Canadian Journal of Physics
Revue canadienne de physique

**Kantowski-Sachs modified holographic Ricci dark energy
model in Saez-Ballester theory of gravitation**

Journal:	<i>Canadian Journal of Physics</i>
Manuscript ID	cjp-2016-0670.R1
Manuscript Type:	Article
Date Submitted by the Author:	23-Jul-2017
Complete List of Authors:	Bhaskar Rao, M.P.V.V.; Vignan's Institute of Information Technology, REDDY, DRK; Andhra University, Waltair, Mathematics Babu, K. Sobhan; JNTU K College of Engg. Narsaraopeta,
Keyword:	Kantowski-Sachs model, Ricci dark energy, Saez Ballester theory, Holographic dark energy, Hybrid expansion law
Is the invited manuscript for consideration in a Special Issue? :	N/A

SCHOLARONE™
Manuscripts

Kantowski-Sachs modified holographic Ricci dark energy model in Saez-Ballester theory of gravitation

M.P.V.V. Bhaskara Rao¹, D.R.K. Reddy², K. Sobhan Babu³

¹ Department of Basic Sciences and Humanities, Vignan's Institute of Information Technology, Duvvada, Visakhapatnam, India.

² Department of Applied Mathematics, Andhra University, Visakhapatnam, India.

³ Department of Mathematics, JNTU College of Engg., Narasaraopeta, India.

²reddy_einstein@yahoo.com

Abstract: We have considered Kantowski-Sachs space-time in the presence of matter and anisotropic modified holographic Ricci dark energy components in the scalar tensor theory of gravitation formulated by Saez and Ballester (Phys. Lett. A 113; 467, 1986) and derived the field equations of the theory. We have used (i) Hybrid expansion law proposed by Akarsu et al. (JCAP, 022, 2014), (ii) a relation between metric potentials and (iii) modified holographic Ricci dark energy density given by Chen and Jing (Phys. Lett. B 679, 144, 2009) to obtain an exact solution of the field equations which describes a Kantowski-Sachs holographic modified Ricci dark energy universe in this theory. Physical and Kinematical parameters are also computed and their physical behavior is discussed.

Keywords: Kantowski-Sachs model, Ricci dark energy, Saez Ballester theory.

1. Introduction

The observations from distant type Ia supernovae confirm that the universe at present, is in an accelerated phase of expansion [1]-[3]. The accelerated expansion can be attributed to an exotic form of energy, known as dark energy. It provides a negative pressure that gives are antigravity effect driving the acceleration. However, the exact nature of dark energy still remains as a mystery [4]. Cosmological constant in the classical FRW model can be a simple candidate for dark energy. But this has fine tuning problem and coincidence problem. Some other candidates proposed to construct dark energy models are quintessence models [5], phantom models [6], K-essence [7] and so on.

There are two major approaches to address this problem of cosmic acceleration either but introducing a dark energy component in the universe and study its dynamics or modifying

Einstein's theory of gravitation termed as 'modified gravity approach'. Hence different modifications of Einstein's theory have been proposed to explain the cosmic acceleration of the universe. Most significant among them are $f(R)$ gravity [8], $f(R, T)$ gravity [9] where R is the Ricci scalar and T is the trace of the energy momentum tensor. Some other alternative theories of gravitation are scalar-tensor theories of gravitation formulated by Brans and Dicke [10] and Saez and Ballester [11]. Here, we focus our attention on Saez – Ballester [11] scalar tensor theory of gravitation.

In Brans-Dicke theory a scalar field ϕ , which has the dimension of the inverse of a gravitational and which interacts equally with all forms of matter, has been introduced in addition to the usual metric tensor field g_{ij} . Subsequently Saez and Ballester proposed a new scalar tensor theory of gravitation. In this theory the metric is compiled with a dimensionless scalar field in a simple manner. This coupling gives a satisfactory description of the weak fields and an antigravity regime appears in this theory. This theory also suggests a possible way to solve the 'missing matter problem' in non – flat FRW cosmologies.

Holographic dark energy is another alternative to the solution of dark energy problem. This is based on the holographic principle. According to this principle the entropy of a system scales not with its volume, but its surface area [12]-[13]. A cosmological version of this principle was proposed by Fischer and Susskind [14] and Cohen et al. [15]. Recently, Grand and Oliveros [16] suggested a new holographic Ricci dark energy model with density $\rho_\Lambda = 3M_{pl}^2 = \eta H^2 + \zeta \dot{H}$. Later, Chen and Jing [17] modified this model by assuming the density of dark energy contains the Hubble parameter H , the first order and the second order derivatives. The expression of the energy density of modified holographic Ricci dark energy is given by $\rho_\Lambda = 3(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1})$. Thus if we take the whole universe into account, then the vacuum energy related to the holographic principle may be viewed as dark energy, usually called holographic dark energy. Hence several authors, in recent years, have investigated holographic dark energy models in anisotropic space-times. This is because of the fact that observational data suggests that the anomalies found in the cosmic microwave background (CMB) simulated increasing interest in anisotropic models. Also, these models will certainly help for a better understanding of the early stages of evolution of the universe. Kiran et al. [18] have discussed minimally interacting holographic Bianchi type-V dark energy models in Saez-Ballester scalar

tensor theory of gravitation. Very recently, Reddy et al. [19] have discussed Bianchi type-IX minimally interacting holographic dark energy model in Saez-Ballester scalar-tensor theory of gravitation. Reddy et al. [20] have discussed Kantowski- Sachs minimally interacting holographic dark energy model with linearly varying deceleration parameter in Saez-Ballester theory. Ramesh and Umadevi [21] have obtained minimally interacting holographic dark energy model in Saez- Ballester theory using linearly varying deceleration parameter proposed by Akarsu and Dereli [22]. Amir and Ali [23] have discussed Ricci dark energy of amended FRW universe in Chern-Simon modified gravity. Ali and Amir [24] have studied holographic dark energy models in Chern-Simon modified gravity. Das and Sultana [25]-[26] have studied Bianchi type-II and VI_0 anisotropic modified holographic Ricci dark energy cosmological models. Santhi et al. [27]-[28] have discussed some Bianchi type modified holographic Ricci dark energy models in general relativity and Brans-Dicke scalar-tensor theory of gravitation. Rao and Prasanthi [29] have studied Bianchi type-I and III modified holographic Ricci dark energy models in Saez-Ballester theory.

The above discussion and the investigation have motivated us to consider the modified holographic Ricci dark energy model in Kantowski-Sachs space – time in the frame work of Saez – Ballester scalar tensor theory of gravitation.

2. Metric and the Field Equations

We consider the Kantowski–Sachs space-time given by

$$ds^2 = dt^2 - A^2 dr^2 - B^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (1)$$

where $A(t)$ and $B(t)$ are the functions of the cosmic time t only.

The field equations given by Saez and Ballester [11] for the combined scalar and tensor fields are

$$R_{ij} - \frac{1}{2} R g_{ij} - w \varphi^n \left(\varphi_{,i} \varphi_{,j} - \frac{1}{2} g_{ij} \varphi_{,k} \varphi^{,k} \right) = - (T_{ij} + \bar{T}_{ij}) \quad (2)$$

and the scalar field φ satisfies the equations

$$2\varphi^n \varphi_{;i}^{,i} + n \varphi^{n-1} \varphi_{,k} \varphi^{,k} = 0 \quad (3)$$

Also we have the conservation equation

$$(T^{ij} + \bar{T}^{ij})_{;j} = 0 \quad (4)$$

where R_{ij} is the Ricci tensor, R is the Ricci scalar, ω and n are arbitrary dimensionless constants and $8\pi G = c = 1$ in the relativistic units. Also the energy momentum tensors for matter T_{ij} and for the anisotropic holographic dark energy $\bar{T}_{ij}$ are defined as

$$\begin{aligned} T_i^j &= \text{diag}[1, 0, 0, 0] \rho_m \\ \bar{T}_i^j &= \text{diag}[\rho_\Lambda, -p_r, -p_\theta, -p_\phi] = \text{diag}[1, -\omega_r, -\omega_\theta, -\omega_\phi] \rho_\Lambda \end{aligned} \quad (5)$$

where ρ_M, ρ_Λ are the energy densities of matter and the holographic dark energy and p_r, p_θ and p_ϕ are the pressures of the holographic dark energy along r, θ and ϕ axes respectively. Here $\omega = \frac{p_\Lambda}{\rho_\Lambda}$ is the equation of state (EoS) parameter of the fluid and

$\omega_r, \omega_\theta, \omega_\phi$ are the EoS parameters in the directions of r, θ and ϕ axes respectively. The energy momentum tensor of holographic dark energy can be parameterized as

$$\bar{T}_i^j = \text{diag}[1, -\omega, -(\omega + \delta), -(\omega + \gamma)] \rho_\Lambda \quad (6)$$

For the sake of simplicity we choose $\omega_r = \omega$ and skewness parameters δ and γ are the deviations from ω along θ and ϕ respectively.

Now the field equations for the metric (1) take the form

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} - \frac{w}{2}\varphi^n\dot{\varphi}^2 = -\omega\rho_\Lambda \quad (7)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{w}{2}\varphi^n\dot{\varphi}^2 = -(\omega + \delta)\rho_\Lambda \quad (8)$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{w}{2}\varphi^n\dot{\varphi}^2 = -(\omega + \gamma)\rho_\Lambda \quad (9)$$

$$2\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}^2}{B^2} + \frac{1}{B^2} + \frac{w}{2}\varphi^n\dot{\varphi}^2 = \rho_\Lambda + \rho_m \quad (10)$$

$$\ddot{\phi} + \dot{\phi} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) + \frac{n}{2} \frac{\dot{\phi}^2}{\phi} = 0 \quad (11)$$

$$\dot{\rho}_m + \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \rho_m + \dot{\rho}_\Lambda + \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) (1 + \omega) \rho_\Lambda + (\delta + \gamma) \rho_\Lambda \frac{\dot{B}}{B} = 0 \quad (12)$$

where overhead dot denotes differentiation with respect to time t .

The following are the physical and geometrical parameters to be used in solving the Saez-Ballester field equations for the space-time given by equation (1). The average scale factor of the Kantowski-Sachs space time is defined as

$$a(t) = (AB^2)^{\frac{1}{3}} \quad (13)$$

Spatial volume is given by

$$V = a^3(t) = AB^2 \quad (14)$$

where $a(t)$ is the average scale factor of the universe.

The average Hubble's parameter H is given by

$$H = \frac{1}{3} (H_1 + H_2 + H_3) = \frac{1}{3} \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \quad (15)$$

where H_1, H_2, H_3 are directional Hubble's parameters in x, y and z directions.

The mean anisotropy parameter Δ is defined as

$$\Delta = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2 \quad (16)$$

where H_i ($i = 1, 2, 3$) represent the directional Hubble parameter .

The scalar expansion θ and shear scalar are given by

$$\theta = \left(\frac{\dot{A}}{A} + 2 \frac{\dot{B}}{B} \right) \quad (17)$$

$$\sigma^2 = \frac{1}{3} \left[\left(\frac{\dot{A}}{A} \right)^2 + \left(\frac{\dot{B}}{B} \right)^2 - 2 \frac{\dot{A}\dot{B}}{AB} \right] \quad (18)$$

3. Solutions of field equations and the cosmological model

From equations (8) and (9), we obtain

$$\delta = \gamma \quad (19)$$

Equations (7)-(11), in view of Eq. (19), reduce to four independent equations in seven unknowns $A, B, \varphi, \delta, \rho_m, \rho_\Lambda$ and ω . Hence we use the following additional conditions to solve the system:

- (i) Shear scalar is proportional to the scalar expansion so that [30]

$$A = B^k \quad (20)$$

- (ii) The hybrid expansion law given by [31]

$$a(t) = (AB^2)^{1/3} = a_1 t^{\alpha_1} e^{\alpha_2 t} \quad (21)$$

and (iii) modified holographic Ricci dark energy density given by [17]

$$\rho_\Lambda = 3(\beta_1 H^2 + \beta_2 \dot{H} + \beta_3 \ddot{H} H^{-1}) \quad (22)$$

Now from Eqns. (20) and (21), we have

$$\begin{aligned} A &= \left(a_1 t^{\alpha_1} e^{\alpha_2 t} \right)^{\frac{3k}{k+2}} \\ B &= \left(a_1 t^{\alpha_1} e^{\alpha_2 t} \right)^{\frac{3}{k+2}} \end{aligned} \quad (23)$$

Also, from Eqs. (11) and (21) the scalar field φ is given by

$$\dot{\varphi} \varphi^{n/2} = \left(\varphi_0 a_0 t^{\alpha_1} e^{\alpha_2 t} \right)^{-3} \quad (24)$$

which on integration can be put in the form

$$\varphi^{\frac{n+2}{2}} = \frac{n+2}{2} \int \left(\varphi_0 a_0 t^{\alpha_1} e^{\alpha_2 t} \right)^{-3} dt + \psi_0 \quad (25)$$

where φ_0 and ψ_0 are constants of integration.

Now the metric (1) can be written as

$$ds^2 = dt^2 - \left(a_1 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6k}{k+2}} dr^2 - \left(a_1 t^{\alpha_1} e^{\alpha_2 t}\right)^{\frac{6}{k+2}} (d\theta^2 + \sin^2 \theta d\phi^2) \quad (26)$$

4. Physical discussion of the model

Equation (26) describes Kantowski-Sachs modified holographic Ricci dark energy cosmological model in Saez- Ballester scalar – tensor theory of gravitation with hybrid expansion law proposed by Akarsu et al. [31].

The following are the expressions for physical and kinematical parameters of the model:

Spatial volume is

$$V = (a_0 t^{\alpha_1} e^{\alpha_2 t})^3 \quad (27)$$

The average Hubble's parameter is

$$H = \frac{\dot{a}}{a} = \frac{\alpha_1}{t} + \alpha_2 \quad (28)$$

The scalar expansion is

$$\theta = 3H = 3\left(\frac{\alpha_1}{t} + \alpha_2\right) \quad (29)$$

The shear scalar is

$$\sigma^2 = \frac{7}{2} \left(\frac{\alpha_1}{t} + \alpha_2\right)^2 \quad (30)$$

The average anisotropy parameter is

$$\Delta = \frac{6(k-1)^2}{(k+2)^2} \quad (31)$$

The deceleration parameter is

$$q = -1 + \frac{\alpha_1}{(\alpha_1 + \alpha_2 t)^2} \quad (32)$$

Fig. 1: Plot of deceleration parameter versus cosmic time t for $\alpha_1 = 0.3$ and $\alpha_2 = 0.7$.

From Eqs. (16) and (28), we have the energy density of modified holographic Ricci dark energy as

$$\rho_\Lambda = 3 \left(\beta_1 \left(\frac{\alpha_1}{t} + \alpha_2 \right) - \frac{\beta_2 \alpha_1}{t^2} + \frac{2\alpha_1 \beta_3}{3t^2(\alpha_1 + \alpha_2 t)} \right) \quad (33)$$

From Eqs. (7), (23) and (33) the EoS parameter of modified holographic Ricci dark energy can be found as

$$\omega = \frac{\frac{6\alpha_1}{(k+2)t^2} - \frac{27}{(k+2)^2} \left(\frac{\alpha_1}{t} + \alpha_2 \right)^2 - \frac{1}{(a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{k+2}}} + \frac{w\varphi_0}{2(a_0 t^{\alpha_1} e^{\alpha_2 t})^3}}{3 \left(\beta_1 \left(\frac{\alpha_1}{t} + \alpha_2 \right) - \frac{\beta_2 \alpha_1}{t^2} + \frac{2\alpha_1 \beta_3}{3t^2(\alpha_1 + \alpha_2 t)} \right)} \quad (34)$$

From Eqs. (8), (23) and (33), we get the skewness parameter as

$$\delta = \frac{\frac{3\alpha_1(k-1)}{(k+2)t^2} - \frac{9(k-1)}{(k+2)} \left(\frac{\alpha_1}{t} + \alpha_2 \right)^2 + \frac{1}{(a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{k+2}}}}{3 \left(\beta_1 \left(\frac{\alpha_1}{t} + \alpha_2 \right) - \frac{\beta_2 \alpha_1}{t^2} + \frac{2\alpha_1 \beta_3}{3t^2(\alpha_1 + \alpha_2 t)} \right)} \quad (35)$$

From Eqs. (10), (23) and (33), we have the energy density of matter as

$$\rho_m = \frac{9(2k^2 + 4k + 1)}{(k + 2)^2} \left(\frac{\alpha_1}{t} + \alpha_2 \right)^2 - \frac{3\alpha_1}{(k + 2)t^2} + \frac{1}{(a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{k+2}}} + \frac{w\phi_0}{2(a_0 t^{\alpha_1} e^{\alpha_2 t})^3} - 3 \left(\beta_1 \left(\frac{\alpha_1}{t} + \alpha_2 \right) - \frac{\beta_2 \alpha_1}{t^2} + \frac{2\alpha_1 \beta_3}{3t^2(\alpha_1 + \alpha_2 t)} \right) \quad (36)$$

The overall density parameter (Ω) is given by

$$\Omega = \Omega_m + \Omega_\Lambda = \frac{\rho_m}{3H^2} + \frac{\rho_\Lambda}{3H^2} = \frac{3(2k^2 + 4k + 1)}{(k + 2)^2} - \frac{1}{(\alpha_1 + t)^2} \left(\frac{3\alpha_1}{k + 2} - \frac{t^2}{(a_0 t^{\alpha_1} e^{\alpha_2 t})^{\frac{6}{k+2}}} - \frac{w\phi_0 t^2}{2(a_0 t^{\alpha_1} e^{\alpha_2 t})^3} \right) \quad (37)$$

Fig. 2: Plot of EoS parameter of MHRDE versus time t for

$\beta_1 = 1.8, \beta_2 = 0.3, \beta_3 = 0.8, w = 2, \alpha_1 = 0.3$ and $\alpha_2 = 0.7$.

Fig.3: Plot of overall density parameter versus time for

$$\beta_1 = 1.8, \beta_2 = 0.3, \beta_3 = 0.8, w = 2, \alpha_1 = 0.3 \text{ and } \alpha_2 = 0.7.$$

Fig. 1 represents the behavior of deceleration parameter with time. It is observed that there is a smooth transition of the universe from early decelerated phase to late time acceleration. Fig. 2 depicts behavior of EoS parameter of MHRDE versus time. It can be seen that the model enters quintessence region from dust model (since $\omega = 0$ initially) and will never cross the phantom divide line (i.e., $\omega = -1$). The variation of average density parameter (Ω) is given in Fig. 3. This shows that Ω initially increases rapidly with time and ultimately becomes equal to one. This shows that the universe becomes flat at late times. All the above results are in good agreement with the observations of modern cosmology.

5. Conclusions

In this paper, we have discussed the modified holographic Ricci dark energy model in Kantowski-Sachs universe in the frame work of scalar-tensor theory of gravitation proposed by Saez and Ballester [11]. We have used the hybrid expansion law and relation between metric potentials to obtain a determinate solution of the field equations. This solution represents the modified holographic Ricci dark energy universe in scalar-tensor cosmology. It is observed that the model is expanding spatially. It can be seen that Hubble's parameter, expansion scalar, shear scalar, scalar field, energy densities of matter and dark energy diverse initially (i.e., at $t=0$) and they all become constant in infinite time. It is well known that scalar fields play a significant role in dark energy cosmology. The results obtained are in accordance with the recent observations of

cosmology and we hope that these results will be useful to throw a better light on our understanding of scalar tensor cosmology.

Acknowledgement: The authors are grateful to the referee for constructive comments which have improved the presentation of this work.

References

- [1]. A. Riess, et al.: *Astron. J.* **116**, 1009 (1998).
- [2]. A. Riess, et al.: *Astrophys.J.* **607**, 665 (2004).
- [3]. S. Perlmutter, et al.: *Astrophys. J.* **517**, 5 (1999).
- [4]. E.J. Copeland, et al.: *Int. J. Mod. Phys. D*, **15**, 1753 (2006).
- [5]. B. Ratra, P.J.E. Peebles: *Phys. Rev. D*, **37**, 321 (1998).
- [6]. R. Caldwell: *Phys. Lett. B* **545**, 23 (2002).
- [7]. T. Chiba, et al.: *Phys. Rev. D* **62**, 023511 (2000).
- [8]. S.M. Carroll, et al.: *Phys. Rev. D* **70**, 043528 (2004).
- [9]. T. Harko, et al.: *Phys. Rev. D* **84**, 024020 (2011).
- [10]. C.H. Brans, R.H. Dicke: *Phys. Rev.* **124**, 925 (1961).
- [11]. D. Saez, V.J. Ballester: *Phys. Lett. A* **113**, 467 (1986).
- [12]. L. Susskind: *J. Math. Phys.* **36**, 6377 (1994).
- [13]. R. Bousso: *JHEP*, **07**, 004 (1999).
- [14]. W. Fischler, L. Susskind: *hep-th/9806039* (1998).
- [15]. A. Cohen, et al.: *Phys. Rev. Lett.* **82**, 4971 (1999).
- [16]. L.N. Granda, A. Oliveros: *Phys. Lett. B*, **669**, 275277 (2008).
- [17]. S. Chen, J. Jing: *Phys. Lett. B*, **679**, 144150 (2009).
- [18]. M. Kiran, et al.: *Astrophys. Space Sci.* **354**, 577 (2014).
- [19]. D.R.K. Reddy, et al.: *Prespace time J.* **7**, 100 (2016).
- [20]. D.R.K. Reddy, et al.: *Prespace time J.* **7**, 300 (2016).
- [21]. G. Ramesh, S. Umadevi: *Astrophys. Space Sci.* **361**, 50 (2016).
- [22]. O. Akarsu, T. Dereli: *Int. J. Theor. Phys.* **51**, 612 (2012).
- [23]. M.J. Amir, S. Ali: *Int. J Theor. Phys.* **54**, 1362 (2015).

- [24]. S. Ali, M.J. Amir: Int. J Theor. Phys. **55**, 5095 (2016).
- [25]. K. Das, T. Sultana: Astrophys. Space Sci., **360**, 4 (2015).
- [26]. K. Das, T. Sultana: Astrophys. Space Sci., **361**, 53 (2016).
- [27]. M.V. Santhi, V.U.M. Rao, Y. Aditya: Can. J. Phys. **95**, 381 (2017).
- [28]. M.V. Santhi, V.U.M. Rao, Y. Aditya: Can. J. Phys. **95**, 179 (2017).
- [29]. V.U.M. Rao, U.Y.D. Prasanthi: Eur. Phys. J. Plus **132**, 64 (2017).
- [30]. C.B.Collins, et al.: Gen. Relativ. Gravit. **12**, 805 (1983).
- [31]. O. Akarsu, et al.: JCAP, **01**, 022 (2014).

For Review Only