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**Lateral compression response of overlapping jet-grout columns with
geometric imperfections in radius and position**

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Lateral compression response of overlapping jet-grout columns with geometric imperfections in radius and position

Abstract

Spatial variability in the radius of a jet-grout column is commonly encountered in practice. Although various prediction models for the column radius are available, they were generally used to predict a nominal radius. The radius variation within a column has been seldom considered. In this study, the intra-column radius variation was simulated as a lognormal stochastic process. This was done based on the existing prediction models where the column radius can be correlated with the undrained shear strength of in-situ soils. A slab consisting of overlapping jet-grout columns was considered. The slab serves as an earth retaining stabilizing structure in a deep excavation. The effects of radius variation on the mass performance of the slab were examined with the finite-element method. In addition, the positioning errors in jet-grout columns were also investigated. Owing to the random nature of the radius variation, the Monte-Carlo simulations were performed to estimate the statistic characteristics of the mass performance of the slab. A strength reduction factor was introduced and tabulated to account for the effects of geometric imperfections in the column radius and the column position. With the strength reduction factor, practitioners could quantitatively evaluate the effects of these geometric imperfections in design considerations. Practical recommendations on the column length and column spacing were also proposed.

KEYWORDS: Jet-grouting technique, Deep excavation, Ground improvement, Statistical analysis, Stochastic process, Finite-element modelling

Introduction

The jet grouting technique has been extensively used to facilitate deep excavation and underground constructions in a wide spectrum of soils (e.g. Coulter and Martin 2006; Flora *et al.* 2011; Yang *et al.* 2011; Modoni and Bzówka 2012; Wang *et al.* 2013; Ochmański *et al.* 2015a; Hu *et al.* 2017). It has been used for fulfilling functions like water-proofing and soil reinforcement. An approximate columnar shape is formed by injecting a high-momentum fluid into permeable or erodible soils through rotating small-diameter nozzles. Then, the injected fluid is mixed with soils after seeping through voids (for pervious soils) or eroding surrounding soils (for fine-grained soils).

A slab consisting of overlapping jet-grout columns in a deep excavation is often designed to serve as an earth retaining stabilizing structure to resist the lateral compressive pressure that is transferred from the retaining walls. Although the unconfined compression strength of core samples is required to exceed a certain value (e.g. 0.6 MPa, see COI 2005), the mass performance of the slab depends on the continuity among individual jet-grout columns. Although the overlapping distance is always designed to ascertain that no zones are untreated, untreated soils are occasionally encountered in practice among the jet-grout columns. These untreated zones can cause sudden piping through untreated zones (Morey and Campo 1999) or significant reduction in mass strength and stiffness, especially when the untreated zones are connected (Liu *et al.* 2015). Many studies reported the presence of untreated zones in such slabs (e.g. Pellegrino and Adams 1996; Shirlaw 1996; Morey and Campo 1999). Morey and Campo (1999) noted that the frequency of the untreated zones increased with depth, especially when the drilling depth exceeded 15 m. In practice, the improvement level of a jet grouting project is likely to be much deeper than 15 m. For

example, the depth was 25-40 m in the Cairo Metro project (Morey and Campo 1999) and around 35 m in the Nicoll Highway project (COI 2005). After an extensive review of the jetting parameters and field procedures, Morey and Campo (1999) summarized that the major sources causing untreated soils among jet-grout columns are the imperfections in the column radius and column position. The former occurs as a result of local variation in the soil characteristics over the column length. The latter results from the unavoidable deviations of the jet-grout column positions from their designated locations during the drilling process.

Many studies can be found on these two types of geometric imperfections. Firstly, the column diameter was observed varying with the layered soil profile (e.g. Croce and Flora 2000; Shen *et al.* 2009; Modoni *et al.* 2016). Flora *et al.* (2013) summarized the diameter variation in jet-grout columns. The results show that the coefficient of variation (COV) in the diameter ranges from 0.04 to 0.29 in clayey soils. This is not surprising since "calabash-shaped" jet-grout columns were often encountered (see Fig. 1(a)). Modoni *et al.* (2016) examined the effect of the diameter variation on the volume of untreated zones. In their study, the diameter of each column is considered as an independent Gaussian random number. As a result, it is likely that perfectly cylindrical columns with small diameters will be generated, which might result in connected, untreated zones from the top to the bottom of the treated layer. In reality, as the column diameter varies with depth (see Croce *et al.* 2014), some parts of the adjacent jet-grout columns may overlap while other parts may not. This creates disconnected, untreated pockets. In this regard, a more realistic simulation may be considering the intra-column variation (Fig. 1(a)). To capture this feature, the column diameter should be described as a stochastic process instead of a random number. This idea was actually originated from

Modoni and Bzówka (2012) who noted that the random deviation from the mean diameter should be as the result of a stochastic process. A stochastic process describes the column diameter as a function of the depth coordinate (see Fig. 1(a)), whereas a random number is independent of the depth coordinate (see Fig. 1(b)). Nevertheless, a realization of the stochastic process can readily reduce to a random number by setting an infinitely large scale of fluctuation (SOF, see definition in Vanmarcke 1983). The idea of using stochastic process or random field to describe geometric imperfection is widely used in literature (e.g. Schenk and Schueller 2007; Chen et al. 2016).

Secondly, the deviation of the column axis from the predetermined direction may lead to untreated zones. Fig. 2 illustrates this deviation. British Standard (BSI 2001) states that *“the deviation of drilling from the theoretical axis should be 2% or less for depth up to 20 m. Different tolerances should apply for greater depths and for horizontal jet grouting”*. Even within this tolerance level, the resulting deviation would not be insignificant, when considering a large drilling depth. Statistical characteristics of the drilling inclination are available in literature (e.g. Croce and Modoni 2007; Flora et al. 2011; Arroyo et al. 2012). Liu et al. (2015) investigated the effect of positioning error on the mass performance of a cement-treated slab as a strut. They found that the error in column positioning was the major influencing factor on the mass performance of the slab.

Despite the increasing popularity in jet grouting, the effect of geometric imperfections on the mechanical performance of a jet-grout underground strut has not yet been extensively studied. To quantitatively examine the mechanical performance, the finite-element method would be a suitable tool to consider the complex geometric boundaries that result from the imperfections in the column radius and the column

position. However, few studies were performed from this standpoint. One possible reason is that a rather fine mesh size is required to capture the geometric imperfections, which renders the calculation practically infeasible and time-consuming. In Liu *et al.*'s (2015) study, the slab consisting of around 160 deep-mixing columns was modelled by the finite-element method, but the radius variation was not considered as it is generally insignificant in a deep-mixing column. However, the radius variation in a jet-grout column is likely to be much more pronounced than that in a deep-mixing column.

As an extension of Liu *et al.*'s (2015) work, the current study examines the feasibility of considering the imperfections in both the column radius and the column position by finite-element analysis. A slab consisting of overlapping jet-grout columns was considered, which serves as an underground strut to resist lateral compressive pressure. The effects of geometric imperfections on the mass performance were investigated by considering a unit-cell of the slab. The column radius was described as a lognormal stochastic process in order to account for the intra-column radius variation. Statistical characteristics of the mass performance were assessed by Monte-Carlo simulations. A strength reduction factor was introduced and tabulated to account for the effects of geometric imperfections. Practical recommendations on the column length and column spacing were also proposed at the end of the study. It is noteworthy that a jet-grout ground also has other functionalities in excavations, including prevention of water inflow in sandy soils at the bottom of excavations, which are out of the scope of this study. Unless otherwise stated, the soils refer to soft clayey soils (e.g. marine clay).

Methodology

Column radius estimation

Various prediction models can be found to estimate the column radius (or diameter) based on mechanical properties of in-situ soils and hydraulic properties of injected fluid (e.g. amongst others, Corce and Flora 2000; Modoni *et al.* 2006; Ho 2007; Flora *et al.* 2013; Shen *et al.* 2013; Ochmański *et al.* 2015b). Ribeiro and Cardoso (2017) conducted an extensive review on the prediction models. Table 1 summarizes four typical prediction models. The hydraulic properties of injection (e.g. velocity of injected slurry at a nozzle) are predetermined by practitioners prior to construction; the variations in these properties are likely to be negligible in contrast to those in the mechanical properties of in-situ soils. For this reason, we postulate that the radius variation is mainly due to the variation in the mechanical properties of in-situ soils. These variations include the undrained shear strength and the unit tip resistance of a cone penetration test. As such, it should be reasonable to correlate the radius with the soil strength as:

$$R = A \cdot (c_u)^{-\zeta} \quad (1)$$

where R is the column radius; A is a deterministic constant accounting for all hydraulic properties of injection; c_u is the undrained shear strength of in-situ soils which accounts for the mechanical properties (other mechanical properties could be converted to c_u); ζ is a constant taking value of 0.25 or 0.5 as suggested by the prediction models listed in Table 1. The inversely proportional relationship between soil strength c_u and predicted radius R in equation (1) was observed in field data from case studies (see Croce *et al.* 2014), and such a relationship is also consistent with Whittle and Davies' (2006)

statement that “*the columns will be much smaller if jetting is carried out within stronger layers...*”. Although we restrict equation (1) to cases of jet grouting in clay, one can readily extend the analysis to coarse grained soils, as long as the prediction models (see Table 1) are extendable.

Statistical characteristics

Table 2 summarizes results from various studies on the spatial variation in the undrained shear strength of in-situ soils. As Table 2 indicates, the horizontal SOF in c_u is often much greater than the vertical value. It is also greater than the column scale. For this reason, only the variation of in-situ soils along the depth direction was considered; that is, laminated soils were considered.

The undrained shear strength of in-situ soils is often assumed to follow the lognormal distribution (e.g. Fenton and Griffiths 2008). This is because of its non-negative property and simple relationship with the normal distribution. Following this assumption, the probability distribution of the column radius also follows the lognormal distribution, as implied by equation (1). Therefore, the COV of the column radius can be related to that of the undrained shear strength (see detailed derivations in Appendix 1):

$$\delta_R = \zeta \cdot \delta \quad (2)$$

where δ_R and δ are the COVs of the column radius and the undrained shear strength of in-situ soils, respectively. Table 2 shows that δ is typically less than 0.5. Since ζ takes a value of either 0.25 or 0.5, equation (2) implies that δ_R is likely to be less than 0.25. This finding is generally consistent with the field data reported by Flora *et al.* (2013) and Croce *et al.* (2014). They reported that the COV of the jet-grout column diameter

typically ranges from 0.04 to 0.29 and from 0.06 to 0.19, receptively. Modoni *et al.* (2016) suggested that the column radius follows the normal distribution. In this regard, the difference between the lognormal and normal distributions would be insignificant in fitting limited data points with a COV being less than 0.25. Furthermore, the lognormal distribution would eliminate the possibility of a counter-intuitive, negative radius. Equation (2) is independent of the mean radius; the latter is the other determining statistic of a lognormal distribution. The mean radius can be evaluated as follows.

Equation (1) leads to a nominal radius R_0 in a deterministic analysis when the COV in c_u is zero. The corresponding nominal column volume ($\pi R_0^2 L$) would be of more practical interest, as it directly determines the cement usage. When the COV in c_u is greater than zero, both the column radius and column volume are random variables. The volume of a column with a varying diameter is considered as an index of cement usage; for comparison, the average volume is set as the nominal column volume (see Fig. 1):

$$E \left[\int_0^L \pi R^2(z) dz \right] = \pi R_0^2 L \quad (3)$$

where $E[\]$ is the expectation operator, and L is the column length. Since $R^2(z)$ is a continuous function, and $E[R^2] = Var[R] + \{ E[R] \}^2$, one has:

$$E[R] = \frac{R_0}{\sqrt{1 + \delta_R^2}} \quad (4)$$

Equation (4) suggests that, in order to maintain a constant cement slurry usage, the radius mean decreases as the variation in radius increases. Fig. 3 shows the deceasing rate.

Since the horizontal variation of c_u is not taken into account, the undrained shear strength can be simulated as a lognormal stochastic process along the depth direction. The mean, variance and autocorrelation of the lognormal stochastic process within the model were assumed to be constant, as the thickness of jet-grout columns is usually only a few meters. A lognormal stochastic process can be translated from a zero-mean and unit-variance Gaussian process through their exponential relationship, which is a commonly used procedure of transformation method (see Liu *et al.* 2013). By virtue of equation (2), both the lognormal stochastic processes for c_u and R can be generated from the same Gaussian stochastic process. This feature ascertains the consistency between the column shape and the surrounding soils around the column. It also enables the compatibility of simulating both jet-grout materials and in-situ soils in the same finite-element model. Fig. 4 demonstrates the simulated stochastic processes for c_u and R . These are translated from the same underlying Gaussian process as illustrated in Fig. 4(a). In particular, Fig. 4(d) depicts one-quarter of a column modelled by finite-elements based on the stochastic processes in Figs 4(b) and 4(c). The autocorrelation function of the Gaussian process is selected as the widely-used, single exponential model (see Vanmarcke, 1983). Since the maximum COV herein is 0.5, the changes in SOF caused by the exponential transformation are negligible (see Liu *et al.* 2013). In other words, the SOFs in Figs 4(a)-4(c) are roughly the same. The underlying Gaussian process with a single exponential autocorrelation function was generated by the Cholesky decomposition method. This method analytically yields prescribed marginal Gaussian distribution and autocorrelation function.

Finite-element analysis

In a ground consisting of jet-grout columns, the columns are often arranged in an

overlapping pattern to act as a strut under the formation level. Fig. 5(a) illustrates a commonly used arrangement pattern (see Soga *et al.* 2004; Modoni *et al.* 2016) as well as a unit-cell of this pattern. Another arrangement of columns will be discussed later on. The column spacing can be described by the ratio s/R_0 , where s is the centre-to-centre distance, and R_0 is the nominal radius as discussed earlier. Because of its symmetry, one-quarter of the unit-cell is sufficient to represent the entire ground. It should be noted that this symmetry assumption may not be fully fulfilled in reality because the soil profile is not perfectly layered as well as some model uncertainties may exist in Eq. (1). A three-dimensional model with s/R_0 equalling 3.33 is shown in Fig. 4(d). The model consists of 62,500 eight-node brick elements with reduced integration. No surcharge on the top face was considered for conservativeness. The model geometric size and mesh size of the model shown in Fig. 4(d) will be used throughout this study. The “soils” and “jet-grout materials” differ from one another due to the assigning of different strength values for each element; the strength can be described as a function of spatial coordinates according to the specific realizations of the lognormal stochastic processes for the column radius and in-situ soils. The model was simulated by the software ABAQUS/Standard version 6.14, which is a commonly used finite-element software. The validation work of modelling soil-cement columns can be found elsewhere (e.g. Liu *et al.* 2014; 2015).

For simplicity, both the in-situ soils and jet-grout materials were simulated as elastic-perfectly plastic materials with Mohr-Coulomb failure criteria. This model is often used in common practice (Modoni *et al.* 2016). Table 3 lists the properties. The elastic-perfectly plastic is unable to reflect the strain-softening behaviour of cement-admixed soils, which occurs after the material reaching its peak strength value;

nevertheless, in a real excavation project, the allowable global strain is generally about 0.5% of the excavation depth. This global strain level could prevent the soil-cement material from reaching the peak strength value.

Considering the low permeability coefficient of both cement-admixed clay and natural clay (typically 10^{-8} - 10^{-10} m/s, see Yu *et al.* 1999; Chu *et al.* 2002; Croce *et al.* 2014), undrained conditions were assumed and a total stress Poisson's ratio of 0.49 was used in the numerical analysis. By doing so, we circumvented the consideration of variations in Poisson's ratio and permeability of the jet-grout materials, as the main objective of this study is to examine the effect of geometric imperfections in radius and position on overall performance. The unconfined compression strength, q_{u0} , of jet-grout materials was taken as constant. This was done in order to isolate the effects caused by the geometric imperfections from those caused by the spatial variation in material properties; the latter has been considered by various investigators (e.g. Liu *et al.* 2015; Modoni *et al.* 2016; Liu *et al.* 2017a). To weaken the effect of the specific strength value of jet-grout materials, all calculated stresses were normalized by q_{u0} . The calculated stress refers to the reaction force from the displacement-controlled surface divided by the surface area.

Results and discussions

The effects of the strength COV of in-situ soils, column length and column spacing were examined. The ranges of those factors were estimated based on the existing literature as summarized in Table 2. The case with the most probable values of these factors was referred to as the *reference case*. Then, parametric studies were performed based on this *reference case* by changing a single parameter while keeping other

parameters constant. The effect of the index ζ in equations (1) and (2) was not considered extensively, which can be done by altering the radius COV as shown in equation (2). To examine this equivalence, two soil profiles were considered: (1) soil strength COV is 0.4 and ζ is 0.25; (2) soil strength COV is 0.2 and ζ is 0.5. Other parameters were set as the same for both cases. The soil profiles for both cases were generated using the same underlying Gaussian process shown in Fig. 4(a). Based on equation (2), the resultant COV in the radius would be 0.1 for both. Thus, essentially there is no difference between these two cases, which is validated by the stress-strain curves for these two cases, as shown in Fig. 6.

Reference case

Table 2 suggests that the in-situ soil strength COV is generally less than 0.5, and a value of 0.3 was selected in the *reference case*. The vertical SOF was selected as 0.5 m; as a result, the ratio of SOF to the column length (2 m) is 0.4. The column spacing ratio s/R_0 was 1.54. The scenario plotted in Fig. 2(c) reflects this s/R_0 value.

Figs 7(a) and 8(a) depict the contours of undrained shear strength for two typical realizations of the *reference case*; the former has a zero COV in radius (i.e. no variation), while the latter has a COV of 0.3 in radius. The top view of the model is essentially one-quarter of the unit cell shown in Fig. 3(b). However, the three-dimensional model cannot be simplified as a plane-strain problem, since the top surface is free. In addition, due to the existence of the natural soils (shown with grey colour), the stress within the model is non-uniform and the top face has the tendency to move upwards. This may justify the failure pattern shown in Fig. 7(b), and non-uniformity in the stress shown in Fig. 7(c). Comparisons between Figs 7(b) and 8(b) indicate that the presence of variation in the radius is likely to affect the failure mode. This effect will be

further examined later on. Both Figs 7(c) and 8(c) show little portion with a positive minimum principal stress, implying that the models were almost always under compression (ABAQUS adopts a tension-positive sign convention). This characteristic ascertains that the Mohr-Coulomb model with a zero friction angle could work properly. This is because those types of materials (i.e. cement-admixed clays) have a much lower tensile strength than the unconfined compression strength (see Pan *et al.* 2015), and the original Mohr-Coulomb model cannot properly consider the tensile failure.

As stated earlier, the radius variation is simulated by a stochastic process. As a result, each realization of the stochastic process yields a different shape of column. Thus, numerous (e.g. hundreds of) realizations have to be simulated so that the statistical characteristics of the overall strength can be evaluated. In this study, 200 realizations were considered for each case studied. The number of 200 is adopted by considering the system level accuracy. The system includes input parameters and output parameters. As for the input parameter of a geotechnical problem (e.g. shear strength of in-situ soils in this study), the data volume is usually small. In addition, as will be shown later, the input data usually have a much greater COV than that of the output data (i.e. simulation results). In this regard, the reliability level of the current study is likely to be less than that required by reliability analysis with a failure probability being less than 10^{-3} (e.g. Wang *et al.* 2011).

Fig. 9(a) shows that the radius variation, in general, adversely affects the mass performance. The realization without variation (termed deterministic analysis in Fig. 9(a)) almost yields the greatest overall strength compared with those of the 200 realizations with radius variation. It is noteworthy that the probability distribution of the overall strength does not necessarily follow the normal distribution. As Fig. 9(b) shows,

a left-skewed distribution may be obtained. That is because the overall strength has an achievable upper limit: the unconfined compression strength of jet-grout materials, q_{u0} . This upper limit occurs when the model purely consists of jet-grout materials.

To measure the spread of variation in the results, the average, minimum and maximum overall strengths were also marked. The minimum overall strength is likely to be of practical interest, as it directly relates to the safety margin of a project. Although 200 repeated calculations (i.e. Monte-Carlo simulations) were performed for each case, the minimum value out of 200 calculations could even overestimate the real minimum overall strength due to the insufficiency in the number of calculations. To account for this bias, the minimum overall strength was adopted as:

$$Q_{\min} = Q_1 - (Q_2 - Q_1) \quad (5)$$

where $Q_{\min}$ is the adopted minimum overall strength, and Q_1 and Q_2 are the least and second-least overall strengths, respectively, out of 200 calculations. The form of equation (5) was often adopted to estimate the lower bound of a random variable without the information of its probability distribution (see Cooke 1979; Liu *et al.* 2016).

Parametric studies

Fig. 10(a) illustrates the effect of soil strength COV on the mass performance. As expected, the soil strength COV adversely affects the mass performance. When the COV is 0.3, for instance, the minimum overall strength is around 83% of the strength when COV is zero; this is out of 200 realizations. Although the variation in soil strength is a natural property and practitioners are unlikely to change its value, a reasonable estimation of the COV value would improve the accuracy in assessing the mass performance. The estimation of the COV in the mechanical properties of in-situ soils

would require both geotechnical and statistical knowledge.

Fig. 10(b) shows the effect of column length on the mass performance. The column length is normalized by the SOF. As the figure shows, when the ratio of column length to SOF is less than four (i.e. the value adopted by the *reference case*), a sharp reduction can be observed in the minimum overall strength. This is because a great SOF is likely to result in a clay layer with a roughly uniform but high value of strength. As a result, the column radius in this layer tends to be small. As an extreme case, when the SOF is infinitely large, a realization of the lognormal stochastic process reduces to a lognormal random number. Then, the minimum overall strength is likely to reduce to the undrained shear strength of in-situ soils. Nevertheless, the effect of SOF is likely to be controlled by altering the column length. The vertical SOF of natural clay is generally less than 6 m, as summarized in Table 2. For a specific project, the vertical SOF of the improvement zone is likely to be much smaller than 6 m. This is because the improvement zone is usually less than 6 m in thickness. A large vertical SOF is likely to result in a deterministic trend, which will be de-trended in calculating the autocorrelation for a specific site. Consequently, the mean strength will be affected instead. Mathematically speaking, the ergodicity of a stochastic process requires an unlimited domain but the improvement zone thickness is limited. Thus, the ensemble average does not necessarily equal the spatial average within the improvement zone. As a result, it would be practically feasible to ensure that the ratio of column length to SOF is greater than four.

The column spacing is likely to be affected by two sources. The first one is the design column spacing without positioning error, as shown in Fig. 2(b). The columns are usually designed to have certain overlapping zones to ensure some safety margin.

Specifically, the s/R_0 ratio is usually designed to be less than 1.414. However, as Fig. 10(c) shows, even the ratio s/R_0 is smaller than this critical value (i.e. 1.414). The overall strength is still likely to be smaller than the unconfined compression strength of jet-grout materials due to the existence of spatial variation in the radius. The second source lies in the positioning errors; that is, the errors due to the off-verticality during the drilling process. The allowable inclination in drilling is around 0.5-2% (see BSI 2001; Stoel 2001; Passlick and Doerendahl 2006; Geo-Institute/ASCE 2009), which is likely to result in a significant deviation in the column position when the improvement zone is often 20-40 m below the ground surface. Fig. 2(a) depicts the worst scenario where all four columns deviate from each other as the depth increases. This is possible since the jet-grout columns are often installed low by low, and system bias in drilling has the tendency to propagate (Liu *et al.* 2015). In this regard, the column spacing is a function of the drilling depth, and the potential untreated zones increase as the drilling depth increases. This is consistent with the field observation reported by Morey and Campo (1999). Based on this worst-case scenario, various s/R_0 ratios were considered and their results are plotted in Fig. 10(c). As this figure shows, the overall strength sharply decreases as the ratio s/R_0 increases. The significant influence of positioning errors on the mass performance implies that the verticality control in drilling is critical in a ground improvement project. However, as reported by Morey and Campo (1999), since the diameter of secondary columns may be significantly reduced due to the existence of “masked zones”, it may be impractical to reduce the volume of untreated zones by tightening up the drilling pattern. As a result, the adverse effects caused by the positioning errors would only be controlled by reducing the maximum allowable inclination and enhancing the supervision of workmanship.

In the current study, the most unfavourable combination was considered; that is, the most pessimistic scenario of column inclination and the lower bound of overall strength. This was done in response to the high requirement of probability of failure in geotechnical engineering. For earth retaining and foundation elements, the probability of failure is generally required to be around 10^{-4} (e.g. Meyerhof 1995; Chen 2016; Fenton *et al.* 2016). This level of probability of failure is based on an overall safety factor ranging from 2 to 3 under the ultimate states design (see Meyerhof 1995).

The height of the finite-element model is 2 m. A column within 2 m is likely to incline about two centimetres. The effect of this level of inclination would be limited. For this reason, the intra-model inclination was not considered. The first four moments of order statistics based on the results of the 200 realizations of all cases were summarized in Table 4.

Engineering implications

In a practical design, a strength reduction factor may be applied on core strength data to account for the effects of geometric imperfections:

$$Q_d = \beta \cdot q_{u0} \quad (6)$$

where Q_d is a representative strength where the effects of geometric imperfections are accounted for; β is the strength reduction factor; q_{u0} is the average unconfined compression strength based on core strength data. The minimum overall strength based on equation (5) was selected as the representative strength, from which the factor β is calculated.

The strength reduction factor β could be determined from the drilling depth and deviation tolerance in drilling; Fig. 11(a) tabulates the results for the rectangular arrangement of columns. In reality, the triangular arrangement of columns (see Fig. 12(a)) is also widely adopted. Figs 12 (b) shows the geometric size, element size and boundary conditions of a unit cell of this type of arrangement. Fig. 12(c) illustrates a typical realization of unit cell under 0.3 COV in undrained shear strength of in-situ soils. Following the procedures for the rectangular arrangement, the cases for the triangular arrangement were analysed, and the results are tabulated in Fig. 11(b). It can be found that, compared with the rectangular layout, the triangular layout generally yields a greater β value when other parameters are the same. This is because, under the same column spacing, the triangular arrangement has a smaller untreated zone; that is, the triangular array has a more compacted arrangement of columns.

At the ground surface (i.e. zero drilling depth), the s/R_0 ratio was set as 1.25. Some technical codes (e.g. JGJ 79-2012) further require that the overlapping distance between two jet-grout columns should exceed 300 mm. This requirement should also be fulfilled at the ground surface, showing as a prerequisite in applying the chart of Fig. 11.

It is noteworthy that, in the current study, the most unfavourable combination was considered; that is, the most pessimistic scenario of column inclination and the lower bound of overall strength. This conservative consideration can be served as a safety margin. Nevertheless, even based on the most unfavourable combination, the results shown in Fig. 11 can still offer guidelines for design. For instance, based on Fig. 11(a), the strength reduction factor is 0.72 for an excavation of 20 m and 1% tolerable column position deviation. Based on Liu et al.'s (2015) result, the spatial variation in strength would further introduce a reduction factor of 0.85. The resultant reduction would be

around 0.6. Considering the average unconfined compression strength of cement-admixed soil is around 1.6 MPa to 2.0 MPa, the overall strength of a slab could be 0.96 MPa to 1.2 MPa, which is still higher than the values often used in design (e.g. 0.6 MPa, see COI, 2005). In this regard, if the columns are arranged in a triangular pattern, where the chart in Fig. 11(b) is applicable, an even greater overall strength can be derived. This finding implies that the strength design of cement-admixed soils may be over-conservative.

Concluding remarks

This study considered a slab consisting of jet-grout columns serving as a strut below the formation level in order to resist the lateral compressive pressure transferred from retaining walls. Two sources of uncertainty resulting in geometric imperfections of the columns were examined: the variation in mechanical properties of in-situ soil, and the off-verticality in drilling during the construction of columns. This study demonstrated the feasibility of employing the finite-element method to account for the geometric imperfections in a quantitative manner. A simple and explicit formula (i.e. equation (2)) was proposed to correlate the column radius COV with the undrained shear strength COV of in-situ soils based on the existing prediction models for column radius. The variations in the in-situ soils and column radius can be simulated as two lognormal stochastic processes. They both can be generated from the same Gaussian stochastic process by virtue of the proposed formula (i.e. equation (2)). This feature ascertains the consistency between the column shape and the surrounding soils around the column, as well as enables the feasibility of simulating both jet-grout materials and in-situ soils in the same finite-element model.

Three main factors affecting the mass performance of the slab were considered in parametric studies: the variation in column radius, column length, and column spacing. Firstly, the variation in column radius adversely affects the mass performance. Secondly, the column length was examined together with the SOF of in-situ soils. The ratio of column length to SOF was observed to have a significant effect on the mass performance. This effect is pronounced when the ratio is less than four, where a sharp reduction in the minimum overall strength can be observed. For this reason, it is recommended that the jet-grout columns be designed with a length more than four times the value of SOF in in-situ soils. Thirdly, the positioning errors are demonstrated as potentially increasing as the improvement depth increases due to the off-verticality during drilling. The increase in positioning errors adversely affects the mass performance. This might only be mitigated by reducing the maximum allowable inclination and enhancing the supervision of workmanship. Charts were developed to evaluate the strength reduction factor that accounts for the effects of geometric imperfections. Furthermore, the main statistics of the Monte-Carlo simulation results were also tabulated so that interested readers could interpret the results in other manners.

Compared with Liu *et al.*'s (2015) work, where a slab scale model has been considered, this study examined the effects of variations in column radius and column positioning. To reflect the variation in the column radius, the mesh size has to be set rather fine which renders a slab scale model impracticable. Instead, unit-cell scale models were used. Owing to this limitation, the spatial variation in material properties of cement-admixed soils (see Namikawa 2016; Liu *et al.* 2017b and 2017c; Toraldo *et al.* 2017) as well as the model uncertainty (see Liu *et al.* 2007) in Equation (1) was not

accounted for. Importance sampling schemes (e.g. subset simulation, see Au and Beck 2001; Au and Wang 2014) may enhance the reliability level, which is the future work of this study. In addition, the results in the current study are applicable to the earth retaining stabilizing structure loaded in compression. Other failure modes (e.g. basal heave failure) will need to be investigated separately. A system reliability approach may be adopted to consider a combination of significant failure modes and will also be the subject of future study.

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Notation list

A	deterministic constant
c_u	undrained shear strength
E	expectation operator
L	column length
Q_1	least overall strength out of 200 Monte-Carlo simulations
Q_2	second-least overall strength out of 200 Monte-Carlo simulations
Q_d	representative strength
$Q_{\min}$	minimum overall strength
q_{u0}	unconfined compression strength of jet-grout materials
R	radius of jet-grout column
R_0	nominal radius of jet-grout column
s	centre-to-centre spacing
Var	variance operator

z	depth axis
β	strength reduction factor
δ	coefficient of variation of in-situ soil undrained shear strength
δ_R	coefficient of variation of column radius
ζ	index in diameter prediction model

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Appendix 1

Taking the natural logarithm of both sides of equation (1) yields

$$\ln(R) = \ln(A) - \zeta \ln(c_u) \quad (A1)$$

where $\ln(A)$ is a constant, and $\ln(c_u)$ is a random variable following the normal distribution because c_u is assumed to follow the lognormal distribution. Thus, $\ln(R)$ also follows the normal distribution. The variance of $\ln(c_u)$ can be calculated as:

$$Var[\ln(c_u)] = \ln[1 + \delta^2] \quad (A2)$$

where δ is the COV of c_u . The variance of $\ln(R)$ can be calculated by considering equations (A1) and (A2):

$$Var[\ln(R)] = \zeta^2 \cdot \ln[1 + \delta^2] \quad (A3)$$

Thus, the COV of R , δ_R , can be analytically calculated based on the variance of $\ln(R)$:

$$\delta_R = \sqrt{(1 + \delta^2)^{\zeta^2} - 1} \quad (A4)$$

In this study, δ was considered to be less than 0.5, and δ^2 is therefore less than 0.25. Bearing this in mind, one can obtain an approximate but straightforward relationship by expanding the term $(1 + \delta^2)^{\zeta^2}$ into a Taylor series and truncating the series at the linear terms:

$$\delta_R \approx \zeta \cdot \delta \quad (A5)$$

Fig. 13 illustrates the relationships given in equations (A4) and (A5), which shows that the difference between the analytical and approximate relationship is negligible for many practical applications. Therefore, the approximate relationship is recommended to be used for simplicity.

Fig. 1. Schematic diagrams of a jet-grout column with (a) varying radius and (b) constant radius

Fig. 2. Illustrations of positioning errors. (a) Inclined columns. (b) - (d) cross-sections at different depths

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Fig. 13. Coefficient of variation (COV) of column radius as a function of COV of in-situ soil strength. Solid curves: analytical solutions, dashed curves: approximate solutions.

Table 1. Theoretical prediction models for jet-grout column diameter, D

Model	Type	Reference	Underlying theory	Applicable range	Notation
$D = \frac{2\Lambda C d_0 v_0}{\sqrt{\frac{\Omega_c g N c_u}{\gamma_f}}}$	Theoretical	Modoni <i>et al.</i> (2006)	The equivalent velocity at a certain distance from centre equals limiting velocity (or the jet equivalent pressure equals the resistive pressure).	Applicable to single fluid scenario; developed different mechanisms for gravel, sandy and clayey soils. The maximum allowable column diameter is predicted.	Λ , Ω_c - calibration factors for jet velocity; C - integration constant; d_0 - diameter of nozzle; v_0 - velocity of the injected fluid at nozzle; g -gravitational acceleration; N - turbulent kinematic viscosity ratio of injected fluid and water; γ_f - unit weight of fluid; c_u - undrained shear strength
$D = 12.5 d_0 \sqrt{\frac{P_i - P_s}{N_c c_u}}$	Theoretical	Ho (2007)	Average stagnation pressure equals ultimate bearing resistance of soils.	Applicable to single fluid scenarios.	d_0 - diameter of nozzle; P_i -pressure inside the nozzle; P_s - geostatic pressure at the depth of nozzle; N_c - bearing capacity coefficient; c_u - undrained shear strength
$D = 2\eta x_L + D_r;$ where $x_L = \frac{\alpha d_0 v_0}{v_L};$ $v_L = \beta \sqrt{\frac{2c_u}{p_{atm}}}$	Theoretical	Shen <i>et al.</i> (2013)	Maximum velocity along the nozzle axis at certain distance from centre equals the limiting velocity.	A generalized method for single, double and triple fluid scenarios.	η - reduction coefficient for injection time; x_L - ultimate erosion distance; D_r - diameter of monitor; α - attenuation coefficient; d_0 - diameter of nozzle; v_0 - velocity of the injected fluid at nozzle; v_L -limiting pressure for soil erosion; β -characteristic velocity; p_{atm} - atmospheric pressure; c_u - undrained shear strength
$D = D_{ref} J^\beta \left(\frac{q_c}{1.5}\right)^{-0.25}$	Semi-theoretical	Flora <i>et al.</i> (2013)	Adjusted from a reference scenario in which all parameters are well-known.	Applicable to single, double and triple fluid. The average diameter is predicted.	D_{ref} - diameter of a standard reference case; J - ratio of specific kinematic energy to its reference value; β - exponent quantifying the influence of jet energy on column diameter; q_c - unit tip resistance (unit: MPa).

Table 2. Statistical characteristics of clay strength

Test type	Strength index	Clay type	Coefficient of variation	Scale of fluctuation (m)		Source
				Horizontal	Vertical	
Cone Penetration Test	Corrected tip resistance	Stiff clay	0.12	-	-	Jaksa <i>et al.</i> (1994)
Cone Penetration Test	Corrected tip resistance	Silty clay		5.0-12.0	1.0	Lacasse & de Lamballerie (1995)
Cone Penetration Test	Tip resistance	Stiff clay	0.02-0.17	23.0-66.0	0.2-0.5	Phoon & Kulhawy (1999)
Cone Penetration Test	Corrected tip resistance	Stiff clay	-	-	0.3-0.7	Cafaro & Cherubini (2002)
Field Vane Test	Undrained shear strength	Sensitive clay	-	23.0	-	DeGroot & Baecher (1993)
Field Vane Test	Undrained shear strength	Clay	-	-	1.0-3.0	Asaoka & Grivas (1982)
Field Vane Test	Undrained shear strength	Sensitive clay	-	-	2.0	Chiasson <i>et al.</i> (1995)
Lab Test: Unconfined Compression Test	Undrained shear strength	Chicago clay	-	-	0.5	Wu (1966)
Lab Test: Triaxial Test & Direct Shear Test	Undrained shear strength	Marine clay	-	-	0.3-0.6	Keaveny <i>et al.</i> (1989)
Lab Test: Unconsolidated Undrained Test	Undrained shear strength	Soft clay	0.10-0.50	-	0.8-6.1	Phoon and Kulhawy (1999)
Lab Test: Vane Shear Test	Undrained shear strength	Soft clay	0.04-0.44	46.0-60.0	2.0-6.2	Phoon and Kulhawy (1999)

Table 3. Parameters used in finite-element analysis

Parameter	In-situ soils	Jet-grouted materials
Constitutive model	Elastic-perfectly plastic model with Mohr-Coulomb failure criteria	
Undrained shear strength, kPa	$c_{us} = 100$ (mean value)	$c_{uj} = 1000$
Friction angle, degree	0	0
Dilation angle, degree	0	0
Young's modulus, kPa	$200c_{us}$	$280c_{uj}$
Poisson's ratio	0.49	0.49

Note: The undrained shear strength of in-situ soils was assumed to follow a lognormal stochastic process.

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Table 4. Summary of parametric study results

Case ID	Input parameters			Statistics of output results			
	δ	L / SOF	s / R_0	Mean	Std / mean	Skewness	Kurtosis
1	0	4	1.54	0.98	0	-	-
2	0.1	4	1.54	0.97	0.01	-0.47	2.62
3	0.2	4	1.54	0.95	0.03	-0.57	2.61
4	0.3	4	1.54	0.92	0.05	-0.55	2.52
5	0.4	4	1.54	0.89	0.06	-0.43	2.38
6	0.5	4	1.54	0.86	0.08	-0.33	2.33
7	0.3	4	2.0	0.68	0.12	0.07	2.63
8	0.3	4	1.82	0.79	0.09	-0.04	2.27
9	0.3	4	1.67	0.87	0.06	-0.31	2.54
10	0.3	4	1.41	0.96	0.03	-0.87	3.01
11	0.3	4	1.33	0.98	0.02	-1.18	3.81
12	0.3	4	1.25	0.99	0.01	-1.61	5.39
13	0.3	0.33	1.54	0.95	0.07	-2.05	8.24
14	0.3	0.5	1.54	0.93	0.08	-1.35	4.30
15	0.3	1	1.54	0.93	0.06	-1.26	4.29
16	0.3	2	1.54	0.93	0.05	-0.58	2.72
17	0.3	10	1.54	0.92	0.03	-0.36	3.09
18	0.3	100	1.54	0.94	0.01	-0.27	2.89

Note: Results are based on the rectangular arrangement of columns. Case 4 is the reference case. δ = coefficient of variation of in-situ soils; L = column length; SOF = scale of fluctuation; s = centre-to-centre distance; R_0 = nominal radius; Std = standard deviation.