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Accelerating Cosmologies with Non-Linear Electrodynamics

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Abstract

In this paper, we consider the framework of non-linear electrodynamics in locally rotationally symmetric Bianchi-I universe model composed of magnetic fluid. It has been shown that an accelerated expansion of universe take place if the non-linear electromagnetic field is a source of gravitational field. In this model, after the big-bang, the universe undergoes inflation and the accelerated expansion, dissipates the initial anisotropy of curvature part without using a selected initial condition. Further validity of generalised second law of thermodynamics in the cosmological model enclosed by apparent horizon is investigated. We also discuss the classical stability of the cosmological model and observational viability of model.

Keywords: LRS Bianchi-I metric; Accelerated universe; Classical stability; Thermodynamics; Second law of Thermodynamics.

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1 Introduction

The current accelerated pace of the cosmic expansion [1–4] can be explained by lambda cold dark matter models that assumes a cosmological constant Λ , and dark energy. The problem at initial cosmological big bang singularity is the problem that bother cosmologist more than the others. Λ CDM model has solved many issues in the cosmology but it is also well known that Λ CDM model suffers from theoretical difficulties like cosmological constant problem [5,6]. To explain the cosmic positive acceleration, exotic form of matter, dark energy has been proposed within the frame work of general relativity [7–9]. Positive acceleration can also be explained through modification of gravity [10, 11]. A different subject of research within the cosmological setting in general relativity (GR) coupled to non-linear electrodynamics (NLE) is related with the chance for NLE to fuel the primordial inflation scenario [12] was proposed in order to solve problems of standard big-bang model (eg. the horizon and flatness problem). It was anticipated that a self interacting scalar field with self-interaction potential as the source of Einstein field equation might cause the universe to expand in an accelerated way. The inflationary universe has been supported by observational evidences [13, 14].

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Cosmological models exploring non-linear electrodynamics may solve problem of early time inflation and singularities without modification of general relativity. In a cosmological setting, these theories have been explored mainly within the so called ‘magnetic universe’ approximation [15–18]. Near the Planck era, NLE models can mimic dark energy and can produce inflation in early universe [19, 20]. The NLE coupled with gravity may give the negative pressure and hence accelerate the universe [16, 17, 21]. The evolution of universe in the Einstein-NLE background prevent the initial big bang singularity [22–25].

Einstein’s equations have been derived by using the first law of thermodynamics $\delta Q = TdS$ together with proportionality of entropy to horizon area of black hole. Here δQ , T are energy flux and temperature seen by the accelerated observer just inside the horizon [26–31].

In literature, physical correction between first law of thermodynamics of apparent horizon and Friedman equation has been established in [28]. In Expanding universe, the generalised second law of thermodynamics (GSLT) states that the total entropy (horizon and matter entropies inside the horizon) is always an increasing function of time. There are several studies in the thermodynamics for dark energy filled universe on horizons [32–40]. A study of GSLT in magnetic universe in light of non-linear electrodynamics in isotropic [41] and anisotropic background [42] has been investigated. Sharif et al [43] have recently discussed the stability of accelerated expansion in light of non-linear electrodynamics.

In this paper, we investigate a homogeneous and anisotropic, locally rotationally symmetric Bianchi-I (LRS B-I) universe under the ‘magnetic universe’ approximation which behaves like perfect fluid with energy density ρ and principal pressure p . The paper is organised as follows: In section 2, we write the cosmological equations and discuss the finite behaviour of cosmological invariants. In section 3, we have investigated about the dissipation of anisotropy and have shown that bounce in this particular NLE model is not possible, although curvature invariants are finite, throughout the cosmological history. We also discuss about the validity of GSLT and have determined the observational parameters. In section 4, we give our conclusion.

2 Cosmological model and equations

The Lagrangian density of non-linear electrodynamics (NLE) is given as [22]

$$L_{em} = -\frac{F}{2\beta F + 1}, \quad (1)$$

where $F = \frac{1}{4}F_{ij}F^{ij} = \frac{1}{2}(\mathbf{B}^2 - \mathbf{E}^2)$, F_{ij} is the field strength tensor, the parameter $\beta^{\frac{1}{4}}$ possess the dimension of length, and electric field strength is maximum at $\frac{1}{\sqrt{\beta}}$, in the model under consideration. βF is dimensionless. The denominator of Lagrangian (1) would be non vanishing because the electric field strength cannot reach the maximum value $\frac{1}{\sqrt{\beta}}$. We take a homogeneous and anisotropic, LRS B-I metric as:

$$ds^2 = dt^2 - a_1^2 dr^2 - a_2^2 (d\theta^2 + \theta^2 d\phi^2). \quad (2)$$

Here, $a_1(t)$, $a_2(t)$ are directional scale factors. The action of general relativity coupled with NLE by Lagrangian density (1) as:

$$S = \int d^4x \sqrt{-g} \left(\frac{1}{2}R + L_{em} \right), \quad (3)$$

where R is Ricci scalar. We take $8\pi G = C = \hbar = k_B = 1$. The corresponding Einstein's field equations and electromagnetic field equations are given by:

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}, \quad (4)$$

$$\partial_i \left(\frac{\sqrt{-g}F^{ij}}{(2\beta F + 1)^2} \right) = 0, \quad (5)$$

The energy-momentum tensor of the electromagnetic field T_{ij} is given [22], by

$$T_{ij} = -\frac{1}{(2\beta F + 1)^2} [F_i^k F_{jk} - g_{ij}F(2\beta F + 1)]. \quad (6)$$

Here T_{ij} has a non-vanishing trace. The scale invariance in NLE model based on L_{em} above is broken, and that allows for negative pressure. The energy density and pressure of NLE field are evaluated by averaging over volume. Following the standard approach given in references [44], we define the volumetric spatial average of a quantity X at a time t as

$$\bar{X} = \lim_{V \rightarrow V_0} \frac{1}{V} \int d^3x \sqrt{-g} X, \quad (7)$$

where, $V = \int d^3x \sqrt{-g}$ and V_0 is a sufficiently large time dependent three-volume. For the electromagnetic field to act as source for cosmological model, we use the average values of electromagnetic fields as follows:

$$\langle \mathbf{E} \rangle = \langle \mathbf{B} \rangle = 0, \langle E_i B_j \rangle = 0, \langle E_i E_j \rangle = -\frac{1}{3}E^2 g_{ij}, \langle B_i B_j \rangle = -\frac{1}{3}B^2 g_{ij}. \quad (8)$$

The symbol $\langle . \rangle$ denotes an average over the volume that is large compared to wavelength of radiation but small compared to curvature of space-time. In the following, we will omit brackets. Under these assumptions, the energy-momentum tensor of EM field associated with L_{em} suggest the following form of energy density (ρ) and pressure (p) [23]

$$\rho = \frac{E^2}{(2\beta F + 1)^2} + \frac{F}{2\beta F + 1}, \quad (9)$$

$$p = \frac{2B^2 - E^2}{3(2\beta F + 1)^2} - \frac{F}{2\beta F + 1}. \quad (10)$$

The average electric and magnetic field squared are denoted by E^2 and B^2 . We consider the case when electric field components E^2 will be assumed vanishing, so the B^2 dominates over E^2 . This case turns out to be relevant in cosmology as long as the averaged electric field E is screened by charged primordial plasma, while the magnetic field lines are frozen [45]. For the case, $E^2 = 0$, we have,

$$\rho = \frac{B^2}{2(\beta B^2 + 1)}, \quad p = \frac{B^2(1 - 3\beta B^2)}{6(\beta B^2 + 1)^2}. \quad (11)$$

In a LRS B-I universe filled with a 'magnetic fluid', having the energy density ρ and isotropic pressure p , the field equations are

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\sigma^2 = -\frac{1}{2}(\rho + 3p), \quad (12)$$

$$\frac{1}{3}\Theta^2 - \sigma^2 = \rho, \quad (13)$$

$$\dot{\sigma} + \Theta\sigma = 0, \quad (14)$$

$$\dot{\rho} + \Theta(\rho + p) = 0, \quad (15)$$

where, $\Theta = \frac{\dot{a}_1}{a_1} + 2\frac{\dot{a}_2}{a_2}$, $\sigma = \frac{1}{\sqrt{3}}\left(\frac{\dot{a}_1}{a_1} - \frac{\dot{a}_2}{a_2}\right)$ are the expansion scalar and shear scalar respectively. Overhead dot denotes the derivative with respect to cosmic time t . The directional Hubble parameters are defined as $H_1 = \frac{\dot{a}_1}{a_1}$, $H_2 = \frac{\dot{a}_2}{a_2}$, the mean Hubble parameter is given by $H = \frac{1}{3}(H_1 + 2H_2)$ and $\Theta = 3H$.

We take the shear scalar to be proportional to expansion scalar, and thus, we have a linear relationship between H_1 and H_2 as $H_1 = nH_2$. This assumption leads to a relation between directional scale factors a_1, a_2 as $a_1 = a_2^n$.

We define spatial volume V as, $V = a^3 = a_1a_2^2$ and thus using above relation, we have $V = a^3 = a_2^{n+2}$ i.e. $a = a_2^{\frac{n+2}{3}}$. Using the definition of spatial volume in equation (15), we have

$$B(t) = \frac{b_0}{a^2(t)} \quad (16)$$

where, b_0 is a positive constant. The above equation shows the sensitivity of magnetic field towards the evolution of average geometry. From equation (11) and (16), we have

$$\rho(t) = \frac{b_0^2}{2(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}})}, \quad p(t) = \frac{b_0^2 a_2^{\frac{4(n+2)}{3}} - 3\beta b_0^4}{6(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}})^2} \quad (17)$$

From equation (17), equation of state parameter can be written as:

$$\gamma = \frac{1}{3} - \frac{4\beta b_0^2}{3\left(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}}\right)} \quad (18)$$

By assuming $\frac{\beta b_0^2}{a_2^{\frac{4(n+2)}{3}}} = y$, we see that for $y = \frac{1}{3}$ we have $\gamma = 0$, for $y = 0$, we have $\gamma = \frac{1}{3}$, for $y = 1$, we have $\gamma = \frac{-1}{3}$. We plot the graph of γ vs y in figure 1.

It follows from equation (17) that there is neither any singularity of energy density nor the pressure at $a_2(t) \rightarrow \infty$. Since $\rho(t)$ and $p(t)$ depends upon n , the behaviour of $\rho(t)$, $p(t)$ and $\gamma(t)$ has been sum upped in Table 1. The Ricci scalar R can be written as

	$\rho(t)$			$p(t)$			$\gamma(t)$		
	$n < -2$	$n = -2$	$n > -2$	$n < -2$	$n = -2$	$n > -2$	$n < -2$	$n = -2$	$n > -2$
$a_2(t) \rightarrow 0$	0	$\frac{b_0^2}{2(\beta b_0^2 + 1)}$	$\frac{1}{2\beta}$	0	$\frac{b_0^2(1-3\beta b_0^2)}{6(\beta b_0^2 + 1)^2}$	$\frac{-1}{2\beta}$	$\frac{1}{3}$	$\frac{1-3\beta b_0^2}{3(\beta b_0^2 + 1)}$	-1
$a_2(t) \rightarrow \infty$	$\frac{1}{2\beta}$	$\frac{b_0^2}{2(\beta b_0^2 + 1)}$	0	$\frac{-1}{2\beta}$	$\frac{b_0^2(1-3\beta b_0^2)}{6(\beta b_0^2 + 1)^2}$	0	-1	$\frac{(1-3\beta b_0^2)}{3(\beta b_0^2 + 1)}$	$\frac{1}{3}$

Table 1: ρ, p and γ as $a_2(t) \rightarrow 0$ and $a_2(t) \rightarrow \infty$.

$$R(t) = \rho - 3p = \frac{2\beta b_0^4}{\left(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}}\right)^2}. \quad (19)$$

Figure 1: Variation of γ verses y .

Further, the Ricci tensor squared ($R_{ij}R^{ij}$) can be expressed as

$$R_{ij}R^{ij} = 2 \left(\frac{\ddot{a}_1^2}{a_1^2} + \frac{3\ddot{a}_2^2}{a_2^2} + \frac{2\ddot{a}_1\ddot{a}_2}{a_1a_2} + \frac{2\dot{a}_1\dot{a}_2}{a_1a_2} + \frac{2\dot{a}_1\dot{a}_2}{a_1a_2} \left(\frac{\ddot{a}_1}{a_1} + \frac{\ddot{a}_2}{a_2} \right) + \frac{\dot{a}_2^4}{a_2^4} + \frac{2\dot{a}_2^2\ddot{a}_2}{a_2^3} \right), \quad (20)$$

Also, we have

$$R_{ij}R^{ij} = \rho^2 + 3p^2, \quad (21)$$

From equations (17) and (21), we have

$$R_{ij}R^{ij} = \frac{b_0^4}{4 \left(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}} \right)^2} + \frac{1}{12} \frac{\left(b_0^2 a_2^{\frac{4(n+2)}{3}} - 3\beta b_0^4 \right)^2}{\left(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}} \right)^2}. \quad (22)$$

For different values of n , expressions for $R(t)$ and $R_{ij}R^{ij}$ are given in the following table. The

	R			$R_{ij}R^{ij}$		
	$n < -2$	$n = -2$	$n > -2$	$n < -2$	$n = -2$	$n > -2$
$a_2(t) \rightarrow 0$	0	$\frac{2\beta b_0^4}{(\beta b_0^2 + 1)^2}$	$\frac{2}{\beta}$	0	$\frac{b_0^4(3\beta^2 b_0^4 + 1)}{3(\beta b_0^2 + 1)^4}$	$\frac{1}{\beta^2}$
$a_2(t) \rightarrow \infty$	$\frac{2}{\beta}$	$\frac{2\beta b_0^4}{(\beta b_0^2 + 1)^2}$	0	$\frac{1}{\beta^2}$	$\frac{b_0^4(3\beta^2 b_0^4 + 1)}{3(\beta b_0^2 + 1)^4}$	0

Table 2: R and $R_{ij}R^{ij}$ as $a_2(t) \rightarrow 0$ and $a_2(t) \rightarrow \infty$.

Kretschmann scalar ($R_{ijkl}R^{ijkl}$) for LRS BI universe can be written as:

$$R_{ijkl}R^{ijkl} = 4 \left((n^2 + 2)(\dot{H}_2 + H_2)^2 + (2n^2 + 1)H_2^4 \right). \quad (23)$$

Since, $(1 + 2n)H_2^2 = \rho$ and $2\dot{H}_2 + 3H_2^2 = -p$, we have $H_2^2 = \frac{\rho}{(2n+1)}$ and $\dot{H}_2 = -\frac{1}{2} \left(p + \frac{3\rho}{(2n+1)} \right)$.

Using these results in equation (23), we have

$$R_{ijkl}R^{ijkl} = 4 \left[(n^2 + 2) \left(\frac{-p}{2} - \frac{3\rho}{2(2n+1)} + \frac{\sqrt{\rho}}{\sqrt{(2n+1)}} \right)^2 + (2n^2 + 1) \frac{\rho^2}{(2n+1)^2} \right]. \quad (24)$$

From equation (24), with $n > -\frac{1}{2}$, we have

$$\lim_{a_2(t) \rightarrow 0} R_{ijkl} R^{ijkl} = \frac{(n^2 + 2) \left((n - 1) + \sqrt{2\beta(2n + 1)} \right)^2 + (2n^2 + 1)}{\beta^2(2n + 1)^2}$$

and $\lim_{a_2(t) \rightarrow \infty} R_{ijkl} R^{ijkl} = 0$. As a result, there are no singularities of Ricci scalar, Ricci tensor squared, nor the Kretschmann scalar as $a_2(t) \rightarrow 0$ and $a_2(t) \rightarrow \infty$.

From equation (11), we have

$$\rho + 3p = \frac{B^2(1 - \beta B^2)}{(1 + \beta B^2)}, \quad (25)$$

Also, we have

$$\rho + p = \frac{2b_0^2 a_2^{\frac{4(n+2)}{3}}}{\left(\beta b_0^2 + a_2^{\frac{4(n+2)}{3}} \right)^2}. \quad (26)$$

In the present model, $\rho + p > 0$, thus, the NEC is always satisfied in the model, but SEC is getting violated when $\beta B^2 > 1$ i.e. when $a_2(t) < \beta^{\frac{3}{4(n+2)}} b_0^{\frac{3}{2(n+2)}}$, and therefore we have accelerated universe scenario.

Adiabatic square sound speed is defined [46], by

$$c_s^2 = \frac{\partial p}{\partial \rho} = \frac{1 - 7\beta B^2}{3(1 + \beta B^2)}, \quad (27)$$

Using equation (16) and (27), we have

$$c_s^2 = \frac{a_2^{\frac{4(n+2)}{3}} - 7\beta b_0^2}{3 \left(a_2^{\frac{4(n+2)}{3}} + \beta b_0^2 \right)}. \quad (28)$$

A cosmological model which is intended to describe the present universe has to meet the bounds on speed at which small perturbation of background energy density propagates as: $0 \leq c_s^2 \leq 1$.

In our cosmological model, with $\beta > 0$, we have $c_s^2 \leq 1$ and with $a_2(t) > (7\beta)^{\frac{3}{4(n+2)}} b_0^{\frac{3}{2(n+2)}}$, we have $c_s^2 \geq 0$. Under these conditions our model is classically stable.

We take the occurrence of bounce at $t = t_b$ by the conditions (i) $\Theta(t_b) = 0$, and (ii) $\dot{\Theta}(t) > 0$ for $t \in (t_b - \epsilon, t_b) \cup (t_b, t_b + \epsilon)$ for small $\epsilon > 0$ [47, 48]. In this cosmological model, which has more than one scale factor a_i , $i = 1, 2$, the above condition should be understood as characterising a bounce in average scale factor $V = a_1 a_2^2$. However, a more generic situation where a bounce can occur in any scale factors a_i , may also be considered. A bounce in a_i will occur at $t = t_b$ (i) $H_i(t_b) = 0$ and (ii) $\dot{H}_i(t_b) > 0$. It is clear that although it may be possible to have a bounce in any one of the scale factor but not the other, this does not lead to a new expanding universe region. We, therefore require that a bounce occur at different times. From equation (12) and (13), at the bounce time $t = t_b$, we have

$$-\frac{2}{3} \dot{\Theta}(t_b) - 2\sigma^2(t_b) = \rho + p$$

Since, $\dot{\Theta}(t_b) > 0$, $\sigma^2(t_b) \geq 0$ and in the cosmological model, $\rho + p > 0$, throughout the cosmological history, therefore bounce in this model is not possible, although R , $R_{ij} R^{ij}$, $R_{ijkl} R^{ijkl}$, ρ and p are finite quantities.

3 General issues

3.1 Dissipation of anisotropy

We next take $D = \frac{\sigma^2}{\rho}$, then the evolution equation for ratio D can be written as:

$$\dot{D} + 3H(1 - \gamma)D = 0, \quad (29)$$

where we take ρ and p are related as $p = \gamma\rho$. Also, we have $1 - \gamma > 0$, for $\beta > 0$.

For an average expanding cosmology ($\Theta > 0$), the solution $D = 0$ of equation (29) is asymptotically stable. Under the condition, $\gamma < 1$, we have $\dot{\Theta} + \Theta^2 > 0$, which yields $\rho > p$. So the cosmology with $\gamma < 1$ in NLE scenario obey the dominant energy condition, further dissipate the initial anisotropy of curvature part without using a selected initial condition. In the asymptotic regime, the magnetic fluid dominates over shear. Therefore, at the late times, the geometry tends to an isotropic model.

3.2 Validity of GSLT

We now investigate the validity of generalized second law of thermodynamics (GSLT) which states that the total entropy (composed of horizon and matter entropies) inside the horizon is always an increasing function of time. We calculate the variation of entropy on the horizon using first law of thermodynamics and variation of entropy inside horizon will be calculated via Gibb's equation.

The radius of apparent horizon [41, 49], is given by

$$R_A = \frac{1}{H}. \quad (30)$$

We consider the net amount of energy crossing through the apparent horizon in time dt [50] as

$$-dE = 4\pi R_A^3 H(\rho + p)dt, \quad (31)$$

and assuming the validity of first law of thermodynamics on apparent horizon ($-dE = TdS_H$), we have

$$\frac{dS_H}{dt} = \frac{8\pi R_A^3 H}{3T} \frac{B^2}{(\beta B^2 + 1)^2}. \quad (32)$$

From the Gibb's equation [51, 52], we have

$$TdS_I = dE_I + pdV, \quad (33)$$

further

$$\frac{dS_I}{dt} = \frac{8\pi R_A^2 H}{3T} \frac{B^2}{(\beta B^2 + 1)^2} (\dot{R}_H - 1), \quad (34)$$

$$\dot{R}_A = \frac{-3\dot{H}_2}{(n+2)H_2^2} = \frac{2}{\beta B^2 + 1}, \quad (35)$$

where, S_I is entropy inside horizon, $V = \frac{4}{3}\pi R_A^3$ is volume of horizon, $E_I = \rho V$ is the internal energy, $T = \frac{1}{2\pi R_A}$ is Hawking temperature. From equation (32), (33) and (35), the rate of change of total entropy becomes

$$\frac{d}{dt}(S_I + S_H) = \frac{8\pi R_A^2}{3T} \frac{B^2(1 - \beta B^2)}{(\beta B^2 + 1)^3}. \quad (36)$$

Figure 2: Rate of change of total entropy of Hubble horizon $\left(\frac{d(S_I + S_H)}{dt}\right)$ versus z for $\beta = 0.25$, $b_0 = 0.5$ and $n = 2, 1.0000212, -0.5$.

We take $1 + z = \frac{1}{a}$ i.e. $1 + z = \frac{1}{\frac{n+2}{a_2}}$ and we use the units $8\pi G = C = h = K_B = 1$. It can be observed from figure 2, that rate of change of total entropy of Hubble horizon is negative up-to a certain stage. So, GSLT is not valid up-to a certain stage and after this stage, it is always valid for this magnetic universe.

3.3 Observational parameters

The deceleration parameter is defined by

$$q = -1 + \frac{d}{dt} \left(\frac{1}{H} \right) = \frac{1 - \beta B^2}{1 + \beta B^2}. \quad (37)$$

Figure 3: Variation of deceleration parameter versus z for $\beta = 0.25, 0.5$, $b_0 = 5$.

With increasing z , q is a decreasing function of time and we have an accelerating universe scenario at current epoch, irrespective of the value of n . The variation of q versus z is given in figure 3.

From equation (11), we have

$$p = \frac{1}{3}\rho - \frac{8}{3}\beta\rho^2 \quad (38)$$

For $|\frac{f(\rho)}{\rho}| \ll 1$ during inflation, the expression for spectral index n_s , the tensor-to-scalar ratio ' r ' and the running of spectral index $\alpha_s = \frac{dn_s}{d\ln k}$ is given as [53],

$$n_s \approx 1 - 6\frac{f(\rho)}{\rho}, r \approx 24\frac{f(\rho)}{\rho}, \alpha_s \approx -9\left(\frac{f(\rho)}{\rho}\right)^2 \quad (39)$$

From above equation (39), with $f(\rho) = -\frac{8}{3}\beta\rho^2$, we have

$$r = 4(1 - n_s) = 8\sqrt{-\alpha_s} = 64\beta\rho \quad (40)$$

The Planck data [54] and WMAP data [55,56] have following values: $n_s = 0.9603 \pm 0.0073(68\%CL)$, $r < 0.11(95\%CL)$, $\alpha_s = -0.0134 \pm 0.0090(68\%CL)$. From equation (40), we obtain the value $\beta\rho \approx 1.562 \times 10^{-3}$ corresponding to these reasonable values of cosmological parameters.

The average anisotropy parameter $A = \frac{1}{3}\Sigma\left(\frac{\Delta H_i}{H}\right)^2$ for the model is $A = 2\left(\frac{n-1}{n+2}\right)^2$. We now constrain the average anisotropy parameter with WMAP data [57] to be $|\sqrt{A}| = 10^{-5}$, which correspond to $n = 1.0000212$ in our model.

From table 1, it can be observed that at the beginning of universe ($a = 0$), for $n > -2$, the model gives $p = -\rho$, the same property as Λ CDM model.

4 Conclusions

A model of nonlinear electromagnetic fields with a dimension parameter β coupled to GR, when the nonlinear electromagnetic field is the source of gravitational field, has been considered in LRS B-I background metric. The scale variance is broken in this model and thus allows for negative pressure. We have considered electromagnetic fields as a stochastic background with the non zero value $\langle B^2 \rangle$. The universe in the model undergoes accelerated expansion. There is absence of singularities at the beginning of universe creation, in energy density, and the cosmological invariants. The magnetic field drives the universe to accelerate because we consider linear equations for electromagnetic fields to be non-linear equations. We take the shear scalar proportional to expansion scalar and thus, we have a relation between directional scale factors. We consider the average anisotropy parameter with WMAP data, which corresponds to $n = 1.0000212$ in our model. The anisotropy in cosmic expansion of this model must be considered as a little perturbation to isotropic behaviour, although this anisotropy annihilates irrespective of value of n .

For LRS BI metric (2), we calculate the relation of $R, R_{ij}R^{ij}, R_{ijkl}R^{ijkl}$ with ρ and p . For FRW metric, relations of $R, R_{ij}R^{ij}, R_{ijkl}R^{ijkl}$ with ρ and p has been obtained in [23]. With general n , our results generalizes the result of FRW metric.

The validity of GSLT is investigated in magnetic universe bounded by apparent horizon. We calculate the time rate of total entropy and its behaviour is shown in figure 2. This shows that GSLT is not always satisfied in the present model. For $n = 1$, our model reduces to FRW model [23]. With n , other than 1, our model generalizes [23], here we conclude that GSLT is not always satisfied in this class of NLE models given by Lagrangian density as in [22].

The model investigated in this article, has finite energy density, curvature invariants but the model does not exhibit bounce [58–60]. Finally we have discussed the behaviour of deceleration parameter

and the spectral index, the tensor-to-scalar ratio, the running of the spectral index calculated are in approximate agreement with *Planck* and *WMAP* data. In this cosmological model, with increasing z , q is a decreasing function of time. For $n > -2$, the model also possesses property of Λ CDM model. We presented here a model of NLE fields coupled to gravity in LRS B-I background which allows us to consider the universe acceleration, with dissipation of curvature anisotropy and having finite energy density, pressure, curvature invariants at beginning and at late times but, the model does not satisfy generalised second law of thermodynamics at every epoch of cosmological history. It is interesting to note that in our model after inflation, the universe decelerates approaching to the radiation era. To solve existing problem of our model, one may introduce fields coupled non-minimally with gravity. We expect to investigate in this direction in future works.

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