Global semi-empirical relationships for correlating soil unit weight with shear wave velocity by void-ratio function

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Global semi-empirical relationships for correlating soil unit weight with shear wave velocity by void-ratio function

By

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ABSTRACT

Numerous studies have attempted to relate shear wave velocity ($V_s$) to the geotechnical properties of soils. However, most correlations were empirically developed only for a particular site or soil type. In this study, we propose a novel approach to incorporate a generalized void ratio function with analytical phase relations for estimating the total unit weight of soils. Based on an extensively compiled soil database, the validation of semi-empirical model was carried out and its performance was also compared against existing $V_s$ – total unit weight correlations. Moreover, a sensitivity analysis of the model input parameters was conducted to assess their significance on total unit weight estimates. It was demonstrated that the proposed semi-empirical model is successful in providing a first-order estimate of the total unit weight of soils based on the $V_s$, without the consideration of the overburden stresses.

Keywords: shear wave velocity, phase relations, unit weight, void ratio function.
INTRODUCTION

Over the past decades, geophysical techniques have been widely employed for site investigation to characterize the physical and mechanical properties of soils because they are able to cover a much wider area more efficiently, compared to conventional borehole drilling. For instance, non-destructive seismic waves (e.g. P-waves, S-waves, Rayleigh waves or Love waves) are used to estimate the elastic properties of the ground by measuring the wave travel-time; shear waves are particularly of greater interest because there is no significant effect of pore water pressure on shear wave propagation and therefore can be directly related to the shear modulus of the soil skeleton at small strains.

In previous studies, shear wave velocity ($V_s$) has been correlated to various geotechnical parameters such as the (1) peak friction angle, $\phi_p$ (Cha and Cho 2007); (2) undrained shear strength, $s_u$ (Blake and Gilbert 1996; Levesques et al. 2007; Likitlersuang and Kyaw 2010; Moon and Ku 2016b); (3) soil unit weight, $\gamma_t$ (Burns and Mayne 1996; Levesques et al. 2007; Moon and Ku 2016a); (4) lateral earth pressure coefficient, $K_o$ (Anand et al. 2011; Fioravante et al. 1998; Ku and Mayne 2013a; Ku and Mayne 2015); (5) compressibility, $C_c$ (Cha et al. 2014); (6) porosity or void ratio (Hussien and Karray 2016; Bui et al. 2010; Foti and Lancellotta 2004; Hardin 1963; Salgado et al. 2000); (7) degree of consolidation (Chang and Cho 2010; Chang et al. 2011); (8) stress history (Yoon et al. 2011; Ku and Mayne 2013b; Ku and Mayne 2014); and (9) degree of saturation (Cosentini and Foti 2014; Heitor et al. 2012; Leong and Cheng 2016; Whalley et al. 2012). Generally, these developed models are empirically calibrated based on a particular soil type and/or region, except for a recent study by Moon and Ku (2016a) who proposed a global correlation model between $\gamma_t$ and a site-specific stress-normalized shear wave
velocity ($V_{sn}$). However, in order to estimate $\gamma_t$ using the $V_{sn}$, there were inconvenient steps for assuming the initial $\gamma_t$ to calculate effective overburden pressure. These involve iterating the overburden stress using the $V_{sn}$ until convergence is reached. More details can be found in Moon and Ku (2016a).

This study proposes a simplified semi-empirical model for estimating the total unit weight ($\gamma_t$) of soils, without considering the confining (overburden) stresses. This novel approach combines $V_s$ with the phase relationships of soils through the void ratio. The results obtained from the proposed model are then compared against the performance of existing $V_s - \gamma_t$ correlations. Sensitivity studies based on an extensively compiled database are also carried out to support the effectiveness of the developed correlation model and determine the significance of the input model parameters on the predicted $\gamma_t$.

**METHODOLOGY**

*Shear wave velocity ($V_s$) – void ratio ($e$) relationship*

Several studies have reported that the shear wave velocity ($V_s$) of soils decreases with increase in void ratio ($e$). This is related to the packing and arrangement of the soil particles within the soil matrix (Hardin and Black 1968; Hardin 1963). The most commonly adopted empirical equation relating $V_s$ to void ratio can be represented as

$$V_s = a \cdot F(e)$$  \hspace{1cm} (1)

where $a$ is a material parameter which is representative of the stress dependency and/or the soil structure and fabric, and $F(e)$ is the void ratio function. Figure 1 illustrates different void ratio
functions from the literature. The trend that can be discerned is that the void ratio functions are
generally expressed in terms of a power function with different exponents, as follows:

\[ F(e) = e^b \]  
(\text{Jamiolkowski et al. 1995; Lo Presti et al. 1997; Shibuya and Tanaka 1996})

\[ F(e) = (1 + e)^b \]  
(\text{Bui et al. 2010; Shibuya et al. 1997})

where \( b \) is an exponent which controls the sensitivity of \( V_s \) to \( e \). From Eq. (2-1) and (2-2), it is
postulated that a generalized power function of void ratio can be expressed as:

\[ F(e) = (c + e)^b \]  
(3)

where \( c \) is an empirical fitting parameter.

Combining Eq. (1) and Eq. (3), we obtain the following generalized equation:

\[ V_s = a \cdot (c + e)^b \]  
(4)

Re-arranging,

\[ e = \left( \frac{V_s}{a} \right)^{\frac{1}{b}} - c \]  
(5)

Eq. (5) will be incorporated with the phase relationship equation in the following section.
Development of semi-empirical model

From the fundamental phase relationships in soil mechanics, the total unit weight ($\gamma_t$) is expressed as

$$\gamma_t = \frac{G_s + S_r \cdot e}{1 + e} \gamma_w$$  \hspace{1cm} (6)

where $G_s$ is the specific gravity of the soil grains, $S_r$ is the degree of saturation, $e$ is the void ratio of the soil matrix and $\gamma_w$ is unit weight of water.

By substituting Eq. (5) into Eq. (6), the void ratio dependency of $\gamma_t$ can be represented through $V_s$. The semi-empirical correlation model for estimating $\gamma_t$ can therefore be written as:

$$\gamma_t = \frac{G_s + S_r \cdot \left(\frac{V_s}{\alpha}\right)^{1/b} - c}{1 + \left(\frac{V_s}{\alpha}\right)^{1/b} - c} \cdot \gamma_w \hspace{1cm} (7)$$

In this study, the proposed semi-empirical relationship, Eq. (7), is examined for a first-order approximation of the $\gamma_t$ using $V_s$.

IN-SITU $V_s$ AND VOID RATIO DATABASE

In order to validate the developed model for estimating total unit weight ($\gamma_t$) using in-situ shear wave velocity ($V_s$), a comprehensive soil database was obtained from Mayne et al. (2009). The compiled data is composed of 155 test sites including various soil types: (1) 35 sands; (2) 8 silts; (3) 3 peat; (4) 7 gravels; (5) 61 intact clays; (6) 3 fissured clays; (7) 3 calcareous clays; (8) 3 clay till; and rock types: (9) 13 weathered rock, and (10) 19 intact rock. More details on the
The dataset are summarised in Table 1. The majority of $V_s$ data (92%) come from downhole-type testing, either in boreholes (DHT) and/or SCPT or SDMT. The remaining are from cross-hole testing (CHT) and a few data are obtained from SASW/MASW tests. For each site, $V_s$ was recorded at different depth for a particular soil layer. Figure 2 shows a compilation of the measured $V_s$ data. The total unit weight and void ratio data were obtained from laboratory testing on the tube samples collected at the same depth as the in-situ $V_s$ measurement.

**MODEL VALIDATION**

*Validation of the generalized equation between $V_s$ and $e* 

Through regression analysis of the compiled database, the general trend between shear wave velocity ($V_s$) and void ratio ($e$) for all geo-materials (i.e. soils and rocks) is plotted in Figure 2, where $N$ refers to the number of data points. The trend is used for validating the proposed relationship, Eq. (4). Figure 2 shows that the equation generally provides a good fit to the data points for a broad range of void ratio (0 to 5) by using the values of $a = 215.64$, $b = -1.043$ and $c = 0$. The coefficient of determination $\text{R}^2$ and the standard error of the dependent variable $\text{S.E.Y}$ were found to be 0.85 and 0.43 respectively. At this point, a reference value of zero is adopted for $c$ so that $a$ and $b$ can be computed through regression analysis.

*Validation of the semi-empirical model* 

The developed semi-empirical model, Eq. (7), is examined for estimating the total unit weight ($\gamma_t$) using $V_s$, based on an assumed specific gravity ($G_s = 2.65$) and water unit weight ($\gamma_w = 9.81 \text{ kN/m}^3$). For simplicity, the soil matrix is also assumed to be fully saturated ($S_f = 1$) below the water table. In practice, this might not always be the case as the degree of saturation of
a submerged soil layer depends on several factors such as its permeability, relative permeability against the adjacent soil layers, capillary forces, among others. The influence of $S_r$ on the model prediction will be assessed in next section.

Figure 3(a) shows the plot of the measured $\gamma_t$ versus the predicted $\gamma_t$ using $V_s$, together with the one-to-one line, representing correspondence between predicted and measured values. For each data point, the measured $\gamma_t$ corresponds to a representative mean value for a particular site calculated by averaging $\gamma_t$ over the depth of the soil layer. This approach helps to even out any measurement errors which might be present in the dataset. Two statistical measures (Pearson correlation coefficient $r$ and root mean square error $RMSE$) are used to assess the performance of the model. The Pearson correlation coefficient $r$ measures the linear correlation between two variables ranging from -1 (all data points lie exactly on a straight line with a negative slope) to +1 (all data points lie exactly on a straight line with a positive slope). The $RMSE$ is a measure of the differences between the two sets of data being compared.

In Figure 3(a), although some deviations are observed, the predictions of $\gamma_t$ using $V_s$ (Eq. 7) are generally close to the line of equality and within the lines of $\pm 1\cdot$Standard Deviation STD (i.e., STD = 1.63 kN/m$^3$). For the comparison, the predicted $\gamma_t$ values using $V_s$ are plotted against those predicted using void ratio $e$ in Eq. (6), as shown in Figure 3(b). It is observed that the scatter of the data points are comparable as demonstrated by the Pearson correlation coefficient $r$ and $RMSE$ values. This gives confidence that the two material parameters ($a, b$) were reasonably determined for all types of soils. It can further be observed that 75% of the data points fall within the lines of $\pm 1\cdot$STD. In particular, the widely scattered data in higher ranges of the predicted $\gamma_t$ are mostly from the intact and weathered rocks. This is because the phase relationship, Eq. (6), is
applicable to a soil matrix idealized as a three-phase (soil, air, water) material. On the other hand, cemented or mechanically bonded rock matrix and the presence of geological features such as faults and discontinuities can have an influence on the overall porosity of the rocks. Thus, the semi-empirical model, Eq. (7), seems to provide a better approximation of the unit weight for soils than rocks. If the intact and weathered rocks are omitted from the plot in Figure 3(a), the STD and \( RMSE \) reduce from 1.63 to 1.21 and from 1.11 to 0.95, respectively. Consequently, intact and weathered rocks will not be considered for further discussion.

**DISCUSSION**

*Comparison with existing \( \gamma_t - V_s \) correlations*

Several empirical models have been proposed in the literature for \( \gamma_t - V_s \) correlations (e.g. Mayne, 2007; Mayne, 2001; Burns and Mayne, 1996), as indicated in Table 2. These correlations were chosen because they were developed using the equivalent soil database. Figure 4 compares the predictive ability of the semi-empirical model against the performance of existing models. In Figure 4(a), the proposed semi-empirical model in this study produces the highest Pearson correlation coefficient \( r \) of 0.89 which gives the best prediction within ±10% of the line of equality. The \( \gamma_t \) values predicted by Mayne (2007), Figure 4(d), generally exhibit more scattering (e.g., 35% of the data points lying outside ±10% of the line of equality) and the highest root mean square error (\( RMSE \)), as compared to them predicted by other three models (i.e., this study, Burns and Mayne (1996) and Mayne (2001)), Figure 4(a-c). This is probably because Mayne (2007)’s correlation adopts the stress-normalized shear wave velocity \( V_{s1} \) which uses a constant exponent of 0.25.
The results in Figure 4 indicate that the proposed semi-empirical model can match and even surpass the predictive ability of the existing models. These models also require an additional input variable namely the overburden stress or depth. On the other hand, Eq. (7) has the advantage of simplicity as it only contains a single independent variable \( V_s \), with all the other model parameters being constant. A parametric study is conducted in the next section to investigate the sensitivity of these model parameters \( (a, b, c, G_s, S_r) \) on the \( \gamma_t - V_s \) relationship.

**Sensitivity study**

**Fitting parameter c**

In the section ‘Shear wave velocity \( (V_s) \) – void ratio \( (e) \) relationship’, it was postulated that a generalized power void ratio function, Eq. (4), can be employed to correlate the void ratio \( (e) \) with \( V_s \), based on different values of parameter \( c \). For example, Eq. (3) simplifies to Eq. (2-1) when \( c = 0 \). Figure 5 shows the influence of varying parameter \( c \) on the proposed semi-empirical model. Generally, the total unit weight \( (\gamma_t) \) data lies in between an upper bound of \( c = 1.0 \) and a lower bound of \( c = -1.0 \). In addition, it is found by regression analysis that \( c = 0.1 \) produces the best-fit curve for the \( \gamma_t \) data. However, adopting a value of \( c = 0.1 \) does not make significant improvement on the fitted curve when compared to the previously adopted value of \( c = 0 \) \( (R^2 \) increases from 0.70 to 0.74 and S.E.Y reduces from 1.19 to 1.18). Therefore, a value of \( c = 0 \) is recommended for the simple application of the proposed semi-empirical equation.
Material parameter \(a\) and exponent \(b\)

It was mentioned earlier that the parameter \(a\) represents the combined effect of the in-situ stresses and structure of a soil. The ideal approach would be to decompose material parameter \(a\) into a stress component and a structure component. In practice, identifying the structure component requires more time and effort because both the structured state of the soil and its remoulded state have to be tested to quantify the amount of structure at a given depth. In the current study, the adopted approach is a more holistic and simplified one and attempts to fit a global relation to the shear wave velocity \((V_s)\) – void ratio \((e)\) data with less computational efforts. Furthermore, using the same database compiled from Mayne et al. (2009), Moon and Ku (2016a) highlighted the prominent stress-dependency of \(V_s\). Thus, the influence of structure on \(V_s\) in this study is not considered to be significant, or as significant as the in-situ overburden stresses.

Based on the regression analysis of the compiled data in Figure 2, it was found that a combination of the material parameters \((a = 215.6\) and \(b = -1.0)\) resulted in the best-fit curve. Figure 3(a) demonstrated that the semi-empirical model could also give a fairly good prediction of the \(\gamma_t\) of soils, except for intact and weathered rocks, using the same material parameters. Herein, the influence of parameters \(a\) and \(b\) on the developed semi-empirical model is further evaluated in terms of the prediction sensitivity. The simplest approach to sensitivity analysis is to vary one parameter at a time repeatedly, while keeping the others fixed (Hamby 1994).

By varying the material parameter \(a\) while fixing the exponent \(b = -1.04\), it is observed in Figure 6(a) that the total unit weight \((\gamma_t)\) data generally lies in between an upper bound of \(a = 80\), and a lower bound of \(a = 610\). In a recent study, Moon and Ku (2016a) reported that the parameters \(a\) and \(b\) are inversely correlated within some ranges (i.e., \(53.3 < a < 2350\), \(-0.1 < b < -

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3.1) for all soil materials. Figure 6(a) shows that the upper and lower bounds of the material parameter $a$ for the current study lie well within the range reported by Moon and Ku (2016a).

Figure 6(b) shows the effect of varying the exponent $b$ on the semi-empirical equation for the $\gamma_t$ estimation. By decreasing the exponent $b$ while keeping the material parameter $a$ constant, the steepness of the semi-empirical curve decreases. This indicates that the $\gamma_t$ predicted using the semi-empirical model becomes less sensitive to changes in the $V_s$. This is not surprising because the exponent $b$ controls the dependency of $V_s$ on the void ratio $e$ in Eq. (1). It is also observed that the exponent $b$ has the most significant influence on the $\gamma_t - V_s$ relationship – as compared to the parameters $a$ and $c$ – as it controls the inflection of the semi-empirical curve. In consideration of the above discussion and the broad range of void ratio investigated in this study, it is envisaged that the use of $a = 215.64$ and $b = -1.04$ with the semi-empirical model would provide a reasonable first-order estimate of the $\gamma_t$ of soils.

Degree of saturation $S_r$

To investigate the effect of the degree of saturation ($S_r$) on the model predictions, the value of $S_r$ was varied from 0 to 1 while keeping all the other parameters constant. Figure 7(a) shows that the semi-empirical curve shifts down as $S_r$ tends to zero. It indicates that the predicted $\gamma_t$ decreases with $S_r$ for a given value of $V_s$, the effect being more significant in the low $V_s$ regime. Furthermore, it can be observed that the best fit curve to the $\gamma_t$ data is obtained by assuming $S_r = 1$, and the fitting worsens with a decrease in $S_r$. Consequently, it is reasonable to assume that soils are fully saturated ($S_r = 1$) under the ground water table, as far as the application of the semi-empirical model is concerned.
Specific gravity $G_s$

In the section ‘Validation of the semi-empirical model’, the semi-empirical model was validated based on the assumed specific gravity of 2.65. For soils, the value of $G_s$ typically ranges from 2.65 to about 2.80. This range of values has been adopted for the sensitivity study. Figure 7(b) indicates that the influence of the specific gravity $G_s$ on the proposed $\gamma_t - V_s$ correlation is almost negligible. Hence, $G_s = 2.65$ is adequate for the semi-empirical model.

SUMMARY AND CONCLUSION

The shear wave velocity ($V_s$) of all geo-materials can generally be expressed as a function of confining overburden stress and/or void ratio ($e$). In this study, the shear wave velocity was expressed in terms of a generalized void ratio function, without consideration of confining overburden stresses. A novel approach is proposed whereby the generalized void ratio function is incorporated with the phase relations to estimate the total unit weight ($\gamma_t$) of soils. The proposed semi-empirical model contains three main calibration parameters in the form of the material parameter $a$, exponent $b$ and the parameter $c$, whose values were determined by performing a regression analysis on an extensive soil database obtained from Mayne et al. (2009).

As an initial step to assess the predictive ability of the model, the predicted $\gamma_t$ values using $V_s$ were compared against the measured $\gamma_t$ and it was found that the performance was satisfactory as a first-order estimate of $\gamma_t$ for soils (not for intact and weathered rocks). The model output was also compared against $\gamma_t - V_s$ correlations by Burns and Mayne (1996), Mayne (2001) and Mayne (2007). It was demonstrated that the semi-empirical model can match and even surpass the predictive ability of the existing models, while having only a single input
variable which is the shear wave velocity. Finally, the sensitivity of the model predictions to the calibration parameters \((a, b\) and \(c)\) as well as the degree of saturation and specific gravity was investigated. It was found that the exponent \(b\) has the most significant influence on the \(\gamma_t - V_s\) relationship compared to the parameters \(a\) and \(c\). The change in specific gravity did not have a significant effect on the model output. In view of the results from the sensitivity analysis and the broad range of void ratio investigated in this study, the use of the recommended input parameters \((a = 215.64, b = -1.04, c = 0, G_s = 2.65\) and \(S_r = 1)\) for the semi-empirical model can provide a reasonable first estimate of \(\gamma_t\) for soils below the ground water table.

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FIGURE CAPTIONS

Figure 1. Some selected void ratio functions from the literature.

Figure 2. General trend between shear wave velocity ($V_s$) and void ratio ($e$) (data obtained from Mayne et al. (2009)).

Figure 3. Comparison of measured $\gamma_t$ and predicted $\gamma_t$ using (a) $V_s$ and (b) void ratio $e$, with the boundaries of ± one standard deviation.

Figure 4. Plot of measured $\gamma_t$ against predicted $\gamma_t$ using (a) semi-empirical model; (b) Burns & Mayne (1996)’s correlation; (c) Mayne (2001)’s correlation; (d) Mayne (2007)’s correlation.

Figure 5. Effect of varying parameter $c$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.

Figure 6. Effect of varying (a) material parameter $a$ and (b) material parameter $b$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.

Figure 7. Effect of varying (a) degree of saturation $S_r$ and (b) specific gravity $G_s$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.
Table 1. Details of collected database: soil type, number of site and data, and range of soil properties ($e$, $\gamma_t$) used for correlation (data from Mayne et al. (2009)).

<table>
<thead>
<tr>
<th>Soil Type</th>
<th>No. of Data</th>
<th>No. of Site</th>
<th>$e$</th>
<th>$\gamma_t$ (kN/m$^3$)</th>
<th>$V_s$ (m/s)</th>
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</thead>
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<tr>
<td>Sands</td>
<td>200</td>
<td>35</td>
<td>0.43-2.15</td>
<td>14.9-22.2</td>
<td>81.5-842.6</td>
</tr>
<tr>
<td>Silts</td>
<td>32</td>
<td>8</td>
<td>0.64-1.43</td>
<td>16.7-20.2</td>
<td>260.9-279.9</td>
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<tr>
<td>Peat</td>
<td>3</td>
<td>3</td>
<td>-</td>
<td>10.4-11.8</td>
<td>20.0-40.7</td>
</tr>
<tr>
<td>Gravels</td>
<td>43</td>
<td>7</td>
<td>0.27-0.70</td>
<td>19.6-22.5</td>
<td>260.9-520.0</td>
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<tr>
<td>Intact Clay</td>
<td>698</td>
<td>61</td>
<td>0.40-6.75</td>
<td>11.2-22.7</td>
<td>25.1-1064.1</td>
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<tr>
<td>Fissured Clay</td>
<td>21</td>
<td>3</td>
<td>0.43-0.84</td>
<td>18.8-21.3</td>
<td>151.3-350.8</td>
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<tr>
<td>Calcareous Clay</td>
<td>18</td>
<td>3</td>
<td>0.95-1.38</td>
<td>16.2-19.7</td>
<td>190.0-473.0</td>
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<td>Clay Till</td>
<td>16</td>
<td>3</td>
<td>0.19-0.56</td>
<td>20.1-24.0</td>
<td>240.9-550.0</td>
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<tr>
<td>Weathered Rock</td>
<td>51</td>
<td>13</td>
<td>0.03-1.13</td>
<td>16.7-26.0</td>
<td>268.1-2000.0</td>
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<tr>
<td>Intact Rock</td>
<td>131</td>
<td>19</td>
<td>0.03-0.71</td>
<td>19.2-26.0</td>
<td>789.7-3789.0</td>
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Table 2. Some empirical correlations between total unit weight $\gamma_t$ and shear wave velocity $V_s$.

<table>
<thead>
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<th>Empirical correlation</th>
<th>Reference</th>
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<tr>
<td>$\gamma_t$ (kN/m$^3$) = $[6.87 \cdot V_s (m/s)]^{0.0227} / [\sigma'_v (kPa)]^{0.057}$</td>
<td>Burns and Mayne (1996)</td>
</tr>
<tr>
<td>$\gamma_t$ (kN/m$^3$) = $[8.32 \log[V_s (m/s)] - 1.61 \log[z (m)]$</td>
<td>Mayne (2001)</td>
</tr>
<tr>
<td>$\gamma_t$ (kN/m$^3$) = $[4.17 \ln[V_{si} (m/s)] - 4.03$</td>
<td>Mayne (2007)</td>
</tr>
<tr>
<td>where $P_a$ is the atmospheric pressure</td>
<td></td>
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\[ V_{si} (m/s) = [V_s (m/s)] / (\sigma'_v / P_a)^{0.25} \]
Figure 1. Some selected void ratio functions from the literature.
Figure 2. General trend between shear wave velocity ($V_s$) and void ratio ($e$) (data obtained from Mayne et al. (2009)).
Figure 3. Comparison of measured $\gamma_t$ and predicted $\gamma_t$ using (a) $V_s$ and (b) void ratio $e$, with the boundaries of ± one standard deviation.
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Figure 5. Effect of varying parameter $c$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.

\[
\gamma_t (kN/m^3) = \gamma_w [G_s + (V_s/a)^{(1/b)} - c]/[(V_s/a)^{(1/b)} + 1 - c]
\]
Figure 6. Effect of varying (a) material parameter $a$ and (b) material parameter $b$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.
Figure 7. Effect of varying (a) degree of saturation $S_r$ and (b) specific gravity $G_s$ on the semi-empirical equation for $\gamma_t$, excluding data for intact and weathered rocks.