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KINEMATIC ANALYSIS OF A NEWLY DESIGNED CABLE-DRIVEN MANIPULATOR

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ABSTRACT

In this paper, the configurations of cable routings for cable-driven manipulator are introduced, and the impact of motion coupling caused by cable transmission routing of $2n$ type cable-driven manipulator is analyzed in details. Based on different configurations of cable routings, the relationship between variation of joints angles and the geometrical sizes of guide pulleys is established with the representation by a matrix for coupled motion. Meanwhile, based on the effects of the motion coupling of cable-driven manipulator, we proposed the condition for the invariance of orientation, which can be achieved by the constraint of the geometrical sizes of guide pulleys and driven wheels. In addition, in order to identify the correctness of analysis for coupled motion, a 3-DOFs planer cable-driven manipulator prototyping model is constructed, and the kinematics and trajectory planning has been solved. Finally, the relationship among actuator space, joint space, and Cartesian space, including the mapping of the motion coupling is experimentally validated. And the property of invariance of orientation is also passed validation by an experiment.

Keywords: cable-driven manipulator; motion coupling; orientation invariance.

RÉSUMÉ

Dans cet article, les configurations des routages des câbles pour le manipulateur à câble sont introduites ainsi que l'impact du couplage de mouvement provoqué par le routage de transmission câblée du manipulateur à câble de type $2n$ est analysé de manière précise. Basée sur de différentes configurations de routage des câbles, la relation entre la variation des angles des articulations et les dimensions géométriques des poulies de guidage est établie par le biais de la représentation par une matrice pour le mouvement couplé. En même temps, en reposant sur les effets du couplage de mouvement du manipulateur à câble, nous avons proposé la condition d'invariance d'orientation, qui peut être réalisée par la contrainte des dimensions géométriques des poulies de guidage et des roues motrices. En outre, afin d'identifier l'analyse pour le mouvement couplé, un modèle prototypique à manipulation à câble à rabot de 3-DOFs est construit et la planification de la cinématique ainsi que de la trajectoire a été résolue. Enfin, la relation entre l'espace des actionneurs, l'espace articulaire et l'espace cartésien, y compris la cartographie du couplage du mouvement, est validée de façon expérimentale. Également, la propriété de l'invariance d'orientation est validée par une expérience.

Mots-clés : Manipulateur câblé; couplage de mouvement; invariance d'orientation.

1 INTRODUCTION

The type of cable-driven transmission has been used in many robotics research today, ranging from robot arms and hands with unique features and performances. With the zeros backlash, low friction, low inertia, and high safety, one famous cable-driven manipulator, the Barrett WAM arm (Townsend and Guertin, 1999; Rooks, 2006), has been applied successfully in many areas, such as patient rehabilitation (Phan et al., 2014) and assembly tasks (Saric et al., 2014). Some cable-driven /tendon-driven manipulators, such as Dexter (De Luca et al., 2005), BioRob (Lens et al., 2010; Lens and von Stryk, 2013), and BDSM (Jong et al., 2008; Lee et al., 2012), perform great advantage to reducing the mass and inertia of robotic arm. And the inherent safety brought by lightweight and the vibration absorption improve the human-friendly. Besides, cable transmission system is innovatively used in the surgical robots for its long distance of force and motion with compact structure, such as Da Vinci (Marescaux and Rubino, 2006), RAVEN (Lum et al., 2009; Hannaford et al., 2013), Microhand (Luo et al., 2009), and so on (Swaney et al., 2015). The tendon-based robotic hands with multi-fingered designed by DLR (Zhaopeng Chen et al., 2010; Borst et al., 2003; Chalon et al., 2011) has been developed through several generations and many requirements are presented, including the mechanical structure, the electronics architecture, and control strategies.

For many cable-driven robots, transmission cables are routed from proximal actuators to its corresponding distal joints. Due to the coupling of multiple degrees of freedom, the kinematic analysis becomes more complicated, which may also be influenced by the mechanical structure design, the types of cable routing, the geometrical size of pulleys and etc. The establishment of kinematic mapping from actuator space to joint space is necessary. Lee, Tsai (Tsai and Lee, 1989; Lee and Tsai, 1991; Chang et al., 2005) and Uyguroğlu (Uyguroğlu and Demirel, 2006) used non-oriented graph and oriented graph technique for the kinematic analysis of tendon-driven robotic mechanisms, respectively. Chao (He et al., 2013) applied the product-of-exponentials formula to general coupled tendon-driven robots and proposed the calculation of kinematics and acceleration. And the QR (Chen et al., 1999) and SVD (Cho et al., 2015) method was also taken into consideration for the representation of cable transmission.

Due to the characteristic that cables can only pull and not push, for n -DOFs cable-driven manipulators, at least $n+1$ cables are required to drive each joint independently. However, that asymmetrically alignment of $n+1$ cables makes it more complicate for the design and control of system transmission, although few actuators is needed. Besides, the design for motion controller requires much highly coordinated, otherwise the tension in each cable will be adjusted as unexpected to damage the transmission device. One uses $2n$ cables and $2n$ actuators to constructed as the open-loop transmission routing, and each joint is driven by two cables with symmetrically alignment. It makes more convenient for tension distribution of cables in arbitrary manner because of the independent tension adjustment of each joint. However, the requirement for more actuators increases the structural design complicate again, which means the matter of the structure design should be considered well. However, n actuators is required if we take the close-loop scheme like the belt-driven or chain-driven. It uses the minimum number of actuators, reduces the complexity in cable routings, and saves the cost of power consumption. By installed some extra tensioning devices for cables, it can also make up the drawback of tension distribution of each joint.

In this paper, we adopt the scheme that $2n$ cables driven by n actuators with close-loop transmission routing to analyse the kinematics of n -DOFs cable-driven manipulator. Firstly, two types of cable-driven routing are introduced, and the impact of motion coupling caused by the cable transmission is analyzed in details. For different configurations of cable routings, the relationship between variation of joint angles and the geometrical sizes of pulleys is established. Meanwhile, we proposed the condition for the orientation invariance of this type cable-driven manipulator based on the motion coupling. In addition, a 3-DOFs planer cable-driven manipulator prototyping model is constructed, and the kinematics and trajectory planning have been solved to identify the correctness of analysis for coupling motion. Finally, simulations and experiments are designed to verify the correctness of the relationship among actuator

space, joint space, and Cartesian space, including the mapping of the motion coupling. And the property of orientation invariance is also passed validation by an experiment.

The rest of this paper is organized as follows. In Section 2, the characteristic of cable-driven structure and two types of cable routings will be described. The kinematic analysis for motion coupling and orientation invariance will be represented in details in Section 3. And Section 4 will show the process of simulations and experiments including the results. Finally, conclusions will be drawn in Section 5.

2 STRUCTURAL REPRESENTATION

2.1 Characteristic of Cable-Driven Structure

For the vast majority typical robotic manipulators, each kinematic joint is actuated directly by single actuator attached to the joint or its neighboring link. Obviously, it can be seen that one actuator corresponds to one joint, and each pair of joint and actuator is independent without coupling. The mapping between actuator space and joint space is easily represented.

However, the remarkable characteristic of cable-driven manipulator structure is that the actuators are positioned farther away from the joints and mounted on the base of the manipulator. The cable transmissions follow the routing from each axis of the joints between the actuators and the controlled joints. Take an example, for the cable of Joint i , the beginning anchor is attached to the winch installed on the output shaft of motor. Then, the cable is routing from each the axis of Joint 1, Joint 2, ..., Joint $i-1$. And the end anchor is attached to the driven wheel installed on the controlled joint shaft (Joint i). Some necessary frictionless idle pulleys are used to form the closed transmission loop, like the belt drive system. It can be used to transmit the motion and force efficiently, and drive the controlled joints to rotate. In contrast, there is a difference between the two transmission schemes. When the rotations happen among the Joint 1, ..., Joint $i-1$, the length of the cable round the corresponding joint axis will be changed, which is related to the rotation angle(s). This effect of cable transmission can be referred as a motion coupling (Jong et al., 2008).

2.2 Two Types of cable Transmission Routing

Fig. 1. shows the planar schematic representation of the two types of cable transmission routing. Fig. 1(a) is the crossed cable routing, and Fig. 1(b) shows the parallel cable routing. The crossed cable routing is used to rotate the driven pulley in the opposite direction of driving actuator. The parallel cable routing is used to rotate the driven pulley in the same direction of driving actuator. In both of two types, motion and force transmission results make one side of cable more tightened compared to the other side.

In order to simplify kinematics analysis, here are some assumptions to be considered:

(a) The sufficient tensions are always existed in cables and the elongation of cables caused by tensions are neglected.

(b) The mass and elasticity of cables are neglected.

(c) No relative sliding occurred between cables and pulleys, wheels or winches.

The different transmission routings will cause the different effects of motion coupling. The symbol $s_{i,j}$ is introduced to represent two types of cable routing that how the driving cable of Joint i is wound from Joint $j-1$ to Joint j . Here, $s_{i,j} = 1$ denotes the parallel routing, and $s_{i,j} = -1$ denotes the crossed routing. Particularly, $j = 0$ represents the index of the actuator, not a joint.

2.3 Analysis of Different Cable Routings

Due to the symmetric arrangement of the driving cables that means each joint has a pair of cables connected, the lengths of two cables on both sides change synchronously with positive or negative rotation of one actuator. Now, let us consider the cable on one side to analyze its length variation during the different motion states. Fig. 2 shows the different motion states of 2-DOFs cable driven manipulator with parallel cable routing.

Here, cable C_i is used to connect motor M_i and Joint i through some pulleys attached to the joints between them. θ_i and q_i denote the rotation angle of the motor M_i and the Joint i , respectively. $l_{i,j}$ denotes the length of the cable C_i on the pulley attached to Joint j , and $l_{i,0}$ denotes the initial length of the cable C_i on the wheel fixed on Joint i .

(a) State 1, the initial position, $\theta_1 = 0^\circ$, $\theta_2 = 0^\circ$.

The initial wrap length of cable C1 on Joint 1 is $l_{1,0}$. The initial wrap length of cable C2 on Joint 2 is $l_{2,0}$, and the length on the pulley attached to Joint 1 are $l_{2,1}$.

(b) State 2, hold motor M1, namely $\theta_1 = 0^\circ$ and drive the motor M2.

When hold motor M1 not to rotate, obviously, the wrap lengths $l_{1,0}$, $l_{2,1}$ on Joint 1 are not changed. And the wrap length variation of cable C2 on Joint 2 can be written as,

$$\Delta l_2 = s_{2,2} \cdot R_2 \theta_2 \quad (1)$$

where R_i denotes the radius of winch installed on motor M_i .

The angle variation of Joint 2 driven by motor directly is

$$\Delta q_2 = \frac{\Delta l_2}{r_{2,2}} = s_{2,1} s_{2,2} \cdot \frac{R_2 \theta_2}{r_{2,2}} \quad (2)$$

(c) State 3, hold motor M2, namely $\theta_2 = 0^\circ$ and drive the motor M1.

When hold motor M1 not to drive, the variation of wrap length of cable C1 is

$$\Delta l_1 = s_{1,1} \cdot R_1 \theta_1 \quad (3)$$

It leads to a rotate angle of Joint 1 written as follows:

$$\Delta q_1 = \frac{\Delta l_1}{r_{1,1}} = s_{1,1} \frac{R_1 \theta_1}{r_{1,1}} \quad (4)$$

The rotation of Joint 1 also causes the change of the wrap lengths of cable C2, and the segment wrapping on the Joint 1. The length variation of the segment can be represented as:

$$\Delta l_{2,1} = s_{2,2} \cdot r_{2,1} \Delta q_1 \quad (5)$$

And the length variation of cable C2 wrapping on Joint 2 changes in the reverse direction:

$$\Delta l'_2 = -\Delta l_{2,1} \quad (6)$$

Again, it means that Joint 2 turns an additional angle relative to the reference coordinate fixed on Joint 1. The additional angle variation can be expressed as:

$$\Delta q'_2 = \frac{\Delta l'_2}{r_{2,2}} = -s_{2,2} \cdot \frac{r_{2,1}}{r_{2,2}} \cdot \Delta q_1 \quad (7)$$

Given the above, we can obtain that the total angle variation of Joint 2 denoted by q_2 contains the angle variation caused by motor M2 directly driving and the additional angle variation brought by the rotation of Joint 1. According to the principle of superposition, the total angle variation of Joint 2 can be written as follows:

$$q_2 = \Delta q_2 + \Delta q'_2 = s_{2,1} s_{2,2} \cdot \frac{R_2 \theta_2}{r_{2,2}} - s_{2,2} \cdot \frac{r_{2,1}}{r_{2,2}} \cdot \Delta q_1 \quad (8)$$

3 KINEMATIC MODELING

3.1 General Expression of n-DOFs Coupling Motion

It is necessary to further study on the general expression of the planar multi-joint cable-driven manipulator. From the previous section, the key kinematic transmission characteristic can be obtained: The movement of Joint i will cause the changing of wrap lengths of those cables (the transmission cables of Joint $i+1$, Joint $i+2$, ..., Joint n) wrapping on Joint i , respectively. We can call this characteristic as motion coupling.

Now, considered the length variation of cable C_i that drives Joint i rotating an angle, the transmission length of cable C_i consists of two parts: one is caused by the rotation of the fore joints (from Joint 1 to Joint $i-1$), and another is caused by the directly driving of its controlled motor M_i . Thus, the rotation angle of Joint i can be split into two parts as well. Based on the analysis of 2-DOFs motion coupling above, some further equations are derived in this section.

Fig. 3. shows the simplified structure of a general n-DOFs cable manipulator. Take the Joint i for an example, the angle driven directly by the motor M_i can be written as,

$$\Delta q_i = \Gamma_{i,1} \cdot \frac{R_i \theta_i}{r_{i,i}} = \Gamma_{i,1} \cdot N_i \theta_i \quad (9)$$

The additional rotation angle brought by the fore joint j , denoted by $\Delta q'_{i,j}$, can be represented as,

$$\Delta q'_{i,j} = -\Gamma_{i,j} \cdot \frac{r_{i,j}}{r_{i,i}} \cdot q_j \quad (10)$$

where $1 \leq j < i$, and the sign function $\Gamma_{i,j}$ is introduced to represent general cable routing of the Joint i from Joint j to Joint i , which can be expressed as

$$\Gamma_{i,j} = s_{i,j} s_{i,j+1} \cdots s_{i,i} \quad (11)$$

Consequently, the sum of the additional angle for Joint i can be written as

$$\Delta q'_i = \sum_{j=1}^{i-1} \Delta q'_{i,j} = -\sum_{j=1}^{i-1} \Gamma_{i,j} \cdot \frac{r_{i,j}}{r_{i,i}} \cdot q_j \quad (12)$$

By combining the Eq. (9) and Eq. (12), the rotation angle variation of Joint i can be represented as

$$\begin{aligned} q_i &= \Delta q'_i + \Delta q_i \\ &= -\sum_{j=1}^{i-1} \Gamma_{i,j} \cdot \frac{r_{i,j}}{r_{i,i}} \cdot q_j + \Gamma_{i,i} \cdot \frac{R_i}{r_{i,i}} \cdot \theta_i \\ &= \frac{1}{r_{i,i}} \left(-\sum_{j=1}^{i-1} \Gamma_{i,j} \cdot r_{i,j} \cdot q_j + \Gamma_{i,i} \cdot R_i \cdot \theta_i \right) \end{aligned} \quad (13)$$

Move θ_i to the left of equal sign and rewrite the Eq. (13) as follows

$$\theta_i = \frac{1}{\Gamma_{i,i} R_i} \left(\sum_{j=1}^{i-1} \Gamma_{i,j} \cdot r_{i,j} \cdot q_j + r_{i,i} \cdot q_i \right) \quad (14)$$

Eq. (14) can be rearranged into a matrix form

$$\Theta_n = C_n \cdot Q_n \quad (15)$$

where

$$\mathbf{Q}_n = [q_1 \quad q_2 \quad \cdots \quad q_n] \in \mathbf{R}_{n \times 1}, \quad \boldsymbol{\Theta}_n = [\theta_1 \quad \theta_2 \quad \cdots \quad \theta_n] \in \mathbf{R}_{n \times 1}$$

$$\mathbf{C}_n = \begin{bmatrix} \frac{r_{1,1}}{\Gamma_{1,1}R_1} & 0 & 0 & 0 \\ \frac{\Gamma_{2,1}r_{2,1}}{\Gamma_{2,2}R_2} & \frac{r_{2,2}}{\Gamma_{2,2}R_2} & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\Gamma_{n,1}r_{n,1}}{\Gamma_{n,n}R_n} & \frac{\Gamma_{n,2}r_{n,2}}{\Gamma_{n,n}R_n} & \cdots & \frac{r_{n,n}}{\Gamma_{n,n}R_n} \end{bmatrix} \in \mathbf{R}_{n \times n} \quad (16)$$

$\boldsymbol{\Theta}_n$ and $\mathbf{Q}_n$ represent the vectors of actuator position and the joint position, respectively. Here, the matrix $\mathbf{C}_n$ is called the motion coupling matrix of the planar cable-driven manipulator. Obviously, once the cable transmission routing is determined, all parameters in $\mathbf{C}_n$ are certain constants and $\mathbf{C}_n$ becomes a nonsingular matrix.

Furthermore, the velocity mapping between actuator space and joint space can be represented as follows:

$$\dot{\boldsymbol{\Theta}}_n = \mathbf{C}_n \cdot \dot{\mathbf{Q}}_n \quad (17)$$

And the mapping of forces and torques can be written as:

$$\boldsymbol{\xi}_n = \mathbf{C}_n \cdot \boldsymbol{\tau}_n \quad (18)$$

where $\dot{\boldsymbol{\Theta}}_n$, $\dot{\mathbf{Q}}_n$, $\boldsymbol{\xi}_n$, $\boldsymbol{\tau}_n \in \mathbf{R}_{n \times 1}$ denote the vector of actuator velocity, joint velocity, actuator output torque, and joint force, respectively.

3.2 The Constraint of Joint Orientation Invariance

In general, the forward kinematics of robotics is used to calculate the position and orientation of the end-effector when the angles of all joints are given. In this section, the orientation of the manipulator will be discussed in details.

Base on the structural characteristic of the n-DOFs parallel manipulator, the orientation of each joint described under the base coordinate system can be represented as follows:

$${}^w q_i = q_i + \sum_{j=1}^{i-1} q_j \quad (19)$$

where $\sum_{j=1}^{i-1} q_j$ denotes the sum of the angle variations from Joint 1 to Joint $i-1$.

Eq. (13) is substituted into Eq. (19) to obtain:

$$\begin{aligned} {}^w q_i &= q_i + \sum_{j=1}^{i-1} q_j = \Delta q'_i + \Delta q_i + \sum_{j=1}^{i-1} q_j \\ &= -\sum_{j=1}^{i-1} \Gamma_{i,j} \cdot \frac{r_{i,j}}{r_{i,i}} \cdot q_j + \Gamma_{i,i} \cdot \frac{R_i}{r_{i,i}} \cdot \theta_i + \sum_{j=1}^{i-1} q_j \\ &= \sum_{j=1}^{i-1} \left(q_j - \Gamma_{i,j} \cdot \frac{r_{i,j}}{r_{i,i}} \cdot q_j \right) + \Gamma_{i,i} \cdot R_i \cdot \theta_i \end{aligned} \quad (20)$$

Now, suppose the following assumption were satisfied:

$$\Gamma_{i,j} = 1, \frac{r_{i,j}}{r_{i,i}} = 1 \quad (21)$$

which leads to

$$s_{i,j} = 1, r_{i,j} = r_{i,i} \quad (22)$$

The conditions mentioned above show that all cable routings are parallel types. In the meanwhile, the pulleys attached to each joint to guide the cables through have the same the radii.

Thus, Eq. (20) can be simplified as follows:

$${}^w q_i = \frac{R_i}{r_{i,i}} \cdot \theta_i \quad (23)$$

where $R_i/r_{i,i}$ denotes the reduce ratio of motor M_i to Joint i .

The coupling matrix $\mathbf{C}_n$ can be decomposed as follows

$$\mathbf{C}_n = \mathbf{N}_n \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \dots & 1 \end{bmatrix} \quad (24)$$

where $\mathbf{N}_n \in \mathbf{R}_{n \times n}$ is a diagonal matrix and each diagonal element is the reduction ratio of the wheel installed on the joint to its winch mounted on the actuator output shaft. And $\mathbf{N}_n$ is written as

$$\mathbf{N}_n = \text{diag} \left\{ \frac{r_{1,1}}{R_1}, \frac{r_{2,2}}{R_2}, \dots, \frac{r_{n,n}}{R_n} \right\} \quad (25)$$

It means that the angle variation of Joint i relative to the base reference coordinate just relates to its own executive motor, while is irrelative to the movements of its fore joints (Joint 1, 2, ..., $i-1$). Furthermore, it also explains that the orientation of the manipulator just controlled by the last joint with its actuator. Suppose a gripper or a wrist joint is installed on the last link as the end effector, then we can easily just calculate the position solution using the main degrees of freedom, and the orientation how to approach or grasp an object just using the degree(s) of freedom of the last one joint or wrist joint. In other words, the kinematics of a multiple-degree-of-freedom manipulator can be divided into two parts: the position and the orientation, which reduces the computational complexity and improves the response speed at real control. The kinematic characteristic described above can be applied suitably in some industrial occasions that need precise orientation control but few computational resource consumptions, such as sorting and welding.

3.3 Example

A 3-DOFs planer cable-driven manipulator is used as an illustrative example. In order to describe the relationship between actuator space and joint space conveniently, the reduction ratios of all joints are set to "1", which means the variation of each joint angle is equal to the variation of its actuator angle. By changing the angle variables of the actuators, the angle variations of the corresponding joints will be responded. Fig. 4. shows the different motion states under the different joint and actuator angles.

In Fig. 4, the first one shows the initial state without joints and actuators rotation, namely $\mathbf{Q}_n = [0, 0, 0]$, $\mathbf{\Theta}_n = [0, 0, 0]$. Compared to the first, the actuators take a rotation, $\mathbf{\Theta}_n = [30^\circ, 0, 0]$ to drive the joints in the second one. The angles of corresponding joints are equal to $\mathbf{Q}_n = [30^\circ, -30^\circ, 0]$. It means Joint 2 has

an inverse rotation corresponding to Joint 1. However, the orientation of Joint 2 does not change under the base reference coordinate. The similar argument can be verified with the rest moments. The relationship between actuator space and joint space satisfy the coupling matrix represented as Eq. (24).

4 SIMULATIONS AND EXPERIMENTS

4.1 Parameters of the Kinematics

The modified D-H parameters for the manipulator are given in Table 1. The parameters of structure are shown in Table 2. Thus, it is simply to compute the forward and inverse kinematics between joint space to Cartesian space of end-effector.

Substituted the parameters into the Eq. (24), the coupling matrix can be written as

$$C_n = \text{diag}\left\{\frac{400}{3}, \frac{100}{3}, \frac{100}{3}\right\} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \quad (26)$$

4.2 Kinematic Modeling

To validate the correctness of the motion coupling proposed, a 3-DOFs planar cable-driven manipulator is used in the simulation and experiment. The structure of the manipulator is presented as shown in Fig. 5.

4.2.1 Forward Kinematic

Given the vector of actuators angles $(\theta_1, \theta_2, \theta_3)$ to solve the position and orientation of the end-effector (p_x, p_y, α) where p_x, p_y denote the position of the end-effector in plane, respectively, and α denotes the orientation. Obviously, α is equal to ${}^w q_n$ mentioned earlier.

By rearranged the Eq. (15), the mapping of actuator space to joint space can be represented as follows:

$$\mathbf{Q} = \mathbf{C}^{-1} \cdot \mathbf{\Theta} \quad (27)$$

By expanded the above Eq. (27), the angle position of joints can be represented as follows:

$$\begin{cases} q_1 = \frac{1}{r_{1,1}} R_1 \theta_1 \\ q_2 = \frac{1}{r_{2,2}} \left(R_2 \theta_2 - \frac{r_{2,1}}{r_{1,1}} R_1 \theta_1 \right) \\ q_3 = \frac{1}{r_{3,3}} \left(R_3 \theta_3 - \frac{r_{3,2}}{r_{2,2}} R_2 \theta_2 - \left(\frac{r_{3,1}}{r_{1,1}} - \frac{r_{3,2} r_{2,1}}{r_{2,2} r_{1,1}} \right) R_1 \theta_1 \right) \end{cases} \quad (28)$$

And based on the homogenous transformation of robotics, we can obtain:

$${}^0_3\mathbf{T} = {}^0_1\mathbf{T} \cdot {}^1_2\mathbf{T} \cdot {}^2_3\mathbf{T} \quad (29)$$

Substituting the Eq. (28) into Eq. (29), the final expression of forward kinematics can be obtained as

$$\begin{cases} p_x = L_1 c_1 + L_2 c_{12} + L_3 c_{123} \\ p_y = L_1 s_1 + L_2 s_{12} + L_3 s_{123} \\ \alpha = q_1 + q_2 + q_3 \end{cases} \quad (30)$$

where c_1, c_{12}, c_{123} is the abbreviation of $\cos(q_1), \cos(q_1 + q_2), \cos(q_1 + q_2 + q_3)$, respectively, and s_1, s_{12}, s_{123} have the same similar abbreviation forms.

4.2.2 Inverse Kinematic

At first, using the knowledge of plane geometry, the joint angle is calculated by

$$\begin{cases} q_1 = \beta \pm \psi \\ q_2 = \arccos \frac{(p_x - L_3 \cos(\alpha))^2 + (p_y - L_3 \sin(\alpha))^2 - L_1^2 - L_2^2}{2L_1 \cdot L_2 \cos(\pi + q_2)} \\ q_3 = \alpha - q_1 - q_2 \end{cases} \quad (31)$$

where

$$\beta = \text{Atan2}(p_y - L_3 \sin(\alpha), p_x - L_3 \cos(\alpha))$$

$$\psi = \arccos \frac{(p_x - L_3 \cos(\alpha))^2 + (p_y - L_3 \sin(\alpha))^2 + L_1^2 - L_2^2}{2L_1 \cdot \sqrt{(p_x - L_3 \cos(\alpha))^2 + (p_y - L_3 \sin(\alpha))^2}}$$

Substituted Eq. (31) into Eq. (15), the inverse kinematics can be represented as follows:

$$\begin{cases} \theta_1 = \frac{r_{1,1}}{R_1} q_1 \\ \theta_2 = \frac{r_{2,2}}{R_2} q_2 - \frac{r_{2,1}}{R_2} q_1 \\ \theta_3 = \frac{r_{3,3}}{R_3} q_3 + \frac{r_{3,2}}{R_3} q_2 - \frac{r_{3,1}}{R_3} q_1 \end{cases} \quad (32)$$

4.3 Simulations

Two cases of the cable-driven manipulator are simulated. One is the path planning of the motion coupling, and another is the validation of the joints orientation invariance under structure size constraint.

4.3.1 Case 1: Path Planning

The geometric path is a planer semi-circle with the radius of 250.0 mm, parallel to the X-Z plane

$$\begin{cases} x = 250 \cos(\pi t / 50) + 355.2 \\ z = 250 \sin(\pi t / 50) + 532.0 \end{cases}, \quad 0 \leq t \leq 50s \quad (33)$$

And the orientation path is set as

$$\alpha = \pi \cdot t / 100 \quad (34)$$

where t is the simulation time with the sampling period of $\Delta t = 1\text{ms}$.

Fig. 6(a) and Fig. 6(b) show the angle change curves of three joints and the corresponding motors. The simulation results verify that the transmission ratios from motors to joints are not the direct reduction ratios of each part, and the angles of distal joints are affected by the additional angles caused by the proximal joints. The angle mapping from joint space to actuator space is in consistence with the coupling mapping as Eq. (26) expressed. Fig. 6(c) shows the kinematic configurations of the manipulator at different time period in Case 1.

4.3.2 Case 2: Orientation Invariance

In order to verify the orientation invariance of the end-effector, we can set the orientation value as

$$\alpha = 0 \quad (35)$$

And other parameters are as the same with Case 1. Aimed at holding the orientation unchanged, the coupled angles of joints need to be compensated according to the motion coupling mapping, respectively. Further, once the sizes of structure design satisfy the relationship as represented in Eq. (22), the coupled angles caused by Joint 1 and Joint 2 is equal to the angle variance of Joint 3 corresponding to the coordinate attached to its fore link. Likewise, the orientation of Joint 3 corresponding to the base coordinate is unrelated to the angle change of Joint 1 or Joint 2. Consequently, in order to control the orientation invariant, we just only hold the actuator of Joint 3 not to drive.

As shown in Fig. 7(a), the angle variance of motor 3 is held at zero, which means motor 3 has no effect on Joint 3. From Fig. 7(b), we can obtain that the angle of Joint 3 has changed corresponding to the coordinate attached to its fore link though the motor 3 is unactuated. The value of its angle variable is equal to the sum of other two fore joints angles during the simulation time:

$$q_3 = q_1 + q_2 \quad (36)$$

which indicates that the orientation of our manipulator is not changed which can be obtained as follows:

$$\alpha = q_1 + q_2 + q_3 = 0 \quad (37)$$

It is more intuitive to see the kinematic configurations at each simulation period as Fig. 7(c) shows.

4.4 Experiment Results

Two experiments are designed to verify the proposed coupling transmission and path tracking under both different conditions. The parameters setup for experiments are the same with simulation.

4.4.1 Experiment 1 Results

To the trajectory planning in Cartesian space of the cable-drive manipulator, Fig. 8(a) and Fig. 8(b) show the angle tracking of three joints and the errors of each, respectively. And Fig. 9(a) and Fig. 9(b) show the position tracking of tool center point(TCP) and errors of the three points, the center points of Joint 2, Joint 3, and TCP.

4.4.2 Experiment 2 Results

Fig. 10(a) and Fig. 10(b) show the angle tracking of three joints and the errors of each, respectively. And Fig. 11(a) and Fig. 11(b) show the position tracking of TCP and errors of the three points, the center points of Joint 2, Joint 3, and TCP. The experimental process of the orientation invariance is shown in Fig. 12. As we can see, when the manipulator tracks the trajectory, the orientation of the end link does not change. The object on the platform shows the phenomenon of orientation invariance obviously. Although, the end joint is not driven by its actuator, the orientation of end joint changes corresponding to its fore link but not changes corresponding to the base coordinate. In other words, the orientation of the end link under the base coordinate is irrelevant to the movements of fore joint(s) but to its own actuator in actually.

4.5 Discussion

From the experimental results, we can see that the motion coupling is existent in the cable transmission. The relationship between joint space and actuator space represented as the coupling matrix is correct. The property of orientation invariance is verified by the Experiment 2, and the tracking errors are within a reasonable range.

5 CONCLUSIONS

In this paper, the configuration of cable routings for cable-driven manipulator is described, and two types of cable routing are introduced and the impact of motion coupling caused by structural characteristics of $2n$ type cable-driven manipulator is analyzed in details. Based on two different cable routings, the relationship between variation of joint angles and the geometrical size of guide pulleys is established. Meanwhile, the matrix of coupled motion effect is represented. The problem of kinematic mapping from joint space to actuator space is solved. Furthermore, the mappings of velocity and force/torque are derived.

In addition, based on the behavior of the coupled motion for the manipulator, we proposed the condition for the orientation invariance which can be achieved by the constraint of the same geometrical size of guide pulleys and driven wheels. The angle variation of one joint or later relative to the base reference coordinate just relates to its own actuator but is irrelative to the movements of fore joints.

Finally, in order to identify the correctness of analysis for coupled motion, a 3-DOFs planer cable-driven manipulator model is developed, and the kinematics has been solved as well. The prototyping model of such a manipulator is constructed. The relationship among actuator space, joint space, and Cartesian space, including the mapping of the motion coupling is experimentally validated. And the property of invariance of orientation is also passed validation by an experiment.

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NOMENCLATURE

$s_{i,j}$	symbol of cable routing type
θ_i	angle of motor M_i (rad)
q_i	angle of Joint i (rad)
Wq_i	angle of joint described under the base coordinate system
Δq_i	direct angle variation
$\Delta q'_i$	additional angle variation
$l_{i,j}$	the length of the cable C_i on the pulley attached to Joint j
$\Gamma_{i,j}$	general cable routing of the Joint i from Joint j to Joint i
Φ_n	vector of actuator angle (rad)
Q_n	vector of joint angle (rad)
C_n	motion coupling matrix
$\dot{\Theta}_n$	vector of actuator velocity (rad/s)
$\dot{Q}_n$	vector of joint velocity (rad/s)
ξ_n	vector of actuator output torque
τ_n	vector of joint force
N_n	reduction matrix

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TABLES

Table 1. D-H Parameters For Manipulator

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L_1	0	θ_2
3	0	L_2	0	θ_3
T	0	L_3	0	0

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Table 2. Structure Parameters

Parameters	Value/(mm)	Parameters	Value/(mm)	Parameters	Value/(mm)
L_1	359.2	$r_{1,1}$	100	R_1	30
L_2	359.2	$r_{2,1} = r_{2,2}$	100	R_2	30
L_3	359.2	$r_{3,1} = r_{3,2} = r_{3,3}$	100	R_3	30

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FIGURE CAPTIONS

- Fig. 1. Two different types of cable transmission routings. (a) is crossed type routing, and (b) is parallel type routing
- Fig. 2. Motion states of a 2-DOFs cable-driven manipulator with symmetric arrangement. The parallel type of cable transmission routing is as an illustration in figure
- Fig. 3. Simplified structure of a general n-DOFs cable-driven manipulator
- Fig. 4. the different motion states under the different joint and actuator angles
- Fig. 5. The prototype of 3-DOFs cable-driven manipulator
- Fig. 6. Simulation results of Case 1: (a) joint angles; (b) motor angles; (c) kinematic configurations
- Fig. 7. Simulation results of Case 2: (a) joint angles; (b) motor angles; (c) kinematic configurations
- Fig. 8. Angle tracking in joint space for Case 1: (a) angle tracking; (b) angle errors curves
- Fig. 9. Position tracking in Cartesian space for Case 1: (a) Position tracking of TCP; (b) Position errors
- Fig. 10. Angle tracking in joint space for Case 2: (a) angle tracking; (b) angle errors curves
- Fig. 11. Position tracking in Cartesian space for Case 2: (a) Position tracking of TCP; (b) Position errors
- Fig. 12. The experimental process of the orientation invariance

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