Membrane effect of geosynthetic reinforcement subjected to localized sinkholes

<table>
<thead>
<tr>
<th>Journal:</th>
<th>Canadian Geotechnical Journal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manuscript ID</td>
<td>cgj-2017-0592.R1</td>
</tr>
<tr>
<td>Manuscript Type:</td>
<td>Article</td>
</tr>
<tr>
<td>Date Submitted by the Author:</td>
<td>24-Dec-2017</td>
</tr>
<tr>
<td>Complete List of Authors:</td>
<td>Feng, Shijin; Tongji University, Department of Geotechnical Engineering Ai, Shu-Gang; Tongji University, Department of Geotechnical Engineering Chen, Hongxin; Tongji University, Department of Geotechnical Engineering</td>
</tr>
<tr>
<td>Is the invited manuscript for consideration in a Special Issue?</td>
<td>N/A</td>
</tr>
<tr>
<td>Keyword:</td>
<td>Geosynthetic, Membrane effect, Arching effect, Maximum strain, Maximum surface settlement</td>
</tr>
</tbody>
</table>
Membrane effect of geosynthetic reinforcement subjected to localized sinkholes

Shi-Jin Feng\textsuperscript{1*}, Shu-Gang Ai\textsuperscript{2}, and Hong-Xin Chen\textsuperscript{3}

\textsuperscript{1*} Professor, Key Laboratory of Geotechnical and Underground Engineering of the Ministry of Education, Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China (corresponding author). E-mail: fsjgly@tongji.edu.cn

\textsuperscript{2} Ph.D. Candidate, Key Laboratory of Geotechnical and Underground Engineering of the Ministry of Education, Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China. E-mail: aishugang@tongji.edu.cn

\textsuperscript{3} Assistant Professor, Key Laboratory of Geotechnical and Underground Engineering of the Ministry of Education, Department of Geotechnical Engineering, Tongji University, 1239 Siping Road, Shanghai 200092, China. E-mail: chenhongxin@tongji.edu.cn
**Abstract:** Membrane effect often occurs in geosynthetic-reinforced structures, where subsoil may have voids or sinkholes. An analytical model is proposed to estimate membrane effect of geosynthetic reinforcement subjected to localized sinkholes. The upper interface friction in subsided area and vertical deformation of supporting soil in anchorage area are considered simultaneously. The maximum geosynthetic strain and the maximum surface settlement, served as key design points, can be determined. Based on the proposed method verified using a full-scale experiment, a parametric study is conducted. The results show that ignoring upper interface friction results in significant undervaluation of maximum geosynthetic strain, and ignoring vertical deformation of supporting soil leads to obvious undervaluation of maximum surface settlement. A practical design framework is also proposed and it is an applicable tool for preliminary design of geosynthetic-reinforced structures especially for cases with soft ground.

**Key words:** Geosynthetic; Membrane effect; Arching effect; Maximum strain; Maximum surface settlement.
Introduction

Geosynthetic-reinforced structures are extensively used in geotechnical engineering. In such an structure where subsoil may have voids or sinkholes, membrane effect often occurs along with soil arching (e.g., Giroud 1981; Kinney and Connor 1987; Giroud et al. 1990; Wang et al. 1996; Han and Gabr 2002; Rogbeck et al. 2003; Briançon and Villard 2008; Abusharar et al. 2009; Lu and Miao 2015; Feng and Lu 2015; Huckert et al. 2016; King et al. 2017). During a subsiding process, geosynthetic interacts with overlying soil: subsided soil deflects geosynthetic and is also prevented from sinking by the deflected geosynthetic at the same time. Such interactions result in load transmission from subsided zone to surrounding less-deformed zone (i.e., arching effect) and load-deformation coordination of deflected geosynthetic (i.e., membrane effect). The former is evaluated to obtain the load acting on geosynthetic over cavity (e.g., Villard et al. 2016; Feng, et al. 2017a, b); the latter directly decides the maximum tensile strain of geosynthetic and greatly influences the surface settlement (e.g., Briançon and Villard 2008; Villard et al. 2016), both of which are key design points. This study will focus on the force and deformation characteristics in membrane effect, which are important for rational design of geosynthetic-reinforced structures.

To evaluate the anti-settling performance of geosynthetic over cavity, ultimate state and some simplification assumptions are adopted. First of all, it is generally assumed that the geosynthetic is fixed at the edges of cavity and its strain is uniform. Based on this, the geosynthetic is assumed to deflect into regular shapes: circular arc (e.g., Giroud 1981; Giroud et al. 1990; Kuo et al. 2005), parabolic curve (Giroud 1995) and combined parabolic-circular curve (Shukla and Sivakugan 2009). Geosynthetic strain calculated by all these methods only depends on the ratio between deflection...
and cavity width, and is not affected by overburden load and even mechanical properties of geosynthetic. Moreover, there does not exist a point with maximum strain in deflected geosynthetic. Obviously, these fly in the face of the real scenario (e.g., Villard and Briançon 2008; Huckert et al. 2016).

Differently, Espinoza (1994) deduced non-uniform strain of deflected geosynthetic with irregular shape based on its force equilibrium conditions. Then, Gourc and Villard (2000) obtained the maximum deflection and maximum tensile force for serviceability design of geosynthetic-reinforced structures subjected to localized sinkholes. But at this point, the slippage between soil and geosynthetic has not been taken into account. Additionally, Kinney and Connor (1987) firstly divided the deflected geosynthetic into two parts (i.e., subsided and anchorage areas) to consider the deformation coordination. Later, Villard and Briançon (2008) analyzed the relative slippage between soil and geosynthetic in anchorage area based on Coulomb friction law. Besides, Feng and Lu (2015) expanded its applicability to geosynthetic reinforcement over two adjacent voids. However, all the above methods ignored the vertical deformation of supporting soil in anchorage area, which may significantly influence the evaluation of anchoring effect of the geosynthetic in this area and then membrane effect (e.g., Yu and Bathurst 2017), especially for soft ground. Furthermore, the relative slippage between overlying soil and geosynthetic in subsided area has rarely been considered by the existing analytical methods, which changes the external load on deflected geosynthetic. Therefore, it is necessary to improve the evaluation model of membrane effect.

This study presents an effective framework for estimating the membrane effect in geosynthetic-reinforced structures under local subsidence. An analytical model is developed first. The model is then verified using a full-scale experiment. Afterward,
the influences of load distribution types, upper interface friction angle, modulus of subgrade reaction and subsidence width on maximum geosynthetic strain and maximum surface settlement are studied using the proposed model. Finally, a practical design method for geosynthetic-reinforced structures is proposed.

**Formulation and solution**

Fig. 1 depicts a geosynthetic-reinforced structure consisting of embankment fill, geosynthetic and supporting soil. Void or sinkhole in the soil substratum results in the occurrence of soil arching in the overlying soil and membrane effect of the deflected geosynthetic (Fig. 2). During the subsidence, the downward moving soil in the subsided area transfers load to the surrounding areas with less deformation, leading to redistribution of stress acting on the geosynthetic. Meanwhile, combination of the embankment fill and the supporting soil deflects the geosynthetic, as shown in Fig. 2. The arching effect, with representation of load transfer mechanism, can be accurately estimated by the improved Terzaghi formulation reported by Villard et al. (2016). The membrane effect, symbolizing force and deformation characteristics of the deflected geosynthetic, can be described by the proposed model in the later part.

**Model characteristics**

The deflected geosynthetic can be divided into two parts (i.e., subsided area and anchorage area in Fig. 2) and then to be analyzed separately. In the process of subsidence, the geosynthetic in the subsided area is stretched with no interaction with the supporting soil and there is no disengagement between the overlying soil and the geosynthetic (Fig. 2), and the upper interface friction is considered in this area. The geosynthetics on both sides of the subsided area play an anchoring role and the
vertical deformation of the supporting soil should not be neglected. To develop the analytical model, some more assumptions are made as follows:

1. The behavior of the geosynthetic serving as a reinforcement is assumed to be linearly elastic (i.e., $T = J \varepsilon$ where $T$ is the tension in the geosynthetic, $J$ is the stiffness for a unit width of the geosynthetic, and $\varepsilon$ is the strain) and the strain of deflected geosynthetic is always less than its allowable tensile strain, $\varepsilon_a$.

2. The interface friction between soil and geosynthetic agrees with the Coulomb friction law (Fig. 3). If the relative displacement between soil and geosynthetic, $u$, is larger than the critical relative displacement when the interface friction is fully mobilized (e.g., Gilbert and Byrne 1996), $u_0$, the frictional stress can be defined as $\tau = \tau_{\text{max}} = \sigma_n \tan \phi$ where $\tau_{\text{max}}$ is the maximum frictional stress, $\sigma_n$ is the normal stress, and $\phi$ is the interface friction angle. Otherwise, $\tau$ is expressed as

$$\tau = \tau_{\text{max}} = \frac{u}{u_0} \sigma_n \tan \phi \frac{u}{u_0}$$

3. A vertical stress-displacement relationship is assumed for the supporting soil:

$$p - q = k_s w$$

where $q$ and $p$ are respectively the stresses on the upper and lower interfaces between soil and geosynthetic, and are assumed to be distributed along the horizontal direction; $k_s$ is the modulus of subgrade reaction; $w$ is the vertical displacement. The overlying and supporting soils of the geosynthetic in the anchorage area are assumed to have no horizontal displacement. Thus, the tensile displacement of geosynthetic can be used as the relative displacement between soil and geosynthetic.

Membrane effect can be deciphered by coupling the forces and deformations of the geosynthetic in subsided and anchorage areas based on the point A in Fig. 2. Thus,
evaluation of membrane effect can be achieved by four steps: (i) determining the vertical stress acting on the geosynthetic; (ii) analyzing the force and deformation of geosynthetic in the subsided area; (iii) analyzing the force and deformation of geosynthetic in the anchorage area; (vi) calculating the maximum surface settlement caused by the void or sinkhole.

**Determination of load applied to the geosynthetic**

The load applied to the geosynthetic can be determined by Terzaghi’s formulation and is generally assumed to be uniform (Giroud et al. 1990; Villard et al. 2000; Briançon and Villard 2008; Lu and Miao 2015; Feng et al. 2017a, b). There are two types of void: a long void and a circular void (Giroud et al. 1990). For a long void with width \( B_L \), the vertical stress acting on the geosynthetic in the subsided area, \( q_{1L} \), can be expressed as

\[
q_{1L} = \frac{B_L \gamma_s}{2K \tan \phi_s} \left[ 1 - e^{-K \tan \phi_s \left( \frac{2H}{B_L} \right)} \right] + q_0 e^{-K \tan \phi_s \left( \frac{2H}{B_L} \right)}
\]  

(3)

where \( K \) is the lateral pressure coefficient and defined as the ratio between the horizontal and vertical stresses; \( \gamma_s \), \( \phi_s \), and \( H \) are the unit weight, internal friction angle and height of the overlying soil, respectively; \( q_0 \) is the surcharge applied on the ground surface. For a circular void with diameter \( B_C \), \( q_{1C} \) can be expressed as

\[
q_{1C} = \frac{B_C \gamma_s}{4K \tan \phi_s} \left[ 1 - e^{-K \tan \phi_s \left( \frac{4H}{B_C} \right)} \right] + q_0 e^{-K \tan \phi_s \left( \frac{4H}{B_C} \right)}
\]  

(4)

Based on discrete-element modeling and full-scale experiment, Villard et al. (2016) proposed an appropriate value for \( K \) (i.e., 1.2 for sand and 1.5 for coarse granular materials) in Eqs. (3) and (4), and pointed out the vertical load (\( q_{1L} \) or \( q_{1C} \)) acting on the geosynthetic is not uniform. From the cavity opening modes (i.e., progressive process of cavity diameter opening and gradual settlement process), there
are two load types (Fig. 4) for a circular void: approximately conical load and inverted parabolic load. For the two types, the total vertical loads carried by the geosynthetic in the subsided area are the same and determined by Eq. (4) and void size. Thus, the non-uniformly load along the horizontal direction, \( q_{IC}(x) \), can be obtained as

\[
\begin{align*}
\text{Approximately conical load:} & \\
q_{IC1} &= \frac{3rq_{IC}}{3+r}, \quad q_{IC2} = \frac{3q_{IC}}{3+r} \\
q_{IC}(x) &= q_{IC1} + q_{IC2} - \frac{2q_{IC1}x}{B_c}
\end{align*}
\]

or

\[
\begin{align*}
\text{Inverted parabolic load:} & \\
q_{IC1} &= \frac{2rq_{IC}}{2+r}, \quad q_{IC2} = \frac{2q_{IC}}{2+r} \\
q_{IC}(x) &= q_{IC2} + \frac{4q_{IC1}x^2}{B_c^2}
\end{align*}
\]

where \( q_{IC1} \) is the maximum value of non-uniform part of the load, and \( q_{IC2} \) is the value of uniform part (see Fig. 4); \( r \) is the ratio \( q_{IC1}/q_{IC2} \); \( x \) is the distance to the middle of cavity.

**Response of geosynthetic in subsided area**

A geosynthetic reinforcement is symmetrically embedded in the supporting soil with a void (Fig. 2). In order to analyze the response of geosynthetic in subsided area to local subsidence, a half part of the geosynthetic and a differential element with length of \( ds \) are analyzed (e.g., Espinoza 1994; Patra and Shahu 2012, 2015), as shown in Fig. 5. Thus, considering force equilibrium in horizontal and vertical directions, the following equations are obtained:

\[
\begin{align*}
dT_{Lh} &= \tau_{iu} ds \cos \beta_L \\
dT_{Lv} &= q_1 dx + \tau_{iu} ds \sin \beta_L
\end{align*}
\]

where \( T_{Lh} \) and \( T_{Lv} \) are the horizontal and vertical components of tension (\( T_L \)), respectively; \( q_1 \) and \( \tau_{iu} \) are the vertical and frictional stresses acting on the upper interface, respectively; \( \beta_L \) is the angle between tangential direction of \( ds \) and the
horizontal direction; $x$ is the horizontal distance from the differential element to the origin of $x$-$y$ cartesian coordinate system (Fig. 5). Then, dividing Eqs. (6a) and (6b) by $dx$ gives

$$\tau_{iu} = \frac{dT_{lh}}{dx}$$  \hspace{1cm} (7a)$$

$$q_i = \frac{dT_{lv}}{dx} - \tau_{iu} \tan \beta_L$$  \hspace{1cm} (7b)$$

And based on geometry of the deflected geosynthetic, the following relationship can be established:

$$T_h = T_l(x) \cos \beta_L(x), \quad T_{lv} = T_l(x) \sin \beta_L(x)$$

$$y'(x) = \tan \beta_L(x) = \frac{dy}{dx}$$

which gives

$$T_{lv} = T_{lh} y'(x)$$  \hspace{1cm} (9)$$

Taking the first-order derivative of Eq. (9) and placing the resulting equation into Eqs. (7a) and (7b), yield

$$q_i = T_{lh} y''(x)$$  \hspace{1cm} (10)$$

Invoking the assumption (1) and the strain-displacement relationship of deflected geosynthetic, the following equations can be given:

$$T_{lh} (x) = J e(x) \cos \beta_L = J \frac{e(x)}{\sqrt{1 + y'^2}}$$

$$u'(x) = e(x) \sqrt{1 + y'^2(x)}$$

which gives

$$T_{lh} (x) = J \frac{u(x)}{1 + y'^2}$$  \hspace{1cm} (12)$$

where $u$ is the tensile displacement of a $s$-length geosynthetic from the origin. Substituting Eq. (12) in Eq. (10), yields
Additionally, by taking the first-order derivative of Eq. (12) and substituting the resulting equation and Eq. (13) into Eq. (7a), yields

\[ u''(x) = 2 \frac{q_{1}}{J} y'(x) + \tau_{1u} \frac{1 + y'^{2}(x)}{u(x)} \]  

(14)

Invoking the assumption (2), \( \tau_{1u} \) can be formulated as

\[
\begin{align*}
\tau_{1u} &= q_{1} \cos^{2} \beta_{l} \tan \phi_{u} \frac{u(x)}{u_{0}} & \text{if } u < u_{0} \\
\tau_{1u} &= q_{1} \cos^{2} \beta_{l} \tan \phi_{u} & \text{if } u \geq u_{0}
\end{align*}
\]  

(15)

where \( \phi_{u} \) is the upper interface friction angle between soil and geosynthetic. Thus, by substituting Eq. (15) into Eq. (14) and combining the resulting expression and Eq. (13), the governing equations representing the force and deformation of deflected geosynthetic in subsided area can be obtained:

\[
\begin{align*}
\begin{cases}
\tau_{1u} &= q_{1} \cos^{2} \beta_{l} \tan \phi_{u} \frac{u(x)}{u_{0}} & \text{if } u < u_{0} \\
\tau_{1u} &= q_{1} \cos^{2} \beta_{l} \tan \phi_{u} & \text{if } u \geq u_{0}
\end{cases}
\end{align*}
\]

(16)

Response of geosynthetic in anchorage area

Similar to the subsided area, the force equilibrium of a differential element, \( dS \), from the geosynthetic in anchorage area (Fig. 6) also yields

\[
\begin{align*}
\tau_{2u} + \tau_{2l} &= \frac{dT_{Rh}}{dX} \\
p_{2} - q_{2} &= \frac{dT_{Rv}}{dX} - (\tau_{2u} + \tau_{2l}) \tan \beta_{R}
\end{align*}
\]  

(17a, 17b)

where \( T_{Rh} \) and \( T_{Rv} \) are the horizontal and vertical components of tension (\( T_{R} \)) for the
geosynthetic in anchorage area, respectively; $q_2$ and $p_2$ are the vertical stresses acting on the upper and lower interfaces between soil and geosynthetic, respectively; $\tau_{2u}$ and $\tau_{2l}$ are the frictional stresses acting on the upper and lower interfaces, respectively; $\beta_R$ is the angle between tangential direction of $dS$ and the horizontal direction; $X$ is the horizontal distance from the differential element to the origin of $X-W$ coordinate system (Fig. 6). For the anchorage area, similar equations to Eqs. (10), (12) and (15) can be reformulated:

$$p_2 - q_2 = T_{Rh} W_\gamma(X)$$

$$T_{Rh}(X) = J \frac{U(X)}{1 + W'^2(X)}$$

$$\begin{cases}
\tau_{2u} + \tau_{2l} = (q_2 \tan \phi_u + p_2 \tan \phi_l) \cos^2 \beta_R \frac{U(X)}{u_0} & \text{if } U < u_0 \\
\tau_{2u} + \tau_{2l} = (q_1 \tan \phi_u + p_2 \tan \phi_l) \cos^2 \beta_R & \text{if } U \geq u_0
\end{cases}$$

where $U$ is the tensile displacement of a $S$-length geosynthetic from the origin of $X-W$ coordinate system; $\phi_l$ is the lower interface friction angle. Then, invoking the assumption (3) for Eq. (18) and substituting Eq. (19) into the resulting equation yield

$$W_\gamma(X) = \frac{k_s W(X)[1 + W'^2(X)]}{J U'(X)}$$

Similarly, by taking the first-order derivative of Eq. (19), substituting the resulting expression and Eq. (20) into Eq. (17a) and then combining these equations and Eq. (21), the governing equations representing the force and deformation of deflected geosynthetic in anchorage area can also be given:
\[
U''(X) = \begin{cases} 
2k_j \frac{W(X)W'(X)}{J} + \frac{q_s (\tan \phi_s + \tan \phi_0)}{J} U(X) + \frac{k_j W(X) \tan \phi}{u_0} \frac{U(X)}{u_0} & \text{if } U < u_0 \\
2k_j \frac{W(X)W'(X)}{J} + \frac{q_s (\tan \phi_s + \tan \phi_0)}{J} + \frac{k_j W(X) \tan \phi}{J} & \text{if } U \geq u_0 \\
\end{cases}
\]

(22)

Next, considering the overall equilibrium at point A in Fig. 2 as the connecting point between the subsided area (Fig. 5) and the anchorage area (Fig. 6), the coupling relationship can be obtained as follows:

\[
\begin{align*}
T_{AL} &= T_{AR} \\
\beta_{AL} &= \beta_{AR}
\end{align*}
\]

(23)

where \(T_{AL}\) and \(T_{AR}\) are the tensile forces of the deflected geosynthetic at point A at the right edge of subsided area (Fig. 5) and the left edge of anchorage area (Fig. 6), respectively; \(\beta_{AL}\) and \(\beta_{AR}\) are the intersection angles on both sides of point A. Moreover, the compatibility of deformation occurs in the deflected geosynthetic and can be expressed as

\[(s_L + s_R) - \left(\frac{B}{2} + L\right) = u_{AL} + U_{AR} \]

(24)

where \(L\) is the initial length of geosynthetic in anchorage area; \(s_L\) and \(s_R\) are the lengths of deflected geosynthetic in subsided and anchorage areas, respectively; \(u_{AL}\) and \(U_{AR}\) are the displacements on both sides of point A, respectively. Additionally, the boundary conditions are as follows: (1) for the subsided area, \(u(x=0)=0, y(x=0)=0\) and \(y'(x=0)=0\); (2) for the anchorage area, \(U(X=0)=0, or U''(X=0)=10^{-12}\) and \(W'(X=0)=0\). Thus, by combining the governing equations (16) and (22), coupling relationship, deformation compatibility and boundary conditions, the membrane effect symbolizing force and deformation characteristics of the deflected geosynthetic over cavity can be evaluated effectively.
**Prediction of maximum surface settlement**

The design of a geosynthetic-reinforced structure depends on the stiffness requirement of geosynthetic and uneven surface settlement criterion. The maximum tensile strain ($\varepsilon_{\text{max}}$) of deflected geosynthetic, which can be calculated by the proposed method in this study, must be less than the allowable tensile strain ($\varepsilon_a$). The ratio ($A_e$) between $\varepsilon_{\text{max}}$ and $\varepsilon_a$ is called the anchorage coefficient (i.e., $A_e = \varepsilon_{\text{max}}/\varepsilon_a$), which reflects the anchoring effect of geosynthetic in anchorage area as well as the membrane effect. For the same geosynthetic reinforcement, geosynthetic strain is proportional to its tensile force, and the larger $\varepsilon_{\text{max}}$ at point A, the greater anchoring effect. On the other hand, the membrane effect of geosynthetic depends on its stretch degree after subsidence, also symbolized by $A_e$. When $A_e = 1$, the membrane effect has been fully developed. In contrast, when the subsoil has no voids or sinkholes, the membrane effect and anchoring effect are not developed (i.e., $A_e = 0$).

In addition, there exists a limitation value of surface settlement ($s_a$), corresponding to one subsidence width ($B_L$ or $B_C$), defined by the RAFAEL method (Villard et al. 2000, 2016; Briançon and Villard 2008) for meeting the surface settlement requirement. The RAFAEL method considers a cylindrical collapse over the cavity, a parabolic shape of the deflected geosynthetic and the ground surface, and soil expansion with a global expansion factor. For a long void with width $B_L$, the maximum surface settlement, $s_{\text{max}}$, can be determined as

$$s_{\text{max}} = f_{\text{max}} - 3H \frac{(C_e - 1)}{2}$$

(25)

where $C_e$ is the expansion coefficient and defined as the ratio between the soil volume ($V_s$) in subsided area after subsidence and the initial soil volume ($V_0$) before subsidence ($C_e = V_s/V_0$); $f_{\text{max}}$ is the maximum vertical displacement of deflected
For a circular void with diameter $B_C$, $s_{\text{max}}$ can also be determined as

$$s_{\text{max}} = f_{\text{max}} - 2H(C_e - 1) \quad (26)$$

**Solution**

The membrane effect is jointly decided by the stresses and deformations of geosynthetic in the subsided and anchorage areas. For geosynthetic in the subsided area, there may exist two kinds of deformation response based on the assumption (2):

(a) $u_{\text{AL}} < u_0$, $[0, B/2]$ governed by Eq. (16 $1+3'$); (b) $u_{\text{AL}} \geq u_0$, $[0, x_C]$ governed by Eq. (16 $1+3'$) and $[x_C, B/2]$ governed by Eq. (16 $2+3'$). And likewise, for the geosynthetic in the anchorage area, there may exist three kinds of responses: (a) $U(X=0) \geq u_0$, $[0, L]$ governed by Eq. (22 $2+3'$); (b) $U(X=0) < u_0$ and $U_{\text{AR}} < u_0$, $[0, L]$ governed by Eq. (22 $1+3'$); (c) $U(X=0) < u_0$ and $U_{\text{AR}} \geq u_0$, $[0, X_C]$ governed by Eq. (22 $1+3'$) and $[X_C, L]$ governed by Eq. (22 $2+3'$). $x_C$ and $X_C$ are the horizontal distances from the point with $u_0$ to the origin for the subsided and anchorage areas, respectively.

An iterative method is adopted for the whole solution, as shown in Fig. 7. To begin with, an arbitrary initial value of $u'(x=0)$ is assumed. The response of the deflected geosynthetic in the subsided area is solved by combining the governing equations Eq. (16) and the boundary conditions (1). Next, arbitrary initial values are assigned to $U(X=0)$, or $U'(X=0)$ and $W(X=0)$. At this point, the response of the deflected geosynthetic in the anchorage area is also solved by combining the governing equations Eq. (22) and the boundary conditions (2). Then, by updating these unknown boundary values and executing iterative calculation for the anchorage area, the coupling condition Eq. (23) is satisfied. Additionally, updating the value of $u'(x=0)$ makes the elongation of geosynthetic to meet the compatibility deformation
condition Eq. (24). Thus, the membrane effect of geosynthetic for local subsidence can be evaluated through the solution procedures.

**Model verification**

This section compares the results of the analytical calculations by this study and the full-scale experimental tests conducted by Villard and Briançon (2008). Table 1 shows the geometry and material properties of soil and geosynthetic. For ease of comparison, the same simplifications from Villard and Briançon (2008) are made here: the arching effect in the overlying soil after subsidence is ignored, and the load over the geosynthetic is assumed to be uniformly distributed (i.e., \( q_1 = q_2 = \gamma H = 8.5 \text{ kPa} \)).  

The modulus of subgrade reaction, \( k_s \), which is close to the elastic modulus \( (E_s) \) as a result of \( \nu = 0.3 \) (Daloglu and Vallabhan 2000), is assumed to be 30 MPa/m. The strain and vertical displacement of deflected geosynthetic are compared, as shown in Fig. 8.

In Fig. 8, the values of strain and vertical displacement of deflected geosynthetic in the anchorage area, calculated by the present method, are close to those of the full-scale experimental tests. It is noteworthy that the vertical deformation of supporting soil after subsidence is well described by the present method, as shown in Fig. 8b. In the subsided area, the results of the present method also match those of the experimental tests reasonably well. Obviously, from Fig. 8a, the maximum strain of deflected geosynthetic occurs at point A. Thus, the method in this study can be used to estimate the membrane effect of deflected geosynthetic over cavity.
Parametric study

Effect of load type on geosynthetic

In this part, three types of distributed load are considered (Fig. 9a): uniform load – UL, conical load – CL and inverted parabolic load – IPL. The load applied to the geosynthetic can be determined: $q_1$ in Eq. (16) = $q_{1C}$ in Eq. (4) or $q_{1C}(x)$ in Eq. (5), and $q_2 = \gamma_s H$. To be consistent with Villard et al. (2016), the experimental test conducted by Huckert et al. (2016) is only adopted for this analysis. The needed parameters are shown in Table 2. And based on the numerical simulations reported by Villard et al. (2016), $K = 1.3$ (Eq. 4) and $r = q_{1C1}/q_{1C2} = 4$ (Eq. 5) were obtained when $B_c/H = 2.2$.

Fig. 9 compares the strains ($\varepsilon$) and vertical displacements ($y$) of deflected geosynthetic between experimental results (Huckert et al. 2016) and analytical calculations (i.e., Villard et al. 2016 and this study). As shown in Fig. 9, ranking the strains and vertical displacements of deflected geosynthetic under the three load types, it is clear that CL > UL > IPL, and the difference is mainly embodied in the subsided area. With the same total load, load UL is uniformly distributed along the geosynthetic, load CL is mainly distributed near the middle of subsided area while load IPL is mainly distributed near point A according to Eq. (5). The different load distributions along the geosynthetic cause the significantly different $\varepsilon$ and $|y|$. Thus, it is crucial to accurately determine load distribution type when considering membrane effect. Additionally, compared with load IPL, the values of $\varepsilon$ and $|y|$ under load CL are closer to the experimental results of Huckert et al. (2016), since the latter is generated by a progressive process of cavity diameter opening similar to the full-scale experiment.

From Fig. 9a, the effect of load type on the variation of geosynthetic strain in the subsided area is more significant for the present method than the analytical method.
proposed by Villard et al. (2016). From the middle of subsided area to its edge, the changing curves (UL, CL, and IPL) calculated by the former are steeper than those by the latter. The main reason is that the upper interface friction in the subsided area is considered by the present method but ignored by the method proposed by Villard et al. (2016). From Fig. 2, the upper interface friction promotes the stretch of geosynthetic relative to point A and the anchoring effect ($A_e$ or $\varepsilon_{\text{max}}$).

In order to further investigate the variations of strain and vertical displacement of geosynthetic in Fig. 9, the variations of relative displacement ($u$ or $U$) between soil and geosynthetic along the interface under different load types are shown in Fig. 10. From the marginal edges of subsided and anchorage areas to the junction point A (see Fig. 2), $u$ and $U$ both gradually increase. And based on the compatibility condition (Eq. 24), the summation of relative displacement is equal to the difference between the length of deflected geosynthetic and the initial length prior to deflection. When $u > u_0$ or $U > u_0$, the interface slippage occurs and thus there exists some critical sliding points ($x_C$ in subsided area and $X_C$ in anchorage area) in Fig. 10. Same as $y$ or $\varepsilon$ in Fig. 9, CL > UL > IPL for $u$ and $U$. Thus, the higher load concentration level at the middle of subsided area leads to the larger relative displacement ($u$ and $U$), stronger membrane effect ($\varepsilon_{\text{max}}$ and $A_e$), greater geosynthetic deflection ($y_{\text{max}}$) and maximum surface settlement ($s_{\text{max}}$).

However, the ratio $r$ in Eq. (5) determining the non-uniformly distributed load, which changes with $B_C/H$, still cannot be obtained by means of an analytical method. Thus, load UL applied to the geosynthetic in the subsided area is adopted for analysis in the following parts. And based on the model verification, the experimental test conducted by Villard and Briançon (2008) is also adopted.
**Effects of width-height ratio and anchorage length**

Fig. 11 shows the variations of maximum geosynthetic strain and maximum surface settlement with width-height ratio and anchorage length. All the required parameters are shown in Table 1 except $L$ and $H$. The expansion coefficient of overlying soil, $C_e$, in Eq. (25) is 1.1, varying between the recommended range (1.03 ~ 1.15) reported by Villard et al. (2016). The arching effect is considered: $q_1$ in Eq. (16) = $q_1L$ in Eq. (3) with $K = 1.3$, and $q_2 = \gamma H$. As shown in Fig. 11, with increasing $B/H$, $\varepsilon_{\text{max}}$ decreases and $s_{\text{max}}$ increases. Under constant $B$, larger $B/H$ means smaller $H$, leading to smaller overlying load ($q_1$ and $q_2$), and then smaller $\varepsilon_{\text{max}}$ and $f_{\text{max}}$. And from Eq. (25), $s_{\text{max}}$ increases with $f_{\text{max}}$ but decreases with $H$. Thus, the increase of $s_{\text{max}}$ is mainly caused by a prevailing effect of $H$ instead of $f_{\text{max}}$. Additionally, from Fig. 11b, $s_{\text{max}}$ approaches zero when $B/H < 1.25$ (i.e., $H > 1.6$ m). Thus, for the cases with greater $H$, the cavity below the geosynthetic does not affect the ground surface.

Then, $\varepsilon_{\text{max}}$ and $s_{\text{max}}$ for different $L$ are also analyzed. The anchoring effect ($A_e$) determined by $\varepsilon_{\text{max}}$ is influenced by the length of geosynthetic in the anchorage area ($L$). When $L$ is close to zero, there is almost no anchoring effect ($A_e \rightarrow 0$). The exertion of anchoring effect ($A_e$) relies on lengthening $L$. When $B/H \geq 2.5$ (i.e., $H \leq 0.8$ m), $\varepsilon_{\text{max}}$ for the case of $L = 1.5$ m is obviously smaller than that of $L = 2.0$ or 2.5 m (Fig. 11a). When $B/H < 2.5$ (i.e., $H > 0.8$ m), $\varepsilon_{\text{max}}$ with different $L$ are almost the same, which reveals that the anchoring effect from the geosynthetic in the anchorage area has been fully developed even for $L = 1.5$ m. When subsidence happens, the anchoring effect of geosynthetic near point A first develops by means of its overlying and underlying soils. The higher overlying soil in the anchorage area, the greater anchoring effect. When $H > 0.8$ m, the anchoring effect has been fully developed and lengthening $L$ has no favoring effect. Similar to $\varepsilon_{\text{max}}$, when $L \geq 2.5$ m for $B/H = 1.0$.  

https://mc06.manuscriptcentral.com/cgj-pubs
4.0, the change in \( L \) has no effect on \( s_{\text{max}} \) (Fig. 11b). Thus, the fixed length of geosynthetic (\( L = 2.5 \) m) can be used for analyses in the following parts.

Fig. 11 also compares the results of three analytical calculations: Villard and Briançon (2008), the modified method with consideration of upper interface friction in the subsided area and the present method. From Fig. 11a, \( \varepsilon_{\text{max}} \) is undervalued by the first method but slightly overvalued by the second method. And in Fig. 11b, \( s_{\text{max}} \) is undervalued by both the first and the second methods. Thus, the present method is very necessary for rational design of geosynthetic-reinforced structures.

**Effects of upper interface friction angle and modulus of subgrade reaction**

Figs. 12 and 13 respectively show the effects of upper interface friction angle and modulus of subgrade reaction on the strain and vertical displacement of deflected geosynthetic. All the required parameters are shown in Table 1 except \( k_s \) and \( \phi_u \).

Under the same load, the upper interface friction in the subsided area is reflected by \( \phi_u \). From Fig. 12, with increasing \( \phi_u \), \( \varepsilon_{\text{max}} \) increases obviously and \( f_{\text{max}} \) decreases slightly. Thus the change in \( \phi_u \) has greater influence on \( \varepsilon_{\text{max}} \) but less influence on \( f_{\text{max}} \).

Observing Fig. 2, it is obvious that the increase of \( \phi_u \) enhances the interface friction on both sides of point A and then promotes the anchoring effect, which enlarges \( \varepsilon_{\text{max}} \). The increase of interface friction in the anchorage area hinders the deflection of geosynthetic, but that in the subsided area is on the contrary. Furthermore, the former plays a dominant role, resulting in the slight decrease of \( f_{\text{max}} \).

According to the assumption (3) or Eq. (2), the modulus of subgrade reaction, \( k_s \), governs the vertical deformation of supporting soil. As shown in Fig. 13, with increasing \( k_s \), \( \varepsilon_{\text{max}} \) increases slightly and \( f_{\text{max}} \) decreases obviously. And the increasing (\( \varepsilon_{\text{max}} \)) and decreasing (\( f_{\text{max}} \)) rates moderate with the increase of \( k_s \). The smaller vertical
deformation of supporting soil caused by larger $k_s$ promotes the anchoring effect of geosynthetic in the anchorage area. However, this hinders the deflection of geosynthetic in the anchorage area, hence leading to smaller $f_{\text{max}}$ with increasing $k_s$. Thus, $\phi_u$ and $k_s$ should be synthetically considered for satisfying the stiffness requirement of geosynthetic and maximum surface settlement criterion.

**Effect of subsidence width**

Fig. 14 shows the effect of subsidence width on the strain and vertical displacement of deflected geosynthetic over cavity. Due to the assumption that the geosynthetic in the subsided area has no supporting soil, $B$ is regarded as the unique parameter representing the local subsidence condition. Same as before, all the required parameters are shown in Table 1 except $B$. As shown in Fig. 14, at any point of geosynthetic, $\varepsilon$ and $|y|$ increase significantly with increasing $B$. For the cases of $B = 1.5, 2.0$ and $2.5$ m, the obtained $\varepsilon_{\text{max}}$ and $f_{\text{max}}$ are $1.37\%, 1.83\%$, $2.24\%$ and $15.77$, $23.06$, $31.02$ cm, respectively. The increase of subsidence width not only enlarges the vertical load acting on the geosynthetic in subsided area but also extends the contact length between soil and geosynthetic, which promotes the membrane effect. Fig. 15 shows the variations of relative displacement ($u$ or $U$) between soil and geosynthetic along the interface for different $B$. From Fig. 15, with increasing $B$, the critical sliding points ($x_C$ in subsided area and $X_C$ in anchorage area) move to the opposite direction, away from the junction point A. Different from $H$, the increase of $B$ only leads to the increase of the overlying load acting on the geosynthetic in the subsided area. The wider subsided area leads to the greater anchoring effect. In conclusion, the change in $B$ has significant influence on $\varepsilon_{\text{max}}$ and $f_{\text{max}}$ or $s_{\text{max}}$, and accurate determination of subsidence width is crucial to the design of geosynthetic-reinforced structures.
Practical design

This section proposes a practical design framework for geosynthetic-reinforced structures under local subsidence including the design charts, design method and an example application. It is noteworthy that the design framework is only applicable for the sheet-shape geosynthetic reinforcements (e.g., geotextiles), but not for the hole-shaped geosynthetic reinforcements (e.g., geogrids). When using the latter, the resistance resulting from the interaction between the soil strike-through within the apertures and the transverse ribs will play a significant role in the reinforcing effect, which is not considered by the proposed method.

First of all, the design charts (i.e., Fig. 16) are proposed based on the present method. The design charts give the maximum geosynthetic strain and maximum surface settlement for a given geometry. During a design process, the maximum geosynthetic strain must be less than the corresponding allowable tensile strain ($\varepsilon_a$), and the maximum surface settlement must meet the maximum surface settlement criterion (less than $s_a$). Only in this way can we obtain the optimal design.

Then, the design method for geosynthetic reinforcement subjected to localized sinkholes is introduced on the basis of design charts. The design consists of determining the stiffness of geosynthetic over cavity for a width-height ratio and a given maximum surface settlement criterion. For a given stiffness of geosynthetic, $\varepsilon_a$ is also known. In general, the anchorage length of geosynthetic is also seen as a key design point and can be determined reasonably by the same way as Fig. 11. To control the computation, a fixed length of geosynthetic is adopted in this section, e.g. $L = 2.5$ m the same as that in the previous analysis. Now, we know all the parameters from Table 1 except the stiffness of geosynthetic. On condition that the interface parameters between soil and geosynthetic are known, we can select a geosynthetic reinforcement
based on a determined $J_S$ (short-term stiffness for geosynthetic). The long-term performance of geosynthetic is considered by adopting the long-term stiffness for geosynthetic ($J_L$), which can be determined as

$$J_L = J_S / (R_C \times R_{ID} \times R_{CBD})$$

by taking into account the reduction factors for creep $R_C$, installation damage $R_{ID}$ and chemical and biological degradation $R_{CBD}$. The specific design can be divided into three steps: (i) find a value of $J_L$ for a given $B/H$ according to Fig. 16b to comply with the maximum surface settlement criterion (i.e., $s_a$); (ii) determine $\varepsilon_{\text{max}}$ of deflected geosynthetic by placing the pair of values (i.e., a given $B/H$ and determined $J_L$ in the first step) into Fig. 16a and check if it is less than $\varepsilon_a$; (iii) repeating the first two steps and find a $J_L$ until $\varepsilon_{\text{max}} < \varepsilon_a$.

Finally, an example application is done based on the proposed design process. The value of $s_a$ is given as 10 cm. For the case of $B/H = 2.0$, we can determine $J_L = 1340$ kN/m$^3$ in Fig. 16b based on the prerequisite of $s_{\text{max}} = s_a = 10$ cm. The value of $\varepsilon_a$ is 2.5% for $J_L = 1340$ kN/m$^3$. Then, we can obtain $\varepsilon_{\text{max}} = 2.46\%$ by using Fig. 16a. We find $\varepsilon_{\text{max}} < \varepsilon_a$ and thus the selected geosynthetic can be served as an applicable reinforcement for the case.

**Conclusions**

In this study, an analytical model is proposed to estimate membrane effect of geosynthetic reinforcement subjected to localized sinkholes. In subsided area, the upper interface friction is considered and the relative slippage between overlying soil and deflected geosynthetic is also analyzed based on Coulomb friction law. In anchorage area, the vertical deformation of supporting soil is considered. The strain of geosynthetic and ground surface settlement under different load types, different anchorage lengths or width-height ratios, different upper interface friction angles or
moduli of subgrade reaction, and different subsidence widths are investigated. The major conclusions are as follows:

(1) Overlying vertical load on geosynthetic and its distribution type with consideration of arching effect significantly influence membrane effect of deflected geosynthetic. In cases when the geosynthetic in anchorage area is well anchored, the larger vertical load applied near the middle of subsided area leads to greater membrane effect.

(2) Anchoring effect first develops near junction point A (see Fig. 2) and then spreads toward the marginal edge of anchorage area during a subsiding process. Increasing the anchorage length can promote the anchoring ability before the anchoring effect is fully developed.

(3) Ignoring upper interface friction results in undervaluation of maximum geosynthetic strain but overvaluation of maximum surface settlement. Anchoring effect is highly influenced by the vertical deformation of supporting soil. Without considering this, the anchoring effect is overestimated, resulting in undervaluation of maximum surface settlement.

(4) Design charts for geosynthetic reinforcement under local subsidence are given based on the present method. A practical design method is proposed and illustrated by an example case. The design framework is an applicable tool for preliminary design of geosynthetic-reinforced structures.

**List of symbols**

- $A_e$ anchorage coefficient
- $B$ subsidence width (m)
- $B_C$ diameter of a circular void (m)
\( B_L \) width of a long void (m)
\( C_e \) expansion coefficient of overlying soil
\( E_s \) Young’s modulus (MPa)
\( f_{\text{max}} \) maximum vertical displacement of deflected geosynthetic (m)
\( H \) height of overlying soil (m)
\( J \) tensile stiffness of geosynthetic (kN/m)
\( k_s \) modulus of subgrade reaction (MPa/m)
\( K \) lateral pressure coefficient of overlying soil
\( L \) anchorage length (m)
\( p_2, q_2 \) vertical stresses on lower and upper interfaces between soil and geosynthetic for anchorage area (kPa)
\( q_0 \) surcharge on ground surface (kPa)
\( q_1 \) vertical stress on upper interface for subsided area (kPa)
\( q_{1C}, q_{1L} \) vertical stresses acting on deflected geosynthetic over a circular void and long void (kPa)
\( q_{1C1}, q_{1C2} \) maximum values of non-uniform and uniform parts of \( q_{1C} \) (kPa)
\( r \) ratio \( q_{1C1}/q_{1C2} \)
\( s \) length of deflected geosynthetic (m)
\( s_{L}, S_R \) lengths of deflected geosynthetic for subsided and anchorage areas (m)
\( s_{\text{max}} \) maximum surface settlement (m)
\( T_L \) tension in geosynthetic for subsided area (kN)
\( T_{Lh}, T_{Lv} \) horizontal and vertical components of \( T_L \) (kN)
\( T_{Rh}, T_{Rv} \) horizontal and vertical components of tension in geosynthetic for anchorage area (\( T_R \)) (kN)
\( T_{AL}, T_{AR} \) left and right tensile forces of point A (kN)
\( u, U \) relative displacements between soil and geosynthetic for subsided and anchorage areas (m)
\( u_0 \) critical relative displacement (m)
\( u_{AL}, U_{AR} \) left and right displacements of point A (m)
\( x, X \) horizontal distances to origin in \( x-y \) and \( X-W \) coordinate systems (m)
\( x_c, X_c \) horizontal distances from point with \( u_0 \) to origin for subsided and anchorage areas (m)
\( y, W \) vertical distances to origin in \( x-y \) and \( X-W \) coordinate systems (m)
\( \beta_L, \beta_R \) angles between tangential direction of \( ds \) and horizontal direction for subsided and anchorage areas (°)
\( \beta_{AL}, \beta_{AR} \) left and right intersection angles of point A (°)
\( \phi_l, \phi_u \) lower and upper interface friction angles (°)
\( \tau_{1u} \) frictional stress on upper interface for subsided area (kPa)
\( \tau_{2l}, \tau_{2u} \) frictional stresses on lower and upper interfaces for anchorage area (kPa)
\( \tau_{\text{max}} \) maximum frictional stress (kPa)

**Acknowledgments**

Much of the work described in this paper was supported by the National Natural Science Foundation of China under Grant Nos. 41572265 and 41661130153, the Shanghai Shuguang Program under Grant No. 16SG19, the National Program for Support of Top-Notch Young Professionals, and the Newton Advanced Fellowship of the Royal Society under Grant No. NA150466. The writers would like to greatly acknowledge all these financial supports and express their most sincere gratitude.
References


List of Table Captions

Table 1. Geometry and material properties of the soil and geosynthetic.

Table 2. Parameters taken from experimental studies conducted by Huckert et al. (2016).
Table 1. Geometry and material properties of the soil and geosynthetic.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>Cavity width, $B_L$ (m)</td>
<td>2</td>
</tr>
<tr>
<td>Height of overlying soil, $H$ (m)</td>
<td>0.5</td>
</tr>
<tr>
<td>Anchorage length, $L$ (m)</td>
<td>4</td>
</tr>
<tr>
<td><strong>Overlying soil</strong></td>
<td></td>
</tr>
<tr>
<td>Unit weight, $\gamma_s$ (kN/m$^3$)</td>
<td>17</td>
</tr>
<tr>
<td>Internal friction angle, $\phi_s$ (°)</td>
<td>30</td>
</tr>
<tr>
<td>Cohesion, $c_s$ (kPa)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Geosynthetic</strong></td>
<td></td>
</tr>
<tr>
<td>Tensile stiffness, $J$ (kN/m)</td>
<td>1100</td>
</tr>
<tr>
<td>Critical relative displacement between soil and geosynthetic, $u_0$ (m)</td>
<td>0.005</td>
</tr>
<tr>
<td>Upper interface friction angle between soil and geosynthetic, $\phi_u$ (°)</td>
<td>30</td>
</tr>
<tr>
<td>Lower interface friction angle between soil and geosynthetic, $\phi_l$ (°)</td>
<td>25</td>
</tr>
<tr>
<td><strong>Supporting soil</strong></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus, $E_s$ (MPa)</td>
<td>30</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td>Surcharge applied on the ground surface, $q_0$ (kPa)</td>
<td>0</td>
</tr>
</tbody>
</table>
Table 2. Parameters taken from experimental studies conducted by Huckert et al. (2016).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Geometry</strong></td>
<td></td>
</tr>
<tr>
<td>Cavity diameter, $B_C$ (m)</td>
<td>2.2</td>
</tr>
<tr>
<td>Height of overlying soil, $H$ (m)</td>
<td>1</td>
</tr>
<tr>
<td>Anchorage length, $L$ (m)</td>
<td>3</td>
</tr>
<tr>
<td><strong>Overlying soil</strong></td>
<td></td>
</tr>
<tr>
<td>Unit weight, $\gamma_s$ (kN/m$^3$)</td>
<td>15.65</td>
</tr>
<tr>
<td>Internal friction angle, $\phi_s$ (°)</td>
<td>36</td>
</tr>
<tr>
<td>Cohesion, $c_s$ (kPa)</td>
<td>0</td>
</tr>
<tr>
<td><strong>Geosynthetic</strong></td>
<td></td>
</tr>
<tr>
<td>Tensile stiffness, $J$ (kN/m)</td>
<td>2988</td>
</tr>
<tr>
<td>Critical relative displacement between soil and geosynthetic, $u_0$ (m)</td>
<td>0.005</td>
</tr>
<tr>
<td>Upper interface friction angle between soil and geosynthetic, $\phi_u$ (°)</td>
<td>23</td>
</tr>
<tr>
<td>Lower interface friction angle between soil and geosynthetic, $\phi_l$ (°)</td>
<td>40</td>
</tr>
<tr>
<td><strong>Supporting soil</strong></td>
<td></td>
</tr>
<tr>
<td>Young’s modulus, $E_s$ (MPa)</td>
<td>19</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu$</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Surcharge applied on the ground surface, $q_0$ (kPa)</strong></td>
<td>0</td>
</tr>
</tbody>
</table>
List of Figure Captions

Fig. 1. Schematic diagram of geosynthetic-reinforced structure.

Fig. 2. Mechanism of collapse during local subsidence.

Fig. 3. Coulomb friction law.

Fig. 4. Two types of load reported by Villard et al. (2016).

Fig. 5. Equilibrium of an elementary section in subsided area.

Fig. 6. Equilibrium of an elementary section in anchorage area.

Fig. 7. Solution procedures of estimating the membrane effect.

Fig. 8. Comparison between experimental and analytical results: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.

Fig. 9. Strain and vertical displacement of geosynthetic for different load types acting on the geosynthetic in subsided area: (a) strain; (b) vertical displacement.

Fig. 10. Variations of relative displacement between soil and geosynthetic under different load types.

Fig. 11. Variations of maximum geosynthetic strain (a) and maximum surface settlement (b) with width-height ratio and anchorage length of geosynthetic.

Fig. 12. Influence of upper interface friction angle between soil and geosynthetic on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.

Fig. 13. Influence of modulus of subgrade reaction on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.

Fig. 14. Influence of width of subsided area on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.

Fig. 15. Variations of relative displacement between soil and geosynthetic with different widths of subsided area.

Fig. 16. Design chart of stiffness of geosynthetic based on its maximum strain and maximum surface settlement.
Fig. 1. Schematic diagram of geosynthetic-reinforced structure.
Fig. 2. Mechanism of collapse during local subsidence.
Fig. 3. Coulomb friction law.
Fig. 4. Two types of load reported by Villard et al. (2016).
Fig. 5. Equilibrium of an elementary section in subsided area.
Fig. 6. Equilibrium of an elementary section in anchorage area.
Fig. 7. Solution procedures of estimating the membrane effect.
Fig. 8. Comparison between experimental and analytical results: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.
Fig. 9. Strain and vertical displacement of geosynthetic for different load types acting on the geosynthetic in subsided area: (a) strain; (b) vertical displacement.
Fig. 10. Variations of relative displacement between soil and geosynthetic under different load types.
Fig. 11. Variations of maximum geosynthetic strain (a) and maximum surface settlement (b) with width-height ratio and anchorage length of geosynthetic.
Fig. 12. Influence of upper interface friction angle between soil and geosynthetic on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.
Fig. 13. Influence of modulus of subgrade reaction on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.
Fig. 14. Influence of width of subsided area on: (a) geosynthetic strain; (b) vertical displacement of geosynthetic.
Fig. 15. Variations of relative displacement between soil and geosynthetic with different widths of subsided area.
Fig. 16. Design chart of stiffness of geosynthetic based on its maximum strain and maximum surface settlement.