THREE ESSAYS IN INTERNATIONAL TRADE

by

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A thesis submitted in conformity with the requirements
for the degree of Doctor of Philosophy
Department of Economics
University of Toronto

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Abstract
Three Essays in International Trade

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2018

This thesis comprises three chapters:

In Chapter I, I develop a general model to study the impact of labour mobility distortions on the gains from trade liberalization. The main feature of the model is that it combines comparative advantage forces with sectoral production misallocation. I use this framework to study the effect of labour market policies affecting the level of occupational mobility distortions on the gains from trade. I first show that differences in market structure across sectors lead to a misallocation of production. A planner is able to solve this misallocation by changing the composition of employment in the economy through changes in the level of labour market distortions. I use the model to study the impact of labour market distortions on the gains from trade from 1992 to 2007 in the US. I find that, relative to observed, optimal distortions lead to a 2.7% increase in the median cost of switching occupations and a doubling of welfare gains from trade.

In Chapter II, I examine whether trade liberalization decreases investment misallocation in India. I provide evidence that, independently of firm heterogeneity, exporters earn more revenue from the extensive margin of production. Also, I show that this difference between exporters and non-exporters is increasing in the degree of external financing needs of the sector in which firms operate. This finding suggests that an increase in trade liberalization has the additional effect of increasing the efficiency of the allocation of investment in new product varieties from less-productive non-exporters to more-productive exporters.

In Chapter III, I propose a model to explain deviations from the source-destination hierarchy prediction in models of trade with heterogeneous firms. I first show that firms do not follow
a pecking order of foreign market entry. To explain these deviations, I show a two-period, partial equilibrium model with asymmetric information in product appeal. Firms know the appeal of their products but consumers are unaware of it. After consuming in the first period, consumers observe a noisy signal and update their expectations. Given the updating process, I show that there is an imperfect sorting of firms into markets.
Acknowledgements

I am grateful to Gueorgui Kambourov and Laura Turner, as well as Peter Morrow, who provided data for the first and last chapter respectively. I would also like to thank my classmates from whom this thesis benefited immensely, specially Scott Orr, Derek Messacar and Jeffrey Chan. Finally, I would like to thank my thesis committee, Professor Daniel Trefler, Professor Peter Morrow and Professor Kunal Dasgupta for helpful advice throughout the program.

SSRHC funding through its Joseph-Armand Bombardier CGS Doctoral Scholarship is gratefully acknowledged.
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Chapter 1

Labour Market Policy and the Gains from Trade

1.1 Introduction

One of the most important mechanisms through which countries gain from trade is the internal reallocation of resources during periods of trade liberalization. Therefore, the effect of distortions to the internal mobility of resources on the outcome of trade integration is a crucial question in the literature. The classical mechanism shows that any reduction in the ability of a country to reallocate resources across broad sectors decreases the gains from trade. The main reason for this is that barriers to mobility reduce specialization in production across countries. According to this mechanism, domestic market policies during periods of trade liberalization should be directed towards reducing barriers to mobility (see, for instance, Kambourova (2009) and Tombe and Zhu (2015)).

The recent literature stresses the importance of accounting for heterogeneous firms as units of production when studying the outcome of trade integration.¹ Including this in the analysis results in richer interactions between internal distortions and the gains from trade that are not fully captured by the classical mechanism. In particular, this mechanism fails to account for

¹For a review of this literature see Melitz and Trefler (2012) and Costinot and Rodriguez-Clare (2013).
what the optimal trade policy literature refers to as the production reallocation externality. Consumers do not account for the fact that sectoral production contracts and expands at the extensive margin. This results in a misallocation of production with relatively low domestic entry in high value-added sectors and high domestic entry in low value-added sectors. Internal mobility frictions could help reduce this misallocation by increasing the share of production in high value-added sectors. Therefore, when the focus of the analysis places firms as the units of production, it is not clear that reducing internal mobility barriers is the optimal policy. The optimal policy in this case depends on the interaction between comparative advantage and the production reallocation externality.

In this paper I study the general equilibrium impact of domestic barriers to labour mobility on the gains from trade in a model that includes both the classical mechanism and the production reallocation externality. In particular, I develop a model in which a planner is able to use occupation-to-occupation mobility distortions as a policy instrument. First, I show the conditions under which an increase in the distortions to mobility during periods of trade liberalization increases the aggregate gains from trade. I then quantitatively assess the model using the US economy as a laboratory to study the relationship between observed and optimal mobility barriers. In particular, I provide sharp characterizations of the outcome of trade under those two scenarios in terms of: the aggregate gains from trade, the within and across group distribution of these gains, changes in the occupation composition of labour, changes in the industrial composition of output.

To develop the model, I expand a standard multi-sector, multi-input, Ricardian-HO model with heterogeneous firms to include the frictional sorting of heterogeneous workers. My benchmark model is in the spirit of Bernard et al. (2007), and Burstein and Vogel (2011). In Bernard et al. (2007), firms operate in two monopolistic competitive sectors with different cost structures. Differences in cost structure are due to differences in which firms combine inputs to

---

2See, for instance, Ossa (2011) and Ossa (2014).
3The optimal trade policy literature uses this externality to show that countries are able to gain from engaging in commercial policy (e.g. Helpman and Krugman (1985), Flam and Helpman (1987), Venables (1987) and Ossa (2011)).
produce across sectors. I expand this framework in three dimensions. First, I redefine inputs of production as occupations and I allow heterogeneous workers to sort into them. Sorting involves a distortion in that there is a wage cost involved in switching occupations. Second, I add sectoral heterogeneity in terms of market structure by modelling different supply and demand side elasticities across sectors. As I explain in detail below, this aspect of the model is crucial in creating the production misallocation. Third, I allow the government to adjust the level of distortions by either subsidizing or taxing labour mobility. I assume that the government funds its labour market policy through lump-sum taxes/subsidies.

I start the analysis by studying a two-sector, two-occupation, two-country micro-founded Roy (1951) model of the labour market. In this model, heterogeneous workers sort into occupations according to their comparative advantage. To make the analysis as simple as possible, I start by assuming that in the benchmark equilibrium there are no costs to the sorting of workers into occupations. I then show that the government has an incentive to subsidize the mobility of workers towards the occupation that increases the production in the sector with the lowest trade elasticity with respect to unit labour costs.

Distortions in the labour market are welfare improving in this model because of the aforementioned production reallocation externality in the output market. I show that within this framework, this externality consists of lower than optimal entry in sectors with low demand and supply side elasticities. Lower elasticity of demand results in higher markups over unit labour costs, reducing firm entry below the optimal level. Sectors with higher firm heterogeneity have a relatively larger mass of highly productive firms, so competition due to marginal entry reduces average sectoral productivity by less than in sectors with high degree of firm homogeneity. By ignoring these differences, consumers effectively ignore that in equilibrium the value of firm entry is higher in the low elasticity sector, which leads to an inefficiently low

---

4I choose to concentrate on occupational distortions because, according to the literature, it is the most important dimension of frictional mobility in the US economy (see, for instance, Kambourou and Manovskii (2009) and Ebenstein et al. (2014)).

5For more on this point see Dixit and Stiglitz (1977) and Helpman and Krugman (1985). In addition, Ossa (2015) states that including sectoral heterogeneity makes the model more flexible, which improves its ability to capture the correct magnitude of the gains from trade.
share of expenditure in that sector. A planner is able to correct this misallocation by changing the sectoral output mix through the direct reallocation of workers across occupations. The main outcome of this policy is to increase firm entry in the high value-added sector, increasing aggregate welfare.

An advantage of the model I propose is that I can calculate the impact of this subsidy over the entire wage distribution. Whether the government subsidy is a Pareto improvement depends on particular functional forms and the value of the model’s parameters. Since I assume homothetic preferences, the increase in aggregate welfare entails a decrease in the consumption price for all workers. However, nominal wages of workers, who before the government intervention were in the occupation that receives the influx of workers, deteriorate. Workers who unambiguously gain the most are those who were close to the marginal worker in the contracting occupation and switch to the expanding occupation after the subsidy. Intuitively, those workers lose the least from switching and still receive a subsidy equivalent to the one that offsets the switching costs of workers who experience greater losses.

In the second part of the analysis, I develop and estimate a structural model of the labour market to quantify the mechanism described in the micro-founded model. The structural model expands the previous model in three important dimensions. First, I follow the recent literature in assuming that in every period the productivity draws of workers are from a common type II extreme value distribution. This assumption allows me to expand the number of occupations and sectors while keeping the model tractable. In addition, it allows me to fully characterize workers’ productivity heterogeneity and switching costs by using data on occupation-to-occupation flows. Also, it allows me to match the empirical observation that occupational gross flows are relatively more important than net flows. Second, I control for observed worker heterogeneity in estimating the mobility costs by adding workers’ educational levels as an additional source of comparative advantage. Third, I increase the model’s flexibility by adding mobility costs to the baseline economy, which, consistently with the previous literature, I estimate to be a significant source of distortions in the US economy.
I use the model to study the impact of labour market distortions on the gains from trade liberalization between the US and fourteen developed countries from 1992 to 2007. To do this, I first estimate the parameters of the model necessary to perform counterfactuals from the US perspective. To estimate the model, I use detailed panel data on the US and cross-sectional data on the foreign labour markets as well as data on aggregate sectoral bilateral trade flows. An advantage of the model is that it is block recursive in the sense that it allows me to separately solve for the partial equilibrium in labour and output markets as functions of prices. The general equilibrium consists of solving for the endogenous prices by combining the equilibrium conditions in both markets. This allows me to calibrate each market separately, using tools which have became standard in the international trade literature. In particular, it allows me to calibrate the model using the method pioneered by Dekle et al. (2007). This method is characterized by calibrating the model in percentage changes from an observed initial equilibrium that perfectly matches the data. The benefit of using this method is the reduction of the data requirements because of a decrease in the number of parameters that I need to estimate.

I begin by assuming that in the baseline model all mobility distortions are due to technological constraints. I find that, conditional on having the same match productivity and educational level, the median switcher across occupational pairs earns 73% less than a stayer. I then calculate the combination of occupational bilateral mobility distortions that maximizes the welfare gains from this trade-liberalizing episode. I find that relative to factual distortions, optimal distortions result in a 2.7% increase in the median mobility cost. In addition, under the optimal scheme there are a large number of bilateral distortions that are above their factual value and a small number significantly below it. The result is that, although the percentage welfare gains from trade more than double (from 0.23% to 0.48% increase in real income), the gains are more unequally distributed. To be more precise the variance of the ex-ante distribution of welfare gains from trade is much higher in the optimal than in the factual equilibrium both within and across educational levels.\(^6\) In terms of production, I find that sectors where labour

\(^6\)The variance of the ex-ante distribution of welfare gains from trade across occupations with factual distortions is 0.2 for high school, 0.1 for college and 0.04 for university. It is 17.4, 46.4 and 73.5 respectively with
has high value-added, such as machinery and equipment manufacturing, expand, while sectors with low value-added of labour, such as textiles, contract.

This paper is not the first paper to study how the gains from trade are affected by internal factor mobility distortions. A closely related paper by Tombe and Zhu (2015), which builds on the urban/trade literature (see, for instance, Allen and Arkolakis (2014) and Redding (2016)), quantifies the impact of spatial and sectoral frictions on the gains from trade in China. I simplify the structure at the intra-national level by considering only one-dimensional mobility frictions. However, I expand their analysis of the impact of international trade on the domestic job composition by considering sectors with different degrees of competitiveness and firms’ relative productivity.7

In addition, there is a connection between the previous trade literature and the mechanism through which distortions affect the total value-added of workers. This effect can be interpreted through two channels. The first channel is Krugman (1980)’s home market effect. This is the standard channel through which changes in tariffs affect the allocation of production in the optimal trade policy literature (see, for instance, Venables (1987), Ossa (2011) and Ossa (2014)). This consists of reallocating production across countries in a way that increases the share of good jobs in the domestic market but decreases it in the foreign market (beggar-thy-neighbour policy). In this paper, there is an additional new effect, which is the expansion of global production in high value-added sectors. This results in an increase in welfare in both countries.8

Both the micro-founded and structural models of the labour market are also related to an extensive theoretical and empirical literature on worker sorting in international trade. Among optimal frictions. Across educational levels it is 0.03 with factual distortions and 0.1 with optimal distortions.

7There is also another crucial difference between my analysis and the analysis in Tombe and Zhu (2015). Their goal is to explain differences in TFP between China and US arising from the misallocation of workers during a period of trade expansion (the specialization effect discussed earlier). I attempt to calculate the level of distortions that yield the highest possible welfare change from trade liberalization and implement it through policy. Therefore, unlike in their analysis, there is a crucial role for government taxation in this paper.

8Ossa (2014) discusses that this channel is also present in the optimal trade policy literature but only as a second order mechanism. Therefore, he argues it is not captured by first order linear approximations to welfare changes. The policy instrument I use here directly affects inter-sectoral production and therefore it is captured by first order linear approximations to welfare changes (such as the one I use below).
papers in the trade literature that use a micro-founded Roy (1951)’s model are Costinot and Vogel (2010), Ohnsorge and Trefler (2004), Ohnsorge and Trefler (2007), Liu and Trefler (2011), Acemoglu and Autor (2011), and Sampson (2014). Consistent with this literature I keep the model tractable by assuming a production technology such that workers endogenously sort only across occupations, even though they could also sort across firms and sectors. However, I depart from these papers in that my main goal is to study sorting decisions in the presence of mobility distortions.

In terms of the structural model of the labour market, this paper is related to a growing literature that uses the result of Eaton and Kortum (2002) regarding the sufficiency of flows to recover parameters of the underlying distribution of worker heterogeneity. Among these papers are Artuç et al. (2010), Hsieh et al. (2013), Lagakos and Waugh (2013), Bryan and Morten (2015), Allen and Arkolakis (2014), Tombe and Zhu (2015), and Cortes and Gallipoli (2015). Even though the approach I use to estimate the parameters of the labour market is similar, the scope of my paper is very different in terms of the main question and analysis. Specifically, I embed the labour market into a Melitz (2003)-Chaney (2008) output market model of international trade to study the impact of labour market frictions when jobs across sectors are heterogeneous in terms of their value-added per worker.

The rest of the paper is organized as follows: In Section 1.2, I introduce the micro-founded model. I first characterize the partial equilibrium in the labour market by deriving a simple sorting rule. I then characterize the output market equilibrium and define the general equilibrium of the model. Thereafter I define the welfare equation and investigate how changes in the labour market affect aggregate welfare and its distribution among workers. Section 1.3 sets up and calibrates the structural model using a simulated method of moments. Then I use the parameter estimates to calculate the optimal level of labour market distortions and the aggregate and distributional impact of moving from factual to optimal. Finally, Section 1.4 concludes. I relegate the data analysis, proofs as well as additional graphs and tables to the appendix.

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9In this respect, the model presented in this paper is a special case of a much general model in Grossman et al. (2014).
1.2 Model

In this section, I introduce the two-sector, two-occupation, two-country model. For simplicity, I assume that there are no labour market distortions in the baseline equilibrium and that the government is able to introduce distortions by taxing/subsidizing labour mobility. This policy affects relative production across sectors by changing the supply of labour to each occupation. An important property of the model is that government policy only affects relative production across sectors.\footnote{Burstein and Vogel (2016), for instance, assumes that different firms employ different types of labour (occupations in my model). In that case, occupational mobility distortions also affect relative production of firms within sectors. I simplify the analysis by studying the impact of labour market distortions across broad industrial sectors.}

1.2.1 Description of Environment

There are two countries in the world: home (h) and foreign (f). Each country, c, is populated by a measure $L_c$ of heterogeneous workers who are endowed with immutable skill $\theta$ and employed in either of two occupations $Z = \{z_1, z_2\}$. Workers’ skills are drawn from a common continuous distribution $F(\theta)$ with country specific compact support $\Theta_c = [\theta_c, \bar{\theta}_c]$. To simplify notation, I call $F_c(\theta)$ the cumulative distribution function with support $\Theta_c$. Additionally, in each country there are two sectors, $S = \{s_1, s_2\}$ wherein a set of firms $\Omega_{cs}$ with exogenous measure $M^e_{cs}$ are able to produce a unique variety. As follows, I discuss the assumptions on the workers’ and firms’ technology, the role of the government and the structure of both the input and output markets.

Workers’ productivity: Workers are both consumers and producers. In terms of production, the productivity of a worker is determined by a match between her skill and an occupation. This is denoted by the mapping $\psi : Zx\Theta_c \rightarrow \mathbb{R}_{++}$. I make the following assumptions regarding the skill distribution and the match specific productivity.
**Assumption 1 (Skills and worker’s productivity)**

(a) Either \( F_h(\theta) \geq F_f(\theta) \) or \( F_h(\theta) \leq F_f(\theta) \) for all \( \theta \in \Theta_h \cap \Theta_f \) (i.e. the skill distribution of one country weakly first-order stochastically dominates the skill distribution of the other country).\(^{11}\)

(b) \( \psi(\cdot,\theta) \) is twice-continuously differentiable in \( \theta \) with:

- \( 0 < \psi(\cdot,\theta) < \infty, \psi'(\cdot,\theta) > 0 \) for \( \theta \in \Theta_c \),
- \( \frac{\psi(z_1,\theta')}{\psi(z_2,\theta)} < \frac{\psi(z_2,\theta')}{\psi(z_2,\theta)} \) for \( \theta' > \theta \) (strict log-supermodularity).

**Production:** Production takes place in heterogeneous establishments as follows. A firm with productivity \( \varphi \) in country \( c \) and sector \( s \) has the following production function:

\[
Q(\varphi) = \varphi \left[ \prod_{n=1}^{2} \left( \int_{\Theta} \psi(z_n,\varphi) l(\varphi, z_n, \theta) d\theta \right)^{\beta_{szn}} \right], \quad \sum_{n=1}^{2} \beta_{szn} = 1, \quad \frac{\beta_{1z1}}{\beta_{1z2}} \neq \frac{\beta_{2z1}}{\beta_{2z2}}, \quad (1.1)
\]

where \( \varphi \) is a Melitz (2003)’s firm specific Hicksean productivity parameter. I follow the literature in assuming that \( \varphi \sim G(A_{cs}, \alpha_s) \), where \( G = 1 - \left( \frac{A_{cs}}{\varphi} \right)^{\alpha_s} \) is the Pareto cumulative distribution with country-sector specific scale parameter \( (A_{cs}) \) and sector specific shape parameter \( (\alpha_s) \). To obtain a finite distribution of firms’ sales in equilibrium, I assume that \( \alpha_s > \sigma_s - 1 \) for \( s = 1, 2 \). In equation (1.1), \( l(\varphi, z_n, \theta) \) denotes the endogenous demand for workers with productivity \( \theta \) in occupation \( z_n \) of a firm with productivity \( \varphi \). Moreover, \( \beta_{szn} \) represents the relative intensity of occupation \( z_n \) in the production of a firm in sector \( s \) and \( \psi(z_n, \theta) \) is the occupation \( (z_n) \)-worker \( (\theta) \) match specific productivity, which I discussed above. As shown in this equation, the production function exhibits constant returns to scale and sectors are relatively more intensive in different occupations.

It is crucial to highlight two properties of the production function that I use to show the sorting equilibrium. The first property is that the skill-occupation match specific productivity
depends on neither firm nor sector specific variables. Second, workers within an occupation are perfect substitutes. That is to say, a firm is indifferent between hiring two bad workers or one good worker, conditional on the total productivity and costs being the same. This would not be the case if, for instance, there were diminishing returns to the number of workers as in Eeckhout and Kircher (2012).\(^\text{12}\)

**Workers’ preferences:** In terms of consumption, workers have preferences for varieties produced in two different sectors.

\[
U_j = \prod_{s=1}^{2} \left( \sum_{c=h,f} M_{cs}^{\varepsilon} \int_{\varphi \in \Omega_{cjs}} q_{cjs}(\varphi)^{\frac{\sigma_s-1}{\sigma_s}} dG_{cs}(\varphi) \right)^{\frac{\gamma_s \sigma_s}{(\sigma_s-1)}}, \tag{1.2}
\]

where \(\Omega_{cjs} \subseteq \Omega_{cs}\) is the endogenous subset of firms from country \(c\) that sells to country \(j\), \(\sigma_s\) is the CES elasticity of substitution of sector \(s\), \(q_{cjs}(\varphi)\) is the aggregate level of consumption in \(j\) of firm \(\varphi\)’s variety from sector \(s\) produced in country \(c\), and \(\gamma_s\) is the share of total expenditure spent in sector \(s\). Equation (1.2) yields the following demand system:

\[
x_{cjs}(\varphi) = E_{js} \left( \frac{p_{cjs}(\varphi)}{P_{js}} \right)^{1-\sigma_s}, \tag{1.3}
\]

where \(P_{js} = \left( \sum_{c=h,f} p_{cjs}^{1-\sigma_s} \right)^{\frac{1}{1-\sigma_s}}\) and \(P_{cjs} = \left( M_{cs}^{\varepsilon} \int_{\varphi \in \Omega_{cjs}} p_{cjs}(\varphi)^{1-\sigma_s} dG_{cs}(\varphi) \right)^{\frac{1}{1-\sigma_s}}\). \tag{1.4}

The first equation is the firm revenue from \(c\) in destination \(j\). In this equation \(E_{js}\) denotes the expenditure of consumers in country \(j\) on varieties from sector \(s\). Since the sectors are nested by a Cobb-Douglas function, this term is \(\gamma_s\) share of the consumers’ total income which I define below. Note that because of this property, I can analyze each sector separately.\(^\text{13}\) The

\(^{12}\)Grossman (2013) surveys the sorting literature in International Trade. It shows how different assumptions on the functional form of the production function lead to different sorting rules. For an analysis of sorting models employed outside the trade literature see survey by Sattinger (1993).

\(^{13}\)See Costinot and Rodriguez-Clare (2013) for a brief discussion of the bias that nesting sectors with an upper Cobb-Douglas utility function has on the implied welfare gains from trade.
last equation is the price index derived from the utility function in equation (1.2).

**Market structure:** Here I define the market structure of both the labour and output markets. In the output market, each firm produces a unique variety (i.e. there is monopolistic competition). To produce and sell in each country a firm must pay sector-source-destination specific variable and fixed costs. The variable cost is a standard iceberg cost where \( \tau_{cjs} = 1 \) if \( c = j \) and \( \tau_{cjs} > 1 \) otherwise. As it is standard in this literature, the fixed cost of entry \( f_{cjs} \) is paid using domestic labour with each occupation in the same proportion as the variable cost of production.\(^{14}\) Throughout the paper, I assume that both the variable and fixed costs are symmetric (\( \tau_{cjs} = \tau_{jcs} \) and \( f_{cjs} = f_{jcs} \)). The total production of a firm with productivity \( \varphi \) is then:

\[
Q(\varphi) = \sum_{j=1}^{2} \mathbb{1}_{q_{cjs}(\varphi) > 0} \left[ \tau_{cjs} q_{cjs}(\varphi) + \varphi f_{cjs} \right].
\]

(1.5)

A firm with higher productivity is able to charge lower prices and sell a higher amount. This increases revenue, operating profits, and total profits.\(^{15}\) I assume that all firms are owned by workers so that all positive profits are returned to them.

The labour market is perfectly competitive. There is a continuum of firms from both sectors that compete for workers in each occupation. In equilibrium, I show that all workers within the same occupation earn the same wage per unit of productivity (effective wages). Following my previous discussion on worker substitutability, firms are then indifferent between hiring any type of worker within each occupation. Since worker-firm matches within an occupation are indeterminate in equilibrium, without loss of generality I will assume that workers and firms meet randomly.

**Government:** The government can manage the stock of workers in each occupation for a

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\(^{14}\)This property is important because it yields a homothetic sectoral total cost function. The main result of this is that the relative share of aggregate employment of each occupation used to produce in each sector is independent of the number of firms and only depends on the relative wage per unit of productivity across occupations.

\(^{15}\)Note that fixed costs of entry are source-destination-sector specific. In the trade literature, it is common to add firm specific fixed costs to reconcile the empirical fact that there is no perfect negative correlation between the relative price that a firm charges domestically and the relative number of markets it penetrates (see, for instance, Eaton et al. (2011)). This relationship holds on average, and since I only consider the aggregate implications of firm heterogeneity, I simplify the model by assuming a common fixed cost.
given relative wage by taxing/subsidizing labour mobility. For expositional purposes, I assume that firms pay mobility taxes or receive mobility subsidies. However, since there is perfect competition in the labour market, it is irrelevant whether the government taxes/subsidizes firms or workers. I define switchers from a distortionless benchmark sorting equilibrium of workers to occupations, which I refer to as the initial equilibrium. So, if a worker with ability $\theta$ is employed in $z_1$ in the initial equilibrium, a firm must pay her $d_{z_1 z_2} \cdot w(\theta)$ to employ her in occupation $z_2$. In the opposite case, it has to pay the worker a wage $d_{z_2 z_1} \cdot w(\theta)$. In equilibrium workers only switch towards one occupation so I make the off equilibrium assumption that mobility taxes/subsidizes are symmetrically opposite: $d_{z_1 z_2} = d_{z_2 z_1} = 1$. This simplifies the analysis by reducing the number of policy variables. In addition, this assumption makes the occupational labour supply schedule continuous. I assume that the government balances its budget every period by either collecting lump-sum taxes from workers or giving workers lump-sum subsidies.

1.2.2 Labour Market Sorting

In this subsection, I derive the occupational labour supply function from the sorting of heterogeneous workers into occupations. To that end, I study the problem of firms deciding which types of workers to hire for each occupation. This is isomorphic to studying the sorting decisions of workers. The reason for this equivalence is that labour markets are perfectly competitive. Under perfect competition each worker receives a tax/subsidy adjusted effective wage equal to the value of its effective marginal product. Since there is a continuum of firms demanding and workers supplying labour in each occupation, wages are taken as given by both firms and workers. If there is a worker who is willing to switch occupations at the current wage schedule, there must be at least a firm willing to hire that person in the new occupation. In this case, the sorting rule for that given wage schedule is out of equilibrium.

The analysis of this subsection applies to both domestic and foreign labour markets so I abstract from country subscripts. In addition, to simplify notation, it is useful to define the
Moreover, labour markets clear at each skill level. For all firms with positive production, \( Q \) is a vector of firm-specific effective marginal product at each skill level.

### Definition 1 (Equilibrium in the labour market)

**An equilibrium in the labour market is a firm specific labour demand function:**

\[ l : \mathbb{Z} x \Omega x \Theta \rightarrow \mathbb{R}_+ \]  \text{ such that given a wage function } w : \Theta \rightarrow \mathbb{R}_+ , \text{ the following holds:}

- For all firms with positive production, \( Q(\varphi) > 0 \), the first order conditions from their cost minimization problem are satisfied:

  \[
  \lambda_{z_1}(I(\varphi)) \psi(z_1, \theta) \leq d(z_1, \theta) w(\theta), \quad l(z_1, \varphi, \theta) \geq 0 \text{ (with c.s } \forall \theta \in \Theta),
  \]

  \[
  \lambda_{z_2}(I(\varphi)) \psi(z_2, \theta) \leq d(z_2, \theta) w(\theta), \quad l(z_2, \varphi, \theta) \geq 0 \text{ (with c.s } \forall \theta \in \Theta). \]

- Labour markets clear at each skill level

\[
 f(\theta) = \sum_{n=1}^{2} \sum_{s=1}^{2} \int_{A_s}^{\infty} l(z_n, \varphi, \theta) dG_s(\varphi). \]

In equation (1.7), \( \lambda_{z_1}(I(\varphi)), \lambda_{z_2}(I(\varphi)) \) denote firm \( \varphi \)'s shadow values of employing an additional effective worker in each occupation.\(^{16}\) Moreover, \( I(\varphi) = \{l(z_1, \varphi, \theta)\}_{\theta \in \Theta}, \{l(z_2, \varphi, \theta)\}_{\theta \in \Theta} \) is a vector of firm \( \varphi \)'s policy functions.

The first order conditions described in equation (1.7) are obtained from the problem of a

\(^{16}\)The shadow values are the result of the firm’s cost minimization problem:

\[
 \lambda_{z_1}(I(\varphi)) = \left\{ \begin{array}{l}
 \frac{w(\theta)}{\psi(z_1, \theta)} \left[ \frac{1}{\beta_{s_1}} \right]^{\beta_{s_1} z_1} \left[ \frac{w(\theta')}{\psi(z_2, \theta')} \right]^{\beta_{s_2} z_2} \frac{1}{\varphi} \\
 \end{array} \right\}.
\]

\[
 \lambda_{z_2}(I(\varphi)) = \left\{ \begin{array}{l}
 \frac{w(\theta)}{\psi(z_1, \theta)} \left[ \frac{1}{\beta_{s_1}} \right]^{\beta_{s_1} z_1} \left[ \frac{w(\theta')}{\psi(z_2, \theta')} \right]^{\beta_{s_2} z_2} \frac{1}{\varphi} \\
 \end{array} \right\}.
\]

with \( l(z_1, \varphi, \theta) > 0, l(z_2, \varphi, \theta') > 0 \) and \( \theta, \theta' \in \Theta \).
firm deciding the total number of each type of worker it employs in each occupation to produce a fixed quantity. These are not derived from the profit maximization problem of the firm, because I ignore output prices, which are endogenous to the firm. As indicated by Sampson (2014), when any type of worker in an occupation performs the same task within the production function, I can separate the firm’s decision regarding the type of workers it employs in each occupation from production and pricing decisions. After I solve for the type of workers a firm employs, I can solve for its price rule, which results in the standard constant elasticity of substitution pricing rule with the marginal cost of production being the effective wages of the types of workers it uses.

Given the structure imposed on the model, I show the following lemma.

**Lemma 1 (Labour market sorting cut-offs)**

In any labour market equilibrium \( \exists \) a unique \( \theta^* \in (\underline{\theta}, \overline{\theta}) \) such that the following holds:\(^{17}\)

1. \( \theta \in \Theta_{z_1}, \Theta_{z_1} = \{ \theta \in \Theta : l(z_1, \varphi, \theta) > 0 \text{ for } \varphi \in \bigcup_{s=1}^{2} \Omega_s \} \) only if \( \theta \leq \theta^* \),

2. \( \theta \in \Theta_{z_2}, \Theta_{z_2} = \{ \theta \in \Theta : l(z_2, \varphi, \theta) > 0 \text{ for } \varphi \in \bigcup_{s=1}^{2} \Omega_s \} \) only if \( \theta \geq \theta^* \),

\[ \frac{w(\theta) d(z_1, \varphi)}{\phi(z_1, \varphi)} = \frac{w(\theta) d(z_1, \varphi)}{\phi(z_1, \varphi)} = w^e_{z_1}, \forall \theta \in \Theta_{z_1}, \]

\[ \frac{w(\theta) d(z_2, \varphi)}{\phi(z_2, \varphi)} = \frac{w(\theta) d(z_2, \varphi)}{\phi(z_2, \varphi)} = w^e_{z_2}, \forall \theta \in \Theta_{z_2}. \]

Properties 3 and 4 indicate that within each occupation the price per unit of workers’ productivity that firms pay equalizes for all workers irrespective of the firm and sector of employment (this includes both wages and government subsidies). This result arises from the perfect substitutability between the number of workers and their skills within occupations (i.e. workers do not have a comparative advantage across firms within occupations). A consequence of this assumption is that, within an occupation, I can separate the marginal effect of hiring workers into a firm and worker components. Therefore, if the wage difference between two workers were bigger than the difference between their productivity, firms would demand only

\(^{17}\)In the appendix, I show that a sufficient condition for \( \theta \in \Theta_{z_1} \) is that \( \theta < \theta^* \) and \( \theta > \theta^* \) for \( \theta \in \Theta_{z_2} \).
the worker with the lowest wage per match productivity. So, for workers with different productivities to be employed in the same occupation, their effective wages must be the same.\footnote{This result requires the substitution between the number of workers and their worker-occupation match productivity to be uni-elastic in both sectors. Relaxing the equal elasticity of substitution between occupations across sectors does not impact lemma (1).}

Across occupations the result is different because there are complementarities between workers’ ability and occupation specific productivity. Assumption (1) states that these complementarities are stronger between high ability workers and occupation $z_2$ (i.e. high ability workers have a comparative advantage in $z_2$). As a result of this, any allocation of workers to occupations that satisfies the labour market equilibrium conditions must feature the sorting of high skilled workers to $z_2$ and low skill workers to $z_1$.\footnote{Lemma (1) is consistent with Costinot (2009) and Costinot (2007), which find that log-supermodularity is a necessary and sufficient condition for sorting. The production function is only log-supermodular across occupations.}

To link the labour market to the output market, I need to determine the relationship between labour supply to each occupation (i.e. $\theta^*$ in lemma (1)) and relative effective wages. The following lemma studies the characteristics of the wage function which I then use to determine a relationship between relative wages and the sorting cutoff.

**Lemma 2 (Properties of the wage function)**

Let

$$S(\theta) = \begin{cases} 
    w(\theta) d(z_1, \theta) & \text{if } \theta \leq \theta^*, \\
    w(\theta) d(z_2, \theta) & \text{if } \theta > \theta^*, 
\end{cases}$$

then $S(\theta)$ is strictly increasing, continuous and differentiable $\forall \theta \in \Theta, \theta \neq \theta^*$. Moreover,

$$\left( \frac{1}{\theta} \right) \lim_{\theta \to \theta^*} S(\theta) = \lim_{\theta \to \theta^*} S(\theta).$$

In lemma (2), $S(\theta)$ denotes the total payment a firm makes to employ a worker with productivity $\theta$. From lemma (1), it directly follows that within each occupation this function inherits the same properties as the match productivity function, which is strictly increasing and twice continuous differentiable. More importantly for the sorting rule is that the difference between
the total payment to the marginal worker in each occupation is equal to the cost a firm incurs to employ an occupational switcher. This property directly leads to the following proposition:

**Proposition 1** *(Sorting rule)*

*The worker’s sorting cut-off productivity is implicitly given by:*

\[
\frac{\psi(z_1, \theta^*)}{d\psi(z_2, \theta^*)} = \frac{w_{z_2}^e}{w_{z_1}^e}. \tag{1.9}
\]

Proposition (1) states that in equilibrium, the relative cost of employing the marginal worker in each occupation is equal. This relationship shows that government policy introduces a distortion in the labour market by creating a wedge between the relative cost per efficiency units across occupations and the relative match worker-occupation productivity of the marginal worker.

I also investigate the distributional effect of changes in \(d\). To that end, I derive from lemmata (1) and (2) a wage function. A direct consequence of these lemmata is that the sorting cut-off \((\theta^*)\) and the wage anchor \((w(\theta))\) are the only endogenous variables I need to solve to obtain the entire wage distribution. The reason for this is that, as I showed in lemma (1), wage differences within occupations in equilibrium are determined exogenously. Figure (1.1) shows the nominal wage function for the case of log-linear worker-occupation match productivity. The slope of the broken red line is given by \(\psi(z_1, \theta)\) and of the solid blue line is given by \(\psi(z_2, \theta)\).

The top panel of the figure displays the distortionless benchmark. In this case, \(w(\theta)\) coincides with \(S(\theta)\), so that nominal wages are equivalent to firms’ marginal revenue product of labour in each occupation. All firms demand zero workers with skills above the cut-off for occupation \(z_1\) and below the cut-off for occupation \(z_2\). The reason for this is that firms’ marginal revenue product of labour is lower than the wage they must pay to employ those workers in that occupation.

The other two panels of the figure show the effect of government subsidies to occupational
labour mobility on the nominal wage function. The discontinuity of the wage function indicates the effect of the subsidy on the nominal wage of the switchers who were previously employed in the other occupation. The analysis of firms’ demand for occupational labour is similar to the one in the distortionless case. The main difference is that the cost to a firm of employing an occupational switcher $\theta$ ($S(\theta)$) differs from the worker’s wage income ($w(\theta)$) by the mobility tax/subsidy.

From the previous analysis, I can define the following occupational effective labour supply, which I use to solve for the model’s general equilibrium:

$$L_{z_1}^e = L \int_{\theta^*}^{\bar{\theta}} \psi(z_1, \theta) dF(\theta), \quad L_{z_2}^e = L \int_{\theta^*}^{\bar{\theta}} \psi(z_2, \theta) dF(\theta). \quad (1.10)$$

1.2.3 Output Markets

In this subsection I take firms’ choice of labour types as given and derive production decisions in each market. I use the perfect substitutability of workers and productivity to write firms’ production costs in efficiency units. Profit maximization leads to the standard constant markups
over marginal cost pricing equation:

\[ p_{cjs}(\varphi) = \frac{\sigma_s}{\sigma_s - 1} \frac{a_{cs} \tau_{cjs}}{\varphi}, \]  

(1.11)

where \( a_{cs} = \left( \frac{w_{e1}}{\beta_{s1}} \right)^{\beta_{s1}} \left( \frac{w_{e2}}{\beta_{s2}} \right)^{\beta_{s2}} \) is the common term of the unit labour cost to all firms in sector \( s \). I substitute equation (1.11) into equation (1.3) to obtain firms’ revenue in equilibrium:

\[ x_{cjs}(\varphi) = \begin{cases} \frac{E_{js}}{p_{cjs}} \left[ \frac{\sigma_s}{\sigma_s - 1} \frac{a_{cs} \tau_{cjs}}{\varphi} \right]^{1-\sigma_s}, & \text{if } \varphi \geq \varphi_{cjs}^*, \\ 0, & \text{otherwise}. \end{cases} \]

(1.12)

Since, as I discussed above, the profit function is strictly increasing in the firms’ productivity, I directly substituted the productivity cut-off \( (\varphi_{cjs}^*) \) in equation (1.12). I now rewrite the price index (equation(1.4)) using the productivity cut-off.

\[ P_{cjs} = M_{cjs}^{\frac{1}{1-\sigma_s}} \frac{\sigma_s}{\sigma_s - 1} \frac{a_{cs} \tau_{cjs}}{\varphi} \left[ E \left( \varphi^{\sigma_s} \frac{1}{\sigma_s - 1} \mid \varphi \geq \varphi_{cjs}^* \right) \right]^{\frac{1}{1-\sigma_s}}, \]

(1.13)

where \( M_{cjs} = M_{cs}^{e} (1 - G(\varphi_{cjs}^*)) \) is the measure of firms from \( c \) selling to \( j \) in sector \( s \). The productivity cut-off is given by the following zero profit condition:

\[ \Pi_{cjs}(\varphi_{cjs}^*) = \frac{x_{cjs}(\varphi_{cjs}^*)}{\sigma_s} - a_{cs} F_{cjs} = 0. \]

(1.14)

Equations (1.9)-(1.14) represent the general equilibrium of the model (formally defined below). To close the model, I need to make an assumption on the distribution of income. Besides labour income, there are profits and government revenue in this model. I assume that each worker receives a share of domestic government revenue and world’s profit proportional to her labour income. Because I follow Chaney (2008) in assuming world profit sharing, there is no balanced trade in equilibrium.
Assumption 2 (Sectoral expenditure)

The total expenditure of country $j$ in sector $s$ is given by:

$$E_{js} = \gamma_s \int_{0}^{\theta_j} y_j(\theta) dF(\theta)$$

where,

$$y_j(\theta) = w_j(\theta) \left[ 1 - \frac{T_j}{\sum_{n=1}^{2} w_{zn} L_{jzn}} - T_j \right]$$

Within country tax/subsidy transfers

and

$$T_j = \left[ \mathbb{1}_{\theta_j \geq \theta_j^b} \left( 1 - \frac{1}{d_j} \right) \int_{\theta_j^b}^{\theta_j^\ast} w(\theta) dF_j(\theta) \right] + \left[ \mathbb{1}_{\theta_j \leq \theta_j^b} \left( 1 - d_j \right) \int_{\theta_j^b}^{\theta_j^\ast} w(\theta) dF_j(\theta) \right].$$

In assumption (2), $T_j$ denotes the lump-sum tax collected to pay for the mobility subsidy and $\theta_j^b$ the sorting cut-off from which government policy is defined.

1.2.4 General Equilibrium

In this section, I first define the equilibrium and then show that it can be parsimoniously solved using a system of aggregate occupation excess demand functions.

Definition 2 (General Equilibrium)

The general equilibrium consists on a vector of labour market cut-offs, $\langle \theta_c^b \rangle_{c=h,f}$, and a set of output market entry cut-off vectors, $\langle \varphi_{cjs} \rangle_{j=1}^{2}$, for $c,j = h,f$ such that given a wage function $w_c : \Theta_c \to \mathbb{R}_+$ and an initial allocation of workers to occupations, $\theta_c^b \in [\theta_c, \bar{\theta}_c]$ in each country $c = h,f$ the following holds:

(a) the first order conditions for the firms’ cost minimization hold (equation (1.7));

(b) workers maximize utility subject to their budget constraint (i.e. the demand system is given by equation (1.3));

(c) firms maximize profits (i.e. firms’ prices are given by equation (1.11), firms’ revenue is given by equation (1.12) and entry decisions are given by equation (1.14));

(d) workers’ total income is given by equation (1.15);

(e) labour markets clear at each skill level (i.e. equation (1.8) holds);

(f) the government’s budget is balanced in each country (i.e. equation (1.16) holds).
A necessary condition for equation (1.8) is that labour markets clear at the occupational level. Because of total cost homotheticity at the sector level, I can aggregate the occupational labour demand of all firms within a sector. I then use this in addition to the occupational effective labour supply previously derived to solve for the endogenous variables that clear the total excess demand for each occupation. Given the assumption of constant elasticity of substitution utility function, and Pareto distributed firm productivity, I show that the value of demand for occupational effective labour in each sector takes a simple form.

Combining (1.12)-(1.14) and the pdf of the Pareto distribution I obtain the following sectoral aggregate revenue equation.\(^{20}\)

\[
X_{cs} = \sum_{j=h,f} \Lambda_{cjs} E_{js}, \quad \text{where} \quad \Lambda_{cjs} = \frac{M_{cs} (A_{cs})^{\alpha_s} (\tau_{cjs} a_{cs})^{-\alpha_s} (f_{cjs} a_{cs})^{\frac{\sigma_s - (\sigma_s - 1)}{1 - \sigma_s}}}{\sum_v M_{vs} (A_{vs})^{\alpha_s} (\tau_{vjs} a_{vs})^{-\alpha_s} (f_{vjs} a_{vs})^{\frac{\sigma_s - (\sigma_s - 1)}{1 - \sigma_s}}}, \quad (1.17)
\]

In equation (1.17), \(\Lambda_{cjs}\) denotes the share of country’s \(j\) expenditure in country’s \(c\) goods from sector \(s\) (i.e. the market penetration ratio). Effective wages enter twice in this term indicating that they affect both the fixed and variable costs of production. The key property of equation (1.17) is that market penetration ratios in each sector only depend on the relative cost of unit labour requirements between the exporting country and all other countries selling in that market. The next lemma uses this equation to define the effective labour demand in each sector.

**Lemma 3 (Total effective labour demand by sector)**

The value of total effective labour demand in each sector is given by the following equation:

\[
D_{cs}^e = \left( \frac{\sigma_s (\alpha_s - 1) + 1}{\sigma_s \alpha_s} \right) X_{cs}. \quad (1.18)
\]

The left hand side of equation (1.18) is equivalent to the payment to all effective factors em-

\(^{20}\)The derivation of this equation from the model’s set up I propose here has became standard in the international trade literature. Therefore, for the sake of space I do not derive it in this paper. I refer interested readers to Chaney (2008) for the one input version of the model and Burstein and Vogel (2011) for the multiple input version of the model (similar to this paper).
ployed in production, including the factors used to pay for the fixed cost. Because of increasing returns to scale, the share of total sectoral revenue firms spend on labour inputs depends on the average size of firms relative to the number (measure) of producing firms. Therefore, in principle, the value of total effective labour demand is different for sectors with a large number of small firms from sectors with a low number of big firms irrespective of whether they have the same aggregate revenue.

In equilibrium with Pareto distribution and CES preferences, the conditional average size of producing firms is constant. When competition in a market is high, only the most productive firms enter that market. However, given the high degree of competition the size of these firms is small. When competition is low, the size of these firms increases. Concurrently, a large number of smaller firms with lower productivity enter the market. In the aggregate, this selection effect completely offsets the competition effect.\footnote{Feenstra (2010a) uses this property to show that he is able to relax the assumption of fixed number of entrants in each sector imposed on the heterogeneous firm model by Chaney (2008) while keeping it analytically tractable. Because of this property, I can easily extend the model of this paper to include free entry. However, given that there is an additional Rybczynski effect from sorting relative to the baseline HO model with heterogeneous firms, a large amount of government subsidy is likely to result in corner solutions. These are computationally costly and not supported by the data. It is also common in the literature to set up a general model and then compute a version without free entry (see, for instance, Ossa (2014)).} Therefore, the number of producing firms is the only variable that affects the total value of labour demand within a sector, which also proportionally affects total revenue in that sector.

I use equations (1.10) and (1.18) and the Cobb-Douglas production function to show the following proposition:

**Proposition 2** (Value of excess labour demand by occupation)

The value of excess demand for effective labour in each occupation is given by:

\[
\sum_{x=1}^{s} \beta_{x} D^e_{c x} - w^e_{c x n} L^e_{c x n} = 0, \text{ for } c = h, f ; n = 1, 2. \tag{1.19}
\]

These equations represent a system of 4 equations and 4 unknowns \(w^e_{c x n}\) for \(c = f, h, n = 1, 2\). By Walras’ law the system can be solved up to a numeraire (one equation is redundant).
To see this we can divide both sides of the equation by $a_{cs}$ and observe that we can solve the system for relative effective wages. To compute the equilibrium, I solve for relative effective wages from these equations and then obtain the labour markets sorting cut-offs from equation (1.9), the output market entry cut-offs from equation (1.14) and the entire wage equation from equation (1.7).

To guarantee the existence of a solution to the system of equations in proposition (2), I need to make the following additional assumption:

**Assumption 3 (Labour supply and demand elasticities)**

\[
\left| \frac{dL_{czn}^e}{d(w_{zn}^e/w_{zo}^e)} \right| \left( \frac{w_{zn}^e/w_{zo}^e}{L_{czn}^e} \right) < \min \left\{ \left( \frac{\sigma_s (\alpha_s - 1) + 1}{1 - \sigma_s} \right) \right\}_{s=1}^S.
\]  

(1.20)

The left hand side of equation (1.20) is the occupational effective labour supply elasticity and the right hand side is the trade elasticity with respect to unit labour costs in each sector. The trade elasticity is directly derived from equation (1.17) by collecting the effective unit labour cost terms ($a_{cs}$). When there is an increase in the effective wage of an occupation the effective demand for that occupation changes for two reasons. First, it decreases because consumers switch to varieties sold from other countries, which are relatively cheaper (substitution effect). Second, workers take advantage of the increase in occupational wages and switch occupations increasing their income (income effect). The above equation is a sufficient condition for the substitution effect to be greater than the income effect. This guarantees that effective demand functions for occupational labour are downward sloping.

**1.2.5 General equilibrium impact of shocks**

I now analyze the general equilibrium effects of shocks. In the context of this model, where there are heterogeneous agents, I first need to define a social welfare function. I compute
aggregate welfare changes in country \( j \) from the equivalent variation of the following function:

\[
W_j = \int \gamma_j(\theta) \cdot dF(\theta) \cdot P_j, \quad \text{where} \quad P_j = \left( \frac{P_{j1}}{\gamma_1} \right)^{\gamma_1} \left( \frac{P_{j2}}{\gamma_2} \right)^{\gamma_2}.
\]  

The numerator is the unweighted sum of workers’ income as defined by equation (1.15) and the denominator is the price index, which is common across workers (homothetic utility function). I choose this social utility function (unweighted sum of workers’ income) to reflect changes in welfare that pass the strong compensating criterion (i.e. efficiency changes). However, the changes that I consider may not be Pareto dominant within the home country, so I also show their distributional consequences.

The following proposition shows the welfare effects of shocks:

**Proposition 3** (Welfare effects of shocks)

The equivalent variation of labour market mobility or trade cost shocks

\[
\left( \langle d_h, \{\tau_{ij}s\}, \{f_{ij}s\} \rangle \rightarrow \langle d'_h, \{\tau'_{ij}s\}, \{f'_{ij}s\} \rangle \right)
\]

is given by:

\[
\ln (\hat{W}_h) = \sum_{s=1}^{S} \frac{\gamma_s}{(1 - \sigma_s)} \ln (\hat{\Lambda}_{hhs}) + \sum_{s=1}^{S} \ln \left( \frac{\hat{X}_{hhs}}{\hat{P}_{hhs} \hat{l}_{hs}} \right), \quad \text{with} \quad l_{hs} = \frac{w_{h2}s t_{h2}s_{1} w_{h2}s t_{h2}s_{2}}{w_{h1}s t_{h1}s_{1} w_{h2}s t_{h2}s_{2}},
\]

where \( hat \) denotes the ratio between the value of the variables after and before the shock. \( l_{hs} \) is the share of the value of total labour employed in producing for the domestic market in sector \( s \). As in Arkolakis et al. (2009), I can infer all general equilibrium effects of shocks in the home country from changes in domestic variables.

The first term is the *terms-of-trade* effect, which I refer to as the classical mechanism. A decrease in the share of expenditure in home produced varieties in sector \( s \) is associated with an increase in the value of production relative to consumption in that sector. The *terms-of-trade* is the weighted sum of these sectoral expenditure shares.. They are weighted by a term that indicates the importance of these changes to the utility of consumers. Sectors that are more important to consumers (i.e. high \( \gamma_s \)) or with lower elasticity of substitution (i.e. high \( \sigma_s \))
receive higher weights.

In equilibrium there cannot be an increase in the share of sectoral expenditure in home production in all sectors (otherwise, equation (1.19) would not hold). Therefore, this term is greater the higher the ability of the country to concentrate production in sectors in which it has a comparative advantage. Distortions that reduce this ability yield lower gains from trade through this term, while distortions in the opposite direction increase the value of this term.22

The second term is the value-added effect, which operates through two different channels. First, any distortion in the labour market reduces workers’ productivity thereby reducing the value-added of production. In this paper, I assume that any government policy in the labour market is distortionary so any change in $d$ away from one negatively impacts welfare through this channel.

The second channel relies on the consumers’ expenditure misallocation. As mentioned above, consumers ignore that reallocating expenditure across sectors results in sectoral changes at the extensive margin (firm creation and destruction). If the value of the marginal firms across sectors is different, this leads to a misallocation of expenditure. Any shock tends to change the equilibrium sectoral composition of output. Therefore, when the relative output of a sector with a higher value of the marginal firm increases, a shock reduces the negative effect of the expenditure misallocation. This results in an increase in the value of production thereby increasing total workers’ value-added and aggregate welfare. The following proposition and simulations explain the conditions under which marginal firms in two sectors differ in value and the mechanism through which government labour market policy is able to exploit these differences to increase welfare.

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22There are actually two effects from increasing the ability to reallocate workers towards the sector in which a country has a comparative advantage. First, a country gains from increasing the degree of specialization. There is a second effect in the opposite direction caused by the decrease in the world price of the country’s export intensive sector. This second indirect effect works through general equilibrium forces and is always smaller than the first direct effect.
Proposition 4 (Misallocation)

Let

\[
\begin{bmatrix}
1 & t_{ijs} \\
\lambda_i & 1
\end{bmatrix} =
\begin{bmatrix}
1 & \infty \\
\infty & 1
\end{bmatrix} \quad \text{for } s = 1, 2,
\]

then in any equilibrium with \(d_h = 1\), \(\beta_{sz} / \beta_{vz} > \beta_{sz1} / \beta_{vz2}\),

and \(\eta_s = \eta\) for \(s = 1, 2\):

- \(\partial W_h / \partial d_h > 0\) iff \(\omega_v > \omega_s\),
- \(\partial W_h / \partial d_h < 0\) iff \(\omega_v < \omega_s\),

where \(\omega_s = \frac{1}{\alpha_s} \left( \frac{\alpha_s \sigma_s - (\sigma_s - 1)}{(\sigma_s - 1)} \right)\) is the elasticity of aggregate price with respect to input costs in each sector.

Proposition (4) simplifies the analysis by analyzing the closed economy case so that the terms of trade effect is always equal to zero.

According to the assumptions, sector \(v\) is relatively more intensive in occupation \(z_2\) so that an increase in \(d_h\) results in an increase in the relative production of this sector (the opposite holds for a decrease in \(d_h\)). The proposition shows that a planner is able to increase welfare by increasing the supply of workers to occupations used more intensively in sectors where the aggregate sectoral price elasticity with respect to input prices is greater. Extensive margin changes amplify the effect of changes in input costs on the aggregate sectoral price index (i.e. unit price of consumption in a sector). A planner is able to reduce the total cost of consumption by reallocating workers to occupations used more intensively in sectors with relatively higher elasticities. As discussed in Feenstra (2010b) and Costinot and Rodriguez-Clare (2013) the price elasticity with respect to input prices is a combination of the entry elasticity and the value of entry to consumers.\(^{23}\)

\(^{23}\)I follow the theoretical trade literature, in particular Arkolakis et al. (2009), in assuming away production gains by equalizing \(\eta_s\) across sectors (see online appendix of Arkolakis et al. (2009)). When \(\eta_s\) are different, sectors also differ in the amount of labour they use to produce a unit of output. In this case, the total amount of labour needed to produce a unit of utility changes when the sector composition of output changes. I include this channel in the numerical model.
The intuition for the results of proposition (4) becomes clearer by studying the effect of changes in $\alpha_s$ and $\sigma_s$ on the sector price elasticity with respect to input costs. In terms of the shape parameter of firms’ productivity distribution, the higher the degree of heterogeneity in firms’ productivity, the higher this elasticity is (i.e. $\frac{\partial \omega_s}{\partial \alpha_s} < 0$). As the degree of firm heterogeneity in productivity increases, the within sector competition that firms face increases because there is a larger measure of highly productive firms. This results in a highly productive marginal firm relative to a sector with lower firm heterogeneity. Therefore, entry in the sector with higher degree of heterogeneity at the cost of exit from the other sector results in an increase in productivity. A similar result holds in terms of consumer side elasticities. As the elasticity of substitution decreases, the elasticity of price index with respect to input costs increases (i.e. $\frac{\partial \omega_s}{\partial \sigma_s} < 0$). This results in a higher price elasticity with respect to input costs because the value of the marginal firm is higher when varieties are less substitutable.

Proposition (4) shows that non-concavities in the impact of input costs on sectoral price indexes are stronger in sectors with higher love for variety and higher firm heterogeneity. Since consumers ignore these non-concavities when making expenditure decisions across sectors, the government can internalize them by directly reallocating labour to those sectors.

Figures (1.2) shows a numerical example using a bounded Pareto distribution for the productivity of workers in both countries. Most of the parameters used in the simulation are obtained by aggregating the numerical model into two sectors and using the calibrated parameters. Particularly important is the fact that the sector with high firm heterogeneity and low elasticity of substitution is relatively more intensive in the high skill occupation ($z_2$). I assume a long run open economy equilibrium with no labour market distortions so any distortion to the left of 1 in the graph is a subsidy to $z_1 \rightarrow z_2$ switchers and in the opposite direction to the right. The distortions within the upper and lower bounds in the graph are the distortions that satisfy assumption (3).

---

24It is worth noting that there is also another effect in the opposite direction as can be seen from $\omega_s$. As firms become more heterogeneous in productivity the measure of firms entering given a reduction in input prices is lower (as indicated by the first term of the equation). This reduces the elasticity of the price index with respect to input costs. However, this effect is more than offset by the productivity effect described above.
The left-hand side graph shows an economy with production misallocation. In particular, as proposition (4) indicates, there is low entry in the sector relatively more intensive in occupation $z_2$ (which I refer to as sector 2). In this case, the optimal labour market policy is to subsidize movers to $z_2$ occupation increasing entry in that sector. This policy raises aggregate welfare through the value-added effect. It is worth noting that under the optimal labour market policy, although welfare is higher, the terms-of-trade are lower, indicating that the home country has a comparative advantage in the sector relatively more intensive in $z_1$ occupation (foreign’s skill distribution first order stochastically dominates the skill distribution of home). The graph on the right-hand side shows an example of an economy with no production misallocation. In this economy, it is always welfare improving to increase the production of the sector in which a country has a comparative advantage. In terms of the distribution of these gains from trade, we can see that this policy benefits the switchers who were very close to cut-off before the policy and therefore are receiving the subsidy to move the most. For these workers, it is not very costly to move and yet they receive a proportional subsidy to their mobility cost equal to workers for whom switching is more costly.
1.3 Quantitative Application

In this section I extend the model to allow for multiple sectors and occupations. However, I maintain the assumption of two countries: \( h \) (US), \( f \) (ROW). Also, I modify the environment in several other dimensions to both match empirical facts and be able to structurally estimate all the model’s parameters necessary for counterfactuals. First, I assume that workers draw match abilities for each occupation, \( \psi(z_n, \theta) = \theta^\mu_{zn} \), from a common distribution. As I discussed above, in equilibrium a worker is always employed in the occupation that pays the highest wage adjusted by switching distortions. To see this, suppose a worker is currently hired at the occupation that pays her the second highest draw. Because of perfect competition in the labour market, a firm would be employing a worker with the same ability in the other occupation and paying her a wage at least equivalent to the highest wage of this worker. In this case the model is not in equilibrium because this firm would prefer to employ the other worker at her current second best wage in this occupation. Therefore, I can rewrite the sorting equilibrium in equation (1.9) for a worker in occupation \( z_o \) at the beginning of the period as:

\[
\text{Max} \left\{ d_{z_o z_n} w_{zn} \theta^\mu_{zn} \right\}_{n=1}^N
\]

In order to match the empirical fact that gross flows are significant, I follow Cortes and Gallipoli (2015) in assuming that workers redraw the occupation specific productivity, \( \theta_{zn} \), in each period \( t \).

Second, I assume that bilateral distortions \( d_{z_o z_n} = d^f_{z_o z_n} d^g_{z_o z_n} \) are the combination of technological mobility frictions \( d^f \) and government mobility subsidies/tax \( d^g \). The main difference between them is that, unlike technological frictions, government distortions affect the value but not the quantity of supply of effective workers. A departure from the previous model is that I reduce the restrictions on distortions by imposing no frictions only for occupational stayers (i.e. \( d_{z_v z_v} = 1 \)). I refer to all deviations of \( d \) from one as distortions.

Third, to capture observed workers’ heterogeneity, I separate workers into different groups indexed by \( u \). The distribution of match specific draws is the same for workers within these draws but different across draws. Every worker is exogenously assigned to a group. As I explain later, I define these groups by educational attainment. In addition, I assume that all
workers within and across groups are perfectly substitutable within each occupation. This assumption makes all the results derived in the previous section to continue to hold.

I assume that individuals within each group draw match productivities from an extreme type 1 distribution. As in Ramondo and Rodriguez-Clare (2013), I assume that draws are correlated across occupations for an individual but independent across individuals. Formally, the vector of match worker-occupation abilities for an individual with educational attainment \( u, \theta_u = [\theta_{zu_1}, \ldots, \theta_{zu_n}, \ldots, \theta_{zu_N}] \) is drawn at time \( t \) from:

\[
F_u(\theta_u) = \exp \left\{ - \sum_{n=1}^{N} T^t_{uz_n} \left( \theta_{zu_n} \right) \frac{\xi}{\mu (1 - \rho)} \right\}^{(1 - \rho)}, \tag{1.23}
\]

where \( T^t_{uz_n} \) is the group-occupation specific location parameter at time \( t \), \( \frac{\xi}{\mu} \) is the common shape parameter of the Frechet distribution and \( \rho \) is the correlation coefficient of draws across occupations. When \( \rho = 0 \), all draws are independent. As \( \rho \) increases, the joint probability of draws with similar values is greater. An important property of equation (1.23) is that it is homogeneous degree \( \frac{\xi}{\mu} \) which, as I show below, allows me to estimate the correlation coefficient separately from the shape parameter of the Frechet distribution.

Given the assumptions on the stochastic process governing the distribution of match productivity, I show in appendix (1.5.1) that I can then rewrite equation (1.10) as follows (as it was the case in the previous section I introduce country subscripts only when I define the model’s general equilibrium):

\[
L^e_{zn} = \sum_{u=1}^{U} \sum_{v=1}^{N} \frac{L_u \Delta u^0_{uz_n}}{\text{Measure of } v \rightarrow n \text{ switches}} \left( T^t_{uz_n} \right)^{(1 - \rho)\mu} \left( d^f_{uz_n} \right) \left( \Delta u_{vzn} \right)^{-\frac{(1 - \rho)\mu}{\xi}} \Gamma \left( 1 - \frac{\mu}{\xi} \right) \left( \frac{\xi}{\mu} \right), \tag{1.24}
\]

\[
\Delta u_{vzn} = \frac{\left( w^e_{zn} d_{zn} \right)^{\frac{\xi}{\mu (1 - \rho)\mu} \epsilon T^t_{uz_n}}}{\sum_{r=1}^{N} \left( w^e_{zr} d_{zr} \right)^{\frac{\xi}{\mu (1 - \rho)\mu} \epsilon T^t_{uzr}}}, \tag{1.25}
\]

\[25\]See, for instance, Hsieh et al. (2013), Cortes and Gallipoli (2015), Artuç et al. (2010), and Lagakos and Waugh (2013).
where $\Delta_{uz}^{0}$ is the proportion of workers with education $u$ employed in occupation $z_v$ at the beginning of the period, $\Delta_{uz,zn}$ is the share of those workers who switch to occupation $z_n$ from the initial allocation of workers to occupations and $\Gamma$ is the gamma function.

The assumptions on the stochastic process lead to very useful theoretical and empirical properties of the labour market equilibrium, which I use to estimate the model’s parameters. When an occupation is relative hard to enter, due to high entry barriers or relative low effective wages, entry into that occupation is low. This is indicated by the first term of equation (1.24). Simultaneously, on average the match productivity of workers conditional on selecting into that occupation is high. As indicated by the second term of equation (1.24), the degree in which the average productivity of workers selecting into an occupation varies is proportional to observable bilateral worker flows. It is also a function of other parameters of the distribution but of no other equilibrium variable. This simplifies the estimation of the labour market parameters because the observed worker occupation-to-occupation flows are a sufficient statistic for selection.

In addition, bilateral worker flows take a very simple ‘gravity’ structure (equation (1.25)). The share of switchers is proportional to the relative mean of the distribution of productivity draws in each occupation, adjusted by the cost of switching. The percentage change in the proportion of switchers caused by a one percent change in relative effective wages adjusted by mobility costs is given by the elasticity of occupational labour supply $\left(\frac{\xi}{(1-\rho)\mu}\right)$. Intuitively, as high productivity draws become more important in production (high $\mu$) or their spread across occupations higher, (low $\xi$ and low $\rho$) effective wage differences become a less important factor in sorting.

Even though this set of assumptions makes the sorting tractable and parameters identifiable using aggregate occupational flows data, it causes some problems for the estimation of occupational frictions. As I show in the next section, I obtain estimates of the bilateral occupational mobility frictions by comparing the average wages of switchers relative to stayers across occupations. Under the assumptions imposed on the stochastic process, in the absence of ed-
ucational groups and frictions, the average wage of switchers is the same as the average wage of stayers. The result of this is that the variation in average wages between workers from different source occupations in a destination occupation is assigned to bilateral distortions. In the micro-founded model I showed that this is not the case (see figure (1.2)). Workers who sorted into different occupations in the starting equilibrium are likely to have different match productivities than stayers. This causes the estimates of the factual distortions to be biased. Moreover, the direction of the bias is not clear as it depends on the unobserved average worker’s ability in the counterfactual occupation.

To correct for this problem I use observed workers’ heterogeneity by assuming that workers with different education levels draw match productivities from separate distributions. Under this assumption, the average worker in each occupation draws match productivities from a different distribution that depends on the composition of education in the occupation of origin. Therefore, even in the case of no frictions, the wage of the average switcher is different than the wage of the average stayer.

The definition of equilibrium in the stochastic model is similar to the definition in the previous section. The only difference is that the labour market equilibrium is defined in terms of shares of workers within educational levels that are employed in each occupation instead of labour markets sorting cut-offs. An important property of this model is that I can rewrite the equilibrium conditions as functions of changes in variables from an original equilibrium reference point. This is particularly helpful in this context because it greatly reduces the dimensionality of the parameter space. This technique pioneered by Dekle et al. (2007) has been

26One of the main reasons for this result is that in each period all workers draw new productivities from the same distribution independently of their original occupation.

27As indicated by Dix-Carneiro (2014), controlling for observable characteristics only partially solves the bias of the estimated frictions because workers also sort according to unobserved characteristics. An alternative specification that solves this problem is to use the micro-founded Roy model in combination with parametric assumptions to separate selection in unobservables from frictions. The problem is that with multiple sectors and occupations this model becomes not tractable without making simplifying assumptions on the general equilibrium effects of worker sorting (see, for instance, Liu and Trefler (2011)). The main goal of this paper is to compare the optimal level of frictions relative to the factual case. In the case where there are no large differences in the amount of worker sorting, the bias in these two cases would approximately be the same. So, the results of this paper can be interpreted in relative terms. Therefore, I choose not to complicate the model in this direction.
widely used in the recent trade literature. Ossa (2014) and Hsieh and Ossa (2015), for instance, use this method in the context of monopolistic competitive markets. Dekle et al. (2007) and Tombe and Zhu (2015) use it in the context of perfectly competitive markets. I combine them by using it with perfectly competitive markets (labour) and monopolistic competitive markets (output).

**Proposition 5 (General equilibrium in changes)**
The solution to the general equilibrium of the model can be expressed in changes as follows (I define changes in world’s total income as the numeraire):

\[
J_{czn} = \hat{w}_{czn} \sum_{u=1}^{U} J_{czzn}^{0} \sum_{\alpha=1}^{N} \Delta_{czzn}^{0} \left( \hat{T}_{czzn} \right)^{\frac{\epsilon_{l}}{\epsilon_{x}} \frac{d_{czn}}{d_{czzn}}} \left( \frac{\Delta_{czzn}}{\Delta_{czzn}} \right)^{1-\frac{1}{\epsilon_{l}}},
\]

\[
\Delta_{czzn} = \left( \frac{\hat{w}_{czn} d_{czzn}}{\sum_{r=1}^{N} \left( \hat{w}_{czzn} d_{czzn} \right)} \right)^{\epsilon_{l}} \Delta_{czzn},
\]

\[
X_{cs} = \sum_{j=h,f} \Lambda_{cjs} \gamma_{s} \frac{\sum_{n=1}^{N} J_{czn}}{\sum_{n=1}^{N} J_{czn}},
\]

\[
\Lambda_{cjs} = \frac{X_{cjs}^{0} \left( \hat{a}_{cs} \right)^{\epsilon_{s}} \left( \hat{c}_{cjs} \right)^{-\alpha_{s}}}{\sum_{v=h,f} X_{cjsv}^{0} \left( \hat{a}_{vs} \right)^{\epsilon_{s}} \left( \hat{c}_{cjs} \right)^{-\alpha_{s}}}, \text{where } \hat{a}_{cs} = \prod_{n=1}^{N} \left( \hat{w}_{czn} \right)^{\beta_{szn}},
\]

\[
\sum_{s=1}^{S} \eta_{s} \beta_{szn} X_{cs} = J_{czn},
\]

where \( \epsilon_{l} = \frac{\epsilon}{(1-\rho)\mu} \) is the occupational labour supply elasticity, \( \epsilon_{s} = -\alpha_{s} + \frac{\alpha_{s}-\sigma_{s}+1}{\sigma_{s}-1} \) is the sectoral export elasticity and \( \eta_{s} = \frac{\sigma_{s}(\alpha_{s}-1)+1}{\sigma_{s} \alpha_{s}} \) is the share of revenue allocated to pay labour costs of production.\(^{28}\) Also, \( J_{czn} \) denotes the value of labour in occupation \( z_{n} \) in country \( c \).

Equations (1.26) and (1.27) are the key equations of the equilibrium in the labour market. The second one of these equations, expresses the worker bilateral flows as functions of changes from an initial allocation of workers to occupations. Because I assume changes from the long-
run frictionless equilibrium, the share of workers in each occupation in the initial equilibrium represents the relative degree of difficulty of sorting into that occupation for all workers with the same education level. So, equation (1.27) represents the changes in the degree of difficulty of sorting into occupation $z_n$ for workers that have previously sorted into $z_o$. This depends on three forces: the relative change in the demand for occupation $z_n$, the relative cost to a worker of having accumulated experience in $z_o$ and not in $z_n$ and the relative degree of technological change that makes everybody in that occupation more productive.

Equation (1.26) represents the total value of effective labour supply to occupation $z_n$. It depends on the bilateral worker flows as follows: If the share of switchers is higher than what it would otherwise be in the absence of shocks, the number of workers switching from $z_o$ to $z_n$ is higher. However, the effective labour supply decreases (with elasticity $\frac{1}{\epsilon}$) because the average switcher has a lower match productivity.

Equations (1.28) and (1.29) rewrite the conditions for the output market equilibrium while equation (1.30) is the aggregate excess demand for occupational effective labour previously discussed. This system of equations can be solved for equilibrium effective wage changes ($N$ equations by substituting them into the last equation and $N$ unknowns).\(^{29}\)

The importance of proposition (5) is that by using observables from the long run equilibrium, I am able to reduce the number of parameters I need to estimate. The parameters needed for determining the labour market equilibrium are: the measure of correlation across draws ($\rho$), the degree of workers’ comparative advantage ($\frac{\xi}{\hat{p}}$), technological bilateral mobility frictions ($d^f$), government bilateral mobility taxes/subsidies ($d^g$) and occupation-education specific technological growth ($\hat{T}$). To solve for the output markets equilibrium, I need to know: the elasticity of substitution across varieties in each sector ($\sigma$), the output supply elasticity in each sector ($\alpha$), the share of expenditure in each sector ($\gamma$), changes in bilateral trade flows in each sector ($\hat{r}$) and the share of total cost in each sector that is spend in each occupation in ($\hat{\beta}$). In the next section, I discuss how I obtain estimates for each of these parameters of the model.

\(^{29}\)Note that in solving the system, I impose Walras’ law by normalizing the change in global income. Therefore, none of the $N$ equations is a linear combination of the others.
1.3.1 Calibration Strategy

I consider seven manufacturing sectors and seven occupations. Tables (1.4) and (1.5) define the sectors and occupations, respectively, that I include in the quantitative analysis. In addition, I use the following three education categories: $u = 1$, at most twelve years of education (High school); $u = 2$, between thirteen and fifteen years of education (College); $u = 3$: sixteen or more years of education (University degree). I combine fifteen different countries, for which reliable labour market data exists, into the foreign country and I calibrate the model to changes in iceberg trade costs between from 1992 to 2007. In appendix (1.5.3), I describe all the data I use to calibrate the model.

Since I consider changes that take place over the span of fifteen years, there is a concern that changes in sectoral revenue are the result of forces other than changes in trade costs such as capital deepening. This is a problem for my analysis to the extent that these forces are included in the factual benchmark but not in the counterfactual equilibrium. As I explain below, I do not solve for the entire general equilibrium to calibrate the model, so I address this problem in the counterfactual analysis section. The only impact for this section is that calibrated iceberg trade costs include differences in technological growth and capital deepening across countries within each sector.

The model is block recursive in the sense that I am able to characterize the partial equilibrium in the labour and output markets separately. Then, I solve for the wages that clear both markets to obtain the general equilibrium. This suggests a calibration strategy that matches partial equilibrium conditions to moments of the data for each market separately. To calibrate the parameters of the home labour market, I use a panel of workers for whom I observe their occupation and hourly wage income in the initial and subsequent periods as well as their educational attainment. For the output markets, I use aggregate sectoral revenue as well as trade flows for each of the sixteen countries (foreign and US). In terms of the foreign labour market, I am not able to estimate the matrix of mobility costs because I observe only a repeated cross-section. Therefore, I assume that occupational labour supply in foreign is at the factual level.
through counterfactuals which I obtain from the cross-section.30

**Home Labour Market**

Before I define the moments of the data I use to calibrate the parameters of the US labour market, I need to define the normalization and parameters set outside the model. I do not observe workers’ match abilities but the total return to their effective labour supply in an occupation. This has two implications. First, I cannot derive effective wages from observed wages so I treat them as an additional parameter to be calibrated. Second, changes in the return to workers within an occupation that arise from an increase in the relative demand for that occupation are observational equivalent to changes that arise from an exogenous increase in the occupation level productivity. That is, I cannot separately identify $\hat{w}_{hz}^e$ from $\hat{T}_{hz}^n$ for all education levels. So, I normalize the exogenous change in productivity to one for all occupations at the high school educational level ($\hat{T}_{hz}^n = 1$ for $n = 1, \ldots, N$). I then identify the exogenous productivity growth for all other education-occupation pairs from this benchmark.31

To further reduce the dimensionality of the parameter space, I obtain these exogenous productivity growth rates directly from the data by calculating the cumulative growth of nominal wage of stayers for different education levels (see figure (1.9)). I show these parameters as well as the set of the model’s parameters from the labour market that maps directly to their counterparts in the data in table (1.8).

I calibrate the remainder parameters by matching the partial equilibrium in the labour market implied by the model to observed moments of the data using *simulated method of moments*. Since all of these parameters are at the occupational level, I aggregate the moment conditions for all the educational levels within occupations. The first moment I target is the total share of

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30I still allow labour to switch sectors as in the HO model.

31An alternative is to set symmetric bilateral mobility frictions, as in Bryan and Morten (2015), and allow the exogenous occupation productivity growth to be different for all education levels. In this case, I would only need to normalize the exogenous productivity growth for one education-occupation pair. Unlike in this paper, symmetry is a more appropriate assumption in Bryan and Morten (2015) because they estimate geographical mobility frictions. However, frictions at the occupational level are unlikely to be symmetric.
switchers from a source to a destination occupation:

$$\Delta_{h,v,z,n} = \sum_{u=1}^{3} L_{huz,v} \Delta_{huz,v,z,n},$$

(1.31)

where \( L_{huz,v} \) is the share of workers within an occupation with education level \( u \).

I obtain the second moment from the equilibrium result that, within an occupation, wages are a linear function of matched ability (i.e. effective wages equalize across occupation - lemma (1), conditions iii, iv). This yields the following equation linking occupational efficiency wages to observable occupational average wages.

$$E_h[w(\theta) | \theta \in \Theta_{z,n}] = \frac{J_{h,z,n}}{\sum_{u=1}^{3} L_{h,u} \sum_{n=1}^{7} \Delta_{huz,o}^{lo} \Delta_{huz,o,z,n}},$$

(1.32)

where \( \Delta_{huz,o,z,n} \) is given by equation (1.27), \( J_{h,z,n} \) is given by equation (1.26).

Equation (1.32) shows the across occupation wage inequality. This inequality is the combination of changes in the return to ability (income) and changes in the composition of workers within occupations (selection). An important property of the Frechet distribution is that in the frictionless case \( d_{hzn,o} = 1 \) for all \( n,o = 1,\ldots,N \) the average wage of workers with the same education level in all occupations is the same. When there is an increase in the relative effective wage of one occupation, there is a positive income effect (workers earn more per unit of ability) and a negative selection effect (less productive workers select into that occupation). With the Frechet assumption these two effects cancel out.

With frictions this result no longer holds because even though all workers draw from the same distribution, the set of friction workers face when sorting into occupations depend on the source occupation. In this case this result extends to the average wage of workers conditional on the source occupation and educational attainment equalizes across destination occupations. I use this result and the assumption that \( d_{huz,v,z} = 1 \) to obtain the second additional home labour
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32 This result is the same as the Head and Ries (2001) index that I use to calculate output market frictions.

\[
\frac{\Delta_{huz,vz,zn} E_{hu}[w(\theta)|z_v \rightarrow z_n]}{\Delta_{huz,n-zn} E_{hu}[w(\theta)|n \rightarrow n]} = \left[ d^f_{hz,vz} d^f_{hz,zn} \right]^f, \tag{1.33}
\]

where \( E_{hu}[w(\theta)|z_v \rightarrow z_n] = \left( w^{z_e}_{hz,v} E_{hu}[\psi(z_n, \theta)|z_v \rightarrow z_n] \right) \) and \( E_{hu}[\psi(z_n, \theta)|z_v \rightarrow z_n] \) is defined in equation (1.24). Equation (1.33) indicates that the difference in total income between two groups of workers from different sources in a destination relative to the relative total income from the same two sources in another occupation identifies the joint cost of switching. The double differencing is important to eliminate the effects of frictions to all the other occupations which in general equilibrium equally affect the total income of switchers from the same source in two different occupations.

Equations (1.31)-(1.33) represent a system of 77 \((N^2 + 7 + \frac{N(N-1)}{2})\) equations which I use to solve the 43 \((N(N-1) + 1)\) unknowns (I calibrate the elasticity of occupational labour supply as one term). In addition, I am able to separate the workers’ comparative advantage from the correlation of productivity across occupations using the following equation, which I directly derived from equation (1.39)

\[
\frac{\text{var}_{hu}[w(\theta)|v \rightarrow n]}{E_{hu}[w(\theta)|v \rightarrow n]^2} = \frac{\Gamma \left( 1 - \frac{2\mu}{\xi} \right)}{\left( \Gamma \left( 1 - \frac{\mu}{\xi} \right) \right)^2} - 1. \tag{1.34}
\]

Given the functional form of the joint distribution of draws (1.23), changes in the correlation coefficient parametrically shift the entire conditional distribution of switchers but leave the relative likelihood of draws constant. Therefore, a higher correlation coefficient increases both the standard error and the mean of the productivity distribution by the same factor.

Output Markets, Trade and Foreign Labour Market

There are some parameters for which there are direct counterparts in the data. These parameters are shown in table (1.9). An advantage of the panel of workers that I use is that it also contains
the sector of employment of each worker which allows me to calculate \( \beta_{szn} \) directly from the data. I also calculate \( \gamma_{cs} \) by measuring the share of revenue from imports and domestic sales in each sector and country using bilateral trade flows data. I use the following equation of relative value of sales between home and foreign across markets, as in Head and Ries (2001), to obtain the changes in iceberg costs in each occupation, \( \hat{\tau}_s \):

\[
\frac{\hat{\Lambda}_{scl}/\hat{\Lambda}_{sjc}}{\hat{\Lambda}_{scc}/\hat{\Lambda}_{sjj}} = \left( \frac{1}{\hat{\tau}_s} \right) e_x.
\]

Finally, I obtain changes in the effective labour supply in each occupation in foreign by first using the model to calculate changes in effective wages in each occupation and then using them to get the amount of effective labour from observed total occupational income. To calculate effective wages, I again use simulated method of moments to calculate foreign occupational effective wages that minimize the difference in changes in the import penetration of foreign varieties into the domestic market in each sector \( \Lambda_{fhs} \) between the data and the model.

These are \( S \) equations in \( N \) unknowns. Since I include the same number of occupations and sectors in the model \( (N = S) \), the system of equations is exactly identified.

### 1.3.2 Results

The calibration exercise delivers reasonable parameters’ estimates for both the labour and output markets. The occupational labour supply elasticity is 1.74. This is higher than in any of the specifications in Artuç et al. (2010), which uses a similar structural model and a different data set to estimate sectoral mobility frictions. The main reason for this difference is that while they calculate short-run elasticities, here I calculate medium-run elasticities by using the first and the last observed occupation for each individual. Table (1.6) shows that the median duration

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33 From the four possible changes in import penetration ratios within each sector \( \Lambda_{cjs} \) for \( c, j = hf \) I can only use one. The reason for this is twofold. First, import penetration ratios sum to one within a market. Therefore, minimizing the difference between data and model for one of the import penetration ratios is equivalent to minimizing it for both. Second, equation (1.35) uses the relative difference between import market penetration ratios across countries so using both of them again is redundant.
between the first and last observed occupation for an individual is significantly higher than one year for all educational levels.\textsuperscript{34}

Important for the counterfactual exercise is the estimation of the factual distortions caused by bilateral mobility costs. I show these estimates in table (1.3). Each off-diagonal element represents the wage of a switcher relative to a stayer, conditional on attaining the same education level and having the same match occupation productivity. An important property of the \textit{Frechet} distribution is that the identification of the model’s parameters is relatively transparent. Understanding how mobility costs are identified is important in interpreting the results in table (1.3). Changes in relative wages between these occupations increase the total income of switchers in one direction and reduce the total income of switchers in the other direction in exactly the same proportion. Moreover, the difficulty of switching to other occupations affects switchers from a source occupation and stayers proportionally. This implies that I can jointly identify the mobility distortions between two occupations from the total income of the two-way switchers relative to stayers in these two occupations (equation (1.33)). To identify each distortion separately, I then use the average wage levels and the share of switchers.

For instance, the joint total income of switchers from professional to sales and from sales to professional relative to stayers in these two occupations is very low. This implies that the joint cost of switching is very high. As indicated by table (1.3) most of the joint cost of switching is due to the high cost to switchers from professional to sales (the wage of a switcher is only 9% the wage of a stayer conditional on having the same education and match productivity). This is largely due to the fact that there are very few switchers from professional to sales relative to the effective wage changes in these two occupations during the period of study. This is likely the result of opportunity costs associated with leaving a high human capital occupation. Since I do not formally model occupational specific human capital accumulation (see, for instance, Kambourov and Manovskii (2009), Dix-Carneiro (2014) and Cosar (2010)), these opportunity

\textsuperscript{34}There is an additional difference between this paper and Artuç et al. (2010). I study mobility at a different level of aggregation. While in their paper they study sectoral mobility, here I study occupational mobility. This, mainly, affects the estimates of mobility costs and to a lesser extent the estimates of the labour supply elasticity.
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<table>
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<th>Clerical</th>
<th>Sales</th>
<th>Trades</th>
<th>Production</th>
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<td>0.35</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 1.1: Factual Bilateral Occupational Mobility Distortions

costs affect the value of the costs of switching I estimate from the model. This is shown in table (1.3), which shows that on average moving from high human capital occupations to low human capital occupations is relatively more costly (the values in upper triangle are lower than in the lower triangle).

In terms of the output markets, in table (1.9) in appendix (1.5.3) I show production and preference parameters. There is a positive correlation between the managerial intensity in production and sectoral high value-added of production as indicated by low supply and demand side elasticities. In particular, other manufacturing is by a large margin the most managerial intensive and also has the lowest elasticities. The large degree of heterogeneity across firms in terms of productivity and product differentiation within this sector is partly due to the fact that this sector serves as a residual for otherwise uncategorized products. However, the analysis of its subcategories also reveals the importance of brand names for products within this category. Among the subcategories there are the production of sporting goods, toys, musical instruments as well as jewellery. Another sector that is also relatively managerial intensive and posses a high degree of differentiation among its products is minerals and metallic products. On the opposite side of the spectrum, there is the chemical, rubber, plastics and fuel sector, which has a high degree of homogeneity among producers and a relatively low share of expenditure on the managerial occupation. The sector with the most producer homogeneity is the wood, cork, pulp and paper sector which is also relatively intensive in production workers.
Figure 1.3: Share of Labour Force by Occupation-Education (data vs model)

Figure 1.4: Average Hourly Wage by Occupation-Education (data vs model)

Figure 1.5: Sectoral Import Penetration Ratio by Market (data vs model)
In addition, the table shows the calibrated values for expenditure shares and change in trade costs. The machinery and equipment sector is the sector with the highest expenditure share in both domestic and foreign countries (this includes personal computers). In terms of change in trade costs, the table shows that the reduction in sectoral tariff barriers between US and foreign from 1992 – 2007 is small. The reason for this is that trade barriers were already very low in 1992. As I show in the counterfactuals this makes the gains from trade in those years very small for both the factual and counterfactual scenarios.

Before using the calibrated parameters for counterfactuals analysis, it is important to verify whether the model delivers adequate approximations to moments of the data. Although I target aggregate moments at the education level, I show how the model performs in matching moments disaggregated by education in figures (1.3) and (1.4). The differences in the sorting across educational levels are the observed initial shares of workers across occupations and the exogenous productivity growth of education-occupation pairs. The first of these figures shows that the model is able to adequately reproduce the shares of workers with a given education across occupations. There is a poorer fit in terms of the average wages across occupations. Particularly, there is a significant difference between the model’s prediction regarding the average wages of sales, trades and production workers with university degrees. For those occupations the model predicts lower than observed occupational average wages. This is the result of the model’s inability to reconcile low shares of labour force with university education with relatively high wages in those occupations. Since these occupations require the lowest human capital, the workers with university degree in those occupations are likely performing some specialized task for which they are paid a premium relative to other workers.

In terms of trade flows, the model is able to closely match the sectoral import penetration ratios of foreign in home but performs poorly in matching the import penetration ratios of home in foreign. The poor match is the result of the assumption of symmetric changes in the iceberg trade costs. This assumption greatly simplifies the calibration without having large implications for the counterfactual analysis. Some of the differences between the observed and
model’s prediction of import penetration ratios are absorbed by lower foreign effective wages throughout the counterfactual analysis. This in effect results in a lower ability of domestic firms to penetrate foreign markets.

### 1.3.3 Counterfactuals

Having calibrated the key parameters of the model, I reintroduce government and calculate the distortions that maximize the aggregate welfare gains from trade denoted in equation (1.21). Unlike in the deterministic model, the reference point from which government policy is evaluated is a positive level of distortions caused by the calibrated mobility costs. The government is able to either increase the distortions by increasing mobility costs through taxation or decrease them by subsidizing occupational mobility.

The goal of the counterfactual is to derive optimal labour market distortions associated with the observed changes in trade barriers from 1992 to 2007. Moving from the long run equilibrium in 1992 to 2007 has two effects on the model. First, there is a reduction in trade barriers represented in the model by a decrease in sectoral iceberg costs. Second, there is an increase in labour mobility costs. This second effect results in workers’ losses, even in the case where there are no changes in the trade barriers, because workers have a lower amount of insurance against bad productivity draws. I am interested in welfare changes that arise from the first effect only, so I cancel the second effect by using as a benchmark a simulated economy with factual mobility costs and 1992 levels of trade barriers.

There is an additional problem when calculating counterfactuals, which I briefly mentioned in the calibration section. This problem is the outcome of ignoring other inputs of production

<table>
<thead>
<tr>
<th></th>
<th>Terms-of-Trade</th>
<th>Value-added</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factual distortions</td>
<td>0.26%</td>
<td>−0.04%</td>
<td>0.23%</td>
</tr>
<tr>
<td>Optimal distortions</td>
<td>0.25%</td>
<td>0.23%</td>
<td>0.48%</td>
</tr>
</tbody>
</table>

Table 1.2: Welfare Gains from Trade
such as capital, which results in wedges between the occupational effective labour demand implied by the model and observed in the data. Since I express the model in changes, the size of these wedges depends on both each sector’s capital intensity and the country’s degree of capital deepening between 1992 and 2007. The difference between the observed and predicted effective labour demand for each occupation depends on the occupational labour-capital complementarity.\textsuperscript{35} This does not create a problem for the calibration because I do not solve for the model’s general equilibrium and the Cobb-Douglas production function makes worker sorting independent of irrelevant alternatives. However, it is a problem for this section because effective wage changes would be affected by these wedges in the counterfactual exercise but are not be affected in the factual equilibrium. To solve this problem, I simulate the economy with all observed changes and I use this as the factual scenario.\textsuperscript{36}

The top row of table (1.2) shows the factual aggregated welfare gains from the decrease in

\textsuperscript{35}Complementarity in this case does not refer to a higher degree of substitutability between occupational labour and capital since the elasticity of substitution between inputs of production is assumed to be one (i.e. Cobb-Douglas production function). It is the correlation between capital and occupational intensities across sectors (this is related to the friends and enemies result in Jones and Scheinkman (1977)). A higher correlation indicates a higher degree of complementarity between capital and labour within that occupation.

\textsuperscript{36}Ossa (2014) uses the same strategy to solve the problem created by unbalanced trade. This solution is not a direct substitute to including capital in the model because it assumes that the value of capital used in each sector is constant through counterfactuals. However, it mitigates the problem in the presence of unobserved capital markets.
trade barriers using equation (1.22). The hat in this case represents the value of the variables obtained from the simulated factual over the simulated benchmark. The table indicates that there are overall welfare gains from trade. These are gains from specialization that arise from the US increasing employment in sectors where it has comparative advantage. These gains are partially offset by an increase in workers’ allocation towards low value-added industries. Figure (1.6) shows the percentage changes in the value of output per industry. The highest increase by a large margin takes place in wood, cork, pulp and paper industry, which is the sector with the lowest degree of value-added per worker.

The second row of the table shows the gains from trade under the optimal mobility costs scheme, which more than doubles the factual gains from trade.\textsuperscript{37} The increase in the gains from trade is achieved by increasing employment in higher value-added sectors relative to the factual equilibrium. In particular, the value of production in industries with high degree of firm heterogeneity, such as other industries as well as machinery and equipment, relative to factual increases, while sectors with more homogenous firms experience a relative decrease. As I show below, there is a relative increase in the median mobility cost so the gains from increasing domestic efficiency come at a cost of slightly lowering the gains from comparative advantage.

Table (1.3) shows the optimal level of occupational mobility distortions.\textsuperscript{38} Under the optimal scheme there are a large number of bilateral mobility costs slightly above, and a small number significantly below, the factual values.\textsuperscript{39} This results in an increase in the median bilateral mobility cost \( c_{zvz}\) of 2.74\%. Figure (1.7) shows the relation between factual and optimal for each bilateral distortion. For those bilateral pairs for which distortions fall to the left of the 45 degree line, the optimal mobility cost is higher than the factual and vice-versa for the bilateral pairs to the right of that line. The figure shows that the change in sectoral out-

\textsuperscript{37}To compute optimal distortions I use the algorithm suggested in Ossa (2014) and Ossa (2016). This consists in maximizing welfare changes (equation 1.21) subject to equilibrium constraints (equations (1.26)-(1.30)).

\textsuperscript{38}The value of \( d \) for movers from clerical to managers, which is greater than 1, indicates that under the optimal scheme the wage of a mover is higher than that of a stayer conditional on the same match productivity.

\textsuperscript{39}Figure (1.10) shows the kernel distribution of bilateral mobility frictions for the factual and counterfactual cases.
Table 1.3: Optimal Bilateral Occupational Mobility Distortions ($d_{z_n z_v}$)

put is achieved mostly by increasing the effective labour supply of managers while reducing the supply of effective workers in the production occupation. This is due to a decrease of costs of switchers moving into those occupations and an increase of costs for switchers moving away from them.

The labour market policy of changing occupational labour supply through mobility frictions has large implications for the distribution of welfare gains from trade. To measure the distributional consequences of this policy, I first show the following proposition:

**Proposition 6 (Ex-ante distribution of welfare effects from shocks)**

The ex-ante expected change in real income from a shock to either the labour market or foreign country to a worker with educational level $u$ conditional on being employed in occupation $z_n$ at the beginning of the period is:

$$
\ln |E_h|/Y|\theta \in \Theta_0| = \frac{(1 - \rho)\mu}{\xi} \ln \left[ \sum^{N}_{r=1} \bar{w}^{e}_{hz;r} d_{hz;r} \Delta^{l_0}_{hz;r} \right] + \ln \left[ \sum^{N}_{r=1} J_{hz;r} \Delta^{l_0}_{hz;r} \right] + \ln \left[ \sum^{N}_{r=1} J_{hz;r} \Delta^{l_0}_{hz;r} \right],
$$

(1.36)

where $T_j = \tilde{w}^{e}_{cz} \sum^{U}_{c=1} \sum^{N}_{o=1} \Delta^{l_0}_{cu o} \left( \tilde{T}_{cu o} \right)^{1/\rho} d^{R}_{cz o} \left( \Delta^{u_{z o}}_{cz o} \Delta^{l_0}_{cz o} \right)^{1 - 1/\rho}$ is the government subsidy written in changes.

I use this equation in combination with the change in price indexes to obtain the *ex-ante* expected real wage changes from reducing trade barriers across different types of workers. Figure (1.8) summarizes these changes for both the optimal and factual cases.

The graph shows that the gains from trade are more unequally distributed under optimal
distortions. The variance of the distribution of real income within educational attainment under factual distortions is 0.2 for high school, 0.1 for college and 0.04 for university, while under optimal it is 17.4 for high school, 46.4 for college and 73.5 for university. This effect is mainly driven by the increase in effective supply of managers. This reduces the real wage of workers who started the period in that occupation independently of their educational attainment. Production workers, on the other hand, benefit from the change, since the labour supply to that occupation is greatly reduced with optimal distortions relative to factual. In terms of across education inequality, the effect is similar, though smaller (the variance increases from 0.03 with factual to 0.1 with optimal distortions). It is also worth mentioning that workers with university degree are the ones that benefit the most from the change in mobility frictions. The main reason is that the difference between exogenous productivity growth in management relative to other occupations is highest for university graduates (see figure (1.9)).

1.4 Conclusion

In this paper I develop and quantitatively analyze a general model of international trade and labour market distortions. The novelty of the paper is that it studies domestic distortions in a model that combines Ricardian-HO forces with sectoral production misallocation due to mo-
nopolistic competition. I show that within this framework, domestic labour mobility distortions could be welfare improving if they increase production in high value-added sectors. I then apply this result to study the impact of occupation-to-occupation mobility distortions on the gains from trade. In the quantitative exercise, I showed that a planner is able to more than double the gains from trade in the US between 1992 and 2007 by using labour market distortions as a policy instrument. This is achieved by expanding employment in sectors such as equipment manufacturing and reducing it in sectors such as textiles relative to the case of no government intervention. A negative consequence of this policy is that although the aggregate gains from trade are greater, they are more unequally distributed both within and across workers with different educational levels.

The reason the gains from trade in the home country increase under government policy intervention is the result of two forces. First, home is able to capture a higher share of high value-added jobs away from foreign countries. This benefits the home market at the expense of foreign markets. Second, there is a global increase the production of high value-added sectors, which benefits all countries. An avenue for further research is to use this framework to separate the share of the gains due to beggar-thy-neighbour from the gains that jointly maximize welfare for all countries. This type of study will likely be concentrated among a few countries since it requires a detailed panel of workers from all the countries in the model.
1.5 Appendix

1.5.1 Derivations

Derivation of Equations (1.24) and (1.25)

Let $\Delta_{u_z, z_n}(\theta^\mu_{z_n}) = \text{Prob}_u \left( d_{z_i, z_n} w^e_{z_n} \theta^\mu_{z_n} \geq \text{Max} \left( d_{z_i, z_o} w^e_{z_o} \theta^\mu_{z_o} \right)_{i=1}^N \right)$ be the marginal probability of the draw $\theta_{z_n}$ in occupation $z_n$ having the highest return for a worker with educational attainment $u$. Since draws are independent across workers, I can then write the share of workers with education $u$ who switch from occupation $z_v$ to occupation $z_n$ ($z_v \rightarrow z_n$) as

$$ \Delta_{u_z, z_n} = \int_0^\infty \Delta_{u_z, z_n}(\theta^\mu_{z_n}) \ d\theta_{z_n}, \text{ where:} $$

$$ \Delta_{u_z, z_n} = \exp \left\{ - \sum_{r=1}^N T^{z_r}_{u_z} \left( \frac{d_{z_v, z_n} w^e_{z_n} \theta^\mu_{z_n}}{d_{z_v, z_r} w^e_{z_r} \theta^\mu_{z_r}} \right)^{1-\rho} \right\} \left( \sum_{r=1}^N T^{z_r}_{u_z} \left( \frac{d_{z_v, z_n} w^e_{z_n} \theta^\mu_{z_n}}{d_{z_v, z_r} w^e_{z_r} \theta^\mu_{z_r}} \right)^{1-\rho} \right)^{-\rho} $$

$$ \frac{\xi}{\mu} (1-\rho) \ T^{z_n}_{u_z} \ (\theta^\mu_{z_n})^{-\frac{\xi}{(1-\rho)\mu}-1} d\theta_{z_n}. \tag{1.37} $$

Simplifying equation (1.37), I obtain equation (1.25)

To derive equation (1.24) I substitute the previous results into the conditional pdf of $v \rightarrow n$ switchers with education $u$:

$$ f_u(\theta^\mu_{z_n} | v \rightarrow n) = \frac{\Delta_{u_z, z_n}(\theta^\mu_{z_n})}{\Delta_{u_z, z_n}}. $$

Using equations (1.37) and (1.25), I obtain:

$$ f_u(\theta^\mu_{z_n} | v \rightarrow n) = \exp \left\{ - \left[ \frac{\Delta_{u_z, z_n}}{T^{z_n}_{u_z}} \right]^{(1-\rho)\mu} \right\} \left[ \theta^\mu_{z_n} \left( \frac{\Delta_{u_z, z_n}}{T^{z_n}_{u_z}} \right)^{(1-\rho)\mu} \right]^{-\frac{\xi}{\mu}} \frac{\xi}{\mu} \left( \frac{\Delta_{u_z, z_n}}{T^{z_n}_{u_z}} \right)^{(1-\rho)\mu} . \tag{1.38} $$

From equation (1.38) we can see that the distribution of abilities in destination $z_n$ conditional
on source $z_v$ for workers with education $u$ is also given by the Fréchet distribution:

$$
\hat{\theta}_{z_n|v \rightarrow n}^u \sim F_u \left( \frac{T_{u z_n}}{X_{u z_v z_n}} \left( \frac{1-\rho \mu}{\xi} \right), \frac{\xi}{\mu} \right), \quad \forall n = 1, \ldots, N.
$$

(1.39)

Therefore, the expected matched productivity conditional on switching from $z_v$ is:

$$
E_u[\psi(z_n, \theta)|v \rightarrow n] = \left( \frac{T_{u z_n}}{X_{u z_v z_n}} \right)^{(1-\rho)\mu} \frac{f_{z_v z_n}}{z_n} \Gamma \left( 1 - \frac{\mu}{\xi} \right).
$$

(1.40)

Weighting this average match productivity by the measure of switchers from each occupation with the same educational attainment and adding over all possible sources and education groups, I obtain equation (1.24).

### 1.5.2 Proofs

**Proof of Lemma (1)**

This proof has three parts. First, I prove conditions 3 and 4. The second part uses these conditions to prove conditions 1 and 2. The last part uses these two results to show that $\theta^* \in (\underline{\theta}, \bar{\theta})$ is unique.

**Part I:** I prove conditions 3 and 4 by contradiction. Suppose that $\exists \theta_1$ and $\theta_2 \in \Theta_{z_n}$ for either $n = 1, 2$ such that $\frac{d(z_n, \theta_1) w(\theta_1)}{\psi(z_n, \theta_1)} > \frac{d(z_n, \theta_2) w(\theta_2)}{\psi(z_n, \theta_2)}$. Then, there exists $m, r \in \bigcup_{x=1}^2 \Omega_z$ such that $l(z_n, m, \theta_1) > 0$ and $l(z_n, r, \theta_2) > 0$. Firms’ cost minimization yield:

$$
\lambda_{z_n} (l(m)) = \frac{d(z_n, \theta_1) w(\theta_1)}{\psi(z_n, \theta_1)} > \frac{d(z_n, \theta_2) w(\theta_2)}{\psi(z_n, \theta_2)} = \lambda_{z_n} (l(r)),
$$

$$
\Rightarrow \quad \lambda_{z_n} (l(m)) \frac{\psi(z_n, \theta_2)}{d(z_n, \theta_2)} > \lambda_{z_n} (l(r)) \frac{\psi(z_n, \theta_2)}{d(z_n, \theta_2)},
$$

$$
\Rightarrow \quad \lambda_{z_n} (l(m)) \frac{\psi(z_n, \theta_2)}{d(z_n, \theta_2)} > w(\theta_2).
$$
Chapter 1. Labour Market Policy and the Gains from Trade

This is a contradiction to cost minimization. A similar contradiction can be derived if we assume \( \frac{d(z_n, \theta_1)w(\theta_1)}{\psi(z_n, \theta_1)} < \frac{d(z_n, \theta_2)w(\theta_2)}{\psi(z_n, \theta_2)} \). Therefore, conditions 3 and 4 hold. A result of this is that effective shadow values equalize across firms in both sectors: \( \lambda_{z_n}(l(\varphi)) = \lambda_{z_n} \forall \varphi \in \bigcup_{s=1}^{2} \Omega_s \).

Part II: I show conditions 1 and 2 throughout the following steps

1. If \( \theta \in \Theta \), then \( \theta \in \bigcup_{n=1}^{2} \Theta_{z_n} \)

   **Proof:** I prove this by contradiction. Suppose \( \exists \theta \in \Theta \) and \( \theta \notin \bigcup_{n=1}^{2} \Theta_{z_n} \), then by the definition of \( \Theta, \Theta_{z_1} \) and \( \Theta_{z_2} \)

   \[
   f(\theta) > \sum_{n=1}^{2} \sum_{s=1}^{2} \int_{A_s} l(z_n, \varphi, \theta) dG_s(\varphi).
   \]

   This is a contradiction to equation (1.8) (labour markets clear at each skill level).

2. Let \( \theta_1, \theta_2 \in \Theta \) such that \( \theta_1 < \theta_2 \). If \( \theta_2 \in \Theta_{z_1} \), then \( \theta_1 \notin \Theta_{z_2} \)

   **Proof:** I prove this by contradiction. Suppose that \( \theta_2 \in \Theta_{z_1} \) and \( \theta_1 \in \Theta_{z_2} \) with \( \theta_1 < \theta_2 \), then by firms’ cost minimization problem

   \[
   \lambda_{z_1} \frac{\psi(z_1, \theta_2)}{d(z_1, \theta_2)} \geq \lambda_{z_2} \frac{\psi(z_2, \theta_2)}{d(z_2, \theta_2)},
   \]

   and

   \[
   \lambda_{z_1} \frac{\psi(z_1, \theta_1)}{d(z_1, \theta_1)} \leq \lambda_{z_2} \frac{\psi(z_2, \theta_1)}{d(z_2, \theta_1)},
   \]

   \[\Rightarrow\]

   \[
   \frac{d(z_1, \theta_1)\psi(z_1, \theta_2)}{d(z_1, \theta_2)\psi(z_1, \theta_1)} \geq \frac{d(z_2, \theta_1)\psi(z_2, \theta_2)}{d(z_2, \theta_2)\psi(z_2, \theta_1)}.
   \]

   By equation (1.6) and the fact that tax/subsidy only applies to switchers

   \[\Rightarrow\]

   \[
   \frac{\psi(z_1, \theta_2)}{\psi(z_1, \theta_1)} \geq \frac{\psi(z_2, \theta_2)}{\psi(z_2, \theta_1)}.
   \]

   This is a contradiction to assumption (1). Similarly, if \( \theta_1 \in \Theta_{z_2} \), then \( \theta_2 \notin \Theta_{z_1} \).
3. By conditions 1 and 2,

- if $\theta_2 \in \Theta_1$ and $\theta_1 < \theta_2$ with $\theta_1, \theta_2 \in \Theta$, then $\theta_1 \in \Theta_1$ (sufficient condition).
- if $\theta_1 \in \Theta_2$ and $\theta_1 < \theta_2$ with $\theta_1, \theta_2 \in \Theta$, then $\theta_2 \in \Theta_2$ (sufficient condition).

4. Since $\Theta_{zn} \subseteq \Theta \subseteq \mathbb{R}_+$ for $n = 1, 2$ and $\Theta$ is compact, both $\inf(\Theta_{zn})$, $\sup(\Theta_{zn}) \in \Theta$.

Define $\theta_{z_1}^* = \sup(\Theta_{z_1})$ and $\theta_{z_2}^* = \inf(\Theta_{z_2})$. Here, I show that $\theta_{z_1}^* = \theta_{z_2}^* = \theta^*$. I study two separate cases

i) Let $\theta_{z_2}^* < \theta_{z_1}^*$. Then, $\exists \theta_1, \theta_2$ such that $\theta_1 \in \Theta_{z_2}$ and $\theta_2 \in \Theta_{z_1}$ and $\theta_1 < \theta_2$ which contradicts 3.

ii) Let $\theta_{z_1}^* < \theta_{z_2}^*$. Then, $\exists \theta \in \Theta$ such that $\theta \notin \bigcup_{n=1}^{2} \Theta_{zn}$ which is a contradiction to 1.

By conditions 3 and 4 we can see that $\theta^*$ is unique. Moreover, I can show the following necessary condition:

$$\theta \in \Theta_{z_1} \implies \theta \leq \theta_{z_1}^*, \quad \theta \in \Theta_{z_2} \implies \theta \geq \theta_{z_2}^*.$$

**Part III:** Given Part II, a sufficient condition for $\theta^* \in (\underline{\theta}, \overline{\theta})$ is that $\Theta_{zn} \neq \emptyset$ for $n = 1, 2$. I show this by contradiction. Without loss of generality, suppose that $\Theta_{z_1} = \emptyset$, by Part II, condition 1, $\Theta_{z_2} \neq \emptyset$ (the same proof holds for the opposite case). Then,

- from firms’ cost minimization, $\forall \theta \in \Theta_{z_2}$, $w(\theta) = 0$,
- for a firm $\varphi \in \bigcup_{s=1}^{2} \Omega_s$ with $l(z_2, \varphi, \theta) > 0$:

$$\lambda_{z_1}(l(\varphi)) \frac{\psi(z_1, \theta)}{d(z_1, \theta)} = \infty > w(\theta).$$

This is a contradiction to cost minimization.
Proof of Lemma (2)

Part I: Here I prove that $S(\theta)$ is strictly increasing, differentiable and continuous at $\theta < \theta^*$ and at $\theta > \theta^*$. Let $\theta_1, \theta_2 \in \Theta_{z_n}$ with $\theta_2 = \theta_1 + \epsilon$ for $n = 1, 2$ and $\epsilon > 0$. Then,

$$S(\theta_1) = \lambda z_n(I(m))\psi(z_n, \theta_1) \quad \text{for some } m \in \bigcup_{s=1}^2 \Omega_s,$$

$$S(\theta_2) = \lambda z_n(I(r))\psi(z_n, \theta_2) \quad \text{for some } r \in \bigcup_{s=1}^2 \Omega_s.$$

Subtracting $S(\theta_1)$ from $S(\theta_2)$ and using the results of lemma (1)

$$S(\theta_2) - S(\theta_1) = \lambda z_n [\psi(z_n, \theta_2) - \psi(z_n, \theta_1)] > 0 \quad \text{(by assumption (1))}. \quad (1.41)$$

Therefore, the wage function is increasing within each occupation. I divide both sides of equation (1.41) by $\epsilon$ and take the limit $\epsilon \to 0$ to obtain:

$$\frac{dS(\theta)}{d\theta} \bigg|_{\theta=\theta_1} = \lambda z_n \psi'_\theta(z_n, \theta_1). \quad (1.42)$$

Since $S(\theta)$ is differentiable, it is also continuous within occupations.

Part II: Here I show that $\frac{1}{\theta - \theta^*} \lim \theta \to \theta^* S(\theta) = \lim \theta \to \theta^* S(\theta)$. Since frictions are symmetrically opposites, I can assume without loss of generality that $\theta^* < \theta^b$ where $\theta^b$ is the benchmark cut-off from which the government subsidies are defined. I prove the lemma by contradiction.

- Assume that $\frac{1}{\theta - \theta^*} \lim \theta \to \theta^* S(\theta) > \lim \theta \to \theta^* S(\theta) \iff \frac{1}{\theta - \theta^*} \lim \theta \to \theta^* \lambda z_1 \psi(z_1, \theta) > \lim \theta \to \theta^* \lambda z_2 \psi(z_2, \theta)$. By continuity of the match worker-occupation productivity (assumption (1)), I can write this as follows:

$$\frac{1}{\theta} \lambda z_1 \psi(z_1, \theta^*) > \lambda z_2 \psi(z_2, \theta^*),$$

By cost minimization:

$$\frac{1}{\theta} \lambda z_1 \psi(z_1, \theta^*) > d(z_2, \theta^*) w(\theta^*),$$
By $\theta^* < \theta^b$  

$$\lambda_{z_1} \psi(z_1, \theta^*) > d(z_1, \theta^*) w(\theta^*).$$

$$(d(z_1, \theta^*) = 1, d(z_2, \theta^*) = \frac{1}{d})$$

This is a contradiction to the cost minimization problem.

• Similarly I can derive the opposite contradiction ($\lambda_{z_2} \psi(z_2, \theta^*) > d(z_2, \theta^*) w(\theta^*)$) by assuming: $\frac{1}{d} \lim_{\theta \to \theta^*} S(\theta) > \lim_{\theta \to \theta^*} S(\theta)$.

The same steps can be followed to show that a similar contradiction holds for the case where $\theta^* \geq \theta^b$.

**Proof of proposition (1)**

Using the results from lemmata (1) and (2)

$$\frac{1}{d} w_{z_1} \psi(z_1, \theta^*) = w_{z_2} \psi(z_2, \theta^*).$$  \hspace{1cm} (1.43)

Reorganizing terms, I obtain equation (1.9). It is worth noting that the continuity of the sorting rule arises because occupation mobility costs/subsidies are symmetrically opposites.

**Proof of lemma (3)**

In this model, labour is the only input of production so I can write the value of total sectoral demand as follows

$$D_{e cs}^e = X_{e cs} - \Pi_{e cs}.$$  \hspace{1cm} (1.44)

I can also write the average source-destination-sector profits as follows

$$\Pi_{e cs} = \frac{X_{e js}^e}{\sigma_s} - a_{cs} F_{e js}.$$  \hspace{1cm} (1.45)
Given the demand system, the value of the demand for two varieties within a source-destination-sector is given exogenously. So, I can express average revenue as follows

$$\bar{X}_{cjs} = \int_{\varphi \geq \varphi_{cjs}} \left[ \frac{\varphi}{\varphi_{cjs}} \right]^{(\sigma_s-1)} X_{cjs}(\varphi_{cjs}) \frac{1}{1-G(\varphi_{cjs})} dG(\varphi), \quad X_{cjs}(\varphi_{cjs}) = \sigma_s a_{cs} f_{cjs}. \quad (1.46)$$

The first term represents a measure of the ‘distance’ of the value of demand of firm $\varphi$ relative to the marginal firm. The second term is the value of demand of the marginal firm which I obtain from equation (1.14). Solving this equation I obtain.

$$\bar{X}_{cjs} = a_{cs} F_{cjs} \frac{\sigma_s \alpha_s}{\alpha_s - (\sigma_s - 1)}. \quad (1.47)$$

Substituting this into equation (1.45)

$$\bar{\Pi}_{cjs} = \frac{\sigma_s - 1}{\alpha_s - (\sigma_s - 1)} a_{cs} F_{cjs}. \quad (1.48)$$

Multiplying both equations by the endogenous measure of source-destination-sector entrants $([1 - G(\varphi_{cjs}^*)] M_{cs}^e)$ and substituting them into equation (1.44) I obtain equation (1.18).

**Proof of proposition (2)**

I combine the first order conditions for firm’s $\varphi$ cost minimization problem (equation (1.7)) to obtain:

$$\frac{L_{z1}^e(\varphi)}{L_{z2}^e(\varphi)} = \frac{w_{z1}^e}{w_{z2}^e}, \quad L_{z1}^e(\varphi) = \int_{\theta \in \Theta} \psi(z_n, \theta) l(\varphi, z_n, \theta) d\theta \text{ for } n = 1, 2. \quad (1.49)$$

Where $L_{z1}^e(\varphi)$ represents the total demand for effective labour in occupation $z_n$ by firm $\varphi$. I then substitute this into the production function to obtain the firm’s value of effective demand for each occupation:

$$w_{z1}^e L_{z1}^e(\varphi) = \beta_{z1} \frac{Q(\varphi)}{\varphi} a_s, \quad w_{z2}^e L_{z2}^e(\varphi) = \beta_{z2} \frac{Q(\varphi)}{\varphi} a_s. \quad (1.50)$$
Since the total cost of labour inputs for this firm is the sum of these equations
\[ D_s^e(\varphi) = w_{z_1}^e L_{z_1}^e(\varphi) + w_{z_2}^e L_{z_2}^e(\varphi) = \frac{Q(\varphi)}{\varphi} a_s, \]
I can substitute this expression back into these equations and combine them with equation (1.10) to obtain equation (1.19).

**Proof of proposition (3)**

The first part of the proof shows that I can express the EV of the home country as a function of the changes in the proportion of income spent in home varieties and the changes in the price of those varieties. This part of the proof is similar to the proof in Arkolakis et al. (2009). In the second part, I solve for percentage changes in total income in the home country.

By the envelope theorem, I can write changes in price index as follows (because I am calculating the EV, I use prices at \( t_0 \)).

\[
dln(P_h) = \sum_{s=1}^{2} \gamma_s \left( \sum_c \Lambda_{chs} a_s dln(P_{chs}) \right) dln(P_{hs}). \tag{1.51}
\]

From the demand system I can write relative changes in the home market penetration as follows:

\[
\frac{1}{1 - \sigma_s} \left[ dln(\Lambda_{hhs}) - dln(\Lambda_{fhs}) \right] = dln(P_{hhs}) - dln(P_{fhs}). \tag{1.52}
\]

We know that: \( \Lambda_{hhs}^0 + \Lambda_{fhs}^0 = \Lambda_{hhs} + \Lambda_{fhs} = 1 \Rightarrow dln(\Lambda_{fhs}) = -\frac{\Lambda_{hhs}^0}{\Lambda_{fhs}^0} dln(\Lambda_{hhs}) \) (this only holds for small changes - i.e linearizing around \( t_0 \) variables). Substituting this into the previous equation, we obtain.

\[
\Lambda_{fhs}^0 dln(P_{fhs}) = \Lambda_{fhs}^0 dln(P_{ccs}) + \frac{1}{\sigma_s - 1} dln(\Lambda_{hhs}). \tag{1.53}
\]
I substitute this equation into equation (1.51) and obtain:

\[ d\ln(P_{hs}) = d\ln(P_{hhs}) + \frac{1}{\sigma_s - 1} d\ln(\Lambda_{hhs}). \] (1.54)

This linearization works for small changes. For large changes we need to integrate over the small changes to obtain an approximation of the welfare changes in the new equilibrium.

\[ \int_{P_{hhs}^0}^{P_{hhs}'} d\ln(P_{hs}) = \int_{\Lambda_{hhs}^0}^{\Lambda_{hhs}'} d\ln(\Lambda_{hhs}). \] (1.55)

Solving for this:

\[ \ln(\hat{P}_{hs}) = \frac{1}{\sigma_s - 1} \ln(\hat{\Lambda}_{hhs}) + \ln(\hat{P}_{hhs}). \] (1.56)

I rewrite the numerator of equation (1.21) as follows

\[ \ln(\hat{Y}_h) = \ln(\hat{Y}_h) + \sum_{s=1}^{2} \ln(\hat{\Lambda}_{hhs}) - \sum_{s=1}^{2} \ln(\hat{\Lambda}_{hhs}), \] (1.57)

\[ \Rightarrow \ln(\hat{Y}_h) = \sum_{s=1}^{2} \ln(\hat{X}_{hhs}) - \sum_{s=1}^{2} \ln(\hat{\Lambda}_{hhs}). \]

I define the value of occupational labour employed in producing for the domestic market in sector as:

\[ w_{e1}^{z1} L_{hhsz1} = \beta_{z1} \eta_s X_{hhs}, \quad w_{e2}^{z2} L_{hhsz2} = \beta_{z2} \eta_s X_{hhs}. \] (1.58)

Using these equations, I rewrite equation (1.57) as:

\[ \frac{w_{e1}^{z1} L_{hhsz1} + w_{e2}^{z2} L_{hhsz2}}{Y_h} = \eta_s \Lambda_{hhs}. \] (1.59)

Lastly, I need to show that changes in country’s \( h \) total income are proportional to changes in the value of labour. This is always true in models with free entry. Here, it might not hold
because there are positive profits besides labour income. However, the combination of Pareto distributed firms’ productivity and Cobb-Douglas utility and production functions results in profits being proportional to labour income. The outcome of this is that all changes in total income are driven by labour income.

By Pareto and CES:

\[
\Pi^w = \sum_{s=1}^{2} (1 - \eta_s) \sum_{i=h,f} \sum_{j=h,f} X_{ij},
\]

(1.60)

By Cobb-Douglas utility function:

\[
X^w_s = \gamma_s (Y_h + Y_f),
\]

\[
\Rightarrow \Pi^w = (Y_h + Y_f) \sum_{s=1}^{2} (1 - \eta_s) \gamma_s.
\]

(1.61)

From the production function:

\[
L^e_{czn} w^e_{czn} = \sum_{s=1}^{2} \eta_s \beta_{szn} \sum_{j=h,f} X_{cj},
\]

(1.62)

By equation (1.61) I can rewrite equation (1.62) as:

\[
\sum_{j=h,f} \sum_{n=1}^{2} L^e_{czn} w^e_{czn} = (Y_h + Y_f) \sum_{n=1}^{2} \sum_{s=1}^{2} \eta_s \beta_{szn} \gamma_s.
\]

(1.63)

So, \( \frac{\Pi^w}{\sum_{c=h,f,n=1}^{2} L^e_{czn} w^e_{czn}} = \frac{\sum_{s=1}^{2} (1-\eta_s) \gamma_s}{\sum_{n=1}^{2} \eta_s \beta_{szn} \gamma_s} \) which is the profit sharing condition. The result is that

\[
\%\Delta(Y_h) = \%\Delta(\sum_{n=1}^{2} w^e_{hn} L^e_{hn}).
\]

I use this and equation (1.56) to obtain equation (1.22).
**Proof of proposition (4)**

I can rewrite small $\Delta$s in welfare for the closed economy as follows:

$$
\frac{d\ln(W_h)}{\sigma_s} = \sum_{s=1}^{2} \gamma_s \omega_s \frac{d\ln(Y_h)}{\sigma_h}. 
$$

(1.64)

By equation (1.63):

$$
\frac{d\ln(W_h)}{\sigma_s} = \sum_{s=1}^{2} \gamma_s \omega_s \ln \left[ \left( \frac{w_{h1}^e}{w_{h2}^e} \right)^{\beta_{sz2}} L_{h1}^e \left( w_{h2}^e \right)^{\beta_{sz1}} L_{h2}^e \right].
$$

(1.65)

In a closed economy equilibrium, the following is a direct implication of lemma (3) and proposition (2):

$$
\frac{w_{h1}^e L_{h1}^e}{w_{h2}^e L_{h2}^e} = M \quad \text{with} \quad M = \eta \frac{\beta_{1z1}}{\beta_{1z2}} \gamma_1 + \frac{\beta_{2z1}}{\beta_{2z2}} \gamma_2.
$$

(1.66)

Substituting equation (1.66) into (1.65) and rearranging terms:

$$
\frac{d\ln(W_h)}{\sigma_s} = \sum_{s=1}^{2} \gamma_s \omega_s \ln \left[ \left( \frac{L_{h2}^e}{L_{h1}^e} \right)^{\beta_{sz2}} \left( L_{h1}^e \right)^{\beta_{sz1}} \right].
$$

(1.67)

In an equilibrium with $d_h = 1$, it must be the case that $\frac{\partial L_{h1}^e}{\partial d_h} = -w_{h1}^e \frac{\partial L_{h2}^e}{\partial d_h}$. I substitute this into the previous equation and solve:

$$
\frac{d\ln(W_h)}{\sigma_s} = \frac{\partial L_{h1}^e}{\partial d_h} \left[ \left( \gamma_1 \omega_1 \beta_{1z1} + \gamma_2 \omega_2 \beta_{2z1} \right) - \left( \gamma_1 \omega_1 \beta_{1z2} + \gamma_2 \omega_2 \beta_{2z2} \right) \frac{\beta_{1z1}}{\beta_{1z2}} \gamma_1 + \frac{\beta_{2z1}}{\beta_{2z2}} \gamma_2 \right].
$$

(1.68)

Noting that $\frac{\partial L_{h1}^e}{\partial d_h} < 0$ and using the assumptions of proposition (4) I obtain the desired results.

**Proof of proposition (5)**

- Derivation of equation (1.26)

By assumption at $t_0 d_{v,n} \equiv 1 \forall v, n = 1, \ldots, N$. Substituting this into equation (1.24), I
obtain:

\[
\Delta_{u_{z_n}}^{t_0} = \frac{\left(w_{z_n}^{e_{t_0}}\right)^{\frac{\xi}{1-\rho \mu}} T_{u_{z_n}}^{t_0}}{\sum_{r=1}^{N} \left(w_{z_r}^{e_{t_0}}\right)^{\frac{\xi}{1-\rho \mu}} T_{u_{z_r}}^{t_0}}.
\] (1.69)

Since this equation is independent of the source, this is equal to the total share of workers in occupation \(z_n\), \(\Delta_{u_{z_n}}^{t_0}\). From equation (1.40), I obtain:

\[
E_{U}^{t_0}[\psi(z_n, \theta)] = \left(\frac{T_{u_{z_n}}^{t_0}}{\Delta_{u_{z_n}}^{t_0}}\right)^{\frac{(1-\rho \mu)}{\xi}} \Gamma \left(1 - \frac{\mu}{\xi}\right).
\] (1.70)

I multiply both sides of equation (1.24) and rewrite it as follows:

\[
J_{e_{z_n}} = \hat{w}_{z_n}^{e_{t_0}} \sum_{u=1}^{3} L_{u}^{t_0} \sum_{o=1}^{N} \left[\Delta_{u_{z_o}}^{t_0} w_{z_n}^{e_{t_0}} E_{U}^{t_0}[\psi(z_n, \theta)] \left(\hat{T}_{u_{z_n}}^{t_0}\right)^{\frac{(1-\rho \mu)}{\xi}} \left(\Delta_{u_{z_n}}^{t_0}\right)^{\frac{(1-\rho \mu)}{\xi}} a_{z_n}^{t_0} \left(D_{u_{z_o}}^{t_0}\right)^{1-\frac{(1-\rho \mu)}{\xi}}\right].
\]

Rearranging I obtain (1.26)

- **Derivation of equation (1.27):**
  I rewrite equation (1.69) as follows:

\[
T_{u_{z_n}}^{t_0} = \left(\frac{\Delta_{u_{z_n}}^{t_0}}{w_{z_n}^{e_{t_0}}\left(\frac{\xi}{1-\rho \mu}\right)}\right) \sum_{r=1}^{N} \left(w_{z_r}^{e_{t_0}}\right)^{\frac{\xi}{1-\rho \mu}} T_{u_{z_r}}^{t_0}.
\] (1.71)

I then substitute this into equation (1.25) to obtain (1.27)

- **Derivation of equation (1.29):**
  I rewrite equation (1.17) as follows:

\[
\Lambda_{c_j s} = \frac{R_{c_j s}^{t_0} \left(\hat{a}_{c_j s}\right)^{-\alpha_s + \alpha_s^{-1}} \left(\hat{\alpha}_{c_j s}\right)^{-1}}{\sum_{v=1}^{2} R_{v_j s}^{t_0} \left(\hat{a}_{v_j s}\right)^{-\alpha_s + \alpha_s^{-1}} \left(\hat{\alpha}_{v_j s}\right)^{-1}}.
\] (1.72)
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Where $R_{cjs}^{0} = M_{c}^{e} (A_{c})^{x_s} \left( a_{c}^{t_0} \right)^{-x_s} \left( t_{c}^{f} \right)^{-x_s} f_{cjs}$. Let $R_{j}^{0} = \sum_{c=1}^{2} R_{cjs}^{0}$. I then divide the numerator and denominator of equation (1.72) by $R_{j}^{0}$ to obtain equation (1.27).

Proof of proposition (6)

From equation (1.40) and the result of lemma (1), I can write the following equation:

$$E \cdot \left[ w(\theta) | z_{n} \right] = \hat{w}_{z_{v}}^{e} d \cdot \epsilon_{z_{v}} \left( T_{u}^{e} \right)^{\frac{(1-\rho)\mu}{\epsilon}} \left( \Delta_{u} \right) \Gamma \left( 1 - \frac{\mu}{\xi} \right). \quad (1.73)$$

Also, because at $t_0$, $\Delta_{u}z_{n} = \Delta_{u}z_{n}$, I can express the average occupational wages in the initial period as follows:

$$E_{u}^{0} \cdot \left[ w(\theta) | z_{n} \right] = \left( \frac{T_{u}^{0} \Delta_{u}^{0} \epsilon}{T_{u}^{0}} \right) \frac{(1-\rho)\mu}{\epsilon} \left( \hat{w}_{z_{v}}^{e} \right) \Gamma \left( 1 - \frac{\mu}{\xi} \right). \quad (1.74)$$

Where because of long-run equilibrium:

$$\Delta_{u}^{0} = \frac{(w^{e,t_0})^{\frac{(1-\rho)\mu}{\epsilon}}}{N} \sum_{r=1}^{N} \frac{(w^{e,t_0})^{\frac{\epsilon}{1-\rho-\mu}}}{T_{u}^{0}}. \quad (1.75)$$

I substitute equation (1.75) into (1.74) to obtain:

$$E_{u}^{0} \cdot \left[ w(\theta) | z_{n} \right] = \left( \frac{\sum_{r=1}^{N} \frac{(w^{e,t_0})^{\frac{\epsilon}{1-\rho-\mu}}}{T_{u}^{0}}}{\frac{(1-\rho)\mu}{\epsilon}} \right) \Gamma \left( 1 - \frac{\mu}{\xi} \right) = E_{u}^{0} \cdot \left[ w(\theta) | v \right]. \quad (1.76)$$

So, I can write

$$E_{u} \cdot \left[ w(\theta) | z_{n} \rightarrow z_{v} \right] = \frac{(\hat{w}^{e})_{z_{v}}^{e} d \cdot \epsilon_{z_{v}} \hat{\Delta}_{u}^{0}}{\frac{(1-\rho)\mu}{\epsilon}}. \quad (1.77)$$

To obtain equation (1.36), I combine this equation with assumption (1.15).
1.5.3 Calibration

Data

I use 1992 as the reference point and I analyze the changes in the model’s variables between 1992 and 2007. As indicated in the model, I consider seven sectors, seven occupations and three educational levels. To simplify the analysis, I aggregate 15 different countries for which reliable labour market data exists between 1992 – 2007 into foreign country. These countries are: Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Israel, Mexico, Netherlands, Spain and United Kingdom. The data that I employ to estimate the parameters of the model are from three different sources: Luxembourg Income Study (LIS), OECD STructural ANalysis (OECD – STAN) database and Panel Study of Income Dynamics (PSID). To make all data sources compatible, I first transform all PSID sectoral data into ISIC – rev3 and occupational data into ISCO – 08. 40 Tables (1.4) and (1.5) show the sectors and occupations I consider alongside their ISIC and ISCO codes.

I obtain occupational average hourly wages and share of manufacturing labour force by occupation for the 15 countries above-mentioned from the LIS. This database provides repeated cross-sections micro data from labour force censuses in those countries. I proxy 1992 and 2007 values with the closest population survey available (the median years are 1994 and 2007). For the starting year, I transform all average hourly wages into 1992 US dollars by first using the Penn World tables 6.1 (the latest available tables with a base year prior to the introduction of the Euro) to transform all local currency into US dollars. Then I deflate them using an annual inflation rate of 2%. I perform a similar analysis for 2007 values but I use the latest Penn World tables instead.

For the calibration, I need total income per occupation in 1992 and 2007 aggregated over all countries $(J_{fz_n})$. I obtain total manufacturing labour supply per country from OECD –

40To do this I need to first transform all census data pre-2001 into 2000 census categories first and then transform it into ISIC. For occupations, I found easier to directly transform all data into SOC categories for which a direct mapping exists to ISCO – 08. All data crosswalks as well as the STATA code are available from the author upon request.
and use this and the other variables obtained from the LIS to calculate these variables (I implicitly assume that all workers work full time, i.e. 40 hour weeks). I do not divide workers by educational level because all workers are perfect substitutes within occupations and, therefore, it is the total occupational labour supply to each occupation in the foreign country that matters for domestic welfare changes.

To estimate all the parameters necessary the for counterfactual analysis from the US perspective, I need a panel of occupational switchers. To avoid the extremely small sample size of the PSID, I proxy 1992 occupations by the first observed occupation of workers in the panel and 2007 occupations by the last observed occupation (I deflate/inflate wages so they are consistent). Table (1.6) shows the summary statistics. To obtain total income per occupation, I use this and OECD – STAN similarly as I use it for foreign countries.

The bilateral trade flows data I use is very standard. The only values from the OECD – STAN that are not ready to use in the calibration are countries own imports (diagonal elements in the export matrix). To do this I first transform total value domestic production for each country into 1992 and 2007 US dollars, correspondingly, and then I subtract total exports to other countries (already in same year US dollars).

<table>
<thead>
<tr>
<th>Industry and code</th>
<th>ISIC-rev 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food, beverages and tobacco</td>
<td>15,16</td>
</tr>
<tr>
<td>Textiles and leather</td>
<td>17-19</td>
</tr>
<tr>
<td>Wood, cork products, pulp and paper</td>
<td>20-22</td>
</tr>
<tr>
<td>Chemical, Rubber, plastics and fuel</td>
<td>23-25</td>
</tr>
<tr>
<td>Minerals and metallic products</td>
<td>26-28</td>
</tr>
<tr>
<td>Machinery and equipment</td>
<td>29-35</td>
</tr>
<tr>
<td>Other manufacturing and recycling</td>
<td>36,37</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Occupation and code</th>
<th>ISCO-08</th>
</tr>
</thead>
<tbody>
<tr>
<td>Managers (Mgt)</td>
<td>1</td>
</tr>
<tr>
<td>Professionals (Prof)</td>
<td>2</td>
</tr>
<tr>
<td>Technicians and Professional Services (Tech)</td>
<td>3</td>
</tr>
<tr>
<td>Clerical Support Workers (Cler)</td>
<td>4</td>
</tr>
<tr>
<td>Sales</td>
<td>5</td>
</tr>
<tr>
<td>Craft and Trades Workers (Trades)</td>
<td>7</td>
</tr>
<tr>
<td>Plant and Machine Operators, Assemblers and Elementary occupations (Prod)</td>
<td>8,9</td>
</tr>
<tr>
<td>Education attained</td>
<td>Number of workers</td>
</tr>
<tr>
<td>--------------------</td>
<td>-------------------</td>
</tr>
<tr>
<td>High School</td>
<td>1724</td>
</tr>
<tr>
<td>College</td>
<td>558</td>
</tr>
<tr>
<td>University</td>
<td>436</td>
</tr>
</tbody>
</table>
## Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Identifier</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Occupational intensity in sectoral reduction</td>
<td>( \mu_{k,n} )</td>
<td>see table (1.9)</td>
<td>Share of worker’s occupation income within sectors (PSID)</td>
</tr>
<tr>
<td>Share of expenditure in each sector</td>
<td>( \gamma_{k} )</td>
<td>see table (1.9)</td>
<td>Share of sectoral revenue within countries (OECD)</td>
</tr>
<tr>
<td>Change in iceberg trade costs</td>
<td>( \tau_{j} )</td>
<td>see table (1.9)</td>
<td>Relative revenue between ( h ) and ( f ) across countries within sectors (OECD)</td>
</tr>
<tr>
<td>Sectoral Demand elasticity</td>
<td>( \sigma_{s} )</td>
<td>see table (1.9)</td>
<td>Hsieh and Ossa (2015)</td>
</tr>
<tr>
<td>Sectoral Supply elasticity</td>
<td>( \alpha_{s} )</td>
<td>see table (1.9)</td>
<td>Hsieh and Ossa (2015)</td>
</tr>
</tbody>
</table>

| Relative exogenous occupational productivity growth College               | \( \hat{T}_{2,c,ap} \) | 0.56 | |
|                                                                           | \( \hat{T}_{2,c,p} \) | 1.95 | |
|                                                                           | \( \hat{T}_{2,c,t} \) | 1.23 | |
| Relative exogenous occupational productivity growth University            | \( \hat{T}_{2,c,cl} \) | 0.98 | Relative increase in nominal wage of stayers |
|                                                                           | \( \hat{T}_{2,c,s} \) | 2.34 | see figure (1.9) |
|                                                                           | \( \hat{T}_{2,c,m} \) | 0.87 |
|                                                                           | \( \hat{T}_{2,c,p} \) | 1.26 |

| Relative exogenous occupational productivity growth University            | \( \hat{T}_{2,c,ap} \) | 0.9 | |
|                                                                           | \( \hat{T}_{2,c,p} \) | 1.5 |
|                                                                           | \( \hat{T}_{2,c,t} \) | 0.72 |

| Relative exogenous occupational productivity growth University            | \( \hat{T}_{2,c,cl} \) | 0.84 | Relative increase in nominal wage of stayers |
|                                                                           | \( \hat{T}_{2,c,s} \) | 1.44 | see figure (1.9) |
|                                                                           | \( \hat{T}_{2,c,m} \) | 0.58 |
|                                                                           | \( \hat{T}_{2,c,p} \) | 0.6 |

Table 1.7: Parameters Set Without Solving for the Model’s GE
Chapter 1. Labour Market Policy and the Gains from Trade

### Home Labour Market:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Identifier</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability correlation across occupations</td>
<td>( \rho )</td>
<td>0.77</td>
</tr>
<tr>
<td>Worker-occupation comparative advantage</td>
<td>( \frac{\xi}{\mu} )</td>
<td>0.4</td>
</tr>
<tr>
<td>Iceberg occupational mobility distortions</td>
<td>( d^{*}_{z;z} )</td>
<td>See table (1.3)</td>
</tr>
</tbody>
</table>

\[
\hat{w}^{\ell}_{h \text{ mgt}} = 1.86 \\
\hat{w}^{\ell}_{h \text{ prof}} = 1.52 \\
\Delta \text{ in occupational effective wages} \\
\hat{w}^{\ell}_{h \text{ tech}} = 2.13 \\
\hat{w}^{\ell}_{h \text{ cler}} = 4.68 \\
\hat{w}^{\ell}_{h \text{ sales}} = 3.37 \\
\hat{w}^{\ell}_{h \text{ trades}} = 2.29 \\
\hat{w}^{\ell}_{h \text{ prod}} = 1.49
\]

### Foreign Labour Market:

\[
\hat{L}^{\ell}_{f \text{ mgt}} = 1.65 \\
\hat{L}^{\ell}_{f \text{ prof}} = 1.82 \\
\Delta \text{ effective occupational labour supply} \\
\hat{L}^{\ell}_{f \text{ cler}} = 0.04 \\
\hat{L}^{\ell}_{f \text{ sales}} = 0.12 \\
\hat{L}^{\ell}_{f \text{ trades}} = 0.48 \\
\hat{L}^{\ell}_{f \text{ prod}} = 0.91
\]

#### Table 1.8: Calibrated Parameters

<table>
<thead>
<tr>
<th>Industry/Occupation</th>
<th>( \alpha )</th>
<th>( \alpha^ \prime )</th>
<th>( \gamma_{h} )</th>
<th>( \gamma_{f} )</th>
<th>( \delta )</th>
<th>Managers</th>
<th>Professionals</th>
<th>Technicians</th>
<th>Clerical</th>
<th>Sales</th>
<th>Trades</th>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food, beverages and tobacco (1)</td>
<td>3.3</td>
<td>6.1</td>
<td>0.13</td>
<td>0.15</td>
<td>0.95</td>
<td>0.16</td>
<td>0.07</td>
<td>0.18</td>
<td>0.06</td>
<td>0.13</td>
<td>0.38</td>
<td>0.64</td>
</tr>
<tr>
<td>Textiles and leather (2)</td>
<td>6.1</td>
<td>9.5</td>
<td>0.02</td>
<td>0.03</td>
<td>0.97</td>
<td>0.06</td>
<td>0.11</td>
<td>0.07</td>
<td>0.02</td>
<td>0.01</td>
<td>0.09</td>
<td>0.64</td>
</tr>
<tr>
<td>Wood, cork products, pulp and paper (3)</td>
<td>10.35</td>
<td>23.5</td>
<td>0.14</td>
<td>0.09</td>
<td>0.99</td>
<td>0.16</td>
<td>0.11</td>
<td>0.12</td>
<td>0.09</td>
<td>0.04</td>
<td>0.1</td>
<td>0.37</td>
</tr>
<tr>
<td>Chemical, Rubber, plastics and fuel (4)</td>
<td>8.07</td>
<td>19.13</td>
<td>0.27</td>
<td>0.21</td>
<td>0.98</td>
<td>0.13</td>
<td>0.2</td>
<td>0.22</td>
<td>0.05</td>
<td>0.06</td>
<td>0.1</td>
<td>0.26</td>
</tr>
<tr>
<td>Minerals and metallic products (5)</td>
<td>3.3</td>
<td>5.8</td>
<td>0.13</td>
<td>0.17</td>
<td>0.95</td>
<td>0.17</td>
<td>0.05</td>
<td>0.13</td>
<td>0.04</td>
<td>0.02</td>
<td>0.16</td>
<td>0.43</td>
</tr>
<tr>
<td>Machinery and equipment (6)</td>
<td>6.3</td>
<td>15.6</td>
<td>0.27</td>
<td>0.32</td>
<td>0.98</td>
<td>0.16</td>
<td>0.17</td>
<td>0.18</td>
<td>0.06</td>
<td>0.04</td>
<td>0.19</td>
<td>0.26</td>
</tr>
<tr>
<td>Other manufacturing and recycling (7)</td>
<td>3.1</td>
<td>3</td>
<td>0.04</td>
<td>0.03</td>
<td>0.91</td>
<td>0.43</td>
<td>0.06</td>
<td>0.14</td>
<td>0.04</td>
<td>0.06</td>
<td>0.1</td>
<td>0.18</td>
</tr>
</tbody>
</table>

#### Table 1.9: Sectoral Parameters/Occupation Intensity in sectoral production
Figure 1.9: Nominal Wage Growth of Occupational Stayers (Intensive Margin)
Figure 1.10: Distribution of mobility costs ($\frac{1}{d_{CS, LNS}}$)
Chapter 2

Multi-Product Firms, Credit Constraints and the Gains From Trade

2.1 Introduction

The degree to which international trade changes aggregate welfare in a country largely depends on the characteristics of the domestic economy. For instance, Fajgelbaum (2013), Itskhoki and Helpman (2015) as well as Kambourov (2009) show that in economies with large frictions to factor mobility the welfare gains from liberalization are lower than in frictionless environments.¹ Frictions to internal factor mobility reduce the ability of countries to take full advantage of comparative advantage and firm selection forces, thereby constraining the gains from liberalization. Undesirable domestic market conditions are also able to amplify the gains from an increase in international trade. An example in the literature is the pro-competitive effects that trade-induced competition has in markets where domestic firms have high levels of market power.² A largely unexplored channel that also amplifies the gains from trade is when

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¹For a short review of the literature on labour mobility frictions and trade see Dix-Carneiro (2012).
²Examples of papers featuring pro-competitive effects are Krugman (1979) and Melitz and Ottaviano (2008). As a side note, Arkolakis et al. (2015) argue that for a large class of models featuring pro-competitive effects, the welfare gains from trade are smaller than in models without these effects. The main object of their study is an *ex-post* comparison of the welfare gains from trade liberalization under different assumptions based on observed changes in trade flows. To reconcile observed changes in trade flows with different models, they needed to allow
liberalization is able to reduce misallocation in investment in new product varieties.

I show evidence that trade liberalization decreases investment misallocation in India by reducing the level of credit constraints that high-productivity firms experience. In particular, I exploit a rich cross-sectional Indian plant-level dataset to show the following: First, I show that, independently of both unobserved and observed heterogeneity, exporters produce more products than non-exporters. Moreover, I show that this difference is higher in sectors where external financing is more prevalent. Second, I show that the mechanism through which exporters are able to increase the scope of production is through higher retained earnings. Higher earnings allow firms to bypass credit constraints by increasing the share of investment that is self-financed. This suggests that a decrease in trade barriers has the additional effect of increasing the allocation of investment in new product varieties from less-productive non-exporters to more-productive exporters.\(^3\)

The recent macroeconomic literature has highlighted the role of credit constraints as barriers to firm growth (see, for instance, Moll (2014)). Studying the interaction of trade with financial constraints to the growth of firms at the extensive margin is important for two reasons. First, increasing the scope of production requires large upfront fixed costs, which are difficult to self-finance. Therefore, the effects of credit constraints should be larger at the extensive margin (adding more products) than at the intensive margin (expanding production of existing products).

Second, Hsieh et al. (2016) show that when the calculation of the welfare gains from trade is adjusted to include the loss of domestic varieties, the gains are significantly reduced.\(^4\) In the case that trade liberalization leads to a reallocation of production at the extensive margin, meaning that more productive firms increase their scope of production, the reduction of domestic varieties is based on comparing the welfare implications of models with and without pro-competitive effects for a given underlying trade shock. Therefore, since models with pro-competitive effects allow for more channels through which a country can benefit from trade, these gains must be larger (see, for instance, Dhingra and Morrow (2012)).

\(^3\)This mechanism requires a plant attribute that is common across products as in Bernard et al. (2011) and Mayer et al. (2014), or within plant correlation of productivity draws across products.

\(^4\)A feature of the heterogeneous firms trade literature is that even though consumers gain access to more varieties, the number of domestic varieties decreases following trade liberalization.
mestic varieties would be smaller. The reallocation from low-productivity to high-productivity firms would be higher and would entail increasing production at the extensive margin. The result would be that, even though there is exit, the average number of varieties of surviving firms would be higher. In this environment with credit constraints, the effect must be an increase in varieties from the no-credit constraint counterfactual, because more productive firms are the ones producing more varieties. Growth of production at the intensive margin must, therefore, be lower.

To provide evidence that trade liberalization reduces credit misallocation, I use a cross section of plants for the 2008-2009 fiscal year collected by the Indian government through its Annual Survey of Industries (ASI). This data set contains detailed plant-level information such as the number of products firms produce, revenue per product, location at the district level, input information, detailed financial data, and share of exports (in revenue terms). To instrument for exporting, I exploit the unique geography of India, where 95% of trade by volume and 70% by value is done through maritime transportation.\(^5\) Using district level data, I interact a measure of transport costs at the firm level with distance to the nearest 10 major exporting ports (handling 91% of the value of maritime trade) to create unique composite product-district instruments.\(^6\) I find that exporting increases the share of revenue that arises from the extensive margin of production by 6.7 times.

I then use a similar method to Rajan and Zingales (1998) to show that most of this difference is due to differences in credit constraints. This method estimates differences between exporters and non-exporters across sectors with different external financing needs. I find that 75% of the differences in revenue from production at the extensive margin are accounted for by comparing plants in industries located in the 75th to plants located in industries at the 25th percentiles of external financing needs.

In the second part of the analysis, I investigate the channel through which exporters are

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\(^6\)For multi-product firms, transport costs are revenue-weighted averages of transport costs at the HS 2007 four-digit product level.
able to increase investment. In particular, I study whether exporters’ collateral constraints are lower or exporters are able to bypass borrowing constraints through higher retained earnings.\textsuperscript{7}

I find that a 1% increase in a firm’s total profit leads to a 64% increase in the probability of being a multi-product firm. Also, once I include profits in the estimating equation, exporting does not statistically significantly change the probability of being a multi-product firm. This suggests that the difference between exporters and non-exporters at the extensive margin arises from differences in the level of retained earnings only.

Recently, the core competency trade literature has shown that, in response to trade liberalization, firms allocate a larger share of their inputs to their best product (see for instance Bernard et al. (2011), Mayer et al. (2014) as well as Eckel and Neary (2010)). This mechanism is a within-firm Melitz (2003) type selection mechanism, in which firms increase their productivity by concentrating production in their most productive variety, and is independent of credit constraints. For a detailed description of this mechanism see Bernard et al. (2011) and Baldwin and Gu (2009). To the contrary, the reallocation described here leads to stronger across-firm resource reallocations, and may or may not be accompanied by within-firm production reallocations. The key mechanism through which my analysis differs from the core competency models is that mine depends on firms not achieving the optimal scope of production due to credit constraints. The reduced form estimates that I present in this paper include both channels. In addition, I show evidence that exporting results in both a stronger reallocation across firms and a higher level of skewness in production within firms across products.

The previous literature on trade and credit constraints studies the effects of financial constraints on trade flows. Manova (2008) and Manova (2012), for instance, study the effects of credit constraints on the sectoral level of trade flows, and find that credit constraints reduce trade in all sectors but more so in sectors with high external dependence. Leibovici (2015), and Caggese and Cuñat (2013) extend the analysis by studying the effect of financial constraints

\textsuperscript{7}Collateral constraints could be lower for exporters because they are able to increase their access to external sources of credit, or because there is less asymmetric information between exporters and lenders (exporting sends a signal that a firm is relatively high productivity).
on the relative trade flows across capital- and labour-intensive sectors as well as the effects on aggregate welfare. In contrast to these papers, I study the effect of trade on credit constraints and the implications for firms’ production at the extensive margin.

This paper is organized as follows: The next section discusses the cross-sectional data I use in the analyses. Section 3 studies the effect of trade liberalization on firms’ production decisions. In particular, it analyses the mechanism through which trade liberalization affects firms’ scope of production.

### 2.2 Data and descriptive statistics

I use a survey of Indian plant-level data for the 2008-2009 fiscal year collected by the government. All factories with 100 or more workers are included in the sample. In addition, a random sample of 20% of the total number of the remaining firms within a state is drawn for each sector. There are a total of 23,006 plants and 43,768 product-plant observations in the sample. From all these plants, 41.64% are multi-product and only 13.3% are exporters. For the analysis, I use data on plants’ location (at the district level), industry (ISIC Rev.4 four-digit level), product (HS 2007 four-digit level), revenue per product (in Rupees), total input costs, total employment and financial information (working capital and total liabilities).

Figure (2.1) shows the proportion of exporting and non-exporting plants by number of products produced. I define a product as a distinct HS 2007 four-digit item that plants produce.\(^8\) The graph shows that around 60% of non-exporters are single-product plants as opposed to 50% of exporters. There are slightly more double-product exporters than non-exporters. As the number of varieties produced increases, the difference between the percentage of exporters relative to the percentage of non-exporters increases. The category for ten products refers to ten or more products (hence, the bunching of plants in the graph).

Table (2.1) expands the analysis of the average exporter relative to the average non-exporter.

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\(^8\)I ignore by-products and combine products with the same HS code into one product.
Each row repeats a regression of the indicated variable on an exporter dummy (equal to 1 if the firm exports) and on industry fixed effects. The table shows that exporters are on average 1.005 log points bigger than non-exporters in terms of total labour, and produce 1.437 log points more revenue. About two thirds of the larger revenue stream is due to higher per-product revenue and a third to more products being produced. In addition, exporters are more likely to produce multiple products and produce on average about 16% more products. In terms of productivity, exporters on average have around 19% more TFP and 57% more revenue per worker.

### 2.3 Effect of exporting on scope

In this section I study the effect of exporting on the scope of production. The first part of the section studies both the causal relationship between exporting and investment in new product varieties, and the role of credit constraints in explaining this relationship. The second part of

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To calculate the margins of revenue I use three different dependent variables. I use $\ln(\sum_{p=1}^{P_i} Y_{pi})$ for the total effect, $\ln(Y_{pi})$ for the intensive margin and $\ln(\sum_{p=1}^{P_i} Y_{pi}) - E[\ln(Y_{pi})|\mathbf{X}]$ for the extensive margin. $Y$ refers to revenue, $p$ refers to product, $i$ to firm and $P_i$ to the total number of products of firm $i$. Also, the expected value used in calculating the extensive margin comes from the intensive-margin regression. It is important to note that the extensive margin is different than the number of products firms produce. The extensive margin measures the contribution of the difference in the number of products to the difference in total plants’ revenue. It is helpful to rewrite the extensive margin as follows:

$$E\left[\ln\left(\sum_{p=1}^{P_i} Y_{pi}\right) - E[\ln(Y_{pi})|\mathbf{X}]\right] = \sum_{p=1}^{10} \text{Prob}(P_i = p|\mathbf{X}) \ln(p) + \sum_{p=1}^{10} \text{Prob}(P_i = p|\mathbf{X}) \left[\ln(E[Y_{pi}|\mathbf{X}, P_i = p]) - E[\ln(Y_{pi})|\mathbf{X}]\right].$$

For a given number of varieties, if the size of each variety in revenue terms is correlated (lower dispersion), then the extensive margin plays a smaller role in total revenue differences. The reason for this is that log revenue differences arise from the differences in revenue in each product produced more than from the number of products produced in this case.

I follow Bernard et al. (2011) in using the multi-factor superlative index of Caves et al. (1982) to calculate $TFP$ (as in Bernard et al. (2011) this is not without problems, as I am measuring $TFP$ of multi-product firms using aggregate inputs at the firm level). I use the following three factors: labour, capital, and material inputs. Unlike in Bernard et al. (2011), I do not need sectoral price deflators because I use a cross-section of firms. Therefore, I use the total value of inputs used at the plant level.
the section studies the mechanism through which exporting affects firms’ credit constraints. In both parts I use an instrumental variable approach aimed at controlling for unobserved firm heterogeneity.

### 2.3.1 Exports and growth at the extensive margin

To study the effect of exporting on firms’ production decisions, I use three different specifications. The most general specification uses revenue at the product-firm level as the dependent variable:

\[
y_{pi} = \alpha_0 + \alpha_1 D_i + X_i \beta + Z_i \gamma + \delta_i + \epsilon_{pi}. \quad (2.1)
\]

In equation (2.1), \(y_{pi}\) is the log value of revenue of plant \(i\) in product \(p\), \(D_i\) is a dummy variable that indicates whether a plant exports, \(X_i\) is a vector of plant characteristics, \(Z_i\) is a vector of domestic market characteristics of the state in which the plant operates (which can potentially be correlated with both exporting and firms’ revenue), and \(\delta_i\) is a vector of industry fixed effects. Since exporting varies at the firm level, \(\alpha_1\) captures the average differences in log revenue for products produced by an exporter relative to products produced by a non-exporter unconditional on whether the product is exported. Therefore, I refer to this as the intensive margin of plant production.

The second specification I use is as follows:

\[
y_i = \alpha_0 + \alpha_1 D_i + X_i \beta + Z_i \gamma + \delta_i + \epsilon_i. \quad (2.2)
\]

This specification is similar to equation (2.1) but I use within firm aggregate log revenue as the dependent variable. That is, I work at the firm level rather than at the firm-product level. This equation captures the total effect of exporting, as it includes both the intensive and extensive margins.

Lastly, to calculate the extensive margin, I use the difference between the total observed
log revenue and the expected log revenue at the product level as the dependent variable \((y_i - E[y_{pi}|\text{covariates}])\). As in equation (2.2) I work at the firm level.

These three specifications measure average revenue differences across exporters and non-exporters, given the covariates as well as the contribution of the extensive and intensive margin of production to those differences. As discussed in detailed in footnote (8), the extensive margin of production is related but not equivalent to the number of products a firm produces. The extensive margin also includes the difference in production skewness conditional on the number of products between exporters and non-exporters. If exporting leads to a high concentration of production in exporters’ best varieties, as in the core competency literature, the extensive margin would also increase. Given that the differences at the extensive margin are fairly large, these are likely to come from both an increase in the number of products and an increase in production skewness.\(^{11}\) To separately estimate these two effects, due to the large number of single-product firms, I would need to use an IV-Poisson model which is estimated using non-linear GMM and has convergence problems in the presence of large number of fixed effects.\(^{12}\) Therefore, for this paper, I am unable to differentiate between these two margins.

To instrument for exporting, I interact distance to the nearest major exporting port in India (handling at least 2% of the total value of maritime exports) with an index of transport costs at the firm level. The index of transport costs at the firm level is the average transport cost of the products a firm produces weighted by revenue shares. For transport costs, I use OECD data on the world’s average ratio of CIF (cost of insurance and freight) to FOB (free on board), which approximately reflects the difference between importing price (which includes shipping costs, such as shipping fees and insurance, as well as import taxes) and exporting price. These data

\(^{11}\)In terms of the core competency literature, I am able to identify a different response of firms to trade liberalization from the non-credit constrained response in Bernard et al. (2011) but not from the response in Mayer et al. (2014). The difference is that in Bernard et al. (2011) the skewness of production arises from firms dropping their worst products only. To the contrary, in Mayer et al. (2014) the production of firms’ best varieties relative to the production of their other varieties increases even when they are produced both before, and after trade liberalization. While the first effect would decrease my measure of extensive margin, the second effect, as it is the case with an increase in the number of products, would increase it.

\(^{12}\)I partially address this problem in the next section, where I estimate the channel through which firms become less financially constrained after exporting.
are at the HS 2007 four-digit product level. This is the best approximation to intra-national transport costs disaggregated at the product level that I am able to obtain.\footnote{For a general discussion and information about these data, see Miao and Fortanier (2017).}

Figure (2.4) shows a map of India with all 614 districts and the major exporting ports (the latter represented by red dots). The size of the dots indicates the share of the value of all maritime exports a port handles annually.\footnote{The largest port in the graph is a combination of the Mumbai and the Jawaharlal Nehru Port, which are located within the same district.} The different shades of blue represent the percentage of all firms within a district that are exporters, with darker blue meaning a larger share of all firms are exporters. Except for a few places where there are special economic zones (most notably around the Delhi area), there is a correlation between the share of firms exporting and the proximity to exporting ports.

Table (2.2) shows the estimation results. Note that by construction each coefficient in the total column is the sum of the corresponding coefficients in the intensive and extensive margin columns. After using transport cost instruments to control for heterogeneity (see discussion below), I find that exporters have 6.7 ($exp(1.909)$) times as much revenue as non-exporters arising from the extensive margin of production.

Another important result that arises from the table is the direction of the OLS bias. Without accounting for the bias, the intensive margin roughly accounts for two thirds of the average revenue differences between exporters and non-exporters, with the remaining one third being accounted for by the extensive margin. However, when I instrument for exporting, the intensive margin becomes negative (though not statistically significant) and the extensive margin becomes larger. This is consistent with a bias arising from differences in the variance of productivity draws between exporters and non-exporters. After accounting for the within-plant conditional average labour productivity, exporters are firms whose productivity draws have a high variance and on average produce very large and very low draws.\footnote{Using either TFP or labour productivity in the estimation equation is a potential source of bias for the other estimates, given the potential mechanical relationship between them and revenue. I find that the multi-collinearity problem between covariates and labour productivity is lower so I choose to include it to control for firm observable characteristics.} With the very large
Chapter 2. Multi-Product Firms, Credit Constraints and the Gains From Trade
draws, firms are able to penetrate foreign markets (positive correlation between variance and exporting), and with the very low draws, firms are not even able to produce for the domestic market (negative correlation between variance and the extensive margin). Also, because firms drop products for which productivity draws are very low, the conditional average size of the varieties being produced is higher for high variance firms (positive correlation between variance and intensive margin). Once I control for differences in unobserved plant level variance, I find that exporting leads to an increase in the number of varieties and a decrease in the average size of these varieties in revenue terms.

The first stage F-test, shown in table (2.2), is large, which indicates that the instrument is highly correlated with exporting. Table (2.3) shows the first stage regression, which reveals that, as expected, total cost of transport to exporting ports is negatively correlated with exporting. The first stage also reveals that there is some correlation between labour productivity and the instrument. This suggests that the exclusion restriction might not be entirely satisfied. As discussed above, the instrumental variable approach in this case does not suffer from a weak instrument problem, so the question in this case is whether there is something to learn from using this instrument. I show in figure (2.3) that the instrument is less correlated with labour productivity (a key firm observable) than the decision of firms to export. These are partial correlation plots with the y-axis variable being the exporting dummy (left panel) or the instrument (right panel), the x-axis being labour productivity and the other controls being the same as in the previous specifications. This shows that the instrument is less correlated than the endogenous variable and, therefore, there is something to learn about the direction of the bias. A large number of firms in India use primary products as inputs into their production processes. Spatially locating near the production of these inputs is potentially more cost effective for exporters than locating near exporting ports, as these inputs are often expensive items to transport, which explains the lower correlation.

It is important to understand the limitations of my instrumental analysis in extrapolating the quantitative results to trade liberalization periods. Within an HS 2007 four-digit product code,
the distance to the nearest port of export is similar to the instrument used in Card (1993) to instrument for college enrolment decisions. Therefore, the instrument I use has a very similar LATE interpretation, in that it only captures the impact of exporting around a Melitz (2003) exporting cut-off, which increases with distance. Since there is not a lot of variation in distance, the coefficient only applies to a small subset of firms and not to firms that are either very productive or very unproductive in producing their best variety.  

2.3.2 Exports and credit constraints

To test whether credit constraints play a major role in the response of firms to exporting, I use the method in Rajan and Zingales (1998). This approach requires one to rank sectors according to firms’ external financing needs. In the context of my analysis I use the following specification:

\[ y_{pi} = \alpha_0 + \alpha_1 D_i + \alpha_2 (D_i E_i) + X_i \beta + Z_i \gamma + \text{IND}_i \delta + \epsilon_{pi}, \]  

(2.3)

In equation (2.3) \( E_i \) refers to the external financial needs of the sector, \( \text{IND}_i \) a set of sector characteristics, and all other variables are the same as in equation (2.1). Using the financial information of firms in the ASI, I measure external dependence as the ratio between liabilities and working capital for the median firm in each four-digit ISIC Rev.4 sector. Figure (2.4) shows the number of products produced by the average exporter relative to the average non-exporter across sectors with different external financing needs. On average, the graph shows that the higher the external dependence, the higher the difference between exporters and non-exporters in terms of the scope of production.

Table (2.4) shows the results of the estimation of equation (2.3). Unlike in other studies that use this method to estimate the effects of credit constraints on firm growth (most notably...

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16This interpretation is only an approximation. A LATE analysis when there are other covariates besides the instrumented endogenous variable is more complicated (see Lileeva and Trefler (2010) for a careful explanation of this point).
Manova (2008), Braun (2003) and Rajan and Zingales (1998)), the treatment in this case is endogenous, so, again, I use firm-level transport costs to instrument for exporting. The direction of the endogeneity of OLS is uncertain, as it is an outcome of two sources. First, the gains from exporting are bigger in sectors with higher financial constraints. This lowers the exporting productivity cut-offs in those sectors, which positively bias $\alpha_2$ in the regressions where the dependent variable is the extensive margin. The reason for this is that the lower the exporting cut-off, the lower the average unobserved variance of exporters relative to non-exporters. This means that the effect of unobserved differences in variance described above is lower with larger external financing needs, which biases the coefficient on the interaction term downward. The bias is the opposite for the intensive margin regression.

Second, as argued by Manova (2008) and Manova (2012), exporting itself requires a large amount of credit, and even more so in sectors where external financing is more important. According to this mechanism, the Melitz (2003) exporting cut-offs are higher in sectors with larger external financing needs, and the coefficient in this case is biased in the opposite direction. Comparing the results of the OLS and 2SLS, the second mechanism dominates.

The last column of table (2.4) shows that the revenue arising from production at the extensive margin is around five ($\exp(1.607)$) times larger for exporters relative to non-exporters, when comparing firms in the 75th percentile sector relative to firms in the 25th percentile sector in terms of external financial needs. This is 75% of the total difference between exporters and non-exporters calculated in the last column of table (2.2). This suggests that the response of firms to exporting in terms of scope of production is due to a decrease in financial constraints. In the following subsection, I explore whether the effect of exporting on firms’ financial constraints is a consequence of exporters being able to obtain more credit per unit of collateral or the increase in retained earnings arising from higher profits.

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17Here I use the levels to instrument for exporting and the interaction between transport costs and the external financing needs of sectors to instrument for the interaction term.

18To calculate this number, I use the coefficient in the first row of table (2.4) times the difference between the observation at the 75th and 25th percentile of the interaction term.
2.3.3 Mechanism

In this subsection I use two different methods to obtain estimates of the mechanism through which trade reduces firms’ credit constraints. First, I use the following linear probability model:

\[ MD_i = \alpha_0 + \alpha_1 D_i + \alpha_2 \text{Profits}_i + X_i \beta + Z_i \gamma + \delta_i + \epsilon_i, \]  

(2.4)

where \( MD_i \) = 1 if firm \( i \) is a multi-product and \( MD_i = 0 \) otherwise. I use two different specification of equation (2.4). In the first specification, \( \text{Profits}_i \) refers to the total value of firm \( i \)'s profit, and in the second it refers to the profit of its largest product in terms of the value of sales. To instrument for profit, I use the changes in the value of world imports from 2002 to 2008 at the product level (HS 2007 four-digit level) from OECD-STAN. As before, I need an instrument at the plant level, so in the case of multi-product firms I use the average of the changes in the value of global demand from all products a plant produces weighted by revenue shares.

This specification suffers from the same problems as the linear count model in terms of out-of-bounds predicted values due to the large presence of single-product firms. To solve this, I also use the following bivariate probit model:

\[ MD_i = \mathbb{1}[\alpha_1 D_i + \text{Cov}_i \alpha_2 > \epsilon_i] \quad , \quad D_i = \mathbb{1}[\beta_1 T_i + \text{Cov}_i \beta_2 > \eta_i]. \]  

(2.5)

In equation (2.5), \( \text{Cov}_i = [\text{Profits}_i, X_i, Z_i, \delta_i] \) and \( T_i \) is the firm level transport cost discussed in the previous section. I assume that the error terms follow a bivariate normal distribution \( (\epsilon, \eta \sim \mathcal{N}_b(0, \sigma_{\epsilon, \eta})) \) and I estimate the coefficients of the latent functions using Maximum Likelihood with a seemingly unrelated regression (SUR) error structure. This approach solves the negative prediction problem of the linear probability model but it does not allow me to instrument for profits.

Tables (2.5) and (2.7) show the results of both methods. After instrumenting for export-
ing, the effects of exports on the probability of being a multi-product firm become statistically insignificant in all specifications. This is regardless of whether plants’ profits are also instrumented (table 2.5) or not (table 2.7). Also, in all specifications except for the one in $ols7$ and $iv7$ (linear probability model with total profit), the average marginal effect of exporting becomes larger when instrumented, which is consistent with the explanation in the previous section. Both total profits and profits of the best performing variety are positive and statistically significant in the bivariate probit model but only total profits is significant after instrumenting in the linear probability model. This could arise from the large variance of the 2sls coefficients or from a weak instrument problem since the results of the F-test for the first stage are particularly small when I instrument for profits of firms’ best varieties (second last row of table 2.5).

2.4 Conclusion

In this paper I provided evidence that exporting is associated with a significant increase in revenue arising from the extensive margin of within-plant production. In addition, applying a similar method to Rajan and Zingales (1998), I showed that a large part of this response of plants to exporting is due to financial constraints affecting investment in new product varieties. The results of this paper suggests that it is optimal for productive plants to produce a larger amount of varieties, but such plants are unable to do so because of credit constraints. Exporting leads to an increase in the amount of retained earnings, which, as in Moll (2014), allows a plant to bypass financial constraints by increasing the level of its collateral.

In future research, I plan to estimate the share of extensive margin production differences between exporters and non-exporters due to the number of varieties produced only. This will help differentiate the response of firms to globalization in markets with frictions from the response in frictionless markets as discussed by the core competency literature. Also, I plan to add a framework to quantitatively assess the welfare effect of the impact of trade liberalization on the efficient allocation of investment across plants.
2.4.1 Figures

Figure 2.1: Firms’ Scope

Number of Products: Exporters vs Non Exporters

![Graph showing the number of products: Exporters vs Non Exporters.](image-url)
Figure 2.2: Share of Exporters by District
Figure 2.3: Correlation between firm level observables and distance to ports
Chapter 2. Multi-Product Firms, Credit Constraints and the Gains From Trade

Figure 2.4: Difference in scope exporters vs. non-exporters across sectors

2.4.2 Tables

Table 2.1: Average plant characteristics

<table>
<thead>
<tr>
<th></th>
<th>Exporter vs Non-exporter</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \ln ) employment</td>
<td>1.005</td>
<td>38.568</td>
</tr>
<tr>
<td>( \ln ) revenue total</td>
<td>1.437</td>
<td>38.788</td>
</tr>
<tr>
<td>Intensive margin</td>
<td>1.045</td>
<td>31.930</td>
</tr>
<tr>
<td>Extensive margin</td>
<td>0.392</td>
<td>10.592</td>
</tr>
<tr>
<td>( \ln ) labour productivity</td>
<td>0.433</td>
<td>18.859</td>
</tr>
<tr>
<td>( \ln ) TFP</td>
<td>0.172</td>
<td>13.133</td>
</tr>
<tr>
<td>( \ln ) num. products</td>
<td>0.141</td>
<td>13.767</td>
</tr>
<tr>
<td>Prob. multiproduct</td>
<td>0.108</td>
<td>13.065</td>
</tr>
<tr>
<td>Obs.</td>
<td>28124</td>
<td></td>
</tr>
</tbody>
</table>

Note: This table reports differences in mean across group of plants controlling for industry fixed effects. The dependent variable is indicated in the leftmost column while the independent variables are industry dummies at the 4-digit ISIC Rev.4 level and a dummy variable indicating whether the firm is an exporter. Enough information to calculate \( \ln \) TFP exists for 23006 firms only.
Table 2.2: Impact of exporting on revenue

<table>
<thead>
<tr>
<th></th>
<th>Total</th>
<th>Intensive</th>
<th>Extensive</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>2SLS</td>
<td>OLS</td>
</tr>
<tr>
<td>Export Dummy</td>
<td>0.958***</td>
<td>1.627***</td>
<td>0.609***</td>
</tr>
<tr>
<td>Ln Age</td>
<td>0.086***</td>
<td>0.087***</td>
<td>0.041***</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>1.163***</td>
<td>1.143***</td>
<td>1.043***</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>-0.011</td>
<td>-0.008</td>
<td>-0.258***</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>0.013***</td>
<td>0.019***</td>
<td>-0.008</td>
</tr>
<tr>
<td>Industry FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>A-P F-test (First Stage)</td>
<td>25.964</td>
<td>47.113</td>
<td>25.964</td>
</tr>
<tr>
<td>Obs.</td>
<td>26349</td>
<td>26349</td>
<td>50371</td>
</tr>
</tbody>
</table>

Notes: OLS std. err. clustered at the firm level. 2SLS std. err. clustered at the composite product-district level. P-value < 0.01 : ***; P-value < 0.05 : **; P-value < 0.1 : *
Table 2.3: Impact of exporting on revenue: First Stage

<table>
<thead>
<tr>
<th></th>
<th>Firm</th>
<th>Product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln distance iv</td>
<td>-0.015***</td>
<td>-0.018***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Ln Age</td>
<td>-0.003**</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>0.029***</td>
<td>0.033***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>-0.014</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>-0.007***</td>
<td>-0.008***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Industry FE</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>26349</td>
<td>50371</td>
</tr>
</tbody>
</table>

Notes: OLS std. err. clustered at the composite product-district level. P-value< 0.01 : *** P-value< 0.05 : ** P-value< 0.1 : *
Table 2.4: Impact of exporting across sectors with different dependence on external financing (No ISIC FE)

<table>
<thead>
<tr>
<th></th>
<th>OLS Total</th>
<th>2SLS Total</th>
<th>OLS Intensive</th>
<th>2SLS Intensive</th>
<th>OLS Extensive</th>
<th>2SLS Extensive</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export Dummy x External Dependence</td>
<td>0.137</td>
<td>3.512</td>
<td>0.081</td>
<td>-0.743</td>
<td>0.056</td>
<td>4.255*</td>
</tr>
<tr>
<td></td>
<td>(0.086)</td>
<td>(2.292)</td>
<td>(0.138)</td>
<td>(1.647)</td>
<td>(0.086)</td>
<td>(2.292)</td>
</tr>
<tr>
<td>Export Dummy</td>
<td>1.056***</td>
<td>-0.511</td>
<td>0.759***</td>
<td>1.577</td>
<td>0.296***</td>
<td>-2.088</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(1.723)</td>
<td>(0.068)</td>
<td>(1.028)</td>
<td>(0.041)</td>
<td>(1.723)</td>
</tr>
<tr>
<td>Ln Age</td>
<td>0.102***</td>
<td>0.101***</td>
<td>0.050***</td>
<td>0.050***</td>
<td>0.052***</td>
<td>0.051***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.007)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>1.126***</td>
<td>1.152***</td>
<td>1.037***</td>
<td>1.021***</td>
<td>0.089***</td>
<td>0.131***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.032)</td>
<td>(0.011)</td>
<td>(0.022)</td>
<td>(0.007)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>External Dependence</td>
<td>0.406***</td>
<td>0.118</td>
<td>0.348***</td>
<td>0.418**</td>
<td>0.058**</td>
<td>-0.299*</td>
</tr>
<tr>
<td></td>
<td>(0.029)</td>
<td>(0.168)</td>
<td>(0.044)</td>
<td>(0.191)</td>
<td>(0.029)</td>
<td>(0.168)</td>
</tr>
<tr>
<td>Ln sector avg firm scope</td>
<td>-0.265***</td>
<td>-0.165*</td>
<td>-1.605***</td>
<td>-1.592***</td>
<td>1.340***</td>
<td>1.426***</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.099)</td>
<td>(0.052)</td>
<td>(0.093)</td>
<td>(0.036)</td>
<td>(0.099)</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>0.003</td>
<td>-0.001</td>
<td>-0.233***</td>
<td>-0.245***</td>
<td>0.237***</td>
<td>0.244***</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.039)</td>
<td>(0.038)</td>
<td>(0.048)</td>
<td>(0.023)</td>
<td>(0.039)</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>0.026***</td>
<td>0.018</td>
<td>-0.005</td>
<td>0.003</td>
<td>0.031***</td>
<td>0.015</td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td>(0.017)</td>
<td>(0.006)</td>
<td>(0.011)</td>
<td>(0.004)</td>
<td>(0.017)</td>
</tr>
<tr>
<td>Industry FE</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Diff-in-diff 75th-25th</td>
<td>0.052</td>
<td>1.327</td>
<td>0.030</td>
<td>-0.273</td>
<td>0.021</td>
<td>1.607</td>
</tr>
<tr>
<td>A-P F-test (interaction)</td>
<td>27.342</td>
<td>46.204</td>
<td>3.971</td>
<td>17.066</td>
<td>3.971</td>
<td></td>
</tr>
<tr>
<td>A-P F-test (export dummy)</td>
<td>3.971</td>
<td>17.066</td>
<td>3.971</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>26349</td>
<td>26349</td>
<td>50371</td>
<td>50371</td>
<td>26349</td>
<td>26349</td>
</tr>
</tbody>
</table>

Notes: OLS std. err. clustered at the firm level. 2SLS std. err. clustered at the product-district level
P-value< 0.01 : *** P-value< 0.05 : ** P-value< 0.1 : *
Chapter 2. Multi-Product Firms, Credit Constraints and the Gains From Trade

Table 2.5: Linear Probability Model

<table>
<thead>
<tr>
<th></th>
<th>ols</th>
<th>2sls</th>
<th>ols</th>
<th>2sls</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln Profit</td>
<td>0.030***</td>
<td>0.641**</td>
<td>0.013***</td>
<td>0.159</td>
</tr>
<tr>
<td>Ln Max Profit</td>
<td>(0.002)</td>
<td>(0.256)</td>
<td>(0.002)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>Export Dummy</td>
<td>0.067***</td>
<td>-1.503</td>
<td>0.084***</td>
<td>0.086</td>
</tr>
<tr>
<td>Ln Age</td>
<td>0.018***</td>
<td>-0.024</td>
<td>0.019***</td>
<td>0.010</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>0.008*</td>
<td>-0.767**</td>
<td>0.030***</td>
<td>-0.165</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>0.045***</td>
<td>0.056*</td>
<td>0.045***</td>
<td>0.052***</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>0.002</td>
<td>-0.039*</td>
<td>0.003**</td>
<td>-0.003</td>
</tr>
<tr>
<td>Industry FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>A-P F-test (export dummy)</td>
<td>8.937</td>
<td>5.489</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A-P F-test (ln profit)</td>
<td>6.120</td>
<td>3.095</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>15391</td>
<td>15191</td>
<td>15392</td>
<td>15163</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is a dummy that equals 1 if the firm produces more than one product. OLS std. err. clustered at the firm level. 2SLS std. err. clustered at the composite product-district level. P-value < 0.01 : *** , P-value < 0.05 : ** , P-value < 0.1 : *
Table 2.6: First Stage Linear Probability Model

<table>
<thead>
<tr>
<th></th>
<th>Total profit</th>
<th>Max Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Export ols20</td>
<td>Profit ols21</td>
</tr>
<tr>
<td>Ln Comp. Prod. W-IMP</td>
<td>0.001</td>
<td>0.040***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.015)</td>
</tr>
<tr>
<td>Ln Max. Prod. W-IMP</td>
<td>-0.021***</td>
<td>-0.126***</td>
</tr>
<tr>
<td></td>
<td>(0.003)</td>
<td>(0.023)</td>
</tr>
<tr>
<td>Ln Min Distance</td>
<td>-0.004***</td>
<td>0.073***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.012)</td>
</tr>
<tr>
<td>Ln Age</td>
<td>0.030***</td>
<td>1.358***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>-0.009</td>
<td>-0.127**</td>
</tr>
<tr>
<td></td>
<td>(0.013)</td>
<td>(0.062)</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>-0.004***</td>
<td>0.043***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Industry FE</td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>26048</td>
<td>15191</td>
</tr>
</tbody>
</table>

Notes: OLS std. err. clustered at the composite product-district level.
P-value < 0.01: ***; P-value < 0.05: **; P-value < 0.1: *
Table 2.7: Bivariate Probit

<table>
<thead>
<tr>
<th></th>
<th>Total Profit</th>
<th></th>
<th>Maximum Profit</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Probit</td>
<td>Biprobit</td>
<td>Probit</td>
<td>Biprobit</td>
</tr>
<tr>
<td></td>
<td>probit1</td>
<td>ame1</td>
<td>probit2</td>
<td>ame3</td>
</tr>
<tr>
<td>Export Dummy</td>
<td>0.184***</td>
<td>0.056***</td>
<td>0.237***</td>
<td>0.073***</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.011)</td>
<td>(0.035)</td>
<td>(0.011)</td>
</tr>
<tr>
<td>Ln Total Profit</td>
<td>0.090***</td>
<td>0.028***</td>
<td>0.038***</td>
<td>0.012***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.002)</td>
<td>(0.006)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln Max profit</td>
<td></td>
<td></td>
<td>0.038***</td>
<td>0.012***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.006)</td>
<td>(0.002)</td>
</tr>
<tr>
<td>Ln Age</td>
<td>0.060***</td>
<td>0.018***</td>
<td>0.064***</td>
<td>0.020***</td>
</tr>
<tr>
<td></td>
<td>(0.008)</td>
<td>(0.003)</td>
<td>(0.008)</td>
<td>(0.003)</td>
</tr>
<tr>
<td>Ln Labour Productivity</td>
<td>0.039***</td>
<td>0.012***</td>
<td>0.110***</td>
<td>0.034***</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.004)</td>
<td>(0.014)</td>
<td>(0.004)</td>
</tr>
<tr>
<td>Ln State GDP per-capita</td>
<td>0.158***</td>
<td>0.048***</td>
<td>0.157***</td>
<td>0.049**</td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.009)</td>
<td>(0.030)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>Rural/Urban population</td>
<td>0.007</td>
<td>0.002</td>
<td>0.010*</td>
<td>0.003</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.001)</td>
<td>(0.005)</td>
<td>(0.001)</td>
</tr>
<tr>
<td>Industry FE</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
<td>yes</td>
</tr>
</tbody>
</table>

Notes: The dependent variable is a dummy that equals 1 if the firm produces more than one product.

Probit std. err. clustered at the firm level. Biprobit std. err. clustered at the composite product-district level

P-value < 0.01 : ***, P-value < 0.05 : **, P-value < 0.1 : *
### Table 2.8: Bivariate Probit “First Stage”

<table>
<thead>
<tr>
<th></th>
<th>Total Profit</th>
<th>Maximum Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ln Min Distance</strong></td>
<td>-0.132***</td>
<td>-0.133***</td>
</tr>
<tr>
<td></td>
<td>(0.041)</td>
<td>(0.041)</td>
</tr>
<tr>
<td><strong>Ln Total Profit</strong></td>
<td>0.159***</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td></td>
</tr>
<tr>
<td><strong>Ln Max profit</strong></td>
<td></td>
<td>0.157***</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.009)</td>
</tr>
<tr>
<td><strong>Ln Age</strong></td>
<td>-0.014</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.014)</td>
</tr>
<tr>
<td><strong>Ln Labour Productivity</strong></td>
<td>0.022</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>(0.025)</td>
<td>(0.024)</td>
</tr>
<tr>
<td><strong>Ln State GDP per-capita</strong></td>
<td>-0.134</td>
<td>-0.134</td>
</tr>
<tr>
<td></td>
<td>(0.120)</td>
<td>(0.120)</td>
</tr>
<tr>
<td><strong>Rural/Urban population</strong></td>
<td>-0.090***</td>
<td>-0.089***</td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.020)</td>
</tr>
<tr>
<td><strong>Industry FE</strong></td>
<td>yes</td>
<td>yes</td>
</tr>
<tr>
<td><strong>rho</strong></td>
<td>-0.040</td>
<td>-0.036</td>
</tr>
<tr>
<td><strong>Obs.</strong></td>
<td>15391</td>
<td>15392</td>
</tr>
</tbody>
</table>

*Notes:* Probit std. err. clustered at the firm level. Biprobit std. err. clustered at the composite product-district level. P-value < 0.01 : ***, P-value < 0.05 : **, P-value < 0.1 : *
Chapter 3

Product Appeal, Information and Trade

3.1 Introduction

Models of trade with monopolistic competition, firm level heterogeneity and market entry costs predict that firms enter export markets according to a specific hierarchy. This hierarchy reflects that some markets are easier to enter than others. It is assumed in these models that firms within the same sector and country share a common hierarchy. The recent literature on international trade has shown that this country-sector specific hierarchy is not observed in the data.\(^1\) The hierarchy prediction fails when confronted with data in two dimensions. First, firms export to fewer destinations than theoretically predicted. Second, firms export to markets that in, theory, are difficult to enter but fail to export to markets that are easier to enter.

In this paper, I present a two-period model of firm heterogeneity that explains deviations from the hierarchy hypothesis. The premise of this model is that there are information asymmetries in international trade. Firms know the appeal of the product they are trying to place but consumers ignore it. Because firms export in many periods, consumers are able to learn through their own and other consumers’ experimentation. After consuming, agents receive a noisy signal, which they use to update their information about the appeal of the firm’s product.

\(^1\)Examples of this literature are: Eaton et al. (2008), Head et al. (2011), and Crozet et al. (2009)
The signal that consumers receive from experimentation is observed in all markets.

There are two important aspects of heterogeneous firm models that result in the country-sector specific hierarchy that this model relaxes. First, there is no interaction among a firm’s market entry decisions. Firm’s total profits are the sum of the profits in each export market plus the profits in the domestic market. Therefore, firm’s market entry decisions are made in isolation from entry decisions in other markets.\(^2\) In this model, the precision of the aggregate signal that consumers receive is proportional to the total population of the subset of markets a firm enters in the first period.

Second, firm and market heterogeneity are complementary in all matches, which leads to monotonicity. That is, the profitability of a firm-destination match is increasing in both the product appeal and the hierarchy of the market for all matches. The model in this paper introduces non-monotonicity: higher hierarchy markets are better for some firms but not for all. Firms strategically use their information advantage to choose patterns of entry in the first period that \textit{ex-ante} maximize their lifetime profits. Therefore, firms choose to enter markets in the first period to jointly maximize the instantaneous profit and level of experimentation.

Instantaneous profits in the first period before experimentation are the same for all firms. The reason for this is that consumers have no information to separate products with low appeal from products with high appeal. However, optimal experimentation is different for firms with different product appeals. Expected profits in the second period are supermodular in firms’ product appeals and aggregate level of experimentation. In equilibrium, there is an imperfect sorting of firms into markets, which is increasing in product appeal and aggregate size of subsets of markets. Firms with the highest product appeal enter all markets because they benefit from a more precise aggregate signal. On the other hand, firms with low product appeal only enter the lowest populated market. The reason for this is that they do not want to reveal the true appeal of their products with high precision. Also, there are firms with medium product appeal

\(^2\)In the Chaney (2008) version of the monopolistic competitive model (short run) there is no interaction among firm’s market entry decisions at all. On the other hand, in Melitz (2003) there is market entry interaction at the country level through the general equilibrium effect on wages. However, this effect is not large enough to generate the lack of entry observed in the trade data.
that enter subsets of markets that are somewhere between these two extremes. The sorting is imperfect because the particular value of the parameter could lead to an equilibrium in which no firm sells to a subset of destinations.

Previous trade literature has modelled deviations from the ranking order by imposing exogenous match specific heterogeneity.\textsuperscript{3} This is an \textit{ad-hoc} solution which is not able to explain which firms deviate from which markets. Within the framework I am proposing, I can study which sets of firms are deviating from the hierarchy prediction and how different industry parameters affect these deviations. As follows, I present a review of the literature related to this paper.

### 3.2 Literature Review

There are several models of trade that use quality heterogeneity and imperfect information to study the patterns of firm entry both at the extensive and intensive margins of trade. Nguyen (2008) proposes a model in which there is demand uncertainty due to incomplete information about the product’s quality. Quality is imperfectly correlated across destinations. Firms choose to enter markets sequentially starting with the highest hierarchy market to reduce the expected loss from uncertainty. As firms gain more information about their quality in all markets, they consider entry in lower ranked markets. In this model, firms enter markets which are relatively costless first. Therefore, there are no deviations from the ranking order due to firm’s maximization behaviour. Deviations arise exogenously after the firm-destination specific random shocks are realized.

Albornoz et al. (2010) also shows sequential exporting in a model with two types of firm heterogeneity: quality and productivity. As in Nguyen (2008), a firm’s profit is uncertain and correlated across markets. Firms enter costless markets first to learn something about their quality in other markets. Akhmetova (2010) uses product quality uncertainty to investigate

\textsuperscript{3}See, for example, Eaton et al. (2008), Head et al. (2011) and Nguyen (2008).
the intensive margin of trade inter-temporally. Firms in each market can choose between two technologies: high variable costs and low fixed costs (testing technology) or low variable cost and high fixed cost. Firms sell less and use the testing technology before incurring large fixed costs of setting a distribution centre in export markets. After learning its quality in those markets, a firm either stops exporting or pays the fixed cost and uses the low variable cost technology. In contrast to this analysis, I propose a model that investigates the pattern of entry of firms only at the extensive margin. In the model presented here, firms induce consumer experimentation through actions at the extensive margin (market entry).

This paper is also related to micro-theory studies that investigate market experimentation in environments with imperfect information, noisy signals and Bayesian updating. In Grossman et al. (1977) there are two separate models. In the first model consumers experiment with quantity in order to dynamically optimize consumption patterns by trading off the instantaneous expected value of consumption with optimal learning level of consumption. In the second model, there is a monopolist who does not know the exact demand for its product and experiment with prices. Trefler (1993) extends the monopolist model to show that the results in Grossman et al. (1977) hold even without the assumption of a normal noise in the likelihood function. Bergemann and Valimaki (2000) study the effects of double sided experimentation (consumers and producers) on socially optimal pricing decisions. Unlike in the previous market experimentation models, in this paper experimentation takes place only at the consumer level. In addition, firms can influence the level of experimentation by choosing different entry patterns (extensive) and not through pricing (intensive).

There are also related papers in the marketing literature that study product launching decisions. In Bhalla (2008) as well as in Liu and Schiraldi (2009) producers know the quality of their product but consumers learn it only through experimentation. Producers control the release of private signals by deciding the sequence in which their product is launched in segmented markets. A product could be launched either sequentially or simultaneously to all markets. The goal of the producer is to choose the optimal launching strategy so as to create
a high quality information herding. In contrast, I consider a trade model with monopolistic competition at the sector level. Unknown products compete with other varieties for which their appeal is known for a share of the market. In addition, there is no herding in my model.

My contribution to the trade literature is twofold. As aforementioned, the first contribution is to generate deviations from the market hierarchy that are endogenous and identify the set of firms that deviate. In addition, this paper contributes to the literature by adapting a model of consumer experimentation to fit a spatial model of trade through simplifying assumptions that keep the essence of both models.

The rest of the paper is as follows: The next section discusses deviations from the hierarchy using a cross section of Chilean firm level data. The third section introduces the theoretical model.

### 3.3 Firm level evidence: A First Look at the Data

In this section, I show that deviations from hierarchy are very pervasive. To that end, I use Chilean firm level exports to nine countries in Europe in 2006. Exports to these European countries represented just over 20% of the total value of Chilean exports in 2006.

Tables 3.1-3.4 show the total number of Chilean firms that exported to each of the nine markets at different level of industry disaggregation. In the first table, I show total entry for all firms in all industries. Subsequently, I show entry at the HS-02,04 and 06 digit level of industry classification. For the disaggregated data, I show entry for the sector with the highest number of total Chilean exporters.

Within each table, the first column indicates the destination country and its rank. These ranks represent the popularity of that destination market among Chilean exporters. In other words, they represent the observed hierarchy for the average firm. I use them as proxy for the real rank of the markets which is unobservable. The second column shows the total number of firms exporting to each destination in 2006. The third column shows the cumulative number of
different firms exporting to that or a lower ranked destination.

Theory predicts that firms will not export to lower ranked markets and fail to export to a higher ranked market. Therefore, the third column shows the amount of entry in each market predicted by theory. The fourth column then shows the percentage deviations from the hierarchy hypothesis in each market. That is, the percentage of firms that enter lower ranked markets but fail to enter that market. The last row of the fourth column is the average deviation over all nine markets.\footnote{This measure of deviations is not perfect. It depends for instance on the number of markets included in the analysis. However, it works well for comparison of entry patterns with different levels of disaggregation conditional on including the same number of markets.}

Table 3.1 shows that on average there are 48% of firms deviating from ranking order when we account for firms in all industries. This result is similar to the one that Eaton et al. (2008) find using data for French manufacturing firms. However, this result is not a direct proof that heterogeneous firm models fail to explain entry patterns observed in the data. In these types of models, the hierarchy is country-sector specific. Therefore, the amount of deviation observed in table 1 could be the result of different sectors having different destination hierarchies. When I disaggregate by sector in table II, III and IV, the average deviations drop by ten percent but still remain high at 36%. We can see that deviations from the hierarchy are a pervasive phenomenon when analyzing market entry. Moreover, unlike what the theory predicts, the deviations from the ranking order do not tend to considerably diminish as we study more disaggregated industry categories. In the next section, I propose a model that aims to explain these deviations by assuming imperfect information in entrant’s product appeal.

### 3.4 A Partial Equilibrium Trade Model with Asymmetric Information

This section assumes a multi-sector version of the Melitz (2003)’s model as developed in Chaney (2008) in the background and derives the pricing and entry decisions of individual
firms for which the appeal of their products is uncertain. All aggregate variables are assumed to be in steady state equilibrium. There are only two deviations from Chaney (2008) regarding aggregate variables. To simplify the model I assume zero profits in equilibrium and equal *per capita* wages across countries.\(^5\) There are \(J\) countries in the world with measure \(L_j\) of identical consumers \(\forall j = 1, \cdots, J\). Consumers are identical both within and across markets. In addition, there are \(H + 1\) different sectors in each country. \(H\) sectors wherein firms sell a differentiated variety and a homogeneous sector. The technology in the homogeneous good sector exhibits constant return to scale. Wages are pinned down by production in this homogeneous sector. In addition, this sector clears bilateral trade flows. I do not model production in this sector and I assume that wages in all markets are equal to one.

In the differentiated sectors each firm sells a distinct variety. All varieties are symmetric except for a utility shifter that I call product appeal. Product appeal is distributed according to a lognormal distribution that is common across sectors and countries. This distribution is common knowledge.

I assume that there is a finite number of new entrants that live for two periods, whose pricing and entry patterns I study (partial equilibrium). Also, I assume that consumers and producers know the appeal of all products except for these new entrants’ product appeal. For these entrants, consumers know only that the appeal of their products is a random draw from the common lognormal distribution. Consumers can inter-temporally change their information set through experimentation. On the other hand, producers know their product appeal.

\(^5\)Chaney (2008) assumes that aggregate firm’s profits from all countries are proportionally redistributed according to the country’s total productivity. This makes entry patterns the same as assuming zero profits and equal wages because the ratio of incomes are still equal to the ratio of total productivity.
3.4.1 Consumers

In each country \( j = 1, \ldots, J \), a representative individual has the following nested Dixit and Stiglitz (1977) CES within a Cobb-Douglas indirect utility function:

\[
v(p, p(v)) = \max_{R, \{x(v)\}_{v \in \Omega_j}} \prod_{h=1}^{H} \int_{v \in \Omega_j} \left[ x(v) \left( \frac{\sigma_h v}{\sigma_h - 1} \right)^{(\sigma_h - 1)} dv \right]^{\frac{\sigma_h}{\sigma_h - 1} \beta_h},
\]

s.t.

\[
pR + \int_{v \in \Omega_j} p(v)x(v) d\omega \leq 1,
\]

where \( \beta_h \) is the sector’s relative importance to the consumer \( \sum_{h=0}^{H} \beta_h = 1 \), \( v \) represents the appeal of the variety, \( \sigma_h > 1 \) is both the price elasticity of demand as well as the elasticity of substitution between varieties within sector \( h \) and \( \Omega_j \) is the set of varieties available in country \( j \). Note that the elasticity of substitution of commodities across sectors is one so \( \beta_h \) of income is spent in each sector. In what follows, I drop the subscript \( h \) because the following analyses apply to any sector.

This utility function representation yields the following demand function in country \( j \):\(^6\)

- Demand function for a variety \( v \) in sector \( h \):

\[
v_{x_j}(p(v), v) = \frac{\beta L_j}{P_j} \cdot \left( \frac{p(v)}{v P_j} \right)^{-\sigma},
\]

where \( P_j = \left[ \int_{v \in \Omega_j} \left( \frac{p(v)}{v} \right)^{1-\sigma} dv \right]^\frac{1}{1-\sigma} \) is the usual sector specific price index.

---

\(^6\)Since the utility function is homothetic, the total demand in country \( j \) is the product of individual demand and the measure of individuals \( L_j \).
Demand function for the homogeneous good sector:

\[ R = \frac{\beta_0 L_j}{p}. \]

**Uncertainty and learning technology**

The distribution of product appeals in each country and sector is common knowledge and is given by \( v \sim \text{LnN}(\mu_0, \sigma_0^2) \). Consumers know the exact product appeal of all firms, except of a finite number of new entrants exporting to country \( j \) from country \( i \). The model is a two period dynamic model so consumers have the opportunity to update their information set after consuming the variety in the first period. The goal of this model is to analyze the entry patterns of these new entrants. To that end, I assume that all aggregate variables are given and constant through time. I also make the following three assumptions that simplify the model.

First, I abstract from analyzing expected utilities by assuming that the number of new varieties is infinitesimally small compared to all the varieties in the market. Therefore, agents consume the new varieties according to the expected level of product appeal as follows:

\[ x_{ij}(p_{ij}(v), v) = \frac{\beta L_j}{p^{1-\sigma}} \phi_0(v) E[v^{\sigma-1}]. \quad (3.1) \]

One way of interpreting this demand function is to think that the number of varieties for which there is uncertainty in their appeal to consumers is so small that consumers behave as if they were risk neutral. It is worth noting that this assumption also shuts down consumers’ inter-temporal utility gains from experimentation. Therefore, consumer’s problem is static in my analysis. The expected product appeal in the first period is:

\[ E_1[v^{(\sigma-1)}] = \int_{-\infty}^{+\infty} v \phi_0((\sigma - 1)ln(v))dv = \exp \left[ (\sigma - 1) \left( \mu_0 + \frac{(\sigma - 1)}{2}\sigma_0^2 \right) \right], \quad (3.2) \]

\(^7\text{Whenever } \sigma \text{ is not followed by a subscript or by } h, \text{ it refers to the elasticity of substitution. Otherwise, it refers to the variance of the quality distribution.}\)
where $\phi_0$ is the probability density function of the normal distribution with mean $\mu_0$ and variance $\sigma^2_0$.

Second, I assume that learning does not change with the amount of the variety consumed but with the number of agents who consume it. To model this, I assume the following learning process. After consuming a variety with appeal $\bar{v}$, consumers observe the following regardless of the amount of consumption:

$$exp[s] = \bar{v} \cdot exp[\epsilon], \quad \epsilon \sim iid N(0, \sigma^2_\epsilon), \quad \sigma^2_\epsilon < \sigma^2_0.$$  

Consumers do not observe the true product appeal after consuming but a noisy signal, $s$. The variance of the noisy signal is smaller than the variance of the prior. This indicates that consumers are able to infer more information from experimentation.

Finally, I assume that a finite number of experimentation outcomes are posted in each market. The number of postings in each market is proportional to its size. For simplicity, I define the measure of consumers, $L_j$ in my previous notation, as the finite number of postings in market $j$. Postings are observed both within and across markets. This number is fixed and cannot be changed by firms’ actions.  

In addition, I assume that consumers ignore the result of their own experimentation and update their expectation according to the posted results. This assumption is not necessary for the following analysis. However, it greatly simplifies the math without affecting the results I obtain. After observing $T$ total postings, consumers update their belief to:

$$v \sim LnN(\mu_1, \sigma^2_1),$$

where $\mu_1 = \rho(T)\bar{s} + (1 - \rho(T))\mu_0$ and $\sigma^2_1 = \frac{\sigma^2_0\sigma^2_\epsilon}{T\sigma^2_0 + \sigma^2_\epsilon}$ ((15) and (16) in Appendix B). The

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8An alternative specification that could be interesting to explore is a specification in which firms choose the number of postings. This could be done by assuming an increasing cost in the number of postings. Like the cost of marketing in Arkolakis (2008), this could have potential implications for the intensive margin of trade. Here, I am interested in extensive margin only so I abstract from this.

9For derivations of the updated mean and variance see Appendix B. For a discussion see Chamley (2004).
expected product appeal is then:

\[
E_2[v^{\sigma-1}|\bar{s}, T] = \exp \left[ (\sigma - 1) \left( \mu_1 + \frac{1}{2} \sigma_1^2 \right) \right].
\]

Substituting for \( \mu_1 \) and \( \sigma_1^2 \):

\[
E_2[v^{\sigma-1}|\bar{s}, T] = \exp \left[ (\sigma - 1) \left( \rho(T)\bar{s} + (1 - \rho(T)) \left( \mu_0 + \frac{1}{2} \sigma_0^2 \right) \right) \right],
\]

where \( \bar{s} = \frac{1}{T} \sum_{i=1}^{T} s_i \) is the average of all \( T \) postings and \( \rho(T) = \frac{\sigma_0^2 T}{T\sigma_0^2 + \sigma^2} \). In these equations \( T \) represents the total number of postings. For instance, if the variety is sold in the first period in markets \( g \) and \( f \) with a measure of consumers \( L_g \) and \( L_f \) correspondingly, then \( T = L_g + L_f \). The larger the amount of experimentation in period one is, the larger the weight, \( \rho(T) \), consumers place on the aggregate outcome of experimentation, \( \bar{s} \), in equation (3.3).

### 3.4.2 Firms

In this subsection, I study the decisions of firms that take the behaviour of consumers derived in the previous section as given. I assume that \( P_j \) is given and study the pricing and entry decisions of a finite number of firms in the differentiated sector. These firms live for only two periods. Also, firms are risk neutral and know the true appeal of their products, \( \bar{v} \). They also know the consumers’ updating process (equation 3.3) and they observe the aggregate outcome of experimentation \( \bar{s} \) after agents consume.

The outcome of experimentation is \textit{ex-ante} random and influenced by the firm entry decision in the first period. By entering bigger markets, a firm reduces the uncertainty consumers have about its product appeal which affects second period’s profits. Therefore, firms’ entry decisions in the first period are influenced by the interaction between present profits and future expected profits.
Technology and timing of the model

Labor, \( l \), is the only input of production. The labor requirement of producing and exporting one unit from country \( i \) to \( j \) \( \forall j \in \{1,...,J\} \) is as follows:

\[
l = \tau_{ij} x_{ij}(p_{ij}, \exp(\psi)) + f_{ij}, \quad \tau_{ij} \left\{ \begin{array}{cl} > 1, & \text{if } i \neq j; \\ = 1, & \text{otherwise}; \end{array} \right.
\]

where \( \tau_{ij} \) is the usual variable iceberg cost and \( f_{ij} \) is the fixed cost of exporting to destination \( j \). I consider entry decisions to a subset of countries in the world \( \Gamma \subset \{1,...,J\} \) for which the firm is considering entering.\(^{10}\) Among these countries there is free flow of information so that everybody observes the outcome of experimentation. These countries are isolated in terms of information flows from other sets of countries. Consumers in those countries do not observe \( \bar{v} \) but they consume according to \( E[v^{(\sigma-1)}] \) as discussed above.

Conditional on entry to market \( j \) in period \( t \), firms in country \( i \) maximize the following profit function:

\[
\Pi^t_{ij}(v) = \max_{p_{ij}} (p_{ij} - \tau_{ij}) x_{ij}(p_{ij}(v), v) - f_{ij}. \quad (3.4)
\]

Pricing decisions in this model are static because they do not affect learning. The reason for this is twofold. First, as mentioned before, the learning technology is defined so that changes in the amount of consumption do not change consumer’s information set. Second, the consumer’s CES utility function leads to positive demand for all commodities available in the market. Therefore, pricing decisions conditional on entry do not change the set of consumers experimenting with the firm’s variety. Uncertainty and consumers’ experimentation in this model affect only the extensive margin of trade.

\(^{10}\)Capital letters represent sets and lower case letters represent individual elements, hereafter.
Prices derived from (4) are as follows:

\[ p_{ij} = \frac{\sigma}{\sigma - 1} \tau_{ij}. \]  

(3.5)

This is the usual monopolistic competitive pricing which is a mark-up over marginal costs. It is important to note that it is a function of neither true nor perceived product appeal. So, all firms follow the same pricing strategy. Total profit function for firms in \( i \) considering to export to \( \Gamma \) in period \( t \) is given by:

\[ \Pi'(I'|\mu_t, \sigma^2_t) = \sum_{j \in I} \Pi'_ij(v). \]

Substituting (2) and (5) into (4):

\[ \Pi'(I'|\mu_t, \sigma^2_t) = \sum_{j \in I'} A_{ij} \exp \left[ (\sigma - 1) \left( \mu_t + \frac{(\sigma - 1)\sigma^2_t}{2} \right) \right] - f_{ij}, \]  

(3.6)

where \( A_j = \left( \frac{\tau_{ij}}{(\sigma - 1)\sigma^2_t} \right)^{1-\sigma} \frac{\beta L_j}{\sigma \sigma^2_t} \) is the expenditure per unit of product appeal from \( i \) in \( j \) and \( I' \) is a choice variable that denotes the entry decisions of firms in period \( t \) which is discussed below.

Figure 3.1 shows the timing of the model. A firm enters at time zero and consumers believe that its quality is the mean of the prior lognormal distribution. The firms observe this and also observe the consumers updating process. However, a priori they do not know the outcome of experimentation, which is the average of random draws (overlines). The firm makes entry and pricing decisions in the first period. Then, in the second period consumers update their belief after experimentation. A firm then chooses prices and entry again at which point consumers consume and firms exit. The value function of a firm after the quality draw is given as follows:

\[ \Pi(v) = \max_{I^1 \in \Theta, I^2 \in \Theta} \left\{ \Pi^1(I^1|\mu_0, \sigma^2_0) + E[\Pi^2(I^2|\mu_1, \sigma^2_1)|T(I^1), \overline{v}] \right\}, \]  

(3.7)
Figure 3.1: Timing of the model

\begin{align*}
\text{ Consumers believe that } v & \sim \text{LnN}(\mu_0, \sigma_0^2) \\
\text{ Agents consume } & \\
\text{ Firms choose markets and prices } & \\
\text{ Firms exit } & \\
\text{ Firms enter the model with a known draw } & \tilde{v}
\end{align*}

\text{s.t.}

\begin{align*}
\mu_1 &= \frac{\sigma_0^2 \text{T} (I^1)}{\text{T} (I^1) \sigma_0^2 + \sigma_\epsilon^2} \bar{s} + \frac{\sigma_\epsilon^2}{\text{T} (I^1) \sigma_0^2 + \sigma_\epsilon^2} \mu_0,
\sigma_1^2 &= \frac{\sigma_0^2 \sigma_\epsilon^2}{\text{T} (I^1) \sigma_0^2 + \sigma_\epsilon^2},
\end{align*}

where \( T(I^1) = \sum_{j \in I^1} L_j, \bar{s} = \sum_{i=1}^{T(I^1)} \frac{s_i}{T(I^1)} \) and \( \Theta = P(\Gamma)(\text{powerset}) \).

The expected value of the second term profit is given as follows:

\begin{equation}
E[\Pi^2(I^2|\mu_1, \sigma_1^2)|\text{T}(I^1), \bar{v}] = \int_{-\infty}^{+\infty} \sum_{j \in I^2} \Pi_{ij}^2 (I^2|\mu_1, \sigma_1^2) \phi \left( \frac{\bar{s} | \text{ln}(\bar{v})}{\sigma_\epsilon^2 / T(I^1)} \right) d\bar{s}, \tag{3.8}
\end{equation}

where \( \phi \left( \frac{\text{ln}(\bar{v})}{\sigma_\epsilon^2 / T(I^1)} \right) \) is the pdf of the normal distribution with mean \( \text{ln}(\bar{v}) \) and variance \( \frac{\sigma_\epsilon^2}{T(I^1)} \), \( I^2 \) is defined below.
It is important to note for the following analysis that profits in both periods are a function of the perceived appeal of a firm’s product and not of its true appeal. The true level of appeal of a firm’s variety affects only the likelihood function.

To solve the model I use backward induction. I start from the second period decisions after the result of experimentation $\bar{s}$ is observed by both producers and consumers.

**Firm’s decision at $t=2$**

In the second period after $\bar{s}$ is observed, a firm chooses which markets it enters. So,

$$j \in I^2 \iff \Pi_{ij}^2 (I^2 | \mu_1, \sigma_1) = A_{ij} \exp[\left(\sigma - 1\right) \left(\mu_1 + \frac{\left(\sigma - 1\right)}{2} \sigma_1^2\right)] - f_{ij} \geq 0.$$ 

As in Melitz (2003) and Chaney (2008) a firm enters a market iff its profit in that market is non-negative. That is,

$$\text{enter iff } \exp[\left(\sigma - 1\right) \left(\mu_1 + \frac{\left(\sigma - 1\right)}{2} \sigma_1^2\right)] \geq \frac{f_{ij}}{A_{ij}},$$

where the right hand side is the expected quality threshold below which a firm decides not to export to country $j$. It is decreasing in the size of the market ($\beta L_j$) because of the decreasing average costs (i.e. increasing returns). It is increasing in the sectoral cost of exporting and the degree of competition (the inverse of $P_j$). It is also increasing in the degree of substitutability, $\sigma$. This is because there is less product differentiation and the composition of the market approaches perfect competition. Under perfect competition only firms with the highest product appeal survive and all others are forced out of the market.

We can write everything in terms of the aggregate outcome of experimentation $\bar{s}$ as follows (I use equation 3.3 in the derivation). So, a firm enters market $j$ iff:

$$\bar{s} \geq \ln(f_{ij}/A_{ij}) \left( T \sigma_0^2 + \sigma_e^2 \right) - \frac{\sigma_e^2}{(\sigma - 1) T \sigma_0^2} \left( \mu_0 + \frac{\left(\sigma - 1\right)}{2} \sigma_0^2 \right),$$

(3.9)
\[
\frac{ds_{ij}^*(T)}{dT}\begin{cases}
< 0 \text{ iff } \ln(f_{ij}/A_{ij}) > (\sigma - 1) \left(\mu_0 + \frac{(\sigma-1)\sigma^2}{2}\right); \\
> 0 \text{ iff } \ln(f_{ij}/A_{ij}) < (\sigma - 1) \left(\mu_0 + \frac{(\sigma-1)\sigma^2}{2}\right).
\end{cases}
\]

To interpret this result we can observe that the updated expected value is a weighted average of the old expected value and the result of experimentation \(s\) (equation 3.3). This updated value increases the weight on \(s\) as the number of observation \(T\) increases. When the initial expected product appeal is below the cut-off, \(s\) has to be high to bring the ex-post expected value above the cut-off. The bigger \(T\) is, the lower \(s\) need to be because the updating formula allocates more weight on \(s\). When the initial expected quality is above the quality cut-off, the opposite is true. Hereafter, I assume that \(\ln(f_{ij}/A_{ij}) < (\sigma - 1) \left(\mu_0 + \frac{\sigma^2}{2}\right)\) \(\forall J \in \Gamma\). That is, the only reason a firm enters any export market in the first period is to increase the second period’s profit by informing or misinforming consumers.

**Firm’s decision at \(t=1\)**

I can rewrite a firm’s value function 3.7 as follows:

\[
\Pi(\bar{v}) = \max_{I' \in \Theta} \left\{ \sum_{j \in I'} A_{ij} E_1[v^{\sigma-1}] + \sum_{j \in \{1, \ldots, J\}} \int_{s_{ij}(T(I'))} A_{ij} E_2[v^{\sigma-1} | \bar{s}, T(I')] \phi \left( \bar{s} | \lg v, \frac{\sigma^2}{T(I')} \right) d\bar{s} \right\},
\]

(3.10)

I need to analyze entry patterns at \(t = 1\) (i.e. \(I^1\)). However, I cannot use standard calculus tools to solve equation (3.10) because \(I^1\) is a discrete choice. Instead, I solve equation (3.10) as follows.

I combine equations (3.8) and (3.9) and I obtain the *ex ante* (before \(\bar{s}\) is observed) conditional on \(\bar{v}\) and \(T(I^1)\) expected profit function as follows:

\[
E[\Pi^2[\bar{v}, T]] = \sum_{j \in \{1, \ldots, J\}} \int_{s_{ij}-\ln(\bar{v})} \left\{ A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma-1)\sigma^2}{2}\right) \right) \right] \right\} \exp \left[ (\sigma - 1) \rho(T) \bar{\epsilon} - f_{ij} \right] \phi \left( \bar{\epsilon} | 0, \frac{\sigma^2}{T} \right) d\bar{\epsilon}.
\]

(3.11)
In equation (3.11), I substituted $\bar{s} = \ln(\bar{v}) + \bar{\epsilon}$. Through a change of variables, I can simplify equation (3.11) as follows:

$$E[\Pi^2 | \bar{v}, T] = \sum_{j \in \{1, \ldots, J\}} \int_{s_j - \ln(\bar{v})}^{\infty} \{A_{ij} \exp(x) - f_{ij}\} \phi(x|\hat{\mu}(\bar{v}, T), \hat{\sigma}^2(T)), \quad (3.12)$$

where $\hat{\mu}(\bar{v}, T) = (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right)$ and $\hat{\sigma}^2 = (\sigma - 1) [\rho(T)]^2 \frac{\sigma^2}{T}$. The derivatives are as follows:

$$\frac{\partial \hat{\mu}(\bar{v}, T)}{\partial \bar{v}} = (\sigma - 1) \frac{\rho(T)}{\bar{v}} \geq 0, \quad \frac{\partial \hat{\sigma}^2(T)}{\partial \bar{v}} = 0,$$

$$\frac{\partial \hat{\mu}(\bar{v}, T)}{\partial T} = (\sigma - 1) \frac{\sigma^2}{(T \sigma_0^2 + \sigma_e^2)^2} \left( \ln(\bar{v}) - \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right),$$

$$\Rightarrow \frac{\partial \hat{\mu}(\bar{v}, T)}{\partial T} \left\{ \begin{array}{ll} \geq 0 & \text{iff } \ln(\bar{v}) \geq \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2; \\
\leq 0 & \text{iff } \ln(\bar{v}) \leq \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2. \end{array} \right.$$}

$$\frac{\partial \hat{\sigma}^2(T)}{\partial T} = (\sigma - 1) 2 \left[ \frac{\sigma_e^2 \sigma_0^2}{(\sqrt{T} \sigma_0^2 + \sigma_e^2)^3} \right] \left( \frac{1}{2} \frac{\sigma_e^2}{\sqrt{T}} - \frac{1}{2} \sigma_0^2 \sqrt{T} \right),$$

$$\Rightarrow \frac{\partial \hat{\sigma}^2(T)}{\partial T} \left\{ \begin{array}{ll} \geq 0 & \text{iff } \frac{\sigma_e^2}{T} \geq \sigma_0^2; \\
\leq 0 & \text{iff } \frac{\sigma_e^2}{T} \geq \sigma_0^2 \text{ (which I assume)}. \end{array} \right.$$}

The ex ante result of experimentation is summarized in equation 3.12. Because the profit function is convex, changes in the amount of consumption $T$ affect profits through both the mean and variance of the distribution. Firms with low product appeal benefit from a low level
of experimentation because this increases both their variance and mean. On the other hand, for firms with product appeal above \( \mu_0 + \frac{(\sigma - 1)}{2}\sigma_0^2 \), different \( T' \)s have two opposing effects. An increase in \( T \) increases the expected value (higher weight on the true quality of the firm) but decreases the variance (less noise). Optimal experimentation for this subset of firms is the \( T \) that maximizes the outcome of these two forces. Note that the effect of \( T \) on the mean is different for different levels of product appeal \( \frac{\partial e}{\partial T} > 0 \) so that not all firms above the cut-off choose the highest populated subsets of markets.

The next lemma and proposition show that for two different levels of experimentation, there is a unique quality \( \bar{v}^* \) for which both forces are equal.

**Lemma 4** \( \forall T \in \mathbb{R}^+, \lim_{\bar{v} \to 0} E[\Pi^2|\bar{v}, T] = 0 \).

**Proof:**

From equation 3.11:

\[
\lim_{\bar{v} \to 0} E[\Pi^2|\bar{v}, T] = \sum_{j \in \{1, \ldots, J\}} \int_{s_{ij} + \infty}^{\infty} \left\{ A_{ij} \frac{1}{\exp[\infty]} \exp \left[ (\sigma - 1)\rho(T)\bar{v} - f_{ij} \right] \right\} \phi \left( \bar{v}|0, \frac{\sigma^2}{T} \right) d\bar{v} = 0.
\]

**Proposition 7** Whenever \( \sigma^2_0 \frac{-8+\sqrt{64+4(\sigma-1)^4\sigma_0^4}}{2(\sigma-1)^2} < \sigma^2_0 \), for any \( T_1 \) and \( T_2 \) where \( T_2 > T_1 \), there exists a unique \( \bar{v}^* \) such that the following holds:

1. \( E[\Pi^2|\bar{v}^*, T_1] = E[\Pi^2|\bar{v}^*, T_2] \);
2. \( E[\Pi^2|\bar{v}^*, T_1] > E[\Pi^2|\bar{v}^*, T_2] \) \( \forall \bar{v} < \bar{v}^* \);
3. \( E[\Pi^2|\bar{v}^*, T_1] < E[\Pi^2|\bar{v}^*, T_2] \) \( \forall \bar{v} > \bar{v}^* \).

**Proof:** See Appendix B

Proposition 7 states that the expected profit in the second period function with different levels of experimentation cross once in the product appeal space (see figure 3.2). Moreover, for firms with product appeal lower than \( \bar{v}^* \), the optimal experimentation is to enter the smaller
market first. On the other hand, for firms with product appeal larger than $\bar{v}^*$, the optimal experimentation is to enter the larger market.

We can define the lifetime value of a firm with product appeal $\bar{v}$ from entering $\Theta_i \in \Theta$ as follows:

$$V_{\Theta_i}(\bar{v}) = \sum_{j \in \Theta_i} A_{ij} \exp \left[ \left( \sigma - 1 \right) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right] - f_{ij} + E[\Pi^2[\bar{v}, T]],$$

where $T = \sum_{j \in \Theta_i} L_j$.

Note that profits in the first period are independent of the true value of the appeal of the variety. The reason for this is that the appeal of each variety is a random draw in period 1. Therefore, consumers prior to consumption have no way of discerning among varieties.

Let there be $n$ distinct markets in $\Gamma$, then $\Theta$ has $2^n$ distinct subsets. Let $\Theta_i$ for $i = 1, \ldots, 2^n$ represent the different subsets starting from the lowest amount of experimentation to the highest.
Chapter 3. Product Appeal, Information and Trade

\( \Theta_1 = \{0\} \) and \( \Theta_{2n} = \bigcup_{j \in \Theta} j \). Then, we can rewrite a firm lifetime value function as:

\[
\Pi(\bar{v}) = \text{Max} \left\{ V_{\Theta_1}, \ldots, V_{\Theta_{2n}} \right\}.
\]

Assume that the fixed costs \( f_{ij} \) is either larger for higher populated subsets of markets or smaller but close.\(^ {11} \) By proposition 4.2, we can then define the policy function \( I^1(\bar{v}) \) as follows:

\[
I^1(\bar{v}) = \begin{cases} 
\text{Enter } \Theta_1 & \text{if } \bar{v} \leq \bar{v}_{1}^*; \\
\text{Enter } \Theta_2 & \text{if } \bar{v} \leq \bar{v}_{2}^*; \\
\vdots \\
\text{Enter } \Theta_{2n} & \text{if } \bar{v}_{2n-1}^* \leq \bar{v};
\end{cases}
\]

where \( \bar{v}_{ij}^* \) is defined as the result of \( V_{\Theta_i}(\bar{v}) = V_{\Theta_j}(\bar{v}) \) and \( \bar{v}_i^* = \text{Min} \left\{ \bar{v}_{ij}^* \right\}_{j > i} \). Note that it could be the case that there is no entry in certain subsets of markets such as when \( \bar{v}_1^* > \bar{v}_2^* \), there is no entry into subset \( \Theta_2 \).

Figure 3.3 shows the result of the previous policy function for the two market case with \( L_2 > L_1 \) (\( T_1 = L_1 \), \( T_2 = L_2 \) and \( T_3 = L_1 + L_2 \)). The case where there is perfect sorting of firms into subsets of markets is shown in the left panel. Firms with high product appeal enter both markets because they benefit from the highest levels of experimentation. Firms with medium-high product appeal enter the more populated of the two markets (market 2). Firms with medium-low product appeal enter only market 1. Firms with the lowest product appeal do not enter any market. In the right panel, I show the case where there is imperfect sorting. In this case no firm enters market 2 only.

This framework is able to generate deviations from the hierarchy prediction as follows: If market 1 is better ranked than market 2, then medium-high quality firms deviate from the hierarchy in the left panel. The reason for this is that they enter market 2 but fail to enter market 1. In the left panel case, there are no deviations from this hierarchy. On the contrary, if market

\(^ {11} \) If it is significantly smaller, then we may have two crossing points in \( \mathbb{R}^+ \).
2 is better ranked, then the firms with the lowest product appeal are deviating in both cases.

### 3.5 Conclusion

This paper presented a framework that explains deviations from the market hierarchy prediction in trade models with firm level heterogeneity. To generate these deviations I present a two-period model of trade with asymmetric information in firm’s product appeal. Firms know the true value of the appeal of their products but consumers are unaware of it. Consumers can dynamically expand their information set in period 2 by experimenting with commodities in period 1. In equilibrium there is an imperfect sorting of firms into markets. Firms with high product appeal enter the most populated set of markets because they benefit from high levels of experimentation. Firms with low product appeal enter the lowest populated market because they benefit from low levels of experimentation. There is also a subset of firms with medium
product appeal that enter subsets of markets that are in between these two extremes. The sorting is imperfect because the value of the parameters could lead to an equilibrium in which no firm sells in a particular subset of markets.
### 3.6 Appendix

#### 3.6.1 Appendix A-Tables

Table 3.1: Total over all industries of number of Chilean firms exporting in 2006 to EU countries that adopted the Euro in 1999

<table>
<thead>
<tr>
<th>Country</th>
<th># of firms</th>
<th># of firms selling to that or lower ranked markets</th>
<th>% selling to lower ranked markets but not to that one</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9) Spain</td>
<td>796</td>
<td>1908</td>
<td>.58</td>
</tr>
<tr>
<td>(8) Germany</td>
<td>670</td>
<td>1521</td>
<td>.56</td>
</tr>
<tr>
<td>(7) Netherlands</td>
<td>614</td>
<td>1276</td>
<td>.52</td>
</tr>
<tr>
<td>(6) France</td>
<td>521</td>
<td>1042</td>
<td>.5</td>
</tr>
<tr>
<td>(5) Italy</td>
<td>510</td>
<td>801</td>
<td>.36</td>
</tr>
<tr>
<td>(4) Belgium</td>
<td>332</td>
<td>479</td>
<td>.31</td>
</tr>
<tr>
<td>(3) Ireland</td>
<td>119</td>
<td>250</td>
<td>.52</td>
</tr>
<tr>
<td>(2) Portugal</td>
<td>93</td>
<td>171</td>
<td>.46</td>
</tr>
<tr>
<td>(1) Finland</td>
<td>89</td>
<td>89</td>
<td>.48</td>
</tr>
</tbody>
</table>

Andorra, Monaco, Austria, Luxembourg and San Marino are excluded

Table 3.2: Total over industry 08 at the HS-2 digit level of number of Chilean firms exporting in 2006 to EU countries that adopted the Euro in 1999

<table>
<thead>
<tr>
<th>Country</th>
<th># of firms</th>
<th># of firms selling to that or lower ranked markets</th>
<th>% selling to lower ranked markets but not to that one</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9) Netherlands</td>
<td>248</td>
<td>359</td>
<td>.31</td>
</tr>
<tr>
<td>(8) Spain</td>
<td>153</td>
<td>285</td>
<td>.46</td>
</tr>
<tr>
<td>(7) Italy</td>
<td>134</td>
<td>242</td>
<td>.44</td>
</tr>
<tr>
<td>(6) Germany</td>
<td>103</td>
<td>201</td>
<td>.49</td>
</tr>
<tr>
<td>(5) France</td>
<td>100</td>
<td>157</td>
<td>.36</td>
</tr>
<tr>
<td>(4) Belgium</td>
<td>74</td>
<td>110</td>
<td>.33</td>
</tr>
<tr>
<td>(3) Portugal</td>
<td>37</td>
<td>51</td>
<td>.28</td>
</tr>
<tr>
<td>(2) Ireland</td>
<td>14</td>
<td>20</td>
<td>.3</td>
</tr>
<tr>
<td>(1) Finland</td>
<td>9</td>
<td>9</td>
<td>.37</td>
</tr>
</tbody>
</table>

Andorra, Monaco, Austria, Luxembourg and San Marino are excluded

08-Edible fruits and nuts, peel of citrus/melons
Table 3.3: Total over industry 2204 at the HS-4 digit level of number of Chilean firms exporting in 2006 to EU countries that adopted the Euro in 1999

<table>
<thead>
<tr>
<th>Country</th>
<th># of firms</th>
<th># of firms selling to that or lower ranked markets</th>
<th>% selling to lower ranked markets but not to that one</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9) Germany</td>
<td>136</td>
<td>208</td>
<td>.35</td>
</tr>
<tr>
<td>(8) Netherlands</td>
<td>109</td>
<td>191</td>
<td>.43</td>
</tr>
<tr>
<td>(7) Belgium</td>
<td>97</td>
<td>176</td>
<td>.45</td>
</tr>
<tr>
<td>(6) France</td>
<td>81</td>
<td>148</td>
<td>.45</td>
</tr>
<tr>
<td>(5) Ireland</td>
<td>79</td>
<td>112</td>
<td>.3</td>
</tr>
<tr>
<td>(4) Finland</td>
<td>40</td>
<td>79</td>
<td>.49</td>
</tr>
<tr>
<td>(3) Italy</td>
<td>35</td>
<td>56</td>
<td>.38</td>
</tr>
<tr>
<td>(2) Spain</td>
<td>23</td>
<td>26</td>
<td>.12</td>
</tr>
<tr>
<td>(1) Portugal</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Percentage deviations</td>
<td></td>
<td></td>
<td>.37</td>
</tr>
</tbody>
</table>

Andorra, Monaco, Austria, Luxembourg and San Marino are excluded
2204-Wines of fresh grapes, grapes must nesoi

Table 3.4: Total over industry 220421 at the HS-6 digit level of number of Chilean firms exporting in 2006 to EU countries that adopted the Euro in 1999

<table>
<thead>
<tr>
<th>Country</th>
<th># of firms</th>
<th># of firms selling to that or lower ranked markets</th>
<th>% selling to lower ranked markets but not to that one</th>
</tr>
</thead>
<tbody>
<tr>
<td>(9) Germany</td>
<td>107</td>
<td>175</td>
<td>.39</td>
</tr>
<tr>
<td>(8) Netherlands</td>
<td>95</td>
<td>158</td>
<td>.4</td>
</tr>
<tr>
<td>(7) Belgium</td>
<td>90</td>
<td>146</td>
<td>.38</td>
</tr>
<tr>
<td>(6) Ireland</td>
<td>79</td>
<td>120</td>
<td>.34</td>
</tr>
<tr>
<td>(5) France</td>
<td>57</td>
<td>92</td>
<td>.38</td>
</tr>
<tr>
<td>(4) Finland</td>
<td>38</td>
<td>68</td>
<td>.44</td>
</tr>
<tr>
<td>(3) Italy</td>
<td>27</td>
<td>47</td>
<td>.43</td>
</tr>
<tr>
<td>(2) Spain</td>
<td>21</td>
<td>24</td>
<td>.12</td>
</tr>
<tr>
<td>(1) Portugal</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Percentage deviations</td>
<td></td>
<td></td>
<td>.36</td>
</tr>
</tbody>
</table>

Andorra, Monaco, Austria, Luxembourg and San Marino are excluded
220421-Other wines, grape must (fermentation arrested in containers 2l or less)

### 3.6.2 Appendix B- Proofs and Derivations

**Part I:** Here I show the derivation of the first two moments of the updated distribution of $ln(v)$ at $t = 2$. Because the distribution is normal, these two moments are enough to fully characterized a firm’s entire product appeal distribution. At period $t = 1$, the probability of the
true value of the firm’s product appeal being $ln(v)$ is:

$$\phi(ln(v)) = \frac{1}{\sigma_0 \sqrt{2\pi}} \exp \left[ -\frac{(ln(v) - \mu_0)^2}{2\sigma_0^2} \right]. \quad (3.13)$$

After a total measure $T$ consume a variety with appeal $\hat{v}$, all consumers receive the following aggregate signal:

$$exp[\bar{v}] \overset{iid}{\sim} LN\left(\hat{v}, \frac{\sigma_\epsilon^2}{T}\right).$$

The likelihood function is then as follows:

$$\phi(\bar{v}|ln(v)) = \frac{\sqrt{T}}{\sigma_\epsilon \sqrt{2\pi}} \exp \left[ -\frac{(\bar{v} - ln(v))^2 T}{2\sigma_\epsilon^2} \right]. \quad (3.14)$$

Using Baye’s rule:

$$\phi(ln(v)|\bar{v}, T) = \frac{\int_{-\infty}^{+\infty} \phi(\bar{v}|ln(v)) \phi(ln(v)) d\nu}{\int_{-\infty}^{+\infty} \phi(\bar{v}|ln(v)) \phi(ln(v)) d\nu}. \quad (3.15)$$

Substituting equations (3.14) and (3.15), we obtain:

$$\phi(ln(v)|\bar{v}, T) = \frac{\exp \left[ -\frac{1}{2} \left( \frac{(ln(v) - \mu_0)^2}{\sigma_0^2} + \frac{(\bar{v} - ln(v))^2 T}{\sigma_\epsilon^2} \right) \right]}{\int_{-\infty}^{+\infty} \exp \left[ -\frac{1}{2} \left( \frac{(ln(v) - \mu_0)^2}{\sigma_0^2} + \frac{(\bar{v} - ln(v))^2 T}{\sigma_\epsilon^2} \right) \right] d\nu}. \quad (3.16)$$

Collecting terms and simplifying we get:

$$\phi(ln(v)|\bar{v}, T) = \frac{1}{\sqrt{2\pi\sigma_1^2(v|T)}} \exp \left[ -\frac{1}{2} \left( \frac{(ln(v) - \mu_1(v|\bar{v}, T))^2}{\sigma_1^2(v|T)} \right) \right].$$
where:

\[
\mu_1[v|\bar{\xi},T] = \rho(T)\bar{\xi} + (1 - \rho(T))\mu_0, \quad (3.16)
\]

\[
\sigma^2_1(v|T) = \frac{\sigma_0^2 \sigma_v^2}{\bar{\xi} \sigma_v^2 + \sigma^2}, \quad (3.17)
\]

with \( \rho(T) = \frac{\sigma_0^2 T}{\bar{\xi} \sigma_v^2 + \sigma^2} \)

**Part II: Proof of Proposition 7** From equation 3.11:

\[
E[\Pi^2|\bar{\nu},T] = \sum_{j \in \{1, \ldots, J\}} \int_{s_{ij} - \ln(\bar{\nu})}^{\infty} \left\{ A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{\nu}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1) \sigma^2}{2} \right) \right) \right] \right. 
\]

\[
\times \exp \left[ (\sigma - 1) \rho(T) \bar{\nu} \right] - f_{ij} \phi(\bar{\nu}|0, \frac{\sigma^2}{T}) \right\} d\bar{\nu}. 
\]

We know by lemma 4 that \( \lim_{\bar{\nu} \to 0} E[\Pi^2|\bar{\nu},T_1] = \lim_{\bar{\nu} \to 0} E[\Pi^2|\bar{\nu},T_2] \) so in order for a single crossing at \( \bar{\nu}^* \), we need:

\[
\frac{\partial^2 E[\Pi^2|\bar{\nu},T]}{\partial \bar{\nu} \partial T} < 0 \text{ for } \bar{\nu} \in (0, \nu') \text{ (submodular);}
\]

\[
\frac{\partial^2 E[\Pi^2|\bar{\nu},T]}{\partial \bar{\nu} \partial T} > 0 \text{ for } \bar{\nu} \in (\nu', +\infty) \text{ (supermodular);}
\]

where \( \nu' < \bar{\nu}^* \).

We can rewrite \( \frac{\partial^2 E[\Pi^2|\bar{\nu},T]}{\partial \bar{\nu} \partial T} \) as:

\[
\frac{\partial^2 E[\Pi^2|\bar{\nu},T]}{\partial \bar{\nu} \partial T} = \frac{\partial^2 E[\Pi^1|\bar{\nu},T]}{\partial \bar{\nu} \partial T} + \ldots + \frac{\partial^2 E[\Pi^j|\bar{\nu},T]}{\partial \bar{\nu} \partial T}.
\]

A sufficient condition is that:

\[
\frac{\partial^2 E[\Pi^j|\bar{\nu},T]}{\partial \bar{\nu} \partial T} < 0, \text{ for } \bar{\nu} \in (0, \nu_j'); \quad (3.18)
\]
\[
\frac{\partial^2 E[\Pi^2_j][\bar{v}, T]}{\partial \bar{v} \partial T} > 0, \text{ for } \bar{v} \in (v_j', +\infty);
\]

(3.19)

\forall j \in \{1, \ldots, J\},

where:

\[
E[\Pi^2_j][\bar{v}, T] = \int_{s_{ij}^*-\ln(\bar{v})}^{\infty} \left\{ A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right) \right] \\
\exp \left[ (\sigma - 1) \rho(T) \bar{e} \right] - f_{ij} \right\} \phi \left( \bar{e} | 0, \frac{\sigma^2_e}{T} \right) d\bar{e}.
\]

Using Leibniz rule and \( \phi \left( \bar{e} | 0, \frac{\sigma^2_e}{T} \right) = \frac{\sqrt{T}}{\sqrt{2\pi} \sigma_e^2} \exp \left[ -\frac{T - \bar{e}^2}{2\sigma_e^2} \right] \):

\[
\frac{\partial E[\Pi^2_j][\bar{v}, T]}{\partial \bar{v}} = A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right) \right] \\
\left\{ (\sigma - 1) \frac{\rho(T)}{\bar{v}} \int_{s_{ij}^*-\ln(\bar{v})}^{\infty} \exp \left[ (\sigma - 1) \rho(T) \bar{e} \right] \phi \left( \bar{e} | 0, \frac{\sigma^2_e}{T} \right) d\bar{e} \right\}.
\]

intensive margin

\[
\left[ \frac{1}{\bar{v}} \exp \left[ (\sigma - 1) \rho(T) (s_{ij}^*(T) - \ln(\bar{v})) \right] - \frac{1}{\bar{v}} f_{ij} \right] \phi \left( (s_{ij}^*(T) - \ln(\bar{v})) | 0, \frac{\sigma^2_e}{T} \right) \right\}.
\]

extensive margin

At the margin the extensive margin is zero because:

\[
A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) s_{ij}^*(T) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right) \right] = f_{ij}.
\]

So,

\[
\frac{\partial E[\Pi^2_j][\bar{v}, T]}{\partial \bar{v}} = A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma - 1)}{2} \sigma_0^2 \right) \right) \right] (\sigma - 1) \frac{\rho(T)}{\bar{v}} R(T, \bar{v}) \geq 0,
\]
where

\[ R(T, \bar{v}) = \int_{s_{ij}^*-\ln(\bar{v})}^{\infty} \exp \left[ (\sigma - 1) \rho(T), e^\bar{\epsilon} \right] \phi \left( \bar{\epsilon}|0, \frac{\sigma^2_\epsilon}{2} \right). \]

The cross derivative is:

\[
\frac{\partial^2 E[J_j|\bar{v},T]}{\partial \bar{v} \partial T} = A_{ij} \exp \left[ (\sigma - 1) \left( \rho(T) \ln(\bar{v}) + (1 - \rho(T)) \left( \mu_0 + \frac{(\sigma-1)}{2} \sigma_0^2 \right) \right) \right] \frac{(\sigma - 1)}{(\sigma^2_0 + \sigma^2_\epsilon) \sigma^2_\epsilon} \ln(\bar{v}) - \left( \mu_0 + \frac{(\sigma-1)}{2} \sigma_0^2 \right) \rho(T)R(T, \bar{v}) +
\]

\[ \frac{\sigma^2_0 \sigma^2_\epsilon}{(\sigma^2_0 + \sigma^2_\epsilon)^2} R(T, \bar{v}) - \rho(T) \frac{ds^*_j(T)}{dT} \exp \left[ (\sigma - 1) \rho(T) (s^*_j(T) - \ln(\bar{v})) \right] \phi([s^*_j - \ln(\bar{v})]|0, \frac{\sigma^2_\epsilon}{T}) +
\]

\[ \rho(T) \int_{s_{ij}^*-\ln(\bar{v})}^{\infty} \exp \left[ (\sigma - 1) \rho(T), e^\bar{\epsilon} \right] \phi \left( \bar{\epsilon}|0, \frac{\sigma^2_\epsilon}{2} \right) \left[ \frac{\sigma^2_0 \sigma^2_\epsilon}{(\sigma^2_0 + \sigma^2_\epsilon)^2} (\sigma - 1) - \frac{\bar{\epsilon}^2}{2\sigma^2_\epsilon} \right] d\bar{\epsilon} \].

We know that \( \frac{ds^*_j(T)}{dT} < 0 \) because \( \ln \left( \frac{f_{ij}}{A_{ij}} \right) > (\sigma - 1) \left( \mu_0 + \frac{(\sigma-1)}{2} \sigma_0^2 \right) \). Therefore, a sufficient condition for equations (3.18) and (3.19) is:

\[
\int_{-\infty}^{+\infty} \exp \left[ (\sigma - 1) \rho(T)e^\bar{\epsilon} \right] \phi \left( \bar{\epsilon}|0, \frac{\sigma^2_\epsilon}{2} \right) \left[ \frac{\sigma^2_0 \sigma^2_\epsilon}{(T\sigma^2_0 + \sigma^2_\epsilon)^2} (\sigma - 1) - \frac{\bar{\epsilon}^2}{2\sigma^2_\epsilon} \right] d\bar{\epsilon} > 0. \quad (3.20)
\]

A sufficient condition for equation 3.20:

\[
\sigma^2_\epsilon < -8 + \frac{\sqrt{64 + 4(\sigma - 1)^4\sigma^4_0}}{2(\sigma - 1)^2}.
\]
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