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1 Particle breakage in granular materials – a conceptual 2 framework

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8 Abstract

9 A simple, yet complete framework is introduced with the aim of modelling grains breakage in
10 soils and crushable granular materials. The evolution of grain breakage is measured using a
11 specific parameter of the grain size distribution. The evolution of this new breakage parameter is
12 related to the applied mechanical work, which allows the predictions to be independent of the
13 stress paths. The correlation function proposed is tri-linear, and is capable of describing the
14 initiation, the development and the stabilisation of breakage. The initial state, coupled with three
15 additional parameters are used to calibrate this function. The three parameters are related to a
16 grain specific quantity representing the strength of the particles that form the granular medium.
17 The theory of fractal fragmentation is adopted, and the final state is considered to be unique, and
18 described by a single parameter: the fractal dimension. When tested against experimental results,
19 this model was able to correctly predict the crushable behavior of a sand.

20 **Keywords :** Particle breakage, granular media, grain crushing, fractal, particle strength, grain
21 size distribution

22 Résumé

23 Un cadre mécanique à la fois simple et complet est introduit dans le but de modéliser la rupture
24 des grains dans les sols et autres milieux granulaires broyables. L'évolution de la rupture des
25 grains est mesurée en utilisant un paramètre caractéristique de la distribution granulométrique.
26 L'évolution de ce nouveau paramètre de rupture est reliée au travail mécanique appliqué, ce qui
27 rend les prédictions indépendantes du chemin des contraintes. La fonction de corrélation proposée
28 est tri-linéaire, et donc capable de décrire l'initiation, le développement et la stabilisation de la
29 rupture. L'état initial, couplé à trois autres paramètres servent à calibrer cette fonction. Les trois
30 paramètres sont reliés à une quantité propre au grain, et qui représente la résistance des particules
31 qui forment le milieu granulaire. La théorie de la fragmentation fractale est adoptée, et l'état final

- 32 est donc supposé comme étant unique, et décrit par un seul paramètre : la dimension fractale.
- 33 Lorsque le modèle est testé et les résultats comparés à des travaux expérimentaux, il devient
- 34 possible d'apprécier la capacité du modèle à simuler le comportement d'un sable.

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Introduction

Granular materials are of particular interest for civil engineers and soil mechanics specialists. Over time, the integrity of these materials varies depending on the mechanical conditions, the nature of the grain, and the structure of the granular packing. For example, significant particle breakage is observed when the material is subjected to high stresses. The same amount of crushing can develop for lower energies when the grains are weak. On the other hand, breakage is more pronounced in a uniformly graded sample when compared to a well-graded sample. Hence, a thorough understanding of the mechanics of these types of materials is essential in order to optimize the lifetime of a structure and its aptitude for service.

When examining particle breakage, much of the attention has been concentrated on the plasticity theories. Breakage is introduced as a hardening parameter (Daouadji et al. 2001; Einav 2007a; Wood et al. 2009), or is calculated from the stress parameters that are computed independently (Hu et al. 2011). In addition, breakage is considered to be a plasticity indicator, and starts with the onset of plasticity, which does not represent the reality of the phenomenon. The development of breakage continues indefinitely (with the loading). These approaches present the disadvantage of neglecting important effects such as the strength of the particles, the initial grading, or the stabilisation of the process. The main objective of this paper is to develop and validate a conceptual model capable of relating the mechanical conditions to the corresponding amount of grain breakage, based on representative parameters of the grains and the initial grading.

This research takes advantage of existing theories and experimental observations to propose a simple mechanical model, able to estimate the development of grain breakage in a soil. The method presented in this paper presents many advantages. It provides a simple way of relating the strength of the particles to the parameters of a function that associates a breakage parameter to the mechanical state. The proposed breakage parameter is capable of describing the evolution of the soil's grading as a function of the imposed loading. This model can be useful for engineers that

60 are looking for a simple technique to estimate breakage based on a basic knowledge of the soil
61 properties and the mechanical conditions.

62 **Background**

63 One of the most promising recent trends in soil mechanics has been the development and
64 application of multiscale techniques to study the mechanical behavior of soils. The grain level
65 properties and interactions become essential parameters in determining the macro-scale behavior.
66 For some problems where the effect of the microstructure is apparent, a multiscale approach is
67 not only interesting, but can become necessary for a complete analysis. For example, when
68 investigating grain crushing in granular materials, the breakage of a single grain is affected by
69 micromechanical conditions like the surrounding grains and the load carried by the force chain.
70 The fracturing of this grain goes on to affect the granular medium in a variety of ways (increased
71 compressibility, decreased peak shear strength ...).

72 In order to study this phenomenon from a constitutive point of view, multiple multi-scale
73 approaches have been proposed, each benefiting from one or more aspects of the effects of
74 breakage. Daouadji et al. (2001), who proposed one of the first constitutive models that consider
75 the effect of breakage, based their investigation on the effect grain breakage has on the critical
76 state line. This observation was the basis for many other constitutive models (Russell and Khalili
77 2002; Hu et al. 2011; Wood et al. 2009). The framework proposed by Einav (2007b) is based on a
78 yielding condition that derives from the definition of Einav's breakage parameter (Einav 2007b),
79 coupled to some other hypotheses.

80 When examining grain crushing and its effects on the behavior of a soil, two main issues have to
81 be addressed. The first issue deals with the definition of a variable capable of measuring the
82 extent of breakage, and the second concerns the ways to couple it to the mechanics of the
83 problem. The grading index chosen should be representative and relatable to the grain size

distribution (GSD), while the mechanical correlation has to be stress path independent for a complete description of the constitutive state of the material.

Particle tensile strength

When considering the failure of a mineral granular material under mechanical loading, it is natural to assume that the nature and properties of the mother rock plays a determining role in defining the strength of the particles. Many authors conducted crushing tests on rocks and rock forming minerals, which allowed them to classify the minerals by order of resistance (Roy 1970; Billam 1971). It is however, difficult to relate the strength of a rock aggregate to the strength of the minerals that are part of its composition. Indeed, many microstructural parameters affect the fracturing of a grain, like the distribution of the micro-cracks and voids, the size of the minerals, their distribution, their size and exact nature. The effect of other mineralogical aspects like the type of bonds between the minerals, the degree of metamorphism and other development and alteration conditions greatly affect the mechanical behavior of the rock particle, and can be problematic to study and model.

One simple way of examining the failure of a rock particle is through simple tests that determine the tensile strength of the grain. By doing so, we consider that the dominant breakage mode is the opening tensile fracture mode. The two shearing fracture modes (II and III) that are responsible for the production of fines through attrition and abrasion are consequently neglected. The tensile strength of a grain is usually measured by Brazilian type techniques like the point load test, or the diametric compression under two parallel platens. For these types of tests, the tensile strength is related to the critical force at failure F_c by (Jaeger 1967):

$$\sigma_c = \frac{F_c}{d^2} \quad (1)$$

where d is a characteristic size of the rock particle. On a force-displacement plot (figure 1), the first peaks are usually caused by the crushing of angular contacts or asperities, or by spalling, and

are usually neglected. The critical force is measured when the particle effectively splits into two or a number of fragments with comparable sizes. On figure 1, this corresponds to diametral breakage at a critical load of 2.7 kN.

Figure 2 is a compilation of the tensile strength of multiple particles of various sizes. The data was compiled by Soga (2005) after the experimental results of Lee (1991), McDowell and Amon (2000) and Nakata et al. (2001). The figure shows that for the same material, larger particles tend to be weaker, while smaller ones are stronger. This is because the larger particles contain more micro cracks and voids, which allows the particles to break more easily.

It should be noted that the breakage of a rock aggregate is affected by multiple factors. The form of the grain can be cited as an example, where important geometrical aspects like the sphericity, roundness and surface texture determine the failure behavior of the grain, and the characteristics of the fragments (Cavarretta 2009). For instance, a 1 mm diameter angular glass particle displays a tensile strength of about 60 MPa while a rounded glass particle has a tensile strength of 220 MPa, i.e., almost 4 times greater (figure 2). This is due to the smaller contact area in angular grains compared to rounded grain, which amplifies the concentration of forces near the contacts, and results in a decreased strength (Salami et al. 2017). Other important aspects that should be considered when researching the fracture mechanics of mineral granular aggregates are the coordination number, the contact dynamics and the presence of corrosive elements (water was shown to accelerate the development of a crack (Atkinson 1987)).

Evolution of GSD by particle breakage

One of the main challenges that arise when considering grain breakage from the packing level is to correctly quantify the amount of particle breakage that occurs in the granular medium. Many authors proposed multiple measures of grain breakage that are generally related to the evolution of the grain size distribution (GSD) (Lee and Farhoomand 1967; Marsal 1973; Hardin 1985;

Einav 2007b). These breakage parameters can then be correlated with mechanical and/or hydraulic parameters through empirical formulas, which would allow modelling of the behavior of soils at extreme conditions, or during extended periods of time.

One simple way to measure the extent of breakage in a soil is to examine the evolution of one of the form factors of the GSD (the most representative example being the coefficient of uniformity C_u (Colliat-Dangus et al. 1988)). The slope of the linear evolution of the GSD in a log-log plot was found to be reasonably representative of this evolution (log refers to the decimal logarithm). Figure 3 is a plot of the evolution of GSD from its initial to an ultimate GSD for an assemblage of gneissic particles subjected to oedometric compression. The classical semi-logarithmic plot of figure 3-a can be transformed to the log-log plot of figure 3-b, where it is reasonable to consider that the GSDs are well represented by a linear function, particularly towards the ultimate state. Assuming knowledge of the largest size fraction d_{100} of the granular packing, the slope of the linear GSDs in the log-log scale is sufficient to fully characterize the GSD by its slope ν .

It should be noted that ν suffers the same limitations as other breakage parameters in the sense that a single parameter cannot fully describe the GSD. Marsal's (1967) index cannot be related to a granular distribution, while Hardin's (1985) parameter limits the changes in the GSD to the higher size fraction by stating that the sizes lower than 0.074mm do not contribute to breakage. Many other shape parameters for the GSD ignore certain size fractions (the coefficient of uniformity, crushing coefficient, Lee and Farhoomand (1967) parameter ...). Other parameters are fundamentally related to a constitutive framework, and would be difficult to adapt to this study (Einav (2007b); Kikumoto et al. (2010) ...). The linear approximation in the logarithmic space offers an acceptable description of the GSD without ignoring any size fraction.

The constitutive models that consider the effect of grain breakage generally adopt a hardening rule that relates a breakage parameter to a stress variable, generally the mean effective stress p or the pre-consolidation stress p_c (McDowell et al. 2002; Einav 2007a; Kikumoto et al. 2010). This

provides a straightforward way to include the effect of breakage through a single mechanical parameter. Since most of these models borrow some aspects of the critical state theory and because the effects of grain breakage on the critical state behavior are well documented (Biarez and Hicher 1994), the stress parameter p is usually a good candidate for a mechanical correlating parameter from a constitutive view. It was however shown that the breakage variable cannot be related to a single stress parameter for all stress paths. The experimental results of Roy (1970) on Quebec limestone (figure 4) show that a considerable amount of grain breakage occurs along the triaxial paths, as confirmed by the important changes in the uniformity coefficient (in the order of 70% when p reaches 12 MPa). The relatively smaller changes in the uniformity coefficient (less than 10%) for that same stress when a comparable specimen is subjected to isotropic consolidation confirm that a single stress parameter, in this case the mean effective stress, does not correlate well enough with the breakage parameter.

In order to couple the mechanical aspects of the soil's behavior with the evolution of grain breakage, many authors proposed a correlation between the amount of breakage and a quantity representing some form of energy input, transformation or dissipation. Miura and O-Hara (1979), who used the surface area as a breakage parameter, established an increasing relationship between the plastic work and the increase of surface area. The function that relates the two parameters was shown to be an 'S' type curve, meaning that breakage begins developing at the earliest stages of the loading, before increasing in rate for the intermediate energies, and then decreasing for the higher energies. The 'S' function exhibits the typical behavior for the development of breakage observed in a crushable granular material, which can be described in three stages:

- An early stage, where the development of grain breakage is negligible;
- An intermediate stage, where the rate of development of grain breakage is at the maximum. The concavity of the 'S' function in this region is subject to much debate, and

the modelling of this evolution takes many forms (see later for a detailed discussion on the importance of a good description of this region for constitutive models).

- A final stage, where grain breakage tends to stabilize. This is consistent with observations of the fragmentation processes in general, that state that the function (size the fragment – number of fragments in this size fraction) converges towards a unique, self-similar distribution.

This three-stage behavior was later confirmed by many authors (Biarez and Hicher 1994; Lade et al. 1996; Daouadji et al. 2001; Russell and Khalili 2002; Einav 2007a; Kikumoto et al. 2010). It is important to note that even though this behavior might look similar to the compressibility curve of a soil (volume-pressure relationship), they represent different concepts that are difficult to relate. The initiation of breakage and its stabilisation are not the result of the first and second inflexions of the compressibility curve (the transition to the normal compression line, and to the critical state line).

Lade et al. (1996) proposed correlating Hardin's (1985) breakage parameter to the total energy input instead of the plastic energy as suggested by Miura and O-Hara. This was justified by the fact that very little elastic energy develops in a specimen with crushable particles. They proposed a simple way of calculating the total work input, which is then used to calculate the energy input for multiple tests on Cambria sand compiled from the literature. The total energy and Hardin's breakage parameter are related through a single hyperbolic function, which is similar to the 'S' function described earlier (a stage where breakage develops rapidly followed by a relative stabilisation of breakage).

The results of Roy, which were used to show that the stress parameter p did not correlate well with a breakage parameter (see figure 4) are reevaluated in light of the findings of Lade et al. (1996). The work input was calculated from the stress-strain curves using the formula proposed

by Lade et al. (Op.Cit.). Figure 5 shows that the work per unit volume correlates well with the coefficient of uniformity (i.e. the evolution of GSD) independently of stress path.

Conceptual framework for the prediction of particle breakage in granular materials

The characteristic behavior described in the previous paragraph is used here as a reference for defining a conceptual framework able to describe and predict the evolution of breakage, independent of the stress path. This is achieved by correlating the breakage parameter (in this case ν , the slope of the GSD in the log-log space) to the input work per unit volume. The work per unit volume is calculated using eqn 2, similar to that proposed by Lade et al. (1996):

$$\frac{W}{V} = \sum_0^f (\sigma_c \dot{\epsilon}_v) + \sum_0^f (\sigma_1 - \sigma_3) \dot{\epsilon}_a \quad (2)$$

where W represents the total work, V the volume of the element considered, σ_c the confining pressure, $(\sigma_1 - \sigma_3)$ the deviatoric stress, $\dot{\epsilon}_v$ the increment of volumetric strain, and $\dot{\epsilon}_a$ the increment of axial strain. The summation is applied between an initial state 0 and a final state f .

According to the behaviour of crushed limestone (Lo and Roy 1973), the evolution of breakage in any granular material can be approximated by three distinct phases in a $\log \nu - \log \left(\frac{W}{V} \right)$ space as illustrated by Figure 6 describing the initiation, development and stabilization of grain breakage as a function of input work. For a given material characterized by an initial GSD, ν_i , negligible grain breakage will occur in the first phase from A to B until the input work reaches a critical value W_{crit} i.e. at the initiation of grain breakage. When the work applied exceeds W_{crit} , ν decreases with increasing work increments. In phase 2 where significant grain breakage occurs, the representative point moves from B to C. During phase 2, the GSD is increasingly spread out over the finer fractions, since breakage causes larger particles to crush into smaller ones. At Point C, the intensity of breakage will stabilize and the GSD reaches an ultimate distribution

characterized by the parameter ν_u . It is assumed that any work increment larger than W_{crit} will not produce further grain breakage (path CD).

In order to characterize the evolution of ν as a function of the applied work, four parameters are required (Figure 7):

- ν_i : which represents the slope of the initial GSD in a log-log plane;
- ν_{ref} : a reference value of ν at a reference value of $\frac{W_{ref}}{\nu} = 0.01 MPa$;
- α : the slope of segment BC;
- ν_u : the ultimate GSD. When ν reaches this ultimate value, a stabilisation of crushing is observed.

The conceptual model applies for any GSD, which satisfies the condition $\nu_i < \nu_{ref}$ and can be described by the following equations:

$$\begin{aligned} W < W_{crit} & \quad \nu = \nu_i \\ W \geq W_{crit} & \quad \log\left(\frac{\nu}{\nu_i}\right) = -\alpha \log\left(\frac{W}{W_{crit}}\right) \\ W_{crit} & = 0.01(\nu_{ref}/\nu_i)^{\frac{1}{\alpha}} \quad (MPa) \end{aligned} \quad (3)$$

The parameters ν_{ref} and α depend on the micromechanical and mineralogical properties of the granular material. The parameter ν_u can be related to the fractal dimension of the material and will be discussed in more details below.

In order to establish a correlation between ν_{ref} and α , and a parameter representing the nature of the rock, an extensive study has been conducted based on published results on the breakable behavior of several granular materials. Since it has been shown that the tensile strength of the rock aggregate is a good indicator of the influence of the nature of the rock on the mechanical behavior of the granular packing, the strength of the size fraction d_{50} (the size corresponding to 50 % passing by mass) was considered as a representative parameter of the mineralogy of the

rock. The values of ν_{ref} and α corresponding to each of the materials analysed herein were determined based on the experimental data and plotted against the tensile strength of the grain size fraction d_{50} (figure 8). The data were compiled from various sources (Al_2O_3 , Quartz, and crushed limestone from Roy (1970), Kenamu sand from (Leclerc 1998), Ottawa sand from (Hagerty et al. 1993), carbonate sand from (Coop et al. 2004) and Cambria sand from (Yamamuro and Lade 1993). The data for Romaine granite were obtained from an experimental investigation conducted for this study. Additional tensile strength tests were also conducted on Al_2O_3 particles and added to the data on figure 2.

It is clear from figure 8 that both ν_{ref} and α can be approximated by a logarithmic function when these parameters are related to the tensile strength of the average particle size d_{50} . This function is decreasing in the case of parameter α and increasing for parameter ν_{ref} . The best fit trend lines yield two empirical formulas given by eqn 4 that allow the calculation of parameters ν_{ref} and α based on the tensile strength of the particle d_{50} , which is noted $\bar{\sigma}_t$:

$$\begin{aligned}\alpha &= -0.061 \ln(\bar{\sigma}_t / \bar{\sigma}_{ref}) + 0.65 \\ \nu_{ref} &= 2.061 \ln(\bar{\sigma}_t / \bar{\sigma}_{ref}) + 2.17\end{aligned}\tag{4}$$

The $\ln$ refers to the natural logarithm. $\bar{\sigma}_{ref}$ is introduced for the purpose of homogeneity, and is considered to be equal to 1MPa. The third parameter of the model, ν_u , will be discussed in more details below. Many other parameters like the evolution of particle shape are neglected by equations (4), which might explain the scatter from the logarithmic trend lines.

Prediction of the evolution of the GSD and fine content

The capabilities of this new conceptual model are tested through the simulation and prediction of the coupled mechanical-crushable behavior of Hostun sand ($d_{max} = 0.37$ mm, $d_{10} = 0.21$ mm, 10% finer than d_{10}) subjected to various stress paths (Colliat-Dangus 1986; Colliat-Dangus et al. 1988). The objective of this analysis is to estimate the extent of breakage in a given soil based on the

mechanical conditions and the particle's properties inferred from the relationships presented in figure 8. The experimental data reported by Colliat-Dangus (1986) comprised five isotropic compression tests on loose and dense samples, two oedometric compression tests on a dense sample, and seven triaxial compression tests on dense samples. The data for Hostun sand are summarized in figure 9. Figure 9a shows the results of the isotropic consolidation tests for loose and dense samples, figure 9b presents the compressibility curves for two oedometric tests on dense sand samples and figure 9c illustrates the stress-strain response of drained triaxial compression tests. No other data on the evolutions of the GSDs, besides the initial GSD (the same for all the tests), was provided but the amount of crushing was measured in terms of a crushing coefficient introduced by the authors. The crushing coefficient C_c was defined as the fraction finer than the characteristic size d_{10} associated with the initial GSD, in a cumulative distribution by mass, divided by 10:

$$C_c = \frac{\% \text{ finer than the initial } d_{10}}{10} \quad (5)$$

The evolution of the crushing coefficient for all the tests analysed herein is presented in figure 10. The parameter ν is calculated using the formula:

$$\nu = \frac{\log(100) - \log(10C_c)}{\log(0.37) - \log(0.21)} \quad (6)$$

The stress-strain curves of figure 9 are used to calculate the mechanical work using the formula (2).

In order to define the parameters of the model, the strength of a particle with a size corresponding to d_{50} is obtained from figure 2 for a characteristic size associated with 50% passing by size, i.e., 0.4mm considering a quartz particle which yields a corresponding strength of 120MPa. Equations (4) provide the values of $\alpha = 0.34$ and $\nu_{ref} = 12.07$ which, in turn, gives the conceptual breakage characteristics shown in figure 11 in the $\left(\log \nu - \log \frac{w}{\nu}\right)$ space.

288 The initial grain size distribution yields an initial value of ν equal to $\nu_i = 4.065$ and gives a value
 289 of the critical work for the initiation of breakage of 0.25 MPa according to eqns (3).

290 Figure 11 is a plot of the experimental data for all the tests considered compared to the proposed
 291 model summarized by eqns (3) and (4). A good agreement is achieved between the experimental
 292 and predicted results.

293 This model, which is based on the work as a measure of the mechanical state, has the advantage
 294 of providing a single mechanical-crushable correlation for all stress paths.

295 **Ultimate grain characteristics**

296 When a granular material is subjected to extremely high stress conditions, the grain size
 297 distribution becomes self-similar, and can be correctly estimated by a fractal distribution. This
 298 means that the fragmentation process become scale invariant. Turcotte (1997) gathered the results
 299 of many experimental studies of the frequency-size distribution of mineral fragments resulting
 300 from high impact events like explosions or projectile impacts. He was able to correctly approach
 301 the grain size-frequency distribution by the fractal power relation:

$$N(d) = f(d^{-D}) \quad (7)$$

302 $N(d)$ = the number of fragments of size d , and D = the fractal dimension of the distribution. It
 303 was shown that the fractal dimension for granular mineral materials has a value of about 2.5. This
 304 relation can then be used to relate the cumulative distribution by mass $F(d)$ for a size fraction, to
 305 the characteristic size of this fraction d , the size of the biggest grain in the distribution d_M , and
 306 the fractal dimension D (Turcotte 1986):

$$F(d) = \left(\frac{d}{d_M}\right)^{3-D} \quad (8)$$

Alternatively, when the fractal distribution is reached, the parameter ν is equal to its ultimate value ν_u . The passing fraction by mass for a size fraction $F(d)$ is by definition, related to the parameter ν_u by the relation:

$$\log(F(d)) = \nu_u \log\left(\frac{d}{d_M}\right) \quad (9)$$

From the two expressions (8) and (9), the parameter ν_u can be related to the fractal dimension D by: $\nu_u = 3 - D$. Rather than using the conventional fractal dimension D , it is proposed herein to use the breakage parameter ν_u . In figure 12, the value of ν at its ultimate state or near it, ν_u , has been plotted against the particle strength for several materials of different mineralogy and different sizes. Similarly to parameters α and ν_{ref} , a unique relationship can be established between ν_u and the average particle strength, $\bar{\sigma}_t$, independently of mineralogy:

$$\nu_u = 0.190 \ln(\bar{\sigma}_t / \bar{\sigma}_{ref}) - 0.14 \quad (10)$$

It should be noted that it is quite difficult to experimentally reach a fractal distribution. Sometimes, a partial stabilisation of the process of breakage results in the faulty assumption that the fractal state is attained, which can lead to an underestimation of the fractal dimensions. This might explain the scatter observed in figure 12.

A working example: Marsal's experiments on Granitic Gneiss

Marsal (1967) was probably one of the first to perform large-scale triaxial tests on rockfill materials (the cylindrical specimen was 113cm in diameter and 250cm in height). The material used in his study was composed of granitic gneiss particles resulting from quarry blasts. Out of the two gradations considered in the original work, the well-graded distribution is considered, where the biggest grain had a characteristic dimension of 200 mm, while the dimension corresponding to a 10% passing was 4mm and the mean particle size was 60 mm. According to figure 2 and from strength data reported by Marsal, the strength of a 60mm particle is about

328 $\bar{\sigma}_t = 3 \text{ MPa}$. The values of α , v_{ref} and v_u corresponding to this value of particle strength are
 329 found to be 0.58, 4.434 and 0.069, respectively.

330 Based on these values, the critical work is $W_{crit} = 0.24 \text{ MPa}$. It is also possible to define a value
 331 of the work W_u , associated with the stabilisation of breakage, and indicating the start of the third
 332 stage of the tri-segmented behavior:

$$W_u = W_{crit} \left(\frac{v_u}{v_i} \right)^{-\frac{1}{\alpha}} \quad (11)$$

333 which yields a value of $W_u = 12.71 \text{ MPa}$.

334 Figure 13 is a plot of the two triaxial tests in the $\left(\log(v), \log\left(\frac{W}{v}\right) \right)$ space reported by Marsal
 335 (1967) and compared to the proposed model (solid line). The first point corresponds to the initial
 336 $v_i = 0.69$ at the reference work $\frac{W_{ref}}{v} = 0.01 \text{ MPa}$, which represents to the initial GSD. Test 1 is
 337 a drained triaxial test with a confining stress of $\sigma_3 = 0.5 \text{ MPa}$. At the end of the test, the shear
 338 stress reached a value of $\sigma_1 - \sigma_3 = 1.6 \text{ MPa}$, and the volumetric strain was $\varepsilon_v = 3\%$ (dilation).
 339 This test did not cause considerable change in the GSD, which is clear from the unchanged value
 340 of v . Test 2 is a triaxial test conducted on a similar initial GSD as test 1, and with a confining
 341 stress of $\sigma_3 = 2.14 \text{ MPa}$. The shear stress developed by the end of this test was $\sigma_1 - \sigma_3 =$
 342 5.4 MPa , which led to higher work development, and resulted in particle crushing. The model
 343 was able to represent the difference between the two tests: the first test did not develop enough
 344 work to cause particle breakage, while the second test went beyond the critical work, hence
 345 developing breakage. It is important to note that none of the two tests came close to the
 346 stabilisation work, which means that the material had an even bigger breakage potential. To reach
 347 the final state where the GSD is self-similar, an additional 12 MPa of applied mechanical work
 348 was needed by the system.

349 **Influence of initial GSD on particle breakage**

The influence of the strength of the particles plays a significant role in defining the behavior of the granular material with respect to breakage under loading. The multiscale approach adopted by this model is representative of the effects of the grain on the behavior of the packing. On the one hand, a grain specific parameter, chosen to be the tensile strength of a representative particle d_{50} , is related to the three parameters of the model α , ν_{ref} and ν_u . This means that the initiation, the development and the stabilisation of breakage are all dependent on the particle strength. On the other hand, the initial grain size distribution is taken into account through the parameter ν_i , which constitutes a measure of the potential of breakage available in the system, and when coupled with the other parameters, is a key factor influencing the behavior of the system.

Tests conducted on Ottawa sand where the behavior of two granular distributions of the same material was compared can be used to illustrate the importance of the initial grain size distribution on the amount of breakage. The parameters of Ottawa are the following:

$$\begin{aligned} d_{50} &= 0.7\text{mm} ; \bar{\sigma}_t = 120\text{ MPa} \\ \alpha &= 0.35 ; \nu_{ref} = 12.04 ; \nu_u = 0.77 \end{aligned} \quad (12)$$

The value for the $\bar{\sigma}_t$ was interpolated from figure 2. The results of one oedometric test (Hagerty et al. 1993), and two oedometric tests (Roberts and de Souza 1958) were analysed. The initial GSD of the test by Hagerty et al. was $\nu_i = 5.12$, while the initial GSDs for the tests by Roberts and de Souza were $\nu_{i1} = 3.38$ and $\nu_{i2} = 1.59$, respectively. For all the tests, the GSDs were measured at different stages of the loading, which explains the number of data points plotted in figure 14.

Let us compare the behavior of the two tests by Roberts and de Souza. The initial parameter ν_{i2} associated with the well-graded packing is smaller than ν_{i1} which is associated with the uniformly graded material. From figure 14, it is clear that the initiation of breakage occurs at a lower value of W/V for uniform Ottawa sand ($W_{crit}/V = 0.1\text{ MPa}$) compared to $W_{crit}/V = 2.2\text{ MPa}$ for well-graded Ottawa sand. This can be explained from a grain level point of view, where breakage

is controlled by two competing parameters: the strength of the grain and the coordination number (McDowell and Bolton 1998). Since this model does not account for the changes in strength associated with the decrease in grain size that accompanies grain breakage, the effect of the coordination number can be used to explain the difference in the critical work for the initiation of crushing. In a well graded material, the bigger and weaker grains are 'shielded' by the smaller and stronger grains. This configuration is the least favorable for the development of breakage, and explains the delayed initiation of breakage. The uniform material is made of grains with comparable sizes and strengths. The coordination number is usually small in this type of distribution, which causes the grain to be in contact with a smaller number of other grains. This mechanical configuration increases the chance of fracture compared with a grain with a higher coordination number. After the initiation of breakage, the two granular distributions behave in the same way, which is consistent with experimental observations.

The data reported above demonstrates also that the proposed model is able to consider adequately the experiments from different authors on the same material under different conditions, since both sets of results follow a unique breakage constitutive equation. This goes to show that the parameters α and v_{ref} are characteristic to the material, and are independent of the initial distribution v_i .

Influence of particle strength

It is also important to analyse the effect of the particle strength on the development of breakage. Figure 15 shows the evolution of the GSDs of different materials with different particle strengths but with the same initial distribution and under identical loading conditions (Lo and Roy 1973). As anticipated, the higher the particle strength, the less breakage for the same amount of input work per unit volume. For the weaker particle, i.e. limestone, the final value of v is 0.25, that of quartz is 0.41 while for the stronger Al_2O_3 it is 1.03.

The particle strength can be affected by its size or its mineralogy. If we compare the parameters α , ν_{ref} and ν_u for two materials with different particle strengths, the stronger material will have a higher ν_{ref} , a lower α and a higher ν_u (figure 16). If we take the example of a rockfill, where the particles are weaker due to their important size, the smaller ν_{ref} and the higher α causes cracking to develop early and more rapidly. The parameter ν_u is also smaller for these type of granular media. In contrast, a laboratory sand (usually siliceous) will have a much higher particle strength than the rockfill, which has the opposite effect of delaying the development of crushing, and decreasing the rate of development of the process.

Conclusions

In this paper, a simple model was proposed with the aim of predicting the evolution of particle breakage based on the mechanical conditions, the initial gradation, and the strength of the grain. A breaking parameter representative of the grain size distribution (GSD) related to the mechanical work per unit volume allows to predict the evolution of GSD during any loading condition.

The proposed conceptual model is representative of many aspects of the soil's behavior that are usually ignored by the existing breakage models. For example, the grain strength plays a determining role in controlling the parameters of the proposed model. The initial gradation is also very important for estimating the critical mechanical state where breakage initiates. The stabilisation of the particle breakage is also considered using the fractal fragmentation theory, and relationships are established between the fractal dimension, the ultimate parameter ν_u and the strength of the particles.

This model was essentially developed for engineering applications. By its use of common parameters and techniques, it differentiates itself from the existing models. Using the mechanical properties and the stress-strain curves of the soil, the user is able to correctly estimate the extent of breakage in the soil, thus controlling its effects on the structure. This model can also be

integrated into a complete constitutive model. One way to do it would be to introduce the tri-linear behavior as a work hardening function.

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Notations

The following symbols are used in this paper:

C_u	=	Coefficient of uniformity
C_c	=	Crushing coefficient
D	=	Fractal dimension
$d ; d_{\%}$	=	Particle size; The size corresponding to the % passing by mass
$F(d)$	=	The cumulative grain size distribution by mass
p, q	=	The triaxial stress parameters
$W ; W_{crit} ; W_u$	=	The applied mechanical work; The critical work ; The ultimate work
α	=	The slope of the breakage development segment in the model
$\nu ; \nu_i ; \nu_{ref} ; \nu_u$	=	The slope of the GSD in the log-log space; initial; reference; ultimate values
$\bar{\sigma}_t$	=	The tensile strength of a particle of size d_{50}

Figure captions

Figure 1. Force-displacement curve for the granitic particles.

Figure 2. Tensile strengths for multiple rocks and mineral particles.

Figure 3. Definition of the new breakage parameter.

Figure 4. Evolution of the coefficient of uniformity C_u as a function of the effective isotropic mean stress p .

Figure 5. Evolution of the coefficient of uniformity C_u as a function of the total input work per unit volume.

Figure 6. Data for crushed limestone (Lo and Roy 1973) as a function of input work per unit volume.

Figure 7. (a) Conceptual Breakage Model for an assemblage of particles (b) Evolution of parameter ν between its initial and ultimate values.

Figure 8. Evolution of parameters α (a) and ν_{ref} (b) as functions of the tensile strengths $\bar{\sigma}_t$ for a variety of minerals.

Figure 9. Mechanical tests on Hostun sand (after Colliat-Dangus (1986) and Colliat-Dangus et al. (1988)) : (a) isotropic compression, (b) oedometric tests and (c) triaxial shear paths (CID).

Figure 10. Evolution of the crushing coefficient for Hostun sand.

Figure 11. Comparison of Model predictions and experimental data for Hostun sand.

Figure 12. Evolution of ν_u as a function of the tensile strength $\bar{\sigma}_t$ for a variety of minerals.

Figure 13. Model Predictions for Marsal's (1967) granitic gneiss.

Figure 14. Effect of the initial grading on the onset of breakage for Ottawa sand.

Figure 15. Final value of ν for different materials at the same initial grading with different particle strengths.

Figure 16. Effects of particle strength on the parameters of the proposed model.

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