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Measuring Quantum Gravity

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Einstein's theory of gravity, known as General Theory of Relativity was discovered in 1915. The theory has survived many experimental tests, and the recent discovery of gravity waves announced in 2016 confirms yet another success. In this article we examine some results from quantum general relativity, and ask the question if the new quantum theory can survive tests in the same way as its classical origins. This article is written for the Ursula Franklin Commemorative Festschrift for Canadian Journal of Physics.

I. INTRODUCTION

Einstein's theory of gravity, General theory of relativity (GR) changed our perception of Newtonian gravity: The theory of gravity is 'not the inverse square law force' but the theory of curvature of space-time according to Einstein. The 'gravitational force' is due to the requirement that particles follow geodesics (paths of minimum length) in curved space-time. This description of gravity, originates from the equivalence principle, "The outcome of any local non-gravitational experiment in a freely falling laboratory is independent of the velocity of the laboratory and its location in space-time. [1]" Einstein, in an effort to define the principle of relativity for accelerating frames, brought forth the concept of freely falling frames. We know and can experience free fall, systems which are moving along geodesics of the gravitational field. In Earth's gravity, a particle falling at 9.8m/s^2 is freely falling. A zero-gravity plane simulates this, and all astronauts in outer space are in free fall. In flat space a freely falling particle moves along a straight line. In curved space-time, these "free fall" geodesics are curved trajectories. We attribute this deviation from the straight path to curvature of space-time, interpreted as the force of gravity. Experimentally gravity is caused

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by matter, and energy (as energy and matter are same due to $E = mc^2$, another equation discovered by Einstein). Therefore, curvature of space-time, should also originate due to matter and energy. To introduce and study curved geometries, Einstein used Riemannian geometry [4], and the ‘Ricci tensor and scalar’ $R_{\mu\nu}$, R and the metric $g_{\mu\nu}$ which measure curvature, and represent geometry. These were equated to ‘matter’ source $T_{\mu\nu}$, the energy momentum tensor, and the equation is now famous as Einstein’s equation.

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \quad (1)$$

Geometry = Matter Energy + Momentum

(where we have set G =Newton’s constant, and we have set the speed of light $c=1$ for brevity in formulas)

The new theory of gravity arising from (1) explained the perihelion precession of mercury to great precision, and predicted new objects, mainly the bizarre black holes which absorb everything including light. These strong gravitating objects are now real observed phenomena and confirmation of their existence comes from gravity waves emitted from the merger of two black holes [3]. What are these black holes? These are objects formed from the collapse of a star, or in the early universe, due to density fluctuations. The event horizon of the black hole is like a spherical one way membrane through which particles fall inside a black hole, and cannot escape back to the exterior region. The particles instead hurtle down to a central singularity, where the laws of physics break down, and energy is no longer conserved [2].

The black holes also bring to the discussion a big puzzle: Black holes have entropy and radiate particles at a temperature [5, 6]. The laws of “black hole thermodynamics” started an entire new era in physics research, and search for a ‘quantum theory’ of gravity. Every entropic system with irreversible time flow was believed to have ‘statistical interpretation’ with large degrees of freedom. One had to find corresponding ‘quantum’ microstructure of a black hole, for this statistical interpretation, otherwise entropy would be a mysterious quantity for a classical system.

To begin the discussion on the search for micro states or quantum states of a black hole, we describe a brief introduction to quantum theory. Quantum mechanics is the theory of particles at the atomic or molecular length scales and beyond. Instead of Newton’s equation which relates causation of motion to mass times acceleration of a particle, in quantum

mechanics we study the Schrödinger equation. This equation describes the dynamics of the wave function of the particle. The modulus square of the wave function of the particle is the ‘probability’ of finding the particle in a given region of the space-time. In quantum mechanics physics is not deterministic, and we can predict only the ‘likelihood’ of the outcome of an experiment using the wave function’s modulus square. The strange result of this new physics of the sub-atomic length scale is that energy is quantized, and the particle is allowed to ‘exist’ in a set of discrete ‘states’ or configurations. A typical example is the example of the atom, where the electron is allowed in a set of discrete shells around the nucleus. If the quantum system is at a finite temperature, and its behaviour is thermal, various measures of entropy exist, including the simplest one, which has entropy as proportional to the logarithm of the number of micro states.

If we try to form an atom of gravitational origin, then the atom’s size should be order of the Planck length 10^{-33}cm , way smaller than the nucleus which is of the order of 10^{-15}cm . This length is beyond the current day accelerators, and we cannot check the existence of gravitational atoms or discretized energy shells of quantum gravity origin. However, what we can check for are indirect evidences of such quantum behaviour of gravitation, which might manifest their effect at much higher length scales. Or, we might find evidences of ‘quantum fluctuations’ to classical gravitational phenomena which are measurable or observable in current day astrophysical data, and or instruments. It is important to test for signatures of quantum gravity in nature, any deviation from predictions of GR should be recorded and analysed. At the current level of observations, the natural phenomena observed are all within the GR limits. However, when an experiment is designed, it is important to know what physical phenomena to look for, the range of parameters one needs to tune to search for new physics. It is with this aim that we discuss new predictions from quantum GR.

In this article we shall examine astrophysical systems, which might be quantum corrected. Albeit the quantum corrections are tiny, we see if these can become manifest in some observational data of the system. This is facilitated by two facts:

- (i) Quantum Gravity is non-perturbative in origin, and thus our ‘quantum corrections’ will be different from linearized perturbations over the classical geometry, predicted from perturbative gravity. Thus experiments might confirm these non-perturbative quantum corrections, even if technically these are mere fluctuations of the classical geometry.
- (ii) Einstein equations are non-linear and time evolving these ‘tiny quantum fluctuations’,

might lead to tangible effects at a future time.

Based on these, we will predict some observational effects. Our article is organized as follows: In the next section we describe the theory of quantum gravity, and its semiclassical limit, using a coherent state description of the system. As there might be readers not familiar with Einstein's GR, we also provide a brief review of all the physical quantities used in the description of a gravitating system. In section three, we study some astrophysical systems where these effects might be relevant. In the fourth section, we shall describe one experiment which is functional, and predict results from quantum gravity. In the fifth section, we shall conclude.

II. SEMICLASSICAL GRAVITY: COHERENT STATES

Einstein gravity is the story of the curvature of space-time, which is caused by presence of matter or energy. The curvature of space-time is measured using the 'metric' and its derivatives. In the following discussion we give a brief introduction to General Relativity.

A. General Relativity

In any space-time, we measure distances using the distance function, given by the metric, in four dimensions, this is a 4×4 symmetric matrix, symbolically $g_{\mu\nu}$ ($\mu\nu = 0, 1, 2, 3$). Here the 0 index stands for time, and the remaining are space labels. Formally

$$ds^2 = \sum_{\mu,\nu} g_{\mu\nu} dx^\mu dx^\nu \quad (2)$$

is the distance between space time points at x^μ and $x^\mu + dx^\mu$.

We are used to distances in Euclidean 3-dimension,

$$ds^2 = dx^2 + dy^2 + dz^2.$$

The Euclidean distance can be written using the metric in the form of (2) with a 'diagonal' matrix with entries (1,1,1). If one is measuring the 'distance' on a circle, then the length is 'curved' and the length along the arc of a circle is

$$ds^2 = g_{\theta\theta} d\theta^2 = r^2 d\theta^2. \quad (3)$$

The metric in this example is 1×1 matrix and has one non-zero component $g_{\theta\theta} = r^2$, thus a function of the ‘radius’. In general a metric in four dimensional space-time can be an arbitrary function of the four coordinates of the manifold coordinate chart, $g_{\mu\nu}(x)$. This matrix is also known as a 2-tensor in differential geometry. We will use the Einstein summation convention, where any ‘index’ repeated in a formula, or which appears twice is being summed over. Thus the metric in four dimensions is written as

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad (4)$$

Under a general coordinate transformation, x^μ transform to another set of coordinates x'^μ , (e.g. in two dimensions $x = r \cos \theta, y = r \sin \theta$ is a coordinate transformation from x, y to r, θ coordinates) the metric transforms as

$$g'_{\mu\nu} = \frac{\partial x^\lambda}{\partial x'^\mu} \frac{\partial x^\rho}{\partial x'^\nu} g_{\lambda\rho} \quad (5)$$

This coordinate transformation can be interpreted as a change in the ‘frame’ in which we observe the physical phenomena. Einstein’s formulation of gravity requires that physics be invariant under such a transformation, and this includes the change in frame to that of an ‘accelerated observer’.

The mathematics of geometry as encoded in the metric had been formulated previously and was known as Riemannian geometry. In the formulation of Riemannian geometry, physical quantities such as vectors, let us say A^μ , are parallel transported along paths on curved geometries to understand the ‘change’, which in flat space-time would be a simple translation. And thus the derivative of a vector along a curved path, is not just the ordinary space derivative, but also includes a ‘Christoffel connection’ which encodes the change induced by the curvature of the geometry (usually a rotation of the vector). This is known as the covariant derivative, given thus: e.g.

$$\partial_\mu A^\nu \text{ (flat)} \rightarrow \nabla_\mu A^\nu = \frac{\partial A^\nu}{\partial x^\mu} + \Gamma_{\mu\lambda}^\nu A^\lambda \text{ (curved)} \quad (6)$$

The three index Christoffel symbol $\Gamma_{\mu\lambda}^\nu$ is a ‘affine’ connection, and locally can be set to zero. In frames in free fall, this is zero, showing that gravity can be replaced with accelerating frames. To find the correct physical quantity, to describe gravity, the curvature of the space-time is encoded in a four indexed geometric quantity known as the Riemann tensor. This measures the change in the vector when it is parallel transported along a closed loop, or say between two points using two different paths.

It can be calculated as the difference or commutator of covariant derivatives of a vector under parallel transport and is of the form:

$$(\nabla_\lambda \nabla_\nu - \nabla_\nu \nabla_\lambda) A^\nu = R^\nu_{\lambda\mu\rho} A^\rho \quad (7)$$

In flat space Riemann tensor's 21 components are all zero. Unlike the Christoffel symbol, the Riemann tensor's components cannot all be set to zero by changing frames in curved geometry. Any non-zero component of the Riemann tensor signifies a non-trivial curvature of space-time. Einstein formulated the dynamics of gravity using the Riemann tensor $R^\nu_{\lambda\mu\rho}$, which can be explicitly derived using derivatives of the metric function, and in order to have a diffeomorphism invariant theory, he used the Ricci scalar $R = g^{\mu\nu} g^{\lambda\rho} R_{\lambda\nu\mu\rho}$ as this remains invariant under the transformations of (5).

The dynamics of this metric is obtained from Einstein action, which is a function of the Ricci scalar, formed from two derivatives of the metric.

$$S = -\frac{1}{16\pi G} \int \sqrt{g} R d^4x + S_{\text{matter}} \quad (8)$$

g is the determinant of the 4×4 matrix of space-time and $\sqrt{g} d^4x$ is the diffeomorphism invariant measure. Equation (8), including all the matter fields existing in nature, is the grand theory of everything. A variation of this action gives the Einstein equations with $T_{\mu\nu} = \frac{\delta S_{\text{matter}}}{\delta g_{\mu\nu}}$ as in equation (1). The entire program of quantum gravity is the attempt to obtain a quantum version of the above action. It is technically very difficult to implement the quantization with the above: one reason being that the Einstein action is non-polynomial in its formulation due to $\sqrt{g}$, in its definition, and we have only successfully quantized quadratic/polynomial actions. There are other problems which we do not have scope to elaborate here, but can be found in discussions of reviews and books in quantum gravity [7]. Whereas matter fields can be quantized explicitly in flat space-time, introducing curvature and thus gravity gives divergent answers for quantum amplitudes. In the next section, we shall discuss an attempt to quantize pure gravity only. A very neat description of kinematical quantization program for Einstein gravity exists, known as Loop quantum gravity, which we shall describe very briefly next.

FIG. 1: Canonical Slicing

B. Loop Quantum Gravity

Einstein gravity can also be formulated as a theory of the ‘tangent space’ to the curved manifold. The soldering forms e^I_μ connect the tangent space to the world volume geometry, the indices I transform in the tangent space, where the metric is flat. Using them:

$$g_{\mu\nu} = e^I_\mu e_{\nu I}.$$

However, in this article we shall use a variant of canonical gravity. In a canonical formulation, the tetrads are broken into the three space tetrads and the space-time, time-time components, of which the latter are Lagrange multipliers. The canonical quantization of gravity in terms of redefined tetrads, is quantized as Loop Quantum Gravity (LQG). This theory is an attempt to quantize gravity using canonical methods. In the next we briefly describe canonical gravity.

Canonical gravity is usually introduced using the ‘ADM variables’, and separates time from space. The time direction is identified using a fiducial normal vector to the spatial

slices n^μ . As the spatial slices are embedded in the 3+1 dimensional space, a new measure of ‘curvature’ is introduced, and that is known as the extrinsic curvature of the slices. This can be defined as a covariant derivative of the normal vector,

$$K_{\mu\nu} = \nabla_\mu n_\nu \quad (9)$$

A pullback of this to three dimensions is

$$K_{ab} = \frac{\partial X^\mu}{\partial x_a} \frac{\partial X^\nu}{\partial x_b} K_{\mu\nu} \quad (10)$$

where X^μ are the world coordinates and x^a ($a, b=1, 2, 3$) are the coordinates on the three dimensional spatial slices. The metric can also be pulled back to the spatial slices:

$$q_{ab} = g_{\mu\nu} \frac{\partial X^\mu}{\partial x_a} \frac{\partial X^\nu}{\partial x_b} \quad (11)$$

The q_{ab} and the K_{ab} form the basic dynamical degrees of freedom for canonical gravity. The momentum conjugate to q_{ab} obtained using the symplectic structure is

$$\Pi^{ab} = \sqrt{q}(K^{ab} - K_c^{ab}). \quad (12)$$

A fundamental Poisson bracket of q_{ab} and Π^{ab} can be lifted to commutator, to obtain a quantization. But this program, known as ”Geometrodynamics” has not yielded results.

Using the ‘soldering forms’ triads of the three manifolds, a more successful quantization has emerged known as LQG.

The three triads, the soldering forms for the three dimensional time slices are labelled as e_I^a , with $a = 1, 2, 3$ such that $e_a^I e_{bI} = q_{ab}$. The new variables, introduced by A. Sen and later developed by A. Ashtekar are [8]

$$E^{aI} = \tilde{e} e^{aI} \quad (13)$$

This is known as the densitized triad, as it is multiplied by the determinant of the matrix e_a^I , $\tilde{e}$. The field redefinition absorbs the square root of the determinant of the metric ($\sqrt{q} = \tilde{e}$), making the action polynomial in the canonical variables. The new ‘affine connection’ is derived requiring that the triad under world volume parallel transport, as well as tangent space rotations, remains invariant.

$$\partial_a e_b^I - \Gamma_{ab}^c e_c^I - \omega_a^{IJ} e_{bJ} = 0. \quad (14)$$

FIG. 2: Tangent Space and Soldering forms

The spin connection; ‘the affine connection’ introduced above is

$$\omega_a^{IJ} = e^{b[I} \nabla_a e_b^{J]} \quad (15)$$

As the tangent space spin connections transform in the spin representation of the $SO(3)$ group, the vectors have a non-abelian ‘internal symmetry’. This can then be utilized for a redefinition of gravity as a non-abelian gauge field

$$A_a^I = \epsilon^{IJK} \omega_a^{JK} + K_a^I \quad (16)$$

($K_a^I = K_{ab} e^{bI}$) where adding the extrinsic curvature gives the non-zero Poisson bracket of the connection with the triads, and the spin connection ensures that the above transforms in-homogeneously using the $SO(3)$ group. In terms of this, the Einstein action is written as

$$S = -\frac{1}{16\pi G} \int \left[E^{aI} \dot{A}_{aI} - NH - N_a H^a \right] dt d^3x \quad (17)$$

where $N = g_{00}$ and $N_a = g_{a0}$ and are Lagrange multipliers (they do not have time derivatives in the Lagrangian). The functions H and H^a which the Lagrange multipliers ‘multiply’ appear as second class constraints of the theory. These constraints ensure that no

time derivatives of N and N^a are generated during the time evolution of the system. The constraints are also the time translation generators (the Hamiltonian) and the spatial diffeomorphism generators of the theory. As these are symmetries of the theory, the system is invariant under the action of these generators. The Poisson brackets of the canonical variables can be lifted to commutators to obtain a kinematical quantization of the system. The Hamiltonian and diffeomorphism constraints can be implemented on the Hilbert space in which the kinematical operators are represented to obtain the physical Hilbert space.

$$\{E^{aI}(x), A_{bJ}(x')\} = \kappa \delta_b^a \delta_J^I \delta^3(x - x') \quad (18)$$

The commutator of the above when written as commutators is: $[\kappa = (16\pi G)]$

$$[E^{aI}(x), A_{bJ}(x')] = i\hbar\kappa \delta_b^a \delta_J^I \delta^3(x - x') \quad (19)$$

As the commutator is distributional, it is customary to smear the variables to obtain a different algebra. In LQG, one introduces

$$h_e(A) = \mathcal{P} \exp\left(\int_e A\right) \quad (20)$$

or the holonomy of the Gauge connection, a path ordered exponential of the integral of the gauge connection on a one dimensional ‘path’ which could be the links of a graph. The densitized triads are similarly smeared over two surfaces. The set of two surfaces, provide a ‘dual discretization’ of the manifold

$$P_e^I = \int_{S_e} *E^I \quad (21)$$

The surfaces are intersected at finite points by the edges of the original graph. Each surface is however intersected by only one edge, and thus the P_e^I is labelled by the edge e . The holonomy and the ‘smeared’ momenta commutator can be found:

$$[P_e^I, h_e] = \sigma(e, S) h_{e_1} \left(\frac{-i\sigma^I}{2} \right) h_{e_2} \quad (22)$$

where $\sigma(e, S) = \pm 1$ depending on the orientation of the surfaces and e_1 and e_2 are the edges into which the edge is subdivided into at the intersection with the surface.

The momentum however can be regularized appropriately to get the algebra in more useful form.

$$P_e^I = -\frac{1}{3} \text{Tr} \left[-\frac{\iota}{2} \sigma^I h_{e_{1/2}} \int *E h_{e_{1/2}}^{-1} \right] \quad (23)$$

where the point of intersection of the edge with the surface is exactly midway. With this regularization the algebra is [7]

$$[P_{e'}^I, h_e] = \delta_{ee'} \frac{-i\sigma^I}{2} h_e. \quad (24)$$

Once the kinematical set up is obtained, this theory can be quantized as a quantum mechanical theory, by representing the above algebra on a Hilbert space. For the LQG SU(2) variables, one uses the ‘spin network states’ built using the j th representation of SU(2) matrices. The fundamental representation of SU(2) is the set of 2×2 unitary matrices with determinant 1. The higher dimensional representation of these fundamental representation is built using a well defined algorithm, such that the matrices are now $(2j+1) \times (2j+1)$ dimensions. The representations are labelled by index j . The fundamental representation has $j = 1/2$. The matrix elements are labelled as $\pi_j(h)_{mn}$ where $m, n = -j..j$. The higher order representations are obtained from the four elements of the two dimensional fundamental representation using formulas defined in [9]. e.g. if

$$h_e(A) = \begin{pmatrix} a & b \\ -\bar{b} & \bar{a} \end{pmatrix} \quad (25)$$

such that $|a|^2 + |b|^2 = 1$, then $\pi_{1/2}(h)_{1/2,1/2} = a$ $\pi_{1/2}(h)_{1/2,-1/2} = b$ $\pi_{1/2}(h)_{-1/2,-1/2} = -\bar{b}$ $\pi_{1/2}(h)_{-1/2,1/2} = \bar{a}$. Each state can also be represented using the ‘vector’ notation $|jmn\rangle$.

The canonical operators act on the state in a way similar to particle quantum mechanics.

$$\hat{h}_e |jmn\rangle = h_e |jmn\rangle \quad (26)$$

i.e. multiplicatively, and given that the graph one dimensional edges on which the holonomy is defined intersect the surface over which the triad is smeared at least once,

$$\hat{P}_e^I |jmn\rangle = i\hbar \frac{\kappa}{4} [\sigma(e_1, S) L_{e_1}^I |jmn\rangle + \sigma(e_2, S) R_{e_2}^I |jmn\rangle]. \quad (27)$$

The point of intersection divides the edge into two edges e_1 and e_2 , and $L_{e_1}^I, R_{e_2}^I$ are the left and right invariant group derivatives defined thus:

$$L^I f(g) = \frac{d}{dt}_{t=0} f(ge^{-i\sigma^I t}) \quad R^I f(g) = \frac{d}{dt}_{t=0} f(e^{-i\sigma^I t}g)$$

FIG. 3: surface intersected by a edge e , divided into e_1 and e_2

and thus the momentum acts as a derivation.

Like the ordinary quantum mechanical system, these operators h_e and P_e^I play the role of the x and p or position and momentum operators respectively. We shall use this to obtain coherent states in analogy with the harmonic oscillator coherent states. Once the kinematical Hilbert space has been obtained the full physical Hilbert space is obtained by imposing the constraints on the quantum states.

C. Coherent states or Semiclassical states

Quantum mechanics is the theory at subatomic length scales, described by some basic axioms, including an uncertainty principle which puts a bound on the accuracy of measurements. Stated by Heisenberg it is

$$\Delta x \Delta p \geq \frac{\hbar}{2} \quad (28)$$

This uncertainty is minimum or the bound is saturated in some special states known as *coherent states*. These states are known as semiclassical states as operator expectation values are closest to their classical values. The coherent state are ‘peaked’ about the classical

value of the operators. These states can be easily constructed using a result due to Hall [10] (and later generalized by Thiemann [11]). In this, the states are obtained as the functional representation of the Heat Kernel.

$$\psi^{\tilde{t}}(x, z) = \frac{\rho_{\tilde{t}}(x - z)}{\sqrt{\rho_1(x)}} \quad (29)$$

Where $\rho_{\tilde{t}}(x - z)$ is the Heat Kernel analytically continued to the complex phase space $z = x - ip$ (a complexification of the phase space point (x, p)). The Heat Kernel is a solution to the Heat dissipation differential equation, and is defined for given manifolds with appropriate boundaries [12]. The Heat Kernel in one dimension is

$$\rho_{\tilde{t}}(x) = (2\pi\tilde{t})^{1/4} \exp[-x^2/2\tilde{t}] \quad (30)$$

This representation of the operator can also be derived using the spectrum of the Laplacian of the theory.

A plot of the above wave function $\psi^{\tilde{t}}(x, z)$, obtained from the Heat Kernel is a Gaussian. A classical limit can be obtained by taking the standard deviation of the Gaussian, proportional to $\tilde{t} \rightarrow 0$, where the function is delta function peaked at $x = x_0$ and signifies the exact ‘classical point.’ $\hbar = \tilde{t}$, is a ‘semiclassical parameter’ in the quantum theory.

In analogy with the Harmonic Oscillator coherent state, we can take the Heat Kernel of a ‘Kinetic operator’ for the LQG Hilbert space [11]. The resultant coherent state has been verified to saturate the quantum gravity uncertainty bound. For quantum gravity we take the Kinetic operator to be $\nabla_\gamma = \sum_e \hat{P}_e^I \hat{P}_e^I$ for a given graph γ comprised of edges e . To make the operator dimensionless, we use an appropriate scaling of P_e^I , as it is smeared over a two surface. We scale it using the length scale, which in case of the black hole example will be taken as $1/r_g^2$, where r_g is the size of the black hole. Using the above scaling the semiclassical parameter which enters the Heat Kernel is $\tilde{t} = l_p^2/r_g^2$, as seen for Planck Length size black holes this is 1, and other wise smaller. For the purposes of our calculations we take $\tilde{t} \sim 10^{-9}$. The complexifier coherent state for one edge e is obtained as a analytic continuation of the Heat Kernel defined thus:

$$\psi^{\tilde{t}}(h_e, g_e) = \rho_{\tilde{t}}(g_e h_e^{-1}) \quad (31)$$

where the coherent state is labelled by h_e, g_e , the two matrices which are the configuration space variable h_e and complexified phase space point in the $SU(2)$ Hilbert space g_e , given

FIG. 4: The coherent state Peak

by:

$$g_e = \exp(iT^I P_e^I) h_e \quad (32)$$

The above when explicitly evaluated can be expanded in the characters of the SU(2) representations as

$$\psi^{\tilde{t}}(g_e h_e^{-1}) = \sum_j e^{-\tilde{t}j(j+1)/2} \chi_j(g_e h_e^{-1}) \quad (33)$$

The Character χ_j is the trace of the matrix in the j th representation. This definition of coherent state has been used by Thiemann et al, and tested for peakedness properties, and Ehrenfest theorems for operators of LQG. The coherent states are defined in terms of the canonical pair, h_e, P_e^I , which are ‘covariant operators’ of the theory. These operators do not commute with the diffeomorphism constraints, and thus are not the observables’ of the system. Building gauge invariant observables for gravity and quantum gravity is a field of research [13], which we are not discussing in this article.

The coherent state defined above is ‘covariant’, it is not defined in the physical Hilbert

space of the theory, but built using the kinematical Hilbert space. However, for the purpose of the paper, we are computing expectation values of covariant operators, and thus these are appropriate for questions mentioned in this article. Gauge invariant observables in quantum gravity have to be obtained before we can discuss expectation values in physical states as discussed in the next section. In the first order approximation, the corrections preserve the gauge ‘inherited’ from the classical geometry. *In the approximation closest to the classical geometry, we can discuss quantum fluctuations of the metric using semi-classical states such as ‘covariant coherent states’ defined here, using the kinematic Hilbert space of the theory.*

III. QUANTUM CORRECTED ASTROPHYSICS

The coherent states are interesting semiclassical states in which we can connect to a ‘classical notion’ of space-time as the expectation value of the operators are closest to their classical values. In the next, we describe the quantum fluctuations to classical black hole geometry.

A. Semiclassical corrections to black hole geometry

In [14], the fluctuations of the geometry was computed in a coherent state, for the Lemaitre slicing of the Schwarzschild black hole. In this article, we discuss this quantum fluctuation, and if the nature of the quantum fluctuation can make a difference to astrophysical observations. The Schwarzschild black hole metric was one of the first known exact solution to Einstein’s equation, obtained for the spherically symmetric systems. This metric represents a space-time which has a curvature singularity at the origin $r = 0$. There is also an event horizon at $r = 2GM$ where M is the mass of the black hole. The event horizon is a spherical surface, where Causality changes, and space-time points within the event horizon are causally disconnected from the outside. Whereas you are allowed to fall inside the black hole, you are not allowed to re-emerge to the outside. This ‘one way membrane’ behaviour gives the horizon a unique structure, and is the reason that there is ‘black hole thermodynamics’.

As, we saw in the previous section, classically a geometry can be represented using a metric or the distance function. For the Schwarzschild metric, spherical symmetry is imposed

by requiring that the metric components are pure functions of r , the radial distance measured from a centre. The metric is also static, and hypersurface orthogonal, in the sense that there are no off diagonal components of the metric. The metric is independent of time, and thus it is known as ‘stationary’. In details written in t, r, θ, ϕ coordinates, this space-time is:

$$ds^2 = - \left(1 - \frac{r_g}{r}\right) dt^2 + \left(1 - \frac{r_g}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (34)$$

At $r=r_g=2GM$ the signature of the time-time component changes to space, and the radial direction acquires the status of ‘time’. Any particle which falls inside has $r = 0$ to its future, and thus ends up at the central singularity. This causality change is the presence of the event horizon, which separates causally disconnected regions. Particles inside the horizon are causally disconnected from those outside. As obvious, the metric in the above coordinates are singular at $r = 2GM$. However, this does not signify that space-time ends there, infact, if we transform to an in falling observers frame, the metric is smooth across the horizon. In the next, we discuss how to obtain ‘quantum fluctuations’ of this classical geometry.

To obtain a canonical quantization of space-time, one must identify the physical degrees of freedom. These for gravity live in constant time slices. If we try to obtain the constant time slices for the Schwarzschild metric, we find that these are curved intrinsically. Moreover the Lapse g_{00} component are non-trivial. The coordinates are singular at the horizon as g_{rr} diverges. To avoid these complications, we use the Lemaitre slicing of the black hole metric. In this frame, ‘time’ is the proper-time of an in-falling observer. The time slices are well defined at the horizon, and extend up to the singularity. Further the time slices have zero intrinsic curvature and the physical degrees of freedom can be defined using planar graphs on a flat geometry [15]. The metric in new τ, R, θ, ϕ coordinates is for the blackhole:

$$ds^2 = -d\tau^2 + \frac{dR^2}{\left[\frac{3}{2r_g}(R - \tau)\right]^{2/3}} + \left[\frac{3}{2}(R - \tau)\right]^{4/3} r_g^{2/3} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (35)$$

The coordinate transformations from the (34) are :

$$\sqrt{\frac{r}{r_g}} dr = (dR \pm d\tau) \quad (36)$$

$$dt = \frac{1}{1 - f^2} (d\tau \pm f dR) \quad f = \left(\frac{2r_g}{3(R - \tau)}\right)^{2/3} \quad (37)$$

The three metric is (or τ constant slices are) completely flat, and one can embed spherical graphs. One can easily see that by observing:

$$ds_3^2 = \frac{dR^2}{\left[\frac{3}{2r_g}(R - \tau)\right]^{2/3}} + \left[\frac{3}{2}(R - \tau)\right]^{4/3} r_g^{2/3} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (38)$$

we define a new coordinate $r'^2 = \left[\frac{3}{2}(R - \tau)\right]^{4/3} r_g^{2/3}$, the metric becomes

$$ds_3^2 = dr'^2 + r'^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (39)$$

The spatial slices are flat and a graph can be embedded into the three slices along the ‘radial’ direction, and along the θ, ϕ lines like latitudes and longitudes of the earth. The details can be found in [15].

The holonomy and the momentum can be obtained using straight forward methods [15], and coherent state constructed using (33), and one obtains the expectation value of the momentum operator as

$$\langle \psi^t | \hat{P}_e^I | \psi^t \rangle = P_e^I \text{cl} \left(1 + \tilde{t} \tilde{f}(P) \right) \quad (40)$$

The function $\tilde{f}$ can be explicitly computed and represents a non-perturbative correction. In fact

$$\tilde{f}(P_e) = \frac{1}{P_e} \left(\frac{1}{P_e} - \coth(P_e) \right) \quad (41)$$

where P_e is the gauge invariant momentum $P_e = \sqrt{P_e^I P_e^I}$. The three metric can be rebuilt, in particular we concentrate on the radial component of the metric: (we divide the quantum measured P_e^I by the surface areas S_e over which the operators E_a^I were initially smeared).

$$q^{r'r'} = \frac{1}{P} \left(\frac{P_{e_r'}}{S_{e_r'}} \right)^2 \left(1 + 2\tilde{t} \tilde{f} \left(\frac{P_{e_r'}}{S_{e_r'}} \right) \right) \quad (42)$$

where $P_{e_r'} = \sqrt{P_{e_r'}^I P_{e_r'}^I}$ is the gauge invariant momentum and $P = \det P_e^I / S_e$. In the limit the $S_{e_r'} \rightarrow 0$, the

$$P_{e_r'} = \frac{2r'^2 \sin \theta \delta \theta \delta \phi}{r_g^2} = \frac{r'^2 \sin \theta S_{e_r}}{r_g^2} \quad (43)$$

Plugging this in, we get

$$q^{r'r'} = \left(1 + 2\tilde{t} \tilde{f} \left(\frac{P_{e_r'}}{S_{e_r'}} \right) \right) \quad (44)$$

When transformed to Schwarzschild coordinates, the perturbed Lemaitre metric will transform as expected. This corrected Lemaitre coordinates when transformed to

Schwarzschild coordinates gives:

$$g^{tt} = \frac{dt}{d\tau} \frac{dt}{d\tau} g^{\tau\tau} + \frac{dt}{dR} \frac{dt}{dR} g^{RR} \quad (45)$$

$$g^{rr} = \frac{dr}{d\tau} \frac{dr}{d\tau} g^{\tau\tau} + \frac{dr}{dR} \frac{dr}{dR} g^{RR} \quad (46)$$

$$g^{rt} = \frac{dr}{d\tau} \frac{dt}{d\tau} g^{\tau\tau} + \frac{dr}{dR} \frac{dt}{dR} g^{RR} \quad (47)$$

where:

$$\frac{dt}{d\tau} = \frac{1}{1-f^2} \quad \frac{dt}{dR} = \pm \frac{1}{1-f^2} \left(\frac{2r_g}{3(R-\tau)} \right)^{2/3} \quad (48)$$

$$\frac{dr}{dR} = \sqrt{\frac{r_g}{r}} \quad \frac{dr}{d\tau} = \pm \sqrt{\frac{r_g}{r}} \quad (49)$$

This gives, in particular the corrections to g^{tr} as

$$g^{tr} = \pm 2 \frac{1}{1-f^2} \left(\frac{r_g}{r} \right)^{3/2} \tilde{t} \tilde{f} \left(\frac{P_{e_r}}{S_{e_r}} \right) \quad (50)$$

This correction is interesting, as quantum mechanically, the spherical symmetry and the static nature of the Schwarzschild metric is broken. This happens as we are using coherent states which are not defined for the reduced phase space of the system, and thus quantum fluctuations are not restricted to preserve the spherical symmetry of the system. The sign of the correction depends on whether the original Lemaitre metric represents a whitehole or a black hole singularity. The black hole metric has the positive sign.

B. The use of covariant states

Next we discuss the use of ‘covariant’ coherent states instead of ‘physical’ coherent states. Let there be a symmetry generated by the constraint operator $\hat{C}$. In quantum mechanics we use Unitary operators to implement the constraints on the operators and the states. Let $U(\hat{C})$ be such an operator, if there exists a physical gauge invariant operator $\hat{A}$, then

$$\hat{A}' = U^\dagger \hat{A} U = \hat{A} \quad (51)$$

The physical state is also such that

$$U |\psi_{\text{phys}}\rangle = |\psi_{\text{phys}}\rangle \quad (52)$$

or the state is invariant under the action of the unitary operator, which implements the constraint in the Hilbert space. Now, let us take an operator, $\hat{B}$ which is not gauge invariant, i.e. $\hat{B}' \neq \hat{B}$, and if we try to obtain the expectation value of a covariant operator in a physical state, i.e.

$$\langle \psi_{\text{phys}} | \hat{B}' | \psi_{\text{phys}} \rangle = \langle \psi_{\text{phys}} | U^\dagger \hat{B} U | \psi_{\text{phys}} \rangle = \langle \hat{B} \rangle \quad (53)$$

As we can see expectation value of a covariant operator remains unchanged in a physical state, which does not make sense. Thus, we can identify only the gauge invariant aspect of operators in a physical state. To see the ‘change’ in a covariant operator, the state itself has to transform in a covariant way to a new state in the Hilbert space, thus $|\psi'_{\text{cov}}\rangle = U|\psi_{\text{cov}}\rangle$. The question therefore arises if indeed there is any meaning in quantum gravity to explore expectation values of gauge covariant operators in kinematical states. The answer to this is yes in the semiclassical approximation. In this, we can still obtain the classical geometry as described above, and the corrections in the same gauge as the classical metric. For a complete quantum system, or a non-perturbatively quantized system, the observables have to be physical to extract any geometrical meaning of the system.

If we observe the diffeomorphism constraint and the Hamiltonian constraint they are:

$$H_a = -D_b [K_a^J E_J^b - \delta_a^b K_c^J E_J^c] \quad (54)$$

$$H = \frac{1}{\sqrt{\det q}} [K_a^L K_b^J - K_a^J K_b^L] E_J^a E_L^b - \sqrt{\det q} R \quad (55)$$

it can be easily seen that this is satisfied by the classical values of $K_a^I = K_{ab} e^{bI}$, E_I^a as shown in [15], and as the expectation values of the operators are constructed to be the classical values at order 0, this is guaranteed to be satisfied. From [15],

$$K_r^1 = \frac{1}{2r} \sqrt{\frac{r_g}{r}} \quad K_\theta^2 = \sqrt{\frac{r_g}{r}} \quad K_\phi^3 = \sqrt{\frac{r_g}{r}} \quad (56)$$

$$E_1^r = r^2 \sin \theta \quad E_2^\theta = r \sin \theta \quad E_3^\phi = r \quad (57)$$

At order t , the corrections to E_J^b and K_a^J are easily seen to satisfy the diffeomorphism constraints as obtained from the coherent state expectation values. This follows from the simple observation that the corrections to E_J^b are proportional to E_J^b (from (40) and K_a^J corrections are parallel in the internal direction to E_a^J [17], making the $H_a = 0$. Quoting

the correction to the holonomy as in [17], we get:

$$\delta h_{AB} = e^{-\tilde{t}/16} e^{-p^2/\tilde{t}} \frac{z_0}{\sinh(z_0)} \frac{\sinh p}{p} \left[g_{AB} \cosh\left(\frac{z_0}{2}\right) + (T^J g)_{AB} \frac{\text{Tr}(T^J g g^\dagger)}{2 \sinh(z_0)} \sinh\left(\frac{z_0}{2}\right) \right] \quad (58)$$

where p is a momentum, and $z_0 = e \tilde{t} p$, where p corresponds to invariant momentum for a given edge of length e . As evident, the corrections to the holonomy h_{AB} are proportional to g_{AB} , the classical matrix. The corrections to the K_a^I are obtained from the observation that

$$h \propto 1 + \int (K_a^I + \Gamma_a^I) T^I dx^a$$

The corrections in the ‘internal’ directions J can be obtained using $\text{Tr}(T^J h)$, and interpreting them as corrections to K_a^J with appropriate regularisations. In the following, we shall discuss only the scheme to identify the corrections in the J th internal directions. If we find $\text{Tr} T^J \delta h$ in the above, then, the corrections are seen to be proportional in internal directions to the original ones. The first term in (58) is easily seen to coincide in the original direction as g . The second term is of the form $(T^K g \tilde{P}^K)$ where we have written $\tilde{P}^K = \text{Tr}(T^J g g^\dagger) \sinh(z_0/2)/2 \sinh(z_0) \propto P^K$ as the holonomy cancels from the $g g^\dagger$. If we compute $\text{Tr}(T^J T^K g \tilde{P}^K)$, we find the only non-zero contribution to the trace is of the form $g^0 \tilde{P}^J$, which is proportional in internal directions to the momenta and hence E^J . (g^0 is the coefficient of g in the identity direction.)

The H_a and the first term of the Hamiltonian H are 0 because of the diagonal nature of the K_a^I and the E_a^I . It is shown in the above discussion that the corrections δK_a^I and δE_a^I are proportional to the original variables, ensuring that at order $\tilde{t}$ the corrected ones are also zero. The Hamiltonian constraint H receives a correction to the $\sqrt{q}R$ term proportional to the $f(P)$, as the intrinsic curvature of the ‘classically flat’ slice gets corrected. This can be easily calculated and verified. This can be cancelled by a ‘correction’ introduced to the Hamiltonian at this order. We can interpret this as a ‘quantum corrected’ source term, or a quantum energy momentum tensor. The details of this will appear in a publication in progress.

The problem of constructing physical observables is a important task, and details of building gauge invariant observables as well as physical states is work in progress [16]. As evident the quantum fluctuation (50) can be interpreted as a ‘strain’ in the background space-time. This was computed to be of the order of 10^{-125} in [17]. The above strain can be compared to the strain produced by a gravity wave [3], which is found to be of the amplitude

10^{-21} . Though the quantum fluctuations might be real, the magnitude is infinitesimal as compared to the gravity wave strain, and thus beyond the sensitivity of current day detectors.

C. Effect of corrections on a Scalar Field

In this paper, we are asking the question: what does this new correction imply for the physics of the system? In [17] this question was discussed for a toy model which simulates scalars on the twisted geometry. In this article, we actually explore the behaviour of a scalar field near the horizon, using numerical programming, and try to see if the new correction makes a tangible difference. We explore the field near the horizon, as $g^{tr} \rightarrow 1$ near the horizon, being a ratio of the semiclassical parameter t and $(1 - r_g/r)$, and as the latter tends to zero near the horizon. However, it is to be noted that in these coordinates, the g^{tr} is still a correction as the metric is singular at the horizon. The ratio of correction to the actual metric is still infinitesimal. To find the behaviour of the scalar field near the horizon, we solve for the scalar field Φ as follows:

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \Phi) = 0 \quad (59)$$

$$\begin{aligned} \frac{1}{\sqrt{-g}} [\partial_t (\sqrt{-g} g^{tt} \partial_t \Phi) + \partial_r (\sqrt{-g} g^{rr} \partial_r \Phi) + \partial_t (\sqrt{-g} g^{tr} \partial_r \Phi) \\ + \partial_r (\sqrt{-g} g^{rt} \partial_t \Phi) + \partial_\theta (\sqrt{-g} g^{\theta\theta} \partial_\theta \Phi) + \partial_\phi (\sqrt{-g} g^{\phi\phi} \partial_\phi \Phi)] = 0 \end{aligned} \quad (60)$$

$$-\partial_t^2 \Phi + 2g^{tr} \partial_t \partial_{r^*} \Phi + \frac{1}{r^2} \partial_{r^*}^2 (g^{tr*} r^2) \partial_t \Phi + \partial_{r^*}^2 \Phi + \left(1 - \frac{2GM}{r}\right) \nabla_{\theta\phi} \Phi = 0 \quad (61)$$

In the last step we have implemented the transformation of the r coordinate to $r^* = r + 2GM \ln(r/2GM - 1)$, known as the Eddington-Finkelstein coordinate. As evident above, the $(1 - 2GM/r) \rightarrow 0$ in the above, and thus we can set the angular part of the equation to zero.

The scalar equation at the horizon is

$$-\partial_t^2 \Phi + 2g^{tr} \partial_t \partial_{r^*} \Phi + \frac{1}{r^2} \partial_{r^*}^2 (g^{tr*} r^2) \partial_t \Phi + \partial_{r^*}^2 \Phi = 0 \quad (62)$$

which we can write as:

$$-\partial_t^2 \Phi + 2g^{tr} \partial_t \partial_{r^*} \Phi + \partial_{r^*} (g^{tr*}) \partial_t \Phi + \frac{2g^{tr}}{r} \frac{dr}{dr^*} \partial_t \Phi + \partial_{r^*}^2 \Phi = 0 \quad (63)$$

Now, to numerically solve the system for $\phi(t, r^*)$, one has to convert the g^{tr} to r^* coordinate. As we have stated earlier, we solve for the scalar equation near the horizon, we shall almost approximate $r \approx r_g$.

$$g^{tr} = \pm 2 \frac{1}{(1 - r_g/r)} \left(\frac{r_g}{r} \right)^{3/2} \tilde{t} \tilde{f}$$

We set $r \approx r_g$ in all the terms, except the one which is singular at the horizon, as that would contribute to the equation non-trivially

$$g^{tr} = \pm N \frac{1}{(1 - r_g/r)}$$

where N is a number. Given the definition of r^*

$$r^* = r + r_g \ln \left(\frac{r}{r_g} - 1 \right)$$

we can obtain this exponential after dividing both sides by r_g ;

$$e^{r^*/r_g} = \left(\frac{r}{r_g} - 1 \right) e^{r/r_g}$$

or

$$e^{r^*/r_g} = \left(1 - \frac{r_g}{r} \right) \frac{r}{r_g} e^{r/r_g}$$

we set $r \approx r_g$, we can approximate the equation as:

$$e^{-r^*/r_g + 1} = \frac{1}{1 - r_g/r}$$

Given this equation, we can rewrite g^{tr} as a function of r^* . For the purpose of mapping this to a numerical grid, one uses the variable x in the interval $[0, 1]$. Note $r^* \rightarrow -\infty$ at the horizon and we map the variable r^* to a finite interval for $x \in [0, 1]$. To achieve this we take $x = (r^* - r_0^*) / (r_M^* - r_0^*)$. To maintain the equation, we scale r^* and t by the same number. We find that the equation is invariant, but we are able to write the equation in $x = (r^* - r_0^*) / (r_M^* - r_0^*)$, such that, $x = 0$ at $r^* = r_0^*$, and $x = 1$ at $r^* = r_M^*$.

$$e^{-x(r_M^* - r_0^*)/r_g + 1 + r_0^*/r_g} = \frac{1}{1 - r_g/r}$$

The equation appears as

$$-\partial_t^2 \Phi + \partial_x^2 \Phi + 2\beta \partial_t \partial_x \Phi + \partial_x \beta \partial_t \Phi = 0 \quad (64)$$

FIG. 5: Numerical Grid

where $\beta = N \exp(-x(r_M^* - r_0)/r_g + 1 + r_0^*/r_g)$. We start with a Gaussian scalar pulse at $t = 0$ and evolve the system in time. We show the graphs for the pulse evolving in time with $\beta = 0$, and then compare with graphs evolving with β non-zero. There is a clear distinction between the two cases, with the Gaussian wave splitting into two waves in the first, and in the second, the split breaks into asymmetrical waves. Whereas this is not surprising, the physical consequence of this might emerge in the calculation of Hawking flux through the horizon and scalar field collapse to form black holes.

D. Numerical Results

Here we discuss the numerical calculations which describes the flow of the scalar field near the horizon. To obtain numerical answers, we discretize the scalar equation near the horizon, using methods of finite difference equations [18]. Here we approximate the time step as k , and space step as h . We take the ϕ field as a discretized $u(x, t)$ defined on a two dimensional grid of points $t_0, t_0 + k, t_0 + 2k, \dots$ and $x_0, x_0 + h, x_0 + 2h, x_0 + 3h, \dots$ as coordinates along the time axis and the space axis. The derivatives are approximated as

$$\partial_t^2 \Phi = \frac{1}{k^2} [u(x, t + k) - 2u(x, t) + u(x, t - k)] \quad (65)$$

The space derivatives as

$$\partial_x^2 \Phi = \frac{1}{h^2} [u(x + h, t) - 2u(x, t) + u(x - h, t)] \quad (66)$$

The mixed derivative is approximated using the following so as to obtain a flow from one time step to another according to the grid shown in the figure.

$$\partial_t \partial_x \Phi = \frac{1}{2kh} [u(x + h, t) - u(x + h, t - k) - u(x - h, t) + u(x - h, t - k)]. \quad (67)$$

Using these approximations in the Equation(63), one obtains the finite difference equation, which is essentially a ‘hyperbolic equation’ as

$$\begin{aligned} u(x, t + k) = & 2(1 - r)u(x, t) - q(x, t)u(x, t) + (r - p(x, t))u(x + h, t) + \\ & (r + p(x, t))u(x - h, t) + p(x, t)u(x + h, t - k) - p(x, t)u(x - h, t - k) \\ & + q(x, t)u(x, t - k) - u(x, t - k) \end{aligned} \quad (68)$$

where

$$r = \frac{k^2}{h^2} \quad q(x, t) = \frac{k}{h} \partial_r g^{rt} \quad p(x, t) = g^{rt} \quad (69)$$

The finite difference equation is then used to numerically evolve the scalar pulse sent as a Gaussian $\exp(-(x - 1)^2/0.002)$ at time $t = 0$ and evolved to 20 time steps. The x-grid is also comprised of 20 grid points, with the scalar field forced to be zero at $x = 0$ and $x = 1$. Thus, we prevent any information entering from the boundaries, as a way to evolve the grid. The plots of the scalar field without the twist field and with the twist field are then obtained using gnuplot and shown as in Figures 4,5,6.

At time $t=2$, the initial Gaussian is shown to break into two peaks, one of which is left moving, and the other which is right moving $\Phi(x + t), \Phi(x - t)$ for both the situations as shown in Figure 4.

At time $t=11$, the oscillations set in, as the pulse propagates to both sides starting from the centre, Figure 5 .

FIG. 6: scalar field at time step=2, evolved using BH metric without twist (on left), and then with twist (on right).

FIG. 7: scalar field at $t=11$, evolved using BH metric without twist (on left), and then with twist (on right).

At time $t=18$, the solution with no twist field propagates as a simple wave, however, there is a non-trivial behaviour change for the scalar field at $x=1$, which has been set as the ‘near horizon’ location Figure 6. We then see the next time steps of the propagating scalar field with just the quantum correction in Figure 7.

The analysis of this is in the observation, that due to the presence of the β function which grows to large values near the horizon, the left-right symmetry of the scalar propagation is broken. The amplitude of the scalar wave near the horizon increases, and because of the boundary condition creates a wave which oscillates at the boundary at $x = 1$. However, the question remains if this will influence Hawking flux through the horizon, or other phenomena

FIG. 8: scalar field at $t=18$, evolved using BH metric without twist (on left), and then with twist (on right).

FIG. 9: scalar field at $t=16, 17, 18, 20$, for BH metric with twist, the y-axis scale is enlarged in the later three steps, to show the near-horizon oscillation.

relevant for the system. In particular if classical accreting material can be modelled using a scalar field, then the correction will produce non-trivial effects on in-falling matter. Note that if we relax the condition that the function is 0 at $x=1$, which is artificially input, then the scalar field amplitude increases to very large values near the horizon, and the oscillation persists. This has been checked using numerical coding, and further interpretations of this will appear as a signature of this might be emergent in a ‘echo’ as predicted from gravitational

FIG. 10: Plot of scalar field without the restriction $\phi=0$ at $x=1$.

waves of black holes. We include some graphs of the code without the boundary condition that the scalar field is 0 at $x = 1$, as in figure (10). The code crashes at the 15th time step. (Note that this code uses some parameters slightly different from the previous code with different boundary conditions.) The crashing of numerical codes at the horizon in Schwarzschild coordinates is not unknown, horizon excision techniques are used. However, the main result is still as affirmed earlier. There is a non-trivial effect of the correction on the scalar field.

If we relax the t parameter to finite values, then the behaviour of the wave shows a speeding shift of the split Gaussian towards $x = 1$, and the bigger oscillations set in earlier time and space step, after which the code gets disrupted. Note that unlike the example of a freely falling scalar field at the horizon, which has a ingoing and outgoing flux at the horizon, the correction, introduces a ASYMMETRY, which grows with time. The Ingoing mode is ‘enhanced’ at the boundary. The sign of the correction can be different depending on the nature of the singularity. If the singularity represents a whitehole, the outgoing wave will be enhanced.

A movie animation of the above time evolution can be plotted. Thus we are able to show conclusively that the scalar field behaviour gets disrupted due to the correction at

the horizon. This is not surprising, as if we observe the wave equation near the horizon, then the terms proportional to β dominate and these signify a solution for Φ which grows exponentially near the horizon, apart from the usual oscillatory behaviour.

IV. TOWARDS EXPERIMENTS

In this section, we show some other immediate consequences of the correction to the metric which might be relevant for the event horizon telescope [19]. Black holes have no direct ‘images’, as they absorb everything including light. However, they accrete materials from their surroundings. These form accretion disks which can be seen, using telescopes. There have been various models of X-ray spectra from accretion disks, and some of them seem to confirm that indeed an event horizon exists. Currently the ‘event horizon’ telescope experiment seeks to observe the event horizon at a much closer angular resolution than previously. A very relevant question which had been asked by theoreticians was what happens if we send a beam of light directly at a black hole? The answer is that the photons reaching unto a certain radius get reflected back/circle the black hole. Light arriving within that radius, get completely absorbed by the black holes’ gravitational field. This absorption cross section is the area of the sphere of with radius equal to the critical radius, which is bigger than the event horizon. This critical radius can be found by solving for the trajectory of a photon, the ‘null geodesic’ in the geometry of the Schwarzschild black hole.

The equation of the trajectory of the null particle is given as

$$\left(\frac{1}{r^2} \frac{dr}{d\phi}\right)^2 + \frac{1}{r^2} \left(1 - \frac{2GM}{r}\right) = \frac{1}{b^2} \quad (70)$$

where b is the impact parameter. This can be derived from the Schwarzschild metric by setting the $ds^2 = 0$, and requiring that the translations along the Killing timeline direction has constant orbits of energy E and that the angular momentum of the trajectories are also constant.

The maximum of the potential can be solved as that is the critical radius for which $dr/d\phi = 0$ or turning points can exist for a given impact parameter b . This critical radius is found to be $b_c = 3\sqrt{3}M$. In this article we show that this gets modified due to the presence of the ‘quantum fluctuation’ of the metric. The potential term gets modified to

$$V(r) = \frac{1}{g} \left[\frac{L^2}{\tilde{q}} + \frac{E^2}{f} + l \right] \quad (71)$$

where

$$f(r) = -\left(1 - \frac{r_g}{r}\right) + h_{tt} \quad g(r) = \left(\frac{r}{r - r_g}\right) + h_{rr} \quad \tilde{q} = r^2 \sin^2 \theta + h_{\phi\phi} \quad (72)$$

The corrections can be computed by the methods of the previous sections using the LQG coherent state. As we saw these corrections are proportional to the semiclassical parameter tF , where F is some function of the Schwarzschild radius.

The critical radius changes to

$$b_c = 3\sqrt{3}GM (1 + \tilde{t}f(P) + ..) \quad (73)$$

This gives rise to a correction to the absorption cross section of the black hole

$$\sigma_{\text{abs}} = 27\pi\sigma_0 (1 + O(\tilde{t})) \quad (74)$$

This is a tangible correction, of the order of 10^{-9} using approximation of [15], is difficult to detect at the current precision levels of the instruments. However, the quantum fluctuations can give rise to interference patterns etc of the ‘image of the spherical black hole’.

V. CONCLUSION

Given that Gravity is very weak, and Planck scale is $10^{-33}cm$, quantum effects of gravity appear at this time elusive for experimental tests. However, for any true theory of nature, the eventual confirmation has to come from nature. In [14], I showed that quantum gravity corrections can be magnified due to instabilities in black hole orbits. It is hard to detect these instabilities in isolation to other instabilities of accreting materials hurtling into black holes. In the current article, I showed that quantum gravity corrections affect matter propagation, such as scalar matter and light. The scale of corrections are yet very tiny, but there might be indirect effects which will be observationally verified. In particular gravity wave observations are probing fluctuations of classical geometry to the order of 10^{-21} . The data published online has plenty of opportunities for quantum gravity experts to test their theoretical predictions [20] (The attention to this work was brought to my notice by Sabine Hossenfelder’s blog backreaction.blogspot.ca).

Recent work has tried to predict phenomena for primordial black holes and link them to Fast radio bursts observable today [21]. Further, Planck scale remnants and Hawking

radiation might also be detectable [22] in the very near future. In this paper a very particular correction in LQG using semiclassical states which are non-perturbative have been explicitly computed. The enhancing of the ingoing wave at a black hole horizon will have consequences for semiclassical phenomena such as Hawking radiation and superradiance. LQG can thus be tested with precision as classical gravity. It is certain that ‘quantum predictions’ will be tested in the near future as gravitational wave window opens up to the cosmos. Whether the various theories like LQG, Causal Set theory, String Theory survive these tests or an entirely new approach to quantum gravity will start is an open question.

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