Laboratory and Analytical Study of Shattering Jelly Roll Smelt

by

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Abstract

During kraft chemical recovery, smelt created in a recovery boiler is shattered into droplets before falling into a dissolving tank. Although a rare occurrence, highly viscous smelt is created when the smelt temperature is close to the smelt freezing temperature, so-called jelly roll smelt. The viscosity of jelly roll smelt is estimated as between $10^4$ to $10^5$ cP. In this research, a laboratory study was conducted and an analytical model was developed to study the shattering of such highly viscous smelt. The study shows that an increasing viscosity undermines the shattering effectiveness by delaying the complete breakup of the liquid stream, and creating large droplets. The analytical model is based on the physics of co-axial atomization, and is shown to fit the lab scale droplet size distribution very well. Given the shattering conditions, the model appears able to estimate the droplet size distribution and maximum shatterable viscosity.
I would like to express deepest gratitude to Professor Markus Bussmann and Professor Honghi Tran for their guidance and support. Thanks to their patience, encouragement and motivation, I am able to successfully finish this research project.

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<th>Description</th>
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<tbody>
<tr>
<td>MEF</td>
<td>Maximum Entropy Formalism</td>
</tr>
<tr>
<td>TAB</td>
<td>Taylor Analog Breakup</td>
</tr>
<tr>
<td>IA</td>
<td>Impingement Angle</td>
</tr>
<tr>
<td>IPF</td>
<td>Inviscid Potential Flow</td>
</tr>
<tr>
<td>ODE</td>
<td>Ordinary Differential Equation</td>
</tr>
<tr>
<td>LPM</td>
<td>Liter Per Minute</td>
</tr>
<tr>
<td>PEG</td>
<td>Poly Ethylene Glycol</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<tr>
<td>CFD</td>
<td>Computational Fluid Dynamics</td>
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<tr>
<td>RTI</td>
<td>Rayleigh Taylor Instability</td>
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## Symbols

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<th>Symbol</th>
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<tr>
<td>( \rho_1 )</td>
<td>liquid density</td>
<td>kgm(^{-3})</td>
</tr>
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<td>gas density</td>
<td>kgm(^{-3})</td>
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<td>( u_1 )</td>
<td>liquid velocity</td>
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<tr>
<td>( A_{\text{object}} )</td>
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<td>m(^2)</td>
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<td>m(^3)s(^{-1})</td>
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</tr>
<tr>
<td>( d_i )</td>
<td>individual droplet diameter</td>
<td>m</td>
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</tr>
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<td>( \mu_1 )</td>
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<td>( \gamma )</td>
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<td>Nm(^{-1})</td>
</tr>
<tr>
<td>( h )</td>
<td>gas phase gap length</td>
<td>m</td>
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Chapter 1

Introduction

The kraft chemical recovery process is an essential part of pulp production. The process recycles the pulping chemicals, and generates energy for production. As an important part of the recovery process, molten smelt is created in a recovery boiler (as shown in Figure 1.1). The molten smelt flows continuously out of the boiler, and is then shattered by steam jets into droplets that fall into a dissolving tank. During normal operation, the smelt is completely molten in the recovery boiler and contains very little solid content. However, in rare circumstances, so called “jelly roll smelt” can be very viscous and can contain a large amount of solid particles. The shattering is then less effective, which can cause a violent reaction in the dissolving tank. In this thesis, the shattering of jelly roll smelt is examined. This chapter introduces the background to this research, presents an overview of the kraft recovery process, smelt shattering, and jelly roll smelt, and the objectives of the research.
1.1 Kraft Recovery Process

Chemical pulping is the most popular wood pulping method in the papermaking industry. The most important advantage of chemical pulping is that it extracts the fine fibers from wood, to give paper products a high strength. The chemical pulping process includes a chemical recovery process, so that nearly all of the chemicals used in kraft pulping are recovered. Figure 1.2 illustrates the kraft recovery cycle.
In the kraft pulping process, trees are first shredded into wood chips and then dissolved in a digester with white liquor. The white liquor is an alkaline solution composed mainly of sodium hydroxide ($NaOH$) and sodium sulfide ($Na_2S$) [1]. Through the dissolution process, wood pulp and weak black liquor are produced. The pulp contains the cellulose fibers from the raw wood, and is further processed for making paper. Weak black liquor contains the remaining organic and inorganic materials from the dissolution. The weak black liquor is first evaporated to eliminate much of the water in the solution, and then sent to a recovery boiler. The recovery boiler burns the organic components in the black liquor, thereby generating steam and energy that is used by the mill. The inorganic components fall onto a char bed at the bottom of the boiler, where they react with carbon dioxide and sulfur to form sodium carbonate ($Na_2CO_3$) and sodium sulfide ($Na_2S$) [2]. Due to the high temperature of the char bed, the resultant chemicals are molten, and called “smelt”. The viscosity, density and surface tension are approximately 5 cP [3], 2000 kg/m$^3$, and 0.21 N/m [4], respectively.

The molten smelt drains from the recovery boiler through multiple spouts; each stream of smelt is shattered by high pressure steam jets into small droplets. The droplets fall into a dissolving tank and dissolve with water to form green liquor. The green liquor is then forwarded to a causticizing plant, where it is reacted with lime to produce white liquor, and the process begins again.
1.2 Smelt Shattering

Molten smelt flows out of the recovery boiler through spouts. Typically, a boiler has four to eight spouts, but the estimated smelt flow rate per spout on many boilers is similar, about 1 L/s [5]. After the smelt falls from the smelt spout, the smelt is shattered by one or two high pressure steam jets. The operating pressure of each steam jet ranges from 50 to 150 psi, and the steam is at its saturation temperature. Figure 1.3 shows a schematic diagram of a smelt shattering operation.

![Schematic diagram of a smelt shattering operation](image)

FigurE 1.3: Schematic diagram of a smelt shattering operation (from [6])

The designs of the smelt shattering nozzles vary from one mill to another. Shattering nozzles are typically custom made. Figure 1.4 shows some typical designs of shattering nozzles.
As shown in Figure 1.4, shatter jet nozzles can be circular (a, b, c) or slits (d, e). The round shattering nozzles produce more concentrated jets, whereas the slit nozzles are less concentrated, but produce steam jets with greater coverage of the smelt stream. Different nozzle geometries will produce different velocity profiles.

The shatter jet disintegrates smelt into droplets, and effective shattering creates small smelt droplets. The smelt shattering effectiveness affects dissolving tank operation. The interaction between smelt and water causes vapor explosions [6]. At the small scale,
the mild explosion caused by an individual smelt droplet interacting with water tends to promote smelt dissolution. Research [6] showed that the droplet size affects the explosion: increasing droplet size increases the possibility and intensity of an explosion. As a result, when a large amount of smelt droplets fall into a dissolving tank within a short duration, the operation is loud and intensive. Nevertheless, the dissolving tank is designed to protect itself from the explosions, the pressure relief vents release pressure from the explosions. In very rare circumstances, the operation is violent enough to damage the dissolving tank, which leads to an emergency shutdown, and can possibly injure personnel. In the last 30 years, such dissolving tank explosions occur approximately once per year in North America [7]. Although such violent dissolving tank explosions are not fully understood, it is found that they are more likely to occur when the smelt is unshattered and/or poorly shattered before falling into the tank [6] [8] [9]. Therefore, the smelt shattering effectiveness significantly impacts the safety of dissolving tank operations.

1.3 Jelly Roll Smelt

During normal operation of a dissolving tank, the smelt is molten in the recovery boiler, with few small solid particles, and so smelt flow is continuous when the smelt is mostly molten. However, occasionally smelt becomes partially frozen as it flows out of the recovery boiler, and the smelt flow will be sluggish and viscous. This smelt flow is often called “jelly roll smelt” [3]. Figure 1.5 shows a spout blockage situation due to a mixture of jelly roll smelt and char. As shown, the jelly roll smelt contains a large amount of frozen particles, which significantly reduces the fluidity of the flow.
Figure 1.5: Spout blockage situation due to viscous flow (from [3])

Figure 1.6 compares the compositions of different types of smelt. Char particles are large, porous and light, and they exist in the jelly roll smelt in some extreme cases.
The formation mechanism and fluidity of jelly roll smelt was studied by Tran et al. [3], who found that when the temperature of smelt in a boiler is close to the smelt freezing temperature, then there is a high chance of jelly roll smelt formation. The sulphidity ($\text{mol} \% \frac{S}{(S+SO_4+CO_3)}$) of the smelt also plays an important role during this process: when the sulphidity of the smelt is high, the smelt has a high freezing temperature, which can be very close to the operating temperature of some recovery boilers ($\approx 760$ degrees Celsius). In such cases, jelly roll smelt can be more easily formed [3], as the viscosity is a function of the smelt temperature. For regular molten smelt, the viscosity is about 5 cP. But as the smelt temperature gets close to its freezing temperature, the viscosity of the smelt increases exponentially. Due to limitations of the apparatus, the maximum measured viscosity from the experiments was approximately 50 cP, however, the viscosity of jelly roll smelt is expected to be much higher [3].

The viscosity of smelt will affect the shattering effectiveness. Intuitively, when the viscosity of a liquid is high, the shattering effectiveness will be low, or it may even be infeasible to disintegrate the liquid. As mentioned, because the shattering effectiveness of the smelt significantly affects dissolving tank operation, a comprehensive understanding of highly viscous liquid shattering is important.

### 1.4 Objectives

This research is a continuation of laboratory studies in smelt shattering conducted by Taranenko [5] and Lin [9]. The particular focus of this work is shattering jelly roll smelt. There are three objectives of this work,
1. Conduct laboratory experiments to examine the effects of key characteristics on jelly roll smelt shattering effectiveness, including liquid viscosity, shatter jet supply pressure, and liquid flow rate.

2. Develop a complementary analytical model to describe the physics of smelt shattering.

3. Use experimental and analytical results to assess the feasibility of shattering jelly roll smelt.

In the remainder of this thesis, Chapter 2 reviews other work on smelt shattering, and the background theory of twin-fluid atomization; Chapter 3 introduces the experimental setup and methodology; Chapter 4 presents the results obtained from the experiments; Chapter 5 shows the derivation of the analytical model to simulate viscous liquid shattering, and a case study is presented; Chapter 6 summarizes this research and lists recommendations for future work; Appendix A presents how the analytical model is evaluated, which was implemented as a MATLAB script.
Chapter 2

Literature Review and Background Theory

Twin-fluid atomization is a process designed to disintegrate one fluid with another; the process usually involves two immiscible fluids, typically a gas and a liquid or two immiscible liquids. Smelt shattering is an example of twin-fluid atomization, as the molten smelt stream is disintegrated by a high pressure steam jet. Twin-fluid atomization is widely used in various industries, including painting and coating, and metallurgy. Due to its many applications, twin-fluid atomization has been extensively studied; however, most research has focused on inviscid liquid atomization. In this work, the effect of liquid viscosity is investigated.

Previously in this research group, Taranenko [5] and Lin [9] studied molten smelt shattering, but did not focus on jelly roll smelt shattering. In this chapter, the work of Taranenko and Lin is first reviewed, and then a literature review on inviscid and viscous liquid atomization is presented.

2.1 Review on Previous Work

Taranenko [5] initiated the molten smelt shattering research in this group by examining how shattering creates smaller droplets, which is presumed to lead to a less violent dissolving tank. He identified key characteristics that affect smelt shattering effectiveness: smelt flow rate, smelt viscosity, shatter jet steam flow rate, and nozzle placement. Taranenko designed and conducted experiments to study these characteristics. The experimental setup was a scaled model, which mimics smelt shattering operations; Figure 2.1 demonstrates the experimental setup of Taranenko.
To mimic smelt shattering in the laboratory, compressed air was used as the shatter jet, and a water-glycerine mixture was used to mimic the molten smelt. By changing the composition of the water-glycerine mixture, the viscosity of the mixture was adjusted. A centrifugal pump was used to circulate the liquid within the system. Liquid was pumped to an upper tank, and flowed down a spout. The vertically oriented compressed air jet shattered the liquid; a tank beneath the spout collected the spray. A high speed camera was used to image droplets at 50 cm below the shattering nozzle, with a focus area of $3.5 \times 3.5 \text{ cm}^2$, with a depth of field of 2 cm. Taranenko [5] used ImageJ to process the images, obtaining the quantity and size of each droplet. From the experiments, he found that increasing the shatter jet velocity and/or decreasing the distance between the shatter jet and liquid stream decreased the mean droplet size, while increasing the liquid flow rate increased mean droplet size. Due to the limitations of the experimental setup, the liquid viscosity was only studied up to 50 cP, and concluded that viscosity does not strongly affect smelt shattering effectiveness within the range.

Lin [9] continued the research on molten smelt shattering, studying other characteristics including shattering nozzle geometry, gas flow rate, gas impingement proximity, and impingement angle. Gas impingement proximity and impingement angle are the distance and the angle between the shatter jet and smelt, as illustrated in Figure 2.2.
Lin used the experimental apparatus of Taranenko, but he improved the image capture and processing techniques. Instead of imaging at a single location, Lin used a matrix technique to image at multiple locations. Each matrix location is $10 \times 10$ cm$^2$, with a depth of field of 2 cm. The matrix is composed of 42 ($6 \times 7$) evenly spaced areas, 10 cm apart from each other [9]. Lin also automated the image processing to automatically count droplets and measure droplet size. Thanks to the imaging matrix technique, Lin was able to study the spatial distribution of droplets, and obtain more accurate droplet size distributions. Lin’s experimental results showed that reducing the impingement proximity, and increasing gas flow rate reduce the mean droplet size. As for the impingement angle ($\theta$), it has two effects. First, the trajectory of the spray is a function of the impingement angle; when the impingement angle is large, the spray is directed towards an area beneath the spout. Second, Lin found that when the impingement angle is small, smaller droplets are created. Finally, from several nozzle geometries, the hollow cone and full cone nozzles yielded the most effective shattering, creating smaller droplets and more uniform droplet size distributions.

Thanks to the work of Taranenko and Lin, smelt shattering was reasonably well understood. However, some important questions remain unanswered. For instance, how does shattering effectiveness change when the liquid viscosity is high (e.g. jelly roll smelt), and how can one extend the experimental results to mill operating conditions? To address these unanswered questions, this thesis examines the physics of smelt shattering, and considers how liquid viscosity affects smelt shattering effectiveness.
2.2 Twin-fluid Atomization

Smelt shattering is an example of twin-fluid atomization, that involves two immiscible fluids, steam and smelt. The physics of a twin-fluid atomization is affected by the angle between the trajectories of a liquid jet and a gas jet. A cross flow atomization (shown in Figures 2.3), has a large angle between the trajectories, which results in a large percentage of energy is dissipated due to impingement and trajectory changes. When the angle is near zero, a liquid jet and a gas jet travel in parallel; that process is co-axial flow atomization (shown in Figure 2.4), which is more effective [9] compared to a crossflow atomization, because less energy is dispersed due to impingement. In this work, smelt shattering is modeled as co-axial flow atomization, neglecting the energy loss due to impingement, and considering a reasonably small impingement angle. Finally, the effect of liquid viscosity can be assessed via this model.

Figure 2.3: Schematic of cross flow atomization (large impingement angle) (from [10])
Twin-fluid atomization is a complicated process, which consists of several intermediate steps. Marmottant et al. [11] conducted extensive studies on co-axial flow atomization, where two immiscible fluids travel parallel to each other and in the same direction. This section reviews their work to explain the fundamental physics of twin-fluid atomization.

A twin-fluid atomization process, whether co-axial or crossflow, typically involves three stages [11][12]: instability growth on the liquid jet surface, transverse instability growth and the formation of ligaments (primary breakup), and the breakup of ligaments into droplets (secondary breakup).
2.2.1 Surface Instability

A surface instability is usually the first stage of an atomization process. Where two parallel immiscible fluids travel at different velocities, they naturally cause an instability [11] on the interface between them, which can be axisymmetric (for a liquid jet) or non-axisymmetric (for a liquid sheet). When a liquid jet travels in a still atmosphere, the liquid experiences the Plateau-Rayleigh instability, which is not a function of gas velocity. The liquid jet breaks up into droplets if the amplitude of the instability reaches a critical amplitude.

If the gas phase not still, but travels parallel to the liquid jet, the Plateau-Rayleigh instability is quickly overcome and becomes an axisymmetric instability on the liquid jet surface, due to shear at the interface between the two phases. This is referred to as a shear instability, which depends on both the gas and liquid phase velocities. During the shear instability, the undulation grows along the moving direction of the liquid jet. The frequency and amplitude of the undulation increase spatially and temporally, but the liquid jet remains axisymmetric until the amplitude becomes critical. Marmottant et al. [11] found that a higher gas velocity increases the shear instability growth rate. In other words, a higher gas phase velocity leads to a faster and more dramatic undulation of the liquid jet.

![Figure 2.6: Axisymmetric surface instability growth (from [11])](image)

2.2.2 Transverse Instability and Ligament Development

An axisymmetric undulation is transformed into a transverse azimuthal undulation when the amplitude of the shear instability reaches a critical value [11]. A transverse modulation is no longer axisymmetric. This is the second stage of the twin-fluid atomization process.
After reaching the critical amplitude in the first stage, the instability starts to grow along the axial direction of the liquid jet, and causes an azimuthal perturbation. According to Marmottant et al. [11], the perturbation is caused by the Rayleigh-Taylor (RT) instability, which is due to the density difference between the gas and liquid phases. Due to the shear instability, the liquid surface moves towards the lighter gas phase, but the acceleration is towards the heavier liquid phase. Thus, the translation and acceleration are in opposite directions, creating a RT instability, which causes the bouncing of the liquid surface.

Figure 2.8 demonstrates both the axisymmetric instability and the transverse instability. The axisymmetric instability is along the direction of motion of the liquid jet, with an amplitude $a$ and a wavelength $\lambda$. The transverse instability is in the radial direction of a liquid jet, forming an azimuthal perturbation on the cross-section of the jet. The
transverse instability has an average peak amplitude of $b$ and a wavelength of $\lambda_\perp$ between each peak.

Each peak caused by the transverse instability grows independently due to the gas drag force, and as a result, the length and the thickness of each peak are different. These peaks become ligaments. The gas pulls each ligament from the liquid column, while surface tension is a resistant force, and Marmottant et al. did not specify the effect of the liquid viscosity. But, according to the Taylor Analogy Breakup (TAB) Model [14], the viscosity acts as a damping force during the elongation process, to slow down the oscillation of the instability.

Once a ligament has elongated to a critical length, surface tension can no longer retain the ligament on the liquid column, and the ligament breaks up. The ligament length and diameter at breakup are referred to as the breakup length and the breakup thickness.

The ligament breakup process can be modelled by a second-order ordinary differential equation (ODE) with the parameters of gas velocity, liquid velocity, liquid surface tension and liquid viscosity. From the ODE solution, the ligament breakup time can be obtained, which is important in this research.

### 2.2.3 Droplet Size Distribution

After breaking off of the liquid column, the ligaments further breakup into smaller segments, and eventually form droplets of different sizes. This is the third stage of an atomization process. It is also called the secondary breakup process. According to the TAB Model [14], secondary breakup follows the same mechanism, where gas drag promotes breaking, surface tension is the resisting force, and liquid viscosity is the damping force. According to the TAB Model, if the gas velocity is sufficiently large, the droplets will break up repeatedly until they reach a smallest possible diameter, which can be calculated knowing a critical Weber number.
The droplet size distribution pattern is significant in many atomization applications, and so extensive studies have been done on droplet size distributions, and how different shattering parameters change the droplet size distribution. Various methodologies have been applied to study droplet size distribution patterns. From the fundamental physics perspective, the maximum entropy formalism (MEF) is popular. For instance, Li and Richard [15] used information entropy to derive a Nukiyama-Tanasawa type distribution function, which is a thin-tail function with one free parameter. Dumouchel [16], on the other hand, used MEF to develop a generalized gamma distribution function with three free parameters. Others have used experimental or phenomenological analysis, and found similar results. Knuth [17] found that the size distribution of large droplets produced in a free-jet liquid expansion follows a linear exponential distribution. And experiments by Marmottant et al. [11] showed that the droplet size distribution in a spray follows a distribution convolution that is between a uniform distribution (primary breakup) and a gamma distribution (secondary breakup).

In fact, the Nukiyama-Tanasawa type distribution function, an exponential distribution and a generalized gamma distribution are all similar, fat-head and thin-tail functions. This indicates that the majority of droplets are small, and only a small portion of droplets are relatively large.

### 2.2.4 Trajectory

The trajectory of droplet is important during a shattering operation. When twin-fluid atomization is not co-axial, a non-zero impingement angle (IA) exists. With this setup, the trajectories of the gas and the liquid jets are diverted from the original trajectories due to impingement.
Mashayek and Ashgriz [18] studied trajectory change due to cross flow impingement, and developed an empirical-numerical model to describe the jet trajectory and deformation after impingement. The model shows that the trajectory of a post-impingement liquid jet depends on the momentum ratio between the liquid jet and the gas jet ($q$) [18]:

$$ q = \frac{\rho_1 u_1^2}{\rho_2 u_2^2} \quad (2.1) $$

As a result, the phase with a greater momentum has the greater impact on the post-impingement trajectory. For instance, if the gas phase has a greater momentum due to its significantly higher velocity, it will push the liquid stream from its original trajectory. As a result, one can adjust the post-impingement trajectory by changing either the gas jet velocity or the liquid jet velocity.

### 2.3 Viscosity Effect

Jelly roll smelt is a partially frozen smelt, which contains a significant amount of solid particles. The main difference between jelly roll smelt and molten smelt is the liquid viscosity. Therefore, we will model the viscosity effect in a twin-fluid atomization process.

Marmottant et al. focused on inviscid co-axial atomization, while Padrino [19] studied the shear instability of a viscous co-axial atomization. Padrino [19] used the same methodology as Marmottant et al. but, introduced a viscosity term into the interfacial boundary condition, that reflects the balance of normal stresses. The detailed derivation of the viscous liquid shear instability is presented in Chapter 5.

According to Padrino [19], the shear instability of a viscous flow is significantly different than of an inviscid potential flow. Figure 2.10 compares the shear instability growth rates of an inviscid potential flow and various viscous flows. The maximum growth rate determines how fast an instability grows: a larger maximum growth rate results in faster instability growth, and a faster breakup process.
For an inviscid potential flow (IPF), there only exists one peak on the growth rate-wavenumber diagram, but, when viscosity is under consideration, there are usually two peaks on each curve. It is interesting to notice that the water-air ($\hat{\mu} = \mu_g/\mu_l = 0.018$) curve has the highest maximum growth rate among all. The maximum growth rate comes from its second peak, while the first peak coincides with the IPF. As the liquid viscosity increases, the maximum growth rate of a shear instability decreases, and when the liquid viscosity is low enough, the secondary peak no longer exits. Therefore, as the liquid viscosity increases, a shear instability will have a smaller maximum growth rate, which indicates a slower primary breakup process.

As discussed in Section 2.2.2, the transverse instability eventually overcomes the shear instability and produces ligaments. The liquid viscosity also plays an important role during the ligament growth process, as it acts as a damping force during the elongation process, to slow down the oscillation of the instability. The same mechanism exists during the secondary breakup process, where ligaments further breakup into smaller droplets.
In conclusion, viscosity damps the undulations caused by instabilities, and therefore slows down the entire liquid breakup process. In theory, given enough time, a liquid column will break up until the droplets are formed. However, in practice, such time is rarely available, and so determining the breakup time is important, which can help to identify whether a viscous liquid steam is has been shattered.
Chapter 3

Experimental Setup and Methodology

In this chapter of the thesis, the experimental setup and methodology are introduced. Note that two different methodologies were used, depending on the liquid viscosity.

3.1 Jelly Roll Smelt Viscosity

To mimic jelly roll smelt shattering in the laboratory, an estimation of the liquid viscosity of jelly roll smelt was needed. Rather than attempt a direct measurement, a different approach was used to obtain an order-of-magnitude estimate of jelly roll smelt viscosity. The flow of various viscous liquids was compared to videos of jelly roll smelt, and the viscosity of jelly roll smelt was estimated based on a best comparison. Water (1 cP), glycerine (1,410 cP), corn syrup (8,000 cP) and a corn syrup-sugar mixture (12,000 cP) were used. Among all, the flow of the corn syrup-sugar mixture was the most similar to the flow of jelly roll smelt. Figure 3.1 compares the flow of a corn syrup-sugar mixture and jelly roll smelt flow.
Of course, the viscosity of jelly roll smelt will depend on the boiler operating temperature and smelt freezing temperature, and so the viscosity of the jelly roll smelt could vary significantly. Thus, jelly roll smelt viscosity is estimated as between $10^4$ to $10^5$ cP.

### 3.1.1 Material Selection

Taranenko [5] used a glycerine-water mixture to study the effect of smelt viscosity. However, as the viscosity of pure glycerine is only 1,410 cP [20], we looked for alternatives.

In other research areas, liquid viscosity is of interest. In studies of geology and volcanos, scientists are interested in the liquid properties of lava, including viscosity. Due to the high temperature of lava, scientists often use analog materials to mimic lava. Various viscous liquids have been used, including corn syrup [21], poly-ethylene glycol (PEG) [22], a gum-rosin homogeneous solution [21], and paraffin oil [23]. Among all, corn syrup is the most commonly used, because it is non-toxic and readily available. Pure light corn syrup has a viscosity of approximately 8,000 cP, which is on the order of jelly roll smelt viscosity. In addition, the viscosity can be further increased by adding sugar crystals to mimic the solid content in jelly roll smelt. Corn syrup has a similar surface tension to water, and its aqueous solutions have a fairly constant surface tension. Preliminary experimental results verified the feasibility of using corn syrup as a viscous fluid, and therefore, it was selected for the experimental study.
3.2 Experimental Design and Setup

The experimental apparatus developed by Taranenko [5] and Lin [9] was utilized for this experimental study. The setup was constructed to study the shattering effectiveness of low viscosity molten smelt. Lin [9] used water to mimic the molten smelt, and Taranenko [5] used water-glycerine mixtures, with the maximum viscosity of 50 cP. As the viscosity of jelly roll smelt is much higher, the previous setup and methodology were modified. Figure 3.2 shows the schematic diagram of the current setup.

The viscosity limitation in Taranenko’s setup was primarily due to the pump capacity; the centrifugal pump used had a viscosity limitation of 50 cP. A Sherwood fixed displacement pump with an Emerson motor (model: S55JXSJZ-6078, power: 0.5 H.P.), with a viscosity limitation of 1,000 cP, was selected as a replacement. Although this viscosity is still significantly lower than the likely viscosity of jelly roll smelt, it is high enough to demonstrate the shattering effectiveness of highly viscous liquids. Therefore, for experiments with a liquid viscosity lower than 1,000 cP, the experimental methodology was similar to that of Taranenko and Lin. To study liquids with viscosity greater than 1,000 cP, a different methodology was used.

Figure 3.3 shows the orientations and positions of the shatter jet and tank in the current setup; these were held fixed for all experiments. One notable change from Taranenko and Lin’s setup was the orientation of the shatter jet. In the current setup, the shatter
jet was oriented at $5^\circ$ from the vertical, and it was only offset by a short distance of 1.5 cm from the spout exit.

![Figure 3.3](image)

**Figure 3.3:** The orientations and positions of the components (not to scale)

This geometry was chosen especially for the high liquid viscosity and/or low flow rate experiments, as such liquid streams tend to flow backwards after leaving the spout (as shown in Figure 3.4). By setting a small offset distance, the gas jet was still close to the liquid stream.

![Figure 3.4](image)

**Figure 3.4:** The flow curves backwards due to low flow rate
3.2.1 Shatter Jet Velocity Profile and Liquid Velocity

The shatter jet nozzle used in the experiments is shown in Figure 3.5. The nozzle is an axisymmetric full cone nozzle, with an inlet diameter of 11 mm and an outlet diameter of 8.5 mm.

The shatter jet nozzle produces a cone shaped velocity profile. The velocity decays from the shatter jet centreline in both the radial and axial directions. Measuring the shatter jet velocity profile is crucial, because the gas velocity significantly influences shattering effectiveness.

A digital manometer (Dwyer Series 45 Mark III) was used to measure the shatter jet velocity profile. A simple platform was designed and built to fix the manometer for the measurements. All measurements were taken at 10 cm from the shatter jet nozzle exit, which was also the impingement distance for all experiments. The measurements
were first taken along the shatter jet centreline, and then the manometer was moved incrementally in the radial direction, to 0, 2.5, 5, 7.5, 10, 12.5, 17.5, 25 and 40 mm from the centreline. Figure 3.6 illustrates the shatter jet velocity profiles, at supply pressures of 50, 100, 200 and 300 kPa.

The profiles shown in Figure 3.6 are approximately exponential functions: the maximum velocity occurs at the centreline of each profile, and the velocity decays along the radial direction. In most experiments, the shatter jet impinged the liquid stream at its centreline with the maximum velocity. However, for liquid viscosities greater than 1,000 cP, there was a small offset distance between the liquid stream and the shatter jet centreline.

The liquid velocity was estimated by measuring the flow rate and flow cross sectional area of the liquid at the spout exit.

\[ u_1 = \frac{Q_{\text{liquid}}}{A_{\text{exit}}} \]  

(3.1)

Images showed that the flow cross sectional area varied little at different flow rates in this study, and therefore an average flow cross sectional area was calculated and assumed constant. The average cross sectional area of the liquid flow was estimated to be 24.5 mm\(^2\). Therefore, the liquid velocities corresponding to 0.6, 2.5 and 5 LPM were 0.41, 1.7 and 3.4 m/s, respectively.
3.3 Low Viscosity Experiments

As mentioned, two sets of experiments were run, for liquid viscosities lower than 1,000 cP, and for high viscosity liquids. In this section, the experimental methodology for the low viscosity liquid study is presented.

3.3.1 Variable Parameters

Various parameters can affect the smelt shattering effectiveness, including gas velocity, liquid velocity, gas phase properties and liquid phase properties; other parameters will be directly or indirectly associated with these primary parameters. For instance, the nozzle geometry, impingement angle and impingement proximity (as defined in Section 2.1) will all affect the gas velocity at the impingement point. The liquid phase properties, as discussed in Chapter 2, including surface tension, viscosity and density, play different roles during the shattering process. The liquid viscosity is the key difference between jelly roll smelt and molten smelt, and is of most interest. Therefore, shatter jet supply pressure (that directly affects the gas phase velocity), liquid flow rate (that defines the liquid velocity), and liquid viscosity are the three parameters in this set of experiments that were varied. Table 3.1 summarizes the variable parameters.

<table>
<thead>
<tr>
<th>Variable Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscosity (cP)</td>
<td>5, 50, 100, 250, 500, 1,000</td>
</tr>
<tr>
<td>Liquid Flow Rate (LPM)</td>
<td>0.6, 2.5, 5</td>
</tr>
<tr>
<td>Jet Supply Pressure (kPa)</td>
<td>50, 100, 200, 300</td>
</tr>
</tbody>
</table>

Table 3.1: The low viscosity experimental parameters

3.3.2 Methodology and Procedure

Utilizing the experimental setup introduced in the last section and illustrated in Figure 3.2, the liquid is pumped from the lower tank to the inclined upper tank; a liquid flow meter measures the flow rate. When the level of liquid in the tank reaches the spout, it begins to flow out.

The shattering nozzle positioned above the spout directs a high pressure air flow to shatter the liquid. The shattering nozzle is connected to a central compressed air supply; the maximum pressure is approximately 345 kPa. The shatter jet pressure is controlled by a ball valve with a pressure gauge. A high speed camera is placed 1 m (horizontally)
and 60 cm below the shatter jet nozzle. A light source is placed under the spout and faces the camera. A light diffuser board is placed close to the light source to soften illumination.

At the liquid viscosity upper limit (1,000 cP), the fixed displacement pump has a maximum flow rate of 0.6 LPM, which is very low. As the liquid viscosity decreases, the maximum flow rate of the pump increases; for example, at 500 cP, the maximum flow rate is 2.5 LPM, and at 100 cP, the maximum flow rate is 5 LPM. Due to this constraint, different flow rates were used at different liquid viscosities. Table 3.2 summarizes the liquid flow rates and liquid viscosities in the experiments.

<table>
<thead>
<tr>
<th>Liquid Viscosity (cP)</th>
<th>Liquid flow rate (LPM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5, 50, 100</td>
<td>0.6, 2.5, 5</td>
</tr>
<tr>
<td>250, 500</td>
<td>0.6, 2.5</td>
</tr>
<tr>
<td>1,000</td>
<td>0.6</td>
</tr>
</tbody>
</table>

**Table 3.2:** Liquid flow rates and liquid viscosities in the experiments

### 3.3.3 High Speed Imaging and Processing

A Mega-Speed greyscale 512×512 pixel high speed camera, with a Sigma DG 28-300 mm 3.5-6.3f lens, was used for imaging. The imaging matrix technique of Lin [9] was used to image droplet distributions over a 6×6 matrix of locations. Figure 3.7 illustrates a schematic diagram of the imaging matrix technique. The centre of the imaging area aligns with the centre of the shattering nozzle. The 36 locations were evenly distributed in space, with each matrix location 5×5 cm square, adjacent to each other. The focal length of the lens was set to 200 mm, with an aperture size of 3.8, resulting in a depth of field of approximately 5 cm, which was the optimal setting for imaging droplets.
The number of images per matrix location is important for obtaining statistically meaningful results. Lin [9] used the confidence interval method to determine the required number of sample images. From the preliminary experiments in the current study, the droplet size distribution had an average standard deviation of approximately 0.5 mm. With this information, Equation 3.2 [9] calculates the required number of images for a 95% confidence interval ($z$-score equals 1.96 when the confidence interval is 95%) with a 5% margin of error:

$$n = \left( \frac{z_{95\%} \times \sigma}{m} \right)^2$$  \hspace{1cm} (3.2)

$m$ is the margin of error, $z_{95\%}$ is the 95% confidence interval $z$-score, and $\sigma$ is the average standard derivation. Equation 3.2 indicates that 384 sample images per location are required; thus, 500 images were taken per location. Thus, the results are statistically significant.

The raw images were processed according to Lin’s method, using ImageJ software [9]. ImageJ is specialized image processing software, that contains many built-in functions, and supports macro and batch processing. Figure 3.8 demonstrates the image processing procedure. For each raw image, ImageJ first filters the out of focus objects (step 1), then uses a contrast threshold to detect the edges of objects on the filtered images (step 2), and then uses built-in functions to determine the geometric information of each object (e.g. area, aspect ratio and roundness) (step 3). Objects with a characteristic length shorter than 0.6 mm are filtered out as noise, since they are too small to be identified by ImageJ [9]. A macro was created to automate this procedure, to automatically extract geometric information of droplets on all sample images at each matrix location.
3.3.4 Data Processing

With the geometric information obtained by ImageJ, individual droplet diameter was calculated.

\[ d_i = \sqrt{\frac{A_{\text{segment}}}{\pi}} \ast 2 \]  \hspace{1cm} (3.3)

The droplets captured by the high speed camera were not usually spherical; in particular, when the viscosity was high, thread-shaped ligaments presented. To consider more general scenarios, \( d_i \) was used to represent the segment size or droplet diameter. For each set of experiments, the arithmetic mean droplet diameter (\( d_{10} \)), droplet size distribution, droplet count spatial distribution, and droplet size spatial distribution were analyzed, using a Python script to extract the data from the ImageJ reports, and to perform the analysis. Note that \( d_{10} \) is an average of \( d_i \), and calculated as

\[ d_{10} = \bar{d}_i \]  \hspace{1cm} (3.4)

The spatial droplet distributions are presented as contour maps. The droplet count contours show the total number of droplets captured at each location in the imaging
matrix, whereas droplet size contours illustrated the mean droplet size at each location. Figure 3.9 is an example of a droplet count contour.

![Figure 3.9: An example of a droplet count contour](image)

### 3.4 High Viscosity Experiments

For the high viscosity liquid experiments (viscosity greater than 1,000 cP), the liquid stream was hardly disintegrated, and therefore, more general shattering patterns and processes were analyzed, instead of the shattering results. A set of images was taken at a fixed location near the spout, that allowed for comparison of shattering processes at different operating conditions. The high speed camera was placed 1 m horizontally from and 10 cm vertically below the shatter jet nozzle, and was used to image an area of $20 \times 20$ centimeters square below the spout. The image centerline aligned with the spout centerline. In the experiments, the focal length was set to 50 mm, with an aperture size of 3.5.

For this set of experiments, the pump was removed from the experimental setup, because of its capacity limitation. To create a flow, for each experiment, spout was blocked by a
stopper, and the tank was filled with a certain volume of viscous liquid, to a level above the spout. After the high speed camera started recording, the stopper was pulled out. Early on, the actual flow rate was similar to the estimated flow rate based on the height of liquid in the tank; as time passed, the flow rate decreased, and eventually stopped as the liquid level in the tank reached the entry to the spout. Therefore, only the first five seconds of a recording was used for analysis. For this set of experiments, the target liquid flow rate was 6 LPM. The liquid viscosities were 1,000 cP, 8,000 cP (pure corn syrup) and 12,000 cP (a corn syrup and sugar mixture), and the methodology was also applied to fluids with lower viscosity for comparison. The shatter jet supply pressures were 50 kPa, 100 kPa, 200 kPa, and 300 kPa. In addition to the shattering patterns, the primary breakup time of the liquid stream was measured.

Table 3.3 summarizes the variables parameters examined in the high viscosity experiments.

<table>
<thead>
<tr>
<th>Variable Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Viscosity (cP)</td>
<td>1,000, 8,000, 10,000</td>
</tr>
<tr>
<td>Jet Supply Pressure (kPa)</td>
<td>50, 100, 200, 300</td>
</tr>
</tbody>
</table>

Table 3.3: The high viscosity experimental parameters
Chapter 4

Experimental Results and Discussion

In this chapter, the experimental results are presented and discussed. The three variable parameters in the experiments: liquid viscosity, shatter jet supply pressure, and liquid flow rate, have different effects on the shattering process and results. As liquid viscosity is the primary variable of this study, it was examined most closely.

As mentioned in Chapter 3, the experimental study was split into low viscosity experiments and high viscosity experiments. In Section 4.1, the results of the low viscosity experiments are presented, and in Section 4.2, the results of the high viscosity experiments.

4.1 Experimental Results for Low Viscosity Liquids

4.1.1 Effect of Viscosity

Liquid viscosity is the main difference between typical molten smelt and jelly roll smelt. During the experiments, six different liquid viscosities were tested: 5, 50, 100, 250, 500 and 1,000 cP. By diluting corn syrup with water, the liquid viscosity was adjusted. When the effects of liquid viscosity were examined, the other representative parameters were held unchanged, including the liquid surface tension.

The images in Figure 4.1 demonstrate how shattering varies with the liquid viscosity (up to 500 cP). For these results, the shatter jet supply pressure is 50 kPa, and the liquid flow rate is 2.5 LPM. Figure 4.2 illustrates the shattering when the liquid viscosity was
1,000 cP, and the shatter jet supply pressure and liquid flow rate are 100 kPa and 0.6 LPM.

Figure 4.1: Shattering results (P = 50 kPa, Q = 2.5 LPM)
As observed in Figure 4.1, liquids with a viscosity lower than 100 cP are shattered into droplets. As the liquid viscosity increases within the range of 5 to 100 cP, the droplet size becomes larger. For the 250 and 500 cP liquids, a relatively low shatter jet velocity is not able to disintegrate the liquid stream; instead, the liquid stream is only perturbed while falling into the reservoir tank. Additionally, notice that the 500 cP liquid is less perturbed than the 250 cP liquid.

In some cases, the viscous liquid stream is shattered into ligaments, rather than into droplets, especially when the liquid viscosity is high while the shatter jet supply pressure is low, so that the viscous liquid stream is only partially disintegrated. Figure 4.3 illustrates ligaments produced in experiments at such conditions.
Figure 4.3: Ligaments formation: (Q = 2.5 LPM)

Ligaments are thread-shaped segments formed from liquid stream breakup. The tips of the ligaments are often spherical droplets, as shown in Figure 4.3. Ligament formation is an intermediate stage during the droplet formation process: the liquid stream is first broken up into ligaments, and then the ligaments are further disintegrated into droplets. When the liquid viscosity is high, this delays the complete disintegration of the liquid stream; a weak shatter jet can only partially disintegrate a liquid stream into ligaments.

Using ImageJ, individual droplet sizes ($d_i$) and arithmetic mean droplet diameter ($d_{10}$) were obtained. Figure 4.4 illustrates the change of mean droplet diameter with respect to liquid viscosity, at two shatter jet supply pressures, for a liquid flow rate of 0.6 LPM. As shown in Figure 4.4, the mean droplet diameter increases as the liquid viscosity increases, although the rate of change is similar between the two.
Each droplet has its own unique size and trajectory, and so the shattering process is stochastic, and is described via probability distributions. However, the size and spatial distributions of droplets will vary with the liquid viscosity.

Figure 4.5 shows the droplet size distributions for two viscosities: 5 cP and 250 cP. Note that, the two histograms were normalized by the probability density function method, so that they have similar scales. This approach was also applied to the other droplet size distribution histograms.
As shown, the droplet size distributions share a similar shape, which is described as a generalized gamma distribution, which has a fat head, and a long but thin tail. This suggests that the majority of the droplets are relatively small. For these two cases, the peaks of the distributions are between 1 and 2 mm. As liquid viscosity increases, the peak shifts towards a larger droplet size, which indicates an increase of mean droplet diameter, and the tail of the distribution becomes longer, suggesting the formation of larger droplets or ligaments.

The droplet spatial distribution also strongly depends on the liquid viscosity. Two sets of contour maps: a droplet count contour, and a droplet size contour, both reveal changes of droplet spatial distribution with respect to liquid viscosity. Table 4.2 illustrates and compares droplet spatial distributions with viscosity.
Table 4.1: Droplet spatial distributions for different viscosities (P = 100 kPa, Q = 2.5 LPM)

<table>
<thead>
<tr>
<th>Viscosity</th>
<th>Droplet Count Contour</th>
<th>Droplet Size Contour (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 cP</td>
<td><img src="image1" alt="Droplet Count Contour" /></td>
<td><img src="image2" alt="Droplet Size Contour" /></td>
</tr>
<tr>
<td>100 cP</td>
<td><img src="image3" alt="Droplet Count Contour" /></td>
<td><img src="image4" alt="Droplet Size Contour" /></td>
</tr>
<tr>
<td>250 cP</td>
<td><img src="image5" alt="Droplet Count Contour" /></td>
<td><img src="image6" alt="Droplet Size Contour" /></td>
</tr>
</tbody>
</table>
As expected, the contour maps in Table 4.1 are roughly symmetric about the vertical centerline, because the shattering process undergoes an axisymmetric instability (to be discussed in Chapter 5). At a low liquid viscosity, the quantity and size of the droplets are relatively uniform over the whole imaging area. As the liquid viscosity increases, larger droplets and ligaments are more likely to fall near the centre of the imaging area, because larger droplets have greater momentum, and so are more resistant to be displaced from the stream centreline.

4.1.2 Effect of Shatter Jet Supply Pressure

The gas velocity at the spout exit strongly affects the shattering effectiveness. In Chapter 3, the relationship between shatter jet supply pressure and the jet velocity profile was presented; they are proportional. In the experiments, the shatter jet impinged the liquid stream at its centreline. In this section, four shatter jet supply pressures were examined: 50, 100, 200 and 300 kPa; their associated gas phase velocities are 43, 76, 132 and 167 m/s, respectively.

Figure 4.6 demonstrates shattering results at the four shatter jet supply pressures.

![Figure 4.6: Shattering results ($\mu_1 = 250$ cP, Q = 2.5 LPM)](image)
Notice that the liquid stream is not shattered when the shatter jet pressure is only 50 kPa. At an air supply pressure of 100 kPa, the liquid stream breaks into long ligaments and a few small droplets. When the shatter jet supply pressure further increases, the shattering produces mostly droplets. Thus, clearly, the greater the shatter jet pressure, the smaller the droplets.

Image processing confirms the observations of Figure 4.6. Figure 4.7 shows the change of mean droplet diameter with respect to shatter jet supply pressure. As shown in Figure 4.7, the mean droplet diameter decreases as the shatter jet supply pressure increases. (Note that the 250 cP liquid could not be disintegrated by the 50 kPa shatter jet.)

![Figure 4.7: Mean droplet size versus air supply pressure, as a function of viscosity](image)

Figure 4.8 demonstrates the droplet size distributions at two experimental conditions. For the droplet size distributions, similar results are seen as in the study of viscosity. The droplet size distributions for different shatter jet supply pressures are also generalized gamma distributions. At higher pressures, the tail of the distribution is thinner and shorter, which indicates a more efficient shattering process. In contrast, when the shatter jet supply pressure is low, the tail of the distribution is fat and long.
Table 4.2 demonstrates droplet spatial distributions at different air supply pressures. When the air supply pressure is low (e.g. 50 kPa), the shattering only creates large droplets concentrated beneath the spout. Increasing the air supply pressure decreases the droplet size, and creates a more uniform droplet spatial distribution. However, a very high supply pressure pushes the shattering trajectory backwards beneath the spout.
<table>
<thead>
<tr>
<th>Air Pressure</th>
<th>Droplet Count Contour</th>
<th>Droplet Size Contour (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>50 kPa</strong></td>
<td><img src="image1" alt="Droplet Count Contour" /></td>
<td><img src="image2" alt="Droplet Size Contour" /></td>
</tr>
<tr>
<td><strong>100 kPa</strong></td>
<td><img src="image3" alt="Droplet Count Contour" /></td>
<td><img src="image4" alt="Droplet Size Contour" /></td>
</tr>
<tr>
<td><strong>200 kPa</strong></td>
<td><img src="image5" alt="Droplet Count Contour" /></td>
<td><img src="image6" alt="Droplet Size Contour" /></td>
</tr>
</tbody>
</table>

*Table 4.2: Droplet spatial distributions versus air supply pressure ($\mu_1 = 5 \, \text{cP}, \, Q = 2.5 \, \text{LPM}$)*
4.1.3 Effect of Liquid Flow Rate

Three liquid flow rates were examined: 0.6, 2.5 and 5 LPM. Due to the limitation of the fixed displacement pump, certain flow rates were selected for certain viscosities. For all liquids, a flow rate of 0.6 LPM was examined. For liquids with a viscosity up to 500 cP, a flow rate of 2.5 LPM was examined. And for liquids with a viscosity up to 250 cP, a flow rate of 5 LPM was also examined.

In general, the effect of liquid flow rate is less significant compared to other factors, but it does affect the liquid velocity. However, for most shattering processes, the liquid velocity is orders of magnitude lower than the gas velocity. As a result, the difference between the two is dominated by the gas velocity.

Figure 4.9 demonstrates shattering results at two different liquid flow rates: 2.5 and 5 LPM, keeping other conditions the same.

Comparing the two images, the most obvious difference is the droplet size. At a higher liquid flow, the shatter jet disintegrates the liquid stream into relatively larger droplets, whereas at a lower flow rate, the droplets are smaller. This is easily explained: as the liquid flow rate increases, the overall momentum of the liquid phase increases, and thus every unit volume of the liquid stream receives less momentum from the gas phase due to the impingement. As a result, larger droplets present. The second observation is that with increasing liquid flow rate, the quantity of shattered fragments is significantly greater, as expected.
Figure 4.10 illustrates mean droplet diameter versus liquid flow rate and gas pressure. As expected, the mean droplet diameter increases as the liquid flow rate increases, although the rate of change with flow rate is relatively small.

![Figure 4.10: Mean droplet size versus liquid flow rate, and as a function of pressure ($\mu_1 = 5$ cP)](image)

Figure 4.11 and Table 4.3 show droplet size distributions and spatial distributions with respect to the liquid flow rate. As shown, the droplet size distributions again follow a generalized gamma distribution pattern. At a higher liquid flow rate, the tail of the distribution is longer and fatter, showing a greater percentage of large droplets. As for the droplet spatial distributions, when the liquid flow rate is 5 LPM, the quantity of droplets per location is almost twice that when the liquid flow rate is 2.5 LPM, and three to six times that when the liquid flow rate is 0.6 LPM. In addition, at a low liquid flow rate, for example 0.6 LPM, the liquid phase momentum is low, and as a result, the droplets are pushed back beneath the spout.
Figure 4.11: Droplet size distributions for 2.5 LPM and 5 LPM ($\mu_1 = 5$ cP, $P = 100$ kPa)
<table>
<thead>
<tr>
<th>Flow Rate</th>
<th>Droplet Count Contour</th>
<th>Droplet Size Contour (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.6 LPM</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>2.5 LPM</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>5 LPM</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

**Table 4.3:** Droplet spatial distributions versus liquid flow rate ($\mu_1 = 5$ cP, $P = 50$ kPa)
4.2 Experimental Results for High Viscosity Liquids

Using a different experimental methodology, described in Chapter 3, the high viscosity liquids were examined. The goal of this set of experiments was to describe the overall shattering patterns, rather than quantify shattering. One of the important parameters during shattering is the primary breakup time, which measures how fast ligaments develop and breakup from a liquid stream; this measure is presented in this section. Also, notice that low viscosity liquids were also examined in this set of experiments, for examining the differences between high and low viscosity liquids.

Two types of materials were used for this set of experiments. The first is corn syrup aqueous solutions, with viscosities of 5, 50, 100, 250, 500, 1,000, and 8,000 cP. The second type is a mixture of pure corn syrup and sugar crystals, which mimic the solid particles suspended in jelly roll smelt. The mixture percentage used was 80 percent corn syrup and 20 percent sugar crystals by weight. A Brookfield Viscometer (Model DV-II+) was used to measure the rheological properties of the mixture; the measurements indicated a viscosity of approximately 12,000 cP, independent of shear rate. Therefore, the corn-syrup-sugar-crystal mixture is a Newtonian fluid with a constant viscosity at room temperature.

For this set of experiments, liquid was first poured into the tank, and then released by pulling a stopper, and the first five seconds of each recording was analyzed. Figure 4.12 presents selected experimental results, which show the transient process of shattering at an imaged area of 20 cm by 20 cm directly below the spout. The figures show several typical shattering processes, at different conditions.
<table>
<thead>
<tr>
<th>Viscosity</th>
<th>Pressure</th>
<th>Initial Stage</th>
<th>Stage 1</th>
<th>Stage 2</th>
<th>Stage 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>500 cP, 200 kPa</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>1000 cP, 200 kPa</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
<td><img src="image7.png" alt="Image" /></td>
<td><img src="image8.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>1000 cP, 300 kPa</td>
<td><img src="image9.png" alt="Image" /></td>
<td><img src="image10.png" alt="Image" /></td>
<td><img src="image11.png" alt="Image" /></td>
<td><img src="image12.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>8000 cP, 300 kPa</td>
<td><img src="image13.png" alt="Image" /></td>
<td><img src="image14.png" alt="Image" /></td>
<td><img src="image15.png" alt="Image" /></td>
<td><img src="image16.png" alt="Image" /></td>
<td></td>
</tr>
<tr>
<td>12000 cP, 300 kPa</td>
<td><img src="image17.png" alt="Image" /></td>
<td><img src="image18.png" alt="Image" /></td>
<td><img src="image19.png" alt="Image" /></td>
<td><img src="image20.png" alt="Image" /></td>
<td></td>
</tr>
</tbody>
</table>

**Figure 4.12:** Shattering of high viscosity liquids ($Q = 6$ LPM)
The first row in Figure 4.12 demonstrates the shattering of 500 cP liquid, with \( P = 200 \) kPa. Each picture in this row shows one of the critical stages during the shattering process. At this experimental condition, the first stage shows a roughly axisymmetric undulation at edges of the liquid stream. As the amplitude of undulation increases, the instability became asymmetric, and the peaks formed during the first stage further develop. The cluster of peaks is like a crown surrounding the liquid column, called a transverse instability. With the help of the shatter jet, the peaks further develop to form ligaments, and then the ligaments experience secondary breakup and form droplets. This is a typical breakup mechanism when the liquid viscosity is low, or when the gas velocity is high.

The second row in Figure 4.12 demonstrates the shattering of 1,000 cP liquid, with \( P = 200 \) kPa. For this condition, the first stage also shows a roughly axisymmetric undulation at edges of the liquid stream. Due to the increased liquid viscosity, the peaks of the undulation are more firmly bonded to the liquid stream, and the growth of the peaks is very slow. Therefore, despite the action of the air flow, the liquid stream remains in this stage. In the second stage, with the help of the shatter jet, the entire liquid stream starts to wander, and form a sine shape wave; the amplitude of the wave is small at the top and large at the bottom of the liquid stream. At the location close to the bottom end of the frame, the liquid stream is pulled into thin threads. In the third stage, some of the thin threads are segmented from the liquid stream to form ligaments, and fall into the tank, mostly near the centre of the liquid stream. As shown, this is a typical breakup mechanism when the liquid viscosity is high, or when the gas velocity is low.

The third row in Figure 4.12 again demonstrates the shattering process of 1,000 cP liquid, with \( P = 300 \) kPa. In this condition, the overall breakup mechanism is similar to the case show in the first row of the figure: axisymmetric shear instability, transverse instability, ligament formation, ligament breakup and droplet formation. Although the liquid viscosity is greater, the increases shatter jet supply pressure compensates for the viscous slowing effect of the high viscosity liquid [11].

The fourth row in the figure demonstrates the shattering of pure corn syrup, with \( \mu_1 = 8,000 \) cP, with \( P = 300 \) kPa. To create a flow rate of approximately 6 LPM for the pure corn syrup, the liquid level in the tank was adjusted accordingly. In this experimental condition, the liquid stream does not breakup, but the entire liquid stream is perturbed and wanders, and forms a sine shaped wave. However, due to the high viscosity, the liquid stream is not disintegrated, and the entire liquid stream falls into the centre of the tank below.
The last row in the figure demonstrates the shattering of a 12,000 cP liquid (pure corn syrup with 20% sugar crystals by weight), with $P = 300$ kPa. The results are similar to the pure corn syrup case.

By using the high speed camera, the primary breakup time was estimated, as presented in Table 4.4.

<table>
<thead>
<tr>
<th>Viscosity (cP)</th>
<th>Shatter Jet Supply Pressure (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>50: 10 ms</td>
</tr>
<tr>
<td>50</td>
<td>100: 35 ms</td>
</tr>
<tr>
<td>100</td>
<td>200: 40 ms</td>
</tr>
<tr>
<td>250</td>
<td>300: 45 ms</td>
</tr>
<tr>
<td>500</td>
<td>N/A: 100 ms</td>
</tr>
<tr>
<td>1,000</td>
<td>N/A: N/A</td>
</tr>
<tr>
<td>8,000</td>
<td>N/A: N/A</td>
</tr>
<tr>
<td>12,000</td>
<td>N/A: N/A</td>
</tr>
</tbody>
</table>

N/A: primary breakup is not observed

**Table 4.4: Primary breakup time versus viscosity and shatter jet supply pressure ($Q \approx 6$ LPM)**

Notice that when the liquid viscosity was very low, the primary breakup occurred too fast to be captured by the high speed camera. (The time interval between two consecutive frames is 5 ms, and for these cases, the primary breakup time was shorter than 5 ms.) On the other hand, when the liquid viscosity is very high, primary breakup was not observed where the camera was focused. From the table, the general trend is that the higher the liquid viscosity, the slower the primary breakup, which once again demonstrates the delaying effect of viscosity.
Chapter 5

An Analytical Model of Viscous Spray Formation

To better understand jelly roll smelt shattering, and the physics of viscous liquid atomization, an analytical model was developed, for each stage of the atomization process. In this chapter, the mathematical formulation is presented, and results of the model are fitted and compared with experimental results, and the model is used to perform a case study. The detailed calculation of the case study is presented in Appendix A, with a MATLAB implementation.

Figure 5.1 presents the structure and calculation procedure of the analytical model, including the inputs to the model, the calculation methodologies, and the outputs generated by the model.
The model considers the three stages of shattering due to a co-axial atomization process: shear instability, transverse instability and ligament development, and secondary breakup. In the first stage, the shear instability dispersion relation function is applied based on Padrino [19], which estimates the maximum growth rate ($\omega_{\text{max}}$) of the shear instability. The estimate of $\omega_{\text{max}}$ is then used to calculate the primary breakup time ($t_{pb}$). In the second stage, the Rayleigh-Taylor instability is applied to describe the transverse undulation growth. The RT instability wavelength ($\lambda_{\perp}$) is used to model ligament development, based on the TAB model. The TAB model leads to estimates of...
the ligament breakup time \((t_b)\) and breakup length. In the third stage, instead of explicitly modeling the secondary breakup process, the droplet size distribution \((p(d))\) and arithmetic mean droplet diameter \((d_{10})\) are modeled; the free parameters in the distribution function were calibrated based on the experimental data. For stages 2 and 3, the model is based on the work of Marmottant et al. [11].

The analytical model was built based on Padrino [19] and Marmottant et al. [11], while it is innovative because:

1. It combined the models of Padrino [19] and Marmottant et al. [11], and modeled the three stages of viscous liquid breakup.
2. It explicitly considered the liquid viscosity in the ligament develop model, while Marmottant et al. [11] used an inviscid liquid assumption.
3. It modified the droplet size distribution model of Marmottant et al. [11]
4. It calibrated the empirical parameters in the model based on the experimental results.
5. It is implemented in MATLAB, and it developed a process to model smelt shattering in the mills, including both molten smelt and jelly roll smelt.

5.1 Assumptions

The analytical model is based on the following simplifying assumptions:

1. Shattering process is due to co-axial atomization. The liquid viscosity is considered.
2. Ideal gas, and incompressible gas flow.
3. The viscosity and surface tension of the liquid are constant throughout the shattering process.
4. The liquid and gas velocity remain constant throughout the shattering process.
5. Due to the high velocity of the gas jet, the gas flow is assumed to be turbulent.
6. Effect of gravity is negligible.
7. Only the temporal growth rate of the instability is of interest.
5.2 Shear Instability

As discussed in Chapter 2, the first stage of the shattering process is a surface undulation caused by the shear instability. As the liquid stream leaves the spout and encounters the gas flow, the velocity difference between the phases causes a shear instability that has different modes, depending on the boundary layer thickness or vorticity layer thickness [11]. The velocity difference between the phases is known as the interfacial velocity:

\[ u_0 = u_2 - u_1 \]  

(5.1)

and the boundary layer thickness \( \delta \) is:

\[ \delta = \frac{37.2 * h}{Re_2^{3/4}} \]  

(5.2)

where \( Re_2 \) is the gas phase Reynolds number:

\[ Re_2 = \frac{h * u_2}{\nu_2} \]  

(5.3)

For the above equations, the parameter \( h \) is the gas phase gap length, \( u_2 \) is the gas phase interacting velocity, and \( \nu_2 \) is the kinematic viscosity of the gas. Due to the high velocity of a shatter jet, the calculation of boundary layer thickness (\( \delta \)) is based on a turbulent flow assumption [11]. With a thin boundary layer, the shear instability follows the Kelvin Helmholtz mode, whereas a thicker boundary layer follows the Rayleigh mode. The criterion for choosing the instability mode is based on a threshold Weber number: when the gas phase Weber number is greater than the threshold Weber number, the instability follows the Rayleigh mod, and in this model, the shear instability is assumed to follow the Rayleigh mode.

Padrino [19] derived a dispersion relation function for the Rayleigh mode shear instability, using the standard linear instability analysis method, by applying a small perturbation boundary layer condition to the liquid jet surface. The liquid viscosity is incorporated into the equations by applying a normal stress equilibrium (between the gas and liquid phases) at the liquid/gas interface. Equation 5.4 demonstrates the Rayleigh mode dispersion relation function obtained by Padrino [19].

\[ e^{-2\eta} = \left[ 1 + (\Omega - \eta) \right] \frac{\Phi + (\Omega + \eta)[2\tilde{\rho} - (1 + \tilde{\rho})(\Omega + \eta) - (1 + \tilde{\mu})\beta_1 \eta^2]}{\Phi + (\Omega + \eta)[2\tilde{\rho} - (1 - \tilde{\rho})(\Omega + \eta) - (1 - \tilde{\mu})\beta_1 \eta^2]} \]  

(5.4)

This function describes the relationship between the shear instability growth rate and the wavenumber. The shear instability growth rate and wavenumber indicate how the
undulation develops with respect to time. With a higher growth rate, the undulation propagates faster, and the amplitude of the perturbation grows faster.

In Equation 5.4, $\Omega$ and $\eta$ represent the dimensionless growth rate and wavenumber:

$$\Omega = \frac{\omega \delta}{u_0}$$  \hspace{1cm} (5.5)

$$\eta = k \delta$$  \hspace{1cm} (5.6)

As only the temporal analysis of the shear instability is of interest, only the complex parts of the growth rate and wavenumber are considered. (Alternatively, a spatial analysis focuses on the real parts of the results, but the temporal analysis will yield good connections with further analysis in this model.)

In Equations 5.5 and 5.6, $u_0$ is the interfacial velocity, $\omega$ is the dimensional growth rate, and $k$ is the dimensional wavenumber.

Returning to Equation 5.4, $\Phi$ is an intermediate term that lumps gravity, and the viscosity and surface tension of the liquid [19]:

$$\Phi = J_1 \eta + \left( \frac{\hat{\rho}}{W e_2} \right) \eta^3 + 2\eta^2 \hat{\mu} \beta_1$$ \hspace{1cm} (5.7)

where,

$$\beta_1 = \frac{i2}{Re_1}$$ \hspace{1cm} (5.8)

$$\hat{\rho} = \frac{\rho_2}{\rho_1}$$ \hspace{1cm} (5.9)

$$\hat{\mu} = \frac{\mu_2}{\mu_1}$$ \hspace{1cm} (5.10)

The gravity, viscosity and surface tension are represented by the liquid phase Richardson number ($J_1$), the liquid phase Reynolds number ($Re_1$), and the gas phase Weber number ($We_2$):

$$J_1 = \frac{(\rho_1 - \rho_2)g\delta}{\rho_1 U_0^2}$$ \hspace{1cm} (5.11)

$$Re_1 = \frac{u_0 \delta}{\nu_1}$$ \hspace{1cm} (5.12)

$$We_2 = \frac{\rho_2 U_0^2 \delta}{\gamma}$$ \hspace{1cm} (5.13)

$\nu_1$ is the kinematic viscosity of the liquid, $\gamma$ is the surface tension between gas and liquid, $g$ is the gravity constant, and $\rho_1$ and $\rho_2$ are the densities of the liquid and gas phases, respectively.
Padrino [19] further showed that Equation 5.4 can be arranged as a cubic function, where the variable is the summation of the dimensionless growth rate and the wavenumber \( \tilde{\Omega} \equiv \Omega + \eta \), equivalent as follows:

\[
a_3\hat{\Omega}^3 + a_2\hat{\Omega}^2 + a_1\hat{\Omega} + a_0 = 0 \quad (5.14)
\]

If \( a_3 \) is non-zero, then both sides of the equation can be divided by \( a_3 \).

\[
\hat{\Omega}^3 + \frac{a_2}{a_3}\hat{\Omega}^2 + \frac{a_1}{a_3}\hat{\Omega} + \frac{a_0}{a_3} = 0 \quad (5.15)
\]

The coefficients of the cubic function are [19],

\[
a_3 = -(1 + \tilde{\rho}) \quad (5.16)
\]

\[
a_2 = a_3 - (1 + \tilde{\mu})\beta_1\eta^2 - (1 - 2\eta)(1 + \tilde{\rho}) + (1 - \tilde{\rho})e^{-2\eta} \quad (5.17)
\]

\[
a_1 = \Phi + (1 - 2\eta)[2\tilde{\rho} - (1 + \tilde{\mu})\beta_1\eta^2] - [2\tilde{\rho} - (1 - \tilde{\mu})\beta_1\eta^2]e^{-2\eta} \quad (5.18)
\]

\[
a_0 = \Phi[(1 - 2\eta) - e^{-2\eta}] \quad (5.19)
\]

Returning to Equation 5.14, considering the assumption of negligible gravity, the fact that the gas-liquid density ratio is small, and the Weber number is large, the term \( \Phi \) can be further simplified as

\[
\Phi = 2\eta^2\tilde{\mu}\beta_1 \quad (5.20)
\]

This cubic function can be solved for three unique roots, as presented below. Note that because the function contains imaginary components, the three roots are complex.

\[
\hat{\Omega}_1 = \frac{-p}{3} + A - B \quad (5.21)
\]

\[
\hat{\Omega}_2 = \frac{-p}{3} + A \frac{-1 - \sqrt{3}i}{2} - B \frac{-1 + \sqrt{3}i}{2} \quad (5.22)
\]

\[
\hat{\Omega}_3 = \frac{-p}{3} + A \frac{-1 + \sqrt{3}i}{2} - B \frac{-1 - \sqrt{3}i}{2} \quad (5.23)
\]

where

\[
p = a_2/a_3 \quad (5.24)
\]

\[
q = a_1/a_3 \quad (5.25)
\]

\[
r = a_0/a_3 \quad (5.26)
\]

\[
A = \frac{(3\sqrt{3}\sqrt{-p^2q^2 + 4q^3 + 4p^3r - 18pqr + 27^2 - 2p^3 + 9pq - 27r})^2}{3 \cdot 2^\frac{1}{3}} \quad (5.27)
\]
Because $\tilde{\Omega} \equiv \Omega + \eta$, Equations 5.21, 5.22 and 5.23 can be expressed in terms of $\Omega$ and $\eta$, and thus, the dispersion relation is obtained. Due to the temporal analysis, the positive imaginary parts of the solutions (Equations 5.21, 5.22 and 5.23) are taken as the resultant growth rate. Of the three solutions of the cubic function, there exists only one positive imaginary growth rate per wavenumber, and therefore, the dispersion relation is unique. Equation 5.29 illustrates how a single solution is selected from the three solutions of the cubic function:

$$\Omega = \max(\text{Imag}(\tilde{\Omega}_1 - \eta), \text{Imag}(\tilde{\Omega}_2 - \eta), \text{Imag}(\tilde{\Omega}_3 - \eta), 0)$$ (5.29)

Typically, the dimensionless wavenumber $\eta$ ranges from $10^{-5}$ to 0.1. Applying this range to Equation 5.29, a corresponding array of dimensionless growth rates can be obtained. Then, based on Equations 5.5 and 5.6 the relationship between dimensional growth rate ($\omega$) and wavenumber ($k$) is identified.

### 5.3 Transverse Instability and Ligament Development

When the undulation amplitude of the shear instability saturates, it is overridden by a transverse instability, which is no longer axisymmetric [11]. The transverse instability refers to the undulation around the cross section of the liquid stream, and it is caused by the density difference between the phases. As the transverse instability develops, the tips of the undulations further grow into ligaments, and with the help of aerodynamic drag, the ligaments grow and eventually breakup. In this section, the transverse instability and the ligament formation process are analyzed.

The “transverse instability starting time” measures how fast the shear instability is overridden by the transverse instability, which is an important portion of the overall breakup time. The transverse instability starting time is usually very short, and difficult to observe and measure from experiments, but it can be roughly estimated from the analytical model.

Observing the shear instability in the Lagrangian reference frame of the liquid, its surface undulation is roughly a simple sine function with respect to time [11]:

$$\xi = -a \times \sin(\omega_L t)$$ (5.30)
where $a$ is the wave amplitude, which typically has a value around 1 mm, $\omega_L$ is the wavenumber in the liquid Lagrangian reference frame, and $t$ is time. This expression describes how a surface undulation of the shear instability changes with respect to time.

To determine the transverse instability starting time, we are essentially interested in when the surface undulation amplitude $a$ reaches a critical amplitude ($a_{cr}$). The key step is to convert the shear instability wavenumber ($k$) to the liquid Lagrangian reference frame pulsation ($\omega_L$). The following approach (Equations 5.31 to 5.33), introduced by Marmottant et al. [11] is used.

The first step is to find the group velocity ($u_c$) of the Rayleigh mode shear instability, which measures how fast an instability propagates in space,

$$u_c = \frac{(\sqrt{\rho_1} \ast u_1 + \sqrt{\rho_2} \ast u_2)}{(\sqrt{\rho_1} + \sqrt{\rho_2})}$$  \hspace{1cm} (5.31)

and then find the wavelength associated with the maximum growth rate of the shear instability.

$$\lambda_{max} = \frac{2\pi}{k_{max}}$$  \hspace{1cm} (5.32)

This wavelength is also taken as a length scale for the calculations in the next steps. Finally, the liquid frame pulsation is obtained as,

$$\omega_L = \frac{2\pi (u_c - u_1)}{\lambda_{max}}$$  \hspace{1cm} (5.33)

and a new Weber number based on the length scale $\lambda_{max}$ is defined as [11],

$$We_{\lambda_{max}} = \frac{\rho_2 u_2^2 \lambda_{max}}{\gamma}$$  \hspace{1cm} (5.34)

As explained previously, when the amplitude of the undulation reaches the critical amplitude, the transverse instability overrides the shear instability, and ligaments start to grow. The method to estimate the critical amplitude is also from Marmonttant et al. [11],

$$a_{cr} = \frac{We_{\lambda_{max}}^{\frac{1}{2}}}{2\lambda_{max}}$$  \hspace{1cm} (5.35)

and therefore, the transverse instability starting time is,

$$t_{cr} = \frac{\pi}{\omega_{max}} + \frac{arcsin(a_{cr}/a)}{\omega_{max}}$$  \hspace{1cm} (5.36)

The transverse instability causes an azimuthal perturbation in the transverse direction of the liquid stream, as illustrated in Figure 2.8. The azimuthal perturbation has an average peak amplitude of $b$ and a wavelength of $\lambda_{\perp}$ between each peak. $\lambda_{\perp}$ is also the
ligament spacing, and is an important length scale for further analysis. The transverse instability is essentially caused by the density difference between the phases, also called the Rayleigh-Taylor (RT) instability. The calculation of the RT instability wavelength is shown in Rayleigh [24], Lewis [25], Taylor [26] and Marmonttant et al [11]:

\[ \lambda_\perp = 2\pi \left( \frac{3\gamma}{\rho_1 g_{\max}} \right)^{\frac{1}{2}} \] (5.37)

where \( g_{\max} \) is the maximum surface acceleration of the Rayleigh mode shear instability. Recall that Equation 5.30 assumes a sine function of the liquid stream surface undulation. The acceleration of the undulation can be calculated by taking the second derivative of Equation 5.30. The maximum surface acceleration occurs when the undulation amplitude is critical,

\[ g_{\max} = a_{\text{cr}} \omega_L^2 \] (5.38)

Due to aerodynamic drag, the ligaments grow in both the axial and transverse directions. The Taylor Analogy Breakup (TAB) model [27] can be used to understand droplet breakup. It models the droplet breakup mechanism as a mass-spring-damper system, which leads to an ordinary differential equation (ODE) to describe the droplet breakup process. In this analysis, the TAB model was used to study the ligament development and breakup processes. Note that in the model of Marmonttant et al [11], the liquid viscosity is not considered in their ligament breakup model. In this work, the viscosity term was incorporated into the model, to model the damping effect of the liquid viscosity.

There are two analysis stages in the ligament development model. In the first stage, the TAB model is used to establish the relationship between ligament length and time. In the second stage, the ligament pinch-off time is calculated. The breakup mechanism in the TAB model contains three components: aerodynamic drag as the driving force, surface tension as the resisting force, and viscosity as the damping force. The TAB model assumes that the ligament volume, frontal surface area, and the area where the ligament connects to the ligament bulk, are all constant.

The ODE is based on force equilibrium applied to the ligament. The aerodynamic drag is,

\[ F_{\text{drag}} = \frac{1}{2} C_{\text{drag}} \rho_2 u_0^2 S \] (5.39)

where \( C_{\text{drag}} \) is the aerodynamic drag coefficient, \( S \) is the frontal area of the ligament, and \( u_0 \) is the interfacial velocity.

The resisting force is,

\[ F_{\text{resisting}} = C_{\text{resisting}} \pi \gamma \] (5.40)

where \( C_{\text{resisting}} \) is the resisting force coefficient.
And the damping force is,

\[ F_{\text{damping}} = \frac{1}{2} \mu_1 C_{\text{damping}} P \]  

(5.41)

where \( C_{\text{damping}} \) is the damping coefficient, and \( P \) is the perimeter of the area connecting the ligament to the liquid column.

Based on the assumptions, the perimeter \( P \), frontal area \( S \) and volume \( V \) of the ligament remain unchanged during the ligament elongation process. These are calculated based on the initial ligament diameter \( d_0 \):

\[ P = 2d_0\pi \]  

(5.42)

\[ S = \frac{d_0^2\pi}{4} \]  

(5.43)

\[ V = \frac{d_0^3\pi}{6} \]  

(5.44)

According to the phenomenological analysis of Marmottant et al. [11], \( d_0 \) is approximately 0.23 times the ligament spacing.

Incorporating an expression for the acceleration of ligament length growth, the force equilibrium equation becomes:

\[ \frac{1}{2} \rho_1 V \frac{d^2 L}{dt^2} = \frac{1}{2} C_{\text{drag}} \rho_2 u_0^2 S - C_{\text{resisting}} \pi \gamma - \frac{1}{2} \mu_1 C_{\text{damping}} P \]  

(5.45)

where \( L \) represents the length of the ligament. The above equation describes ligament growth with respect to time, considering aerodynamic drag, surface tension, and viscosity.

The next step is to determine when the ligament breaks off from the liquid column, and the breakup length of the ligament. Due to viscosity, the breakup process is delayed by a significantly increased capillary time [11]. Therefore, to estimate the ligament pinch-off time, the capillary instability of the ligament must be considered, based on the initial diameter of the ligament \( d_0 \).

Based on studies of the capillary viscosity instability by Chandrasekhar [28] and Eggers [29], a dispersion relation function is given as follows,

\[ \omega_i = \sqrt{\frac{\gamma}{\rho_1 d_0^2}} \left( \sqrt{4k_i(1-k_i^2)} + 4.5Re_{d_0}^{-2}k_i^4 - 3\sqrt{2}Re_{d_0}^{-1}k_i^2 \right) \]  

(5.46)
\( \omega_i \) and \( k_i \) are the dimensional wave growth rate and dimensionless wavenumber, respectively, and \( Re_{d_0} \) is the Reynolds number based on \( d_0 \),

\[
Re_{d_0} = \sqrt{\frac{d_0 \gamma}{2 \rho_1 v_1^2}} \tag{5.47}
\]

Typically, the dimensionless wavenumber of the capillary viscosity instability ranges from 0 and 1.

The pinch-off time, which measures how fast the ligament pinches off from the liquid column, equals the inverse of the maximum growth rate of the capillary viscosity instability \([11]\).

\[
t_b = \frac{1}{\omega_i^{\text{max}}} \tag{5.48}
\]

Finally, the primary breakup time measures the overall time from when the primary undulation starts, to when the ligament breaks off of the liquid column. It is simply the sum of the traverse instability starting time \((t_{cr})\) and the ligament breakup time \((t_b)\).

\[
t_{pb} = t_{cr} + t_b \tag{5.49}
\]

### 5.4 Droplet Size Distribution

Droplet size distributions have been extensively studied. The droplet size distribution of an atomization process has been found to follow a generalized gamma distribution, as confirmed by different approaches, including the Maximum Entropy Formalism (MEF) \([16]\) and phenomenological analysis \([11]\). In this section, the droplet size distribution due to smelt shattering is examined.

Ligament formation and droplet breakup is a convolutional process: ligaments are first segmented from the liquid stream, and the ligaments then breakup into smaller pieces; following the same breakup mechanism, the small pieces further breakup until the critical Weber number is reached. The final droplet size distribution consists of two parts: the ligament size distribution \((p_L(d_l))\), and the size distribution of droplets with respect to their parent ligament size \((p_B(\frac{d}{d_l}))\). The two distributions follow different distribution functions, but are coupled by \([11]\),

\[
p(d) = \int_{d_l=0}^{\infty} p_L(d_l) p_B \left( \frac{d}{d_l} \right) \frac{d(d_l)}{d_l} \tag{5.50}
\]

Marmottant et al. \([11]\) proposed a uniform distribution function for the ligament size distribution \(p_L(d_l)\); however this is a weak assumption for which there is a lack of
supporting evidence. In this study, a normal distribution is assumed for the ligament size distribution $p_L(d_l)$, consisting of a mean ligament diameter $d_0 = 0.23\lambda_\perp$ and a standard deviation of $d_0/10$. Note that this standard deviation was calibrated based on the experimental results, with a standard deviation of $d_0/10$, the model results reasonably well match with the experimental results.

$$p_L(d_l) \sim N(d_0, d_0/10)$$  \hfill (5.51)

The ligament breakup process is a convolutional process, as each convolution during this process is assumed to be identical and have the same probability density distribution. Such a convolutional process can be expressed by a gamma distribution function \cite{11}. The distribution of the final droplet size with respect to the parent ligament size distribution ($p_B(d/d_l)$) is,

$$p_B\left(\frac{d}{d_l}\right) = \frac{b^n}{\Gamma(n)} \left(\frac{d}{d_l}\right)^{(n-1)} e^{-b \frac{d}{d_l}}$$  \hfill (5.52)

There are two free parameters in Equation 5.52, $b$ and $n$, which are from the generalized gamma distribution function. $b$ and $n$ control the rate and shape of the gamma distribution, and can be estimated by fitting model generated results to experimental results. It was found that $b$ and $n$ depend on the operating conditions. When the liquid viscosity is high, the value of $n$ must be high to fit the experimental results, whereas $b$ is inversely proportional to the liquid flow rate. From numerical experiments, $n$ ranges from 10 to 20 and $b$ ranges from 3 to 10. The two parameters were calibrated based on the experimental results: $n$ equals 15, and $b$ equals to 9.

Finally, the arithmetic mean droplet size was calculated by integrating the final droplet size density distribution function.

$$d_{10} = \int_0^\infty p(d) d(d)$$  \hfill (5.53)

\section{Model Validation}

To verify the analytical model, various comparisons were performed between the experimental results and results generated by the model with similar conditions. In this section, an example of the comparisons is shown. The results generated by the model were compared with the experimental results conducted at $P = 100$ kPa, $\mu_1 = 100$ cP, and $Q = 2.5$ LPM. The experimental conditions were converted to the model inputs, which are shown in Table 5.1.
65

<table>
<thead>
<tr>
<th>$\rho_1$</th>
<th>$\rho_2$</th>
<th>$\mu_1$</th>
<th>$\mu_2$</th>
<th>$v_2$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1370 g/cm$^3$</td>
<td>1.22 g/cm$^3$</td>
<td>100 cP</td>
<td>1.85e$^{-2}$ cP</td>
<td>1.57e$^{-3}$ m$^2$/s</td>
<td>0.08 N/s</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$u_1$</th>
<th>$u_2$</th>
<th>Gas Phase Gap Length (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.7 m/s</td>
<td>78 m/s</td>
<td>0.025 m</td>
</tr>
</tbody>
</table>

Table 5.1: Material properties and operating conditions for validating the analytical model

5.5.1 Results of Shear Instability

At the first stage of the model, the shear instability was analyzed based on the inputs. The shear instability dispersion relation function, as an intermediate stage output, is shown in Figure 5.3.

![Figure 5.3: Dimensional shear instability dispersion relation](image)

Based on the shear instability analysis, the maximum growth rate was found. The maximum growth rate is the most unstable one, that leads to further breakup processes. For the presented example, the maximum growth rate was 926.0 s$^{-1}$. 
5.5.2 Results of Transverse Instability and TAB Model

Based on the maximum shear instability growth rate ($\lambda_{max}$), the “transverse instability starting time” ($t_{cr}$) was estimated. In this example, $t_{cr}$ was approximately 6.35 ms. Subsequently, the TAB model was used to evaluate ligament growth with respect to time, as shown in Figure 5.4.

![Figure 5.4: Ligament length growth with time](image)

The capillary viscosity instability model was then applied to estimate the ligament breakup time ($t_b$), which is the inverse of the maximum capillary instability growth rate ($\omega_i^{max}$). Figure 5.5 shows the capillary viscosity instability dispersion relation for the example case.
$t_b$ is approximately 6.06 ms and the primary break time ($t_{pb}$) is 12.41 ms, which indicates that ligaments breakup from the liquid stream very rapidly.

One additional modeling was done, assuming $Q = 6$ LPM while other conditions are the same, the result showed that $t_{pb}$ is 14.20 ms. Comparing with the experimental results presented in Chapter 4, at $\mu_1 = 100$ cP, $P = 100$ kPa, and $Q = 6$ LPM, the primary breakup time was roughly 45 ms. Considering the experimental errors, and the simplifying assumptions of the analytical model, the analytical prediction reasonably matches the experimental result.

### 5.5.3 Results of Droplet Size Distribution Model

Based on the previous information, the droplet size distribution model was applied to this case study. The model generates the base distribution structure, which was fitted to the experimental data to determine the shape and rate of the distribution function. Figure 5.6 compares the normalized histogram from the experiment, at $P = 100$ kPa, $\mu_1 = 100$ cP, and $Q = 2.5$ LPM, and the probability density function generated by the analytical model. The model predicts a mean droplet size of 1.67 mm.
The comparison shows that the predicted base distribution function fits the experimental data well, and the mean droplet sizes are nearly the same. This comparison gives confidence to the analytical model, and suggests that it might be used for predicting droplet size distributions of actual smelt shattering operations.

5.6 A Case Study using the Analytical Model

To obtain insight into smelt shattering at mill operating conditions, the analytical model was applied to examine an actual shattering operation. The material properties of molten smelt were input into the model to predict the mean droplet diameter and droplet size distribution. Additionally, the model was used to assess the sensitivity of the results on the liquid viscosity, in order to estimate the maximum viscosity that can be shattered. Note that the detailed calculation process of this case study is shown in Appendix A.

5.6.1 Estimation of Shatter Jet Velocity

At most pulp mills, steam is used to shatter smelt. Compared to an air jet, steam has a higher density but lower viscosity. The speed of sound in saturated steam is also higher compared to air.
To obtain the velocity profile of a steam jet. CFD analysis was used to predict the velocity profile. In this case, a cone-shaped shatter jet nozzle with 1.5 cm inlet radius and 1 cm outlet radius was assumed, which injects 25 psi saturated steam into a 0.5 m by 0.2 m domain. The speed of sound in the saturated steam is approximately 480 m/s [30]. The results of the simulation are illustrated in Figures 5.7 and 5.8: the first diagram shows the steam velocity contour, and the second plot shows the centreline velocity of the steam jet.

![Velocity contour of the steam jet](image-url)

**Figure 5.7**: Velocity contour of the steam jet
Assuming the impingement point is 40 cm below the shatter jet nozzle, the centerline velocity of the steam jet is approximately 120 m/s. The density and viscosity of the steam were assumed constant, at 1.51 kg/m$^3$ and 1.33×10$^{-5}$ Pas.

### 5.6.2 Droplet Size Distribution of Shattered Smelt

To calculate the mean droplet diameter and droplet size distribution, other parameters are required. The density of molten smelt is approximately 2000 kg/m$^3$, the viscosity is 5 cP, and the surface tension is 0.21 N/m [4]. The liquid flow rate was assumed to be 1 liter per second, corresponding to a liquid velocity estimated to be 5 m/s. Table 5.2 summarizes the parameters used in this case study.
Applying this data to the analytical model, the mean droplet diameter and the droplet size distribution of the molten smelt stream were predicted. As mentioned in Section 5.4, there are two free parameters in the droplet size distribution model, which are the rate and shape of the gamma distribution (Equation 5.52). The free parameter \( n \) relates to the liquid viscosity; in this case study, it was assumed to be 15, the same as in the model validation. On the other hand, the free parameter \( b \) relates to the liquid flow rate. Due to a much higher flow rate of smelt, \( b \) was assumed to be 5, which is smaller than in the model validation. The estimated mean droplet diameter was approximately 2.8 mm, and the droplet size distribution function is plotted in Figure 5.9.

![Figure 5.9: Prediction of molten smelt droplet size distribution](image-url)
5.6.3 Maximum Shatterable Viscosity

The liquid viscosity of jelly roll smelt was estimated to be between $10^4$ and $10^5$ cP, and can be up to 100,000 cP. In this section, a sensitivity test is performed, and the primary breakup time is of interest. As introduced earlier, the primary breakup time is a measure of how fast ligaments form and breakup from the liquid column. For a certain operating condition, if the smelt primary breakup time is shorter than the time it takes for the smelt to fall into the dissolving tank, then the smelt is at least shatterable into ligaments. Thus, by this sensitivity test, the relationship between primary breakup time and viscosity can be identified, as well as the maximum viscosity that can be shattered.

A range of viscosities were tested, from 1 cP to 150,000 cP, keeping other conditions unchanged, at the values mentioned previously. Figure 5.10 is a plot of liquid viscosity versus primary breakup time. Note that the primary breakup time increases as the liquid viscosity increases, as expected. Also, it was found that when the smelt viscosity is lower than approximately 125,000 cP, the primary breakup time is a real and positive value. However, beyond 125,000 cP, the model produces complex traverse instability starting times ($t_{cr}$), which means the smelt stream is too viscous to experience the transverse instability, or form ligaments during shattering. Thus, for this case, 125,000 cP is the theoretical maximum liquid viscosity, but note that the breakup time is then infinite, implying that the smelt steam must fall from an infinitively high position.

Figure 5.10: Primary breakup time versus viscosity
For a more reasonable falling distance of 3 meters, the falling time is approximately 0.6 seconds. Thus, the smelt should have a primary breakup time shorter than 0.6 seconds, in order to be shattered into ligaments. From Figure 5.10, the maximum shatterable viscosity is roughly 80,000 cP.
Chapter 6

Conclusions and Future Work

In this section, the findings of this research are summarized, and recommendations are presented for future work.

6.1 Conclusions

Jelly roll smelt is a partially frozen smelt, which has a tremendously high liquid viscosity compared to molten smelt. The liquid viscosity of jelly roll smelt was estimated to be between $10^4$ and $10^5$ cP, depending on the smelt temperature and the smelt freezing temperature.

Experiments were conducted on a quarter-scale experimental apparatus, to study viscous smelt shattering. A corn syrup-sugar mixture was used to mimic jelly roll smelt. By using the imaging matrix technique developed by Lin [9], arithmetic mean droplet diameter, droplet size distributions and droplet spatial distributions were obtained as a function of liquid viscosity, shatter jet supply pressure (gas phase velocity), and liquid flow rate (liquid phase velocity). An analytical model was also presented to explain the physics of viscous liquid shattering, and predict the primary breakup time and the droplet size distribution for a given shattering operating condition. A summary of the effects of various parameters is presented below.

Effect of Liquid Viscosity

Liquid viscosity has a damping effect on smelt shattering, it delays the disintegration of the smelt stream. As shown in the experiments, as liquid viscosity increases, the shattering effectiveness decreases, forming larger droplets, and the droplet trajectories are concentrated to a small area beneath the spout. When the liquid viscosity increases
to a certain threshold, ligaments present instead of droplets. Since the droplet size distribution is a generalized gamma distribution, the majority of the droplets are in relatively small size range. A maximum shatterable viscosity exists for each particular operating condition, indicating the maximum liquid viscosity that can be disintegrated at least into ligaments. Given an operating condition, the droplet size distribution and the maximum shatterable viscosity can be estimated by using the analytical model.

**Effect of Shatter Jet Supply Pressure**

Shatter jet supply pressure directly affect of the gas phase velocity. Both the experimental data and the analytical model show that the gas velocity is an important parameter during shattering, as it drives the breakup of the smelt stream. A higher shatter jet supply pressure improves the shattering effectiveness by decreasing the droplet size, and can be used to compensate for the damping effect of liquid viscosity. However, an increasing shatter jet supply pressure also increases the gas phase momentum, so that the trajectory of shattered droplets is towards the boiler wall, beneath the spout.

**Effect of Liquid Flow Rate**

The liquid velocity is proportional to the liquid flow rate. Both the experimental data and the analytical model show that a higher liquid flow rate increases the droplet size, however only by a little.

### 6.2 Future Work

As liquid viscosity is the primary difference between molten smelt and jelly roll smelt, this research focused on the effect of liquid viscosity during smelt shattering. Besides liquid viscosity, molten smelt and jelly roll smelt are different in terms of surface tension. Surface tension is a temperature dependent variable, which increases as the smelt temperature approaches its freezing temperature. Thus, the study of the surface tension effect during shattering is recommended as future work.

To apply the model in practice, it is important to obtain the steam jet velocity profile. Due to the limited scope of this research, the nozzle design used in the analytical study was simplified, and may not be representative in practice. In the future, comprehensive simulations of steam shatter jet velocity profiles are recommended.

For shattering operations in pulp mills, it would be most practical to adjust shatter steam pressure to breakup jelly roll smelt flow. Thus, in the next step of this research, the maximum shatterable viscosity predicted by the model could be associated with the
shatter jet pressure, so that mills could adjust shatter jet steam pressure based on the estimate of the smelt viscosity.

Finally, the interaction between molten smelt and water has been studied by Jin [31], who showed that the interaction triggers vapor explosions. In the future, similar experiments should be conducted to examine the interaction between jelly roll smelt and water.
Appendix A

Analytical Model Calculation and MATLAB Script

In this Appendix, the calculation procedures and details of the analytical model are shown. The MATLAB script implemented for the case study is presented as an example.

A.1 Model Input and Parameter Calculation

The first step of the calculation process is to define the model inputs and parameters, including material densities, viscosities and surface tension, and fluid velocities. In the case study, this information is listed in Table 5.2. Based on Equations 5.1 to 5.3, the boundary layer thickness of the shatter jet is found, and then the shear instability mode can be determined accordingly. The following code indicates the model inputs and calculates the various parameters.

clear; close all; clc

%% Define Basic Properties and Mode Selection
u1 = 5; % liquid velocity
u2 = 120; % steam velocity
u0 = u2 - u1; % interfacial velocity
h = 0.05; % steam nozzle gap
v2 = 0.88*10^-5; % kinematic viscosity of steam
re2 = h*u2/v2; % Re for steam

blt = 37.2*h/(re2^(3/4)); % Boundary Layer Thickness
rou1 = 2000; % density of smelt
rou2 = 1.51; % density of steam
rou = rou2/rou1; % density ratio
mu1 = 5/1000; % dynamic viscosity of smelt
mu2 = 1.33*10^-5; % dynamic viscosity of steam
mu = mu2/mu1; % dynamic viscosity ratio
surftension = 0.21; % surface tension steam/smelt
we_threshold = rou^0.5; % threshold weber number
we2 = rou2*u0^2*blt/surftension; % weber number based on gas shear-layer thickness

% Shear Instability Mode Selection
if we2 <= we_threshold
disp ('The Weber number is ', num2str (we2),', choose Kelvin Helmholtz instability')
else
disp ('The Weber number is ', num2str (we2),', choose Rayleigh mode')
end

re1 = u0*blt*rou1/mu1; % Re based on liquid shear-layer thickness
re2_blt = u0*blt/v2; % Re based on gas shear-layer thickness

% Effect of Gravity
J1 = (rou1-rou2)*9.81*blt/(rou1*u0^2); % Richardson number
if J1 >= 10^-4
disp ('The Richardson number is ', num2str (J1),', gravity effect strong')
else
disp ('The Richardson number is ', num2str (J1),', gravity effect weak')
end

A.2 Shear Instability

When shattering begins, the smelt experiences an axisymmetric undulation caused by the shear instability. As mentioned in Chapter 5, the high velocity of the steam jet results in a thick boundary layer, and therefore the shear instability is the Rayleigh mode. This section solves the shear instability dispersion relation function (Equation 5.4), and identifies the maximum shear instability growth rate. The following MATLAB script solves Equation 5.4 based on the cubic function method (Equation 5.5 to 5.29).

Note that function ‘cubic’ calculates the coefficients of the cubic function, and function ‘cubicfun’ determines the three unique solutions.

%% Shear Instability Rayleigh Mode:
beta = 2*i1/re1; % Equation 5.8
eff = logspace(-5, 0.6, 200000);% Define range of dimensionless wavenumber
k = eff/blt; % Range of dimensional wavenumber
phi = 2*(eff.^2)*mu*beta; % Equation 5.20
xsol = zeros (200000, 3); % Create empty matrix for 3 unique solutions
solimg = zeros (20000,1); % Create empty array for imaginary part of the final dimensionless growth rate
sol = zeros (20000,1); % Create empty array for final dimensionless growth rate

% For each dimensionless wavenumber, solve for the 3 unique solutions of
% the cubic function, find the positive imaginary component of the
% solutions, and establish dispersion relation function
for i = 1:200000
[p, q, r] = cubic (rou, mu, beta, eff(i), phi(i)); % coefficient of cubic function
[x1, x2, x3] = cubicfun (p, q, r); % 3 unique solutions of cubic fun
xsol (i, 1) = x1 - eff(i); % Dimensionless growth rate 1
xsol (i, 2) = x2 - eff(i); % Dimensionless growth rate 2
xsol (i, 3) = x3 - eff(i); % Dimensionless growth rate 3

% Find the which solution has positive imaginary part (Equation 5.29)
if imag(x3 - eff(i)) >= 0
    solimg (i) = imag(x3 - eff(i));
sol (i) = x3 - eff(i);
elseif imag(x1 - eff(i)) >= 0
    solimg (i) = imag(x1 - eff(i));
sol (i) = x1 - eff(i);
elseif imag(x1 - eff(i)) && imag(x3 - eff(i)) <= 0
    solimg (i) = imag(x2 - eff(i));
sol (i) = x2 - eff(i);
end
end

[maxsolimg , indsol] = max ( solimg );
disp ([ 'The maximum dimensionless disturbance is ' , num2str(maxsolimg)])
disp ([ 'The maximum dimensionless wavenumber is ' , num2str(eff(indsol))] )

% Plot the Dispersion Relation Function
figure
plot (eff , solimg )
axis ([0 0.1 0 2*10^-3])
xlabel (' Eta ( Dimensionless Wavenumber ) ')
ylabel (' Omiga ( Dimensionless Growth Rate ) ')
title (' Dimensionless Dispersion Relationship ')

w = sol*u0/blt; % Covert the dimensionless growth rate to dimensional
wimg = imag(w); % Find the imaginary part of dimensional growth rate
[maxw , indw ] = max ( wimg ); % Maximum dimensional growth rate
disp ([ 'The maximum disturbance is ' , num2str (maxw ), ' s^-1'] )
disp ([ 'The maximum wavenumber is ' , num2str(k(indw)) , ' m^-1'])
figure
plot (k, wimg )
axis ([0 500 0 1500])
xlabel ('k ( Wavenumber ) s^{-1} ')
ylabel ('w ( Growth Rate ) m^{-1} ')
title (' Dimensional Dispersion Relationship ')

% 'cubic' finds the coefficients the cubic function
function [p, q, r] = cubic (rou , mu , beta , eff , phi)
a3 = -(1+rou);
a2 = 2*rou - (1+mu)*beta*eff^2 - (1-2*eff)*(1+rou) + (1-rou)*exp(-2*eff);
a1 = phi + (1-2*eff)*(2*rou - (1+mu)*beta*eff^2) - (2*rou - (1-mu)*beta*eff^2)*exp(-2*eff);
a0 = ((1-2*eff)-exp(-2*eff))*phi;
p = a2/a3;
q = a1/a3;
r = a0/a3;
end

% 'cubicfun' finds the 3 unique solutions the cubic function
function [x1, x2, x3] = cubicfun (p, q, r)
term2 = 3*3^0.5*sqrt(-p^2*q^2+4*q^3+4*p^3*r-18*p*q*r+27*r^2);
term1 = -2*p^3+9*p^2*q-27*p*q^2;
A = (((term1+term2)^(1/3))/(3*3^0.5))/((term1+term2)^(1/3));
B = (-p^2 + 3*q)/(9*A);
x1 = -p/3 + A - B;
x2 = -p/3 + (-1-(3*0.5)*1i)/2*A - (-1+(3*0.5)*1i)/2*B;
\[ x_3 = \frac{-p}{3} + \frac{(-1+(3^{-0.5})\times i)}{2} A - \frac{(-1-(3^{-0.5})\times i)}{2} B; \]

end

A.3 Transverse Instability and Ligament Development

When the amplitude of the shear instability saturates, the transverse instability starts, and ligaments form and develop. The following MATLAB script shows the calculation of the transverse instability starting time and ligament spacing, and the modeling of ligament growth (TAB model). Note that function ‘modTAB’ models the ligament development (Equation 5.39 to 5.45), and function ‘viscosityslowing’ calculates the ligament breakup based on the capillary instability (Equation 5.46 to 5.48).

%% Stage 2: Transverse Instability
%% R-T instability, ligament spacing and ligament birth time
uc = (sqrt(rou1)*u1 + sqrt(rou2)*u2)/(sqrt(rou1)+sqrt(rou2));
% group velocity, wave propagation velocity
lambda_m = 2*pi/k(indw);
% maximum wavelength
% Surface Acceleration:
w_L = 2*pi*(uc-u1)/lambda_m;
% liquid frame undulation pulsation
amp_wave = 1/1000;
% typical wave saturation amplitude, (may change in future)
we_lambda = rou2*u2^2/(surf tension/lambda_m);
% Weber number based on max wavelength
amp_c = 0.5*we_lambda^(-1/3)*lambda_m;
% Critical amplitude for ligament birth
t_ligbirth = pi/w_L + asin(amp_c/amp_wave)/w_L;
% Time from the beginning (shear instability) to ligament formation,
% the velocity is so small comparing to acceleration
disp(['The ligament will start to grow after ', num2str(t_ligbirth), 's'])
ligspacing = 2*pi*(3*surf tension/(rou1*amp_c*w_L^2))^(0.5);
% Ligament Spacing

%% Ligament Development/TAB Mode Elongation
Cdrag = 10; % Drag coeff
d0 = 0.23*ligspacing; % Initial Ligament Diameter
Cdamp = 10; % Viscosity Damping Coeff
Ck = 8; % Surface Tension Coeff
tspan = linspace(0,0.1,500); % Time Interval to Solve ODE
ic = [0 0]; % Initial Condition to Solve ODE
[t, l, v0] = modTAB(rou1, rou2, u0, Cdrag, d0, Cdamp, mu1, surf tension, Ck, tspan, ic);
% Calculating Viscous Slowing Breakup time (capillary time) based on Marmottant
[max_w_vis, time_cap, x, Omega_vis] = viscosityslowing(d0, surf tension, rou1, mu1);
disp(['The ligament breakup time is ', num2str(time_cap), 's'])

%% Function 'modTAB'
function [t, l, v0] = modTAB(rou1, rou2, u0, Cdrag, d0, Cdamp, mu1, surf tension, Ck, tspan, ic)
% Assumptions: 1. The ligament volume remains unchanged. 2. The ligament
% frontal surface area remains unchanged, since when elongated, the width
% becomes smaller. 3. The area connecting liquid bulk is unchanged. The
% model is analog of spring-mass-damper system (TAB model)
\[ r_0 = \frac{d_0}{2}; \]
\[ s = \pi d_0^2/2; \]
\[ v_0 = \pi d_0^3/6; \]
\[ c = \frac{\rho_2 u_0^2 + C_{\text{drag}} s}{(\rho_1 v_0)}; \quad \% \text{s/v = surface area/volume} \]
\[ a = 2 C_{\text{damp}} \mu_1 r_0 \pi / (\rho_1 v_0); \quad \% \text{p/v = perimeter/volume} \]
\[ b = 2 \sigma_{\text{surf}} C_k \pi / (\rho_1 v_0); \quad \% \text{p/v = \pi/volume} \]
function \[ d \text{ldt} = \text{odefun}(t, 1, a, b, c) \]
\[ \text{dldt} = \text{zeros}(2,1); \]
\[ \text{dldt}(1) = \text{l}(2); \]
\[ \text{dldt}(2) = -a*\text{l}(2) - b*\text{l}(1) + c; \]
end
\[ [t, \text{l}] = \text{ode}45(@(t, \text{l}) \text{odefun}(t, \text{l}, a, b, c), \text{tspan}, \text{ic}); \]
end

%% Function 'viscosityslowing'
function \[ [\text{max}_w_{\text{vis}}, \text{time}_{\text{cap}}, x, \Omega_{\text{vis}}] = \text{viscosityslowing}(d_0, \sigma_{\text{surf}}, \rho_1, \mu_1) \]
\[ \text{re}_d_0 = \sqrt{d_0 \frac{\sigma_{\text{surf}}}{(2 \rho_1 (\mu_1/\rho_1)^2)}}; \quad \% \text{Equation 5.47} \]
disp (\['\text{The viscosity Reynolds number is ', num2str(re_d0),']\])
\[ x = \text{linspace}(0, 1, 500); \quad \% \text{Identify range of capillary instability wavenumber} \]
\[ \Omega_{\text{vis}} = \sqrt{4 x^2 (1-x^2) + 4.5 \text{re}_d_0^{-2} x^4} - \frac{3}{2} \sqrt{2} \text{re}_d_0^{-1} x^2; \]
\[ w_{\text{vis}} = \Omega_{\text{vis}} \sqrt{\frac{\sigma_{\text{surf}}}{\rho_1 d_0^3}}; \]
% Find the growth rate with respect to the wavenumber (Equation 5.46)
figure
\[ \text{plot}(x, w_{\text{vis}},'\text{LineWidth}',3) \]
\[ \text{axis}([0 1 0 200]) \]
xlabel (\('\kappa_{\text{i}} (\text{Dimensionless Wavenumber})', 'FontSize', 24)\)
ylabel (\('\omega_{\text{i}} (\text{Dimensional Growth Rate}) s^{-1}', 'FontSize', 24)\)
\[ \text{xt} = \text{get}(\text{gca}, 'XTick'); \]
\[ \text{yt} = \text{get}(\text{gca}, 'YTick'); \]
\[ \text{set}(\text{gca}, 'FontSize', 28) \]
title (\('\text{Capillary Viscosity Instability Dispersion Relation}', 'FontSize', 28)\)
\[ [\text{max}_w_{\text{vis}}, \text{x_max_index}] = \text{max}(w_{\text{vis}}); \]
\[ \text{time}_{\text{cap}} = 1/\text{max}_w_{\text{vis}}; \quad \% \text{Equation 5.48} \]
end

A.4 Droplet Size Distribution

After ligaments break up from the liquid stream, the ligaments experience secondary
breakup, and droplets form. The MATLAB script below calculates the droplet size
distribution (Equation 5.50 to 5.53). Note that the function ‘drop dist mill’ is the major
calculation, and is called from the main function.

%% Ligament Size (in diameter) Distribution
\[ [\text{pd}, \text{mean}_{\text{droplet size}}] = \text{drop dist mill}(d_0); \]
disp ([\'The average final droplet size is ', num2str(mean_droplet_size), ' mm\'])

%% Function 'drop_dist_mill'
function \[ [\text{pd}, \text{mean}_{\text{droplet size}}] = \text{drop dist mill}(d_0_{\text{avg}}) \]
% Create empty array to store results, the function calculates the probability for the final droplet size within the range of 0 to 10 mm
sizei = length (0:0.00001:0.01);
pd = zeros (sizei, 1);
j = 1;

% For the final droplet size within the range of 0 to 10 mm
for d = 0:0.00001:0.01
    num = 15; % Gamma n (constant)
    % call plpb to create the distribution function (Equation 5.50)
    fun = @(d0) plpb(d0, d0_avg, num, d); % Integral (Equation 5.50)
    p = integral (fun, 0, Inf, 'ArrayValued', true);
    pd (j) = p;
    j = j + 1;
end

% Fit the probability data point into a polynomial distribution function so that we can integral to find the mean droplet size
x = 0:0.01:10;
fit_coeff = polyfit (x', pd ./1000, 9);
fit_function = polyval (fit_coeff, x');
size_fun = @(x_inter) (x_inter.*polyval(fit_coeff, x_inter));
mean_droplet_size = integral (size_fun, 0, 10); % Equation 5.53

% Plot the distribution function
plot (0:0.01:10, pd ./1000, 'LineWidth',3)
xlim([0,10])
xlabel (' Diameter (mm)', 'FontSize', 24)
ylabel (' Frequency (mm^{-1})', 'FontSize', 24)
h2 = legend ([('Model: The mean droplet size is ', num2str(round(mean_droplet_size,2)), ' mm')]);
set(h2, 'FontSize', 20)
x = get(gca, 'XTick ');
y = get(gca, 'YTick ');
set(gca, 'FontSize', 28)
title ('Droplet Size Distribution', 'FontSize', 28)

% This function creates the distribution function (Equation 5.50)
function f = plpb (d0, d0_avg, num, d)
b = 5; % Gamma b (constant)
sigma = d0_avg/10; % standard deviation of the ligament distribution
pl = normpdf (d0, d0_avg, sigma); % ligament distribution (Equation 5.51)
% Droplet distribution with respect to their parent liagment
pb = (b*num/gamma(num))*(d/(d0))^(num-1)*exp(-b*(d/(d0)));
%f = pl*pb/(d0);
end
Bibliography


