Exploring the Extent of Interactions in Activity-Based Models: A Critical Examination of Intra-Household and Spatial Interactions Through Choice Modelling

by

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Civil Engineering

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Abstract
This research increases the overall understanding of a subset of interactions that influence patterns of behaviour: intra-household interactions and spatial interactions. Understanding these interactions and their influence on travel behaviour is essential to developing policy sensitive models. These models provide much of the prerequisite information for evidence based planning. This understanding is framed through a review of existing best practices in travel demand modelling and a discussion of how independent models can be integrated within a larger framework.

The thesis presents four empirical models that incorporate either one or both interactions. A model of joint household mode choice which incorporates decisions regarding joint travel and vehicle allocation is estimated. To accomplish this, the model uses a novel choice set formation technique. This is the first modelling to accomplish these tasks simultaneously. Models of transit station location choice are estimated using a newly proposed model structure, the spatially weighted error correlation model. These spatial models are used to compare the difference between independent (transit accessed by driving alone) and joint (transit accessed by being dropped off) spatial choices. The presented model outperforms existing models used to capture
the same trend and provides new behavioural insights. A model of chauffeur allocation and location choice for daycare trips is estimated. This model incorporates spatial constraints in this decision through the application of a stochastic frontier model to generate task allocation specific spatial choice sets. This allows for a reduction in the sampling frame for spatial choices. Finally, a model of high school escort and household mode choice is estimated using a parallel constrained choice logit formulation. This empirical investigation generalizes the parallel constrained choice logit to allow for any number of household members to be considered. Concurrently, this empirical analysis highlights a flaw within an existing “best practice” modelling framework.
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List of Publications

The following published and worker papers have been reproduced with modification as the basis for Chapters 4 through 7 in this thesis:

Chapter 4 Household Mode Choice

Chapter 5 Park and Ride vs. Kiss and Ride Station Location Choice

Chapter 6 Daycare Location and Drop Off/Pick Up Allocation Choice
Weiss, A., Habib, K.M.N. 2017. Who’s picking up the child from daycare? Understanding intra-household dynamics in drop-off and pick-up task allocations for household with dependent children. Transportation Research Record (Forthcoming)

Chapter 7 Household Mode and Student Escort Choice
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Chapter 1
Introduction

This thesis presents an examination of interaction in travel demand models, with a focus on interactions within the household and across space. The rationale for addressing intra-household interaction and spatial issues is that accounting for these factors provides a better understanding of the travel behaviour of individuals. This increased understanding and incorporation of that understanding within models of travel demand will lead to better forecasts and therefore better policy decisions and infrastructure investments. The understanding of these issues is accomplished through a review of the existing best practices and methods as well as a set of innovations directly relating to these points. Specifically, this thesis provides empirical investigations, novel applications and expansion of existing best practices for understanding interactions within the household and spatial issues in models of travel demand. The primary argument is that to understand individual level travel behaviour, the analyst must understand the context of individual behaviour. This context comes from numerous factors, though two critical components are spatial and household interactions.

This chapter presents an overview of the dissertation, providing some basic background information, an outline of the research objectives and relevance, a review of the data used for the analysis and a roadmap for the remainder of the thesis.

1.1 Background on Activity-Based Models

The importance of understanding the underlying reason for why an individual decides to travel from one point to another is not a new concept and has been well established within the travel demand literature. This has spurred the movement away from trip based approaches that were initially developed in the 1950s towards an activity based approach. McNally and Rindt (2008) review the limitations associated with trip based approaches, which can be summarized as follows:

- Trip based models consider the trip as the unit of demand rather than the reason for making the trip (activities at the end of the trip)
- Trip based models lack connectivity between subsequent trips made by the same individual. This connectivity is both spatial and temporal.
- Trip based models will often use misspecified model structures (e.g. ignoring constraints on behaviour, limited behavioural underpinnings, etc.)
- Trip based models typically rely on individual-level random utility maximization without considering alternative approaches to explain behaviour.

The alternative approach to this conventional trip based method of understanding travel behaviour is one that uses an activity based framework. The concept of linking travel to activity participation is postulated by Mitchell and Rapkin (1954) who also called for a comprehensive framework for understanding travel behaviour. The theoretical foundation of the framework currently in use today stems from the following papers and books:
  - Hägerstrand (1977) who presents the concept of space time prisms for constraining activity participation.
  - Chapin (1974) who discusses the factors that influence activity participation, namely a sequential process of motivation, choice and outcome. This framework is then used to identify patterns in activity participation across time and space.
  - Fried et al. (1977) who present a psychological analysis of the motivations for travel behaviour, while examining how social structures (at the household level and within society) constrain these motivations.
  - Brail and Chapin (1973) who present some initial analysis on the observed patterns of 1960’s heads of household within the metropolitan United States,
  - Cullen and Godson (1975) who discuss the underlying patterns of behaviour, most notably, the allocation of time and the order of scheduling activities.

These studies and others have created the activity based framework currently in use today where:
  - The choice to travel is a result of the desire to participate in activities
  - Sequences of connected trips (trip chains) are analyzed rather than single independent trips
  - The household and other social structures influence and constrain trip making and activity participation
• Spatial, temporal and transportation related constraints are included in the model specification
• Activities are scheduled to fit within the general set of constraints presented above.

This thesis aims to address a subset of characteristics within existing activity based approaches, by providing an advancement on existing practices. The main goal of this thesis is to improve on analysis examining intra-household interaction, though this was later expanded to include analysis examining spatial constraints and spatial interactions along with household interactions. This objective is motivated by the existing limitations surrounding the incorporation of these interactions in constraints within the current best practices for activity based modelling. While most if not all activity based frameworks recognize the importance of incorporating interactions at the spatial and household level within their development, this implementation is fragmented and lacks consistent focus. As will be discussed in Chapters 2 and 3, these limitations are the result of two main concerns.

The first concern relates to issues surrounding computational tractability whereby the model framework can be estimated and applied using a standard personal computer. This concern is not the main point of this thesis though as personal computers continue to become faster and more powerful, these limitations may fall away. Working within this first constraint, a second and more pressing concern emerges.

The second concern stems from no consistent unified framework for understanding spatial and household issues within an activity based framework. While some activity based models may address issues surrounding certain household interactions, they simplify or overlook other aspects. Other modelling structures do provide methods for addressing these concerns but rely on simplifying assumptions or heuristic methods which have questionable policy sensitivity as they are not grounded in an underlying behavioural theory. Finally, the clear majority of these structures are simplistic in their development and do not take advantage of the more complex and robust modelling techniques that are available within the literature. Based on these limitations with existing practices for addressing household and spatial interactions, the subsequent section will outline the research objectives of this thesis.
1.2 Research Objectives

As noted above, the primary and overarching research objective is to improve the realism of models predicting individuals’ travel behaviour by understanding the factors which influence this behaviour. While numerous factors exist, this thesis narrows in on factors in the household and spatial constraints. While there exist methods for addressing these issues within the transportation literature, the consensus is that the transportation demand modelling community has spent insufficient time and resources examining these issues (de Palma et al. 2014). This thesis will aim to bridge some of this gap by presenting a set of empirical investigations whose objective is to address issues of how household interactions and spatial constraints/interactions influence travel behaviour. More formally, the key research objectives of this thesis are:

1. Review the state of the art in random utility modelling for household decision and location choice models.
2. Identify weakness and limitations within these best practice models.
3. Propose alternative modelling frameworks (either previously developed or developed through the course of this thesis) to address the limitations and weaknesses with existing best practices.
4. Test the alternative modelling frameworks through empirical case studies based on their effectiveness with standard and/or best practice models.
5. Present a conceptual framework for the integration of the presented models within a broader model of travel and activity demand.

It should be noted at this point that this work also aims to use conventional data sources to capture these behavioural trends. Some studies do make use of specialized data collection processes to understand the dynamics of household behaviour though most comprehensive models of travel demand make use of a more standard household travel survey. These travel surveys are more general but lack the richness of a data collection project designed specifically to capture behavioural insights on household or spatial interaction. This is not a research contribution per se but narrows the scope of the analysis to what behavioural insights can be obtained using standard data sources (a conventional revealed preference household travel survey).
One of the key benefits of the analysis in this thesis is the broad applicability of the presented models and methods. The approach used in this thesis is a modular one, whereby specific behavioural aspects or sets of behaviours are isolated from a full set of patterns over the course of a day. This approach is consistent with numerous existing demand modelling frameworks and makes the application and integration of the empirical investigations presented in this thesis generalizable across numerous modelling contexts. Moreover, the modelling structures that are used in this thesis can be applied and adapted to different choice contexts which fall outside of the scope of the forthcoming analysis. This means that there are numerous practical applications of the modelling structures presented in this thesis both with respect to the specific choice contexts examined and more generally.

Moreover, while not initially part of the goal of this analysis the forthcoming empirical investigations provides an interesting insight into the current practices within the field. Specifically, a model formulation which has been regularly applied and accepted within the demand modelling literature is found to be theoretically inconsistent with underlying concepts of random utility maximization. This finding, while not presenting an exciting new method for capturing interactions, raises an important point which is often forgotten when estimating and applying model structures. The goal of any modelling exercise is to present models which are intuitive and appropriate for the context of their application. When models are incorrectly applied outside of their appropriate context, the resulting conclusions from these applications may and often will be wrong, which can lead to misinformed policy and infrastructure investment decisions. This research highlights these existing shortcomings for a specific model structure that is misapplied and provides alternative theoretically consistent models that capture similar behavioural patterns.

Finally, this research aims to expand the application of robust modelling structures to understanding these behaviours as a methodological contribution. Current standard practice for incorporating household interactions in models of travel demand typically use very simple and/or theoretically inconsistent methods for modelling these behavioural outcomes. This thesis aims to use state of the art approaches that have seen limited applications generally and for the examination of intra-household interactions. Furthermore, this thesis presents a new modelling framework for capturing spatial interactions as a means of addressing limitations associated with
existing models for capturing interactions in spatial choice. This will frame this thesis as a useful reference for understanding the application of advanced modelling practices for intra-household and spatial interactions but also more generally within the realm of travel demand modelling.

1.3 Research Relevance

The research is relevant within a theoretical and applied context. Theoretically it presents a series of methods for capturing spatial and household interactions that have not previously been used. While three of the four of these methods are new applications of existing modelling structures, this work also presents a new methodological contribution for understanding spatial correlation. The novel applications and the new method are valuable contributions to the state of the art in demand modelling. The applications and method will provide future researchers with a framework for more robust or other interesting applications of these approaches within the transportation or broader decision making literature. Furthermore, the application of these methods provides interesting empirical findings regarding the appropriate application of these advanced modelling structures. The basis for this understanding the different modelling structures applied in this thesis is present in the literature review (Chapter 2). After the basics of understanding are established, the empirical applications (Chapter 4 to 7) of these structures provides interesting and new findings that raise new questions regarding the analyst’s choice between different modelling structures. Finally, the analysis in this thesis provides a thorough review of the existing state of the art/practice in techniques for household and spatial interaction modelling. From this review, the thesis points towards several potential avenues that warrant further investigation and/or application within the literature. Four of these avenues are examined in greater detail throughout the empirical investigations presented in this thesis.

From a more practical standpoint, the empirical investigations within this thesis provide the basis for the integration of models of intra-household interaction within an individual level model of travel and activity demand. This integration will allow these integrated models to answer questions that conventional trip based models are incapable of answering. The inability of these conventional models to accurately predict the outcome of a given policy may result in the limited efficacy of the policy in question as well as the failure to anticipate unintended negative consequences, such as societal inequity and/or decreased transportation system performance. This thesis provides a means for addressing these concerns through the integrations of the
empirical models presented within this thesis within a practice ready modelling frameworks. Furthermore, while there has been a gradual shift towards practice ready activity based models, many public agencies still rely on trip based methods for their analysis. The models presented in this thesis are in many cases flexible and applicable to both more general activity based approaches as well as conventional trip based models.

While there have been efforts to implement techniques like those proposed in this thesis within frameworks of travel, the existing practices are limited in their scope, as they either use inappropriate methods or drastically simplify some or all the behavioural processes under consideration. These simplifications are often done due to concerns over computational tractability but the advent of more powerful personal computers makes these concerns less pressing. This thesis expands on many of the techniques used in current state of the art modelling frameworks. In several cases, a comparison between the application of the existing techniques and the new models or applications presented in this thesis is discussed. Where applicable, this discussion is done based on improvements in model fit and the ability to capture the impact transportation policies.

At a more basic level, this thesis provides further motivation for advancing the practical applications of advanced modelling structures. This stimulus is provided by highlighting and reinforcing the limitations with existing trip based practices and highlighting potential solutions to these challenges. This thesis contributes to the growing movement towards activity based modelling structures and more generally an activity based method for thinking about travel demand modelling.

1.4 Data Used for the Analysis

Before moving forward with the thesis, there are a few housekeeping issues that must be addressed, namely, the source of the data used for this analysis. The demand data used for this analysis comes from a subset of households extracted from the 2011 Transportation Tomorrow Survey (TTS) for the Greater Toronto and Hamilton area (GTHA), Canada. The TTS is a 5 year predominantly telephone based survey which collects information on household composition and trip making for all household members over the age of 11. Approximately 5% of the population is surveyed. A discussion of the techniques used to process the data for the empirical analysis
done in this thesis can be found in appendix A. Figure 1.1 outlines the location of the region with respect to the rest of the province of Ontario.

Figure 1.1 Map of the GTHA (taken from Metrolinx, 2008)

The TTS survey data is supplemented with a multimodal static traffic assignment model that is developed at the University of Toronto using the 2011 TTS data and is used by numerous GTHA planning agencies (including most notably, the City of Toronto). This demand model produces travel times and travel costs for personal automobiles and transit from origin to destination traffic analysis zones within the region. This model splits the region into 2299 traffic analysis zones (TAZs) with an additional 76 transit station zones which are used for transit with automobile access. As noted in Miller et al. (2015) this framework does not distinguish between transit modes (regional versus local, bus versus subway, etc.) at the mode choice level, treating these
modes as the generic transit mode. As will be noted below this creates a unique challenge for certain multimodal travel times and costs.

1.5 Thesis Outline

This thesis is structured as follows:

1. Chapter 1 provides an overview of the thesis and a discussion of to the research objectives and relevance of understanding intra-household and spatial interactions in models of travel demand.

2. Chapter 2 reviews the literature relevant to the subsequent analysis presented in Chapters 4 through 7.

3. Chapter 3 presents a conceptual framework for the development of this thesis. This chapter discusses the underlying theories and hypotheses that shaped the analysis of the empirical investigations and a set of suggestions for implementing these empirical investigations within a broader framework of travel demand modelling.

4. Chapter 4 presents a trip based model of household mode choice. The model makes use of an a priori specification of the choice set to define household vehicle allocation and joint travel options. This chapter also outlines an interesting finding regarding the application of nested logit models relative to the mixed error component logit, namely that significantly different correlation patterns between alternatives can be found between the two models. The difference between the models is attributed to the error component logit capturing heteroskedasticity across the errors of the alternatives, and therefore (potentially) providing greater behavioural insights.

5. Chapter 5 presents a model of station location choice for park and ride and kiss and ride travel, outlining the difference between these choices. Moreover, this chapter outlines the development of a new modelling structure, the spatially weighted error correlation model. This new model formulation captures both spatial correlation and spatial heteroskedasticity. This model structure is shown to statistically outperform conventional modelling structures for capturing spatial correlation in models of discrete choice.

6. Chapter 6 presents a model of daycare location and task allocation choice for two parent households. This chapter provides a link between spatial and household considerations and presents a new method for constraining spatial choices, the stochastic frontier model. The stochastic frontier model presents not only a novel way for capturing spatial constraints in
task allocation, but also a means for generating more realistic consideration choice sets for spatial choice models.

7. Chapter 7 presents a joint model of mode choice for high school students and adults living in the same household. This model also explicitly considers the choice of an adult to escort the student to school. This chapter examines an existing method for capturing household interactions (the multi linear logit) and provides an overview of the theoretical inconsistencies of this method while also applying an alternative approach.

8. Chapter 8 provides concluding comments, reiterates the contributions of the thesis and discusses potential avenues for future research.
Chapter 2
Literature Review

The literature that is relevant to this thesis can be subdivided into two overarching sections: intra-household interactions and spatial interactions. As will be highlighted in subsequent chapters, these considerations are in fact intertwined and when efforts are made to examine them jointly, there are significant improvements towards our understanding of underlying behaviour. That said, the literature for these two main categories will be examined independently.

Within the literature there have been several different studies reporting investigations into intra-household interactions and their influences on travel behaviour. Ho and Mully (2015b) present an overview of four main research topics within this field: household auto ownership and mode choice (2.1), household task and time allocation (2.2), activity generation and scheduling (2.3) and long term decisions such as residential location choice (2.4). For the purposes of this thesis, longer term decisions such as residential location choice are outside of the primary scope of the analysis so these issues will only be examined briefly. Within these research areas, there are several different methods for examining these topics, including, implicit/exogenous variable inclusion (2.5), unitary decision making models (2.6), group decision making models (2.7) and a brief overview of other structures (2.8). Generally, any given paper reviewed within this section will fall into at least one of the topics and make use of at least one of the methods though there is potential for significant overlap (where a paper examines multiple topics or multiple methods). As such, papers may be reviewed multiple times, first in terms of analysis of their topics and relevant findings and then to understand their methods. To avoid repetition, an attempt has been made to focus only on the aspects of any given paper that are related to the section in which it is being discussed.

Equally relevant are the considerations of spatial interactions or more generally understanding spatial constraints in models of travel demand. Methods to understand spatial constraints within the context of travel demand are reviewed in Section 2.9. These discussions lead to a more formal need to understand the context of spatial choices and therefore lead to a more general
discussion of spatial interaction and the methods and models that are used to capture these interactions. The discussion on spatial interactions is presented in Section 2.10.

2.1 Household Mode Choice Models

There are two main challenges associated with modelling household mode choice behaviour: the first relates to understanding how household members make tradeoffs with regards to the allocation of the vehicles and the second is identifying the case for joint travel.

An example of an initial modelling attempt which, while innovative at the time, addresses neither of these concerns effectively is that of Badoe (2002). Badoe presents an initial joint modelling structure which argues that household members will select travel behaviours that maximize the total household utility, as opposed to each member aiming to maximize his or her utility independently. Using a simple two-mode (drive transit) structure, the resulting model has 4 possible modes (mode pairs) and is thus a drastic oversimplification of the choice scenarios faced by households, who may select minor modes or choose to travel jointly. Furthermore, the chosen model structure fails to capture automobile allocation in auto deficient households (so a household with a single car has a non-zero probability of both household members driving). Despite these deficiencies, the mode pair framework which is postulated by Badoe is an intuitive and attractive method for considering household mode choice. A modelling framework which extends this structure is presented in Chapter 4 of this thesis. More recent studies have circumvented many of these initial issues with the framework of Badoe, though they typically drop the mode pair definition and will examine either vehicle allocation or joint travel independently without considering both decisions simultaneously.

2.1.1 Vehicle Allocation

The issue of capturing the vehicle allocation process in a demand model at the household level is an interesting challenge that has been tackled using numerous behavioural approximations (more on this in Section 2.3). Models which explicitly capture these behavioural processes have recently come to the forefront. Two of the most prevalent approaches are presented below:

Anggraini et al. (2008; 2012) examined the vehicle allocation process in auto deficient households. Their earlier work focuses on the allocation for work tours and their more recent
work expands this concept to focus on non-work tours. The analysis uses a decision tree induction method and assumes that vehicles can be allocated to any driver within the household or not be allocated at all. Both studies find that the most important factors influencing vehicle allocation include trip purpose (work trips have vehicle allocation priority), gender (men are more likely to be allocated the vehicle than woman), the presence of children (children living in the household increased the probability of auto allocation to a household member) and accessibility measures to the destination (a more accessible destination makes vehicle allocation to an individual less likely). These works provide insight into how the allocation process occurs but again does not consider the question of mode choice.

Habib (2014) presents an empirical model investigating vehicle ownership/sufficiency, vehicle allocation and mode choice for two worker households. This work focuses on the specific context of reverse commuting and vehicle allocation in auto deficient households. Given the complexity of this decision structure, a three-level nested choice structure is proposed with the long-term decision of vehicle ownership being the upper nest, the choice of vehicle allocation for auto deficient households with one car as the second nest and then individual mode choice as the third and final nest. While this approach explicitly captures the choice of vehicle allocation at the household level, it does not consider the household choice of joint travel.

2.1.2 Joint Travel and Activity Participation

In the context of travel demand, joint travel behaviour has different facets, making a general analysis of joint travel challenging. As a rule, all joint travelling behaviour shares the following characteristic: two or more individuals travelling together for some portion or all a trip. Of interest to modellers of travel demand are joint trips which occur by automobile (although understanding joint travel by other modes is an interesting avenue for understanding household dynamics). Joint travel by automobile is the primary focus of joint travel analysis because of the potential for reduction in single occupancy vehicle usage and vehicle kilometers travelled.

Differences in joint trip making behaviour are identified based on the degree to which the trip is performed jointly (partially or completely) and the purpose of the trip for the driver and the passenger (joint activity participation, driver facilitating the passenger’s activity, different activities as the same destination, etc.). These differences are formally outlined in Gliebe and
Koppelman (2005), who present a typology of joint activity-travel patterns and then model the choice of patterns at the household level.

Models such as those of Gliebe and Koppelman (2005) provide a strong overview of joint travel and activity participation but exclude examination of the choice of mode for these trip patterns. Many models will use post processing for a given set of household activity patterns to allocated mode choice selections to the household travel patterns. Examples of this process include the set of papers of Ho and Mully (2013a; 2013b; 2015a), who present tour based mode choice models nested on the choice of joint or separate activity-travel patterns. These models then explicitly account for joint travel through a household level choice regarding the possibility for joint travel in the upper nest and individual mode choices, conditional on the choice of joint or individual tours. Unfortunately, this proposed model structure is missing the explicit consideration of vehicle allocation for the independent travel tours. The authors are also vague on the how the explicit allocation of vehicles (facilitator/chauffeur versus passenger) occurs within their modelling structure as they do not explicitly include a separate passenger mode, instead of grouping both modes into the generic car option. While not a problem for complete joint travel (even with separate activity participation at the same location), this formulation collapses when considering partial joint travel (drop-off and pick-up behaviour). Because of these shortcomings, this modelling structure lacks the internal consistency required for explicit implementation within an agent and activity based travel demand model.

Miller et al. and Roorda et al. (2005; 2006) explicitly tackle the question of mode choice through a simulation-based approach. Both works present a tour based mode choice model, which uses a three-step sequential approach to determine vehicle allocation and joint travel/activity opportunities. These models first perform unconstrained mode choice at the individual level. Mode choice is then simulated and potential conflicts with respect to vehicle allocation are identified. The vehicle is then ultimately awarded to an individual based on maximizing total household utility. Finally, a procedure to identify any possible opportunities for drivers to serve or facilitate other household members as passengers is performed. Joint travel opportunities are only deemed to be feasible should they increase the total household utility. Conceptually this approach satisfies the requirements of combining mode choice and joint travel however, the sequential procedure implies that all mode choice decisions are made in this manner. The counter
argument against this approach is therefore that it is possible that individual mode choice is predicated on the initial choice to engage in joint travel or a joint activity. Furthermore, there are some concerns with the treatment of vehicle allocation and joint travel. These decisions are predicted based on a set of simple rules that do not explicitly utilize a random utility maximizing (RUM) approach at the household level. Instead the summation of the individual systematic component of the utility across household members is examined. The choice to allocate the vehicle to an individual is done to maximize this summation of individual utilities. This means that the vehicle allocation decision is not an explicit decision that is modelled, but instead a byproduct of the simulation technique used for practical applications/forecasting. While certain allowances and assumptions must be made when developing operational or practice ready models, a more thorough examination of the choice of joint travel and auto allocation seems prudent.

More recently Vyas et al. (2015) present a method for considering joint travel at the tour level for large multi person households. This method accommodates the choice set explosion problem (discussed in Chapter 3, Section 2) by examining pairs of travelers independently. The presented model uses a set of binary choice models for joint travel for each unordered pair of individuals in the household. Multi person joint travel and activity arrangements are determined based on looping through these pair wise joint participation models. The iterative looping is stopped when a probabilistically consistent result emerges (i.e. if person A participates with person B and person C, then person B and person C must also participate). The authors note that this method can results in behaviorally inconsistent results: namely cases where two adults participate in a joint activity leaving a preschool aged child at home. Though non-household child care (i.e. babysitters) exist, the authors do not consider this as a possibility and instead present an alternative modelling framework to address this “limitation”. They present an addition, which considered the probability of joint activity participation given that an individual is not participating in a joint activity. The pairs of household members are now ordered and each household member is looped through iteratively. During each iteration, the impact of their choice to participate or not on the decision of all other household members is explicitly considered. This structure allows the choice to jointly participate in an activity to be a function of who is participating in the activity and who is not, thus increasing the behavioural realism of the choice. While a model which does consider the possibility of non household member child care would be
ideal, this would involve either specialized surveys or traditional household travel surveys including questions related to in home child care services. Furthermore, this model only examines joint travel and does not explicitly examine vehicle allocation.

These limitations with existing models contributed to the development of a trip based household level mode choice model which accounts for vehicle allocation and joint travel.

2.1.3 Transit Kiss and Ride (and Park and Ride)

A weakness of the existing literature is the consideration of joint travel for multimodal trips. The most common example of such a mode is the transit auto passenger access (kiss and ride) mode. In this mode, a family member will drive the transit rider to the station and drop them off at the station before continuing to their destination. This mode is typically very niche with a small percentage of the population using it, though there is a growing interest in encouraging individuals to find alternate access modes to regional transit due to the limitations associated with parking, as noted by Schank (2002). Other than Schank’s analysis, there exist limited works which have included kiss and ride travel modes as a viable option and those that do likely use a similar if not identical model for park and ride station choice. With that said, there is a significant amount of analysis in the literature directed towards understanding the factors that influence park and ride station location choice. The importance of understanding the park and ride mode stems from the goal of many transit agencies of attracting suburban drivers towards transit, particularly in regions with extreme congestion or expensive parking in employment hubs (Dickins, 1991). As far back as the late 1970s, efforts have been undertaken to model park and ride and kiss and ride as distinct alternatives from transit during mode choice modelling (Demetsky and Korf, 1979). More recent studies examining park and ride include the use of a multimodal network equilibrium assignment model, which simultaneously considers the choice of mode, route path, transfer point and parking location choice (Li et al. 2007). This type of holistic approach is attractive, however, requires substantial calibration for a given region and has difficulty in capturing decision maker specific attributes in the decision process. It should be noted that the multimodal traffic/transit assignment model discussed in Section 1.4 of this thesis uses a similar framework to that proposed by Li et al.
Despite these concerns, the analysis done by Li et al. shows that the number of parking spaces at the station and the quality/frequency/fare of the transit service are key in determining the choice of park and ride over unimodal alternatives. Other studies find other factors have an important amount of influence on these choices. As noted by Schank (2002), the need to provide parking at commuter rail stations can be challenging for numerous transit agencies. The combination of the substantial infrastructure investment required for parking coupled with the limited or complete lack of parking fees charged by many agencies exacerbates the problem. Schank’s detailed examination of station and demographics on influencing kiss and ride choice shows that station characteristics (including parking availability and enforcement of parking regulations) play a much larger role than demographics towards encouraging kiss and ride with the notable exception of gender. He finds that households with at least one woman were much more likely to engage in kiss and ride trips.

While these general trends regarding the behaviour of park and ride and kiss and ride are relevant, a more focused analysis of station choice behaviour is of specific interest to this discussion. The works of Kastrenakes (1988) and Lythgoe and Wardman (2004) examine station attributes and travel time/transit service attributes in determining station choices. Kastrenakes finds that while travel times to the station and services at the station (frequency of service and travel time to the destination) are significant, station characteristics showed counterintuitive signs in the model. Conversely Lythgoe and Wardman find parking availability and other station attributes are of key importance in selecting station locations. The work of Mahmoud et al. (2014) uses access distance and a combination of station characteristics for station choice without any consideration of service characteristics of either the access to the station or the transit trip from the station. Finally, Chen et al. (2014) present a location based service for assisting commuters in selecting an optimal park and ride station using a multi-criteria decision making model. Considering projected arrival time at any given park and ride station, the model determines the likelihood of the user finding a parking spot. This analysis suggests that the choice of park and ride station is a complex process, which is dependent not only on the geographic location of the travellers’ origin and destination but also departure time in terms of finding a parking.
It should also be noted that several studies examine the choice of access mode and station choice jointly, typically using a nested logit structure. These studies are summarized in the work of Mahmoud et al. (2014) and are not discussed further here for the sake of brevity and as they typically do not provide any further insight into the station choice issue. An alternative approach to the disaggregate choice modelling structures discussed above involves the use of direct demand models for transit stations (Gutiérrez et al. 2011). These models predict the number of riders attracted to each station directly, rather than capturing the demand through a more conventional four stage or activity based process. While an attractive approach due to the simplicity associated with the analysis, these models are typically stand alone and have no capacity to iteratively integrate within a broader model of travel demand, thus limiting their applicability.

2.2 Household Task Generation and Allocation

This is a broad subject area and the details of how activities are generated and scheduled at an individual level is still a very active area of research. As a result, the focus on this section will be to provide a broad overview of these theories at an individual level with a much greater focus on how interactions within the household influence these decision processes. Generally, there exist two main methods of considering the generation and allocation of tasks: the time allocation model and the discrete choice model, each of which will be discussed in turn.

2.2.1 Time Allocation Models

The seminal paper on the theory of time allocation (Becker, 1965), presents a discussion on the underlying economic theory behind time allocation. Becker presents a model of time allocation formulated using a utility maximizing approach. This structure assumes that an individual either work to gain income and therefore financial utility or spends their time on the creation of commodities.

Examples of commodities include sleeping, eating, seeing a movie, reading, etc. The commodities created by an individual have a utility and a cost associated with their production. The cost of a commodity is a key for defining the utility maximizing model. While the commodities discussed above may have costs for goods associated with them, there are also costs associated with the time used by the individual in creating those commodities (spending some
amount of time to eat a meal or watch a movie). A monetary cost associated with time expenditure can be determined if foregone earnings associated with time spent on commodity creation (time that would otherwise be spent working) are considered. If the budget constraint associated with this is based on maximum hypothetical income (the income that would be achieved if all an individuals’ time is spent working), then Becker’s approach can be used as a comprehensive theory of time allocation. Individuals choose to balance their time budget between the time spend at work to gain income and the ability to spend time on commodity generation which provides utility. This suggests a tradeoff or interdependence between the time spent at work to gain income and the time spent on commodity creation.

This is an obvious simplification of the process of time allocation. Other factors, such as the working hours required by firms or accurately defining the utility of a given commodity make applying this method of understanding time allocation impractical without further adjustments. For practical applications, structures which incorporate the constraints of time allocation are required, though the fundamental concepts initially presented by Becker are sound.

The concept presented by Becker form the constraints for time allocation for utility maximization and as such presents a conceptual framework for the development of more empirical models of time allocation. Within the realm of transportation demand modelling, numerous examples of time allocation models exist. Zhang et al. (2002, 2005a) and Zhang and Fujiwara (2006), present a model of task and time allocation based on a multi linear group utility function at the household level. As is discussed in Section 2.5, and to a greater extent in Chapter 7, the application the multi linear utility function formulation is questionable in this context however the time allocation component of the model is still reasonable. More generally, these studies are limited in the description and classification of activities. Particularly troubling is the treatment of drop-off and pick-up activities, which are bundled in the general “out of home allocated activities”, thereby presenting the possibility that a child is not picked up from daycare. Other examples of this formulation include Kato and Matsumoto (2009) who include a Tobit structure to allow for no time to be allocated to a given activity type. The Kato and Matsumoto model also considered financial or budgetary constraints within the formulation. Gliebe and Koppelman (2002) also use a time allocation approach examining a 2-person household, though they did not use the multi linear component, instead opting to consider the household the decision making unit. They use
structural constraints within the likelihood function to ensure that the time spent on independent activities for all household members are equal. This approach is analogous to the parallel constrained choice logit discussed in greater detail below. The work of Srinivasan and Bhat (2005) presents two models of household maintenance activity participation within nuclear families. This model uses a seemingly unrelated regression model to determine the amount of time spent by each spouse has spent on chores in and around the house. Seemingly unrelated regression applies a bivariate normal correlation between the error terms of two linear functions, in this case for the time spent on household chores. The second model presented in this paper is a joint discrete continuous model for shopping allocation (male head shops, female head shops, joint shopping, no shopping) using a mixed logit formulation) and the shopping duration is estimated using a hazard duration model. Transforming the individual error terms for each model into normal random variables and applying a joint distribution between all the now normal error components of each model allows for joint estimation of the mixed logit and hazard duration models using a maximum simulated likelihood approaches. Finally, Bhat et al. (2013) present a model of time allocation which uses a multiple discrete continuous extreme value model to control for the size of the choice set when considering joint versus independent activities at the household level. This model is challenging as it only predicts the time spent on a given activity type rather than predicting explicit activity episodes. To split these time allocations into individual episodes, a set of sub processes must be undertaken. These sub processes are applied after the initial model is estimated and applied, which imposes a sequentially on the decision making process.

Based on these papers, there have been significant contributions towards an understanding of time allocation modelling at the household level within the literature. These models not only provide insight into the behavioural process of household time allocation but also evidence that some amount of household interaction occurs within travel (and therefore activity) demand. Unfortunately, models of time allocation provide limited insights into the scheduling of activities and provide limited insights into the understanding of very short activities (i.e. dropping off or picking up a dependent). Furthermore, these dependent escort trips typically have both a drop off and a pick up task, which adds further complexity to the models used to predict the allocation of these tasks. To address the later of these concerns, we next examine task allocation models which use a discrete choice framework.
2.2.2 Task Generation and Allocation Models

A foil to the time allocation models discussed above is a framework grounded in models of discrete choice. These model structures take as their basis part of the underlying theory of activity based analysis with an explicit focus on activity generation. For more information on the underlying theory of the generation of activities themselves, the interested reader is referred to the works of Bhat and Koppelman (1993) or more recently Habib (2007) for a review of the underlying theory of activity generation.

More practically, what these models typically employ is a series of discrete choices regarding first long run decisions (home and work location, vehicle ownership, etc.) followed by short run (generation of individual activities, generation of household activities, allocation of vehicles to household members) process which can be modelled using a discrete choice context. Decisions regarding task and resource allocation to household members are particularly relevant to the work presented in this thesis. Both Wen and Koppelman (1999, 2000) and Vovsha et al. (2004) present modelling frameworks which address these concerns. Both papers estimate discrete choice models of general household maintenance activity generation and then jointly model the allocation of these activities to individual household members. These models also consider the selection of travel patterns given the allocated activities at the individual level. Due to the requirements of tractability in the paper of Vovsha et al. (2004), the authors do not distinguish explicitly between maintenance activities, meaning that dropping of a child at daycare is analogous to doing the household’s grocery shopping. This creates some challenges in terms of ensuring behavioural realism, namely ensuring that dropped off dependents are picked up at some point during the day.

2.2.2.1 Models for Chauffeur Task Allocation

Models of escort task allocation which address escort task consistency have been developed, with Vovsha and Peterson (2005), who provide an update on their earlier work by accounting for the drop-off and pick-up tasks as well as the allocation of the escort task to a potential household chauffeur. This model is further extended by Gupta et al. (2014) to account for households with multiple children and potential chauffeurs, though the authors note that this generalization results in an explosion in the size of the choice set. The authors suggest estimating the drop-off and pick-up choices separately with the output of the other choice as an input in the model. For the application of the model for policy analysis or forecasting, the authors use an iterative sampling approach (discussed in greater detail in Chapter 3). Gupta et al. also note that a potential future
avenue of research could be the application of mode choice for other household members alongside the escort decision though leave this for future work. This comprehensive analysis would provide a direct link between the challenges associated with task allocation and resource allocation, as vehicles are implicitly allocated during the task allocation process. Finally, worth noting in this section is the model of Ermagun and Levinson (2016), who attempt to apply a multi linear logit group decision making model to the process of school escort decisions. Unfortunately, as will be seen in Section 2.5 and in Chapter 7, the multi linear logit model is inappropriate for use in a discrete choice context.

These models for chauffeur task allocation present several interesting challenges and provide greater potential for research contribution relative to the time allocation models discussed above. The linkage of escort decisions to household mode choice provides an interesting research context and would provide a considerable improvement towards our understanding of how school travel decisions occur and moreover how these interactions influence the mode choice patterns of non-chauffeuring household members (i.e. questions regarding vehicle allocation in auto deficient households). This is particularly relevant as previous studies for the Toronto area have completely overlooked modelling escort decisions for students, focusing only on mode choice between active and inactive modes (e.g. Mitra et al. 2010). Other than this study, there have been limited published works focusing on this issue directly for the GTHA, though the comprehensive models used for transportation planning likely do deal with these issues in some ad hoc manner. For example, the comprehensive GTA model, (TMG, 2015) used by numerous planning agencies in the region, does not explicitly consider daycare to nearly this level of detail (due in large part to tractability and the lack of explicit trip records for individuals under the age of 11). That said, much like the initial work of Vovsha et al. (2004), these trips are likely bundled into a comprehensive maintenance activity, meaning that while internal household consistency is not maintained, aggregate trip patterns will likely be accurate. This disaggregate inconsistency creates some interesting challenges in terms of understanding the impact of certain policies. These challenges are discussed in greater detail in Chapter 6. There is also a probable connection between spatial constraints and household interaction. Understanding how the home location and work locations of potential chauffeurs influences the possible location and the allocation of child care or school services is an interesting concept that has been for the most part overlooked or indirectly captured in the literature.
2.3 Long Term Decisions

While not the primary focus of this thesis, a brief overview of the literature on intra-household interactions with respect to long term decisions is pertinent. It should be noted that while the context of the decision is different relative to the short-term decisions discussed above, the methods (which are discussed in subsequent sections) to address these concerns are identical. Also, worth noting is that long term household decisions are typically modelled in the context of a land use transportation interaction model. In these models, there is a linkage between short and long term decisions whereby short term travel decisions influence long term decisions and vice versa. The most commonly studies decision contexts are residential location and vehicle ownership decisions at the household level each of which are briefly overviewed in turn.

Picard et al. (2013) present a model of residential location choice which explicitly accounts for heterogeneity in value of travel time between spouses within a household. Their model assumes both spouses pick their work locations independently and then make explicit tradeoffs towards jointly selecting a home location based on their own personal preference and relative weight (discussed in greater detail below). This is a substantial step forward as most household location models (e.g. Guo and Bhat (2007) or Lee and Waddell (2010)), do not explicitly consider these tradeoffs directly, or assume equal weighting for household members.

The primary medium term decision analyzed in the literature is questions regarding vehicle ownership. Zhang et al. (2009) present a latent class model for different group utility functions for understanding household vehicle ownership. Their formulation accounts for different decision strategies both between households and more directly between household members. Again, this formulation makes use of a multi linear logit model, which is inappropriate for the discrete choice context. Conversely, Roorda et al. (2008) jointly consider a vehicle transaction (both disposal and acquisition) model which explicitly links short run decisions (i.e. expected maximum utility from mode choice) with the decision to add to, remove from or maintain an existing vehicle fleet. The vehicle ownership question is also examined by the previously discussed paper of Habib (2014), who looks at the number of household vehicles owned where the expected utility from the vehicle allocation and mode choice decisions for a given number of owned vehicles are fed into the utility of the ownership decision.

While the analysis in this thesis does gloss over these topics, they represent intra-household
interactions and furthermore, represent significant contributing factors to the interactions, lending credence to examining these decisions through an integrated land use transportation model. The methods and approaches presented in both the conceptual framework (Chapter 3) and the empirical investigations (Chapters 4 through 7) could be extended into a land use transportation interaction model.

The topics of this literature review now shift away from behavioural patterns and trends of intra-household interactions and moves towards the study of methods for capturing these trends.

2.4 Implicit/Exogenous/Proxy Variable Specification

The clear majority of models of travel demand focus on the individual and not on the household as the decision making unit. It is hard to argue that these models represent a gross misspecification or misunderstanding of the behavioural process as ultimately, it is typically the individual who makes the final decision about how to travel, where to go, etc. However, it should be very clear based on the discussion above that an individual is not operating in a vacuum and is influenced by other factors, most notably other household members. More specifically, it is reasonable to assume that the individual is operating under a set of constraints that the household (among other agents) places on that individual.

To account for this in individual models of travel behaviour the typical method is the inclusion of household level variables within the individual’s utility. The most common example of this is the inclusion of the number of household vehicles in the utility of auto dependent modal options in a mode choice model (Bekhor and Shifman, 2010, Habib and Weiss, 2014, among many others). This allows for households with fewer cars to have a lower probability of driving, thereby reducing the likelihood that members of an individual household will violate vehicle availability constraints. On an aggregate level, this is sufficient to ensure that total vehicle demand (measured in terms of vehicles kilometers travelled, etc.) will be reasonable but is only a crude approximation of the actual behavioural process. More subtle applications include the inclusion of gender dummies to highlight the tendency for husbands to take precedence in vehicle allocation or the inclusion of variables such as the number of children in the household, to account for the chauffeur escort requirements necessitating driving. As with the number of household vehicles, these methods continue to suffer from the same limitations of achieving an
accurate outcome at the aggregate without capturing the underlying behavioural process driving these decisions.

2.5 Unitary Decision Making Models

As an alternative to considering the individual as the decision maker and indirectly capturing household interactions, an approach which considers the household as the decision making unit is possible. This approach defines a single household level utility without considering any of the explicit negotiation that occurs within the household to make this choice. This utility can either be defined based on household level attributes or can be treated as an unweighted additive function of individual household member’s utilities. This creates a fine line between a unitary model and a group decision model (discussed in Section 2.5). Ultimately, the nomenclature for unitary versus group decision making model is somewhat irrelevant, as techniques to induce group interactions into unitary models are common.

The most prominent example of the unitary approach is the previously mentioned and oft cited work of Wen and Koppelman (1999, 2000). Their paper examines household maintenance task generation, allocation of tasks to household members, vehicle allocation and then tour structure all considering a single household’s utility. Other papers that can be said to loosely fall into this category included the paper of Badoe (2002) for household mode choice also uses this structure, where the utility for the household mode choice pair is treated as the unweighted sum of each individual household member’s utility.

These types of models are also often known as unitary models and are broadly used in the economics literature, though a movement towards a more formal understanding of the underlying interactions between individuals which create the group’s decision has been recommended (Alderman et al. 1995). With that said, unitarily models are also significantly simpler to specify and estimate using conventional software packages. Furthermore, in the case of a democratic decision making process, whereby each group member is allocated an equal weight to the group decision, the unitary model mirrors the more complex group decision making models discussed below. Finally, unitary decision can represent either a starting off point for more in depth analysis or a medium for more endogenous methods for capturing interaction without specifying a more complex group decision making model.
2.6 Group Decision Making Models

As an alternative to the unitary decision making model, group decision making models consider that each group member contributes their own personal needs to the group’s decision. These models can be specified to assume that group members may have different relative importance, may exhibit altruism or even show a desire for an equitable solution. Within this section of decision making models, there are two main approaches, weighted utility models and endogenous interaction models, each of which are discussed in turn.

2.6.1 Weighted Utility Models:

Corfman and Gupta (1993) present an insightful chapter on group decision making which provides an overview of mathematical models of group choice, providing an excellent reference for models of group decision making. They first outline that a trivial solution does not typically exist in group decisions, as there is often conflict between the equity (everyone gets approximately the same benefit) of the outcome across all decision makers and the efficiency (the total benefit received is maximized) of the outcome. Next, they outline the work of Harsanyi (1955), who presents an additive weighted utility model for group choice when the utilities are cardinal but unknown to the decision makers. In Harsanyi’s model, the total utility of the group's decision is simply the sum of each group member’s preference, weighted according to how important they are relative to the other group members. As noted much later by Diamond (1967) and then more formally by Keeney and Kirkwood (1975), this formulation does not account for social equity across the group. Consider the following example outlined in Table 2.1 (adapted from Diamond) with two individuals with equal weights (A and B) and 3 possible choices:

<table>
<thead>
<tr>
<th>Choice Alternative</th>
<th>Group benefit = $\sum \text{utils} \times w$ (assuming equal weight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choice 1: Person A gets 10 utils and person B gets 0 utils.</td>
<td>$10 \times 0.5 + 0 \times 0.5 = 5$ utils</td>
</tr>
<tr>
<td>Choice 2: Person A gets 7 utils and person B gets 3 utils.</td>
<td>$7 \times 0.5 + 3 \times 0.5 = 5$ utils</td>
</tr>
<tr>
<td>Choice 3: Person A gets 5 utils and person B gets 5 utils.</td>
<td>$5 \times 0.5 + 5 \times 0.5 = 5$ utils</td>
</tr>
</tbody>
</table>

Here based on Harsanyi model, the group should be indifferent between the choices, however, this seems highly unfair to person B. Instead, assuming the group is equity seeking, the group should choose alternative 3 as it provides an equal outcome for all group members. This choice
problem initially comes across as quite simple is in fact complex and the model proposed by Harsanyi is not sufficient to capture the nuances of this or other group choice contexts. Some means of determining to what degree the group aims to balance equity and optimality is desirable to properly address group decision making.

These concerns have been addressed by Keeney and Kirwood (1975), who present a multi linear group utility formulation which is outlined more formally in Chapter 7 of this thesis. This formulation adds an additional set of terms to account for potential inequity. Unfortunately, the multi linear group utility function formulation is inappropriate for the discrete choice context due to theoretical inconsistencies. The explanation for this inapplicability is straightforward: the true value of the utility in the discrete choice context is not knowable (as noted in Train (2009), among others) thereby making the application of this model inappropriate. A more formal discussion of these concerns and an example outlining the deficiencies of the multi linear model for a discrete choice context is presented in the model formulation section of Chapter 7.

Despite the fundamental flaw associated with this modelling structure, the multi linear logit model has been brought into the world of travel demand modelling by Junyi Zhang and colleagues. These applications included the time use context (Zhang et al. 2002; Zhang et al. 2005; Zhang et al. 2006) and the discrete choice context (Zhang et al. 2009a, Zhang and Fujiwara, 2009). These works no longer considered cardinal, scaled utilities, as random utility models all make use of ordinal utilities. The failure to recognize the assumptions regarding the cardinality of utilities ended up making the application of these models incorrect for a random utility context. Limited applications of this model structure have been used, with Ermagun and Levinson (2016) being the notable exception. Finally, the works of Zhang and colleagues and other applications of the multi linear logit model formulation have exclusively examined the case of the multinomial logit (MNL) model, leaving nested formulations for later work (Zhang, 2009). Chapter 7 will provide an overview of the multi linear logit model formulation in the context of escort decisions and mode choice, outlining its limitations and unsuitability for discrete choice modelling.

That said, the weighted utility model without the interaction terms has been used by numerous members of the transportation demand modelling commuting as well as by practitioners and
academics within other fields within econometrics and more regularly in household economics. Browning and Chiappori (1998) present a model which is equivalent to the weighted utility model of Harsanyi, though derived under very different conditions. This explains why Harsanyi’s model holds for models of discrete choice, the multi linear logit is no longer appropriate.

The general weighted utility model is applied by Dosman and Adamowitz (2006), who present an application of this model in environmental economics. More recently, Hensher et al. (2017) apply this model, while also considering that the weighting of utilities may be alternative specific. Other sporadic applications of this model structure exist, though they are few and far between, likely because there is limited commercial software capable of estimating these model. Also, worth noting is the model of Gliebe and Koppelman (2005) for joint travel patterns, also follows this structure though with an interesting twist. Rather than simply multiplying each group member’s utility by a weight, this model structure allows for individual decisions to be made conditional on a group decision. The group decisions utility is treated as the weighted logsum term of each group member’s choice conditional on that group decision. As noted by Ben-Akiva and Lerman (1985), this logsum term represents the expected maximum utility from that choice. The logsum term is multiplied by each group member’s weight as is the case with the standard weighted utility model making this log sum model a relatively straight forward extension of the weighted utility formulation. The Gliebe and Koppelman model structure is known as the parallel constrained choice logit (PCCL) formulation and has up until this point seen limited applications practically and within the research literature. The lack of applications is unfortunate, as the PCCL has the potential to directly capture the interaction between group and individual decisions.

Alternative methods to address group dynamics in decision making have been proposed, as initially suggested by Brewer and Hensher (2000). These works use a form of stated choice experiment called an interactive agency choice experiment. This form of analysis requires an explicit data collection process whereby the choices of agents in the group are provided to other group members iteratively as a means of capturing the negotiation of group desires. This approach will then typically apply weighting factors like those of a standard weighted group decision making formulation. There are numerous studies using this methodology (mostly done
by David Hensher and colleagues) but this approach is cumbersome due to the specific data collection requirements for the analysis.

At this point it is pertinent to highlight that aside from the Gliebe and Koppelman PCCL model, weighted group decisions models typically assume a single choice made by the household and do not consider joint individual and household decisions (where the individual makes a choice based on the group choice). This suggest that the PCCL model has significant strengths for tackling complex joint group/individual decisions. Also, worth noting is that the clear majority of these studies only consider two group members, typically adults. While a select few models do expand their consideration up to more than two adults, most academic applications view the two-adult context as the most straightforward and leave generalizations of the model structure for future research or for practice ready implementations. This is generally because the generalization of these models to larger groups is computationally expensive without providing any additional research insights into group dynamics. We now quickly turn our attention to endogenous group interaction models:

2.6.2 Endogenous/Error Component Interaction Models

Aside from the weighted group utility function, other approaches have been applied throughout the literature. The two most common random utility maximizing discrete choice approaches are through the inclusion of dummy variables or through error correlation approaches. Prominent examples of each are examined in turn.

Bradley and Vovsha (2005) apply a structure for the choice of daily activity patterns at the individual level. The household’s set of daily activity patterns is the choice under consideration in their application. The estimation procedure uses an additive structure for the household utility, whereby individual’s utilities for a given daily activity pattern are added to the household utility. This model does include an interaction term whereby groups of activity patterns which have a joint activity in them are viewed as either desirable or undesirable, depending on the sign of the parameter for the interaction term. The inclusion of the interaction term brings this model structure significantly closer to a group utility model, though this resembles an exogenous or proxy input to the model designed to capture the increased utility gained from joint participation.
Scott and Kangalorou (2002) present a trivariate ordered probit model for non work activity episode generation. In this model three correlated choices are made: the number of personal independent discretionary activities for each of the two adults and the number of joint activities that occur between the household heads. By using a trivariate ordered probit, the number of each type of activities can be correlated. The error correlation is what induces the household interaction though it does not explicitly capture the behavioural process by which the decision is made (i.e. bargaining process). The model is also limited from a more practical standpoint, as it does not consider different activity types. A more robust model would decompose the model general non work activity type into a more detailed set of alternatives.

It should be noted that in both these models’ intra-household interaction is induced and not directly captured. In the case of the Bradly and Vovsha model, the interaction is captured through the addition of a dummy term for joint participation in the systematic utility. Conversely, the Scott and Kangalorou model correlates the errors of the decisions for number independent and joint activities to capture the tradeoffs between joint and individual activity participation. The dummy term method is an ad hoc approach, analogous to the implicit interactions which is discussed in Section 2.4. The error correlation approach is more robust as it allows for the joint consideration of both individual and group decisions. Also, worth noting is that the correlation of decision errors is analogous the coupla based approach which is discussed in Section 2.7. With that said, these models are something of a hybrid between the unitary model and a true weighted group decision making model.

2.7 Other Models of Household Interaction

While not explicitly relevant to this discussion, several other possible methods for examining and capturing household interaction exist. The following studies, while not a comprehensive list, illustrate several the most common methods:

1. Structural equation modelling to determine the relationship between the behaviours of household members. Examples of these approaches include Golob and McNally, (1997) and Kang and Scott (2010). These modelling approaches provide detailed insights into the relationships that influence travel behaviour outcomes at the household level. Unfortunately, the forecasting capacity of these models is nonexistent as they are used
predominately for their exploring the causal relationships between personal attributes and behavioural outcomes.

2. Seemingly unrelated regression (Srinivasan and Bhat, 2005), based models, whereby the error component for the regressions of two (or more) continuous dependent variables are correlated using a multivariate distribution. This is analogous to the trivariate ordered probit of Scott and Kangalorou (2002) but for a continuous variable rather than a discrete ordered variable.

3. Coupla based structures which induce correlation between choices across decision makers (Ermagun et al. 2015). These models use a function to inset a multivariate distribution across the errors of a set of alternatives or choices given that these errors have known marginal distributions. These methods are not used in this thesis, though they present an interesting potential avenue for future research and application for capturing intra-household interactions.

4. The application of the multiple discrete continuous extreme value (MDCEV) model for modelling a set of discrete and continuous choices, used by Bhat and Sen (2006) to estimate a model of household vehicle ownership and vehicle usage. This approach is an extension of the unitary household model, though it could be extended to include model structures such as the weighted utility structure or more generally the PCCL discussed above. More recently, Bhat et al. (2013) provide a MDCEV for joint household activity scheduling, taking advantage of the capacity of the MDCEV to consider multiple alternatives simultaneously to reduce the size of the choice set. While not addressed in this thesis directly, the MDCEV model is a powerful empirical tool that could be used to understand complex intra-household interactions.

5. Rule based approaches perform household level decisions based on a set of rules or hierarchy of decisions (Roorda et al. 2006). In their paper, rules regarding vehicle allocation are established based on simulated utilities as discussed in Section 2.1.2.

6. Simulation based approaches use a sequence of clustering and sequential simulation to allocated travel patterns to individuals within a household (Pribyl and Goulias, 2005). Alternatives to this approach include the model of Vyas et al. (2015), which uses a simulation based framework based on marginal models of joint travel participation for large households.
Although these methods are not explicitly important to the subsequent empirical investigation, they do provide a set of alternative approaches to the unitary and group decision making models discussed above. In many cases, the application of these model structures is appropriate for either behavioural reasons as could be the case for the MDCEV model or for computational tractability for operational models as is the case for the model of Roorda et al. (2006) or Pribyl and Goulias (2005). The MDCEV model has several interesting possibilities for linking the advancements in discrete choices made with respect to intra-household interaction with the advances in time use allocation. The work of Bhat et al. (2013) is a very strong step in the right direction, however further research on potential applications of this modelling structure is required.

This marks the end of the discussion on intra-household interaction. The next two sections will now focus on issues regarding spatial constraints and spatial interactions.

### 2.8 Spatial Constraints in Models of Travel Demand

The concept of proposing spatial constraints to models of travel demand is built off the space time prism concept of Hägerstrand (1977). Space time prisms are methods for tracking an individual’s movement through space and time. For an assumed time budget and a given spatial location (or locations if the desired end point is different from the start point), it is then possible to constrain the feasible locations an individual can travel to over a period of time. This structure is used to great effect within the FAMOS modelling structure (Pendyala et al. 2005) though specifically within the PCATS sub module within this structure. Specifically, Kitamura et al. (2000a, 2000b) apply a stochastic frontier model to estimate the start and end times of morning and evening travel. If work start and end times are given (or assumed), this allows the analyst to form reasonable space time prisms for the commute to and from work. Although nomenclature within the literature is space time prism, a space time ellipse is a more accurate geometric representation of the process. This is because the ellipse as it provides a better visual representation of the viable locations that can be travelled to for a given time budget (assuming uniform travel time in all directions) where the start and end points for the space time prism are the loci of the ellipse. A visual depiction of this visual representation is found in Figure 2.1.
The use of space time prisms provides increased realism in choice set generation but also has the added benefit of reducing the choice set of spatial choices to a more manageable size. More generally, Ortuzar and Willumsen (2011) define three basic approaches for choice set determination: deterministic rules (which would include the stochastic frontier approach), explicit elicitation of the consideration set during data collection and random sampling. Rashidi et al. (2012) more recently present a method for creating what could be called a stochastic rule-based approach. In their work, they jointly estimate a model for maximum commuting distance and residential location choice, using a hazard duration model to predict the frontier beyond which an individual would be unwilling to locate. Also of interest is the approach of Lemp and Kockelman (2012), who provide an interesting iterative approach to obtain more realistic choice set structure by using the previous iterations model to define sampling rates for the subsequent iteration. Over a series of iterations, the likelihood of achieving the true choice set improves drastically, mimicking a deterministic approach.

One of the very few studies that examined spatial and household interactions jointly is that of Ettema et al. (2007). In their paper, the authors examine the impact of locations on time scheduling using a model of time allocation. While their discussion does examine on general task allocation (through time allocation to household tasks), their representation of these trends is

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**Figure 2.1 Visual Representation of Space Time “Prisms” as an Ellipse**
challenging as escort task have minimal actual time allocated to them, meaning that a more comprehensive method of analysis is likely required.

2.9 General Spatial Interactions

General spatial interactions move beyond the impact of locations on travel patterns towards a more comprehensive examination of the process by which spatial locations are selected. The main behavioural process of interest here is the concept that spatial alternatives may be correlated with each other, suggesting that simple modelling structures like the multinomial logit are not appropriate. This concern stems from the first law of geography initially postulated by Tobler (1970, pp 234):

“everything is related to everything else but near things are more related than distant things”.

The initial interpretation of this law is that outcomes originating from proximate locations will likely be correlated, whereas outcomes originating from disparate locations will be less correlated. As an example, in the context of trip generation models, the number of trips coming from two proximate zones is hypothesized to be correlated. Other potential examinations of this sort relate to decisions from proximate zones. This approach typically uses a spatial autoregressive error or spatial lag approach initially proposed by Anselin (1988).

In the context of choosing between spatial choices, this trend has a similar, though subtly different interpretation: alternatives which are close together are more correlated and will act as strong substitutes for each other, whereas disparate alternatives will be poor substitutes for each other. A visual depiction of this trend is represented in Figure 2.2.
Figure 2.2 Graphical Representation of the Trend of Tobler’s Law

Numerous modelling techniques and approaches have been attempted over the years to capture this fundamental issue. Boluc et al. (1997) propose a multinomial probit for doctors’ choice of first practice location in Quebec. The proposed model suggests proximate alternatives can be correlated using a SAR process analogous to the Anselin (1988) approach. This approach proved promising relative to an uncorrelated IID probit in terms of policy analysis. Subsequent works, such as that proposed by Miyamoto et al. (2004) expand on this proposed method by utilizing a SAR approach within the systematic portion of the utility and the error component within a mixed logit structure. Traditional spatial autoregressive models within the realm of discrete choice aim to capture correlation between proximate decision makers (see Sidharthan et al. 2011 for an example). Instead the applications of Bolduc et al. (1997) and Miyamoto et al. (2004) aim to capture the correlation between alternatives. Unfortunately, these spatial autocorrelation approaches require complex matrix manipulation to achieve the covariance matrix of the alternatives. If the choice set of each decision maker is unique, then the matrix manipulation must be performed for each decision maker separately, though this is a much simpler process for generic choice sets. In the cases of Bolduc et al. (1997) and Miyamoto et al. (2004) the spatial choice set is the same across all decision makers, making the application of the SAR approach appropriate. On top of concerns regarding individual specific choice sets, SAR based models
also require simulation based estimation approaches, making them doubly computationally expensive. This makes testing alternative model structures very challenging to accomplish quickly, particularly for large decision maker specific choice sets. These limitations suggest an alternative means of addressing these issues should have significant value.

The alternative non-probit/mixed approaches that can be used to address the issue of IIA are generalized extreme value (GEV) models. Bhat and Guo (2004) present a mixed spatially correlated logit (MSCL) model that combines the mixed and GEV approaches. The MSCL accounts for random taste variation across individuals through the mixed component of the model while capturing spatial correlation using a closed form GEV structure. The main limitation with this approach is that spatial correlation is only captured through contiguous dummy variables at the traffic analysis zone level. This suggests that alternatives that do not share a contiguous border are not correlated, a fundamentally incorrect (though computationally simplifying) assumption. Furthermore, given that some spatial choice contexts do not allow for the alternatives to share any borders an MSCL model is not appropriate in these contexts. Spatially distinct alternatives such as schools or transit stations are typically dispersed such that it is unlikely that any two stations would be contiguous at the traffic analysis zonal level, thereby suggesting that an alternative method for capturing the spatial correlation between these types of alternatives is required. A more recent study by Sener et al. (2011) expands on the MSCL model, presenting a generalized version of the model, the GSCL, which allows the correlation to be extended across all possible zone pairs, relaxing the simplifying assumption discussed above. This approach is a highly flexible and general method for addressing the correlation between spatial alternatives but it is still shackled by the limited associated with the homoskedastic nature of the correlated error components. As will become clear in subsequent sections, this is a potential limitation of the GSCL, particularly if there is a given spatial reference point for the decision maker. The importance of a hetero versus homoskedastic error component for spatial choices is an interesting avenue for future research.

2.10 Chapter Conclusions

This chapter provides a comprehensive overview of models of intra-household interaction, spatial interaction and spatial constraint within models of discrete choice with a specific focus on applications of these models for travel demand. This review has highlighted several gaps, which
this thesis aims to address through the empirical investigations in Chapters 4 through 7 respectively. In summary, these gaps consist of the following points:

1. There is no comprehensive econometric framework for jointly considering the household vehicle allocation process and joint versus independent travel choices at the household level. Instead, these issues have been addressed in turn/separately and then included in comprehensive activity based modelling frameworks. Model structures which address these concerns jointly have not yet been developed. A choice modelling framework to address this concern is presented in Chapter 4. The empirical investigation in this chapter also highlights known differences between error component mixed logit and nested logit models.

2. Models capturing the correlation between spatial alternatives in location choice models have been proposed but these models are homoskedastic. This creates challenges associated with understanding the true pattern of behaviour in respect to spatial choices where the alternatives are heteroskedastic. A new choice model formulation for spatial choices which addresses spatial correlation and spatial heteroskedasticy is presented in Chapter 5.

3. Task allocation at the household level is a well-researched subject though there are underlying spatial implications of these decisions that are poorly understood. Other constraints such at the temporal level have been briefly discussed by Gupta and Vovsha (2013) who found evidence suggesting that some degree of coordination of evening home arrival time occurs between household members. This finding suggests that spatial constraints do play a role in this decision process. This argument is further supported by Vuk et al. (2016), who note the importance of time spent at home for all household members. These findings suggest that constraints at both a spatial and temporal level play a key role in determining household interaction. Given the focus in the existing literature on temporal constraints and coordination, Chapter 6 presents a method for including spatial constraints when modelling the household tasks allocation decision process. The model presented in Chapter 6 also uses a method for reducing the consideration set for spatial choices. This method has the potential to greatly increase the accuracy of location choice models.
4. Task allocation and joint household mode choice models has been identified as an interesting avenue for future analysis by Gupta et al. (2014). The parallel constrained choice logit of Gliebe and Koppelman (2005) is adopted and applied to model these decisions jointly in the empirical investigation of Chapter 7. This chapter also identifies a theoretical inconsistency with the multi linear logit of Zhang et al. (2009a).
Chapter 3
Conceptual Framework

As noted in the introduction and the literature review, there is significant evidence to support that a conventional travel demand modelling structure is limited by its aggregate and trip based nature. Unfortunately, more advanced modelling frameworks have their own set of challenges relating to application complexity, leading to tradeoffs between the two approaches. This chapter provides a conceptual framework for dealing with a subset of the limitations of the conventional modelling framework through the introduction of more complex modelling techniques. These techniques shift the perspective of model away from the individual as an independent entity. An overview of the specific empirical applications presented in this thesis, is provided, addressing how these fit into an activity based framework. Finally, the more general themes that are developed through this thesis are outlined.

3.1 Travel Demand Modelling Background

There is a common aphorism used in statistics and econometrics attributed to Professor George Box (1979):

“all models are wrong but some are useful”.

This aphorism can be interpreted to say that models are approximations of the true processes which they are developed to represent. Some models are better representation of some portion of the process in question, whereas others can be viewed as very crude approximations that provide little to no value over no information at all. This raises several very important questions when dealing with models:

- What is the process that the analyst is trying to represent?
- How accurately must the process be represented?
- What are the consequences for misrepresenting this process?
In the context of models of travel demand, the processes that are being represented are the factors which induce travel behaviour. The aspects of travel behaviour that are of concern are left to the discretion of the modeller performing the analysis.

The early history of models of travel demand coincided with post war auto centric development and as such the main goals of these forms of analysis were improving the automobile output (in terms of VKT, average speeds, etc.) of the roads in the transportation network. The needs and interests of travel demand have evolved over time to the present, where a more holistic and nuanced method of analysis is now needed. This has prompted the movement away from the trip based format towards a more activity based framework. This adoption of a holistic approach has come at a cost of more complex and computationally expensive modelling structures with more detailed data requirements. These tradeoffs are summarized in Figure 3.1.

Figure 3.1 Model Outputs vs. Complexity

In many respects, the development of more complex models of behaviour simply consists of considering more aspects of the behaviour. In conventional trip based models, the focus is on four principal questions: is a trip made, where is the trip going, by what mode is the trip made and by what route does the trip maker travel? Activity-based models attempt to answer these questions but may also add further complications. Behaviours including the number of trips performed by an individual, the sequence of scheduling of these trips, when each of these trips
start, joint travel and activity participation, etc. can all be considered along with the four primary questions of trip based approaches. Furthermore, activity based models aim to understand the underlying motivation for making this set of connected trips, which adds further nuances and complexity to the data requirements and the underlying model structures used to predict this behaviour. The returns on investing additional time and resources into the development of these complex models are behaviourally rich and policy sensitive model outputs which provide decision makers with a more complete picture of the behavioural processes in question.

While adding in more behavioural processes into the modelling structure is one way of achieving higher quality outputs, another is through the increase in the complexity of the model structures themselves. While complex model structures may or may not require any additional data relative to their basic counterparts, the formulation and application of these models are typically more time consuming relative to simpler modelling structures. This implies a tradeoff as these complex models are often capable of capturing more nuanced behavioural patterns that are lost within simple modelling structures. Furthermore, if the simple model structure is unable to capture the actual behavioural pattern in question, the model may provide inaccurate predictions. When these predictions are used to inform policy or infrastructure investment decisions, the biased model is worse than useless as it may be harmful to the overall prosperity of an urban region.

An example of this comes from the well-known red bus blue bus choice context for commuting mode choice. Consider an individual selecting between driving to work and taking a red bus service. Both options are assumed to be equally attractive, which suggests that the individual has an equal probability of selecting either of them (50%). Consider then that a new blue bus service is introduced that is, for the purposes of this hypothetical example, identical to the red bus. The expected pattern of behaviour is that the probability of driving would remain constant (50%) and the probability of taking either the red bus or the blue bus would be equal (25%). If this scenario is modelled using an MNL model, this expected pattern of behaviour would not be predicted. Instead because of the IIA property, the model would predict equal modal probability (33%) for all three modes. If the blue bus is a publicly funded infrastructure project designed to reduce single occupancy vehicle travel, then the use of the MNL would drastically over predict the cost
effectiveness of the blue bus infrastructure investment. This represents a waste of the resources of the urban region on an ineffective policy.

To obtain a more clear and useful understanding of the behavioural process, it is not sufficient to simply apply either joint decisions or advanced modelling structures independent of each other – they must be applied in conjunction. This is not a new argument. There has been simultaneous development of advances in modelling methodology as well as activity based modelling framework over the last 50 years. This thesis aims to make contributions to both fronts as they are equally and symbiotically relevant and important to the advancement of the field of travel demand modelling.

3.2 Framework for Model Implementation

In most applications of econometrically constructed activity based models, the models are implemented using a modular method whereby models for specific decisions are applied sequentially and in some cases iteratively. The wide spread adoption and extensible nature of modular model components makes modular models attractive due to their generality. As a result, the empirical models presented in this thesis follow this modular framework. This section presents a practical discussion of the process that could be used to implement any set of modular components within a broader activity based framework. It should be stressed that the main research objective of this thesis is not the implementation of behavioural models but instead the development of the models or modelling structures themselves. As a result, while the techniques presented in this chapter are used to inform the research done in this thesis, the implementation of this framework fell outside of the scope of the research contribution.

Aside from the modular nature of most activity based models, the clear majority of models of travel demand make use of a top down approach as seen in the CEMDAP model (Bhat et al. 2004) or the CT-Ramp family of models (Paleti et al. 2017). This top down approach defines the overall structure of the daily activity pattern (defined as the number and structure of tours performed throughout the day) before fleshing out the details of these trips (locations, secondary activity purposes, etc.). This top down approach has led to the argument that these models are simply “tour based” rather than trip based, where the unit of demand is still travel, rather than the underlying activity that is occurring.
Alternative structures attempt to integrate many of the decisions into a single unified framework. Models which take this unified approach include the CUSTOM framework described by Habib and Hui (2017), the integrated framework of Pinjari et al. (2014) or the MDCEV-SimAGENT approach (Bhat et al. 2013, Goulias et al. 2012) This bottom up approach models the activities first and then subsequently links the activities to travel patterns. The main concern with this approach is that as the number of activity details increases, the size of the choice set becomes exponentially larger, which leads to longer estimation times.

Even at the level of a single complex decision such as the school escort (drop-off and pick-up) decisions discussed in Gupta et al. (2014), the number of possible alternatives can explode quite quickly. Gupta et al. note that for even for what can be considered reasonable household structures, the number of potential drop-off and pick-up escort alternatives becomes infeasibly large. This becomes problematic as the computational time/power required to estimate and run models with large choice sets can make practical applications of these model infeasible.

Furthermore, models which consider this number of alternatives run the risk of having biased model results due to “choice-set misspecification”. Choice set misspecification occurs when the choice set the analyst defines does not match what the decision maker considers. When alternatives that are not actually considered by the decision maker are included in the decision maker’s choice set, the decision maker will have a non-zero probability of selecting this unconsidered alternative. This introduces bias because the decision maker does not consider this alternative in reality, meaning it actually has a zero probability of being selected. Alternatively, when alternatives which are actually considered by the decision maker are excluded from the choice set the same problem occurs, but in reverse. Choice set misspecification is a challenge for models with large choice sets as it is incorrect to assume that a decision maker considers all possible alternatives considered in the model. This challenge with large choice sets is even more prevalent when considering multi person households. An additional person exponentially increases the number of alternatives to be considered as all possible alternative pairs must now be fully considered. Unfortunately, some degree of misspecification is unavoidable due to the analyst not ever fully knowing what is considered by the decision maker unless the choice set is elicited from the respondent during data collection. As explicit elicitation of the choice set is not
a common practice, efforts should be made to specify a “reasonable” choice set at the discretion of the analyst. Practical techniques to obtain reasonable choice sets in the context of spatial locations are discussed in Chapter 2, section 7 and in the subsequent set of subsections.

3.2.1 Tricks for Reduction in Computational Burden

Numerous techniques have been suggested within the literature and are discussed here for the purposes of understanding how modular models can be integrated within a comprehensive demand modelling framework.

3.2.1.1 Choice Set Simplification Through Sampling

As noted by McFadden in his seminal paper on household location choice (1978), the multinomial logit (MNL) model exhibits a positive conditioning property, whereby this model can be estimated on a sample of alternatives from the full choice set. To obtain unbiased parameter estimates, McFadden proves that a simple correction can be added to the utility of each alternative. By applying a sampling based approach, it is hypothetically possible to reduce the computational intensity of the size of the choice set, thereby reducing the estimation time and the simulation time associated with the model structure. Note that this approach is only valid if the individual is considering all possible options in the universal choice set (the set of all possibly considered alternatives) due to the same issues outlined above.

This approach has been employed by Habib and Hui (2017), among others with a reasonable amount of success for location choice modelling, although it requires relatively complex corrections to the utility structure if the identical and independent error term assumption of the MNL model is relaxed (see Chapter 6 for more details on this correction). If more complex modelling structures are used, then the correction becomes even more complicated. Alternatives to the pure sampling approach could involve analyst imposed (i.e. deterministic) rules and restrictions or a more intelligent approach such as those outlined in Section 2.7 of this thesis. The work in Chapter 6 of this thesis examines the possibility of using stochastic individual specific rules through the application of stochastic frontier models for the context of location choice.

3.2.1.2 Modular/Linear Application of Models

As noted above, the many behavioural models follow a modular framework, whereby the order in which decisions are made is assumed \textit{a priori}. This allows models of individual behaviour to
be estimated for a given set of observed behavioural inputs. An easy example of this is the question of household task generation and the allocation of those tasks to household members and then the allocation of the household’s vehicle to a household member. In a modular application structure, the number of household tasks is generated first for a given household structure. Then, once these tasks have been generated for the household, they are allocated to a given individual. Finally, based on the allocations of tasks, the vehicle is then allocated to a household member. The utility for each vehicle allocation alternative would take as an input the number of tasks that the individual in question is given based on the previous choice. This is a sequential process analogous to what is proposed by Wen and Koppelman (1999, 2000), though they estimate sections of this process jointly where the size of the choice set is reasonable. A visual depiction of this process is outlined in Figure 3.2

![Sequential Process for Task and Vehicle Allocation](image)

**Figure 3.2 Sequential Process for Task and Vehicle Allocation**

Using this sequential approach, a model predicting the number of household tasks is estimated based on a dataset of observed household tasks. Next, using the same or a similar dataset, a model predicting the allocation of tasks to household members is estimated. Separate allocation models for the number of tasks (i.e. a one-task allocation model, a two-task allocation, etc.) can be estimated or a generic allocation model for all possible numbers of household tasks is estimated. Finally, a model of vehicle allocation is estimated, again using the vehicle allocation values observed in a dataset. Each of these models will be estimated using a set of sociodemographic variables, which may include the observed behaviours that other models in the
sequence are predicting. For example, the observed task allocation at the household level can be treated as an input into the vehicle allocation model (an individual with more tasks allocated to them may more likely to be allocated the vehicle). Alternatively, the number of tasks generated at the household level will directly influence the number of tasks that need to be allocated.

To apply the three models, a simulation based approach is used. with the number of generated tasks being fed into the allocation model and the allocation of tasks being fed into the vehicle allocation model.

The primary criticism of this approach, whereby the allocation of the vehicle is done after the tasks are allocated, is that a substantial assumption made by the modeller. It is possible that for this household the vehicle allocation occurs first, followed by the allocation of tasks, though what is more likely is that the decision occurs jointly. This can be accomplished by estimating this process jointly using a combined choice set method, whereby a single model considering all 3 dimensions of the choice is estimated (the approach of Wen and Koppelman (1999, 2000)). As noted above, this process provides a detailed representation of the choice process, relaxing the assumed sequential nature, at the cost of estimation and simulation time due to the size of the choice set. In this simple example, it is reasonable to estimate this process jointly, though as more and more decisions are added onto these, the time to estimate and apply the model grows. The analyst must also be conscious of potential choice set misspecification problems that arise from including too many alternatives in each decision maker’s choice set.

3.2.1.3 Gibbs Sampling Approach

To obtain a computationally tractable modelling structure Gupta et al. (2014) propose a modular approach. In their case, the morning period mode and escort choice for a student is treated as an input for the evening period model for the same choices. Conversely, the evening period choice is treated as an input during the estimation of the morning period choice. For application of the model a Gibbs sampler method is employed, where the morning and evening period models are applied using Monte Carlo simulation iteratively: where the output from one is treated as the input for the other. This approach is repeated iteratively until the aggregate mode shares are relatively stable between iterations. For the purposes of the example provided above regarding vehicle and task allocation, a similar approach could be employed, as is outlined in Figure 3.3.
Initially, three separate models are estimated for task generation, task allocation and vehicle allocation. These models will take (where applicable) observed outcomes of the other decisions as variables in the utility. For example, the vehicle allocation model will take the observed number of household tasks allocated to an individual as a variable, influencing the propensity for a vehicle to be allocated to that individual. To apply the model, the process in Figure 3.3 is used, whereby a Monte Carlo simulation algorithm is employed. The simulation algorithm uses an iterative process. To start the simulation, outcomes from the vehicle allocation model are assumed and used as inputs into the task generation model. The simulated task generation output is then used as an input into the task allocation model. The output of the task allocation model is then used to simulate a new outcome for the vehicle allocation. This framework can be applied to any set of $n$ interrelated decisions, where $n$ is an integer greater than two.

This process is repeated over numerous iterations for all members of the sample/forecasting population until the aggregate shares of each decision reach a state of relative equilibrium. More generally, it would be possible to move beyond a binary or three level Gibbs samplers to a more robust model structure. This simulation framework would be applied to an entire set of behavioural models, capturing all aspects of the daily travel behaviour. While not performed within this thesis, the application of this type of model structure following a model formulation like that proposed by Bhat et al. (2004) for his CEMDAP formulation could be a very interesting avenue for future research. Ideally, this implementation would adapt the empirical models developed in this thesis as structures included within the framework. While some adaptation/minor re-estimation would be required, this framework is the recommended method of integrating the independent empirical investigations developed in this thesis within a comprehensive activity based model. It should again be stressed that this thesis does not perform this explicit integration. This section of the thesis is only proposing a method for integrating the presented models within a broader framework of travel demand.
While a modular approach is desirable for computational tractability, the joint approach provides increased behavioural realism that may be lost in the simulation of joint decisions. As a result, a hybrid between a modular and joint model system strikes a balance between realism and practicality. Using this hybrid joint-modular approach, certain individual decisions could be modelled jointly, while treating all other decisions, including household level decisions as endogenous inputs to be estimated using separate model structures. Effectively a joint model would act as an independent or modular decision in the Gibbs Sampling framework.

A possible example for this is the CUSTOM framework proposed by Habib and Hui (2017). The CUSTOM model is a unique formulation of the MDCEV structure. CUSTOM is an individual activity generation and scheduling model which predicts the duration, location and number of activities that are performed through the course of a day. The current CUSTOM framework performs the complete activity schedule over the course of an entire day without considering the implications of intra-household interactions. These interactions can be inserted directly into the CUSTOM framework either by using the sequential estimation approach (either before or after
the individual behaviours have been simulated from the CUSTOM model) or preferably with a Gibbs Sampler type approach where the simulated behavioural outputs are used as endogenous inputs.

The process of integrating the task and vehicle allocation process above would make use of the following procedure:

1. Estimate all models (CUSTOM and task allocation) independently. When estimating the CUSTOM model the choice set will be constrained as a method to ensure that the individual who is observed to be allocated a task performed that task. For example, if an individual is observed performing a daycare pick up task, that individual must perform the daycare pickup task before they return home permanently for the day. Other constraints such as drop-off and pick-up times could also potentially be added to this structure. Alternatively, a simpler approach would be the standard exogenous dummy variable included in the utility for household activity type. The downside of the exogenous dummy variable approach is that it would only ensure behavioural realism at an aggregate level rather than truly capturing the behavioural constraint. This is because the exogenous dummy variable for task allocation only implies a higher likelihood that the individual will perform the task rather than the individual being constrained to do so by the model structure. Conversely the task allocation model would include such variables as the observed number of trips or whether any trips were in the central business district. This specification allows the Gibbs sampler to jointly simulate between the results of CUSTOM.

2. Simulate for forecasting purposes, following the same structure outlined in Figure 3.3. The starting values are assumed (either based on averages in the population or zeros to start) and the results from one of the models are obtained using Monte Carlo simulation. These results are then fed into the other model structure and new values are simulated. This process repeats itself until aggregate convergence is reached.

The main differences between this more complex process and the much simpler process discussed in Figure 3.3 are the complexity of the models and how the integration occurs if the choice set restriction approach is used. Aside from these two differences, this approach is very similar to the standard Gibbs Sampling process proposed in Figure 3.3.
3.2.1.4 Other approaches

Bhat et al. (2013), note that the MDCEV model has the capacity to consider the choice of multiple alternatives without having to enumerate through all possible combinations, which makes the choice set for joint estimation of different choice constructs significantly faster. The main concern with the MDCEV is that there is limited commercial software to estimate these types of modelling and their estimation is complex. While theoretically this is not a constraint, applications of this model structure have been limited to academic settings. That said, the CUSTOM framework of Habib and Hui (2017) also follows an MDCEV structure but does not explicitly take advantage of these strengths of the MDCEV as applied by Bhat et al. (2013). This is because the model proposed by Bhat et al. is only used for time allocation to activities whereas the CUSTOM framework is a joint individual activity generation and scheduling model. Because the CUSTOM can perform scheduling, along with time allocation, the order in which the decisions occur is also predicted. In the model proposed by Bhat et al. the order is not predicted, thereby allowing multiple discrete alternatives to be predicted simultaneously (rather than sequentially).

Alternatively, many studies use a brute force method of joint estimation in cases where the choice set is still reasonable. Joint choices can easily exceed one hundred options, though models with up to several hundred alternatives are routinely estimated and applied. These models are not necessarily realistic as individuals do not typically consider any more than five or ten alternatives at a given time. Because many of the alternatives are not actually considered in models with large choice sets, these models will inevitably provide somewhat biased estimation results.

3.2.2 Implementation Overview

The processes discussed above are not explicitly implemented within this thesis but instead act as a roadmap for the future implementation of the presented models. It is important to stress that the models presented in this thesis are methods and means to address limitations associated with existing practices as discussed in the literature review. The subsequent empirical investigations are also somewhat methodological in nature, meaning that more focus is placed on their development relative to their potential applications for policy analysis. The next four chapters of this thesis provide modular empirical works that provide detailed insights into the specific behavioural patterns of household and spatial interaction. The empirical models in this thesis can
be implemented within a broader activity based framework as discussed above (sequentially or with Gibbs sampling) with some minor adjustments to fit the context of the model with which they are being implemented. Furthermore, these models can be adapted and adjusted as standalone models for understanding and addressing the implications of intra-household interactions, spatial interaction and/or spatial constraints on individual and household travel behaviour.

The diagram in Figure 3.3 outlines the points of implementation within a traditional activity based modelling structure.

**Figure 3.4 How Analytical Contributions of the Thesis Fit in an Activity Based Framework**

As will become clear when examining the specific chapters within this thesis, many of the models that are presented make use of a trip based framework as opposed to “tour based” structures that account for a sequence of trips made by the same individual. This simplification is done for several reasons, most notably that the extension to tour based models is relatively straightforward. Existing models, such as those of Ho and Mulley (2015a) have estimated a single mode for a given tour structure and it could be argued that their model is analogous to the trip based approach used within this thesis. A more robust tour based approach would involve
estimating the outward segment and the inward segment of a tour (as is proposed by Gupta et al. 2014 and more recently by Hasnine and Habib 2017). These models can either be estimated jointly or separately as discussed above. While this would have been possible in the context of this thesis, to reduce the data processing requirements and estimation complexity, a trip based view is used. The trip based models in this thesis do incorporate trip chaining behaviour, considering linked trips by the same mode as a single trip. As such, it is reasonable to label these as complex trips relative to what is done in conventional modelling approaches (e.g. Bekhor and Shiftan, 2010, Habib and Weiss, 2014, among many others). This simplification does not detract from the validity or the insights of the proposed empirical investigations and in fact provides valuable methodological additions to agencies and planning authorities who continue to use trip based approaches.

3.3 Overview of Empirical Investigations

As noted in Figure 3.4, each chapter/empirical investigation in this thesis touches on two aspects of a generic activity based modelling framework. This section provides a brief introduction to each chapter with respect to how each empirical investigation feeds into an activity based modelling framework. It should be noted that chapters are presented in an order that is temporally linear according to when the empirical investigation was performed. For the reader, this means that there is a logical progression in terms of topic but also an increase in methodological complexity. This is particularly relevant when comparing Chapters 4 and 7, which examine very similar behavioural processes using different approaches. The approach applied in Chapter 7 exhibits considerably more modelling complexity than the approach applied in Chapter 4. This is not to discredit the work of Chapter 4 as there is often a desire to find balance between model complexity and behavioural realism. The simple parsimonious solution to a problem can be viewed as more desirable. This is particularly true when examining applications outside of academia.

The reader is reminded that these empirical applications represent case studies and while there are obvious linkages between chapters, they are independent works. These chapters represent both the author’s knowledge about intra-household and spatial interaction, as well as the evolution of the author as a researcher. It should also be noted that these empirical investigations attempt to strike a balance between methodological contribution and practical applications for
policy analysis. That said, the discussions in this thesis do focus more heavily on the methods themselves rather than the potential for applying these presented models. A brief discussion of how each chapter fits into an activity based modelling framework follows.

3.3.1 Household Mode Choice
The empirical investigation in this thesis starts with an attempt to address the interaction between task and resource allocation. This chapter focuses on the allocation of vehicles to individuals in a household and the task of escorting another travelling adult if they are allocated this vehicle. Given the task and resource allocation, the choice of modes for both adults are modelled jointly and simultaneously. This empirical investigation is presented in Chapter 4.

3.3.2 Park and Ride vs. Kiss and Ride Station Location Choice
The subsequent empirical investigation in this thesis builds on the mode choice analysis in the previous section, examining a subcomponent of a specific type of joint trip: the transit station location choice for transit drop-off or kiss and ride. This analysis examines how the station location choice differs from the traditional transit drive access or park and ride trips in terms of incorporating spatial interactions and intra-household interactions. This empirical investigation is presented in Chapter 5.

3.3.3 Daycare Location and Drop Off/Pick Up Allocation Choice
The next empirical investigation looks at more complex task allocations relative to joint travel between adult household members: escorting tasks for dependents. Specifically, this application looks at the daycare location and the allocation of escort duties for children who use daycare or preschool. The model structure uses a two-stage process whereby constraints on the location of the daycare for a given escort decision are applied followed by the choice of chauffeur (for drop-off and pick-up tasks) using the predefined choice sets for each allocation. This empirical investigation is presented in Chapter 6.

3.3.4 Household Mode and Student Escort Choice
The final empirical investigation aims to generalized issues surrounding mode choice and task allocation, providing a foil for the investigation presented in Chapter 4. This application examines school travel mode choice for high school students, commuting travel mode choice for
household adults and the escort decisions (which adult, if any, escorts the student) jointly. This empirical investigation is presented in Chapter 7.

3.4 Overarching Themes

Over the course of the development of the empirical models in the subsequent chapters, several overarching themes became apparent. These themes do not relate directly to issues surrounding spatial or household interactions but nevertheless present interesting general conclusions regarding the application of models of travel demand broadly.

3.4.1 Importance of the Choice Set

Models of discrete choice are framed as a decision maker selecting a single option from a bundle of distinct alternatives, where the bundle has at least two alternatives. This set of alternatives is defined as the choice set for the analysis. It is of utmost importance that the choice set for a given choice context is defined properly as either the exclusion of a considered but unselected alternative or the inclusion of an unconsidered alternative leads to biased estimates of the parameters for the choice model. This is a form of model misspecification, which is prevalent and often overlooked in modelling frameworks. Furthermore, the specification of the choice set is an important tool for several other reasons more directly related to the focus of this thesis. Namely, it is capable of capturing household interactions and constraints as seen in Chapter 4, spatial interactions as seen in Chapter 5 and spatial constraints as seen in Chapter 6. These lessons are not limited to issues surrounding spatial or household interactions. These findings act as encouragement to other demand modellers to consider solving issues surrounding capturing complex behaviour through novel choice set specification.

3.4.2 Importance of Considering Multiple Degrees of Interaction

The empirical investigations in this thesis are focused on interactions at either the household or spatial level, though more generally, each empirical interaction also includes some component of interaction between different aspects of the activity based framework. While this thesis proposes a modular method for incorporating these interactions within models of travel demand, there is always merit to endogenously incorporating the joint and interactive nature of these decisions within the modelling framework. The nuanced interaction patterns are challenging if not impossible to capture using the sequential or simulation-based approaches discussed in Chapter 3, Section 2. As a rule, a balance must be struck between increasing the behavioural detail and
having tractable and usable models, where the degree of balance is subject to the computational powers available to the analyst and the application of the empirical investigation in question.

### 3.4.3 Importance of Using Advanced Models

As an additional point to the discussion regarding joint modelling, the increased behavioural detail associated with advanced models is also of importance. The traditional multinomial logit (MNL) model is highly limited and simplistic in its ability to capture nuanced behavioural patterns associated with making choices. This limitation stems from the well documented independence of irrelevant alternatives (IIA) property of the MNL. This property simply states that the ratio of probabilities between two alternatives remains constant even when the characteristics of a third option are changed. This implies that all alternatives are uncorrelated which is an unrealistic assumption when dealing with similar alternatives. A practical example of the limitations of this property is illustrated in the well-known red bus blue bus problem outlined in numerous econometric/discrete choice texts and discussed earlier in this chapter. To account for this, models which circumvent this assumption by either adding correlation or heteroskedasticity to the error terms of each alternative are introduced. This relaxes the identical (in variance) and independent error distribution property of the MNL, which is the foundation for the IIA property.

To accomplish this task using discrete choice models, two main methods are typically employed either independently or together. The first is from the family of generalized extreme value (GEV) models (of which the MNL is a special case), which can allow the analyst to capture correlation patterns between alternatives. Within the GEV family, the nested logit formulation is the most popular. This structure groups common alternatives into nests. Alternatives within these nests do exhibit IIA, though alternatives not in the same nest now exhibit non-proportional substitution patterns. This means that an improvement in an alternative will draw proportionally more from nested alternatives than from non-nested alternatives. This approach can be extended into other structures, such as cross nested logits where alternatives belong to multiple nests. This approach has the benefit of maintaining a closed form likelihood function, which allows for standard maximum likelihood estimation techniques to be used.
Conversely, the other commonly used approach is the mixed logit formulation. This family of approaches incorporates a multivariate normal distribution into the standard MNL structure by adding an additional normally distributed error component to the utility. This creates a change in the derivation of the choice probability as the distribution for the difference in utility no longer follows a closed form. This means that simulation based approaches, such as those discussed in Train (Chapter 6 and Chapter 9, 2009) must be used. While not elaborated on here, this approach is known as maximum simulated likelihood. The mixed logit model can approximate the substation patterns of GEV type models but also has the added characteristic of introducing heteroskedasticity, meaning the variance of the alternatives is no longer uniform. As will be discussed in Chapters 4 and 5, this can lead to improved results relative to the conventional GEV approach.

### 3.4.4 Importance of Model Identification

Another key concept that is important to gain from the reading of this thesis is the importance of understanding the method of analysis that is being used. As will be noted in Chapters 4 and 5 and to a much greater extent in Chapter 7, the application of models without fully understanding the underlying reason for their application can cause the analyst to draw misleading conclusions. George Box’s famous aphorism about the correctness and usefulness of models is very important to remember in these contexts. Models which are not derived using fundamental economic/statistical principals or models which are not correctly identified can create large problems. Keeping in mind that these models are ultimately being used at the very least to gain insights about travel behaviour and more and more are being used to inform large scale infrastructure decisions is important.

### 3.5 Chapter Conclusions

This chapter presents a broad overview of the integration of the empirical works of this thesis within a broader travel demand modelling framework. The development of the subsequent empirical applications was guided by preliminary thinking on this framework. At this point, this thesis moves from background information on the subject matter towards a more empirical and analytically focused discussion of a specific set of behavioural processes.
Chapter 4
Household Mode Choice

This chapter presents an innovative two adult household mode choice model, which aims to capture the implicit tradeoffs made by individuals and the household. Tradeoffs with respect to vehicle allocation and joint travel opportunities are examined. This understanding is achieved through the examination of the choice of mode pairs at the household level rather than examining individual choices separately. The choice set is defined \textit{a priori} as a means of capturing decisions regarding vehicle allocation and joint travel. The presented models are developed using a traditional household travel survey, which does not explicitly collect data regarding the tradeoffs made at the household level. To capture these tradeoffs, the survey data and corresponding system performance measures (from a multimodal traffic/transit assignment model) are processed to uncover the implied individual travel time increases and household travel cost savings that are available to households who engage in joint travel. The implications of these issues on several different travel demand management policies are also briefly discussed.

4.1 Introduction

One of the primary motivations for considering intra-household interactions in travel demand models is the recognition that travel demand management policies may have subtle and unforeseen consequences at the household level that are not captured using conventional travel demand modelling approaches.

To highlight the importance of these interactions, the following cases are interesting. First, the case of a one worker, two adult household with only one car where the worker typically commutes by car to work. The non-worker could have several out of home activities throughout the day and uses transit to travel between them. Now, consider the situation where a stringent parking restriction policy coupled with high parking cost policy is imposed at work locations to discourage private car use. It is possible that this policy could have the reverse effect. While the parking policies may discourage the commuter from driving to work, the non-worker would then
have access to the vehicle and could choose to make numerous discretionary shopping and personal maintenance trips throughout the day. These auto trips would otherwise not occur (or occur during a less congested time period after the non-worker’s spouse returned from work) due to the non-worker not having access to the vehicle before the parking price increase. Ho and Mulley (2015a) present another insightful example regarding the effectiveness of high occupancy vehicle (HOV) lanes for encouraging carpooling and/or joint travel. They suggest that the congestion reduction benefit of policies such as HOV lanes may be over predicted in conventional models, as not all households are capable or willing to travel jointly due to spatial separation between destinations or the inability to coordinate departure time choice.

Ideally, models of travel demand will be able to capture these intricacies and tradeoffs to provide better information for policy decisions. Unfortunately, traditional four stage trip based models are inherently limited in their ability to capture these interactions. These limitations have encouraged the push towards a comprehensive activity based modelling approach. Activity based modelling explicitly considers travel of individuals throughout the day and as such can be extended to capture the relationships between individuals within the same household. Given the nature of activity based models, much of the literature within this field has been focused on specifically understanding how intra-household interaction influences the formation of individual activities, be it household level maintenance activities (Wen & Koppelman, 2000) or complete daily activity patterns (Bradley & Vovsha, 2005). In a comprehensive activity based modelling framework, the output of these activity pattern generation models can then be fed into a tour based mode choice model.

Tour based mode choice models present an interesting and under-researched aspect of intra-household interaction with only a few examples within the literature which explicitly tackle issues of intra-household interaction. Issues surrounding the question of vehicle allocation and joint travel are often dealt with in an ad hoc or sequential manner (as noted in Chapter 2 Section 1.1). The issues of vehicle allocation and joint travel are often addressed this way due to limited data or information regarding the explicit decision making process that occurs at the household level for these decisions. While traditional trip diaries are collected at the household level, the trip record data is often collected such that each trip maker is treated as a separate entity. As
such, information such as joint travel and activity arrangements are often hidden within the individual trip records.

This chapter presents an extension of our understanding of intra-household interaction at the household level. This is accomplished through the presentation of a joint morning peak period mode choice model. This model explicitly captures the details of the vehicle allocation process and the choice to engage in joint escort travel at the household level. The model is developed using a traditional household travel diary, which has been cleaned to reveal joint interactions between household members. While the model presented in this chapter follows a trip based approach, the modelling framework could be easily extended to account for tour based behaviour using the techniques discussed in Chapter 3. The remainder of this chapter is divided as follows. Section 4.2 provides a discussion of a joint mode choice modelling framework designed to tackle these issues. Section 4.3 outlines the data in addition to the TTS survey that is used for the case study. Section 4.4 provides the results of the empirical models developed for the case study with a discussion of their significance. The chapter concludes with a review of its contribution to the travel demand intra-household interaction literature as well as a brief overview of the main findings and potential avenues for future analysis.

### 4.2 Model Formulation

Before the explicit model formulation is presented, it is useful to consider the intricacies of joint household decision structures relative to the more conventional individual decision making process. In typical choice modelling approaches, the individual is assumed to select an alternative from a set of alternatives determined a priori based on their own personal characteristics. It is possible to define this predetermined set of alternatives as the person specific choice set. Within the context of household level mode choices, the determination of the choice set for an individual is no longer dependent on that individual’s personal characteristics alone. Instead, the availability of certain alternatives is also dependent on the behaviour and choices of other individuals living in the same household. This is particularly important for two cases:

1. The availability of the drive alone mode in auto deficient households. In households where there are more travelers than there are vehicles, the question of vehicle allocation and vehicle availability add additional complexity to the choice scenario. Specifically, within the context
of a two-adult-one-vehicle household, the vehicle may be allocated to one person, the other person or neither individual. The outcome of the allocation process will drastically shift the choice set of both household members as those who are not allocated the vehicle will no longer have vehicle dependent modes as options within their choice set. Conventional approaches typically employ two methods for accounting for vehicle allocation. Vehicle allocation/availability is often implicitly assumed and the number of household vehicles is included in the utility equation (see Chapter 2 Section 3). This accounts for the decreased probability of availability (and therefore selection) of driving modes for that individual. Other models will explicitly model the vehicle allocation process and then directly assume a drive travel mode for the individual who has been allocated the vehicle (as is done in Habib, 2014). This approach does not consider vehicle dependent modes such as park and ride and fails to capture the propensity for auto deficient households to make joint trips.

2. The availability of joint passenger travel. In multi person households, there is often inherent benefit associated with joint activity participation (which falls outside the scope of the analysis in this chapter) and/or joint travel. Joint travel may result in a reduction in overall household travel costs (and under special circumstances, travel time) while providing household members with the intrinsic benefit of each other’s company during the trip. Despite these benefits, within the context of joint driver passenger travel both household members may be required to compromise. These compromises can come in the form of adjusting departure times for either or both household members or selecting alternative and complementary discretionary activity locations for either or both parties. Furthermore, in almost all cases the household member who is driving must be willing to accept detours and the increased travel time associated with facilitating the passenger. These detours mean that this form of joint passenger travel is altruistic in nature because the driver is making a sub optimal choice to assist the passenger. Conventional person level mode choice models are unable to directly capture the choice to travel jointly or separately and will instead often use post-processing approaches to identify potential joint travel opportunities.

The failure to understand and accurately model these two intra-household decision contexts may cause significant issues with model validity and predictive accuracy. This becomes particularly important in policy and forecasting situations. Incorrect behavioural assumptions regarding how
these processes occur can lead to over or under predictions of trip counts for certain modes. This skews the results from a given policy or forecasting exercise. As such, it is imperative that these issues be considered when modelling travel mode choices. The following model structure attempts to address these concerns explicitly through a joint simultaneous household mode choice model.

4.2.1 Choice Set Formulation

Consider the following scenario: both adults in a 2-adult household aim to travel during the same general time-period, for example, the morning peak. Each individual household member has the same unconstrained universal choice set of travel mode options. If the number of alternatives in each individual household member’s choice set is equal to N, then the number of potential modal combinations or mode pairs is equal to $N^2$. This formulation mirrors the model structure initially proposed by Badoe (2002). Badoe’s structure is expanded on in three main ways:

1. Adding in minor modes such as transit with park and ride and non-motorized travel.
2. Constraining the choice set based on personal and household characteristics. This process allows the household level choice set definition to account for the issue of automobile allocation within the model structure.
3. Explicitly modelling the choice to travel jointly or individually through the creation of joint travel mode pairs. This process involves distinguishing between inter-household carpooling and intra-household joint auto passenger travel.

Based on these improvements relative to the work of Badoe, a hypothetical choice set can be thought of as what is shown in Tables 4.1 and 4.2:
Table 4.1 Individual Mode Pair Choice

<table>
<thead>
<tr>
<th>Mode/Mode</th>
<th>Drive All Way</th>
<th>Interhousehold Carpool</th>
<th>Public Transit with Walk Access</th>
<th>Public Transit with Park and Ride (Drive) Access</th>
<th>Non-Motorized Travel</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive All Way</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interhousehold Carpool</td>
<td></td>
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<tr>
<td>Public Transit with Walk Access</td>
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<td></td>
</tr>
<tr>
<td>Public Transit with Park and Ride (Drive) Access</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-Motorized Travel</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Table 4.2 Joint Mode Pair Choices

<table>
<thead>
<tr>
<th>Mode/Mode</th>
<th>Drive All Way</th>
<th>Public Transit with Park and Ride (Drive) Access</th>
<th>Auto Passenger Drop Off</th>
<th>Public Transit Auto Passenger/ Kiss and Ride</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive All Way</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Transit with Park and Ride (Drive) Access</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Auto Passenger Drop Off</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Public Transit Auto Passenger/ Kiss and Ride</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Each cell in Tables 1 and 2 represents a hypothetical mode pair that the household could select. In total, there are 25 possible individual mode pairs and 8 possible joint mode pairs, resulting in a total of 33 different alternatives. The black cells in Table 4.2 represent modal pairs that are either already considered in Table 4.1 or are impossible within the choice context mode pairs (i.e. two individuals cannot both be dropped off at a transit station). It should also be noted that these mode pairs do not have an explicit order associated with them; for example, the transit drive access-transit walk access mode pair is effectively the same in terms of utility specification as the
transit walk access-transit drive access mode pair. This specification does not mean the utilities are equal as the overall value of the utility is dependent on the individual characteristics of the trips and the trip makers. The rationale for this specification is that it does not force the modeller to assume a hierarchy of decision making within the household, whereby the choice of an individual is conditional on the earlier choice of another person. To achieve this flexibility, the modeller needs to impose equality restrictions on the alternative specific constants (ASCs) of mirrored alternative pairs. Furthermore, it is possible to further argue that the ASCs of each mode pair are composed of two individual alternative components. Therefore, it is possible to specify the utility of a modal pair as follows:

\[
U_{a-b} = ASC_a + ASC_b + f(a) + f(b) + f(i) + f(j) + f(h) + \epsilon_{a-b} = V_{a-b} + \epsilon_{a-b}
\]

Where \(U_{a-b}\) is the total utility mode pair in question, \(f(a)\) and \(f(b)\) are functions of the attributes of modes \(a\) and \(b\) respectively, \(f(i)\) and \(f(j)\) are functions of the household member selecting mode \(a\) and \(b\) respectively, \(f(h)\) is a function of the attributes of the household (i.e. the presence of children) and \(\epsilon_{a-b}\) is an additive error component about whose distribution assumptions are made about to obtain different model structures. As a final comment on the development of the ASCs, joint mode pairs were assumed to have a single ASC value rather than the composite two ASCs for individual travel mode pairs. As above, mirrored mode pairs (i.e. “joint drive all way – drop-off” and “joint drop-off – drive all way”) were constrained to have the same ASC value in the utility specification. This ASC specification was not used in the work of Badoe (2002), who had two different alternative specific constants for his drive-transit and transit-drive mode pair alternatives, which seems to be a minor misspecification of the utility function. The \textit{a priori} specification of the choice set allows vehicle allocation decisions and joint travel decisions to be endogenously modelled implicitly without having to define unique choice contexts. Using this structure even simple multinomial logit type models can be used to capture complex intra-household decisions.

As an aside, this structure follows a unitary decision making model ads discussed in Chapter 2 section 5. The main criticism of the unitary decision structure is that it does not capture the group dynamics to the same extent as the group decision models presented in the subsequent section in
the literature review. While the household mode choice structure presented above could be easily extended to consider group dynamics (as done in Chapter 7) this was not done here. The omission of this complexity is justified as this chapter provides an example of how a complex set of behavioural processes (i.e. vehicle allocation and joint travel decisions at the household level) can be modelled using simple techniques and careful specification of the choice set. The models presented in this chapter are all estimated using commercial black box software and do not require explicit definitions of the likelihood function. This is an important distinction, as many practitioners do not possess the knowledge or background to define their own likelihood functions and estimate using a more flexible software package. That being said, the other chapters in this thesis do make use of more complex structures which require explicit definitions of the likelihood function.

4.2.2 Joint Travel

It is also pertinent to explicitly examine the treatment of joint travel using the mode pair choice set framework. The framework only examines joint travel and does not explicitly concern itself with joint versus separate activity participation. The exclusion of joint activity participation is motivated by the choice context under examination. In the context of this study, the study time-period is a weekday morning during the peak period, when most travel is for commuting (an inherently independent activity type). While it would be possible to jointly model activity type, location and access mode, this work falls well outside of the scope of this analysis.

As further simplifying assumption, this framework only considers joint travel between the two adults travelling and as such does not explicitly consider escort trips for dependents. Methods to address escort decisions for dependents are discussed in greater detail in subsequent chapters. The model presented in this chapter does however implicitly capture the tradeoff between individual and joint travel times and costs by explicitly accounting for the potential detour (and increase in travel time) associated with facilitating a passenger by either dropping them off at their destination or at a transit station. This is a fundamental contribution and expands substantially on existing best practices in the literature (i.e. Ho and Mulley, 2015a).
Based on this formulation, it is possible to apply any standard discrete choice modelling formulation. For empirical investigation a standard multinomial logit, a nested logit and a mixed nested logit (error component) model are estimated and presented.

4.3 Data Description

As noted in the introduction, the modelling exercise in this chapter and the other empirical investigations in this thesis make use of the TTS data from 2011. The TTS data is collected at the household level but does not explicitly collect information regarding joint travel or explicit vehicle allocation behaviour. This means that it is not directly clear from the TTS trip records if a vehicle is available to a household member or if the household vehicles have been allocated to another household member. Furthermore, the trip records do not explicitly state that household member A travelled as a passenger in a vehicle with household member B acting as the driver. These challenges required that the TTS trip records be processed to identify cases of household auto deficiency and joint travel. To make this process simpler, all the TTS records are first skimmed to only leave the municipalities for which travel times and costs are available. Then to further simplify the analysis households that met the following criteria were examined:

1. The household only has two adults (members over the age of 18). This criterion is used to simplify the analysis of joint travel and vehicle allocation. While these opportunities exist for multi person households or for household heads and dependents, the simplest case of interaction between two household heads is used as the basis for this analysis.
2. The household has at least one car. This criterion is used to fix the analysis on households where questions regarding vehicle allocation and joint automobile travel are concerns. In households without vehicles, both questions become trivial.
3. Both household members travel during the morning peak period (between 6:00 and 9:00 a.m.).
4. At least one of the household members travel during the morning peak period is for a work-related purpose. Criteria three and four are used to frame the discussion of travelling during the morning peak commuting period. Examining non-commuting households would potentially skew the analysis.
5. Neither adult household member returned home during the morning peak period. This criterion is used to exclude short discretionary or passenger facilitation trips from the analysis to streamline vehicle availability (i.e. avoid the case where person A drops of child, returns home and then person B takes the vehicle).

Based on these criteria, a joint household level mode choice model of morning peak period travel which captures vehicle allocation and joint travel opportunities can be established. A sample of 12450 households is used for the estimation process.

4.3.1 Identification of Vehicle Allocation and Joint Travel

As noted above, the choice set for each household can be constructed to ensure vehicle allocation and joint travel is considered within the model framework. Households that have one car had their choice sets constrained such that mode pairs where both household members drove for all or part of their journey were not available for that household. This process means that the task of explicitly determining which household member is allocated the vehicle is relatively straightforward (if they were observed driving).

Identifying and classifying observed cases of joint travel is significantly more involved. As noted in Table 4.2, there are four possible joint travel arrangements with eight possible alternatives to represent the two possible driver/passenger combinations at the household level. Because of the nature of the TTS, joint travel between household members is not explicitly recorded. This means the process discussed in Appendix A needs to be performed. In general, for joint travel to have occurred all trips needed to start at the same time from the same location as noted in Appendix A. Beyond these standard criteria, the specific identification criteria for each of these 4 modal pairs are as follows:

4.3.1.1 Drive All Way – Drop Off:
In this mode pair, the mode for one individual is drive and the mode for the other individual is passenger. Next, two possible sub cases exist.

a) The first case occurs when the driver makes two trips and the passenger makes a single trip. The purpose of the driver’s first trip is “facilitate passenger” and the destination location for the driver’s first trip is the same traffic zone as the destination location for the passenger’s
trip. The travel time and cost for subsequent trip made by the driver are added to their trip
time and cost and the subsequent destination is treated as the final destination for the driver.
b) The alternate case occurs when both the driver and passenger make a single trip. In this case,
the destination purpose for the driver is not facilitating a passenger and the destination for
both individuals is the same then that also constitutes a joint travel pattern. This constitutes a
case where either non work joint activity participation occurs, or both household members
work at the same location/traffic zone.

4.3.1.2 Drive All Way – Kiss and Ride:
In this mode pair, the mode for one individual is auto drive and the mode for the other individual
is transit with auto passenger access mode. The driver must make two trips where the driver’s
first destination purpose must be “facilitate passenger”. These conditions are likely sufficient
however a final check is imposed. As noted in Chapter 1, Section 4, all subway and go train
stations are coded within their own unique traffic zones within the assignment model. A mapping
between the station zone and the standard traffic assignment zone is generated using GIS
software (by simply overlaying/intersecting the station zone centroid with the standard traffic
zone polygon). Then if driver’s destination location (the standard traffic zone) corresponded
with the station centroid zone, the trip is confirmed to be a joint drive all way - kiss and ride trip. As
above, the subsequent trip’s travel time and cost were added for the driver’s trip and their
subsequent destination is treated as the final destination.

4.3.1.3 Park and Ride – Kiss and Ride:
In this mode pair, the mode for one individual is transit with auto drive access and the mode for
the other individual is transit with auto passenger access. There exist two possible cases for this
mode though the first occurred exclusively in the TTS sample. In the first case, the only
requirement is that the traffic zone for the station for the park and ride and kiss and ride travelers
are identical. In the second case, the driver would first make an auto drive trip with a destination
purpose of “facilitate passenger” to the traffic zone where the transit station for the kiss and ride
traveler is located. The driver would then proceed to their station of choice. The second case had
zero observations in the sample and is not expected to be a realistic behaviour as most
households selecting this mode pair would simply select the same station.
4.3.1.4 Park and Ride – Drop Off:
In this mode pair, the mode for one individual is transit with drive access and the mode for the other individual is an auto passenger. The transit with drive access individual must first make a driving trip with facilitating a passenger as the destination purpose to the same location as the passenger’s trip. Then the driver would make a transit trip with the access mode set to drive. Unfortunately, there were only 4 observations across the sample of this modal pair within the TTS data. The main reason for the lack of observations of this household commuting pattern is likely due to the impracticality of this behaviour. Because most transit stations with park and ride are suburban, there exist limited employment opportunities within short driving distance of these locations making cases where this mode is practical few and far between. As such, this mode is excluded from the analysis.

4.3.2 Mode Split and Descriptive Statistics
Before estimation, the mode split of the sample is examined to determine cases where minor modes should be excluded or combined. The carpool mode has in total fewer than 100 observations, with practically insignificant counts for the mode pairs containing carpooling. As such these records are removed and the carpool mode is considered unavailable for the remaining households. While this is an obvious simplification, it is a reasonable one for the context of this analysis. The primary objective of this study is intra-household joint travel and the data used for the analysis has limited information regarding inter household ride sharing behaviour. Alternative methods to address inter household ride sharing could be examined although that analysis would likely require an alternative data source with more information about this sort of travel. Park and ride and non-motorized mode pair combinations are also dropped due to there being less than 10 observations in the dataset. This is reasonable given that individuals who live in a location where walking is a viable means of reaching their destination are far more likely to live within walking distance of transit and thus their household partner likely does not need to drive to access transit. Finally, as noted above, park and ride drop-off mode pairs are also excluded due to low observation counts. The elimination of these modes is again reasonable as most park and ride stations are suburban or rural, which means that there will be limited employment opportunities in these zones making this type of travel pattern unlikely. Moreover, the park and ride driver would not be making use of the automobile for most of the day as they would leave the vehicle at the transit station, suggesting that the passenger who is dropped off
should take the car, making the drive all way kiss and ride mode pair more reasonable. Based on these simplifications the final counts and modal share percentages are outlined in Table 4.3.

**Table 4.3 Modal Counts and Modal Share for Mode Pairs**

<table>
<thead>
<tr>
<th>Mode Pair</th>
<th>Count</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive All Way Drive All Way</td>
<td>6311</td>
<td>50.69%</td>
</tr>
<tr>
<td>Drive All Way Transit Walk Access</td>
<td>1568</td>
<td>12.59%</td>
</tr>
<tr>
<td>Drive All Way Transit Drive Access</td>
<td>1028</td>
<td>8.26%</td>
</tr>
<tr>
<td>Drive All Way Non-Motorized Travel</td>
<td>390</td>
<td>3.13%</td>
</tr>
<tr>
<td>Transit Walk Access Transit Walk Access</td>
<td>412</td>
<td>3.31%</td>
</tr>
<tr>
<td>Transit Walk Access Transit Drive Access</td>
<td>57</td>
<td>0.46%</td>
</tr>
<tr>
<td>Transit Walk Access Non-Motorized Travel</td>
<td>125</td>
<td>1.00%</td>
</tr>
<tr>
<td>Transit Drive Access Transit Drive Access</td>
<td>93</td>
<td>0.75%</td>
</tr>
<tr>
<td>Non-Motorized Travel Non-Motorized Travel</td>
<td>117</td>
<td>0.94%</td>
</tr>
<tr>
<td>Drive All Way Drop Off*</td>
<td>2198</td>
<td>17.65%</td>
</tr>
<tr>
<td>Drive All Way Kiss and Ride*</td>
<td>56</td>
<td>0.45%</td>
</tr>
<tr>
<td>Transit Drive Access Kiss and Ride*</td>
<td>95</td>
<td>0.76%</td>
</tr>
<tr>
<td>Total</td>
<td>12450</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

*Joint mode-pairs*

Consistent with other studies in the region, a set of individual modal availability conditions were applied; namely:

- Drive all way is only available if the individual owns a driver’s license.
- Transit alternatives (kiss and ride, park and ride, transit walk access) are only available if the total travel time is under 150 minutes.
- The non-motorized option is only available for travel distances under 5 kilometers.

Furthermore, as discussed above, the choice set is further constrained based on household vehicle ownership. Auto deficient households are not able to select mode pairs where both individuals travelled separately by car (transit drive access and drive all way options). A summary of the key (though not exhaustive) variables within the data set can be found in Table 4.4.
Table 4.4 Descriptive Statistics of the Sample

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Count</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average Number of Children</td>
<td>0.7</td>
<td>---</td>
</tr>
<tr>
<td>Max Number of Children</td>
<td>5</td>
<td>---</td>
</tr>
<tr>
<td>Number of Auto Deficient Households</td>
<td>4083</td>
<td>32.8%</td>
</tr>
<tr>
<td>Average Number of Household vehicles</td>
<td>1.78</td>
<td>---</td>
</tr>
<tr>
<td>Average Age of Individuals</td>
<td>44.9</td>
<td>---</td>
</tr>
<tr>
<td>Mode of Age of Individuals</td>
<td>40</td>
<td>---</td>
</tr>
<tr>
<td>Number of Individuals who are Men</td>
<td>12364</td>
<td>49.7%</td>
</tr>
<tr>
<td>Number of Individuals with Driver’s Licenses</td>
<td>23772</td>
<td>95.5%</td>
</tr>
<tr>
<td>Number of Individuals with Transit Passes</td>
<td>3250</td>
<td>13.1%</td>
</tr>
<tr>
<td>Number of Individuals Employed in the Manufacturing Sector</td>
<td>2131</td>
<td>8.6%</td>
</tr>
<tr>
<td>Number of Individuals Employed in the General Office Sector</td>
<td>4582</td>
<td>18.4%</td>
</tr>
<tr>
<td>Number of Individuals Employed in the Professional Sector</td>
<td>10856</td>
<td>43.6%</td>
</tr>
<tr>
<td>Number of Individuals Employed in the Retail Sector</td>
<td>5752</td>
<td>23.1%</td>
</tr>
<tr>
<td>Number of Individuals who are Unemployed</td>
<td>1569</td>
<td>6.3%</td>
</tr>
<tr>
<td>Number of Individuals who are Full Time Workers</td>
<td>22634</td>
<td>90.9%</td>
</tr>
<tr>
<td>Average Straight Line Distance to Destination (Km)</td>
<td>12.8</td>
<td>---</td>
</tr>
<tr>
<td>Average Distance Between Destinations of Both Household Members (Km)</td>
<td>13.1</td>
<td>---</td>
</tr>
</tbody>
</table>

One of the key limiting factors associated with the TTS is the lack of income data at an individual and household level. This is particularly important given the potential for income effect on mode choice behaviour (Jara-Díaz & Videla, 1989). Prior studies, which use this dataset (or previous iterations of it), have overcome these issues by estimating employment category (general office worker, retail workers, professional workers and manufacturing) specific income variables. This approach has two main issues with it, the second issue being specific to the context of the study. First, employment categories are broad and as such may encompass a wide range of incomes. The study of Habib and Weiss (2014) for the same study area using a 2006 dataset found that only the manufacturing cost parameter had a substantial difference from the other three employment categories. The second issue arises from the joint model formulation discussed above. In this formulation, both household members have individual employment categories. While it would be possible to apply individual specific cost coefficients based on employment categories, the validity of this approach is questionable if the household members share their income. If income sharing does occur then there are 20 possible cost coefficients that could be estimated (given 4 categories of employment and the requirement that at least one household member be employed and travel to work). As it is generally not feasible to explicitly estimate this number of cost parameters, several different aggregated categories were tested, including parameters for dual worker and single
worker households and different parameters dependent on the number of children. Unfortunately, none of these categories proved to be significantly different from each other and as such the investigation of income effects for this sample is postponed for future research.

Another concern associated with the TTS, like many RP data, stems from the lack of explicitly defined household roles. The specific roles of household members may have a strong influence the decision making process. Though it is possible to make inferences regarding the household structure, the trend of decreasing numbers of traditional married husband and wife couples makes this process more challenging. This is exacerbated by the recent trend in Canada for more young adults to continue or return to living in their parental home and to put off forming unions or partnerships and forming their own families has the potential to induce emerging travel behaviour scenarios at the household level. Further information on the specific roles of household members could potentially be inferred through their individual contribution to household income thus highlighting the importance of individual and household income information.

4.3.3 Transit with Auto Access Times and Costs

Chapter 1 Section 4 outlines the traffic assignment model used for this analysis and the challenges associated with transit access mode. These challenges stem from taking a route choice perspective on the access mode to transit. This means that the existing time and cost matrices used for this analysis do not explicitly have transit drive access times and costs. To circumvent this, a simple station location choice model (using the multinomial logit formulation) is estimated and applied to all mode pairs where a vehicle is used for transit access. The parameter estimates for this model are presented below and form the basis for the initial modelling work done on station location choice in Chapter 5. It should be noted that the work in Chapter 5 is motivated by the inadequacies of this model at capturing spatial dynamics. The limitations of this model and methods used to correct these deficiencies are discussed in greater detail in Chapter 5.
Table 4.5 MNL Station Location Choice

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Estimates (T-Stats in parenthesis)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>-3.357 (13.08)</td>
</tr>
<tr>
<td>Adjusted Rho-Squared</td>
<td>0.419</td>
</tr>
<tr>
<td>AIC</td>
<td>6.732</td>
</tr>
<tr>
<td>Expected drive time from home to the transit station (produced by traffic assignment model)</td>
<td>-0.0928 (-13.08)</td>
</tr>
<tr>
<td>Expected transit wait, walk, transfer and in vehicle time from the station to the destination (produced by the traffic assignment model)</td>
<td>-0.0284 (-13.61)</td>
</tr>
<tr>
<td>Expected travel cost (fare + drive cost) for the entire trip (produced by the traffic assignment model)</td>
<td>-0.489 (-10.77)</td>
</tr>
<tr>
<td>Parking cost at the station (set to zero for kiss and ride travelers irrespective of station cost)</td>
<td>-1.468 (-30.29)</td>
</tr>
<tr>
<td>Dummy variable for if the station is on the subway</td>
<td>1.715 (7.18)</td>
</tr>
<tr>
<td>Natural logarithm of the parking lot capacity</td>
<td>0.386 (9.21)</td>
</tr>
<tr>
<td>Dummy variable for if the station has a washroom</td>
<td>0.864 (8.17)</td>
</tr>
<tr>
<td>Dummy variable for if the station-home-work angle is greater than 90</td>
<td>-0.0631 (-1.04)</td>
</tr>
<tr>
<td>Natural logarithm of the frequency of trains at the station per hour</td>
<td>0.827 (9.59)</td>
</tr>
</tbody>
</table>

This model is developed using the 4 closest stations to the home location and the selected station of all transit auto access records in the TTS (including those not in two adult households). As will be noted in Chapter 5, this cut off is implemented as the clear majority of the recorded transit auto access trips were to one of the five closest stations to the individual’s origin. This resulted in 3603 TTS records being used to generate this model. The model is then used to estimate expected park and ride and kiss and ride travel times for all the 12450 households in the mode choice subsample. In cases where one of the individuals is observed accessing transit with an automobile this is not required as the observed station is used. Otherwise the 5 closest transit stations to the individual’s home are determined. The station choice model is then used to determine the probability of an individual selecting each of the five stations. Then the travel times and travel costs for each of those five stations were weighted using the probability that the individual would select the station and then summed to obtain an expected travel time and cost by car and transit for the individual in question. Finally, for the two modes where the household members jointly accessed transit by car (joint park and ride), the driver’s transit travel times and transit costs from the station to their destination are used to determine the utilities and then the probabilities for the station choice (as opposed to the sum of the driver and the passengers times and costs). This is obviously a limiting assumption, though understanding the dynamics of
household station choice when both individuals are using transit is complex and would require additional observations to understand the dynamics of this choice.

4.3.4 Consideration of Non Escort Trip Chaining

One final issue with the TTS data is its trip based format, which creates challenges with respect to modelling the mode for trip chaining behaviour. The proposed modelling structure requires an analysis of the entire morning peak period trip chain to determine if there are any facilitate passenger trips corresponding to dropping off the other adult within the individual’s observed schedule. These escort trip chains are treated as singular trips in the model and the travel times and costs are summed for these trips. Aside from escort trip chaining there can be other trip chaining behaviour that occurs without joint travel (e.g. dropping a child off at school or daycare, stopping to drop-off dry-cleaning, etc.). The consideration of these intermediate trips is not the purpose of this analysis as the model estimated within is a trip based formulation rather than a tour based formulation. As such, the following structure is imposed within the data generation stage to obtain reasonable estimates.

1. All trips made by each individual household member during the morning peak period are examined.
2. If the individual makes a trip to work, then the analysis for that individual household member is finished (i.e. it is assumed that they have reached their destination for the morning peak period)
3. If no work trip is observed during the morning peak period then the destination of the last trip the individual made before the end of the peak period is treated as their destination.
4. If the household member returns home during the morning peak (completes a tour), then that individual and their household record are removed from the analysis. This is done to simplify concerns regarding automobile availability (i.e. to avoid the case where in an auto deficient household, individual A used the automobile from 7:00 to 8:00 to drop-off children at school and then return home and then individual B uses the automobile to travel to work).

With these considerations made, it is possible to move on to the results of the model estimation exercises.
4.4 Model Estimation Results

This section presents three models, a multinomial logit, a nested logit and an error component mixed logit. The results of these modelling exercises are presented in Tables 4.7 through 4.9 respectively. The models are estimated using the Nlogit 5 package (Greene, 2012). The overall goodness of fit of the models were determined by using the well know rho squared test which finds the difference between one and the ratio of the log likelihood of the model in question and either the constant only model (representing the modal share of the dataset) and the null model (representing the case where each alternative is equally probable and the choice is selected at random). The goodness of fit results of this modelling framework are presented in Table 4.6 The multinomial logit model is relatively well known and as such will not be derived here, the structures for the nested logit and mixed logit (using the random parameter and error component formulations) require some further details.

Table 4.6 Model Goodness of Fit Statistics

<table>
<thead>
<tr>
<th></th>
<th>Number of observations</th>
<th>Number of alternatives</th>
<th>Log likelihood null</th>
<th>Log likelihood constant only</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>-37669.62497</td>
<td>-20151.20509</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>MNL</th>
<th>Nested</th>
<th>Error Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log likelihood of estimated models</td>
<td>-11195.79</td>
<td>-11181.02</td>
<td>-11029.32</td>
</tr>
<tr>
<td>Rho Squared against Constant Only</td>
<td>0.444</td>
<td>0.445</td>
<td>0.453</td>
</tr>
<tr>
<td>Rho Squared against Null Model</td>
<td>0.703</td>
<td>0.703</td>
<td>0.707</td>
</tr>
<tr>
<td>Number of parameters</td>
<td>23</td>
<td>26</td>
<td>27</td>
</tr>
<tr>
<td>AIC</td>
<td>22437.58</td>
<td>22414.04</td>
<td>22112.64</td>
</tr>
</tbody>
</table>

4.4.2 Nested Logit Discussion

The nested logit structure employed here follows the well-known structure discussed in numerous discrete choice analysis texts (e.g. Train, 2009). The purpose of this model is to capture the correlation between alternatives (and corresponding non-proportional substitution patterns). This is accomplished by relaxing the independence of the error components of each alternative so that they are correlated. As noted in Chapter 8 of Koppelman and Bhat (2006), one method for accomplishing this relaxation is by simply decomposing the error term for each alternative into an alternative specific component and a component which is common amongst alternatives which share a nest. This common component creates a covariance (and therefore correlation) between the nested alternatives. That said, the total error (which is composed of the
sum of the common and individual components for each alternative) is still identically distributed following a Gumbel distribution (scale parameters equal to one), which means that the variance of each composite error is still equal to $\pi^2/6$. This means that the variance of each component of the error must be less than this value. As noted in Hensher and Greene (2002), this independence creates questions regarding the normalization of the scales for the common and individual components. Most econometric text books (Train 2009, Koppelman and Bhat, 2006) recommend Hensher and Greene’s random utility form 2, whereby the variance of the individual component is scaled. The scale is constrained to be between one and positive infinity though most commercial software will estimate the inverse of the scale (also known as the logsum/inclusive value parameter) instead, which is constrained to be between zero and one). For the model presented in Table 4.8, numerous specifications were tested though the specification which proved the most significant involves creating a nesting structure following the tree pattern outlined in Figure 4.1:

![Figure 4.1 Nested Logit Correlation Tree Structure](image)

**Figure 4.1 Nested Logit Correlation Tree Structure**

- DAW = Drive All Way
- TWA = Transit Walk Access
- TDA = Transit Drive Access
- NMT = Non Motorized Travel
- DO = Drop Off
- KR = Transit Kiss and Ride/Pasenger Access
- j = Joint Travel Prefix
The probability for any given alternative $i$ and corresponding nest $j$ in this tree diagram is defined as follows:

$$P_i = \frac{\left( \exp\left( \frac{V_i}{\lambda_j} \right) \right) \times \left( \sum_{k \in C_j} \exp\left( \frac{V_k}{\lambda_j} \right) \right)^{\lambda_j - 1}}{\sum_{n \in J} \left( \sum_{m \in C_n} \exp\left( \frac{V_m}{\lambda_n} \right) \right)^{\lambda_n}}$$

Which can be decomposed into the product of two logits (see Train 2009 Chapter 4 for details) as follows:

$$P_i = P_{i|j} \times P_j$$

$$= \frac{\exp\left( \frac{V_i}{\lambda_j} \right) \times \exp\left( \ln\left( \sum_{k \in C_j} \exp\left( \frac{V_k}{\lambda_j} \right) \right) \times \lambda_j \right)}{\sum_{k \in C_j} \exp\left( \frac{V_k}{\lambda_j} \right) \times \sum_{n \in J} \exp\left( \ln\left( \sum_{m \in C_n} \exp\left( \frac{V_m}{\lambda_n} \right) \right) \times \lambda_n \right)}$$

Where $V_i$ is the utility of alternative $i$, $\lambda_j$ is the nesting coefficient for the nest $j$ (constrained between zero and 1), $C_j$ is the set of alternatives in nest $j$ and $J$ is the comprehensive list of nests. The nested logit model collapses to the standard multinomial logit when the nesting coefficient equals one.
Table 4.7 Multinomial Logit Model Household Mode Pair Choice

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Estimate</th>
<th>Std. Err</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive all way constant.</td>
<td>0.48223</td>
<td>0.07951</td>
<td>6.06</td>
</tr>
<tr>
<td>Transit walk access constant.</td>
<td>-0.48861</td>
<td>0.1304</td>
<td>-3.75</td>
</tr>
<tr>
<td>Transit park and ride constant.</td>
<td>-0.87565</td>
<td>0.11478</td>
<td>-7.63</td>
</tr>
<tr>
<td>Non-motorized travel constant.</td>
<td>0.33131</td>
<td>0.12464</td>
<td>2.66</td>
</tr>
<tr>
<td>Joint drive all way - drop-off constant.</td>
<td>0</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>Joint drive all way - kiss and ride constant.</td>
<td>-4.01448</td>
<td>0.16686</td>
<td>-24.06</td>
</tr>
<tr>
<td>Joint drive transit access constant.</td>
<td>-3.93655</td>
<td>0.24572</td>
<td>-16.02</td>
</tr>
<tr>
<td>Coefficient for the natural logarithm of the angle between the destination of person one, the home and the destination of person two for joint drive all way drop-off</td>
<td>-0.6759</td>
<td>0.02066</td>
<td>-32.72</td>
</tr>
<tr>
<td>Coefficient for the natural logarithm of the origin destination distance for non-motorized travel.</td>
<td>-0.31128</td>
<td>0.01397</td>
<td>-22.29</td>
</tr>
<tr>
<td>Dummy coefficient for auto deficiency (one car household) for drive all way - drop-off.</td>
<td>2.0042</td>
<td>0.06853</td>
<td>29.25</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual has a transit pass for transit walk access and transit park and ride</td>
<td>3.18996</td>
<td>0.06512</td>
<td>48.99</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for being escorted as a passenger (excluding joint park and ride)</td>
<td>-0.91807</td>
<td>0.14885</td>
<td>-6.17</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for transit walk access and transit park and ride</td>
<td>-0.47032</td>
<td>0.05894</td>
<td>-7.98</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for escorting as a driver (excluding joint park and ride)</td>
<td>0.2794</td>
<td>0.14694</td>
<td>1.9</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for non-motorized travel</td>
<td>0.38699</td>
<td>0.08634</td>
<td>4.48</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is over the age of 65 for non-motorized travel</td>
<td>-1.55698</td>
<td>0.23013</td>
<td>-6.77</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual lives in Toronto for transit walk access.</td>
<td>0.49977</td>
<td>0.05254</td>
<td>9.51</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual makes a non-joint travel intermediate stop before travelling on to their destination for drive all way.</td>
<td>0.99566</td>
<td>0.0624</td>
<td>15.96</td>
</tr>
<tr>
<td>Generic coefficient for the in-vehicle travel time by automobile for any trip using an automobile</td>
<td>-0.05206</td>
<td>0.00207</td>
<td>-25.13</td>
</tr>
<tr>
<td>Generic coefficient for the in-vehicle travel time for transit trips across all transit vehicles</td>
<td>-0.0168</td>
<td>0.00168</td>
<td>-10.02</td>
</tr>
<tr>
<td>Generic coefficient for the parking cost at the destination for the any auto drive trip</td>
<td>-0.62108</td>
<td>0.01847</td>
<td>-33.63</td>
</tr>
<tr>
<td>Generic coefficient for the travel cost (excluding parking cost)</td>
<td>-0.07721</td>
<td>0.0083</td>
<td>-9.3</td>
</tr>
<tr>
<td>Generic coefficient for wait time for transit trips</td>
<td>-0.05752</td>
<td>0.00431</td>
<td>-13.34</td>
</tr>
<tr>
<td>Generic coefficient for walk time for transit trips</td>
<td>-0.21029</td>
<td>0.00869</td>
<td>-24.19</td>
</tr>
</tbody>
</table>
Table 4.8 Nested Logit Model Household Mode Pair Choice

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Estimate</th>
<th>Std. Err</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive all way constant.</td>
<td>0.48073</td>
<td>0.07278</td>
<td>6.61</td>
</tr>
<tr>
<td>Transit walk access constant.</td>
<td>-0.48822</td>
<td>0.1184</td>
<td>-4.12</td>
</tr>
<tr>
<td>Transit park and ride constant.</td>
<td>-0.75632</td>
<td>0.11004</td>
<td>-6.87</td>
</tr>
<tr>
<td>Non-motorized travel constant.</td>
<td>0.35471</td>
<td>0.11503</td>
<td>3.08</td>
</tr>
<tr>
<td>Joint drive all way - kiss and ride constant.</td>
<td>-4.01534</td>
<td>0.16597</td>
<td>-24.19</td>
</tr>
<tr>
<td>Joint drive transit access constant.</td>
<td>-3.82807</td>
<td>0.22731</td>
<td>-16.84</td>
</tr>
<tr>
<td>Joint Drive All Way - Drop Off Constant</td>
<td>0</td>
<td>---</td>
<td>---</td>
</tr>
</tbody>
</table>

Coefficient for the natural logarithm of the angle between the destination of person one, the home and the destination of person two for joint drive all way drop-off

Coefficient for the natural logarithm of the origin destination distance for non-motorized travel.

Dummy coefficient for auto deficiency (one car household) for drive all way - drop-off.

Dummy coefficient for if the individual has a transit pass for transit walk access and transit park and ride.

Dummy coefficient for if the individual is male for being escorted as a passenger (excluding joint park and ride).

Dummy coefficient for if the individual is male for transit walk access and transit park and ride.

Dummy coefficient for if the individual is male for escorting as a driver (excluding joint park and ride).

Dummy coefficient for if the individual is male for non-motorized travel.

Dummy coefficient for if the individual is over the age of 65 for non-motorized travel.

Dummy coefficient for if the individual lives in Toronto for transit walk access.

Dummy coefficient for if the individual makes a non-joint travel intermediate stop before travelling on to their destination for drive all way.

Generic coefficient for the in-vehicle travel time by automobile for any trip using an automobile

Generic coefficient for the in-vehicle travel time for transit trips across all transit vehicles

Generic coefficient for the parking cost at the destination for the any auto drive trip

Generic coefficient for the travel cost (excluding parking cost)

Generic coefficient for walk time for transit trips

Generic coefficient for wait time for transit trips

Inclusive Value Parameters

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Estimate</th>
<th>Std. Err</th>
<th>T-Stat (relative to 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both household members use a car nest</td>
<td>0.87429</td>
<td>0.0223</td>
<td>-5.637</td>
</tr>
<tr>
<td>Household member one uses a car nest</td>
<td>0.87429</td>
<td>0.0223</td>
<td>-5.637</td>
</tr>
<tr>
<td>Household member two uses a car nest</td>
<td>0.87429</td>
<td>0.0223</td>
<td>-5.637</td>
</tr>
</tbody>
</table>

As alluded to above, numerous nesting structures were attempted before arriving at the specification outlined in Figure 4.1. Specifically, the alternatives in the uncorrelated nest (where the nesting coefficient is constrained to one) were combined with the existing nests (where
applicable) and new nests (e.g. joint travel nests). These hypothetical models were non-consistent with utility maximizing (nesting coefficients greater than 1). A potential (though as will be shown below, wrong) explanation for the joint trips not being correlated with each other (or the vehicle allocation nests) is that joint travel arrangements are very specific and will often require more coordination to be reasonable for both household members (e.g. coordinated start times, likelihood of needing coordinated meeting times for the return leg after work). It is possible that the coordination required to make these trips function is too different from other joint travel options as to make them poor substitutes for each other and therefore result in them being uncorrelated. That said, one exception to this argument is the joint park and ride and kiss and ride mode pairs where the coordination between the two choices is nearly identical. A nest containing these two options is estimated but again proved to be inconsistent with utility maximizing. One potential reason for this finding is that this mode pair has a relatively low number of observations in the sample used for this analysis.

The other non-correlated alternatives were also attempted to be grouped as a nest (or set of sub nests) though they were also found to have nesting coefficients inconsistent with utility maximization. One general finding that can come from this result is that vehicle allocation determines the substitution patterns between an individual’s modes in a household. Put more plainly, modes that require a vehicle are much more substitutable relative to modes which do not require a vehicle, which is an intuitive finding.

Of note in the nested logit model is the common nesting coefficient across all nests. An earlier model was estimated with two nesting coefficients (one for both household members using a car and one for the individual cases) though a t-statistic calculated between the difference of the two parameter estimates found their difference to be statistically equivalent to zero so the presented model contains one nesting parameter. A similar unconstrained model and test was performed on the male dummy variable for transit walk and park and ride access with the same finding.

The test for whether two parameters are statistically different from each other is as follows:

\[
\frac{(\beta_a - \beta_b)}{\sqrt{(SE_a)^2 + (SE_b)^2 - 2 \times COV_{ab}}}
\]
Where the $\beta$s are the parameter estimates, the $SE$s are standard errors and $COV_{ab}$ is the covariance between the parameter estimates.

4.4.3 The Error Component Mixed Nested Logit (ECMNL)

An alternate to the nested logit model for capturing correlation between alternatives is the error component mixed logit structure (Walker et al. 2007). The examination of this structure is motivated by the desire to examine potential cross nested correlation patterns between alternatives though it’s application provides an interesting alternate finding. Along with considering complex correlation patterns, the error component mixed logit structure also allows for the explicit consideration of different variances across alternatives. This heteroskedasticity occurs because the error is now decomposed into an IID type I Gumbel extreme value component (which has equal variance) and an unconstrained normal component (unequal variance). Each correlated/nested alternative will have a common normally distributed error component added to them. This additive format further relaxes the assumption of identical and independently distributed error components by not only adding a common error component to induce correlation but by also inducing heteroskedasticity (difference in variance) making the composite errors for each alternative distinct (in variance).

Before getting into the specifics of the model estimation it is pertinent to discuss the identification issues for the error component logit. This is important as an unidentified model may appear identified when using simulation based approaches for maximum likelihood (Chiou and Walker, 2007). These models are problematic as they may provide counter intuitive results leading the analyst to draw incorrect conclusions. For the case of the error component logit, Walker et al. (2007) identify a general procedure for the number of parameters that can be identified and a set of specific rules for different sub specifications. In their work, they identify several conditions which must be met for identification. First, the order condition provides a required but not sufficient means of determining the number of parameters that can be estimated in a mixed logit or probit. The number of parameters $S$ must adhere to the following:

$$S \leq \frac{J(J - 1)}{2} - 1$$
Where S is the maximum number of parameters to be estimated and J is the number of alternatives. This condition provides a very easy check in the context of this analysis as there are 20 alternatives so the order condition constraint is 189 parameters.

The second condition is the rank condition requirements, which are much more elaborate and require a great deal of manual computation. For a general nested logit model, Walker et al. (2007) derive the general rule for rank conditions: all nesting parameters can be identified for 3 or more nests but only one can be identified for 2 nests. A quick application of the error component logit to the previously used nesting structure finds that the model performs poorly relative to the closed form nested structure. This finding is unsurprising given that the error component mixed logit does introduce heteroskedasticity so it does not directly replicate the nested logit. This finding suggests that the correlated alternatives in the original nested model have lower variance than the alternatives in the uncorrelated nest. As a result, several alternate specifications were attempted before settling on the structure outlined in Figure 4.2. The model results for the error component nested logit using this structure are outlined in Table 4.9. As with other studies using the mixed logit formulation, the log likelihood function is approximated using a technique known as maximum simulated likelihood. The simulated log likelihood approach is used because the probability function is no longer closed form (due to the integral of the normal component of the error term not having an analytical expression). In this approach, the average of a set of simulated probabilities/likelihoods is used as the likelihood function. The simulated probabilities are calculated from simulated utilities which are defined by taking draws from the normal distribution and adding the systematic component of the utility to the draw. These simulated utilities are then used to calculated the simulated probability. This represents an unbiased estimator of the actual choice probability whose variance decreases as the number of draws increases. High variance estimates of the probability may result in unstable parameter estimates, therefore, models with up to 1000 simulated draws using a quasi-random Halton sequence (Bhat, 2001) are tested. The results from the 500 draw and the 1000 draw model runs were practically identical, suggesting that the 500-draw model is sufficient.

As the correlation structures between the nested and mixed logit vary drastically, there is a key conclusion that is derived from this analysis. Namely, there are potential challenges in identifying correlation using a standard nested logit structure when alternatives are
heteroskedastic. An appropriate approach is to use an error component logit structure alongside nested logit structures to determine if correlation (and heteroskedasticity) are being missed by the standard nested logit.

Figure 4.2 Final Error Component Mixed Logit Correlation Structure

While it is also possible to test cross nesting models with the ECMNL, the rank conditions for these models have no general rule and must be derived on a model by model basis. Unfortunately for this context, the number of alternatives makes performing the test incredibly cumbersome. For example, Walker et al. (2007) examined models with at most five alternatives whereas in the application presented in this chapter, twenty alternatives are considered. Ideally, an automated computer process to perform these checks could be developed but as none exists currently the examination of cross nesting is left for future work.
Table 4.9 Error Component Mixed Nested Logit Household Mode Pair Choice

<table>
<thead>
<tr>
<th>Parameter Description</th>
<th>Estimate</th>
<th>Std. Err</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive all way constant.</td>
<td>0.60175</td>
<td>0.0525</td>
<td>11.46</td>
</tr>
<tr>
<td>Transit walk access constant.</td>
<td>-1.49931</td>
<td>0.14686</td>
<td>-10.21</td>
</tr>
<tr>
<td>Transit park and ride constant.</td>
<td>-1.46138</td>
<td>0.10994</td>
<td>-13.29</td>
</tr>
<tr>
<td>Non-motorized travel constant.</td>
<td>0.20203</td>
<td>0.12226</td>
<td>1.65</td>
</tr>
<tr>
<td>Joint drive all way - drop-off constant (fixed to 0).</td>
<td>0</td>
<td>***</td>
<td>***</td>
</tr>
<tr>
<td>Joint drive all way - kiss and ride constant.</td>
<td>-5.59416</td>
<td>0.22973</td>
<td>-24.35</td>
</tr>
<tr>
<td>Joint drive transit access constant.</td>
<td>-5.41048</td>
<td>0.34796</td>
<td>-15.55</td>
</tr>
<tr>
<td>Coefficient for the natural logarithm of the angle between the</td>
<td>-0.89788</td>
<td>0.03672</td>
<td>-24.45</td>
</tr>
<tr>
<td>destination of person 1, the home and the destination of person 2 for</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>joint drive all way drop-off</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coefficient for the natural logarithm of the origin destination distance</td>
<td>-0.31712</td>
<td>0.01583</td>
<td>-20.04</td>
</tr>
<tr>
<td>for non-motorized travel.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for auto deficiency (one car household) for drive</td>
<td>2.76733</td>
<td>0.12186</td>
<td>22.71</td>
</tr>
<tr>
<td>all way - drop-off.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for if the individual has a transit pass for transit</td>
<td>3.53899</td>
<td>0.06921</td>
<td>51.14</td>
</tr>
<tr>
<td>walk access and transit park and ride</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for being dropped off at a transit</td>
<td>-1.31419</td>
<td>0.21953</td>
<td>-5.99</td>
</tr>
<tr>
<td>station (kiss and ride) by the other household member</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for being dropped off by the other</td>
<td>-1.71423</td>
<td>0.10561</td>
<td>-16.23</td>
</tr>
<tr>
<td>household member</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for non-motorized travel</td>
<td>0.45416</td>
<td>0.09891</td>
<td>4.59</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is male for transit walk access</td>
<td>-0.27484</td>
<td>0.06472</td>
<td>-4.25</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual is over the age of 65 for</td>
<td>-1.57661</td>
<td>0.23236</td>
<td>-6.79</td>
</tr>
<tr>
<td>non-motorized travel</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dummy coefficient for if the individual lives in Toronto for transit walk access.</td>
<td>1.05919</td>
<td>0.08045</td>
<td>13.17</td>
</tr>
<tr>
<td>Dummy coefficient for if the individual makes a non-joint travel intermediate stop</td>
<td>1.11938</td>
<td>0.07983</td>
<td>14.02</td>
</tr>
<tr>
<td>before travelling on to their destination for drive all way.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic coefficient for the in-vehicle travel time by automobile for any</td>
<td>-0.05711</td>
<td>0.0022</td>
<td>-25.99</td>
</tr>
<tr>
<td>trip using an automobile</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic coefficient for the in-vehicle travel time for transit trips across all</td>
<td>-0.02016</td>
<td>0.00179</td>
<td>-11.27</td>
</tr>
<tr>
<td>transit vehicles</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic coefficient for the parking cost at the destination for the any</td>
<td>-0.66413</td>
<td>0.01958</td>
<td>-33.93</td>
</tr>
<tr>
<td>auto drive trip</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Generic coefficient for the travel cost (excluding parking cost)</td>
<td>-0.07565</td>
<td>0.01107</td>
<td>-6.84</td>
</tr>
<tr>
<td>Generic coefficient for walk time for transit trips</td>
<td>-0.05405</td>
<td>0.00421</td>
<td>-12.85</td>
</tr>
<tr>
<td>Generic coefficient for wait time for transit trips</td>
<td>-0.16928</td>
<td>0.00959</td>
<td>-17.66</td>
</tr>
</tbody>
</table>

**Error Component Standard Deviations**

<table>
<thead>
<tr>
<th>Estimate</th>
<th>Std. Err</th>
<th>T-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person A drops off person B</td>
<td>1.88502</td>
<td>0.12144</td>
</tr>
<tr>
<td>Person B drops off person A</td>
<td>1.88502</td>
<td>0.12144</td>
</tr>
<tr>
<td>Joint drive transit access</td>
<td>1.61127</td>
<td>0.31014</td>
</tr>
<tr>
<td>Auto independent mode pairs</td>
<td>1.66521</td>
<td>0.09815</td>
</tr>
<tr>
<td>Auto dependent non-joint mode pairs (fixed at zero)</td>
<td>0</td>
<td>***</td>
</tr>
</tbody>
</table>

As with the previous nested logit, the facilitator/facilitated mode pairs were constrained to have equal error variances. Train (2009) notes that the correlation between two alternatives in the same nest \( n \) can be calculated using the following formula:
Where $\sigma^2_n$ is the variance of the common error component and $\frac{\pi^2}{6}$ represents the scaled variance for the type I Gumbel extreme value error component.

Given the improvements in model fit provided by the error component mixed logit relative to the nested logit in terms of log likelihood, rho squared, and AIC, it is reasonable to state that statistically the error component logit provides greater insights into the behaviours being modelled. More generally, this finding suggests that an error component logit is a distinct model from the nested logit, rather than an approximation of it (as has been previously noted by Brownstone and Train, 1998). This is highlighted by the distinctly different correlation patterns across alternatives achieved when using the error component model versus the homoskedastic nested logit. In this case both models provide different behavioural outcomes (based on correlation patterns) and the error component model provides improved fit relative to the nested model. This suggests then there is substantial impetus to make use of the model with the higher statistical fit for forecasting and policy analysis, less the forecasts be biased due to the restrictive homoskedastic nature of the nested logit.

### 4.5 Discussion of Results

In terms of the model results, the error component model structure is a complete reversal of the behaviour found in Figure 4.1 and Table 4.8 in terms of correlation but the parameter estimates for the systematic component of the utility remain relatively stable across all three models.

The value of travel time savings for time driving, transit in vehicle time, transit walk access/egress time and out of vehicle travel time are calculated for all 3 models by taking the ratio of the respective time and cost coefficient and converting into dollars per hour (from dollars per minute). The results of this calculation are presented in Table 4.10. These values are all relatively close to each other across all models. The values of time savings are generally slightly higher than what we would expect (with the exclusion of transit in vehicle time). These high values can be attributed to the higher income of two worker households, which make up a large portion of the sample used for this analysis. The relative values of time savings for waiting and
the travel options is also very high, though does show the expected trend (i.e. waiting is costed at a higher rate than travelling modes).

Table 4.10 Values of Travel Time Saving for Mode Pair Choice Models

<table>
<thead>
<tr>
<th>Value of time ($/Hour)</th>
<th>MNL</th>
<th>Nested</th>
<th>ECNL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Auto travel VOT</td>
<td>50.81</td>
<td>40.44</td>
<td>45.30</td>
</tr>
<tr>
<td>Transit in Vehicle VOT</td>
<td>17.11</td>
<td>13.40</td>
<td>15.99</td>
</tr>
<tr>
<td>Transit Wait VOT</td>
<td>164.35</td>
<td>162.10</td>
<td>134.26</td>
</tr>
<tr>
<td>Transit Walk VOT</td>
<td>48.14</td>
<td>44.97</td>
<td>42.87</td>
</tr>
</tbody>
</table>

4.5.1.2 General Parameter Interpretation

A consistent set of individual level variables are included in the systematic portion of the utility across all models. Based on existing research a dummy variable for being over the age of 65 is included in utilities for non-motorized travel, capturing the reluctance of older individuals to walk or bike, likely due to mobility issues that come along with growing older. Both transit walk and drive access had a positive coefficient for a dummy variable indicating if the individual possesses a transit pass. The model framework used in this chapter is unable to determine explicit mobility tool sharing at the level of transit passes, though this is certainly a possibility, particularly in households with part time workers or schedule heterogeneity over the course of a work week. This is an interesting avenue for future research. Of interest at the household level is the natural logarithm of the angle created by the two work locations and the home location as outlined in figure 4.3.
A dummy for households that are auto deficient is added to all joint travel utilities to capture the increased likelihood of joint travel. Coefficients for the dummy variable for if the individual is male are also included and showed intuitive results, namely that men are more likely to be facilitators than passengers for joint travel, men are less likely to take transit and men are more likely to use non-motorized travel (walking and biking). Individuals with homes located in the much denser and transit oriented City of Toronto proper are more likely to take transit. The parking cost (coefficient and variable are added to all drive all way modal options) is a deterrent to driving relative to non-auto centric modes, though a substantial proportion of individuals in the sample had free parking at their workplace/destination suggesting that under current conditions if free parking is available for employees driving becomes considerably more attractive for them. Unfortunately, the magnitudes of the parameter estimates have limited interpretive value on their own. As a result, many studies look at the relative values between parameter (as is done in Table 4.10 for value of travel time saving), or look at elasticities and marginal effects. Given the large number of alternatives in this study and the existing contributions to household travel behaviour and understanding the challenges associated with
capturing correlation between alternatives under the presence of heteroskedasticity, this analysis is left for future work.

4.5.2 Policy Implication

While not included in the presented application, it would be possible to utilize high occupancy vehicle (HOV) and single occupancy vehicles (SOV) assignment models to obtain a more complete picture of the time increases and/or cost savings associated with joint travel. This process would involve including an updated traffic assignment model which included HOV and SOV travel times and costs between each origin destination pair. Independent trips would use the SOV travel times and costs whereas the trips which are joint would use the HOV travel times. Once the passenger has been dropped off the driver would have to return to using SOV lanes and would therefore use SOV travel times in the utility for any subsequent travel. This is a very straight forward approach for understanding the potential implications of implementing HOV lanes and could be used to answer numerous questions regarding the efficacy of their introduction.

Specifically, questions regarding reduction in the number of single occupancy vehicle trips, vehicle kilometers travelled and corresponding environmental impacts could all be addressed using this framework. It is also worth noting is the integration of this model with the models presented in subsequent chapters can provide deeper insights into the impact of a wide array of different policies. Robust outlines of the integration processes and the potential policy implications of these integrations are provided in Chapters 5 and 6.

4.6 Chapter Conclusions

This chapter presents an innovative and novel approach to addressing concerns regarding vehicle allocation and joint trip making within the context of joint household mode choice for trips during the morning peak period. The primary contribution of the chapter is the *a priori* specification of the choice set as a means of capturing complex household decisions such as vehicle allocation and joint travel. The chapter’s findings contribute to a more robust understanding of how intra-household dynamics influence the travel behaviour of individuals. These findings are particularly insightful given that this analysis made use of RP Data where dynamics at the household level are not explicitly outlined in the survey data. Given the
significant percentage of households who are either auto deficient and/or chose to make joint trips in the morning peak period within the sample, the findings of this chapter are promising. The relevance of the geospatial findings relating to the detours associated with joint travel has strong implications for medium and long term decisions at the household level, such as home and work location as well as auto ownership. Ultimately, a model such as this could be used (with logsum parameters) to obtain a more robust understanding of these long-term decisions based on how they influence tradeoffs at the household level on a day to day basis.

Though only mentioned briefly, there exist several interesting policy applications for this model structure, most notably to examine the effectiveness of HOV lanes. It is possible that the increase in travel time for a driver associated with making a detour to drop-off a passenger can be reduced or in some extreme cases eliminated entirely. This is because the use of HOV lanes for the joint travel portion of the trip would result in travel time savings relative to independent travel. Given the high number of observed joint intra-household trips and the relatively low number of carpool trips in the dataset, this model structure presents significant opportunities for testing the viability of HOV lanes in the GTHA and potentially elsewhere.

In addition to these policy implications, this analysis has provided some insights into potential situations where the application of an error component mixed nested logit model may provide superior results relative to a conventional nested logit model. It is quite clear based on the increased goodness of fit that the ECNL model outperforms the conventional nested logit, though a more formal understanding of situations where the ECNL model would provide more behavioural realism is an interesting future research avenue. That said, in this context the behavioural outcomes of using the nested logit over the ECNL are potentially disastrous due to the different substitution patterns between the two models.

Finally, the current model structure is applied as a trip based mode choice model for the morning peak period, however it is possible that the model structure could be adapted to fit a more comprehensive tour based mode choice model for integration into an activity based modelling framework. As much of the work in intra-household interaction looks at activity based modelling frameworks, it seems logical that the presented model structure could be adjusted to match the requirements of a tour based mode choice model.
Looking forward, the next chapter of this thesis provides a more detailed analysis on transit trips with automobile access. Following the conclusion of this portion of the thesis, the station location choice model used to generate expected travel times and travel costs is determined to be too simplistic in terms of its capacity to capture correlation between alternatives. These limitations lead to the development of a new spatial correlation model which forms the basis of the next chapter.
Chapter 5
Park and Ride vs. Kiss and Ride Station Location Choice

The previous chapter looked at a specific choice context where choice travel could occur, though this chapter provides a much more detailed examination of a specific joint travel decision: the choice of kiss and ride station location choice. Through the course of investigating this specific choice context, another key modelling challenge arose: capturing spatial correlation between alternatives. As such this chapter aims to address two main goals: investigate the difference in modelling park and ride and kiss and ride station location choice and the empirical testing of a new method for capturing spatial correlation between alternatives. The proposed method, while initially designed to capture spatial correlation also found patterns of spatial heteroskedasticity across alternatives. The general finding of this analysis was that alternatives that are close together are more correlated and alternatives that are farther away from the decision maker’s home or reference location are more random. This investigation provides additional insights into how station choice occurs for such complex trips at the household and spatial levels. These findings indirectly lead to the hypothesis outlined in Chapter 6, namely that there is a spatial dimension to household interactions.

5.1 Introduction

Sustainable modes of travel are often difficult to model because they are frequently multimodal and/or may consist of auxiliary choices beyond the mode itself. Both park and ride (parking at the transit station and riding transit) and kiss and ride (being dropped off at the transit station) trips have these complexities. These modes are intrinsically multimodal, using both the personal automobile and transit services to complete the trip and the choice of transit access station has equal importance relative to the choice of the mode itself. Understanding how park and ride and kiss and ride transit station choices occur allows transit agencies to forecast boardings at new or existing stations or quantify the factors influencing the choice of one station over another. Furthermore, station choices play key roles in determining the travel time and cost associated with the choice of both the drive access and main mode, which are key inputs into any travel
demand modelling framework. Understanding how transit station choices for these trips occur can be used to shape transit policy and ultimately encourage transit ridership.

Significant efforts have been made towards understanding factors impacting station attractiveness at a pedestrian level (Higgins and Kanaroglou, 2016) (Jun et al. 2015) and the competition between park and ride and other access modes more specifically (Lin et al. 2014). Conversely, studies examining kiss and ride travel are significantly rarer within the literature. As kiss and ride trips often occur with a household member dropping off the transit rider (as opposed to a non-household member performing this task), there is an obvious intra-household travel behaviour associated with these trips. As such, understanding the intricacies of kiss and ride travel requires an understanding of intra-household interactions. The failure to capture such intricacies may lead to incorrect predictions in forecasting these low in frequency but increasingly important trips.

It is also important to obtain realistic substitution patterns between station locations when examining these choices. Any new station (or station improvement) will see boardings from two sources: new riders and existing riders who choose a new or different access station. It is imperative to accurately forecast from which stations the existing riders are coming as this allows transit agencies to plan service frequencies along transit lines and required parking infrastructure at a given station. The fundamental independence of irrelevant alternatives (IIA) assumption, which stems from the identical and independently distributed error terms inherent with logit models, may create biased forecasts in these situations. Specifically, this assumption means that if an attribute of one alternative is changed, all other alternatives will experience a uniform percentage change in probability (Train, 2009). While in some circumstances, this is a reasonable assumption; there are specific cases (such as location choice problems) where this is fundamentally incorrect. Because of the limitations of the IIA assumption, numerous alternative model specifications have been developed, which aim to circumvent this fundamental limitation. These models include the nested logit model, the generalized extreme value and the mixed logit model. The mixed logit model is particularly interesting due to the generality of the structure and the opportunity for unique sub models to be developed.
The summary of these issues is as follows: there is an interest in furthering the understanding of park and ride and kiss and ride station choice for two contexts. First, what are the implications of ignoring the intra-household interactions for kiss and ride trips and second, how can substitution patterns between distinct spatial alternatives in such choice contexts be captured. This chapter addresses these questions through an application of a park and ride and kiss and ride station location choice model for the Greater Toronto and Hamilton Area. The remainder of this chapter is organized as follow. First, a new model structure, the spatially weighted error correlation (SWEC) choice model is presented. Second, an overview of the TTS data used for an application of the model is discussed. Third, the models used in the application are presented and analyzed both in terms of substitution patterns and in terms of intra-household interaction for kiss and ride trips. Finally, the chapter concludes with an overview of the key findings and possible avenues for further research using the presented model structure.

5.2 Model Formulation

Given the limitations associated with the mixed structures for capturing spatial interdependence outlined in the previous section, a spatially weighted error correlation (SWEC) logit model is proposed. The proposed application is a variation on the widely known and used mixed logit model. This structure utilizes a Cholesky decomposition of a multivariate variance-covariance matrix of the errors of each alternative. This allows the draws used for the maximum simulated likelihood estimation procedure to occur from a set of standard univariant normal distributions. As such, the general model structure follows the standard error correlation structure using a Cholesky decomposition to represent the correlation between alternatives:

\[ U_{it} = V_{it} + \eta_{it} + \varepsilon_{it} \quad \forall \; i \in J \]

Where \( U_{it} \) equals the utility of alternative \( i \) for individual \( t \), \( V_{it} \) equals the systematic, linear-in-parameter component of the utility and \( \eta_{it} \) and \( \varepsilon_{it} \) terms represent the two components of the random disturbance/error term. The \( \varepsilon_{it} \) component of the error term is type I Gumbel extreme value which is identically and independently distributed across all alternatives \( J \) with each error having a mean of zero. This component of the error structure leads to the well-known multinomial logit formulation. Conversely the \( \eta_{it} \) component represents a normally distributed
term with a mean of zero. Unlike the $\varepsilon_{lt}$ term, the normally distributed terms are jointly
distributed using a multivariate normal distribution:

$$
\begin{bmatrix}
\eta_{1t} \\
\vdots \\
\eta_{Jt}
\end{bmatrix} = \text{Multivariate Standard Normal with Covariance } \Omega = 
\begin{bmatrix}
\sigma_{11} & \cdots & \sigma_{1l} \\
\vdots & \ddots & \vdots \\
\sigma_{J1} & \cdots & \sigma_{ll}
\end{bmatrix}
$$

As mentioned above, to facilitate estimation using a maximum simulated likelihood approach,
drawing directly from the joint distribution is impractical, particularly as the choice set increases
in size. As a result, the covariance matrix is decomposed into a lower triangular
Cholesky matrix $T$. This allows us to reconsider the above equation using the following structure:

$$
U_{lt} = V_{lt} + \xi_{lt} + \varepsilon_{lt} \forall i \in J
$$

Where:

$$
\begin{bmatrix}
\xi_{1t} \\
\vdots \\
\xi_{Jt}
\end{bmatrix} = T \ast \begin{bmatrix}
S_1 \\
\vdots \\
S_f
\end{bmatrix}
$$

and $S_i$ is a standard normal random variable and $T$ is a lower triangular Cholesky matrix of
factors (parameters and variables) to be specified and estimated by the modeller. This
formulation allows the modeller to make random draws from a standard normal distribution for
the estimation process.

Before elaborating further on the specification, it is important to highlight some key
issues/opportunities that arise from the use of the Cholesky matrix. First, as noted by Brownstone
and Train (1999) the mixed logit is a highly flexible model structure, which can be used to
approximate any other model structure (most notably in their case, the nested logit). This
approximation is accomplished through specification of the Cholesky matrix. The ability of the
mixed logit to accurately approximate other closed form model structures is countered by the
work of Munizaga and Alvarez-Daziano (2002), who note the heteroskedasticity associated with
utilizing a mixed logit formulation (compared to the nested logit which is homoskedastic). This
finding agrees with the work presented in Chapter 4, where the error component and nested
logits that were found to provide the best fit for the data were considerably different from each other due to the introduction of heteroskedasticity.

Though not relevant to this discussion, Munizaga and Alvarez-Daziano also note that in their empirical exercise, their heteroskedastic mixed logit does manage to achieve similar market shares to the nested logit. The reason we observe heteroskedasticity in the mixed logit is highly intuitive: we are adding more and more draws to the utilities lower down the Cholesky. Note that this is only the case if a full specification of the Cholesky is undertaken. When the off diagonals are not specified (as is the case in the error component nested logit), heteroskedasticity also occurs. This is because each nesting error will have a different estimated standard deviation and therefore variance. If the standard deviation of the errors for two alternatives are different, then by definition, heteroskedasticity follows.

Bhat and Srinivasan (2005) showed how homoskedasticity can be imposed, whereby the diagonal of the Cholesky is equal to the square root of one minus the sum of the squared values of the off diagonal for that given row. Unfortunately, if the sum of the squared values of the off diagonal for any row exceeds one, the model estimation process collapses; an issue which becomes increasingly likely as the number of alternatives to be considered increases. As a result, it may be appropriate to consider a heteroskedastic case for the SWEC model or more generally, mixed logit models with full Cholesky matrix specification. Empirical tests should be done for the given choice context as the data or behavioural context in question may be better represented by a homoskedastic model. As per the discussion in Chapter 4, if the choice context is indeed heteroskedastic then this structure of the mixed logit may result in improvements in model fit and different behavioural findings relative to a homoskedastic GEV structure.

Given the heteroskedasticity associated with the mixed logit, several questions must first be addressed. First, given that the order in which the alternatives are considered influences the variance of the error of that alternative’s utility, what is the most appropriate sequence for the alternatives? In the case of park and ride and kiss and ride station location choice (or many other spatial choices), an interesting possibility emerges. Not only are alternatives correlated based on the distances between them but it is feasible to consider heteroskedasticity based on the relative distance from the location of the decision maker. Specifically, if the alternatives are farther away
from the decision maker then there is more randomness (and therefore higher variance) associated with the choice of this alternative. If this hypothesis is correct, then ordering the alternatives relative to their distance from the origin of the decision maker should produce an improved model fit relative to considering a random order of alternatives. Secondly, what is an appropriate parameterization of the diagonal and off diagonal cells of the Cholesky matrix?

$$T = \begin{bmatrix} \pm 1 & 0 & 0 \\ ? & ? & 0 \\ ? & ? & ? \end{bmatrix}$$

The choice of the structure of the Cholesky matrix should aim to capture the heteroskedastic trend and the correlation/covariance between more proximate alternatives. To accomplish this within the context of a Cholesky matrix is extremely challenging as there is no direct interpretation of the variables present in the Cholesky matrix. Specifically, the diagonals and off diagonals have influence on the variance and covariance of several different alternatives, clouding what the appropriate specification might be. This implies that a full parameterization of the Cholesky matrix may be required. The main concern with a full parameterization is one of practicality that the number of parameters explodes as the number of alternatives increases (at a rate of $J \times (J - 1) - 1$ where $J$ is the number of alternatives). Furthermore, as noted by Walker et al. (2007) and discussed in the previous chapter, the number of parameters that can be estimated in the error component of mixed logit models is typically constrained by rank and order conditions, potentially making a full parameterization infeasible. The derivation of the rank and order conditions for the two more complex forms of the five alternative SWEC can be found in Appendix B. Five alternatives are used as the empirical analysis in this chapter make use of that number of alternatives.

As noted in appendix B, the SWEC model is identified with a single parameter on the diagonal. As will be discussed below, more complex formulations of the SWEC further parameterize this diagonal based on variables specific to both the individual and alternative in question. To weight the spatial correlation between alternatives, the off diagonals are then specified as an estimated parameter times some function of the distance between the two stations represented by that off diagonal position. To capture Tobler’s (1970) rule, we set this function to be the inverse of some function of the distance between stations. The inverse of distance is used because this functional
form will create higher correlation for distances which are closer together. The number of off
diagonal parameters that are estimated is debatable, though the basic SWEC model will use a
single parameter for all off diagonals, with more advanced forms of the model adding more
parameters. Finally, the diagonal terms should be specified as a single set of parameterized
values, which can then be interacted with variables from the dataset at the modeler’s discretion.

In the empirical example presented below this parameterization occurred using the drive access
time to the station, producing significant improvements in model goodness of fit. This
interaction allows the variance of alternatives to vary (heteroskedasticity) while still maintaining
an identified model structure (see Appendix B). This discussion is relatively ambiguous which
leaves the Cholesky structure as a very general formulation. As a rule, the Cholesky structure
should at the very least follow the following parameterization, though further complications such
as those discussed above can be included:

$$
T = \begin{bmatrix}
    b_d & 0 & \cdots & 0 \\
    b_w \left[ \frac{1}{f(d_{21})} \right] & b_d & \ddots & \vdots \\
    \vdots & b_w \left[ \frac{1}{f(d_{12})} \right] & \ddots & 0 \\
    b_w \left[ \frac{1}{f(d_{12})} \right] & b_w \left[ \frac{1}{f(d_{12})} \right] & \cdots & b_d
\end{bmatrix}
$$

Where $T$ is the Cholesky matrix which is being used $b_d$ and $b_w$ are the parameters for the diagonal and
off diagonal respectively and $f(d_{ij})$ is some function of the distance between alternatives $i$ and $j$ (row
and column of the matrix).

At this point is important to note that the SWEC parametrizes the Cholesky matrix of the
variance covariance matrix and not the full matrix itself. This is done to facilitate deriving the
simulated log likelihood function as it is difficult to make draws from a multivariate normal
distribution. The result of using the Cholesky matrix is that the variance covariance matrix will
be some function of the functions specified in the Cholesky. This means that to determine the
variance covariance matrix of the alternatives (which is the goal of specifying the Cholesky), the
analyst will need to multiply the choleksy by its transpose.
Ultimately, the appropriate SWEC model structure for a choice context must be determined empirically. Numerous partial parameterizations/specifications of the Cholesky matrix following the guidelines above should be attempted. Ultimately, the parameterization which provides the optimal fit (in terms of Akaike information criterion (AIC) and/or Adjusted Rho squared) should be selected. Section 4 provides a detailed overview of the different parameterizations attempted in this study.

5.3 Data Description

As with the other empirical investigations in this thesis, this analysis made use of the TTS dataset for the City of Toronto. The analysis is restricted to considering commuting trips using the park and ride and kiss and ride modes. The choice set for each commuter is defined as the 4 closest unselected transit (either GO rail or subway) stations to their home location based on drive access time and the station selected by the respondent irrespective of the drive access time. For forecasting purposes, the 5 closest stations can be used. A map outlining the location and some key characteristics of the stations is presented in Figure 5.1. This is done as the clear majority of the park and ride and kiss and ride trips are selected from one of the 5 closes stations, which supports the theory that the consideration set for station location choice is formed based on proximity to home. In the cases where the selection falls outside of this region, the selected station is treated as the farthest away (i.e. alternative 5). It should be noted that 82% of all station choices in the pooled model are from the set of the four closest stations, suggesting that this decision rule is reasonable for capturing the consideration set. Expanding the number of closest stations considered provides diminishing returns in terms of capturing a higher percentage of the populations choice. Furthermore, expanding the consideration set to a larger number of stations could introduce bias as the consideration set may then include stations not actually considered by the respondents. Future work would consist of using a more robust rule based method for consideration set formation, however, for the purposes of the subsequent modelling exercise, the proposed method is deemed to be sufficient.
Four possible types of trips were observed in the TTS data used for this analysis:

1. **Pure park and ride trips**, where the commuter drove and parked at the station by him or herself (3026 records).

2. **Carpool – passenger transit access trips**, where a non-household member drove the transit rider to the station, (413 records). It is assumed that the clear majority of these trips involve both the driver (non-household member) and passenger taking transit together for their trip rather than the driver continuing to their final destination by car. This make this case analogous to trips in category four. Unfortunately, because of the limitations of the TTS (no travel information for non-household members), this information is not available.

3. **True household kiss and ride trips**, where a household member dropped off the commuter at the transit station before driving to a different location (577 records).

Figure 5.1 Distribution of Park and Ride and Kiss and Ride Stations Across the GTHA
4. Joint park and ride, where both household members drove to a transit station and both rode transit to their destination (191 records).

Of these four classes of trips, this study is explicitly interested in cases one, three and to some extent four. Category two trips had insufficient driver information to estimate the kiss and ride model and thus are excluded from that analysis. The fourth case of trips present a different set of tradeoffs from conventional inter-household kiss and ride trips and while interesting, had too few records to develop a separate model. As these trips are not able to integrate smoothly with the more conventional case three trips, these trips are excluded from the analysis. Despite this exclusion, joint drive access and to an even greater extent, joint drive egress from transit, present an interesting avenue for future research. There are also a small number of trips for which no travel time or cost data existed due to the origin not being included in the multimodal traffic assignment model used for time and cost generation. Finally, a small number of trips were treated as erroneous due to the reported station not having explicit parking or drop-off facilities. While not strictly impossible (the passenger could have been dropped off at the road or even at the station parking lot), these trips were removed from the analysis due to their relative low number and the complications that these trips present. Specifically, the lack of direct drop off facilities at the station make the walk time for these trips to access the train platform much more variable than the walk time for those dropped off at a designated facility.

From this subsample, two datasets were generated. First a pooled dataset whereby the park and ride and kiss and ride trips were mixed though a dummy variable for parking cost at the station. This parking cost dummy variable is used to distinguish between park and ride and kiss and ride options. In total, the pooled dataset contains 3603 records. A kiss and ride only dataset is also used, allowing a comparison of how a pooled model (conventional practice, in demand modelling) compares to the kiss and ride only model. This kiss and ride only dataset contains 577 records.

As noted in the introduction, the assignment model used to generate travel times and travel costs does not differentiate between regional rail and local/subway transit at the mode choice level. Instead the choice of transit service is done at the route choice level creating generic time and cost matrices for transit. While there are arguments for and against this approach, the
implications of this for this analysis are that the type of transit service used at a park and ride or kiss and ride station are unclear. For example, an individual might park at a GO rail station but ride the local bus to their final destinations. The TTS survey data does include information regarding the selected service for the chosen station, though this information is obviously not available for the unselected station. This means that we cannot be explicitly sure of the actual transit service that is used at any given station (other than the selected station). Despite these limitations there is a high probability that the subway or GO rail service is selected, prompting the inclusion of service frequencies in the utility function for each station. In cases where both services exist at a single station, it is assumed the cheaper and higher frequency subway service is preferred. To capture the differences between the services, the frequency of service (at an hourly rate) for the subway (25.5 trains an hour) and specific GO rail lines (ranging from 4 to 17 trains an hour) are included in the station utility. A summary of the descriptive statistics of the variables that are used in the model estimation process is presented in Table 1.

Table 5.1 Descriptive Statistics of Study Area and Park and Ride/Kiss and Ride Dataset Used for Estimation

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average drive access time for pooled dataset (park and kiss and ride)</td>
<td>12.39</td>
</tr>
<tr>
<td>Average transit travel time (walk, wait and in vehicle) for pooled dataset</td>
<td>87.80</td>
</tr>
<tr>
<td>Average travel cost for pooled dataset</td>
<td>4.84</td>
</tr>
<tr>
<td>Average Work-Home-Station Angle for Pooled Dataset (See Figure 3-a)</td>
<td>80.13</td>
</tr>
<tr>
<td>Average distance between any two station for pooled dataset</td>
<td>13.63</td>
</tr>
<tr>
<td>Average drive time for household in kiss and ride dataset</td>
<td>32.33</td>
</tr>
<tr>
<td>Average transit travel time in kiss and ride dataset</td>
<td>85.02</td>
</tr>
<tr>
<td>Average household travel cost for kiss and ride dataset</td>
<td>5.83</td>
</tr>
<tr>
<td>Average angle sum for pooled dataset (See Figure 3-b)</td>
<td>121.70</td>
</tr>
<tr>
<td>Average distance between any two station for kiss and ride dataset</td>
<td>13.19</td>
</tr>
<tr>
<td>Average parking cost across all TTC subway stations</td>
<td>4.65</td>
</tr>
<tr>
<td>Percentage of all stations with washroom</td>
<td>92</td>
</tr>
<tr>
<td>Average lot capacity across all stations</td>
<td>1184.4</td>
</tr>
</tbody>
</table>

5.4 Model Estimation Results

The modelling exercises presented here aim to accomplish two main goals. First, the models are an application of the proposed SWEC model structure and a comparison of the models to the traditional MNL formulation. As discussed earlier, the SWEC model is a relatively generalized model, whereby the specification of the diagonal and off diagonal terms in the Cholesky will impact both the variance and covariance across multiple alternatives. Several different
specifications of the SWEC in the context of park and ride choice are presents. The models presented are described in Table 2.

The second goal relates to capturing the intricacies and tradeoffs at the household level with respect to kiss and ride travel. Two datasets are used in the estimation of the SWEC models. First a pooled dataset, containing the park and ride and kiss and ride trips is used. In this dataset, no information regarding the driver of the kiss and ride commuter is included in the utility formulation. The second dataset is the kiss and ride only subset of the pooled dataset. In this dataset, the characteristics of the subsequent trip of the driver are included. The models are compared both in terms of general goodness of fit and based on a discussion of the variables included in both models. The results for the pooled model and the kiss and ride only model are presented in Tables 5.3 and 5.4 respectively. Each of the SWEC model structures is estimated in GAUSS (Aptech Systems, 2015). The maximum simulated likelihood estimation technique with a scrambled Halton sequence for the random number generation (Bhat, 2003) is applied for maximum simulated likelihood. Models two and three were tested with 50, 100 and 200 and 300 draws and seemed to reach convergence after 200 draws. The 300 draw model results are presented. Models four, five and six took longer to reach convergence, providing a reasonable degree of convergence after 800 draws. The results for the 800 draw models are presented for these models. For other empirical investigations using the SWEC structure, robust testing of the number of draws required is recommended.
Table 5.2 Description of models included in the analysis

<table>
<thead>
<tr>
<th>Model Index</th>
<th>Model Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1*</td>
<td>Standard MNL</td>
</tr>
<tr>
<td>2</td>
<td>Randomly scrambled the order of the alternatives, then estimated SWEC with single parameter on the diagonal of the Cholesky and a single parameter divided by a function of distance on the off diagonal</td>
</tr>
<tr>
<td>3</td>
<td>SWEC with ordered alternatives based on proximity to the origin with a single parameter on the diagonal</td>
</tr>
<tr>
<td>4*</td>
<td>SWEC with ordered alternatives based on proximity to the origin with a single parameter multiplied by the difference in access time</td>
</tr>
<tr>
<td>5*</td>
<td>SWEC with ordered alternatives based on proximity to the origin with a single parameter multiplied by the difference in access time. The off diagonal of the Cholesky is then given a unique column parameter</td>
</tr>
<tr>
<td>6**</td>
<td>SWEC with ordered alternatives based on proximity to the origin with a single parameter multiplied by the difference in access time. The off diagonal is multiplied by a dummy for if the column and row are either subway or Go Transit Stations</td>
</tr>
</tbody>
</table>

*Models with * are also estimated for the kiss and ride only station choice
**Model 6 is estimated only for kiss and ride station location choice

More formally and for the reference of the reader, models 3 through 5 make use of the following Cholesky factors:

Model 3:

\[
T_4 = \begin{bmatrix}
  b_d & 0 & \cdots & 0 \\
  b_w \left[ \frac{1}{\sqrt{(d_{21})}} \right] & b_d & \ddots & \vdots \\
  \vdots & b_w \left[ \frac{1}{\sqrt{(d_{i1})}} \right] & \ddots & 0 \\
  b_w \left[ \frac{1}{\sqrt{(d_{j1})}} \right] & b_w \left[ \frac{1}{\sqrt{(d_{j2})}} \right] & \cdots & b_d 
\end{bmatrix}
\]
Here, as above, $T$ is the Cholesky matrix which is being used for estimation, $b_d$ and $b_w$ are the parameters for the diagonal and off diagonal respectively and $f(d_{ij})$ is some function of the distance between alternatives $i$ and $j$ (row and column of the matrix). For models 5 and 6 $td_j$ is the from the origin (home location) to the park and ride station $j$. 

<table>
<thead>
<tr>
<th></th>
<th>$b_d$</th>
<th>$0$</th>
<th>$\ldots$</th>
<th>$0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_{w1}\left[ \frac{1}{\sqrt{(d_{21})}} \right]$</td>
<td>$b_d * (1 + td_2 - td_1)$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td></td>
</tr>
<tr>
<td>$b_{w2}\left[ \frac{1}{\sqrt{(d_{ij})}} \right]$</td>
<td>$\vdots$</td>
<td>$0$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b_{w1}\left[ \frac{1}{\sqrt{(d_{j1})}} \right]$</td>
<td>$b_{w2}\left[ \frac{1}{\sqrt{(d_{j2})}} \right]$</td>
<td>$\ldots$</td>
<td>$b_d * (1 + td_j - td_1)$</td>
<td></td>
</tr>
</tbody>
</table>
Table 5.3 Parameter Estimates for the Pooled Park and Ride and Kiss and Ride Models (T Stats in parenthesis)

<table>
<thead>
<tr>
<th>Attribute/Model</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
<th>Model 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>-3357</td>
<td>-3354</td>
<td>-3295</td>
<td>-3012</td>
<td>-3008</td>
</tr>
<tr>
<td>Adjusted Rho-Squared</td>
<td>0.419</td>
<td>0.421</td>
<td>0.432</td>
<td>0.481</td>
<td>0.481</td>
</tr>
<tr>
<td>AIC</td>
<td>6732</td>
<td>6732</td>
<td>6612</td>
<td>6045</td>
<td>6043</td>
</tr>
<tr>
<td>Expected drive time from home to the transit station (produced by traffic assignment model)</td>
<td>-0.0928 (-13.08)</td>
<td>-0.0937 (-12.932)</td>
<td>-0.1142 (-12.31)</td>
<td>-0.2551 (-13.394)</td>
<td>-0.2641 (-12.691)</td>
</tr>
<tr>
<td>Expected transit wait, walk, transfer and in vehicle time from the station to the destination (produced by the traffic assignment model)</td>
<td>-0.0284 (-13.61)</td>
<td>-0.0285 (-13.523)</td>
<td>-0.0276 (-10.847)</td>
<td>-0.0279 (-7.84)</td>
<td>-0.0284 (-7.709)</td>
</tr>
<tr>
<td>Expected travel cost for the transit trip (fare + drive cost to the station) for the trip (produced by the traffic assignment model)</td>
<td>-0.489 (-10.77)</td>
<td>-0.5038 (-10.886)</td>
<td>-0.4621 (-8.186)</td>
<td>-0.7009 (-8.159)</td>
<td>-0.778 (-8.537)</td>
</tr>
<tr>
<td>Parking cost at the station (set to zero for kiss and ride travellers irrespective of station cost)</td>
<td>-1.468 (-30.29)</td>
<td>-1.4776 (-30.306)</td>
<td>-1.6577 (-29.812)</td>
<td>-2.314 (-17.901)</td>
<td>-2.3679 (-17.229)</td>
</tr>
<tr>
<td>Dummy variable for if the station is on the subway</td>
<td>1.715 (7.18)</td>
<td>1.7814 (7.297)</td>
<td>1.959 (6.511)</td>
<td>1.5741 (3.804)</td>
<td>2.0847 (4.819)</td>
</tr>
<tr>
<td>Natural logarithm of the parking lot capacity</td>
<td>0.386 (9.21)</td>
<td>0.3938 (9.295)</td>
<td>0.5184 (9.5)</td>
<td>0.6651 (9.096)</td>
<td>0.6832 (8.932)</td>
</tr>
<tr>
<td>Dummy variable for if the station has a washroom</td>
<td>0.864 (8.17)</td>
<td>0.8648 (8.04)</td>
<td>0.9413 (6.384)</td>
<td>1.2918 (7.267)</td>
<td>1.4119 (7.346)</td>
</tr>
<tr>
<td>Dummy variable for if the station-home-work angle is greater than 90</td>
<td>-0.0631 (-1.04)</td>
<td>-0.0609 (-0.992)</td>
<td>-0.208 (-2.539)</td>
<td>-0.3991 (-4.046)</td>
<td>-0.4599 (-4.314)</td>
</tr>
<tr>
<td>Natural logarithm of the frequency of trains at the station per hour</td>
<td>0.827 (9.59)</td>
<td>0.8311 (9.53)</td>
<td>0.9675 (8.92)</td>
<td>1.0243 (7.343)</td>
<td>1.0385 (7.122)</td>
</tr>
<tr>
<td>Off diagonal Cholesky parameter (enters Cholesky as ( \frac{b}{\sqrt{d_{ij}}} ) where ( i ) is the column index, ( j ) is the row index, ( d_{ij} ) is the distance between alternatives ( i ) and ( j ) and ( b ) is the estimated parameter)</td>
<td>---</td>
<td>0.5442 (1.6)</td>
<td>3.7125 (14.627)</td>
<td>3.0299 (8.526)</td>
<td>C1 3.4797 (8.12)</td>
</tr>
<tr>
<td>Diagonal Cholesky (parameter times one plus the difference in access travel time between the first alternative and the alternative in the row in question for models 4 and 5)</td>
<td>---</td>
<td>0.162 (1.305)</td>
<td>0.6764 (4.458)</td>
<td>0.2116 (15.474)</td>
<td>C2 3.8435 (6.434)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>C3 2.1733 (2.85)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>C4 2.8938 (3.792)</td>
</tr>
</tbody>
</table>

*Note, C1 through C4 represent the column specific off Diagonal Cholesky Matrix parameters*
Table 5.4 Parameter Estimates for Kiss and Ride Only Models (T Stats in parenthesis)

<table>
<thead>
<tr>
<th>Attribute</th>
<th>Model</th>
<th>1</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td></td>
<td>-389</td>
<td>-374</td>
<td>-367</td>
</tr>
<tr>
<td>Adjusted Rho-Squared</td>
<td></td>
<td>0.5746</td>
<td>0.597</td>
<td>0.605</td>
</tr>
<tr>
<td>AIC</td>
<td></td>
<td>790</td>
<td>765</td>
<td>750</td>
</tr>
<tr>
<td>Expected Drive Time from Home to the Transit Station plus the drive time for the driver to their subsequent destination (produced by traffic assignment model)</td>
<td></td>
<td>-0.081(-8.61)</td>
<td>-0.2359(-6.567)</td>
<td>-0.1465(-6.723)</td>
</tr>
<tr>
<td>Expected Transit Wait, Walk, Transfer and in vehicle time from the Station to the destination (produced by the traffic assignment model)</td>
<td></td>
<td>-0.032(-4.33)</td>
<td>-0.0539(-3.786)</td>
<td>-0.041(-3.767)</td>
</tr>
<tr>
<td>Expected travel cost (fare + drive cost) for the entire trip, including the subsequent drive trip (produced by the traffic assignment model)</td>
<td></td>
<td>-1.682(-9.09)</td>
<td>-2.7164(-7.723)</td>
<td>-2.047(-8.204)</td>
</tr>
<tr>
<td>Dummy variable for if the station is on the subway</td>
<td></td>
<td>4.180(10.00)</td>
<td>5.0164(7.379)</td>
<td>3.5529(6.684)</td>
</tr>
<tr>
<td>Dummy variable for if the station has a washroom</td>
<td></td>
<td>1.283(5.02)</td>
<td>2.1672(5.932)</td>
<td>1.6574(5.084)</td>
</tr>
<tr>
<td>Natural logarithm of the frequency of trains at the station per hour</td>
<td></td>
<td>1.242(4.81)</td>
<td>2.8267(5.652)</td>
<td>2.0822(5.962)</td>
</tr>
<tr>
<td>Off Diagonal Cholesky Parameter (enters Cholesky as ( \frac{b}{\sqrt{(d_{ij})}} ) where ( i ) is the column index, ( j ) is the row index, ( d_{ij} ) is the distance between alternatives ( i ) and ( j ) and ( b ) is the estimated parameter)</td>
<td></td>
<td>---</td>
<td>0.7341(0.956)</td>
<td>1.0072(0.935)</td>
</tr>
<tr>
<td>Diagonal Cholesky (parameter times one plus the difference in access travel time between the first alternative and the alternative in the row in question for models 4, 5 and 6)</td>
<td></td>
<td>---</td>
<td>0.2099(4.541)</td>
<td>0.1509(5.652)</td>
</tr>
</tbody>
</table>

5.5 Discussion of Results

When examining the model results, several interesting trends emerge. The overall goodness of fit of the models improves as further complexity is added to the Cholesky. Furthermore, in the pooled model, the angle dummy parameter becomes significant as the spatial correlation and heteroskedasticity between alternatives are captured. As discussed in the model formulation section, the order in which the utilities are considered has significant relevance to the overall model fit. This is illustrated by the comparison between models two and three in Table 5.3 (the pooled table). Remembering that model two randomly scrambled the alternatives and model 3 ordered them from closest to furthest away we find that model three outperforms model two in terms of adjusted rho squared and AIC. The AIC can be calculated as follows:
\[ AIC = 2 * k - 2 * \ln (L) \]

Where \( k \) is the number of estimated parameters and \( L \) is the model’s likelihood function at convergence.

This comparison of the rho squared and AIC values for models 2 and 3 suggests that the order in which the utilities are considered has significant impact on the model outcome when using the SWEC formulation. Alternatives which are expected to have higher variance should be considered later/lower in the Cholesky matrix as failure to do so may result in a loss of model fit based on this analysis. More generally, unless the homoskedastic correction suggested by Bhat and Srinivasan (2005) is applied, this finding can be extended to other mixed logit models where the off diagonal of the Cholesky is fully parametrized. This is because the variance is equal to the sum of the square of the value in each row. Moving down the Cholesky matrix, there are more values and therefore the variance is expected to be larger. It should be noted that the homoskedastic correction of Bhat and Srinivasan (2005) requires that the sum of the squares of the off diagonal parameters be less than 1 for every row/alternative in the Cholesky matrix. As the matrix becomes larger, this restriction becomes harder to accomplish and the attempt to impose homoskedasticity even with the five-alternative dataset resulted in the model crashing as the sum of the squares exceeds 1. Attempts to introduce this homoskedastic correction for the sample used in this analysis resulted in no convergence.

Conversely it could be argued that the homoskedastic correction is undesirable. The concept that alternatives that are father away have a higher variance is highly intuitive. This finding is can be used to describe a second law of geography (to accompany Tobler’s (1970) first law) for spatial choices:

“When making a choice from a given reference point, options that are far away are more random than options that are close by”.

In this context there is more uncertainty (randomness) associated with higher variance choices within the model, which reflects the higher degree of uncertainty associated with the more distant decision. As the decision makers consider alternatives farther and farther away from their current
location, it could be hypothesized that they are less clear on the specific attributes of the surrounding area (roads, shops, etc.). An alternative explanation for this trend is that stations that are farther away from the commuter will typically be suboptimal relative to closer stations. This suggests that commuters that do select those stations have a reason for their selection that is not captured in the systematic component of the utility. As a result, treating these alternatives as more random (i.e. higher variance) is a plausible and consistent approach that results in improved models fit.

This finding is further supported by comparing models four and five to models one two and three. Note that the more detailed parameterization of the Cholesky matrix allows for further understanding of the heteroskedasticity and covariance between different alternatives. This is accomplished by having the diagonal of the Cholesky vary based on the drive access time to the transit station (minus the first alternatives drive access time to act as a reference). The increased goodness of fit of this specification further confirms that alternatives which are farther away from the home of the decision maker have higher variance. The variance covariance matrix of the errors is obtained by multiplying the Cholesky matrix by its transpose. This means that any cell in the Cholesky matrix can impact the variance of and covariance between several different alternatives. As an attempt to account for these impact of the covariance, column specific off diagonal parameters are used in model five. Unfortunately, this specification provided almost no improvement in fit relative to model four, suggesting that model four is the preferred option.

It should also be noted that the signs of the parameter values for the cholesky are effectively meaningless assuming all the parameters share the same sign. This is because each of the parameters (or the corresponding functions associated with each of these parameters) is multiplied by a draw from a standard normal distribution, which can be thought of as being constrained between -3 and 3 with an expected value of zero. This means that the negativity of the parameters has no impact on the actual value being added to the distribution. Negative covariance (and therefore correlation) can occur if the signs of some parameters are positive and others are negative, which is not an expected behavioural outcome.

An interesting finding associated with the kiss and ride only models is that the proposed SWEC models (model four) only resulted in marginal improvements relative to the standard MNL
Furthermore, it should be noted that the off diagonal parameters estimated for model four were insignificant, further raising concerns regarding the SWEC formulation for kiss and ride choice. This can be attributed to several factors, most notably the lower sample size used for the kiss and ride only model. An alternative behavioural explanation is that the correlation between spatial alternatives is not as pronounced due to the complexity associated with the kiss and ride choice. It is possible that there are other extenuating factors that cause certain alternatives to be selected over others, that make what would normally be highly correlated alternatives uncorrelated. One possibility is that those that engage in kiss and ride trips are single vehicle households and are thus constrained in terms of household budget for travel expenses. This becomes a factor due to the variable price scheme present in the region: the TTC subway is $3.00 CDN flat fare while the Go transit service is typically much costlier for a similar length trip. This could suggest that subway stations could be significantly less correlated with proximate go stations for this segment of the population and vice versa. To account for this, the formulation of model six is applied, which provides SWEC based distance decay covariance only between stations which are either both on the subway or both on the GO rail network. As can be seen when comparing the results between models five and six (which only allows correlation between subway stations or GO Train stations), this seems to be a reasonable assumption and a plausible explanation for the improvement in goodness of fit. Despite these improvements in model fit the off diagonal parameter is still insignificant for model six. This can again be attributed to the smaller sample size for the kiss and ride only analysis.

In terms of parameter specific values, all the parameters reported had the correct sign and were significant. Travel time for the access to the station proved to have a larger negative parameter value relative to the transit travel time from the station to the destination. This is somewhat intuitive as commuters using these services may make use of the train portion of their trip to read or potentially even catch up on sleep, activities they are less able to do as drivers. Both are negative values, suggesting that stations that are easy to get to and stations with provide quick transit service to the commuters’ destinations are important. First, as can be seen in Figure 5.2, the angle variable, $\theta$, in the pooled model represents the angle created by connecting straight lines to the home from the station and the final destination. This variable captures the relative direction of the station between the home and the workplace. If this angle is above 90 degrees, then the access to the station results in the commuter moving away from the final destination to
access the transit station, which has a negative impact on the overall attractiveness of the station. In the context of kiss and ride trips, the angle variable (and an additive angle value that considered the same value for the driver, considering their final destination) were found to be insignificant and thus not reported. This further supports that kiss and ride choices involve considerable tradeoffs between the driver and transit rider which require further consideration to capture.

![Diagram](image)

**Figure 5.2 Angle Between the Final Destination of the Transit Rider, the Home and the Transit Station**

Continuing with the theme of examining the impact of the kiss and ride trip on the driver, the kiss and ride only model updates several the variables initially included within the more standard pooled model. Travel time is updated such that the access time to the station is counted as double. This is done due to the presence of two individuals in the vehicle (the driver and the passenger). Furthermore, the drive time and travel cost variables are also incremented to reflect the time and cost of the subsequent trip made by the driver after dropping off the passenger. This
type of specification captures a more complete picture of household expenditure in terms of time and cost based on different kiss and ride station options. In the presented models, single generic household drive time and travel cost coefficients are used and are presented in Table 5.4 as the final model specifications. A potential extension would be to include different individual weights for household members as is applied in Chapter 7. Overall the kiss and ride only model improves drastically (in terms of overall goodness of fit) on the pooled model for capturing kiss and ride travel behaviour due to its ability to capture inherent household level tradeoffs. This improvement is attributed to the kiss and ride only model explicitly considering the total household level resource expenditure on the trip. This formulation explicitly considers the households desire to minimizing the total detour relative to each household member’s final destination.

A limitation of this analysis is the absence of variables related to the schedules of each transit user and in the case of kiss and ride trips, the driver dropping off the passenger. The model does not consider the implications of the return trip. While not performed in the analysis presented here, the travel times and travel cost variables could be updated to include the sum of the values for the morning and evening peak period trips. For the kiss and ride model, a joint choice (either directly through the modelling framework or using a Gibbs sampling approach) of the morning station and evening station choice could be performed as kiss and ride trips are not constrained to occur at the same station because the vehicle used to access the station is not parked at the station. As the primary focus of this chapter is on spatial correlation and heteroskedasticity, this task is left for future work.

5.5.2 Potential Policy Analysis
The practical applications of this modelling framework involve integrating such models into a comprehensive model of travel demand – particularly within the mode choice component of such a model. Effectively, models of station location choice and mode choice could be applied iteratively within a traffic assignment model to determine both the change in mode share and ridership attraction at a station by station level. At a significantly more basic level, the SWEC model could be used to capture potential ridership shifts from minor or temporary changes due to construction or other factors. This could provide transit operators with a sense of the impact of
such changes as the temporary closing down a parking lot at a station for maintenance or construction. While this short-term closure would certainly result in a shift away from park and ride as a mode for some commuters, a certain component of the demand would shift from the closed station to other surrounding stations. Because of the IIA property of the MNL, the MNL would provide biased predictions of the magnitude of this shift. Conversely, the SWEC would provide a more realistic picture of the impact of this demand. For further robustness, a mode choice model (such as the one presented in Chapter 4) could be either estimated jointly or sequentially with the station location choice. In this framework, the expected maximum utility of the station choice would be fed into the choice for both park and ride and kiss and ride transit. This would allow the policy tool to capture both the shift in demand from a given station as well as the potential shift in demand away from park and ride and kiss and ride transit. Conversely, the same process can be applied to understand shifting ridership because of station improvements.

From a practical standpoint, this application of the mixed logit formulation for policy analysis follows a similar method to the estimation procedure. A set of draws from a set of standard normal distributions are generated for each alternative and the probability of each station for each individual is calculated. These probabilities can then be summed to produce aggregate predictions for station ridership. This process is repeated for changes to a given station’s characteristics by adjusting the corresponding variables in the station’s utility. The inclusion of the normal error draws will produce the desired non-proportional substitution patterns, thereby providing a clearer picture of the proposed change.

### 5.5.3 Correlation Plots

One of the primary goals of this analysis is to generate non-proportional substitution patterns based on the specification of the error components of choice model. To test the effectiveness of this approach, examining the correlation matrix of the error components can normally be undertaken. Under typical applications of the random error component logit, a single covariance matrix is generated. However, for the case of the SWEC model structure, each individual record will have a unique covariance and correlation matrix, as these matrices are dependent on the specific choice set (and home locations for models four, five and six) for the individual in question. The implications of this make it nearly impossible to effectively examine the
correlation matrix. In the case of models four five and six, two individuals with identical choice sets who live in different locations may exhibit different correlation matrices due to the impact of the diagonal parameter and drive access time variable. Furthermore, the correlation and covariance of two alternatives are highly dependent on where each alternative is in the choice set: if for one individual the two alternatives are the first and third closest and for another they are the second and the fifth, the covariance (and as a result, correlation) will be further skewed by the Cholesky matrix transformation. That said, the policy application discussed in Section 5.5.2 could provide a practical means to test the non proportional substitution patterns inherent with the SWEC. That said, the correlation plots for each observation can be generated using the following procedure:

1. Generate the variance covariance matrix for the alternatives by multiplying the lower triangular Cholesky matrix $T$ by its transpose $T'$. 
2. To determine the off diagonal correlation between any two alternatives, take the covariance of those alternatives and divide this value by product of the square root of the variance of each of the alternatives.

$$COR_{ij} = \frac{COV_{ij}}{\sqrt{VAR_i} \ast \sqrt{VAR_j}}$$

This calculation was performed for each observation in the data set. Once these correlation values have been generated they can be plotted against distance. It should be noted that based on the specific choice set of the respondent each respondent will typically have a unique variance and covariance matrix. This trend is compounded further when examining models 4, 5 and 6 listed in Table 5.2, as the origin location will influence the variance covariance matrix. Practically what this means is that the correlation between two alternatives is not just a function of the distance between them but also based on the relative location of the alternatives in the cholesky and the geographic location of the home location (for models 4 through 6). This becomes very evident when examining the correlation versus inter-alternative distance plot for all considered alternative pairs from model 5 using the pooled data set (shown in figure 5.3. This plot shows that while the general trend of decreasing correlation across alternatives is consistent, there is significant heterogeneity across the correlation between alternative pairs that have the
same distance. This heterogeneity is an unintended mathematical by-product of the use of specifying the Cholesky to introduce correlation and heteroskedasticity.

Figure 5.3 Correlation versus Inter-Alternative Distance for Model 4 (Pooled Dataset)

Before diving into behavioural implications, a closer inspection of the correlation versus distance plot reveals further interesting trends. Namely, the degree of correlation is dependent on where in the Cholesky the first of the two alternatives is considered. Alternatives which are considered higher up or earlier in the Cholesky are less correlated and alternatives that are considered lower down or later in the Cholesky are more correlated. This trend is quite visible when considering the distance versus correlation plot shown in Figure 5.4 which colour codes a subsample of the alternative pairs in Figure 5.3. This is again a mathematical by-product of the Cholesky specification rather than a behavioural trend like Tobler’s law that an analyst is trying to capture. That said, the SWEC model provides significant improvements in fit over conventional approaches suggesting that the SWEC is capturing more of the behaviour in the model structure.
Figure 5.4 Impact of the Index of the Closer Alternative on the Correlation

Given the order in which the alternatives are specified within the cholesky, the lower correlation between alternative pairs where one alternative in the pair is closer to the individual perhaps suggests that decision makers make explicit tradeoffs between alternatives that are close by. Conversely, pairs of alternatives that are farther away are treated more generally, which would explain why these alternatives are more correlated. Specifically, alternatives that are both farther away are inherently better substitutes for each other because the decision maker explicitly considers alternatives in order. As more alternatives are added to the consideration set, they are grouped (i.e. considers the first and second closest and then all the rest). While this is an interesting hypothesis, it is still speculation; the underlying behavioural cause of the improvement in fit are still not entirely clear. Further investigation into how spatial correlation is perceived from a given reference point would provide increased clarity onto this subject. Further testing of the SWEC formulation using simulation techniques for particular contexts or other case studies should provide greater insights into whether these findings are simply an anomaly within the dataset or a more general trend. Furthermore, it should be noted that adding additional parameters to the model structure should at worst maintain the existing overall fit of the model (relative to a specification without the parameters). Further analysis into if the improvements in
fit from the SWEC result in differences in forecasting results are an important next step in justifying the use of the model over conventional practices.

5.6 Comparison with the GSCL

The discussion in section Chapter 2 section 9 presents a closed form method for dealing with spatial correlation between alternatives, the GSCL of Sener et al. (2011). This formulation makes use of a GEV structure to induce correlation between the error terms of different alternatives. The basis of this model is the pairwise combinatorial logit of (Chu, 1981, Koppelman & Wen, 2000, 2001), where each alternative is correlated with each other alternative. Based on this structure, the probability of selecting an alternative I can be defined as follows:

\[ P_i = \sum_{j \neq i} \left( \frac{(\alpha_{i,j} \times e(V_{ij})^{1/\mu})}{(\alpha_{i,j} \times e(V_{ij})^{1/\mu} + (\alpha_{j,i} \times e(V_{ji})^{1/\mu})} \right) \times \frac{\left( (\alpha_{i,j} \times e(V_{ij})^{1/\mu} + (\alpha_{j,i} \times e(V_{ji})^{1/\mu}) \right)^{\mu}}{\sum_{k=1}^{I-1} \sum_{l=k+1}^{I} \left( (\alpha_{k,kl} \times e(V_{kl})^{1/\mu} + (\alpha_{l,kl} \times e(V_{lk})^{1/\mu}) \right)^{\mu}} \]

The first ratio term represents the probability of selecting alternative i given that the nest ij is selected for all possible alternative pairs ij. The second ratio term represents the probability of selecting the nest ij from all possible nests. The \( V_i \) variable represents the utility of the alternative i. The \( \mu \) variable represents the nesting parameter, which reflects the degree of independence between the alternatives within a nest. This variable is generally constrained greater than 0 and less than or equal to 1. When \( \mu \) reaches one, the alternatives are completely independent and the model collapses to the standard multinomial logit. The \( \alpha_{l,ij} \) variable represents the allocation of alternative i to nest ij, where the sum of these allocation parameters for a given alternative across all possible nests must equal 1. In their paper, Sener et al. (2011) suggest numerous different formulations for specifying this allocation parameter, with almost all model structures utilizing a multinomial logit formulation. Through an empirical investigation, the allocation formula used for the model shown below is as follows:

\[ \alpha_{l,ij} = \frac{\exp \left( \left( \frac{1}{\ln(d_{ij})} \right)^b \right)}{\sum_{k \in C, k \neq i} \exp \left( \left( \frac{1}{\ln(d_{ik})} \right)^b \right)} \]
The $b$ term is an estimated parameter that is expected (and constrained) to be positive using the exponent of a parameter. The $d_{ij}$ variable is the distance between alternatives $i$ and $j$. This structure suggests that as alternatives become farther away, the level of correlation between them decreases.

As a means of comparing the GSCL with the SWEC, a GSCL model is estimated on the pooled dataset and is presented in Table 5.5. As can be seen when comparing Tables 5.3 and 5.5, the SWEC models 5 and 6 vastly outperform the GSCL, though the GSCL is an improvement over the conventional MNL based on a log likelihood ratio test with degrees of freedom equal to 2 and a 95% confidence interval. The log likelihood ratio test for the MNL and the GSCL is calculated as follows:

$$
= -2(LL_{constrained} - LL_{unconstrained})
= -2(-3357 - -3245)
= 224
$$

This test is used because traditional t-statistics cannot be used for both the scale and allocation parameters. This is because the scale and allocation parameters do not have reasonable reference points (typically zero or one). Given that the calculated statistic greatly exceeds the Chi square statistic for 2 degrees of freedom and 95% confidence (5.991), we can safely reject the constrained MNL model and accept the GSCL.
Table 5.5 GSCL Pooled Model Results

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Likelihood</td>
<td>-3245</td>
<td></td>
</tr>
<tr>
<td>Adjusted Rho-Squared</td>
<td>0.439</td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>6511</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable Name</th>
<th>Parameter Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected drive time from home to the transit station (produced by traffic assignment model)</td>
<td>-0.2022</td>
<td>-10.387</td>
</tr>
<tr>
<td>Expected transit wait, walk, transfer and in vehicle time from the station to the destination (produced by the traffic assignment model)</td>
<td>-0.0595</td>
<td>-9.794</td>
</tr>
<tr>
<td>Expected travel cost for the transit trip (fare + drive cost to the station) for the trip (produced by the traffic assignment model)</td>
<td>-1.1253</td>
<td>-7.514</td>
</tr>
<tr>
<td>Parking cost at the station (set to zero for kiss and ride travellers irrespective of station cost)</td>
<td>-3.4016</td>
<td>-15.029</td>
</tr>
<tr>
<td>Dummy variable for if the station is on the subway</td>
<td>3.6393</td>
<td>4.927</td>
</tr>
<tr>
<td>Natural logarithm of the parking lot capacity</td>
<td>0.7398</td>
<td>7.705</td>
</tr>
<tr>
<td>Dummy variable for if the station has a washroom</td>
<td>1.7525</td>
<td>6.846</td>
</tr>
<tr>
<td>Dummy variable for if the station-home-work angle is greater than 90</td>
<td>0.0303</td>
<td>0.226</td>
</tr>
<tr>
<td>Natural logarithm of the frequency of trains at the station per hour</td>
<td>2.0737</td>
<td>8.314</td>
</tr>
<tr>
<td>Scale parameter treated as the parameter $\alpha$ in the equation $\frac{1}{1+e^{\alpha}}$</td>
<td>0.9724</td>
<td>10.183</td>
</tr>
<tr>
<td>Allocation Parameter treated as the parameter $b$ in the equation $e^{b}$</td>
<td>1.8538</td>
<td>6.129</td>
</tr>
</tbody>
</table>

Given the closed form nature of the GSCL model, and the lower estimation time, there is some merit to its application, though its inability to capture heteroskedasticity and the improved goodness of fit of the SWEC suggest that when possible, the SWEC should be estimated. This means that there is no direct answer as to the preferred modelling structure for spatial choices as the correct structure is context specific. That said, when possible the SWEC provides significant improvements over the GSCL.

5.7 Chapter Conclusions

This chapter presents a useful new model structure, a spatially weighted error correlation model. The SWEC model uses a mixed logit formulation and introduces correlation between all alternatives. This is achieved through a parameterization of the Cholesky factor using the inverse of the distance between spatial alternatives. This parameterization structure approximates the general correlation pattern of close alternatives being more correlated than those that are more distant. This model structure is applied to two interrelated choice contexts: park and ride and kiss
and ride station location choice. The models were estimated using a household travel survey for the GTHA and transit station characteristics of park and ride and kiss and ride facilities in the region. Other potential applications of this approach include decisions such as specific school location (high school 1 vs. high school 2) or vacation location choice.

The proposed park and ride model builds on existing studies in the region, though the use of explicit time and cost variables over distance. The model is estimated using a pooled park and ride and kiss and ride dataset. Conversely, the kiss and ride model utilizes household level variables, including the travel time and cost incurred by the driver while driving their passenger to the transit station and then for their subsequent trip to their destination. This approach implicitly captures the bargaining that occurs at the household level (a transit station that is more convenient for the rider may be an inconvenient detour for the drive). This method of capturing household interaction is an obvious improvement over a naïve pooled park and ride – kiss and ride approach. Furthermore, given that kiss and ride is often overlooked within the literature, this chapter provides a starting point for an understanding of kiss and ride travel behaviour.

Specifically, the results of this chapter spark a set of further questions regarding kiss and ride behaviour at the household level. Is the relative importance of the driver versus the passenger weighted? Do individual or household sociodemographic factors play a role in determining how the household negotiation occurs? As these concerns were not explicitly addressed in this study, there is certainly room for further investigation into the household dynamics of kiss and ride behaviour. More generally, issues surrounding station choice and departure time were alluded to briefly but not discussed further. For park and ride users, departure time is linked with the likelihood of finding parking at a given transit station. The choice to leave later may result in having to use a less favourable transit station. Understanding interaction between departure time and station choice for park and ride users is essential to understanding these complex multimodal trips. An approach like that proposed by Habib et al. (2012), which links parking type choice with activity start time could be extended to examine station choice and departure time choice for park and ride and kiss and ride trips. This sort of approach would allow modellers to move beyond using crude capacity totals within their models and would provide a better indicator of the availability of parking at the station. Finally, there are likely some limitations associated with the level of detail included in terms of the difference between park and ride and kiss and
ride station choice. These limitations are a consequence of park and ride trips requiring the transit rider to finding a parking spot and then walking to the station which results in additional walk time. On the other hand, kiss and ride trips makers are often dropped off right at the front door of the station, making their initial walk to the transit service much faster. In other cases, kiss and ride drop-off and pick-up areas may involve much lengthier walk from the actual transit service. This means that these kiss and ride trips likely have walk times that might be more comparable to park and ride trips. To account for these factors, station specific walk times for both park and ride and kiss and ride trips could be included in the utility for each station. This level of details is unfortunately outside of the scope of the data that is available and thus an examination of these trips must be left for future research.

Despite these questions surrounding park and ride and kiss and ride behaviour the applications of the SWEC model presented here have key policy implications for modelling these trips. The ability of the SWEC to capture both heteroskedasticity and correlation patterns between alternatives results in significant improvement in model fit and forecasting accuracy. These advancements over traditional methods for modelling transit station location choice (typically MNL) are significant. The proposed model will not only circumvent the IIA assumption associated with the MNL but also is more appropriate for long range forecasting due to the congestion sensitivity gained from using travel time in the place of distance (relative to the work of Mahmoud et al. 2014, who used straight line distance in the place of travel time). Finally, the model joins a set of alternative model structures, which are used to tackle IIA issues for spatial choices more generally. Despite the growing number of options for addressing these issues, there are still significant research gaps to be addressed. An examination of the capacity of these models to capture alternative correlation for different choice contexts and a comparison of their strengths and weaknesses is an interesting avenue for future research.

The past two chapters provide a detailed look at the general travel patterns of adults in the context of the other adult household members for commuting travel. These modelling exercises lend credence to the idea that there is a connection between the travel behaviour of individuals and the needs of the household that cannot be captured using conventional individual specific modelling approaches. These concepts tie directly back to the theory of considering group dynamics in models of individual decisions. These group dynamics are either considered directly
or as exogenous inputs into the modelling structure. Looking forward, the next two chapters will look at understanding the relationship between the travel patterns of adults and the needs of household dependents, namely children. These exercises will also provide new insights into potential new methods for understanding these household dynamics, as well as incorporate spatial constraints into the modelling framework.
Chapter 6
Daycare Location and Drop Off/Pick Up Allocation Choice

This chapter presents a joint model for the allocation of drop-off and pick-up responsibilities and the choice of the daycare location for two adult households with dependent children. This analysis aims to capture the tradeoffs, which occur at the household level in terms of daycare location selection and drop-off and pick-up allocation. The chapter utilizes a stochastic frontier modelling approach to predict feasible locations for each possible allocation pair. The frontier model predicts the maximum distance an individual would be willing to travel within a time budget constraint. This travel time prediction is then applied to each individual household member for generating feasible location sets given two endogenous anchor points (typically home and work locations). This method can more generally be said to constrain the consideration set for spatial choices to a more realistic set of alternatives. The chapter then presents a joint econometric choice model of task allocation and daycare location with heterogeneous sampling correction factors for each possible allocation. Captured within the model are the choice of who performs the drop-off and pick-up activities and the corresponding location that is selected for the daycare.

Much of the work in this chapter, particularly the discussion of using stochastic frontiers was informed through discussions with James Lamers, who applied this modelling framework to his own Masters research work.

6.1 Introduction

There is an implicit and strong relationship between the day-to-day travel patterns of individuals and their medium/longer term decisions (Pinjari et al. 2011). This relationship is bidirectional and decisions might not be made sequentially. A change in short term decisions (e.g. switching mode in response to a new transit line or a new road) may ultimately pressure or stress an individual or a household into making a much longer term change, e.g. changes in car ownership or even home location. Conversely, a change in a long-term decision like where to live will obviously affect a wide variety of short-term travel decisions. Furthermore, as noted in Chapter
2, most conventional modelling practices typically assume day-to-day travel decisions are made at the individual level, whereas longer-term decisions are often assumed to involve a larger decision making unit: the household. The work in Chapter 4 and to a certain extent in Chapter 5 in this thesis provide a set of means to address these concerns. Unfortunately, many travel demand models simply ignore long-term decisions (such as vehicle ownership or residential location) treating them as exogenous inputs. Such simplifications can create biased predictions. Therefore, capturing the tradeoffs between short-term individual decisions and long-term planning is of great importance to obtaining accurate predictions from models. These concerns have been recognized by the travel demand modelling community. In the last 10 years, there has been a movement towards integrated modelling of these short and long term decisions. This chapter aims to continue moving this trend forward by considering a specific medium term decision as it pertains to intra household interactions.

This chapter investigates a set of medium term decisions that have been overlooked in much of the travel demand literature: choices regarding child care or daycare service. These trips are often completely ignored or combined with school escort trips as in Bhat et al. (2013), which, while a large step in the right direction, still falls short in explicitly considering daycare trips as a separate entity. While newborn children are not usually candidates for daycare, numerous studies (Haby et al. 2000, Irwin et al. 2005) suggest that there are benefits to enrolling children in daycare services before the age of two. Within the North American context, children up to the age of 5 or even 6 may be enrolled in a preschool daycare type program before they are old enough to start school. Furthermore, trips to and from daycare make up a small but relatively important component of both morning and evening peak travel. These trips create unique challenges for policy makers who look for ways to deter single occupancy vehicle (SOV) use for commuting. These challenges arise because there are often difficulties associated with accessing daycare services without access to a vehicle. As a result, any model that excludes daycare-related trips will potentially and systematically overpredict mode switching away from SOV use.

This issue is exasperated when considering the case of the two adult households, as is outlined in (Bhat et al. 2003). Specifically, the authors argue that a policy designed to keep drivers off the road would force a family to reallocate the task of dropping-off their child at school from the commuter in the household to the non-commuter, thereby creating an additional vehicle trip.
Furthermore, this reallocation might then encourage the non-commuter to make subsequent shopping trips throughout the day (with access to a car). Finally, the lack of consideration of individuals who use daycare services creates the possibility for unintended social inequity. Social inequity could occur when a policy that penalizes drivers will simply increase the cost for individuals who must drive to access daycare services without causing the desired change in behaviour. A discussion of the means to capture these concerns using the modelling framework presented in this chapter and in Chapter 3 is presented in section 6.4.

This chapter aims to capture the first half of the reciprocal relationship between short and long-term decisions through the presentation of a model of task allocation and location choice for daycare drop-off and pick-up trips for two adult households. The presented analysis also showcases an advancement on the generation of location choice sets through a stochastic frontier model approach. The stochastic frontier model approach generates a maximum travel time during both the morning and evening to determine feasible locations for a given drop-off and pick-up allocation between two household adults. These feasible locations are then used to determine the allocation of drop off and pick up tasks. This method is proposed as a means of addressing concerns regarding the biased results obtained when using simple random sampling (SRS) for spatial location choices.

The remainder of the chapter is organized as follows: first, the framework for the modelling of day care task allocation is presented. Second, an overview of additional data sources and the TTS subsampled used for the analysis is discussed. Third, the results of the modelling exercises are presented and the implications of these results are discussed. Finally, a summary of the work done and potential avenues for future work is outlined.

6.2 Model Formulation

The empirical models which are developed in this chapter use a two-stage modelling approach. First, this section presents the underlying theory behind the stochastic frontier model. Next, the application of the stochastic frontier model for the generation of the location choice sets is presented. Finally, a heterogeneous sampling correction logit model is presented, which is used to capture locations, drop-off and pick-up allocations and daycare use.
6.2.1 Use of Stochastic Frontiers

Stochastic frontier models are commonly used in economics literature originally proposed in the 1970s (Aigner et al. 1977). The stochastic frontier model approach assumes that the maximum production (or minimum expenditure) of an economic firm is a latent variable and any given firm will attempt to approach this latent value but cannot due to inefficiencies that are unobservable to the modeller. Because of the unobservable nature of these inefficiencies, they are treated as a positive random component. This modelling structure can be expanded to consider numerous different phenomenon. For the purposes of this chapter, the stochastic frontier model approach is expanded to consider the maximum amount of time (production) an individual is willing to spend travelling. The stochastic frontier model is typically formulated as follows:

\[ y = \beta'x + v - u \]

Where: \( y \) is the observed output (in this case travel time over a trip chain), \( \beta'x \) is a set of estimated parameters times a set of independent variables, \( v \) is a stochastic normally distributed error component with mean zero and unknown variance \( \sigma_v^2 \) and \( u \) is a positive inefficiency component, which is commonly treated as the absolute value of a normal distribution with mean zero and variance \( \sigma_u^2 \). This is the normal half normal form of the model and is the most commonly used structure, though alternative distributions may be used for the inefficiency component. Finally, the model structure typically follows a Cobb-Douglas or translog formulation, which is applied here. This means that the natural logarithms of the dependent travel time variable and the independent continuous variables are taken where appropriate. This provides significant improvement on model fit relative to the unlogged or partially logged versions of the model.

Stochastic frontier models are applied in several contexts within the transportation field. Kitamura et al. (2000a, 2000b), use stochastic frontier models to predict the frontier for departure times in the FAMOS PECATS scheduling model reviewed above. Pinjari et al. (2016) also used stochastic frontier models to generate the budgets for an MDCEV model for time spent travelling on a vacation. Aside from these two applications, the mainstream transportation demand modelling community has not used stochastic frontier models.
6.2.2 Model Application

The model application follows a three-step process outlined here and discussed in greater detail subsequently. The three steps are as follows:

1. Generate feasible daycare locations for a.m. and p.m. time-periods at the individual level
2. Determine overlap between feasible locations for a given allocation of drop-off and pick-up tasks. These overlaps create the constrained choice sets for a given drop-off and pick-up allocation.
3. Model the choice of drop-off or pick-up allocation and the location of the daycare based on the constrained choice sets

Each of these steps will now be outlined in greater detail

6.2.2.1 Step 1: Individual Choice Sets

The stochastic frontier approach is used to generate the expected maximum travel time an individual is willing to spend travelling during a time period using the following assumptions:

- Individuals spend an observable amount of time travelling to complete their activities during a relatively short time-period (the morning (a.m.) and evening (p.m.) peak).
- The maximum time they are willing to travel is latent and unobservable permitting the use of a stochastic frontier model.
- Explicitly, the amount of time a person is willing to travel during a time-period is set to be equal to the time they are observed to travel plus some positive inefficiency term, \( u \).
- From this maximum/frontier travel time, we determine a set of feasible locations where a set of activities (in this case, daycare drop off and pick up tasks) can occur based on the scheduled/observed start and end locations of that short time-period. This frontier time constrains the consideration set to a more realistic bundle of spatial alternatives relative to considering the full or universal choice set.

These start and end locations are defined as anchor locations. This feasible set is generated by only including locations where the travel time for the start location - candidate location- end location trip chain falls below the predicted frontier. This can allow modellers to significantly reduce the number of considered spatial alternatives from the universal choice set and the possibility of choice set misspecification associated with including unconsidered alternatives. This is analogous to defining the boundary of the space time prism/ellipse shown in Chapter 2
It should be noted that in some cases the anchor locations considered in this framework are not explicitly constrained to be home and work, home and home, or work and home. This three anchor points represent the expected pattern of behaviour for daycare drop off and pick up. A cursory review of the data found that the clear majority of the travelers’ schedules did follow this framework, suggesting the reasonableness of this approach. Note that this approach does not explicitly fix the departure time for a given trip. Instead the only constraint implied by the model is that each of the trips start times occur within the specified time period. This means that an individual could feasible perform 3 trips (lasting 10 minutes in total) from 8:15 to 8:45 a.m. and each of these trips would be included in the morning peak period (6 to 9 a.m.).

Practically, there are issues associated with the use of the stochastic frontier models for defining the boundary of the space time prism in this manner. The predicted frontier is treated as the sum of a parameterized component $\beta'x$ and a normally distributed error component $v'$. The $v'$ term has an expected value of zero but can be positive or negative, which may (and invariably will in some cases) result in the predicted travel time frontier being less than the observed travel time, thereby excluding the observed location from the choice set. Furthermore, one of the key assumptions undertaken in the frontier modelling exercise is that the time frontier is a function of how many trips/activities each individual household member is making/performing. This is because more activities mean more trips and therefore typically more time spent travelling between trips. Unfortunately, we only have the revealed allocation behaviour of the household (i.e. person A dropped off and person B picked up) and thus cannot generate the frontiers for unobserved allocation. To account for these concerns, the following cases were established for the appropriate application of the frontier model for the generation of the boundaries of the space time prism:

1. The individual is scheduled/observed to make zero trips during the time-period. In this case, the number of trips is increased from zero to two (the trip to the daycare and the return to where they started). The zones that are considered feasible for this individual are based on the travel time from their origin at the start of the time-period to the daycare zone plus the travel time to return to their home zone. These individuals will typically be travelling from home to the daycare and then back to home, though there is a possibility that some adults could have been planning to stay late at work and thus were not scheduled to travel over that
time-period. To achieve behavioural realism, these individuals are forced home once they pick-up the child from daycare, rather than returning to work with the child.

2. The individual is scheduled to make a single trip during the time period. In this case, the number of trips is increased from 1 to 2 and the feasible set of zones considered for the daycare trip are determined based on the origin to daycare to destination travel times being less than the projected frontier. This is the classic interpretation of the space time prism/ellipse approach.

3. The individual is scheduled to make 2 or more trips during the time-period. In this case, the number of trips is increased by 1. We do not intend to explicitly model the scheduling/trip chaining process (i.e., in what order the new drop-off or pick-up activity is included, which alters the feasible location set). Therefore, to simplify the calculation procedure for this proof of concept study, the same approach used for case 2 is applied (origin to daycare to destination travel time less than the frontier). This is a simplification of the process for generating feasible locations, though generalizing this process would require some information regarding the scheduling process for each household member. As a result, this simplification is implemented for tractability during the data preparation stage. A more general solution is left for future work.

4. The individual already makes a daycare trip. In this case, as discussed above, rather than explicitly using the predicted frontier, the observed travel time plus the predicted inefficiency is used as the frontier. This approach ensures that the observed alternative is considered feasible. The predicted inefficiency can be calculated as follows, though most commercial software will perform these calculations internally.

\[
E(u|v-u) = \left(\frac{\sqrt{\sigma_u^2 + \sigma_v^2} \times \frac{\sigma_u}{\sigma_v}}{1 + \left(\frac{\sigma_u}{\sigma_v}\right)^2}\right) \times \left[\frac{\phi(w)}{1 - \Phi(w)} - w\right]
\]

where \( w = \left(\frac{(v - u) \times \frac{\sigma_u}{\sigma_v}}{\sqrt{\sigma_u^2 + \sigma_v^2}}\right) \)
Remembering that \( v \) is the normal distribution and \( u \) is the half normal distribution it is possible to substitute \( y_i - \beta' x_i \) for \( v - u \) which results in individual specific inefficiency components across all \( i \) observations. The other values of the estimation process are calculated as components to be estimated during the maximum log likelihood estimation. For reference \( \Phi(w) \) is the standard normal cumulative density function evaluated at \( w \) and \( \phi(w) \) is the standard normal probability density function evaluated at \( w \).

### 6.2.2.2 Assumptions and limitations of the stochastic frontier approach

There are two major assumptions worth noting within this analysis. First, drop-off trips occur in the a.m. peak period while pick-up trips occur in the p.m. peak and second, all drop-off pick-up trips occur by car. The clear majority of trips do fit these assumptions though approximately 10% of the sample for which we applied the model had one or more discrepancies from this standard. Given this small percentage and the correlation of travel time by any mode with distance, the standard framework is applied to these discrepancies. This is a large simplifying assumption and there are potential scenarios which may cause it to be incorrect, for example:

- individuals may be willing to spend less time travelling so they leave earlier (before the a.m. peak or later (after the p.m. peak) to avoid congestion,
- daycare services may not operate for the full day, meaning that drop-off or pick-up times may fall outside of the predetermined a.m. peak drop-off and p.m. peak pick-up times.

Conversely, the argument that daycares operate during traditional business hours as a means of allowing dual income or single parents to work, presents a compelling counter argument to half day daycare scenario. This is further supported by the relatively low number of temporal discrepancies observed in the sample. This method will also potentially over predict the size of the choice set for individuals who do not have access to a car as travel time by other modes will typically reduce the ability to access other zones within the predicted time frontier. This creates a larger challenge in terms of determining auto availability, though as will be discussed in the allocation model, an endogenous dummy variable for number household vehicles (like that discussed in Section 2.3) is applied to allocation utilities to encourage single parent escort allocation.
Potential longer term solutions to these issues could be the development of mode specific frontiers and a time of day choice model for drop-off and pick-up daycare trips though these tasks are left for future research projects. These approaches could be integrated within an individual model of travel demand, where a vehicle allocation outcome predicted from another model is inputted into the drop-off pick-up task allocation model (and vice versa), through a Gibbs sampling approach. It should also be reiterated that in the case of the subsequent analysis, these cases were not common, with approximately 10% of households who make daycare trips having one or more of the three possible discrepancies. This means that while not ideal, the proposed modelling framework provides a reasonable prediction of behaviour.

6.2.2.3 Step 2: Allocation Constrained Choice Sets

A predicted frontier is generated for each household member and time-period within the sample. Next, the start and end locations (at the TAZ level) of each household member for the a.m. and p.m. time-periods are determined. Given this information, the travel times from the start location to a possible daycare and then from the possible daycare to an end location are extracted from a travel time matrix for the region. These travel times are summed for each household member and time-period across all possible daycare locations. The actual travel time value is then compared against the predicted frontier for that individual and time-period. If the travel time value is less than the predicted frontier, that alternative location is considered feasible for the time-period.

Once each household member has an exhaustive set of locations for both time-periods, the choice sets for each of the four possible allocations can be determined. The allocation choice sets are determined by comparing the alternatives in the a.m. drop-off set to the p.m. pick-up set. Alternatives that are common across the sets are kept and used as the choice set for that allocation alternative. For example, consider the case where person A has the a.m./drop-off feasible set of locations \{5,17,22,33\} and person B has the p.m./pick-up location set \{5,13,22,36\}. The feasible universal set for the allocation of person A doing the drop-off and person B doing the pick-up would then be \{5,22\}. This approach is applied to all allocations, generating four location choice sets. A visual depiction of this process is outlined in Figure 6.1.

In this example, household member A is allocated the drop-off responsibility and household member B is allocated the pick-up responsibility. The dark blue area represents the overlap between their respective choice sets and is therefore used to determine the feasible daycare locations for this specific drop-off and pick-up allocation scheme.
6.2.2.4 Step 3: Allocation and Location Choice Models

Before considering the formulation of the model, we first outline the literature regarding sampling alternatives. As noted by McFadden (1978), when using a sample of alternatives to estimate a multinomial logit model, it is possible to obtain consistent estimates through the addition of a sampling correction term to the utility of each alternative (based on the positive conditioning property). This sampling correction term is equal to the natural logarithm of the sampling rate. McFadden also notes the uniform conditioning property: that if the sampling rate is constant for each household member, it cancels out in the likelihood function. McFadden’s work is further extended by Guevara and Ben-Akiva (2013), who present an innovative method to account for sampling error using nested logit structures (and more generally using GEV structures). These issues become highly relevant for the forthcoming case due to the heterogeneity involved in the sampling rate across different allocations for any given household.

To facilitate estimation, a random draw of up to 10 alternatives is taken for each allocation (allocations with less than 10 feasible alternatives had all their alternatives drawn). This approach results in heterogeneous sampling rates across allocations and as such McFadden’s uniform conditioning property does not hold. Fortunately, the positive conditioning property...
does continue to hold so each allocation within a household has a unique sample correction factor. This factor is equal to either zero if the full consideration set is less than 10 or the log of one divided by the size of the full consideration set generated through the stochastic frontier approach. Note that because sampling without replacement occurs, each alternative will have a unique sampling correction term whereby the size of the consideration set is slowly reduced by one for each alternative that has already been sampled. This further correction is not required if sampling with replacement is performed. As outlined by McFadden (1978), this correction is all that is needed for the MNL, however, for the nested logit there is a substantially more involved process. Specifically, Guevara and Ben-Akiva (2013) note that the log sum term within the nested logit formulation must also be corrected to reflect the true size of the choice set. There are various means of applying this correction, most notably a resampling of alternatives to calculate the log sum term. This resampling is done to avoid the bias associated with utilizing the original sample (which must contain the chosen alternative).

The heterogeneous sampling correction logit (HSCL) is presented as follows:

\[
P_{ia} = \frac{\exp \left( V_{ia} + \ln \left( \frac{1}{a_{\text{size}} - i^*} \right) \right)}{\sum_{j^*} \exp \left( V_{j^*a} + \ln \left( \frac{1}{a_{\text{size}} - j^*} \right) \right)}
\]

Where \(a\) is an allocation of drop-off and pick-up duties to household members, \(i\) is the alternative location, \(i^*\) and \(j^*\) are the index of the alternative location based on the order in which the sample is drawn and \(a_{\text{size}}\) is the size of the consideration set for allocation \(a\). The attempted nested logit formulation assumes the upper level nests are the four feasible allocations of drop-off and pick-up tasks and the lower level choices conditional on the upper level nests are the locations for that given allocation. To account for sampling bias in the locations, the logsum term is recalculated based on a completely different sample relative to the one used to calculate the probability. Again, the interested reader is referred to Guevera and Ben-Akiva, (2013) for the details on both the rational and the execution of this correction.
6.3 Data Description

As with previous studies in this thesis, this analysis used the TTS 2011 data and the corresponding traffic assignment model. Two different subsamples are used in the estimation of the models presented in section 6.4. Subsample 1 consisted of all households with exactly two adults and at least one child under the age of 6 for the stochastic frontier model generation. This allowed the frontier models to be generated for the morning and evening peak periods for a given number of trip purposes and trip numbers. For the estimation of the frontier models, only those adult household members who made trips are used (resulting in 10394 individuals in the a.m. and 12906 individuals in the PM). This sample is further reduced to only those households who used daycare for the allocation and location model. This subsample consisted of 1093 households, which represents 0.6% of the full TTS. Table 1 provides an overview of many of the statistics associated with this subsample. The sample is used to generate predicted frontiers from the stochastic frontier model discussed and presented below, as well as the nested logit model.

Table 6.1 Daycare Location and Allocation Choice Model Sample Statistic

<table>
<thead>
<tr>
<th>Individual Statistics</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>% who are male</td>
<td>49.68%</td>
</tr>
<tr>
<td>Average age</td>
<td>36.8</td>
</tr>
<tr>
<td>% with driver's license</td>
<td>98.21%</td>
</tr>
<tr>
<td>% with transit pass</td>
<td>10.58%</td>
</tr>
<tr>
<td>% work type manufacturing</td>
<td>8.84%</td>
</tr>
<tr>
<td>% work type general office</td>
<td>16.25%</td>
</tr>
<tr>
<td>% work type profession</td>
<td>44.28%</td>
</tr>
<tr>
<td>% work types retail/services</td>
<td>21.38%</td>
</tr>
<tr>
<td>% unemployed*</td>
<td>9.25%</td>
</tr>
<tr>
<td>% employed full time*</td>
<td>80.04%</td>
</tr>
<tr>
<td>% employed part time*</td>
<td>4.67%</td>
</tr>
<tr>
<td>% part-time students*</td>
<td>2.61%</td>
</tr>
<tr>
<td>Household Statistics</td>
<td>Value</td>
</tr>
<tr>
<td>Average number of household vehicles</td>
<td>1.83</td>
</tr>
<tr>
<td>% live in a detached house</td>
<td>77.75%</td>
</tr>
<tr>
<td>% live in an apartment</td>
<td>9.62%</td>
</tr>
<tr>
<td>% live in a townhouse</td>
<td>12.64%</td>
</tr>
<tr>
<td>average number of children under 7</td>
<td>1.44</td>
</tr>
<tr>
<td>average number of children between 7 to 12</td>
<td>0.15</td>
</tr>
<tr>
<td>Average number of teenagers (13-18)</td>
<td>0.05</td>
</tr>
<tr>
<td>Number of households who make a drop-off and pick-up Daycare trip</td>
<td>1093</td>
</tr>
</tbody>
</table>

*Some individuals chose not to report their full or part-time status for school/employment so percentages do not add to 100 for these four categories.

This study utilized the TTS data for travel behaviour (allocation of drop-off and pick-up responsibilities and location of daycare services) and as a proxy for surrounding land use. Specifically, TTS records are aggregated at the TAZ level to obtain population and employment
Retail land use density is captured through proxy by the number of expanded shopping activities recorded in the TTS at each TAZ. Each TTS record is expanded according to an individual specific expansion factor. These expansion factors are calculated by comparing age distribution across the population at a relatively disaggregate census spatial unit (a forward sortation area). This land use information is further supplemented with data from the enhanced point of interest database (EPOI). The EPOI file is a Canadian database of geocoded businesses and points of interest. Geocoded points for child care facilities and schools in the GTHA were extracted and overlaid across the TAZ file to generate TAZ specific counts. A total of 4295 childcare locations and 7885 schools are contained within the study area.

6.4 Empirical Model Results

As the proposed method has two components, this discussion will focus first on the results of the stochastic frontier model and subsequently on the results for the nested logit.

6.4.1 Stochastic Frontier Results

The results for the stochastic frontier models are found in Tables 6.2 and 6.3 for the a.m. and p.m. frontiers respectively. All parameters showed relatively intuitive signs with the employment category and the home-work/work-home travel time values essentially canceling each other out. We also note the high value of the log likelihood ratio test for both models, essentially rejecting the OLS model in favor of the stochastic frontier model. The main weakness of this sort of approach stems from the limited ability to capture what makes any person travel for longer. While we attempt to address these concerns with a set of spatial dummy variables, we recognize this as an approximation at best. Aside from adding in a significantly larger number of dummy variables, potential improvements to the stochastic frontier approach might involve some form of spatial autocorrelation between proximate observations or a more robust definition of the production/travel time function. Worth noting are the model outputs for the error terms. As the means and standard deviations for these components are not available separately, the ratio between the zero-mean component and the inefficiency component standard deviations is defined as lambda and sigma being the square root of the sum of the variances for the inefficiency component and the zero-mean component.
Table 6.2 Morning Peak Period Stochastic Frontier Max Travel Time During Period Model

<table>
<thead>
<tr>
<th>Time-period</th>
<th>AM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-Likelihood ratio test against OLS</td>
<td>635</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Socio-demographic Variables</th>
<th>Parameter</th>
<th>Std. Err.</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.77744</td>
<td>0.05188</td>
<td>34.26</td>
</tr>
<tr>
<td>Dummy male gender</td>
<td>0.11585</td>
<td>0.01636</td>
<td>7.08</td>
</tr>
<tr>
<td>log of number of vehicles +1</td>
<td>0.10002</td>
<td>0.03713</td>
<td>2.69</td>
</tr>
<tr>
<td>number of 7 to 12 year olds</td>
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<td>Dummy general office job</td>
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<tr>
<td>Dummy professional job</td>
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<td>0.03425</td>
<td>-12.42</td>
</tr>
<tr>
<td>Dummy retail/service job</td>
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<td>0.03483</td>
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<td>dummy part time worker</td>
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<th>Parameter</th>
<th>Std. Err.</th>
<th>T-Stat</th>
</tr>
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<tbody>
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<td>number school based trips</td>
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</tr>
<tr>
<td>number daycare trips</td>
<td>0.18177</td>
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<td>8.59</td>
</tr>
<tr>
<td>number facilitate passenger trips</td>
<td>0.17236</td>
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<td>11.41</td>
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<tr>
<td>number trips to home</td>
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<td>0.02728</td>
<td>15.33</td>
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<tr>
<td>number shopping trips</td>
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<td>11</td>
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<tr>
<td>number work trips</td>
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<td>0.02568</td>
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<tr>
<td>number other trips</td>
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<table>
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<th>T-Stat</th>
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<td>0.04343</td>
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<td>0.11829</td>
<td>0.02091</td>
<td>5.66</td>
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<tr>
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<td>0.02885</td>
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<td>OD pair inside of Toronto to planning district 1</td>
<td>0.09203</td>
<td>0.03993</td>
<td>2.3</td>
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<td>OD pair inside of Toronto to outside Toronto</td>
<td>0.29242</td>
<td>0.03405</td>
<td>8.59</td>
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<tr>
<td>Log of Home Work Travel Time without any additional stops +1</td>
<td>0.39186</td>
<td>0.00776</td>
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<th>T-Stat</th>
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<tr>
<td>Sigma (=\sqrt{\sigma_u^2 + \sigma_v^2})</td>
<td>1.11685</td>
<td>0.0008731</td>
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Table 6.3 Evening Peak Period Stochastic Frontier Max Travel Time During Period Model

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<tr>
<th>Time-period</th>
<th>PM Log-Likelihood ratio test against OLS</th>
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<tr>
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<table>
<thead>
<tr>
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<th>t-Stat</th>
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<td>Dummy male gender</td>
<td>0.13217</td>
<td>0.01485</td>
<td>8.9</td>
</tr>
<tr>
<td>log of number of vehicles +1</td>
<td>0.30031</td>
<td>0.04497</td>
<td>6.68</td>
</tr>
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<td>number of 7 to 12 year olds</td>
<td>-0.03104</td>
<td>0.01043</td>
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<td>Dummy manufacturing job type</td>
<td>-0.30138</td>
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</tr>
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<td>Dummy general office job type</td>
<td>-0.32449</td>
<td>0.03023</td>
<td>-10.73</td>
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<td>Dummy professional job type</td>
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<td>-12.09</td>
</tr>
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<td>Dummy retail/service job type</td>
<td>-0.31322</td>
<td>0.02748</td>
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<td>dummy part time worker</td>
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<td>0.02739</td>
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<table>
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<th>Trip Count Variables</th>
<th>Parameter</th>
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<th>t-Stat</th>
</tr>
</thead>
<tbody>
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<td>0.10463</td>
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</tr>
<tr>
<td>number daycare trips</td>
<td>0.18952</td>
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<td>9.37</td>
</tr>
<tr>
<td>number facilitate passenger trips</td>
<td>0.22958</td>
<td>0.01221</td>
<td>18.8</td>
</tr>
<tr>
<td>number trips home</td>
<td>0.13351</td>
<td>0.01498</td>
<td>8.91</td>
</tr>
<tr>
<td>number shopping trips</td>
<td>0.2245</td>
<td>0.01557</td>
<td>14.42</td>
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<tr>
<td>number work trips</td>
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<tr>
<td>number other trips</td>
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<table>
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</tr>
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<td>0.01694</td>
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<td>19.32</td>
</tr>
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<td>OD pair inside of Toronto to planning district 1</td>
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<td>0.08014</td>
<td>4.66</td>
</tr>
<tr>
<td>OD pair inside of Toronto to outside Toronto</td>
<td>0.43051</td>
<td>0.0245</td>
<td>17.57</td>
</tr>
<tr>
<td>Log of work home travel time without any additional stops +1</td>
<td>0.31447</td>
<td>0.00641</td>
<td>49.08</td>
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</table>

<table>
<thead>
<tr>
<th>Variance parameters for compound error</th>
<th>Parameter</th>
<th>Std Err</th>
<th>t-Stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lambda = \sigma_u / \sigma_v</td>
<td>2.46274</td>
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<td>54.76</td>
</tr>
<tr>
<td>Sigma = \sqrt{\sigma_u^2 + \sigma_v^2}</td>
<td>1.17049</td>
<td>0.005844</td>
<td>20029.94</td>
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</table>

6.4.1.2 Allocation and Location Choice Results

As alluded to above, two models were estimated, a heterogeneous sampling correction logit (HSCL) and a nested logit with heterogeneous sampling correction. Table 6.4 presents the results for the HSCL. The parameter estimates are intuitive in terms of sign and are all significant. The rho squared against the constant only model is 0.10 and 0.14 against the null model.

Examining specific model parameters presents several interesting findings. For the choice of location, the positive sign for the number of listed daycare facilities and to a lesser extent the number of schools listed in the zone suggests an increase in child care options (schools may run daycare programs) for the parents. Furthermore, these variables may potentially be a proxy for land use characteristics that are particularly appealing for child care (reduced speed limits, green...
space, etc.). We also note that positive coefficients for density suggest parents are interested in trip chaining to other activities around the daycare. This is supported by the positive coefficient for the number of shopping trips observed in the TTS for that zone. Conversely, parents could be concerned about the daycare location being in a highly dense employment area such as the central business district (CBD). The positive coefficient for the distance from the CBD in conjunction with the other parameter values suggests parents strike a balance between overly busy areas and the ability to trip chain to other amenities. Of more interest are the variables which were excluded from the location choice model: most notably travel time. Various forms of the travel time variable (flat value, natural logarithm, etc.) were attempted though the coefficient ended up being insignificant and/or the incorrect (positive) sign. A potential explanation for this finding is that the use of the stochastic frontier to eliminate infeasible alternatives from the choice set creates a much more competitive alternative space for many individuals (with respect to travel time).

In terms of the allocation of drop-off and pick-up tasks to a single individual (AA, BB in the Table), we found that the drop-off and pick-up duties were more likely to fall to full time workers if their spouse works part time or is unemployed. This is likely because full time workers will be able to chain their trips to and from work. Another interesting set of findings are that men and individuals working in manufacturing less likely to be allocated the drop-off and pick-up tasks. These results are consistent with the previous literature on escort trips for school. Namely, traditional gender roles are still prevalent and those working shift work may be less flexible in terms of work start times, thus complicating escort duties. Transit pass ownership had a negative parameter for escort allocation which further supports the assumption that most escort trips occur by car. Households with only a single car are more likely to allocate drop-off and pick-up tasks to a single individual. Although this analysis does not look explicitly at vehicle allocation it is likely safe to assume that the driver is allocated these tasks. This is further supported by the individual drivers’ license dummy (equal to one if the other adult in the household does not have a drivers license) which indicates that the individual with the driver’s license is more likely to be allocate the drop-off and pick-up tasks. Several variables were tested for split allocation of drop-off and pick-up tasks (for example, the difference in age between the adults). Unfortunately, the dataset is limited in terms of variables which capture why a split allocation occurs. The only variable that is significant in this regard is a dummy for both adults
being full time workers. This result suggests a shift in the allocation of breadwinning and childcare responsibilities away from traditional gender roles. With that said, this trend is still countered by the significantly lower likelihood for men to be allocated the drop-off and pick-up tasks.

A nested logit formulation is estimated, following (Guevara and Ben-Akiva, 2013), where the locations were correlated for a given allocation. Unfortunately, the model results were not consistent with utility-maximizing theory (nesting coefficient greater than 1). One possible future research avenue might be a reversal of the nesting structure, with the possible allocations correlated for a given location. This testing would not require the Guevera and Ben-Akiva correction as the locations are now above (though it would still require the utilities to have a positive conditioning correction). To further complicate the execution of this analysis, this approach would also require a complete redevelopment of the data used for the estimation as each the locations will have to be redrawn independent of the allocations. Once a location is drawn, the possible allocations for that location based on the frontier overlap would then have to be determined. This is a substantial data processing task and as such is not included in this analysis. This analysis does only use a sample of ten location alternatives for each allocation which is a significant limitation. Unfortunately, due to estimation and data preparation computation time constraints larger sampling rates for the nested logit model are not tested.
Table 6.4 HSCL Model Results

<table>
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<th>T-Stat</th>
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<tr>
<td>Rho Squared against constant only model</td>
<td>0.10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rho Squared against null model</td>
<td>0.14</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Location Specific Variables</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Number Daycare Facilities in zone</td>
<td>0.1038</td>
<td>0.0131</td>
<td>7.931</td>
</tr>
<tr>
<td>Number Schools in Zone</td>
<td>0.0486</td>
<td>0.0101</td>
<td>4.82</td>
</tr>
<tr>
<td>ln(employment + resident counts)</td>
<td>0.0815</td>
<td>0.0122</td>
<td>6.705</td>
</tr>
<tr>
<td>ln(distance from CBD)</td>
<td>1.1334</td>
<td>0.1249</td>
<td>9.076</td>
</tr>
<tr>
<td>ln(shopping trip counts)</td>
<td>0.0827</td>
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<td>Dummy for location in Home Zone</td>
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<tr>
<td><strong>Single Allocation Variables (AA,BB)</strong></td>
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<td>Dummy male gender</td>
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<tr>
<td>Dummy having a transit pass</td>
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<td>Dummy the other adult has no driver's license</td>
<td>2.6826</td>
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</tr>
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<td>Dummy household has only one car</td>
<td>1.0089</td>
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<tr>
<td><strong>Split allocation Variables (AB,BA)</strong></td>
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<tr>
<td>Dummy for both adults are full-time workers</td>
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<tr>
<td>Constant Split allocation</td>
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6.4.2 Potential Policy Analysis

As noted in the introduction of this chapter, there are numerous potential negative implications for not considering the daycare drop off and pick up allocation task procedure in models of travel demand. Issues surrounding the under prediction of vehicle kilometers travelled derived from policies designed to curb single occupancy vehicle use are of concern. Unfortunately, the modelling exercise presented within this chapter is not independently capable of capturing these behavioural trends. That said, when this modelling framework is applied in conjunction with a mode choice model such as that presented in Chapter 4, substantial policy insights could be gained. The model in Chapter 4 could be expanded to include variables regarding number of scheduled escort tasks (or other household maintenance tasks) in the utilities of drive centric modes. Concurrently, a dummy for the mode used to access the primary activity (typically a work activity) for each adult could be included in the allocation model for drop off and pick up. These models could then be applied iteratively as discussed in Chapter 3 to obtain a more accurate representation of the impact of various policies on SOV trips. The integration of these models into a broader activity based framework would provide significantly greater insights relative to a conventional trip based framework. Trip based models typically do not consider intermediate stops and trip linkages, making daycare drop off and pick up trips on the way to or from work difficult or impossible to model. Furthermore, trip based models do not consider the
linkage between drop off and pick up trips, which produces behaviourally inconsistent
disaggregate results. Practically, this means that a drop off trip does not explicitly result in a pick
up trip to the same location. Conversely, the model presented in this chapter can circumvent the
limitations associated with trip based models when combined with a broader activity based
scheduling framework.

6.5 Chapter Conclusion

Much of the above discussion has focused on the implementation of the various model structures
designed to capture the behaviour of individuals when it comes to daycare task allocation and
location. While this is all very fascinating, it is also important to remember the reason for
jumping through all these hoops. Ideally, this chapter will have improved the understanding of
how long-term decisions impact day to day household decision. This is a substantial step forward
from conventional escort decision models, which typically only examine the allocation of the
task without explicitly considering how the location of the task constrains the process. This is a
fundamental contribution to understanding escort task allocation. Moreover, the use of the
stochastic frontier model presents an interesting method of specifying a more realistic
consideration set for spatial alternatives which has seen limited applicability in travel demand
modelling.

Unfortunately, this analysis is still missing a few key components. Most notably, the question of
daycare usage is not examined in this chapter. A hypothetical model might aim to capture the
choice to use or not use daycare alongside the existing behavioural components that are already
captured within the modelling though this may require a more robust data collection process.
While some work has been done on daycare usage for the City of Toronto (Lamers, 2017), there
are some challenges associated with integrating the two modelling structures jointly. Most
notably there are some serious computational requirements to both generate and estimate the data
set that would be used for this analysis. A second limitation of the analysis in this chapter relates
to several of the simplifying assumptions. These assumptions are related to the drop off and pick
up travel mode and the time when these trips occur. Relaxing these assumptions would require
significant increases in the complexity of the modelling framework.
Fortunately, most of the observed trips fit into the established constraints (automobile usage and drop off and pick up timing). Even so, estimating a joint mode, allocation, location and drop off and pick up time period model for daycare drop-off and pick-up trips is a possibility. Furthermore, while this process does capture the day-to-day choice of allocation duties, it does not currently fit neatly into a broader model which considers factors like vehicle allocation and the scheduling process. To capture this behavioural process, the models presented above (or an expanded model to capture the choice to use daycare) could be integrated with a comprehensive activity based scheduler. This would provide insights into how the choice of daycare location and allocation of drop-off and pick-up trips changes as hypothetical changes are introduced to the transportation-land use system. As alluded to in Chapter 3, this form of iterative forecasting would give us deeper insights into the impact of our policies with respect to reducing SOV trips. Finally, alternative nesting structures (allocations correlated for a given location) could be tested.

On top of these structural improvements and applications, this chapter focuses on spatial constraints based on exogenous information regarding the location of an individual at a given time-period. This method attempts to capture some component of both spatial and temporal constraints, though the temporal constraints are more of a byproduct of the consideration of spatial issues. Conversely, approaches such as that used by Kitamura et al. (2000a, 200b) is the reverse of the method presented in this chapter. In their paper the temporal constraint is directly modelled and the spatial constraints are a byproduct of this process. This is an interesting juxtaposition and the application of both approaches on the same data would create an interesting testbed for examining the two techniques for different applications.

While these avenues are promising, that does not take away from the insights gained from the presented analysis. The application of stochastic frontier models for generating space prisms is a highly novel and practical concept. Stochastic frontiers provide a useful method for reducing the universal location choice set for many other out of home activities outside of daycare.

With the integration of daycare location and drop-off and pick-up task allocation complete, this thesis now turns to another examination of chauffeur responsibility: school aged children. As has been discussed, these two choices exhibit significantly different behavioural patterns as school aged children have the possibility of independent travel, particularly as they get older. Chapter 7
will examine these behavioural patterns in detail while providing a novel method for jointly estimating chauffeur decisions. Chapter 7 expands the existing practice within the literature by also considering household level mode choice for all commuting household members.
Chapter 7

Household Mode and Student Escort Choice

This chapter presents an expanded and generalized formulation of a discrete group decision making model. The empirical model is developed for modelling high school student escort and travel mode choices. The development of the model uncovers issues with an existing group decision model, the multi linear logit. As an alternative to the multi linear logit, the chapter proposes an expanded formulation of parallel constrained choices logit model. The proposed model explicitly considers the mode choice of the students and commuting adults within the household and how these decisions change in response to household escort decisions. The model is generalized such that any number of adults can be accommodated within the modelling structure whereas previous studies have been limited to either two adults or two workers. The key findings of the analysis include a certain degree of altruism (or dedication) from adults to household children. This altruism is captured indirectly within the modelling framework based on a lower relative weight for the adults in the household relative to the children.

7.1 Introduction

Within activity based models of travel demand, there is often an underlying assumption regarding the structure of how an individual’s daily schedule is developed. Specifically, an individual’s work activity is viewed as the most important and all other activities are shaped by this activity (Bowman and Ben-Akiva, 2001). While there is much merit to this concept and in many cases, it could be argued to be true, there are other considerations throughout the course of a day that may play an equal if not greater role in constraining the scheduling process. There is convincing evidence that interactions between household members play a pivotal role in shaping an individual’s daily choices, which in turn create an individual’s travel pattern (Gupta and Vovsha, 2013; Bhat and Pendyala, 2005; Zhang et al. 2009; Timmermans and Zhang, 2009). Moreover, Ho and Mulley (2015a, 2015b) and Akbari and Habib (2015) suggest that decisions in both the short and long term that are conventionally viewed as being made by a household unit are in fact composite decisions involving multiple household members who make tradeoffs as a means to achieve a reasonable household level utility. Ignoring these trends could lead to
irrational and biased results which could then seriously compromise the validity of any forecasts or decisions made based on the model results. As such, incorporating the means and methods by which households negotiate into models of travel demand is of upmost importance within the field of transportation planning and forecasting (Glieber and Koppelman, 2005).

As discussed in the literature review in Chapter 2, household interactions are addressed by travel demand modellers using three main techniques. The focus of this chapter is on the examination of models of group decision making, which are outlined in Section 2.6. As a quick refresher, these models consider a group decision making approach whereby household utility is a function of each household member’s utility for a given choice context. Within the literature, various modelling structures make use of different group decision making frameworks, including:

- a. a weighted group utility structure
- b. a multi linear utility structure
- c. an iso-elastic utility structure

Structures b and c have been reviewed extensively by Junyi Zhang and colleagues within the context of transportation and take their basis in formal group decision theory (Zhang et al. 2009). These three approaches present a representative sample of the approaches used to address concerns regarding intra-household interaction and have been applied broadly outside of the transportation context. Each of these models has their own strengths and weaknesses and can be used to capture specific behavioural trends when applied correctly.

That said, there exists little to no consensus within the field of transportation or other fields regarding the appropriate approach to take to capture household interactions. Therefore, it is prudent to empirically investigate each relevant approach for a given decision context or utilize a latent class approach (as done by Zhang et al. 2009).

Unfortunately, the multi linear logit (MLL) model as presented by Zhang and colleagues has some significant issues with respect to its applicability for discrete choice modelling. These issues will be formally presented below and call into question the work done with the multi linear model within a discrete choice context. These issues were encountered when attempting to extend the existing work of Ermagun and Levinson (2016), who examined escort and mode
choice decisions for school aged children using an MLL structure. Given these considerations, this chapter has three main objectives:

1. Capture decisions regarding the mode choice for high school students and commuting adults living in the same household. This analysis also considers the issue of the choice of escort decision within the model structure (e.g. does an adult drive the student to school and if so, which adult in a multi adult household?). This is particularly relevant as most existing literature has limited their analysis to two parent households (with a few notable exceptions).

2. Identify the limitations of the work of Zhang et al. (2009) and Ermagun and Levinson (2016) and present an alternative model structure which is consistent with random utility-maximizing theory.

3. Address the concerns raised by Gupta et al. (2014) for the development of a comprehensive mode choice model at the household level that can consider escort decisions for dependents.

Looking forward, the remainder of the chapter will be organized in the following four sections. First, a formal presentation of the multi linear utility structure, an overview of the limitations of this structure and then an overview of the parallel constrained logit model is shown. Next, an overview of the study area and dataset used for the analysis. Third, the presentation of the results from the application of the modelling structures and a discussion on the implications of these result is presented. Finally, a concluding section overviewing the contributions of this work and potential avenues for future research.

7.2 Model Formulation

The conventional weighted utility model for group decisions follows the following basic weighted additive structure:

\[ V_{g\varphi} = \beta x_g + \sum_{i=1}^{J} w_i * V_{i\varphi} \]

Where \( V_{g\varphi} \) is the groups utility for selecting alternative \( \varphi \), \( w_i \) is the individual \( i \)'s weight and \( V_{i\varphi} \) is the utility of individual \( i \) for selecting alternative \( \varphi \). \( \beta x_g \) are a set of parameters and group variables respectively reflecting the overall group’s preferences regarding alternative \( \varphi \) and \( J \) is the total set of all individuals in the group. The sum of the weights across all individuals must
equal one to ensure the estimation is bounded and the model is identified. To meet this requirement the weight terms are defined as follows:

\[ w_i = \frac{e^{\delta x_i}}{\sum_{j=1}^{J} e^{\delta x_j}} \]

Where \( \delta \) is a vector of parameters to be estimated and \( x_i \) are a set of sociodemographic variables for group member \( i \). As noted in Chapter 2, this structure is limited because it fails to account for the group’s relative desire for equity versus optimality. To account for this, Keeney and Kirkwood (1975) derived an additional set of terms that can be introduced into the utility function so that it becomes the a multi linear formulation:

\[
V_{g\varphi} = \beta x_g + \sum_{i=1}^{J} (w_i * V_{i\varphi}) + \alpha \\
* \sum_{i=1}^{J} \sum_{j=i+1}^{J} (w_i * V_{i\varphi} * w_j * V_{j\varphi}) + \alpha^2 \\
* \sum_{i=1}^{J} \sum_{j=i+1}^{J} \sum_{k=j+1}^{J} (w_i * V_{i\varphi} * w_j * V_{j\varphi} * w_k * V_{k\varphi}) + \cdots \\
+ \alpha^{J-1} (w_i * V_{i\varphi} * w_j * V_{j\varphi} * \ldots * w_j * V_{j\varphi})
\]

In this form, there are now two components to the group utility, a weighted additive component which is identical to the initial equation and a set of interaction components. The interaction components include a new parameter \( \alpha \), which according to the literature, represent the degree to which a group prefers equity or inequity in their decision making. A value less than zero suggests that the group prefers an alternative which provides an unequal distribution of benefits across group members. Unfortunately, this model is incapable of determining which group member receives those benefits, which could indicate a degree of altruism from other group members. Conversely a value greater than zero suggests that the group favors alternatives that provide a more equal weighted utility value across group members. This is done by multiplying the product of all possible combinations of weighted utilities by \( \alpha \) to the power of \( (N-1) \), where \( N \) is
the level of interaction taking place. When comparing two group utilities that are equal based on
the weighted additive component the interaction will determine which alternative is preferred.
The reason this structure works is because the product of the weighted utilities that are closer
together in value will result in a higher value thereby increasing the utility for a positive $\alpha$ and
decreasing the utility for a negative $\alpha$. Note that this is not necessarily true for groups with three
or more individuals and large negative values of $\alpha$. This is because the three level interaction
will to be positive (rather than negative) because $\alpha^2$ will always be positive. The three level
interaction would then skew the results of the analysis. Keeney and Kirkwood note in their
derivation that the interaction term $\alpha$ is constrained to be no smaller than negative one, thereby
circumventing this issue. That said, no applications of the MLL in the transportation field discuss
this constraint. Fortunately, these applications have only included two level interactions. That
said, there are other more substantial problems with the MLL.

Unfortunately, the multi linear utility model is fundamentally flawed for models using random
utility maximizing for one very simple reason: the absolute value of the utility included in the
interaction component is irrelevant (and unknowable) and only the difference in utility matters.
This is further supported by the following example:

Option 1: A gets utility of -12 B gets utility of -8

$\rightarrow$ Group weighted additive utility $= \frac{1}{2} \times -12 + \frac{1}{2} \times -8 = -10$

$\rightarrow$ Interaction $= \frac{1}{2} \times \frac{1}{2} \times -12 \times -8 = 24$

Option 2: A gets utility of 4, B gets utility of 8

$\rightarrow$ Group benefit $= \frac{1}{2} \times 4 + \frac{1}{2} \times 8 = 6$

$\rightarrow$ Interaction $= \frac{1}{2} \times \frac{1}{2} \times 4 \times 8 = 8$

Option 3: A gets utility of 0, B gets utility of 4

$\rightarrow$ Group benefit $= \frac{1}{2} \times 0 + \frac{1}{2} \times 4 = 2$

$\rightarrow$ Interaction $= \frac{1}{2} \times \frac{1}{2} \times 0 \times 4 = 0$
In this example, the difference in utility between individual A and B is uniformly 4, which would suggest that the level of equity across all three options is uniform. Option 2 provides the highest level of weighted additive utility and Option 1 has the lowest weighted additive utility. When calculating the interaction component of the utility (assuming $\alpha = 1$), it becomes quite clear that the magnitude of this interaction component varies quite drastically across all three options. Option 1, which has the lowest weighted additive utility, has the highest interaction term. Based on an interaction parameter equal to one, Option 1 becomes the most desirable of the three options. Conversely, for an interaction parameter of negative one, option 3 is now the preferred option. Given that all three alternatives in the example above are equally (in)equitable, these are highly counter intuitive results. The implications of this hypothetical example are that the magnitude of the interaction parameter can no longer be interpreted as a measure of the group’s desire for equity or inequity. This finding makes the application of the multi linear logit model needless from a behavioural standpoint as it does not actually capture group (in)equity considerations.

This counter intuitive result can be explained as follows. The original derivation by Keeney and Kirkwood (1975) is derived using cardinal utility functions, rather than the ordinal utility functions used in a discrete choice context. Keeney and Kirkwood’s derivation also notes that the group utility and the individual utilities must be scaled from zero to one for their application of the multi linear model, which is obviously not the case in the discrete choice context. Discrete choice utilities are ordinal and their bounds of the individual and group utility functions are all real numbers. Therefore, while a novel concept, the multilinear utility function is fundamentally unsuitable for the discrete choice context. This calls into question all model structures that have used this application.

Given these limitations, the analysis in this chapter reverts to the standard weighted utility model but introduces a structural addition to account for joint choices at both the individual and group level. As originally proposed by Gliebe and Koppelman (2005), this chapter makes use of a parallel constrained choices logit (PCCL) to jointly model the choice of mode for all commuters (travelling to either work or school). This structure uses a structural nested logit formulation. The PPCL defines upper-level group decisions (does escort occur and if so which household adult
acts as the chauffeur) as the trunks of a nesting structure. The branches of these trunks represent individual household member mode choices. This nesting is not done to capture substitution patterns between alternatives, but rather to ensure that the probability of a parent escorting and a student being escorted by said parent are equivalent (or that the probability that no escort occurs at all being equal for all household members). This structural nesting implies that the nesting coefficient used to determine correlation patterns may not be significantly different from one. This is completely acceptable depending on the choice context, though efforts should be made to test for correlation across individual outcomes for a given group decision. The formulation and application of the PCCL in this thesis uses a generalized form of the model which considers any number of group members rather than the two originally proposed by Gliebe and Koppelman. Because of the structural nesting, an individual’s utility for a group outcome can be expressed as follows:

\[ V_{i\varphi} = \gamma x_i + \frac{\ln \left( \sum_{m=1}^{C_{i\varphi}} e^{\mu V_{im}} \right)}{\mu} \]

Where \( C_{i\varphi} \) is the set of modes available to individual \( i \) for the given group choice \( \varphi \), \( \gamma x_i \) are a set of parameters and variables influencing the preference of the individual \( i \) on group decision \( \varphi \) irrespective of their individual decision and \( \mu \) is a scaling factor. This allows us to define the joint probability function for the selection of a given group decision \( \varphi \) and a set of individual decisions \( m' \) as follows:

\[ P_{\varphi m'} = \frac{e^{V_{g\varphi}}}{\sum_{\theta \in C_{g}} e^{V_{g\theta}}} \prod_{i \in I} \left( \frac{e^{\mu V_{im}}}{\sum_{m=1}^{C_{i\varphi}} e^{\mu V_{im}}} \right) \]

\[ \quad = \frac{e^{b X_{\varphi} + \sum_{i=1}^{I} (w_i + (V_{i\varphi}))}}{\sum_{\theta \in C_{g}} e^{V_{g\theta}}} \prod_{i \in I} \left( \frac{e^{\mu V_{im}}}{\sum_{m=1}^{C_{i\varphi}} e^{\mu V_{im}}} \right) \]
Note that here, $C_g$ represents the choice set of options available to the group and $bx_\varphi$ represents the household level (versus individual) utility (parameters and variables respectively) associated with choice $\varphi$. $V_{g\varphi}$ is treated as the weighted sum of the individual utilities for alternative $\varphi$ plus the household utility gained from this choice.

It is now possible to apply this formulation to the decision structure of escorting students to school and then the subsequent mode choice for the student and all possible adult escorts travelling on to work. This decision structure is outlined in Figure 7.1. This choice context creates several interesting challenges. First, note that while student school attendance is typically compulsory, not all adults go to work. To account for this, two possible utility outcomes for adults are defined: the case where the adult travels to work and the case where the adult does not travel to work. If the adult travels to work, their individual choice set $C_{i\varphi}$ is a selection of modes from their available choice set and their escort choice involves travelling to the student’s school and then on to their work location. Conversely, if the adult does not travel to work and does not escort the student then the model assumes that no mode choice occurs (choice set of one alternative – no travel). If the non working adult does escort the student then they return home after said trip. This is an obvious simplification of the analysis (e.g. adults may drop-off students and then proceed to a shopping activity). This simplification is implemented to reduce the complexity of the data generation and processing although the model will still function as intended with non-work/non-escort travel included in the model framework.

Regrettably, the model formulation also grossly simplifies the behaviour of multi-children households for computational tractability. Namely, multi student households are treated as independent and students are not permitted to act as chauffeurs to their siblings. These simplifications are implemented because as the number of children in the household increases, the number of possible escort outcomes grows exponentially and adds potential new student chauffeurs. As such, all students in the same household are treated as independent observations.
One possible technique to account for this is the use of sampling of potential escort outcomes for multiple students, however, this is left for future research. Alternatively, students from the same household could be treated as a panel using a mixed logit formulation to induce correlation between the students.

This structure contains similar logic to the mode choice model proposed in Chapter 4, particularly with respect to the technique of using the reduction in auto availability for a given chaperone outcome. Specifically, in a one car household, if a chaperone is chosen to escort the student to school by car, then other household members do not have access to the vehicle. This approach does not explicitly allow for the level of detail described in Chapter 4 as it employs individually defined choice sets rather than the group choice set approach used in that chapter. As a result, while the example above is capable of explicitly constraining the choice set, households with more two or more cars that are auto deficient (households with more drivers than cars) cannot have their choice sets constrained properly for vehicle allocation using this formulation. This is primarily due to the generalized (in terms of the number of household members examined) format employed here, whereas Chapter 4 only examined two adult households, making the choice set constraint much easier to determine and employ.
Figure 7.1 Choice Tree for Joint Mode and Escort Choice Model

*student carpooling could be joint student travel (another student in the same household drives) or intra household carpooling

**adults carpooling could be inter or intra household carpooling
7.3 Data Description

As with the other studies in this thesis, this chapter makes use of the TTS survey data. For the purposes of this analysis, only households from the City of Toronto proper (excluding surrounding municipalities) with at least one student between the ages of thirteen and seventeen inclusive (the typical age for high school) who make a trip to school were included in the analysis. The surrounding municipalities are excluded for two main reasons. First, to create a more reasonably sized data set as the model runs can be quite long on a typical desktop computer. Second, the study area is constrained to the City of Toronto as this study area dovetails with an existing database of high schools for the city. After cleaning the data and removing records with incomplete or erroneous behaviour, 4197 students remained. To simplify the analysis, students who lived in the same household were treated as separate records. This simplification was used because multi-student households would result in an explosion in the number of escort possibilities. The analysis did not explicitly model joint travel between adult household members for the same reason. Joint travel between two adults in the household is instead included as a generic passenger mode which is indistinguishable from carpooling with someone from outside the household. This model could easily be extended to consider multi-student households and joint travel between adults in the future though the time to convergence for a model of this complexity would increase in turn. Table 7.1 provides a distribution of household, student and adult attributes and behaviours.

The TTS data is supplemented with an inventory of high schools in the City of Toronto with information regarding the characteristics of all schools located in each zone. Specifically, the school board status (public, Catholic, private) has a significant impact on the escort probability of a given student. In total, there are 151 number of schools spread across 625 traffic zones. Due to the relatively high density of schools in the region, there are several zones that contain multiple schools. In these cases, the characteristics of the school are summed (i.e. a zone with both a public and a private school is treated as public and private school simultaneously, school enrolment summed, etc.). Moving forward, a more detailed record of which specific school a student travels to would reduce this bias, though, given the limitations of the TTS data, this aggregation is unavoidable for this analysis. Finally, the results of the multimodal aggregate traffic assignment model developed using the TTS data is also included to generate travel time
and cost values for driving and transit modes for all individuals. These times and costs were for the morning peak period (from 6 to 9 a.m.), which is generally when most trips to school occur.

Table 7.1 High School Students in Toronto Descriptive Statistics

<table>
<thead>
<tr>
<th>Household and Individual Descriptive Statistics</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of household vehicles</td>
<td>1.34</td>
</tr>
<tr>
<td>Percentage of single student households</td>
<td>73.2%</td>
</tr>
<tr>
<td>Average number of household potential escorts (household has at least one car and escort has a driver's license)</td>
<td>1.7</td>
</tr>
<tr>
<td>Average student age</td>
<td>15.45</td>
</tr>
<tr>
<td>Average adult age</td>
<td>45.3</td>
</tr>
<tr>
<td>Percentage of male students</td>
<td>52.5%</td>
</tr>
<tr>
<td>Percentage of male potential escorts</td>
<td>53.2%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Student Mode Distributions</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Drive</td>
<td>0.74%</td>
</tr>
<tr>
<td>Auto passenger (carpool with someone other than an adult family member)</td>
<td>3.98%</td>
</tr>
<tr>
<td>Walk</td>
<td>30.12%</td>
</tr>
<tr>
<td>Bike</td>
<td>1.45%</td>
</tr>
<tr>
<td>Transit</td>
<td>42.58%</td>
</tr>
<tr>
<td>Escorted by an adult family member</td>
<td>21.13%</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Adult Mode Distributions</th>
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</thead>
<tbody>
<tr>
<td>Drive (Not Escort)</td>
<td>28.06%</td>
</tr>
<tr>
<td>Auto passenger (either joint or intra-household carpool)</td>
<td>2.22%</td>
</tr>
<tr>
<td>Walk</td>
<td>1.18%</td>
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<tr>
<td>Bike</td>
<td>1.07%</td>
</tr>
<tr>
<td>Transit</td>
<td>10.14%</td>
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<tr>
<td>No travel</td>
<td>44.89%</td>
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<tr>
<td>Escort (then drive to work)</td>
<td>4.70%</td>
</tr>
<tr>
<td>Escort (then return home)</td>
<td>7.74%</td>
</tr>
</tbody>
</table>

7.4 Empirical Model Results

Through the course of the analysis, numerous models were estimated with a focus on obtaining the correct specification of the nesting structure. As is the case in Gliebe and Koppelman’s initial paper, numerous nesting coefficients were found to be equal to one. In the presented model formulations, to constrain the nesting coefficient to be larger than one the nesting coefficient is specified as follows:

\[
\mu = 1 + \exp (bx)
\]

Because of this structure, standard t-statistics cannot be used to determine the significance of the nesting parameter. Instead, log likelihood ratio tests are used relative to the model with no nesting coefficients where parameters were deemed to have a statistical improvement on the
model if the ratio test exceeded the critical value of the chi-square table at 5% significance level. In the case of the models presented below there is no statistical evidence to suggest any nesting. Fortunately, the PCCL still functions when the nesting parameters collapse to 1 as nesting coefficients greater than 1 simply suggest that travelers view the modes within that nest as reasonable substitutes for each other relative to other nests. As there is no reason for the modes for a given escort case to be considered reasonable substitutes to each other relative to modes in another nest, the lack of significance of this parameter across all nests is quite reasonable. The PCCL model is still powerful even with non-significant nesting coefficients as it constrains the choice probability for individual and group decisions to be theoretically consistent and equal across individuals. The parameter estimates and t-statistics for the estimates along with the general goodness of fit statistics for the final version of the PCCL model are found in Table 7.2. The presented model was estimated using the Gauss programming language (Aptech Systems, 2015).
<table>
<thead>
<tr>
<th>Parameter Category</th>
<th>Variable Description</th>
<th>Estimate</th>
<th>T-Statistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Escort Alternative Specific Constants</td>
<td>Auto drive</td>
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<td>4.622</td>
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<td>Auto passenger</td>
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<td>Walk</td>
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<td></td>
<td>Bike</td>
<td>0.352</td>
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<td></td>
<td>Transit</td>
<td>2.089</td>
<td>9.531</td>
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<td></td>
<td>No travel (escorts only)</td>
<td>1.461</td>
<td>5.172</td>
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<td>Level of Service Variables</td>
<td>Generic in vehicle travel time</td>
<td>-0.018</td>
<td>-6.210</td>
</tr>
<tr>
<td></td>
<td>Generic out of vehicle travel time</td>
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<tr>
<td></td>
<td>Travel cost (adults only)</td>
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<td></td>
<td>In distance walk</td>
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<tr>
<td></td>
<td>In distance bike</td>
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<tr>
<td>Socio-Demographics on Adults’ Mode</td>
<td>Number of household vehicles in the drive utility</td>
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<tr>
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<td></td>
<td>Dummy variable for male in transit utility</td>
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<td></td>
<td>Dummy variable for male in walk utility</td>
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<td>Dummy variable for male in bike utility</td>
<td>1.408</td>
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<td>Dummy for if the adult has a transit pass in transit utility</td>
<td>3.538</td>
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<tr>
<td>Socio-Demographics Students’ Mode</td>
<td>Dummy for if household has more drivers than vehicles auto driver</td>
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<td>-6.420</td>
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<tr>
<td>Choice</td>
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<td>Variables Influencing Adult Escort</td>
<td>Dummy variable for if there are 2 or more teens in the household</td>
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<td>Utilities</td>
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<td>Variables Influencing Student Escort</td>
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<td>Influencing Escort Decisions</td>
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<td>McFadden Rho Squared (against null)</td>
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<tr>
<td></td>
<td>McFadden Rho Squared (against constant)</td>
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<tr>
<td></td>
<td>Number of observations used</td>
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</tr>
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</table>
7.5 Discussion of Results

The discussion of the results of this empirical modelling exercise is divided into two sections: a discussion of elasticities of select parameter estimates, and a discussion of more general findings and behavioural outcomes derived from examining the model results.

7.5.1 Elasticities

The magnitude of the parameter values for discrete choice models have no interpretive value independently. Instead, econometricians typically turn to elasticities and marginal effects or ratios between parameters to calculate indices such as the value of travel time savings. In the case of the parallel constrained logit, the calculation for the elasticities is significantly more involved relative to the well-known multinomial logit model marginal effect and elasticity calculation. This complication arises from the inclusion of the structural nesting term and the inclusion of the weight of the log sum component. Consider a single variable $x$ which directly influences the utility of an individual’s choice. The probability of the household outcome and individual outcome as a function of that variable $x$ on said individuals’ choice can then be expressed as the following marginal probability function:

$$P_{ij} = \left( \frac{e^{a + \frac{w_i}{\mu} \ln(e^{\mu(bx+c)} + d) + f}}{e^{a + \frac{w_i}{\mu} \ln(e^{\mu(bx+c)} + d) + f} + g} \right) \times \left( \frac{e^{\mu(bx+c)}}{e^{\mu(bx+c)} + d} \right)$$

Where:
- $a$ = A constant term reflecting the total group decision utility for alternative $\varphi$
- $w$ = The weight for the individual $i$
- $\mu$ = The nesting coefficient
- $b$ = The coefficient for the variable $x$
- $x$ = The variable for which we are interested in determining the elasticity or marginal effect
- $c$ = A constant reflecting the remainder of the utility for alternative $j$
- $d$ = A constant term reflecting the sum of the exponent of the remainder of the modal utilities available to individual $i$
- $f$ = A constant term reflecting the weighted sum of the exponent of the utilities for the remainder of the individuals for a given group decision
\[ g = \text{A constant term reflecting the sum of the exponent of the utility of all other possible group decision outcomes.} \]

The first bracketed term represents the probability of the group decision, which in our case is the decision regarding who, if anyone, is allocated the escort task for the student. The second term represents the probability that for individual \( i \), the mode \( J \) is selected. In this equation, we ignore the impact of the modal choice of all other household members and only consider the impact of the continuous variable on the individual’s choice. This greatly simplifies the task of calculating the elasticities relative to considering a “full elasticity” as the number of modal combinations becomes exceedingly large as the number of possible chauffeurs grows. This is identical to the formulation provided by Gliebe (2004) though it is derived here as a formal reference which is not shown in the initial work.

Taking the partial derivative of this function with respect to the variable in question \( x \), the resulting function or marginal effect can be expressed as follows:

\[
\frac{dP_{ij}}{dx} = b \left( d \mu e^{\frac{a + w}{\mu} \ln(e^{\mu(bx+c)+d}) + f + g \mu(bx+c) + d g \mu} \right) \left( e^{\frac{a + w}{\mu} \ln(e^{\mu(bx+c)+d}) + f + g} \right) \left( e^{\mu(bx+c)+d} \right)^2
\]

When simplified we arrive at the following equation

\[
\frac{dP_{ij}}{dx} = \left( b \left( d \mu e^{\frac{a + w}{\mu} \ln(e^{\mu(bx+c)+d}) + f + g \mu(bx+c) + d g \mu} \right) \right) \left( e^{\mu(bx+c)+d} \right)^2
\]

\[
= b \left( \mu \ast (1 - P_{m|\varphi}) \ast (P_{\varphi}) + w \ast (1 - P_{\varphi}) \ast (P_{m|\varphi}) + \mu \ast (1 - P_{m|\varphi}) \ast (1 - P_{\varphi}) \right) P_{ij}
\]

\[
= b \left( \mu \ast (1 - P_{m|\varphi}) + w \ast (1 - P_{\varphi}) \ast (P_{m|\varphi}) \right) P_{ij}
\]

The elasticity is then calculated by multiplying this expression by the variable \( x \) and dividing by \( P_{ij} \). As noted above, this is the same result achieved by Gliebe (2004) indicating that the N person PCCL model functions identically to the original 2-person model.
This equation appears much more reasonable however it should be noted that this elasticity only applies for certain utilities. If an individual’s choice is invariant across nests then the g term (the remainder of the base for the escort decisions) defined above is no longer a constant but instead a function of the variable for the modal alternative for which the elasticity is being calculated. In the analysis presented in this chapter, the above equation is only valid for a few cases:

- The students’ modal choice given that no escort decision is made.
- The adult's utility for escorting the student (here there is no modal choice as the parent will always drive).

The remaining modal and escort alternatives require an extension of the g constant in the above formula. These complications suggest that an analytical approach to the elasticity calculations could be undertaken to approximate the elasticity as follows. The marginal probability function outlined is calculated at multiple values of x ranging from $0.9 \times x$ to $1.1 \times x$ at intervals of 0.01 for all continuous variables. The difference in probability between that value and the base probability at $1 \times x$ is then divided by the base probability for scaling purposes and is then plotted, with the slope of the line representing the elasticity. For dummy variables, the marginal probability is calculated with the variable equal to 1 and then with the variable equal to zero. To generate unbiased aggregate estimates, a probability weighted sample enumeration approach is used (Hensher et al. 2015), whereby each of these values is then weighted by the marginal probability of the actual observation (the value of the marginal probability at x). Figures 7.2 to 7.5 represent the elasticity plots for select continuous variables of interest for students and chaperones for the PCCL model.
Figure 7.2 Transit Elasticities

Figure 7.3 Non-Motorized Elasticities
Figure 7.4 Auto Driver and Auto Passenger Elasticities

Figure 7.5 Probability of Chauffeuring/Being Escorted

Percentage Change in Probability of Choosing Driving or Auto Passenger

Percentage Change in Variable Values

Percentage Change in Probability of Chauffeuring/Being Escorted

Percentage Change in Variable Values

Percentage Change in Vehicle Travel Time

Percentage Change in Vehicle Cost
7.5.2 General Findings

The models above provide numerous insights into the travel behaviours and patterns of adults and students. Of these findings, the following are particularly interesting and/or relevant:

Cost perceptions between students and adults:
Numerous model specifications were attempted before settling on the ones seen above. One of the findings that came out of this testing is the exclusion of the travel cost variable in the student mode choice utility functions. This exclusion was done after testing a generic cost coefficient and two specific cost coefficients (one for students and one for adults). Both model structures produce insignificant or incorrect signs for the cost coefficients and provide insignificant improvements in overall fit relative to the presented model. One possible hypothesis for this finding is the student may not be responsible for paying for the cost of travel (i.e. it is the parent’s responsibility to pay). This suggests that students are insensitive to the monetary cost as they do not directly experience the cost itself. Another plausible explanation is the concept that students are forced into a certain modal choice (i.e. too far to walk, not reasonable to be chaperoned means that the student must take transit). This concept is further supported as many schools provide free transit to students. Officially, the public-school board provides transit funding (the school provides TTC tickets) for students that live more than 4.8 kilometers from their school (TDSB, 2014), though anecdotal evidence suggests that Catholic and private/independent schools will also provide these services to students on a case by case basis.

Gender/age differences in student driving/escorting likelihood:
While most of the estimated parameters were significantly different from zero, there were a few that showed interesting behavioural trends without exceeding the critical value for the hypothesis test. Of these, the fact that male students were less likely to drive than their female counterparts suggest that parents may recognize the well-reported trend of young male drivers being significantly more likely to be involved in a motor vehicle accident (Constantinou et al. 2011). As such, parents may be less inclined to allocate a household vehicle to a male student relative to a female student. The other variable that is not significant but shows an interesting trend is the dummy for age under 16 for escort likelihood. This variable is included to capture the trend that
parents may be more protective of younger students. Younger age cutoffs are also tested and produced similar insignificant parameter estimates. While not examined here, a more detailed examination of how the age of the student interacts with other variables on the likelihood of being escorted is an interesting avenue of future research. For example, the presence of an older sibling may suggest a more relaxed attitude to independent travel. Parents of these student could be more comfortable with the concept of the students travelling independently (or under the supervision of the older sibling). Unfortunately, the modelling presented in this chapter is unable to fully capture older student escort behaviour, though it could be expanded to examine these escort possibilities. Finally, gender is also tested in terms of its impact on escorting utility but is also found to be insignificant. The general expectation is that girls are more likely to be escorted than boys due to safety concerns, though this hypothesis was rejected based on the insignificance of the parameter.

*Higher weighting for students:*
While not explicitly capturing the desire for group equity discussed above, the weighted model structure does provide a clear indication that the student’s needs are viewed as the most important by the group. While not directly capturing the trend, the higher student weight suggests parents may experience altruism for their children, which agrees with the conventional understanding of family dynamics (Becker, 1991). The weight parameter for students suggests that the household values the student utility 167% more than any given adult in the household (calculated by taking the ratio of the weights of the student and any adult household member). This finding implies that a decrease in adult utility can be offset by a proportionally smaller increase in child utility. Unfortunately, while the general trend of altruism is captured, there currently exists no explicit method for directly modelling the altruistic behaviour of household members towards each other (i.e. specifying an individual’s utility as a function of the utility of other group members). A model which captures the explicit level of altruism a given household member feels for other members would provide further stratification of altruism throughout the household and thus may provide new insights into household behaviour.

One possible example of a deeper insight from this trend is brought up in Becker’s (1991) seminal work. Becker suggests that parents have conflicting desires relative to the desires of their children (i.e. as discussed above, they do not want their child driving the car as they fear they
will get into an accident whereas the child wants to drive as it is more convenient, fun, etc.). Unfortunately, the PCCL model is incapable of truly capturing these behavioural patterns (i.e. the parents’ utility for a group choice is also some function of their children or of other household members). A model that can capture these behavioural trends within a discrete choice framework would represent a considerable advancement relative to existing practices.

*Households with No Viable Escorts*

Interestingly, when households have no viable escorts (households that do not own a car or have no adults with a drivers licence), students are significantly less likely to carpool to school (either with a non-family member or a sibling under the age of 18). There are several potential explanations for this finding. Potential explanations include the following examples:

- First, this finding could be an indication of the household being located in an area with an urban form which promotes more sustainable forms of travel (e.g. the family home is near the subway). This would cause household members to forgo vehicle ownership because of the high level of accessibility without a vehicle. This high level of accessibility would directly transfer to the student for their trip to school.

- Second, this finding can be explained by the household being located in an economically disenfranchised area. This would suggest that the household owns no vehicles and that the surrounding households are also less likely to own vehicles, thus severely limiting the capacity for inter-household ride sharing.

### 7.6 Chapter Conclusions

The factors and considerations that influence an individual’s travel decision throughout the course of a day are broad and extend beyond the scope of what is typically considered in traditional travel demand models. Parents will often make sacrifices for their children and will intentionally make sub-optimal choices if those choices improve the welfare of their children. Models which can capture these trends and behavioural tradeoffs are essential to our understanding of the impact of a wide range of transportation policies moving forward. With that said, caution must be undertaken when developing the advanced model as applying a model incorrectly could lead to disastrous results. There must be a set of checks and balances in place to ensure that the model functions properly and is not providing the analyst with counterintuitive or meaningless results. This chapter acts as one of those checks and correctly identifies the
inapplicability of the multi linear logit model for group discrete choice applications within a random utility maximizing framework. While only applicable to the MLL model, hopefully, the analysis here will serve as a general reminder about the risks of applying a modelling frameworks without fully understanding how the model structure works.

This chapter also provides an extension to existing models of a group decision, generalizing the parallel constrained choices logit model from only allowing 2 individuals to any number. This work is, to the best of our knowledge, the first generalized application of the PCCL to groups larger than two. This is an intuitive and useful modelling structure which allows group choices and individual choices to be jointly modelled. The PCCL is not, however, a silver bullet in terms of modelling household interactions as it still is unable to explicitly capture household desires for equity or to directly capture the altruistic behaviour of household members. These limitations do present some avenues for future research, most notably the development of a model capable of capturing individual altruism.

On top of the development of a model of individual altruism, the model presented here could be adapted and expanded to consider:
- Incorporating non-commuting travel for adults within the household as part of the mode choice model
- Examining multi-student households directly without separating them into individual records
- Examining travel to school during the morning peak period and the return to home (or more generally, incorporating a tour-based model logic within the framework

Despite these potential expansions moving forward, the analysis within this chapter does make two substantial contributions: the identification of the inapplicability of the MLL and the generalization of the PCCL to any number of individuals.
Chapter 8
Conclusions, Research Contributions and Avenues for Future Work

This chapter is structured as follows:

- A summary of the thesis in section 8.1.
- A review of the methods and primary conclusions of the empirical investigations in Chapters 4 through 7 in section 8.2.
- A discussion on the overall contributions of this research in section 8.3.
- A discussion of potential avenues for future work in section 8.4.

8.1 Thesis Summary

This thesis begins with an overview of activity based modelling frameworks and a discussion of the research objectives, the relevance of the proposed research and some background on the study area and data used for empirical evidence. Chapter 2 of this thesis presents a review of the relevant literature, focusing on intra-household interactions, spatial interactions and spatial constraints within the context of activity based modelling. Chapter 3 presents a conceptual framework for integrating models of intra-household interaction and spatial interaction within comprehensive models of travel demand.

Chapters 4 through 7 present empirical investigations on specific aspects of intra-household interaction. Chapter 4 presents a household level unitary model of mode choice, considering specification of the choice set as a means of addressing vehicle allocation and joint travel. Chapter 5 presents a model of station location choice for park and ride and kiss and ride travel, proposing a new model, the spatially weighted error correlation (SWEC) logit model which captures spatial correlation and spatial heteroskedasticity. Chapter 6 presents a model of daycare drop-off and pick-up task allocation and location choice of daycare, utilizing an innovative method for linking spatial constraints with intra-household interaction and more generally a method for producing more realistic consideration choice sets for spatial choices. Finally, Chapter 7 presents a generalized model of household mode choice, considering the mode of all
commuting adults and a student within the household while also considering the choice for an adult to escort the student. Chapter 7 also identifies a theoretical inconsistency within an existing model of group decision making.

8.2 Research Overview

One of the primary motivations of switching towards an activity based modelling framework is the increased realism of the predictions relative to a conventional trip based structure. This increase in prediction realism results in an improvement in the information used to direct policy decisions and infrastructure investment. Increased realism and accuracy in the predictive capacity of a model can better inform decisions about road/congestion pricing, HOV or high occupancy toll (HOT) lanes, the addition of a new transit line or a host of other possibilities. While activity based modelling is an area of active research, one of the current weaknesses within many operational activity based models is the limited consideration of the impact of intra-household interaction on individual travel. This is a very broad aspect of travel demand modelling as it can be argued that virtually all choices associated with an individual’s daily travel pattern are either directly or indirectly influenced by interactions within the household. When these intra-household interactions are coupled with other interactions or constraints (such as the spatial issues discussed in this thesis) the task of accurately approximating travel behavioural patterns becomes even more challenging. To accommodate these challenges, this thesis provides an overview of a set of methods for including models which address household and/or spatial interactions as modules which can be incorporated within a comprehensive model of travel demand.

The accommodation of these interactions is based on a conceptual framework, where the population is stratified based on the household structure and a set of models are estimated and applied for each of these strata. While numerous models can be estimated regarding a wide range of travel behaviours, this thesis addresses questions regarding the allocation of tasks and resources to individuals within the household. This thesis focuses predominantly on questions regard the household vehicle allocation process and the chauffeuring of either other household adults or dependents. It is found through an extensive review of the existing literature that tasks requiring a drop-off and a pick-up (typically of a dependent, though ride sharing between adults also falls into this category) are complex tasks to model. These chauffeur tasks require a high
degree of spatial and temporal coordination (e.g. agreement to leave or meet at the same time, travel to and from the same place), making these tasks much more complex than the allocation of a single trip task, such as performing the household grocery shopping for the week. Given these complications, escort/chauffeur tasks were examined in detail in this thesis, both from the perspective of joint travel between adults and the escort decisions for dependents.

The background information regarding the limitations of existing practices for capturing task and resource allocation leads to the empirical investigation presented in Chapter 4. In this chapter, the joint choice of morning peak period mode is modelled for two adult household (both auto sufficient and auto deficient). This model uses a unitary decision making approach, whereby the household’s utility is defined as the unweighted sum of each household member’s utility for their own independent mode. Building from the model proposed initially by Badoe (2002), this structure expands the modal definitions drastically and corrects a minor error in utility specification of the original model structure. This structure allows for the choice of vehicle allocation and joint travel to be implicitly captured within the choice set using a standard discrete choice modelling format. This formulation permits a set of complex joint decisions (vehicle allocation and joint travel) to be addressed using a simple and unified framework, without having to resort to simulation based methods or complex structural nesting structures as has been done previously. Moreover, two additional complex model structures are estimate: a nested logit and an error component nested logit. The differences between the nesting structures found in these two models highlight the challenges associated with selecting an appropriate model structure. In the empirical context of Chapter 4, it is found that the alternatives exhibited a heteroskedastic error structure, which the nested model is unable to directly capture. This suggests that a nested and error component structure should be tested to provide a better understanding of correlation and heteroskedasticity across alternatives.

The investigation in Chapter 4 opens many interesting sub questions regarding intra-household interaction leading to the work performed in Chapter 5. Examining joint multimodal trips is the most prominent of these sub questions and forms the basis for the analysis in Chapter 5. Multimodal transit trips are notoriously complex and require a detailed set of questions regarding the different trip sub legs, including the access mode to transit and access transit station. As Chapter 4 examines the choice of access mode, Chapter 5 moved on to examining station
location choice for kiss and ride trips. This analysis provides a comparison between station choice for park and ride and for kiss and ride trips, highlighting the differences that arise between these two choice contexts. From an intra-household perspective, park and ride station choice is purely based on individual utility maximization (i.e. pick the station that best suits the need of the individual travelling), whereas kiss and ride station choice requires tradeoffs between the driver’s needs (a station closer to their final destination) and the passenger’s needs (a station with frequent service towards their destination). These concerns are then coupled with conventional issues regarding the interaction between spatial alternatives, namely that spatial alternatives exhibit a degree of correlation inversely proportional to the distance between them. These issues surrounding spatial correlation support the development of a new model structure, the spatially weighted error correlation logit model. Using this general model structure, a series of models are estimated, considering a pooled park and ride and kiss and ride data set as well as a kiss and ride only data set. The findings of this study are that the kiss and ride model showed significant differences from the park and ride model, suggesting that park and ride and kiss and ride records should be treated independently. Furthermore, the results of the kiss and ride model suggest that spatial correlation between kiss and ride stations is not as pronounced as it is for the park and ride model, which is possibly explained by the increased importance of competing household interests for the decision.

Shifting gears, Chapters 6 and 7 focuses the analysis on questions regarding the allocation of escort tasks for dependents. Chapter 6 focuses first on preschool aged children, who are not sufficiently autonomous to travel on their own and therefore require an escort to and from daycare services. Given that the existing literature typically examines these daycare trips as analogous to school aged trips, a specific examination of daycare only travel is performed, highlighting the differences between school travel and daycare travel. This analysis also builds off the conclusions of Chapter 5, by explicitly incorporating decisions regarding the location of the daycare into the analysis through a joint modelling framework. This leads to the hypothesis that the allocation of the daycare drop-off and pick-up tasks are based on the given location of the daycare. Conventional modelling structures would capture this by using a simple random sample of all alternatives in the region, though this is not realistic and goes against one of the fundamental axioms of activity based modelling, namely the incorporation of spatial constraints. Specifically, this chapter applies the Hägerstrand (1970) concept of space time prisms to
generate location choice sets which are specific to a given task allocation outcome. While the application of the stochastic frontier model in Chapter 6 is used in other applications for space time prisms, the empirical investigation in Chapter 6 applies this model structure spatially, rather than temporally, as has been done in the past. This alternate application provides an interesting foil to conventional practice within activity based models of travel demand.

Finally, Chapter 7 provides an application and generalization of the parallel constrained choice logit for household mode and school escort decisions for high school students. This chapter expands the existing approaches for intra-household interaction by generalizing the number of household members considered in the model to any number. Moreover, this chapter builds on the discussion in Chapter 4, by using choice set restrictions at the individual level to account for vehicle allocation in auto deficient households. The behavioural interpretation of the results from this modelling exercise includes a general trend of altruism or sacrifice on the part of household adults for their children as represented by the increased relative importance of the student’s utility in the group utility function. This chapter represents a methodological advancement on the work of Chapter 4, in terms of the techniques used and the complexity of the decision making process. Concurrently, Chapter 7 also identifies a theoretical inconsistency with the multi linear logit model. This modelling structure uses a multiplicative form derived under the assumption of cardinal utilities that no longer functions as intended when ordinal utilities (which can be negative) are used.

8.3 Research Contributions

The research contributions of this thesis cover several different aspects of travel demand modelling. Generally, the thesis provides broad conclusions regarding the analysis of intra-household and spatial choice behaviours in the context of travel demand modelling. The thesis also provides a general framework for the integration of these topics into models of travel demand. Finally, the thesis provides new modelling structures and novel applications of existing modelling techniques. The detailed description of the contributions of this thesis are as follows:

1. This thesis presents practical and applicable models of household mode choice. These models are easy to estimate using commercial or general statistical software and do not require specialized formatting (i.e. likelihood function definition). This is particularly true of the
general framework and MNL model presented in Chapter 4, where complex intra-household
behaviours can be captured through an appropriate choice set specification.

2. This thesis provides an interesting set of discussions on the applicability of the mixed logit
model for capturing correlation patterns between alternatives. Empirically the results of the
analysis performed in this thesis suggest that jointly capturing heteroskedasticity along with
correlation patterns provides superior model fit and behavioural interpretation relative to
capturing correlation alone.

3. This thesis extends existing literature on the application of mixed logits, noting that the order
in which alternatives are considered has a profound impact on the outcome of the modelling
exercise when a full Cholesky matrix for the error correlation structure is applied. The thesis
also presents a set of guidelines for approaching the order of alternatives for spatial choices
which follows an intuitive and interpretable structure.

4. This thesis presents an interesting new modelling structure, the spatially weighted error
correlation model for spatial choices. The SWEC model takes the insights and contributions
from points 2 and 3 in this list and presents a model structure capable of capturing detailed
information about spatial choices in terms of their correlation and in terms of a pattern of
heteroskedasticity. The correlation patterns follow Tobler’s (1970) first law of geography and
the heteroskedastic pattern is expressed such that alternatives that are father away from the
decision maker have larger variances.

5. This thesis presents a conceptual second law of geography based off the findings from the
analysis done using the SWEC: “When making a choice from a given reference point,
options that are far away are more random than options that are close by”.

6. The thesis presents an interesting application of stochastic frontier model for space time
prisms where the space is modelled for a given time-period. This presents an interesting foil
to the approach that has been used elsewhere in the literature (Kitamura et al. 2000a, 2000b),
who consider the temporal vertices for travel as opposed to the boundaries of the spatial
prism (or more accurately, ellipse).

7. This thesis identifies a severe theoretical inconsistency inherent within an existing and
recently applied modelling structure, the multi linear logit model. The MLL model has been
applied in numerous peer reviewed publications though it is inappropriate for ordinal utilities
used for random utility maximizing behaviour. This thesis is the first work to identify these
concerns.
8. This thesis contributes to the generalization of the parallel constrained choice logit model to incorporate any number of group members into its structure.

9. This thesis provides a novel application of the parallel constrained choice logit to capture escort decisions at the household level and mode choice decisions at the individual level. This contribution addresses a concern identified by notable contributors in the field of children escort decisions (Gupta et al. 2014).

As a general conclusion, this thesis also provides further support and evidence of intra-household interaction and its importance in demand modelling. This supports the maintenance of some degree of household information in travel surveys as opposed to moving towards a totally individual level approach to data collection. A loss of the level of detail regarding household interactions obtained by conventional household travel surveys would severely limit model development and provide analysts and decision makers with an incomplete picture of the behavioural process. As new technologies for data collection become available, the agencies which collect and maintain this data will have to ensure that this information is collected in some capacity.

8.4 Future work

While this thesis does provide notable contributions to intra-household interaction and spatial interactions, the work in this thesis opens several interesting avenues for future research. The primary next step of this framework would be to execute some of the model integration discussed in Chapter 3. That said, there exist other sub projects that should also be undertaken such as:

1. Expanding the model developed in Chapter 4 to incorporate more than two adults. This creates a challenge as the size of the choice set can balloon exponentially as more adults are added, suggesting a need for balance. It is safe to assume that most households have either one or two heads, meaning that the proposed modelling framework is for the most part adequate.

2. Expanding the model developed in Chapter 4 to incorporate a tour based logic within the framework. Tour based logic in the modelling structure could be incorporated either directly by modelling the outbound (morning peak) and inbound (evening peak) trip chain mode choice simultaneously, sequentially or by using a Gibbs sampling framework for independently estimated inbound and outbound trips. Tour based constraints could also be
included indirectly (i.e. dummy variables for the presence of intermediate stops on either the morning or evening peak legs. The inclusion of these variables would provide further insights into the rationale behind vehicle allocation and joint travel decisions at the household level.

3. Jointly modelling the choice of station location and within a broader model for general household mode choice. This would be an important improvement to the models presented in this thesis because several transit station location choice models use this joint framework. This would provide a clearer understanding of how station improvements impact of station ridership levels and transit mode share changes because of station improvements.

4. Expanding the analysis of daycare location and escort task allocation to incorporate decisions regarding the use of daycare within the model structure, either sequentially or jointly.

5. Comparing the application of the stochastic frontier model in this thesis to the application of Kitamura et al. (2000a, 2000b). This comparison could be done in terms of overall model fit for a specific decision context (i.e. daycare location choice) and then more generally in terms of the appropriateness of either model structure for different contexts within a comprehensive model of travel demand.

6. Examining and exploring different model formulation for a model of household altruism for the joint household mode and escort model applied in Chapter 7.

7. Examining different policy applications for the modelling structures:
   a. Using the household model in Chapter 4 to predict the effectiveness of introducing high occupancy vehicle or high occupancy toll lanes.
   b. Using the station location choice models in Chapter 5 to examine the impact on station attraction rates of increasing or decreasing parking capacity or increasing parking cost at transit stations.
   c. Using the models of escort task allocation in Chapters 6 and 7 to understand how various travel demand management policies have the potential to influence travel patterns and vehicle allocation in unusual ways. A prime example of this is presented in Bhat et al. (2002), who outline how a road pricing policy has the potential to result in more automobile travel due to a restructuring of household roles.

8. Using the models presented in this thesis for different potential applications outside of those considered. Specifically, the SWEC models in Chapter 5 can be applied to a broad array of spatial decisions and the stochastic frontier models in Chapter 6 can be expanded to consider feasible location for a broad array of non-mandatory activities. Potential spatial decisions for
the SWEC model include school location, grocery store location, or vacation location choice models. The frontier models used in Chapter 6 could be expanded to constrain the spatial choice set for a broad array of discretionary activities either using the modelling context presented in Chapter 6, or a more general activity based formulation. More generally, the PCCL model in Chapter 7, or a simpler weighted group utility model could be expanded and applied to a broad range of spatial and intra-household interactions in both the short term and long term. Short term examples include joint activity participation and long term examples include vehicle ownership and residential location choice.

9. Incorporating broad new data sources into the analysis, including, but not limited to, passive smart phone GPS data collection. These smart phone data collection tools could be used to passively collect information regarding joint travel based on inferred social network. Alternatively, a more formal definition of the social network between all household members could be employed, which would reduce the error associated with passively inferring the relationship between individuals travelling or performing activities together. While not directly related to the analysis in question, the development of a passive data collection smart phone application capable of collecting this level of information is also relevant.

10. Performing and analyzing the data from a multi day survey, which would provide a clearer picture of how interactions within the household vary over the course of a week. While no recent multi day survey is currently available for analysis, a proper investigation into how tasks are allocated over the course of a week based on household scheduling could provide a richer understanding of the intra household behavioural process. This is particularly true for households with part time workers or workers with flexible schedules. Out of home household maintenance tasks may be allocated based on the individual’s daily schedule (i.e. fewer tasks on a heavy work day, more tasks on a lighter or no work day). Given the limitations of the TTS, this is not a process that is feasible to consider in this thesis. Should a dataset of this nature be collected, the models presented within this thesis could be adapted to meet the needs of multi day analysis. Given the generality of the models in this thesis, this application to multi day survey data could be accomplished with only minor adjustments to the presented models. Specifically, the easiest method for accomplishing multi day analysis would be using a panel format (where each day represents one response in the panel), though more complex methods could also be explored.
11. Considering the implications of emerging technologies and services on joint travel, with a focus on autonomous vehicles and car/ride sharing services. Specifically, the consideration of these technologies and services require the inclusion of new modal alternatives within the models presented in this thesis. That said, the model structures themselves are fundamentally sound and will continue to provide detailed behavioural insight even with the introduction of these new technologies.

12. Perform further empirical evaluations and validation exercises of the presented modelling frameworks and structures to further cement their importance relative to existing practices.

13. Applying the presented models within an activity based model of travel demand and using this integrated model to test the impact of various policies both in the short and long term.
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Appendix A – Data Processing Tool

The TTS data is stored in a database, consisting of a series of tables that are linked by identification numbers (IDs) at the household, person and trip level. These tables can be linked to create a detailed representation of an individual’s schedule. As the analysis presented below requires an examination of trips where two people travel together and in some cases, the subsequent trip made by one of those individuals, a process to isolate these trips is used. Using the Java programming language, a series of classes (Household, Person, Trip, Transit Trip Details) are used to process the TTS database tables. Each of these classes were again linked to the other classes using the IDs present in the TTS database. Using this structure, it is possible to identify joint trips made by persons living in the same household by examining trip departure times, origin locations, destinations and modes in a method like that of Kang and Scott (2011), though developed independently for the specific context of the TTS. Furthermore, given the trips were stored in a list format, obtaining the subsequent trip for one or both individuals in question became relatively trivial.

The pseudo code used for this process is as follows:

```
For every Household h within the sample
    For every Person p within the Household h
        For every Trip t made by Person p
            If Trip t is made using the auto passenger mode (either as the main or access mode) then
                Store These trips in a Holder for Possible Joint Travel
            End If
        End For (Trips)
    End For (Persons)
End For (Households)

For every Person p within the Household h
    For every Trip t made by Person p
        For every Trip t* in the Holder for Possible Joint Travel
            If t* and t are a match for joint travel
                Store these trips together for joint travel and mark them as joint
            End If
        End For (Possible Joint Trips)
    End For (Trips)
End For (Persons)
End For (Households)
```
Appendix B – Identification Check for the SWEC

The check for the identification of error component logits follows the method laid out in the paper of Walker et al. (2007). This check of identification is performed for models four and five in Table 5.2 in this thesis. This identification check uses some simplifying assumptions regarding the nature of the SWEC to minimize the algebraic complexity of the analysis. The main assumption is that all intra-zonal distances are equal to 1. When these distances are different, the complexity of the variance covariance matrices used in this identification check will be more complex though the results will be identical (i.e. the model is identified). This is because the distances act as non-variable factors in the equations presented in step 4 and 5, making the results identical from this simplified case.

**Identification for Model Four**

**Step 1:** Calculate the normal component’s covariance matrix:

\[ C \cdot C' = \text{COV}_{\text{norm}} \]

\[
\begin{array}{ccc|ccc}
 a & 0 & 0 & 0 & 0 & a & b & b & b \\
 b & a & 0 & 0 & 0 & 0 & a & b & b \\
 b & b & a & 0 & 0 & 0 & 0 & a & b \\
 b & b & b & a & 0 & 0 & 0 & 0 & a \\
\end{array}
\times
\begin{array}{ccc|ccc}
 a & b & b & b & a & 2b^2 + a^2 & b^2 + ba & 2b^2 + ba \\
 b & a & b & b & a & 2b^2 + a^2 & b^2 + ba & 2b^2 + ba \\
 b & b & a & b & a & 2b^2 + ba & 3b^2 + a^2 & 3b^2 + ba \\
 b & b & b & a & a & 2b^2 + ba & 3b^2 + ba & 4b^2 + a^2 \\
\end{array}
= \\
\begin{array}{ccc|ccc}
 a^2 & b & b^2 + a^2 & b^2 + ba & b^2 + ba \\
 b & b^2 + ba & 2b^2 + a^2 & 2b^2 + ba & 2b^2 + ba \\
 b & b^2 + ba & 2b^2 + ba & 3b^2 + a^2 & 3b^2 + ba \\
 b & b^2 + ba & 2b^2 + ba & 3b^2 + ba & 4b^2 + a^2 \\
\end{array}\]
**Step 2:** Add Gumbel variance to diagonal to create mixed logit covariance

\[
COV_{ML} =
\begin{array}{c|c|c|c|c}
 a^2 + \frac{g^2}{u} & b & b & b \\
 b & b^2 + a^2 + \frac{g^2}{u} & b^2 + ba & b^2 + ba \\
 b & b^2 + ba & 2b^2 + a^2 + \frac{g^2}{u} & 2b^2 + ba \\
 b & b^2 + ba & 2b^2 + ba & 3b^2 + ba \\
 b & b^2 + ba & 2b^2 + ba & 3b^2 + ba \end{array}
\]

**Step 3:** Transform the variance covariance to utility difference form using the transformation outlined in Walker (2001). A linear transformation matrix \( L \) is used as follows:

\[
COV_{AML} = L \ast COV_{ML} \ast L'
\]

\[
L =
\begin{bmatrix}
1 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 & -1 \\
\end{bmatrix}
\]

When this calculation is performed, our new utility difference matrix \( COV_{AML} =
\begin{array}{c|c|c|c|c}
 2a^2 + 4b^2 - 2ab + \frac{2g^2}{u} & a^2 + 3b^2 - ab + \frac{g^2}{u} & a^2 + 2b^2 - ab + \frac{g^2}{u} & a^2 + b^2 - ab + \frac{g^2}{u} \\
 a^2 + 3b^2 - ab + \frac{2g^2}{u} & 2a^2 + 3b^2 - 2ab + \frac{2g^2}{u} & a^2 + 2b^2 - ab + \frac{g^2}{u} & a^2 + b^2 - ab + \frac{g^2}{u} \\
 a^2 + 2b^2 - ab + \frac{g^2}{u} & a^2 + 2b^2 - ab + \frac{g^2}{u} & 2a^2 + 2b^2 - 2ab + \frac{2g^2}{u} & a^2 + b^2 - ab + \frac{g^2}{u} \\
 a^2 + b^2 - ab + \frac{g^2}{u} & a^2 + b^2 - ab + \frac{g^2}{u} & a^2 + b^2 - ab + \frac{g^2}{u} & 2a^2 + b^2 - 2ab + \frac{2g^2}{u} \\
\end{array}
\]

**Step 4:** Now we identify each unique term in this matrix and place it into a vector. Note that if we have heterogeneous intra-alternative distances, it is feasible that every (symmetric) off
diagonal cell would be unique, however in this case we note that there are only 3. As each of the
diagonals is unique in this case, we are left with a vector of 7 terms

\[
\begin{align*}
2a^2 + 4b^2 - 2ab + \frac{2g^2}{u} \\
2a^2 + 3b^2 - 2ab + \frac{2g^2}{u} \\
2a^2 + 2b^2 - 2ab + \frac{2g^2}{u} \\
2a^2 + b^2 - 2ab + \frac{2g^2}{u} \\
a^2 + 3b^2 - ab + \frac{g^2}{u} \\
a^2 + 2b^2 - ab + \frac{g^2}{u} \\
a^2 + b^2 - ab + \frac{g^2}{u}
\end{align*}
\]

**Step 5:** We take the partial derivative of these equations, with respect to each unknown term
(treating the term b ab as its own unique unknown term) and then place the resulting partial
derivatives in a matrix as follows:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a²</td>
<td>b²</td>
<td>ab</td>
<td>g²</td>
<td>u</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

**Step 6:** Calculate the rank of this matrix. The rank minus 1 is the number of identifiable
parameters. The rank of this matrix is 3, meaning that both a and b are identified.

This solution in appropriate for model 4 in Table 5.2.

Unfortunately, this identification check is only correct for model 4. For model 5, which is the
most complex form of the SWEC model estimated, the same approach is used, as follows:
Identification for Model Five

**Step 1:** Calculate the normal component’s covariance matrix:

\[ C \ast C' = COV_{norm} \]

\[
\begin{array}{cccc}
 a & 0 & 0 & 0 \\
 b & a & 0 & 0 \\
 b & c & a & 0 \\
 b & c & d & a \\
 b & c & d & e \\
\end{array}
\]

\[
\begin{array}{cccc}
 a & b & b & b \\
 0 & a & c & c \\
 0 & 0 & a & d \\
 0 & 0 & 0 & a \\
 0 & 0 & 0 & a \\
\end{array}
\]

\[
\begin{array}{cccc}
 a^2 & b & b & b \\
 b & a^2 + b^2 & b^2 + bc & b^2 + bc \\
 b & b^2 + ac & a^2 + b^2 + c^2 & b^2 + c^2 + ad \\
 b & b^2 + ac & b^2 + c^2 & a^2 + b^2 + c^2 + d^2 + ae \\
 b & b^2 + ac & b^2 + c^2 + ad & a^2 + b^2 + c^2 + d^2 + ae \\
\end{array}
\]

**Step 2:** Add Gumbel variance to diagonal to create mixed logit covariance

\[ COV_{ML} = \]

\[
\begin{array}{cccc}
 a^2 + \frac{g^2}{u} & b & b & b \\
 b & a^2 + b^2 + \frac{g^2}{u} & b^2 + bc & b^2 + bc \\
 b & b^2 + ac & a^2 + b^2 + c^2 + \frac{g^2}{u} & b^2 + c^2 + ad \\
 b & b^2 + ac & b^2 + c^2 + ad & a^2 + b^2 + c^2 + d^2 + \frac{g^2}{u} + ae \\
 b & b^2 + ac & b^2 + c^2 + ad & a^2 + b^2 + c^2 + d^2 + ae \\
\end{array}
\]

Transform the variance covariance to utility difference form using the transformation outlined in Walker (2001). A linear transformation matrix \( L \) is used as follows:
\[ COV_{AML} = L \ast COV_{ML} \ast L' \]

\[
L = \begin{bmatrix}
1 & 0 & 0 & 0 & -1 \\
0 & 1 & 0 & 0 & -1 \\
0 & 0 & 1 & 0 & -1 \\
0 & 0 & 0 & 1 & -1
\end{bmatrix}
\]

When this calculation is performed, our new utility difference matrix \( COV_{AML} = \)

\[
\begin{array}{c}
2a^2 + b^2 + c^2 + d^2 + e^2 - 2ab + \frac{2g^2}{u} \\
a^2 + c^2 + d^2 + e^2 - \frac{2g^2}{ac + \frac{g^2}{u}} \\
a^2 + d^2 + e^2 - ad + \frac{g^2}{u} \\
a^2 + e^2 - ae + \frac{g^2}{u}
\end{array}
\]

\[
\begin{array}{c}
a^2 + c^2 + d^2 + e^2 - 2ab + \frac{2g^2}{u} \\
a^2 + c^2 + d^2 + e^2 - 2ac + \frac{2g^2}{u} \\
a^2 + d^2 + e^2 - 2ad + \frac{2g^2}{u} \\
a^2 + e^2 - 2ae + \frac{2g^2}{u}
\end{array}
\]

\[
\begin{array}{c}
a^2 + c^2 + d^2 + e^2 - \frac{2g^2}{u} \\
a^2 + d^2 + e^2 - ad + \frac{g^2}{u} \\
a^2 + e^2 - ae + \frac{g^2}{u}
\end{array}
\]

\[
\begin{array}{c}
a^2 + e^2 - ae + \frac{g^2}{u}
\end{array}
\]

\[
\begin{array}{c}
a^2 + e^2 - 2ae + \frac{2g^2}{u}
\end{array}
\]

**Step 4:** Now we identify each unique term in this matrix and place it into a vector.

**Step 5:** We take the partial derivative of these equations, with respect to each unknown term and then place the resulting partial derivatives in a matrix as follows:
Step 6: This matrix has a rank of 7, meaning that 6 parameters are identified. Given that we have 5 parameters in the normal component, we could hypothetically also identify the scale for the Gumbel error term for model 6 in Table 5.2 this model is identified.

<table>
<thead>
<tr>
<th>$a^2$</th>
<th>$b^2$</th>
<th>$c^2$</th>
<th>$d^2$</th>
<th>$e^2$</th>
<th>$ab$</th>
<th>$ac$</th>
<th>$ad$</th>
<th>$ae$</th>
<th>$g^2_u$</th>
</tr>
</thead>
<tbody>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>-2</td>
<td>0</td>
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<td>1</td>
<td>1</td>
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<td>-1</td>
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