Investigating the Combined Impact of Cognitively Guided Instruction and Backward Design model in Mathematics on Teachers of Grade 3 Students

by

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A thesis submitted in conformity with the requirements for the degree of Doctor of Philosophy
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This study investigated the effectiveness of combining the Backward Design model and Cognitively Guided Instruction (CGI) in elementary school mathematics classrooms. The study employs a case study model of four Grade 3 Elementary teachers at two different schools and the impact this intervention has on teachers’ mathematics teaching practices and student performance in this area. This intervention utilizes Wiggins and McTighe’s (2005) Backward Design model as a framework to plan, teach, and assess a unit on multiplication, while embedding Carpenter et al.’s (2014) CGI to examine its effectiveness in improving teacher practice and student performance and achievement in elementary mathematics.

The findings of this study demonstrate that the delivery of this intervention through professional development can be effective, provided that teachers dedicate adequate time, support, and resources to the implementation of the intervention. Moreover, upon successful implementation of this intervention, the findings of this study suggest: (1) teacher perception of improved student performance; (2) increased student engagement in mathematics; (3) changed teacher practices that include more diverse activities and assessment measures; (4) increased teachers’ knowledge, both pedagogically and in the mathematics content area; and (5) that using the intervention and teaching grade 3 students computational skills through problem-solving questions is a more effective means to teaching students each skill separately.
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Chapter 1: Introduction

Simply put, most policy efforts to improve classroom teaching focus on teachers rather than teaching, attending mostly to who is in the classroom instead of what they do when they get there. Most policy work aimed at improving teaching has focused on recruiting better teachers: increasing the qualifications for teachers, making the certification processes more rigorous, and improving the salaries and working conditions for teachers. Little attention has been paid to the methods these teachers will use to promise better student learning. (Stigler & Hiebert, 2009, p. xii)

1.1 Research Context

Student performance in elementary mathematics in Ontario, as weighed by large-scale measurement tests, has been decreasing (EQAO, 2017). In fact, according to the Ontario Education, Quality, and Accountability Office (EQAO), the number of students in grades 3 and 6 mathematics who have met or exceeded the standard has experienced decreasing performances (EQAO, 2017). In Ontario, achievement is measured against the provincial standard which is considered a Level 3 or grade of B (70-79%). According to the Ontario Ministry of Education, students working at a Level 3 “have achieved all or almost all of the expectations for that grade … [and they] will be prepared for work in the next grade” (MOE, 2005, p. 19).

In the Toronto District School Board (TDSB), grade 3 students have achieved an approximate 70% average (ranging from 67% to 71%) of students reaching or exceeding a Level 3 or higher in mathematics between 2012 and 2016 on the EQAO assessment (EQAO, 2017). Grade 6 TDSB students have seen a decrease of 7% over the same five-year period, to 55% (EQAO, 2017).

More alarming is the outcomes for these students on the grade 9 Mathematics Provincial Assessment results. Only 44% of the students in grade 9 Applied mathematics course achieved the provincial standard of a Level 3 or higher on the assessment (EQAO,
2017), which is a further decrease of 3% since 2014. During this same period, 83% of students in the Academic course type achieved the provincial standard, which is a 2% decrease since 2014 (EQAO, 2017). Furthermore, more than two times the number of students enrol in the Academic course type than Applied (EQAO, 2017), and although the outcomes for students in both streams are declining, the achievement gap between these courses is concerning.

One of the outcomes of this poor performance in early mathematics is that fewer students are enrolling in higher-level mathematics courses at the secondary level (People for Education, 2013). The students that do enrol are opting to for Applied-level mathematics streams, instead of Academic-level mathematics courses, reducing their likelihood of graduating high school (People for Education, 2013). This makes Ontario’s future in mathematics seem dim.

According to the 2010 Programme for International Student Assessment (PISA) results, which tracks the performance of international 15 year olds, this is a concern because:

The skills and knowledge that individuals bring to their jobs, to further their studies and to our society, play an important role in determining our economic success and overall quality of life. The shift of knowledge-based economy driven by advances in information and communication technologies reduces trade barriers, and the globalization of markets has precipitated changes in the type of knowledge and skills that the present future economy requires…Elementary and secondary education systems play a central role in laying a solid base upon which subsequent knowledge and skills can be developed. Students leaving secondary education without a strong foundation may experience difficulty accessing the postsecondary education system, the labour market and they may benefit less when learning opportunities are presented later in life. Without the tools needed to be effective learners throughout their lives, these individuals with limited skills risk economic and social marginalization. (Knighton, Brochu, & Gluszynski, 2010, p. 9)
With such dire outcomes regarding the current mathematics performances of Ontario students, and with the outcomes having such heavy importance, it is imperative that our education system takes action to address this problem.

1.2 Rationale of the Study

A large number of elementary school teachers struggle in their teaching, planning, and assessment of mathematics (Hill, Ball & Schilling 2004). Although many of these teachers are aware of their limitations, they often feel a lack control in taking action to address them (Leithwood, 2006; Tye & O’Brien, 2002). This is primarily due to a lack of support (Handal & Herrington, 2003), as well as differences in mathematical knowledges (Hill, Rowan & Ball, 2005). These challenges have resulted in a number of debates about the role of professional development for teachers (Ferguson, 2006; Leithwood, 2006; Marshall & Sorto, 2012; Nishimura, 2014; Telese, 2012) and the role of teacher education in preparing teachers to teach mathematics in elementary classrooms (Downy & Cobbs, 2007; Krawec & Montague, 2014; Miller et al., 2006; Steele et al., 2013).

Some teachers have a high degree of pedagogical knowledge (Lannin et al., 2013) and some teachers have limited mathematical content knowledge (Hill, Rowan & Ball, 2005), while others lack mathematical pedagogical content knowledge (An & Wu, 2012; Ball, Hill & Bass, 2005). For the purposes of this study, pedagogical knowledge is used to refer to teacher’s knowledge of content and students, content and curriculum, and content and teaching (Ball et al., 2008) while content knowledge, or subject matter knowledge, is referred to as common content knowledge, specialized content knowledge and horizon content knowledge (Ball et al., 2008). In other words, pedagogical knowledge is used to refer to knowing how to teach whereas content knowledge is knowing what to teach in
general. Thus, pedagogical content knowledge is the integration of these two areas, where a teacher uses their experience and soundly developed teaching of mathematics alongside their knowledge and understanding of student development (Ball et al., 2008).

Generally speaking, it has been my observation as an educator that teachers understand how to support and manage a classroom or possess sound pedagogical knowledge. Instead, teachers lack adequate mathematical knowledge, or content knowledge, and require support in order to successfully deliver this knowledge to their students. Some teachers might require support to improve their mathematical content knowledge in order to improve their overall pedagogical content knowledge as well as improve their teaching practice to support all learners in mathematics.

Support student learning in the area of mathematics is further compounded by teachers’ limited mathematics content knowledge and their ability to plan for learning (Ding & Carlson, 2013), particularly planning for assessment (Ball, Hill & Bass, 2005). If teachers lack the content knowledge and are uncertain of a particular concept they are teaching, planning and assessing this concept will be even more challenging. Selecting and analyzing appropriate evidence to measure student knowledge of said concept is limited. Teachers are restricted in their ability to support and guide students in their understanding.

These issues have likely contributed to the downward trend of mathematics performance, as measured provincially (EQAO, 2014), and nationally (Knighton, Brochu, & Gluszynski, 2010). Students who perform poorly in mathematics are at an increased risk of social and economic marginalization as adults (Knighton, Brochu, & Gluszynski, 2010), increased risk of not entering or dropping out of post-secondary education (Jia & Maloney,
and experience “employment difficulties and difficulties in many common day-to-day activities” (Geary, 2011, p. 250).

Knowing its significance and repercussions, considerations for what can be done to help elementary students perform better in mathematics is critical. Ways in which we can correct the downward trend in mathematics performance amongst all students, and where and how to start in breaking the negative cycle of poor performance leading to decreased enrollment, are larger areas that require further investigation. In order to do this, we must first address ways in which we can assist teachers in this rapidly changing and expanding environment to acquire the appropriate knowledge and skills to increase their ability to plan, teach, and assess mathematics in a climate in which they feel under-valued, overwhelmed and inundated with competing initiatives.

Stigler and Hiebert (2009) suggest that, in order to evaluate students effectively, we need to evaluate teaching and what is currently taking place in classrooms. In addition, we need to start by looking at the teacher and evaluating our teaching, planning, and assessment processes and methods.

1.3 Purpose of the Study

The purpose of this study is to better understand effective ways to plan, teach, and assess students in elementary mathematics. More specifically, this study will investigate how student learning in mathematics can be enhanced by combining Cognitively Guided Instruction (CGI) with the Backwards Design (BD) model and the effect this combination has on planning, teaching, and assessing elementary mathematics.

I have been teaching in a classroom since 2008 and I have experienced first-hand many of the barriers and challenges in the classroom, such as time and behavioural issues. I
have also witnessed other teachers suffering from a lack of content knowledge and thorough planning for delivering and implementing mathematics lesson and unit plans that are tailored to the students’ individual needs. The overarching dilemma educators experience in teaching mathematics is that they struggle with meeting the specific needs of their students.

Upon further investigation of these issues, I discovered that many teachers rely on teaching strategies and ideas that are based on how they were taught. They are therefore constricted and limited with the type and number of approaches that they can implement. The intervention proposed and explained later in this paper, will utilize some strategies that teachers are familiar with, but also provide new, practical strategies and approaches. It is believed that these new strategies will provide a more balanced approach to their numeracy instruction.

The importance of the proposed research centres on its contribution to an understanding of how to improve student learning in mathematics through focused attention on the planning, teaching, and assessment process teachers utilize. By creating an approach that explores how planning, teaching, and assessing processes interact and inform one another, teachers will have a better informed and more succinct way of approaching their students’ learning in mathematics.

Improving teaching practice is an ongoing and necessary component of the educational field (Guskey, 2003; Hertzog & O’Rode, 2011). As society develops and changes, so does the world around us and therefore so should our teaching practices. It is imperative that teachers and researchers in education seek out ways in which to assess our effectiveness as educators (Demetriou, Charalambous, & Kyriakides, 2006; Kane et al., 2011), and find ways to improve student learning in all areas.
Understanding how we can better support teachers in their mathematics classroom has many implications for a variety of stakeholders. Primarily, if a sustainable and manageable intervention can be implemented, changes in teacher training may be required. If this intervention is successful, it has the potential to increase student performance and achievement in mathematics. The proposed intervention is intended to increase student achievement in grade 3 mathematics and prove to be manageable and effective for teachers. Teachers will find the intervention to be accessible and provide them with a set of tools or a framework upon which they can reflect and review, in order to plan mathematics units and lessons in the future.

Students will find that their understanding of mathematics, with regards to computations, fluency and higher order thinking, will increase as a result of the targeted activities planned by their teachers (Carpenter et al., 2014). It is likely that students will begin to use a greater number and selection of strategies when solving word problems and, due to their increase computational proficiency; they will find that their solutions are more efficient than they were previously.

1.4 Research Questions

The research questions that will guide my study are:

1. How can student learning in mathematics be enhanced by combining Cognitively Guided Instruction (CGI) with the Backward Design model?

2. What factors affect the implementation of a program that combines CGI and Backward Design?

1.5 Background of the Researcher

Since the beginning of my career in teaching, I always enjoyed teaching mathematics. I have invested my time, effort, and resources in developing programming for
students that is accessible to all and helps students achieve their highest potential in this area. Admittedly, I have likely focused my attention on mathematics to the extent that it has limited my ability to improve knowledge as a teacher in other academic areas.

Graduating from the Bachelor of Education program at York University with distinction and an award of teaching excellence with Primary and Junior (grades K-6) qualifications, I immediately began to invest my time and energy in learning about teaching mathematics. I took Additional Qualification courses (AQ) and Additional Basic Qualification courses (ABQ) that provided me a more in-depth learning on teaching mathematics. It also provided me the opportunity to learn and experience teaching mathematics to different populations, both in regards to age and demographics. It allowed me to better understand how the Ontario Curriculum is developed from the Kindergarten program to Grade 12 as a continuum.

Teaching mathematics in all grades is unique and different from teaching other subject areas. First, the culture and climate around mathematics is different than that of other subject areas, such as literacy. Although in my experience, teachers, students, and parents believe that mathematics is an important area of study, they often find many of the specific content irrelevant. Furthermore, with a change in our curriculum focus, from computations to higher order thinking, many people in Ontario’s current education system are being raised by parents, and taught by teachers, who believe that mathematics is only about computations and have a difficult time understanding the importance of higher-order thinking skills. According to Marshall and Swan (2010):
parental involvement in education is positively associated with student achievement (Desforges & Abouchaar, 2003), yet... having parents help their children with mathematics can be problematic. Teachers may not feel comfortable asking parents to help with their children’s mathematics, and the parents themselves are often uneasy about helping. (p. 25)

From Grades 1 to 11, students are mandated to enrol and pass mathematics courses each year. Although the deliveries of these courses are entirely dependent upon the teacher’s direction, the content typically builds upon the previous year’s curriculum. Some years require the teaching of new concepts whereas other years, the concepts are taught to a more specific degree.

As an elementary teacher having taught in several urban TDSB schools since 2008, I have taught mathematics to students in Kindergarten to students in Grade 8. Throughout this time, I have worked with a large number of teachers. I have held many different positions with my school board including classroom teacher, special education teacher, and mathematics lead teacher, all of which have required me to collaborate and provide instructional leadership to other teachers; my colleagues. Many teachers I have worked with from the elementary panel have admitted to struggling with it. More specifically, these teachers self-identified as struggling with their lack of mathematical content knowledge as well as their teaching, planning, and assessment, in this area.

Although many of these teachers are aware of their issues, they often feel a lack of power or control in taking action to fix them. This is largely due to the lack of support provided to them by their employer and the high demands of the classroom. Over the years, I have worked with a number of these teachers, in a variety of different capacities, to help them create sustainable programming that will meet their needs as well as the needs of their students.
It is through working with these teachers that I recognized that much of the education to prepare teachers for teaching elementary is insufficient. In my experience, teachers often have a high degree of pedagogical knowledge and some level of mathematical content knowledge, but they often lack mathematical pedagogical knowledge. In other words, teachers understand how to support and manage a classroom and they have some understanding of the concepts they are currently teaching in mathematics, but they require support on how to successfully deliver this knowledge to all of their students.

To further add to the problem, with limited understanding in mathematics content knowledge, the teachers lack the ability to identify links to higher and more advanced curriculums, as well as the ability to predict future problems students might encounter. Therefore, the teachers are unable to appropriately provide current support to students to guide them in their understanding.

1.6 Format of the Thesis

The thesis is organized into five separate chapters. In Chapter One, an introduction to the research is provided in order to explain its purpose, context, and rationale of the study. It states the intended research questions and how they will be addressed. I then describe my personal background and how it relates to and assisted me in arriving at the study.

Chapter Two focuses on the current literature in the area of elementary mathematics, as it relates to the study. On focus is highlighting how mathematics education has changed over time and the problems that we are currently faced with in order to introduce the problem being researched, the necessity for the study, and how it contributes to the specific and general knowledge in curriculum development and learning in mathematics. More specifically, I discuss current issues in education, two researched methodologies that have
been used in elementary mathematics; Cognitively Guided Instruction and Backward Design model, how they are related to planning, teaching, and assessment, how they can be combined, and how they will interact.

In Chapter Three, I discuss the intended methodology used in the study. I explain the research design and methods, outline the proposed intervention, and explain the purpose of recruitment of the participants. This chapter outlines how the data analysis will be completed, any necessary ethical considerations, and outline and potential limitations and issues that may be encountered.

Chapter Four describes the findings of the thesis based on the data collected for the four case studies. Chapter Five provides an interpretation of the findings and how they relate to teacher practice, student learning, and to the overall educational system and structure.
Chapter 2: Literature Review

As my research interests lie with a better understanding of teacher practice in mathematics, it is necessary to understand what current research says about this area. In this chapter, I will provide a brief history explaining how math has changed and evolved over the last several decades. Following this is an overview of the three most important issues concerning teachers in today’s classrooms and a discussion on their impact on student achievement in mathematics. This will introduce the topic of current research regarding teaching practice in elementary mathematics and its three main categories: planning, teaching, and assessing and evaluating, before specifically exploring two specific teaching methods, Cognitively Guided Instruction (CGI) and Backward Design, their purposes and foci, and their intended combined results.

2.1 A Brief Summary of Mathematics Education: From Traditional to Reform

Over the last several decades, a long-standing disagreement in the literature of mathematics education has taken place with two very distinct sides. One faction of researchers believe that mathematics needs to be explicit and focus more on traditional, basic mathematical facts, computations, and formulas (Baker, Gersten, & Lee, 2002; Bryant et al., 2008; Doabler & Fien, 2013; Fuchs et al., 2006; Kroesbergen, 2004; Kroesbergen & Van Luit, 2003; Mabbott & Bisanz, 2008; Pool et al., 2012; Toll & Van Luit, 2012). Some of these traditionalists believe that the reform mathematics approach lacks organization and clarity for a number of students (Baxter, Woodward, & Olson, 2001). Thus, they believe that an increased focus on assisting students to address these social challenges or increase their basic understanding of mathematics is necessary before “students are to benefit from reform-based mathematics instruction” (Baxter, Woodward, & Olson, 2001, p. 529).
The other faction of researchers, identified as reform mathematics researchers, believe that mathematics needs to be more student-based and exploratory (Carpenter et al., 2014; Carpenter et al., 2000, Carpenter et al., 1999; Fosnot, 2005; Moscardini, 2014; Ritchhart, Church, & Morrison, 2011; Small, 2009; 2012). Students need opportunities to think creatively and problem-solve without the constant interruption and interference of a traditional teacher immediately correcting them (Carpenter et al., 2014; Carpenter et al., 1991; Smith, 1996).

However, as the research has evolved, there is an understanding that both traditionalist and reform practices are necessary for teaching mathematics in the 21st century (Schoenfeld, 2004). Current literature has explored a variety of different techniques, from both the teacher and student perspectives, that combine both traditional and reform teaching practices on planning, teaching, and assessment. However, very little research has been conducted that explores the effects of improving teaching practice in all three areas, simultaneously: planning, teaching, and assessing. Through my education and experience, I believe that it is imperative that all three areas need to be researched together, as they inform one another and work together to promote student learning.

2.2 Key Elements in Elementary Mathematics Education in Relation to this Study

2.2.1 Teachers’ Mathematical Content Knowledge

Pedagogical content knowledge and content knowledge “are key components of teacher competence that affect student progress” (Kleickman et al., 2013, p. 90). Mathematical content knowledge, or subject matter knowledge, is referred to as common content knowledge, specialized content knowledge and horizon content knowledge (Ball et al., 2008). Teachers with less mathematics education possess less mathematical knowledge
and have a more difficult time assisting students (Marshall & Sorto, 2012) and supporting them with simple tasks, such as homework skills (An & Wu, 2012). Overall, teachers’ content knowledge “has a larger role in predicting student achievement than mathematics pedagogical knowledge” (Telese, 2012, p. 102). This is likely due to the fact that, although teachers have a strong understanding of their students and the content and curriculum is that needs to be taught, teachers need to know that they require greater understanding of the actual mathematical concepts and a variety of different strategies to achieve a reasonable or accurate result for said concept. “Teaching … requires understanding … [of a concept] that goes beyond the kind of tacit understanding … needed by most people” (Ball et al., 2008, p. 400). “Teachers must know the subject they teach” (Ball et al., 2008, p. 404).

Although Ontario teachers receive training to support their pedagogical content knowledge, Ontario Ministry of Education has continued to release documents to support teachers in developing their formal content knowledge (Ontario Ministry of Education, 2011). Without content knowledge and pedagogical content knowledge, teachers’ effectiveness in planning, teaching, and assessing mathematics is limited (Ball, Hill, & Bass, 2005; Blomeke, Suhl, & Kaiser, 2011), in all grade levels including the younger grades (Ball et al., 2008).

Younger students, who struggle with acquiring new mathematical concepts, suffer most from teachers with limited training and pedagogical content knowledge (Ball et al., 2008). When teachers lack pedagogical knowledge, content knowledge, and/or pedagogical content knowledge, it is challenging for them to identify a child’s particular needs and create supportive interventions (Ball et al., 2008).
2.2.2 Teacher Anxiety and Previous Experience

According to a large body of research, teachers’ fear and anxiety of mathematics and its teaching adversely effects how mathematics is being taught in today’s classrooms (Ashcraft & Krause, 2007; Beilock et al., 2009; Hoffman, 2010; Turner et al., 2002). Teachers prefer to teach in the manner in which they were taught (Ball et al., 2008; Dunn & Dunn, 1979). Unfortunately, for many teachers, this means that they revert to the approaches and strategies to mathematics they were taught (Oleson & Hora, 2014), which may reduce the opportunities for creative and higher-level thinking for their students. These approaches rely heavily on rote understanding and the memorization of facts and information in mathematics almost at the exclusion of meaningful understanding of why and how operations and procedures work. Students are asked to complete a series of tasks, which often become repetitious and meaningless. These types of activities rarely provide students with prospects to think and problem-solve. Instead, these planned activities provide students with the chance to practice their memorization of formulas of standard algorithms.

2.2.3 Teachers’ Attitudinal Concerns

Another issue in our current educational climate is that teachers feel overwhelmed and inundated with the number of initiatives they are expected to implement in their classrooms, which negatively impacts their perceived working conditions (Conley & Woosley, 2000; Lewis, 2009; Valli & Buese, 2007). According to Leithwood (2006), positive working conditions for teachers are imperative for student success. However, in the educational climate in Ontario, teachers are feeling overwhelmed with the workload volume and complexity (Leithwood, 2006; Leithwood & McAdie, 2007). More specifically, teachers have a growing concern about the number of students in their classrooms, the number of
hours spent planning and assessing students unnecessarily in our increased meritocracy, and
with the demand for ‘data’ (Leithwood, 2006).

Furthermore, Leithwood (2006) notes that a large portion of teachers have
experienced a decrease in the level of autonomy they feel they have in their work
environment and are lacking in their “sense of community [and] collaborative culture” (p. 50). Teachers feel they have less control over their classrooms and professional development (Leithwood, 2006). Thus, even teachers who recognize and identify mathematics as an area in need of additional professional development, believe that there is little support they can access unless it is completed entirely on their own time, outside of the workplace and pay for it personally (Broad & Evans, 2006). This creates a number of issues such as an issue with equity where newest and lowest paid teachers who may require this training are less able to access it.

Despite their training and experiences, without additional support and formal professional development, teachers find it challenging to create and implement a mathematics program that enhances student learning in elementary classrooms (Bruce & Ross, 2008). Creating and implementing a mathematical program that is balanced and considers all aspects of the teaching process from planning, to delivery, to assessment, is not only important, but essential. Furthermore, this study will provide additional support to Ball, Hill, and Bass’ (2005) findings that teacher training and preparation has an important impact on student performances in mathematics, even in the earliest grades.

2.2.4 Mathematics Professional Development

One way to address the concerns educators currently face in mathematics classrooms is to provide them with Professional Development (PD). Given that the intended
intervention is essentially on-going teacher training sessions on mathematics problem-solving and changes in planning, teaching and assessment, and given the fact that PD is a form of curriculum reform, PD is an important topic of discussion.

Most researchers support the idea that teachers benefit from PD opportunities to develop and refine their teaching practice (Marshall & Sorto, 2012; Nishimura, 2014; Telese, 2012). Ferguson (2006) claims there are five challenges to effective PD: “introducing new activities in ways that inspire buy-in; balancing principal control with teacher autonomy; committing to ambitious goals; maintaining industriousness in pursuit of those goals; and effectively harvesting and sustaining the gains” (p. 48). In addition to these challenges, Hunzicker (2011) argues that PD needs to “consider the needs, concerns, and interests of individual teachers along with those of the school or district” (p. 177).

In order to be effective, PD needs to be “supportive, job-embedded, instructionally focused, collaborative and on-going… [and] is most effective when teachers have multiple opportunities to interact with information and ideas over several months” (Hunzicker, 2011, p. 178). This is supported by Patel et al. (2012), who discovered that PD was more successful when participants had opportunities to practice and solidify concepts being taught, and when one or more follow-up activities took place (Nishimura, 2014).

With regards to PD in mathematics, Telese (2012) found that PD should be directed at increasing teachers’ knowledge of curriculum and instructional methods, and knowledge of student learning. The National Research Council (2001) states that:

If… students are to develop mathematical proficiency, teachers must have a clear vision of the goals of instruction and what proficiency means for the specific mathematical content they are teaching. They need to know the mathematics they teach as well as the horizons of that mathematics—where it can lead and where their students are headed with it.
Therefore, PD in mathematics should increase teachers’ Content Knowledge (Hill, Ball, & Schilling, 2004; Marshall & Sorto, 2012; Telese, 2012).

Furthermore, many researchers believe that PD in mathematics works best when it takes the form of a coaching model, or a system where a more experienced individual supports another in the planning, teaching, assessment and reflection thereof of their students in a non-judgemental or evaluative manner (Kennedy & Shiel, 2010; Nishimura, 2014; Tschannen-Moran & McMaster, 2009). A coaching model or system provides multiple opportunities to engage with the subject content (Nishimura, 2014; Patel et al., 2012), while likely increasing teachers’ comfort levels and Content Knowledge (Hill, Ball, & Schilling, 2004; Marshall & Sorto, 2012; Telese, 2012). Unfortunately, not all PD is effective.

PD is traditionally delivered ineffectively in a teacher-directed fashion where participants passively sit and receive information (McLesky & Waldron, 2002). PD may also not be effective due to the teacher’s absence from the classroom as it may be detrimental to student achievement (Telese, 2012). Thus, evaluating PD, by considering its purpose and challenges, is necessary.

2.3 Student Achievement in Elementary Mathematics

Recognizing that student performance is decreasing in mathematics is not a new problem. This fact has been highlighted by the number of different researchers investigating ways to improve teaching and enhance learning in this area (Carpenter & Fennema, 1991; Carpenter et al., 2014; Carpenter et al., 2000; Carpenter et al., 1999; Carpenter et al., 1989; Franke & Kazemi, 2001; Ontario Ministry of Education, 2011).
A significant portion of the literature on student performance in mathematics focuses on improving student achievement through changing or altering teacher approach. In other words, a great deal of research has questioned how teachers are contributing to students’ decreasing performance in mathematics. Researchers, such as Hill, Rowan, and Ball (2005), have provided findings that demonstrate that both Pedagogical Knowledge and mathematics Content Knowledge are largely connected to student performance in this area. Thus, students of teachers whose knowledge in teaching pedagogy and mathematics content knowledge is “in the top quartile …, showed gains in their [mathematics’] scores that were equivalent to that of an extra two to three weeks of instruction” (Ball, Hill, & Bass, 2005, p. 44). Similar and related findings support the idea that teachers with a higher knowledge in teaching pedagogy and mathematics content knowledge are linked to higher student performance (Ball & Rowan, 2004; Ball et al., 2008; Moyer-Packenham et al., 2008; Schulman, 1986; Shulman, 1987).

Lefebvre and colleagues (2011) found similar results to Ball, Hill and Bass’ research (2005), despite having an entirely different focus. Lefebvre, Merrigan, and Verstraete (2011) investigated the effects of public subsidies and private schools on students’ mathematics scores in a Canadian setting. They discovered that teacher quality was likely a significant contributing factor. Since “the working environment is superior in private schools because teachers do not have to deal with problem students and good teaching is rewarded by observing a large number of students” (Lefebvre, Merrigan, & Verstraete, 2011, p. 91), it is assumed that teachers have more time to dedicate to planning for student learning and implement their teaching knowledge of mathematics content and pedagogy. Furthermore, Lefebvre and colleagues found that private schools alone or teacher performance did not
account for the increase in student performance in mathematics. They provided evidence to suggest that private schools use a mix of groupings, where students of different abilities work together in mathematics classes, accounted for a large portion of the enhanced student performance.

Lefebvre, Merrigan, and Verstraete (2011) suggest that peer effects in mathematics are significant, but more so for the lower performing students. They also hypothesize that increasing student performance in mathematics can be done by monitoring students and their grades by authority figures at school and home, by decreasing delinquent behaviour, by setting high expectations and valuing excellence, and by implementing and valuing a high level of discipline. Although Lefebvre and colleagues (2011) study did not directly investigate how and why student performance in mathematics is declining, it offers many findings as to how it might be corrected.

Bassani’s study (2006) offers yet another perspective on the declining performance in mathematics scores in Canada. According to her research, this decline can be explained through Social Capital Theory, which believes that:

a variety of factors (i.e., social [relationships], human [investing in self], financial [income], cultural [cultural beliefs and behaviours] and physical [material projects] capital) contribute to young people’s well-being, though social capital is believed to be of tantamount importance. (Bassani, 2006, p. 382)

Based on these findings, in order to enhance student performance in mathematics, we must increase a student’s social capital either at school, at home or both. According to Bassani (2006), this can be done by increasing students’ study time with one or both parents, or by increasing the level of teacher-student educational interaction.

Stack (2006) conducted research on declining mathematics scores. She feels that, in Canada, students’ mathematics scores are decreasing, but there is uncertainty about the
magnitude of this decrease. Since many of the test scores used to measure students’ mathematics scores globally are presented using social statistics, the interpretation of them should be done with caution:

The issue is not statistics or testing but the interpretation of them. The issue is that statistics provide the media with a simple mechanism of appearing to report reality, which is made into a story with the use of emotional anecdotes and/or expert quotas that lend verisimilitude to the numbers. (Stack, 2006, p. 64)

2.4 Current Teaching Practices in Elementary Mathematics: Teaching, Planning and Assessment

According to Sullivan et al. (2013), teaching can be summarized as a process with three active parts: teaching, planning, and assessment and evaluation. When people refer to teaching, they generally are referring to the teaching process and therefore are including the planning and assessing of student work as well.

2.4.1 Teaching

Teaching refers to the delivery of the content to the student and encompasses ideas, such as what is being taught, lesson duration, modelling of skills, and the language used (Archer & Hughes, 2011). It is the active time teachers are working with students in order to help them acquire knowledge (Ma, 2010). Although it is clearly linked to planning, teaching is better explained as what a teacher is doing in the classroom to help students gain knowledge. “[T]eaching mathematics consist[s] of a relatively limited repertoire of methods that focus on helping students acquire isolated skills” (Stigler & Hiebert, 2009, p. 11), and is influenced by a number of factors, including geographic location.

A large body of research is dedicated to support teachers with the teaching of elementary mathematics. Strategies to garner students attention and increase student engagement, such as use of manipulatives (Carbonneau, Marley, & Selig, 2013; Marley & Carbonneau, 2014; Moyer, 2001; Reimer & Moyer, 2005; Scarlatos, 2006), use of
technology (Craig, 2000; Dawson et al., 2013; Reimer & Moyer, 2005), creating mathematics goals (Federici, Skaalvik, & Tangen, 2015; Hannula, 2006), establishing a positive mathematics environment (Gilbert et al., 2014; Spinner & Fraser, 2005; Tarr et al., 2008), as well as many others, have been explored and investigated.

Unfortunately, little research exists that investigates interventions that target effective teaching practices in elementary mathematics combined with the planning and assessing aspects. Instead, most of the research focuses on isolated aspects of the teaching process and thus ignores additional important factors.

2.4.2 Planning

Planning encompasses the thoughts and actions that help prepare a teacher to teach. It includes the collection of necessary materials, organization of space, consideration of approach, understanding of students’ prior knowledge, deliberation about the goal and the tasks needed arrive there, as well as the amount of time required (Wiggins & McTighe, 2009). Planning can take place any time before and during the lesson and may take the shape of lesson or unit plans, or more basic plans for a specific activity.

Planning effective mathematics lessons require Content Knowledge, Pedagogical Knowledge, and Pedagogical Content Knowledge. Sullivan et al. (2013) found that many teachers lack a well-developed and thorough mathematics Content Knowledge, which limits their ability “to articulate the important ideas in that topic” (p. 457). The concept that Content Knowledge is imperative to the successful planning of effective mathematics instruction is supported by other researchers (Ball, Hill, & Bass, 2005; Dunekacke, JenBen, & Blomeke, 2015; Franke & Kazemi, 2001).

Niess (2005) found that elementary mathematics teachers were less focused on their
students and more on their own teaching practice. To increase teacher’s focus on student learning, they encouraged teachers to consider how “students [will] understand the concepts in the…instructional activity” (Niess, 2005, p. 521). Ding and Carlson (2013) found that teachers’ ability and quality of planning lessons increased alongside their “growth in work examples, representations, and deep questions” (p. 359). The more experienced teachers became while working with and trying to understand student thinking in student work samples, they better their judgements and understanding of student work and knowledge became.

Although planning is often discussed alongside teaching or assessment, it is important to consider its relationship to both of these processes, simultaneously. Many researchers discuss planning and assessment together, but neglect to discuss the act of teaching process (Wiggins & McTighe, 2009). They fail to include how the actual act of teaching influences students and teachers, and its many implications for previous and future plans and assessments.

### 2.4.3 Assessment and Evaluation

According to the Ontario Ministry of Education (2005):

Assessment is the process of gathering information from a variety of sources (including assignments, day-to-day observations and conversations/conferences, demonstrations, projects, performances, and tests) that accurately reflects how well a student is achieving the curriculum expectations in a subject… Evaluation refers to the process of judging the quality of student work on the basis established criteria, and assigning a value to represent that quality. (p. 18)

Assessment and evaluation are topics in mathematics that have undergone an extensive amount of research over the past two decades. Researchers argue that assessments in mathematics should measure students’ “skills and procedures of mathematics … [as well as their understanding of] the concepts, processes, and problem solving applications” (Kulm,
Suurtamm and Koch (2014) explain the need “to incorporate a range of assessment practices that are responsive to student thinking and promote student learning” (p. 263), while educating others to help change current assessment practices. Although there is no definitive consensus on how to assess and evaluate students in mathematics, a general agreement that assessment in this area is crucial and a differentiated approach is necessary in order to accurately capture students’ understanding.

The Ontario Ministry of Education (2010) developed a mandatory document to guide Ontario teachers in their assessment and evaluation of students. Growing Success (Ontario Ministry of Education, 2010), provides teachers with a wealth of information and a variety of different techniques for assessment. According to this document, teachers in Ontario are to break down their assessment practices into three different categories: assessment for learning, assessment as learning, and assessment of learning.

Research on assessing mathematical knowledge found that focusing on the wrong evidence could mislead a teacher’s understanding of student knowledge (Smith & Smith, 2006). By over-emphasizing a concept, teachers may misinterpret student results. Jitendra, Dupuis and Zaslofsky (2014) found that mathematics assessments “typically involve[] computational fluency … with limited attention to problem solving or conceptual tasks” (p. 242). They argue for the use of curriculum-based measurement, a framework that supports teachers with assessment of student work.

An and Wu (2012) discovered that the quality of assessments increase as teacher Pedagogical Content Knowledge increases. They argue that, when assessing student work, teachers should engage “in the inquiry process of the 4 steps of identifying errors, analyzing reasons for the errors, designing approaches for correct, and taking action for correction” (p. 242).
717). By doing so, teachers are more likely to increase their own Pedagogical Content Knowledge as well as their knowledge of students’ thinking (2012) and increase their ability to accurately grade student work.

Although not exhaustive, current literature on assessment in mathematics argues that teachers need to consider many things when assessing student knowledge: the type of evidence they intend to collect that demonstrates student learning (Smith & Smith, 2006; Wiggins & McTighe, 2009), the creation of assessments that measure a variety of different tasks (Jitendra, Dupuis, & Zaslofsky, 2014), and the process in which they undergo during the evaluation process (An & Wu, 2012; Wiggins & McTighe, 2009).

Despite all of the research in this area, limited understanding about how assessment and evaluation affects both planning and teaching, simultaneously. Although Wiggins and McTighe (2009) discuss how assessment and planning are closely related and linked and the Ontario Ministry of Education (2010) discuss how assessment is linked to planning and how it is linked to teaching separately, very few researchers discuss how assessment and evaluation is linked to teacher planning and the physical act of teaching combined. Since it is this act; the act of connecting teaching, planning, and assessment together, that will assist teachers and students to create more effective mathematics classrooms, it is imperative to discuss an intervention that includes all three components of the teaching process.

2.5 Two Frameworks for Teaching, Planning and Assessing Elementary Mathematics

2.5.1 Cognitively Guided Instruction

Cognitively Guided Instruction (CGI) is a term coined by Carpenter et al. in their 1989 study and is described as an intervention “to help teachers understand how children develop [mathematical] … concepts and provide them the opportunity to explore how they
might use that knowledge for instruction” (p. 504). CGI focuses on the process each student partakes in while exploring mathematical concepts and evaluates thinking as opposed to solutions (Carpenter et al., 2014).

By adjusting the teachers’ focus to students’ thinking and processes, and away from prescribed teaching formats, “teachers knew more about individual students’ problem-solving processes” (Carpenter et al., 2014, p. 499), which led to more targeted and enhanced student learning (Carpenter et al., 2014; Moscardini, 2014).

Carpenter et al. (1989) argue that four underlying and guiding principles of CGI contribute to its effectiveness. They believe that:

- instruction should develop understanding by stressing relationships between skills and problem solving, with problem solving serving as the organizing focus of instruction … and that instruction should be organized to facilitate students’ active construction of their own knowledge with understanding … [E]ach student should be able to relate problems, concepts, or skills being learned to the knowledge that he or she already possesses [and] … because instruction should be based on what each child knows, it is necessary to continually assess not only whether a learner can solve a particular problem but also how the learner solves the problem. (p. 505)

Student learning increases when new information and concepts are situated in a context that can be manipulated or is familiar (Fosnot, 2005; Carpenter et al., 2014; Moscardini, 2014; Musanti, Celedón-Pattichis, & Marshall, 2009; Ritchhart, Church, & Morrison, 2011; Stigler & Hiebert, 2009).

A cornerstone assumption of this research is that the level of student mathematical understanding is determinable. Thus, teachers must develop not only knowledge of the curriculum, but a knowledge of their students, their progress, and their misunderstandings, and plan accordingly (Carpenter et al., 2014; Carpenter et al., 1989; Fosnot, 2005; Moscardini, 2014; Musanti, Celedón-Pattichis, & Marshall, 2009; Stigler & Hiebert, 2009; Ritchhart, Church, & Morrison, 2011).
CGI has many implications for teaching practice; however, it can be challenging to implement, as it may be foreign from how teachers were taught. Based on their initial study, teachers using CGI had an increased knowledge of their students and their mathematical understanding, and student word-problem performance improved (Carpenter et al., 1989). Notably, the researchers did not notice a difference between addition and subtraction computational performance between CGI and non-CGI classrooms, suggesting that this method allows opportunities for students to simultaneously develop foundational skills. Their study was limited to elementary classrooms and simple word-problems.

Based on other research, CGI appears to be an effective strategy for teaching students mathematics, including the instruction of more complex concepts. After learning and implementing CGI in classrooms, Moscardini (2014) found that teachers had an increased awareness of their students and their understanding. As a result, they were better able to support student learning. CGI also had a positive impact on teacher learning and student understanding in a variety of different classrooms in the U.S (Baek, 2004). These findings, “offer[] possible opportunities for teacher educators to re-conceptualize teacher education” (p. 421) in other settings. Christenson and Wager (2012) argue that CGI’s effectiveness is partially due to its ability to meet the needs of all learners. They believe that CGI increases elementary student participation and provides a framework for teachers to implement differentiation (2012).

According to Wistrom (2012), CGI is effective because it requires teachers to tailor the learning to meet students’ individual needs and learning. Wistrom provides multiple examples of how to do this and by doing so, a common theme emerges (see Appendix S). Although the illustration to follow is explained using addition as an example, it is
application to planning for all other areas as well. First, Wistrom (2012) assumes teachers have an understanding of students’ knowledge and areas of need. With regards to addition, knowing if a student grasps the idea that addition requires the adding of parts to form a new whole is important. Once a teacher has determined this, understanding how to encourage further growth and understanding is simple. Wistrom deconstructs addition into three categories: result unknown, change unknown, start unknown. These categories act as a continuum or stages of development to support student learning. Once students have understood and mastered one stage, the teacher can provide learning opportunities in the next stage.

Following teachers’ understanding of students’ initial knowledge, Wistrom (2012) provides tips on using Cognitively Guided Instruction. After determining the point between student knowledge and area of need, she encourages educators to develop meaningful and contextual mathematical stories. These stories can be suited to each student so that they are relevant, applicable, and engaging for them but that they also utilize questioning effectively to target the specific level in which they need to work.

2.5.2 Backward Design model

What enabling knowledge … and skills… will students need in order to perform effectively and achieve desired results? What activities will equip students with the needed knowledge and skills? What will need to be taught and coached, and how should it best be taught, in light of performance goals? What materials and resources are best suited to accomplish these goals? (Wiggins & McTighe, 2005, p. 18)

The Backward Design model (Wiggins & McTighe, 2005) emphasizes “teaching for understanding”. It is best described as a planning framework that, unlike traditional teaching methods that starts at the beginning and works towards a goal, begins with the intended outcome and then selects appropriate forms of assessment (Wiggins & McTighe, 2005).
After the goals and assessments are determined, supporting resources can be collected and a teaching plan can be created and implemented (Wiggins & McTighe, 2005).

**Figure 2: UbD: Stages of Backward Design (Wiggins & McTighe, 2005, p. 18)**

To support this notion of an effective mathematics classroom, Wiggins and McTighe (2005) argue that the implementation of the principles of Backward Design can be most effectively utilized in order to create an ideal environment that is meaningful and targets students’ specific and immediate needs. This is supported by Kelting-Gibson’s (2005) research arguing that, when Backward Design model is implemented in a mathematics classroom, student achievement and performance is increased.

According to Wiggins and McTighe (2000), Backward Design “starts with the end – the desired results … and then derives the curriculum from the evidence of learning (performances) called for by the standard and the teaching needed to equip students to perform” (p. 13). Wiggins and McTighe (2005) summarize their findings of using a Backward Design model stating that it is important to first “identify [the] desired results [then determine acceptable evidence [and finally plan] learning experiences and instruction”
It allows teachers and students to continually strive to achieve a specific, measureable goal, and compare where they are in the process to achieving that goal. The framework utilizes three steps: identify, determine, and plan (see Figure 2).

Three questions to guide the user in identifying their desired result: “What should students know, understand, and be able to do? What content is worthy of understanding? What **enduring** understandings are desired?” (Wiggins & McTighe, 2005, p. 17). These questions support the teacher in selecting appropriate outcomes and aids them providing opportunities to students for ‘purposeful surveys’ instead of basic ‘coverage’ (Wiggins & McTighe, 2005, p. 18).

**Figure 3 Backward Design model for Planning – Wiggins & McTighe (2005)**

Once the desired results have been identified, teachers need to determine how to measure them and the evidence(s) used for proof of understanding. Often teachers assume a paper-and-pencil test will provide evidence of understanding but instead, this stage
encourages teachers to explore the evidence critically. Furthermore, it requires teachers to question how the evidence is connected to student learning. It is imperative to evaluate the assessment and to ensure it is not impacted by other unrelated constructs. Following this, teachers are asked to begin planning. Although typically the stage most teachers start at, it is necessary to plan the previous two stages first to ensure that learning is ‘clear and purposeful’.

Although its developers believe that the BD model is effective for planning educational endeavours, it is important to explore other research on this model. Kelting-Gibson (2005) found that teachers who used BD outperformed their peers using basic and traditional Curriculum Design. The students achieved their objectives more efficiently and effectively, and that the teachers using BD were better able to “demonstrat[e] knowledge of content and pedagogy, demonstrat[e] knowledge of students, select suitable instructional goals, demonstrat[e] knowledge of resources, design coherent instruction, and assess student learning” (Kelting-Gibson, 2005, p. 26).

Inoue and Buczynski (2011) argue that Backward Design is necessary in order to implement inquiry pedagogy in current mathematics classes. After doing a case analysis of preservice teachers teaching inquiry-based mathematics lessons to elementary students, Inoue and Buczynski (2011) found that the preservice teachers are unprepared to answer many of the “diverse, unexpected [student] responses” (p. 10). With their knowledge of Backward Design, they believe had students effectively utilized this framework in their planning of the lesson, that this could have been avoided and that they would therefore be able “to give pedagogically meaningful responses to [the] students” (p. 10).

McTighe and Brown (2010) refute the criticism that BD only supports standard-
driven goals and does not allow for differentiated instruction. “[D]ifferentiation and standards [can and should] coexist … [and teachers should remain] responsive to individual students” (p. 234).

Unfortunately, not all of the research regarding Backward Design is conclusive and supportive. Grooms (2010) explored Backward Design and its ability to change educator attitudes and teaching strategies. At the end of her study, although there was no conclusive or definitive support that Backward Design improved teachers’ attitudes, she did find that there was an “elevation in the educator’s level of awareness concerning principles of efficacy and understanding of backward design curriculum” (p. iii).

2.6 Creating an Effective Mathematics Classroom

Based on Schoenfeld’s work (2014), it is possible to create an effective classroom, both for teachers and students. Schoenfeld (2014) learned from his research, “that consisted of a decade-long series of design experiments aimed at understanding and enhancing students’ mathematical problem solving” (p. 404), that problem-solving is one of the most effective ways to improve student mastery of all mathematical concepts. Thus, Schoenfeld argues that teachers need to play many roles in the classroom, including both the role of a researcher where they design and create interventions to test out their understanding of how to support and enhance student learning and investigate ways about how students best learn, as well as the role of teacher where they implement the researcher’s design plan and observe and measure it effectiveness.

Schoenfeld (2014) essentially argues that teaching in today’s classroom and climate requires teachers’ roles to be intertwined with the roles of a researcher. In other words, he believes that:
research and practice can and should live in a productive synergy, with each
enhancing the other, and … that research focused on teaching and learning in a
particular discipline can, if carefully framed, yield insights that have implications
across a broad spectrum of disciplines. (p. 404)

This is precisely what I intend to do in my study; model and encourage teachers to take on
both the role of researcher and teacher so that they can come to an understanding that
supports teaching and learning, as well as theory and practice. Commenting on his life’s
work, Schoenfeld (2014) discusses his research findings and how it is imperative that:

The extent to which the mathematics discussed is focused and coherent and to which
connections between procedures, concepts, and contexts (where appropriate) are
addressed and explained. Students should have opportunities to learn important
mathematical content and practices and to develop productive mathematical habits of
mind. (p. 407)

Although this is not a complete review of all of the research regarding mathematics
and student performance, it highlights the key themes in current literature about declining
student achievement in mathematics in a Canadian setting. In order to rectify this issue, the
broad research argues that it is imperative to explore ways to improve teaching, so that
student learning in mathematics can be enhanced (Carpenter et al., 2000; Hill, Rowan &
Ball, 2005; Moscardini, 2014; Moyer-Packenham, Bolyard, Kitsantas, & Oh, 2008). This
can be done via changing instruction, such as improving teacher knowledge (Ball & Rowan,
2004; Ball, Hill, & Bass, 2005; Ball, et al., 2008; Lefebvre, 2011; Moyer-Packenham et al.,
2008; Schulman, 1986; Shulman, 1987) or by altering teachers’ foci or approaches (Bassani,
2006).

Bassani (2006) offers a different lens with different qualities that can be used to
measure student performance that are linked to achievement in academic areas, such as
mathematics. These alternative qualities should also be considered in any type of
intervention aimed at enhancing student performance. Regardless, when evaluating student
performance on any scale, it is important to consider Stack’s (2006) warning and consider the significance and meaning of the statistics during the interpretation.

2.7 The Combined Intervention

Using Schoenfeld’s framework combining research and practice, I intend to investigate the effects of combining Wiggins and McTighe’s (2005) BD model with Carpenter et al.’s (2000) CGI intervention, in elementary mathematics classrooms in Ontario. By supporting teachers to learn, understand, and implement BD and then providing them PD on CGI, I can describe how these efforts impact student achievement. After using BD and preparing for the selected mathematics unit, teachers will introduce the unit with a multi-step problem inducing higher-level thinking, in order to collect data on immediate student understanding, and then use CGI to support student learning.

I think it is necessary to combine both of these models as frequently we explore the planning for mathematics and the delivery and assessment of it in a compartmentalized manner. Although this is important for research purposes and to better understand each aspect of the teacher process, all of these processes coexist. It is important that we piece them together in order to maximize our effectiveness in mathematics classrooms.

Schoenfeld’s (2014) framework argues that:

research and practice can and should live in productive synergy … [and] that research focused on teaching and learning in a particular discipline can, if carefully framed, yield insights that have implications across a broad spectrum of disciplines. (p. 404)

created out of research on practice, they meet both of Schoenfeld’s conditions and can be used in understanding teacher practice in mathematics.

All three researchers agree that posing mathematical problems better support student learning, in an elementary setting. Furthermore, by integrating Backward Design with CGI, an intervention for the planning, teaching, and assessment of mathematics can be created that supports student learning, encourages higher-level thinking, and is accessible to all teachers and learners.

According to the National Council of Teachers of Mathematics, problem solving “refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing students’ mathematical understanding and development” (NCTM, n.d.). Better understood as reform mathematics teaching, Herra and Owens’ (2001) describe the new mathematics reform as “advocat[ing] changes in content and pedagogy. A significant difference from the new math movement is … [the] emphasis on mathematics content and instruction suitable to all students, not only the college bound” (p. 89). All three of the frameworks and models in my study: Schoenfeld’s framework for powerful classrooms; Wiggins and McTighe’s (2005) Backward Design model; and Carpenter et al.’s (2000) CGI instructional framework, utilize a balanced mathematical format and style.

At the beginning of a unit, students will be posed with a problem that requires the combination of problem-solving skills, as well as specific mathematics content that has not been taught, and asked to arrive at a reasonable solution (Carpenter et al., 2014). Students will be provided time to work in small groups to try to deconstruct the problem and identify areas of weakness or need, as well as to identify specific content in which they may need to learn. A whole-class discussion will then take place to share different ideas and thoughts.
This discussion should eventually lead to a course of action for the teacher and students on how to move forward on solving the end of unit problem. The teacher would plan a set of specific activities that would help the students learn the required skills necessary to answer the problem posed in the first class.

Overall, this intervention will consider current and relevant teacher concerns for teaching mathematics, provide them with resources to improve their practice, and establish a pragmatic process linking planning, teaching, and assessment. This approach will allow for a deeper, richer understanding of student knowledge, which will in turn likely lead to increased student performance.
Chapter 3: Methodology

3.1 Methodology Overview

As the goal of my study is to investigate the effectiveness of an intervention aimed at enhancing student mathematical understanding, it is important to address the concerns stated in the literature: elementary teachers have a knowledge deficit, as well as limited time to improve their teaching practice. By combining Carpenter et al.’s (2014) knowledge of Cognitively Guided Instruction with Wiggins and McTighe’s (2005) knowledge of Backward Design model, I developed materials and worked with teachers to implement an intervention that supported them in improving their mathematical content knowledge and enhance student learning in grade 3 mathematics.

Incorporating quantitative measures and qualitative methods while exploring factors that contribute to effective teaching, planning, and assessment, requires the use of a mixed method design (Creswell & Plano Clark, 2007; Greene, 2006). My explanatory design (Baxter & Jack, 2008) followed a two-phase approach: initially beginning with the qualitative research, collecting information on the current teaching practices, student responses, and teacher and student knowledge. This was completed through the use of open-ended questions in semi-structured interviews, predetermined tasks provided to the students, and collection of field notes recorded during classroom observations and interviews.

After the collection of the qualitative data, the quantitative component measured the level of change after the intervention. It utilized an intermethod mixing data collection strategy (Johnson & Turner, 2003), and incorporated interviews and classroom observations to gain a meaningful perspective. My research followed the purpose of expansion (Greene, 2006), as not only are my methods sequential, the results from my qualitative component are quantified with quantitative measures.
3.2 Research Design

Case studies provide the best opportunity to investigate the effectiveness of this combined CGI and Backward Design approach to mathematical teaching and learning. As supported by many researchers, case studies permit the researcher to deeply investigate and analyze a variety of factors, intrinsically and extrinsically related and interacting, for a specific purpose of study (Flyvbjerg, 2006). This allows the researcher to learn about specific phenomenon in a particular context (Flyvbjerg, 2006). “[T]he case study produces the type of context-dependent knowledge that research on learning shows to be necessary to allow people to develop … [I]n the study of human affairs, there appears to exist only context-dependent knowledge” (Flyvbjerg, 2006, p. 221).

In addition to Flyvbjerg’s (2006) work on case studies, Yin (2014) argues that, although challenging, case studies are ideal for investigating social science research that is looking at ‘how’ and ‘why’ particular phenomenon are taking place. “The more that your questions seek to explain some present circumstance … the more that case study research will be relevant. The method also is relevant the more that your questions require an extensive and ‘in-depth’ description of some social phenomenon” (Yin, 2014, p. 4). Thus, case studies allow for better understanding of “complex social phenomena… [by allowing] investigators to focus on a ‘case’ and retain holistic and real-world perspective[s]” (Yin, 2014, p. 4).

Although using case studies seemed to be an excellent match for my study, it was not without its limitations. According to Merriam (2009), generalizing information from a case study to a larger population can be problematic. Since the case study is often focused on one example, either a small group or a particular event, it risks not being representative of other
situations, demographics, or populations (Merriam, 2009). That being said, Eisner (1991) and Stake (2005) argue that this limitation can be considered a strength, as it allows the researcher to provide its audience (i.e., teachers) with an extremely detailed description of events, which can then be used as a template to follow in their current situation or environment (i.e., classroom).

This case study research utilized case studies in order to investigate the research questions.

Using case studies for this research was important as it provides the opportunity to develop an understanding of a complex issue. Since teaching practice is directly related to student performance, and since there are a plethora of variables that influence both teachers, students, and the classroom, case studies allow for a contextual understanding and analysis of the effectiveness of this mathematics intervention. As one of the purposes of case studies is to evaluate an issue in its natural environment, by design it includes all of the authentic variables that would occur naturally. Thus, it takes into consideration a variety of different effects that could have the potential to bias the results, which might otherwise be ignored or neglected using a different methodology.

To further support the validity of the data using case studies, teachers in the research tracked student progress individually as well as measuring student progress in mathematics by administering the EQAO large-scale provincial test. These results are compared to their classroom results as individuals, cohort results compared to previous years and the five-year cohort average, and between the different classes and schools. This provided additional evidence as to whether or not the stated intervention had any effect, which will help answer a research question.
3.3 Research Paradigm

3.3.1 Social Constructionism

Social Constructionism as defined by Andrews (2012), means “view[ing] knowledge as constructed as opposed to created… [it] is concerned with the nature of knowledge and how it is created and as such, it is unconcerned with ontological issues … [Thus] meaning is shared” (p. 39) and is dependent upon a particular context. Developed primarily from the Progressive Reform Movement, social constructionism is an epistemological lens available to help the researcher understand and explain a particular phenomenon (Golafshani, 2003).

3.3.2 Implications for Social Constructionism in Mathematics

Knowledge must be constructed and develop based on a child’s interest and beliefs (Vygotsky, 1978). Using this lens, Vygotsky (1978) developed a theory, the ‘Zone of Proximal Development’, that argues that students learn best when pushed slightly outside their current knowledge zone. Thus, curriculum should be student-centred and based on prior knowledge so that it can increase students’ level of understanding.

Vygotsky’s theory is important to consider when investigating elementary school mathematics activities as it speaks to how students will make meaning out of knowledge. Students are likely to build on their prior knowledge (Booth, 2011) and require planning and teaching dedicated to achieving this. Thus, teachers need to be cognizant of students’ knowledge in all aspects of teaching, including planning, teaching, and assessment.

Furthermore, Vygotsky’s (1978) Theory of Development and SC theory have implications for teachers and their development. Teachers acquire new knowledge based on their existing knowledge (Bada, 2015). When developing new knowledge on how to teach, plan, and assess, it is important to take this into consideration.
Since knowledge is socially constructed, it can be challenging to measure. Specifically, measuring the effectiveness of a teaching intervention can be challenging without considering how the teacher was previously delivering curriculum to students and what they understood prior to the delivery of the intervention. The research question suggests the exploration of an intervention that enhances student mathematical performance through altering teaching practice. It therefore required the collection of different teachers’ understandings and their reflections on this method, as well as students’ reactions to the strategy.

Since this construction of reality is at the centre of the research and since teachers and students may construct reality differently, using a social constructivists approach permits multiple realities to be investigated. These potential differences between teachers and students are important as they provide a better understanding of how different teachers and students respond to the intervention. It also creates the opportunity to apply the results more generally.

For my study, measuring the effectiveness of a teaching strategy required the collection of different teachers’ understandings and their reflections on this method, as well as many students’ responses to the strategy. Since this construction of reality is at the centre of my research and since teachers and especially students may construct their realities differently, using a social constructivists approach will allow for these different realities to be investigated and eventually be combined and evaluated for trends and themes. These potential differences between teachers’ actions and perceptions and between student responses are important as they permit me to better understand how different teachers and
students respond to the strategy and will create the opportunity to apply the results of the proposed study more generally.

3.4 The Intervention

3.4.1 Overview

The intervention referred to in this study combines the frameworks from Wiggins and McTighe’s (2005) Backward Design model and Carpenter et al’s (2015) Cognitively Guided Instruction (CGI). The intentions of this intervention will be described first; followed by a description of how the knowledge was shared with the teachers and tracked within this study.

3.4.2 Combining BDm and CGI

The purpose of using the Backward Design model, while infusing CGI, was twofold. First, it creates a pragmatic and targeted approach to teaching mathematics in a framework that is child centred and accessible for all students. Second, since both models have been shown to increase student success, combined they should be most effective. Thus, the intention of this intervention was to improve student achievement in mathematics through the education of teachers on the use and implementation of this combined framework.

To properly measure the effectiveness of this intervention, it is imperative to assess both computational fluency and problem solving skills. As such, a pre- and post-assessment of students’ computational skills and word-problem solving abilities will be used to compare their growth and maturation as related to the intervention. Following the completion of the study, EQAO data was used to triangulate and evaluate the findings of this research.

Over the course of a three-month period, participating teachers from both schools volunteered to meet together approximately once a week in order to: (i) learn about the proper implementation of these frameworks; (ii) explore different scenarios, situations, and
challenges that have been experienced by others when implementing these frameworks; and (iii) begin the initial planning for the implementation of these frameworks within their own unit.

Each training session for participating teachers lasted one hour and began with a brief word problem that incorporated a different mathematical concept or calculation. These concepts began with multiplication and progressed to more challenging concepts, such as functions and ratios.

This opening word problem served many purposes. First, it acted as a model for each participant during the teaching in their own classroom. Second, it provided recent experience that they could share when teaching their own students. Third, as a demonstrated need by the research of the National Research Council (2001) and supported by many others (Hill, Ball, & Schilling, 2004; Marshall & Sorto, 2012; Telese, 2012), initiating the professional development session with a word problem helped each participating teaching develop mathematical thinking at their level in a safe way.

The first three sessions were focused on the Backward Design model. During the first session, after solving the opening word problem, I provided an overview of Backward Design model (Appendix H - BDm Presentation). We reviewed the model’s structure, purpose and organization and studied two different examples of units – one that had used the Backward Design framework and one that did not. Teachers were provided time to compare and analyze each example before participating in a brief discussion comparing the two.

During the second and third sessions, the opening word problem increased in difficulty. It was apparent that all of the participants were more willing to engage with the
problems as compared to the first session. The teachers were more comfortable and willing to discuss and consider the problem as a group.

During these second and third sessions, the focus turned to planning a unit using the Backward Design model. We explored questions that provided intentionality and guidance to help keep the teachers focused on this new way of planning. Teachers were also provided time to begin the planning of their own units. During this initial planning, the teachers collectively decided it was in their interest to address the same skill. Given that they all were required to teach multiplication and had yet to do so, combined with the fact that there were samples of units incorporating this skill in existence, they agreed to all base their units in this topic.

During these two sessions, discussions focused more heavily on the assessment and delivery component of their units. Teachers shared their concerns regarding how to adequately assess individual students and plan a lesson when they were uncertain if all students were ready to move forward. This led to a natural yet brief introduction about CGI and how it could be used to address this concern. Teachers were asked to further consider their own current knowledge and what they might do to solve this problem.

The next four sessions focused on CGI. Each session again began with a brief word problem that required the use of a different mathematical concept or calculation. At this point, teachers were now more accustomed to this approach and were more willing and able to commence work on the problem at the outset of each session. The first of these sessions provided an overview for CGI and explained its basic premise and purpose (Appendix F - CGI Presentation).
The second and third sessions on CGI provided more time to delve into the many tutorial videos provided with the CGI textbook and manual. Some of these videos were intended only to observe CGI in practice. Other videos were used to facilitate discussions among the group. The video would be partially shown and then stopped in order to afford each teacher time to generate a response that they felt best embodied their understanding of CGI.

Teacher responses to these videos were generally consistent with intended CGI outcomes. These sample videos allowed teachers to focus their time and attention on a single problem and avoid the many other distractions that inevitably exist in a classroom. It also provided each with time to think and consider each problem together with colleagues. However, as observed by several teachers participating in these sessions, the environment in the videos was contrived. In particular, the examples shown in the videos highlighted only one specific issue at a time and neglected to address how a teacher might address or manage competing questions or demands, such as follow-up questions or other issues or conflicts occurring in the classroom.

The final session required teachers to review their unit template and ensure that their plan reflected a CGI approach to teaching. Teachers were also provided guiding questions from the CGI textbook as well as student-focused literature. Topics covered by these materials included: (i) How does this support all students’ learning?; (ii) Is this student centred?; (iii) How do I know that all of the students understand the concept/lesson?; and (iv) How do I support the students who are not yet grasping the material while simultaneously aid students who are ready to move forward?
Following this final session, participating teachers completed their unit plans and submitted them to me prior to implementing their plan in their own classrooms. Some teachers sought out my support and other qualified individuals to assist in completing their unit plans.

Each participating teacher’s unit plan was intended to contemplate the following: (i) the plan would focus on problem solving and word problems as set out by the authors of CGI; (ii) the plan would embed CGI expectations while using the Backward Design templates; and (iii) students would be assessed on their multiplication calculations as well as their problem solving skills at both the commencement and completion of the unit in order to measure the students’ progress (Appendix E – Backward Design model Sample & Appendix T - Sample Teacher Pre- and Post-Word Problem).

I created a standardized assessment that was applied to measure students’ calculations skills in a pre- and post-assessment to match curriculum expectations (Appendix J – Sample Student Calculation Drill). This assessment was intended to measure the effectiveness of the transfer of knowledge from word problems to calculations as indicated from the CGI research (Carpenter et al., 2015). The focus on multiplication was selected by the participants as they shared it was a major focus in their curriculum and they felt it was wise to spend their time focused on this area. They also identified that this was the first time many students would be learning this skill and that it had been identified as an area of need at this school by the administration.

The calculation assessment consisted of 60 different single-digit multiplication questions ranging from 0x0 to 9x9 on one side of a single sheet of paper. These questions included in the pre- and post-assessment were identical. However, the order of the questions
was randomized in both assessments. Notwithstanding curriculum requirements that students only calculate single-digit multiplication questions up to 7x7, 10% of the questions included in the assessments were more challenging in order to allow for differentiation of student performance. Student answers were scored as correct or incorrect and tracked by the participating teacher. As per usual practice, raw scores on this assessment were converted to a percentage and the corresponding level and/or grade were provided.

To measure students’ ability in higher order thinking skills and problem solving skills, all teachers agreed to individually develop their own pre- and post- word problem assessments. The purpose of this is to understand how well students performed on word problem tasks, to track their progress, and to compare it to their calculation assessment results to determine if there is a connection, as suggested by Carpenter et al. (2015). The decision for participating teachers to develop their own word-problems was made based on the principles of CGI that recognizes each student and classroom as unique entities. In order to support students at their individual level and to provide an appropriate context while relating it to prior knowledge, it was in the best interest of the students for teachers to create their own word problems using the CGI framework and based on the expectations outlined in the Ministry of Ontario’s Mathematics Curriculum.

Word problems were scored by each individual participating teacher using their knowledge of the Mathematics Curriculum and the Achievement Chart. The Achievement Chart categorizes student results into one of five overall sections: “R” indicating that the student has not demonstrated any knowledge related to a task; Level 1 or D, indicating the student attempted the task but demonstrated limited knowledge; Level 2 or C, indicating that the student completed the task and demonstrated some knowledge; Level 3 or B, the
expected end performance level indicating the student demonstrated a considerable knowledge or understanding; and Level 4 or A, indicating that the student demonstrated a knowledge that is above and beyond of what is expected of them at this time.

It is common practice for teachers to provide students with levels as well as an additional indicator of where on that level the student is performing. If the student is performing at a low level, mid-level, or high level of that category, this is represented with a minus sign, a space, or a plus sign after the level respectively. It is necessary for the purposes of this study to create a conversion scale in order to measure student growth throughout the intervention. Thus, a 12-point scale was created and used to reflect and measure student achievement based on all of the potential outcomes teachers might provide, as shown below.

**Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart**

<table>
<thead>
<tr>
<th>Level/Grade</th>
<th>0/R</th>
<th>1-/D-</th>
<th>1/D</th>
<th>1+/D+</th>
<th>2-/C-</th>
<th>2/C</th>
<th>2+/C+</th>
<th>3-/B-</th>
<th>3/B</th>
<th>3+/B+</th>
<th>4-/A-</th>
<th>4/A</th>
<th>4+/A+</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point Value</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
</tbody>
</table>

**3.4.3 BDm and CGI in the Classroom**

Although each teacher’s interpretation of the combined intervention was slightly different, due to variable factors such as differences in students, knowledge, and classroom environment, their purpose and goal were identical: to explore the effectiveness of this intervention on student mathematical achievement. Once teachers had created their plans, it was intended that they start their unit by determining the end goal, or big idea, that they hoped students would achieve. At this point, the participating teachers should have already collected a large portion of the resources needed and determined the evidences they would use to measure students’ progress towards the final goal. In other words, teachers are expected to have collected or created several different word problems that would support...
student learning throughout the unit, specifically geared towards achieving fluency and accuracy in calculating single-digit multiplication as well as solving higher-order-thinking word problems using multiplicative reasoning. This is rather different from what teachers are currently doing in their classrooms for three reasons: (i) teachers in this study often plan day-to-day and respond to immediate student need; (ii) teachers who do select an end goal do not typically consider the various steps required to achieve this goal, but instead plan as described in step (i); (iii) each individual lesson and activity is not directly and explicitly linked to the desired final goal or overall outcome.

Traditionally, teachers in the study plan mathematics lessons on a day-to-day or short-term basis. This is not entirely consistent or generalizable to all elementary teachers in the field by any means; however, there is some support that teachers planning of mathematics at the elementary level is inadequate or insufficient (Sullivan et al., 2012).

Although many teachers refer to the Ontario Ministry of Education’s curriculum to guide them in this process, they tend to select a small goal that they would like students to achieve and work towards that or insufficiently plan for the entire unit (Sullivan et al., 2012). Employment of the Backward Design framework ensures that lessons are “not so small that it leads to isolated lessons and overly discrete learnings, yet not so large that it seems overwhelming and too broad to guide day-to-day teaching” (Wiggins & McTighe, 2015, p. 275). This is important because “[w]hen the planning, teaching, and assessment of students is viewed in such limited terms, it is impossible to ensure that student learning and their learned knowledge is transferrable, worthy, linked to a big idea instead of simply the small goal at hand, and that it is anchored in credible and useful evidence” (Wiggins & McTighe, 2015, p. 50).
Planning mathematics on a short-term basis creates a challenging learning environment for students trying to advance in this subject area. Furthermore, this type of short-sighted planning creates a situation in which teachers may focus too much on the specific tasks and respond accordingly to these limited student needs, which are not based in the context of a larger goal and do not provide students with the necessary larger context and broader skills. For example, if a teacher were to plan a lesson or group of lessons on teaching students the standard algorithm for single-digit subtraction, instead of considering a larger goal, context, and purpose in which this could be situated, students may fail to learn or understand the formula for a variety of reasons: the goal is not connected to prior knowledge; possible misunderstandings regarding the minuend and the subtrahend; and/or students may provide evidence that they understand how to use the formula but are unable to transfer it to any other context when the information is presented in a different format.

Short-term planning creates an environment where teachers respond to the immediate need and provide answers that will address needs of the moment, but often will not support student learning in a larger capacity.

Another issue that the Backward Design framework resolves is the situation that arises when a teacher selects an end goal but fails to adequately plan for students to achieve this goal. In some instances, the goal is too large and overwhelming. However, in my experience, teachers of mathematics often select goals that are too small and limiting. Although both of these instances have different challenges and issues, they both lead to limited student knowledge, learning, and understanding.

When there is adequate planning to achieve the desired result, the evidence collected may not appropriately measure student learning and understanding. Without this key piece
of knowledge, it is impossible to measure student understanding, creating a troubling situation: connecting new learning to prior knowledge becomes extremely challenging; communicating to parents, teachers, and students about student understanding is inaccurate; future planning is based on limited or poor information, creating a cycle of sub-par learning activities.

Employment of the Backward Design model ensures that each task, lesson or section of learning is linked to a larger goal. For the specific unit in the intervention, participating teachers were asked to use the curriculum to select a goal that links the learning of problem-solving skills to multiplication calculation skills. Thus, when teaching a lesson, the desired goal or focus of that lesson is explicitly linked, for both students and teachers, to the larger goal that they are trying to achieve.

For example, this is how the Backward Design model was intended to be used in this intervention: teachers were to select their overall desired goal - for students to acquire multiplication calculation skills through the use of problem-solving skills focused in the area of multiplication. Once teachers found a way to articulate this goal, they determined the variety and pieces of evidence that would be used to measure student learning and understanding. To measure students’ multiplication calculation skills, a standardized assessment was adopted both pre- and post-unit, as discussed above. To measure students’ problem-solving skills, teachers created their own word problem for pre- and post-assessment. Once this was complete, teachers were tasked with the most important aspect of the Backward Design framework: selecting or creating sub-tasks along with their appropriate measures. Teachers varied significantly in this process; this will be discussed and explained in each specific case study.
3.5 Research Method

My study will consisted of four grade 3 classroom teachers at two different schools. I examined their specific practices in teaching mathematics and, through an intervention to support their teaching instruction, tracked how effective the intervention is on student achievement, as measured by teacher created assessments and EQAO, an independent, large-scale assessment. Prior to and throughout the intervention, teachers were interviewed in order to explore their background knowledge in teaching and mathematics content, beliefs, and attitudes as well as their rationale for their teaching practices in mathematics. Combined with my observations of their implementation of the intervention in teaching mathematics, rich and purposeful information allowed for a deep analysis of the interventions effectiveness.

3.6 Participants

3.6.1 Teachers

Teachers were asked to participate in the study via email (Appendix N – Teacher Initial Email Recruitment Letter). Letters were emailed to some grade 3 teachers in an urban school board. Participation in the study was voluntary and teachers could withdraw from the study at any point in time, with no repercussions. Teachers received additional support in planning and assessing mathematics in their classrooms; however, no monetary gifts were supplied. On occasion, it was necessary to provide some of the teacher participants with specific classroom materials or resources. Once teachers were selected for the study, a formal consent letter was provided outlining specific details of the study and its purpose (Appendix R – Teacher Consent Form). This consent asked participating teachers to provide
permission to audio and/or video record their classroom, provided permission is obtained from all students’ parents as well (see 3.52 Students).

3.6.2 Students

As students were previously assigned to teachers and I have no control over this process, students in the respective teachers’ classes participated in the study insofar as they were completing the mathematics work. As it was necessary to collect student data, including but not limited to assessment data, work samples, and audio and visual recordings, a parental consent letter was sent to each students’ parents and collected prior to the beginning of the study (Appendix Q – Student Parent Consent Form). Consent was obtained to allow me to work with each student in the classroom, to photograph or copy the student work with no identifying characteristics, to share EQAO results with me as they pertain to the study, and consent to be audio and video recorded. Students who do not provide all levels of consent participated in the study to the extent in which permission is obtained.

All student names and data were encrypted so that it cannot be traced to a particular individual and no personal information of any student will be shared. Any collected materials were coded in a fashion that links it to a particular teachers’ classroom but cannot be identified by members outside of this study, as a particular piece of a student’s work.

3.7 Procedure

In my study, each teacher was asked to participate in a semi-structured interview (Appendix J – Teacher Survey Sample). Some of the questions, such as length of teaching and number of secondary level mathematics courses taken, were provided in advance to provide the teachers an opportunity to plan and prepare answers. The goal of the initial interview was to identify important and relevant information such as level of education,
experience in teaching, experience in teaching mathematics, attitudes as a teacher and
towards mathematics, as well as insights to his/her current mathematical thinking, practices,
and content knowledge. Interviews took place at a location convenient to each particular
teacher.

Following the initial interviews, I observed each participant teaching mathematics
for several lessons. During this time, I completed observational notes about the teacher to
combine with interview data during the analysis of the study. This formed the foundation for
the type of mathematics education they have provided to their students and provides some
ideas as to whether this teacher supports traditional mathematics education, reform
mathematics education, or utilizes ideas from both. It also provided other important
information such as the amount of time spent on mathematics classes, amount of time spent
on specific activities, types of activities regularly used, and how students are assessed both
formally and informally.

After a teacher was observed in their classroom for several lessons, I meet with each
teacher individually to discuss their experiences. It is at this time that the participants
participated in eight professional development sessions regarding the intervention and how
to effectively use Cognitively Guided Instruction in mathematics with a Backward Design
model (Wiggins & McTighe, 2005) (Appendix H - BDm Presentation & Appendix F - CGI
Presentation). They were provided resources, support, and time with the planning and
implementation of this intervention, as necessary. During these professional development
sessions, the participating teachers were given details and parameters regarding the pre- and
post-assessment for students. Each teacher was asked to plan a unit and develop two similar
word problems that espouse the qualities and goals of the unit, which will be used during the intervention.

Once the teachers were prepared for the unit, I observed and video recorded them throughout their implementation of the intervention in the classroom. I made observational notes on both the teacher and students at that time, including notes and samples of student work. The teacher and I continued to meet during this stage for additional support.

After the units were completed, a semi-structured interview was held with each teacher individually to collect information regarding their perspective on the intervention and to discuss the results of the students. I collected, analyzed, and compared the data collected from the end of unit assessment and from EQAO results. This information was used to make various comparisons, such as pre- and post-performance for individual students, comparison between different class performances, and cohort comparisons on previous EQAO assessments.

3.8 Data Collection

The research project began in October 2016. The teachers and students in this project had the opportunity to develop a rapport, build community, and learn mathematics from their teacher in their classroom setting as usual, without the intervention. This provided some insights as to the current teachers’ approach to teaching mathematics, prior to the intervention. Data collection took place beginning October 2016 until June 2017. The EQAO mathematics student assessment was independently administered and scored in May 2017. These scores were analyzed with other data collected in the classroom in November 2017.
Data were collected from four main sources: 1) classroom observations; 2) teacher interviews, surveys, and online questionnaires (TKAS); 3) student work; and 4) EQAO grade 3 student mathematics assessment.

3.8.1 Classroom Observation

Classroom observations took place to observe lessons prior to the intervention and then throughout the study regularly to observe how the intervention is being implemented and to observe both teachers’ and students’ reactions, attitudes, and changes.

Observations were made by me through field notes and video recordings. Field notes were used to provide evidence of behavioral changes, such as changes in facial expressions and body language. Audio-visual recordings provided the opportunity for me to review the events that have taken place. Given that the classroom is a busy environment where individuals act simultaneously, it is unlikely I was able to notice every detail. Thus, audio-visual recordings allowed me to take notes while ensuring that details do not go unobserved.

3.8.2 Teacher Interviews, Surveys and Guided Support and Planning

Interviews took place both pre- and post-intervention with each teacher. Throughout the study, additional meetings took place to discuss planning, collectively establish goals, and evaluate student work. These sessions were tape-recorded and transcribed. Field notes were written to collect responses and behavioral information.

The teacher survey was administered prior to the beginning of the intervention. The survey contains work from the Teachers’ Mathematics Knowledge for Teaching (Hill, Ball & Schilling, 2004), which collects information on teachers’ mathematical content knowledge, mathematical understanding, and measures their “knowledge for teaching mathematics in several content areas and teaching domains” (University of Michigan, 2017).
The initial interview utilized semi-structured questions to assess teachers’ current level of implementation on the teaching style being used in the intervention (e.g., exploring their current understanding of BD and CGI, their use of it, how it is implemented when used and collect data on teacher information such as number of years teaching, supposed comfort level teaching mathematics, and previous exposure to learning and teaching mathematics.

3.8.3 Student Work

Student problem-solving work samples were collected to evaluate the effectiveness of the intervention. It will be assessed primarily by the teachers with occasional consultation with me using the Ontario Curriculum, the Ministry of Ontario’s Growing Success document, and based on the gained knowledge from the intervention.

3.9 Data Analysis

Data analysis was divided into two phases: the qualitative data analysis and the quantitative data analysis. Qualitative data from the classroom were coded into various themes such as beliefs, attitudes, and practices and analyzed for patterns, trends, and changes. Likewise, notes from teacher semi-structure interviews with open-ended questions were coded in a similar manner.

Quantitative data were analyzed in order to compare a variety of different factors: teacher changes pre- and post-intervention, student work changes for each class pre- and post-intervention, connections between student performance and teacher mathematical knowledge, and to compare previous and current cohort performance on EQAO’s mathematics grade 3 student assessment. Teacher changes were compared using information collected during initial interviews and surveys and final interviews after the intervention.
Data on students will be collected in three following ways: recordings from the classroom, student work samples including the pre- and post-assessment, and the EQAO mathematics assessment results. Students’ pre- and post-assessment performance was compared to analyze any changes. Furthermore, an analysis comparing classroom student cohorts was compared between one another while considering other factors such as teacher knowledge and socio-economic measures. Finally, EQAO scores were analyzed for individual performance on specific tasks related to the mathematics unit taught in the intervention and compared to their individual classroom score, to compare the various classes in the proposed study while considering factors such as teacher’s individual mathematics knowledge ranking from Hill, Ball, and Schilling’s (2004) assessment, and to compare entire school grade cohorts to previous school grade cohorts and the school’s average scores over the past five and ten years.

3.10 Ethical Considerations

I received approval from the Office of Research of Ethics of the University of Toronto, as well as The Toronto District School Board’s External Research Review Committee, prior to beginning any research or making contact with any teachers. Pseudonyms for teachers and schools were used to ensure confidentiality and any specific details that could link the school, teacher, or students to this study, will be omitted.

All teacher participants of the study were provided an overview of the study and letter of consent. The overview will contain information about the goal of the study, length and time it is expected to take place as well as their role and duties if they choose to participate. The formal consent letter informed them of any benefits and risks they assumed during the research project and asked them to sign and return it prior to beginning the study.
The consent letter also acknowledged that teachers may withdraw at any point in time for any reason.

3.11 Scope and Limitations

Although I made every effort to ensure that all data were collected accurately and that they are interpreted fairly and reasonably, the study has some limitations. Namely, the study itself is limited in the number of teacher participants. With five teachers involved in the study, it limits the breadth of experience that may better be captured. However, with such a small number, it allowed me to develop meaningful connections with each teacher, time to support teachers in learning about, planning for, and implementing the intervention, as well as more time to collect data in each classroom, which lead to a more in-depth understanding of the intervention in each classroom. Furthermore, with five teachers, it involved approximately 100 students, which is a reasonable sample size when exploring the effectiveness of the intervention from a student performance perspective. Additional limitations will be addressed in Chapter 5.

3.12 Concluding Remarks

Undeniably, mathematics scores are decreasing, according to five-year trends both within the province and based our international PISA assessments (Knighton, Brochu, & Gluszynski, 2010). The teaching system in Ontario is becoming increasingly less competitive in the global economy and thus, teaching mathematics as we have been, requires a change. Based on a large body of literature and research, we have an increased understanding of how students best learn and conceptualize mathematics at all levels of education.
However, there is a gap in the literature that both satisfies and links all aspects of the teaching process from planning, to teaching, to assessing and evaluating student work. Arguably, it is necessary to be competent in all areas of the teaching process to be an effective teaching in our 21st century education system. My research helped to provide evidence for the need to combine two well-documented and well-researched frameworks that address and focus on different aspects in the teaching process. I believe that the best way to enhance student learning in mathematics is by developing a balanced numeracy program; however, this program must be grounded in research that already exists.
Chapter 4: Findings

This chapter details the teaching practice of the grade 3 intervention, as well as the students’ response to it. Four different classrooms in two different schools were observed. I will commence my discussion with a description of the school contexts of both the teachers and the students. This introduction will provide a foundation on which to consider the unique situations of those involved, as well as to compare the similarities and differences between them. Case study discussions will include an overview of the academic background of each teacher participant; commentary on each participating teacher’s attitude towards instructing mathematics at the elementary level; a depiction of each participating teacher’s mathematical knowledge, approach and beliefs prior to and following the intervention; students’ response to and performance on the various measures used in the intervention. Each case study will conclude with a summary of the findings per classroom.

4.1 Norden Junior Public School

Norden Junior Public School is a medium-sized, Kindergarten to Grade 8 school with just under 500 students registered at the time of the study. The student population is almost equally comprised of boys and girls, at 49% and 51% respectively (TDSB, 2017). Although 85% of the students were born in Canada, 11% of the total student population is categorized as English Language Learners (EQAO, 2017) while 30% of the population learns a language other than English first at home (EQAO, 2017). Approximately 81% of the student body has only ever attended Norden Junior Public School, while 19% has previously attended one or more other schools (EQAO, 2013). Of note, during the school year in which the data was collected, Norden Junior Public School experienced a significant
enrollment increase. This was due to a large development made available to new immigrants and families who have experienced domestic violence.

Norden Junior Public School sits amidst a culturally diverse community. The school has representation from over 10 different religions, with members comprised of differing and varying family structures, incomes, and educational backgrounds. Although a limited number of families earn more than $100,000 per year (less than 20%), the majority of families (approximately 50%) earn $60,000 or less. Approximately 10% of these families earn less than $15,000 annually.

Over 20% of the student population is raised by single parents and over 60% of the family population has two or more children. Approximately 38% of the parent population has a high school diploma or less education, while the remaining population has some form of additional post-secondary education or apprenticeships.

Based on the most recent EQAO data, approximately 25% of the total student population was identified as having Special Needs, of these identified students, 14% wrote the EQAO assessment. The attitudes of the students who attend Norden Junior Public School, according to the EQAO Mathematics Student Questionnaire, have seen an improvement over the past five years. According to the 2016-2017 EQAO Grade 3 Student Attitudes survey, 51% of students reported that they always like mathematics (Appendix A - Grade 3 EQAO Attitudinal Trends – Norden Junior Public School). Conversely, 16% of students reported that they never like mathematics (Appendix A - Grade 3 EQAO Attitudinal Trends – Norden Junior Public School).

Similarly, the previous year’s assessment had 46% of students always liking and 19% of students reporting that they never like mathematics (Appendix A - Grade 3 EQAO
Attitudinal Trends – Norden Junior Public School). This is a slight increase in student attitude towards mathematics from five years ago, where the 2011-2012 assessment indicated that 40% of students always liked mathematics and only 19% reported to never like mathematics (Appendix A - Grade 3 EQAO Attitudinal Trends – Norden Junior Public School). Throughout the five years before this study’s intervention, there was a 3% decrease in students who reported never liking mathematics and an increase of 11% of students who reported always liking mathematics (Appendix A - Grade 3 EQAO Attitudinal Trends – Norden Junior Public School).

During the five years prior to this intervention, Norden Junior Public School’s student performance on the Grade 3 Mathematics Assessment also showed a decreasing trend. In the 2011-2012 EQAO Assessment of Mathematics, 64% of grade 3 students at Norden Junior Public School met or exceeded the provincial standard (Appendix B - Grade 3 EQAO School Math Achievement 5-Year Trend - Norden Junior Public School). With the exception of a significant increase in the 2012-2013 assessment data, where 81% of students met or exceeded the provincial standard, there has been a steady decrease in student performance. By the 2015-2016 assessment, only 54% of students met or exceeded the provincial standard, indicating a 10% decrease in students who met this expectation (Appendix B - Grade 3 EQAO School Math Achievement 5-Year Trend - Norden Junior Public School). It is important to note that in the 2012-2013 Assessment data, 15% of the students were exempted and not reported in the findings.

4.1.1 Case Study of Genevieve

Genevieve joined Norden Junior Public School at the start of the year in which the study took place. She is currently in her twelfth year of teaching, all of which have been at
grades 1 to 6. Genevieve has taught core mathematics to her class every year. However, she only taught grades 1 to 3 during the first three years of her career. Genevieve also noted that her recent experiences in teaching have been with the older grades and, in addition to moving schools and communities, it has been many years since she has taught in the Primary division.

Genevieve earned a Bachelor of Arts degree followed by a Bachelor of Education degree. She has taken a couple of formal mathematics courses but has completed no additional course work in mathematics since university. She taught a combined Grades 2/3 class during the year of the study, a return to the Primary division, after many years spent in Intermediate classrooms.

4.1.1.1 Teacher Attitude

Genevieve was very willing and open to sharing her perspective on teaching primary mathematics. During the pre-interview, Genevieve stated that she typically enjoys teaching mathematics. After sharing her initial positive feelings, she immediately began to share her worries and concerns:

I like it, but I wish I was more confident... I wish I knew more about, like, manipulatives. That is something that I would like because I know more of the old-school ways, so we did not have all these types of manipulatives. So I feel like I am doing [the students] a disservice by not using all of those different kinds. (Interview, April 10)

Although eager, open, and willing to teach and learn about instructing mathematics, Genevieve shared some concerns about her teaching and its effectiveness. She demonstrated this through elaborating on her teaching style and her approach to mathematics instruction. She stated that she was more familiar and comfortable using traditional methods of teaching. She believes her teaching focuses more on:
Speed, facts, five minutes and how many can you do [drills]. I know a lot of people are against that because what if the kids cannot work quickly… [I focus] more [on] the traditional types [it is] what I go to because it was the way that I was taught and it worked very well for me. (Interview, April 10)

By the end of the unit, Genevieve shared that she still really enjoyed teaching mathematics but continued to have some apprehension:

I have always really liked it but I am still nervous to do it. This unit we did was very helpful…First of all, doing a split grade, doing a pathway and having a place to go, was really helpful to me. And being the first time, well not the first time but a long time since doing a 2/3, I needed direction and this was helpful in setting up a great math program because there is so much to cover. (Interview, June 19)

Genevieve also shared that, while she performed fairly well in mathematics as a child, she did not fully understand many of the specific mathematical concepts until she had to teach them. She feels that a stronger understanding of manipulatives and how to use them would serve to improve the effectiveness of her teaching practice, allowing her to better meet the needs of her students. As her knowledge and confidence increase, so does theirs, resulting in a firmer grasp on the part of both teacher and students. Genevieve also believes that had she used manipulatives as a child, she likely would have better understood the concepts rather than simply memorizing the process: “I know if I could understand the manipulatives better, my lessons would be more useful to the kids to really understand it” (interview, April 10).

In her pre-attitudinal survey, Genevieve scored an overall 2.9 out of a possible 4 in regards to her attitude towards teaching mathematics (Appendix J – Teacher Survey Sample). By the end of the study, this score had positively increased by 0.3, resulting in a new total of 3.2 out of 4 (Appendix J – Teacher Survey Sample). On the survey, Genevieve consistently indicated that she felt that “mathematics is important,” scoring its importance as a 4 out of 4 on both the pre- and post-attitudinal assessment. Genevieve’s survey also shows
that her biggest improvements were a result of the time she spent planning her mathematics lessons and units as well as her increased comfort-level with the material. As reflected on the survey, both her confidence in and enjoyment of teaching mathematics improved by one entire point. However, although her overall attitudinal score increased, when asked to rate the generalized statement of “I like mathematics,” Genevieve indicated that this area had decreased over the course of the study, falling from a 3 to a 2.

### 4.1.1.2 Teacher Knowledge

Genevieve demonstrated a lot of resourcefulness in teaching mathematics. She shared the multiple strategies she uses to collect resources and to learn a variety of approaches to teaching. Genevieve describes how she prepares to teach mathematics:

> I always start with the curriculum first, you have to follow that. And I look online, I look in textbooks, I look to what I was taught when I was younger. Next door, [teachers] give me tons of ideas and we are sharing constantly. [I speak] with other colleagues as well so [preparing to teach mathematics] involves a little bit of everything if you want to do it well. (Interview, April 10)

After participating in the study and learning about CGI and the Backward Design model, Genevieve shared some new pedagogical insights she has developed into teaching mathematics and employing this combined intervention:

> There is so much to cover in math, you need to break it down, and sit down and plan it. This method makes so much more sense than what I have done in the past, with good intentions, but I have never been able to catch up to what I was supposed to get to. But sitting there and thinking about it and using this method, I definitely see the benefits of it. Once you do it once, it becomes more familiar, but then you can also keep [the knowledge], use it and follow it next year. (Interview, June 19)

She also indicated that her feelings about teaching have grown with experience, but that she sometimes feels conflicted between her new thoughts and her previous opinions:
My own teaching philosophy part [has changed]. What part is memorization and what part is multiplication? You have to memorize. You cannot avoid it but that goes against so many different things. But obviously if you cannot memorize, you have to be able to solve a multiplication question. I really do feel like [the students] can because I, for the very first time, understood where I was going throughout the unit. And talking to you throughout it was great. I forget the doubles, the skip counting, all of that stuff. Instead of me worrying about memorizing, which is what I originally practiced, having practiced skip counting and these different techniques you helped me with and that I discovered in the planning of the unit, it made me realize there is not just two different ways to get to it. So that really opened my eyes. (Interview, June 19)

Genevieve also shared her attitude, knowledge and experience around using the combined Backward Design model and CGI:

It did not take that long ... I could do it on my own. It was straight forward. I also like that you can, especially for grade 3 or 6 or any grade really, have that opening question and then that closing question is good, putting the start and the finish together like that and then the steps to get there, it’s not time consuming, it’s not adding on work to do. And at first I did actually think that, I thought ‘oh great, now I have to plan a unit before.’ It is so, I do not want to say general, but it is enough. Whereas before, I would have thought I need to go lesson by lesson, but it’s not. Instead I am going to try skip counting, I’ll try double, I am going to try all these different ways. So that when I look back at it, I can say I can try this one now but this did not work. I can try this one instead because I need to get to this end result. And it is helpful for the kids too because they can do the same. And it is really good too because these are EQAO-like questions that our kids need to be prepared for. (Interview, June 19)

At the beginning and the end of the intervention, Genevieve completed the online survey to measure her mathematics knowledge for teaching both pre, post, and overall. In terms of overall, Genevieve scored a 52.6% on her knowledge assessment (Figure 9: Teacher Knowledge for Teaching Compared to Student Performance). For the purposes of this study, Genevieve’s knowledge score has been classified in the Low Knowledge Range. Of the participants, Genevieve’s score was the only one to show a mild (5.5%) decrease from the beginning to the end of the intervention, falling from 55.5% to 50% after
completion. This is consistent with the findings on the attitudinal survey when Genevieve demonstrated a decrease in her response to *I like mathematics*.

Genevieve stated that she learned a lot through this experience. She demonstrated growth throughout the study, in regards to content knowledge, knowledge for teaching mathematics, and her overall teaching approach. This growth was reflected during classroom observation in numerous ways. Towards the end of the unit, she had introduced many new activities and improved her practice by starting the lesson with a student activity or question, grouping students based on academic ability, and using alternative forms of assessment that did not require tests or writing (such as a conference). Genevieve commented on these enhancements in her post interview.

It was not possible to observe the changes in Genevieve’s mathematical content knowledge. It is possible that Genevieve believes that the improvements in her teaching practice reflect an augmented understanding of mathematics. It is also possible that Genevieve acquired further knowledge from the various mathematical questions worked on in the professional development sessions. However, no evidence was collected to support this claim.

**4.1.1.3 Lesson Planning and Implementation**

Prior to completing any of the training, Genevieve admitted having limited knowledge of both the Backward Design model and CGI instruction. She had some idea of what the Backward Design model is and could briefly describe its purpose, but she was unable to articulate what CGI entails or to provide any examples.

Prior to the intervention, Genevieve’s lesson plans were not well–organized. It was common for her to use curriculum documents as a checklist to ensure that she met all of the
Genevieve planned in a forward-thinking fashion. She would teach one or two lessons, stop to reflect, and then plan for the upcoming session based on the outcome. Although this method demonstrates some level of a child-centred approach in that it adapts to students’ needs on an on-going basis, it lacks the structure and guidance provided by the Backward Design model. Genevieve felt that, prior to the intervention, the units often took more time than she had anticipated, or more time than she was able to adequately afford before needing to move on to a new concept or unit.

Prior to attending the professional development session, Genevieve’s mathematics program used a limited range of activities, which consistently culminated in a paper-and-pencil task or assessment. Genevieve used worksheets to teach, practice, consolidate, and assess students’ computational skills and would borrow word problems directly from the Math Makes Sense or Nelson textbooks she had available. She did frequently ask students to work in small groups or pairs as a way to help them learn and build skills. When questioned about how and when student groupings were determined, she shared that it was often of-the-moment, based on social dynamics within the classroom, and only an occasional thought around students’ academic understanding.

Genevieve’s mathematics lessons were rather predictable and formulaic. She consistently asked the grade 2/3 class to come to the carpet, where she would begin the math session by teaching a lesson about an idea or concept. She would dismiss the younger students first, sending them back to their desks to work while she continued to build on the idea for the grade 3 students. Occasionally she would ask students to contribute by raising their hand, otherwise limiting student contribution. Upon completion of the lesson, the grade 3 students would be sent back to their desks to begin working. During the work period,
Genevieve would circulate, offering encouragement and answering questions. Students were frequently observed getting up from their work space to collect materials, such as blocks or counters as tools to assist them with their work. There was no direct instruction observed that gave students permission to do this, as Genevieve explained she had previously instructed the students to seek out appropriate materials as needed without asking.

Upon completing the assigned task, students were expected to hand in their work to Genevieve, who would most often mark it for correctness. If no changes were required, students would move on to free time while their peers finished up. If corrections were needed, Genevieve would indicate which questions needed to be redone.

Genevieve regularly implemented diagnostic assessments at the beginning of a new mathematics unit in order to determine the level of the students’ understanding prior to tackling new concepts. These assessments were all written tests, which focused heavily on computational skills. The higher-order-thinking questions, or word problems, on these assessments often read similar to a number sentence, and thereby did not demonstrate a need for critical thinking, communication, or application of knowledge. Answers were either correct or incorrect; there was limited opportunity for students to develop their own approach or strategies.

After completing eight professional development sessions, four centered on the Backward Design model and four on CGI, Genevieve was better able to describe both of these interventions. Hence, she began to plan out her unit using these two frameworks. Her planning using the Backward Design model was detailed and organized, and reflecting many of the goals intended by this framework. In her plans, Genevieve included her essential questions, the required student understandings, and her intended established goals, as guided
by the Ontario Mathematics curriculum (Appendix G – Backward Design model Template and Appendix E – Backward Design model Sample). Genevieve also organized her unit by first stating the desired outcome, then proceeding to design appropriate measures to track student progress of this outcome, followed by identification of the collected or created tools and resources that would support the students throughout.

Genevieve appeared engaged and open to new ideas during the planning process. She was often the first person to ask for clarification and shared many ideas during our discussions. Following one of the sessions on CGI, Genevieve came back to the group to share an article about CGI that she had found online, hoping it would help her colleagues as it had her.

Genevieve used the Backward Design model to plan her entire unit prior to its initiation. Her overall goal was for students to “accurately use multiplicative reasoning to solve multiplication word problems that are relevant to students and that reflect their lives and experiences.” Genevieve included the pre- and post-assessment on multiplication in her unit plans as well as a pre- and post-assessment for word problems. Her organizational method involved grouping the unit based on strategy and selecting a word problem as evidence of student learning. Genevieve selected five different approaches to solving multiplication problems and used these approaches as her sub-areas of learning: multiple addition, arrays, standard algorithm, friendly numbers, and using pictures. Each sub-area was, by nature, explicitly linked to her overall goal. Throughout the unit, Genevieve emphasized this connection to her students.

Genevieve utilized the Backward Design model effectively. She first selected the overall goal or desired result, in line with the intention of students “accurately us[ing]
multiplicative reasoning to solve multiplication word problems that are relevant to [them] and that reflect their lives and experiences.” She went on to determine the appropriate evidence, namely the various word problem tasks and multiplication assessment. Finally, she planned learning experiences and instruction that would support students in achieving the desired goal. She identified the sub-areas that focused on different approaches to solving multiplication questions, while continually linking them to the larger goal.

Genevieve’s unit plan provided some examples and attempts to infuse CGI in her teaching practice. For example, Genevieve included her own students’ names in the word problem assessments. This strategy worked to raise engagement levels and to provide an approachable and relatable context for the students in her classroom.

Throughout the implementation of the unit, Genevieve required some support by me, via two-way discussions and my posing probing questions. This support helped her to ascribe to the intended expectations of the intervention. Genevieve continued to follow her unit plan as initially set out, straying from this plan only when required and in order to meet student needs. She regularly sought out support from individuals that were knowledgeable in the applicable areas and who were able to provide her with guidance and direction.

With my support, Genevieve improved her ability to track individual student progress on a regular basis. I modelled various strategies that could be used to track student progress in the classroom, such as employing a class list to track strategies and tools used; creating a class chart to record observations and student statements; using an iPad or camera to capture productivity that was not in written form; setting-up effective student-teacher conferences with recorded feedback. Initially, Genevieve had used a lot of small group instructions and tasks in her classroom. Although Genevieve believed that small group
organization was effective, as evidenced by student performance on final assessments, she was left feeling uncertain about individual student ability and knowledge during the unit. To rectify this possible lack of individual understanding, Genevieve developed individual follow-up activities and set up brief student-teacher conferencing to provide this missing information. She began to implement these follow-up strategies regularly in her program.

As a result, she was better able to intentionally group students in a manner she felt would be most effective. She was also able to better support students’ acquired knowledge as she had a stronger understanding of how each student was performing in an individual context. She was more aware of students who were struggling and could identify their specific challenges. She was able to support those who needed help while continuing to plan for those who could work more independently. As she had done prior to the intervention, Genevieve continued to consider social dynamics when grouping students. However, she now began to place a more significant emphasis on students’ mathematical skills and academic abilities when establishing pairs or small working groups.

Her lessons became more student-focused; Genevieve was observed frequently talking to individual students throughout the activities. Often the lesson within this unit allowed for student choice so that students could individually select the question they were going to work on, while remaining focused on the same mathematical concept or strategy as a group.

Alongside modelling from me, Genevieve decided to record her observations of students instead of simply relying on her memory. She took to jotting down ideas in a logbook. I provided her with samples of what other teachers had done in the past and
modelled a lesson on multiple-addition to demonstrate how I would record student responses throughout the lesson. She asked for a copy of this template and eventually created her own.

Genevieve began to ask more questions that required students to think independently. She allowed more thinking time to students. Most notably, in the last few activities of the unit, Genevieve’s responses to questions evolved from providing the answer to suggestions of *keep trying* or *ask a friend*. This led to further topic-centered communication amongst students, who came to view each other as resources, a step beyond the traditional classroom model where the teacher is considered the gatekeeper of knowledge. She also started to address the class as a whole with a question, followed by discussion in pairs or small groups and culminating in a sharing of ideas. This strategy allowed for an increase in the number of students who were able to participate in class discussions and provided a platform on which to share ideas. This was a large stride forward from her previous practice, where students were required to raise their hand to respond individually.

**4.1.1.4 Description of the Class**

Genevieve taught an active grade 2/3 combined-grade class this year. Although all of the students completed the tasks, activities, assessments, and questions that Genevieve selected or designed, only the grade 3 students were tracked for the purposes of this study. In total, Genevieve had 19 students in her class. During the study, three new students were added for a total of 22 students. Of these students, 12 were in grade 3 and therefore tracked in this study. Of the 12 grade 3 students, eight identified as boys and four as girls. Nine of the students had attended this school since Kindergarten, one student joined in grade 1, and two of the students were new to the school this school year.
Genevieve and her current students participated in an additional school initiative focused on numeracy skills in the classroom prior to the intervention. Genevieve shared that she used this initiative to learn about new and innovative classroom mathematics activities and how to integrate mathematics into other subject areas. Genevieve completed and submitted her plans more than four weeks prior to implementation. Her unit was expected to take approximately five weeks long. By its completion date, the unit had run for seven weeks.

4.1.1.5 Class Performance

Although Genevieve had not provided any direct instruction on multiplication to her class prior to the introduction of the unit, a number of students admitted to having been exposed to the concept through various means, either at home, via other teachers, or in communication with friends and siblings.

4.1.1.5.1 Multiplication Calculations

Collectively, Genevieve’s grade 3 students scored an average of 4% on their multiplication drill, or 2.6 correct responses out of 60 total questions, prior to the Intervention (Appendix I – Sample Student Calculation Drill). Student responses ranged from 0 out of 60 possible correct answers to 11 correct responses.

On the post-drill after the Intervention, Genevieve’s grade 3 students scored an average of 13.34%, with a range of three correct responses to 47 correct responses out of 60 possible points (Figure 5: Genevieve’s Student Results). The assessment was developed to measure student performance at the grade 3 level and the questions were based on the Ontario Ministry of Education’s Grade 3 Mathematics curriculum expectations. Every 10% increase in scores is equivalent to one grade. The average class increase of 13%, represents
an entire grade level, over the course of the intervention. The results from one student were excluded from these findings because the student joined after the pre-multiplication assessment drill of the intervention. This student’s performance, based on final results, was significantly higher than that of his peers. The inclusion of his data in the study would positively skew or bias the results. Furthermore, without access to a measure of his pre-performance, it is impossible to measure his growth throughout his time in the intervention.

4.1.1.5.2 Multiplication Word Problem Solving Results

On average, Genevieve’s students scored a reported Level 2/C letter grade on their initial Multiplication Word Problem Assessment that she created and interpreted, or 5.1 points out of a possible 12 according to her interpretation of the students’ results (Figure 5: Genevieve’s Student Results). Responses ranged from zero points to eight points out of 12, or a Level R to a Level 3/B letter grade.

On the post Multiplication Word Problem Assessment, the class scored a reported average of 7.4 points out of 12, or a Level 3-/B- letter grade (Figure 5: Genevieve’s Student Results). These responses ranged from three points to 11 points out of 12, or a Level 1+/D+ letter grade to a Level 4/A letter grade level. The Multiplication Word Problem Assessment results show an average increase of 2.3 points, or the equivalent of two steps on the 12-point scale (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart).

It is important to note that the pre- and post-assessment word problems were designed by Genevieve and Siryna and that the pre- and post-assessments were very similar but not identical. They required the students to select an appropriate strategy and solve for a 7x7 equation.
4.1.1.7 EQAO

Genevieve’s students scored an overall average of 2.33 out of a possible score of 4 on the Grade 3 EQAO Mathematics Assessment (EQAO, 2017). All 12 of her grade 3 students participated in the week-long assessment that was administered in her classroom in May 2017. Students’ scores in mathematics on this assessment ranged from an overall Level 1 to Level 4.

4.1.2 Case Study of Mary

Mary joined Norden Junior Public School at the beginning of the school year. At the start of the study, Mary was beginning her tenth year of teaching, five of which had been in the public school system, in the elementary panel. At the time of the study, Mary had taught primary mathematics for less than five years.
Before teaching in the public school system, Mary spent two years teaching in a private setting. She shared that this had helped her learn and grow as a teacher in general but acknowledged that it was a very different environment from her current setting.

Within the public school board, Mary had most recently taught in Special Education classes where she often did not teach mathematics on a regular basis. Most of time, she did not teach mathematics at all. Prior to this, Mary was employed as an occasional teacher where “most of my math teaching, for about five years, was done as a supply teacher” (Interview, February 24). She explained that she felt this was different from being a permanent classroom teacher for various reasons.

Mary earned a Bachelor of Science degree in Sri Lanka. After taking a break from her studies to raise her children, Mary earned her Bachelor of Professional studies in the United States of America, which also gave her certification to teach in the elementary panel. Mary’s entire post-secondary education included very little formal mathematics. Although she was required to take a research methods course in pursuit of her Science degree, Mary did not believe that this qualified as a mathematics course. Aside from this, Mary has taken no formal mathematics courses at the post-secondary level, nor has she completed any additional courses focused in Mathematics, since becoming a teacher. Mary taught a combined Grades 3/4 class during the year of the study.

4.1.2.1 Teacher Attitude

Before the study, Mary stated that mathematics was a subject area she enjoyed from both a teaching and learning perspective:

I really like it. I have started recently only but I really enjoy teaching math. Maybe because I like math, that’s probably why. When I see students really grasps the concepts of it, it makes me do more teaching…and so far, I think I am enjoying it and I am finding ways to understand it and enjoy it, too. (Interview, February 24)
Although she enjoys mathematics and feels it is a subject area in which she is comfortable, Mary also shared that many frustrations and challenges in teaching the subject area remain. Outside of this unit, Mary admitted that she continues to plan her mathematics lessons from one day to the next, and that they do not garner the same results as were shown during the intervention (interview, Feb 24).

Upon completion of the study, Mary shared that learning and implementing the intervention had had a positive impact on her: “I actually enjoyed [this intervention]. I felt that I accomplished something, especially in this particular unit. It was so satisfying. It made me feel good” (interview, June 16).

According to her initial attitudinal survey, Mary scored a 3.2 out of a total score of 4 (Appendix J – Teacher Survey Sample). After her participation in the study, Mary’s post-attitudinal survey score increased to a 3.8 out of a possible 4, an overall increase of 0.6 points.

Although Mary’s overall attitude towards mathematics has improved, her attitudinal survey indicates that the largest improvement took place with regards to the amount of time she spent on her planning. In her pre-attitudinal survey, Mary rated the question I spend a lot of time planning for mathematics classes as a two, whereas in her post-survey, she rated this as a four. This is consistent with observations made prior to, during, and towards the end of the unit. Initially, Mary shared that her planning took place day-to-day and was influenced heavily by the Math Makes Sense textbook that she would use to guide each mathematics unit. Although her initial plans for the unit involved in the intervention initially relied heavily on questions from this textbook, over time, Mary’s lessons began to shift away from
those textbook questions. Instead, she began seeking out different resources and creating or designing some of her own questions for the students to complete.

Mary’s survey responses also indicate an improvement to the questions *I like teaching mathematics* and *I feel comfortable teaching mathematics* following the completion of the study. Mary rated the questions *I like mathematics* and *I feel mathematics is important* as a perfect four in both the pre- and post-attitudinal surveys. Interestingly, Mary’s comfort level teaching mathematics appeared to have increased throughout the study. However, because she provided a perfect score in the initial survey, it was impossible to reflect any improvement from the survey results.

It is possible that Mary’s increased content knowledge in mathematics influenced her comfort level in teaching the subject. It is also possible that her limited knowledge at the beginning of the study impacted the initial rating of her comfort level in teaching mathematics. It is likely that Mary was unaware of many missed opportunities to engage students or that she lacked the content and pedagogical knowledge of different teaching approaches, leading her to feel naively comfortable in teaching mathematics at that time.

By the end of the unit, Mary was regularly observed circulating the classroom, crouching down at students’ desks, asking encouraging questions, and providing positive feedback around processes and progress. This hands-on approach led to her spending less time on teacher talk. This is significantly different from Mary’s initial approach to teaching mathematics, where she sat at her desk or in a chair following her laboured teacher-directed lesson, and then instructing students to show her their work so she could indicate if it was correct or incorrect. Now, Mary’s body language and verbal expression indicate a positive change in her attitude towards teaching mathematics.
4.1.2.2 Teacher Knowledge

Mary’s overall knowledge for teaching score in mathematics was 44.7%. For the purposes of this study, her score was classified in the Limited Knowledge Range (Figure 9: Teacher Knowledge for Teaching Compared to Student Performance). Mary saw growth in regards to her knowledge for teaching over the course of the training and intervention, improving from 42.9% to 47.1%. This is likely a result of her active participation and completion of content knowledge tasks during the professional development sessions, as well as through her own research when planning and implementing the intervention.

Mary demonstrated pedagogical knowledge of teaching mathematics by stating that she seeks out a variety of different resources to support her teaching in the classroom:

Mostly I prepare on a daily basis. I start with what we did the previous day and then look for other resources to help me plan the next lesson until we are done. Sometimes I will find resources online or use the textbook and sometimes I will ask my colleagues for ideas. (Interview, February 24)

Mary also provided some insights around the challenges in teaching and how these relate to her pedagogical knowledge of teaching mathematics:

Though I prepare at the beginning of the lesson, as I teach, I end up changing my lesson from what I notice from the student needs. I would say I prepare on a daily basis, sometimes I even prepare then and there as I teach. So, it is an on-going thing for me. I enthusiastically prepare every day. But sometimes, as I am teaching, I realize that none of them get it. That is my big challenge or hurdle. Mostly, in my room especially, the behaviour interrupts [our lesson] as well. Then I need to simplify my lesson for them, which sometimes prolongs the unit. Then some of the students who really get the idea get frustrated because they get the idea already. But some of the students are not ready to move on. I find that is a challenge because I need to finish the unit but some of them are ready but some of them are lingering on. (Interview, February 24)

Mary seems unclear of definitive starting and stopping points in the units she taught. She is also unsure of how to support all types of learners achieve her pre-determined goals, which were established as guided by the Ontario curriculum.
Mary also shared that she feels weak in her ability to assess student learning in mathematics. She attributed this to the types of tasks she selects for students:

The thing that I would like to add is the assessment. Which is something that I am not sure of either, because I do not rely on paper and pencil that much, [instead] I go on a daily assessment. So, what I think, [and] this is from my past experience, is that students really get agitated when they are being told they are going to be tested on something. Rather to measure their actual knowledge, I think it is better to see when they are functioning in a normal environment. I find out that when I say there is going to be a test, they are under a lot of pressure. I basically do not depend on paper and pencil that much. But then again, I myself am not sure if that, whether I should be doing that and then sometimes I am not certain if they understand the concept or not [when it is not a paper and pencil activity]. (Interview, February 24)

At the end of the unit, Mary continued to face challenges around implementing CGI in her classroom, time management, and assessing student knowledge:

[The students] still struggle to understand. Not everybody, though, so I felt like ‘ok, am I giving enough to the kids who already got it? Am I giving enough time to the students who really need to learn and understand the concept?’ That was the biggest struggle I had, I always felt confused and was not sure if I was giving enough to everyone and their understanding. I mean, I knew whether they understood or not but did I provide enough that they could gain from the unit. (Interview, June 16)

After completing the unit, Mary was able to somewhat accurately describe the Backward Design model and its process. However, she was unable to articulate any of the ideas or principles about CGI:

The Backward Design model is easy when you really get it. You start with the end and then just plan how to get there. I really liked it and it was so helpful because I had everything all ready [at the beginning of the unit]. CGI - oh wow, I still do not know or I am having a mental block! (Interview, June 16)

Mary’s pedagogical knowledge appeared to have improved following the completion of the unit. While Mary initially used many paper-and-pencil tasks, activities, and assessments, at the end of the unit, different tasks with different approaches were seen to be implemented more regularly. Students were observed employing various tools to capture and record their ideas, such as computers and iPads, instead of constantly drawing on
written responses to questions. However, Mary’s approach to assessment remained the same. She continued to use paper-and-pencil assessment tasks and allowed for little to no differentiation in the approach to the task or assessment of the task, despite having varied learning needs amongst her various students.

**4.1.2.3 Lesson Planning and Implementation**

Prior to the unit, Mary had no formal written plans and would often organize her lessons day-by-day. She stated that this allowed her more opportunity to respond to student needs, enabling her to spend more time on concepts students were struggling with and move onto new ones when students were ready.

Prior to completing the intervention, Mary had some knowledge of the Backward Design Model. However, she had no knowledge regarding CGI. Mary shared that her interpretation of the Backward Design model was that it was about starting with an end result, then going back to determine a path to get there.

During the professional development sessions, Mary appeared engaged and would share ideas when called on. She readily joined in with the group when completing mathematical questions but required considerable guidance from me or her colleagues when she was working on her own lesson plans.

When questioned about her unit plans and ideas, Mary often stated that it was a lot to take in and that she would think about it more at a later time. I asked Mary lots of questions to probe her thinking. I also provided suggestions on the different types of word problems Mary might use in her classroom to support the learning needs of her students. However, Mary felt more comfortable using the word problems provided in the textbook she was using.
After attending all of the professional development sessions offered around the Backward Design model and CGI, Mary showed some improvement in her ability to articulate and use the Backward Design model. Conversely, she still required support in articulating and adhering to CGI practices. She continued to work on planning her unit template.

Mary used the Backward Design model to plan her unit shortly before its initiation. Her overall goal was for students to “use a variety of strategies to solve a problem involving multiplication of one-digit numbers”. She included the pre- and post-assessment to measure students’ multiplication calculations in her unit plans. However, she only included one-word problem intended for use in assessment of student learning at the end of the unit. Instead of selecting evidence tasks or samples of tasks that would be used to measure student learning throughout the unit and connect to the overall desired goal, Mary preferred to use anecdotal observations to determine student understanding.

Mary organized her unit into three sections but did not explain how they were connected to the overall goal or big idea: (i) multiplication procedures; (ii) place value and properties of operations and how to use them to solve problems; (iii) the relationship between multiplication and division. She did not provide any tasks or activities designated for each section. She also did not make clear how she planned to communicate to her students the overall goal, or the connection of the selected sub-areas to the overall goal. Thus, Mary attempted to effectively use the Backward Design model. She selected the overall goal or desired result, generally one where students would “use a variety of strategies to solve a problem involving multiplication of one-digit numbers”. She did not determine appropriate evidence apart from a final word problem task and the pre- and post-assessment.
for multiplication calculations. She also did not include in her plans the learning experiences and instruction that would support students in achieving the desired goal.

Mary’s plans were, however, easy to follow and clearly discussed the intended end results. As no problems were described or outlined in the plans, it is impossible to determine how Mary intended to infuse CGI throughout the unit. When questioned, she referred to using word problems throughout the unit. However, she did not provide these to me prior to beginning the unit or before any lesson. Furthermore, many of the forms of evidence that were included on her unit template referred to paper-and-pencil tests, desk work, quizzes, anecdotal observations, or reference to a traditional learning model.

Many of the activities Mary used throughout her unit appeared to be quite similar to those she had used prior to the intervention. This is also true for the structure of her lessons. Mary would often start with a teacher-directed lesson. She would explain an idea, provide one or two examples, and then ask the students to go to their desks to try to replicate her examples. Meanwhile, she circulated, providing support in the form of indicating correct or incorrect answers.

Towards the end of the unit, Mary appeared more willing to try new ideas and activities. She was introduced to “Parallel Tasks” (Small, 2009), which asks the teacher to create two questions or problems that require the exact same skill but with varying difficulty, so that the student can then select the question he or she feels is most appealing. Although initially reluctant, Mary was willing to try this approach and noted that her students did in fact respond more positively. This led to a small change in the activities Mary used throughout her unit and allowed Mary to reconsider the structure of her lessons.
In the last few lessons of the unit, Mary agreed to try starting her lesson by giving her students a quick problem that introduced the skill or strategy she wanted them to learn. This was met with some success and Mary was willing to try using this strategy again, but perhaps not for every lesson.

Although there appeared to be some change in Mary’s teaching performance, moving from a traditional approach to a more student-focused approach, most of the activities were paper-and-pencil based. Few manipulatives were provided outside of anchor charts or explanatory lists and students were regularly assessed via writing activities. Additionally, Mary used very few word problems throughout her unit, aside from the initial and final word problem used to assess the students.

4.1.2.4 Description of the Class

Mary taught a combined grade 3/4 class with a total of 24 students. Although all of the students completed the tasks, activities, assessments, and questions that Mary selected or designed, only the grade 3 students were tracked and included for the purposes of this study. Of the total number of students, 13 were enrolled in grade 3. Eight of these were male and five were female. No new students joined Mary’s class during the implementation of the intervention, but it must be noted that one of the Grade 3 female students moved to a Special Education program within the building during the final week. Both Mary and the Special Education teacher felt it would be in the student’s best interest to complete the mathematics unit with her original class and transition out of the regular classroom when the unit finished. Her data has been included in all of the findings.
Mary completed and submitted her unit plans to me four school days prior to the implementation. She did not include a timeline for the completion of the unit. Her unit lasted four weeks.

4.1.2.4.1 Class Performance

Some direct mathematics instruction around multiplication had taken place in the classroom prior to the beginning of the intervention. This was largely due to this being a combined grade 3/4 class, where the concept of multiplication had arisen while problem-solving in other mathematical areas for the grade 4 students. Additionally, two grade 3 male students reported that they had already begun working on their multiplication facts at home with their parents.

4.1.2.4.2 Multiplication Calculations

Prior to the intervention, Mary’s students scored an average of 14.4% correct answers on their multiplication drill, or 8.6 out of 60 possible correct responses (Appendix I – Sample Student Calculation Drill). Students’ initial correct responses ranged from 0% to 91.7%, or from 0 to 55 out of 60 correct responses.

At the completion of the intervention, Mary’s students scored an average of 27.7% correct answers on their multiplication drill, or 16.6 correct responses out of a possible 60 (Figure 6: Mary’s Student Results). The final assessment range was also large, ranging from a score of 1.7% to 100%, or from 1 to 60 correct responses out of a possible 60 questions.

Overall, Mary’s students improved their calculation accuracy on the multiplication drill by 13.3%, or eight correct responses, the equivalent of one full letter grade, during the course of the intervention (Figure 6: Mary’s Student Results).
4.1.2.4.3 Multiplication Word Problem Solving Results

Mary’s class scored a reported average of a Level 2/C letter grade on the initial Multiplication Word Problem Assessment created and interpreted by Mary, or a score of 5.4 out of a possible 12 on the step scale (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores for this initial assessment ranged from a Level 1/D letter grade to a Level 3/B letter grade.

On the post Multiplication Word Problem Assessment created and interpreted by Mary, Mary’s class scored a reported average of a Level 2+/C+ letter grade, or a score of 6 points out of a possible 12 (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores for this assessment ranged from a Level 1/D letter grade to a Level 4/A letter grade.

Figure 6: Mary’s Student Results

<table>
<thead>
<tr>
<th>NAME</th>
<th>Pre X (/60)</th>
<th>Pre %</th>
<th>Post X (/60)</th>
<th>Post %</th>
<th>% Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>JP</td>
<td>11</td>
<td>18.3%</td>
<td>19</td>
<td>31.7%</td>
<td>13.3%</td>
</tr>
<tr>
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<td>28.3%</td>
<td>5.0%</td>
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<tr>
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<td>55</td>
<td>91.7%</td>
<td>60</td>
<td>100.0%</td>
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</tr>
<tr>
<td>NCC</td>
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<td>8.3%</td>
<td>24</td>
<td>40.0%</td>
<td>31.7%</td>
</tr>
<tr>
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<td>1.7%</td>
</tr>
<tr>
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<tr>
<td>CDH</td>
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<td>1.7%</td>
<td>7</td>
<td>11.7%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Avg.</td>
<td>8.6</td>
<td>14.4%</td>
<td>16.6</td>
<td>27.7%</td>
<td></td>
</tr>
</tbody>
</table>

Average Change: 13.3%
The Multiplication Word Problem Assessment results suggest a minimal improvement in the class average, increasing from a Level 2/C letter grade to a Level 2+/C+ letter grade overall, or a step increase of 0.56 points on a 12-point scale (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). It is important to note that the pre- and post-assessment word problems were selected and adapted by Mary with support from the researcher from a textbook in her classroom. Both the pre- and post-assessments were very similar but not identical. Both assessments required students to select an appropriate strategy and solve for a 7x7 equation.

4.1.2.6 EQAO

Mary’s students scored an overall average of 1.15 out of a possible score of 4 on the Grade 3 EQAO Mathematics Assessment (EQAO, 2017). All 13 of her grade 3 students participated in the week-long assessment that was administered in her classroom in May 2017. Students’ scores in mathematics on this assessment ranged from an overall Level 0 to Level 2.

4.2 Forest Elementary School

Forest Elementary School is an alternative elementary small-sized school, with classes ranging from Kindergarten to Grade 8. There were approximately 200 students total registered at the time of the study. Founded less than 10 years ago, Forest Elementary School was designed to respond to the lack of environmental and outdoor education currently offered in Ontario, using a holistic, student-centred and student-based approach (TDSB, 2017).

Housed within another local elementary school, Forest Elementary School is unique from many other public schools for two main reasons: first, enrollment in the school is done
via lottery; secondly, programming is based in outdoor education. In order to enroll in the 
school, parents must submit an application to have their child entered into a lottery. A 
limited number of spots are entered each year to fill vacancies in the upper grade levels and 
a set number of enrollments are made available for the outdoor kindergarten program.

The student population and community of Forest Elementary are not as diverse as 
many other public schools found in the public school board. However, Forest Elementary 
School is equally comprised of boys and girls. Although approximately 49% of its parent 
population has a high school diploma or less, over 20% of the parents earn in excess of 
$100,000 annually; over 50% of the parent population earns more than $60,000 annually; 
less than 10% of the population earns $15,000 or less annually.

Approximately two-thirds of the school population and community speak English, 
while a mere 2.5% of the population have a first language other than English. Less than 20% 
of the student population at Forest Elementary School are identified as Students with Special 
Education Needs. According to the most recently available EQAO data, 18% of the primary 
students at Forest who wrote the EQAO Assessment of Mathematics were identified as 
students with Special Education Needs (EQAO, 2017).

Forest Elementary School’s students’ attitudes and performance on EQAO have been 
inconsistent. This is primarily due to limited participation in the EQAO Assessment over the 
past five years. According to one of the participating teachers, a number of parents will opt 
out, refuse to participate, or withdraw their child during standardized testing. He believes 
that this is due to this specific community’s value system and feelings about large-scale 
assessment; standardized testing stands in contrast to the principles, morals, and values that 
characterize Forest Elementary School.
The teacher further shared evidence to support this claim in the form of an anonymous student’s past report card. There were two versions of this report card. One version was sent home; one was filed in the student’s Ontario Student Record folder. The two versions were almost identical, except that the one that went home was void of letter grades. The only commentary provided on this version was the teacher’s remarks regarding the student’s progress and next steps. The version that was filed was complete with subject letter grades. The teacher explained that the ungraded report card was representative of the culture of the school: Forest Elementary holds that marks are not supportive judgements to measure student growth and are therefore not generally shared with the students. Instead of letter grades, teachers provide constructive feedback on strengths and ideas for future improvement.

According to the EQAO Grade 3 Student Attitude Survey, Forest Elementary School’s students have demonstrated a mild improvement in their beliefs and attitudes towards mathematics (Appendix C - Grade 3 EQAO Attitudinal Trends – Forest Elementary School). Five years ago, in the 2011-2012 assessment year, 18% of students responded that they never like mathematics, while in the 2015-2016 assessment year, only 4% of students responded likewise (Appendix C - Grade 3 EQAO Attitudinal Trends – Forest Elementary School). The percentage of students who respond negatively to mathematics has decreased steadily over the last five years, for a total change of 14%.

The number of students who responded that they always like mathematics also increased during this time period. In the 2011-2012 assessment year, 18% of students responded that they always like mathematics, whereas 50% of students responded likewise in 2015-2016. This represents a 32% increase in students who report a positive attitude
towards mathematics (Appendix C - Grade 3 EQAO Attitudinal Trends – Forest Elementary School).

With regards to Forest Elementary School’s performance on the EQAO Assessment of Mathematics, it is challenging to report any trends prior to the intervention due to lack of available information. Although it appears that there has been a significant increase (20%) in the number of students who met or exceeded the provincial standard in mathematics, this value is not reliable because there is no data available from the 2012-2013 assessment year (due to the limited number of participants) and no available data for the 2014-2015 assessment year (due to work stoppages).

However, when you explore the ten-year trend of performance, there is a decreasing trajectory which begins in 2010-2011, where 67% of students met or exceeded the provincial standard; this number went down to 48% in the following assessments, fell again to only 31% in the next available assessment (Appendix D - Grade 3 EQAO School Math Achievement 5-Year Trend – Forest Elementary School).

4.2.1 Case Study of Siryna

Siryna has been teaching for four years. All of her experience has been as a Long-Term Occasional teacher in the public school board. She reported that she has been teaching elementary mathematics for three out of her four teaching years.

Siryna completed a Bachelor of Music degree prior to completing her Bachelor of Education degree. Siryna stated that she did not enrol in any mathematics courses during her post-secondary studies, although she did spend some time learning about teaching mathematics at the elementary level during her Bachelor of Education degree.
Since the completion of her post-secondary education, Siryna has enrolled in and successfully completed two teacher-directed courses in mathematics, aimed at improving her teaching practice in this area. These courses were designed and led by primary teachers who hold a particular interest in the area of mathematics. According to Siryna, both courses focused on how to teach and assess primary mathematics. Neither of these courses was recognized by the Ontario College of Teachers for further accreditation. Siryna taught a grade 3 class during the year of the study.

4.2.1.1 Teacher Attitude

Siryna disclosed that mathematics is a subject area that is “growing on her,” but that she finds it challenging both as a learner and as a teacher:

I feel like teaching math to a primary classroom can be quite challenging … in the primary level there are a lot of different achievement levels in the classroom. Also, it can be difficult because I have been an LTO; there is not always the same manipulatives or resources in the classroom. Sometimes we have do not have enough and have to share, like textbooks and stuff. Otherwise, it is probably my third favourite subject to teach because I personally did not like math, but now after going back to Teacher’s College and teaching it to younger kids, I am enjoying it more now. (Interview, January 12)

Siryna shared that she feels she has had a lack of training to teach mathematics, especially since she does not have a lot of experience in learning mathematics as a student:

Being in so many different classrooms and getting moved a lot or only being in a classroom for short, extended times, it is really challenging to get better. I mean, I try. I have taken some courses and they help, but I do not get enough time to practice what I have learned. Also, I feel like I know the math often but it is really hard to explain it when a student does not get it. (Interview, June 16)

Siryna was eager to share her ideas and thoughts during the intervention sessions she attended and was consistently willing to ask questions. During the initial problem solving activities, Siryna appeared comfortable talking to the other teachers in the study and asking for their ideas. She was comfortable making a mistake and could be seen laughing in
enjoyment when she saw other teachers’ solutions that appeared to use a better approach or a more efficient strategy to arrive at an answer. Siryna mentioned that she was interested in learning about mathematics on a number of occasions. She was also very expressive of her appreciation for the help she received from the other participants.

Upon completing the study, Siryna shared that she felt more confident in her ability to teach, plan and assess mathematics:

Generally, after having someone coming in to observe me and someone to consult with, I feel more confident. It was nice to have somebody check in and say ‘these are things you are doing really well so keep doing that.’ But also to have someone to say ‘maybe you should do this a little differently’ or ‘have you thought about this’? Just, sort of, pointing out things that could change for the better. (Interview, June 16)

According to her pre-attitudinal survey, Siryna scored a 3.2 out of a possible 4.0 score (Appendix – Teacher Survey Sample). Siryna indicated that mathematics is an “extremely important subject area,” but that she was somewhat uncomfortable with teaching it at times. Siryna’s attitude towards mathematics had improved by the end of the study, according to both her attitudinal survey and her post-interview. On her post-survey, Siryna scored a 3.8 out of a possible 4.0, an overall attitudinal increase of 0.6 points. Almost all of Siryna’s responses improved from her pre- and post-surveys, rising from three points to four points, with the exception of her response to the statement of I feel mathematics is important, which was rated a perfect four on both surveys.

Notably, neither Siryna’s pre- or post-attitudinal survey reflected a change in response to the question I feel comfortable teaching mathematics. Siryna rated this statement a three out of four on both the pre- and post-survey, despite having orally stated that her comfort-levels had increased after the intervention. This might be related to the limitations of the survey scale, or could be due to Siryna’s change in knowledge, as she may feel more
comfortable with the general teaching of mathematics but has also learned new ideas or concepts in some areas, which she is not altogether confident in as of yet. However, Siryna’s overall improvement does reflect a positive change in attitude toward mathematics, which is likely to influence her comfort-level in teaching this subject area.

4.2.1.2 Teacher Knowledge

Siryna scored an overall knowledge for teaching mathematics score of 56.2%, ranking in the Low Knowledge Range for the purposes of this study (Figure 9: Teacher Knowledge for Teaching Compared to Student Performance). Siryna demonstrated an increase in content knowledge of 7.4% from her initial score, arriving at a final score of 60%. This increased mathematical knowledge is likely a result of learning new concepts, as well as reviewing and increasing her understanding of other concepts during the professional development sessions. Another potential contributing factor to her increased knowledge score is the independent work she did while planning and delivering the intervention.

During one of the professional development sessions, the mathematical concept of the distributive property arose. At first, Siryna could not remember what this term meant, but was reminded when one of the other teachers wrote down an equation to explain it to her.

Example:

\[ 6 (2 + 4) = X \]

Siryna initially used her knowledge of BEDMAS to solve the problem, summing 2 + 4 in the brackets before multiplying that sum by 6 to arrive at an answer of 36. Although correct, the other teacher explained to her that you may not always know the value within the brackets or that you may not be able to simplify the brackets to a numerical value. He then showed
her how to execute the distributive property calculation, which Siryna nodded at and said she somewhat remembered doing in her studies long ago.

Most interestingly, the other teacher then demonstrated how this works in the example above by multiplying 6 by each value in the bracket and then adding them together. Following along, Siryna commented that she had never understood why the distributive property worked, and only did it because she was instructed to do so. Siryna gained a new and deeper mathematical understanding that likely contributed to her mathematical content knowledge increase.

Siryna demonstrated a high level of pedagogical knowledge about teaching mathematics before the intervention. She was familiar with how mathematics is organized in the curriculum documents; she shared a number of different strategies on how to acquire appropriate tasks, questions, and activities; and she demonstrated a working knowledge around students’ varying needs in this area:

I prepare for math by strand, not that there is not cross-strand teaching…I start by reading the [curriculum] and then I decide which one will be my [focus]. Then I decide the order ideally, but sometimes that changes depending on the class, like their previous knowledge or that one concept takes longer than I thought it would. Then I use textbooks to look for ideas or activities. I also prepare by using YouTube or Khan Academy. I also have a textbook about Teaching Mathematics in the Elementary School. (Interview, January 12)

Later, she said:

Sometimes, figuring out student evaluations or assessments, and trying to find their logic and reasoning [is difficult]. I felt less confident with that but I think because I talked to the students a lot and got them to explain it, I actually think that is something I am good at, but it can still be a challenge. (Interview, June 16)

By the end of the unit, Siryna’s teaching practice had changed in not only in the area of mathematics, but in other curriculum areas as well. Siryna found it useful to integrate mathematical language into other curriculum areas. For example, it was observed that Siryna
started to include specific mathematical language on her Word Wall in the classroom. This demonstrates a positive change in both Siryna’s perception of mathematics and her approach and knowledge around the subject.

Another way Siryna integrated mathematics into other subject areas was through discussion and language. Although the concepts were not related to this unit, Siryna infused conversations about area and perimeter into her dialogue and classroom activities. Specifically, when Siryna would ask her students to come sit on the carpet, she would indicate if she preferred them to sit on the area of the carpet or perimeter. When this observation was brought to her attention, Siryna stated:

Oh yeah, I started using some of the math words as instructions because I am worried that the students might forget what they mean and EQAO is coming up. Also, these are things that they need to know anyways and it does not take me any more time to tell [the students] to come to the edge of the carpet area or perimeter. In fact, when I first started doing it, it was by accident but the students thought it was fun, kind of like a game! (Class observation, May 17)

Both of these examples demonstrate an increase in Siryna’s knowledge about teaching mathematics, which is likely related to an increase in her mathematical content knowledge as well.

4.2.1.3 Lesson Planning and Implementation

Siryna joined the study later than the other participants as the initial teacher in the classroom took an unexpected medical leave. She was provided the same information as the other participants in regards to expectations and ability to remove herself and classroom from the study. Due to this delayed start, Siryna did not attend all of the professional development sessions with the other teachers. Instead, she joined during session four, when discussions about CGI began, and missed the group professional development sessions on the Backward Design model.
To provide Siryna with the missing information from the Backward Design model sessions, I conducted a 90-minute session for her after school. During this time, we reviewed the Backward Design model and went through the presentation, as had been done with the larger group. However, since she was the only individual in attendance, there was little opportunity for her to learn from other people’s questions, ideas, and discussions. I attempted to compensate for this by sharing some of the insights gained in previous sessions. Before she joined, Siryna was also aware that the group had collectively decided to plan their unit based in multiplication.

Prior to completing the professional development sessions, Siryna was unaware of the Backward Design model or CGI. After completing the sessions, Siryna appeared enthusiastic about both parts of the intervention, stating that the Backward Design model made ”so much more sense,” and uniquely comparing CGI to her knowledge and understanding of differentiated learning and instruction.

Siryna used the Backward Design model to plan her entire unit prior to its initiation. Her overall goal for students was to “demonstrate an understanding of multiplication and to use it to solve problems from real-life situations” Siryna included the pre- and post-assessment on multiplication in her unit plans and attached her pre- and post-assessment for word problems. Siryna organized her unit into five different sub-areas based on multiplication solving strategies: repeated addition; using known number facts; partitioning numbers; reversing equations; arrays. Each of these sub-areas was taught individually and linked directly to her overall goal. To support this in the classroom, Siryna wrote and posted the goal so that students could see it at all times. During the introduction of a new strategy,
she would refer to this overall goal and explain how students could use this information to understand and solve multiplication problems.

Siryna used the Backward Design model effectively. She selected an overall goal or desire first: students would demonstrate an understanding of multiplication and use it to solve problems from real-life situations. Next, she determined appropriate evidence to be collected throughout the unit. She selected several word problems that would measure student learning and understanding and set up conferences where students could share their knowledge and complete mini quizzes. By selecting sub-areas that focused on different multiplication-solving strategies, Siryna succeeded in offering thoughtful learning experiences tailored to her specific students in order to support them in achieving the desired goal.

In her unit plans, Siryna infused CGI principles and made numerous connections between new concepts and previously-learned ideas (Appendix G – Backward Design model Template). For example, Siryna wrote under student understandings that “students will know how to use repeated addition [because] addition is related to multiplication” (Siryna Unit Plan). Siryna’s template is clear and its goals are easy to understand. She highlights precisely what students will learn and what they will accomplish by the completion of her unit. Siryna did not include her initial and final word problem within her template, but instead attached them to the template. This was done because she wanted to discuss the formatting of the problem to ensure it was also student-friendly before committing to it.

Many of the activities and much of the assessment evidence in Siryna’s plans were flexible. She provides lots of different choices, and many of her activities allow students to demonstrate their knowledge and understanding in a way that does not require writing, if the
student so desires. Siryna also makes reference to other mathematical teaching methods that she learned in Teacher’s College regarding differentiated instruction that support CGI learning, such as parallel tasks and student-created problems.

Because Siryna joined the study later than the other participants, the amount of time spent observing her prior to the professional development sessions was limited. Thus, it is difficult to compare the observations made throughout the unit to her prior approach to teaching. However, based on our conversations, her interviews, and her planning notes, a few ideas are available.

Siryna previously did not complete a lot of formal planning. She admitted to this and shared that, as a new teacher, she sometimes felt overwhelmed by all of the paperwork. She tended to plan her mathematics lessons day-to-day, using the curriculum as a guide. Siryna originally stated that she was unfamiliar with the Backward Design model but, after learning about it, embraced it enthusiastically in its entirety: “I have even started using this template to plan a Science unit!” (Interview, January 12)

Siryna was unable to describe or define what CGI was prior to the professional development session. However, upon completion of the sessions, she was able to connect her knowledge about CGI to differentiated instruction and to discuss many different strategies and activities she hoped to try throughout her unit.

During the implementation of her unit, Siryna used a variety of different activities that varied from the traditional teaching method. She did so in a manner that appeared completely natural. In almost all lessons, students were provided a choice in either the problem they wanted to solve, the tools they wanted to use to solve it, or the peer they wanted to work with.
Although she appeared familiar with implementing various tasks and tools in her mathematics teaching practice, Siryna was able to improve her ability to monitor students individually throughout the unit by including new assessment methods and increasing the frequency of their use. Instead of waiting for a formal opportunity to assess the students, as she had previously done, Siryna would circulate throughout the classroom and pose questions to clarify student thinking. Siryna used her iPhone to document many of these observations, in an effort to help herself remember what had happened, since she did most of her work and planning electronically.

Siryna would also use games as means of assessing student knowledge and understanding. Throughout the unit, Siryna played a game of Around The World, where students would compete one-on-one to answer a single-digit multiplication question as quickly as possible. If they were first to answer and correct, they would continue on in the competition. Otherwise, they would be asked to sit down and either observe or act as a judge. When questioned how this game was used to learn about students, Siryna explained that she could pre-determine which students would initially be paired, and she could track correct and incorrect answers Siryna also indicated that she found it more useful to find out who got the answers wrong so that she could set aside time to explore this issue with them privately or in a small group. She also insisted on sharing that the class celebrated students’ efforts and that they would only give awards to students who had tried their best or who demonstrated improvement in the game over time.

Later in the intervention, Siryna could be seen more comfortably responding to student questions and ideas. She also veered away from always providing an answer, instead referring students to seek out another student or asking a specific question aimed at keeping
the asker on-track. This is different from earlier in the unit when Siryna would often jump in quickly to model a solution or point out an error immediately. By the end, Siryna had taken to questioning all students’ solutions regardless of correctness and would seldom point out an error unless the student had been given time to review or it had been overlooked by a peer.

4.2.1.4 Description of the Class

Siryna taught a grade 3 class of 19 students. Nine of these students were male and ten female. Two of the students were listed as having exceptional needs and received Special Education support almost daily. One new student joined the classroom a week into the intervention. Siryna chose to catch this new student up on the unit, so his data is included in these findings. Finally, one student arrived shortly before the beginning of the intervention and had limited English skills. Because this student did complete all of the work, his information was included in the study.

Although Siryna joined the study later than the other participants, she completed her plans over six weeks in advance of starting the unit. Siryna’s timeline stated that her unit would last for five full school weeks; the duration of the unit turned out to be six weeks.

4.2.1.4.1 Class Performance

Some direct mathematics instruction around multiplication had taken place in the classroom, prior to the beginning of the intervention. This instruction was led by Siryna. Additionally, one male grade 3 student reported that he had been enrolled in a mathematics after-school tutoring program and was familiar with some of the multiplication facts.
4.2.1.4.2 Multiplication Calculations

Prior to the intervention, Siryna’s class scored an average of 20% on the multiplication drills, or 12 correct answers out of a possible 60 (Appendix I – Sample Student Calculation Drill). The students’ initial correct responses ranged from 0% to 88.3%, or from zero to 53 correct responses. At the completion of the intervention, Siryna’s students scored an average of 26% on the multiplication drill, or 15.6 correct responses out of 60 possible questions (Figure 7: Siryna’s Student Results). The post-multiplication drill assessment had a range of 1.7% to 100%, or one to 60 correct responses.

Overall, Siryna’s class showed an average improvement in their multiplication calculations of 6%, or ten additional correct responses out of a possible 60 (the equivalent of half a letter grade) over the course of the unit (Figure 7: Siryna’s Student Results).
Figure 7: Siryna’s Student Results

<table>
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<th>NAME</th>
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**Average Change**

5.90%

4.2.1.4.3 Multiplication Word Problem Solving Results

Siryna’s class scored a reported average of a Level 2/C letter grade on the initial Multiplication Word Problem Assessment, created and interpreted by Siryna, or a score of 5.4 out of a possible 12 on the step scale (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores on this assessment ranged from a Level 0/R letter grade to a Level 3+/B+ letter grade.

On the post Multiplication Word Problem Assessment, also created and interpreted by Siryna, Siryna’s class scored a reported average of a Level 3/B letter grade, or a score of
7.6 points out of a possible 12 (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores for this assessment ranged from a Level 0/R letter grade to a Level 4/A letter grade.

The Multiplication Word Problem Assessment results suggest a significant improvement in the reported class average, increasing from a Level 2/C letter grade to a Level 3/B letter grade overall, or a step increase of 2.2 points on a 12-point scale or the equivalent of almost one full letter grade improvement (Figure 4:12-Point Conversion Scale for Levels from Achievement Chart. It is important to note that the pre- and post-assessment word problems were designed by Genevieve and Siryna and that the pre- and post-assessments were very similar but not identical. They required the students to select an appropriate strategy and solve for a 7x7 equation.

4.2.1.5 EQAO

Siryna’s students scored an overall average of 2.47 out of a possible score of 4 on the Grade 3 EQAO Mathematics Assessment (EQAO, 2017). All 19 of her grade 3 students participated in the week-long assessment that was administered in her classroom in May 2017. Students’ scores in mathematics on this assessment ranged from an overall Level 2 to Level 3.

4.2.2 Case Study of Calvin

Calvin has been teaching at Forest Elementary School for two years and is currently in the fourteenth year of his teaching career. Calvin’s full teaching career has been spent in the elementary panel of the public school board, where he has taught grades 1 through 6 in three different schools. Of his fourteen years of teaching, only four were spent in the
Primary grades. Furthermore, Calvin has been the core mathematics teacher to his homeroom for eleven years of his fourteen-year career.

Prior to completing his teaching degree, Calvin co-completed an undergraduate and graduate degree in the Arts. During these degrees, Calvin did not complete any mathematics courses, with the exception of a research methods course for his graduate work. Since becoming a teacher, Calvin has completed several additional qualification courses, which have certified him to teach in all divisions in the public school board. None of these courses, however, were related to the field of mathematics.

4.2.2.1 Teacher Attitude

Calvin demonstrated an overall positive and eager attitude towards teaching and learning mathematics prior to starting the study:

I generally feel good about teaching mathematics. It is interesting, for one thing, and continues to be interesting fourteen years on and obviously there is a lot of change on how people have taught math in recent years. It is a lot of fun to think about that and how to engage [the students]. (Interview, December 19)

After planning and implementing the intervention, Calvin stated that, “it felt really good” (Interview, June 19). He felt his students had shown a lot of improvement, and that he himself had learned some different and useful approaches and activities for mathematics:

I would definitely use the planning for this unit again as well as the homogenous pairings. My learning from this is a product of both working [in this study] as well as the [school initiative], but yes, one hundred percent it’s been very valuable, I think. (Interview, June 19)

According to his pre-attitudinal survey, Calvin scored 3.6 out of a possible 4.0 score (Appendix J – Teacher Survey Sample). By the study completion date, Calvin’s attitude towards teaching mathematics had improved, according to both his attitudinal survey and his
post-interview. On his post-survey, Calvin scored a 4.0 out of a possible 4.0, an overall attitudinal increase of 0.4 (Appendix J – Teacher Survey Sample).

4.2.2.2 Teacher Knowledge

Calvin achieved an overall knowledge for teaching mathematics score of 83.3% and ranked in the High Knowledge Range for the purposes of this study (Figure 9: Teacher Knowledge for Teaching Compared to Student Performance). Calvin demonstrated an increase in knowledge for teaching mathematics of 9.7% from his initial score of 78.3%, resulting in a final score of 88%.

This increased mathematical knowledge for teaching is likely a result of Calvin’s participation in this initiative, as well as other mathematics initiatives throughout the year, both inside and outside of school. Another potential contributing factor to Calvin’s increased mathematical knowledge score is the independent work he completed during his preparation for and delivery of this unit. Calvin was specifically asked to complete some of the mathematics questions himself to practice arriving at a solution using a variety of techniques.

Calvin demonstrated a high level of mathematics content knowledge as well as pedagogical knowledge prior to and throughout the intervention. He was already aware of the concept of students with exceptional needs in general and had strategies on how to address these needs. He stated his thoughts as such:

It is important to have questions that kids can access easily and hopefully have some differentiation [in the teaching practice]. Delivering a math program that meets the needs and extreme ends of the class, both the low end and high end, are imperative … teaching math in a way, you know, the real world connections are not just isolated to math class so it is not that bad in terms of finding a question that has some meaningful context to kids, but maybe try to find a way to connect that meaningful context to other areas of the curriculum. (Interview, December 19)
Calvin was familiar with planning for mathematics, as well as how to develop a program that is student-centered or student-focused:

Preparation [for mathematics] needs to be rooted in the curriculum…I do not use a single program so I try to draw on different resources and try to get specific curriculum expectations boiled down to sort of one or two questions. (Interview, December 19)

Calvin indicated that his knowledge and understanding of learning and teaching mathematics have changed over the course of his teaching career, which, in turn, has affected how he teaches math:

Being rooted in the curriculum documents is different than the way people taught math, or at least the way I experienced teaching math for myself and my colleagues 10 years ago. [Back then] people were using a single textbook program. But now definitely using the curriculum document is essential, especially when teaching a grade knowing that the standardized assessment exists… One of the challenges of teaching in today’s primary classroom, among many, include [time management, language and ] ensuring that some of those basic numeracy skills do not get short-tripped or our priorities are somewhere else. (Interview, December 19)

Over the course of the study and school year, Calvin indicated a change in his classroom organization, structure, and assessment in mathematics. He attributed this to the research study and to his interest in learning about mathematics, as well as acknowledging the influence of another initiative he was involved with:

I feel differently about teaching mathematics … It has been interesting this year, but I for sure feel differently about it. I am really intrigued by this idea of homogenous pairings and using diagnostic assessments … [this intervention] has been super productive for this class. Finding themselves in a homogenous pair during problem solving activities, providing, I hope anyway, with their homogenous pair an access point to the subject matter we are working through. (Interview, June 19)

Calvin shared how his knowledge about planning and organizing mathematics in a Primary classroom has changed, noting that he had acquired some new knowledge during the study:

I am sort of intrigued because I know it is not what I expected…[homogenous groupings have] been so successful for the kids in this room, but you know as people say in math, a variety of different pairings would be beneficial for kids, but I am sort
of a doubter of that right now. I would never have done it before but it really worked. I am sort of intrigued by that. For me, the biggest learning has been that putting two lower kids together is more productive than putting a low kid with a high kid, for the high kid. Taking the high kid is almost granted either way [that they will learn] but if you get that high kid with another high one, they are able to push each other, especially if you pay some attention to not just mathematical ability but the type of learner. You get a couple of these kids together and they are doing some pretty cool and Forest things. (Interview, June 19)

After completion of the intervention, Calvin was successfully able to describe the Backward Design model:

Start with the end result that you want, whether it is a curriculum expectation or learning goal, and then work backward from there to take apart the steps that need to happen to get the students to that point. You know, teaching with the end point in mind. (Interview, June 19)

When asked, Calvin was unable to define the concept of CGI. However, he consistently referred to this approach in his planning, activities, and discussions around his teaching goals in mathematics.

4.2.2.3 Lesson Planning and Implementation

Prior to completing the professional development, Calvin had some working knowledge regarding the Backward Design model and limited knowledge about CGI. He was able to describe the Backward Design model and shared how it could be used for planning. Although initially he was unable to discuss CGI, after it was briefly introduced, Calvin noted that it was a method based on developing individual student knowledge. He made further connections to other teaching practices that he felt were similar. After attending all of the professional development sessions, Calvin was able to clearly articulate the Backward Design model as well as highlight some of the foundational principles of CGI.

During the professional development sessions, Calvin was reserved and quiet. He initially only spoke when asked a question. He would consistently provide a response when
asked but did not initially volunteer his own ideas. Towards the fourth session, Calvin began to join in the conversation more often and shared his ideas without prompt. He asked insightful questions that were on-topic and encouraged the group to think about many different situations that might arise throughout their units.

In his plans, Calvin was very brief and concise (Appendix G – Backward Design model Template). He highlighted the end goal for his students: that they will recognize multiplication as the combining of equal groups. He also provided a synopsis of the different activities he would use to support students in achieving this understanding: mathematics journals; word problems, both independent and in small groups; games or oral language activities. He also included a short summary of the various forms of evidence he would use to measure students’ understanding: mathematics journals; self and peer assessments; word problem assessments; student-teacher conferencing; parallel tasks; tests. Although Calvin’s plans were limited in detail, they included the overall desired goal, the evidence he would collect to measure student learning/understanding, and a basic description of the activities and learning experiences he would implement to support the students in achieving the overall goal.

Calvin preferred to share his vision of the unit during our meetings instead of including all of the details in his unit plan. Calvin discussed how he would use the pre-assessments, the standardized multiplication drill, and the two initial word problems he was using and to create a global score to group student homogenously. As Calvin was keen on improving his teaching practice in mathematics, he was simultaneously exploring how different groupings can be used to support student learning. He was also using this unit to learn more about this first-hand. Calvin then organized his students into groups based on
their similar abilities, knowledge, and understanding; these groups were maintained throughout the unit.

During the implementation of his unit, Calvin employed a variety of teaching techniques and strategies. He also provided a numerous unique learning activities. These activities varied from the traditional work sheet and oral drills to word problems developed in context with student lives and experiences. Calvin used a few minutes of almost every class to talk about computational skills, although this was never a main focus. Most of Calvin’s lessons started with a question or problem, which the students were asked to solve either independently or in groups.

Although no instruction was provided, students selected a variety of different ways to show their knowledge. Some students would quickly seek out the iPad, using tailored programs to show their work, while others would create models on the carpet out of blocks; some chose to write on paper or in their mathematics journals. This was no different in the pre-observation of Calvin’s class, where many students would seek out different tools to solve the problem.

After speaking with other teachers in the study, Calvin shared that he was concerned about meeting the needs of some of the lower-performing groups. He independently sought out support, where he learned about Parallel Tasks, which he implemented halfway through his unit. Instead of providing all students with exactly the same question, Calvin began to create questions that varied in difficulty but covered the same mathematical concept, known as Parallel Tasks (Small, 2005). Students were provided with two or three choices in the word problem that they wanted to solve; all word problems were related to multiplication and the context was often very similar, if not identical.
Students who felt less comfortable with the multiplication aspect of the problem could select the question with less items or lower numbers; students who required more of a challenge could choose the problem they felt would best suit their skills. In-line with the principles of CGI, this allowed all students to work at their own level, developing their skills on par with their current understanding.

Upon completion of the study, Calvin shared that the idea of Parallel Tasks was yet another strategy that he would like to continue to include in his practice regularly. He felt that it was highly effective and, in addition to students feeling more at ease and comfortable when having a choice, students' performance was better overall.

Notably, an increase in student engagement in mathematics in Calvin’s class was observed. On several occasions, after mathematics class time ended, many students requested more time to continue working on the activities. On one occasion, after the bell had rung indicating it was time for recess, two female students asked to remain inside to finish their debate over the solution to the problem. In another instance, one of the students asked for a few extra minutes during recess to finish explaining her solution to her partner. In addition to using personal time to continue working on mathematics, increased student engagement was also observed. Students began to ask for more detailed explanations of a concept, as well as for permission to use different materials to support their learning in mathematics. In one specific instance, a student asked to speak to Calvin about a concept he had recently been studying, that of decomposing numbers. The student understood how to apply the strategy so that he could add larger numbers more easily but was unclear as to why the solution worked and whether there were occasions when it would not work. An increase
in student engagement was seen in many of the students in Calvin’s class, in many different forms.

Calvin used the same assessment methods during the intervention as he did prior to the intervention to record student progress: tests; student-teacher conferences; student self-assessments. However, during this unit, Calvin also asked his students to assess their partner on his/her ability to complete the problem, to select appropriate tools, and to work appropriately. He tracked these scores almost daily. After the initial few peer assessments, Calvin decided he would first have some students share the solutions to the problem at hand before asking them to assess each other and themselves. Once this change was implemented, he perceived the students’ self and peer assessments to be much more accurate and useful.

4.2.2.4 Description of the Class

Although teachers at this alternative school were asked to focus on holistic education, Calvin taught a grade 3 class of 23 students. Of these students, 11 were male and 12 were female. Only two of the students joined the school after Kindergarten, while the majority of the students had attended Forest Elementary School since Kindergarten. All of the students in the classroom had enrolled and had started the class in September. No new students joined the class and no students were demitted from the class throughout the year.

Four of the students in the class were identified formally as having exceptional learning needs and received daily Special Education instruction, both in and outside of the classroom. However, their focus was not directly related to mathematics. This support did not interfere with their attendance or participation in any of the classroom activities or assessments.
Calvin’s unit plans for this class were completed and submitted eight weeks prior to implementation and he intended to spend two school weeks on the unit. The actual unit lasted four weeks. This was due to Calvin’s decision to provide the students with more time to develop and practise their multiplication facts, in the context of rich and meaningful word problems.

**4.2.2.4.1 Class Performance**

Although limited direct instruction regarding multiplication had taken place in the classroom prior to the beginning of the intervention, Calvin indicated that he had worked with two of his students on this skill since the beginning of the year. The reasons for this were different in each case, but primarily due to these students having an advanced skill and knowledge level in comparison to their peers.

**4.2.2.4.2 Multiplication Calculations**

Prior to the intervention, Calvin’s class scored an average of 18.3% on the multiplication drill, or 11.0 correct responses out of a possible 60 (Appendix I – Sample Student Calculation Drill). The students’ initial correct responses ranged from 1.7% to 50%, or from one to 30 correct responses out of a possible 60.

At the completion of the intervention, Calvin’s class scored an average of 30.2%, or 18.1 correct responses out of a possible 60 (Figure 8: Calvin’s Student Results). The post-assessment had a range of 3.3% to 76.7%, or two to 46 correct responses out of a possible 60.

Calvin’s class demonstrated an average improvement in their multiplication calculations of 11.9%, or almost 7.1 additional correct responses, on the timed multiplication drill.
4.2.2.4.3 Multiplication Word Problem Solving Results

Calvin’s class scored a reported average of a Level 3-/B- letter grade on the initial Multiplication Word Problem Assessment, created and interpreted by Calvin, or a score of 7.1 out of a possible 12 on the step scale (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores on this assessment ranged from a Level 1+/D+ letter grade to a Level 4/A letter grade.

On the post Multiplication Word Problem Assessment, also created and interpreted by Calvin, Calvin’s class scored a reported average of a Level +/-B+ letter grade, or a score of 8.5 points out of a possible 12 (Figure 4: 12-Point Conversion Scale for Levels from Achievement Chart). Scores for this assessment ranged from a level 2+/C+ letter grade to a Level 4/A letter grade.
Figure 8: Calvin’s Student Results

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The Multiplication Word Problem Assessment results indicate a significant improvement in the class average, from a Level 3-/B- letter grade to a Level 3+/B+ letter grade overall, or a step increase of 1.4 points on a 12-point scale (Figure 4: 12-Point Conversion Scale for Levels of Achievement Chart). It is important to note that the pre- and post-assessment word problems were designed by Calvin and that the pre- and post-
assessments were very similar but not identical. They required the students to select an appropriate strategy and solve for a 6x7 equation.

4.2.2.6 EQAO

Calvin’s students scored an overall average of 2.68 out of a possible score of 4 on the Grade 3 EQAO Mathematics Assessment (EQAO, 2017). 19 out of 23 of his grade 3 students participated in the week-long assessment that was administered in her classroom in May 2017. Students’ scores in mathematics on this assessment ranged from an overall Level 1 to Level 4. Only the participating students’ scores were used in the above calculations.

**Figure 9: Teacher Mathematical Knowledge for Teaching Compared to Student Performance**

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<th>Student Multiplication Calculation Raw End Score</th>
<th>Increase in Student Multiplication Calculations (Percentage)</th>
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<td>16.6</td>
<td>13.3%</td>
<td>6</td>
<td>1.15</td>
</tr>
<tr>
<td>Siryna</td>
<td>56.2%</td>
<td>2.2</td>
<td>15.7</td>
<td>5.9%</td>
<td>7.6</td>
<td>2.47</td>
</tr>
<tr>
<td>Calvin</td>
<td>83.3%</td>
<td>1.3</td>
<td>18.1</td>
<td>11.9%</td>
<td>8.5</td>
<td>2.68</td>
</tr>
</tbody>
</table>
Chapter 5: Discussion and Interpretation of Findings

Introduction

In this chapter, I will review the research questions presented in Chapter One and explore the effectiveness of the intervention. The intervention combined the Backward Design model (BDm) and Cognitively Guided Instruction (CGI) on multiplication skills in four different grade 3 classrooms, in two different schools. I will then discuss the various factors that influence the success of this combined intervention and recommend areas for future research.

5.1 Research Questions

My thesis focuses on two research questions, as stated in Chapter One:

1. How can student learning in mathematics be enhanced by combining Cognitively Guided Instruction (CGI) with the Backward Design model?

2. What factors affect the implementation of a program that combines CGI and Backward Design?

5.2 Question 1: How can student learning in mathematics be enhanced by combining Cognitively Guided Instruction (CGI) with the Backward Design model?

There were three key ways in which student learning in mathematics was enhanced based on the described intervention: increased student performance and outcomes; improved student attitude toward mathematics; increased level of student engagement in mathematics; increased time.

5.2.1 Increased Student Performance and Outcomes

In general, almost all of the students increased their ability to solve mathematical questions related to multiplication. They were also generally all able to increase their computational fluency with multiplication, as measured by the pre- and post-assessments. Although natural maturation needs to be taken into account, it is reasonable to assume that
the intervention was impactful due to the increase in the number, variety, breadth, and depth of strategies students used to achieve their solutions when problem-solving. All participating teachers focused student assessment on use of strategies: the sophistication and complexity of the strategy; its appropriateness for the context; its effectiveness. Since student performance in this area was enhanced, it is reasonable to suggest that the improvement is related to students’ increased strategy use and implementation.

After collecting and analyzing all of the data, it is necessary to reflect on larger themes and trends. Most notably, there appears to be a connection between a teacher’s mathematical knowledge for teaching and that teacher’s students’ achievement in mathematics, which is supported by a large body of research (Hill, Rowan & Ball. 2005; Marshall & Sorto, 2012; Moyer-Packenham et al., 2008; Oleson & Hora, 2014; Pool et al., 2012; Telese, 2012).

Based on these results, Genevieve’s class had the greatest overall increases in average student performance in both the Word Problem Performance and Multiplication Calculations. Calvin’s class had the highest overall end achievement in both of these areas (Figure 9: Teacher Knowledge for Teaching Compared to Student Performance). In addition to having the highest scores in both areas, Calvin had the highest overall mathematical content knowledge.

Mary had the lowest increase in student achievement on Word Problems, which is likely related to her reluctance to try new tasks, to effectively implement CGI principles, and to use alternative assessment means to measure student performance on an on-going basis.

Most interestingly, Genevieve, Mary and Siryna had similar levels of Knowledge for Teaching, ranging from an overall score of 44.7% to 56.2% (Figure 9: Teacher Knowledge
for Teaching Compared to Student Performance). In all of these classrooms, the student overall average end achievement scores in Word Problem activities ranged 1.6 points on a 12-point scale, from 6 to 7.6 points, or from Level 2/C+ letter grade to a Level 3/B letter grade.

In regards to student performance on the multiplication assessment, Genevieve, Mary and Siryna’s classes had similar final results, ranging from a score of 13.6 to 16.6 correct answers out of 60 possible questions. In comparison, Calvin’s class had the highest overall average student performance on the Word Problem Assessment, at an 8.5 or a Level 3+/B+ letter grade. Calvin’s class had the highest overall average student score on the Multiplication Assessment, with 18.1 correct answers out of a possible 60.

The highest increase in overall average student performance on the Multiplication Assessment and Word Problem Assessment took place in Calvin’s class, where students increased by 14% in correct responses on the Multiplication Assessment and a 2.3 point increase, or the equivalent of an average student increase from a Level 2/C letter grade to a Level 3/B letter grade on their Word Problem Assessments. Again, it must be noted that, when interpreting these results, the word problem assessments were designed by each individual teacher. The word problems varied from class to class and a direct comparison is not entirely possible. All of the word problems were assessed in comparison to the Ontario Mathematics Curriculum, using the same standards and expectations.

In regards to student performance on the EQAO assessment, Forest Elementary School outperformed Norden Junior Public School in all measures. At Forest, 70% of grade 3 students met or exceeded the provincial standard on the Grade 3 EQAO Mathematics Assessment whereas on 32% of students met or exceeded the provincial standard at Norden
Junior Public School. The overall average student score for grade 3 Forest students was Level 2.68 out of a possible Level 4 while the overall average student score for grade 3 Norden Junior Public School students was 2.05.

Once again, Calvin’s students had the highest overall average on the Grade 3 EQAO Mathematics assessment of 2.68 out of 4, while Siryna and Genevieve had similar scores of 2.47 and 2.33 respectively. Mary’s students had the lowest overall average on the Grade 3 EQAO Mathematics assessment scoring a 1.15 out of a possible 4.

There appears to be a connection between teacher knowledge and student performance in mathematics in terms of elementary students learning multiplication that requires more research for a clearer understanding (Appendix K - Student Overall Average Multiplication Drill Raw End Score Organized by Teacher MKT, Appendix L - Overall Average Student Increase in Word Problem Solving Organized by Teacher MKT, & Appendix M - Overall Average Student End Performance On Word Problems and Multiplication Organized by Teacher MKT).

Interestingly, only Mary allocated time planning and engaging her students regularly in the traditional multiplication drill work. All participating students regularly participated in higher-order thinking in varying degrees based on problem-solving questions that required them to use multiplicative reasoning. All teachers introduced the standard algorithm for multiplication questions. However, aside from Mary, no teacher required their students, in a regular classroom activity, to practice this in a repetitive nature. Despite the lack of practice in the classroom, almost all of the students increased their speed and the accuracy of their multiplication skills, as demonstrated by the difference in their pre- and post-assessment scores. This provides support for the effectiveness of the intervention as well as Carpenter et
al.’s (2015) theory that students are more likely to acquire new skills when taught in a familiar context. They are also more likely to remember or transfer this skill into new or unfamiliar situations when their learning is authentic and meaningful to that individual student.

In all of the classrooms, as the students learned about the concept of multiplication, they also learned a variety of different strategies, which could be used to calculate multiplication questions. They were regularly asked to investigate ways to solve a multiplication problem that was situated in a familiar context. As a result, they were able to learn at their own level; the learning that they completed was meaningful to each and every student. The result likely contributed to an increase in the retention of knowledge and the ability to transfer that knowledge into a new learning context.

To improve their word-problem-solving abilities, students were required to do the following: increase the number of strategies used to approach problem-solving; improve their ability to communicate their own ideas; and appropriately choose a suitable strategy for a particular context or word problem. Given that each class demonstrated some level of improvement, it is reasonable to infer that many students increased their abilities in all of these areas.

Additionally, the Grade 3 EQAO Mathematics Assessment provides support for the effectiveness of the intervention as a whole. Although there was an overall decrease in student achievement on the mathematics portion of this assessment at the provincial and board levels, there was great disparity in student achievement among the various classrooms. Comparatively, Forest Elementary Public School out-performed its previous
year’s performance, its five-year trend performance, as well as Norden Junior Public School’s grade 3 performance.

At Forest Elementary Public School, in Calvin’s class, 70% of students met or exceeded the provincial standard. In the previous year, 68% of students met or exceeded the provincial standard. Although this marks only a 2% improvement, it represents a significant variance (20% increase) from its five-year trend average of 50%. This year-over-year variance and increase in five-year trend average provides evidence that, as an entire cohort, this intervention contributed positively to student achievement in mathematics.

At Norden Junior Public School, there was an overall 22% decrease from its previous year’s achievement on the Grade 3 EQAO Mathematics Assessment from 54% of students meeting or exceeding the provincial standard to 32%. Additionally, Norden Junior Public School under-performed in comparison to its five-year trend, where 64.5% of students met or exceeded the provincial standard.

In Siryna and Genevieve’s classes, overall student performance was Level 2.47 and Level 2.33, respectively. Comparatively, Mary’s overall class average on the same assessment was Level 1.15. Furthermore, Siryna and Genevieve had 42.1% and 50% of their students meet or exceed the provincial standard, respectively, whereas Mary had 7.6% of her class meet that standard; the equivalent of one student. Thus, while there was an overall decrease in student achievement in mathematics on the EQAO Assessment, these outcomes were not consistent across all of the classrooms. The classrooms that adhered more to the intervention saw student achievement scores and performance significantly greater than the students in Mary’s class, where the adherence and implementation of the intervention was limited.
Despite the fact that the students in this study were comprised of grade 3 cohorts from two very different elementary schools, the intervention successfully supported student learning in both schools and in all classroom settings.

5.2.2 Student Attitude Toward Mathematics

Although no formal measure of student attitude was conducted at the beginning of the study, field observations were conducted. Teacher interviews provided some insights into students’ general attitudes towards mathematics. However, given that each teacher was not provided a standardized guide on how to observe and gauge student attitude towards mathematics, it is impossible to provide a quantitative analysis in change or differences over time. Nevertheless, qualitative analysis combined with past and present EQAO data provides some insights as to how student attitudes toward mathematics might have improved in this study.

Initially, students appeared no more or less interested in mathematics than in any other academic subject, although a few students indicated that their favourite subject was mathematics. In many of the classes, students would ask about different parts of their schedule and when certain subjects would take place in that day. There were no recordable observations where students asked about mathematics, however there are several instances in which students asked about recess, physical education, music, computers, and reading. This provides some indication that students generally were not excited or waiting in eager anticipation to start mathematics that day.

During the pre-observations in the classrooms at mathematics time, many notes were made regarding student attitude towards mathematics. On several occasions, a number of students were observed groaning when the teacher asked them to transition to mathematics.
In two of the classes, students shared that they “hated math” and one student questioned, “Why do we even have to study math?” When asked to initiate a task in mathematics, students would ask to be excused to go to the washroom or to get a drink; some distractedly asked an off-topic question instead of beginning the task. During the task, especially during group work sessions, students spent equal amounts of time talking with their peers about off-topic subjects as they did on their work. Notably, all of these observations took place at Norden Junior Public School. Combined, these observations provide some insights as to student attitudes toward mathematics prior to the intervention at Norden Junior Public School.

At Forest Elementary School, students appeared indifferent toward mathematics. They would initiate tasks just as quickly as they would with other subject areas and they remained on-task more often than not. Forest Elementary School students were observed to have a more positive attitude toward mathematics than their peers at Norden Junior Public School.

In addition to attitudinal observations, data was collected by EQAO and compared year over year. At Norden Junior Public School, student attitude towards mathematics has remained relatively stable over the past five years, with over 80% of students sometimes or always liking mathematics (EQAO, 2017). Interestingly, there has been in increase in students who never like mathematics in recent years. This category has seen a significant increase from 12% of students reporting a negative attitude five years ago, to 18-19% of students reporting the same in recent years. However, in their most recent year, 84% of students reported on EQAO that they sometimes or always liked mathematics; an increase of
There was also a decrease of students who reported that they never liked mathematics, falling to 16%.

At Forest Elementary School, student attitude toward mathematics has been on the rise. Five years ago, 83% of students reported that they sometimes or always liked mathematics; in 2016, this number rose to 96% (EQAO, 2017). Forest Elementary has also seen a consistent decrease in the number of students who report that they never like mathematics, a difference of 18% in 2012 to 4% in 2016. Most recently, student attitude has had a mild negative increase with 89% reporting that they sometimes or always like mathematics and 11% indicating that they never like mathematics (EQAO, 2017).

Again, it is impossible to concretely attribute the general improvement in student attitude reported on EQAO to the intervention delivered in the classroom, but it is reasonable to believe that it was a contributing factor for three reasons. First, student attitude in general in all of the classes initially appeared indifferent; similar reports by individual teachers supported this. However, throughout the unit, the amount of discussion and the number of questions asked during mathematics class and about mathematics time increased, as reported by many of the teachers. In three of the classrooms, students began to ask when they would get to do math that day; they were excited to talk about it, work with their group, and get started. It was observed and also reported by teachers that this was a positive change in behaviour for students, indicating an increase in student attitude towards mathematics.

Second, as noted in my observational field notes, the amount of time spent on-task for students throughout the intervention increased in all of the classes. It was witnessed that students spent less time avoiding tasks by seeking out distractions, such as finding a material, asking to go to the washroom, or wandering around the room. Most students were
much more eager and willing to initiate the activity immediately or very shortly after instructed to do so as the unit progressed. Although some of this eagerness can certainly be attributed to the students feeling more prepared as the unit progressed, it suggests that student attitude toward mathematics in general had improved.

Third, it is reasonable to assume that the favourable change in the types of tasks, activities, and assessments also had a positive influence on student attitude. Genevieve and Mary used paper-and-pencil-based and teacher-directed activities prior to the intervention. Although some tasks veered outside of this in varying rates in the different classrooms, most activities followed a pattern where the students had to observe the teacher’s lesson and examples, participate in discussion, and then attempt to complete related tasks either in a group or independently.

During the intervention, the types of activities became more relevant to the students, providing a richer and more relatable context. Students were better able to work at their own pace and the teacher more aptly provided planning and support for this due to his/her new focus and understanding of assessment. Thus, if students are working at their independent learning pace and experiencing more success, as demonstrated by their achievement and by the nature of this intervention, it is reasonable to assume that they would have a better attitude. The observations of the study, comments of the teachers, and results of EQAO support this idea.

5.2.3 Increased Student Engagement in Mathematics

Similar to student attitudes, student engagement in mathematics also appeared to increase. This was observed by the number and quality of questions that were asked during
student work time, the number of tasks or questions that students completed, and the
duration of time for which students would remain on task.

Initially, in all of the classrooms, several students would ask off-topic and work-
avoidance questions at the beginning of mathematics class. Students would ask to use the
washroom or grab a drink, or they would ask to participate in some other activity unrelated
to mathematics. During the intervention, the number of students who would ask to leave
during mathematics class decreased.

Furthermore, there were several instances in which the mathematics class time ended
and the students requested more time to continue working on the activities. In Calvin’s class,
two students were debating their solutions to a word problem about a trip to the park they
had done a day earlier. When the recess bell rang, they had not come to an agreement, so
they asked Calvin if they could have a few more minutes inside at recess. One of the girls
had an idea as to how to prove her solution to the other and wanted the opportunity to see it
through. In this example, students were asking a question that related to the structure of the
task at hand, whereas in other examples observed later in the units, students would ask for
deeper explanation of a concept or to use different materials to help them solve the problem.
This was one of many examples that demonstrated increased student engagement in
mathematics.

Additionally, the number of tasks or questions that students completed and the
amount of time they spent on-task also increased. Many of the teachers would provide the
students up to 25 minutes to complete a task in mathematics, once assigned. Their
justification for this was that if you leave the students to work for too long, they lose focus
and begin to participate in off-topic tasks, such as socializing. Instead, toward the end of
their units, many teachers planned lessons that would take up to 80 minutes. Of the 80
minute lesson, it was observed that 30 to 40 minutes of this time was dedicated to student
work time. It is suspected that the increased social aspect of many of the activities allowed
students to focus on tasks for a longer duration than was previously effective. Regardless,
this increase in the amount of time that students would remain committed to mathematics
tasks is suggestive of increased student engagement.

The design of the intervention itself directly impacted student engagement. With a
strong sense of direction and clear understanding for each unit and its direction or goals, the
participating teachers were better able to deliver a CGI model in their classroom, which led
to more personalized instruction. This was especially true for Genevieve, Siryna, and
Calvin’s classes, as the activities they implemented were more in-line with CGI
expectations. CGI instruction allowed for differentiation of the tasks, and the types of tasks,
as well as how students were assessed, which better met the student at their individual level
and needs. Students who feel confident and comfortable in a classroom are more likely to
have an increased level of engagement since they feel able to attempt the task and more
often than not experience some level of success (Taylor & Parsons, 2011). Thus, it appears
that student engagement in this study increased as a result of teachers having a solid unit
plan to guide their instruction, including assessment pieces and goals, so that they were
better able to personalize the learning for each student. This positive change furthered
student success and achievement, and ultimately motivated their engagement.

5.2.4 Time

In addition to being an essential contributing factor, time was also found to be a
beneficial result of the intervention. Explained in detail below, the benefit of time was seen
in three key ways: increased teacher time and efficiency, increased learning time for students, and increased time spent on individual tasks by students.

5.2.4.1 Increased Teacher Time and Efficiency

As the unit had pre-established goals along with an intended path to achieve these goals, teachers were provided a clearer foundation for their daily planning and teaching. These pre-established goals also reduced the amount of reactionary, or day-to-day, planning that often saw teachers getting lost, missing information, not connected to the overall goal, or not using time effectively. With this more defined plan, teachers had an increased level of time each day to reflect on the happenings and outcomes of the individual lessons. They also had more time during the lesson to respond to and meet student needs. This positive change in structure ultimately created cohesive, individualized flow for each of the students.

Teachers had pre-determined goals and collected resources that were designed to meet and support the learning of these goals, allowing for a more effective use of everyone’s time.

Participating teachers pre-determined and planned larger mathematical goals with connected sub-goals at the beginning of their respective units. Thus, they were able to spend less time planning daily activities for mathematics. This increased preparedness permitted teachers to dedicate a greater focus on observing students and facilitating learning. As a result, teachers became more aware of individual learning needs and were better able to provide support in a timely manner. They could prompt students with appropriate questions to re-direct their thinking; they could redirect students to collaborate with a peer, tool, or manipulative; they could meet with students one-on-one to discuss any confusion. This was especially true in the classrooms of Genevieve, Siryna, and Calvin.
When these teachers were able to spend more time observing and interacting with their students during the learning time, their awareness of student understanding increased, as did the students’ learning. This in-depth understanding of student knowledge enabled teachers to continue to unfold their unit plans more effectively, adapting pre-determined tasks to better meet the current needs of their students. In one instance, in Genevieve’s class, the teacher ended up using her understanding of student learning to replace a written activity with a game. These adaptations were often more time efficient for two reasons: students had an appropriate challenge and experienced higher engagement, leading to more on-task time and effective mathematics conversations; teachers were able to move on from concepts once students were ready, instead of spending unneeded time on skills or ideas that students had already mastered.

In addition to students’ increased knowledge and use of problem-solving strategies, teachers also gained new knowledge and understanding of approaches and strategies for teaching mathematics. In all of the classrooms, teachers sought out and implemented new methods for documenting and assessing student understanding. In one case study, Siryna started to regularly use her iPhone to record and document student work and responses. She stated that using the iPhone allowed her more freedom and flexibility during the teaching process, as it was quick and simple. She further stated that she found this to make assessment much easier.

Another example of increased knowledge that led to a change in teaching approach was apparent in Calvin’s classroom. In this class, Calvin remembered his use of composing and decomposing of numbers. He taught the students how to use a graphic organizer for this to support multiplication. Calvin introduced the graphic organizer to perform the calculation
to a group of students who were much more advanced than their peers. Calvin stated that he never typically used graphic organizers for mathematics, but he was willing to try to explore the outcomes with the students.

In Mary’s case study, despite being somewhat resistant to implementing the intervention, she agreed to try using technology to support student learning. These students were able to show their ideas without having to create a written record, which appeared to increase their engagement as well as the teacher’s interpretation of those students’ abilities. Although this did not change the teacher’s practice as to how she chose to measure student knowledge on the final task (Mary insisted on assessing these problems based solely on what was included on the paper), it did appear to have a positive impact on her teaching practice towards the end of the unit.

5.2.4.2 Increased Learning Time

The intervention also appears to have led to an increase in student learning time in the classroom. As this unit required teachers to focus on combining skills and teaching them simultaneously, students had significantly more opportunities to practice and consolidate their mathematical understanding by participating in more tasks. Typically, students would participate in one to two mathematical tasks a day that focused on one specific skill or idea. Instead, in this unit students participated in more tasks and spent more time on those tasks in the classroom. Specifically, the intervention asked teachers to use problem-solving as a way to teach multiplicative reasoning skills, instead of using time to teach both independently. Based on Carpenter et al’s (2015) research, the approach of teaching students through problem-solving is more likely to assist students in retaining the knowledge and being able to transfer it to new situations. This allowed teachers to spend more time on rich,
contextually-meaningful word problems that required multiplicative reasoning, instead of traditional drills or worksheets.

In short, this increased time spent on higher-order thinking tasks permitted students to participate in more of these kinds of tasks, instead of squandering time on the practice of basic multiplication skills. It was predicted that students would transfer this knowledge to their basic multiplication skills, when required. This, in turn, created opportunities for more authentic teaching, questioning, and discussion, as well as additional time for student assessments. These opportunities directly contributed to students’ enhanced learning, as time was used more effectively and more purposefully. This improvement was most evident in Genevieve’s and Calvin’s classrooms, where both teachers adhered to the intentions of the intervention most closely.

5.2.4.3 Time On-Task

Not formally considered in this study is the total amount of time students spent on the unit as well as the duration of every single lesson. Although anecdotal observations and tracking did take place regarding time spent on tasks and the timing of most lessons within this unit, this form of time-tracking was not completed on other units for comparison. The amount of time students spent on the tasks and the amount of time they had to spend with the intervention impacted its effectiveness. However, more research should be done specifically focused on this area. Furthermore, these studies might want to consider the time of day or time of school year in which the intervention is delivered to investigate whether this impacts its effectiveness as well.
5.2.5 Increased Teacher Understanding of Individual Student Performance

As part of the intervention planning and implementation, teachers were asked to find effective ways to measure individual student performance on an on-going basis, outside of the traditional paper-and-pencil tasks. Instead, teachers were asked to explore and incorporate different means to measure student knowledge and understanding throughout this unit, such as conferencing, technology and games, and orally through presentations.

Initially, most teachers found this to be challenging, as they were used to grouping students together and assessing the group work but were not entirely sure of each individual students’ understanding/progress. Although challenging, Genevieve, Siryna and Calvin embraced this aspect of the unit. In all three of these case studies, overall student performance in all measured areas increased significantly, with student performance on word problems increasing 1.3 to 2.3 letter grade levels.

One exception to this increase was in Siryna’s classroom, where student achievement in multiplication skills increased by 5.9%. Genevieve and Calvin classes increased 11.9% and 14%, or by approximately twice as much. However, Siryna also had the highest pre-performance in the multiplication drill assessment, which can explain why her students’ improvement was not as significant as that of the other two classes. Regardless, these increases are attributed to each teacher’s adherence to the intervention and their use of student-focused instruction to teach multiplicative reasoning through word problems. Teachers had more time in their classrooms to spend on higher-order-thinking word problems, which led their students to an increased understanding of multiplicative reasoning, thus increasing overall achievement. Teachers themselves gained new knowledge and understanding of approaches and strategies for teaching mathematics.
In all of the classrooms, teachers sought out and implemented new methods for documenting and assessing student understanding. Siryna started to regularly use her iPhone to record and document student work and responses. She stated that using the iPhone allowed her more freedom and flexibility during the teaching process, and that she found it to be quick and simple, making assessment much easier.

Another example of increased knowledge that led to a change in teaching approach was found in Calvin’s classroom. In this class, Calvin remembered his previous successful use of composing and decomposing of numbers and taught the students how to use a graphic organizer for this to support multiplication. Calvin introduced the graphic organizer to perform the calculation to a group of students who were much more advanced than their peers. Calvin stated that he never typically used graphic organizers for mathematics but he was willing to try to explore the outcomes with the students. As a result of the reduced time that Calvin spent on daily planning, and by increasing the time spent facilitating learning in the classroom, this teacher gained a better understanding of a particular group of students that may have otherwise been under-serviced or gone unnoticed. Furthermore, with his increased understanding, Calvin was better able to plan and implement a strategy to support those students’ learning.

Mary, despite being somewhat resistant to implementing the intervention, agreed to try using technology to support student learning. These students were then able to show their ideas without having to create a written record, increasing their engagement in the task. Mary’s familiarity with those students’ individual abilities also increased. Although this did not change her practice on how to measure student knowledge on the final task, it did have an impact on her teaching practice towards the end of the unit.
5.3 Question 2: What factors affect the implementation of a program that combines CGI and Backward Design?

Although there exists an indeterminable number of factors that influence teaching and learning in a classroom, four in particular are key to the implementation of a Backward Design unit that incorporates CGI-based teaching principles: teacher knowledge; teacher attitude; professional development; time.

5.3.1 Teacher Knowledge

Teacher knowledge was the largest contributing factor to the success of the intervention in this study. Four teachers over two different schools participated, each with a unique background and knowledge set in mathematics content and pedagogy. In all of the case studies, evidence collected indicates that there is indeed a connection between teacher knowledge and student achievement.

5.3.1.1 Mathematical Knowledge for Teaching

Teacher mathematical knowledge for teaching was measured both pre- and post-study using an established online mathematical assessment (Learning Mathematics for Teaching, 2005; Learning Mathematics for Teaching, 2008; University of Michigan, 2018). It is important to note that this assessment is not intended for evaluative purposes but, for the purposes of this study, was simply used to measure any differences in understanding and the impact on student outcome.

Genevieve’s, Mary’s, and Siryna’s mathematical knowledge for teaching was considered low’ however, Mary had the lowest knowledge score of 44.7%. Her students were the only group to be affected by their teacher’s performance; this group had significantly lower results than those of the other class averages. Students in Mary’s class had an average increase in word problem scores of only 0.56 of a level, or by less than half
of a grade level. This is a significant difference in comparison to all of the other case studies. This also represents the lowest overall student average end raw score on the Word Problem Assessment. The average score for Mary’s class was a 6, which is equivalent to a Level 2/C letter grade. This score represents 1.4 to 2.5 points less than the other classes involved in the study.

Interestingly, Mary’s class had the second highest improvement in students’ overall average multiplication assessment scores. Mary’s students scored 16.6 correct answers out of 60 possible questions, the second highest post-intervention score, or an improvement of 13.3% from the post assessment, which is also the second largest improvement. Initially, this evidence seems contradictory to the study’s findings, calling into question the effectiveness of the intervention and its dependence on teachers’ mathematical knowledge. There is, however, a potential explanation for this.

It is likely that Mary’s students performed at a higher level in the area of multiplication calculations because they focused more on lower-level thinking activities and traditional paper-and-pencil tasks. Traditional classrooms often produce students who can complete many quick and accurate basic computations (Kroesbergen, 2004; Smith & Smith, 2006). Unfortunately, however, research has demonstrated time and again that these computations are often memorized and not fully understood or conceptualized by the student (Carpenter et al., 2015, Carpenter et al., 2000; Smith & Smith, 2006). This knowledge does therefore not transfer well to new or unfamiliar situations, leaving students challenged to advance in the area of mathematics as they mature. Students taught in a traditional manner are often unable to relate new concepts to previous concepts due to their lack of understanding (Carpenter et al., 2015; Smith & Smith, 2006).
It is likely that Mary’s students were encouraged by her to participate and practice their multiplication calculations due to their increased achievement with this skill. However, a lack of conceptual understanding amongst these students is also probable, based on their poor performance and limited increase in their Word Problem Assessments. Students can memorize algorithms, but in order to fully understand them and transfer that knowledge, it is imperative that they learn these methods in meaningful ways, which will help them to develop a true understanding.

Another connection between teacher knowledge and student performance appeared upon examination of the data around the average student final achievement on both the Word Problem Assessment and the Multiplication Drill Assessment. Mary, who had the lowest mathematical knowledge for teaching score, also had student average outcomes that were the lowest reported of all of the case studies. Calvin, the teacher with the highest overall mathematical knowledge for teaching, had the highest reported average student scores in both areas.

Based on this evidence and supported by Ball et al. (2008), it appears that teacher mathematical knowledge for teaching is a significantly important factor in relation to the described initiative in this study, as well as to teaching mathematics in an elementary classroom in general.

Teachers must know the subject they teach … Teachers who do not themselves know a subject well are not likely to have the knowledge they need to help students learn this content … [Furthermore] teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work and choosing powerful ways of representing the subject so that it is understandable to students. (Ball et al., 2008, p. 404)

It is encouraged that more research in this area be conducted in order to better understand this connection and to explore possible ways to resolve this matter in the future.
5.3.1.2 Content and Pedagogical Knowledge

Although reasoned through observation only and not quantifiably measured in this study, teacher content and pedagogical knowledge improved and was also a contributing factor to the implementation of the designed intervention. Pedagogical knowledge, or the knowledge of teaching, was a factor in all three stages of the teaching process: planning, teaching, and assessment. Content knowledge, or the knowledge of mathematics processes and concepts, also impacted all three stages in the teaching process.

5.3.1.2.1 Planning

All of the teachers in this study admitted to generally planning on a day-to-day basis, as opposed to adopting a pragmatic, unit-based approach. Although every teacher had some awareness of what was involved in the design of a unit plan prior to the intervention, none of the participating teachers, with the exception of Calvin, were comfortably familiar with the Backward Design model. However, once teachers prepared their unit plans according to the intervention instructions, their daily planning appeared to be much more focused and goal-oriented.

Most of the teachers were able to seek out and implement a variety of non-traditional, non-paper-and-pencil tasks and activities to reflect the different learning needs of all of the students in their classroom. The teachers’ pedagogical knowledge allowed them to critically analyze new teaching approaches and to select those that they felt would be most effective for their specific students. The teachers learned to predict and determine if the task was engaging; consider how students enjoyed the task; understand how to structure and organize the task as geared to their specific classroom space; reflect on how to adapt the task to the needs of their students.
It can be assumed that the general population of teachers, without access to the tools and support provided by the intervention and with low or limited pedagogical knowledge, would likely struggle to select appropriate tasks and activities while planning a full unit. This, in turn, could lead to repetitive tasks that are unlikely to support or be effective in student learning. As mentioned by many of the teachers in the study, it is likely that inexperienced teachers will resort to the types of activities and strategies they themselves did at as students, as this is what they are familiar with. These would be taught somewhat blindly, without a solid grasp of the mathematical ideas and, limiting their usefulness.

All of the teachers struggled with using the Backward Design model; the four participating educators each had a difficult time initiating their plans in this format. With encouragement from the researcher, however, most teachers seemed to relax once the framework was underway.

Genevieve and Siryna took a similar approach in planning their units. They selected similar overall goals and then continued to establish sub-goals that would support their students in achieving the desired result. They determined the relevant evidence that would be used to measure student learning and understanding before selecting a variety of non-pencil-and-paper activities to deliver in the classroom. As evidence, Genevieve and Siryna both used word problems for pre- and post-assessment purposes, as well as the pre- and post-mathematics drill to assess student understanding of computations. They also planned and included student-teacher conferences, group work, and oral presentations as evidence of understanding throughout the unit.

During the implementation of the Backward Design model in their classrooms, both Genevieve and Siryna adhered to their plans. They demonstrated their ability to be reflective
in their teaching practice and sensitive towards their students’ needs. As a result, it is considered that they were both successful in delivering the intervention appropriately and according to its intentions.

Mary was highly challenged by the Backward Design model, and therefore struggled to initiate and complete her plan using the Backward Design framework. Upon completion, Mary included a pre- and post-assessment of student computation knowledge that was provided to her by me at the beginning of the unit. She failed, however, to include a pre-assessment for measuring students’ understanding and knowledge around word problem skills. Mary selected an overall goal for her students and three sub-goals. Her sub-goals were not directly linked to the overall goal, which was confusing, leaving me to question how these sub-goals would be connected or used to support learning and understanding of the larger goal.

Adding to the confusion, Mary did not include samples or descriptions of the activities she planned to use to collect evidence of students’ understanding or to support student learning. Instead, she provided a list of the types of tasks she thought she might use. These were not in line with CGI principles but rather focused on more traditional activities such as worksheets, paper-and-pencil tasks, and textbook work/questions. These activities were very similar to the types of tasks she had used during previous mathematics teaching, demonstrating an unwillingness or inability to adjust her strategies.

During the implementation of the Backward Design model in her classroom, Mary continued to focus on day-to-day planning. Although she did implement pre- and post-assessments for both calculations and word problems, it was unclear how the activities she selected could be used to support student learning around word problems. Towards the end
of the unit, Mary showed flexibility in considering my suggestions and incorporating new activities, such as the Parallel Task, to support student learning of word problem skills. She also agreed to try changing the structure of her lessons in an effort to reduce teacher-directedness and become more student friendly, following the expectations of CGI. On the contrary, Mary’s collection of evidence was derived solely from paper-and-pencil tasks such as tests and quizzes.

It is difficult to determine exactly how closely Mary followed her own plans, as her plans were missing certain necessary information. Throughout the unit, Mary sparingly demonstrated her potential to implement new tasks and forms of assessment. However, given the limited instances in which this happened, it is considered that she did not deliver the intervention appropriately and according to its intentions.

Mary’s challenges with the Backward Design model appear to be connected to challenges highlighted by Wiggins and McTighe (2015). Although she did not directly make the claims, Mary appeared to lack the necessary knowledge on Backward Design, its importance and true purpose, and its use (Wiggins & McTighe, 2015). Furthermore, Mary’s actions seem to suggest that time may have been an issue and that she simply felt overwhelmed and was unable to adequately learn and complete a Backward Design plan.

Calvin, on the other hand, completed his Backward Design model in a brief yet concise and efficient manner. He then requested to meet with me in his classroom to explain how the plan would be put into action. He established a clear overall goal and described the evidence he would collect to demonstrate student learning and understanding. Calvin included a variety of forms of evidence, including but not limited to: word problems assessed by rubrics; student-teacher conferences; and self and peer assessments. He then set
out a series of varied tasks that would support student learning by integrating information
drawn directly from students’ lives and experiences.

During the implementation of the Backward Design model in his classroom, Calvin
adhered closely to his plan. He demonstrated his ability to be reflective of his teaching
practice, to respond to student learning, and to be aware of all of his students’ needs. As a
result, it is considered that he was successful in delivering the intervention effectively and
according to its intentions.

5.3.1.2.2 Teaching

The success of the implementation of the intervention requires pedagogical
knowledge, mathematical content knowledge, and an awareness of the link between both of
these areas. Teachers must be familiar with a variety of different strategies and techniques
for managing a class and able to understand the purpose of these. Furthermore, teachers
must be able to differentiate where and when a technique is appropriate and how it will
achieve the desired outcome. Teachers with a high level of pedagogical knowledge will have
this foundation and understanding; they are more likely to be able to manage the varying
academic and social needs of a classroom. Thus, a teacher’s pedagogical knowledge impacts
how he/she will manage a classroom, and his/her ability to deliver a lesson or structure an
activity. This, in turn, will influence the effectiveness of the intervention.

Teachers with considerable pedagogical knowledge are better able to implement
practices that are consistent with the principles of CGI (Carpenter et al., 2015). Teachers are
also required to have a certain level of mathematical content knowledge and knowledge for
teaching mathematics in order to employ the Backward Design model when planning out a
unit. Teachers are required to have knowledge not only around the topics they are teaching,
but also in regard to how the topical concepts are connected. Teachers must understand how to decompose a large mathematical concept into smaller ideas, as well as how to connect the larger concept to previously learned and future-taught material. In the classroom, teachers must to be able to hold this knowledge and transfer it to students in the moment. As the classroom is a busy and dynamic place, teachers require broad, in-depth knowledge that will allow them to answer and respond to student queries; direct students to appropriate resources; facilitate discussion and learning. Teachers must be proactive in connecting students’ prior knowledge with present and future mathematical concepts.

5.3.1.2.3 Assessment

Understanding which assessment is best suited for a particular student also requires pedagogical knowledge. The ability to diversify assessment can impact the delivery of a unit planned through the lens of the Backward Design model and utilizing CGI instruction. In this study, Genevieve, Siryna, and Calvin were open and willing to include a variety of non-traditional assessment at many different points throughout their respective units. In each of these cases, student academic achievement improved significantly in terms of ability to effectively solve word problems and capacity to quickly and accurately calculate multiplication questions.

Conversely, Mary initially struggled with incorporating a variety of assessment techniques; additionally, her understanding of student knowledge was limited. Mary’s struggle with productive classroom management, effective lesson planning, and suitable assessment tools was detrimental to her teaching practice, as these combined obstacles had the effect of stifling her students’ achievement. From an observational perspective, Mary lacked the appropriate pedagogical knowledge to find, select, and implement appropriate
assessment tasks in her practice. This limited her understanding of student knowledge and negatively impacted the effectiveness of her experience with the overall intervention.

This is different than the other three cases, where the participating teachers were able to productively incorporate appropriate assessment tasks into their practices. This achievement led to an increased understanding of individual student knowledge, which positively impacted the effectiveness of the intervention.

5.3.2 Teacher Attitude

An individual teacher’s attitude is capable of influencing an entire classroom population. There are a number of reasons for this. Teachers whose attitudes are affable towards change and innovation are more likely to seek out and implement new activities, tasks, and assessments, diversifying material and connecting with student lives by remaining on-trend. Open-mindedness and positive thinking are qualities that can lead a teacher to approach student learning in a meaningful, fruitful way.

This study has shown that there is a connection between teacher attitude (including but not limited to the practice and learning of mathematics) and student performance. When teacher attitude was ranked at the highest level, student achievement was also at its highest; when teacher attitude was ranked lower, student performance also decreased. Given the possibility of conflation and a myriad of additional factors that were not accounted for in this study, it is impossible to conclude that teacher attitude is directly linked to student achievement. However, this initial finding lends itself to the recommendation that future research further explore this area.
5.3.3 Professional Development

The intervention required both sound mathematical content knowledge and solid pedagogical knowledge that not all participating teachers possessed. Attempts were made to provide professional development to address mathematical content knowledge and to help participating teachers to understand the Backward Design model and CGI. Although the sessions planned appeared to be effective in regard to the Backward Design model, more development on CGI was necessary for all teachers in this study. In order for this intervention to achieve its optimal level of success, it is imperative that participating teachers are properly trained in CGI methods, via an experienced teacher, and are provided the appropriate supports to explore this content area.

5.3.4 Limitations

In general, time was a limitation of this study, which impacted the effectiveness of the intervention. In addition to likely benefitting from more time spent learning about the intervention, it would also be advantageous to see this intervention taught in different grade-level classrooms and with a higher number of participating teachers. Although it is reasonable to believe that this intervention would be successful in other grade-levels, more time to explore this theory is required.

Participating teachers used this intervention for one single unit in their classrooms. It would be advantageous to explore how successful this intervention would be when employed in the same classroom throughout multiple units during the school year. This would allow more time for teachers to become familiar with the knowledge required by the intervention, as well as provide more time to consolidate this knowledge while teaching.
Additionally, participating teachers had limited time to dedicate to the Professional Development portion of the unit. There was also limited time available for planning the unit once the learning was complete. The concept of time and its relationship to teachers’ learning and planning should be explored.

As teachers chose to stay with the study and not withdraw, it is possible their desire for student improvement and the success of the intervention biased their judgement and, in turn, the results. Although this must be taken into consideration, it is not believed to have influenced the study’s results as teacher’s judgement of student performance and knowledge was compared to a third party assessment, EQAO, and the majority of student results were comparable on each assessment.

Not initially considered to be an influencing factor in the study, class size and class composition are two variables that all participating teachers commented on, expressing their belief that these did impact effectiveness. It is reasonable to conclude that class size has an impact on the success of this intervention. The study requires intense student focus and tracking during each and every lesson. As the number of students increase in a classroom, the teacher has an increased workload and less time and resources available to spend on each individual student. This reduced time would make it more challenging to deliver the intervention at its highest potential, thereby impeding its effectiveness. As class size increases, one-to-one and small group teacher-to-student time will inherently decrease. If teachers are unable to meet with all students and observe and question their knowledge, meeting the expectations of CGI will be challenging if not impossible.

Class composition also impacts the effectiveness of the intervention. Teachers use the term class composition to describe the diversity of students in their class in terms of
varying strengths and needs. The term encompasses classroom dynamics; inter-personal relationships; special educational needs; independence levels; executive functioning abilities. Genevieve, Siryna, and Mary all expressed concerns over the increasing number of students in their respective classrooms with special education needs. Each of these teachers shared that these students required much more time, support, and resources to enhance their learning. Not all students require the same amount of attention and support during the intervention, and so class composition needs to be taken into consideration during future research regarding this intervention.

One final limitation to note is regarding the word problem assessments. As teachers created their own word problems suited to their classroom, a direct analysis of the results will yield information and data that is not directly comparable. The created word problems were not standardized. Although each teacher used the same expectations to write their word problems, and that within each classroom the pre- and post-word problem assessment was very similar if not identical allowing a comparison for individual student progress, comparing the results to other classes needs to be done with caution. It should be noted that Siryna and Genevieve co-created their assessment problems.

Bias in the post-assessment from the pre-assessment and natural maturation also must be taken into consideration during the analysis of this study. Given that the students were exposed to both a similar pre- and post-assessment for the word problems, students may have potentially been biased and absorbed some of the information during the pre-test. Furthermore, students may have remembered the task and performed better on the post-assessment as a result of this memory and not a change in understanding or knowledge. This is unlikely and would have been noticed by the teacher as they track student progress.
throughout the unit and an unexplained increase in the final task would likely have been explored.

Finally, as all of the students are growing and developing, natural maturation must be taken into consideration when analyzing these results, as there is no way to account for this in the study’s design. That being stated, given that students in different classes showed varying improvement and given that their growth outperformed previous students’ growth as measured by a number of other assessments, it is reasonable to suggest that this intervention was impactful.

5.3.4.1 Implementation

Genevieve and Calvin chose to plan, deliver, and assess their respective units drawing only from the Backward Design model. Both of these teachers implemented their units as closely to the CGI framework as possible. In short, these two teachers appropriately planned out their respective units and taught multiplication through problem-solving, while simultaneously maintaining a student-focus. There are strong similarities between the experiences of both of these teachers and their students, despite being from two entirely different schools with vastly different populations. Their success with the intervention provides significant evidence that a combination of the BDm and CGI frameworks is effective in reaching all types of learners; students are able to grasp hard, tangible skills through problem-solving when the teaching is student-focused and the material is set within a context that is meaningful to the students.

Genevieve’s class achieved the highest percentage increase in overall average student performance on multiplication calculations. She also reported the largest overall increase in word problems related to this skill. Calvin, although he did not have the largest
gains, ascribed closely to the program and posted the highest end-scores for multiplication calculations and overall scores for word problems. The take-away from this is that a program tailored to student needs, with clearly defined goals, is highly effective in teaching mathematics to younger students.

Interestingly, Mary, who was initially reluctant to implement the unit as intended, had the lowest score in word problem-solving, despite the fact that her class was similar in composition and variables to that of Genevieve. This low achievement in word problem-solving is attributed to a lack of student-focused tasks and inadequate CGI implementation throughout the entire unit. Although Genevieve’s students had the second largest percentage increase in multiplication calculations, this group also wound up with the lowest end-score in this area. These students did not have rich, meaningful word problems to explore and use to consolidate their understanding of multiplicative reasoning, leaving them unable to successfully transfer these skills to situations that required simple calculations.

5.4 Major Findings

This study investigated the effectiveness of combining the Backward Design Model (Wiggins & McTighe, 2005) with Cognitively Guided Instruction (Carpenter et al., 2015) as an instructional framework for teaching elementary mathematics to Grade 3 students in various public educational settings. The study explored whether this framework was effective; the study also examined the contributing factors that supported the framework. The major findings can be summarized as follows:

1. **Perceived Improved Student Performance**

When implemented properly, Calvin, Genevieve and Siryna all believed the intervention led to improved student performance and increased achievement, both in
calculations and problem-solving abilities related to multiplication. This was cited throughout their post-lesson and post-intervention interviews. Although the increase in student performance varied between participating classrooms, student performance and achievement improved in three out of four classrooms. In Mary’s classroom, where the intervention was not implemented as intended as this teacher chose to continue teaching in a more traditional style, student achievement in problem-solving was limited. This provides evidence to support the three teachers’ perceptions that the combination of Backward Design model and CGI does contribute to an increase in student performance.

The effectiveness of the intervention on student performance did not appear to be influenced by the school environment. This intervention had a positive impact on student achievement, regardless of the different approaches to teaching mathematics modelled by the two schools.

2. Increased Student Engagement

The intervention appears to have demonstrated an increase in student engagement due to two key attributes: more appropriate and differentiated task selection and increased teacher awareness. Although no formal measure of student engagement took place, a number of examples have been cited to support this claim. Student engagement increases when learners participate and complete tasks successfully. This intervention increased student engagement by providing students with timely work that suited the particular events of the moment. The idea of differentiating desk work, based on the principles of the intervention, is a key factor in improving student engagement with mathematics. Furthermore, as teacher awareness of both student learning and mathematical content
increased, participating teachers were better able to select, plan, and implement activities tailored to the learners in their respective classrooms.

3. Change in Teacher Teaching Practice

All four participants showed changes in their teaching practice during this intervention. These teachers were asked to explore, using problem-solving methods, the operation of multiplication. This is a step away from the traditional means of teaching multiplication. In doing so, most of the participating teachers supplemented their instruction with new and innovative strategies; most also enhanced the boundaries of the intervention by utilizing technology. Rather than beginning their planning with a selection of relevant activities and tasks, teachers were encouraged to first choose an overall goal, followed by the determination of suitable evidences geared towards student understanding. This finding has implications in the area of mathematics instruction and all areas of the curriculum.

4. Increased Teachers’ Mathematical Knowledge

Mathematical knowledge for teaching increased for all but one of the participating teachers. Teacher knowledge appears to be linked to student achievement in mathematics. Therefore, the relationship between a teacher’s knowledge for teaching mathematics and that of her students is an important component to include in future interventions of elementary mathematics classrooms.

As the participating teachers gained new knowledge around approaches and tasks related to mathematics, their pedagogical knowledge also appeared to have increased. It is important that an intervention provide varied learning tasks and activities, as differentiation is a necessary characteristic of effective mathematical instruction.
5. Teaching Computational Skills Through Problem-Solving

In each of the case studies, student achievement in multiplication increased. However, the amount of increase varied from case to case. Student multiplication raw end-scores varied from 13.6 to 18.1. It must be noted that only Mary selected a traditional approach, spending a significant amount of time teaching and practicing multiplication questions; the other teacher participants opted to teach this skill via word problems. Thus, it can be concluded that teaching calculation skills through the use of word problems has a similar outcome as teaching these same skills explicitly.

Although the study found that calculation skills can be taught in an equally-effective manner via word problems or traditional repetitive practice, the study also provides evidence that the word problem approach has a greater impact on student achievement in other areas of mathematics. In the case study of Mary, where word problems were not used to teach calculation skills and these skills were instead taught separately, improvement in student achievement on word problems was 0.46 points on a 12-point scale. This is significantly less than the results of each of the three other case studies, where overall student performance on word problems increased by four to six times more, ranging from 1.3 to 2.3 points on the 12-point scale.

Throughout the implementation of this study, it became apparent that other external factors affect teachers who use the newly-designed framework. These external factors have been represented in this framework in an attempt to provide an overall conceptual framework for teaching mathematics. This framework can be used to help identify teachers’ strengths and challenges, creating the opportunity for teachers to seek out appropriate support for improvement. Furthermore, the framework offers guidance and insights as to
how some obstacles may have been conceived, how to overcome these roadblocks, and how to build on established strengths.

5.5 Implications for Future Research

This study involved a limited sample of teachers and students, in addition to a limited timeframe. Future research into how effective this intervention would be using additional teachers and students in different settings is necessary. Based on the findings of this study, it is possible that there are other influential factors that can contribute to the effectiveness or trials of this intervention.

This study involved limited time and ability relegated to the delivery of professional development. It would be beneficial to revisit this study with a group of teachers who have more time to commit to professional development, specifically on CGI. It would also be useful to investigate how effective this intervention could be if teachers were educated by a CGI instructor, or even had access to official CGI training workshops. As these advantages would likely lead to an increased knowledge and understanding of this framework, student performance in these classes would then also require further investigation.

Future research is necessary to explore how effective this intervention could be in regard to developing other mathematical calculation skills, such as addition, subtraction, and division. Based on the findings of this study, as well as research conducted by Carpenter et al. (2015), an investigation into the identification of different factors would be beneficial.

5.6 Recommendations

The Ontario Ministry of Education’s Mathematics Curriculum K-8 should be revised to reflect current understandings in teaching and learning of mathematics. This study sheds new light on the importance of encouraging teachers to provide students with more time to
explore mathematical concepts and learn about calculations via the investigation method. A revision of the Mathematics Curriculum to reflect this understanding and to provide teachers with guide is a next step towards implementing this practice in classrooms across the province.

Despite the extremely limited sample of teachers involved in this study, it is of grave concern that teachers in the Ontario elementary panel are ill-equipped and under-prepared to teach mathematics. Three out of the four teachers in this study had limited or insufficient mathematical content knowledge. It is known that mathematical content knowledge is linked to student achievement (Ball, Hill, & Bass, 2005; Campbell et al., 2014; Hill, Rowan & Ball, 2005; Marshall & Sorto, 2012; Moyer-Packenham et al., 2008; Oleson & Hora, 2014; Pool et al., 2012; Telese, 2012). It is therefore imperative that we consider how to address this concern.

It is recommended that the public education system re-evaluate how they determine who teaches mathematics classes at the elementary level. It is worth investigating the outcome of hiring teachers with high mathematical content knowledge as well as a strong foundational knowledge for teaching mathematics to teach the subject on a rotary-style basis, similar to the model used for French language instruction.

It is further recommended that post-secondary institutions that qualify teachers address this lack of mathematical content knowledge and knowledge for teaching mathematics with educators-in-training. Bachelor of Education programs may want to consider a requirement that education students enroll in a university-level course that is based in the study of mathematics; these institutions could alternatively increase the
prerequisites for admission into the Primary/Junior program to reflect a more appropriate level of mathematics knowledge.

It is imperative that we re-examine the current educational system, specifically in regard to the framework in which we organize teachers and their teaching role. We must search for ways to best serve the elementary students in mathematics classes, pairing them up with teachers who are fully prepared to teach this subject area. Two recommendations to achieve this idealistic setting include: increasing all teachers’ mathematical knowledge prior to entering or during Bachelor of Education degree program; adjusting the current educational structure to require those teaching mathematics to have specific qualifications reflecting the appropriate level of knowledge. In other words, the Ministry of Education may consider mandating mathematics as a specialized area that requires an established set of knowledge, not unlike French as a second language or the secondary school structure.

5.7 Conclusion

This research documents the effects of the combined intervention using the Backward Design model and CGI instruction. It highlights the complexity of the classroom as a space of learning and acquiring knowledge, and reinforces the idea that teaching must be pragmatic, intentional, and adaptable to this type of environment.

One of the most important findings of this research is confirmation that higher-order-thinking problems can be used to teach computational skills equally as effectively as traditional teaching means. More importantly, as the focus is on word problems and higher-order-thinking skills, student achievement in this area increased significantly more than in classrooms that maintain traditional teaching methods. As a result, current teaching practices are called to incorporate more higher-order-thinking activities that can provide students with
a rich and meaningful context, in order that they may effectively learn and develop mathematical computational skills that will transition with them into other areas.

In order to plan for a fundamental shift in the way we currently teach mathematics, teacher knowledge is the first step to address; it is necessary that teachers possess suitable knowledge to teach mathematics effectively. Currently, many teachers in the elementary panel do not possess enough mathematical content knowledge to create, seek out, analyze, and adapt higher-order problem-solving questions in their mathematics classes. These types of questions are necessary for the development and mastery of both critical-thinking skills in mathematics and computational fluency. Without the appropriate knowledge, teachers are ill-equipped and under-prepared to meet the needs of and to support all types of learners.
References


Appendices

Appendix A - Grade 3 EQAO Attitudinal Trends – Norden Junior Public School (EQAO, 2017)

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<td>I like mathematics</td>
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<td>19 35 46</td>
<td>18 41 38</td>
<td>19 35 46</td>
<td>16 33 51</td>
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<td>I am good at mathematics</td>
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<td>4 58 35</td>
<td>3 41 54</td>
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<td>9 49 42</td>
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<td>I am able to answer difficult mathematics questions</td>
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<td>19 62 15</td>
<td>10 49 38</td>
<td>15 48 37</td>
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<td>I do my best when I do mathematics activities in class</td>
<td>2 22 75</td>
<td>0 23 73</td>
<td>5 13 82</td>
<td>4 22 74</td>
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<td>I read over the problem first to make sure I know what I am supposed to do</td>
<td>12 28 56</td>
<td>12 23 62</td>
<td>8 26 67</td>
<td>4 22 74</td>
<td>9 39 52</td>
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<td>I think about the steps I will use to solve the problem</td>
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<td>4 50 42</td>
<td>8 51 38</td>
<td>7 50 41</td>
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<td>We talk about the mathematics work I do in school</td>
<td>28 38 31*</td>
<td>15 15 66*</td>
<td>13 18 68*</td>
<td>22 28 37*</td>
<td>15 27 59*</td>
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N - Never  S - Sometimes  A - Always  N/A - Not Answered

*For the purposes of this study, the categorizations “1 to 3 times a week” and “Every day or almost every day” were amalgamated.
Appendix B - Grade 3 EQAO School Math Achievement 5-Year Trend – Norden Junior Public School (EQAO, 2017)

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### Appendix C - Grade 3 EQAO Attitudinal Trends – Forest Elementary School (EQAO, 2017)

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<td>N  S  A</td>
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<td>I like mathematics</td>
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<td>NR NR NR</td>
<td>16 53 26</td>
<td>4 46 50</td>
<td>11 42 47</td>
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<tr>
<td>I am good at mathematics</td>
<td>0 53 47</td>
<td>NR NR NR</td>
<td>11 47 42</td>
<td>4 46 50</td>
<td>5 37 58</td>
</tr>
<tr>
<td>I am able to answer difficult mathematics questions</td>
<td>24 59 12</td>
<td>NR NR NR</td>
<td>21 47 32</td>
<td>4 62 35</td>
<td>5 58 37</td>
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<td>I do my best when I do mathematics activities in class</td>
<td>0 6 94</td>
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<td>I think about the steps I will use to solve the problem</td>
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<td>NR NR NR</td>
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<td>8 54 38</td>
<td>5 42 47</td>
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<td>We talk about the mathematics work I do in school</td>
<td>12 41 41*</td>
<td>NR NR NR</td>
<td>16 32 53</td>
<td>8 19 62</td>
<td>5 26 63*</td>
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</table>

N - Never  S - Sometimes  A - Always

*For the purposes of this study, the categorizations “1 to 3 times a week” and “Every day or almost every day” were amalgamated.
Appendix D - Grade 3 EQAO School Math Achievement 5-Year Trend – Forest Elementary School (EQAO, 2017)

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<td>17</td>
<td>3</td>
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<tr>
<td>At or Above Provincial Standard</td>
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<td>71</td>
<td>68</td>
<td>67</td>
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<td>65</td>
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</table>

*NR – Not Reliable to report (due to the limited number of students)
### Stage 1 - Desired Results

**Established Goals**

1. Relate one-digit multiplication, and division by one-digit divisors, to real-life situations.
2. Create basic representations of simple mathematical ideas.
3. Communicate mathematical thinking orally, visually, and in writing, using everyday language, a developing mathematical vocabulary, and a variety of representations.

**Understandings**

- Addition is related to multiplication and multiplication is related to division.
- How to solve problems using a variety of strategies, and demonstrate an understanding of multiplication and division.

**Essential Questions**

- What number(s) or number patterns do you recognize?
- What related facts can help you?
- How many different ways can this problem be solved?

**Non-Essential questions:**

- How could you start solving the problem?
- What might the missing number(s) be?
- Do you see a pattern?
- How can this be broken down into a simpler problem?

**Students will know...**

- How to use repeated addition.
- How to use known facts to solve unknown.
- How to use partitioning strategies.
- Reversing equations.
- How to make an array.

**Students will be able to...**

- Relate multiplication of one-digit numbers and division by one-digit divisors to real life situations, using a variety of tools and strategies (e.g., place objects in equal groups, use arrays, write repeated addition or subtraction sentences).
- multiply to 7 x 7 and divide to 49 ÷ 7, using a variety of mental strategies (e.g., doubles, doubles plus another set, skip counting).

<table>
<thead>
<tr>
<th>Stage 2 - Assessment Evidence</th>
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<tbody>
<tr>
<td><strong>Performance Tasks</strong></td>
</tr>
<tr>
<td>-writing equations for arrays/diagrams</td>
</tr>
<tr>
<td>-writing equivalent addition and multiplication equations</td>
</tr>
<tr>
<td>-using counters to make groups</td>
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<tr>
<td>-criteria: curriculum achievement chart</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Stage 3 - Learning Plan</th>
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</thead>
<tbody>
<tr>
<td><strong>Learning Activities</strong></td>
</tr>
<tr>
<td>Where the unit is going and what is expected: big idea posted in an easy to see place, repeated each lesson, success criteria given</td>
</tr>
<tr>
<td>Hook all students and hold their interest: create engaging minds on activities and use of technology</td>
</tr>
<tr>
<td>Equip students, help them explore key ideas: model different strategies</td>
</tr>
<tr>
<td>Revise and rethink: ask questions to prompt students and give multiple chances/assessments</td>
</tr>
<tr>
<td>Evaluate their work (students self assess): give students opportunity to present work and self assess orally or on paper using success criteria</td>
</tr>
<tr>
<td>Tailored to different needs and abilities: parallel tasks, accoms and mods if/when needed</td>
</tr>
<tr>
<td>Organized to maximize student engagement: lessons planned in advance, monitor whether or not students are engaged and adjust if necessary</td>
</tr>
</tbody>
</table>
Appendix F - CGI Presentation

Cognitively Guided Instruction (CGI)

CGI is defined as an instructional methodology that emphasizes the understanding of mathematical concepts. It is based on the belief that students develop mathematical understanding through problem-solving, exploration, and reasoning. The goal of CGI is to help students develop flexible problem-solving strategies and deepen their understanding of mathematical concepts.

1. Develop Understanding
   Instruction should develop understanding by strengthening relational knowledge and problem solving, with problem solving serving as the organizing focus of instruction.

2. Construction of Knowledge
   Instruction should be organized to foster students' active construction of their own knowledge with understanding.

3. Link Prior Knowledge
   Each student should be able to relate problems, concepts, or skills being learned to their existing knowledge that they already possess.

4. Instruction Based on Student Knowledge
   Since instruction should be based on what each student knows, it is necessary to continually assess not only where a learner can solve a particular problem but also how the learner solves the problem.

CGI and Multiplication

CGI strategies for multiplication include drawing pictures, using manipulatives, and applying the distributive property. Understanding multiplication requires students to be able to decompose numbers and use the properties of operations to solve problems.

CGI Strategies for Problem Solving

1. Direct Modeling
   Problems are solved by physically representing the problem using objects, drawings, or equations. Direct modeling is useful when students are first learning problem-solving strategies.

2. Adding/Subtracting
   Problems are solved by adding or subtracting numbers. This strategy is useful when students are able to solve problems using basic operations.

3. Number Facts
   Problems are solved by using number facts. Students are encouraged to recognize and use known number facts to solve problems.
Number Facts
- most abstract way of thinking
- requires students to access prior and understood facts
- facts are learned best in a context that students often use in every day life, e.g., a numerical order
- commutative properties support learning

Number Fact Examples
1. Multiplication
2. Measurement Division
3. Partitive Division

Next Steps...
Using the CGI framework, go back and look at your 8-8M Model and explore how you can create tasks/problems that provide your students context and help you understand their understanding and potential next steps. Keep in mind your big idea and how your students are set up and developing tasks to support their learning based on what they already know.
Appendix G – Backward Design model Template

<table>
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<tr>
<th>Title</th>
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<td>Topic</td>
<td>Grade:</td>
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**Stage 1 - Desired Results**

- Established Goals

<table>
<thead>
<tr>
<th>Understandings</th>
<th>Essential Questions</th>
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<td>Students will know...</td>
<td>Students will be able to...</td>
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**Stage 2 - Assessment Evidence**

- Performance Tasks
- Other Evidence

**Stage 3 - Learning Plan**

- Learning Activities
Appendix H - BDm Presentation

Backward Design Model (BDM)

Structured into three stages:
1. Identifying Desired Results
2. Determining Acceptable Evidence
3. Planning the Learning Experiences/Instructions

1. Identifying Desired Results

- Describe the learning outcomes in measurable terms
- Make sure the outcomes are connected and aligned with the standards
- Consider the prior knowledge and skills of the students

2. Determining Acceptable Evidence

- Identify the assessments that will be used to evaluate student learning
- Ensure that the assessments are valid and reliable
- Align the assessments with the learning outcomes

3. Planning the Learning Experiences/Instructions

- Design instructional activities that will help students achieve the desired results
- Include strategies for differentiating instruction
- Provide opportunities for students to apply their learning

TIPS (Before)

- Be open-minded and flexible
- Create a similar beginning and an end assessment
- Follow the 5D sequencing strategy (teaching should be done, students should understand and be able to do it! How will we know if students have achieved the desired results? What activities best support this learning?)

TIPS (During)

- Consider supportive questions for students:
  - What are you doing? Why are you doing it?
  - Don’t expect the process to be linear
  - Adjust as necessary, and keep your goal.
- Include a variety of strategies and approaches as well as varied learning activities
- Explore a range of transferable skills, concepts, and strategies

MISCONCEPTIONS

- Although the three stages present a linear design, it is not meant to be sequential or absolute.
- The framework and accompanying template is from the teacher perspective, not the learner.
- This is a guide to help design units, rather than a rigid blueprint.
- Effective planning begins with asking questions about the desired end result.
- Mathematical instruction does not have a fixed design. Although math rules are important, it is essential that students understand the big ideas.

Next Steps

Today, using the BD Model and Template, you are tasked with designing a Grade 3 Number Sense (1 unit) that teaches multiplication skills. The content and ensuring understanding or big ideas, are entirely up to you!
## Appendix I – Sample Student Calculation Drill

Name: ____________________  
Score: ________

Teacher: ____________________  
Date: ________

### 3 Minute Drill

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Teacher Survey

NAME ____________________     CODE __________________

Part 1 – Background/History
1. How many years have you been teaching?

2. How many years have you been teaching in the elementary panel?

3. How many years have you taught mathematics?

4. How many years have you taught mathematics in a primary grade (gr 1-3)?

5. How many mathematics university courses have you taken in total?

6. How many additional qualification courses have you taken in mathematics (AQ, ABQ, etc.)?

Part 2A – Attitudes (Multiple Choice)
Please rate the following on a 1 – 4 scale (where 1 represents strongly disagree and 4 represents strongly agree)

1. I like mathematics
   1  2  3  4

2. I like teaching mathematics
   1  2  3  4

3. I feel comfortable teaching mathematics
   1  2  3  4

4. I spend a lot of time planning for mathematics classes
   1  2  3  4

5. I feel mathematics is important
   1  2  3  4
Part 2B – Attitudes (Short Answer)

1. How do you feel about teaching mathematics to a Primary Classroom?

2. How do you prepare to teach mathematics?

3. What challenges do you face when teaching mathematics?

4. How do you ensure you meet all of the curriculum expectations when teaching mathematics?

5. Is there anything else you would like to add?
Appendix K - Student Overall Average Multiplication Drill Raw End Score Organized by Teacher MKT

**Overall Average Multiplication Drill Raw End Score Organized by Teacher MKT**

- **Mary (44.7%)**: 16.6
- **Genevieve (52.6%)**: 13.6
- **Siryn (56.2%)**: 15.7
- **Calvin (83.3%)**: 18.1
Overall Average Student Increase in Word Problem Solving Organized by Teacher MKT

Teacher | Overall Average | Multiplication Drill Raw End Score (/60)
---------|----------------|--------------------------------------
Mary (44.7%) | 0.56 |
Genevieve (52.6%) | 2.3 |
Siryna (56.2%) | 2.2 |
Calvin (83.3%) | 1.4 |
Appendix M - Overall Average Student End Performance On Word Problems and Multiplication Organized by Teacher MKT
Re: Investigating the Combined Impact of Cognitively Guided Instruction and Backward Design Model in Mathematics on Teachers of Grade 3 Students

Dear Teacher:

My name is Robert Walters and I am currently completing my Doctoral Studies at the Ontario Institute for Studies in Education (OISE) at the University of Toronto. My Thesis Supervisor is Professor Doug McDougall and my area of focus is on mathematics education. I am interested in recruiting volunteers to participate in a study designed for exploring the effects of combining two previously tested and proven mathematics Interventions in Grade 3 classrooms. You are receiving this email because your Principal has indicated interest in participating in this research and you meet the conditions for the study (i.e., you currently teach at a school in which it is possible to research the entire Grade 3 cohort and you are currently teaching mathematics to Grade 3 students). The Intervention will combine Cognitively Guided Instruction (CGI) and the Backward Design model in hopes of finding a more effective practice for teaching and delivering mathematics education in the elementary panel. CGI focuses on the process each student partakes in while exploring mathematical concepts and evaluates thinking as opposed to solutions (Carpenter et al., 2014), while Backward Design is a framework that starts at the beginning and works towards a goal, beginning with the intended outcome and then selecting appropriate forms of assessment (Wiggins & McTighe, 2005). Thus, teachers will have the opportunity to learn about each individual Intervention and work with the Researcher on combining and implementing them in their classrooms.

The study will begin in early October 2016 and is expected to be completed no later than June 2017. No compensation of any form is provided for participating in the study. There are no known risks for participating in the study and teachers who volunteer in the study will be allowed to withdraw at any point, without explanation or any consequence.

By volunteering to participate in this study, you agree to work with the Researcher to plan an Intervention in mathematics based on two researched teaching Interventions (to be explained at a later date). Participation will require the following:

- brief and regular meetings with the Researcher before and throughout the Intervention
participation in a pre- and post-survey and semi-structured interview
planning/writing/obtaining two similar higher-level-thinking problems in the form of word problems, at the beginning of the unit, that are related to the mathematical concept(s) being taught during this unit, using the knowledge learned from the Intervention
observation of you teaching in your classroom before and during the Intervention to record behaviours, attitudes, and interactions of both teacher and students, including audio-visual recordings
sharing of student information (with consent) with the researcher in the form of letter grades and work samples

PLEASE NOTE: All information collected will be kept strictly confidential and secure and no identifying markers or characteristics will be published, in any capacity. Only non-identifiable pseudonyms will be used for teachers and their schools in any reporting of the findings in the thesis dissertation and professional publications or presentations. Students’ work will be identified only by code. This code will use a number to identify the student’s teacher and then each individual teacher will create a unique number for each student in their class. This number will not be shared with the researcher in order to protect the students’ identities.

During the study, the Researcher is willing to provide support in terms of how to implement the Intervention, to discuss previous and future ideas and experiences, and to ask questions and provide feedback if requested. However, the Researcher is not responsible for providing any letter grades or academic feedback. Teachers will be responsible for their students’ letter grades and for providing feedback to both students and parents. The focus of this study is on the effectiveness of the Intervention and teachers are NOT being evaluated during this study.

Prior to beginning the Intervention in any classroom, informed consent from each student and their subsequent parent/guardian with varying levels or types of consent, will be obtained. The consent will allow the Researcher to explain the Intervention to the students and guardians/parents, examine their work and outcomes for the purposes of the study, and observe and possibly audio-visually record them for the purposes of the researcher’s dissertation. A consent letter for students will be written, copied, and provided to each teacher.

If you are interested in participating in this study or if you have any questions, concerns, or require more information about the study, please do not hesitate to contact the Researcher, Robert Walters via email at Robert.Walters@utoronto.ca. You are also welcome to contact the researcher’s Thesis Supervisor, Dr. Doug McDougall, at doug.mcdougall@utoronto.ca, or via email at doug.mcdougall@utoronto.ca.
If you have any questions regarding your rights as a research participant, please contact the University of Toronto’s Office of Research Ethics at 416-946-3273 or via email at ethics.review@utoronto.ca, or REB at TDSB via email EERC@tdsb.on.ca or phone 416-394-4949.

Sincerely,

Mr. Robert Walters
Teacher – PhD Candidate
Curriculum, Teaching, and Learning - OISE/University of Toronto
252 Bloor St. West    Toronto, ON   M5S 1V6
Appendix O – TKAS Pre- and Post-Assessment Email

Assessment Name: SAMPLE
Administrator Name(s): Robert Walters
Date Windows:
- Pre-test 11/13/2016 - 11/14/2016
- Post-test 11/15/2016 - 11/16/2016
Program Code: ######

Dear Participating Teacher:

Below is a link to the Pre-Assessment (TEST 3):
https://lmt.isr.umich.edu/assessment/SessionLogin.aspx

Once you have opened the link, you will be prompted to enter a Program Code (found below). Once done, please create a profile using your TDSB log in by clicking on the "first-time user" link.

Log-In: YOUR EMAIL
PASSWORD: Determined by user
PROGRAM CODE: ######

Once you have completed the assessment, TKAS will ask prompt you for personal information. This part of the assessment is also entirely optional and you are not required to share any information you do not wish to disclose.

Thank you for using the TKAS system!
Appendix P – Classroom Observation Guide

**Investigating the Combined Impact of Cognitively Guided Instruction and Backwards Design model**

Classroom Observation Template

**Pre-observation Conference**
1. What is the specific goal/learning outcome you are trying to achieve today? (Review lesson plan/template together)

2. Are there any individual students whom you are aware are going to require special assistance? If so, how do you intend to meet these needs?

3. How do you think your lesson is using CGI and Backward Design that we learned about?

4. Is there anything specific that you want me to look for?

**During-observation** (To be electronically recorded)

**Post-observation Conference**
1. How do you think it went?

2. Do you think the goal/learning outcome was achieved by all?
   - If so, how do you know?
   - If not, are you aware of whom may need more support? Have you thought about what this may look like?

3. Would you do anything differently? If so, please explain.

4. Give feedback from your notes. This discussion should centre on what you saw and heard.

5. Now that you have had this feedback, what are you going to do next to improve or change your teaching practice, in relation to CGI and the BD model? Give the teacher an opportunity to talk about what he/she might do in the future/following lesson.
Possible Evidence

Cognitively Guided Instruction (CGI)
- During the lesson, Teacher is able to provide support to different students in various ways (i.e., asking probing questions, having them gather different tools/manipulatives/referring to another student’s ideas/work, etc.)
- Teacher demonstrates an understanding of individual students’ needs and is able to articulate the need and possible ways to support them
- Teacher shows flexibility and adapts to whole class and individual student needs within the lesson while attempting to move towards the established learning goal of the lesson and unit
- Students are grouped in ways that would facilitate growth for all
- When a student responds incorrectly, Teacher finds a way to restore dignity to the student
- Student work is posted and different approaches are highlighted
- Teacher feedback is honest, but offered in a positive tone
- Teacher does not immediately indicate whether or not an answer is correct
- Teacher relates the process/solution of the task to mathematical ideas previously encountered
- Teacher makes deliberate connections to prior knowledge
- Student questions support and drive the lessons and task
- Teacher listens to student answers and encourages exploration of understanding, errors and misconceptions
- Teacher’s tone and body language does not influence student responses negatively
- Teacher asks probing questions to start the lesson, deepen thinking and understanding
- Students are regularly encouraged to clarify their understanding so teacher can support learning
- Teacher openly states that she believes that each individual student can do math successfully
- Teacher welcomes a variety of approaches to mathematical situations
- Criteria or samples of work are discussed and posted to show expectations
- Teacher goes beyond rules and helps students make sense of the math in a meaningful way

Backward Design model (BDm)
- Teacher is able to explain the progression of the lesson and the activities
- The progression of the lesson appears logical with sound reasoning/judgement to lead students to better understanding of a concept/idea and towards a larger goal
- Teacher is able to explain the process/expectations in a meaningful way to others including students
- **Teacher is able to describe how the individual lesson fits in with the overall goal of the unit**
- Teacher shows familiarity with the outcomes/expectations and can articulate a rationale for deciding which to focus on and when
- Teacher can demonstrate and justify a range of tasks/assessments in a given unit of study
- Pretests or other forms of diagnostic assessment are used to either alter the content taught or inform differentiation of instruction
- Students are regularly assessed at various points during the unit to ensure understanding, connections, and retention of knowledge/concepts/processes
- Teacher chooses strategies wisely, for example, uses interviews in situations where appropriate data cannot be gathered in other ways

Effective Implementation
- Teacher is prepared to teach the lesson with the appropriate tools and materials required
- Teacher shows familiarity with the use of key concepts for guiding questions (e.g., for whole class, small groups, and individual students)
- Teacher has an intended plan prior to lesson
Observations
Appendix Q – Student Parent Consent Form

Study Name: Investigating the Combined Impact of Cognitively Guided Instruction and Backward Design Model in Mathematics on Teachers of Grade 3 Students

Dear Parent/Guardian:

Your son’s/daughter’s teacher has volunteered to participate in a mathematics study aimed at improving mathematics education and instruction, beginning as early as October. This study will take place during the school day and will replace or supplement the mathematics instruction that they are currently receiving. Thus, your child may notice that some of the activities in the mathematics classes may be different. Although participation in the class activities by your child is mandatory as it is part of the instructional day, it is based on the Ontario Ministry of Education’s Grade 3 Mathematics Curriculum.

As there are a number of classes participating in this study and since the implementation and delivery of the mathematics curriculum is determined solely by their teacher, it is impossible to determine what time lessons will take place, the duration of each lesson, and the time it will take to complete a unit. That being said, your son/daughter’s class is expected to participate in the study for no less than 6 weeks, and possibly longer depending on the teacher’s discretion.

Attached is a consent form indicating the three different levels and types of consent you may provide to the researcher regarding your child. The levels of consent are explained in detail below; however, if you require more information please contact the researcher directly. You may withdraw consent or change the level of consent any time until the completion of the data collection in the classroom by contacting the Researcher via email at Robert.Walters@utoronto.ca

PLEASE NOTE: All information collected is strictly confidential and your son/daughter will NOT be identified in any summaries or findings. The reason for collecting this information is for data analysis, educational purposes, and the researcher’s thesis dissertation.

Consent Level 1: During the study, the Researcher will be present in your child’s class for some of the mathematics lessons. This is to provide feedback to the teacher on their implementation as well as to assess the effectiveness of the Intervention. Although your child’s participation in the class activities is not optional, the degree to which the Researcher works with your child and reviews your child’s recorded information is. During the class, it might prove useful for the Researcher to observe and speak with your child about the tasks they are assigned and their attitudes, feelings, and ideas. Please initial beside Consent Level 1 on the attached form to provide consent for the Researcher to speak with your child about their thoughts, feelings, attitudes, and ideas related to the activities in the study. Even with consent, your child has the right to not answer any or all questions he/she does not wish to answer. If you do not wish for the Researcher to speak with your child in the classroom, he will not initiate any contact with your child and, in the event your child speaks to him, your child will be redirected to his/her teacher.

Consent Level 2: In addition to collecting information on your child’s performance on the tasks assigned and grades as well as any attitudinal and behavioural changes, it would be useful for the
Researcher to occasionally collect copies or samples of your child’s work/performance tasks/assessments and notes on their thoughts, questions, ideas, and progress, related to the mathematics unit of study, for the purposes of their research. All collected information would be coded. Field notes recorded by the Researcher about a child would be coded with a unique number only known to the Researcher while work samples would be coded with a unique identifying number that is only known to your child’s teacher, during the study. Although the Researcher will be able to identify the teacher or class in which the sample came from, they will not know your child’s identity until after the data collection of the study is complete. It is also important to note that the Researcher will have NO input to your child’s evaluation in the unit; it will be evaluated by their teacher. Any samples/assessment/grades collected are for data analysis and the researcher’s thesis dissertation and will NOT be used in any other publications. If you consent to the Researcher collecting periodic samples of your child’s work, pre and post-assessment grades/tests, and assigned tasks for their use of his thesis dissertation and NOT any other publications, initial beside Consent Level 2a. Given that your child will be asked to participate in Education Quality and Accountability Office (EQAO) assessment, the research is requesting permission to see your child’s results. These results will be used as a comparison to other assessments used in the class room during this unit. With your consent, these results will be obtained by the Principal of the school when they are available and will be kept in strict confidence. Used for data analysis purposes, your child’s identity and results will not be shared in any capacity in which they can be identified. If you consent to the Researcher reviewing your child’s EQAO results (related to mathematics), initial beside Consent Level 2b.

Consent Level 3: Given the busy nature of the classroom and the number of students, the Researcher would like to audio and/or visually record the classroom during the activities being observed for this study. Any audio-visual recordings will be stored securely according to the University of Toronto’s and the Toronto District School Board’s standards and used only for the researcher’s thesis dissertation. They will not be published or shared anywhere else. To provide consent for your child to be audio-visually recorded, initial beside Consent Level 3 on the attached form. If you do not wish for your child to be audio-visually recorded during the study, arrangements will be made in accordance with TDSB’s Media Release policies and actions will be taken to ensure that your child is not captured on the audio-visual recordings.

If you have any questions, concerns, or require more information about the study, please do not hesitate to contact the Researcher of the study, Robert Walters via email at

Dr. Doug McDougall via phone [Redacted] or via email at [Redacted], if you have any additional concerns or concerns that you feel have not been appropriately addressed. If you have any questions regarding your son’s/daughter’s rights as a research participant, please contact the University of Toronto’s Office of Research Ethics via phone 416-946-3273 or via email at ethics.review@utoronto.ca, or REB at TDSB via email EERC@tdsb.on.ca or phone 416-394-4949.

Sincerely,

Mr. Robert Walters
Teacher – PhD Candidate - OISE/University of Toronto
Curriculum, Teaching, and Learning
252 Bloor St. West
Toronto, ON   M5S 1V6
Email: [Redacted]
Parent/Guardian Consent Form (Appendix Q)

Study Name: Investigating the Combined Impact of Cognitively Guided Instruction and Backward Design Model in Mathematics on Teachers of Grade 3 Students

Researcher: Robert Walters (Robert.Walters@utoronto.ca)

University of Toronto Thesis Supervisor: Dr. Doug McDougall (Doug.McDougall@utoronto.ca)

I understand that my child will be participating in the research study stated above and will be observed in their classroom by the researcher. I understand that they will be evaluated only by their teacher.

Please initial beside each and every statement you agree with and give permission to:

<table>
<thead>
<tr>
<th>Consent Level 1</th>
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<tbody>
<tr>
<td>_____ I give consent for the Researcher to speak and work directly with my child, in the classroom, for the purposes of this study. Even with my consent, my child has the right to refuse to answer any or all questions he/she does not wish to answer.</td>
</tr>
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<th>Consent Level 2</th>
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<tr>
<td>_____ a. I give consent for the researcher to photographed/copy (with no identifying characteristics) work samples, tests, and other tasks or assignments done by my child related to the mathematics unit of study.</td>
</tr>
<tr>
<td>_____ b. I give consent to release my child’s EQAO results to the researcher for data analysis purposes. My child’s results will NOT be shared with anyone else and will be used in a manner that will not expose their identity.</td>
</tr>
</tbody>
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<tr>
<th>Consent Level 3</th>
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<tbody>
<tr>
<td>_____ I give consent for my child to be included audio-video recordings of class activities for the purposes of the Researcher’s thesis dissertation.</td>
</tr>
</tbody>
</table>

Child’s Name _______________________________________________________

Child’s Teacher _____________________________________________________

Parent/Guardian Name (Print) ________________________________________ / (Sign)____________________

Contact Information _______________________________________________

Please keep a copy of this consent form.
Dear Teacher:

Thank you for agreeing to participate in this study aimed at investigating a combined intervention in mathematics as explained in the recruitment letter (Appendix D). Your participation is voluntary, and you have the right to withdraw at any time without any explanation or consequence. You also have the right to decline to answer any questions or participate in any of the procedures/tasks without any consequences. In the event you do withdraw from the study, you have the right to ask the researcher to destroy and/or all of the data collected on you and/or your class. In order to withdraw, you are required to email the researcher indicating you wish to be withdrawn from the study and whether or not you wish for any/all of the data collected on you or your class to be destroyed. This includes information that has been collected orally, audio-visually, and via surveys both for you and your students. Please note: there is no compensation for participation in this study.

In order to meet the criteria to participate, you must currently be teaching grade 3 mathematics. In addition to meeting this criterion, the reason you have been asked to volunteer is because the researcher is looking to include all teachers teaching the grade 3 cohort from entire schools.

By volunteering to participate in this study, you agree to work with the researcher to plan an intervention in mathematics based on Cognitively Guided Instruction (CGI) and Backward Design model. CGI focuses on the process each student partakes in while exploring mathematical concepts and evaluates thinking as opposed to solutions (Carpenter et al., 2014), while Backward Design model is a framework that, unlike traditional teaching methods that start at the beginning and work towards a goal, begins with the intended outcome and then selects appropriate forms of assessment (Wiggins & McTighe, 2005). This study will require you to meet and communicate with the researcher regularly, in approximately 20 minute durations with the exception of the initial and final interview and survey, before and throughout the implementation of the intended mathematics unit. You will also be asked to plan/write/obtain two similar higher-level thinking problems in the form of a word problem, at the beginning of the unit, that are related to the mathematical concept(s) being taught during this unit. You will then be required to, with support of the researcher, map out smaller goals to help you achieve the larger overall goal you selected and plan a unit based on this information. Before beginning the unit, the researcher will provide you support on implementing CGI and how to support students’ explore mathematical concepts and how to evaluate their thinking and the process. The researcher will also observe you teaching in your classroom before and during the intervention to record behaviours, attitudes, and interactions from both teacher and students. The time duration of the unit will be determined by you; however, it is the researcher’s experience that a unit lasts between 6-8 weeks. During the study, the researcher is willing to provide support in terms of how to implement the intervention, to discuss previous and future ideas and experiences, and to ask questions and provide feedback if requested; however, the researcher is not responsible for providing any grades or academic feedback. Teachers will be responsible for students’ grades and for providing academic feedback to students and parents. At no time will the teachers’ actions be evaluated.

Prior to beginning the study, it is necessary to collect information about you related to your teaching practice, mathematical knowledge, and attitudes. This information is important as it will provide some insights on how the intervention can be used in future classrooms and possibly identify other key factors related to the success of this intervention. This information will be collected in two stages: a semi-structure interview and an online survey. Each component will take approximately 30-50 minutes. All information collected is strictly confidential and no identifying markers or characteristics will be published in any capacity. Only the researcher and his thesis supervisor will have access to any information collected on you or your classroom with the exception of the teacher survey. Since the survey is drafted, tracked, and administered online through TKAS, TKAS personnel will have access to this information; however, you have the option to not share your personal data with them by leaving any fields or responses blank. Please note that you log in using an identifying code familiar only to the researcher, so your information in
TKAS is anonymous. Following the completion of the study, you will receive an email inquiring if you are interested in reviewing a summary of the findings.

Prior to beginning the intervention with any classroom, informed consent from each student and their subsequent parent/guardian will be obtained. The consent will allow the researcher to discuss the intervention with the students, examine their work and outcomes for the purposes of the study, and observe and possibly audio-visually record them for the purposes of the researcher’s dissertation. A consent letter will be written and copied and provided to each teacher. If you have a child or family in which English is not their first language and they require a translated copy of the consent and outline of the study, please make the researcher aware of this in person or via email. It would be helpful if you could let the researcher know this information as soon as possible as well as provide him with the language in which it would be best to correspond with those families.

As the researcher intends to use audio-visual recording to capture the activities and behaviours in the classrooms, it is necessary to explain this to the students and guardians/parents. In the event that consent is not obtained for an individual student for audio-visual recordings or to speak with the researcher, it will be necessary to implement a classroom plan to support and respect this request. This plan can be based upon similar school plans where students attend assemblies with audio-visual recordings but do not grant permission to be audio-visually recorded. Any electronic recordings will be done on a secure device that is password protected. After each session, it will be transported on the secured device from the school and transferred to a secured, encrypted USB storage device in accordance with the University of Toronto’s standards. Audio-visual recordings will be used for data analysis purposes and potentially for the researcher’s dissertation and therefore will not be shared with any participants in the study or in any presentations or publications. They will be kept in strict confidence and destroyed according to the University of Toronto’s standards 5 years following the completion of the study.

If you have any questions, concerns, or require more information about the study, please do not hesitate to contact the researcher of the study, Robert Walters. You are also welcome to contact the researcher’s Thesis Supervisor, Dr. Doug McDougall at [redacted] or via email at [redacted]. If you have any questions regarding your rights as a research participant, please contact the University of Toronto’s Office of Research Ethics at 416-946-3273 or via email at ethics.review@utoronto.ca, or REB at TDSB via email EERC@tdsb.on.ca or phone 416-394-4949.

Please sign the attached form, if you agree to participate in the research study outlined above. A secure drop-box has been placed in your school’s office for you to return your consent letter. A copy of this will be provided to you for your records. Once consent has been provided, the researcher will contact you via email to set up the initial semi-structured meeting and survey. Thank you very much for your help.

Sincerely,

Mr. Robert Walters
Teacher
Curriculum, Teaching, and Learning - OISE/University of Toronto
252 Bloor St. West   Toronto, ON   M5S 1V6

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**Teacher Consent Form (CGI and Backward Design Model)**

I acknowledge that the topic and methods of this research study have been explained to me and that any questions that I have asked have been answered to my satisfaction. I understand that I can withdraw at any time without consequence.

- [ ] I consent to participate in this study as described above.
- [ ] I consent to the audio recording of my classes for the purposes of this study (for data analysis purposes only)
- [ ] I consent to the video recording of my classes for the purposes of this study (for data analysis purposes only)
I have read the letter provided to me by Robert Walters and agree to participate in the study for the purposes described.

Signature: __________________________________________________________

Name (printed): __________________________________________ Date: __________________

Mr. Robert Walters
Teacher – [Name]
Curriculum, Teaching, and Learning - OISE/University of Toronto
252 Bloor St. West  Toronto, ON  M5S 1V6
Ph: [Phone number]  Email: [Email]

Please keep a copy of the Consent Form for your records.
Appendix S – Winstrom (2012) CGI Addition Sample Questions

Below, Wistrom (2012) provides the following support to teachers with students focused on learning addition:

- Addition Problems involve a direct or implied action in which a quantity is increased by a particular amount. They are sometimes referred to as "Joining" problems in CGI literature. There are three types of basic addition problems:

1. Result Unknown 15+32+___

2. Change Unknown 15+___=47 (a slightly more difficult problem)

3. Start Unknown ___+32=47 (the most difficult of the basic addition problems)

Below are some sample story problems for each type. These problems can be changed to include familiar names, actions or situations with your students. Be creative! The numbers in the problems can also be altered, depending on the child's mathematical abilities. Feel free to try different combinations or even several numbers to make the problem more challenging.

**Result Unknown:**

- You have 4 books to read. You get 3 more books from the library. How many books do you have now?
- Robin has 98 toy cars. Her parents gave her 45 more toy cars for her birthday. How many toy cars will she have then?
- There are 320 boys in a club. 29 girls join the club. How many children are in the club altogether?
- Jane gave 10 pieces of candy to Sam. Fred gave Sam 6 pieces of gum. Mary gave Sam 12 pieces of gum. How many pieces of gum does Sam have now?

**Change Unknown**

- Kristin had 17 apples. How many more apples will she need to have 39 apples all together?
- Eric has 72 golf balls. He finds some more in the basement. Now Eric has 183 golf balls. How many golf balls did Eric find in the basement?
- Columbus saw 25 whales in the ocean. How many more whales will he need to see to make it 3 dozen whales all together?
- There were 450 kids in the school. Some more kids came to the school. Now there are 920 kids at the school. How many more kids came to the school?
Start Unknown

- Clifford has some bones. Emily gave him 23 more bones. Now Clifford has 46 bones. How many bones did Clifford have to start with?
- Mrs. Wistrom saw some ducks. Then she saw 156 more ducks. All together, Mrs. Wistrom saw 234 ducks. How many ducks did she see to begin with?
- Mom had baked some cookies. Then she baked 4 dozen more cookies. All together, mom has baked 58 cookies. How many cookies did mom bake to begin with?
- Jeff had some leaves in a bag. Then he found 16 more leaves, and put those in the bag. Now Jeff has 76 leaves. How many leaves did Jeff find to begin with?
Appendix T – Sample Teacher Pre- and Post-Word Problem

A. Student wants to buy a new DVD that costs $48.00. In order to save money, Student has decided to walk her neighbour’s dog for a week. Her neighbour has promised to pay Student on the last day. If Student is paid $7 per day, will she have enough money to purchase the DVD at the end of the week? Explain your thinking.

B. The custodian needs to set up the chairs in the gym for our class concert. He plans to arrange the chairs in rows of 7. Each row holds 7 chairs, 3 on the left side of the isle and 4 on the right. If there are 50 parents attending the concert, will there be enough chairs? How do you know?

C. The custodian needs to set up the chairs in the gym for our class concert. After the stage is pulled down, there is enough room for 7 rows. If each row must have the same number of chairs, how many chairs must there be per row in order to seat 50 parents.