THE ROLE OF COGNITIVE FUNCTIONS AND ENGLISH LANGUAGE PROFICIENCY IN ARITHMETIC ACHIEVEMENT: DOES L1/L2 STATUS MATTER?

by

Emily Safronsky

A thesis submitted in conformity with the requirements for the degree of Master of Arts
Department of Applied Psychology and Human Development
Ontario Institute for Studies in Education
University of Toronto

© Copyright by Emily Safronsky (2014)
Abstract

The current research examined the role of different cognitive functions as well as the role of English language proficiency in the arithmetic performance of 91 Grade 5 Native English Speakers (EL1) and 172 Grade 5 English Language Learners (ELL). Cognitive profiles of normally achieving (NA) children and children at-risk (AR) for a mathematics learning disability (MLD) were examined in each language group. Findings revealed that nonverbal reasoning, working memory, and processing speed significantly predict Grade 5 arithmetic performance, regardless of language background. English language proficiency did not predict arithmetic performance in either language group. Children at-risk for MLD exhibited similar cognitive profiles and processing deficits regardless of language group. Results suggest that English language proficiency does not place ELL children at a greater risk for poor achievement in arithmetic. Implications for the assessment and diagnosis of MLD in children who are learning English as a second language are discussed.
Acknowledgements

First and foremost, I would like to thank Dr. Esther Geva, my supportive, patient, and insightful research advisor. Without her attentive guidance at each step of the way, this thesis would have never been completed! I look forward to continuing to develop as a scientific researcher with Dr. Geva as my supervisor. I must also acknowledge Dr. Geva for supplying archival data from the ESL At-Risk project. I would also like to thank Dr. Fataneh Farnia who kindly offered her support and assistance in getting me on the right track with this thesis. In addition, I am thankful to Dr. Judy Wiener, my second reader, who always passed along relevant research articles that she came across. I am further thankful to the members of the Geva Lab who were willing to provide valuable input and suggestions to my thesis.

Finally, I would like to thank my parents for always helping me achieve my goals, and my family and friends for their continuous support throughout my Master’s studies.

Support for the collection of this data was provided to Dr. Geva by the Social Sciences and Humanities Research Council of Canada (SSHRC).
# Table of Contents

Abstract .................................................................................................................................................. ii

Acknowledgements ............................................................................................................................... iii

Table of Contents ................................................................................................................................. iv

List of Abbreviations ............................................................................................................................ v

List of Tables ......................................................................................................................................... vi

List of Figures ........................................................................................................................................ vii

List of Appendices ............................................................................................................................... viii

Introduction ........................................................................................................................................... 1

Methodology .......................................................................................................................................... 14

Participants ........................................................................................................................................... 14

Predictor Measures .............................................................................................................................. 16

Results .................................................................................................................................................. 19

Language Group Differences .............................................................................................................. 19

Intercorrelations Among Predictors Variables and Dependent Variable ...................... 21

The Domain-General Cognitive Deficit Hypothesis ......................................................... 23

Cognitive Profiles and Processing Deficits of Children At-Risk for MLD ......... 26

Discussion ............................................................................................................................................ 31

The Domain-General Cognitive Deficit Hypothesis ............................................................ 31

The Role of Language Proficiency in Mathematics ...................................................... 37

Cognitive Profiles of EL1 and ELL Children At-Risk for MLD ................................ 39

Limitations ........................................................................................................................................... 40

Implications for the Assessment and Diagnosis of MLD in ELL Children ........ 41

References ........................................................................................................................................... 44
## List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EL1</td>
<td>Native English Speaker</td>
</tr>
<tr>
<td>ELL</td>
<td>English Language Learner</td>
</tr>
<tr>
<td>ESL</td>
<td>English as a second language</td>
</tr>
<tr>
<td>L1</td>
<td>First Language</td>
</tr>
<tr>
<td>L2</td>
<td>Second language</td>
</tr>
<tr>
<td>LD</td>
<td>Learning Disability</td>
</tr>
<tr>
<td>MLD</td>
<td>Mathematics Learning Disability</td>
</tr>
<tr>
<td>RD</td>
<td>Reading Disability</td>
</tr>
<tr>
<td>WM</td>
<td>Working Memory</td>
</tr>
<tr>
<td>RAN</td>
<td>Rapid Automatized Naming</td>
</tr>
<tr>
<td>PA</td>
<td>Phonological Awareness</td>
</tr>
<tr>
<td>WRAT</td>
<td>Wide Range Achievement Test- Revised</td>
</tr>
<tr>
<td>PPVT</td>
<td>Peabody Picture Vocabulary Test- Revised</td>
</tr>
</tbody>
</table>
List of Tables

Table 1. Descriptive statistics and Group Comparisons for Native English Speakers (EL1) and English Language Learners (ELL) ..........20

Table 2. Correlations among arithmetic, cognitive, and language predictor variables for EL1 and ELL groups .................................................................22

Table 3. Hierarchical Regression Analyses predicting WRAT Math performance separately for EL1 and ELL group - Summary table .........................25
List of Figures

Figure 1. Z-score cognitive profiles of normally achieving children and children at-risk for MLD in both language groups............................................30
List of Appendices

Appendix A. Descriptive statistics and language group comparisons for each cognitive predictor, arithmetic measure, and language measure by math performance group- Summary Table……………………………………………………………………..53

Appendix B. Descriptive statistics and math performance group comparisons for each cognitive predictor, arithmetic measure, and language measure by language group- Summary Table………………………………………………..54
Over the last few decades, clinicians and educators have become sensitized to issues involved in the assessment and diagnosis of learning disabilities (LD) in English Language Learners (ELL) (Geva, Yaghoub-Zadeh, & Shuster, 2000). Furthermore, research has demonstrated that many of the cognitive skills underlying specific learning disabilities can be reliably assessed in ELL children (Geva & Wiener, 2014) and many cognitive skills transfer between a child’s first (L1) and second (L2) language (Geva et al., 2000). For example, ELL children develop and acquire word-level reading skills similarly to their native English speaking (EL1) peers, and reading skills can be reliably predicted by the same underlying cognitive-linguistic processes as in EL1 students (Geva et al., 2000). Numerous longitudinal studies have consistently found that, just as is the case with EL1 children, phonological processing abilities (phonological awareness, phonological memory, and rapid automatized naming) are reliable and helpful in predicting basic reading skills in ELL children. Although it is now possible to reliably disentangle the root proponents of reading difficulties in ELL children, these students continue to be more academically vulnerable than EL1 children and are persistently over or under identified in terms of specific learning disabilities and special education in other academic subject areas (Geva & Farnia, 2012).

One specific academic area that remains relatively untouched by second language research is mathematics. Although some studies have examined word problem-solving skills in ELL children, limited research has focused specifically on the foundational skills of mathematics such as arithmetic and basic computation. Similar to literacy, conceptual understanding and skill acquisition in numeracy (i.e. basic competencies in mathematics)
is highly predictive of future academic and professional success (Geary, Hoard, Nugent, & Bailey, 2012a; Andersson & Östergren, 2012; Koponen, Aunola, Ahonen, & Nurmi, 2007). Further, numeracy and arithmetic are functional academic skills that are needed to manage common tasks in everyday life such as paying bills, keeping track of time, shopping for groceries, and budgeting (Andersson & Östergren, 2012; Vukovic & Siegel, 2010; Koponen, et al., 2007).

Comparable to reading disabilities (RD), approximately 5-8% of school-aged children have serious difficulty acquiring basic computational and arithmetic skills (Geary, 2004). Although prevalence studies indicate that math learning disabilities (MLD) are just as common as RD, limited and inconsistent literature exists on the origins and underlying cognitive functions of mathematic skills in comparison to reading (Andersson & Östergren, 2012). Moreover, very few studies have focused specifically on the extent to which certain cognitive processes predict arithmetic proficiency in ELL children. Of growing interest is whether language abilities can explain learning difficulties in mathematics, regardless of language group. For example, some research findings indicate an existing relationship between phonological awareness and number competencies, phonological awareness and basic arithmetic computation, as well as grammar ability and early numeracy (Kleemans, Segers, & Verhoeven, 2011; Krajewski & Schneider, 2009; Simmons & Singleton, 2008). Further, because math difficulties are often co-occurring with reading difficulties, research suggests that language ability and processing may influence math performance (Kleemans et al., 2011). However, it remains unclear whether second language proficiency places ELLs at a greater risk for poor achievement in basic mathematics (Vukovic & Lesaux, 2013).
The main focus of this thesis is to examine the underlying cognitive skills that predict arithmetic performance in EL1 and ELL children, and to investigate whether the relations between different cognitive processes and performance in arithmetic differ between EL1 and ELL children by comparing the cognitive profiles of children who are at-risk for MLD from both language groups (EL1 vs. ELL). Finally, this research will help to disentangle the role of language proficiency in foundational mathematics, in order to inform practice on the assessment and identification of MLD in ELL children.

**The Nature of Mathematical Learning Disabilities (MLD)**

Mathematics is an academic subject area consisting of multiple components (e.g., arithmetic, algebra, probability, geometry) and is typically taught in a hierarchical sequence, beginning with basic counting, single-digit calculations, and progressing to more complex problem solving (Koponen et al., 2007). According to Geary (2004), children who are at-risk for MLD can be described as having a deficit in the ability to represent or process information in one or all of these mathematical domains. However, children who experience difficulties in mathematics often fail to develop and acquire age-appropriate skills in basic numeracy and arithmetic taught in the early school years (Anderson, 2010; Geary 2004). Arithmetic, like other mathematical domains, is a multicomponential skill that requires conceptual knowledge, procedural knowledge, factual knowledge, and problem solving ability (Andersson, 2010). Children who struggle to acquire arithmetic skills may show weaknesses in one or all of these components. For example, a student with poor conceptual knowledge may have trouble understanding mathematical operations, while a student with procedural deficits may have trouble remembering, understanding and applying specific procedures and strategies with
flexibility (Andersson, 2010). However, one of the most important components, which allows children to progress in the hierarchy of mathematics, is factual knowledge. Factual knowledge refers to storing and efficiently retrieving the association between the operation and the answer to simple arithmetic problems from long-term memory (Andersson, 2010; Geary 1993). Effective and automatic arithmetic fact retrieval becomes an essential skill and allows the child to solve more complex, multi-step problems with efficiency (Andersson, 2010).

Because children at-risk for MLD exhibit difficulties primarily in the areas that are foundational for more complex, higher-order mathematical cognition (e.g., fact retrieval and single-digit arithmetic) these children often struggle across all mathematical domains, and show persistent difficulties as they progress through school (Geary, 2011; Andersson, 2010). For example, children with MLD are slower at retrieving and solving single-digit calculations, commit more atypical errors (e.g. 3 X 5= 30), use immature counting strategies (e.g. counting out loud, counting on fingers, using manipulatives), and have trouble applying operations in multi-step problems (e.g. carrying, borrowing), in comparison to their normally achieving peers (Vukovic & Siegel, 2010; Andersson 2010; Geary, 2004; Jordan, Hanich, & Kaplan, 2003). Although children with MLD progress at a similar rate to their normally achieving peers, the gap in math achievement remains significant even after 7 years of formal schooling (Gersten, Jordan, & Flojo, 2005). Overall, children with MLD continuously use ineffective strategies and cannot shift from procedure-based problem solving to memory-based problem solving, as do typically achieving students (Geary, 2004).
Underlying Cognitive Processes of Arithmetic Skills in EL1 Children

Researchers investigating the underlying causes of MLD continue to debate whether MLD originates from a domain-specific deficit in number processing, or whether these difficulties are a result of deficits in general cognitive functions (Andersson, & Östergren, 2012; Koponen, et al., 2007; Geary 2004). The two main hypotheses are briefly discussed below.

Research investigating the *domain-specific* hypothesis suggests that humans are born with an innate sense of number and quantity (Butterworth, 2010; Andersson, & Östergren, 2012). This inherent ability to manipulate numbers forms the foundation for the development and the acquisition of the number system that is used to develop skills in arithmetic (Butterworth, 2010). Research arguing in favor of the domain-specific hypothesis suggests that children who experience severe difficulties in mathematics have a core deficit in this innate foundational capacity for numbers (Butterworth, 2010). Within this domain-specific framework, a number of variations and hypotheses exist (see Geary, 2004, or Andersson and Östergren, 2012 for a review). However, they all predict that children with MLD have problems with symbolic magnitude processing (Andersson, & Östergren, 2012). To date, research on MLD within the domain-specific framework remains inconsistent and inconclusive.

A more widely regarded theory highlights the *domain-general deficit* hypothesis. Geary (2004) proposed that although children may exhibit deficits in various components of arithmetic, there may be general, underlying, cognitive processes that allow them to acquire and use these skills (i.e. fact retrieval, conceptual understanding, and procedural knowledge). Both short-term and longitudinal studies have revealed consistent findings...
supporting the domain-general hypothesis. However, few studies have considered multiple areas of cognitive functioning in one study. The various cognitive processes related to difficulties in mathematics are discussed below.

**General Intelligence.** General intelligence or cognitive ability allows a child to think logically and to use mental operations to solve novel problems (Geary et al., 2012a). For example, general cognitive ability allows children to draw inferences, problem-solve, transform information, and to form conceptual understanding in complex domains such as mathematics (Campos, Almeida, Ferreira, Martinez, & Ramalho, 2013). Some research has shown that general intelligence explains differences in math performance to a greater extent than other cognitive functions (Deary, Strand, Smith, & Fernandes, 2007). For example, Deary et al. (2007) found that general intelligence explained up to 60% of the variance on a national mathematics test in school-aged children. However, other researchers argue that general intelligence does not significantly predict arithmetic performance over and above other cognitive processes such as working memory (Campos et al., 2013). Further research is needed to determine the role of general cognitive or reasoning ability.

**Working Memory (WM).** Research consistently shows that children with MLD have some form of working memory deficit; meaning that they have difficulty in tasks that require them to store and manipulate arithmetic information simultaneously (Geary, 2004). However, there is some inconsistency about the role of WM throughout the elementary grades. Some research has shown that the influence of WM may decrease over time. For example, Geary, Hoard, Byrd-Craven, Nugent, and Numtee (2007) suggest that WM is most important from kindergarten through the third grade, while Fuchs et al.
(2006) found that working memory was no longer a significant path to arithmetic computation in the third grade. Additionally, Vukovic and Siegel (2010) found that WM is an important component of an early screening battery but may not predict arithmetic performance throughout the elementary years. One explanation may be that as basic facts become more automatized and efficiently retrieved in the later grades, the WM load involved in carrying out basic computations is reduced when solving more complex arithmetic problems. At the same time, findings regarding the role of different WM mechanisms in arithmetic (i.e. central executive, phonological loop, and visual-spatial sketch pad) remain mixed (Geary et al., 2012a; Geary et al., 2007; Swanson & Jerman, 2006; Geary, 2004). For example, some studies have shown that the central executive component of WM is crucial for basic fact retrieval (Geary, et al., 2007), while other studies indicate that verbal WM and visual-spatial WM are core deficits in children with difficulties in arithmetic (Berg, 2008). Geary et al., (2007) further suggests that specific WM components may influence different areas of mathematics.

Regardless of specific areas of deficit, research is consistent in showing that WM deficits significantly disrupt the execution of mathematical procedures. In addition, WM deficits may be the source of counting procedure errors in children with MLD (Geary, 2004). For example, children may lose track of their counting causing them to over-count or under-count, which, in turn, results in errors when solving arithmetic problems (Geary, 2004). Further, Geary (1990) indicated that children with MLD continue to use finger counting as a strategy because representing numbers on fingers greatly reduces the working memory demands when solving arithmetic problems. Children with MLD have also been shown to have poor performance on Digit Span tasks, which are reliable
measures of WM (Gersten et al., 2005). Overall, research supports the notion that children with MLD show weaker performance on WM tasks than their normally achieving peers, and that children with MLD exhibit a domain-general cognitive deficit in WM (Geary, 2004).

*Short-term memory.* Research on the role of short-term memory (that is, the ability to hold information in memory for a few seconds) in MLD is less consistent than WM research (Silver, Ring, Pennett, & Black, 2007). Some studies suggest that children with MLD show impaired performance on short-term memory span tasks that involve the recall of numerical information (Passolunghi & Siegel, 2001), while other work suggests no short-term memory differences between normally achieving children and children with MLD (Passolunghi, 2011; Bull & Johnston, 1997). To illustrate, Geary, Hoard, and Hamson (1999) found that children at-risk for MLD had significantly lower backward digit span (considered a WM task) performance than normally achieving children but no group differences emerged for forward digit span, a short-term memory task. Geary et al. (1999) suggest that verbal short-term memory may be important for arithmetic and number fact recall when its role is to attend to, retain, and manipulate verbal information but not when it is simply used for simple passive repetition of verbal information. Again, additional research is needed to determine the role of short-term memory in arithmetic.

*Phonological Awareness.* Research outcomes regarding the role of phonological awareness (PA) in math performance remains mixed. Several studies have found that phonological awareness (i.e. the ability to manipulate phonemes in words) assessed in kindergarten is significantly correlated with mathematical competencies in the primary grades (Alloway, et al. 2005; Hecht, Torgesen, Wagner, & Rashotte, 2001; Geary, 1993).
Further, Leather and Henry (1994) found that 31% of the variance in arithmetic performance at age 7 is predicted by phonological awareness assessed at the same age (as cited in Krajewski & Schneider, 2009). More recently, Krajewski and Schneider (2009) found that phonological awareness facilitated development of early numeracy skills. Specifically, phonological awareness was particularly important for articulating numbers in counting procedures and for basic arithmetic computations. However, PA was not found to predict other areas of mathematics that did not involve the manipulations of phonological codes. Although phonological abilities may be predictive of math performance in the early years when the number word sequence is not yet automatic, it may be less important for more complex, higher-order mathematical competencies in the later grades (e.g. word problem-solving, geometry, data analysis) because these domains are less reliant on the manipulation of phonological codes (though reading math problems may relate to phonological awareness). Krajewski and Schneider (2009) suggest that once children have learned and memorized the number word sequence by heart, phonological abilities may be less important in predicting arithmetic achievement.

*Processing Speed.* Processing Speed represents the ability to process simple or highly learned information quickly, accurately, and effortlessly (Vukovic & Siegel, 2010). Processing speed is a core area of deficit for children with MLD (Cowan & Powell, 2014; Passolunghi, 2011; Bull & Johnston, 1997). Rapid Automatized Naming (RAN) is a common processing speed task that assesses a child’s ability to accurately and quickly retrieve highly learned verbal codes (e.g. numbers, letters, colours, and common objects) from a long-term memory (LTM) store (Norton & Wolf, 2012). In addition, children with math difficulties are significantly slower than normally achieving children.
in naming speed (Koponen et al., 2007). In a meta-analytic review, Swanson and Jerman (2006) found that children with math difficulties were slower on rapid naming tasks than both normally achieving peers as well as children with reading disabilities. These results suggest that RAN may be an important processing component that underlies math difficulties, and a cognitive component that may differentiate MLD from RD. Relatedly, it is well-documented that slow basic fact recall and retrieval is a defining feature of MLD (Geary, Hoard, & Bailey, 2012b; Geary, 2004). Because both basic calculation and rapid naming rely on rapid retrieval of information from LTM, it has been suggested that slow naming speed may be an index of inefficient basic fact retrieval in children with MLD (Koponen et al., 2007; Swanson & Jerman, 2006; Geary, 2004). Although a substantial portion of the research suggests that processing speed is an important cognitive process that underlies arithmetic performance and overall mathematics achievement, some researchers question whether processing speed plays a unique role over and above other underlying cognitive skills such as working memory in the later elementary years (Vukovic and Siegel, 2010; Berg, 2008). For example, Berg (2008) suggested that processing speed may lose its importance once the retrieval of number representation from LTM becomes more automatic and once children become more proficient in arithmetic. Additionally, research has shown that individual differences in processing speed are more pronounced in younger children than in older children, and may therefore play a greater role in explaining differences in arithmetic achievement in the earlier years (Kail, 1991).
The Role of Language Ability in ELL Math Performance

Oral language is said to play an important role in the development of mathematical skill and mathematical cognition (Cowan & Powell, 2014; Vukovic and Lesaux, 2013). However, it is unknown whether oral language plays a direct role in the development of these skills, or whether language ability is simply important because it is the primary method of mathematical instruction in the elementary years (Cowan & Powell, 2014). Although the role of certain linguistic processes (i.e. phonological processing) in math performance has been investigated in some studies, there is a dearth of research that examined the role of language proficiency in math performance.

Previous studies have suggested that in addition to the cognitive processes discussed above, general language ability may also be highly important for mathematical cognition and skill development (LeFevre, et al., 2010; Fuchs et al., 2006; Spelke & Tsivkin, 2001). For example, LeFevre et al. (2010) found that in addition to phonological awareness, vocabulary knowledge explained unique variance in arithmetic performance in Grade 2. LeFevre et al. (2010) further suggest that language ability may be essential for early numeracy tasks because these tasks rely on the knowledge of the symbolic number system (e.g., writing and naming Arabic digits). In a study conducted by Spelke and Tsivkin (2001), addition and subtraction facts were taught to bilingual adult students in either English or Russian. Newly learned arithmetic facts were more efficiently retrieved in the language of training. These findings suggest that arithmetic facts may be stored verbally, and indicate that arithmetic efficiency may be dependent on language ability (Spelke & Tsivkin, 2001). A recently conducted study has further suggested that linguistic ability is a significant predictor of the development of early numeracy skills.
(Kleemans, et al., 2011). Kleemans and colleagues (2011) examined the cognitive and linguistic precursors to early numeracy in kindergarten, in both first (L1) and second language (L2) learners. They report that L2 learners scored lower than the L1 learners on all linguistic and early numeracy tasks. However, no differences were detected amongst the relationships between cognitive precursors and early numeracy skills for L1 versus L2 learners (Kleemans, et al., 2011). These important findings suggest that although language ability may influence math performance, EL1s and ELLs who experience difficulties in mathematics exhibit very similar cognitive profiles, and these similar cognitive profiles may, in turn, play a greater role in mathematics than language proficiency.

On the other hand, a longitudinal study that looked at first grade through fourth grade math achievement in second language learners found varying results (Vukovic & Lesaux, 2013). In this study, 6-9 year-old EL1s and ELLs were assessed in arithmetic, algebra, data analysis, and probability. Analyses revealed that, general language ability (i.e. receptive vocabulary and oral listening comprehension) did not explain arithmetic difficulties in either language group (EL1 and ELL). Further, findings indicated that second language proficiency did not explain unique variance in ELL math performance. Vukovic and Lesaux (2013) concluded that language ability was not involved in either arithmetical procedures or algebraic reasoning, but influenced how children made meaning of mathematical content in both language groups. This study further suggested that although other cognitive systems, beyond language, are likely more responsible for math achievement in school-aged children, the existing relationship between language ability and math performance is similar for both EL1 and ELL children.
Overall, limited research has examined how English Language Learners are acquiring, developing, and using their basic mathematical skills. As in reading disability studies, the important question remains whether English language proficiency plays a significant role in explaining foundational mathematic difficulties (i.e. difficulties in arithmetic) experienced by ELLs.

The Current Study

I have not been able to identify research that has specifically examined the extent to which nonverbal reasoning, working memory, short-term memory, phonological awareness, processing speed, and English language proficiency predict basic computational arithmetic skills in EL1 and ELL children. The current study examined these cognitive processes and basic arithmetic achievement in 172 ELL Grade 5 children in comparison to 91 EL1 Grade 5 controls. Based on a review of the literature, it was hypothesized that these cognitive processes would significantly explain unique variance in arithmetic performance (Geary, 2004; Geary et al., 2012a; Andersson & Östergren, 2012; Swanson & Jerman, 2006). Based on findings from Vukovic and Lesaux (2013), it was hypothesized that arithmetic performance would be better explained by these cognitive processes, than by general language ability for both EL1s and ELLs. Specifically, it was hypothesized that although English proficiency (as measured by receptive vocabulary knowledge) would be weaker in the ELL sample, it would not be predictive of poor math performance in either language group. Further, based on findings from reading disability research, it was hypothesized that ELL children at-risk for MLD in this sample would exhibit similar cognitive profiles and processing deficits to their EL1 counterparts.
The main objectives of the current study were to:

1) Investigate the domain-general hypothesis and determine whether nonverbal reasoning, working memory, short-term memory, phonological awareness, and processing speed predict basic computational arithmetic skills in both language groups (EL1 and ELL).

2) Determine whether English Language Proficiency (as measured by receptive vocabulary knowledge) significantly predicts arithmetic performance in children.

3) Determine whether the cognitive profiles and processing deficits of children at-risk for MLD are similar in EL1 and ELL children.

Methodology

Participants

The data used for analyses of this study were selected from a larger, multi-cohort longitudinal project that was conducted by previous researchers in our laboratory. The longitudinal project, launched in 1996, involved students from 12 different schools located in 4 separate boards of education in ethnically and linguistically diverse regions of the Greater Toronto Area. The original study recruited 614 participants over four successive cohorts of Grade 1 students who were each followed for 6 years. Participating schools were asked to identify students with known developmental disabilities or impairments (e.g., speech and language impairments, hearing impairment, Autism Spectrum Disorder), and they were excluded from the initial phase of the project. Mathematics achievement data were only available for the beginning of Grade 5.
The final sample of the current study involved 263 Grade 5 students from various language backgrounds, including students with English as a first language (EL1), and English Language Learners (ELLs) who were exposed to a home language other than English or French. The ELL sample consisted of 172 students whose home languages were Punjabi ($n = 68$), Portuguese ($n = 58$), Tamil ($n = 28$), Cantonese ($n = 3$), Hindi ($n = 4$), Gujarati ($n = 3$), Urdu ($n = 6$), and Mandarin ($n = 2$). The EL1 sample consisted of 91 monolingual English-speaking students. At time of testing (winter of Grade 5) the mean age of the EL1 group was 10 years, 10 months ($SD = 3.60$ months), and the mean age of the ELL group was 10 years, 8 months ($SD = 3.66$ months) ($t(261) = 3.47$, $p < .01$). While individual level demographic data was not available, the 2006 Canadian Census from Statistics Canada provides relevant demographic information at the school level. Based on the 2006 Canadian Census, which is reported by municipality, the majority of EL1 and ELL participants were identified as living in low to middle socioeconomic status neighborhoods. The median family income of each of the areas from which the 12 schools were recruited, was considerably below the median of the corresponding city. Parental education in these communities ranged from no high school diploma (20%) to having a high school diploma (26%), a college diploma (23%) or holding a university degree (31%) (Statistics Canada, 2006). All participants were recruited from the same schools and drawn from the same classrooms.

All 263 students selected for the current study had parental consent to participate, had no known disability, and had lived in an English-speaking country for a minimum of four months at the onset of the study, when they were in Grade 1. This 4-month criterion
was included to ensure that all ELL students had sufficient knowledge of the English language and were able to understand English instructions throughout testing.

**Predictor Measures**

Based on the numerous studies and reviews described above, relevant cognitive measures which are associated with arithmetic achievement were selected from the larger test battery.

*General Intelligence.* The Raven’s Standard Progressive Matrices (Raven’s PM) is a standardized measure that was used to assess nonverbal, general reasoning ability (Raven, Raven, & Court, 1998). Because of potential residual second language factors, I specifically assessed individual differences in nonverbal ability. Raven’s PM is one of the most widely used measures of nonverbal reasoning in culturally and linguistically diverse minority groups (Raven, et al., 1998). It consists of 60 multiple choice questions where the student must identify a missing part in visual puzzles. The 60 puzzles are divided into five sets (A, B, C, D, and E) of 12 items each. The test requires minimal verbal abilities and no verbal response on the part of the examinee. Research suggests that the Raven’s PM can be administered to minority children with limited English proficiency and that it is more or less a valid measure of their cognitive ability and general intelligence (Raven et al., 1998). Raw scores were used in the analyses. According to Raven et al. (1998) the measure has internal consistency coefficients of .80.

*Working memory.* The Backward Digit Span portion of the Digit Span subtest (Wechsler Intelligence Scale for Children, Third Edition [WISC-III]; Wechsler, 1991) was used to measure individual differences in working memory. The task assesses intake, maintenance, processing and retrieval of information. Children are required to recall,
reverse order, a series of orally presented digits that increase in set size. The test developers report an internal consistency reliability of .75 for the age of the children in the study.

*Verbal short-term memory.* Verbal short-term memory was assessed with the Forward Digit Span portion of the Digit Span subtest (Wechsler Intelligence Scale for Children-Third Edition [WISC-3]; Wechsler, 1991). For the Forward portion of the Digit Span task, children are required to listen to a series of digits and then to recall the digits in the same order. The orally presented digits increase in set size and children receive two trials at each span length. The test developers report an internal consistency reliability of .79 for the age of the children in the study.

*Phonological Awareness.* Phonological Awareness was measured using an adapted version of the Auditory Analysis Skills Task (Rosner & Simon, 1971). This task consists of 25 items, forming three sets (with each set gradually increasing in difficulty). In the first set, the students were asked to delete one syllable from either the beginning or ending position of a spoken word (e.g. ‘Say “sunshine”’; ‘Say “Sunshine” without saying “shine”’). The second set required deletion of the first or last single phonemes in one-syllable words (e.g., ‘Say “hand”’; ‘Say “hand” without saying the ‘/h/’). The third set involved deletion of single phonemes in first or last consonant blends in a word (e.g., ‘Say “stop”’; Say “stop” without “/s/”). Raw scores were used for analyses. The internal consistency for this test was .92 and .90 for EL1 and ELL groups, respectively.

*Processing speed.* The rapid letter-naming task from Denckla and Rudel’s (1976) Rapid Automatization Naming Test (RAN) was used to measure individual differences in retrieval speed of highly learned verbal information. This task includes five letters, each
appearing 10 times in a random order. Accuracy and time (in seconds) to name all 50 items were recorded. The raw score (speed of naming) was calculated as the time (in seconds) it took the child to name all 50 items accurately.  

**Receptive vocabulary.** The Peabody Picture Vocabulary Test—Revised (Dunn & Dunn, 1981) was used to measure students’ receptive vocabulary in English. This receptive vocabulary task was used as an indicator of general language ability for both language groups, as well as a measure of English language proficiency. In this task, participants see four pictures, and are asked to point to the picture that matches a word said by the tester. Because the Peabody Picture Vocabulary Test—Revised task was not normed on ELL students, raw scores were used for analyses. Test developers report a split-half reliability of .80 (ranging from .67 to 68) and a test-retest reliability of .78.

**Dependent Measure**

**Arithmetic Computation.** The Arithmetic subtest of the Wide Range Achievement Test-Revised (WRAT) (Wilkinson, 1993) was administered to assess informal and formal basic written computation and calculation skills as well as overall mathematics ability. This timed test consists of 40 computation problems and students are given 15-minutes to complete the test. The computation items involve both single-digit and multi-digit addition, subtraction, multiplication, and division problems. Items also include basic operations involving fractions and algebraic equations (i.e. solving for x). Instructions (given in English) were to solve as many problems as they could within the given time limit. The reported reliability for the arithmetic subtest is .89 for the age of participants.

---

1 The phonological processing measure (i.e. Auditory Analysis Skills Task) and the Denckla and Rudel’s (1976) RAN measure, were precursors to the Elision and RAN subtests of the CTOPP. The CTOPP was not yet available when the current data about these processing skills were collected.
Results

Independent samples $t$ tests were used to compare the raw scores of EL1 students and ELL students on the arithmetic, cognitive, and language measures. Next, a correlation matrix for all measures was calculated separately for EL1 and ELL children. Further, separate hierarchical regression analyses were used to determine which variables were uniquely related to arithmetic performance in each language group. Finally, raw scores were standardized, based on the whole sample (EL1 and ELL), in order to compare the cognitive profiles of normally achieving (NA) and at-risk (AR) children in the EL1 and ELL language groups.

Language Group Differences

Table 1 summarizes the descriptive statistics (means, standard deviations, group differences) of the cognitive, linguistic, and arithmetic measures. Group comparisons revealed that EL1 and ELL students were not significantly different on any measure with the exception of PPVT scores. As might be expected, EL1 students ($M=110.25$, $SD=14.47$) outperformed ELL students and had significantly higher PPVT scores than ELL students ($M=99.97$, $SD=14.28$), $t(261)=5.80$, $p < .01$. These results indicate that EL1 children had better knowledge of English vocabulary than ELL children and confirm the ELL designation. Due to the differences in English language proficiency (as measured by PPVT), subsequent analyses were conducted separately for each language group.
Table 1. Mean Raw Scores, Standard Deviations, and Group Comparisons for Native English Speakers (EL1) (n= 91) and English Language Learners (ELL) (n=172).

<table>
<thead>
<tr>
<th>Measures</th>
<th>ELI</th>
<th>ELL</th>
<th>Test of Group Differences</th>
<th>Cohen’s d</th>
</tr>
</thead>
<tbody>
<tr>
<td>WRAT Math</td>
<td>31.14 (3.94)</td>
<td>32.06 (4.39)</td>
<td>-1.66</td>
<td>-.21</td>
</tr>
<tr>
<td>Raven’s PM</td>
<td>36.02 (8.12)</td>
<td>34.06 (8.25)</td>
<td>1.84</td>
<td>.23</td>
</tr>
<tr>
<td>Digit Span (forward)</td>
<td>8.51 (1.95)</td>
<td>8.19 (2.14)</td>
<td>1.19</td>
<td>.15</td>
</tr>
<tr>
<td>Digit Span (backward)</td>
<td>4.69 (1.44)</td>
<td>4.92 (1.79)</td>
<td>-1.07</td>
<td>-.13</td>
</tr>
<tr>
<td>RAN Letters</td>
<td>23.14 (5.07)</td>
<td>22.27 (4.98)</td>
<td>1.35</td>
<td>.17</td>
</tr>
<tr>
<td>Auditory Analysis Skills (PA)</td>
<td>19.07 (5.60)</td>
<td>20.06 (5.10)</td>
<td>-1.46</td>
<td>-.18</td>
</tr>
<tr>
<td>PPVT</td>
<td>110.66 (14.00)</td>
<td>99.98 (14.02)</td>
<td>5.80**</td>
<td>.72</td>
</tr>
</tbody>
</table>

*Note.*

**. *t* test significant at the .01 level.
Intercorrelations Among Predictors Variables and Dependent Variable

Table 2 displays the correlations among age in months, the predictor variables (Raven’s PM; Backward Digit Span; Forward Digit Span; RAN Letters; Auditory Analysis Skills (PA); and PPVT) and the dependent measure (WRAT Math) with the EL1 correlation matrix presented above the diagonal and the ELL correlation matrix presented below the diagonal. There was no significant correlation between the age of participants and the dependent measure (WRAT Math) in both language groups. Further, age was not significantly correlated with any of the predictor variables, with the exception of a weak correlation with RAN Letters in the ELL group ($r = -.17$). Any significant correlations among predictor variables in both language groups were weak to moderate, ranging between $r = -.24, p < .05$ and $r = .46, p < .01$ in the EL1 group, and between $r = .18, p < .05$ and $r = -.46, p < .01$, in the ELL group. All predictor variables were significantly correlated with arithmetic performance, which indicated that the data were suitable for examination through multiple linear regression. The correlations between the predictor variables and the dependent variable (arithmetic performance) were all weak to moderate, ranging between $r = .25, p < .05$ and $r = .48, p < .01$ in the EL1 group, and between $r = .15, p < .05$ and $r = -.37, p < .01$, in the ELL group. Similarly to Vukovic and Lesaux (2013), although PPVT and WRAT Math were significantly correlated, the observed correlations between the arithmetic measure and the language measure for both language groups were not strong.
Table 2. Correlations among age, arithmetic, cognitive, and language predictor variables for Native English Speakers (EL1) (n= 91) above the diagonal and English Language Learners (ELL) (n=172) below the diagonal.

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (months)</td>
<td>1.00</td>
<td>.06</td>
<td>.07</td>
<td>.09</td>
<td>.07</td>
<td>.09</td>
<td>.05</td>
<td>.09</td>
</tr>
<tr>
<td>WRAT Math</td>
<td></td>
<td>1.00</td>
<td>.48**</td>
<td>.36**</td>
<td>.45**</td>
<td>-.31**</td>
<td>.25*</td>
<td>.38**</td>
</tr>
<tr>
<td>Raven's PM</td>
<td></td>
<td></td>
<td>1.00</td>
<td>.29**</td>
<td>.30**</td>
<td>-.01</td>
<td>.32**</td>
<td>.40**</td>
</tr>
<tr>
<td>Digit Span (fw)</td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>.46**</td>
<td>.24*</td>
<td>.30**</td>
<td>.25**</td>
</tr>
<tr>
<td>Digit Span (bw)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>-.33**</td>
<td>.34**</td>
<td>.18</td>
</tr>
<tr>
<td>RAN Letters</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
<td>-.25*</td>
<td>-.18</td>
</tr>
<tr>
<td>Auditory PA</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>.18</td>
<td>.27*</td>
</tr>
<tr>
<td>PPVT</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.00</td>
</tr>
</tbody>
</table>

*Correlation is significant at the p<0.05 level (2-tailed).

**Correlation is significant at the p<0.01 level (2-tailed).

Note.
**The domain-general hypothesis: Variables that predict basic computational arithmetic skills in EL1 and ELL children**

To investigate the contribution of predictor variables to performance on the arithmetic task, I conducted two hierarchical regression analyses separately for each language group (EL1 and ELL). Preliminary analyses were conducted to ensure no violation of the assumptions of normality, linearity, and homoscedasticity. Further, I examined the histograms and scatterplots to identify outliers and reviewed the distribution of the variables. When missing values were found due to data-entry error (n=2), the raw data was used for reference and the missing values were reentered (n=1). If missing data could not be obtained (n=1), the corresponding case was excluded from subsequent analyses. Because age (in months) was not found to correlate with the dependent measure, it was removed from subsequent analyses. The final analyses included 91 EL1 children and 172 ELL children.

Based on the domain-general hypothesis, all cognitive measures were entered into the hierarchical regression prior to the language measures (PA and PPVT-R). As a first step, I entered Raven’s PM raw scores to control for general cognitive ability (nonverbal reasoning) (Step 1). Based on previous research which indicates that working memory plays a significantly important role in arithmetic achievement, Backward Digit Span was entered in Step 2, and Forward Digit Span was entered as a measure of verbal short-term memory in Step 3. RAN Letters was entered in Step 4, and Auditory Analysis (PA) was entered in Step 5. PPVT was entered in the final step (Step 6).
Table 3 summarizes the essential results from the hierarchical regression analyses separately for each language group. Overall, for the EL1 sample, the final model was statistically significant $F(6, 84) = 9.42$, $p < .001$ indicating a good fit for the data, and explained 40.2% of the variance in arithmetic performance. The results of the regression analysis indicated that of the 6 cognitive predictors entered, 3 were uniquely predictive of arithmetic performance in the EL1 group. At the first step, Raven’s PM accounted for 22.7% of the unique variance with a standardized beta of .39, $p = .00$ in the final model. In Step 2, Backward Digit Span accounted for an additional 10.1% of the unique variance with a standardized beta of .22, $p = .03$ in the final model. The contribution of Forward Digit Span (entered in Step 3) was not significant. RAN Letters entered in Step 4 accounted for an additional unique 3.8% of the variance with a standardized beta of -.22, $p = .02$ in the final model. The contribution of Auditory Analysis Skills (PA) entered in Step 5, and of PPVT in the final step was not significant and did not add to the final model. Therefore, nonverbal reasoning ability, working memory, and rapid automatized naming were significant predictors of Grade 5 arithmetic performance in the EL1 group. Examination of the beta weights in the final step of the model suggested that nonverbal ability was in fact the most important predictor in the model in the EL1 group.

In the ELL group, the final model was statistically significant and explained an overall 24.2% of the variance, $F(6, 165) = 8.79$, $p < .001$. Again, the results of the regression analysis indicate that the same 3 cognitive variables that were found to be significant in the EL1 group were also significant in predicting arithmetic performance in the ELL group.
Table 3. Hierarchical Regression Analyses predicting WRAT Math performance separately for Native English Speakers (EL1) and English Language Learners (ELL) using Raven’s PM, Digit Span (backward), Digit Span (forward), RAN Letters, Auditory Analysis Skills (PA), and PPVT.

<table>
<thead>
<tr>
<th>Predictor Variables</th>
<th>EL1 (n=91)</th>
<th>ELL (n=172)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R^2$</td>
<td>$\Delta R^2$</td>
</tr>
<tr>
<td>Step 1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Raven’s PM</td>
<td>.23</td>
<td>.23**</td>
</tr>
<tr>
<td>Step 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Backward Digit Span</td>
<td>.33</td>
<td>.10**</td>
</tr>
<tr>
<td>Step 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Forward Digit Span</td>
<td>.34</td>
<td>.01</td>
</tr>
<tr>
<td>Step 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RAN Letters</td>
<td>.38</td>
<td>.04*</td>
</tr>
<tr>
<td>Step 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auditory Analysis Skills (PA)</td>
<td>.38</td>
<td>.00</td>
</tr>
<tr>
<td>Step 6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPVT</td>
<td>.40</td>
<td>.02</td>
</tr>
</tbody>
</table>

Note.
**. $p<0.01$ level (2-tailed).
*. $p<0.05$ (2-tailed).
$\beta^a$ = Standardized Beta Coefficient in the final step of the model.
At the first step, Raven’s PM significantly accounted for 6.5% of the variance. However, in the ELL group, Raven’s PM did not uniquely contribute to the final model, $t(165)=1.94$, $p = .054$, $\beta = .15$. In Step 2, Backward Digit Span accounted for an additional unique 5.3% of variance with a standardized beta of .18, $p = .03$ in the final model. The contribution of Forward Digit Span in Step 3 was not significant. In Step 4, RAN Letters accounted for an additional unique 10.4% of the variance with a standardized beta of -.31, $p = .000$ in the final model. Finally, as in the EL1 group, the contribution of Auditory Analysis Skills (PA) in Step 5 and PPVT in the final step, were not significant and did not add to the final model. Again, as in the case of the EL1 group, nonverbal ability, working memory, and rapid automatized naming were significant predictors of Grade 5 arithmetic performance in the ELL group. Although nonverbal ability played a significant role in predicting arithmetic performance in the overall model, its unique contribution in the ELL group was less evident when the contribution of working memory and rapid automatized naming was included in the model. Altogether, a higher percentage of the variance was explained in the EL1 group than in the ELL group.

**Cognitive profiles and processing deficits of children at-risk for MLD**

To determine the extent to which the developmental profiles of EL1 and ELL children who are normally achieving in arithmetic or those who are at-risk for MLD are similar, children’s raw scores were converted to standardized Z-scores and performance subgroups were compared in each language group. To identify and create at-risk for MLD (AR) and normally achieving (NA) subgroups, several procedures have been suggested in learning disability research. For example, Branum-Martin, Fletcher, and Stuebing (2013) caution against the use of cutpoints to determine subgroups. They
suggested that like reading, math disabilities are dimensional and math achievement scores represent continuous data. Therefore, when patterns emerge from applying cutoffs to indicate group differences, they are often a result of the cutpoint and the correlational structure of the data (Branum-Martin, et al., 2013). In order to avoid some of the issues noted by Branum-Martin and colleagues (2013) with regard to the use of cut-off scores to define subgroups, a cluster analysis can be conducted to identify different arithmetic groups. Therefore, as a first step, a K-means Cluster analysis (Bartelet, Ansari, Vaessen, and Blomert, 2014; Antonenko, Toy, & Niederhauser, 2012) was conducted separately for each language group. However, this clustering procedure did not yield distinct AR and NA arithmetic performance subgroups in either language group. Likewise, Geary et al. (2012a), who applied similar clustering procedures, were unable to identify independent performance groups in their sample of monolingual children ranging from kindergarten through Grade 5. Geary et al. (2012a) acknowledge that children classified as AR and those classified as NA reflect variation along a continuum, in this case a continuum of arithmetic performance, but they argue that cutpoints are helpful for the purpose of group categorization and for the purpose of subgroup comparisons. They further suggest that using a dimensional approach as proposed by Branum-Martin et al. (2013) may not be useful for the development of special curricula and intervention approaches that may be needed for children with learning disabilities, and that categorizing students based on cutpoints may be more informative and useful for both research and applied purposes. At the same time, it must be acknowledged that the dimensional approach may be useful when issues of eligibility for special education are discussed. For example, children who perform at the lower end of the Average range
(e.g., 29th percentile) may benefit from the same intervention that is provided to those who qualify for an LD designation (e.g., 25th percentile or below).

For the purpose of subgroup comparisons, and based on previous MLD research (Geary et al., 2012a; Geary 2004), cutpoints were used in the present study to identify the performance subgroups. As a first step, EL1 arithmetic scores were standardized based on the EL1 sample, and ELL arithmetic scores were separately standardized based on the ELL sample. Following, EL1 and ELL arithmetic scores were also standardized based on the whole sample. This step revealed that designation as normally achieving or as being at-risk for MLD, did not differ whether the calculations of arithmetic standard scores were done separately for the EL1 and ELL groups, or on the whole sample. Therefore in subsequent analyses, raw scores were converted to standardized z-scores based on the whole sample (N=263). The at-risk group consisted of children whose scores on the arithmetic task were -.75 standard deviations below the whole sample mean (i.e. < 21st percentile), and the normally achieving group consisted of children who scored -.67 standard deviations above the whole sample mean (i.e. > 25th percentile) on the arithmetic measure. Based on the above mentioned classification, in the EL1 group, 29 students were classified as at-risk for MLD and 62 students were classified as NA. In the ELL group, 42 students were classified as at-risk for MLD, and 130 as NA. A chi-square test indicated that the prevalence of at-risk for MLD did not differ by language group, $X^2 (1, N = 263) = 1.68, p=.195$. 
Figure 1 illustrates the Z-score based cognitive profiles of EL1 and ELL groups who were either at-risk for MLD or normally achieving. Figure 2 includes the significant predictor variables (i.e., Raven’s PM, Backward Digit Span, RAN Letters) as well as the language measure (i.e., PPVT). The objective is to underscore the cognitive and language profiles of EL1 and ELL children who are at-risk for MLD and those who are not. As illustrated in Figure 1, the cognitive profiles of EL1 and ELL children with no difficulties in simple arithmetic were similar.

Independent samples t tests were used to compare the standardized z scores of AR and NA children in both language groups, on the arithmetic, cognitive, and language measures. Group comparisons revealed that the EL1 and ELL normally achieving groups had comparable performance on Backward Digit Span and RAN Letters. On the other hand, nonverbal ability, as measured by Raven’s PM, was found to be significantly higher in normally achieving EL1s than normally achieving ELLs (see Appendix A). Further, normally achieving EL1s scored significantly higher on the PPVT than normally achieving ELLs. These expected findings confirm the existing “second language status” of the students in the normally achieving ELL sample.

Likewise, the cognitive profiles of EL1 and ELL children at-risk for MLD were similar and both language groups showed processing deficits in the same domain-general cognitive functions. As might be expected on the basis of the regression analyses, regardless of language group, EL1 and ELL at-risk groups had significantly lower Raven’s PM, Backward Digit Span, and RAN Letters scores than the respective normally achieving groups (NA EL1>AR EL1 and NA ELL>AR ELL).
Figure 1. Z-score cognitive profiles of children who are normally achieving and those who are at-risk for MLD in both language groups.
As with the normally achieving groups, at-risk EL1s scored significantly higher on the PPVT than at-risk ELLs, again, confirming the ELL designation of the at-risk ELL group. Although the PPVT was not a significant predictor of arithmetic performance, regardless of language group, children designated as at-risk for MLD had significantly lower PPVT Z-scores than the respective EL1 and ELL normally achieving groups (see Appendix B).

**Discussion**

The overarching goal of this research was to determine whether English language proficiency places ELL children at a greater risk than EL1 children, for difficulties with arithmetic calculation in the mid-elementary grades, or whether in both language groups, domain-general cognitive processes could predict individual differences in arithmetic skills. Findings from the current study support the notion of domain-general cognitive deficits in children with difficulties in arithmetic and basic computation, which are not tied to language background. Secondly, in contrast with some previous work (Kleemans et al., 2011; LeFevre et al., 2010; Spelke and Tsivkin, 2001) general language ability did not predict arithmetic performance in either language group. Finally, the current results suggest that English language proficiency does not place ELL children at a greater risk for poor academic achievement in arithmetic and that the same cognitive screeners can be used to reliably identify ELL students at-risk for MLD. Each of these findings is discussed below.

**The Domain-General Cognitive Deficit Hypothesis**

The current results support the domain-general cognitive deficit hypothesis. Of the cognitive measures used (namely, nonverbal reasoning, working memory, verbal
short-term memory, phonological awareness, and processing speed), three processes, nonverbal reasoning, working memory, and processing speed assessed in Grade 5 predicted Grade 5 arithmetic performance in both language groups. Unlike other studies (e.g., Cowan & Powell, 2014; Vukovic & Siegel, 2010; LeFevre et al., 2010) phonological awareness did not play a significant role in predicting arithmetic performance in either language group. Thirdly, although the same cognitive processes were found to be significant in predicting arithmetic performance in both language groups, the importance of each significant predictor differed between language groups.

Consistent with previous research, nonverbal reasoning played a significant role in explaining differences in performance on the standardized mathematics task but the proportion of variance explained was not identical in the two groups. In the EL1 group, nonverbal reasoning explained uniquely 22.7% of the variance in arithmetic performance and was found to be the most important predictor in the model. However, in the ELL group, although nonverbal reasoning explained 6.5% of the variance in arithmetic performance, when working memory and processing speed were included, it became non-significant. These results are consistent with findings from Campos et al. (2013) who also found that nonverbal reasoning ability (as measured by Raven’s Progressive Coloured Matrices) was not predictive of arithmetic performance once WM and processing speed were accounted for. However, the current results suggest that while nonverbal ability may be a strong indicator of arithmetic performance for native English speakers, other cognitive functions may be more important when identifying at-risk second language learners. Although nonverbal reasoning tasks are said to be more reliable than verbal reasoning tasks when assessing ELL children, cross-cultural differences continue to exist
(Rosselli & Ardila, 2003; Tzuriel & Kaufman, 1994). It is therefore possible that these results are due to differences in exposure to nonverbal tasks such as puzzles in the various cultures in the current sample. Moreover, there is also the possibility that MLD is a heterogeneous disorder, as recently suggested by Bartelet and colleagues (2014). In a longitudinal study of arithmetic skills, these researchers performed clustering procedures on Grade 3 through 6 monolingual students with low mathematics achievement, and found support for the notion that MLD is a heterogeneous disability. Bartelet, et al. (2014) suggest that different cognitive subtypes of MLD exist (e.g., spatial difficulties subgroup, access deficit subgroup, no numerical cognitive deficit subgroup, a garden-variety subgroup) and therefore, it is likely that nonverbal ability may predict arithmetic performance in one MLD subtype but not in another. Additional research is needed to investigate the role of nonverbal reasoning in second language learners who may belong to different MLD subtypes.

Overall, based on the current results, nonverbal ability seems to be important in predicting arithmetic performance in the mid-elementary years in both EL1 and ELL children. This finding supports previous work that has demonstrated that general cognitive ability is directly involved in mathematical achievement (Cowan & Powell, 2014; Kleemans et al., 2011; Deary et al., 2007) and research showing that children diagnosed with a nonverbal learning disability (NLD) often have major deficits in arithmetic achievement (Rourke, 1995). Although it is not yet clear why children with NLD have arithmetic deficits, research suggests that spatial working memory may explain the arithmetic deficit in children with impaired visuospatial/nonverbal abilities (Mammarella, Lucangeli, & Cornoldi, 2010; Rourke, 1995). Again, additional research is
needed to examine cross-cultural differences in nonverbal reasoning and it’s role in academic achievement.

The Backward Digit Span task is considered to be a working memory measure that assesses the ability to store information and carry out cognitive processes simultaneously (Sattler, 2008b). In line with the bulk of WM research reviewed earlier, performance on the Backward Digit Span task uniquely predicts arithmetic performance in both EL1 and ELL language groups. Since the language demands of this task are minimal, it is perhaps not surprising that this ability to hold factual information in memory and at the same time carry out cognitive operations, such as computation, predicts arithmetic performance regardless of language background. This finding replicates the Geary et al. (1999) results in showing that the Forward Digit Span task does not play an important role in predicting arithmetic performance once the Backward Digit Span task is taken into account. These findings are consistent with the passive/active hypothesis of verbal memory proposed by Cornoldi and Vecchi (2003). This hypothesis suggests that children at-risk for MLD are expected to have average passive verbal memory (i.e. verbal short-term memory) but impaired active verbal memory (i.e. verbal working memory) (cited in Passolunghi, 2011). Together, these findings support the notion that WM is a core cognitive deficit in children at-risk for MLD. In this study only verbal WM was assessed, and in future studies, it may be beneficial to disentangle the various components (i.e. central executive, phonological loop, and visual-spatial sketchpad) of WM to determine whether specific aspects of WM predict arithmetic performance to a greater degree in various language groups (Geary et al., 2009).
With regard to processing speed, the findings converge with the bulk of research conducted with monolinguals (Cowan & Powell, 2014; Passolunghi, 2011; Bull & Johnston, 1997) and extend them to ELLs. The present study suggests that processing speed, specifically RAN, independently predicts arithmetic performance in the later elementary years, regardless of language background. In fact, the findings indicate that RAN is the most essential predictor of arithmetic performance in the ELL group. Because of colinearity among the cognitive functions, it is difficult to determine whether speeded processing helps reduce the load on WM, thereby increasing arithmetic efficiency, or whether processing speed is directly involved in arithmetic performance. Studies conducted by both Cowan and Powell (2014) and Vukovic and Siegel (2010) suggest that basic calculation and fact retrieval depend on the speed of response and therefore it is likely that processing speed plays a direct role in arithmetic achievement. The current results support those findings and the plausibility of a direct relationship between processing speed and arithmetic performance in the mid-elementary years, regardless of language background.

Unlike previous research (e.g. Vukovic & Siegel, 2010; Fuchs et al., 2006), in the current study, phonological awareness did not predict arithmetic performance in either language group. This finding, which is consistent with Krajewski and Shneider (2009), suggests that phonological awareness may be less important for predicting mathematical skill in the later elementary grades. Further, in the Vukovic and Siegel (2010) and the Fuchs et al. (2006) studies, phonological decoding (i.e. pseudoword decoding) was used as a measure of phonological awareness, and was found to predict arithmetic skill. On the other hand, in the present study, a direct measure of phonological awareness ability was
used. This direct PA task may be less confounded with other component processes such as memory for orthographic clusters and related decoding skills. It is possible that decoding tasks, rather than direct measures of phonological awareness, are more likely to predict arithmetic performance due to a possible relationship between mathematics and reading ability. Relatedly, while not significant in the present study, it is important to acknowledge that in fact, there were significant albeit low correlations between PA and arithmetic performance in both the EL1 and ELL groups. Considering these correlational findings, it is possible that there is another mediating variable (e.g. word-level reading) in the relationship between mathematical skill and phonological awareness, regardless of language group, that should be investigated in future research. Regardless, the current study suggests that a direct phonological awareness task may be less confounded with decoding skills and therefore less related to arithmetic skill. In other words, when a purer and less confounded measure of PA is used that does not involve decoding, it does not play a major role in predicting specific mathematical difficulties.

Overall, while the current results suggest that arithmetic difficulties may be attributed to deficits in several cognitive processes, it is important to note that (a) a higher percentage of the variance was explained in the EL1 group than the ELL group, and (b) that there was a substantial amount of unexplained variance in both language groups. Additional research is needed to examine the role of individual level demographic characteristics (e.g. parental education, socioeconomic status) as well as the role of attention, motivation, and instructional strategies, in arithmetic achievement (Geary et al., 2012a; Passolunghi, 2011; Passolunghi & Siegel, 2001). Moreover, the current study did not examine the domain-specific hypothesis. It is therefore possible that specific number
processing capabilities (e.g. symbolic magnitude processing) may also play a role in predicting arithmetic achievement in the mid-elementary grades (Andersson, & Östergren, 2012). Future studies should consider examining both MLD hypotheses in various language groups.

**The Role of Language Proficiency in Mathematics**

The second objective of the current study was to examine the role of receptive vocabulary knowledge as a measure of English language proficiency, in arithmetic and computational math performance in the mid-elementary years. Congruent with findings from Vukovic and Lesaux (2013), and unlike word problem solving in mathematics (Fuchs et al., 2006; Fuchs, Fuchs, & Compton, 2014), language ability is not directly involved in how students performed on arithmetic computation, regardless of language group. These findings support the notion that although language is the primary method of teaching and therefore partially related to the learning of mathematical skills and dealing with arithmetic word problems, language proficiency is not responsible for how well children are able to manipulate numbers and perform algorithms in operational mathematic subjects like arithmetic and algebra (Vukovic and Lesaux, 2013). In future studies, it may be important to examine how cognitive functions and language impact other, more complex, areas in mathematics (e.g., data analysis and probability, geometry, and so on).

On the other hand, these findings are somewhat opposing previous work. For example, Kleemans and colleagues (2011) compared native Dutch speaking kindergarten students to Turkish and Moroccan L2 learners on a variety of cognitive, linguistic, and mathematical measures. Turkish and Moroccan immigrants in the Netherlands tend to
live in native-language speaking homes, where children only begin to learn Dutch (i.e. the primary language of instruction) once they enter formal schooling (Kleemans et al. 2011). Their results revealed that native Dutch speakers outperformed their second language learner counterparts on all of the cognitive, linguistic, and numeracy measures and that language skills were highly correlated with early numeracy. Unlike Kleemans et al. (2011), in the current study, EL1 and ELL Grade 5 students performed similarly on the arithmetic task, as well as on all of the cognitive measures, with the exception of Raven’s PM. As predicted, in addition to nonverbal ability, the main difference between the groups involved a vocabulary measure of English language proficiency. These contradictory results may be understood in terms of other factors such as age of arrival in the receiving country, the age of the children, parental education, socioeconomic status, and culturally-relevant practices. Moreover, the Kleemans et al. (2011) study focused on early numeracy skills in kindergarten students who had not yet received formal reading or mathematics instruction. It may therefore be plausible that linguistic processes are more heavily related to numeracy in the pre-school period, when basic literacy and numeracy knowledge is just beginning to simultaneously emerge. The present results show that arithmetic achievement does not seem to depend on language proficiency in the mid-elementary period (Grade 5) in the case of ELLs whose English language skills are more established. Because of the correlational nature of the study, it cannot be concluded that language proficiency does not have a causal impact on arithmetic performance, however, it is reasonable to suggest that as students progress through formal schooling in the mid-elementary years and achieve more advanced English language skills, language ability may be of less importance for arithmetic achievement.
Finally, it is important to note that although language ability did not play a significant role in determining individual differences in arithmetic achievement, scores on the receptive vocabulary task were significantly correlated with scores on the arithmetic measure in both language groups ($p < .01$), (refer to Table 2). These results indicate that there may, in fact, be a relationship between language and arithmetic computation. However, consistent with Vukovic and Lesaux (2013), this relationship is similar for both EL1 children and ELL children. Future studies should explore this relationship longitudinally, in order to determine the exact role of language in various areas of mathematics. Recent work involving mathematical word problem solving in monolinguals (e.g. Fuchs et al., 2006; Fuchs, et al., 2014), suggests that this should be a fruitful area of further study. Overall, while basic cognitive-linguistic processes are related to more effective arithmetic computation skills in EL1s and ELLs, having poorer English language proficiency should not place ELL students at a greater risk for impaired achievement in arithmetic and basic computation.

**Cognitive Profiles of EL1 and ELL Children At-Risk for MLD**

The final objective of the current study was to describe the cognitive profiles and cognitive deficits of children at-risk for MLD who were either EL1 or ELL. Firstly, the results demonstrated no performance differences on the cognitive measures between normally achieving EL1 and ELL children, with the exception of Raven’s PM. The normally achieving EL1 students outperformed the normally achieving ELL students on the measure of nonverbal ability. As suggested earlier, performance on the nonverbal task may perhaps be attributed to cross-cultural and cross-linguistic differences. At the same time, this EL1 advantage was not replicated in the case of the at-risk for MLD groups.
Secondly, as shown in Figure 1, it is evident that children who are at-risk for MLD exhibit similar deficits on the same cognitive functions, regardless of their home language backgrounds. On the whole, EL1 and ELL children who were categorized as at-risk for MLD showed no differences in their performance on any of the tasks, with the exception of the anticipated difference in PPVT Z scores. Regardless of language group, children who were at-risk for MLD, had lower nonverbal reasoning skills, slower naming speed, poorer working memory and receptive vocabulary, than normally achieving EL1 and ELL children. In other words, children who are at-risk for MLD will likely exhibit weaker WM, slower processing speed, and somewhat impaired nonverbal reasoning ability. Irrespective of language status, the arithmetic performance of ELLs who may be at-risk for MLD can be understood and predicted by considering the same cognitive processes that underlie mathematical difficulties in EL1 children.

Limitations

In addition to a number of limitations mentioned throughout this discussion, it is important to address some important caveats of this research study. Most importantly, although the children were selected from four successive cohorts, the current study was based on one year of schooling (Grade 5). Research has shown that there are children who struggle in mathematics at one time point, but not at another (transient MLD), though some experience persistent difficulties in mathematics (persistent MLD) (Vukovic & Lesaux, 2010). It is therefore possible that the findings may vary when cognitive processes and arithmetic performance are examined longitudinally. Further, the evaluation of achievement in mathematics was based solely on arithmetic performance. Although research suggests that arithmetic and basic computation are core impairments in
children with MLD, it may be beneficial to examine other areas of mathematics such as word problems to determine whether ELL children are at-risk of falling behind in other mathematical domains, especially word problems that involve more language skills. Finally, although the data were collected across 12 different schools in low SES neighborhoods, specific individual level demographic information was not available. Such information could shed additional light on the role of parental education, home language, and socio-economic status, in the development of arithmetic skills. Finally, factors such as attention, motivation, and instructional approaches have been shown to be related to achievement in mathematics (Geary et al., 2012a; Passolunghi, 2011; Passolunghi & Siegel, 2001). Future studies should include such behavioural measures in addition to cognitive measures when assessing arithmetic performance in various language groups.

**Implications for the Assessment and Diagnosis of MLD in ELL Children**

In line with reading disability research, the results of the current study demonstrate that the typical cognitive functions associated with foundational mathematical difficulties in EL1 children are similarly associated with these arithmetic difficulties in ELL children. These findings propose that working memory, RAN, and nonverbal reasoning measures can be used to reliably screen both EL1 and ELL children for difficulties in foundational mathematics throughout the elementary grades. Specifically, the study also revealed no differences in arithmetic performance between normally achieving EL1 and normally achieving ELL students, nor did it reveal differences in arithmetic performance between at-risk EL1 and at-risk ELL students. This finding suggests that once English language skills are more established in the mid-
elementary grades, it is possible to use standardized arithmetic tasks and norms when assessing ELL arithmetic achievement. Further, the results indicate that language ability may not play an essential role in predicting the arithmetic computation skills in EL1 and ELL children alike. Although language ability may be important for higher-order mathematical cognition and mathematical word problems (Fuchs et al., 2006; Fuchs, et al., 2014; Vukovic & Lesaux, 2013), weaker English language proficiency and ELL status should not place ELL children at a greater risk for poor achievement in basic computational or operational mathematics when children have attended school on a regular basis. These findings are important for researchers, teachers, school professionals, and clinicians who are trying to disentangle the role of language proficiency in the assessment and diagnosis of learning disabilities in mathematics in ELL students. That being said, individual differences on these cognitive measures can guide teachers and school professionals on determining which students are struggling in mathematics simply due to poor English skills or because of more pervasive underlying learning difficulties. Just like EL1 children, ELL children can have specific learning disabilities in mathematics. Moreover, the current research suggests that ELL children can be assessed and identified even when full English language proficiency has not been attained yet.

These results also shed light on the future directions for the development of evidence-based arithmetic interventions. Although working memory and RAN training programs have been shown to improve these cognitive functions (Westerberg & Klingberg, 2007), it is yet to be determined whether these types of cognitive interventions can lead to increases in the acquisition and effective use of arithmetic skills. Future studies are needed to determine whether specific WM and RAN training programs will
lead to gains in arithmetic calculation. However, based on the current findings, evidence-based interventions that are developed to ameliorate arithmetic calculation should be equally effective for both EL1 and ELL school-aged children based on the similar cognitive profiles that emerged for both language groups, though this question deserves additional attention.
References


doi:http://dx.doi.org/10.1016/j.lindif.2012.05.004

Andersson, U. (2010). Skill development in different components of arithmetic and basic cognitive functions: Findings from a 3-year longitudinal study of children with different types of learning difficulties. Journal of Educational Psychology, 102(1), 115-134. doi:http://dx.doi.org/10.1037/a0016838


doi: [http://dx.doi.org/10.1016/j.tics.2010.09.007](http://dx.doi.org/10.1016/j.tics.2010.09.007)


doi: [http://dx.doi.org/10.1037/a0034097](http://dx.doi.org/10.1037/a0034097)


doi:http://dx.doi.org/10.1016/j.cogdev.2009.10.001


http://search.proquest.com/docview/61874883?accountid=14771


doi: [http://dx.doi.org/10.1111/j.1467-8624.2010.01508.x](http://dx.doi.org/10.1111/j.1467-8624.2010.01508.x)


doi: [http://dx.doi.org/10.1177/0022219409355482](http://dx.doi.org/10.1177/0022219409355482)

Annual Review of Psychology, 63, 427-452. doi:http://dx.doi.org/10.1146/annurev-psych-120710-100431


doi: [http://dx.doi.org/10.1016/j.jep.2013.02.002](http://dx.doi.org/10.1016/j.jep.2013.02.002)


doi: [http://dx.doi.org/10.1111/j.1540-5826.2009.00298.x](http://dx.doi.org/10.1111/j.1540-5826.2009.00298.x)


Appendix A

Table 4. Means (standard deviations), and group comparisons for each cognitive predictor, arithmetic measure, and language measure.

<table>
<thead>
<tr>
<th>Measures</th>
<th>Normally Achieving (N=192)</th>
<th>At-Risk for MLD (N=71)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EL1(n=62)</td>
<td>ELL(n=130)</td>
</tr>
<tr>
<td>PPVT</td>
<td>.60 (.83)</td>
<td>-.11 (.89)</td>
</tr>
<tr>
<td>Raven’s PM</td>
<td>.40 (.85)</td>
<td>.03 (1.00)</td>
</tr>
<tr>
<td>Backward Digit Span</td>
<td>.13 (.83)</td>
<td>.14 (1.06)</td>
</tr>
<tr>
<td>RAN Letters</td>
<td>-.09 (.90)</td>
<td>-.22 (.93)</td>
</tr>
<tr>
<td>WRAT Math$^a$</td>
<td>.35 (.63)</td>
<td>.54 (.70)</td>
</tr>
</tbody>
</table>

*Note.*

$^a$. The WRAT Math (i.e., arithmetic measure) variable was used to define NA and AR subgroups in each language group

**. $p<0.01$ level (2-tailed).

*. $p<0.05$ (2-tailed).
### Table 5. Means (standard deviations), and group comparisons between Normally achieving students and At-Risk students within the EL1 and ELL groups.

<table>
<thead>
<tr>
<th></th>
<th>Normally Achieving</th>
<th>At-Risk for MLD</th>
<th>Test of Group Differences</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>EL1</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPVT</td>
<td>.60 (.83)</td>
<td>.17 (1.07)</td>
<td>(t(89)=-2.12^*)</td>
</tr>
<tr>
<td>Raven’s PM</td>
<td>.40 (.85)</td>
<td>-.37 (1.05)</td>
<td>(t(89)=-3.75^{**})</td>
</tr>
<tr>
<td>Backward Digit Span</td>
<td>.13 (.83)</td>
<td>-.56 (.74)</td>
<td>(t(89)=-3.87^{**})</td>
</tr>
<tr>
<td>RAN Letters</td>
<td>-.09 (.90)</td>
<td>.55 (1.11)</td>
<td>(t(45.89)=2.74^{**})</td>
</tr>
<tr>
<td>WRAT Math(^a)</td>
<td>.35 (.63)</td>
<td>-1.31 (.35)</td>
<td>(t(86.50)=-16.29^{**})</td>
</tr>
<tr>
<td><strong>ELL</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PPVT</td>
<td>-.11 (.89)</td>
<td>-.66 (1.01)</td>
<td>(t(170)=-3.36^{**})</td>
</tr>
<tr>
<td>Raven’s PM</td>
<td>.03 (1.00)</td>
<td>-.43 (.92)</td>
<td>(t(170)=-2.66^{**})</td>
</tr>
<tr>
<td>Backward Digit Span</td>
<td>.14 (1.06)</td>
<td>-.25 (1.05)</td>
<td>(t(170)=-2.09^{\dagger})</td>
</tr>
<tr>
<td>RAN Letters</td>
<td>-.22 (.93)</td>
<td>.42 (1.04)</td>
<td>(t(170)=3.76^{**})</td>
</tr>
<tr>
<td>WRAT Math(^a)</td>
<td>.54 (.70)</td>
<td>-1.30 (.32)</td>
<td>(t(151.77)=-23.51^{**})</td>
</tr>
</tbody>
</table>

**Note.**

\(^a\) The WRAT Math (i.e., arithmetic measure) variable was used to define NA and AR subgroups in each language group.

\(^\dagger\) **. \(p<0.01\) level (2-tailed).

\(^*\) * \(p<0.05\) (2-tailed).