Spatial Skills Activities in the Middle School Mathematics Teachers' Toolkit: The Impact of Spatial Skill Activities on Mathematical Thinking

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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Abstract

This study investigated the impact of the incorporation of spatial skill activities into their pedagogical repertoire by middle school mathematics teachers on the mathematical reasoning of their students. Current research suggests that success in spatial reasoning is a strong predictor of success in STEM fields, and is known to be strong in successful mathematicians (Newcombe, 2013; Tepylo & Moss, 2013; Uttal et al., 2012). In this study, teachers replaced starter activities three or more times per week with hands-on spatial skill problems that required mental rotation and spatial visualization to solve. While there have been many studies linking spatial skill to success in mathematics, there are few studies that attempt to establish a relationship through intervention in a middle school classroom. Findings suggest that incorporating spatial skill activities into mathematics lessons had a positive impact on both the teachers’ reflective practice and the students’ learning skills. While there was some improvement in mathematical reasoning, it was not possible to definitively attribute this to the spatial skill activities.
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Writing a dissertation later in life, while balancing work and family is a lonely business. Frequent travel meant that often I had to connect with colleagues and advisors via Skype or other long-distance forms of communication.

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Chapter 1 - Introduction

The purpose of this study is to understand the impact of incorporating spatial skill activities into the middle school mathematics classroom on the mathematical reasoning of these students and the pedagogy of their teachers.

My interest in the impact of spatial skill reasoning on mathematical reasoning began back in the early nineties when I was working as a Middle School Mathematics teacher at Branksome Hall School, and writing a paper for my Master’s degree. At that time, I was teaching an enriched Mathematics course for girls. I wondered if there was a common characteristic that the girls in my enriched class all shared. I questioned what it was that made mathematics easy for me? When I discovered that I would be able to work with girls identified as gifted in mathematics (they scored in the 97th percentile or higher on the TIMMS test) I was hopeful that I might get an answer. Not only were these adolescent girls talented young mathematicians, but they also possessed excellent analytical skills and could probably help me design a survey that might elicit the answer. The fact that they were of an average age of twelve proved to be helpful in answering why this is the age that many girls switch from being comfortable with mathematics to becoming anxious and struggling.

What did we discover? The girls and I learnt that we had had a small handful of shared experiences as young children and that we were all attracted to, and adept at, certain types of puzzles. It turned out that we had all been exposed to both gross motor and fine motor spatial activities as pre-school children and that we were all particularly comfortable with spatial tasks (Younger, 2009). Over the next seventeen years, I had an opportunity to explore these questions both formally as part of my professional duties bringing about changes in mathematics education in developing countries, and informally as “fun” activities, warm-ups,
and breaks during my traditional mathematics classes. My observations led me to think that there may be a causal relationship between spatial skill and mathematical reasoning.

Fast forward to the years 2004 to 2013, I had the incredible opportunity of being a part of educational reform in the United Arab Emirates. During my time there, I was involved in training government mathematics teachers in modern pedagogy for mathematics and Assessment for, as and of learning with a focus on mathematics; I also helped design mathematics curriculum. While working with the teachers and their Emirati students, I began integrating my spatial activities and continued my inquiry into the relationship between spatial reasoning and mathematics. The teachers enjoyed having their students play with the games, and during collegial conversations discussed how these games had inspired them to think of alternative ways of introducing mathematics content. I watched the teachers change their teaching style by engaging their students through spatial skill activities.

I asked the following questions: Could it be that spatial skill was an important key to mathematical success? Could spatial skill be trained? What would happen if we trained “weak” math students in spatial skills? Would they become more confident and successful mathematicians? Do teachers spontaneously expand their teaching repertoire when they discover the benefits of spatial skill activities?

This chapter outlines the plan of the study and the research questions, and situates the research within the current literature on spatial reasoning and its implications for improving mathematics education.

1.1 Spatial Reasoning

Research literature throughout the twentieth century has defined spatial reasoning in a variety of subtly different ways (Harle & Towns, 2010; Mohler, 2009). Mathematical
researchers explain that when spatial reasoning is used in the application of mathematical reasoning it “refers to the application of visual and spatial representations (e.g., diagrams, physical or dynamic graphical models, and mental imagery) and processes (e.g., composing, decomposing, mental and hands-on moving)” (Sinclair, Mamolo, & Whiteley, 2011).

However, this explanation only describes one of six spatial factors defined by Kimura (2002), who also includes spatial orientation, spatial location memory, targeting, disembedding, and spatial perception. Gardener (1991) considered visuospatial reasoning to be one of seven distinct human bits of intelligence in his original books, and still includes it today as he adds additional intelligence constructs to his list. Others find spatial reasoning to be a unique form of intelligence distinguishable from verbal reasoning and logical reasoning: a subset of non-verbal reasoning (Fibonicci, 2016). In general, spatial abilities are considered to be a form of mental activity that enables individuals to create spatial images, and to manipulate them in solving various practical and theoretical problems (Hegarty & Waller, 2005).

Park, Lubinski and Benbow (2010) define spatial ability as “a capacity for mentally generating, rotating and transforming visual images” (Recognizing Spatial Intelligence; para. 3). They stress the importance of developing this type of reasoning and warn that the “neglect of spatial ability in school curricula” (Recognising Spatial Intelligence; para. 9) not only ignores the population of students with spatial reasoning strengths, but fails to develop these strengths which are known to be an important factor for success in the scientific and technical workforces.

Spatial reasoning can be taught and improved upon (Newcombe, 2013), yet this type of thinking is not commonly included in the teaching and learning of mathematics at the middle school level. In the primary and elementary levels, activities that involve gesturing, playing
with blocks and puzzles, storytelling, working with drawings, animations, and video games are useful in developing spatial reasoning skills (Hawes et al., 2015). As Kinach (2012) outlines:

> despite the overwhelming support for spatial thinking as a worthwhile educational activity, tensions within the school curriculum have resulted in downplaying geometry and elevating either numeracy (primary level) or algebra (secondary level). (p. 536)

Performance in mathematics at the middle school level has not yet reached a level that satisfies all constituent parties such as educators, parents, students and political bodies. “The mathematics of the elementary and middle school curriculum is not trivial, and the underlying concepts and structures are worthy of serious, sustained study by teachers “(Kilpatrick et. Al, 2001, p. 373). Students and their middle school teachers need to expand their mathematical repertoire in order to increase the opportunities for improved performance.

With today’s twenty-first century focus on careers in the fields of engineering and technology, students are expected to emerge from middle school mathematics classes with a consolidated understanding of mathematical concepts and processes, that allows them to further expand the depth of their understanding of secondary mathematics. An evolving global society demands both experts and technicians proficient in skills required for Science, Technology, Engineering and Mathematics (STEM) careers, with mathematics being an essential requirement for all STEM fields. Spatial ability has been shown to be a reliable indicator of future success in STEM fields (Wai, Lubinski & Benbow, 2009).

The deliberate use of spatial teaching in education is sparse (Newcombe, 2010). Ongoing research by UNESCO into the value of encouraging teachers to expand their teaching tool kit to include spatial activities has been further supported by a 2014 document produced by the Ontario government titled “Paying Attention to Spatial Reasoning”. This research has slowly been filtering its way into the administrations of Ontario public schools, and has
resulted in an increased awareness of the impact of spatial reasoning skills on students’ increased understanding of mathematical skills (UNESCO, 2012; Sinay & Nahornick, 2016).

Many middle school mathematics subject specialist teachers themselves were, for the most part, educated in a system that did not include spatial reasoning other than basic concepts related to geometry and measurement. “Too many teachers of mathematics remain professionally isolated, without the benefits of collaborative structures and coaching, and with small opportunities for professional development related to mathematics teaching and learning” (NCTM, 2014, p. 3).

Teachers should be exposed to the relationship between spatial reasoning and success in Science, Technology, Engineering and Mathematics, known as (STEM) fields, to consolidate an understanding of the link between spatial reasoning and mathematical reasoning (Newcombe, 2010). These teachers should be encouraged to use these activities in their classrooms. Including educators in the research process both validates their knowledge and allows for an environment that will promote an understanding of the link between spatial reasoning and mathematical reasoning.

1.2 Impact on Students

Students need to think about and use mathematics in a meaningful way; simply exposing students to topics is not enough (Growing Success, 2010). Cobb and Yackel (1998) state that “the tools with which students interact profoundly influence the mathematical ways of knowing that they develop [and] has come to orient our approach to instructional design” (p. 188). The addition of spatial skill tasks allows the students to consider mathematics from a more holistic viewpoint, adding to and possibly replacing rote algorithms and memorized facts as a transferable skill for problem-solving (Wright et al., 2008).
The precise mechanism for how spatial ability is related to mathematical reasoning is an evolving question; however, incorporating these tasks should complement and exercise the “three main components to the reasoning process: conjecturing, generalising and justifying… [which] are interrelated and students can move through them in any order” (Lannin et al., 2011, p. 1). These moments of insight are more likely to occur through actively engaging—physically, intellectually and emotionally—with the process of learning, rather than through the passive acceptance and reproduction of a prescribed set of algorithms to solve an abstract and possibly irrelevant problem (Cobb, 1994; Dewey, 1916).

The act of discovery occurs at the point in the learner’s efforts at which he grasps the organizing principle embedded in a concrete instance or in a series of instances and can, therefore, transform this information: the learner can see the relationship of the facts before him, he can understand the causes of the phenomenon, and he can relate what he sees to his prior knowledge. This point in the learner’s efforts is also referred to as “the moment of insight. (Taba, 1963, p. 311)

Therefore, curriculum design must take into consideration learning experiences that help “learners [to] get at the structure, or at the laws and principles of a subject, by allowing them to discover’” (Taba, 1963, p. 308) that “aha” moment in the process of learning. These learning experiences might involve interacting with spatial games and activities designed to activate learner spatial reasoning to complement mathematical reasoning. Recommendations for developing student understanding include creating a semi-structured series of challenges based on established learning trajectories that encourage equal quality playing time with spatial-manipulative toys with spatial discussions that will benefit all children (Clements & Sarama, 2009; Hawes et al., 2015).

“Taken together, evidence points to spatial thinking as a strong contributor to both learning processes and learning outcomes” (Hawes et al., 2015 p. 60). Students given the
opportunity to engage with spatial skill tasks while studying mathematics in the middle school classroom are likely to develop and enhance positive learning habits, as well as improved reasoning skills.

1.3 Research Questions

This study is designed to answer the following questions:

- How does spatial skill development affect learning skills amongst middle school mathematics students?
- How does spatial skill development in middle school mathematics affect mathematical reasoning?
- How do teachers’ pedagogy change when using this strategy?

1.4 Significance of the Study

Spatial reasoning is an important skill for everyday tasks, and it has been shown to be necessary for learning in Science, Technology, Engineering, and Math (STEM) domains (Uttal & Cohen, 2012; Wai, Lubinski & Benbow, 2009). Importantly, spatial ability is not fixed; rather, it can be developed through experience (Uttal et al., 2013).

More developed spatial reasoning skills have been shown to foster a positive attitude in students towards mathematics, including improved confidence in mathematical ability beginning at elementary school and continuing throughout high school and post-secondary school (Eccles et al., 1993)

Spatial ability not only plays a unique role in assimilating and utilising pre-existing knowledge but also plays a unique role in developing new knowledge. Without spatial ability, the psychological architecture supporting creative thought and innovative production is incomplete. (Kell, Lubinski, Benbow & Steiger, 2013, p. 1)

This study will contribute to research in the area of spatial skill development and mathematical reasoning by observing, recording and categorising the impact of this development on students’ mathematical reasoning through the incorporation of three-
dimensional and two-dimensional games designed to develop and enhance spatial reasoning into middle school mathematics lessons.

These activities will take place as part of a regular mathematics lesson, administered by the classroom teacher to a normal spectrum of students at this level, thereby putting theory and meta-analysis into practice.

Middle school mathematics teachers will be introduced to additional teaching strategies that should have a positive impact on their students’ learning of mathematics and attitudes towards it.

1.5 Background of Researcher

I started out my professional career as a Statistician at a time when statistical software was limited, and required the use of mainframe computers. When analysing data for clients, I always found it easier to develop a full picture by using a response surface analysis in concert with interpretations of the numerical results. At the time, I did not realise that I was bringing spatial reasoning into play in order to understand and explain the results. Later, I became a secondary school mathematics teacher determined to share my passion for mathematics as both science and art with my students. Early on in my teaching career, it became apparent that the traditional method of teaching mathematics was not adequate for all of my students.

I began to experiment with different pedagogical practices such as use of colour, diagraming, and gross motor activities with the students to understand concepts; I even had my husband build a 3m x 3m tangram for my students to explore. I noticed that, with these activities, more students were able to grasp the concepts, and their engagement and positive attitude towards mathematics improved. The most obvious improvement was amongst the students who were struggling with the grade-appropriate content, with some improvement
observed amongst the middle group. The stronger mathematics students were already, or quickly became, adept at the spatial tasks, and little change in their performance was seen.

A desire to more fully understand the process of learning mathematics led me to return to university for a M.Ed. degree. My research focused on the relationship between physiological brain development and how girls learn math. Admittedly, being the early nineteen-nineties, these were very early days in the fields of neuroscience and education, but this experience did raise my curiosity as to how valuable exposure to spatial skill activities is in the early years of life for future success in mathematics. I began my first steps down a path that is now eighteen years on in researching the relationship between spatial skill reasoning and the understanding of mathematical concepts.

Following on the results of my study with talented young female mathematics students, which found a common theme of early exposure to both fine motor and gross motor activities, and the observation that my highly functioning mathematics students appeared to already be adept at spatial skill games, I asked the administration at the school if I could work with a couple of classes of girls who were struggling with mathematics at the grade nine and ten level. These classes were small, with six to ten students. I was given permission to use my spatial games on a daily basis, as well as to use a variety of teaching strategies that were not commonly used at this level of mathematics instruction.

The girls played with blocks, used colour, engaged in interactions with huge geometric figures, played with the spatial games and created three-dimensional rotating polyhedrons from nets that they drew with a compass and protractor. Concurrently, the grade nine and ten Ontario mathematics course was taught. Most of the girls were pretty negative about mathematics at the beginning of the school year, and it took a fair amount of convincing to
establish that initial engagement with the activities. Once they started having success, many areas improved. The girls were more receptive to learning, more willing to try novel approaches, and were less negative about their own self-images as mathematics students. All of the girls successfully completed the course, some passing math comfortably for the first time in their school life.

I was intrigued. Was this the effect of the games and activities that I believed in so strongly? Was it because it was a small group? Was it because I was so determined to show these girls that math could be pleasurable, and that they were capable? Was it just an artefact of their development, and that the “light bulb had finally turned on”? I could not answer that question, but I wanted to.

I experimented with weak math students, incorporating spatial skill activities into their lessons, and found both improved results and attitudes in learning mathematics. I took these experiments to the Middle East, where I was involved in a five-year project to improve the education of Emirati public school students through training their mathematics teachers, rewriting their curriculum and developing effective assessment strategies and practices. In every case, the use of the spatial skill activities improved the students’ learning experiences and comprehension.

I saw improvement in both males and females, in middle school students, secondary school students, and college level students. I wondered if these results were merely a product of my enthusiasm and passion for mathematics. When I returned to Canada, I applied for acceptance to OISE to put this idea that had been rattling around in my head for most of my adult life to a rigorous test.
1.6 Format of Thesis

The dissertation is divided into five chapters. Chapter 1 provides an introduction, overview of research questions and reasoning behind the development of this study. Chapter 2 contains a literature review. Chapter 3 describes the research paradigm, methodology, and method as well as presenting some of the results of the pilot studies that led to the design of this study. Chapter 4 examines the qualitative and quantitative findings. Chapter 5 interprets the findings and reviews the implications of the findings and their possible contributions to mathematics education.
Chapter 2 – Literature Review

2.1 Introduction

The literature review will summarise the current literature on the instruction of spatial skill and spatial reasoning as a teaching strategy to enhance middle school mathematics students’ learning experience. Specifically, it will show that there is a relationship between spatial reasoning and mathematical reasoning, and that being adept in these areas is a strong predictor of success in the STEM fields. Pre-service teacher education, as well as continuous professional development that may impact the evolution of the educator’s instructional practice, will be discussed as an outcome of including spatial skill instruction as part of the mathematics curriculum. This section will conclude with a discussion of learning habits that may be improved by engaging in these spatial skill activities, and their importance for success with middle school mathematics curriculum content.

2.2 Spatial Skill

Spatial skill development is not a new technique and has been shown to be valuable in the training of post-secondary Science Technology Engineering and Mathematics (STEM) students (Uttal & Cohen, 2012). Spatial ability has been defined in a variety of different ways by various researchers, but most descriptions can be categorised into three types of knowledge based on cognitive functions: spatial perception, mental rotation and spatial visualisation (Linn & Petersen, 1985).

Spatial perception is a person’s ability to sense the size, shape, movement and orientation of objects (Hegarty & Waller, 2005). Hegarty (2005), in a talk to students interested in pursuing a Master’s degree in Spatial Studies, simplifies spatial thinking into two broad areas: thinking about space (paper folding, mechanical thinking) and using space to think
(graphing, “spatial: habits of mind”). Spatial perception is considered to be a natural process of the brain, and is a function of depth perception and the navigation of one’s body through space (Eckersley, 2012). By contrast, mental rotation and spatial visualisation require the brain function of imagining and predicting how an object might appear if it is moved linearly, laterally or rotationally. For this study, spatial skill will refer to the ability to engage in spatial habits of mind: pattern recognition, spatial description, visualisation, spatial concept use, and spatial tool use (Kim & Bednarz, 2013).

Spatial skills involve an individual’s capacity to “mentally compare, manipulate, and transform visual, non-linguistic information” (Kersh, Casey, & Young, 2008, p. 234), tasks such as mentally manipulating objects, paper folding, mental rotation of 3D images, cross-sectioning tasks, and matching two-dimensional nets to three-dimensional objects. Research demonstrates that mathematicians have an aptitude for these tasks (Newcombe, 2013; Uttal et al., 2012). Spatial reasoning and aptitude with spatial skills can be taught and improved upon (Uttal et al., 2013).

Lubinski and Benbow (2006) established that there is a relationship between spatial skill and mathematical thinking. Whether strength in spatial reasoning leads to strength in mathematics, or vice versa, has not been fully determined. Numerous longitudinal studies have shown that individuals with stronger performance on spatial reasoning tests are more likely to enter STEM fields (Lubinski & Benbow, 2006; Shea, Lubinski, & Benbow, 2001; Wai, Lubinski & Benbow, 2009). Additionally, high performance in spatial tests in middle school and secondary school is a strong predictor of STEM careers, and has more predictive value than verbal and mathematics scores (Tepylo & Moss, 2013).
Despite a periodic history of recommendations to include spatial reasoning as a curricular discipline by panels and committees formed to rewrite mandated ministry education (Whiteley, 1999), spatial reasoning has not been meaningfully incorporated into classroom teaching of science or mathematics (Newcombe, 2010; NCTM, 2015). Mathematics teachers should, therefore, be exposed to the relationship between spatial skill and mathematical reasoning (Sorby, 2009; Sorby & Baartmans, 2005). They must be encouraged to incorporate spatial skills development, in addition to the units of geometry and measurement, as a useful teaching strategy to improve their students’ mathematical reasoning (Battista et al., 1997).

2.3 Mathematical Reasoning

The definitions of mathematical reasoning are as diverse as the subject itself. Graham and Cuoco (2010) define a reasoning habit as “a productive way of thinking that becomes prevalent in the processes of mathematical inquiry and sense making” (p. 4). Many mathematics teachers consider reasoning to be the act of thinking through higher order problems that involve multiple steps (Ma, 1999; Wright et al., 2008). By contrast, Martin (2009) explains that mathematical reasoning can be thought of as the process of drawing conclusions on the basis of evidence or stated assumptions…Sense making can be defined as developing an understanding of a situation, context, or concept by connecting it with existing knowledge. (p. 2)

The American based National Council of Teachers of Mathematics (NCTM), defines mathematical reasoning and sense making as:

drawing logical conclusions on the basis of assumptions and definitions. Sense making involves developing an understanding of a situation, context, or concept by connecting it with other knowledge. Reasoning and sense making are closely interrelated. (p. 4)
The NCTM goes on to outline the importance of the direct teaching of mathematical reasoning from Kindergarten and continuously throughout a student’s school experience, stating that it must regularly be practiced to ensure it becomes a habit.

For this study, the NCTM definition of mathematical reasoning, which includes a greater diversity of cognitive skills required to solve a mathematical question, will be used.

2.4 The relationship between spatial skill and mathematical reasoning

An increase in spatial reasoning ability may lead to improved mathematics achievement and mathematical thinking in geometry as well as other mathematical topics (Casey, Nuttall, & Pezaris, 2001; Delgado & Prieto, 2004; Doyle, Voyer, & Cherney, 2012; Tolar, Lederberg, & Fletcher, 2009). In fact, the relationship between spatial reasoning and mathematics appears early in development and continues to persist (Mix & Cheng, 2012). In a study conducted by Casey et al. (2001), adolescents’ performance on spatial skills involving mental rotation and other spatial-mechanical competencies were found to be significantly correlated with student scores on parts of the Trends in International Mathematics and Science Study (TIMSS).

Hawes et al. (2015) describe three main areas of mathematics where the literature has conclusively described spatial reasoning as an important cognitive aspect of comprehension.

They describe some areas of mathematics such as geometry, measurement and linear algebra as inherently spatial, while other topics such as geometry, algebra, mental arithmetic, word problems and advanced mathematics are described as relying heavily on the ability to mentally rotate both two-dimensional and three-dimensional objects (p. 61)

Kytalla and Lehto (2008) argue that the limited use of spatial skill in the context of the learning process of mathematics material may be a contributing factor to poor performance in not only apparently spatial topics such as geometry, measurement, and trigonometry, but also
in more seemingly linear topics such as numeracy. The difficulty lies in designing a study that tests whether improving spatial skill will, in fact, significantly impact mathematical reasoning outside of, or in addition to, routine mathematics instruction (Grattoni, 2007). If it is possible to use spatial interventions to improve mathematics learning, more nuanced analyses are needed (Mix & Cheng, 2012).

It is not clear that spatial training would need to transfer to other spatial tasks to have an impact on math. There may well be productive connections between spatial training and math, even if these do not transfer to other spatial tasks because the transfer could occur at a very specific process level. (Mix & Cheng, 2014, p.4)

The literature proposing a link between spatial reasoning and mathematical reasoning has been around for more than thirty years. The recent attention given to the STEM fields has encouraged educators to investigate possible causal links that would improve success in these areas. Spatial reasoning seems to be a common factor. Investigations into the relationship between spatial reasoning and mathematical reasoning have moved from the collection of correlating historical data to the classroom, where students were assessed for their spatial abilities as well as their comprehension of mathematics. Spatial skills have been shown to rely on neuronal networks partially shared with mathematics (Tosto et al., 2014). Many studies helped to set the stage for this study, which took classroom investigation one step further into researching the possibility of improving the spatial reasoning of middle school students through spatial skill games, and how these might impact their mathematical reasoning.

In a 2001 study by Casey, Nuttall, and Pezaris, 187 eighth-grade students were studied to determine if there was a relationship between their existing spatial reasoning and their success on the TIMMS instrument. The focus of this study was to investigate gender
differences in mathematical success. However, they were also able to make some links between spatial reasoning and mathematical reasoning.

In order to assess spatial reasoning, three tests were administered at the end of the eighth-grade year over a period of two class times. The three tests consisted of: the Vandenberg mental rotation test, the Water levels test and the mechanical reasoning subtest of the Differential Aptitude Test Battery. The Water levels test involves showing the child a picture of an upright glass half-filled with water. The child is then shown pictures of tilted glasses and asked to draw a line, which represents how the surface of the water would look in these glasses. In the Vandenberg test, a three-dimensional object is presented, and the subject is asked to select one or two objects from a set of choices that represents a rotation of the original image (see figure 1).

Figure 1. Vanderberg Test

![Vanderberg Test Example](http://journal.frontiersin.org/article/10.3389/fpsyg.2016.00333/full)

The Mechanical Reasoning test measures the ability to understand the basic mechanical principles of machinery, tools and motion. Each item consists of a pictorially presented
mechanical situation and a simply worded question. Items require reasoning rather than special knowledge.

In a 2004 study by Delgado and Prieto, four hundred and fifty-five thirteen-year-old students were assessed. Three measures were used: the Lexical Access Test, the Mental Rotations Test and a Mathematics Test. The purpose of the study was to assess gender differences in spatial reasoning and success in geometry. No gender differences were found, but success on the mental rotation test was shown to be a good predictor of success in geometry and word problems.

In a 2007 Master’s thesis by Grattoni, sixty-three grade ten students were surveyed to see if they used pictorial images as a problem-solving technique for Trigonometry. The participants were given three spatial tests: the cube comparison test, the paper-folding test and the hidden patterns test. The cube comparison test required identifying a matching cube after it had been rotated a number of times. The paper folding test involved having the student observe a piece of paper being folded a number of times and then having holes punched in it. A description of where the holes will be when the paper is unfolded must be made. The third test required the students to identify a given figure that may or may not be embedded in a geometric pattern.

These students were also administered a survey designed to elicit details of their own methodology for solving trigonometry problems, and whether they imagined pictures when problem-solving. Grattoni (2007) found that successful problem solvers did rely on spatial reasoning.

Kytalla and Lehto (2008) conducted research on the relationship between active and passive visual-spatial working memory (VSWM), and non-verbal fluid intelligence predicted
success in mathematics performance in a standard school setting. One hundred and thirty-five students aged fifteen to sixteen from four different Finnish comprehensive schools participated. The tests were administered during class time by the researchers, as well as by their mathematics teachers over five sessions. The VSWM task was broken down into three parts: passive simultaneous, passive sequential and active VSWM, and was administered during the first session after the students were allowed to practice.

During the second session, the students completed the Raven Matrices to measure fluid intelligence. The teacher conducted the third and fourth session and two 45-minute grade appropriate Finnish mathematics curriculum content tests were completed. Finally, a Finnish Mathematics Test designed to assist grade proficiency was administered at the end of the study.

The results exhibited the significant contribution of non-verbal intelligence in maths performance. Moreover, our findings are subject to suggest that, along with fluid intelligence, different visuospatial skills predict performance in different mathematical domains. The present results suggest that both passive visuospatial storing (measured with short term storage tasks), active visuospatial processing (mental rotation) and fluid intelligence are crucial in maths skills. Different (visuospatial) working memory components seem to be required in different mathematical domains or task types. (Kytalla & Lehto, 2008, p. 89)

Tolar, Lederberg, and Fletcher (2009) investigated the relationship between cognitive abilities, arithmetic skills and achievement in the domain of algebra. The participants were college students who completed a number of tasks to measure algebra achievement, the Scholastic Assessment in Mathematics (SAT-M), structural equation modeling, working memory, three-dimensional spatial ability and computational fluency.

While it was found that computational fluency had a strong effect on algebra reasoning, 3-D spatial ability proved to have a stronger impact on SAT-M scores. This affirms the theory that solid spatial reasoning skills have a positive impact on SAT-M scores.
More recently, Mix and Cheng (2014) tested the hypothesis that mental rotation training could improve mathematics performance in fifty-eight six- to eight-year-old primary school students. The students were initially assessed for a range of numeracy and mathematics skills before being randomly divided into two groups. One group received mental rotation training, based on a previous study by Ehrlich, Levine and Goldin-Meadow (2006), which had demonstrated improved spatial ability in this age group. The second group practised their crossword puzzle solving skills.

Both groups were administered pre- and post-tests designed to measure proficiency in mental rotation, spatial relations and mathematics. The results were quite conclusive and showed that even a single session of spatial training led to significant improvement in some mathematics problems (Mix & Cheg, 2014).

Hawes, Moss, Caswell and Polisczucuk (2015), in conjunction with the University of Toronto Department of Neuroscience, investigated whether spatial thinking could be changed and how development of spatial skills could improve mathematics performance. They randomly assigned sixty-one six- to eight-year olds to a six-week intervention that was either mental rotation emphasis or literacy emphasis. iPads were used as part of the daily classroom routine for the students to practice their assigned intervention.

Pre- and post-tests were administered one week before and one week after the study period. While the students in the mental rotation emphasis group demonstrated measurable improvements in mental rotation over the literacy emphasis group, transferrance to mathematical reasoning was not evident.

Although we found no evidence that spatial training led to gains in children’s calculation skills, the current study was short in duration and narrow in the spatial skills trained (i.e., mental rotation). More comprehensive and sustainable intervention
approaches are recommended to better take advantage of the historically tight relationship between spatial thinking and mathematics. (Hawes, Moss, Caswell & Polisczuk, 2015, p. 67)

The design and methodology of these studies contributed to the development of the pilot study, and subsequently the main study that is this dissertation.

2.5 The training and education of mathematics teachers

Student success in mathematics has always been a desired outcome by commercial societies. Ministry-mandated content, assessment strategies and pedagogical techniques are regularly reviewed, revised, and rewritten, but the ongoing struggle associated with classroom mathematics instruction continues. Typically:

traditional approaches had worked for no more than 5% to 15% of the students; what was needed was a challenging mathematics curriculum that prepared every student to think mathematically—to develop the foundations in mathematical reasoning, concepts, and tools needed for advanced mathematics education as well as enlightened living in the age of technology. (NCTM, 21st Century Mathematics, 2000, p. 9)

Students’ failure to achieve desired mathematics learning outcomes has been laid at the feet of teachers, curriculum developers, students’ motivation, and gender differences as well as parents’ cultural differences (Hersh & John-Steiner, 2011; Matteson et al., 2011; Pepin & Trouche, 2011).

Novice and experienced teachers are held accountable for their teaching practices, student content mastery, creative and meaningful assessment technique, and for allowing cultural biases and gender bias to influence their teaching (Hill-Jackson, Lewis, & McLaren, 2010; McClam, 2005; McPherson & Anid, 2014). Teachers have been accused of reluctance to incorporate “new” methodologies such as Information and Communication Technology (ICT), differentiation, problem-based learning, Assessment for, as and of learning in addition to a
plethora of other failings often attributed to the people we hope are contributing to the betterment of mankind through educating our children (Noyes, 2004).

Selden and Selden (1995) describe the challenges of pre-service as follows:

The way teachers explain mathematics depends, to a large extent, on the conceptual grasp they acquire in their college classes. In addition, they often teach as they were taught, modeling themselves after their college mathematics teachers as much as their K-12 teachers…. [Teacher training] should provide prospective teachers with an opportunity to strengthen their pedagogical content knowledge, in particular, help with relating various representations (models) and applications to algorithms and procedure. (p. 1)

Post-secondary faculties of education programs may not provide adequate apprenticeship situations that would allow aspiring teachers to consolidate and internalise effective pedagogical strategies necessary for the broad diversity of learners that they will find in their classroom (Monroe et al., 2010). Teachers who work in middle schools as subject specialists bring with them their confidence issues as well as their pre-conceived ideas of what teaching and learning look like within their chosen discipline (Kiefer et al., 2013). “Novice teachers are challenged with balancing theory with student instruction” (Carr, 2013, p. 17). Elementary and middle school pre-service teachers who will be required to teach all subjects may not be comfortable with their own grasp of mathematics, and consequently may have a negative attitude towards teaching mathematics (Latterall & Wilson, 2016).

Mathematics is the most international of all curriculum subjects, and mathematical understanding influences decision making in all areas of life—private, social, and civil. Mathematics education is a key to increasing the post-school and citizenship opportunities of young people, but today, as in the past, many students struggle with mathematics and become disaffected as they continually encounter obstacles to engagement. It is imperative, therefore, that we understand what effective mathematics teaching looks like—and what teachers can do to break this pattern. (Anthony & Walshaw, 2009, p. 6)
Pre-service, novice and experienced teachers who struggle to find enthusiasm while teaching mathematics for a variety of reasons need support in the forms of mentorship and meaningful professional development in order to improve their own attitudes towards mathematics and its instruction (Craft, 2002). Failing to provide a support system to teachers may result in them taking the path of least resistance while operating autonomously in their isolated classrooms, and is likely a key ingredient in why students today continue to perform at mediocre levels with the current curriculum content (Hiebert & Grouws, 2007).

The administration of successful schools creates environments where novice teachers are encouraged to be keen, committed and excited about embarking on their lifetime career, and open to learning techniques and welcoming of regular, meaningful feedback (Curtis, 2011; Henry et al., 2013). New teaching and learning techniques that teachers are encouraged to use, such as incorporating spatial skill training into mathematics education, must convince both novice and experienced teachers that they work and will have a beneficial impact on their students (Harwell, 2003). “What is needed is a carefully crafted system of teacher education that begins in the K-12 mathematics experience and continues all the way to on-going, well-designed, appropriately focused in-service professional development” (Milgram, 2005, p. 414).

2.6 Pedagogical approaches to teaching and learning mathematics

The modern mathematics teacher must bring to bear all the tools available in order to provide a learning environment conducive to the successful consolidation of mathematics (Bishop et al., 2012). These teachers must be expert lesson planners, knowledgeable in various learning styles, clever with differentiation techniques, masters of the material content, proficient at using Assessment for, as and of learning, and able to modify their own delivery
system according to the classroom dynamic they see before them on any given day (Growing Success, 2010; Ontario Ministry of Education. 2005; Sinay & Nahornick, 2016).

In terms of learning outcomes, model-based feedback was superior to verbal feedback alone, models served as a learning scaffold rather than a crutch, and learning with model-based feedback was resilient over a 7-day delay. Finally, concrete and virtual models were equivalent in promoting learning, and action-congruence of the interface did not affect learning. (Stull & Hegarty, 2016, Abstract)

Teaching mathematics can be very easily, but not necessarily effectively, done using the traditional teacher-led “chalk and talk” method (Wakely, 2004). Tyson (2015), in her article discussing how the application of the philosophy of John Dewey to her own teaching practice, explains that “A balance between teacher-led ‘chalk and talk’ and independent learning [is] found to create the best learning environment, avoiding student frustration at not being able to progress but stimulating engagement at the same time” (p. 1).

Dewey (1902), Montessori (1964), and Piaget (1960) argued that independent learning opportunities were necessary for a constructivist approach to teaching and learning. Constructivist teaching and learning is a theory that explains that people construct their understanding and knowledge of the world through experiencing things and reflecting on those experiences (Richardson, 2003). "In the constructivist approach, we look not for what students can repeat, but for what they can generate, demonstrate, and exhibit" (Brooks & Brooks, 1999, p. 15). Students and teachers interact with a concept, asking questions, exploring its properties and deciding to accept or reject preconceived ideas about it (Grenon-Brooks, 2002).

The learning and teaching of mathematics lend themselves well to constructivist approaches. Dewey (1902) rejected the notion that students were merely vessels to be filled up with information. He felt that “learning is active; it involves reaching out of the mind; it involves organic assimilation starting from within” (p. 9). Similarly, Bruner (1960) felt that
instructional settings should be designed to allow learners to extrapolate their ideas, fill in the
gaps and transfer skills to the novel as well as familiar inquiries.

Constructivist mathematics pedagogies can be described as “approaches to teaching,
towards which one initially aspires and which then become fundamental aspects of the
teachers’ praxis discipline of mathematics” (Richardson, 2003, p. 1626). There is much to be
said for including rote memorization of facts; one memorizes the alphabet to read, for example,
and applies conventional algorithms to acquire solutions to mundane problems (Brean, 2014).
However, to solve higher order, complex, and creative mathematical inquiries, a variety of
teaching and learning strategies must be employed. Cobb (1994) states “that mathematical
learning should be viewed as both a process of active individual construction and a process of
enculturation (socialisation) into the mathematical practices of [a] wider group or society” (p.
13).

It may be that the ideal learning environment does not require a teacher or leader, but
rather access to resources and tasks that naturally stimulate the innate curiosity of young
people to figure out why and how something works (Dewey, 1903; Taba, 1963). Offering
opportunities for learners to engage with skills that may seem unrelated to the discipline as
prescribed, such as mathematics, but encourages those desirable and transferable skills of
executive function exercised through spatial skills, are all components of constructivist
learning that support a strong curriculum (Montesorri, 1964).

In general, teachers tend to naturally adopt one of five types of teaching style: Expert,
Authoritative Leader, Personal Model, Facilitator and Delegator (Boundless, 2017). Each of
these teaching styles can be effective when teaching the various topics in mathematics, but
when considering the learning styles of the students it becomes necessary for the modern
teacher to step out of his or her comfort zone and adopt the other four styles, when appropriate, for the content and the diversity of the learners (Anthony & Walshaw, 2009).

Many mathematicians, teachers of mathematics, and teachers of general subject matter know that the process is as important, if not more important, to the learning of mathematics than acquiring a series of algorithms to come up with the “correct” answer (Ontario Principals Council, 2009; Wiggins, 2014). Some teachers view the process as just that, a series of algorithms, and “take matters of selection as either a fundamentalist reproduction of literal basics, the pursuit of self-evident truth, [rather than a] logical and organic extension… of disciplinary knowledge” (Deng & Luke, 2008, p. 67). At the same time, there is also a strong movement towards incorporating the understanding of disciplines into curriculum development within a broader and more plural conception (OECD, 2012; Ontario Elementary Mathematics Curriculum, 2009; NACCCE Report, 1999).

Schwab (1964) places the understanding of the students above discipline itself. It is the understanding rather than merely the memorization of content that warrants and recommends the incorporation of activities such as spatial skill games into the learning of mathematics. Interacting with these games should allow for the development of understanding and problem-solving in addition to the exercising of higher order thinking skills that will complement and enhance the acquisition of mathematical content knowledge (Cobb, 1994).

2.7 Teacher Change

Some of the key components of change management in school improvement with respect to teacher’s pedagogy is to recognize and acknowledge the individual’s expertise and professional skill, convince the teacher that the new strategy will have a positive impact on their student’s learning and demonstrate the use of that strategy with the teacher’s own students.
Rather than telling the teacher to incorporate a new strategy, trust that the teacher’s professional curiosity will be aroused when that strategy is being used with their own students with a positive impact (Munro, 2005). These ideals of change management are also effective strategies when mentoring teachers to facilitate reflective practice.

Ofsted (2004), the UK-based Office for Standards and Education, states “The most distinctive characteristic of … very good teachers is that their practice is the result of careful reflection” (p. 10). “Reflection on classroom experiences, however, has been shown to be effective in changing teachers' beliefs” (Stipek et al., 2001, p. 224). Highly effective teachers regularly reflect on the learning outcome value of their lessons and the teaching strategies that they have employed, and modify subsequent learning opportunities accordingly (Darling-Hammond & Youngs, 2002). In addition:

teachers are obliged to conduct themselves in a way that offers experiences that are responsive to the nature of the learner, that are within the purposeful and deliberately conceived educational scope of the school, and that satisfy a standing ethic of professionalism. (Hlebowitsh, 2010, p. 508)

Exposing pre-service, novice, and experienced teachers of mathematics to activities that develop spatial skill should have a two-fold impact by expanding the teachers’ pedagogical repertoire and assisting those students whose spatial skills are weak by developing a supporting and transferable form of reasoning known to be strong in those who are considered successful in mathematics (Newcombe & Shipley, 2012).

Involving those same teachers in the research process honours their expert knowledge and helps them become more influential agents in the improvement of the teaching and learning of mathematics. Recently, researchers “were able to see the benefits of a model of
professional development where teachers are involved in research based on their current classroom context rather than on a prepackaged intervention or a narrow agenda” (Esmonde & Caswell, 2010, p. 253).

Change requires the commitment of all the constituent parties, not just the theorists or the teachers but the entire set of people affected by, and able to effect, a change in education.

It is not enough for people to come together in dialogue in order to gain knowledge of their social reality. They must act together upon their environment in order critically to reflect upon their reality and so transform it through further action and critical reflection…. This can be done only by means of the praxis: reflection and action upon the world in order to transform it. (Freire, 1970, p 51)

Teachers immersed in praxis would bring their philosophical thoughts to every decision as they make them, adapting their actions in a classroom to ensure they continue to encourage the learning their students are undertaking. This situates the learning as a conversation between learner and teacher rather than as a teacher carrying out their plans, which were crafted in the hypothetical world of being ‘good in theory’ (Collins et al., 1987).

Freeman et al. (2012) explain that “it is through critical and reflective thinking that they can begin to create alternative conceptions of how things ought to be” (p. 4). Designing a curriculum that includes a curriculum for teachers, as teachers and of teachers requires an involvement of teachers in the research process. Similarly, ensuring that the curriculum meets the needs and envisions the future requirements of students also requires involving students in the research process.

Engaging in a curriculum that promotes problem-solving as a skill and a discipline will blend the content knowledge that we believe to be critical for student learning and success into a system of twenty-first-century skills. Content will be taught, enhanced, reflected upon and understood through the act of transformative skills such as problem-solving in multi-modal
ways: linear, spatial, lateral, literal and holistic. In this way, we will affirm both the facilitator of the learning and the participant in the learning in the “process of becoming” (Freire, 1970, p. 84).

Mathematics will cease to be a collection of content from the disparate mathematical realms but a vehicle for understanding curriculum as a whole. “If theory does not exist to provide practical solutions to everyday problems, why does it exist” (Pinar, 1992, p. 230)? Such methods might allow us to participate in “school reform in ways that do not hypostatize the present, but rather, allow our labour and understandings to function as do those in psycho-analysis, to enlarge the perception and deepen the intelligence of the participants” (Pinar, 1992, p. 234).

While teachers do need to have a “profound understanding of fundamental mathematics” (Ma, 1999) in order to make sound pedagogical decisions to support learning (Ball, Hill, & Bass, 2005), they also need to be aware of and understand the variety of modalities required to elicit internalization of not just concepts, but the process of problem-solving that leads to mathematical reasoning in addition to, and more importantly above, a repertoire of concepts (Orey, 2010). The collection of ideas should include visual forms throughout the [study of] mathematics, at all levels, to include more of the students within the tent of those who are good at math and to enrich the range of approaches of all students. Mathematics needs to be taught in an inclusive way that helps the visually strong people to connect to the core of mathematics and also see themselves as empowered users and creators of mathematic[s]. (Whiteley, 1999, p. 17)

This is an opportunity to “reconceive the study of teaching so that we honour the complexity, the sensitivity, and the joyousness of those who do it well” (Grumet, 1989, p. 16). For those who do it well, incorporate what they know, and are willing to learn about diverse
strategies for effective learning of mathematics, they will, in the process, demonstrate the value of mathematical reasoning.

2.8 Desirable learning habits

Dweck (2007) found that when she taught “a group of elementary and middle school children who displayed helpless behaviour in school that a lack of effort (rather than lack of ability) led to their mistakes on math problems, the kids learned to keep trying when the problems got tough” (p. 2). Through reinforcing, praising and teaching perseverance skills, students were able to focus more on process than ability, and were more likely to view failure as a stepping-stone to the solution rather than a blow to their self-worth.

Students praised for intelligence preferred to continue working on easier tasks, while students praised for effort chose to progress to more challenging tasks (Mueller & Dweck, 1998). Desirable habits such as perseverance, collaboration, and acceptance of failure as a stepping-stone to problem-solving are reliant on executive function (Diamond & Lee, 2011).

“For teachers to become effective learners, they need specific attitudes and skills, including persistence, understanding of the transfer of training, understanding of the need for theory and the ability to use peers productively” (Joyce & Showers, 2002, p. 1). Teachers and their students need to develop and enhance the skills necessary for positive learning experiences.

Students demonstrate growth in thinking abilities by increasing their use of alternative strategies of problem-solving. They collect evidence to indicate their problem-solving strategy is working, and if one strategy does not work, they know how to back up and try another. They use systematic methods of analyzing a problem, knowing ways to begin, knowing what steps must be performed, what data need to be generated or collected. This is what I'm meaning by perseverance. (Costa, 1989, p.3)
“The term executive function is used to describe the capacity that allows us to control and coordinate our thoughts and behaviour. These skills include selective attention, decision-making, voluntary response inhibition and working memory” (Blakemore & Choudury, 2006, p. 301). While executive function is defined in a variety of ways depending on whether it is approached from the field of education, psychology or neuroscience (Meltzer, 2007), the general idea is very similar across fields. It may be that the enhancement of spatial skill and its resultant impact on mathematical reasoning are due to an improvement in the executive function necessary for higher order mathematical reasoning (Bull & Lee, 2014; Bull & Scerif, 2001).

Miyake et al. (2001), in their study of how visuospatial, working memory and executive function are related, found that different types of spatial skills such as mental rotation (3-dimensional manipulation), paper folding (2-dimensional manipulation) and spatial orientation placed different demands on executive functions. Importantly, they found that spatial visualisation tests and mental rotation tests “necessitate the management of task-specific goals and sub-goals as well as the scheduling and coordination of different cognitive processes … [which in addition to] performing mental transformations seems to require an active resistance to perceptual interference from the test stimuli” (Miyake et al., 2001, p. 625).

Bull and Scerif (2001) found that executive function was a strong predictor of mathematical ability. “Cognitive skills such as working memory, perseverance and inhibition had the most impact on variance” (Bull & Scerif, 2001, p. 283). Many mathematics teachers, when asked what factors are most likely to impair a student’s ability to perform well on problem-solving tasks, are liable to suggest behaviours such as “stick-to-itiveness” and flexibility. As a novice mathematics teacher myself, I often noticed that students were quick to
give up when the “rule” they had learned did not apply in the same manner as it had been taught even though they were on the right track. At that time, I was not aware that executive function was variable and could be improved. “Results from the Wisconsin Card Sorting Test (WCST) clearly show that the main difficulty for children of lower mathematical ability is with inhibiting a learned strategy and switching to a new strategy” (Bull & Scerif, 2001, p. 284). If spatial skill activities can enhance executive functions such as working memory and inhibition, it seems likely that it will also have a positive effect on mathematical reasoning.

Timing the intentional use of spatial skill activities to the period in adolescence when the brain is once again receptive to stimuli that promote or inhibit synaptic pruning may have additional benefits. Spatial skill enhancement during adolescence may also prevent the all-too-familiar scenario of students who felt competent with elementary school mathematics curriculum beginning to flounder and doubt their abilities as they struggle with the transition to the more abstract reasoning required as they encounter middle school mathematics curriculum.

An interesting speculation based on the results of the studies discussed in this review is that puberty represents a period of synaptic re-organization and as a consequence, the brain might be more sensitive to experiential input at this period of time in the realm of executive function (Blakemore & Choudury, 2006, p. 307).

Further study of the relationship between spatial skill, executive function, and mathematical reasoning will be an important step forward.
Chapter 3 – Methodology

3.1 Introduction

In this chapter I describe the methodology for the study. I begin by describing the research design and the process involved with the evolution of the study’s methodology. The design, implementation and the results of the Pilot Study are included in this section as this process was instrumental in the streamlining of the experimental design for the main study. Subsequently, I describe the data collection and analysis methodology of the main study. Finally, I discuss ethical considerations.

3.2 Research Design

This study meets the description and criteria of a mixed methods design through a case study. “Good mixed methods studies incorporate rigorous methods of data collection, analysis, and interpretation of the quantitative and the qualitative design” (Creswell, 2013, p. 552). In particular, it follows a multi-stage evaluation design that begins with a strictly qualitative approach, progresses to a mixed approach that emphasises qualitative and introduces quantitative analysis through pre-, and post-testing, which then leads to a final phase that enriches the qualitative data through statistical summaries of periodic skills tests in spatial and mathematical reasoning (Creswell & Clark, 2011).

Yin (2014) explains that, in order to get a full picture of how a treatment or intervention works in a real classroom, it is necessary to use a case study. “The study would cover relevant events, including the implementation of the treatment and how the treatments appeared to have altered the classroom teaching and learning” (Yin, 2015, p. 216). The quantitative data in this case would serve to enhance the qualitative data, while the qualitative data would give meaning to the statistical results.
A mixed method case study approach was selected as the best method for this study because it allowed for a complete picture of how the intervention was impacting both the teachers and the students. This methodology meant that the teacher participants were able to impact on the design through their feedback and reflections on the how the intervention was being implemented as well as its efficacy. When the first-year pilot study participants unanimously felt that some form of measurement of impact was needed both to evaluate student change as well as to validate their own observations of student change, this methodology allowed for that.

Mixed method research works particularly well for case study research as it allows the researcher to take the rich empirical data yielded from case studies and apply either quantitative or qualitative methods to the data. In this manner, qualitative data can be quantised or quantitative data can be qualitized to extract meaning from the data sets that might otherwise be hidden. (Mills, Durepos & Wiebe, 2010, p. 562)

The research project consists of eleven case studies in three schools, seven teachers in a two-year pilot study and four teachers in the main study. The qualitative data was collected from interviews, classroom observation, field notes and teacher journals. The quantitative data was collected from pre- and post-testing of students’ spatial skill development and mathematical reasoning development. The pilot study represented an exploratory case study to allow a dynamic design that should emerge from the data. The case studies were analysed through coding of the teacher interviews and searching for common themes. “Such a task calls for sensitivity to the nuances in data, tolerance for ambiguity, flexibility in design, and a large dose of creativity” (Strauss & Corbin, 1998, p. 34).

One of the fundamental philosophical assumptions of qualitative research is that individuals construct their reality as they interact with their social worlds (Merriam, 1998).
this study teachers were encouraged to be participants and co-researchers. Yin (2014) describes a case study as an “empirical inquiry that investigates a contemporary phenomenon (the “case”) in depth and within real-world context” (p. 16). The data collection methods involve interviews, teachers’ journal observations and multiple sources of observable data and behaviour from classroom visits. The interviews and videos were transcribed.

One of the greatest strengths of using a progression of case studies for this investigation was that the research method itself allowed for the development of a more rigorous theory. The rigorous, panoptic approach over time also contributed to establishing meaningful results. Another strength is that it appeals to my preference for detailed and systematic data collection, using ongoing analysis to continually refine the method and allow for examination of the data from a variety of perspectives.

Strauss and Corbin (2015) feel that, “while quoting makes for fascinating reading, it doesn’t lead to theory or provide the reader with any framework for making sense of those readings” (p. 376). In-depth coding of the transcripts as well as including some relevant measuring instruments can “often provide additional findings of interest” (Strauss and Corbin, 2015, p. 373).

The data from the first year of the pilot study was used to identify themes and categories to develop questions that would allow for meaningful comparisons and emerging refinements for the main study. The second year of the pilot was designed to allow for investigation and control of variation and development of a process that is situated in current literature, relevant and practical to classroom research and will further develop pertinent questions and processes to be investigated in the third phase. The main study was expanded to
include quantitative data collection to augment the meaning and interpretation of the qualitative data.

3.3 Pilot Study

The pilot study was designed to test procedures and data collection techniques, and was inspired and informed by results of similar studies I had conducted while working on educational reform in the United Arab Emirates. Two Toronto private schools were selected based on my connection with them as a former staff member. The administrations at both of these schools were familiar with my past research that had been conducted at the first school, investigating common themes amongst gifted female middle school mathematics students, and later applying the principles of spatial skill training to a small group of struggling grade ten mathematics students. As a result, they knew that their students’ parents were receptive to these types of interventions.

The teachers’ input, views and observations of the process, development and the impact on their students’ mathematical reasoning were recorded through interviews at the beginning, at regular intervals throughout, and at the conclusion of year one and year two of the pilot study. These interviews (see Appendix B) were both structured and semi-structured. They were generally conducted face-to-face but, on occasion, were conducted via Google Hangout when either distance or time available during the school day demanded it. Teachers’ observation notes, as well as classroom observation notes, were collected and kept with the transcribed interviews. Initially, all transcriptions were done by the researcher and coded by hand.

The spatial games replaced starter-type activities that are already part of the teachers’ lesson program (typically called “Minds On” in the three-part lesson). In the second year of the
pilot study, students wrote a pre-test and post-test to measure both changes in spatial reasoning and changes in age-appropriate mathematical reasoning. These instruments (see Appendix A) were implemented during the second year of the pilot study to establish whether meaningful results could be collected, and to receive feedback from the participants in order to refine and improve upon them.

Table 1. Pilot Study Participants

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Age</th>
<th>Gender</th>
<th>Teaching as a second occupation</th>
<th>Years teaching</th>
<th>University/College Degree</th>
<th>Grades taught</th>
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<td>F</td>
<td>Y</td>
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<td>F</td>
<td>N</td>
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<td>7-11</td>
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<td>F</td>
<td>Y</td>
<td>5</td>
<td>B.Sc. Neuroscience, B.Ed.</td>
<td>7</td>
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<td>F</td>
<td>Y</td>
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The pilot study served a pivotal role in the design on the methodology of the main study. The teachers in the pilot study served a dual role as both case study participants and co-researchers. Their interviews were designed to not only test the interview process but to elicit their opinions on how best to conduct the implementation of the games as well as the interview process for the greatest rigour in data collection. At the end of the first year of the pilot study, the teacher participants felt that some form of quantifiable test needed to be implemented to support their intuitions and observations. As a result, the second year of the pilot study included a pre- and post-test to measure both spatial reasoning and mathematical reasoning.
The teachers tested a few instruments before recommending ones that they felt would elicit the most information from their students. At the end of the second year of the pilot study, the teachers reflected on how well both the interview process and the pre/post tests described what they were seeing in their classrooms. This resulted in the design for the main study.

### 3.3.1 Qualitative data collection pilot study

Face-to-face, semi-structured interviews were used to collect the data, and these interviews were audiotaped (with prior consent to do so by each meeting participant). I observed classroom interactions between teacher participants and students to learn more about the implementation of the new activities, how the students were engaging with the games and how the teachers were managing this intervention. The date, time, and location of each meeting was determined in communication with each participant, based on the participant’s availability within the timeframe of the study, and at a location that was convenient to each participant. The data collected during the interviews focused on each member’s experiences with using spatial skill activities in their middle school mathematics classes.

I kept field notes during the teacher interview process in order to identify the possible impact on middle school mathematics content, as well as to identify whether the teachers felt their participation and contribution to the study had an impact on their pedagogy. The practice of interviewing the teachers in the pilot study was used to refine the interview process and improve the structure of the questions.

### 3.3.2 Pilot Study – School A

School A is a coeducational secondary school that employs mathematics specialist teachers from grade seven through twelve. The school reports that more than 40% of their
students are on Independent Education Plans (IEPs) and in many classes, the students on IEPs comprise the bulk of the student body. Class sizes are small, ranging from 10 to 18 students.

The teaching staff is young, and intentionally comprised of newly graduated teachers. As a result, the school has a weekly program of staff development and a sophisticated mentoring program to aid these novice teachers. The administration is also young, and actively encourages the staff to participate in professional development, including teacher research. Five teachers from this school participated in the pilot study. They were all under thirty years of age, mathematics specialists, and keenly interested in the research process. Their students were in grade seven, eight and nine regular and enriched classes, as well as one grade eleven remedial class.

The teachers had some strong opinions regarding the impact of engaging in spatial games on the students’ learning habits. Specifically, they noted that their students seemed more willing to take risks, were less threatened by failure, and were more determined to complete a task regardless of its difficulty level.

3.3.2.1 EJ.

EJ graduated with a Bachelor of Chemical and Bioengineering degree and subsequently earned a Bachelor of Education degree. Before joining school A, EJ was the director of a science and engineering outreach program at an Ontario University. EJ continues to involve herself as an outdoor education teacher during her summers. When not in the classroom, EJ enjoys indoor and outdoor rock climbing, camping, canoeing, hiking, kayaking, and snowboarding.
EJ is a creative teacher who spends a great deal of personal time on lesson development. Her lessons tend to be kinesthetic, active and engaging. EJ regularly uses manipulatives and gross motor activities to improve her students’ understanding of mathematical concepts. She was skeptical that the spatial games would be anything other than a “fun” activity. Because of her practiced use of hands-on activities, it was not much of a stretch for her to include spatial skill activities as a regular part of her curriculum delivery.

EJ’s Science background made her a valuable participant, as she approaches concept introduction much like action research. Her interview responses were thoughtful, carefully considered and holistic in detail.

3.3.2.1.1 The typical lesson with the games

EJ chose to work with a smaller selection of the games available in order to streamline her setup. Her students played with the Bloc by Bloc games (shown on the right). These games came with seven three-dimensional pentominoes and a set of challenge cards. On one side of the card was a picture of the completed task and on the other side of the card two images of the partially completed task were given as clues.

EJ allowed fifteen minutes for her students to work on the task. They were given free choice to select a card. Initially the students worked in pairs and began the task using the clues. As they became more proficient, the students attempted to build the solid using only the completed image. Students were assigned partners. EJ circulated through the room offering leading questions to assist the students when necessary, but generally allowed the students to work on their own.
3.3.2.1.2 EJ’s views of the impact on students.

EJ observed several learning skills amongst her students that she felt may be attributed to engaging with the spatial games. EJ reported that her students surprised her in how they approached problem-solving the games. Some of her students preferred to work in pairs or small groups, while others worked individually. Initially, many of her students demonstrated some level of frustration when out, but once they had developed strategies either on their own or through working as a team, they worked through the problems with a perseverance that was not as observable prior to their engagement with the spatial skill activities. EJ said that one of her students would use negative self-talk when he could not immediately solve a task such as “stupid, not smart…” but would be very pleased with himself once he did persevere and solve a task within the allotted 20-minute period, to the point where he asked if he could take the games home.

3.3.2.1.3 EJ’s views on her own changing pedagogy.

EJ did not really feel that being involved in the pilot study or interacting with the games substantially changed the way she went about preparing and delivering mathematics curriculum. She did find that the interview sessions provided her with an opportunity to reflect on her own teaching and to discuss observations of student learning behaviour that was interesting to her.

I think this research process helps to make me more aware of what I am doing in the classroom, and [I] like thinking about thinking about spatial problems. It was already on my radar before, and it is good that this is another medium, or another means that we are working on thinking because this is important but not in the curriculum, so we would not spend the same time on it. If it was not part of the curriculum, I think it would be a little bit different. I think that the accountability, for me, is important.
EJ also felt pressured by the time constraints for delivering the required curriculum and, while she liked to have active lessons and to use manipulatives, the sheer density of the grade seven mathematics curriculum and the shortened time to get through it that is common in the private school system meant that these types of lessons were often forgone in favour of a quick and easy chalk and talk lesson. That is not to say that she did not recognize the value of using the spatial skill games to enhance students’ ability to problem solve:

because some students have so much trouble seeing 3d and understanding that, that I think being able to touch it and see it in real life is valuable. I think that a mix of both is important because if you have a tool that is going to be more scaffolded in its approach, that is huge for the students that have no strategy. … I mean really your goal is to engage the students, and I think having more mediums of being able to have them work and practise thinking, like the more ways, the better. Because the more ways you present it, the more ways the brain has to work. Your brain is going to work a little differently to touch something and use it.

3.3.2.1.4 EJ’s views on implementing the spatial skill activities.

EJ said that she often struggles to do justice to the enormous amount of curriculum to be delivered in the depth that parents expect. The private school year is shorter than the public school year, and has many interruptions for the required field trips, school events, community service outings and add-ons that are all part of the independent educational institutional experience. As a result, EJ found that when time was short, her fifteen-minute time slot allotted to spatial games would be spent catching up on curriculum or preparing for an upcoming assessment or revisiting a concept that was not as well-learnt as she had hoped.

EJ said that “It has been really interesting. The good thing about this is that it forces you to make time. How I have been phrasing it, I have only been able to do two actual sessions with it.” In the end, EJ was only able to implement the activities on average once per month rather than once per week due to the practicalities of her work environment. It was not that it
was difficult to set up and run with the students, especially as the students viewed the spatial skill activities as a break from the regular routine, it was simply a matter of time. The teacher needed time to collect the Toy Box from the shared resource centre. She needed time to plan with the other teachers for scheduled use of the games. She also needed to find time in her heavily packed, carefully designed curriculum that was regularly monitored by her teacher mentor.

3.3.2.2 EP.

EP graduated with a B.A. degree in Geography and a B.Ed. degree in Geography and Mathematics. EP has been teaching at school A for eight years. She is also coordinator of school life and runs leadership programs, extracurricular clubs and serves on several school committees.

EP joined the pilot study midway through the first year after returning from maternity leave. Not a natural mathematician, by her own admission, EP plans her lessons in a systematic, linear and functional manner. She does incorporate some interesting activities into her lessons but, in general, her teaching style is a modern version of “chalk and talk” using co-written and shared lessons written by other staff on an electronic whiteboard. EP was excited to participate in the study as she had a genuine desire to add some additional teaching strategies to her repertoire as well as to explore mathematics from a different perspective.

3.3.2.2.1 The typical lesson with the games.

EP inherited a class that had already been exposed to weekly fifteen-minute interactions with the games. The students had acquired a preference for playing with certain games and had also established regular partners to work on these games. EP found that allowing this to continue meant that there was little interruption in routine and that she simply had to present
the toy box to the students, and they and their partners would select the games they wished to work with. In addition to the games shown above, EP also used the online game BLOXORZ, as a brain break or reward for students who had completed their work early. This was easily implemented as all of her students worked with laptops.

Figure 4. Online Spatial Game Bloxorz (Appendix D)

3.3.2.2 EP’s views on the impact on students.

EP had recently returned from maternity leave part way into the second term of the school year. The grade eight class that she took over was having some difficulty adjusting to a new teacher mid-year, and she was finding it difficult to elicit enthusiasm from the students for her teaching style. EP was quite pleased that the students would become more animated when they were engaged with the spatial games. Her class had been practicing with the games with the previous supply teacher.

One of the guys in our class, who was so reluctant to take a note, always wants to get up and get drinks of water, that sort of thing during the lesson, really not engaged to do any sort of "Chalk and Talk" or even if it is one-on-one, working through a problem, was so engaged when I brought out the toys and actually had to be asked numerous times to put it away when the time was over. Just wanted to keep doing puzzle after puzzle, and I think like I said last time, I think it is because he could see the immediate result. He could sort of see the goal. He knew what he was working toward, and he just wanted to keep going and working on it. It was just so interesting to be like, "Wow, this sort of tactile problem solving you are really into, whereas anything that is rote or that sort of thing, totally uninterested in participating." So that was really cool to see.
EP made full use of the prewritten lessons that School A developed to be shared by the mathematics teachers. These lessons were easily accessed from the school server, and were projected on the Smartboards and the students’ connected laptops in all of the classrooms. EP stood at the front of the classroom and talked through the slides with her students. Although the technology made for a more appealing series of images, the teaching style was not all that different from the traditional teacher standing at the front of the room writing the lesson on the blackboard and talking at the students. She found that a break from her very prescribed “chalk and talk” lessons that were written by her teaching group had quite a positive impact on many of her students.

And then just even some of the kids were working together on the tangrams and some of the block-building puzzles, and just even the energy between the kids. "Oh no, put that one there. No, put that one there," and just sort of the verbal problem solving that was going on was also really interesting, because some of the kids just sat there and were very on their own working at things, but the kids that were talking it out with one another were coming up with a lot of interesting possible solutions and sort of, I guess, strategies to take to solve the problems. I really liked opening the class up like that.

Prior to participating in this study EP was worried that she would lose control of her class and that an active lesson would make classroom management quite difficult. Not only did she find that classroom management was not an issue, but she found that the students were engaging in learning strategies without her direct intervention. “So, the fact that we actually got into them this past week, I saw the benefit of it, and I also saw how it could engage the kids, which was really cool.”

3.3.2.2.3 EP’s views on her own changing pedagogy.

Although EP is a relatively young teacher, she has developed a process for lesson planning and delivery that she is quite comfortable with. Because mathematics is not her area of expertise, she prefers to use pre-written lessons that have been developed by the
mathematics team at her school. These lessons are delivered via an interactive whiteboard and are stored on a central server that both students and teachers can access. They have the value of consistency and can easily be followed by a student who has missed a class. EP is committed to her students and is open to expanding her pedagogical toolkit. However, she views deviations from the traditional form of teaching as a form of sleight of hand to “trick” the students into thinking.

And so, it is a trick, like if you can almost trick the kids into seeing these problems and puzzles as games and something fun, but at the same time it is developing other skills, that is the head fake. It is almost like if I could find that spot in math, how to do that. It is sort of what Dan Meyer does with his 3-act problems.

EP is aware of modern practices that have been shown to be successful for improving mathematical reasoning. She regularly attends professional development events both within the school setting and outside of the school setting. She would like to implement some of the strategies she has learned.

I would definitely be interested in exploring if there is something out there in terms of my ability to develop that kind of task. I do not know if it is in me, because I think that takes sort of a more creative, broader scope of math than I probably have the skill set for, but I would definitely be interested in trying to bring that more into my classroom, because it is so much more engaging than how I have taught in the past.

EP is fortunate in that she works in a school that has a systematic and supportive professional development program in place. The teaching staff meet weekly on Wednesday mornings to share teaching strategies of their own and techniques that they have learned at conferences. Every teacher has a mentor, and they meet regularly to discuss individual teaching needs. Every discipline has a team that also meets to develop lesson plans, assessments, and to plan curriculum delivery. It should be easy for EP to find the support she needs to try new things in her classroom.
The pressure of keeping up with all the expectations and requirements that a private school teacher must fit into the day in order to satisfy administration and parent clients can be overwhelming. “I used to be so competent, and now I just feel less competent…” EP sighs as she outlines what she needs to accomplish each day. As a result, creative lesson planning often takes a backseat to the familiar chalk and talk.

3.3.2.2.4 EP’s views on implementing the spatial skill activities.

EP felt that the timing of this type of intervention was important both with respect to when it occurred during the school year, and the frequency of implementation during curriculum delivery.

I think we need to maybe start this at the beginning of the year when we feel like we have all the time in the world to do our curriculum, and then we can just block in 15 minutes at the start of class every week for 2 months. If they know that start of class on Friday is puzzle time or whatever, then I think that that will work out much better.

EP found the actual implementation of the games for the fifteen-minute interventions pretty straightforward. In part, this was because she was returning mid-way through the year from maternity leave and the previous teacher had already been using the spatial activities with the students.

If you took two weeks or so, and you had a couple 20-minute meetings, and you did a run-through of the games, I think that that would be pretty effective, because then you know what you are looking for when you are walking around the room. They are so self-contained as it is. My kids in my class were exposed to them already, right, so it was easy for me. I mean, they know how Penguins on Ice work, and they just go. But maybe for some of the other teachers that would be beneficial to have a couple sessions where we just get together, and we play and explore them.

EP did find that she gravitated towards using the spatial activities that she was able to solve easily on her own or that required minimal teacher guidance as she felt somewhat insecure when students struggled with a task that she struggled with herself.
3.3.2.3 EK.

EK studied Neuroscience as an undergraduate, where she completed a thesis examining stress and Parkinson’s Disease. She later went on to earn her Bachelor of Education degree and Specialist in Special Education. After interning at school A, EK moved to a private boys’ school where she coordinated support and accommodation for diverse learners in their Learning Enrichment Centre. Since returning to school A, EK has been a member of the Student Success Centre team, working closely with teachers, parents, and students to meet the needs of middle school learners. She has also created blocked math programs with the grade seven and grade eight panel, to ensure personalization and differentiation. The value of the blocked program means that all grade seven and eight classes have their math lessons at the same time. This allows for students and teachers to work in and with different classes, as well as allowing classes to work together on larger scale activities.

EK’s dedication to her students’ individualized learning needs means that she employs a variety of teaching strategies to ensure that every student in her class is engaged. EK was excited to add spatial skill games to her weekly mathematics fun days, but had some doubts as to whether it would have an impact on students’ learning. As a result, if EK was short of time in her curriculum delivery the spatial games were shelved, and her classes did not have an opportunity to interact with the activities on a regular basis.

3.3.2.3.1 EK’s views on the impact on students.

EK found that she struggled with classroom management while the students were interacting with the games, and as a result she would cut short activities if the students could not behave in a manner acceptable to her. Her initial enthusiasm for using these activities as a
form of differentiation and a scaffold for improving mathematical reasoning was dampened early on in the pilot study.

I want to say, I think they got better, persevering. Which is, for me, half the battle. They got better at continuing to try when they were frustrated.

When asked whether she felt that having her students engage in these activities was having an impact on their learning skills in any way, she reflected that the scheduling of her mathematics lessons at the end of the day was challenging for her grade seven students and may have inhibited or negated any positive effect on learning that might have happened in a different environment.

Well, some positive things but not as many as I would have hoped, so I think time of day was challenging for them, as well. It is not even really the task; it is more like all the parameters around the task that they are challenged by, which says a lot about them, too, right? Because obviously, they have got the great ability. It is more the little things that are throwing them off.

In general, EK felt that she could not see the use of the spatial skill activities as a means of enhancing mathematical reasoning. The games were fun but were more of a breather for the students.

It is kind of like a nice add-on, but one thing I try to do with my class is doing a brain break. It is something I have tried to put in place of that, with mixed success, but it is not a result of the games or the idea, it is in the execution. It has not really been done well on my part.

3.3.2.3.2 EK’s view on her own changing pedagogy.

EK is quite proud of her lesson planning and ability to deliver a strong lesson. She feels quite confident in her success with assisting students that struggle with mathematics. EK is ambivalent about whether involvement in the research process changed the way she taught, but she did acknowledge that it made her reflect on how important managing the social dynamic in her classroom is for learning, and for her comfort zone as a teacher.
It makes me question how I am structuring my day and structuring the big picture. It also, where my priority is right now. Part of that might be because it is the end of the year and it is a reflective time. You are thinking about all the good things and all the things I should have done and were not doing, and all that stuff. I'm not sure if it is the timing or the study, but giving more opportunities to them to play more and less structure - do I like that or not like that? How can I be more like that but still keep control, and all that kind of stuff, management.

EK did not see engaging in the spatial skill activities as serious mathematical learning and is unlikely to engage in this type of learning activity with her students outside of this pilot study, at least, not at this stage in her teaching career. EK admits that she did not really give the games a fair chance and frequently eliminated the activity when she was pressed for time. She does feel that being involved in the research process started her down the path of reflecting on how she is structuring her lessons, but whether including spatial activities is something she will embrace is doubtful.

I think it does affect the way or make me think about the way I'm structuring the actual lesson because I do believe in the whole learning cycle model: here is a skill, try it together, try independently, all that kind of stuff. I do believe in the success of that, so where does this fit in?

3.3.2.3.3 EK’s views on implementing the spatial games.

There were many aspects of implementing the spatial games that EK found quite challenging. The Toy Box was shared amongst five teachers, and it was necessary to sign out the games and return them. EK is involved in many aspects of school life in addition to being a mathematics teacher, and found that she would leave sign-out time to the last minute or forget to return the Toy Box to its shared location. In addition, EK found that the students needed quite a bit of supervision to put the toys back together correctly and return them, complete with all the pieces, to the Toy Box. This was a frustration that led her to minimise her use of the games.
Student behaviour and manners are important to EK, and part of her teaching legacy. She prefers an orderly classroom. The noise and activity that accompanied the students’ interaction with the games were not conducive to EK’s view of what a successful lesson looks like.

3.3.2.4 MC.

MC holds a B.Sc. (Hons.) in Mathematics and French, and a Master of Teaching degree from OISE. While at OISE, she focused her research on math anxiety experienced by secondary school students, and the implications of this for educators. Before coming to school A, MC lived in France and taught ESL at the secondary school level. MC loves being part of school A, as it is a collaborative, innovative and student-focused community.

MC has excellent research skills, which she applies in her interactions with her students. She regularly reflects on and discusses positive and negative outcomes of lesson delivery in her own classroom with her colleagues. As a result, her colleagues will come to her with a mathematics teaching issue they might be having so that she can dissect, discuss and in partnership come up with a solution. This made MC a valuable asset in the pilot study phase of this investigation. She did not wait for interview sessions to discuss what she was seeing but provided regular feedback and observations when we crossed paths in the staff room.

3.3.2.4.1 MC’s views on the impact on students.

MC was quick to see the value of the spatial skill activities for reducing anxiety in problem-solving. This is likely due to her own extensive research in the area of math anxiety.

She noted a number of observed improvements in learning behaviour.

I think the biggest thing is problem-solving and perseverance. There is an answer; we know there is an answer, coming at it different ways, trying different strategies. Doing it methodically like, “Okay, try that, that did not work so now try that another way;
remember what it is for.”

MC felt that the non-threatening process of solving a puzzle and the desire to find an answer encouraged the students to take risks in articulating their problem-solving strategies while working in groups to solve a problem.

I sometimes think in group work [when solving the spatial activities] there is a game mentality when you are working with your buddies, I think there is a little bit less buddy work in algebra work. Traditionally there is not as much. I also think that it is way more tangible for them so I would imagine it feels like there is only so many ways to try this thing, one of them has to be right. Even though the same is true in algebra, they seem to have a good understanding because it is physically in front of them.

MC’s research in mathematics anxiety has allowed her to engage in a fair amount of literature review on the impact of anxiety on perseverance. She believed that the games encouraged students to practise perseverance and that each success allowed the student to take a greater risk and pursue completion of a more challenging spatial game.

Interviewer: Do you think that perseverance is a transferable skill that you can train them?
MC: I think so. I totally do, I think confidence comes from overcoming a task that is just hard enough to persevere through and from that success breeds success, right? If you get that feeling once it is kind of like an addiction, you want it again and again. You feel like you can get it again and again.

MC also noted that including the spatial skill activities as part of her teaching repertoire had a positive impact on her relationships with her students. The activities allowed her to see some of her students from a different perspective as they found a task in the mathematics class that they could both enjoy and succeed at.

I think part of it you get to see your kids in a new light, so it just shakes up your whole … You try not to pigeonhole your kids, but it shakes that up a little bit which is a good thing. I think those kids that do not get to show off that skill regularly enough get to show off that skill a little bit more regularly. Gives them a little bit more to talk about with me like, “Hey, this is what I tried,” Or, “This is what we did.” It makes math easier to talk about for them.
3.3.2.4.2 MC’s views on her own changing pedagogy.

MC’s practice as an educator involves action research both to inform her own professional curiosity in relation to how to make mathematics more accessible, and as part of her own ongoing personal education. Her involvement with the research process in this study was familiar ground and did not effect much of a change on her pedagogical practice. It did affirm some of her pre-existing beliefs.

I had a sense that the way I have my kids in my brain it would totally shift by what I saw. I did not know how it was going to shift I just knew that when I present them with this some were going to absolutely run with it and I was going to be shocked. Some of them would not run with it, but I would also be shocked.

3.3.2.4.3 MC’s views on implementing the spatial games.

Some of them had easier problems, and got them right away and were solving, and got through multiple problems ... Whereas some of them started off and the engagement wavered in the beginning, because it was overwhelming and they did not really know what to do. So we got them started on a strategy, and by the end they were [the] ones who were more satisfied. So that is an interesting kind of piece of that time-on-task, and how that affects engagement and end-satisfaction.

As with the other teachers in the study, the practical realities of distributing and retrieving the games at the beginning and the end of the fifteen-minute session presented a challenge for MC. To offset this problem in the second year of the study, when she was working with a different group of students, she spent some time at the beginning training the students.

I explicitly showed them how to put all the games away as I handed them out. Tried to do almost the full lesson of how to use them before they got them and that was part of the routine so when your game is packed up and on your desk completely then you can leave. I just implemented that as part of the routine a little bit stronger.

This meant that the students knew what the expectations were, and were able to spend more of the fifteen-minute time slot actually engaging in problem-solving with the activities.
MC established a weekly routine that allowed the students to take the initiative on “games day” to enter the classroom, select their chosen activity, and begin problem-solving, often before the teacher arrived. As a result, MC’s classes interacted with the games more frequently over the two-year pilot study than the other participants’ classes. The only area that was logistically a challenge was administering the diagnostics. At the beginning of the year, it was not that difficult to find a longer time slot to have the students focus on writing the pre-test, but at the end of the year, with the pressure of preparing students to write their culminating tasks, MC found that the post-test was delayed and delayed. She was finally able to administer it on an optional study day, and very few of her students actually wrote the post-test.

3.3.3 Pilot Study – School B

School B is an all-girls private school that has over 125 years of history in Toronto. The students in the secondary school are following the MYP and IB Diploma program. The teaching staff is experienced, and some have taught in international schools. The administration is well-established, hierarchical, and experienced.

Professional development takes place both inside and outside of the school, and teachers are required to participate in their subject specialization, and encouraged to acquire professional development in supporting areas such as learning skills, gifted programs, and mediation. Class sizes have between 22 and 27 students. Two teachers participated in the study, although one dropped out in the first year. Their students were in grade seven and eight advanced and enriched mathematics classes.

3.3.3.1 SB.

SB is a thirty-three-year-old middle school Science and Mathematics teacher with nine years of experience. He completed an undergraduate degree in Biology, followed by a
Master’s of Education. He is currently working on his Doctorate of Education. SB spent several years supply teaching at a variety of schools, which he describes as a valuable experience, before coming to Toronto to work for School B. SB is very confident in his abilities as a teacher. He invests quite a bit of time in his lesson planning, and has a great depth of knowledge in modern pedagogical practices such as differentiation and Assessment for, as and of learning. He often brings his Science background into the math classroom and is quite involved with the Robotics club.

SB participated in this study in part because of his own doctoral studies, and in part because he felt it would further his career at School B, which rewards its teachers for active professional development practices. Despite his dedication and desire for excellence, SB is not a particularly creative teacher, preferring to follow the three-part lesson in a somewhat rigid manner. He found the spatial games frustrating, as this type of thinking was not his standard approach to mathematics.

3.3.3.11 SB’s views on the impact on students.

SB felt that the spatial games were a fun break from the daily routine, but did not really see much educational value to them. When pressed on making a connection between the games and specific curriculum, he did see some value, but if asked to recommend including spatial games as a means of enhancing students’ learning skills, problem-solving skills or understanding of mathematics curriculum in middle school he would have said “no”.

Probably in things to do with geometry, especially when you think about they do not do it until grade ten, or at least for us until grade ten, they do it earlier in other schools, similar triangles. Students have such trouble thinking about two triangles and the fact that if you just rotate one of them, you can then see the similarity between that and the other one. When the girls do that sort of game where they are constantly manipulating and moving a shape around and then seeing it from all sides and noticing how it fits, I think that that
is the sort of area where they really see a connection.

SB did feel strongly that the ability to rotate objects mentally should have a positive impact on mathematical problem solving, but did not see the value at the middle school level. He also felt that the chosen games were not the best for making a connection to the middle school curriculum. SB suggested the intentional development of:

some sort of a visual spatial puzzle that requires thought and problem-solving. Those are the things that are going to create what you want to see in them when they do the mathematical work later on.

When discussing a possible connection between the games and desirable learning skills such as perseverance, SB did not see any evidence of this type of connection, but did go on to suggest that perhaps that was because the type of clientele his school attracts and accepts already have strong learning skills in place.

3.3.3.1.2 SB’s views on his own changing pedagogy.

SB works at a private school that has upwards of five classes in each grade grouping; all subjects are taught by a team of subject specialists. Correspondingly, the timing of lesson delivery and the amount of content that must be covered in each lesson is clearly delineated prior to the school year beginning. This leaves little time for individual teacher creativity, considering the large amount of curricular material that must be covered in the shortened school year. Nevertheless, SB was able to get through about 1 hour and 45 minutes of games practice, or about six sessions, throughout the school year.

When asked if he felt that using the spatial games in his lessons changed his pedagogy, he replied:

To be honest? Not really, not specifically, and I do not think that it is a bad thing. I just think that it’s given me a great lens through which to view what effect this type of mind challenge could have on their learning, but in terms of how I would actually go about
teaching them right now, other than incorporating these kinds of games in more, I do not really know.

SB does not have the flexibility at his current school to make any changes to his pedagogy. Pedagogical change takes time and repeated practice of new techniques before they take hold (OECD, 2012). This was not possible for SB, given the very dense content he was expected to share.

Now that we have been doing this and I see how connected they are to the visuals that could be something I could incorporate more in the future. Up until now, no, it probably would have been very numerically and almost algorithmically based.

3.3.3.1.3 SB’s views on implementing the spatial games.

SB has a very clear idea of what a good mathematics lesson looks like, and places much importance on classroom management for the success of the lesson. While he is tolerant of noise and activity in his Robotics club, mathematics lessons should be orderly, particularly when working with middle school students.

Trying to keep up with all of them has been a bit of a challenge because I do not know who is at what point in the process yet. I think that has been a challenge. Then, it is very difficult, and as a teacher, especially a seventh-grade teacher, you like to lay out for them some boundaries, some steps, like let us all be working on this, but that is not always best.

The use of the games in his classroom meant that he occasionally had a more difficult time bringing the girls back on-task after the warm-up. Even when, during the games, he tried to make the warm-ups a teachable moment and discuss strategies or starting points, his students were more interested in pursuing the solution to their own puzzles.

The actual logistical implementation of it was not a challenge at all. It was the gathering of data that was difficult. The challenge was being able to keep an eye on the whole class and see what different students were doing while being able to give advice and create
areas of thought for a student to pass to the next level without giving them the answer or taking over completely.

I think that was the challenge behind it. It was that whole idea of how do I set up a system whereby I can take in the most data that I can as a teacher, so I know what is going on?

SB’s science background is revealed in the above statement. He felt confident that he could be an asset to this pilot study, and made a concerted effort to collect as much data as possible. This effort posed the greatest challenge to him as he was not offered a concrete method of data collection other than recording his observations and discussing them during the interviews.

I think that prescribing for the teachers in the study how to do it would lead to better results.

The games themselves were not all that appealing to SB’s ideas of what spatial skill activities should look like. When pressed on what might have been more appropriate SB felt that “as a mathematical exercise, [he thought] it would be best to limit it to games that have a connection to mathematics.” In other words, he could not see a connection between the chosen spatial skill activities and the grade seven mathematics curriculum. He suggested a phone app tile game such as 2048, or a Rubix cube as being more useful.

3.3.4 Quantitative data collection Pilot Study

Similar middle school classrooms were selected from two private schools in Toronto, Ontario. In total, there were seven classes, including three grade seven classes, two grade 8 classes, one grade nine class, and one remedial grade 11 class.

In each class, the teacher identified a spectrum of ability levels, and assigned aliases to the students. The classes received spatial skill training using “Toy Boxes” that had a variety of
age-appropriate spatial reasoning games (see Appendix D). Some teachers chose to have their classes work primarily with three-dimensional, rotating block building games, while other teachers had their classes focus mainly on the two-dimensional pentominoes and maze manipulating games, and still other teachers encouraged their students to try all of the games at least once. The interventions took place, in general, one to two times per week for fifteen minutes throughout the school year during the pilot study, and on a daily basis for fifteen minutes or more over a ten-week period.

Data was collected through the administration of a spatial reasoning and a mathematical reasoning instrument as a pre-test, and post-test. The pilot study instrument was created by modifying two existing instruments used for the middle school age group, and has been developed by the author with the input of the teachers involved in the pilot study.

The spatial reasoning portion was modelled after the age-appropriate instruments found on the SILC site: http://spatiallearning.org/index.php/testsainstruments, with an emphasis on tasks that measure success with mental rotation. The mathematical reasoning portion is constructed similarly to a 2009 diagnostic tool developed at OISE (UOIT PJ math diagnostic) to measure grade-appropriate mathematical skill in conjunction with diagnostic tools found on the Ontario Education site: http://ontariomathresources.ca/teacher-resources/assessment-resources. The diagnostic problems are similar to those found in the EQAO Grade six benchmark tests as well as intermediate mathematics textbooks used at the two schools. Questions related to reasoning were selected and modified to try to remove bias due to the timing of curriculum delivery.
3.3.4.1 Quantitative Findings – Pilot Study

The pre-test was written by 106 students, and 72 students wrote the post-test. The simple statistical analysis shows no measurable change in spatial reasoning, and slightly quantifiable change in mathematical reasoning. Results from the pre-tests and post-tests show a moderate improvement in both mathematical and spatial reasoning, although not statistically significant. The relationship between mathematical reasoning and spatial reasoning could not be tested. In order to allow for a more meaningful comparison, the spatial games would need to be implemented on a more regular and more rigorous basis than was carried out in the pilot study. It would be expected that there would be some relationship between spatial reasoning and mathematical reasoning but, in both tests conducted during the pilot study, they appeared to be independent.

Figure 5. Box Plots of All Students in the Pilot Study

The data, while not normally distributed, was reasonably symmetrical about the median for both tests in both spatial reasoning and mathematical reasoning, which lends validity to these conclusions. In the pre-tests, nearly ten percent of the students had very high spatial reasoning with very low mathematical reasoning scores, or very high mathematical reasoning with very low spatial reasoning scores, which was reduced to 5% of the students completing
the post-test. The box plots are more symmetrical in the post-test as well, suggesting that there was a subtle shift in both spatial reasoning and mathematical reasoning towards the mean.

Figure 6. Students with Extreme Differences and Spatial and Mathematical Reasoning

Examination of the outliers – students with high mathematical reasoning and low spatial reasoning, or students with high spatial reasoning and low mathematical reasoning, indicates that there are fewer outliers at the end of the intervention. Students’ mathematical reasoning may have improved over the school year.

Pre- and post-test samples for both mathematical reasoning and spatial reasoning were compared using a 1-tail T-test. While the data is not normally distributed, it is reasonably symmetrical about the means with a similar variance, which allows reasonable interpretation through a T-test.

Table 2. Testing the null hypothesis that the means are not the same.

<table>
<thead>
<tr>
<th>T-Test</th>
<th>Change in Means of Spatial Reasoning</th>
<th>P(0.345) &gt; 0.05, therefore not significant</th>
</tr>
</thead>
<tbody>
<tr>
<td>T-Test</td>
<td>Change in Means of Mathematical Reasoning</td>
<td>P(0.049) &lt; 0.05, therefore possibly significant</td>
</tr>
</tbody>
</table>
This result indicates that while there is no apparent difference in the pre- and post-test for spatial reasoning, there may be a difference in the pre- and post-test for mathematical reasoning.

Descriptive statistics of the pre- and post-test results support the T-Test results in that the means are likely not significantly different. However, examination of the results at the first and third quartile indicate that there was some upward movement of the weakest and average students that may be attributed to the spatial skill activities. This result supports the qualitative data from the teacher interviews.

Students in the first quartile showed an improvement in both math and spatial reasoning of 4-5 percentage points, while students in the 75th percentile demonstrated an improvement of 13% in mathematical reasoning and 20% in spatial reasoning.

Table 3. Descriptive Statistics Pilot Study

<table>
<thead>
<tr>
<th></th>
<th>Mean (μ)</th>
<th>Standard Deviation (σ)</th>
<th>Variance σ²</th>
<th>Q1 – 25th percentile</th>
<th>Q3 – 75th percentile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Test Math</td>
<td>37.4</td>
<td>29</td>
<td>860</td>
<td>15</td>
<td>55</td>
</tr>
<tr>
<td>Post-Test Math</td>
<td>44.4</td>
<td>26</td>
<td>686</td>
<td>20</td>
<td>68</td>
</tr>
<tr>
<td>Pre-Test Spatial</td>
<td>45.5</td>
<td>28</td>
<td>784</td>
<td>23</td>
<td>59</td>
</tr>
<tr>
<td>Post-Test Math</td>
<td>43.8</td>
<td>27</td>
<td>721</td>
<td>27</td>
<td>79</td>
</tr>
</tbody>
</table>

Anecdotal evidence of previous interventions using spatial skill activities suggests that students who began with strong mathematical reasoning skills also had strong spatial reasoning skills, and little change was evident. By contrast, students with weak mathematical reasoning skills had far more variable spatial reasoning skills, and tended to show the most upward movement in both mathematical reasoning and spatial reasoning (this is an area that needs to
be quantitatively tested). Students who fell in the “box” or the middle 50th percentile also had more variability in both mathematical and spatial reasoning. This group tended to show moderate improvement.

It is possible that the change in the mathematical reasoning may be attributed to the mathematics instruction the students received as part of their regular curriculum experience.

However, a significant number of unanswered mathematical reasoning questions in the pre-test, which were answered in the post-test, may be contributing to an artificial inflation. In short, we cannot conclude that there was any change in both the spatial reasoning and the mathematical reasoning of these students.

3.3.5 Implications of Pilot Study Results

The results of this pilot study indicated that a more rigorous design was needed for the primary study.

The instruments need to be re-evaluated for reliability.

1. The spatial reasoning test is two-dimensional while the games are 3-dimensional.
2. A student/teacher code for identification should be implemented and carefully monitored.
3. The timing and implementation of the treatment needed to be better regulated.
4. Students need to be encouraged to answer every question to the best of their abilities.
5. A mid-study test should be added.
6. A control group would strengthen results.

Analysis of the teacher interviews suggests that time is an important factor in the effectiveness of implementing the study.

1. Interventions need to be tied into the curriculum.
2. The administration should be encouraged to support the teachers in implementing the interventions.

3. Classes should be divided into two groups, one being a control, to remove teacher effect.

4. Students and educators should receive ongoing support and encouragement throughout the study from the researcher.

Design recommendations one through three were implemented in the main study, and effectiveness recommendations one and four were made in the main study. The remaining recommendations were not possible to implement due to time constraints, ethical considerations, and the timing of the interventions with respect to curriculum delivery.

3.4 The Main Study

Initially, two local schools were approached. Both administrations were interested in participating in the study. At the time, the local schools were in a work-to-rule situation, and the teachers were unable to include any extras in the classroom, such as research. By the time the work-to-rule situation had ended, only one school was still interested in participating. The principal of the school approached the grades four to eight teachers to solicit interest. The principal had a particular interest in spatial reasoning, which helped.

At that point, I was invited in to present information to the staff about the study, and to answer any questions the faculty might have. Following this presentation, I returned for a more specific presentation to a smaller group of teachers who were interested in implementing spatial skill games into their curriculum. Of that group, four teachers, including three males and one female agreed to participate in a ten-week intervention using spatial skill activities in their classroom.
Table 4. Main Study Participants

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Age</th>
<th>Gender</th>
<th>Teaching as a second occupation</th>
<th>Years teaching</th>
<th>University/College Degree</th>
<th>Grades taught</th>
<th>Current Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>BL</td>
<td>47</td>
<td>male</td>
<td>Yes</td>
<td>20</td>
<td>English and Music, B.Ed.</td>
<td>6-8</td>
<td>7-8</td>
</tr>
<tr>
<td>JR</td>
<td>52</td>
<td>male</td>
<td>No</td>
<td>30+</td>
<td>Environmental Studies, B.Ed.</td>
<td>7-8</td>
<td>8</td>
</tr>
<tr>
<td>CS</td>
<td>Mid-fifties</td>
<td>male</td>
<td>Yes</td>
<td>7</td>
<td>Theology, B.Ed.</td>
<td>4-6</td>
<td>4/5</td>
</tr>
<tr>
<td>KD</td>
<td>33</td>
<td>female</td>
<td>Yes</td>
<td>5</td>
<td>Social Services Worker Diploma, Masters of Teaching</td>
<td>4-6</td>
<td>5/6</td>
</tr>
</tbody>
</table>

### 3.4.1 Data Collection

The method of data collection evolved based on results from the pilot study. The qualitative data was first collected, and then the pre- and post-tests were administered to support the interpretation of the quantitative data.

#### 3.4.1.1 Qualitative data collection Main Study.

The data collection methods involved a series of bi-weekly structured and semi-structured interviews, teachers’ journal observations shared via email, and multiple sources of observable data and behaviour resulting from classroom visits.

The interviews were carried out twice during the six weeks leading up to the intervention, and then every two weeks throughout the intervention period, culminating in a final exit interview. These interviews were usually face-to-face and carried out in the teacher’s classroom. Occasionally interviews were carried out via Skype when weather prevented either the teachers or the interviewer from travelling to the school location. There were three “snow days” during the ten-week intervention period that required re-scheduling and Skype interviews. The interviews were transcribed first by the researcher, and again by a professional transcription service in order to ensure that nothing was missed.
The pre-intervention interviews were semi-structured, and were geared towards information sharing and practicing with the spatial games. The interviews during the intervention were structured with each interview representing one of five sections: background information of the participant relevant to the study, outcomes of the research experience on the participant’s development as a teacher, specific aspects of the research process and their impact on the participant’s development as a teacher, challenges with the research experience, and recommendations for design improvements for similar studies in the future. The structured questions can be found in Appendix B. The exit interview was unstructured.

Classroom visits were carried out when it was convenient to the teacher participants. Some of the teachers welcomed regular planned and unplanned visits, while one teacher preferred keeping visits to a minimum. During classroom visits, I interacted directly with the students as they engaged in the spatial skill activities in addition to observing how the teacher interacted with the students. These observations were recorded in a journal (Taylor, Bogdan & DeVault, 2016). Photographs were taken with permission.

These observations were geared towards developing a holistic sense of how the games were being used as an intervention strategy by the teachers. They helped to contextualise the data from the interview process.

Observation of a field setting involves:

- prolonged engagement in a setting or social situation
- clearly expressed, self-conscious notations of how observing is done
- methodical and tactical improvisation in order to develop a full understanding of the setting of interest
- imparting attention in ways that are in some sense standardised
- recording one’s observations

3.4.1.2 Quantitative Data Collection Main Study

Similar middle school classrooms were selected from one public school in small town, Ontario. In total there were four classes: one grade 4/5 class, one grade 5/6 class, one grade 7/8 class, and one grade eight class.

The teacher identified a spectrum of ability levels, and assigned aliases to the students. The classes received spatial skill training using “Toy Boxes” that had a variety of age-appropriate spatial reasoning games (see Appendix D). Some teachers chose to have their classes work primarily with three-dimensional, rotating block building games, while the other teachers had their classes focus mainly on the two-dimensional pentominoes and maze manipulating games, and still other teachers encouraged their students to try all of the games at least once. The interventions took place on a daily basis for fifteen minutes or more over a ten-week period during the main study.

Data was collected through the administration of a spatial reasoning and a mathematical reasoning instrument as a pre-test, and post-test. There were two main study instruments that were built and refined based on the outcome of the pilot study instrument. These instruments were structured to be age appropriate for grades four to six and grades seven and eight.

The spatial reasoning portion was modelled after the age appropriate instruments found on the SILC site: http://spatiallearning.org/index.php/testsainstruments, with an emphasis on tasks that measure success with mental rotation. The mathematical reasoning portion is constructed similarly to a 2009 diagnostic tool developed at OISE (UOIT PJ math diagnostic) to measure grade-appropriate mathematical skill in conjunction with diagnostic tools found on the Ontario Education site: http://ontariomathresources.ca/teacher-resources/assessment-resources. Additional mathematical reasoning questions were selected using the Math
Math Mammoth is a series of homeschooling textbooks tied to the American Common Curriculum. It was developed by Maria Miller, a mathematics and statistics specialist with an interest in creating meaningful homeschooling resources that allow for greater depth of understanding of grade-appropriate curriculum. The diagnostic problems are similar to those found in the EQAO Grade three and six benchmark tests, as well as junior and intermediate mathematics textbooks used at the three schools. Questions related to reasoning were selected and modified to try to remove bias due to the timing of curriculum delivery.

3.5 Data Analysis

3.5.1 Quantitative data analysis

The quantitative data was analysed using basic exploratory statistical methods. Descriptive statistics and analysis of variance were the first steps in making sense of the data. Descriptive statistics included mean, median, mode, variance, standard deviation and quartiles. The data was also graphed as scatter plots to determine whether regression analysis could be performed. Box plots were created and compared within schools and classrooms as well as across schools and teachers. This lead to $t$-tests to compare means to determine if changes were significant. Analysis of variance in the form of $F$-tests was carried out across teachers, within schools and across schools.

The quantitative data consisted of pre-test and post-test results that were scored both categorically and numerically. A rubric based on the “Thinking” portion of the Ontario mathematics curriculum assessment criteria was used to categorise the quality of the answers.
The intent was to measure three areas: change in spatial skill, change in mathematical reasoning, and the impact of engaging in spatial skill activities on mathematical reasoning. The first two were relatively straightforward to measure.

The sample size per year in the pilot study was about 90 students. In the main study, the sample size was approximately 120 students collected from four classes of about 32 students in each. Absent students did not write the tests at another time. It was a challenge to accurately measure the impact of spatial skill development on mathematical reasoning with such a small sample size.

Although I am experienced in a variety of Statistical software such as SPSS, SAS/STAT, and Fathom, the analysis was completed with EXCEL, which has a very useful statistical add-in that can perform most descriptive statistics with simple data such as this.

3.5.2 Qualitative data analysis

The interview data from both the pilot study and the main study were combined to examine common themes found by classifying recurring responses (Creswell, 2013). A matrix of categories was developed and supported by graphic illustrations such as mind maps and flow charts (Yin, 2014). I used nVivo11 to code the data. The pilot study interview data was coded by hand. The interview data was repeatedly revisited to refine the coding process. In addition, several key literature pieces were coded similarly in order to strengthen the validity of the coding.

Performing line-by-line coding is especially important at the beginning of a study because it enables the analyst to generate categories quickly and to develop those categories through further sampling along dimensions of a class’s general properties, a process of
sampling known as “theoretical sampling” (Strauss & Corbin, 1990, p. 119). “It is coding that starts the chain of theory development” (Bowen, 2008, p. 139).

A case study requires an exhaustive analysis of interview data to ensure that a central theme and its supporting sub-categories emerge with confidence. Repeated interviews of the pilot study teachers over a two-year period, combined with information provided by the teachers from the main study provided a rich set of data.

The data from the pilot study was used as a starting point to identify themes and categories and develop questions that would allow for meaningful comparison and emerging refinement for the subsequent phases. The interview data from the pilot study was transcribed and coded. Once the main study interviews were coded, the interview data from the pilot study was revisited and the themes were refined, presenting a clear picture of similarities and differences amongst the teachers based on their experience, school and students’ socio-economic background.

3.6 Ethical Considerations

The pilot study was approved by the University of Toronto’s ethics committee and has met or exceeded the required stipulations to ensure confidentiality of participants and data as well as to ensure that all members are aware of their rights and have chosen to take part. To ensure rigour, a committee will review the data analysis and writing of this study, as it is part of a dissertation.
Chapter 4 – Case Studies

4.1 Introduction

In this chapter, I will discuss both the qualitative and quantitative findings of the main study. The findings from the pilot study, which were used to further refine the design of the main study, are discussed in the Methodology section. While the pilot study was undertaken to drive the design of the main study, the participants in the pilot study were demographically different from the participants in the main study on a number of levels. As a result, a comparative analysis of the results of the pilot study versus the main study will also be discussed. The cross-case analysis yielded several themes related to the participants’ perception of the impact of including spatial games in their practice, on their students, and on themselves.

Coding of the interview data from the pilot study and the main study as well as journal notes and observation led to the development of three key themes. These themes are 1) Impact on students with sub-categories of learning habits, mathematical reasoning and enjoyment; 2) Teacher Change with sub-categories of experience as a student, early professional style and current style; and 3) Implementation of spatial games in the classroom.

4.2 Main Study – School C

The main study was conducted at School C in a small town in rural Ontario, whose population has a mean income of about $24 000 (Stats Canada). Twenty-six percent of the adult population did not complete a high school diploma. The unemployment rate is slightly greater than the provincial average at 6.1%. School C is an elementary school, with a population of approximately 300 students from JK to grade eight, comprised of mostly bussed-in rural students. Ninety percent of the population speaks English as a first language, and one
percent speaks French as a first language, with the remaining percentage being made up of German and Dutch (Mennonite) families.

The school is older, with a declining enrollment, and is on its large geographical district’s list of schools to be closed. The local factory, which employs a large percentage of the students’ parents, has made an offer of money to refurbish the school to keep it open. The faculty comes from the surrounding villages and towns. Many of the teachers were educated in the local schools.

4.2.1 The typical lesson with games

The teachers at school C administered the games activities in the same manner during each session. The number of times that the students were exposed to the games per week varied amongst the four teacher-participants.

A student leader was appointed who was responsible for collecting the Toy Box (see Appendix D) and laying out the games on a table. Students were given free choice to select a game, and were encouraged to work with a different partner during each session. If students gravitated towards the same game each session, the teacher would suggest that they attempt a different one.

The teacher allowed for fifteen to twenty minutes for the students to interact with the games and often partnered with a student or circulated through the room to offer encouragement. When the fifteen-minute time slot was finished the students packed up the games and returned them to the Toy Box.

Most of the teachers were able to implement the spatial games activities on a daily basis throughout the ten-week study.
4.2.2 JR.

JR is a middle school teacher with twenty-seven years of experience. He comes from a rural area and attended school in the community. He knows his students well as he has lived most of his life in the community. Although his undergraduate degree was in Environmental Studies, JR has both a keen interest in teaching mathematics and solving math puzzles in addition to exploring and using diverse teaching strategies to engage all of the members of his classroom. Thoughtful, open-minded, genuinely interested in his students, JR quietly keeps his students engaged and on-task with carefully crafted lesson plans.

JR was a wealth of information during interview sessions with his insightful observations and ability to make connections to a wide range of aspects of his students’ lives, as well as his own learning path as both a student and a classroom teacher.

4.2.2.1 JR’s views of the impact on students.

JR immediately saw the positive impact the games had on students that he recognised as having strong mathematical skills, but who were unable to demonstrate them in a traditional school setting.

I think these games are part of that piece where some kids are the brightest math kids because they can do all the problems and they can follow through, but there are other kids that are great at math, and we are not asking them questions to show that.

JR is the most experienced teacher in both the pilot study and the main study. His dedication to his students and ability to connect the material he teaches to the realities of his particular cohort is impressive. JR put a lot of thought into how these games had an impact on his students, not only in terms of their understanding of mathematics but also in terms of their general learning skills and life skills. He is cognizant of the fact that most of his students are likely to find employment in the local factory, the trades, or on their family farms.
I do think that physical hands-on part is really important. I think these kids are plugged in enough. I think they actually need to ... You are not going to fix a car online; you are not going to bake online, you are not going to fix somebody’s teeth online. You need to be able to manipulate your hands and develop that fine motor skill. To move things around and shape them from a different perspective. They are just carpenters, masons, any trades person, somebody that is doing hair. Cutting this side, what is it going to look like when you get to the other side? Is it going to be symmetrical or not? I think the hands-on piece is really important.

JR saw a lot of value in the use of the spatial activities for developing planning skills that would lead to better problem-solving. He felt the novelty of the problems led to an enthusiasm to pursue the solution that might not be as evident when working on a novel mathematics problem from the grade eight curriculum. JR could not say that he saw a confirmed improvement in perseverance and problem solving after ten weeks, but he felt quite positive that he would see these skills when he continued to utilise these activities after the study’s completion.

I think that when you do that spatial reasoning stuff with the kids, they are looking at constructs or problems may be from a perspective that is totally new to them. That idea of trying to in their brain they are kind of figuring ahead maybe the next step and looking at what it could be. I think when they are doing that, I think that is a skill that could be transferred over to logical reasoning and things where you would use it formerly because they are looking down the road and going, "This is the end product?" Knowing how to get there.

JR was pleased to see that during the spatial activities the students were attacking the problems both as small groups and, more importantly, as individuals. JR worries about students’ access to outside help through phones, computers and other digital devices, as well as his school’s emphasis on group work for most tasks and assessments; “add to that is how much group work there is. They [the students] have lost their own independence and knowing that they have that skill.” He feels strongly that our culture and societal expectations are reducing self-reliance and the ability to be a self-starter. JR saw that when the students interacted with
the spatial activities, they gained confidence in their ability to work through a puzzle on their own even when a starting point was not always obvious.

JR also noted that the students tended to verbalise their strategies for solving problems and would share these ideas with him and their classmates. “What [he] would be hoping for as the teacher, that [this type of dialogue] is going to improve their communication skill so that you can see how they are thinking.”

JR was pleasantly surprised at the learning skill bi-products for his students that he saw as they worked through the spatial skill puzzles, so much so that he continues to use these types of activities in his classroom.

4.2.2.2 J R’s views on his own changing pedagogy.

JR describes himself as pretty flexible in his teaching. As the most experienced teacher in this study, he seemed open and willing to try new things and expand his pedagogical tools. JR is always looking to make connections between mathematical content and his students’ lives; in collegial conversations he often considers how he can use information about their lives in his teaching and activities, and how this has an impact on student comprehension and retention of new knowledge.

JR is an excellent example of the reflective practitioner. His teaching techniques and methodology are in a constant state of evolution. JR’s involvement in this research did not change his approach to teaching, but did provide him with another tool to investigate and use in enhancing his students’ learning.

4.2.2.3 J R’s views on implementing the spatial games.

“I do like math puzzles and challenges and stuff like that.”
JR enjoyed the whole process of implementing the spatial games. He developed a system on the first day to ensure that students were in charge of selecting and distributing the games as well as returning them, complete with all pieces, to the Toy Box at the end of the session. JR often participated either in competition with another student, or as a partner with a student to solve the puzzles in the time allowed. He felt that he was able to model the value of failure as a process to learning, and that he was also able to build relationships with his students through the spatial skill activities.

I think it is a piece where they can see me struggling, too. It is sort of like, "Oh, he does not get everything." Well, they know that already, but they can see that you have got to think through things and persevere.

JR also identified expert students for each of the games in the Toy Box and would have these students explain how to start the games or assist other students whose stick-to-it-iveness was waning. But, like all teachers, JR did identify the same primary challenge of time.

I think certainly finding the time with the curriculum and trying to manage what it is supposed to be, what we perceive we are supposed to be getting through and what that all looks like can be problematic.

4.2.3 BL.

BL is a teacher who has been teaching Intermediate grades for 20 years. His undergraduate degrees are Honor’s Music Education and English, with a minor in Business Administration, which he obtained from Wilfred Laurier University. BL has teaching credentials in Primary, Junior, and Intermediate from Brock University, with additional qualifications in Drama, Grade 7/8 Mathematics, and Technology Integration in the Classroom. In his spare time, he enjoys a variety of gaming, including board games, roleplaying, and video games. BL is also actively involved with community music programs, running an after-school
music program for young people now in its 10th year, as well as involvement in a community choir. In 2014, he received the Prime Minister’s Award for Teaching Excellence.

BL is a confident, high-energy, enthusiastic teacher whose students are active participants in his lively and fun lessons. BL had prior experience in classroom intervention experiments and embraced participating in this study. He was especially keen to take part as he had spent the past few years experimenting with games as a teaching strategy and feels strongly that using games in the classroom to support the curriculum also has beneficial effects on learning habits.

4.2.3.1 BL’s views of the impact on students.

The timing of the ten-week intervention with BL’s mathematics program was fortuitous, as he was just completing a unit on Algebra and beginning a unit on Measurement, specifically area and perimeter of compound figures. Traditionally this unit had been challenging for the bulk of BL’s class, which is usually made up of a high percentage of students on IEPs.

I have got some of those students who are operating at phase two, which is the equivalent of roughly grade two, where they are in currently grade eight. That is a significant gap in understanding. However, in an opportunity to be using games such as what would have been brought in, it does activate, I think, kind of a different part of their mind. It is not a mathematical numbers sense of the area. Instead, it is more visual/spatial, and I think that they can potentially have more success and have a better growth mindset as well.

BL was pleased to see that his weaker students could find success in the measurement unit by applying their spatial understanding from interacting with the games on dissecting compound figures into simpler figures such as triangles, squares and rectangles. BL felt that the spatial skill games’ impact was most evident in his struggling students, had some
observable impact on his moderately weak students, and felt little change was noticeable with his successful and highly successful students.

While BL saw that his weaker students were making better decisions in their measurement and were better able to articulate how to approach the problem, they continued to struggle with the final stage of applying formulas.

I think that while I have done this 20 years, I think I am seeing, already, better results from having had exposure to these games than I have seen in the past as a whole. There are still students, the ones who are weak, who still have a hard time making that transition from visual to paper, but I'm still ... I would say that I have seen growth.

When reflecting on how the spatial games exercises might have an impact on his students’ comprehension of algebra, BL felt that there would be some transference, but he could not, with confidence, say that he actually saw any during this ten-week intervention.

Again, with algebra, we are keeping things equal and in balance. I do not know that I have seen a huge connection between equal and in balance in terms of using the tools as a spatial game, but it is all just about making those pathway connections within the brain, so every little piece will make another pathway.

BL was able to see positive benefits in other areas of his program as well. Each day BL poses a problem of the day that can be from any subject that he is teaching that week. These problems are somewhat optional, and students write an explanation about how they would go about solving it. “I would say easily 85 to 90% of the students were able to find success, and I partly attribute that to their experience of working with a spatial tool.” In particular, he found that students were more willing to take risks and wrote more details about their approach even if they were not able to find the solution. He attributes this to the student’s experience of problem-solving with the spatial activities. “It is a different kind of focus. It is not a focus where you are struggling and getting angry and giving up, but instead where you are going to try and puzzle and get to a solution.”
BL feels strongly that developing good group work skills is essential for his particular group of students. Many of his learning opportunities are geared towards group work, and his classroom management skills also emphasize the importance of being a good citizen, which promotes the success of the group. He was pleased to see that many of the spatial games could be used to promote working together towards a solution wherein each member of the group can actively participate. “I really work toward a collaborative environment in the room, and it was just another piece that helped for that collaboration to happen.”

BL was pleased with the impact on his students from participating in this study, and has added several of the spatial games to his collection of other games that he regularly uses to promote learning.

4.2.3.2 BL’s views on his own changing pedagogy.

BL describes his own experiences as a student in the mathematics classroom as uninspiring, with teachers who mainly used the “chalk and talk” method.

I have always loved math. Yeah. I struggled in high school, and I am a real computer guy. I am really into technology. As I became an adult, I thoroughly love math. It is actually my favourite subject to teach. One of my favourite things to work in.

His own boredom as a student encouraged him to explore different avenues of pedagogical practice. “I want to find a rich environment for my students. I want them to be engaged. I want them to enjoy themselves in all subjects.”

When asked if his experience in this study changed his teaching practice in any way, BL replied that his participation was really more of an extension of his quest for personal excellence as an educator. “I would not say it has changed my methodology in that I always want kids to be hands-on anyway with what they have been doing.” BL was already using
games as part of his teaching technique and had been exploring spatial reasoning as part of his administrator’s in-house professional development. It was a simple segue to add spatial games to his daily practice.

BL’s principal has organised several in-house professional development/staff meetings around the “Paying Attention to Spatial Reasoning” document handed down by the Ontario government. As a result, this ten-week intervention was timely in terms of this particular school’s mandate for improving mathematics. Unfortunately, not all of the staff is as enthusiastic as BL is when it comes to adding new skills to their toolkit.

I know that one of my colleagues had said ... I was talking about some of the things that I'm doing in my class. He said, "You know, you must spend a lot of time looking for resources on your own time." I said, "Well, yeah, I do because I see that as part of the job."

Interviewer: That particular teacher ... If the administration came down and said, "You will do this in your classroom, would he do it?"
BL: Until the administrator left.

In the final exit interview, BL did identify a few areas where including these spatial skill activities had an impact on his planning and execution of lessons. But for the most part, the greatest impact was an affirmation that he was on the right track with his own reflective practice.

Well, I guess a little bit of an improvement in my own lesson planning in terms of being able to connect to something that the kids have done. When I have been, for example, we are looking at the area of trapezoids and moving into three-dimensional figures, that type of thing. I'm able to make connections to what we have been doing through the games and just have that as another piece of my repertoire of things that I can use as a connection for the students.

I would not say it significantly did not impact my practice other than attributing some specific time to students to engage in play.

It changed how I do things in [starter activities] …and playing and [choosing skill specific] games. It changed some of my process.
BL felt that for change to take place it would require collegial conversation and support. He found that the discussions that he had with the other three participants, the principal, and me were the most valuable aspects of the study for him. His very last comment in the final interview: “When teachers work together and collaborate and try and make some plans, that is when you start to see some change.”

4.2.3.3 BL’s views on implementing the spatial games.

BL is already a gamer himself and actively uses a variety of games in his classroom as a teaching strategy. His pre-existing games formed a wide cross-section including board games as well as online games such as Minecraft, Dungeons and Dragons and other strategy games in a variety of formats. His students were accustomed to using these activities as warm-ups and brain breaks, so the addition of the spatial skill activity was not a novel experience.

“They look forward to coming in and doing that each day. I do occasionally need to make a choice for them of what activities I want them to be focusing on. Otherwise, they tend to gravitate towards the ones that can be competitive, which is fine.” It was important that the games served a learning purpose and, while BL felt the competition aspect increased perseverance and desire to complete the exercises, he would consciously direct specific students to specific games that he felt would develop a skill they needed.

I really have not found a lot of problems with the implementation phase; I'm glad to say. The students have been able to engage in multiple games. I guess if anything it might be that occasionally I had to prompt them to try something new, but for the most part the students that I'm working with are willing to try something new, to please me as much as they can, and this is the way that they felt that they could do that successfully by having a variety of choice. So, I see students quite often choosing different activities each day, and if they hear me say I want everyone to do new partners, they are with new partners. If I ask somebody to do a new activity, they are on to a new activity. So that has not been really a challenge.
Interviewer: Do you think that is because you have already set up a sort of classroom management and classroom expectations prior to the study?
BL: I would like to hope so. Yes. I have very high expectations of my students, and it is ... I hope that I see it reflected in the study.

The three-part lesson including a meaningful starter is part of BL’s repertoire. When reflecting on going forward with using the spatial activities as part of his mathematics lesson BL commented:

I think that it is a great warm-up in math just to get a minds-on type of thing. I like it as a 10-15 minute at the start of an hour period. It is still kind of ends up with a typical 40-minute or traditional 40-minute period of math afterwards, and about three times a week I think that that is a good use of their time as well. So, in a perfect world, having my own set of tools, it'd be three times a week of 15-20 minutes at the start of the math period.

In fact, the principal of the school encouraged the parents’ association to purchase class sets of spatial games for the school based on the feedback from the teachers involved in the study.

4.2.4 CS.

CS is a second-career teacher after spending many years as a Minister in Northern Ontario. CS’s wife, who is a teacher, may have influenced this choice as he claims that it was a career that seemed quite undesirable to him when he was a young man. CS is a traditional “chalk and talk” teacher who is rule-oriented, and ranks good behaviour high on his list of desirable traits in a student. He is dedicated to his students and spends a great deal of time developing and making his own learning resources for them. CS’s professional background is evident in his desire to help his students become good citizens who work hard to please him and meet his high standards. His colleagues appreciate his desire to help, and value his advice.
CS manages his classroom in a traditional manner, with students sitting in single-file columns, facing the front board. He found the active, kinesthetic aspect of administering the spatial games difficult, in that the students were noisy and active as they worked together to solve these problems.

4.2.4.1 CS’s views on the impact on students.

CS found it difficult to identify how the spatial games were having an impact on his grade four and five students in part because he felt tired and discouraged that this cohort had so many behavioural issues. CS participated in the study because he felt, as a senior member of staff, that he should set a good example for the younger staff. He had some hope that having his students engage in the spatial skill activities might reveal some “magic trick” to inspire his students to learn. CS reported that he could not see a connection to mathematics or to problem-solving skills.

I think that is the struggle is one is not always sure what expectations are we meeting with this, and how can we use it? Not only in developing their skills but as well as part of our assessment and progress towards meeting some of the curriculum expectations.

CS felt that the ten-week intervention had little impact because his students were too rigid to try another approach to learning, having been educated in the traditional method of “here is the model, this is what you do, step one, step two, step three, and follow it because that seems to be how they are being taught math ...” Although CS felt that this methodology was unlikely to be inspiring, it was exactly how he conducted his math lessons, preferring to insist on uniform procedures for solving problems with little encouragement to estimate or consider whether the procedure provided the correct answer.

I have always found that the students here tend to be less interested in puzzles and math-related games. The instant gratification. That tends to be very predominant. I find with the younger classes even more so. That lack of perseverance. I have tried to give
them a reward in order to persevere, and it is a bit of a struggle to get them to really see the enjoyment of persevering and really trying to figure it out.

CS reflects that he has seen a change in student behaviour with the increasing use of electronic devices. He finds that while they seem to like playing with games, they are quickly bored and simply look up the answer.

Interviewer: You do have a number of very weak students, which seems to be a bit of a theme at this school. IEPs identify all sorts of things, but usually as a teacher when you interact with the students you see more personal reasons why these kids might be struggling. Is there something that stands out for these kids? Maybe not stands out, but is common to the kids?
CS: The lack of ability to focus.
Interviewer: Right. Where do you think that comes from?
CS: A number of them either daydream or space out when they are with an adult.
Interviewer: Okay.
CS: That is a common thing. Otherwise, they tend just to be very much focused on the quick action like in a video game, or watching something on television. Very, very quick.

CS expressed typical teacher frustration when asked about his struggling students.

There was one outstanding student in his class who I asked about based on her near-perfect scores on both the spatial portion and mathematical portion of the pre-test. He attributed her success to her solid family home and parental support and felt the school environment had little, if anything, to do with it.

In part, CS feels that the reason the students feel unsuccessful with the spatial activities is that they do not have consistent success with them. They are not developing strategies, preferring instead to use a guess and check approach.

No, they do enjoy it. They are not seeing their development because they are not moving towards really persevering and getting it. I know, when you build a shape with the building blocks [Bloc by Bloc], once you have got it done, it does not mean that you can the next day repeat the challenge because there is so much involved in it. I think that is part of it, too. They like to be successful at a task and feel that they can repeat that success. They are finding that that is not true.
CS rarely demonstrated questioning to elicit student reflection on how to develop a strategy. It may be that he did in other lesson situations, but was too tired and disillusioned to engage with the students while they were interacting with the games, preferring instead to sit at his desk and mark papers. In CS’s opinion, including spatial skill activities had no impact on his students in terms of learning or skill development.

4.2.4.2 CS’s views on his own changing pedagogy.

CS was quite blunt about the impact this intervention had on his pedagogy. He could not see any improvement in his students, he found the implementation messy, and it took him out of his comfort zone.

That is who I am. I like to have things neat and organised, so we can use the tools well when we need them. I found that because they are going from place to place, it was a difficulty for me.

CS is highly unlikely to change his style from his self-described “sage on the stage”. Intellectually, he would like to, but as he is nearing the end of his professional life he knows that he is unlikely to make any methodological changes.

One, most teachers will take an activity and repeat it, and look for and hope for the learning that they think they did, and want to see the child learn, but they are not necessarily creative in developing that. The other thing is that they are not seeing how it changes the way students learn. It is like working in small groups if they actually see a big change in the dynamics and in the learning with the children, they would continue on with it. If not, they will just discard it and go back into their comfort zone.

CS has engaged in quite a bit of professional development over his second career in teaching, and has brought the experiences of his first career as a minister to bear on his teaching career. However, he has rarely, if ever, found techniques or strategies from these professional development opportunities to bring back to his classroom. He did admit that he
enjoyed his course in 21st-century learning because it was progressive, but the ideas were simply not practical, in his opinion, for grade four or five classrooms.

4.2.4.3 CS’s views on implementing the spatial games.

CS found quite a lot to be disappointed with when it came to be implementing the spatial games. He is a very methodical man and had planned his thrice-weekly fifteen-minute sessions down to the second. The first challenge was that the Toy Box was shared amongst the four teachers, and they needed to communicate with each other to schedule when they were using it. The second challenge was that he often received the Toy Box in a disorderly manner. CS tried to fix this problem by asking for extra boxes and packaging to keep the different types of games grouped and the pieces all in one place. “It took a long time for them to start to actually try to begin solving the problem of the puzzles because so often there was a lot of chaos, first of all, because they thought it was just a fun activity.”

Distributing the toys in his room meant that the students were noisy and not necessarily sitting in their assigned seats. They did not always like the toys they were given, but if they were left to choose the situation grew even more chaotic. The effort required to use leading questions to direct the students to develop problem-solving strategies was overwhelming. “They do not even talk about how they make connections and why they think their answer is reasonable.” If a student did not want to engage in the exercise, they were simply left to sit for the fifteen-minute slot.

“Part of their problem, I find, is it is the way that they are ... They do not have an ability to actually calm down and concentrate well.” In fact, the students did want to engage with the activities and did seem to want the adults in the room to discuss their strategies with them. They appeared thrilled when they solved a problem and excitedly shared their success with
their classmates. The scene that I saw when I was observing the class was quite different from CS’s view, as he was more focused on behaviour and order.

CS did feel that the intervention would have been more successful if there had been at least two adults or possibly older students in the room “who are supporting the kids and are guiding the kids and getting them on task and getting them thinking through that whole process and helping them, supporting them, also would show the students, as well as the teacher, a good way of interacting”. CS’s large class of younger students, many of whom he felt were quite immature, and many of whom were on IEP’s, may have made the practicalities of a study like this overwhelming even to a teacher as experienced and as committed as him.

4.2.5 KD.

KD is a second-career teacher who grew up in the local community and attended the school where she now teaches. She graduated with a diploma in social services and worked in the Catholic School Board as an educational assistant. Realising that she wanted more from her involvement in education, she returned to school and earned a Master’s of Teaching. KD is deeply committed to her students, and to understanding how learning occurs at every level. Before taking part in this study, she had performed some action research in her classroom, also investigating spatial reasoning.

KD freely admits that mathematics is a struggle for her, and worries that her students might suffer as a result. From the ten weeks, I spent with her; I would suggest that quite the opposite is true. Her struggles with mathematics have meant that her lesson design is geared towards understanding and mastery. She uses a variety of teaching techniques and is always investigating alternative methods for introducing curriculum content. KD has a large class of
grades four to six students, many of whom are on IEPs for learning and behavioural issues. Her students enjoy her lessons, which tend to be a bit on the noisy side, but in a positive way.

4.2.5.1 KD’s views on the impact on students.

KD’s previous experience with including spatial games as a learning tool in her classroom meant that she had more time than other participants to reflect on how these activities might have an impact on her students’ learning.

The major thing that I have found is through the learning skills, even more so than mathematical skills so far, right? The big thing is perseverance and challenging themselves because a lot of them do not do that on a regular basis.

KD’s experience in previous action research, inspired by the Ontario document “Paying Attention to Spatial Reasoning”, meant that she was looking for connections between the students’ practice with the spatial activities, and its impact on both their mathematical reasoning and their learning skills. KD noticed an improvement in perseverance amongst her weaker students, but also with the students that did not find the curriculum particularly challenging.

There is some that were unlikely to challenge themselves that before they would take the easy route out because math is quite easy for these two people I’m thinking about. When all of a sudden it was not easy for them they would give up like that, no problem right? Now I’m finding with these that they are tending to stick with it and challenge themselves, take the cube ones [Bloc by Bloc] that they know is a bit harder for them instead of the easier puzzles.

She felt that the games gave all her students a non-threatening way to fail and try again and ultimately be successful.

Using the spatial games as regular Minds On pieces at the beginning of the mathematics lesson meant that the students were more enthusiastic when they made the
transition from another subject or learning opportunity to mathematics. KD mused that in the past when she announced that it was time for a mathematics lesson:

it [was] constant groaning. For a long time. [Now] they were excited because they knew that they got to start out with puzzles because I would often use it as that Minds On piece. So, they were actually excited to start math because they knew that was coming. That was great.

This new enthusiasm for the lesson meant better-sustained focus during the introduction of new topics, with more of her diverse class engaged than in the past.

Also, with each other, a lot of kids liked working in pairs when they were working on these puzzles. I often saw an increase in their communication, so that was good to see.

KD felt intuitively that there should be a connection with the various spatial games and specific content that she was delivering, but was not sure whether what she was seeing was because of her positive beliefs, or an actual effect of the games.

I find it is hard for me to measure. I think that it has definitely impacted their geometry. But in other strands of math I have not seen, or at least I cannot directly connect any improvement to, the spatial reasoning activities.

KD is looking forward to teaching new topics in the grade four to six curricula, and is foreseeing a connection there.

I said geometry, definitely the ability to see figures from different perspectives. I thought of an example where one of the students had a cube [Bloc by Bloc], one of the cube ones, right? They were trying to fit the piece in but not even rotating it to see that it actually looked different from a different perspective, right? I taught them how to do that, and I'm anticipating that when we get into transformations and stuff like that more in class that they will do a lot better on it because of this.

4.2.5.2 KD’s views on her own changing pedagogy.

KD is a committed reflective practitioner. She keeps a learning journal and regularly writes down her observations on how her lessons had an impact on her students. KD is acutely aware of her own impact on her students’ learning environment and works hard to create a
Joyful classroom. Mathematics is not a natural topic for her, and she reports that she finds it difficult to think of creative means for lesson development. When asked whether her participation in this study changed the way she thought about teaching mathematics she replied with an enthusiastic affirmative.

The way I think about teaching math, yeah. Before I really thought things and, first of all, teaching one thing at a time and so like geometry, that would just be one unit that I really did not go back and revisit constantly. Then also teaching things in different ways. Some things are harder for me than others because I learned it in a certain way and so I can teach it in a certain way. I also did not recognise the importance of spatial reasoning and other units in math. It definitely changed my view on that as well.

KD did not require any persuasion to participate in this study when her principal canvassed the staff, but she did confess that these add-ons to her already heavy load as a teacher, and her own personal desire to be seen as competent and knowledgable, were daunting. ‘I do think a big part of this, and in my mind, I'm thinking of, I think a big part of it is trust and anxiety and the thought of having to do yet another thing.” She felt that while the other staff members who had clearly enjoyed the hands-on professional development that her principal had been conducting with SOMA blocks (see Appendix E) could see the benefits of including these spatial activities in their day-to-day teaching, few were likely to try it due to insecurity and time constraints.

KD felt that her participation in the study and her experience with using the spatial games with her students had several other added benefits to her teaching practice.

Probably better differentiation of [material] in the lesson coming up with more hands-on stuff for them to do and trying to use different tools, manipulatives, to help them with their everyday learning.
Having another tool at her disposal for differentiation was a gift for her very diverse student body. The games gave KD a starting point to develop creative lesson introductions and kinesthetic lessons.

I think that it was just the implementation of the games as Minds On activities and, like we talked about before, implementing more hands-on things in everyday math lessons where I do not think I did that as much before. It was more so paper, pencil. Although we did do some hands on, it was not as often as I did it. I think I’m implementing it a lot more smoothly now whereas before it was something I really had to think about. So, I think that is the biggest difference.

KD has taken her experience from this study and has continued her own action research using these games. She is currently trying to devise a quantitative method to measure progress in spatial reasoning and mastery in grades four to six mathematics content that removes or minimises teacher effect.

4.2.5.3 KD’s views on implementing the spatial activities.

KD had been using similar spatial games in her classroom for the previous year as part of an ongoing professional development action research project to raise the level of the students’ understanding of geometry and measurement. The interaction with the puzzles was more of an add-on rather than an integrated part of the mathematics lesson and was usually treated as a reward for those students who completed their work in a timely fashion.

When I had the games last year, what I did was when you have done your work grab a puzzle, and you can work on it. The problem with that was that I found that some kids, those kids who never get the work done will never get a chance to play with the puzzles. What I like doing was using them as that Minds On piece in class, so it was the first thing I did at the beginning of math class.

As KD’s students were already familiar with some of the spatial games from prior experience, it was not that difficult to train her students to start each math lesson with a fifteen-minute warm-up with the Toy Box activities. KD did find it challenging having to share the
class set with her three study mates, preferring to keep the Toy Box in her classroom so that if additional opportunities to have her students interact with the games arose, she could have them do so on an ad-hoc basis.

We talked about having each individual classroom have their own box; so that would be an improvement in itself. I think the big thing for me would be time. 10 weeks is too short I think. I just thought that it was too little time to see an improvement. Also being able to run our own diagnostics, even if it were the diagnostics that you gave us, but being able to see those scores.

KD echoed the other study participants’ desire to be able to see some form of measurable improvement. While she felt that it was beneficial to the students, she was not sure that she could trust her own judgement when she imagined so many favourable implications of this study. KD feels that assessment for learning is an important aspect of lesson planning and curriculum development. “I think that the diagnostics would be a big part of that for me so that I knew where to go next. Because that is where I'm at right now is where do I go next because I do not know where the improvement [is].”

4.3 Quantitative Findings – Main Study.

Four classes of students at this school participated in the pre-test and post-test while taking part in a ten-week intervention where they engaged in spatial games for fifteen minutes, usually daily. Ninety-nine students wrote the pre-test, and ninety-two students wrote the post-test. As the students ranged from grade four to eight, there was a Junior pre/post-test and a Senior pre/post-test for mathematical reasoning. The spatial reasoning portion was the same for each grade cohort. The mathematical reasoning portion on the post-test had a few additional questions to include mathematics material learned during the ten-week period. All classes were studying grade specific measurement and data management (probability) at the time of the intervention.
Interestingly, the students tended to have stronger spatial reasoning skills than mathematical skills even at the very beginning of the study.

Table 5. Descriptive Statistics for All Classes Combined

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<thead>
<tr>
<th></th>
<th>Spatial</th>
<th>Math</th>
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<tr>
<td></td>
<td>Pre</td>
<td>Post</td>
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<tr>
<td>Mean</td>
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<tr>
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<td>Q3</td>
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Table 6. Measurement of Change in Reasoning from Pre-Test to Post-Test

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<thead>
<tr>
<th></th>
<th>t-Test Spatial</th>
<th>P=0.38</th>
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<tbody>
<tr>
<td>t-Test Math</td>
<td>P=0.0057</td>
<td>P&lt;0.05</td>
<td></td>
<td>Significant Change</td>
</tr>
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</table>

Although the spatial reasoning results did not appear to change due to only a slight increase in the mean results, the median results did rise by 10%. The mathematical reasoning results do show a significant change with the mean results increasing from 47% to 63%. The mathematical reasoning change from pre-test to post-test also showed a large gain where the 50th percentile results increased by 18%. While these results are not strong, they do suggest that having the students engage in the spatial games for fifteen minutes each day does not have
a negative impact on the middle school students’ mathematical reasoning, and may have a positive effect.

Table 7. Descriptive Statistics by Grade Grouping

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<tr>
<th></th>
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<th>Gr 4-6</th>
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<td></td>
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<td>77%</td>
<td>47%</td>
<td>74%</td>
<td>60%</td>
<td>62%</td>
</tr>
</tbody>
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When the data is broken down by grade cohorts, four to six and 7/8, we can see that while a positive change in mathematical reasoning is clearly evident in the 7/8 group and possibly evident in the grades four to six group, there also appears to be a possible positive change in spatial reasoning amongst the younger cohort.

Table 8. \( t \)-Test Grades 7/8

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</tr>
</thead>
<tbody>
<tr>
<td>( t )-Test Spatial</td>
<td>P=0.38</td>
<td>P&gt;0.05</td>
<td>No Change</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t )-Test Math</td>
<td>P=0.0071</td>
<td>P&lt;0.05</td>
<td>Significant Change</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Table 9. \( t \)-Test Grade 4-6

<p>| | | | | | | |</p>
<table>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>( t )-Test Spatial</td>
<td>P=0.06</td>
<td>P&gt;0.05</td>
<td>Possible Change</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( t )-Test Math</td>
<td>P=0.09</td>
<td>P&gt;0.05</td>
<td>No Change</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

Analysis of the individual teacher’s class results suggests that there may be a slight teacher effect. This is impossible to quantify given the data available.
In these tables, it is evident that the class results and behaviour are similar for these two teachers, despite their very different pedagogical styles.

Measurable change in both spatial reasoning and mathematical reasoning in the younger grade group is more moderate, with CS’s class resembling the grade 7/8 behaviour. KD’s class, which had been involved in spatial intervention action research prior to this study, shows little change other than a reduction in outliers and greater concentration about the mean.

Comparing means of pre-test versus post-test for each teacher reveals some differences amongst the teachers.

Table 10. *t*-Tests by Teacher Participant

<table>
<thead>
<tr>
<th></th>
<th>JR</th>
<th>BL</th>
<th>KD</th>
<th>CS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change in Spatial Reasoning</td>
<td>0.378</td>
<td>0.25</td>
<td>0.289</td>
<td>0.044</td>
</tr>
<tr>
<td><em>t</em>-Test P P&lt;0.05</td>
<td>Not significant</td>
<td>Not significant</td>
<td>Not significant</td>
<td>significant</td>
</tr>
<tr>
<td>Change in Mathematical Reasoning</td>
<td>0.0003</td>
<td>0.001</td>
<td>0.388</td>
<td>0.047</td>
</tr>
<tr>
<td><em>t</em>-Test P P&lt;0.05</td>
<td>significant</td>
<td>significant</td>
<td>Not significant</td>
<td>significant</td>
</tr>
</tbody>
</table>
Towards the end of the study, students played with a Bloc-by-Bloc activity. This game is a three-dimensional puzzle consisting of 3-D pentominoes. The students are presented with the seven pentominoes and a puzzle diagram to solve. This activity is considered the most difficult of the games and often takes more than twenty minutes to solve using clues and working in pairs. Frequent practice does improve results, but it does appear to be age-related. These results reflect the students working as individuals without using any of the clues. All students completed the task in fifteen minutes, while a few could complete the activity in less than three minutes.

Figure 10. Comparison of Grade Groupings on Bloc by Bloc Test

Comparison of grades 4-6 versus grades 7/8 reveals that their results, as a whole, are highly correlated (Pearson = 0.94), and the grade 7/8 group is consistently faster. $t$-Test = 0.038. Examination of box-plots, to estimate normality, shows symmetric results for the older students and skewed results for the younger students.

Analysis of variance amongst the classes within schools revealed no significant differences. An $F$-test performed, comparing the pilot study schools with the main study schools, did reveal a significant difference on a 1-tailed test. However, as the student bodies of the pilot study schools were considerably less diverse due to the private school setting versus
the public school setting, it is unlikely that this difference is due to the outcomes of the pre- and post-tests measuring spatial and mathematical reasoning.

Categorical data analysis of quality of response demonstrated no measurable change or even relationship between the pre-test and post-test, with the exception of one class whose responses on the proportional reasoning question improved in almost all cases.

4.4 The findings when comparing the Pilot study to the Main study

There were a few differences between the participants in the pilot study and the main study that may have had an impact on their views. The teachers in the pilot study were all young, novice teachers teaching at independent schools with students who came from a high socio-economic background and had strong parental support. In contrast, the teachers in the main study were older and more experienced, two of whom were nearing retirement. The student body came from a more challenged socio-economic background and had little parental support.

The class sizes in the pilot study were considerably smaller than the class sizes in the main study, allowing for longer and more frequent interactions between students and teacher participants during a mathematics lesson. The pilot study teachers were all subject teachers who saw their student less than three hours per week, while the main study teachers were with their students all day, every day. Both School A and School C had a large percentage of their students on IEPs, while School B’s students were required to write entrance exams for acceptance and were working towards MYP diplomas.

In terms of job security, the faculty at School A were on yearly long-term occasional contracts with few benefits; the faculty at School B were on renewable contracts, while the faculty at School C, in the main study, were all hired by their district school board and were
members of the teachers’ union with all of the benefits that come with being a public-school teacher. The main study faculty had the uncertainty of not knowing where they would be working at the end of the school year, as their school was one of many slated to close at the end of June 2017.

Teachers in School A and School C expressed similar positive observations with respect to increased perseverance on tasks, greater student engagement, and improvement in communication skills between students and between student and teacher when problem-solving. The students at School B did not demonstrate any obvious changes in learning habits according to their teacher, who also suggested that it was possible that this particular group came to him with well-established learning skills.

One teacher at both School A and School C could not see any connection between the spatial skill games and an impact on student learning skills. Interestingly, both of these teachers had concerns about losing control of the class while the activities were going on and saw the spatial games as more of a fun break rather than as exercises with any educational value. Similarly, the teacher at School B did not see any immediate connections to learning skills and worried about the loss of control when his students were engaged with the spatial activities during the intervention time.

Every teacher in the pilot study described seeing their students in a different light after the exercises, but only one teacher in the main study alluded to this theme. Perhaps novice teachers have not met a large enough cross-section of students at this early stage in their career to recognise the large variety of learning styles in existence? Perhaps this feeling in the pilot study teachers resulted from them having a more homogeneous student body? Perhaps more
experienced teachers are not surprised to see students behave differently when presented with a different learning experience, and did not feel it was worth mentioning?

When it came to reflecting on their own pedagogy, the participants in both the pilot study and the main study responded similarly. All felt that their involvement encouraged them to think a bit more about their mathematics lessons and the value of using hands-on activities, but only one reported any significant change. An equal number in both the pilot study and the main study felt their teaching beliefs were affirmed by the research process.

All of the pilot study subject teachers expressed concern about the lack of time available in their curriculum for using the spatial skills as an add-on, while the classroom teachers in the main study did not find time to be an overwhelming challenge. Interestingly, three out of four teachers in the main study continue to use the activities in their classroom and have shared them with their colleagues, while only one out of the original seven has reported that they still use the activities as a mind break after the study ended.
Chapter 5 – Discussion and Implications

This chapter will be in five parts. The first part will discuss how the quantitative data has an impact on the meaning and interpretation of the teachers’ beliefs. The second part will try to situate the findings within the context of teaching mathematics at the junior-intermediate level. The third section will address the implications of the results. The fourth section will be a discussion of recommendations for future study. Finally, a consolidation of the main findings will be presented.

5.1 Evaluation of the quantitative data.

Although the quantitative data in the pilot study was not collected with the same rigour as in the main study, it is worth mentioning that it did suggest that there might be a slight improvement in mathematical reasoning over the course of the intervention. Unfortunately, these results have little meaning, as the spatial games were not administered in the same manner, or with the same frequency by all of the participants in the pilot study. In addition, one would expect an improvement in mathematical reasoning in an educational setting over a two-year period regardless of any intervention. The main outcome of the quantitative data from the pilot study was to reinforce the importance of a well-designed process of intervention, as well as supervision by the researcher to ensure that the study was being conducted as planned. These results did lead to design changes for the main study.

The quantitative data in the main study revealed some interesting information about the students whose teachers were participants in the study. At the beginning of the study, many of the students had stronger than expected spatial reasoning skills. It is possible that this can be attributed in part to their socio-economic background of coming from a predominantly rural area where these students would have engaged in gross motor and fine motor activities while
assisting their parents with chores both around the farm and in the shop. In addition, there is some evidence to suggest that a portion of students identified as weak in mathematics may only appear weak due to the predominant teaching methodology, which does not include spatial reasoning or spatial enhancement of mathematics lessons (Paying attention to spatial reasoning; 2014, Newcombe, 2013; Shea, Lubinski & Benbow, 2001).

Examination of the results of the pre-test versus the post-test when all the students are grouped together shows that there is a measurable increase in mathematical reasoning over a very short ten-week period. There may be some improvement in spatial reasoning as indicated by a slight increase in group means, although this was not statistically significant. It is possible that the two classes whose teachers were previously involved with action research in spatial reasoning may have had less room to move due to their prior interactions with spatial games. It is not possible to attribute the improvement in mathematical reasoning to the spatial games, but there is enough evidence to recommend pursuing spatial skill training as a possible candidate for enhancing mathematical reasoning in middle school students. These results also support the teachers’ intuition that their students’ reasoning skills appeared to improve throughout the study period.

Of the four teachers at School C, only CS felt that the spatial games were nothing more than a fun activity, and that he could not see any positive impact or change in his students. In fact, CS’s class saw the most measurable movement of the four classes with the following results:

Table 11. CS grade 4/5 class

<table>
<thead>
<tr>
<th>Test Type</th>
<th>P-value</th>
<th>Significance</th>
</tr>
</thead>
<tbody>
<tr>
<td>t-Test Spatial Reasoning</td>
<td>P(0.044)&lt;0.05</td>
<td>significant</td>
</tr>
<tr>
<td>t-Test Mathematical Reasoning</td>
<td>P(0.047)&lt;0.05</td>
<td>significant</td>
</tr>
</tbody>
</table>
Given CS’s frustration with the behaviour of his students during these interventions, and his negative attitude towards the spatial awareness games, it is unlikely that teacher effect is responsible for these improvements over the ten-week intervention. KD’s students did not see any measurable change in either spatial reasoning or mathematical reasoning despite her anecdotal evidence to suggest that she felt they were becoming more adept at both.

The two Intermediate teachers had very similar results showing measurable improvement in mathematical reasoning and little or no change in spatial reasoning. Both of these teachers reported that they did not see much change in their students’ mathematical reasoning skills that they could attribute to the intervention, other than learning attributes such as perseverance and communication.

Both of these classes did start off with exceptionally strong spatial reasoning results whose means were much higher than their mathematical results. The spatial reasoning behaviour versus mathematical reasoning behaviour on the post-test shows both a more uniform distribution of spatial reasoning and mathematical reasoning, and similar means between the two types of thinking. The spatial reasoning instrument needs to be revised and expanded to be a better measurement tool. At present, spatial reasoning instruments that have been validated are difficult to find. It is likely that there are developmental factors that affect an age-group’s aptitude on a spatial reasoning task. The American Spatial Intelligence and Learning Center (SILC) website provides a comprehensive collection of validated spatial reasoning tests which are identified as being for children or adults. However, there are no developmental-level-specified reasoning tests in their data bank.

While it is not possible to accurately measure quantitative results between the pilot study and the main study it is worth noting that students from the pilot study versus students in
the main study show similar distributions amongst the pre- and post-tests of mathematical reasoning with elevated spatial reasoning pre-test results amongst the main study students versus the pilot study students. It is possible that the rural children have more natural access to spatial skill opportunities than their urban counterparts (Childs, 1983), although there is a dearth of current literature to answer this question. It may be that the relative poverty of the students in the main study versus the affluence of the pilot study students may have an impact on early spatial skill opportunities as well. It may also simply be an artefact of the fact that the principal of the main study school has been pursuing professional development with her staff in line with the 2014 “Paying Attention to Spatial Reasoning” document that has had an impact on how her faculty are structuring their mathematics lessons.

5.2 The findings with respect to teaching middle school mathematics

5.2.1 How does spatial skill development affect learning skills amongst middle school mathematics students?

Spatial skill development appears to have a positive impact on some important learning skills. Most of the teachers in the study reported that the use of the games to improve spatial reasoning encourages students to persevere.

The spatial activities do not have obvious solutions and do not offer a “one size fits all” starting point. Students generally begin with a guess and check approach, and through repeated trial and error begin to develop strategies to solve the problem (Ontario Math Curriculum, 2009). This methodology is essential to both problem solving, and refining approaches to solving a novel question. Students may struggle to find a starting point when solving mathematical problems, which can prove to be a barrier to finding the next step of guess and check (Moursund, 2016). The non-threatening aspect and low cost of failure with
the spatial games allows students to experiment with a variety of starting points. Many of the games lend themselves well to collaboration, allowing students to see and participate with classmates who are not worried about finding the right starting point the first time.

Once committed to the activity, the students often struggle to find the solution but once they do, their sense of accomplishment is intoxicating. In part, this is because they have not been given a formula for how to solve the problem, but have developed their own strategy, albeit not necessarily an elegant strategy to begin with. That ownership of developing their own strategy to win is self-affirming of their own ability. “Winning” the puzzle encourages the students to try to repeat this accomplishment, and with each repetition, they are consciously or unconsciously refining their strategy to get from finding the starting point to the end game. Suddenly, there is value and reward for perseverance.

Communication skills are another important learning skill, as well as a life skill. The ability to articulate one’s process for finding a solution allows identification of the stepwise nature of problem-solving and leads to consolidation of methodology. The interactive nature of the spatial skill games that can be played individually, as a group or as a competitive game naturally encourages students to discuss with each other how to solve the problem. This verbal communication allows the teacher to understand the student’s thought process, and provides an opportunity to assist with refining both the process and the explanation of the process.

Words such as “thinking, trying, perseverance, collaboration, engagement, connections” and more were within the top 100 words generated by nVivo 11 from all of the interviews, teacher journal data, observation journal data and email correspondence with the teacher participants. All are indicators of how this intervention had an impact on the participants’ students.
It is encouraging to see “thinking” as the most common word, with several learning skill descriptors appearing with high frequency.

5.2.2 How does spatial skill development in middle school mathematics affect mathematical reasoning?

The teachers in both the pilot study and the main study found it difficult to say with confidence that they saw any direct links to the curriculum they were teaching at the time. Two of the teachers in the main study reported that they saw a better grasp of measurement, particularly when the students were working with compound figures.

Teachers and educators who participated in a focus group to evaluate the spatial games and make links from specific games to middle school mathematics content were able to link the spatial skill activities with several middle school mathematics curricular expectations.

The two-dimensional maze puzzles were linked to probability:

I find the snake game I played with a useful companion for building and strengthening problem solving skills in mathematics. In particular, in the probability strand. This game would lead into building conceptual understanding in computer science since students would learn to create algorithms in solving problems, including modelling and creating games. (Doctoral student – OISE)
These same puzzles were also linked to patterning skills, as well as promoting a step-wise approach to problem-solving.

The game I played with during the focus group meeting was the maze… A link to the curriculum would be combinations and permutations. At the grade 5 level, which is the grade I teach, I would see this activity to be one that promotes problem-solving, reasoning and patterning skills. (Grade 5 teacher, Ph.D Education/Math)

Interestingly, while few of the classroom teachers in the study made connections to algebraic reasoning with the games, previous research that I conducted in the Middle East, members of the focus group, as well as a current OISE study by Hawes et al. (2015), identified these types of games as useful methods for training algebraic reasoning. Specifically, the maze games Penguins (a type of pentomino puzzle) and Qbits were useful.

With the Penguin Game (not sure of the proper name) - students are asked to solve puzzles with an increasing level of difficulty. As the difficulty level increases, students are asked to place puzzle pieces into a formation to solve it. At first, there are many/multiple answers to each puzzle, and the pieces fit together rather easily. But as the difficulty increases, the game adds "conditions", which is directly linked to math, specifically algebra. They must solve a puzzle while meeting a certain condition (e.g., the Penguins in certain places). Furthermore, as it continues to increase in difficulty, in order to be successful with the game, students are required to progress from using the traditional and rudimentary "guess and check" or "trial and error" method to more advanced algebraic reasoning (e.g., selecting an odd piece formation and working backwards, knowing what something is by eliminating what it is not). Overall, the Penguin Game authentically replicates algebraic math questions in a fun and engaging way. (Teacher, TDSB)

Most teachers and participants in the focus group reported that they found the link to measurement and geometry fairly straightforward to see.

[Bloc by Bloc]
Grade 7: Geometry and Spatial Sense – transformations (the hints could be useful for this topic)
Grade 8: Geometry and Spatial Sense
Grade 8: Patterning and Algebra
(Chinese exchange student, Doug McDougall Graduate Student Meeting)
One member of the pilot study identified a link between fractions and the three-dimensional and two-dimensional pentomino-type puzzles. She felt that her students were better able to understand the division of fractions after they had interacted with these games. However, she had also undergone some professional development in using an online program [http://www.visualfractions.com/](http://www.visualfractions.com/) to promote the understanding of multiplying and dividing fractions. This program requires a fair amount of spatial reasoning to use.

Figure 12. Example of Visual Aid for Dividing Fractions

![Visual Aid for Dividing Fractions](image)

The pre- and post-tests in both the pilot study and the main study showed a quantifiable increase in mathematical reasoning. The results of the main study may have had less of an impact from both teacher effect and maturity effect as the intervention was conducted over a much shorter period than the pilot study. The spatial games were implemented, usually, daily for fifteen minutes over a ten-week period and were the only intervention while two topics of mathematics content were being delivered. These quantitative results supported the teachers’ beliefs that they could see an impact on mathematical reasoning despite being unable to specify exactly from what it resulted.

The literature on the relationship between spatial reasoning and mathematical reasoning predicts that spatial reasoning is a trainable skill (Newcombe, 2010) and recognises that the
STEM fields are populated by people who have strong spatial reasoning skills (Lubinski & Benbow, 2006), implying that there is a relationship between spatial reasoning and mathematical reasoning. This study does not contradict either of these assertions.

5.2.3 How does teachers’ pedagogy change when using this strategy?

The teachers did not feel that they had changed their pedagogy in any significant manner but did acknowledge a number of subtle influences that engaging in these activities had such as adopting techniques that allow a more holistic perception of their students’ abilities and strength in mathematics, lesson planning and design to complement curriculum delivery, the value of incorporating spatial activities as a means of enhancing concept consolidation in both the spatial and apparently less spatial curriculum content, and rethinking what constitutes effective content delivery. These experiences are part of a teachers’ experience of living the reality of curriculum through its impact on their students learning (Olsen, 2012).

The teachers’ experience with incorporating the spatial activities into their mathematics lessons was generally positive. Three teachers struggled with concerns about control issues that were at odds with how they viewed successful lesson delivery. Every teacher commented that the games made them rethink the importance of using manipulatives, as well as how certain topics could be enhanced and/or better understood through the use of spatial reasoning practice.

Some of the teachers felt that after this experience they needed to re-think their own practice at both a lesson planning and a lesson delivery level. A few felt pushed outside of their comfort zone in terms of their self-confidence in their own mastery of the mathematical content, but admitted that this was a positive thing that gave them a chance to reflect on how their students might be experiencing their lessons. In addition, some of the teachers
acknowledged that they could directly teach learning skills through the spatial reasoning activities. These teachers reported that they felt that this approach would benefit their students’ comprehension of the content, and that this method of acquiring learning skills seemed to be fun, effective and transferable. The caveat in this outcome is that all of these teachers volunteered to participate in this study, and as a result were more receptive to pedagogical change. When asked whether they felt their colleagues would respond in the same manner, the answer was unanimously negative, citing time, effort, and the desire of their colleagues to follow the path of least resistance. When the teacher participants were asked if it was possible to influence these colleagues, and whether it was worth their while to try, they unanimously replied the intervention would have to show measurable improvement and be easy to implement.

5.3 Implications

This study has an impact on both the teaching of mathematics at the middle school level and the design of future, in-depth and broader investigations of the relationship between spatial skill development and mathematical reasoning in a classroom setting.

The findings in this study support and confirm findings in similar studies investigating the impact of using spatial skill activities by middle school mathematics teachers. Richardson and Stein (2008), note that the preservice teachers who were conducting their practicums at the grade eight level frequently mentioned an improvement in communication, use of mathematical language while engaging in the spatial activities, and an improved ability to articulate process. They concluded that

The majority of teachers have had minimal experiences with spatial tasks as part of their own K–12 mathematics curriculum; thus, their spatial ability is underdeveloped.
The importance of thinking spatially is being increasingly recognized (National Research Council, 2006). Not only do we believe it is important for our preservice teachers to develop their own spatial abilities, we also think it is equally important for them to present the in-class activities to the middle-grades or elementary students. (p. 107)

The inclusion of spatial games as a “Minds On” type of activity addresses several mandates and concerns in government educational documents, such as math anxiety, learning skills for problem-solving, meeting the needs of a diverse group of learners, and improving comprehension of the mathematical concepts of measurement and geometry.

These types of activities are simple to implement, enjoyed by the students, and have a positive impact on student learning. Teachers will appreciate the minimal prep time required for incorporating spatial skill activities, especially when it is weighed against improved habits of mind for the students and sets the stage for a readiness to learn mathematical topics such as measurement and geometry.

The inclusion of spatial skills as a method for teaching mathematics may be beneficial if incorporated into curriculum design. Teachers’ colleges, administrators, teachers, parents and educational institutions should consider promoting professional development to help mathematics educators realise the value of spatial reasoning as material content to be taught concurrently with middle school mathematics programs. The Ontario government document “Paying Attention to Spatial Reasoning” has value and potential to improve Ontario students’ results in mathematics.

The NCTM document “Principals and Standards for School Mathematics” includes the instruction of spatial reasoning and spatial understanding for students from Primary to Secondary level. Most spatial reasoning references are linked to geometry and measurement,
but the transferability of spatial reasoning is also mentioned in sections outlining the teaching of algebra, matrices and numeracy.

In the 2014 NCTM document “Principles to Actions”, eight principles of effective teaching are outlined. Three of those principles may be achieved through allowing students to engage in spatial reasoning activities such as the games in this study.

1. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

2. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

3. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships. (page 10)

While recommendations for the teaching of spatial reasoning are present in current curriculum documents, the techniques for explicitly teaching it are not always clear. “Teachers need to understand what spatial thinking is, and what kinds of pedagogical activities and materials support its development” (Newcombe, 2010, p. 33). This study offers a simple teaching strategy for practicing spatial reasoning that can easily be incorporated in the middle school mathematics lesson plan.

A common theme throughout this study was that teachers would incorporate a new teaching tool into their repertoire if it was not too much of a burden on prep time, and they believed that it would improve their students’ learning outcomes. Adding spatial games to the middle school mathematics teacher’s toolkit addresses both requirements for a teacher to adopt a new tool.
5.4 Future study

It might be possible to establish a causal relationship between spatial reasoning and student success in mathematics through a carefully designed study or intervention that is carried out in middle schools. The study teacher-participants recommended some improvements when they were asked to reflect on what they might change to improve future similar studies.

Although there were some mixed opinions on the ideal length of study, the majority of the teachers felt that a longitudinal study that follows a cohort of students from grade four until grade eight could help us learn about the long-term implications of teaching in this way. Many of the teacher participants felt that the shorter length of their study was appropriate as an initial investigation to see if there was a possible relationship. “I think usually a six-week study is a good amount of time. This was a little longer than I normally would have put it into to truly see if there is an initial difference or not” (BL, School C). Teachers in the main study reported that they believed that they would see more concrete evidence of both improvements in spatial reasoning and improvement in mathematical reasoning by following a cohort of students for at least two school years. “Personally speaking, I would love to have done something like this over a two-year period” (JR, School C). Future studies might follow a cohort of students for two or more years.

The teachers were hoping for quantifiable evidence to support a link between specific games and curriculum content, and would have liked to use those corresponding games at the time of its related content. The teachers instinctively felt that some of the games were better suited than others for making the transfer from 3D game to middle school curriculum, but would have felt more confident if statistically supported classroom-based results were available.
to give credence to their connections. The focus group made some good links, but a broader
diversity of mathematics specialists and educators would lend more weight to the game-content
connections. In addition, all the participants would have liked a more concrete measure of
success through statistical analysis. Future studies might incorporate carefully designed
diagnostics to measure both spatial reasoning and mathematical reasoning, controlled for
teacher effect, student maturation and curriculum content.

I do not know if they’ll see the value of it. That is what I worry about, is them seeing
the value. Really, in the end, you need to see statistical proof that this has made a
difference and have administration as well prompt and encourage that to happen. (JR,
School C)

This is difficult to achieve when working with real students in real classrooms with real
teachers, as this type of environment makes controlling confounding variables quite
challenging. Nevertheless, a carefully thought out design using control groups and established
diagnostic instruments across a large number of schools would be helpful in confirming the
importance of including spatial reasoning as part of the mathematics curriculum.

In both the pilot study and the main study the teachers commented that the fifteen
minutes “Minds On” approach needed to be implemented at a minimum of three times per
week to see an impact on improvement of spatial reasoning. The more rigorous approach to
using this intervention on a regular basis in the main study as opposed to the unmethodical use
in the pilot study demonstrated the importance of frequent access to the games. The teachers in
the main study were only required to use the intervention three times per week, but quickly saw
the games’ value and increased the use of the games to a daily basis. Many felt that daily
intervention was ideal if the games were only to be used in a short ten-week study such as this
one. It would be interesting to conduct a research project where the teachers use the materials daily.

Teachers reported that ease of access made the regular use of the spatial activities more likely. Several teachers liked to use the activities as rewards for work finished quickly to occupy the student’s time in a meaningful way while classmates were completing seat-work. Other teachers liked to have spontaneous access to the games for “brain breaks”. Most of the study participants suggested that classrooms or, at the very least, teachers should have dedicated sets of spatial activities to facilitate ease of compliance. Future study should consider the ease of access to materials for an intervention to maximise the likelihood that teachers will readily implement these activities.

Teachers commented that the involvement and attitude of the administrators played an important role in how committed the teacher participants felt they could be to frequent use of the spatial games. In the pilot study the teachers felt that if they had been allowed more planning time to meaningfully incorporate these activities into their mathematics lessons that they might have used them more often and more regularly. In the main study the teachers embraced the intervention with the full knowledge that their administrator was fully on board with the idea that spatial reasoning was a valuable skill to develop for supporting the learning of mathematics. Future study should consider the impact of external players such as administrators, colleagues and parents on the comfort level of the teachers when implementing an intervention concurrently with the delivery of curriculum content.

The principal at School C had already been conducting professional development with her staff, using spatial reasoning to improve their schools’ measurement and geometry scores on the EQAO grades three and six benchmarks. As a result, she was very receptive to having
this study conducted in her classrooms. “[The principal] saw a much-improved engagement when she observed the students during the activities. She also felt that, compared to other studies, this one showed real impact that was easy to implement.” (MY, Journal notes) During the study, the principal encouraged her staff to use SOMA blocks in their starters, and by the end of the study, she had convinced her Parents Association to use half of their budget for purchasing additional class sets of the spatial games activities (Principal School C, email correspondence). Administrators involved in this type of study should be encouraged to allow for more planning time to support teachers to incorporate these types of activities into their daily lessons.

5.5 Conclusions

This study contributes to the field of teaching and learning by providing evidence to support the use of a simple to implement spatial skill intervention in the classroom, which enhances both the teachers’ toolkit and the students learning skills for middle school mathematics. The findings support the literature that recommends the use of spatial skill activities to complement the comprehension of middle school mathematics curriculum (Hawes et al., 2015; Kytello & Lehto, 2008; Uttal et al., 2013).

The findings when taken in context of their evolution through a pilot study, which formed the building blocks for the main study should be seriously considered. The participants throughout the course of this investigation represented a large cross section of those involved with the educational process in Ontario. The teacher participants spanned the continuum of newly graduated young professionals to reaching retirement master practitioners. The students attended schools in large city centres as well as local village schools. The schools themselves were both independent and government funded with large heterogenous classroom populations
and small, homogenous classes of either predominantly IEP students or carefully, selected talented single sex students. Despite this large variety of participants and the relatively short ten-week intervention of the main study, several common themes did emerge. These main findings represent a consolidation of these themes.

1) Teachers could see the value of using spatial skill activities in the middle school classroom and felt that this intervention should be explored in a more extensive and rigorous study.

2) Teachers noticed that spatial skill activities appear to have a positive impact on learning skills necessary for mathematics problem solving.

3) The use of spatial skill activities appears to encourage teachers to reflect on how to expand their teaching repertoire to improve mathematics problem-solving in their students.

4) Middle school students who engage in spatial skill activities three or more times per week appear to develop skills which enhance their mathematical problem-solving.
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Appendix A

Pilot Study Diagnostic

Look at the net and decide which of the 3D models is correct. Circle or place a checkmark on your choice.

Name: ____________________
Place today’s date on the line. Make a dash and write the number.

Sept.   June

Draw a picture of a CUBE and a RECTANGULAR. Be sure to label it with as much information as possible.

<table>
<thead>
<tr>
<th>CUBE</th>
<th>RECTANGULAR PRISM</th>
</tr>
</thead>
</table>

Is a CUBE a kind of a RECTANGULAR PRISM? Explain.

If a number plus two is less than five, is the number less than four or is the number greater than four? How do you know?
Can you place 6 X’s on a Tic Tac Toe board without making three in a row in any direction?

If you ride your bike to the store at 20 km/hr, how fast must your ride your bike back home again to average 25km/hr for the whole trip? Assume you take the same route both ways and do not include your time in the store.

If “n” is any EVEN number, which of the following is NOT TRUE about “n+1”.

1. Odd
2. Even
3. Prime
4. Equals “n-1”
At what time(s) did Sue stop and admire the scenery? How fast was she driving between 4 pm and 5 pm?

Sally and Sam were answering a question on a math test. Sally determined that the female killer whale which weighs about 7 500Kg was 16 000 times heavier than a female Russian rat which weighs about 450g. Sam said it was only 15 times heavier. Who is closer to being right and why?

If I ate 2/5 of this chocolate bar, and you at 1/3 third of what is left, how many little squares would be available for someone else?
Look at the net and decide which of the 3D models is correct. Circle or place a checkmark on your choice.
Place today’s date on the line. Make a dash and write the number.
Look at this picture of a CUBE and a RECTANGULAR PRISM. Be sure to label it with as much information as possible.

Is a CUBE a kind of a RECTANGULAR PRISM? Explain.

If a number plus two is less than five but greater than Zero, what could the number be? Explain.
Can you place 6 X’s on a Tic Tac Toe board without making three in a row in any direction?

If you ride your bike to the store at 20 km/hr, how fast must your ride your bike back home again to average 25km/hr for the whole trip? Assume you take the same route both ways and do not include your time in the store.

If “n” is any EVEN number, which of the following is NOT TRUE about “n+1”

1. Odd
2. Even
3. Prime
4. Equals “n-1”
At what time(s) did Sue stop and admire the scenery? How far from home was Sue at 3:00pm?

If I ate 2/5 of this chocolate bar, and you ate 1/3 third of what is left, how many little squares would be available for someone else?

Hint: 1) Place X’s in the boxes that make up 2/5 of the chocolate bar.
2) Shade in how much you ate?
3) How many squares of the whole chocolate bar are left?
School C Pre-Test Diagnostic Grades Four to Six

Look at the net and decide which of the 3D models is correct. Circle or place a checkmark on your choice.

Name: ________________________

What is the mirror reflection of the shape in the image?
Place today’s date on the line. Make a dash and write the number.

Sept. ____________________________ Dec

Look at this picture of a CUBE and a RECTANGULAR PRISM. Be sure to label it with as much information as possible.

<table>
<thead>
<tr>
<th>CUBE</th>
<th>RECTANGULAR PRISM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Is a CUBE a kind of a RECTANGULAR PRISM? Explain.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
Find the value of the triangle.

\[
a. \quad 414 + \triangle = 708 \\
\triangle \text{ is } \underline{\quad \quad} \\

\]

\[
b. \quad \triangle - 339 = 485 \\
\triangle \text{ is } \underline{\quad \quad} \\
\]

Can you place 6 X’s on a Tic Tac Toe board without making three in a row in any direction?

Write the time that the clock shows and then time ten minutes later:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

10 min. later

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

138
Divide, but CROSS OUT the impossible expressions.

\[
\begin{array}{ccc}
   \text{a. } 17 \div 1 = \underline{\quad} & \text{b. } 17 \div 17 = \underline{\quad} & \text{c. } 1 \div 1 = \underline{\quad} \\
   17 \div 0 = \underline{\quad} & 0 \div 0 = \underline{\quad} & 0 \div 1 = \underline{\quad}
\end{array}
\]

If I ate 2/5 of this chocolate bar, and you ate 1/3 third of what is left, how many little squares would be available for someone else?

Hint: 1) Place X’s in the boxes that make up 2/5 of the chocolate bar.
      2) Shade in how much you ate?
      3) How many squares of the whole chocolate bar are left?
School C Post-Test Grades Seven and Eight

Look at the net and decide which of the 3D models is correct. Circle or place a checkmark on your choice.
If you use the following four pieces to build a cube, which image would result?

Look at the following “originals” and circle its match on the right.
What is the date and year of your birthday? ________________

Place your birthday on the line

Jan. 1st

Dec.

Describe in words how you knew where to place your birthday on the line?
Use your pencil to divide these shapes into simpler figures such as triangles, squares, rectangles and so on.

What are all the possible numbers that are greater than zero, less than twelve, and can be divided by three? Explain how you figured this out.

A farmer sells potatoes in sacks of various weights. The table shows the price per weight.

<table>
<thead>
<tr>
<th>Weight</th>
<th>5 lb</th>
<th>10 lb</th>
<th>15 lb</th>
<th>20 lb</th>
<th>30 lb</th>
<th>50 lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$4</td>
<td>$7.50</td>
<td>$9</td>
<td>$12</td>
<td>$15</td>
<td>$25</td>
</tr>
</tbody>
</table>
Are these two quantities in proportion? Explain how you can tell that.

What size bag should you buy for the best price?

If I ate 2/5 of this chocolate bar, and you ate 1/3 third of what is left, how many little squares would be available for someone else?
If you closed your eyes and picked one of the marbles below what are the chances of:

a) Picking a yellow marble?

b) Not picking a red marble?

c) Picking a red or a blue marble?
Look at the net and decide which of the 3D models is correct. Circle or place a checkmark on your choice.
If you use the following four pieces to build a cube, which image would result?

Look at the following “originals” and circle its match on the right.

Original | Image
What is the date and year of your birthday? ________________

Place your birthday on the line

Jan. 1\textsuperscript{st} \hspace{1cm} Dec.

Describe in words how you knew where to place your birthday on the line?
Use your pencil to divide these shapes into simpler figures such as triangles, squares, rectangles and so on.

Round the prices to the nearest dollar. Use the rounded prices to estimate the total bill.

- crackers $1.28, cheese $8.92, jam $3.77, butter $9.34.

Write the numbers in order from the smallest to the greatest.
217,200 227,712 27,200 227,200
Write the area of the *whole* rectangle as a SUM of the areas of the *smaller* rectangles. Lastly, add to find the total area.

\[
\text{Area} = 8 \times 127
\]

\[
= \_\_\_ \times \_\_\_\_\_ + \_\_\_ \times \_\_\_ + \_\_\_ \times \_\_\_
\]

\[
= \_\_\_
\]

How many times does \( \times \) go into \( \_\_\_\_\_\_\_\_ \)?

Answer Yes or No and show why.

<table>
<thead>
<tr>
<th>a. Is 5 a factor of 60?</th>
<th>b. Is 7 a divisor of 43?</th>
</tr>
</thead>
<tbody>
<tr>
<td>___, because ___ \times ___ = ___.</td>
<td>___, because ___ \div ___ = ___.</td>
</tr>
<tr>
<td>c. Is 96 divisible by 4?</td>
<td>d. Is 34 a multiple of 7?</td>
</tr>
<tr>
<td>___, because ___.</td>
<td>___, because __.</td>
</tr>
</tbody>
</table>
Appendix B

Rubric for Assessing “Thinking” on Pre- and Post-Tests

Table 12. Rubric for Assessing Mathematical Thinking

<table>
<thead>
<tr>
<th>Categories</th>
<th>Level 1</th>
<th>Level 2</th>
<th>Level 3</th>
<th>Level 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thinking: The use of critical and creative thinking skills and/or processes*</td>
<td>- uses planning skills with limited effectiveness</td>
<td>- uses planning skills with some effectiveness</td>
<td>- uses planning skills with considerable effectiveness</td>
<td>- uses planning skills with a high degree of effectiveness</td>
</tr>
<tr>
<td>Use of planning skills</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- understanding the problem (e.g., formulating and interpreting the problem, making conjectures)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- making a plan for solving the problem</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use of processing skills*</td>
<td>- uses processing skills with limited effectiveness</td>
<td>- uses processing skills with some effectiveness</td>
<td>- uses processing skills with considerable effectiveness</td>
<td>- uses processing skills with a high degree of effectiveness</td>
</tr>
<tr>
<td>- carrying out a plan (e.g., collecting data, questioning, testing, re-inventing, modelling, solving, forming conclusions)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>- looking back at the solution (e.g., evaluating reasonableness, making convincing arguments, reasoning, justifying, proving, reflecting)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use of critical/creative thinking processes** (e.g., problem-solving, inquiry)</td>
<td>- uses critical/creative thinking processes with limited effectiveness</td>
<td>- uses critical/creative thinking processes with some effectiveness</td>
<td>- uses critical/creative thinking processes with considerable effectiveness</td>
<td>- uses critical/creative thinking processes with a high degree of effectiveness</td>
</tr>
</tbody>
</table>
Appendix C

Interview Process – Main Study

Pre-Interview

- Welcome and thank participant.
- Review the purpose of the study and review the Informed Consent Form.
- Request participant sign two copies of the Informed Consent Form.
- Provide the participant with one signed copy of the Informed Consent Form.
- Share the expected timing, structure of the interview (a series of questions) and a reminder that the interview will be audiotaped.
- Provide the participant with a list of the interview questions.
- Ask if the participant has any questions, issues or concerns.

The interview questions are divided into the 5 sections:

1. **Background** information of the participant relevant to the study,
2. **Outcomes** of the research experience on your development as a teacher,
3. Specifics aspects of the research **process** and their impact on your development as a teacher,
4. **Challenges** with the research experience, and
5. **Recommendations** for future studies on spatial skills interventions.

After checking in with the participant to determine if they are feeling ready to proceed, turn on the audio recorder (if they confirm they are ready), and begin asking the interview questions.
Interview Questions

1. Background of participant
   1.1. Describe your experiences as a student in the mathematics classroom (e.g., teacher’s style of lesson delivery, specific mathematics skills that you excelled in)
   1.2. Describe the types of games and activities you were involved in as a small child (Lego, played soccer, etc.)
   1.3. Describe your classroom teaching experience since graduation (e.g., date, place of employment, grade levels).
   1.4. Describe any professional development you have experienced in mathematics education that has changed your teaching practice.

2. Outcomes
   2.1. What skills do you feel you developed through the process of including spatial activities in your lessons?
   2.2. How are these skills applicable to your practice as a teacher? (May break down this question by the skills participant noted in response to question 2.1.)
   2.3. How did the use of spatial skill activities impact on the classroom dynamic?
   2.4. What sort of impact, if any, did you see on the mathematical reasoning of your students that might be attributed to the spatial skill activities?

3. Process
   3.1. How did the specific aspects of the research process impact your practice as a teacher?
   3.2. How did the specific aspects of the research process impact on your students’ interaction with the spatial skill activities?

4. Challenges
   4.1. What challenges did you experience when engaging in the research process?
   4.2. What challenges did the students in your class experience when engaging with the spatial skill activities?

5. Recommendations
   5.1. What practical recommendations would you make for future research in this area?
   5.2. What sort of support would you need or would you envision a teacher would need in order to easily participate in a study of this sort.
   5.3. Are there any other recommendations you would like to share related to the impact of spatial skill activities on mathematical reasoning or using these activities as part of your teaching repertoire?
   5.4. Is there anything else you would like to share related to this study?
Pilot Study Interview Questions

1. Skills Development

1.1. What skills do you feel you developed through the process of including spatial activities in your lessons?
1.2. How are these skills applicable to your practice as a teacher? (May break down this question by the skills participant noted in response to question 2.1.)
1.3. How did the use of spatial skill activities impact on the classroom dynamic?
1.4. What sort of impact, if any, did you see on the mathematical reasoning of your students that might be attributed to the spatial skill activities.

2. Process

2.1. How did the specific aspects of the research process impact your practice as a teacher?
2.2. How did the specific aspects of the research process impact on your students’ interaction with the spatial skill activities?
2.3. Were you able to implement the activities on a weekly basis? Explain.

3. Challenges

3.1. What would have made it easier to ensure regular use of the activities?
3.2. What challenges did you experience when engaging in the research process?
3.3. What challenges did the students in your class experience when engaging with the spatial skill activities?
Appendix D

Teacher Handout of Toy Box contents

Below is a handout that comes with the “Toy Box” for the teachers. The presumed connections to mathematics curriculum are derived from focus groups and are not proven.

Spatial Skill Activities

Marcy Cook Tile Problems – These are work sheets that are best done if the student uploads onto their tablets and uses “smart note” or “one-note” to write the answers in the boxes.

These puzzles address most basic mathematical concepts but require the students to place the digits 0-9 in each of the open boxes exactly once to make each of the statements true. The students need to draw on their numeracy skills as well as develop organised problem solving strategies. There are often multiple solutions to each puzzle.

Students should solve these problems individually. The first ones finished should attempt alternative solutions or give hints (not show) their classmates on how to problem solve. End the task after ten minutes even if students do not complete the task. You may or may not want to have another student demonstrate the answer. Often it is better to leave it unsolved as many students will continue to attempt to solve it on their own and feel very pleased with themselves when they do.
**Bloc-by-Bloc**

This activity is quite challenging and should be completed in groups of up to three. At first students should attempt the puzzles by using the clues. As they become more practiced they should attempt the puzzles by simply looking at the task. Initially you may have to give direction on how to manipulate the polyhedrons so that they are oriented the same way as in the diagrams. This activity takes around 15 minutes to solve.

This activity stimulates several areas of the brain and requires the use of both hemispheres as well as the hippocampus. As a result practice with this game improves several areas of Mathematics, in particular higher-order algebra problems solving, geometry, visualising functions as well as converting working memory to long term memory.

**On-line Games:**

http://www.coolmath-games.com/0-bloxorz/

Allow students exactly seven minutes to attempt this game and record what level they achieve. First time players tend to reach Level 3-7. Subsequent attempts should always begin at level 1 to test improved skill. Students can be encouraged to practise at home. This game activates many areas of the brain and appears to improve understanding of functions including topics such as the Normal distribution (I am not really sure why).

**Tangrams**

This is a popular activity for students to improve their spatial problem solving. This activity stimulates the hippocampus for improved memory consolidation as well as activating several areas in the right side of the brain. Students who
practise this activity show improved skill in geometry, measurement and problem solving. This activity can be done individually or in pairs (as a timed competition). Allow ten minutes max, for a problem. Do not allow students to look at the solution until the time is up.

**Maze**

This activity is best done individually or in pairs. Do not allow the students to look at the answer on the back of the page. Puzzles 1-5 are pretty simple. Puzzles 11+ are very challenging. As these puzzles were designed for all ages, for our students, insist that they solve the problem with no dead ends. In other words the path cannot end on open space. All puzzles can be solved this way even though the answers on the back of the page do allow for dead ends. This activity usually takes about ten minutes. Students who practise this activity show improved skill in most Middle School mathematics topics as it activations multiple areas of the brain. In particular, algebra skills are enhanced.

**Ship Shape**

This activity is surprisingly challenging even for students with strong spatial skills in place. Students select a card and must arrange the pentominoes to match. This is a combination of Tangrams and Bloc by Bloc. Allow ten minutes to solve. This task is best done individually by may be done in pairs.

**Penguins**
This activity increases in difficulty from very simple puzzles to extremely challenging puzzles quite rapidly. It is best performed individually or in pairs. Students must choose a puzzle diagram and arrange the transformable pentominoes in the grid so that the penguins are in the correct location.

Students seem to be particularly drawn to this game but often stay at the 2nd level rather than pursuing the more difficult tasks. Allow ten to fifteen minutes for completion of the task.

There is quite a bit of spatial reasoning research involving elementary school children that has been conducted with pentominoes including some ongoing studies at the Jackman Institute of Child Study in concert with OISE.

Contents of the Tox Box

- 10 Bloc by Bloc
- 6 Penguins on Ice
- 3 Go Getter Mazes
- 2 IQ blocks

- These games have been used extensively and have shown clear impact on improvement of spatial reasoning.

✓ 2 Shape by Shape
✓ 1 Kataminoes
✓ 2 Q-bits
✓ 2 Cool Circuits
✓ 3 Shapeometry
✓ 2 Tangrams

✓ These games should have a positive impact on spatial reasoning but have not been used extensively enough yet

1 bag of extra cards and pieces for Bloc by Bloc
The Marcy Cook games are very effective “starters” for grade 4-8 students.

Bloxorz – online game is fun and addictive but you need to watch that the kids are not using the “cheats”
Also try to get them to start at the first level every time and see if they get to the higher levels faster. http://www.mathplayground.com/logic_bloxorz.html

Figure 13. Photograph of the Contents of the Toy Box
Appendix E

SOMA Blocks PD

The principal of School C had been conducting ongoing professional development around the importance of spatial reasoning. She had shared a report with her staff that the schools’ results in measurement and geometry were disturbingly low. As a result she had encouraged her teachers to look at ways to improving the lesson delivery in these topics from primary through to intermediate level.

Figure 14. SOMA Blocks


During the ten-week intervention, one of her PD sessions involved the use of SOMA blocks. These blocks are similar to the Bloc by Bloc game in the Toy Box. The SOMA cube was original invented in 1936 by Piet Hein and has been used since then in a variety experiment in psychology and education.

The seven Soma pieces are six polycubes of order four, and one of order three:
Figure 15. List of SOMA pieces

- **Piece 1**, or "V".
- **Piece 2**, or "L": a row of three blocks with one added below the left side.
- **Piece 3**, or "T": a row of three blocks with one added below the center.
- **Piece 4**, or "Z": bent triomino with block placed on outside of clockwise side.
- **Piece 5**, or "A": unit cube placed on top of clockwise side. Chiral in 3D.
- **Piece 6**, or "B": unit cube placed on top of anticlockwise side. Chiral in 3D.
- **Piece 7**, or "P": unit cube placed on bend. Not chiral in 3D.


This particular PD session was very popular with the staff. They were asked to go away and come up with lessons that could incorporate this cube into their mathematics teaching. The four teachers in the study at school C all brought up this PD session, discussed their observations of how the staff and faculty participated in the session and how they might incorporate the SOMA cube into their teaching.
Appendix F

Informed Consent Form

Date: ___________________

Dear mathematics faculty,

The purpose of this study is to understand how the addition of spatial skill activities might contribute to mathematical thinking in middle school students, and the possible impact the research experience has on a mathematics teacher’s practice in the classroom. The four participants selected for this study will be currently employed as teachers.

This study will be carried out under the supervision of Professor Doug McDougall in the Curriculum, Teaching and Learning Department of the Ontario Institute for Studies in Education/University of Toronto.

The data for this study will be collected through face-to-face interviews of approximately fifteen minutes to half an hour. During the interview you will be asked questions about your experience with implementing spatial skill activities in your class, whether you observed a change in your students’ mathematical reasoning and if it impacted on your teaching practice in general.

The interviews will be audio taped and later transcribed to paper. The information obtained in the interview will be kept in strict confidence and stored at a secure location. All information will be reported in such a way that individual persons, schools, school districts, and communities cannot be identified. All raw data (i.e. transcripts, field notes) will be destroyed five years after the completion of the study.
You may at any time refuse to answer a question or withdraw from the interview process. You may request that any information, whether in written form or audiotape, be eliminated from the project. At no time will value judgments will be placed on your responses nor will any evaluation be made of your effectiveness as a teacher. Finally, you are free to ask any questions about the research and your involvement with it and may request a summary of the findings of the study.

If you have any questions, please feel free to contact me at 416-898-2371 or at martha.younger@mail.utoronto.ca. You may also contact the Principal Investigator, Dr. Doug McDougall at 416-978-0035. Finally, you may also contact the University of Toronto’s Office of Research Ethics for questions about your rights as a research participant at ethics.review@utoronto.ca or 416-946-3273.

Thank you in advance for your participation.

Please sign the attached form, if you agree to be interviewed. The second copy is for your records. Thank you very much for your help.

Yours sincerely,

Martha Younger (Co-Investigator)

Curriculum Studies and Teacher Development
OISE/University of Toronto
Telephone: 416-898-2371
Email: martha.younger@mail.utoronto.ca
By signing below, you are indicating that you are willing to participate in the study, you have received a copy of this letter, and you are fully aware of the conditions above.

Name: ________________________________

Signed: ______________________________

Date: ________________________________

Please keep a copy of this form for your records.