10
23.03	2.90	2.90	43.45
23.03	8.69	8.69	63.79
23.03	5.79	63.79	65.17

Table A7b. Mills and Zimmerman

f_c	σ_3	σ_2	σ_1
26.93	0.00	0.00	26.93
26.93	0.62	50.69	50.69
26.93	0.62	45.86	45.86
26.93	1.24	49.66	53.10
26.93	1.24	50.69	50.69
26.93	2.90	62.76	62.76
26.93	2.90	62.76	62.76
26.93	1.45	1.45	41.52
26.93	2.90	2.90	41.76
26.93	4.83	5.24	56.28
26.93	4.83	5.24	61.59
26.93	7.24	7.24	71.24
26.93	7.24	7.24	71.17
26.93	1.45	21.72	58.90
26.93	2.90	36.21	66.14
26.93	1.45	21.72	61.59
26.93	2.90	28.97	68.83

Table A7c. Mills and Zimmerman

f_c	σ_3	σ_2	σ_1
36.10	0.00	0.00	36.10
36.10	0.62	47.10	48.28
36.10	0.62	51.66	53.10
36.10	1.24	60.34	61.31
36.10	1.24	50.69	51.24
36.10	2.90	67.59	68.83
36.10	2.90	66.62	68.34
36.10	1.45	1.45	54.55
36.10	2.90	2.90	59.38
36.10	1.45	21.72	63.24
36.10	2.90	28.97	72.90
36.10	4.83	5.24	64.00
36.10	4.83	4.83	73.10
36.10	1.45	26.55	66.41
36.10	2.90	28.97	68.83
36.10	2.90	2.90	54.34
36.10	1.45	1.45	51.66

Appendix-B

Calculation of criterion parameter value:

1. Drucker Pragger Criteria

$$f(I_1, J_2) = 0 \quad (\text{B.1})$$

$$\text{or, } \alpha I_1 + \sqrt{J_2} - k = 0 \quad (\text{B.2})$$

where, α and k are material constant or criterion parameters.

and,

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3 \quad (\text{B.3})$$

$$J_2 = \frac{1}{6} \left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 \right] \quad (\text{B.4})$$

The criterion parameters can be calculated considering two failure states i.e.

a. Uniaxial compressive strength:

$$\sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (\text{B.5})$$

b. Equal biaxial strength:

$$\sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (\text{B.6})$$

Considering first failure state:

$$I_1 = f_c \quad (\text{B.7})$$

$$\text{and, } J_2 = \frac{f_c^2}{3} \quad (\text{B.8})$$

putting the value of I_1 and J_2 in equation A.2.2, we get,

$$f_c \left(\alpha + \frac{1}{\sqrt{3}} \right) - k = 0 \quad (\text{B.9})$$

Similarly, considering second failure state:

$$I_1 = 2 f_{bc} \quad (\text{B.10})$$

$$\text{and, } J_2 = \frac{f_{bc}^2}{3} \quad (\text{B.11})$$

putting the value of I_1 and J_2 in equation A.2.2, we get,

$$f_{bc} \left(2\alpha + \frac{1}{\sqrt{3}} \right) - k = 0 \quad (\text{B.12})$$

Equation A.2.9 and A.2.12 can be solved by considering the standard correlation between equal biaxial strength and uniaxial compressive strength as:

$$f_{bc} = 1.15 \cdot f_c \quad (\text{B.13})$$

Using equation A.2.13 and A.2.12 we get,

$$f_c \left(2.30\alpha + \frac{1.15}{\sqrt{3}} \right) - k = 0 \quad (\text{B.14})$$

Solving equation A.2.9 and A.2.14, we get

$$\alpha = -0.0666$$

$$k = 0.5107 \cdot f_c$$

2. Bresler-Pester Criteria

$$f(\tau_{oct}, \sigma_{oct}, f_c) = 0 \quad (\text{B.15})$$

$$\frac{\tau_{oct}}{f_c} = a - b \cdot \frac{\sigma_{oct}}{f_c} + c \cdot \left(\frac{\sigma_{oct}}{f_c} \right)^2 \quad (\text{B.16})$$

where, a , b and c are material constant.

and, σ_{oct} and τ_{oct} are octahedral normal and shear stress defined as:

$$\sigma_{oct} = \frac{I_1}{3} \quad (B.17)$$

$$\text{and, } \tau_{oct} = \sqrt{\frac{2}{3} J_2} \quad (B.18)$$

The criterion

parameters can be calculated considering three failure states i.e.

a. Uniaxial compressive strength:

$$\sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (B.19)$$

b. Uniaxial tensile strength:

$$\sigma_1 = \sigma_2 = 0; \sigma_3 = -f_t \quad (B.20)$$

c. Equal biaxial strength:

$$\sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (B.21)$$

Considering first failure state:

$$\frac{\sigma_{oct}}{f_c} = \frac{1}{3} \quad (B.22)$$

$$\text{and, } \frac{\tau_{oct}}{f_c} = \frac{\sqrt{2}}{3} \quad (B.23)$$

putting the values in equation A.2.16, we get,

$$a - \frac{b}{3} + \frac{c}{9} = 0.4714 \quad (B.24)$$

Similarly, considering second failure state:

$$\frac{\sigma_{oct}}{f_c} = \frac{-f_t}{3f_c} \quad (B.25)$$

$$\text{and, } \frac{\tau_{oct}}{f_c} = \frac{\sqrt{2}}{3} \frac{f_t}{f_c} \quad (B.26)$$

putting the values in equation A.2.16, we get,

$$a + \left(\frac{b}{3} - \frac{\sqrt{2}}{3} \right) \frac{f_t}{f_c} + \frac{c}{9} \left(\frac{f_t}{f_c} \right)^2 = 0 \quad (\text{B.27})$$

Similarly, considering third failure state:

$$\frac{\sigma_{oct}}{f_c} = \frac{2}{3} \frac{f_{bc}}{f_c} \quad (\text{B.28})$$

$$\text{and, } \frac{\tau_{oct}}{f_c} = \frac{\sqrt{2}}{3} \frac{f_{bc}}{f_c} \quad (\text{B.29})$$

putting the values in equation A.2.16, we get,

$$a - \left(\frac{2b}{3} + \frac{\sqrt{2}}{3} \right) \frac{f_{bc}}{f_c} + \frac{4c}{9} \left(\frac{f_{bc}}{f_c} \right)^2 = 0 \quad (\text{B.30})$$

Equation A.2.27 and A.2.30 can be further simplified by considering the standard correlation between equal biaxial strength, uniaxial tensile strength and uniaxial compressive strength as:

$$f_t = 0.10 \cdot f_c \quad (\text{B.31})$$

$$\text{and, } f_{bc} = 1.15 \cdot f_c \quad (\text{B.32})$$

Using equation A.2.31 and A.2.27 we get,

$$a + 0.033b + \frac{0.01}{9}c = 0.047 \quad (\text{B.33})$$

Similarly using equation, A.2.32 and A.2.30, we get

$$a - 0.766b + 0.587c = 0.542 \quad (\text{B.34})$$

Now on solving equation A.2.24, A.2.33 and A.2.34, we get

$$a = 0.099$$

$$b = -1.532$$

$$c = -1.245$$

3. Hsieh-Ting-Chen (HTC) Criterion

$$f(I_1, J_2, \sigma_1) = 0 \quad (B.35)$$

$$\text{or, } a \frac{J_2}{f_c^2} + b \frac{\sqrt{J_2}}{f_c} + c \frac{\sigma_1}{f_c} + d \frac{I_1}{f_c} = 1 \quad (B.36)$$

where, a , b , c and d are material constant.

The criterion parameters can be calculated considering three failure states i.e.

a. Uniaxial compressive strength:

$$\sigma_3 = \sigma_2 = 0; \sigma_1 = f_c \quad (B.37)$$

b. Uniaxial tensile strength:

$$\sigma_1 = \sigma_2 = 0; \sigma_3 = -f_t \quad (B.38)$$

c. Equal biaxial strength:

$$\sigma_3 = 0; \sigma_1 = \sigma_2 = f_{bc} \quad (B.39)$$

d. Confined triaxial compression strength:

$$\sigma_2 = \sigma_3 = f_{pc}; \sigma_1 = f_{cc} \quad (B.40)$$

Considering first failure state:

$$I_1 = f_c \quad (B.41)$$

$$\text{and, } J_2 = \frac{f_c^2}{3} \quad (B.42)$$

putting the value of I_1 , σ_1 and J_2 in equation A.2.36, we get,

$$0.333a + 0.577b + c + d = 1 \quad (B.43)$$

Similarly, considering second failure state:

$$I_1 = -f_t \quad (B.44)$$

$$\text{and, } J_2 = \frac{f_t^2}{3} \quad (\text{B.45})$$

putting the value of I_I , σ_I and J_2 in equation A.2.36, we get,

$$\frac{a}{3} \left(\frac{f_t}{f_c} \right)^2 + \frac{b}{\sqrt{3}} \left(\frac{f_t}{f_c} \right) + d \left(\frac{-f_t}{f_c} \right) = 1 \quad (\text{B.46})$$

Similarly, considering third failure state:

$$I_I = 2 f_{bc} \quad (\text{B.48})$$

$$\text{and, } J_2 = \frac{f_{bc}^2}{3} \quad (\text{B.49})$$

putting the value of I_I , σ_I and J_2 in equation A.2.36, we get,

$$\frac{a}{3} \left(\frac{f_{bc}}{f_c} \right)^2 + \frac{b}{\sqrt{3}} \left(\frac{f_{bc}}{f_c} \right) + c \left(\frac{f_{bc}}{f_c} \right) + 2d \left(\frac{f_{bc}}{f_c} \right) = 1 \quad (\text{B.50})$$

Similarly, considering fourth failure state:

$$I_I = 2 f_{pc} + f_{cc} \quad (\text{B.51})$$

$$\text{and, } J_2 = \frac{(f_{cc} - f_{pc})^2}{3} \quad (\text{B.52})$$

putting the value of I_I , σ_I and J_2 in equation A.2.36, we get,

$$\frac{a}{3} \left(\frac{f_{cc} - f_{pc}}{f_c} \right)^2 + \frac{b}{\sqrt{3}} \left(\frac{f_{cc} - f_{pc}}{f_c} \right) + c \left(\frac{f_{cc}}{f_c} \right) + d \left(\frac{f_{cc} + 2f_{pc}}{f_c} \right) = 1 \quad (\text{B.53})$$

Equation A.2.46, A.2.50 and A.2.53 can be further simplified by considering the standard correlation as:

$$f_t = 0.10 \cdot f_c \quad (\text{B.54})$$

$$f_{bc} = 1.15 \cdot f_c \quad (\text{B.55})$$

$$\text{and, for } f_{pc} = 0.8 \cdot f_c, \quad f_{cc} = 4.2 \cdot f_c \quad (\text{B.56})$$

using above mentioned correlation, Equation A.2.46, A.2.50 and A.2.53 are transformed as:

$$0.00333a + 0.0577b - 0.1d = 1 \quad (\text{B.57})$$

$$0.4408a + 0.6639b + 1.15c + 2.30d = 1 \quad (\text{B.58})$$

$$3.853a + 1.963b + 4.2c + 5.8d = 1 \quad (\text{B.59})$$

Now on solving equation A.2.43, A.2.57, A.2.58 and A.2.59, we get

$$a = 2.002$$

$$b = 16.806$$

$$c = -9.128$$

$$d = -0.236$$

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